STUDY AND ANALYSIS OF AUTOMATED ORDER PICKING SYSTEMS

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Study and Analysis of Automated Order Picking Systems

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ABSTRACT

Order picking is an essential part of order processing in warehousing and distribution operations and can be performed using manual, automated, or semi-automated systems.

This thesis analyzes two automated systems, which include carousel and AS/RS (automated storage and retrieval system). The main goal of this research is to develop mathematical models to compare the performance of both systems under random and class-based storage assignments.

Simulation models are used to validate the reliability of mathematical models. The outputs of mathematical and simulation models are consistent indicating carousel system with class-based assignment has the highest throughput.

Economic analysis is used to estimate the payback periods required to convert from manual to AS/RS and carousel systems. The economic analysis shows that converting from manual to AS/RS with class-based assignment has the shortest payback period.

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CHAPTER 1. INTRODUCTION

1.1. Background

Warehouses can operate under different departments in the supply chain and also can be used for different purposes. However, the main reason for having a warehouse is storage. When there is no demand the time difference between supply and demand is solved by storing finished products. Manufacturers can accomplish production economies without being distracted by inadequate demand at the time of production. Customers do not have to wait for a production cycle to get the product. Instead, they can have it shipped directly from the warehouse. Also, a warehouse accumulates products from different manufacturers and merges the shipment to one consumer. The customer need not receive five separate boxes if he orders ten different products, and the manufacturer does not have to send five shipments if products are ordered by five customers. Figure 1.1 illustrates the various departments in a warehouse. The basic departments are Receiving, Storage, Order Picking and Shipping.

The general process used in most warehouses is as follows. When incoming products arrive, the warehouse will acquire them and unload them at the receiving dock. Their quantity and quality will be checked; and at the same time, labels or tags might be attached to

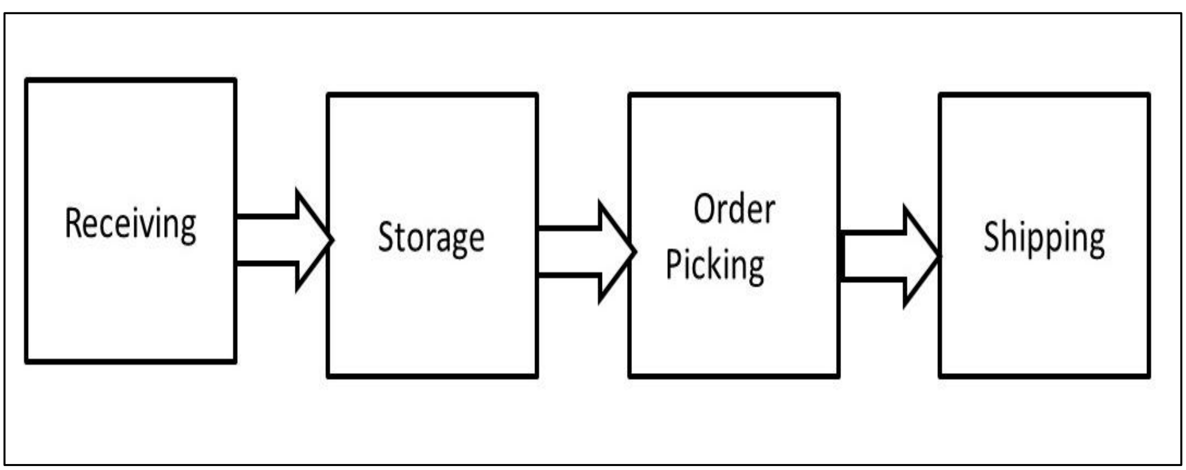


Figure 1.1. Basic Departments in a Warehouse

them to enable tracking. After recording their profiles, they will be placed in the storage area. The product will stay there until the customer orders it. Upon receiving a customer order, the warehouse will process it and generate a picking list that indicates the product, quantity, and location in the warehouse. Therefore, the picker will know where to pick products.

Order picking is the process of fulfilling customer requests through picking items from storage. According to Kong and Masel (2008) among all warehousing operations, order picking makes up approximately 55% of the cost. This is because order picking is a labor intensive activity for traditional manual warehouses. Therefore, order picking is a very important activity in warehouse management, and the order picking system should be planned wisely and controlled efficiently. If the cost of order picking is decreased, the operation costs of the warehouse will decrease significantly. That is the reason why order picking is a crucial topic in warehouse management, and therefore a lot of research has been done on it.

The different order picking technologies are shown in Figure 1.2. order picking is divided into three types: 1) manual order picking system 2) semi automated and 3) automated order picking systems.

1) Manual order picking is a labor intensive operation in warehouse; in this type of order picking system the pickers to parts process is involved. There are the basic forms include picking by article (batch picking) or pick by order (discrete picking). In the case of picking by article, multiple customer orders (the batch) are picked simultaneously by an order picker. Many in-between forms exist, such as picking multiple orders followed by immediate sorting (on the pick cart) by the order picker

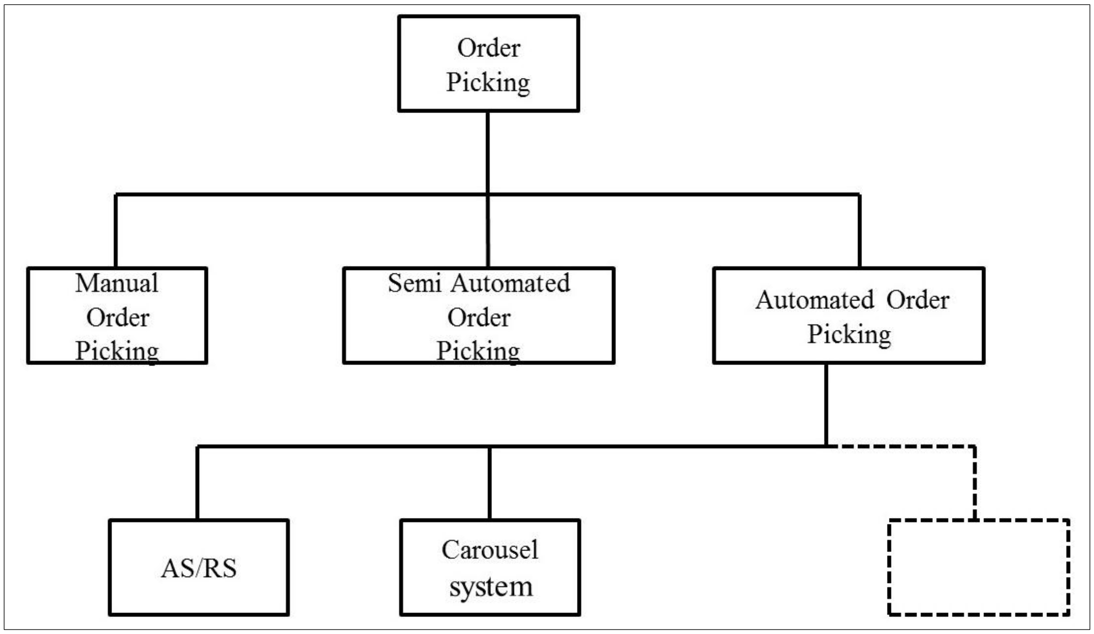


Figure 1.2. Order Picking Technologies

(sort-while-pick) or the sorting takes place after the pick process has finished (pick-and-sort). This type of order picking system is also called as low-level order picking system.

2) The semi-automated is another order picking system which also involves the same process (picker to parts) as manual order picking systems, in this type order pickers travel to the pick locations on board of a lifting order-pick truck or crane. The crane automatically stops in front of the appropriate pick location and waits for the order picker to perform the pick. This type of system is called a high-level or a man-aboard order-picking system.

3) The automated system follows the parts-to-picker systems, which include automated storage and retrieval system (AS/RS), and carousel System. In this systems mostly aisle-bound cranes that retrieve one or more unit loads (pallets or bins; in the latter case the system is often called a mini-load) and brings them to a pick position (i.e. a load/unload station). At this position the order picker takes the required number of

pieces, after which the remaining load is stored again. This type of system is also called a unit-load or end-of-aisle order-picking system. The two main automated order picking systems are automated storage and retrieval system (AS/RS) and carousel system.

The automated storage and retrieval machine can work under different operating modes: single, dual and multiple command cycles. The single-command cycle means that either a load is moved from the load/unload station to a rack location or from a rack location to the load/unload station. In the dual-command mode, first a load is moved from the depot to the rack location and next another load is retrieved from the rack. In multiple command cycles, the storage/retrieval (S/R) machines have more than one shuttle and can pick up and drop off several loads in one cycle. For example, in a four command cycle, the S/R machine leaves the depot with two storage loads, stores them and returns with two retrieved loads. Figure 1.3 represents the simple AS/RS system, which is currently used in warehouses.

Carousel is an automated storage and retrieval system. A carousel system is one of two types vertical or horizontal carousel, a vertical carousel provides for closed loop automatically controlled rotation of the basic storage unit because storage is vertical, such systems are popular when conserving floor space. Although automatic insertion and extraction of individual items or loads is possible, it is not as common as it is with horizontal carousel applications. A horizontal carousel is a carousel that consists of a fixed number of adjacent storage columns, or bays, that are mechanically linked to either an Overhead or floor mounted drive mechanism to form a complete loop. Figure 1.4 represents the horizontal carousel system, which is used in warehouses. Table 1.1 is a summary of comparing the characteristics of AS/RS and carousel system.

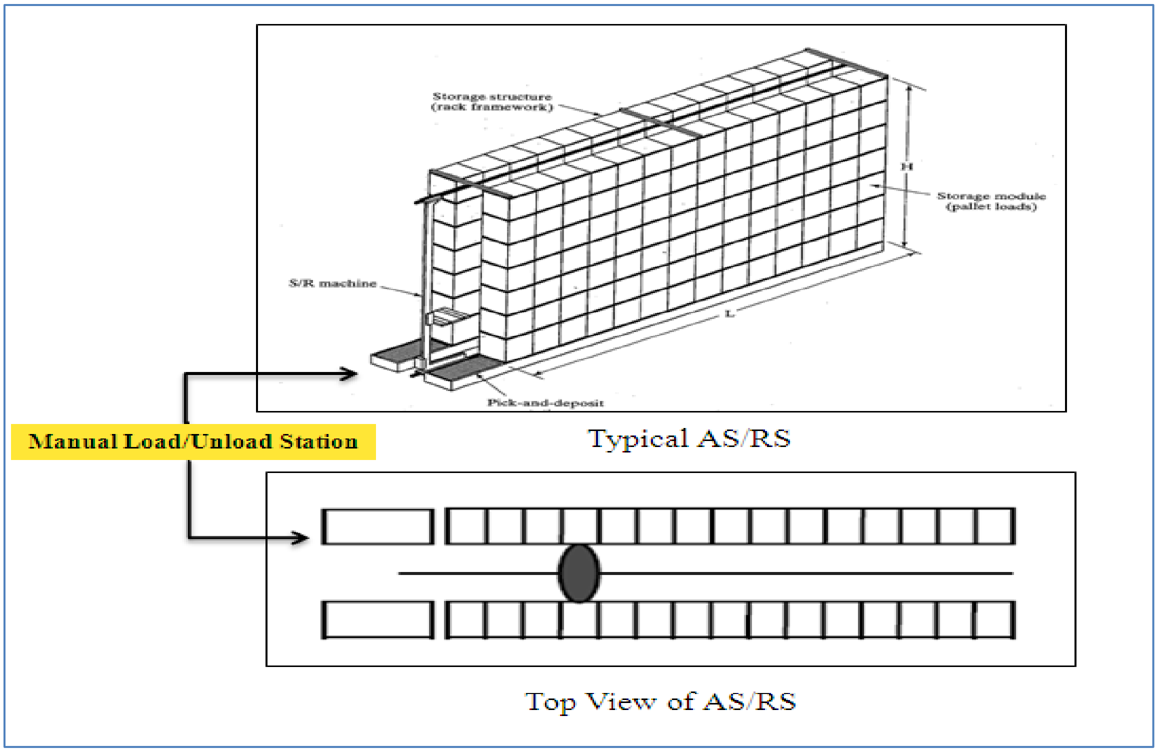


Figure 1.3. AS/RS, adapted from Groover (2007)

