MULTI-OBJECTIVE OPTIMAL PHASOR MEASUREMENT UNITS PLACEMENT IN POWER SYSTEMS

A Dissertation
Submitted to the Graduate Faculty
of the
North Dakota State University
of Agriculture and Applied Science

By

Vahidhossein Khiabani

In Partial Fulfillment of the Requirements
for the Degree of
DOCTOR OF PHILOSOPHY

Major Department:
Industrial and Manufacturing Engineering

November 2013

Fargo, North Dakota

North Dakota State University
Graduate School

Title

Multi-objective Optimal Phasor Measurement Units Placement in Power Systems

By

Vahidhossein Khiabani

The Supervisory Committee certifies that this ***disquisition*** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

DOCTOR OF PHILOSOPHY

SUPERVISORY COMMITTEE:

Dr. Kambiz Farahmand
Chair

Dr. Kendall E. Nygard

Dr. Jing Shi

Dr. Jun Zhang

Approved:

January 10, 2014 — Date

Canan Bilen-Green — Department Chair

ABSTRACT

The extensive development of power networks has increased the requirements for robust, reliable and secure monitoring and control techniques based on the concept of Wide Area Measurement System (WAMS). Phasor Measurement Units (PMUs) are key elements in WAMS based operations of power systems. Most existing algorithms consider the problem of optimal PMU placement where the main objective is to ensure observability. They consider cost and observability of buses ignoring the reliability aspect of both WAMS and PMUs. Given the twin and conflicting objectives of cost and reliability, this dissertation aims to model and solve a multi-objective optimization formulation that maintains full system observability with minimum cost while exceeding a pre-specified level of reliability of observability. No unique solution exists for these conflicting objectives, hence the model finds the best tradeoffs. Given that the reliability-based PMU placement model is Non-deterministic Polynomial time hard (NP-hard), the mathematical model can only address small problems. This research accomplishes the following: (a) modeling and solving the multi-objective PMU placement model for IEEE standard test systems and its observability, and (b) developing heuristic algorithms to increase the scalability of the model and solve large problems. In short, early consideration of the reliability of observability in the PMU placement problem provides a balanced approach which increases the reliability of the power system overall and reduces the cost of reliability. The findings are helpful to show and understand the effectiveness of the proposed models. However the increased cost associated with the increased reliability would be negligible when considering cost of blackouts to commerce, industry, and society as a whole.

ACKNOWLEDGEMENTS

I would like to express the deepest appreciation to my advisor, Professor Kambiz Farahmand, for his support and advice during my PhD study. I am grateful to him for his valuable guidance and encouragement.

I would like to thank Professor Kendall Nygard for his support, guidance and encouragement during my research. I also would like to thank Dr. Jin Shi, and Dr. Jun Zhang, for their comments and careful evaluation of the dissertation. I am grateful to all of them for having patiently gone through the dissertation and the constructive suggestions which greatly improved its presentation.

I would like to place on record, my sincere and heartfelt thanks to Professor Alagar Rangan who always provided me with encouragement and support in several ways. I am deeply indebted to him who always stood by me in times of need.

I am what I am today because of my father and words fail me in expressing my love and gratitude to him. I express my gratefulness to my lovely wife for her never-ending love, support, understanding, and patience. I owe them everything in life and it is natural that I dedicate this dissertation to them.

Finally, I would like to thank all my instructors, graduate students, friends and office staff of my department, who played a constructive role in my academics. They also made my life as a member of the NDSU family, a very interesting and memorable experience.

DEDICATION

To my Father & Wife

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABBREVIATIONS

BUS………...Binary Unit System

GA…………Genetic Algorithm

GPB………...Goal Programming Based

GPS…………Global Positioning System

IEEE………...Institute of Electrical and Electronics Engineers

NP-hard……..Non-deterministic Polynomial-time hard

OPP…………Optimal PMU Placement

PDC…………Phasor Data Concentrator

PMUs……….Phasor Measurement Units

RBP………...Reliability-Based Placement

SCADA…….Supervisory Control and Data Acquisition

WAMS……..Wide Area Measurement System

NOMENCLATURE

$A_{i,j}$…………...binary connection matrix of the system

b…………….redundancy level for each bus

c ……………total number of PMUs resulting from minimization problem

C …………...number of PMUs available

f_i ………......total number PMUs of covering i^{th} bus

I_{ij} …………...current phasor of line i - j

$i(t)$ …………sinusoidal current function

I_{max} …………amplitude of current

n …………… number of the buses in the network

$N(i)$ …………set of PMUs placed on buses j that are adjacent to i^{th} bus.

