CAVITATION INCEPTION AND CONTAMINANT EFFECTS

A Paper
Submitted to the Graduate Faculty
of the
North Dakota State University
of Agriculture and Applied Science

By
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In Partial Fulfillment of the Requirements
for the Degree of
MASTER OF SCIENCE

Major Department:
Mechanical Engineering

December 2013

Fargo, North Dakota
Title

Cavitation Inception and Contaminant Effects

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The Supervisory Committee certifies that this disquisition complies with North Dakota State University's regulations and meets the accepted standards for the degree of

MASTER OF SCIENCE

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ABSTRACT

Within a liquid, it is possible to lower the pressure to a point in which the nuclei gas bubbles will cause cavitation inception. During this drop in pressure, at constant temperature, the bubble tension will increase to the inception pressure. The difference between vapor pressure and the pressure where inception occurs is the tensile strength of the liquid. Frankel introduced the calculation of the potential tensile strength of a liquid. Studies show a drastic difference in the calculated potential tensile strength of a liquid and the tensile strength obtained through experimentation. Environmental effects or contaminants affect cavitation inception. Studying the mechanics of a bubble will help one to predict if inception, and later cavitation, may occur and to what extent.
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LIST OF SYMBOLS

$r$.................................length of bubble radius, radial coordinate

$\theta$.................................polar angle

$\phi$.................................azimuth angle

$u_x$.................................velocity and direction

$P$................................Pressure

$g_r$.................................gravitational force in the radial direction

$R$................................Radius at rupture

$t$................................time

$\nu$.................................kinematic viscosity

$\rho$.................................density

$R_c$.................................Critical Radius

$S$................................Surface Tension

$\sigma$.................................molecular diameter potential, cavitation number

$\epsilon$.................................depth of potential well

$\gamma$.................................surface tension

$a$.................................the internal radius of a tube

$b$.................................the external radius of a tube

$G$.................................bulk moduli

$n$.................................bulk rigidity

$T$................................Temperature

$V$................................Volume

$\Delta P$.................................Potential Tensil Strength
$C_p$……………………………………………………………………………………………………Pressure coefficient

$U_\infty$………………………………………………………………………………Velocity of flow far from bubble

$\theta_A$…………………………………………………………………………………………Advancing Contact Angle

$\theta_R$…………………………………………………………………………………………Reducing Contact Angle

$2\alpha$…………………………………………………………………………………………Angle of Crevice Pocket

$N$………………………………………………………………………………………………Gas Constant
1. INTRODUCTION

Cavitation occurs when vapor cavities within a liquid rupture due to a sudden decrease in the pressure of the liquid. Cavitation is generally an undesirable effect because the collapse of vapor bubbles produces shock waves so severe that damage to a surface may occur. This effect can be seen occurring in nature as well as in engineering applications. Some of these damaging effects can be seen in Figure 1.1. The destructive effects of cavitation can be used in the destruction of corrosive materials or in ultrasonic medical devices to remove unwanted tissue. The cavitation effect may also be beneficial as it is in supercavitation which occurs at the low pressure tip of high speed submerged water craft. A gas bubble at the leading edge (naturally or mechanically occurring) allows the development of a pressure bubble so large that it will encompass the entire body of the vessel thus reducing drag. We understand the effects that cavitation can cause. This paper will focus on what affects cavitation inception and bubble dynamics.

Figure 1.1: Examples of cavitation damage. [28][8][9][10]
Inception occurs at a gas nucleation point within a liquid. This nucleation may be heterogeneous or homogeneous. The isothermal liquid is subjected to a decrease in pressure until the pressure falls below the vapor pressure of the liquid. The difference between the decreasing pressure and the vapor pressure where rupture occurs is the tensile strength of the liquid. The potential tensile strength of a liquid may be calculated as shown by Frankel.

Experiments have been done to determine, empirically, the tensile strength of a liquid. Literature shows an extreme difference between the potential tensile strength and experimental results. The effects on cavitation inception will be addressed. After inception occurs, the bubble will grow and collapse. Also, within a flowing liquid, the cavitation number can describe the potential for cavitation.

The mechanics of the bubble can be described using the Rayleigh-Plesset Equation. Derivation of this equation can be done by applying the Three Laws of Conservation: conservation of mass, conservation of momentum and conservation of energy. What happens during a sudden large drop of pressure can be determined by application of the Rayleigh-Plesset Equation.
2. LITERATURE REVIEW

Many studies have been done in an effort to understand cavitation inception. These studies have been experimental or through simulation. In an effort to quantify the tensile strength of a liquid, the potential tensile strength may be calculated. This potential, however, only exists in an ideal world under conditions which cannot be achieved. Experimenters have attempted to quantify tensile strength through the use of application of static stresses and dynamic stress or through the simulation of these using modeling. Conclusions have been drawn from these results which suggests contaminants. These contaminants must be considered if we are to better understand inception that ultimately leads to destructive cavitation.

2.1. Potential Tensile Strength and Cavitation Number

Frenkel\textsuperscript{[13]} introduced the concept of calculating the potential strength of a pure liquid. When considering two molecules within that liquid with any distance between them, the potential energy will be as shown in Figure 2.1.