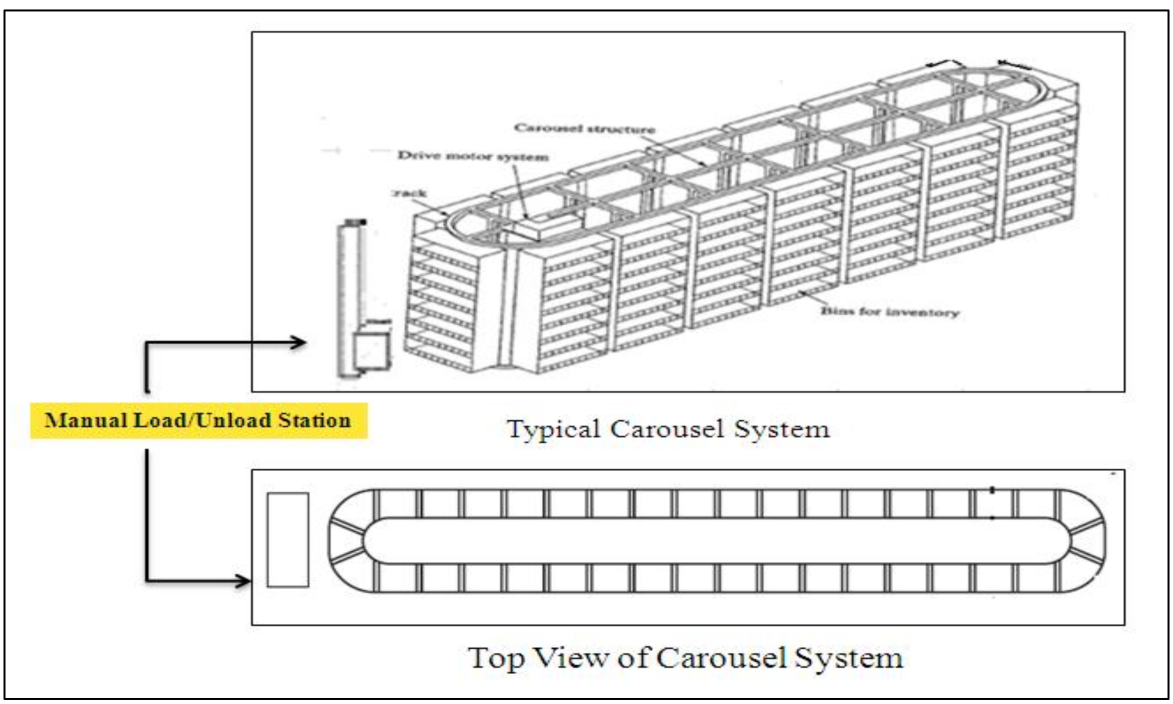


Figure 1.4. Carousel System, adapted from Groover (2007)

Table 1.1. Summary of Characteristics for both AS/RS and Carousel Systems

Characteristics	**AS/RS**	**Carousel system**
Storage structure	Rack arrangement to provision pallets or shelf arrangement to provision tote bins.	Bins suspended from overhead conveyor columns.
Motions	Linear motions of S/R.	Revolution of overhead carousel conveyor columns around oval track.
Storage/retrieval operation.	S/R machine travels to compartments in rack structure.	Carousel revolves to bring bins to load/unload station.
Replication of storage Capacity	Multiple aisles, each consisting of rack structure and S/R machine.	Multiple carousels, each consisting of oval track and suspended bins.

1.2. Motivation

The main problem for further refining the automated order picking systems is because they are becoming important part of warehouse systems, among automated order picking systems the technologies that are being used are AS/RS and carousel and did not found literature that compares the performance of these two systems. These are some of the factors that motivated to conduct research on these systems.

1.3. Research Objective

More warehousing and distribution companies are adopting AS/RS and carousel systems. There is a need to have tools that can help decision makers in evaluating the performance of these systems. One measure of system performance is throughput, which is the rate the systems can complete orders per unit of time. The storage policies such as random and class-based have influence on system performance.

The main goal of this research is to develop a mathematical model for comparing the performance of AS/RS and carousel system under two different storage assignments, namely random and class-based storage assignments. The measure of performance is throughput, which is the number of SKUs (stock keeping units) retrieved per unit of time.

The specific objective for this thesis is to develop a mathematical model that can be used to determine and compare the performance of the systems.

1.4. Thesis Outline

This thesis is organized as follows. This Chapter has presented the background, motivations and objective of this research. Chapter 2 reflects the literature review related to AS/RS and carousel systems and concludes with a statement about the void in research related to these systems.

Chapter 3 explains the development of the mathematical models that can be used for evaluating the automated systems performance that are the focal point of this thesis.

Chapter 4 addresses the validation of the mathematical model, which is performed using simulation model developed using Arena simulation software (Version 13.90.00000). Chapter 4 also included economic analysis comparing investment requirements and payback period for converting from manual to automated order picking systems.

Chapter 5 includes the conclusions derived from the study and recommendation for future research.

CHAPTER 2. LITERATURE REVIEW

This section introduces the different order picking technologies and summarizes the research and accomplishments in AS/RS and Carousel systems.

2.1. Automated Order Picking Systems

According to De Koster R et al. (2007) there are two types of order picking systems, first is picker to parts which is also called as manual order picking systems and other is Parts to picker systems which comprises of automated storage and retrieval systems (AS/RS) and carousel systems. An AS/RS contains many different kinds of systems that depend on the size and volume of items and the storage method. A mini load AS/RS and a unit-load AS/RS are commonly discussed since they can reduce labor cost and increase space utilization. Carousel systems are efficient automated warehousing systems and they can be categorized into "three kinds: horizontal, vertical, and rotary racks" was given by Park et al. (2003).

A horizontal carousel is a carousel that revolves horizontally and a vertical carousel is a carousel that revolves vertically. Rotary racks are carousels that have self-regulating rotating racks. The benefit of carousel systems is low cost because it is cheaper than other systems such as mini load AS/RS. Here in this thesis AS/RS and horizontal carousel system will be considered.

2.2. AS/RS (Automated Storage and Retrieval System)

Automated storage and retrieval systems have been extensively used in distribution and manufacture locations since their introduction in the 1950s. Roodbergen (2009) have presented an important outline of all significant issues concerning AS/RS design and control in both production and distribution environments and some of the literature related to AS/RS design and control is discussed in this section. An AS/RS usually consists of racks assisted by cranes running through aisles among the racks. An AS/RS is capable of holding pallets without the interfering of an operator and the position where operator stands is known as load/unload station where the crane picks and drops off loads. Thus the system is fully automated. Both in manufacture and distribution environments, AS/RSs are used for storing the items in storage and for retrieving those items from storage to accomplish an order. According to Roodbergen (2009) in United States there was a substantial increase in usage of AS/RS by distributions centers during 1994 to 2004, due to the fact that usage of AS/RSs has an upper hand over manual picking systems. Some of those benefits are saving in manual labor costs and floor space, increasing consistency and condensing error rates.

Apparent drawbacks are high investment costs of about $634,000 for a single aisle AS/RS Zollinger (1999), less flexibility and higher investments in control systems about $103,000, Zollinger (1999). While designing an AS/RS, many physical design and control issues have to be addressed in the correct way to fully take advantage of all its benefits. Kusiak (1985) describes design and operational decision problems for flexible manufacturing systems with an emphasis on automated guided vehicles and AS/RSs. The author discusses design, storage and batching procedures for AS/RSs. Johnson and Brandeau (1996) discuss stochastic models for the design and control of automated guided vehicles and AS/RSs. Manda and Palekar (1997) discuss certain papers on travel time models for AS/RSs and storage assignments.

2.2.1. Types of AS/RS

A great number of construction choices exist for AS/RSs, but according to Roodbergen (2009) there are three types of AS/RSs. The basic form of an AS/RS has only one crane in each aisle, which cannot leave its aisle and which can carry only one unit-load at a time. Material handling in this instance is by unit-load only; manual labor is not involved to handle single items. The racks in the basic form are fixed and single-deep, which means that the crane can directly access every load. This type of AS/RS is referred to as a single aisle AS/RS. The second type of AS/RS consists of multi-shuttle cranes that are this type of crane can carry two or more loads at same time and the cranes, which carry two loads, are called as dual shuttle cranes. The third type of AS/RS is a special type of AS/RS, which is called as autonomous vehicle storage and retrieval system. This type of AS/RS includes horizontal and vertical travel. Vehicles travelling horizontally over rails through aisles and lifts are used to transfer loads vertically.

2.2.2. Advantages of AS/RS

1. Savings in labor costs and floor space
2. Increased consistency
3. Reduced error rates

2.2.3. Disadvantages of AS/RS

1. High investments costs
2. Less flexibility

2.2.4. Travel time for AS/RS

Hausman et al. (1976) were some of the first to present travel time models for a single-shuttle unit-load AS/RS. These authors have proposed estimates for single command scheduling in square-in-time continuous racks. Random, full turnover, two- and three-class-based storage assignment policies were considered and extended those results by also considering interleaving times resulting from a first-come-first-served (FCFS) dual command scheduling policy.

The authors introduce variable b as the shape factor of the rack to deal with rectangular racks. Other authors such as Eynan and Rosenblatt (1994) since then continue the research of Hausman et al. (1976), by studying different control policies, configurations of AS/RS and operational characteristics. Instead of an FCFS-policy a nearest-neighbor (NN) policy can be used to schedule requests. Recursive procedures and closed-form expressions by Kouvelis and Papanicolaou (1995) have been proposed for n-class-based storage and full turnover storage.

2.2.5. Storage assignment for AS/RS

Hausman et al. (1976) and Graves et al. (1977) have described few storage assignments; among them the following five are the storage assignments, which are commonly used. These rules are:

- Dedicated storage assignment
- Random storage assignment
- Closest open location storage assignment
- Full-turnover-based storage assignment
- Class-based storage assignment

For the dedicated storage method each product type is assigned to a fixed location. Replenishments of that product always occur at this same location. The main drawback of dedicated storage is its high space requirements and low space utilization.

Furthermore, for each product type enough space must be kept to accommodate the highest stock level that may take place. Most advantages of dedicated storage, such as locating intense products at the bottom or matching the layout of stores, are related to non-automated order-picking areas and are not as interesting for AS/RSs. For random storage all empty locations have an equal chance of having an incoming load assigned to it. If the nearby open location storage is applied, the first empty location that is encountered will be used to store the products.

This typically leads to an AS/RS where racks are full around the load/unload station and gradually emptier towards the back (if there is excess capacity). The full-turnover storage policy determines storage locations for loads based on their order frequency. Often requested products get the easiest available locations, usually near the load/unload station. Slow-moving products are located farther away from the load/unload station. An important assumption for this rule is that the turnover frequencies need to be known beforehand. Heskett (1963) presents the cube-per-order index (COI) rule, which is a form of full-turnover storage. The COI of a load is defined as the ratio of the load's required storage space to the number of request for this product per period. The COI rule assigns loads with the lowest COI to the locations closest to the load/unload station.

Malmborg and Bhaskaran (1990), give a proof of optimality for this rule while taking into account the non-uniqueness of the COI layout if dual command scheduling is used. Malmborg and kumar (1989), show that the COI-rule is optimal for person aboard AS/RSs with respect to order-picking costs if there are fixed inventory levels and a fixed balanced assignment of order pickers to items. However, according to Lee (1992) the COI-rule cannot be applied for person-on-board systems due to the fact that an order usually consists of more than two independent items at different locations. Therefore, the author develops a new heuristic that outperforms the COI-rule.

According to Roodbergen (2009) for practical purposes it is easiest if a full-turnover policy is combined with dedicated storage. The problem is that demand frequencies change constantly and also the product to assortment is usually far from constant. Any change in frequency and any addition of a new product to the system may require a large amount of repositioning of loads to bring it back in line with the full-turnover rule. To prevent excessive repositioning, a new storage allocation is in practice typically calculated once per period. To reduce space requirements and periodic repositioning while maintaining most of the efficiency gains, class-based storage can be used.

2.3. Carousel System

According to Litvak and Vlasiou (2010) a carousel is an automated storage and retrieval system, which is often used in modern warehouses. It consists of a number of columns, which are connected together, and these columns revolve in a closed circle. These authors also described that carousels are commonly used for storage and retrieval of small and medium-sized items, such as health and beauty products, repair parts of boilers, parts of vacuum cleaners and sewing machines, books, shoes and many other goods. Hassini and Vickson (2003) stated that carousels are highly flexible, and come in a wide range of structures, sizes, and types. They can be horizontal or vertical and rotate in either one or both directions.

Although both unidirectional (one-way rotating) and bidirectional (two-way rotating) carousels are encountered in practice, the bidirectional types are the most regular (as well as being the most proficient). One of the main benefits of carousels is that, rather than having the operator travel to an item (as is the case in a warehouse where items are stored on shelves); the carousel rotates the items to the operator. While the carousel is travelling, the operator has the time to perform other tasks, such as pack or label the retrieved items, or serve another carousel. This carry out enhances the operational effectiveness of the warehouse.