P [B] ………..probability of failure of all PMUs observing i^{th} bus

p_j…………….reliability of each PMU

q_j ……………probability of failure of j^{th} PMU

q_j…………….probability of failure of j^{th} PMU and

R ……………system wide level of reliability of observability for whole network

R_{min} …………minimum reliability of observability requirement for whole network

r……………..minimum reliability of observability requirement of each bus required to meet system wide reliability level

r_i…………….reliability of observability level for i^{th} bus

θ_I …………..phase of current

θ_V …………...phase of voltage

$\sum A_{i,j} x_j$ ………total number PMUs of covering i^{th} bus

$R_i(t)$ …………reliability of the i^{th} item

V_j …………….voltage phasor of i^{th} bus

$v(t)$ …………...sinusoidal voltage function

V_{max} ……….amplitude of voltage

w ………..…angular frequency of voltage and current

X_k …………...kth sample of the input signal taken over one period

X …………….the fundamental frequency component of the Discrete Fourier transform

x_i …………….the binary decision variable vector, which will acquire value one if a PMU is installed on the i^{th} bus and zero otherwise

Z_{ij} ……………impedance of line $i - j$

CHAPTER 1. INTRODUCTION

Recent research in electric power systems is focusing on novel monitoring and control techniques based on the concept of Wide Area Measurement System (WAMS) (Phadke and de Moraes, 2008 and Liu et al., 2009). The WAMS based applications have significant potential in improving power system security, operation, control, and modeling. Phasor Measurement Units (PMUs) which provide time synchronized measurements of voltage and current phasors are key elements of WAMS (Phadke, 1993). The synchronization in PMUs is achieved via signals available from the Global Positioning System (GPS) (Phadke, 2008). This ability of a PMU to calculate synchronized phasors will improve the performance of state estimators. This feature makes PMUs one of the most important measurement devices in power system protection and control (Novosel et al., 2008 and Dua et al., 2008).

Phasor Data Concentrators (PDCs) gather the measured data by PMUs and time stamps the data before sending it to the monitoring and control center in WAMS. This data is arranged chronologically using the time stamp. A bus is an electrical conductor, which serves as a conducting pathway for continuous connection of the loads and the sources of electric power between different parts of a power network. Transmission between buses is made through lines in the power network. A bus is called observable when the voltage phasor at that bus is estimated, and the power system is called observable if the measurement sets and their distributions are sufficient for solving the current state. By placing a PMU[1] at a bus in a power system, one obtains: (a) the voltage phasor (magnitude and phase) at that bus; and (b) current phasors in all branches (i.e. lines) that are incident on that bus.

---

[1] This dissertation only considers multi-channel PMUs.

Power system is called observable if the measurement sets and their distribution are sufficient for solving the current state of the power system. Availability of the voltage phasor of a bus and the entire incident line currents, the voltage phasor at adjacent buses can be calculated using the Ohm's law:

$$V_j = V_i - Z_{ij} I_{ij} \tag{1}$$

where Z_{ij} is the impedance of line i - j. Therefore, a PMU placed at a bus makes that bus and all buses adjacent to it observable (Dongjie et al., 2004 and Denegri et al., 2002). Hence providing a precise model for the power systems, the addition of the PMU to the strategic buses makes those buses and all of their neighboring buses observable (Dongjie et al., 2004 and Denegri et al., 2002). This implies that a network can be made observable with a lesser number of PMUs than the number of buses. As a result, the objective of the PMU placement is to obtain system wide observability by using the minimum number of PMUs. The use of PMUs at each bus will lead to a simplified linear state estimator (Phadke and de Moraes, 2008). Several algorithms have thus been proposed for optimal placement of PMUs to ensure observability. A graph theoretic procedure to find a minimal (not necessarily the minimum) PMU placement was reported in (Baldwin et al., 1993). An integer linear programming (ILP) approach to solve this problem was proposed by Xu and Abur (2004), and subsequently extended by Gou (2008) to address the cases of redundancy, partial observability and pre-existing conventional measurements. An exhaustive binary search algorithm for PMU placement is presented in Chakrabarti and Kyriakides (2008) . The optimal PMU placement problem is shown to be Non-deterministic Polynomial time hard (NP-complete) (Brueni and Heath, 2005). A systematic ILP approach for phasing of PMU placement considering failure of a single PMU and modeling zero-injection buses was developed by Dua et al.(2008). Kavasseri and Srinivasan (2010) considered reducing the number of PMUs needed for system observability through judicious placement of the power-flow measurements.