![Figure 2.1: Intermolecular potential. \textsuperscript{[3]}](image-url)
The potential tensile strength, $\Delta P$, of a liquid to overcome the potential energy may be determined using the equation:

$$\Delta P = \frac{2S}{R_c}$$

Where $S$ is the surface tension and $R_c$ is the critical radius. Once the critical radius of a bubble is reached it will continue to expand. If it does not reach this size, it will shrink. When determining the potential tensile strength we use a typical surface tension for the liquid and assume the critical radius to be the typical intermolecular distance for that liquid. Water is used in the following example.

$$\Delta P = \frac{2 \times .005 \text{lb}}{\frac{3.28 \times 10^{-9} \text{ft}}{f t}} = 1441 \text{ atm}$$

This shows that a bubble in water should be able to withstand tension of 1441 atm. However, experiments show results far below this as will be shown later. \[3\]

The most common occurrence of cavitation is in flowing systems in regions where the pressure is at or below the vapor pressure. The pressure coefficient, $C_p$, of a steady single phase flow is shown to be:\[3\]

$$C_p(x_i) = \frac{p(x_i) - p_{\infty}}{\frac{1}{2} \rho U_{\infty}^2}$$

Where $C_p$ and $p$ are its minimum we have $C_{pmin}$. If a flow has an overall pressure decrease or the flow velocity is increased so that some point's pressure approaches the vapor pressure the cavitation number, $\sigma$, which is dimensionless, may be employed.\[3\]

$$\sigma = \frac{p_{\infty} - p_v(T_{\infty})}{\frac{1}{2} \rho U_{\infty}^2}$$
Where $p_v$ is the vapor pressure at the given temperature. Cavitation inception occurs most often where: $\sigma_l = -C_{pmin}$ where the nucleus pressure is equal to the vapor pressure of the liquid.

However, if the liquid has been degassed, inception will occur below $-C_{pmin}$ (Figure 2.2) and the pressure of the nucleus will be less than the vapor pressure for a period of time. For a liquid containing a lot of contaminant gas, the cavitation number will be above $-C_{pmin}$. The pressure of the nucleus will always be below the vapor pressure at this point (more negative). In a degassed liquid $-C_{pmin}$ is reduced by $\frac{\Delta P_c}{2\rho_L U_\infty^2}$. Where there is a lot of contaminant gas $\Delta P_c$ may be negative. For a nucleus to grow to the critical radius a finite amount of time must take place. This is known as the residence time effect.\(^{[3]}\) A more in depth analysis of effects of contaminants on cavitation number will be shown in section 2.5.

![Figure 2.2: Pressure distribution of a streamline.\(^{[3]}\)](image-url)
2.2. Static Stresses

Marcelin Berthelot\textsuperscript{[14]} (1850) was the first to perform static tensile stress experiments. A single glass cylindrical tube was used. At room temperature, the tube was nearly completely filled with liquid which left a small volume to be occupied by air and the liquid vapor. The tube was then heated so the liquid could expand faster than the tube thus expanding to fill the tube entirely. This forced the air into the liquid. The tube was then allowed to cool gradually. The liquid then would adhere to the wall but still remained filling the entire tube at the lower temperature until it reach a critical temperature in which bubble cavities formed within the liquid. He determined that the tension in the water at this point was 50 atm. It was also noted that cavitation began at the wall.\textsuperscript{[5]} The use of a sealed tube containing fluid is to this day termed a Berthelot tube.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{berthelot.png}
\caption{Schematic of Berthelot's tube.\textsuperscript{[5]}}
\end{figure}

Meyer\textsuperscript{[15]} (1911) employed his own modification to the Berthelot tube. While sealed at one end, the other sealed end was drawn into a thin spiral and a mirror was attached at the top. (Figure 2.4) The starting angle of the tube spiral was measured. As pressure inside the tube
changes, the spiral would coil tighter or looser. A light beam reflecting off the mirror gave the change to the angle of the spiral from start until bubbles. As a result, Meyer was able to find the volume of the liquid and get useful Pressure, Volume, Temperature data. The results of his test showed a tension in the water of 34 atm.

Figure 2.4: Schematic of Meyer apparatus.\[^{[5]}\]

Temperly & Chambers\[^{[16]}\] (1946-1947) repeated experiments done by Berthelot and obtained very similar results. They then employed the addition of steel chips within the glass tube wall. They found that the link between water and gas was stronger than the link between water and steel. They were able to derive an equation to describe the link between the water and its weak link.\[^{[5]}\]

\[
F = \left( \frac{b^2 - a^2}{a^2 \left( \frac{1}{G} + \frac{1}{n} \right)} \right) \frac{\Delta V}{V_0} - p_v
\]

Where \(F\) is the tension calculated. \(a\) and \(b\) are the internal and external radii of the tube in its unstressed condition, \(G\) is the bulk moduli of the tube material, \(n\) is the bulk rigidity of the tube
material. \( V_o \) is the unstressed external volume of the tube, \( \Delta V \) is the sudden increase in external volume of the tube at the break of the water. The success of this equation is dependent on accurate measurement of \( \Delta V \).\(^5\)

Hayward\(^{17}\) (1964) first plotted the change in negative pressure of a Berthelot tube. He first sealed a tube which had strain gauges attached to the side and filled it with liquid in a constant temperature bath. He then added an empty tube to the bath which had strain gauges mounted to the side. The gauges were bridged to each other and the difference in measure was the pressure in the sealed tube. With this he was able to chart the path of negative pressure of 20 bar.