2.3.1. Types of carousel system

Carousel systems can be horizontal or vertical and rotate in either one or both directions, that is unidirectional (one-way rotating) and bidirectional (two-way rotating) was given by Hassini and Vickson (2003).

2.3.2. Advantages of carousel system

1. The picker travels to an item.
2. While the carousel is travelling, the operator has the time to perform other tasks.
3. The design has considerable versatility.

2.3.3. Disadvantages of carousel system

1. Carousel processes only small or medium type of SKUs.

2.3.4. Travel time for carousel system

All research papers mentioned so far that investigated travel time models of carousel systems have assumed average uniform velocity of the carousel. In other words, the main assumption is that the carousel travels with constant speed and the acceleration from the stationary position (when a pick is performed) to its full speed, as well as the deceleration from the maximum speed to zero speed, are neglected when estimating the travel time of the carousel. Guenov and Raeside (1989) gave some empirical indications that the error persuaded when neglecting acceleration and deceleration of an order picking vehicle is indeed negligible.

Hwang et al. (2004), however, developed approaches for retrieving that take into consideration the variation in the speed of the carousel. For unit-load automated storage and retrieval systems there are several travel-time models that consider the speed profiles of the storage and retrieval robot. Unlike the unit-load automated storage and retrieval systems, nearly all the existing travel-time models for carousel systems assume that the effects of the variation in speed are negligible. Again in Hwang (2004) the authors try to bridge this gap in the literature. They assume that the items are assigned randomly on the carousel and derive the expected travel times for both single and dual command cycles. These authors have verified their suggested models by comparing the results with expected travel times of discrete racks.

Egbelu and Wu (1998) have studied the problem of pre-positioning the carousel in expectation of storage or retrieval needs in command to improve the average response time of the system. The dwell point selection problem can be referred by choosing the right initial point of a carousel in expectation of an order. This approach becomes applicable when the items are assigned under the organ pipe arrangement. In this situation the dwell point should be chosen to be the location of the most popular item. Spee (1996) is apprehensive with mounting design measures for carousels. He states the simple conditions for designing carousels and comments on the optimal storage design. Namely, he is interested in finding the correct number of picking robots and the right number and dimensions of a carousel so that the investment is minimized, provided that the size of the orders that need to be retrieved is given.

McGinnis et al. (1986) studies some of the design and control issues relevant to rotary racks. A rotary rack is an automated storage and retrieval system that strongly resembles carousels. In fact, conceptually, a rotary rack is simply a carousel, where the only difference is that each level or shelf of this carousel can rotate independently of the others. The author concludes that, while rotary racks appear to be a simple generalization of conventional carousels, the control strategies that have been shown effective for carousels do not appear to be as effective for these systems. Rotary racks can be viewed as a multiple-carousel system (where each level is considered as a sub-carousel) with a single picker.

2.3.5. Storage assignment for carousel system

The basic storage assignments used for carousel system are:

- Random storage assignment
- Class-based storage assignment
- Organ pipe arrangement

The performance of a carousel system depends wholly upon the way it is loaded and the order frequency of the items placed on it. An effective storage system may decrease drastically the travel time of the carousel. Numerous approaches have been followed in practice to store items on a carousel. The simplest approach is to place the items randomly on the carousel. Random storage assignment has been examined extensively by Hwang and Ha (1991) and Litvak (2001) and various performance characteristics have been derived under the assumption that the items are uniformly distributed on the carousel.

One way to better obtain the throughput of a carousel system is to implement a storage policy other than the random storage assignment policy. Hwang and Ha (1994) have studied the two class-based storage, which is a storage scheme that divides the items in two classes based on their demand frequency. The items with a higher turnover are randomly assigned to one continuous region of the carousel, while the less frequently asked for items occupy the complementary region. The authors demonstrate by simulation that the two-class-based storage can reduce significantly the expected cycle time, both in the single-command cycle and in the case of dual-command cycle.

Authors Hwang and Ha (1994) also observed the effects of the two-class-based storage policy on the throughput of the system, and present a case where there is a 16.29% progress of this assignment over the random storage assignment. Another storage scheme is suggested by Stern (1986). Assignments are made using a maximal adjacency principle, that is, two items are placed closely if their probability of appearing in the same order is high. The author evaluates this storage assignment analytically by using a Markov chain model he develops. The organ pipe arrangement for a carousel system is introduced in Lim et al. (1985) and is proven to be optimal in Beng (1995) under a wide variety of settings.

The organ pipe arrangement has been widely used in warehouses. In carousel systems, the organ pipe arrangement places the item with the highest demand in an arbitrary bin, the items with the second and third highest demands in the bin next to the first one but from opposite sides, and sequentially all other items next to the previous ones, where the odd-numbered items according to their frequency are grouped together and placed next to one another in a decreasing order from the one side of the most frequent item and similarly the even-numbered items are grouped together and placed to the other side.

2.4. Summary of Literature Review

The literature reviewed provides better understanding of automated order picking systems. Table 2.1 summarizes the literature by topics and Table 2.2 summarizes the literature that has the common objectives. Some of the literature reviewed included performance measure of AS/RS or carousel systems but none of them included the comparison of the systems performance against each other. Furthermore the literature that include AS/RS and carousel performance measures used over simplistic models which did not include parameters such as operator pick time and retrieval/storage time that impact the systems performance. None of the literature included economic analysis of systems showing the investment requirements and payback periods resulting from switching from manual to AS/RS and carousel order picking systems.

Table 2.1. Summary of Literature Review by Topics

Topics	**Literature**
Automated order picking systems	De Koster et al. (2007), Park et al. (2003)
AS/RS design	Zollinger (1999), Roodbergen (2009), Johnson and Brandeau (1996), Kusiak (1985)
AS/RS travel time	Hausman et al. (1976), Eynan and Rosenblatt (1994), Kouvelis and Papanicolaou (1995)
AS/RS storage assignment	Graves et al.(1977), Heskett (1963), Malmborg and Bhaskaran (1990), Malmborg and Krishna (1989)
Carousel design	Hassini et al.(2003), Litvak and Vlasiou (2010)
Carousel travel time	Egbelu and Wu (1998), Guenov and Raeside (1989), Hwang et al.(2004), Spee (1996), McGinnis et al.(1986)
Carousel storage assignment	Beng (1995), Hwang and Ha (1994), Litvak (2001), Hwang and Ha(1991), Stern (1986), Lim et al.(1985)

Table 2.2. Summary of Literature Review by Common Focus

References	**Common Focus**	**Methodology Used**
Bhaba and Sobhan (1995)	Review of travel time models in automated storage/retrieval systems	These aspects include improving throughput rate, changing retrieval sequencing rules, using order batching algorithms, applying various dwell point strategies and increasing the storage/retrieval machine capacity
Hwang and Ha (1991)	Cycle time models for single/double carousel system	Based on a randomized storage assignment policy, cycle time models are developed for single and dual commands.
Hwang and Ha (1994)	For reducing expected cycle time, the two-class-based storage assignment was used	Simulation

CHAPTER 3. METHODOLOGY

In this chapter, the details of the considered automated order picking systems will be presented. The assumptions and details according to the proposal will be described. The mathematical model for system throughput, which can be defined as the number of orders retrieved per unit time, will be carried out for four situations respectively. Here throughput is explained in terms of pickup time. The pickup time for AS/RS system is sum of crane travel time for retrieving order, crane pick time to retrieve order from rack, crane travel time to return load/unload station and operator pick time. The pickup time for carousel system is sum of time taken by a carousel for indexing the columns holding the order to front end location, storage/retrieval robot travel time for retrieving order, storage/retrieval robot retrieval time, operator pick time, storage/retrieval robot travel time for returning order and storage/retrieval robot deposit time. The storage systems associated with AS/RS and carousel systems include:

a. Random storage Assignment: In random storage the items are placed into storage in the nearest available open location; that is, when an order received for given SKU, the stock is retrieved from storage according to FIFO (first in first out) policy.

b. Class-based storage Assignment: class-based dedicated storage is based on Pareto principle (80-20 Rule) that is roughly 80% of the effects come from 20% of the causes. In a class-based storage assignment the storage system is divided into several classes according to activity level. The classes A containing 20% of SKUs, which make up (80%) of items ordered, class B containing 30% of SKU make up (15%) of items ordered and class C containing 50% of SKU make up (5%) of items ordered.

Table 3.1 summarizes the evaluations for different arrangements of assignments and systems.

Table 3.1. Evaluations for Different Arrangements of Assignments and Systems

<table>
<tr><td rowspan="2">Storage Assignment</td><td colspan="2">Systems</td></tr>
<tr><td>AS/RS</td><td>Carousel</td></tr>
<tr><td>Random Assignment</td><td colspan="2" rowspan="2">System Throughput</td></tr>
<tr><td>Class-based Assignment</td></tr>
<tr><td colspan="3">System throughput: Operating hours/pickup or cycle time.
Pickup or cycle time: It is the sum of operator pick time, machine retrieval/deposit time and machine travel time.</td></tr>
</table>

In this section the considered automated order picking systems are described in detail and a mathematical model is developed for estimating the throughput of both systems under different storage assignments which include random and class-based storage assignments.

3.1. AS/RS

The AS/RS that is being modeled consists of single aisle with each side containing H× V locations (H horizontal locations and V vertical locations). There are total N numbers of slots each containing single bin; the storage rack will contain one bin in it.

In an AS/RS a crane (mechanical storage/retrieval device) transports storage bins, containing items, to and from a load/unload station. This system will have load/unload located at one end of the aisle. The crane will operate in dual operating mode.

3.1.1. The typical order picking process in AS/RS

The typical order picking process in AS/RS is shown in Figure 3.1. When the first order is (i=1) placed, the crane from load/unload station goes directly to storage location retrieves, the first order P_1 and comes back to load/unload station. When processing the second order (i>1), the crane at load/unload station goes to storage location to return the first order R_1 and after that the crane goes for retrieving second order P_2. As such in processing the order i the crane first returns i-1 order then it retrieves the order i from storage location.

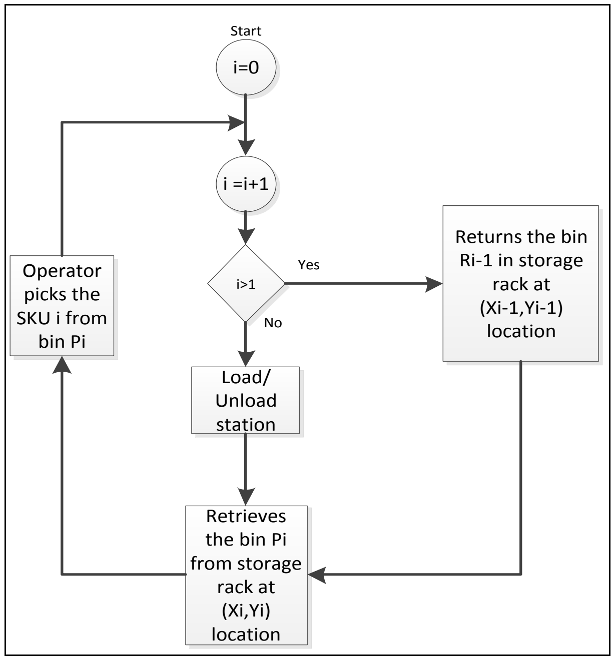


Figure 3.1. Typical Order Picking Process in AS/RS

3.1.2. Pickup or cycle time for AS/RS

Pickup or cycle time is the sum of crane travel time, crane retrieve or storage time and operator pick time. The pickup time or cycle of a crane in AS/RS under single command cycle is shown in Equation 3.1.

Pickup time for first order is $T_{Pi} = CT_i + CP_i + OP_i + RT_i$ (3.1)

Where CT_i = crane travel time for retrieving first order

CP_i = crane retrieval time

RT_i = crane travel time to return to load/unload

OP_i = operator pick time

For first order retrieval travel time = return travel time.