Given a placement that is optimal with respect to cost, it is both of interest and importance to compute how reliable the arrangement is. It is intuitively clear that protecting against loss of observability under failures, such as transmission line faults, bus faults, outages, or metering failures, will require a level of redundancy with additional PMUs. In the foreseeable future, the power system operations will be increasingly depend on PMUs. It is thus natural to consider and assess the reliability of a group of networked PMUs when it is a part of WAMS, especially when several critical functions are entrusted to PMUs.

The reliability of WAMS can be determined through the identification of its components such as PMUs, communication systems, and the central computation unit then by estimating reliability of all of these components as a group. The overall WAMS reliability can be increased by using a backup of its subsystems and/or components (Marek et al., 2010). WAMS structure normally consists of PMUs installed in different locations around the power grid connected to PDC through a regional network. These groups of PMUs and PDC are connected to the Monitoring Center of WAMS through a Synchronous Digital Hierarchy (SDH) wide area communication network (Liu et al., 2009). Figure 1.1 shows the structure of WAMS.

Li (2005) established a repairable reliability model of the power system based on a series parallel structure model and the Markov state-space method. Liu et al. (2009) proposed reliability modeling of WAMS using the Markov process.

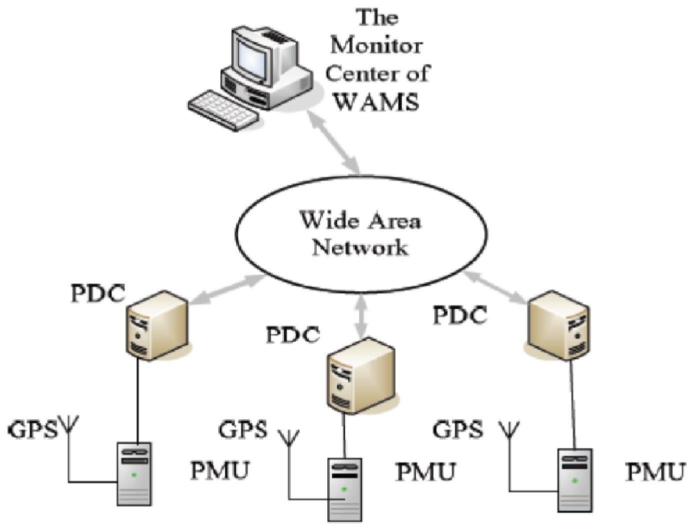

Figure 1.1. Structure of WAMS (Liu et al., 2009).

A reliability estimation model for a single PMU was proposed in a paper by Yang et al. (2009). The model develops a series-parallel structure for a single PMU viewed as a collection of seven subcomponents and identifies the most critical component within the structure of a PMU.

However, the literature on analyzing the reliability of a network of PMUs placed to achieve a certain objective, say observability, is scant. The existing PMU placement models consider cost and observability of buses ignoring the reliability aspect of both WAMS and PMUs. For example, optimal placements of PMUs that are robust against line (branch) or bus (node) failures have been proposed (Aminifar et al., 2010). However, such placement solutions call for additional or redundant PMUs to cover any bus or line failure. The existing PMU placement models require redundancy factor as an input to solve the placement problem. However, the redundancy factor is provided by the experts based on purely subjective assessment and, hence, there exists no means to assess or estimate the redundancy factor objectively. With traditional placement models, it is difficult to quantify the additional benefits from a reliability standpoint. On the other hand, reliability-based models essentially focus on achieving a desired reliability of any given system while satisfying other requirements such as cost and observability. Reliability-based models have been used widely in system design optimization to improve system functionality and reliability. However, reliability models have not been exploited in existing optimal PMU placement based algorithms.

