SCHEDULING OF APPLIANCES BASED ON SENSITIVITY TO DYNAMIC PRICING IN A SMART GRID

A Paper
Submitted to the Graduate Faculty
Of the
North Dakota State University
Of Agriculture and Applied Science

By

Barsha Saha

In Partial Fulfillment of the Requirements
For the Degree of
MASTER OF SCIENCE

Major Department:
Computer Science

September 2013

Fargo, North Dakota

North Dakota State University
Graduate School

Title

Scheduling of Appliances Based on Sensitivity to Dynamic Pricing in a Smart Grid

By

Barsha Saha

The Supervisory Committee certifies that this ***disquisition*** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

MASTER OF SCIENCE

SUPERVISORY COMMITTEE:

Dr. Kendall Nygard
Chair

Dr. Brian Slator

Dr. Saeed Salem

Dr. Sanku Mallik

Approved:

9/30/2013
Date

Dr. Brian M. Slator
Department Chair

ABSTRACT

The work described in this paper models the scheduling of smart home appliances of different types of consumers based on their sensitivity to dynamic price changes. The price sensitivity is established on a scale of 0 to 100 percent. We have used Linear programming to optimally schedule a set of residential appliances under variable peak pricing in order to minimize the consumer's energy bill. This paper shows different scheduling models for the home appliances for different categories of price sensitive consumers. It takes into consideration that all consumers have access to perfect information such that they can alter their usage and thereby benefit from the associated cost savings. We also analyze the monthly electricity cost/expenditure against price sensitivity and define a function for the same.

ACKNOWLEDGEMENTS

I would like to thank Dr. Kendall Nygard for his continuous support throughout my research work. Without his guidance and advice it would not have been possible for me to achieve my academic goal. I am very proud to be a part of Dr. Nygard's Smart Grid Research Group and I thank everyone in the team.

I would also like to thank Dr. Brian Slator, Dr. Saeed Salem and Dr. Sanku Mallik for agreeing to serve on my graduate supervisory committee.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

A smart grid is a modernized electrical grid that uses information and communications technology to gather and act on information, such as information about the behaviors of suppliers and consumers, in an automated fashion to improve the efficiency, reliability, economics and sustainability of the production and distribution of electricity. Dynamic pricing is an integral part of a smart grid. The dynamic price of electricity determines the price of electricity for a time period based on the demand in that given time period.

Time-based or dynamic pricing is a form of pricing where consumers are charged different rates for a service depending on the time of day, month, or even season. In some cases, consumers can be given price information in real time. In the case of the smart grid, utilities can monitor the load on the grid as well as the cost of generation of electricity and can transmit this information to customers. This can lead to a more stable and efficient grid.

The objective of this paper is to develop and test a model for the optimized scheduling of residential appliances under variable peak pricing for consumers with different price sensitivity to minimize their cost of consumption. The consumers respond based on imperfect information about real-time price. With imperfect information, a consumer predicts the price of electricity based on an assumed pattern of price. Based on the estimated pattern of price, a consumer makes decisions to reduce consumption and/or shift a load. This paper formulates an optimal framework for scheduling of home appliances using the concept of linear programming. This paper also categorizes consumers on their percentage of price sensitivity. It models how a consumer belonging to a specific range of price sensitivity would optimally schedule his home appliances to minimize his overall cost consumption. And finally, it establishes a relationship between cost and price sensitivity and how it can be used to solve real life challenges.

The structure of this paper is as follows. Chapter 2 describes the related literature in smart grid and dynamic pricing. Chapter 3 describes the optimal scheduling of home appliances under variable peak pricing. Chapter 4 describes the model for the scheduling for different categories of price sensitive consumers. Chapter 5 describes the Implementation detail of the model. In chapter 6, experimental results and benefits for consumers and suppliers are analyzed and quantified and the corresponding utility function is established. In chapter 7, future research is presented.

2. LITERATURE REVIEW

The smart grid is a compilation of concepts, technologies, and operating practices intended to bring the electric grid into the 21st century. Smart grid concepts and issues are difficult to address because they include every aspect of electric generation, distribution, and use. Dynamic pricing is a key benefit for many utilities in their business case for Smart Grid/AMI deployment. Many of the benefits, particularly on the customer side of the meter, are driven by rate design. Dynamic pricing has garnered much interest in the country during the past decade since it has the potential for lowering customer energy costs by mitigating the need to install expensive peaking capacity.

2.1. Smart Grid

A smart grid is an electricity network that incorporates a suite of information, communication and other advanced technologies to monitor and manage the transport of electricity from all generation sources to meet the varying electricity demands of end users.[14] It not only promotes the distributed generation of green electrical power through sources such as wind turbines and solar panels, but also more closely and accurately monitors the grid through the use of devices such as smart meters and phasor measurement units[19]. These improvements along with advanced visualization, and artificial intelligence technologies help to manage the electrical grid in a more efficient and reliable manner. The process of “smartening” the electricity grid, which has already begun in many regions, involves significant additional upfront investment, though this is expected to reduce the overall cost of electricity supply to end users over the long term. Smart grid technologies are evolving rapidly and will be deployed at different rates around the world, depending on local commercial attractiveness, compatibility with existing technologies, regulatory developments and investment frameworks.

2.1.1. Why do We Need to Smarten Electricity Grids?

Electricity systems worldwide face a number of challenges, including ageing infrastructure, continued growth in demand, shifting load patterns (including changes resulting from the increased use of electric vehicles) [14], the need to integrate new sources of supply and the variability of some sources of renewables-based supply. Smart-grid technologies offer a cost-effective means of helping to meet these challenges and, in so doing, contribute to the establishment of an energy system that is more energy efficient, more secure and more sustainable. They do this by:

- Enabling and incentivizing consumers to adjust their demand in real time to changing market and network conditions.
- Accommodating all generation sources and storage options.
- Tailoring power quality to customer needs.
- Optimizing the utilization and operating efficiency of generation, transmission and distribution assets.
- Providing resiliency to unplanned supply disruptions and outages.

2.2. Dynamic Pricing

Dynamic pricing is one of the most important advancements proposed with the Smart Grid. In [17] the possible impact in California of multiple dynamic pricing strategies is analyzed including, peak-time rebate (PTR) and real-time pricing (RTP). California could find that dynamic pricing can be quite effective with time of use pricing lowering peak loads by up to 6 percent and critical peak pricing lowing peak loads by up to 20 percent.

Dynamic pricing is a form of time-of-use (TOU) pricing where prices during the peak period on a limited number of days can vary to reflect market conditions on a day-ahead or day-of basis. One popular variant of dynamic pricing is critical-peak pricing (CPP) in which prices during the top 40-150 hours of the year rise to previously specified levels designed to recover the full capacity and energy cost of power plants that run primarily during those hours [3]. During all other hours of the year, prices are lower than existing rates by an amount sufficient to leave the bill unchanged for a customer whose load shape mirrors that of the rate class.

2.2.1. Features of Dynamic Pricing

- Reflects the marginal cost of providing electricity, which varies by time of day.
- Under dynamic pricing, either the price of electricity and/or the time it will be activated are uncertain.
- Compared to flat (non-time varying) rates, dynamic pricing can clip off the highest peak loads during the year which can account for 7 to 17 percent of system load.

2.2.2. Time-based Pricing

Time-based pricing can be divided into—but not limited to—several categories. [13] They include:

- Time-of-use (TOU) pricing: Prices are set in advance—typically one day—and do not change frequently. Thus, the rates paid by consumers during these periods are known by consumers in advance. This allows them to respond to these price signals and manage energy usage.
- Critical Peak Pricing: This pricing scheme only comes into play during certain peak days when demand is at its greatest; otherwise, customers pay TOU pricing. The prices represent the actual cost of electricity generation.

- Real time pricing/Dynamic pricing: Prices changes hourly or, in some exceptional cases, every few minutes. In some regions of the US (parts of New England and California), the real time market operates in five minute intervals [14].
- Peak Load Reduction Credits: Customers with large loads can enter into pre-established peak load reduction agreements. If they meet these requirements, they receive credits towards their next bill.

2.3. Linear Programming

Linear programming (or linear optimization) is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear relationships. Linear programming is a specific case of mathematical programming (mathematical optimization).

Linear programming uses a mathematical model to describe the problem of concern. The adjective linear means that all the mathematical functions in this model are required to be linear functions. The word programming does not refer here to computer programming; rather, it is essentially a synonym for planning. Thus, linear programming involves the planning of activities to obtain an optimal result, i.e. a result that reaches the specified goal best (best according to the mathematical model) among all feasible alternatives. Although allocating resources to activities is the most common type of application, linear programming has numerous other important applications as well. In fact, any problem whose mathematical model fits the very general format for the linear programing model is a linear programming problem. Job scheduling is also another problem where linear programming has been used extensively.

2.3.1. AMPL

AMPL is a language for large-scale optimization and mathematical programming problems in production, distribution, blending, scheduling, and many other applications. Combining familiar algebraic notation and a powerful interactive command environment, AMPL makes it easy to create models, use a wide variety of solvers, and examine solutions. Though flexible and convenient for rapid prototyping and development of models, AMPL also offers the speed and generality needed for repeated large-scale production runs.

AMPL closely resembles the symbolic algebraic notation that many modelers use to describe mathematical programs, yet it is regular and formal enough to be processed by a computer system[18]. It is particularly notable for the generality of its syntax, for the variety of its indexing operations, and for the similarity of its expressions to the algebraic notation customarily used in the modeler's form. It offers a variety of types and operations for the definition of indexing sets, as well as a range of logical expressions. AMPL draws considerable inspiration from the XML prototype language but incorporates many changes and extensions.

3. OPTIMAL SCHEDULING OF HOME APPLIANCES

In this section, we provide a written description of the appliance scheduling problem that will be mathematically formulated as a linear programming problem in the following section. We will be designing a smart Scheduler/Controller for the residential appliances, scheduling their time of use during the day, to optimize the overall price consumption under variable peak pricing.