In this thesis the pickup time for first order is neglected. The pickup or cycle time for remaining orders will have a change in its formula because for remaining orders greater than one crane need to perform one extra operation during retrieval of second order, that is it needs to return first order and the formula for pickup or cycle time is shown in Equation 3.2 which shows the pickup time for remaining orders where i > 1.

$$T_{pi} = CT_{i-1} + DT_{i-1} + CT_i + CP_i + RT_i + OP_i \quad (3.2)$$

CT_{i-1}= crane travel time to return order i-1

DT_{i-1}= crane depositing time

CT_i = crane travel time for retrieving order i

CP_i= crane retrieval time

RT_i= crane travel time to return to load/unload station

OP_i= operator pick time

3.1.3. AS/RS storage assignments

In Figures 3.2, the storage rack structures of AS/RS in random and class-based storage assignments are shown. Figure 3.2 shows the side view of AS/RS storage rack structure. The top portion of the figure shows the random assignment of SKUs to storage locations. The bottom portion of the figure shows class-based assignment of SKUs. The SKU locations are generated by Excel software. The yellow, green and blue colors in storage rack represents the class A, class B and class C type SKUs.

3.2. Carousel System

The carousel system that is being modeled will have C number of columns and D number of storage bins per column and is served by a single storage/retrieval robot, which carries only one bin (unit load) at a time. The carousel length and height, its rotating speed, and the vertical speed of the storage/retrieval robot are known. The load/unload station is located at the bottom front corner of the carousel. The storage/retrieval robot can move in a vertical direction while the carousel rotates. The carousel used here is a bidirectional.

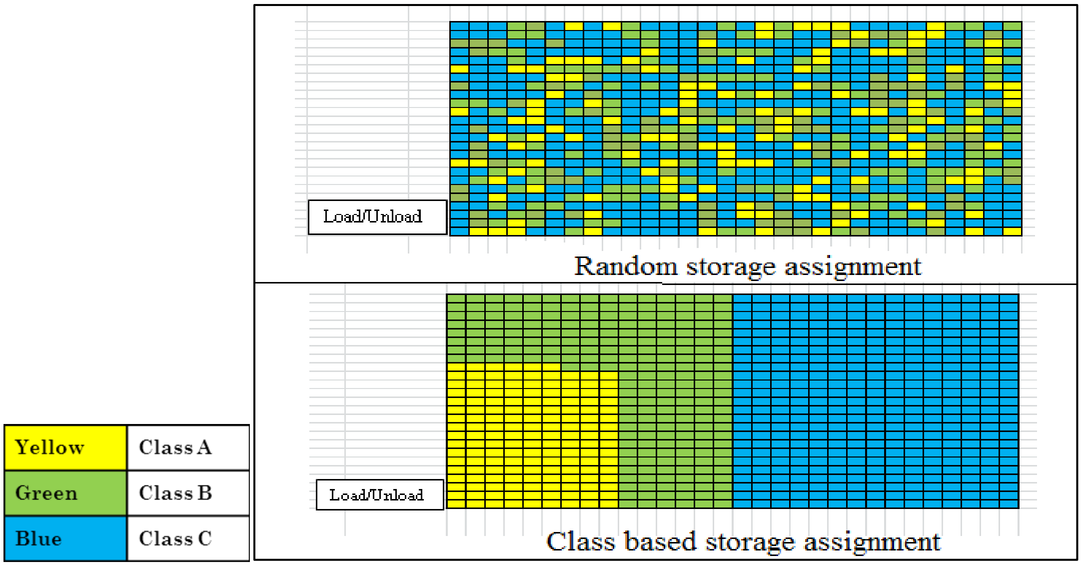


Figure 3.2. Random and Class-Based Storage Assignment for AS/RS

3.2.1. The typical order picking process in carousel system

The typical order picking process in carousel system is shown in Figure 3.3. When the first order is (i=1) placed, the carousel rack indexes the column holding bin to load/unload station. Then (storage/retrieval) robot at load/unload station goes to carousel rack retrieves the first order P_1 to load/unload station then robot returns the first order R_1 and after that the Carousel rack indexes the column holding second order. Then robot goes retrieves second order P_2 then it returns second order R_2. As such in processing the order i, carousel rack indexes the column containing order i then robot retrieves order i from carousel and returns the order.

3.2.2. Pickup or cycle time for carousel system

Pickup or cycle time is the sum of carousel indexing time, robot travel time, robot retrieve or storage time, and operator pick time. The pickup time or cycle of carousel system under single command cycle is shown in Equation 3.3.

$$T_{qi} = MI_i + MT_i + MP_i + MR_i + OT_i \tag{3.3}$$

This is the pickup time for first order Where:

MI_i= time taken by a carousel for indexing the column holding the order i to pick column location

MT_i = robot travel time for retrieving orders i

MP_i = robot retrieval time

MR_i= robot travel time in returning to load/unload station

OT_i= operator picks time

In this thesis as discussed before the pickup time for first order is neglected. The pickup or cycle time for remaining orders will have a change in its formula because for remaining orders greater than one robot need to perform one extra operation during retrieval of second order that is it needs to return first order and the formula for pickup or cycle time is shown in Equation 3.4.

$$T_{qi} = MT_{i-1} + ST_{i-1} + MI_i + MT_i + MP_i + MR_i + OT_i \quad (3.4)$$

Where:

MT_{i-1}= robot travel time for returning order i-1

ST_{i-1}= robot deposit time

MI_i= time taken by a carousel for indexing the column holding the order i to pick column location

MT_i = robot travel time for retrieving order i

MP_i= robot retrieval time

MR_i= robot travel time in returning to load/unload station

OT_i= operator picks time

3.2.3. Carousel system storage assignments

In Figure 3.4 the storage rack structures of the carousel in random and class-based storage assignments are shown. Figure 3.4 shows the structure of carousel when it is unwrapped into a rectangle form such that the circumference of the carousel becomes the width of a rectangle. The top portion of the figure shows the random assignment of SKUs to storage locations. The bottom portion of the figure shows class-based assignment of SKUs. The SKU locations are generated by Excel software. The yellow, green and blue colors in storage rack represents the class A, class B and class C type SKUs.

3.3. Assumptions

The assumptions for AS/RS and Carousel are:

3.3.1. The assumptions used for AS/RS

1. The AS/RS will have only one crane in its aisle.
2. It is a single shuttle type.
3. It is a dual command cycle.
4. It carries only single unit load at a time.
5. The speed of the crane is constant.
6. The racks in AS/RS system are stationary and single-deep.

3.3.2. The assumptions used for carousel system

1. The carousel system is served by only one storage or retrieval robot.
2. It is a bi-directional system.
3. It carries only single unit load at a time.
4. The rotating speed of carousel and vertical speed of the robot are known.
5. The load/unload station is located at the bottom front corner of the carousel.

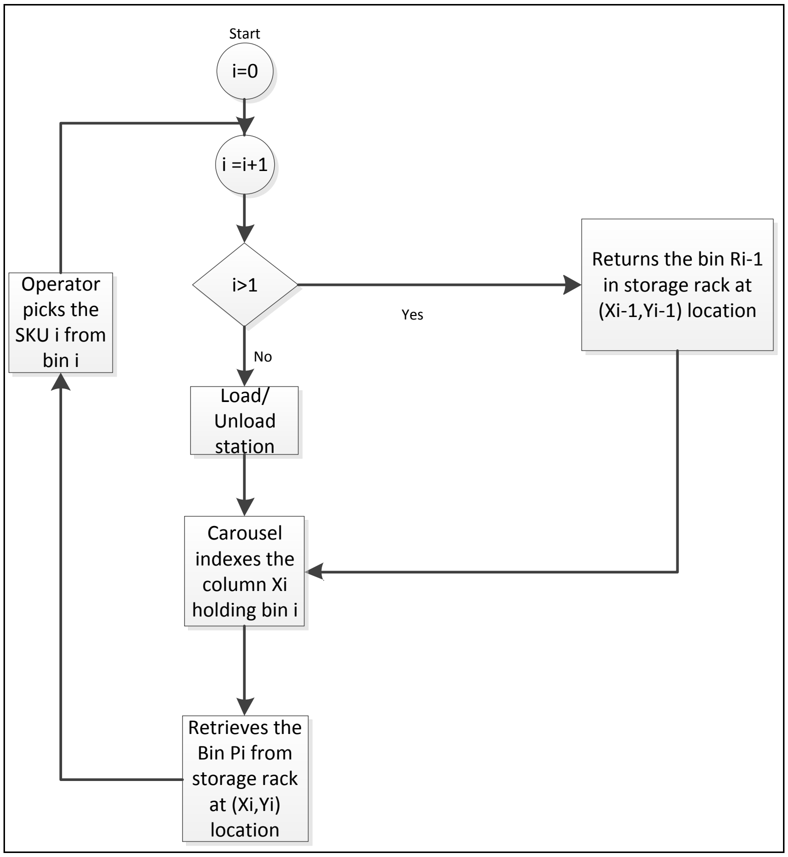


Figure 3.3. Typical Order Picking Process in Carousel System

3.3.3. Combined assumptions for both AS/RS and carousel systems

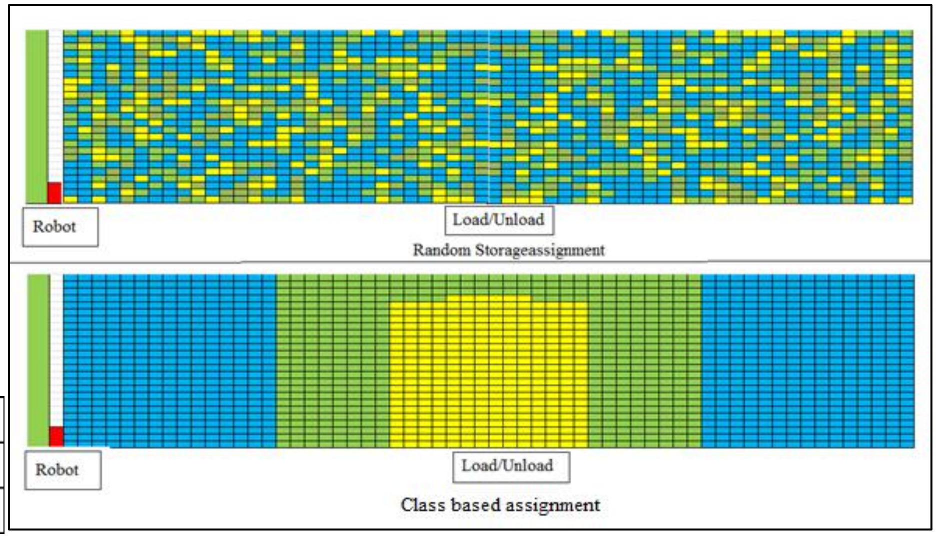
1. Both systems have the same number of storage locations.
2. Both systems have the same number and types of SKUs.
3. Both systems are used to process the same types of orders.

3.4. Notations

The notations are used for the variables considered for analysis of throughput of automated order picking systems. The notations used are listed in Tables 3.2 and 3.3.

3.5. Throughput

Throughput of a system can be defined as the number of orders retrieved per unit of time.



Yellow	Class A
Green	Class B
Blue	Class C

Figure 3.4**.** Random and Class-Based Storage Assignment for Carousel System

3.5.1. Throughput of AS/RS under random storage assignment

From Equation 3.2 travel time model for AS/RS where they have given expected time for AS/RS as:

$$\mathbf{T_{pi} = CT_{i-1} + DT_{i-1} + CT_i + CP_i + RT_i + OP_i}$$

The time taken by AS/RS crane to reach a horizontal location containing the SKU i-1 can be represented as:

$$Th_{i-1} = \frac{Dh_{i-1}}{Sh} \quad (3.5)$$

The time taken by AS/RS crane to reach a vertical location containing the SKU i-1 can be expressed as shown in Equation 3.6:

$$Tv_{i-1} = \frac{Dv_{i-1}}{Sv} \quad (3.6)$$

Therefore to reach pick location i-1 the time taken by a crane is expressed as:

$$\mathrm{C}\,T_{i-1} = \max\langle Th_{i-1},\ Tv_{i-1}\rangle \quad (3.7)$$

Similarly the time taken by crane from pick location i-1 to i and from pick location i to load/unload station is represented in Equations 3.8, 3.9 respectively:

Table 3.2. Notations used for AS/RS

Symbols	Description
Sh	Speed of crane in horizontal direction
Sv	Speed of crane in vertical direction
Dh_{i-1}	Horizontal distance travelled by crane from load/unload station to pick location i-1, i = 1,2, 3------N
Dv_{i-1}	Vertical distance travelled by crane from load/unload station to pick location i-1, i = 1,2, 3------ N
Dh_{i-1}	Horizontal distance travelled by crane from pick location i-1 to i, i = 1,2, 3---N
Dv_{i-1}	Vertical distance travelled by crane from pick location i-1 to i, i = 1,2, 3------ N
Rh_i	Horizontal distance travelled by crane from pick location i to load/unload station, i = 1,2, 3-- N
Rv_i	Vertical distance travelled by crane from pick location i to load/unload station, i = 1,2, 3--- N
P_k	Probability of kth class of fast-moving SKUs in class-based storage assignment per order, $k = 1,2,3$
CT_{i-1}	Crane travel time to return SKU i-1, i = 1,2, 3------ N
DT_{i-1}	Crane depositing time at rack which contains SKU i-1, i = 1,2, 3------ N
CT_i	Crane travel time to rack which contains SKU i, i = 1,2, 3------ N
CP_i	Crane retrieval time at rack which contains SKU i, i = 1,2, 3------ N
RT_i	Crane travel time to return to load/unload station, i = 1,2, 3------ N
OP_i	Operator pick time, i = 1,2, 3------ N
TP_i	The cycle time/pickup times for one SKU under random storage assignment, i = 1,2, 3-- N
$E[T_i]$	The expected travel time under dual command cycle
$E[T_{pi}]$	The expected time for one SKU under random storage assignment
T_a	Throughput of AS/RS under random storage assignment
T'_{pi}	The cycle time/pickup times for one SKU under class-based storage assignment
$E[T_o]$	The expected time for one SKU under class-based storage assignment
T'_a	Throughput of AS/RS under class-based storage assignment

$$CT_i = \max\langle Th_i,\ Tv_i\rangle \tag{3.8}$$

$$RT_i = \max\langle Rh_i,\ Rv_i\rangle \tag{3.9}$$

The expected cycle time for picking a unit load is given by:

E [TP_i] = expected travel time to pick a unit load + machine deposit/retrieve time + operator pick time.