Rees and Trevena\(^7\) (1964) performed the same experiments as Berthelot using a steel tube in place of a glass tube. The results were similar to those of Temperly and Chambers with a glass tube containing steel chips. This gives evidence that the tensile strength can be affected by the material the water is contained in.\(^7\)

Chapman\(^{18}\) (1975) performed Berthelot tube experiments using a pressure transducer. A stainless steel tube was used (Figure 2.5). A strain-gage pressure transducer was attached to the tube in such a way that it became the end wall of the tube. Deionized water was used. The tube was sealed at such a temperature that the test liquid would be compressed. The tube was placed in a water bath which was cooled slowly. The transducer voltage was measured and pressure changes charted. During the first cooling cycle, no tension was developed, so it was cyclically heated (\( .5^\circ C \) warmer each time) and cooled until tension was created. Liquid tension is generated up to 35 bar.
Henderson and Speedy\textsuperscript{[19]} (1980) performed tension tests using a very thin Pyrex tube. The tube was drawn into a spiral form, filled with water and sealed. The ends aligned on the same axis and the spiral was at the same angle throughout. A mirror was attached to one end as it was in the Meyer experiment to measure the angle of twist of the tube as Pressure changed. The outer diameter of the tube was a thin 0.5 mm and the inner diameter was 0.25 mm (Figure 2.6). This created a very small test volume of liquid. Tension of up to 160 bar was reached using this method. One attribute for this tension was that such a small volume of liquid reduces the number of available nucleation sites.
Overton\cite{20} (1982) performed experiments using an evacuated tube. A Berthelot tube was first evacuated down to 10^{-3} \text{ mbar} and then a degassed liquid was introduced. The liquid was degassed using a vacuum system. The tube was subjected to a number of temperature and pressure cycles, as done by Chapman, to develop tension. The first cycle observed breaking tension of 30 \text{ bar}. Once 40 \text{ bar} was reached there was a steady increase, per cycle, up to 68 \text{ bar}.

Briggs\cite{21} (1950) performed a number of experiments with Berthelot type tubes. In one experiment he drew fresh tubes, cut into sections, with liquid and submerged them in warm silicone oil. As the liquid in the short tube is superheated, it develops negative pressure 1 \text{ atm} below vapor pressure. The temperature is increased to reduce cohesive forces. The saturation pressure at rupture is equal to the tension. In another experiment, Briggs draws a fresh U tube and fills with liquid. The lower half is submerged in a warm bath to raise the test liquid temperature. If cavitation did not occur, the tube was withdrawn and the experiment was repeated until a warm enough temperature was reached to cause cavitation. The tension for this experiment reached 50 \text{ atm}. In yet another experiment, Briggs obtained his best results. A Z-shaped tube was drawn and filled. It had a pin through the geometrical center of the Z and was mounted parallel to the ground and allowed to spin around its center. The centrifugal pressure was calculated to be

\[ \frac{1}{2} \rho \left( \frac{l}{2} \right)^2 \omega^2 \]

Where, \( l \), is the column length, \( \rho \), is the liquid density and \( \omega \) is the angular velocity. He was sure to use boiled water in very clean tubes. He was able to reach tensions of up to 270 \text{ atm}. 

2.3. Dynamic Stresses

There are two ways that a liquid can be subjected to tension by dynamic stressing. One way is that a compression pulse can be created within the liquid which is later reflected off a boundary to create a tension pulse. Another way is by creating a tension pulse beginning in the liquid.[5]

Cole[23] performed dynamic stress by creating underwater explosions. This deep water explosion would produce a positive pressure pulse. This is followed by a rapidly falling pressure over time and distance. This velocity propagation can be approximated well using acoustic theory. Tension is created when the pulse impinges on a boundary such as the waters free surface, animal life or a ship hull. A sink is considered above the free surface. Its distance above the surface is the same as the charges distance below the surface. The pressure at a given point, a far distance from the explosion is given by the equation:[5]

\[
P = P_a + \frac{A}{R_1} e^{-\frac{t}{c} - \frac{R_1}{c}} - \frac{A}{R_2} e^{-\frac{t}{c} - \frac{R_2}{c}}
\]

Where \( R_1 \) is the distance from the point to the charge and \( R_2 \) is the distance from the point to the sink. \( c \) is the speed of sound through water, \( A \) depends on the explosive. A pressure gauge at the point reads a tension of about 15 bar is reached at time:[5]

\[
t = \frac{R_2 - R_1}{c}
\]

After which time, \( t \), there is a drastic negative drop in pressure resulting in "mush" readings. This is because cavitation is occurring. The free surface effects can be described based on the depth of the charge. For a very deep charge, the only observable effect is a momentary darkening of the surface. For shallower depths, there is a spray dome producing a white ring
within a dark ring. The formation of these "pluming" drops (not of the plume itself) can be traced.[5]

There is a certain depth below the surface at which the pressure becomes negative. This layer can be calculated from the above pressure equation because the tension is now known. A thin layer at this depth will be thrown upwards. This creates a new free surface which acts as a reflector for the original pulse. Another layer forms at a below this free surface and is thrown upwards. This cycle repeats. The topmost layer decelerates when it is thrown upwards while the layer below it is at the vapor pressure for water. The deformation of the top layer as it decelerates is known as the Taylor instability. This instability is responsible for the layering of a water drop. As stated earlier, an explosion in regular water showed a tension of 15 bar. Later it was shown that an explosion in sea water yielded a tensile strength of 40 bar.[5]

![Figure 2.7: Pressure record at a point below the free surface.][5]