3.1. Problem Setup

The significant role played by a number of electrical appliances in our daily lives is quite undeniable. Man is undoubtedly dependent on different types of home and kitchen appliances like air conditioners, LCD TVs, heaters, vacuum cleaners, coolers and so forth. All these gadgets are known to lessen our burden and make life easier. The different types of electrical appliances used in our daily lives include tube light, refrigerators, water heaters, room heaters, air conditioners, coolers, fans, washer, dryers, geysers, LED lights and so forth. Out of these home appliances, for some of the appliances like washer, dryer, water heater etc., we can schedule their time of use during the day. In this problem, we would be referring to only those appliances whose time of use can be scheduled. Thus, we choose a set of five such commonly used residential appliances and optimally schedule them under variable peak pricing in order to minimize the customer's energy bill also considering consumer’s preferred time of use during the day.

3.1.1. Assumptions

We chose 5 appliances at home whose time of use can be scheduled, each having the following power consumption (kW) and the following time taken (Minimum and Maximum) in hours for its operation and also consumer's preferred time of use during the day as shown in Table 1.

Table 1. Problem Setup

Serial No.	Appliances	Power Consumption/hr. (KW)	Time taken to complete operation (hrs.)	Consumer preferred time of use
1	Washer	0.51	2-3	9 a.m. – 8 p.m.
2	Dryer	3.40	1-2	9 p.m. – 8 p.m.
3	Dish Washer	1.50	2-3	9 a.m. – 5 p.m. & 8 a.m. – 10 p.m.
4	Water Heater	0.48	1-2	4 a.m. – 6 a.m. & 6 p.m. – 9 p.m.
5	Water Softener	0.002	1-2	7 a.m. – 10 p.m.

We divide the time of day to 1 hour slots ranging from 1-24 when electricity rates $\boldsymbol{C_t}$ vary per hour as shown below in Table 2.

Table 2. Time Slots and Cost/Hour

Time slots	t	C_t (USD/KWh)
12:00 a.m. - 1:00 a.m.	1	0.041
1:00 a.m. - 2:00 a.m.	2	0.038
2:00 a.m. - 3:00 a.m.	3	0.036
3:00 a.m. - 4:00 a.m.	4	0.037
4:00 a.m. - 5:00 a.m.	5	0.038
5:00 a.m. - 6:00 a.m.	6	0.039
6:00 a.m. - 7:00 a.m.	7	0.054
7:00 a.m. - 8:00 a.m.	8	0.057
8:00 a.m. - 9:00 a.m.	9	0.058
9:00 a.m. - 10:00 a.m.	10	0.069
10:00 a.m. - 11:00 a.m.	11	0.07
11:00 a.m. - 12:00 p.m.	12	0.068
12:00 p.m. - 1:00 p.m.	13	0.068
1:00 p.m. - 2:00 p.m.	14	0.068
2:00 p.m. - 3:00 p.m.	15	0.065
3:00 p.m. - 4:00 p.m.	16	0.065
4:00 p.m. - 5:00 p.m.	17	0.063
5:00 p.m. - 6:00 p.m.	18	0.095
6:00 p.m. - 7:00 p.m.	19	0.094
7:00 p.m. - 8:00 p.m.	20	0.072
8:00 p.m. - 9:00 p.m.	21	0.063
9:00 p.m. - 10:00 p.m.	22	0.056
10:00 p.m. - 11:00 p.m.	23	0.049
11:00 p.m. - 12:00 a.m.	24	0.048

We then number each of the residential appliances for linear programming formulation as shown below in Table 3:

Table 3. Appliances

Appliances	a
Washer	1
Dryer	2
Dish Washer	3
Water Heater	4
Water Softener	5

3.2. Decision Variables

In this section we define all the decision variables to be used for formulating linear programming for the optimal scheduling of the home appliances.

$\boldsymbol{C_t}$ is the cost of electricity at time slot $\boldsymbol{t}$ in **USD/KWh**

$\boldsymbol{P_a}$ is the electric power consumption of the appliance $\boldsymbol{a}$ in **Kilowatt**

$\boldsymbol{X_{at}}$ is the binary decision variable where $\boldsymbol{X_{at}} \in \{0,1\}$, $\boldsymbol{X_{at}} = 1$ if and only if appliance $\boldsymbol{a}$ can be turned on during the time slot **t** according to consumer preference.

$\boldsymbol{CP_{at}}$ is the consumer preference matrix determining at which time of the day, consumer prefers to use his appliance. Below is a consumer preference matrix (Table 4).

Table 4. Consumer Time Preference Matrix for 0% Price Sensitive Consumer

A		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
p	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
p	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
l	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
i	4	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
e	5	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0

$\boldsymbol{Peakpower_t}$ is the Peak energy usage allowed per hour.

$\boldsymbol{MaxTime_a}$ and $\boldsymbol{MinTime_a}$ are the maximum and minimum time taken to complete operation by an appliance $\boldsymbol{a}$.

3.3. Cost Function

The objective of the proposed scheduler is to minimize the total electricity cost for operating the appliances. In this paper the cost calculation is based on a given 24-hour ahead electricity tariff (e.g. USD per kWh).

Let $\boldsymbol{C_t}$ denote the electricity tariff for time slot t, $\boldsymbol{P_a}$ denote the electric power consumption of the appliance a and $\boldsymbol{X_{at}}$ denote the binary decision variable where $\boldsymbol{X_{at}} \in \{0,1\}$, $\boldsymbol{X_{at}}$ = 1 if and only if appliance a can be turned on during the time slot t according to consumer preference otherwise 0. Thus, the total electricity cost for running all appliances is

$$\text{Minimize } \sum_{t=1}^{24} \sum_{A=1}^{5} \; C_t P_a X_{at}$$

3.4. Constraints

To ease the description, the constraints are organized into three groups – energy/power constraints, timing constraints and consumer preference constraint.

3.4.1. Energy Constraint

The summation of the energy/power consumption of all the appliances running at a particular hour should not exceed the Peak energy usage allowed per hour. If $\boldsymbol{P_a}$ is the electric power consumption of the appliance $\boldsymbol{a}$ in **Kilo,** $\boldsymbol{X_{at}}$ is the binary decision variable where $\boldsymbol{X_{at}}$ $\in \{0,1\}$, $\boldsymbol{X_{at}}$ = 1 if and only if appliance $\boldsymbol{a}$ can be turned on during the time slot **t** according to consumer preference and $\boldsymbol{Peakpower_t}$ is the Peak energy usage allowed per hour, then the following constraint (1) can be enforced:

$$\sum_{a=1}^{5} P_a \, X_{at} \leq PeakPower_t \qquad \forall t$$

3.4.2. Time Constraint

The appliance must be operated in between the minimum and maximum time of operation of the appliance. If $\boldsymbol{MaxTime_a}$ and $\boldsymbol{MinTime_a}$ are the maximum and minimum time taken to complete operation by an appliance $\boldsymbol{a}$ and $\boldsymbol{X_{at}}$ is the binary decision variable where $\boldsymbol{X_{at}} \in \{0,1\}$, $\boldsymbol{X_{at}} = 1$ if and only if appliance $\boldsymbol{a}$ can be turned on during the time slot **t** according to consumer preference, then the following constraint (2) is enforced:

$$MinTime_a \le \sum_{t=1}^{24} X_{at} \le MaxTime_a \qquad \forall a$$

3.4.3. Consumer Preference Constraint

The time of use of an appliance should be made considering the consumer's preferred time of use of the appliance throughout the day. If $\boldsymbol{X_{at}}$ is the binary decision variable where $\boldsymbol{X_{at}} \in \{0,1\}$, $\boldsymbol{X_{at}} = 1$ if and only if appliance $\boldsymbol{a}$ can be turned on during the time slot **t** according to consumer preference and $\boldsymbol{CP_{at}}$ is the consumer preference matrix determining at which time of the day, consumer prefers to use his appliance, then the following constraint (3) can be enforced:

$$X_{at} \le CP_{at} \qquad \forall a, \forall t$$

Other constraints that can be incorporated are an uninterruptible operation constraint and a sequential processing constraint considering consumer's ease of appliance usage. The uninterruptible operation constraint refers to the uninterruptible operation of an appliance while it is scheduled. When an appliance is scheduled by the scheduler, it should be allotted sequential timeslots for its operation. The sequential processing constraint refers to the sequential processing between two appliances that are dependent on each other for their operation, for example a dryer cannot operate before a washer. The timings of a washer and a dryer should be scheduled such that a dryer always runs after the washer has completed its operation.

3.5. Linear Programming Formulation

To sum up, the proposed minimum electricity cost appliance scheduling problem can be summarized into

$$\text{Minimize } \sum_{t=1}^{24}\sum_{A=1}^{5} C_t P_a X_{at}$$

Subject to

Power Constraint (1):

$$\sum_{a=1}^{5} P_a\ X_{at} \le PeakPower_t \qquad \forall t$$

Time Constraint (2):

$$MinTime_a \le \sum_{t=1}^{24} X_{at} \le MaxTime_a \qquad \forall a$$

Consumer Preference Constraint (3):

$$X_{at} \le CP_{at} \qquad \forall a, \forall t$$

4. SCHEDULING APPROACH BASED ON PRICE SENSITIVITY

Price sensitivity is the degree to which the price of a product or service affects consumer's purchasing or usage behaviors. Another way of explaining price sensitivity is, "the consumer demand for a product is changed by the cost of the product. The degree of price sensitivity varies from service to service and from consumer to consumer. There are different methods of defining levels of price sensitivity. One of them is in terms of percentage. If we can categorize consumers based on a price sensitivity on a scale of 0% to 100%, then we can also model a residential appliance optimal scheduling for minimum cost consumption for each of those category of consumers. The purpose of such an approach would be, giving consumer the ability to optimally schedule their home appliances according to their price sensitivity level and pay minimum electricity bill. Also the various electricity supplier companies can study the behavior of the various levels of price sensitive consumers and anticipate corresponding electricity consumption.

4.1. 0% Price Sensitive Consumer Definition

A 0% price sensitive consumer is one who does not worry about his electricity bill, he uses his home appliances whenever he wishes to without considering the variable peak pricing for different hours of the day.