According to Hausman et al. (1976) the expected travel times for both single and dual command cycles of an AS/RS are shown in equations 3.10, 3.11.

Table 3.3. Notations used for Carousel

Symbols	Description
V_c	Speed of carousel
V_m	Speed of robot
Y_{i-1}	Distance travelled by robot from load/unload station to pick location i-1, i = 1,2, 3------N
Y_i	Distance travelled by robot from pick location i-1 to i , i = 1,2, 3----- N
R_i	Distance travelled by robot from pick location i to load/unload station , i = 1,2, 3------N
X_i	Distance travelled by carousel in indexing the column containing SKU i to load/unload station, i = 1,2, 3------N
MT_{i-1}	Machine travel time for returning SKU i-1 , i = 1,2, 3------N
DT_{i-1}	Machine deposit time, i = 1,2, 3------ N
MI_i	Time taken by a carousel for indexing the column holding the SKU i to pick column location, i = 1,2, 3------N
MT_i	Robot travel time for picking SKU i , i = 1,2, 3------N
MP_i	Robot retrieval time, i = 1,2, 3------N
MR_i	Robot travel time in returning to load/unload station, i = 1,2, 3------N
OT_i	Operator pick time, i = 1,2, 3------ N
T_{qi}	Cycle time/pickup time for picking one SKU under random storage assignment, i = 1,2, 3------N
$E[T_i]$	The expected travel time under dual command cycle
$E[T_{qi}]$	The expected time for one SKU under random storage assignment
T_c	Throughput of carousel under random storage assignment
T'_{qi}	Cycle time/pickup time for picking one SKU under class-based storage assignment, i = 1,2, 3------N
$E[T'_o]$	The expected time for one SKU under class-based storage assignment
T'_c	Throughput of carousel under class-based storage assignment

$$\mathrm{E}[T_i] = \frac{1}{N}\sum_{i=1}^{N}(CT_i + RT_i) \tag{3.10}$$

$$\mathrm{E}[T_i] = \frac{2}{N(N-1)}\sum_{i-1=1}^{N-1}\sum_{i=2}^{N}(CT_{i-1} + CT_i + RT_i) \tag{3.11}$$

Therefore the expected cycle time for processing an order can be rewritten as:

$$\mathrm{E}[T_{pi}] = \frac{2}{N(N-1)}\sum_{i-1=1}^{N-1}\sum_{i=2}^{N}(CT_{i-1} + CT_i + RT_i) + DT_{i-1} + CP_i + OP_i \tag{3.12}$$

Average throughput of a system time can be described as the cycle/pickup time for processing one order. Though, in command to have a perfect knowledge of deciding which system has the best performance, the cycle time needs to be converted into system throughput in an hourly standard. Throughput of AS/RS for random assignment is expressed as shown in Equation 3.13.

$$T_a = \frac{opearating\ hours\ per\ day}{E[T_{pi}]} \quad (3.13)$$

3.5.2. Throughput of AS/RS under class-based storage assignment

As the percentage of every class in one order has been taken into consideration because of class-based storage assignment, the SKUs allocation is optimized to save travel time, which can be seen in Figure 3.3. Three classes of SKUs in one order are considered for different percentages, the new total picking time can be expressed as:

$$E[T_0] = \sum_{k=1}^{3} P_k \, \mathrm{E}\left[T'_{pi}\right] \quad (3.14)$$

Where

$$\mathrm{E}\left[T'_{pi}\right] = \frac{2}{N(N-1)} \sum_{i-1=1}^{N-1} \sum_{i=2}^{N} (CT'_{i-1} + CT'_i + RT'_i) + DT'_{i-1} + CP'_i + OP'_i \quad (3.15)$$

The throughput for AS/RS under class-based storage assignment is given in Equation 3.16:

$$T'_a = \frac{opearating\ hours\ per\ day}{E[T'_{pi}]} \quad (3.16)$$

3.5.3. Throughput of carousel system under random storage assignment

According to Hark Hwang and Jae-Won Ha (1991) there are two aspects of the Carousel system, which differ from, typical unit-load AS/RS. One is that while pickup/discharge operations are being performed at the load/unload station by a storage/retrieval robot; the carousel may rotate at the same time. The effect is to make storage locations closer to the load/unload station than they are in a comparable unit-load

AS/RS. The other aspect is that the position of each rack opening relative to the load/unload station may change from cycle to cycle. Thus the model of unit-load AS/RS cannot be directly applied to the carousel system. The expected cycle time for carousel system is from Equation 3.4.

$$\mathbf{T_{qi} = (MT_{i\text{-}1} + ST_{i-1} + MI_i + MT_i + MP_i + MR_i + OT_i)}$$

In Equation 3.17 the times taken for machine from load/unload station to pick location i-1 is shown:

$$MT_{i-1} = \frac{Y_{i-1}}{V_m} \tag{3.17}$$

Similarly the time taken for machine to travel from pick location i-1 to i and pick location i to load/unload station are given in Equations 3.18, 3.19 respectively:

$$MT_i = \frac{Y_i}{V_m} \tag{3.18}$$

$$MR_i = \frac{R_i}{V_m} \tag{3.19}$$

The Equation 3.20 shows the carousel indexing time:

$$MI_i = \max\left(\frac{X_i}{V_C}, \frac{Y_i}{V_m}\right) \tag{3.20}$$

According to Hwang et al.(2004) the expected travel time for carousel is:

$$\mathrm{E}[T_i] = \frac{2}{N(N-1)} \sum_{i-1=1}^{N-1} \sum_{i=2}^{N} (MT_{i-1} + MI_i + MT_i + MR_i) \tag{3.21}$$

Expected cycle/pickup time for one SKU:

$$\mathrm{E}\left[T_{qi}\right] = \frac{2}{N(N-1)} \sum_{i-1=1}^{N-1} \sum_{i=2}^{N} (MT_{i-1} + MI_i + MT_i + MR_i) + ST_{i-1} + MP_i + OT_i \tag{3.22}$$

Throughput of carousel for random assignment is expressed as shown in Equation 3.23

$$T_c = \frac{opearating\ hours\ per\ day}{\mathrm{E}[T_{qi}]} \tag{3.23}$$

3.5.4. Throughput of carousel system under class-based storage assignment

Three classes of SKUs in one order are considered for different percentages, the new total picking time can be expressed as: E [T'o] = $\sum_{k=1}^{3} P_k \, \mathrm{E}\left[T'_{qi}\right]$ (3.24)

Where:

$$\mathrm{E}\left[T'_{qi}\right] = \frac{2}{N(N-1)} \sum_{i-1=1}^{N-1} \sum_{i=2}^{N} (MT'_{i-1} + MI'_i + MT'_i + MR'_i) + ST'_{i-1} + MP'_i + OT'_i \quad (3.25)$$

Equation 3.26 gives the throughput of carousel for class-based storage assignment:

$$T'_c = \frac{opearating\ hours\ per\ day}{\mathrm{E}[T'_o]} \quad (3.26)$$

3.6. Summary

The major difference appears especially when it comes to the travel time of the systems in processing one order. In order to provide a clear view of all the outputs and inputs in four different systems, Table 3.4 shows the formulae for expected cycle time and throughput and Table 3.5 summarize the formulae for basic inputs.

Table 3.4. Summary of Outputs

		System	
		AS/RS	**Carousel**
Random	**Expected cycle time**	$E[T_{pi}] = \frac{2}{N(N-1)} \sum_{i-1=1}^{N-1} \sum_{i=2}^{N} (CT_{i-1} + CT_i + RT_i) + DT_{i-1} + CP_i + OP_i$	$E[T_{qi}] = \frac{2}{N(N-1)} \sum_{i-1=1}^{N-1} \sum_{i=2}^{N} (MT_{i-1} + MI_i + MT_i + MR_i) + ST_{i-1} + MP_i + OT_i$
	Throughput	$T_a = \frac{operating\ hours\ per\ day}{E[T_{pi}]}$	$T_c = \frac{operating\ hours\ per\ day}{E[T_{qi}]}$
Class-based	**Expected cycle time**	$E[T_o] = \sum_{k=1}^{3} P_k E[T'_{pi}]$	$E[T'_o] = \sum_{k=1}^{3} P_k E[T'_{qi}]$
	Throughput	$T'_a = \frac{operating\ hours\ per\ day}{E[T_0]}$	$T'_c = \frac{operating\ hours\ per\ day}{E[T'_o]}$

Table 3.5. Basic Inputs and Formulae for AS/RS and Carousel System

System	Inputs	
AS/RS	$CT_{i-1} = \max\langle Th_{i-1}, Tv_{i-1}\rangle$	Crane travel time from load/unload to the previous SKU storage location
	$CT_i = \max\langle Th_i, Tv_i \rangle$	Crane travel time from load/unload to the SKU current storage location
	$RT_i = \max(Rh_i, Rv_i)$	Crane travel time from SKU current storage location to load/unload
	$Th_{i-1} = \frac{Dh_{i-1}}{Sh}$	Time taken by crane in horizontal direction to reach previous SKU storage location
	$Tv_{i-1} = \frac{Dv_{i-1}}{Sv}$	Time taken by crane in vertical direction to reach previous SKU storage location
	DT_{i-1} = Constant value	Crane deposit time value at previous SKU storage location
	CP_i = Constant value	Crane retrieval time value at current SKU storage location
	OP_i = Constant value	Operator pick time value
Carousel	$MT_{i-1} = \frac{Y_{i-1}}{V_m}$	Robot travel time from load/unload to the previous SKU storage location
	$MI_i = \max(\frac{X_i}{V_C}, \frac{Y_i}{V_m})$	Carousel indexing time for current SKU
	$MR_i = \frac{R_i}{V_m}$	Robot travel time from SKU current storage location to load/unload
	ST_{i-1} = Constant value	Robot deposit time value at previous SKU storage location
	MP_i = Constant value	Robot retrieval time value at current SKU storage location
	OT_i = Constant value	Operator pick time value

CHAPTER 4. ANALYSIS AND RESULTS

This chapter includes three sections. The first presents the mathematical models to compare performance of AS/RS and carousel. Excel (Version 14.0.6029.1000) is used to perform the mathematical operations. In the second section similar comparisons are done between these two systems by using simulation software. At last, an economic analysis is presented to evaluate the cost effective and the sensitivity analysis is conducted to test the relation between the gross profit per order and payback period.

4.1. Scenario Description

Figure 4.1 shows the SKU flows in both systems. In the AS/RS, at the beginning of retrieval operation, if it is a first order in sequence then the crane goes to storage rack location where the first SKU is stored from there it retrieves SKU and returns it back to load/unload. If it is a second order then the crane first returns the first SKU at its location and from there it moves to the other location in storage rack where it retrieves the second SKU and comes back to load/unload station.

In this thesis the SKU locations are selected from a data table and this table is generated for identifying (X, Y) coordinates of SKU, where X represents the row and Y represents the column of storage rack. Suppose there is an order for two SKU's which are at (X_{i-1}, Y_{i-1}) and (X_i, Y_i) positions in a storage rack. In this case the crane performs a single command operation while retrieving the first SKU from (X_{i-1}, Y_{i*1}) and while retrieving the second SKU, the crane first returns the SKU1 at (X_{i-1}, Y_{i*1}) then retrieves the second SKU at (X_i, Y_i); that is, the crane is operated in dual command cycle.