Couzens and Tevena[22] performed two different types of dynamic experiments to determine tensile strength by conversion of a pressure pulse into a tensile pulse. In the first, a steel tube filled with liquid was closed at each end with free pistons. A lead bullet was fired at one piston which created a pressure pulse in the liquid which acted upon the opposite piston. Within this system, the magnitude and duration of the pulse pressure and its reflection can be
calculated. The negative pressure at which rupture occurs may be determined. The tension of the water in these experiments was about 9 atm.\textsuperscript{[7]}

In the second experiment, the liquid filled tube had a piston only at one end and was allowed to remain open at the other end. Again a bullet was shot at the piston end. The pulse was reflected at the open end and was measured using a Duralumin pressure bar. Stress waves were measured by strain gages. With this experiment he obtained results of 8 to 14 atm.\textsuperscript{[7]}

![Diagram](image)

Figure 2.8: Couzens and Trevena single piston apparatus schematic.\textsuperscript{[5]}

Carlson and Henry\textsuperscript{[24]} (1973) performed dynamic testing with a method termed the cell method. Test liquid was confined to a cell between a solid plate and a stretched aluminized Mylar film. A stress pulse was produced of an electron beam towards the solid plate. Since a solid and liquid of the same acoustic impedances had been chosen, the pulsed compression stress wave moved into the liquid. This wave was reflected off the Mylar film which gave the film surface motion. This motion was able to be measured to determine the negative stress generated in the liquid. The breaking tension of glycerol was shown to be 600 bar. In addition, this
experiment was able to cause breaking within the liquid itself, some distance from the surface of the solid or the film.

Figure 2.9: The cell method.\cite{5}

Lackme\cite{25} (1978) used a method in which a tension pulse was generated directly. His Experiment configuration is shown in the Figure 2.9. A and B are cylindrical aluminium bars. B rests directly on the liquid. A tension pulse was created by dropping the ring shaped weight, R, to fall onto the bottom plate. The pulse wave traveled through A, through the liquid and into B. A strain gauge mounted on both A and B and tensions in each could be measured. The amplitude of the pulse could be changed simply by changing the weight of the ring. Tensions of 5 bar were measured, but this was limited by the size of the apparatus.
Overton and Trevena[26] (1981) developed a similar configuration as shown in Figure 2.11. The tube was half filled with water. The tube could be pulled down against rubber tensioned supports and released and then stopped by a buffer a distance, z, away. The water would, of course, continue upward and create a pressure pulse with cavitation. The pressure change was monitored using a piezoelectric transducer. Photographic records were taken as well. By changing z, the velocity of the tube could be changed which resulted in a change of amplitude of the pulse. After the initial pulse where cavitation occurred, a series of similar pulses with reduced amplitude followed as seen in Figure 2.12.
Figure 2.11: Apparatus used by Overton and Trevena.\textsuperscript{[5]}

Figure 2.12: Pressure records beyond the cavitation threshold.\textsuperscript{[5]}
2.4. Simulation Testing

Tsuda et al\(^4\) studied the growth of a bubble using dynamic simulations. During molecular dynamics simulation, they used the Lennard-Jones potential to describe the interactions between gas molecules. This is given as

\[ \phi(r) = 4\varepsilon \left\{ \left(\frac{\sigma}{r}\right)^{12} - \left(\frac{\sigma}{r}\right)^6 \right\} \]

where \(\sigma\) and \(\varepsilon\) are potential parameters corresponding to the molecular diameter and the depth of the potential well. The number of molecules, the volume of the computational domain and the total internal energy were all kept constant.

Simulation of a cavitation bubble was achieved by preparing a decompressed metastable liquid. The volume of a bubble nucleus is calculated by determining the average radius within a cluster of molecules. The time evolution of the volume for the 5 largest molecules were shown in Figure 2.13. This shows that all fine nuclei began to grow at approximately the same rate. However, over time, one nucleus predominately grew while the others shrank in size. They present a theory known as Ostwald ripening. The competition mechanism between the larger and smaller domain depends on the change of the supersaturation ratio in the bulk fluid with coarsening of the domains.\(^4\)

Figure 2.14 shows that the competing growth was due to the change of the critical size with the bulk pressure change which was caused by the growth. A bubble larger than the critical size will continue to grow while pressure is decreased. A bubble smaller than this will shrink. The critical radius is given by the Young-Laplace equation:

\[ R_c = \frac{2\gamma}{p_v - p_L} \]
Where $\gamma$ is the surface tension and $p_v$ and $p_L$ are the saturation pressure and the liquid pressure.\cite{4}

![Graph](image1.png)

Figure 2.13: Evolution of time for 5 largest molecules.

![Graph](image2.png)

Figure 2.14: Critical volume with pressure.\cite{4}

2.5. Theories of Contaminant Effects

Experimenters have obtained such a large variety of results from experiments. This suggests that there are weaknesses within the liquid to allow rupture. Contaminants act as weaknesses within a liquid and limit the tension the liquid can support. There are several types
of weaknesses that can occur in a system to cause inception. The junction between a solid and a liquid boundary, on small micro particles in the liquid, micro bubbles of contaminant gas in solid boundary crevices and cosmic radiation are all examples of weaknesses within a liquid.[3]

The boundary between a solid and a liquid boundary is a common place for nucleation. This will happen one of three ways as shown in Figure 2.15. A hydrophobic surface is one that repels water and a hydrophilic surface is one that is easily wetted. For the hydrophobic surface, the tensile strength is $2\sin\theta/R$. The maximum $\theta$ is equal to $\pi$ where the tensile strength is zero. The hydrophilic surface acts similar to homogeneous nucleation. At the microscopic level, a surface appears very rough. Nucleation may occur in the conical cavities of this "rough" surface. This bubble will grow to fill the cavity at pressures above the vapor pressure.[3]