4.1.1. Time Preference for Appliances for a 0% Price Sensitive Consumer

A 0% price sensitive consumer has specific time preference, he uses the appliance whenever he wishes to, so we have 1's only for those time intervals when he wants to use the appliance. This is how the consumer preference matrix looks like for a 0% price sensitive consumer as shown in Table 5. Also refer to Table 2 for time slots and cost/hour, and Table 3 for appliances.

Table 5. Consumer Time Preference Matrix for 0% Price Sensitive Consumer

A		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
p	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
p	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
l	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
i	4	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
e	5	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0

4.2. 100% Price Sensitive Consumer Definition

A 100% price sensitive consumer is one who is very careful about his electricity bill. He will shift his entire load to off peak hours in order to have maximum benefit of a lower price, he uses his home appliances only when the variable peak pricing for electricity is lowest for the hour of the day that can be even at midnight to save cost. He will lower overall consumption by turning off extra light bulbs and turning off the air conditioners in the room that is not being used.

4.2.1. Time Preference for Appliances for a 100% Price Sensitive Consumer

100% price sensitive consumers do not have any time preference, so we have all 1's for all the 24 hours for the time preference matrix. This is how a consumer time preference looks like for a 100% price sensitive consumer as shown in Table 6.

Table 6. Consumer Time Preference Matrix for 100% Price Sensitive Consumer

A		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
p	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
p	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
l	3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
i	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
e	5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

4.3. 10% Price Sensitive Consumer Definition

A 10% price sensitive consumer is one who has 10% more flexibility in time preferences for using home appliances than 0% price sensitive consumers. He is not completely rigid about using a particular home appliance at a certain point of time during the day and he has 10% more flexibility of the time of use giving him more option of accessing lower peak value hours.

4.3.1. Time Preference for Appliances for a 10% Price Sensitive Consumer

0% price sensitive consumers have specific time preference, so we had 1's only for those time intervals when he wants to use the appliance. 10% price sensitive consumers will have 10% more time flexibility during the day. This is how a consumer time preference looks like for a 10% price sensitive consumer as shown in Table 7.

Table 7. Consumer Time Preference Matrix for 10% Price Sensitive Consumer

A		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
p	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0	0	0
p	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
l	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	0	0
n	4	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
s	5	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0

4.4. 90% Price Sensitive Consumer Definition

A 90% price sensitive consumer is one who has 90% more flexibility in time preferences for using home appliances than 0% price sensitive consumers. He is not rigid about using a particular home appliance at a certain point of time during the day and he has 90% more flexibility of the time of use giving him a lot more option of accessing lower peak value hours.

4.4.1. Time Preference for Appliances for a 90% Price Sensitive Consumer

0% price sensitive consumers have specific time preference, so we had 1's only for those time intervals when he wants to use the appliance as shown in table 7. 90% price sensitive consumers will have 90% more time flexibility during the day. This is how a consumer time preference looks like for a 90% price sensitive consumer (Table 8).

Table 8. Consumer Time Preference Matrix for 90% Price Sensitive Consumer

A		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
p	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	1
p	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1
l	3	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
n	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0
s	5	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

4.5. Consumer Time Preference Matrix for other Percentage of Price Sensitive Consumers

The Consumer time preference matrix for all other percentages (20%, 30%, 40%, 50%, 60%, 70%, 80%) of price sensitive consumers are shown below in Tables 9 to 15:

Table 9. Consumer Time Preference Matrix for 20% Price Sensitive Consumer

A		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
p	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0
p	2	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
l	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	0
n	4	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
s	5	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	0	0	0	0	0	0	0	0

Table 10. Consumer Time Preference Matrix for 30% Price Sensitive Consumer

A		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
p	1	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	0	0	0	0
p	2	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
l	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1
n	4	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
s	5	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0

Table 11. Consumer Time Preference Matrix for 40% Price Sensitive Consumer

A		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
p	1	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
p	2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
l	3	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1
n	4	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
s	5	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0

Table 12. Consumer Time Preference Matrix for 50% Price Sensitive Consumer

A		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
p	1	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
p	2	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
l	3	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1
n	4	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
s	5	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0

Table 13. Consumer Time Preference Matrix for 60% Price Sensitive Consumer

A		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
p	1	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
p	2	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	1	1	1	1
l	3	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
n	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0
s	5	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0

Table 14. Consumer Time Preference Matrix for 70% Price Sensitive Consumer

A		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
p	1	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
p	2	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1
l	3	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
n	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0
s	5	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

Table 15. Consumer Time Preference Matrix for 80% Price Sensitive Consumer

A		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
p	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0
p	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0	1	1	1	1
l	3	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
n	4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	0	0	0	0
s	5	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0

5. IMPLEMENTATION DETAIL

The application developed as a part of this project is an optimization model using linear programming. I have used AMPL as the modeling language. AMPL is a comprehensive and powerful algebraic modeling language for linear and nonlinear optimization problems, in discrete or continuous variables. In this section, the LP formulation of the optimal scheduling of residential appliances has been transformed to AMPL code.

5.1. LP Formulation to AMPL Code

5.1.1. Decision Variables

The corresponding decision variables have been used in AMPL, the table 16 below shows the decision variables used in LP formulation and the corresponding variable used in AMPL.

Table 16. Decision Variables Used in LP Formulation and AMPL

Decision Variables		
LP Formulation	**AMPL code**	**Purpose**
C_t	Cost[t]	The cost of electricity at time slot t in USD/KWh
P_a	Appliancepower[t]	The electric power consumption of the appliance a in Kilo
X_{at}	onOff[a][t]	Binary decision variable where Xat ∈{0,1} }, Xat = 1 if and only if appliance a can be turned on during the time slot t according to consumer preferrence.
CP_{at}	ConsumerPreference[a][t]	The consumer preference matrix determining at which time of the day that the consumer prefers to use his appliance
$Peakpower_t$	PEAKPower[t]	The Peak energy usage allowed per hour
$MaxTime_a$, $MinTime_a$	MaxTime[a], MinTime[a]	The maximum and minimum time taken to complete operation by an appliance.

5.1.2. Direct Formulation

The natural approach to program this LP formulation in AMPL is entered using syntax that is very close to the algebraic expressions of the LP formulation above. To start we need to create a .mod file, for example scheduler.mod, in which we write the AMPL code. In general, we can divide the code in three main parts as shown below:

5.1.2.1. Part 1: Declaration of Variables (variables, parameters, sets etc.)

The decision variables to be used for formulating linear programming for the optimal scheduling of the home appliances are declared below. Refer to Table 16 for the purpose of each decision variable.

```
set Time;

set Appliance;

param Cost{t in Time} >= 0;

param AppliancePower{a in Appliance} >= 0;

param PEAKPower{t in Time} >= 0;

param MaxTime{a in Appliance} >= 0;

param MinTime{a in Appliance} >= 0;

param ConsumerPreference{a in Appliance,t in Time} >= 0;

var onOff {a in Appliance, t in Time} binary;
```

5.1.2.2. Part 2: Objective Function (name and mathematical expression)

The objective of the proposed scheduler is to minimize the total electricity cost for operating the appliances. In this paper the cost calculation is based on a given 24-hour ahead electricity tariff (e.g. USD per kWh).

Let Cost[t] denote the electricity tariff for time slot t, AppliancePower[a] denote the electric power consumption of the appliance *a* and onOff[a,t] denote the binary decision variable where onOff[a,t] ∈{0,1}, onOff[a,t] = 1 if and only if appliance *a* can be turned on during the time slot t according to consumer preference otherwise 0. Thus, the total electricity cost for running all appliances is:

```
minimize cost_of_consumption: sum{t in Time, a in Appliance} Cost[t] *
    AppliancePower[a] * onOff[a,t];
```

5.1.2.3. Part 3: Constraints (names and corresponding mathematical expressions)

5.1.2.3.1. Energy Constraint

The summation of the energy/power consumption of all the appliances running at a particular hour should not exceed the Peak energy usage allowed per hour. If **AppliancePower[a]** is the electric power consumption of the appliance ***a*** in **Kilo, onOff[a,t]** is the binary decision variable where **onOff[a,t]** ∈{0,1}, **onOff[a,t]** = 1 if and only if appliance ***a*** can be turned on during the time slot **t** according to consumer preference and **PEAKPower[t]** is the Peak energy usage allowed per hour, then the following constraint can be enforced:

```
subject to PowerConstraint{t in Time}:
sum{a in Appliance} AppliancePower[a] * onOff[a,t] <= PEAKPower[t];
```

5.1.2.3.2. Time Constraint

The appliance must be operated in between the minimum and maximum time of operation of the appliance. If **MaxTime[a]** and **MinTime[a]** are the maximum and minimum time taken to complete operation by an appliance ***a*** and **onOff[a,t]** is the binary decision variable where **onOff[a,t]** $\in\{0,1\}$, **onOff[a,t]** = 1 if and only if appliance ***a*** can be turned on during the time slot **t** according to consumer preference, then the following constraint is enforced:

```
subject to TimeMaxConstraint{a in Appliance}:
    sum{t in Time} onOff[a,t] <= MaxTime[a];


subject to TimeMinConstraint{a in Appliance}:
    sum{t in Time} onOff[a,t] >= MinTime[a];
```

5.1.2.3.3. Consumer Preference Constraint

The time of use of an appliance should be made considering the consumer's preferred time of use of the appliance throughout the day. If **onOff[a,t]** is the binary decision variable where **onOff[a,t]** $\in\{0,1\}$, **onOff[a,t]** = 1 if and only if appliance ***a*** can be turned on during the time slot **t** according to consumer preference and **ConsumerPreference[a,t]** is the consumer preference matrix determining at which time of the day, consumer prefers to use his appliance, then the following constraint can be enforced:

```
subject to ConsumerPreferenceConstraint{a in Appliance, t in Time}:
    onOff[a,t] <= ConsumerPreference[a,t];
```

5.1.3. Data File

The corresponding data file or .dat file is formulated.