The differences between AS/RS and carousel system could be seen as: 1) the storage rack in carousel is not stationary as in AS/RS. So the carousel indexes the column.

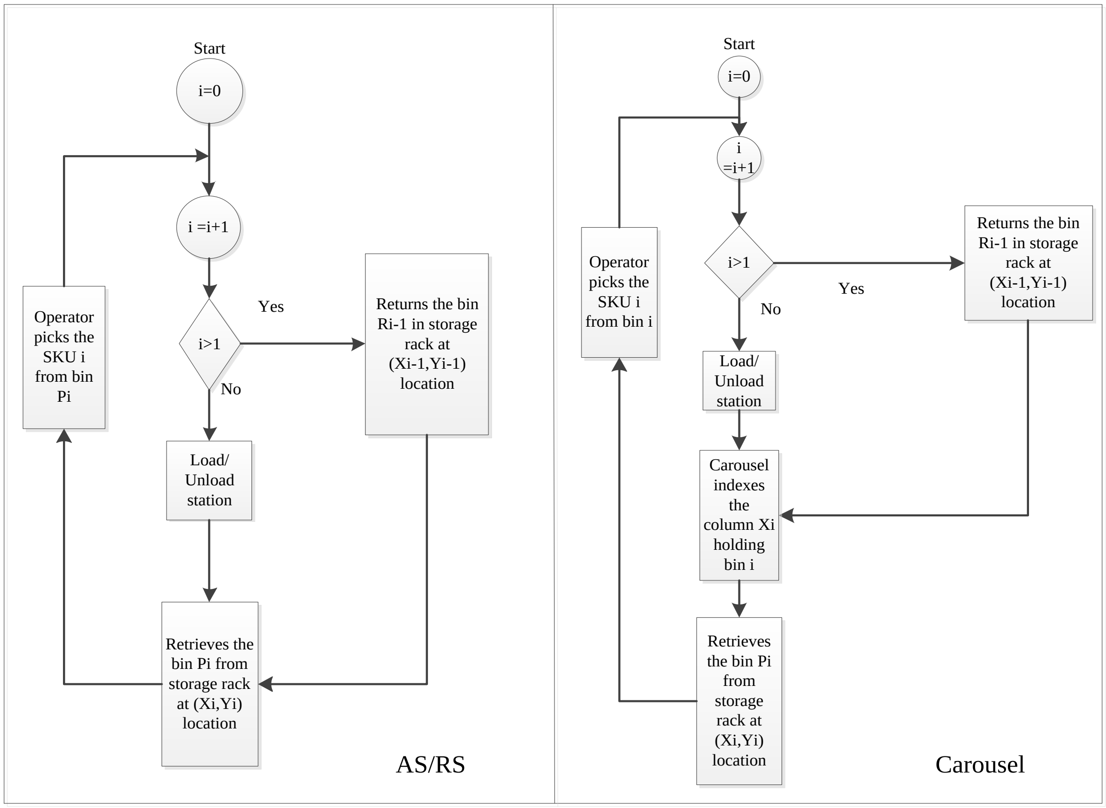


Figure 4.1. SKU Flows in AS/RS and Carousel Systems

which holds the required SKU to front end location. From there the vertical storage/retrieval robot picks the SKU. In this process when order is placed, if it is a first order in sequence then carousel indexes the column holding the first SKU to front end location then the SKU is retrieved by storage/retrieval robot. If it is a second order in sequence storage/retrieval robot returns first SKU, then carousel indexes column holding the second SKU to front end location and again the storage/retrieval robot retrieves SKU from that position and comes back to load/unload.

The same procedure of AS/RS for selecting SKU location is followed here; that is, the SKU locations are selected from a data table and this table is generated for identifying (X, Y) coordinates of SKU, where X represents the row and Y represents the column of storage rack. Suppose there is an order for two SKU's which are at (X_{i-1}, Y_{i-1}) and (X_i, Y_i) positions in a storage then the carousel performs a single command operation while retrieving the first SKU from (X_{i-1}, Y_{i-1}) and while retrieving the second SKU robot first returns the SKU1 at (X_{i-1}, Y_{i-1}) $then$ carousel indexes the column X_i holding second SKU and is retrieved by robot that is operated in dual command cycle.

4.2. Case Study

Table 4.1 provides a summary of the input parameters that are used as inputs to both mathematical and simulation models. All the values are based on practical situations, which can be referenced from AFT–system specifications (2012), Schaefer carousel system (2011), Hwang and song (2004).

Table 4.1. Input Parameters and Value used in the Case Study

		System	
		AS/RS	**Carousel**
Inputs	**Height**	H = 25ft[1]	H= 25ft
	Overall Length	W=100ft[1]	W=100ft
	Speed of crane in horizontal direction	S_h = 180ft/min[1]	N/A
	Speed of crane in vertical direction	S_v =90ft/min[1]	N/A
	Speed of carousel	N/A	V_c=90ft/min [2]
	Speed of robot	N/A	V_m=160ft/min [3]
	Number of storage racks	1500[2]	1500[2]
	Number of rows	25	N/A
	Number of columns	30	60
	Number of bins in each column	N/A	25
	Probability of lth level of fast-moving SKUs in level loading per order	P_1=0.8 P_2=0.15 P_3=0.05	P_1=0.8 P_2=0.15 P_3=0.05
	Crane deposit/retrieval time	4 sec	N/A
	Robot deposit/retrieval time	N/A	4 sec
	Operators pick time	6 sec	6 sec

[1] data is adapted from AFT - system specifications (2012). [2] data is adapted from Schaefer Carousel system (2011). [3] data is adapted from Hwang and Song (2004).

The sequence of calculation will be followed, as the expected number of SKUs needs to be picked. Also, the average system throughput time includes travel time, machine retrieval or storage time and operator picking time taken for average number of SKU's.

The main outputs of the systems can be seen in Table 4.2. All the calculations are contributing to the final system throughput. Comparisons between each output also can be seen in Figures 4.2, 4.3 and can obtain the conclusion that carousel system with the class-based assignment method has the best system performance in finishing an order in about 29.81 seconds and the best throughput with capacity of processing almost 120.75 orders per hour, this as expected is the most throughput.

The average system throughput can be also explained as the cycle time to process one order. In the AS/RS system, the difference between random assignment and class-based is about 24 seconds in this case study. However, when considering carousel, the processing time difference between random assignment and class-based is about 16 seconds. The processing time for carousel in class-based assignment is much less than the remaining systems. In class-based assignment, by choosing AS/RS, the cycle time will be improved by approximately 62.37% compared to random assignment, as well as in carousel; the cycle time will be improved by approximately 55.21% compared to random assignment.

Table 4.2. Main Outputs of Mathematical Model

	Systems Performance			
	AS/RS		**Carousel**	
	Random	**Class-based**	**Random**	**Class-based**
Expected cycle time (sec)	61.93	38.14	45.56	29.81
Throughput (orders/hour)	58.12	94.37	79	120.75

The reason for the difference in cycle time in random and class-based systems can be explained by the assigning of fast moving SKU's near to load/unload station. As a result, by adapting this method, the total picking time in class-based assignment is shown to be comparatively less than the same order components in random assignment. Since the orders are being handled instantaneously, the average cycle time needs to be presented in order to measure the system throughput in one system.

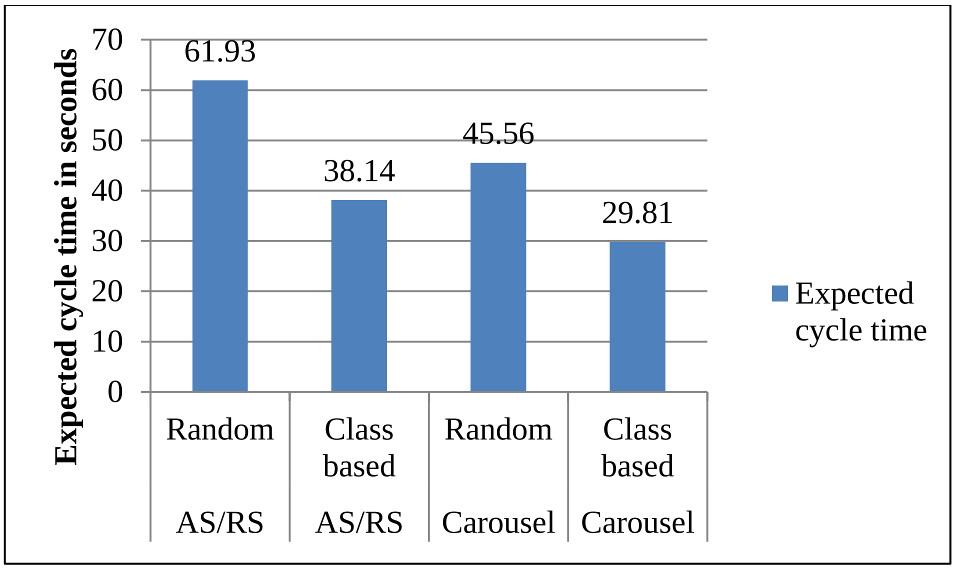


Figure 4.2. Cycle Time Resulting from Executing Mathematical Models

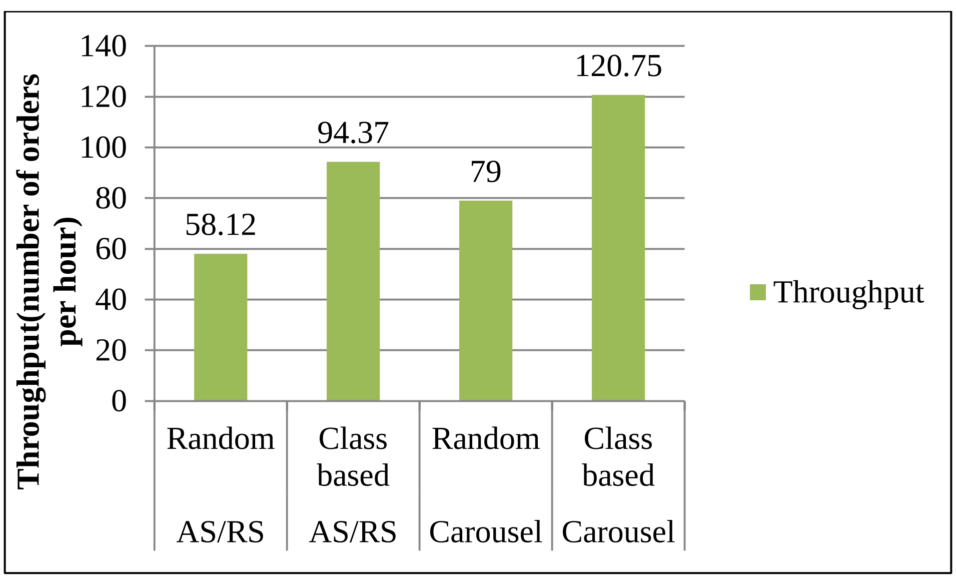


Figure 4.3. Throughput Resulting from Executing Mathematical Models

There will be nearly 120.75 orders to be fulfilled in carousel with class-based in one operating hour without external limits considered. In addition, the outputs demonstrate that the storage policy serves an important function in increasing the system throughput.

4.3. Validating Mathematical Model

As stated at beginning of this chapter, that Excel software is used to perform the mathematical operations. In this section simulation models were developed to validate mathematical models. Simulation models were generated by using Arena simulation software according to parameters of mathematical models. The results generated by the simulation models to estimate cycle times for AS/RS and carousel systems with different storage assignments are shown in Appendix A, Table A.1- A.4.

Table 4.3 compares the mathematical and simulation models outputs. This table is also shown in graphic form in Figure 4.4 and 4.5, each respectively showing the cycle times and throughput for the systems. According to these figures carousel system with class-based assignment has the shortest cycle time and largest throughput among the systems.