![Figure 2.15: Nucleation at surface weaknesses.][3]

The surface roughness will not only provide conical crevices for nucleation. Surface roughness affects the degree of turbulence in a flow. At a microscopic level, the boundary layer of separation may be affected which affects the pressure and velocity field. A bubble in the vortex of the flow will be subjected to lower pressures than that of the bulk fluid. This will cause
localized cavitation. Also, as cavitation occurs, damage is often done to the surfaces near the flow. This will increase the surface roughness of the wall and result in even more low pressure vortices.\[3\]

Other potential nucleation sites are in the liquid bulk at small particles and micro bubbles. Free stream nuclei seem to be more prevalent than surface nuclei. The most accurate data has been through the use of holograms which can be reconstructed at micro levels and inspected. It is still difficult, however, to determine if these nuclei are micro bubbles or solid particles. It is also difficult to determine, at this small size, which sites will actually grow to cavitate and which will not.\[3\]

In order for contaminants to affect the tensile strength of a liquid, it must affect physical properties of a liquid such as viscosity, density, surface tension, thermal properties, etc. Tap water contains between 50000 and 100000 of solid particles per cubic centimeter.\[7\] In theory, if a liquid is held above a pressure higher than the vapor and gas pressure combined, no gas bubbles can exist. Degassing should show an increase in tensile strength, and it does, but not to the extent one would like.\[3\] Several experimenters (Numachi, Crump, William and McNulty, and Ziegler) performed hydrodynamic testing using venturi nozzles. They generally found that at lower gas contents, pressure at inception fell.\[7\]

Three explanations have been proposed for the lack of increase in tensile strength during degassing. First is the Harvey nucleus which describes a bubble in a crevice. Harvey\[27\] explains that a gas pocket in a crevice will act according to the stability of that gas pocket. A stable gas pocket will hold the gas in the crevice. Another theory is cosmic radiation constantly produces nuclei. This energy, which initiates nucleation, is produced by collision between high energy particles and liquid molecules and is always occurring. The last is a concept that describes the
bubble being surrounded by an organic skin. Older publications site this as unlikely; however, advances in surface rheology have shown small amounts of contaminant can create large surface elastic effects. [3]

Figure 2.16 gives a good visualization of just how much contaminants can affect the cavitation number. These water tunnel experiments were done around the world with different types of water from the region or with differently treated water from the same region. This charts cavitation numbers ranging from about .3 by Penn State and up to above 1.0 in Tokyo. Figure 2.17 shows water from the same region treated differently. From the charts on the left of Figure 2.17, we can see that untreated tap water shows a large number of nucleation points when compared to degassed or filtered tap water. In addition, the nucleation sites generally showed bubbles of larger radius. The graph on the right of Figure 2.17 shows these waters flowing through a 3, 4.5 and 6 cm pipe and so the cavitation number can be graphed against Reynolds number. [3]

Figure 2.16: Water tunnel experiment cavitation numbers. [3]
Figure 2.17: Water within the same facility and flow through 3, 4.5 and 6 cm pipes.\textsuperscript{[3]}
3. DERIVATION OF THE RAYLEIGH-PLESSAT EQUATION USING CONSERVATION THEORIES

The conservation of mass and the conservation of momentum may be used to derive the Rayleigh Plesset equation which is used to describe individual bubble dynamics. The liquid is said to have constant density, viscosity, and Temperature. The liquid pressure is dependent on time. Within the bubble, temperature, pressure and radial velocity will be a function of radius and time. We choose a spherical polar coordinate system.

3.1. Conservation of Mass

We employ the conservation of mass using the continuity equation:[1]

\[
\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (u_{\theta} \sin \theta) + \frac{1}{r \sin \theta} \frac{\partial}{\partial \phi} u_{\phi} = 0
\]

Where \(r\) is the radius of the bubble and \(\theta\) and \(\phi\) are the tilt of the bubble polar and azimuth direction. Both are assumed to be zero. Likewise \(u_r, u_{\theta}\) and \(u_{\phi}\) are the velocity components of the bubble wall in the radial, vertical spinning and horizontal spinning direction. Assuming there is only velocity in the radial direction, the continuity equation reduces to:

\[
\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 u_r) = 0
\]

Separating variables we have:

\[
\frac{1}{r^2} \left( r^2 \frac{\partial u_r}{\partial r} + u_r \frac{\partial r^2}{\partial r} \right) = 0
\]

\[
\frac{\partial u_r}{\partial r} + \frac{u_r}{r^2} \frac{\partial r^2}{\partial r} = 0
\]

\[
\int \partial u_r + \frac{u_r}{r^2} \int_{r^2 = R^2} \partial r^2 = 0
\]
\[ u_r(R) + \frac{u_r}{r^2} R^2 = 0 \]

\[ u_r(R, t) = \frac{R^2 dR}{r^2 dt} \]

3.2. Conservation of Momentum

Next we employ the use of the Navier Stokes equation to describe Newton's Second Law:

\[
\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} + \frac{u_\phi}{r \sin \theta} \frac{\partial u_r}{\partial \phi} - \frac{1}{r} (u_\theta^2 + u_\phi^2) = -\frac{1}{\rho} \frac{\partial P}{\partial r} + g_r \\
+ v \left( \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 u_r}{\partial \phi^2} \right) u_r - \frac{2u_r}{r^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} - \frac{2 u_\theta \cot \theta}{r^2} - \frac{2}{r^2 \sin^2 \theta} \frac{\partial u_\phi}{\partial \phi} \]