First in the data file we need to set the Time in accordance to Table 17.

Table 17. Time Slots and Cost/hour

Time slots	**t**	C_t
12:00 a.m. - 1:00 a.m.	1	0.041
1:00 a.m. - 2:00 a.m.	2	0.038
2:00 a.m. - 3:00 a.m.	3	0.036
3:00 a.m. - 4:00 a.m.	4	0.037
4:00 a.m. - 5:00 a.m.	5	0.038
5:00 a.m. - 6:00 a.m.	6	0.039
6:00 a.m. - 7:00 a.m.	7	0.054
7:00 a.m. - 8:00 a.m.	8	0.057
8:00 a.m. - 9:00 a.m.	9	0.058
9:00 a.m. - 10:00 a.m.	10	0.069
10:00 a.m. - 11:00 a.m.	11	0.07
11:00 a.m. - 12:00 p.m.	12	0.068
12:00 p.m. - 1:00 p.m.	13	0.068
1:00 p.m. - 2:00 p.m.	14	0.068
2:00 p.m. - 3:00 p.m.	15	0.065
3:00 p.m. - 4:00 p.m.	16	0.065
4:00 p.m. - 5:00 p.m.	17	0.063
5:00 p.m. - 6:00 p.m.	18	0.095
6:00 p.m. - 7:00 p.m.	19	0.094
7:00 p.m. - 8:00 p.m.	20	0.072
8:00 p.m. - 9:00 p.m.	21	0.063
9:00 p.m. - 10:00 p.m.	22	0.056
10:00 p.m. - 11:00 p.m.	23	0.049
11:00 p.m. - 12:00 a.m.	24	0.048

set Time := 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24;

Next. set the Appliance in accordance to the following table (Table 18):

Table 18. Appliances

Appliances	a
Washer	1
Dryer	2
Dish Washer	3
Water Heater	4
Water Softener	5

```
set Appliance := 1 2 3 4 5;
```

Then we set the Cost parameter also according to Table 17.

```
param: Cost :=
1       0.041
2       0.038
3       0.036
4       0.037
5       0.038
6       0.039
7       0.054
8       0.057
9       0.058
10      0.069
11      0.070
12      0.068
13      0.068
14      0.068
15      0.065
16      0.065
17      0.063
18      0.095
19      0.094
20      0.072
21      0.063
22      0.056
23      0.049
24      0.048;
```

We also set the AppliancePower parameter in accordance to Table 19 below:

Table 19. Problem Setup

Serial No.	Appliances	Power Consumption/hr. (KW)	Time taken to complete operation (hrs.)	Consumer preferred time of use
1	Washer	0.51	2-3	9 a.m. – 8 p.m.
2	Dryer	3.40	1-2	9 p.m. – 8 p.m.
3	Dish Washer	1.50	2-3	9 a.m. – 5 p.m. & 8 a.m. – 10 p.m.
4	Water Heater	0.48	1-2	4 a.m. – 6 a.m. & 6 p.m. – 9 p.m.
5	Water Softener	0.002	1-2	7 a.m. – 10 p.m.

param: AppliancePower :=
1	0.51
2	3.40
3	1.50
4	0.48
5	0.002;

We assume the PEAKPower for each hour is 4.57 KW. And at no time should the total load go above this peak power for a given hour.

param: PEAKPower :=
1	4.57
2	4.57
3	4.57
4	4.57
5	4.57
6	4.57
7	4.57
8	4.57
9	4.57
10	4.57
11	4.57
12	4.57
13	4.57
14	4.57
15	4.57
16	4.57

```
17        4.57
18        4.57
19        4.57
20        4.57
21        4.57
22        4.57
23        4.57
24        4.57;
```

We set the MaxTime parameter in accordance to the column 4 of Table 19.

```
param:  MaxTime :=
1         3
2         2
3         3
4         2
5         2;
```

We set the MinTime parameter in accordance to the column 4 of Table 19.

```
param:  MinTime :=
1         2
2         1
3         2
4         1
5         1;
```

We set the ConsumerPreference parameter in accordance to Table 20 below:

Table 20. Consumer Time Preference Matrix for 0% Price Sensitive Consumer

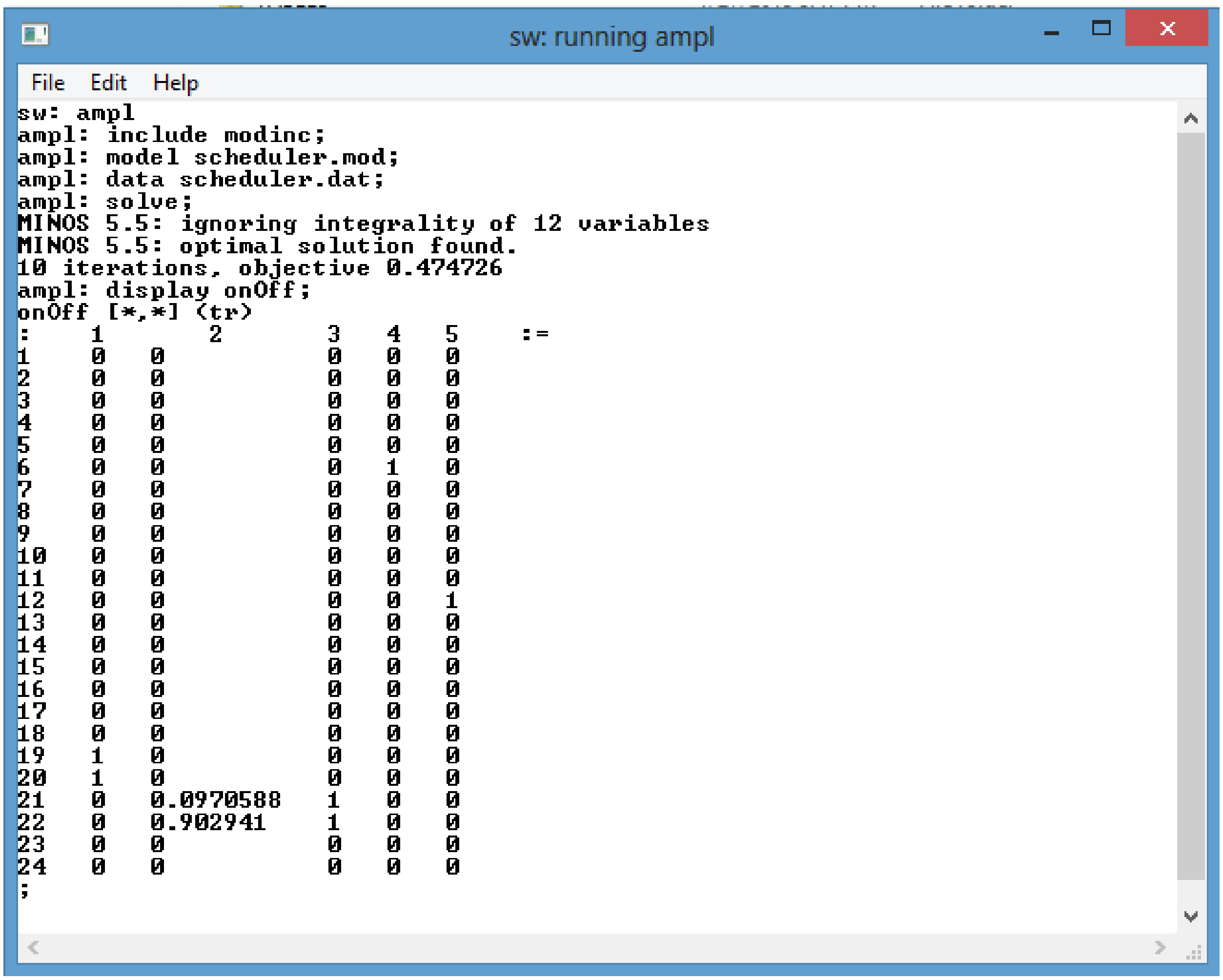
A		**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**
p	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	0
p	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0
l	3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0
i	4	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
e	5	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0

```
param  ConsumerPreference:  1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 :=
                1                 0 0 0 0 0 0 0 0 0  0   0   0   0    0   0   0   0   1   1  1   0   0   0   0
                2                 0 0 0 0 0 0 0 0 0  0   0   0   0    0   0   0   0   0   0  0   1   1   0   0
                3                 0 0 0 0 0 0 0 0 0  0   0   0   0    0   0   0   0   0   0  1   1   1   0   0
                4                 0 0 0 0 0 1 1 0 0  0   0   0   0    0   0   0   0   0   0  0   0   0   0   0
                5                 0 0 0 0 0 0 0 0 0  0   1   1   0    0   0   0   0   0   0  0   0   0   0   0;
```

This is the parameter which changes for each level of price sensitivity according to Tables 6 through 15, and we note down different results/output for different levels of price sensitivity.

5.1.4. Output for 0% Price Sensitivity

After running the .mod file using the consumer preference matrix for 0% price sensitivity, the output below is shown in Figure 1.




```
sw: ampl
ampl: include modinc;
ampl: model scheduler.mod;
ampl: data scheduler.dat;
ampl: solve;
MINOS 5.5: ignoring integrality of 12 variables
MINOS 5.5: optimal solution found.
10 iterations, objective 0.474726
ampl: display onOff;
onOff [*,*] (tr)
:     1        2        3    4    5     :=
1     0    0            0    0    0
2     0    0            0    0    0
3     0    0            0    0    0
4     0    0            0    0    0
5     0    0            0    0    0
6     0    0            0    1    0
7     0    0            0    0    0
8     0    0            0    0    0
9     0    0            0    0    0
10    0    0            0    0    0
11    0    0            0    0    0
12    0    0            0    0    1
13    0    0            0    0    0
14    0    0            0    0    0
15    0    0            0    0    0
16    0    0            0    0    0
17    0    0            0    0    0
18    0    0            0    0    0
19    1    0            0    0    0
20    1    0            0    0    0
21    0    0.0970588    1    0    0
22    0    0.902941     1    0    0
23    0    0            0    0    0
24    0    0            0    0    0
;
```

Figure 1. Output

From the output displayed for onOff we can show the optimal scheduling of the appliances for 0% price sensitive consumer as shown in Table 21 below and further graphically in Figure 2.