Table 4.3. Main Outputs of Mathematical and Simulation Model

	Systems Performance			
	AS/RS		Carousel	
	Random	Class based	Random	Class based
Mathematical Model Output				
Expected cycle time (sec)	61.93	38.14	45.56	29.81
Throughput(orders/hour)	58.12	94.37	79	120.75
Simulation Model Output				
Expected cycle time (sec)	64.86	37.99	45.42	29.32
Throughput (orders/hour)	55.5	94.75	79.25	122.75

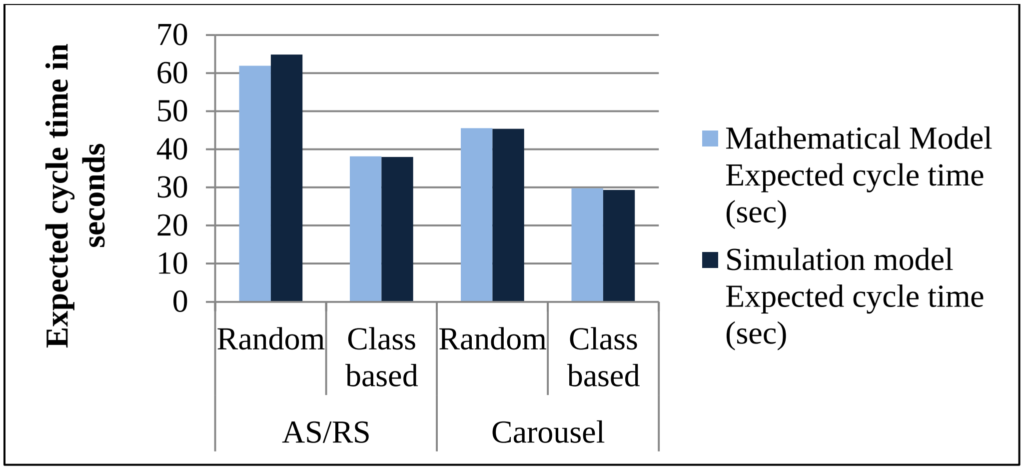


Figure 4.4. Cycle Time for Four Systems in Mathematical and Simulation Models

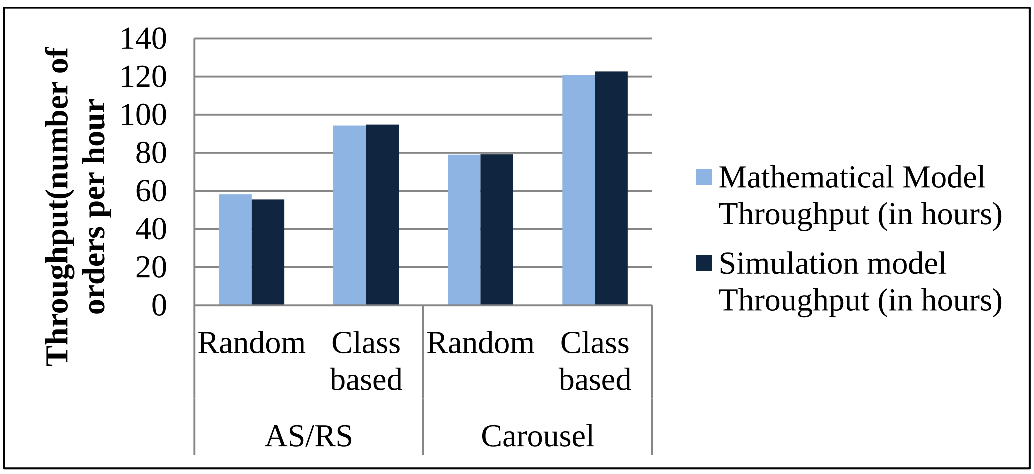


Figure 4.5. System Throughputs for Four Systems in Mathematical and Simulation Models

4.4. Simulation Model Verification and Validation

In order to determine if the simulation model is correctly representing the conceptual model, there are several ways to verify and validate a simulation model, such as find the Real outcomes for the model or calculate the extreme examples to test the simulation model Song (2010). For the purpose of verifying the simulation model, calculation was done using sample data, which represent the extreme situation. The extreme data included inter arrival time and order processing times. Table 4.4 shows the results of simulation model using extreme data. The extreme data values are shown in Appendix B, Tables B1-B4.

Table 4.4. Simulation Model Verification and Validation Summary

	Systems Performance Throughput (orders/hour)			
	AS/RS		Carousel	
	Random	Class-based	Random	Class-based
Extreme Simulation Model	57.97	94.53	78.83	122.28
Extreme Mathematical Model	57.97	94.53	78/83	122.28

The "Extreme Simulation Model" performances shown in Table 4.4 are as a result of running the simulation model using the extreme case input parameters. Using the mathematical model that the author has developed and the extreme case data, manual calculations were performed to estimate the systems' performance. The "Extreme Mathematical Model" performances shown in Table 4.4 are as a result of executing the manual calculations. Based on the verification and validation, the conclusion can be drawn as the simulation model is providing the reasonable test for the systems.

4.5. Economic Analysis

In the real operation of a warehouse, cost is one of the most important issues to consider besides the system throughput, which will impact the organization choices. In the economic analysis, discussion is mainly focused on the shortest payback period.

The gross profit per year can be expressed as:

Gross profit per year = Number of orders processed annually× Gross profit per order.

However, additional cost is required to convert from random to class-based storage. To convert from random to class-based SKU assignment requires some investment; SKU reallocation. The investment estimation is presented in Table 4.5 adapted from Zollinger (2001) and Bastian Solution's (2012).

Table 4.6 presents the annual gross profit for the four situations of systems respectively by operating 255 days, 8 hours per day, $6 profit per each SKU in order to calculate the annual gross profit among the systems.

The simple payback period for each kind of investment is shown in Table 4.7. Payback period is the time required for recovering investment required for alternative systems and can be expressed as: Payback Period = $\frac{\text{Investment}}{\text{Annual Cash Inflows}}$. Calculations for payback periods in Table 4.7 are shown in Figure 4.6. According to Table 4.7 changing from manual to AS/RS with random SKU assignment have the longest payback period.

Table 4.5. Investment Summary

	AS/RS	**Carousel**
System Cost (one time cost)	$435,500[1]	$724,921[2]
SKU Reallocation (recurring cost)	$47,360	$47,360

[1] Adapted from Zollinger (2001) [2] adapted from Bastian Solution's (2012)

Table 4.6. Annual Gross Profit Comparison

		AS/RS		**Carousel**	
Output	**Manual**	**Random**	**Class-based**	**Random**	**Class-based**
Throughput (orders/hour)	40 [2]	58.12 [1]	79 [1]	79 [1]	120.75 [1]
Throughput [3] (orders/year)	81,600	118,564.8	192,514.8	161,160	246,330
Gross profit per year	$489,600	$711,388.8	$1,155,089	$966,960	$1,477,980

[1] output of mathematical models
[2] adapted from literature
[3] based on 8 hours / day and 255 days/year
Gross Profit on the basis of $6 profit per SKU

Table 4.7. Payback Period Summary

System Conversion	**Payback Period in Years**
(a) Manual to AS/RS Random	1.96
(b) Manual to AS/RS Class-based	0.70
(c) Manual to Carousel Random	1.51
(d) Manual to Carousel Class-based	0.77

$$\text{(a) Payback Period} = \frac{435{,}500}{(711{,}388.8 - 489{,}600)} = 1.96 \text{ years}$$

$$\text{(b) Payback Period} = \frac{435{,}500}{(1{,}155{,}089 - 489{,}600 - 47{,}360)} = 0.70 \text{ years}$$

$$\text{(c) Payback Period} = \frac{724{,}921}{(966{,}960 - 489{,}600)} = 1.51 \text{ years}$$

$$\text{(d) Payback Period} = \frac{724{,}921}{(1{,}477{,}980 - 489{,}600 - 47{,}360)} = 0.77 \text{ years}$$

Figure 4.6. Calculations for Payback Periods

Conversions from manual order picking to carousel system with random SKU assignment (random) is about one and half year and conversions from manual order picking to carousel system with class-based is about nine months. The shortest payback period belongs to the conversion from manual to AS/RS with class-based. It can be concluded that the investment in SKU reallocation, which is required for switching to class-based assignment, helps to improve the automated systems throughput, gross profit, and payback periods.

4.5.1. Sensitivity analysis

In this section, the sensitivity analysis will be presented for the effect of gross profit per order to simple payback period. Figure 4.7 shows the sensitivity analysis of gross profit ranging from $4 to $ 12. The ranking of gross profit, from least to most, are manual order picking, AS/RS (random), AS/RS (class-based), carousel (random), carousel (class-based).

Since the time scale for conversion from manual to automated order picking system that is manual order picking to AS/RS (random), manual order picking to AS/RS (class-based), manual order picking to carousel (random), manual order picking to carousel (class-based) is shown in the small scales in Figure 4.8 and it shows the sensitivity analysis of payback period that by only AS/RS will take the longest payback period.

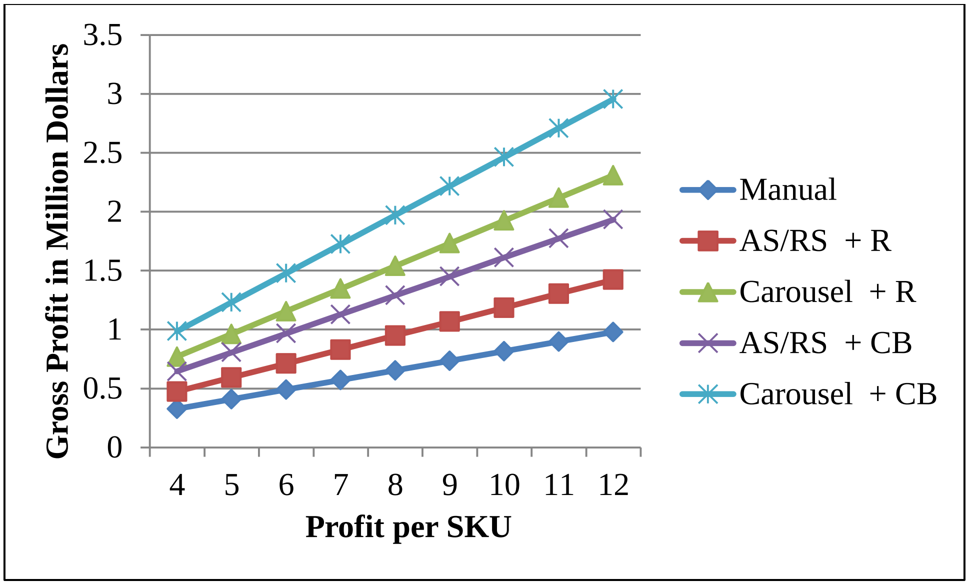


Figure 4.7. Sensitivity Analysis of Gross Profit

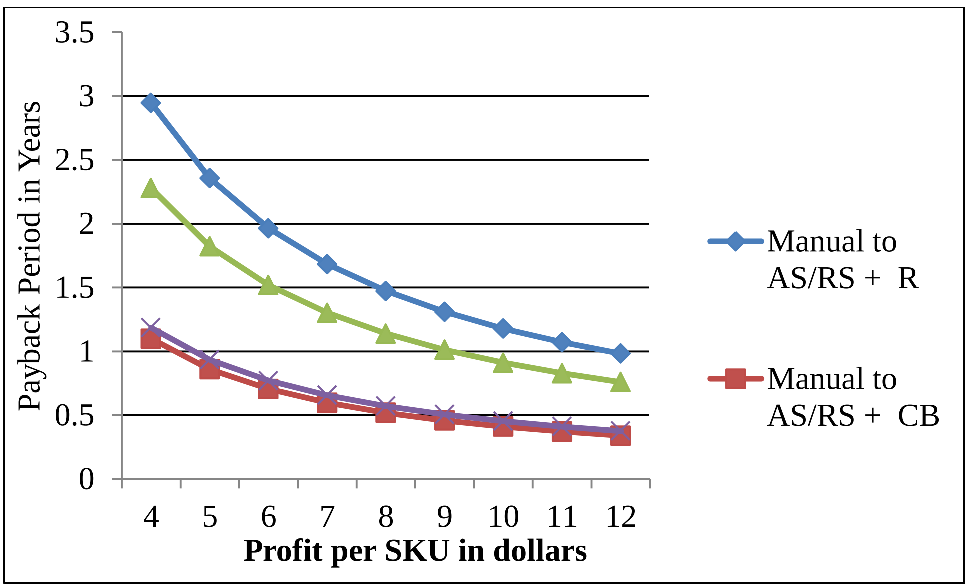


Figure 4.8. Sensitivity Analysis of Payback Period

Furthermore, the change from manual to AS/RS (class-based) has the shortest payback period, which indicates the system and storage policy implementation has better system performance. As profit per SKU increases payback period will decrease for all systems. The more profit earned per order, the less payback period will cost. In both figures AS/RS and carousel with random assignment both are represented by + R and AS/RS and carousel with class-based assignment is represented by + CB.