Because there is only radial expansion of the bubble we will only assume there is momentum in the radial direction: Additionally, gravitational forces are ignored. The equation reduces to:

\[
\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + v \left( \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) - \frac{2u_r}{r^2} \right)
\]

Rearranging for Pressure we have:

\[-\frac{1}{\rho} \frac{\partial P}{\partial r} = v \left( \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) - \frac{2u_r}{r^2} \right) + \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} \]

Substituting the continuity equation into the momentum equation we have:

\[-\frac{1}{\rho} \frac{\partial P}{\partial r} = \frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} = \frac{R^2 d^2 R}{r^2 dt^2} - \frac{2R^4}{r^5} \left( \frac{dR}{dt} \right)^2 + \frac{2R}{r^2} \left( \frac{dR}{dt} \right)^2 \]
It is interesting to note that the viscous force completely cancels in substitution.

Integrating

$$-\frac{1}{\rho} \int_{R}^{\infty} dP = \int_{R}^{\infty} \frac{R^2}{r^2} \frac{d^2R}{dt^2} - \frac{2R^4}{r^5} \left( \frac{dR}{dt} \right)^2 + \frac{2R}{r^2} \left( \frac{dR}{dt} \right)^2 dr$$

$$-\frac{1}{\rho} (P_\infty - P_R) = \left[ -\frac{2R^2}{r} \frac{d^2R}{dt^2} + \frac{2R^4}{4r^4} \left( \frac{dR}{dt} \right)^2 - \frac{2R}{r^2} \left( \frac{dR}{dt} \right)^2 \right]_R^{\infty} = R \frac{d^2R}{dt^2} + 3 \left( \frac{dR}{dt} \right)^2$$

3.3. Force Balance

To further reduce this equation, a force balance is necessary:

![Free body diagram](image)

Figure 3.1: Free body diagram force balance.

From the free body diagram we have a pressure force balance

$$\sum P_B + \sigma_{rr} - \sigma_s = 0$$

$P_B$ is the pressure of the bubble, $\sigma_N$ is the normal stress of the bubble acting on the liquid and $\sigma_s$ is the shear stress at the bubble boundary. Deformation law for a Newtonian fluid shows:\[1]\]

$$\sigma_{rr} = -P + K \varepsilon_{rr}$$

Where $K = 2\mu$ and $\varepsilon_{rr} = \frac{\partial u_r}{\partial r}$
\[ \sigma_s = \frac{2S}{R} \quad \sigma_{rr} = -P_R + 2\mu_L \frac{\partial u_r}{\partial r} \]

\[ P_B - P_R + 2\mu_L \frac{\partial u_r}{\partial r} - \frac{2S}{R} = 0 \text{ where } r = R \]

\[ P_R = P_B - \frac{4\mu_L}{R} \frac{dR}{dt} - \frac{2S}{R} \]

Substituting into the momentum equation we have:

\[ \frac{1}{\rho} \left( P_\infty - P_B + \frac{4\mu_L}{R} \frac{dR}{dt} + \frac{2S}{R} \right) = \frac{R^2}{r^2} \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 \]

\[ \left( \frac{P_\infty - P_B}{\rho} + \frac{4\nu_L}{R} \frac{dR}{dt} + \frac{2S}{\rho R} \right) = \frac{R^2}{r^2} \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 \]

The Rayleigh Plesset equation becomes:

\[ \frac{P_B - P_\infty}{\rho} = R \frac{d^2 R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 + \frac{4\nu_L}{R} \frac{dR}{dt} + \frac{2S}{\rho R} \]
4. BUBBLE MECHANICS

It can be useful to study the mechanics of the liquid as well as the bubble. The metastable region of a phase diagram can be reached in two different ways. One of these is by superheating the liquid, the other is by a liquid under tension. The metastable phase is the phase in which the liquid is taken into the vapor phase, but vaporization does not occur. Vaporization can occur by either evaporation which can be quickened by increasing temperature, or by cavitation which can be quickened by dropping pressure. A simple phase diagram describes this process. (Figure 4.1)

![Phase diagram showing two paths to metastable region](image)

Figure 4.1: Phase diagram showing two paths to metastable region.\[^{[11]}\]

Harvey proposed that when a container is filled with liquid, there are tiny crevices filled with gas pockets within the surface of the container. (Figure 4.2) These crevices occur due to the natural roughness of the container wall. Harvey described the situation in which a gas pocket in a crevice is determined to be stable. That is, it will continue to exist in the crevice. This will occur if the surface is hydrophobic which means it is imperfectly wetted. There reaches a point, with large enough pressure reduction, where the pocket will no longer be stable and will be
drawn out of the crevice to form a cavity. When there is high positive pressure in the liquid, the gas pocket will compress into the crevice. After this happens, an even greater negative pressure is required to overcome the pocket equilibrium in order for the gas to escape.