This motivates the consideration of the optimal placement problem from a reliability standpoint. In the proposed multi-objective models, the reliability of an individual PMU and the desired system level reliability are factored as inputs into the model. The redundancy factors (which call for additional PMUs) are thus dictated by these inputs subject to the system topology. The solution thus achieves complete observability while meeting or exceeding a specified level of reliability. In fact the reliability-based PMU placement model is NP-hard, therefore the mathematical model cannot address large scale problems by exact solution approaches. Hence to increase the scalability of the model, we developed a genetic algorithm (GA) approach based on binary encoding for multi-objective placement of PMUs in power network.

1.1. Reliability-Based PMU Placement

The placement of a PMU at any given bus allows direct measurement of the voltage phasor at that bus and computation of the voltage phasors at neighboring buses. Further, the reliability of power system observability depends on the reliability of PMUs covering each bus. A bus is said to be observable if the voltage phasor at that bus is known. Therefore, the power system will be fully observable if all buses are covered with one or more PMUs. On the other hand, if none of the PMUs were redundant, the failure of any one PMU would result in the complete loss of system observability, which could result in system failure. This understanding allows us to consider that from the reliability point of view, buses are connected in series structure. Further, in case of redundancy, each bus is covered by more than one PMU and these redundant PMUs will be treated as parallel connected. Thus a parallel structure model can be used to estimate the reliability of bus observability. Hence we defined the reliability of observability of the system as (Khiabani et al., 2012):

$$R = \prod_{i=1}^{n} \left(1 - q_j^{\sum_{j=1}^{n} A_{i,j} x_j}\right) \tag{2}$$

where q_j represents the probability of failure of j^{th} PMU, x_i is the binary decision variable vector, which will acquire value one if a PMU is installed on the i^{th} bus and zero otherwise, $A_{i,j}$ is the binary connection matrix of the system which can be directly obtained by transforming the bus admittance matrix's entries into binary form and $\sum A_{i,j}\, x_j$ is the total number of PMUs covering i^{th} bus. We formulated the optimal PMU placement problem as a two- stage optimization model from a reliability standpoint where redundancy levels for all buses in the system are the same (Khiabani et al., 2012a). However, maintaining separate objectives of minimizing the cost and maximizing the reliability results in infeasible solutions in some cases. This is because the formulation in (Khiabani et al., 2012a) overlooks combinations which could result in better solutions. Given the twin and conflicting objectives of cost and reliability, we presented a multi-objective optimization formulation that maintains full system observability with minimum cost while exceeding a pre-specified level of reliability in (Khiabani et al., 2012b). This is achieved in the formulation by relaxing the assumption of identical redundancy levels (bus reliabilities(r)) at all buses in the system. The resulting formulation clearly dictates the placement of additional PMUs to achieve a specified level of overall reliability. Later we developed a goal programming multi-objective optimization formulation consisting of two goals (Khiabani et al., 2013a). The first goal is to maintain full system observability while aiming for a pre-specified level of reliability. The second goal is to minimize cost by placing less PMUs. Also zero-injection buses were incorporated into the model.

The previously discussed multi-objective optimization (Khiabani et al., 2012b) and goal programming multi-objective optimization (Khiabani et al., 2013a) models, consider minimizing the number of PMUs to reach full system observability maintaining a pre-specified level of reliability both relaxing the existence of a limited number of PMUs. However in practice the resources could be limited because of the high price of purchasing and installing the PMUs. In this case, the decision maker will decide to allocate the limited recourses either to the strategic locations or to cover the maximum possible buses. Therefore we considered the PMU placement problem from a maximum covering standpoint (Khiabani et al., 2013b). In the proposed model, the number of existing PMUs is factored as inputs into the model. The maximum coverage thus dictated by this input is subject to the system topology.