CHAPTER 5. CONCLUSIONS AND FUTURE RESEARCH

In this conclusion chapter, the main results of this thesis research will be summarized. A section illustrating the major contributions of this thesis research to academic warehouse management research follows the summary of main results. Also, during the progress of obtaining the results, several opportunities for future research will be presented in the last section.

5.1. Main Results of the Thesis Research

Mathematical models were developed that can help with evaluating the performance of AS/RS and carousel automated order picking systems under random and class-based storage assignments.

Simulation models were developed and used to verify the validity of mathematical models. The output of mathematical and simulation models measuring systems' performance were consistent. Mathematical and simulation models showed that compared to other systems under study, carousel system with class-based assignment has the shortest cycle time and the highest throughput. In contrast, AS/RS with random storage assignment had the longest cycle time and the smallest throughput.

Economic analysis was performed to compare investment requirements and payback periods for converting from manual to AS/RS and carousel systems. Based on the profit margin, required SKUs, and the order volume data used for this thesis, the economic analysis showed that converting from manual order picking to AS/RS with class-based assignment has the shortest payback period.

5.2. Contributions of the Thesis Research

This thesis evaluates the performance of two systems, AS/RS and carousel system with two different SKU assignments, random assignment and class-based assignment. From the view of both academia and practice, this thesis study makes the following contributions.

First, the models developed in previous studies were over simplistic and did not include parameters such as operator pick time and retrieval/storage time that impact the systems performance. In this thesis both operator pick time and retrieval/storage time were considered.

Second, this thesis compares the performance of AS/RS and carousel systems. The literature reviewed included measuring performance of AS/RS or carousel, but not comparing the performance of the two systems against each other.

Finally, none of the previous studies included economic analysis of these systems. This thesis included the economic analysis, which compared the investment requirements for converting from a manual to automated order picking system and the payback periods.

5.3. Limitations and Future Research

In this thesis the systems' performance was measured based on the assumption that each order is made of one SKU. Future researches can include studying the impacts of having more than one SKU per order and the requirement that an entire order is processed before processing the next order.

Future research can also include having multiple shuttles and aisles in an AS/RS and multiple carousels as opposed to single shuttle and single carousel, which was the focus of this thesis.

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APPENDIX A. THE SIMULATION RESULTS

Table A.1. Simulation Result for AS/RS with Random Assignment

	Cycle time	**Throughput per hour**	**Throughput per day**
1	67.13287	53.625	429
2	64	56.25	450
3	62.33766	57.75	462
4	64.28571	56	448
5	62.7451	57.375	459
6	63.71681	56.5	452
7	66.82135	53.875	431
8	67.28972	53.5	428
8	63.01969	57.125	457
10	63.57616	56.625	453
11	62.47289	57.625	461
12	65.45455	55	440
13	64	56.25	450
14	62.6087	57.5	460
15	64.28571	56	448
16	65.60364	54.875	439
17	67.28972	53.5	428
18	64.57399	55.75	446
19	67.92453	53	424
20	63.57616	56.625	453
21	63.71681	56.5	452
22	67.60563	53.25	426
23	64.57399	55.75	446
24	70.58824	51	408
25	62.7451	57.375	459
26	66.66667	54	432
27	66.05505	54.5	436
28	63.2967	56.875	455
29	65.15837	55.25	442
30	64.42953	55.875	447
Average	64.91837	55.50417	444.0333

Table A.2. Simulation Result for AS/RS with Class-Based Assignment

	Cycle time (sec)	**Throughput per hour**	**Throughput per day**
1	38.45127	93.625	749
2	37.64706	95.625	765
3	37.74574	95.375	763
4	35.25092	102.125	817
5	40.16736	89.625	717
6	35.12195	102.5	820
7	39.34426	91.5	732
8	38.6059	93.25	746
9	36.04506	99.875	799
10	38.50267	93.5	748
11	39.72414	90.625	725
12	36.36364	99	792
13	38.4	93.75	750
14	38.1457	94.375	755
15	37.84494	95.125	761
16	36.82864	97.75	782
17	37.06564	97.125	777
18	38.29787	94	752
19	38.97158	92.375	739
20	37.99472	94.75	758
21	39.39808	91.375	731
22	40.7932	88.25	706
23	41.14286	87.5	700
24	40	90	720
25	35.64356	101	808
26	37.1134	97	776
27	38.97158	92.375	739
28	38.4	93.75	750
29	35.73201	100.75	806
30	38.34887	93.875	751
Average	38.06875	94.725	757.8

Table A.3. Simulation Result for Carousel with Random Assignment

	Cycle time (sec)	**Throughput per hour**	**Throughput per day**
1	46.08	78.125	625
2	44.44444444	81	648
3	44.65116279	80.625	645
4	43.37349398	83	664
5	42.228739	85.25	682
6	42.04379562	85.625	685
7	47.76119403	75.375	603
8	46.82926829	76.875	615
9	42.66666667	84.375	675
10	44.85981308	80.25	642
11	47.36842105	76	608
12	42.98507463	83.75	670
13	44.30769231	81.25	650
14	45.64183835	78.875	631
15	45.42586751	79.25	634
16	45	80	640
17	46.67747164	77.125	617
18	44.72049689	80.5	644
19	47.92013311	75.125	601
20	45.14106583	79.75	638
21	45.28301887	79.5	636
22	46.82926829	76.875	615
23	47.44645799	75.875	607
24	52.17391304	69	552
25	42.2907489	85.125	681
26	48.73096447	73.875	591
27	45.21193093	79.625	637
28	44.72049689	80.5	644
29	46.4516129	77.5	620
30	45.64183835	78.875	631
Average	45.49689633	79.29583333	634.3667

Table A.4. Simulation Result for Carousel with Class-Based Assignment

	Cycle time (sec)	**Throughput per hour**	**Throughput per day**
1	29.26829268	123	984
2	29.5687885	121.75	974
3	29.20892495	123.25	986
4	26.91588785	133.75	1070
5	30.67092652	117.375	939
6	26.86567164	134	1072
7	30.57324841	117.75	942
8	30.34773446	118.625	949
9	28.34645669	127	1016
10	29.41777324	122.375	979
11	29.8136646	120.75	966
12	28.65671642	125.625	1005
13	30.18867925	119.25	954
14	29.93762994	120.25	962
15	30.37974684	118.5	948
16	28.82882883	124.875	999
17	28.77122877	125.125	1001
18	29.29806714	122.875	983
19	30.06263048	119.75	958
20	28.15249267	127.875	1023
21	30.41182682	118.375	947
22	30.60573858	117.625	941
23	30.44397463	118.25	946
24	29.87551867	120.5	964
25	27.50716332	130.875	1047
26	29.03225806	124	992
27	29.93762994	120.25	962
28	30.25210084	119	952
29	28.0155642	128.5	1028
30	29.75206612	121	968
Average	29.37024103	122.7375	981.9

APPENDIX B. EXTREME DATA VALUES

Table B.1. Extreme Data Values for AS/RS with Random Assignment

Arrival time (minutes)	Start time (minutes)	Process time (minutes)	Ending time (minutes)	Waiting time (minutes)	Order
0	0	1.035	1.035	0	1
1.035	1.035	1.035	2.07	0	2
2.07	2.07	1.035	3.105	0	3
3.105	3.105	1.035	4.14	0	4
4.14	4.14	1.035	5.175	0	5
5.175	5.175	1.035	6.21	0	6
6.21	6.21	1.035	7.245	0	7
7.245	7.245	1.035	8.28	0	8
8.28	8.28	1.035	9.315	0	9
9.315	9.315	1.035	10.35	0	10
10.35	10.35	1.035	11.385	0	11
11.385	11.385	1.035	12.42	0	12
12.42	12.42	1.035	13.455	0	13
13.455	13.455	1.035	14.49	0	14

Table B.2. Extreme Data Values for AS/RS with Class-Based Assignment

Arrival time (minutes)	Start time (minutes)	Process time (minutes)	Ending time (minutes)	Waiting time (minutes)	Order
0	0	0.619355	0.619355	0	1
0.6356	0.6356	0.619355	1.254955	0	2
1.2712	1.2712	0.619355	1.890555	0	3
1.9068	1.9068	0.619355	2.526155	0	4
2.5424	2.5424	0.619355	3.161755	0	5
3.178	3.178	0.619355	3.797355	0	6
3.8136	3.8136	0.619355	4.432955	0	7
4.4492	4.4492	0.619355	5.068555	0	8
5.0848	5.0848	0.619355	5.704155	0	9
5.7204	5.7204	0.619355	6.339755	0	10
6.356	6.356	0.619355	6.975355	0	11
6.9916	6.9916	0.619355	7.610955	0	12
7.6272	7.6272	0.619355	8.246555	0	13
8.2628	8.2628	0.619355	8.882155	0	14
8.8984	8.8984	0.619355	9.517755	0	15
9.534	9.534	0.619355	10.153355	0	16
10.1696	10.1696	0.619355	10.788955	0	17
10.8052	10.8052	0.619355	11.424555	0	18
11.4408	11.4408	0.619355	12.060155	0	19
12.0764	12.0764	0.619355	12.695755	0	20
12.712	12.712	0.619355	13.331355	0	21
13.3476	13.3476	0.619355	13.966955	0	22
13.9832	13.9832	0.619355	14.602555	0	23

Table B.3. Extreme Data Values for Carousel with Random Assignment

Arrival time (minutes)	Start time (minutes)	Process time (minutes)	Ending time (minutes)	Waiting time (minutes)	Order
0	0	0.7953	0.7953	0	1
0.7593	0.7593	0.7953	1.5546	0	2
1.5186	1.5186	0.7953	2.3139	0	3
2.2779	2.2779	0.7953	3.0732	0	4
3.0372	3.0372	0.7953	3.8325	0	5
3.7965	3.7965	0.7953	4.5918	0	6
4.5558	4.5558	0.7953	5.3511	0	7
5.3151	5.3151	0.7953	6.1104	0	8
6.0744	6.0744	0.7953	6.8697	0	9
6.8337	6.8337	0.7953	7.629	0	10
7.593	7.593	0.7953	8.3883	0	11
8.3523	8.3523	0.7953	9.1476	0	12
9.1116	9.1116	0.7953	9.9069	0	13
9.8709	9.8709	0.7953	10.6662	0	14
10.6302	10.6302	0.7953	11.4255	0	15
11.3895	11.3895	0.7953	12.1848	0	16
12.1488	12.1488	0.7953	12.9441	0	17
12.9081	12.9081	0.7953	13.7034	0	18
13.6674	13.6674	0.7953	14.4627	0	19

Table B.4. Extreme Data Values for Carousel with Class-Based Assignment

Arrival time (minutes)	Start time (minutes)	Process time (minutes)	Ending time (minutes)	Waiting time (minutes)	Order
0	0	0.49602162	0.49602162	0	1
0.49069	0.49069	0.49602162	0.98671162	0	2
0.98138	0.98138	0.49602162	1.47740162	0	3
1.47207	1.47207	0.49602162	1.96809162	0	4
1.96276	1.96276	0.49602162	2.45878162	0	5
2.45345	2.45345	0.49602162	2.94947162	0	6
2.94414	2.94414	0.49602162	3.44016162	0	7
3.43483	3.43483	0.49602162	3.93085162	0	8
3.92552	3.92552	0.49602162	4.42154162	0	9
4.41621	4.41621	0.49602162	4.91223162	0	10
4.9069	4.9069	0.49602162	5.40292162	0	11
5.39759	5.39759	0.49602162	5.89361162	0	12
5.88828	5.88828	0.49602162	6.38430162	0	13
6.37897	6.37897	0.49602162	6.87499162	0	14
6.86966	6.86966	0.49602162	7.36568162	0	15
7.36035	7.36035	0.49602162	7.85637162	0	16
7.85104	7.85104	0.49602162	8.34706162	0	17
8.34173	8.34173	0.49602162	8.83775162	0	18
8.83242	8.83242	0.49602162	9.32844162	0	19
9.32311	9.32311	0.49602162	9.81913162	0	20
9.8138	9.8138	0.49602162	10.3098216	0	21
10.30449	10.30449	0.49602162	10.8005116	0	22
10.79518	10.79518	0.49602162	11.2912016	0	23
11.28587	11.28587	0.49602162	11.7818916	0	24
11.77656	11.77656	0.49602162	12.2725816	0	25
12.26725	12.26725	0.49602162	12.7632716	0	26
12.75794	12.75794	0.49602162	13.2539616	0	27
13.24863	13.24863	0.49602162	13.7446516	0	28
13.73932	13.73932	0.49602162	14.2353416	0	29
14.23001	14.23001	0.49602162	14.7260316	0	30