![Figure 4.2: (a) Stable gas pocket in crevice (b) Negative pressure forms a cavity (c) Positive pressure forces the gas to compress into the crevice.][3]

The conditions for a stable gas pocket and description of the transient interface is shown in Figure 4.3 as described by[7] Figure 4.3a shows the gas pocket with a liquid saturated with gas. The equilibrium with this radius has a contact angle of $\theta_e > \frac{\pi}{2} + \alpha$. The angle of the crevice is said to be $2\alpha$. In Figure 4.3b, the liquid is undersaturated with gas. The gas will then dissolve into the liquid. The angle in which the liquid advances is known as the advancing contact angle, $\theta_A$. Also, the radius will be reduced. In Figure 4.3c, the liquid is supersaturated with gas. This gas will diffuse from the liquid into the gas pocket causing it to grow. The angle of contact will be the reducing contact angle, $\theta_R$ and the radius will increase. The gas pocket is said to be stable if

$$\theta_A > \frac{\pi}{2} + \alpha > \theta_R$$
When determining the pressure within the bubble, there will likely be gas contaminants as well as thermal effects. The pressure within the bubble becomes:

\[ P_B(t) = P_v(T_B) + P_{G_0} \left( \frac{T_B}{T_\infty} \right) \left( \frac{R_0}{R} \right)^3 \]

Where \( T \) is temperature. If there are no thermal effects

\[ P_B(t) = P_v(T_\infty) + P_{G_0} \left( \frac{R_0}{R} \right)^3 \]

Where \( P_{G_0} \) and \( R_0 \) are the partial gas pressure and radius at time 0. The Rayleigh Plesset equation with no thermal effects becomes
\[ \frac{P_v(T_\infty) - P_\infty(t)}{\rho_L} + \frac{P_c}{\rho_L} \left( \frac{R_0}{R} \right)^3 = R \frac{d^2R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 + \frac{4\nu_L dR}{R \ dt} + \frac{2S}{\rho_L R} \]

Rayleigh gives the equation of motion of the bubble wall as\[^{[12][7]}\]:

\[ -R \frac{dU}{dt} - \frac{3}{2} U^2 = \frac{P_v - P_B}{\rho} \]

\[ P_B = P_v - \frac{2S}{R} + \frac{NT}{R^3} \]

Substituting in we have:

\[ \rho R \frac{dU}{dt} + \frac{3}{2} \rho U^2 = P_v - P_\infty - \frac{2S}{R} + \frac{NT}{R^3} \]

For a bubble in equilibrium

\[ f = 0 = (P_v - P_\infty) - \frac{2S}{R} + \frac{NT}{R^3} \]

The bubble will become unstable where the Critical Radius, \( R_c \) is:

\[ R_c = \frac{\partial f}{\partial R} \geq 0 \]

\[ R_c = \left( \frac{3NT}{2S} \right)^{\frac{1}{2}} = \frac{-4S}{\frac{3}{3}(P_\infty - P_v)_c} \]

Using the equation for \( f \) the pressure drop vs radius can be plotted (Figure 4.4). We assume \( P_v \) to be constant. We will use water at room temperature as it was used to calculate the potential tensile strength. \( S \) for water at room temperature is \( .005 \text{ lb/ft}^3 \). \( NT \) in this case will be \( 1.33 \times 10^{10} \text{ lb in.} \) For \( f \), \( S \) is dependent upon temperature, and \( N \) is the mass of the gas which remains constant for a particular gas.\[^{[7]}\]
Figure 4.4: Radius change with change in $P_\infty$.

Likewise, the critical pressure vs critical radius of a bubble can be plotted as seen in Figure 4.5. For a radius, $R$ will increase, excess pressure causing growth will increase and the bubble will rupture, theoretically, at $R_c$.

Figure 4.5: Critical pressure versus critical radius.
We again consider the Rayleigh Plesset equation to determine the expansion of a bubble over time. We restate it here for convenience:

\[
\frac{P_v(T_\infty) - P_\infty(t)}{\rho_L} + \frac{P_{G_0}}{\rho_L} \left( \frac{R_0}{R} \right)^3 = R \frac{d^2R}{dt^2} + \frac{3}{2} \left( \frac{dR}{dt} \right)^2 + \frac{4\nu_L}{R} \frac{dR}{dt} + \frac{2S}{\rho_L R}
\]

We will consider the bubble to be inviscid and \( P_B = P_v \).

\[
P_{G_0} = P_\infty(0) - P_v(T_\infty) + \frac{2S}{R_0}
\]

Integrating twice we find the equation for time, \( t \), to be\([3]\):

\[
t = R_0 \int_0^R \left( \frac{2(P_v - P_\infty)(1 - x^{-3})}{3\rho} + \frac{2S(1 - x^{-2})}{\rho R_0 x} + \frac{2P_{G_0}}{x^3 \ln x} \right)^{-\frac{1}{2}} dx
\]

This equation may be done in two parts. First is for a radius up to the size of the critical radius. For this example we use the critical radius at the largest pressure drop shown in Figure 4.4. The initial pressure of the system was atmospheric pressure. \( P_v \) is the vapor pressure of the bubble and is \( .3631 \) psi. The pressure at this critical radius is \( 33.33 \) psi below the vapor pressure and the corresponding critical radius is \( .0002 \) inches. Once a critical radius is reached, the bubble will continue to grow. If it is not reached, it will shrink or diffuse. Using the trapezoidal rule in 4 steps, the time vs. radius may be plotted (Figure 4.6). Using the same approach, the radius growth can be plotted for a radius above the critical radius (Figure 4.7). The program used to plot the figures were done using Matlab and are attached in Appendix A.
Figure 4.6: Radius growth over time before the critical radius.