Table 21. Optimal Scheduling Output for 0% Price Sensitive Consumer

Time	Washer	Dryer	Dishwasher	Water Heater	Water Softener
1					
2					
3					
4					
5					
6				4	
7					
8					
9					
10					
11					
12					5
13					
14					
15					
16					
17					
18					
19	1				
20	1				
21		2	3		
22		2	3		
23					
24					

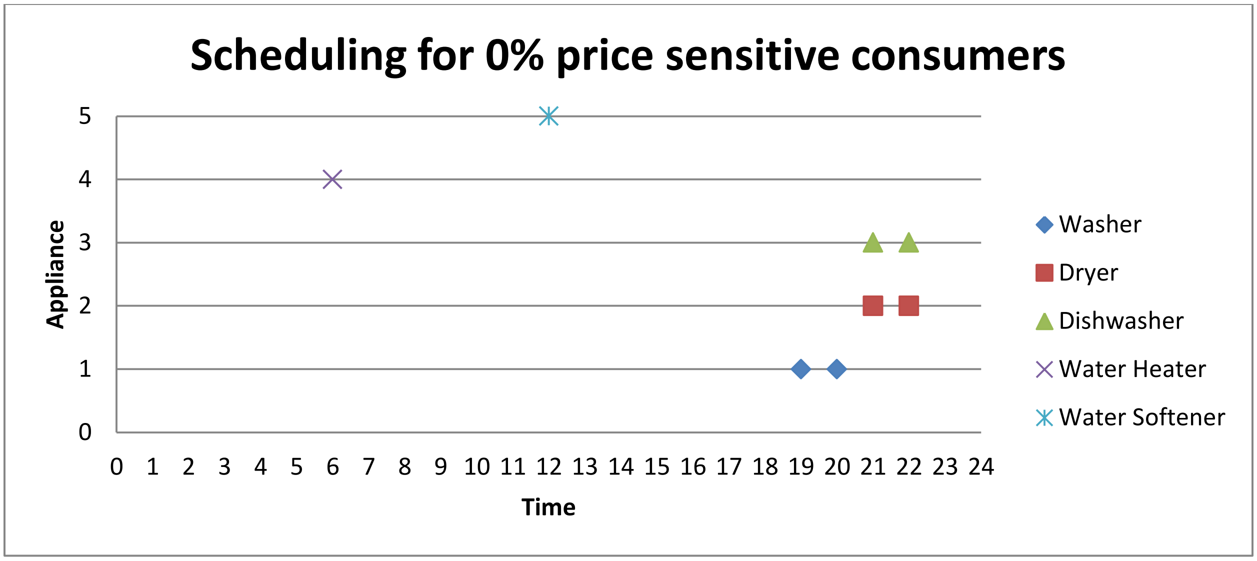


Figure 2. Scheduling for 0% Price Sensitive Consumer

5.1.5. Code Run for Different Price Sensitive Consumers

The AMPL code is run for different price sensitive consumers (0% to 100%) by changing the consumer time preference matrix according to Table 5 through 15. For each percentage of price sensitivity, the AMPL code is executed and results are noted.

6. EXPERIMENTAL RESULT ANALYSIS

The results based linear programming formulation in AMPL for optimal scheduling of residential appliances for different price sensitive consumers are discussed in this chapter. In the software all of the concepts and methodology discussed in the earlier chapters are applied. The consumer behavior, appliance scheduling and savings based on the levels of sensitivity to price. The analysis shows that all of the categories of consumers can use the scheduling model to optimally schedule their home appliances and benefit by the lowest cost consumption.

6.1. Experimental Results for 0% Price Sensitive Consumers

In this section, the optimal scheduling with minimum cost is modeled for different percentage of price sensitive consumers.

6.1.1. Experimental Results for 0% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 0% price sensitive consumer is **$0.474726**. Hence, his monthly electricity bill would come around **$14.24178** for a month. The optimal scheduling after running the scheduler.mod for 0% price sensitive consumer is graphically shown below in Figure 3:

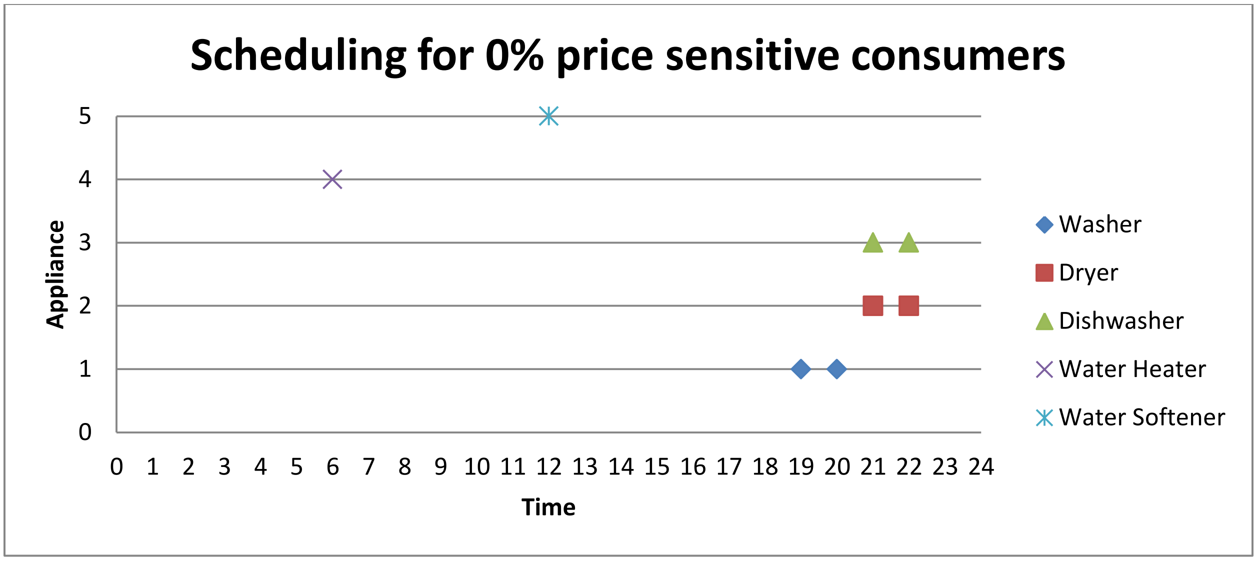


Figure 3. Scheduling for 0% Price Sensitive Consumer

6.1.2. Experimental Results for 10% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 10% price sensitive consumer is **$0.425356**. Hence, his monthly electricity bill would come around **$12.76068** for a month. The optimal scheduling after running the scheduler.mod for 10% price sensitive consumer is graphically shown below in Figure 4:

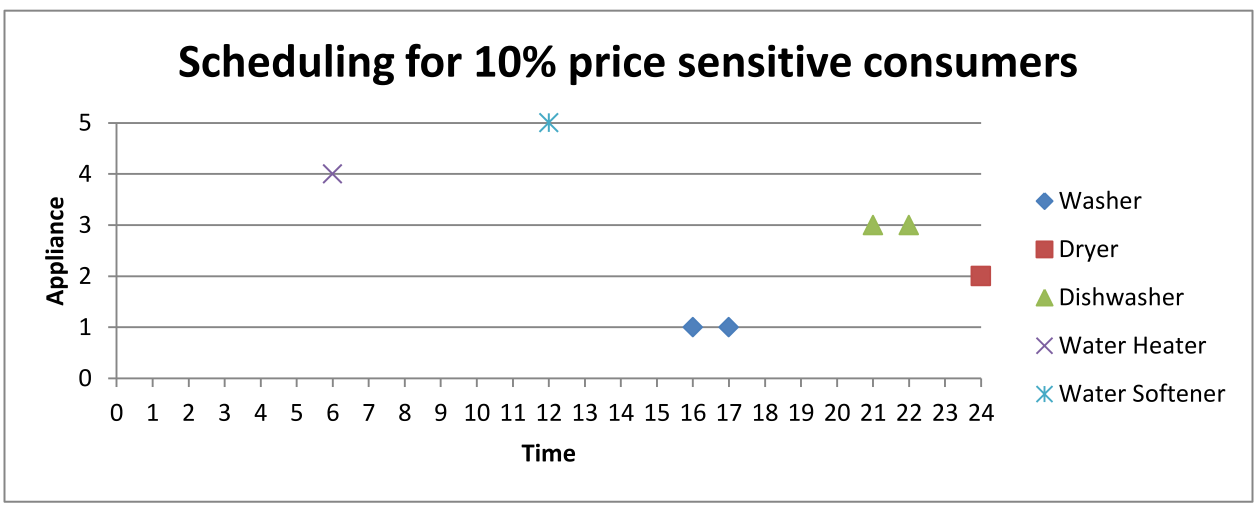


Figure 4. Scheduling for 10% Price Sensitive Consumer

6.1.3. Experimental Results for 20% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 20% price sensitive consumer is **$0.36987**. Hence, his monthly electricity bill would come around **$11.0961** for a month. The optimal scheduling after running the scheduler.mod for 20% price sensitive consumer is graphically shown below in Figure 5:

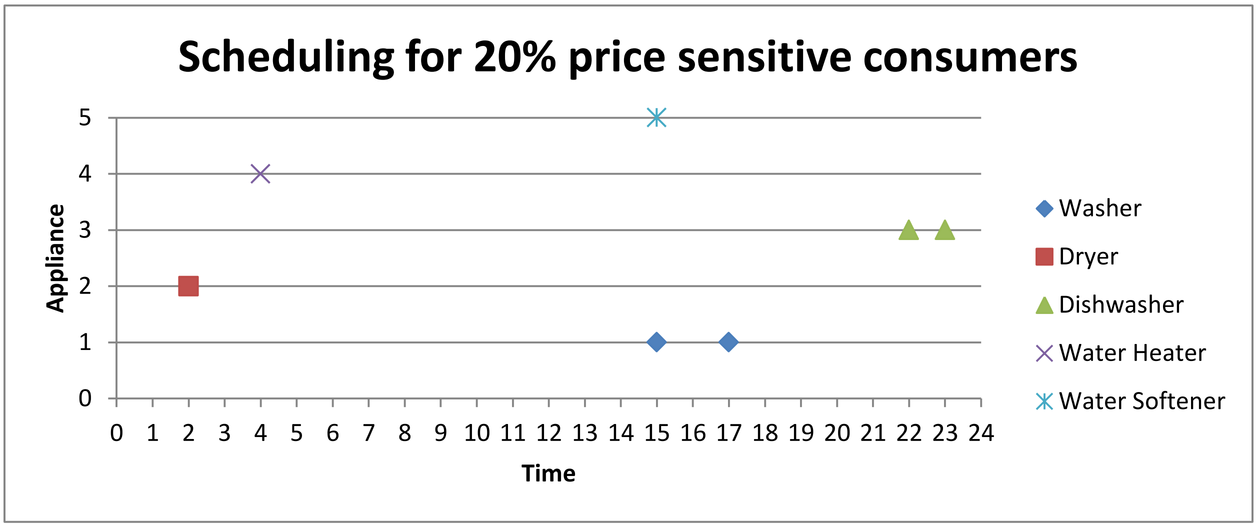


Figure 5. Scheduling for 20% Price Sensitive Consumer

6.1.4. Experimental Results for 30% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 30% price sensitive consumer is **$0.350586**. Hence, his monthly electricity bill would come around **$10.51758** for a month. The optimal scheduling after running the scheduler.mod for 30% price sensitive consumer is graphically shown below in Figure 6:

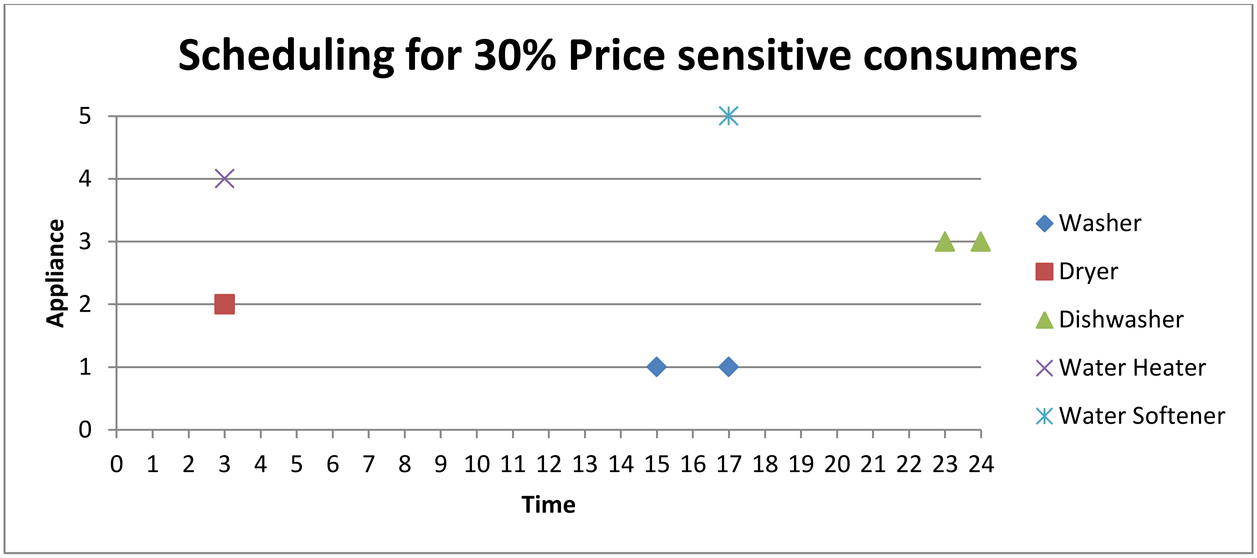


Figure 6. Scheduling for 30% Price Sensitive Consumer

6.1.5. Experimental Results for 40% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 40% price sensitive consumer is **$0.350576**. Hence, his monthly electricity bill would come around **$10.51728** for a month. The optimal scheduling after running the scheduler.mod for 40% price sensitive consumer is graphically shown below in Figure 7:

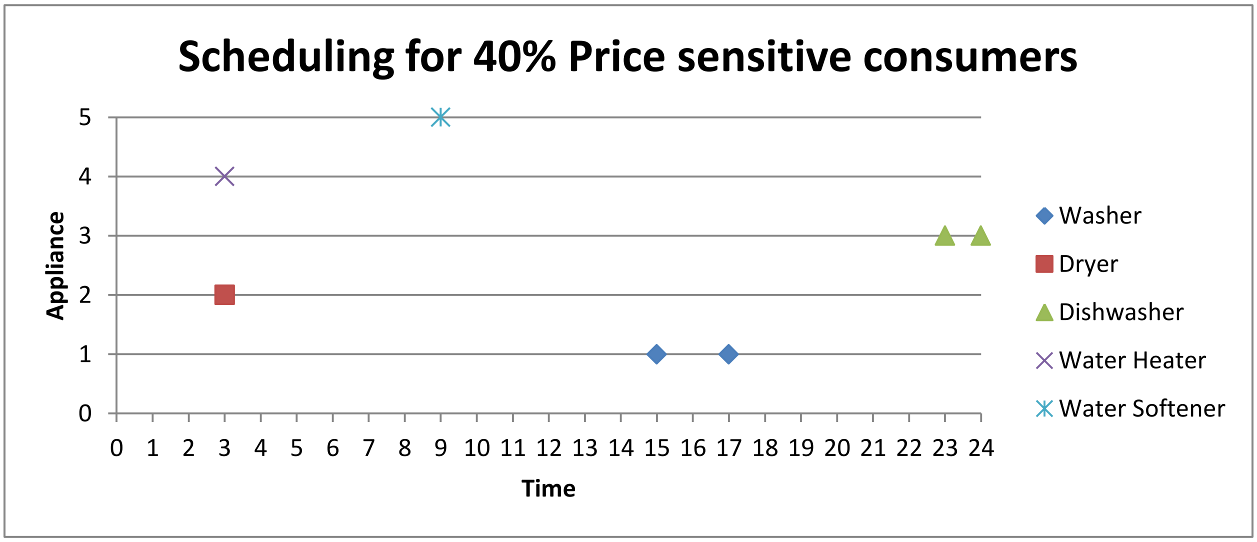


Figure 7. Scheduling for 40% Price Sensitive Consumer

6.1.6. Experimental Results for 50% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 50% price sensitive consumer is **$0.343944**. Hence, his monthly electricity bill would come around **$10.31832** for a month. The optimal scheduling after running the scheduler.mod for 50% price sensitive consumer is graphically shown below in Figure 8:

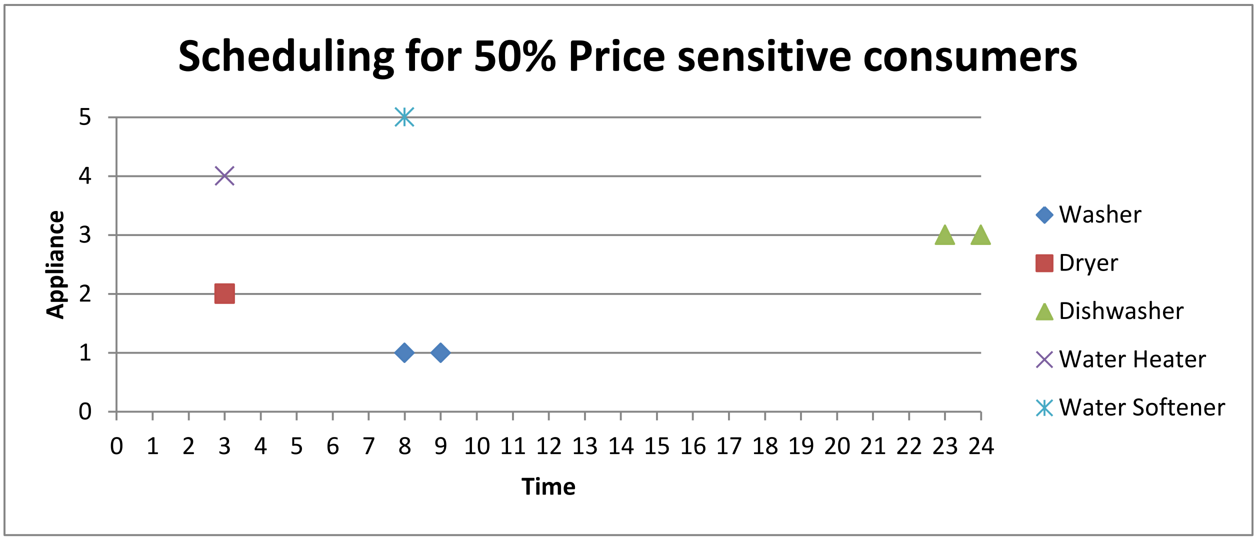


Figure 8. Scheduling for 50% Price Sensitive Consumer

6.1.7. Experimental Results for 60% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 60% price sensitive consumer is **$0.332724**. Hence, his monthly electricity bill would come around **$9.98172** for a month. The optimal scheduling after running the scheduler.mod for 60% price sensitive consumer is graphically shown below in Figure 9:

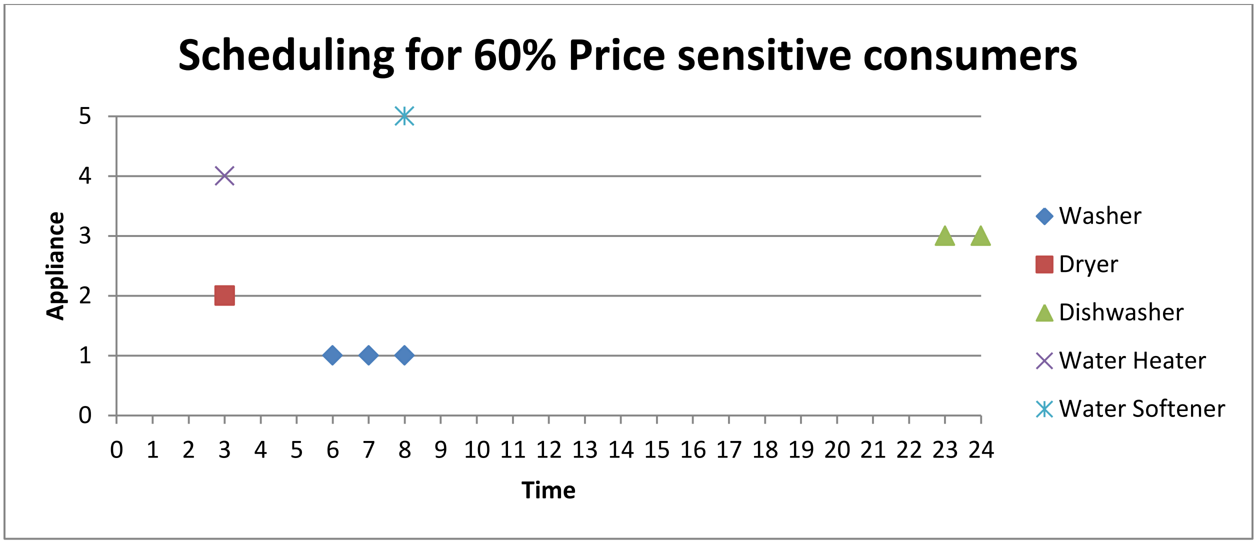


Figure 9. Scheduling for 60% Price Sensitive Consumer

6.1.8. Experimental Results for 70% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 70% price sensitive consumer is **$0.323538**. Hence, his monthly electricity bill would come around **$9.70614** for a month. The optimal scheduling after running the scheduler.mod for 70% price sensitive consumer is graphically shown below in Figure 10:

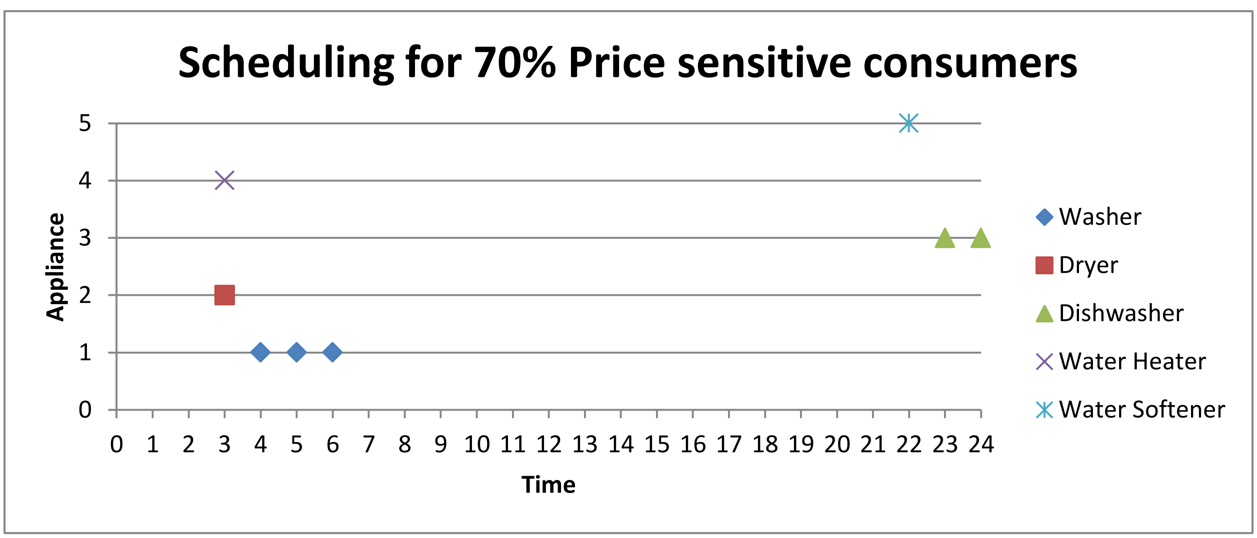


Figure 10. Scheduling for 70% Price Sensitive Consumer

6.1.9. Experimental Results for 80% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 80% price sensitive consumer is **$0.307486**. Hence, his monthly electricity bill would come around **$9.22458** for a month. The optimal scheduling after running the scheduler.mod for 80% price sensitive consumer is graphically shown below in Figure 11:

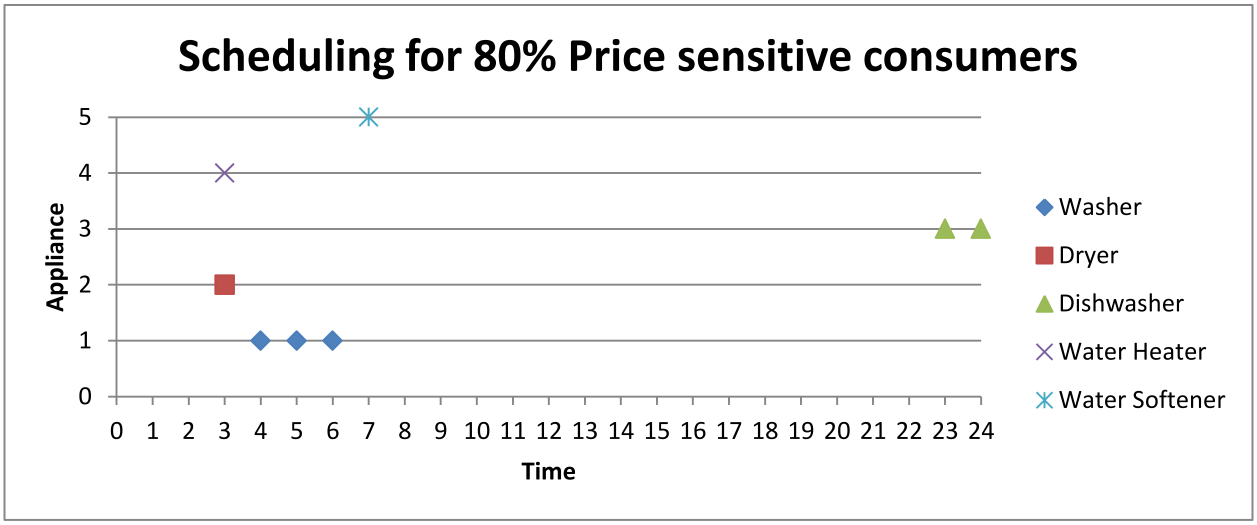


Figure 11. Scheduling for 80% Price Sensitive Consumer

6.1.10. Experimental Results for 90% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 90% price sensitive consumer is **$0.289482**. Hence, his monthly electricity bill would come around **$8.68446** for a month. The optimal scheduling after running the scheduler.mod for 90% price sensitive consumer is graphically shown below in Figure 12:

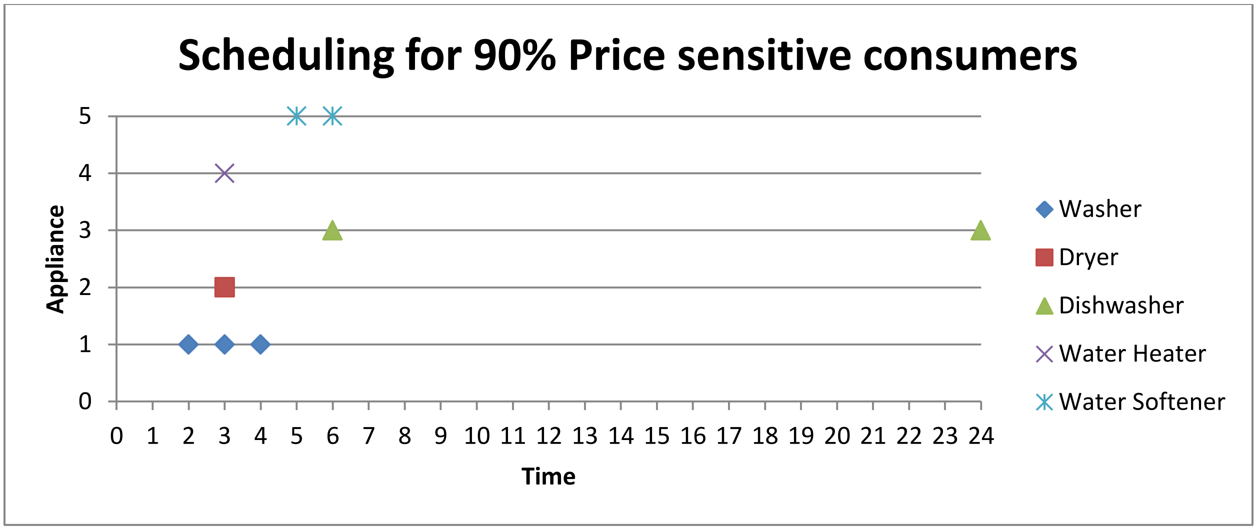


Figure 12. Scheduling for 90% Price Sensitive Consumer

6.1.11. Experimental Results for 100% Price Sensitive Consumers

Using Linear programming for Scheduling of appliances, we find that the Cost of electricity (bill) for 100% price sensitive consumer is **$0.287804**. Hence, his monthly electricity bill would come around **$8.63412** for a month. The optimal scheduling after running the scheduler.mod for 100% price sensitive consumer is graphically shown below in Figure 13:

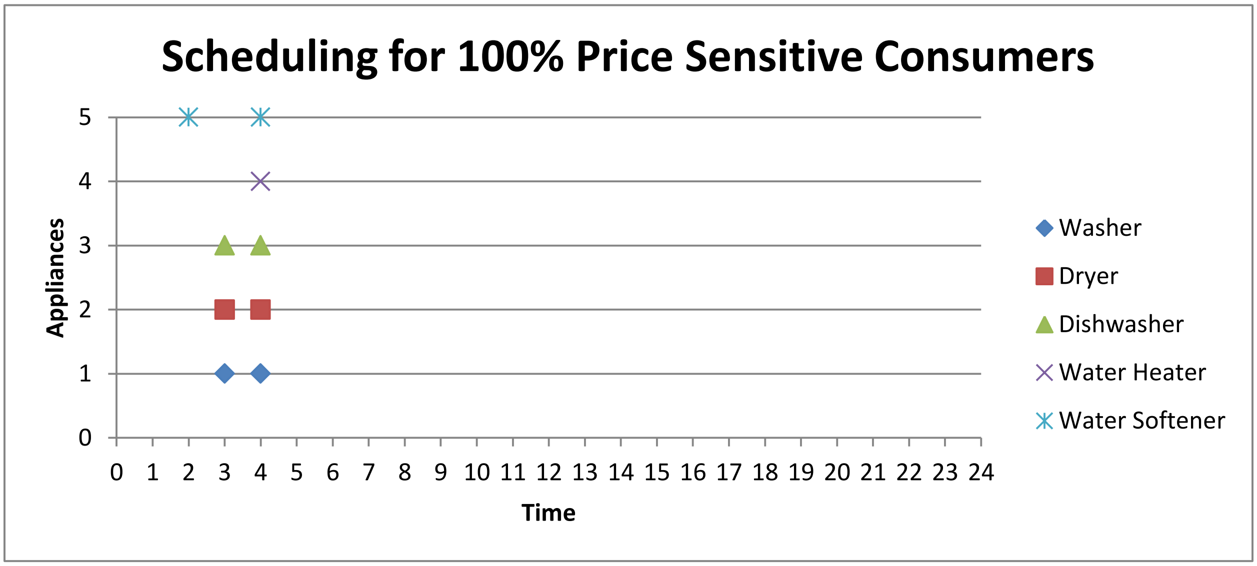


Figure 13. Scheduling for 100% Price Sensitive Consumer

6.2. Analysis of Cost Against Price Sensitivity

In this section we analyze the cost of electricity consumption against percentage price sensitivity of a consumer. Table 22 lists the values:

Table 22. Monthly Electricity Cost/Expenditure Against Percentage of Price Sensitivity

% Price Sensitivity	Cost(USD)/day	Monthly Cost(USD)
0	0.474726	14.24178
10	0.425356	12.76068
20	0.36987	11.0961
30	0.350586	10.51758
40	0.350576	10.51728
50	0.343944	10.31832
60	0.332724	9.98172
70	0.323538	9.70614
80	0.307486	9.22458
90	0.289482	8.68446
100	0.287804	8.63412

We see that the monthly expenditure/cost of a consumer on electricity decreases exponentially with the increase in the percentage of price sensitivity as shown in Figure 14. 0% to 20% of price sensitivity could be considered as Low, 30% to 80% can be considered as medium and 90% to 100% can be considered as high price sensitivity where consumer makes maximum savings.

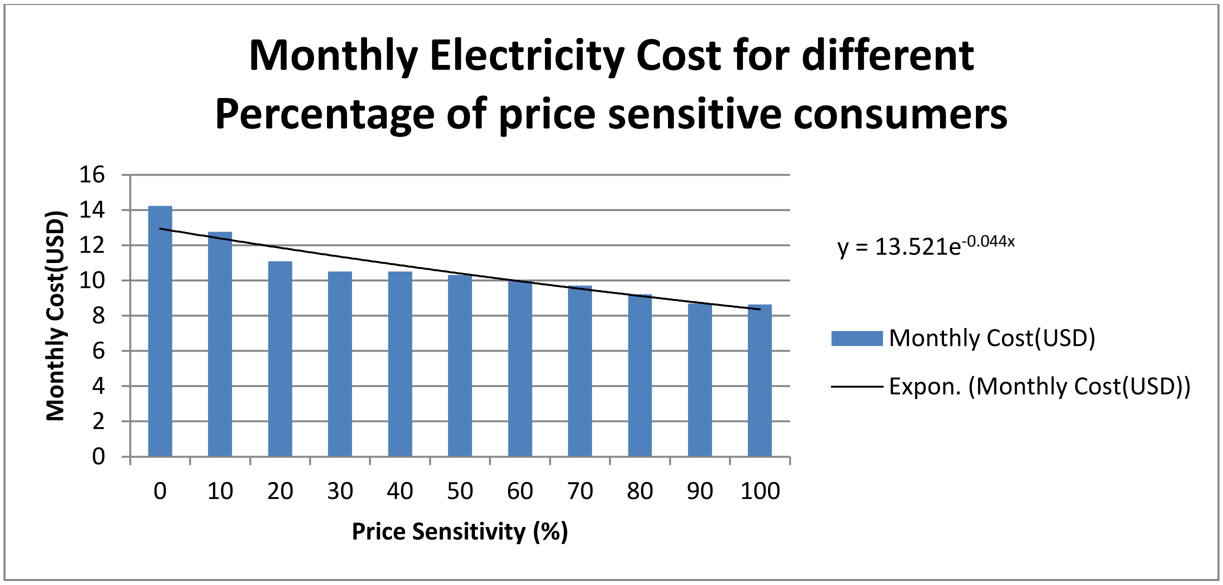


Figure 14. Monthly Electricity Cost/Expenditure Against Percentage of Price Sensitivity

After analyzing the data, we can define the following exponential function for monthly cost vs. price sensitivity.

Let C_m be the monthly cost/expenditure on electricity

Let p be the price sensitivity of consumers

Then we can define a function:

$$\boxed{C_m = 13.521e^{-0.044p}}$$

7. CONCLUSION AND FUTURE WORK

In this paper, we used a linear programming model to optimally schedule a set of residential appliances and minimize the electricity cost consumption for various percentages of price sensitive consumers. In order to achieve that, first we defined the constraints namely energy, time and consumer time preference and then use the constraints to minimize the cost thus optimally scheduling a set of residential appliances.

We conducted experiments by applying the optimal scheduling model for different percentage of price sensitive consumers ranging from 0% to 100%. The results showed how the cost consumption decreased exponentially with the increase in price sensitivity.

In this paper we had taken into consideration, that the consumer had perfect information regarding the cost of electricity for each hour throughout the day, on the basis of which he modified his behavior and time of use of the appliances throughout the day. However, too much perfect information available to the consumer could have an adverse effect on the system. Since if the consumer has exact information regarding the price of electricity for each hour in advance, he will try to be more price sensitive in order to make maximum saving, and as a result, more consumers may use their appliances at the low peak hours, which in turn would increase energy load, and further increase the price of electricity for such hours. How the system would behave in response to very high price sensitivity and what precautions can be taken to prevent such adverse effects is an important concern. Also, with perfect information provided, one question is whether or not consumers would try to be in the high price sensitivity range. This can be a good scope for future research.

REFERENCES

[1] Kin Cheong Sou, James Weimer, Henrik Sandberg, and Karl Henrik Johansson. Scheduling Smart Home Appliances Using Mixed Integer Linear Programming. 2011 50th IEEE Conference on Decision and Control and European Control Conference (CDC-ECC) Orlando, FL, USA, December 12-15, 2011.

[2] Md Mahbubur Rahman Khan (2012). The Modelling of consumer responses to dynamic pricing in a smart grid, (26-33).

[3] Ahmad Faruqui (2010). The Ethics of Dynamic Pricing, (2-7).

[4] Allcott, H. (2011). Rethinking Real-time Electricity Pricing. Resource and Enegy Economics , (820-842).

[5] Bapat, T., N. Sengupta, S. Ghai, V. Arya, Y. Shrinivasan and D. Seetharam (2011). User-sensitive Scheduling of Home Appliances. GreenNet'11 , (43-48).

[6] Chao, H. (2010). Price-Responsive Demand Management for a Smart Grid World. The Electricity Journal , (7-20).

[7] Du, P. and N. Lu (2011). Appliance Commitment for Household Load Scheduling. IEEE Transactions on Smart Grid , (411-419).

[8] Faruqui, A. and S. Segici (2009). Household Response To Dynamic Pricing Of Electricity—A Survey of the Experimental Evidance, (1-53).

[9] M.T. Jonsson and T.P. Runarsson. Optimizing the sailing route for fixed groundfish survey stations. International Council for the Exploration of the Sea, 1996.

[10] Faruqui, A. and S. Sergici (2010). Household Response to Dynamic Pricing of Electricity: A Survey of 15 Experiments. Journal of Regulatory Economics, (193-225).

[11] Faruqui, A., R. Hledik, and J. Tsoukalis (2009). The Power of Dynamic Pricing. The Electricity Journal, 2009 Elsevier, (42-56).

[12] Nygard, K., S. Bou Ghosn, M. Chowdhury, D. Loegering, R. McCulloch and P. Ranganathan (2011). Optimization Models for Energy Reallocation in a Smart Grid. 2011 IEEE Conference on Computer Communications Workshops (INFOCOM WKSHPS), (186 - 190).

[13] Dynamic Pricing and the stability of smart grid, http://www.leonardo-energy.org/dynamic-pricing-and-stability-smart-grid

[14] Trevor Morgan. Smart Grids and Electric Vehicles: Made for each other? (8-10), 2012

[15] Roozbehani, M., et al., "Volatility of Power Grids under Real-Time Pricing," ArXiv e-prints, June 2011.

[16] Ziming Zhu, Jie Tang, Sangarapillai Lambotharan, Woon Hau Chin, Zhong Fan. An integer linear programming based optimization for home demand-side management in smart grid January 2012.

[17] A. Faruqi, R. Hledik and J. Tsoukalis, "The Power of Dynamic Pricing," *The Electricity Journal,* vol. 22, no. 3, pp. 42-56, 2009.

[18] Robert Fourer, David M. Gay and Brian W. Kernighan, "A Modeling Language for Mathematical Programming." Management Science 36 (1990) 519-554.

[19] Ryan McCulloch (2012), A distributed and autonomous power market for the future smart grid: design, simulation and testing, (1-13).