Figure 4.7: Radius growth following the critical radius.
5. CONCLUSION

There is still much to be learned from cavitation inception. Studies have been able to show general trends of what affects tensile strength of a liquid but the results are more qualitative than quantitative. As seen in Table 1, which is a summary of the results from the experimenters discussed in section 2.2, conclusions can be drawn and questions may come to mind. There were three types of tubes used. The glass tubes showed higher tensile strengths than the tubes with steel chips or made of steel. This may be due to the different surface roughnesses between these types of tubes.

Table 5.1: Comparison of experimental results of water.

<table>
<thead>
<tr>
<th>Experimenter</th>
<th>Arrangement</th>
<th>Result (atm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berthelot (1850)</td>
<td>Sealed glass tube containing water and air</td>
<td>50</td>
</tr>
<tr>
<td>Meyer (1911)</td>
<td>Glass tube with a spiral end</td>
<td>34</td>
</tr>
<tr>
<td>Temperly &amp; Chambers (1946)</td>
<td>Tap Water in an ordinary glass tube</td>
<td>30-50</td>
</tr>
<tr>
<td>Hayward (1964)</td>
<td>Glass tube with strain gages in a bath</td>
<td>20</td>
</tr>
<tr>
<td>Rees &amp; Trevena (1964)</td>
<td>Tap water in a steel tube</td>
<td>10-30</td>
</tr>
<tr>
<td>Chapman (1975)</td>
<td>Pressure sealed tube. Heated and cooled repeatedly to cause tension. Cavitation occurred within fluid itself</td>
<td>Up to 35</td>
</tr>
<tr>
<td>Henderson and Speedy (1980)</td>
<td>Very thin spiral tube with only 2-4 mm3 of liquid</td>
<td>160</td>
</tr>
<tr>
<td>Overton (1982)</td>
<td>Degassed liquid in a pressure sealed tube</td>
<td>45-68</td>
</tr>
<tr>
<td>Briggs (1950)</td>
<td>Centrifugal spinning z tube using distilled water and a brand new fresh tube that was very clean</td>
<td>277</td>
</tr>
</tbody>
</table>
Henderson and Speedy employed the use of the very thin spiral tube. This allowed for only a small quantity of water, reducing the number of nucleation sites.

The type of water affects results as show in section 2.5. Overton showed that degassing, thus reducing the number of nucleation sites, the liquid has an effect. Briggs obtained the best results with the use of his z tube. The centrifugal force forced the gas bubble to accumulate in one area. In addition he use a very clean tube which was freshly drawn and distilled the water.

The Rayleigh Plesset equation can be derived using the conservation theories and shown in section 3. Using this equation, Reyleigh describes the equation of motion for the boundary of a bubble wall. This was show graphically in section 4. These relationships are still the theoretical results. As stated by Plesset, "these theories do not account for stability of the interface, the compressibility of the liquid, the effect of energy flow into or out of the bubble, and the physical conditions within the bubble."[12]

The advanced in computer technology have allowed for modeling to describe what happens to a bubble radius after inception as seen in section 2.4. Using this modeling technique, they were able to trace a bubble from inception. They determined that bubbles that do not reach a critical radius will not continue to grow and later rupture. They will instead be absorbed into the liquid as they shrink. Simulation will be important in future studies but may see some obstacles when it comes to scaling at such a microscopic level.
REFERENCES


11. Phase change to metastable region. Graph. Phase Diagrams of Pure Substances.


<http://www.chemguide.co.uk/physical/phaseeqia/phasediags.html>


<http://www.forddoctorsdts.com/articles.html/_/dtsarticles/cavitation-erosion-r27>
function rayleigh
pv=.3631; % psi
pinf=-33.33; % psi
rho=(62.31/1728); % lb/in^3
S=.005/12; % lb/in
R0=2e-5;
x=linspace(4,50,50);
collapse=.915*(rho*R0^2/(14.7-pv))^.5
Pg=(14.7-pv)+2*S/12/2e-5;
t1=.03967;
t2=1./sqrt(616*(1-(.33.*x).^3)+116*(1-(.33.*x).^2)/(.33.*x)+(2*Pg./(.33.*x.^3)).*log(.33.*x));
t3=1./sqrt(616*(1-(.67.*x).^3)+116*(1-(.67.*x).^2)/(.67.*x)+(2*Pg./(.67.*x.^3)).*log(.67.*x));
t4=1./sqrt(616*(1-x.^3)+116*(1-x.^2)./x+(2*Pg./(x.^3)).*log(x.^-1))
f1=t1;
f2=2.*t2;
f3=2.*t3;
f4=t4;
t=2.5*(f1+f2+f3+f4)*R0;
R=x*R0;
plot(t,R)
xlabel('time (s)')
ylabel('Radius, in')
title('Radius over time after Critical Radius is reached')

function rayleigh1
pv=.3631; % psi
pinf=-33.33; % psi
rho=(62.31/1728); % lb/in^3
S=.005/12; % lb/in
R0=1.667e-6
Rc=.0002
x=linspace(2,130,98);
collapse=.915*(rho*R0^2/(14.7-pv))^.5
Pg=(14.7-pv)+2*S/12/R0;
t1=.04029;
t4=1./sqrt(616*(1-x.^3)+((2*S)/(rho*R0))*(1-x.^-2)./x+(2*Pg./(x.^3)).*log(x.^-1))
f1=t1;
f4=t4;
t=(150-2)/98*(f1+f4)*Rc;
R=x*R0;
plot(t,R)
xlabel('time (s)')
ylabel('Radius, in')
title('Plot of Radius over time before Critical Radius is Reached')