1.2. Increasing the Scalability of the Model

The reliability-based PMU placement and multi-objective models are able to solve small size problems such as IEEE 14, 30 and 57. However, larger problems could be tackled but not in a timely manner. As the problem size increases, the complexity of the system increases exponentially rendering the problem mathematically unsolvable. As noted earlier, the PMU placement model is NP-hard and cannot be solved using exact algorithm for large size problems. Further the addition of the second objective, which is maximization of reliability of observability, makes the NP-complete optimal PMU problem even more complex and renders it unsolvable for large scale problems using exact solution approaches. Therefore, a genetic algorithm approach based on binary encoding for multi-objective optimal PMU placement problem consisting of two main goals to tackle large scale problems (Khiabani et al., 2013c) was developed.

CHAPTER 2. LITERATURE REVIEW

2.1. Background

The sources of electrical power are usually connected by a network or a transmission system that distributes the power to the various load centers. A power network usually consists of generators, transformers, loads, circuit breakers, and buses. Buses or nodes are the points of connection in the power network (Bergen and Vittal, 2000). In other words, a bus is an electrical conductor that serves as a conducting pathway for continuous connection of the loads and the sources of electric power between different parts of a power network. There are two basic types of buses. PQ buses are nodes that have both constant real and reactive injections that represent load buses without voltage control, and PU buses are connected to a generator represent generation buses with voltage control, which have constant voltage value. Transmission between buses is made through lines or branches in the power network. Each branch has two terminal buses which may be shared by one or more other branches in the network.

In order to estimate the system state, power system state estimator uses a set of available measurements such as voltage phasor, magnitude and phase, and current phasor. Power system is called observable if the measurement sets and their distribution are sufficient for solving the current state of the power system. Given a set of measurements and their locations, the power network observability analysis will determine if a unique estimate can be found for the system state (Abur and Exposito, 2004). A bus is said to be observable if the voltage phasor at that bus is known and the power system is said to be observable when all the buses are observable.

In steady state most power system voltages and currents are (at least approximately) sinusoidal of time with the same frequency. Phasor is complex number that contains the amplitude and phase angle in formation of a sinusoidal function. Its concept can be developed relating the exponential function to the trigonometric functions using Euler's identity.

$$e^{\pm j\theta} = Cos\,\theta \pm j\,Sin\,\theta \tag{3}$$

Assuming that the Voltage and Current both are sinusoidal waveform represented by a unique complex number known as phasor with angular frequency of *w*. Considering a sinusoidal signal for Voltage and Current we have:

$$v(t) = V_{\max}\,Cos(wt + \theta_V) \tag{4}$$

$$i(t) = I_{\max}\,Cos(wt + \theta_I) \tag{5}$$

where V_{max} and I_{max} are real numbers called the amplitude, and θ_V and θ_I are called the phase of voltage and current, respectively. Now using (3) the phasor representation is as follows:

$$v(t) = \frac{V_{\max}}{\sqrt{2}} e^{j\theta} = \frac{V_{\max}}{\sqrt{2}}\,(Cos\,\theta + j\,Sin\,\theta) \tag{6}$$

$$i(t) = \frac{I_{\max}}{\sqrt{2}} e^{j\theta} = \frac{I_{\max}}{\sqrt{2}}\,(Cos\,\theta + j\,Sin\,\theta) \tag{7}$$

In power system the PMU is a device capable of measuring the synchronized voltage and current phasor. Synchronization is obtained by same time sampling of all measurements using common reference signal provided by a GPS. PMU separates the fundamental frequency component and calculates its phasor representation applying the Discret Fourier Transform. If X_k's are the *N* samples of the input signal taken over one period, then the phasor representation is given by (Phadke and Thorp, 2008):

$$X = \frac{\sqrt{2}}{N}\sum_{k=1}^{N} X_k e^{\frac{-jk2\pi}{N}} \qquad (8)$$

where X is the fundamental frequency component of the Discrete Fourier Transform.

Phasor calculations demand accuracy of more than one millisecond. GPS is capable of providing a one microsecond second signal at any location around the world. Figure 2.1 depicts the block diagram of the PMU . The anti-aliasing filter, produces a phase delay from the input waveform frequency depending upon the filter characteristics. The phase locked oscillator converts the one pulse per second signal provided by a GPS receiver into a sequence high speed timing pulses used in the waveform sampling. The phasor microprocessor executes the Discrete Fourier Transform phasor calculations. For the final stepthe phasor is time stamped and uploaded to a collection device known as a data concentrator (IEEE Working Group H-8, 1998).

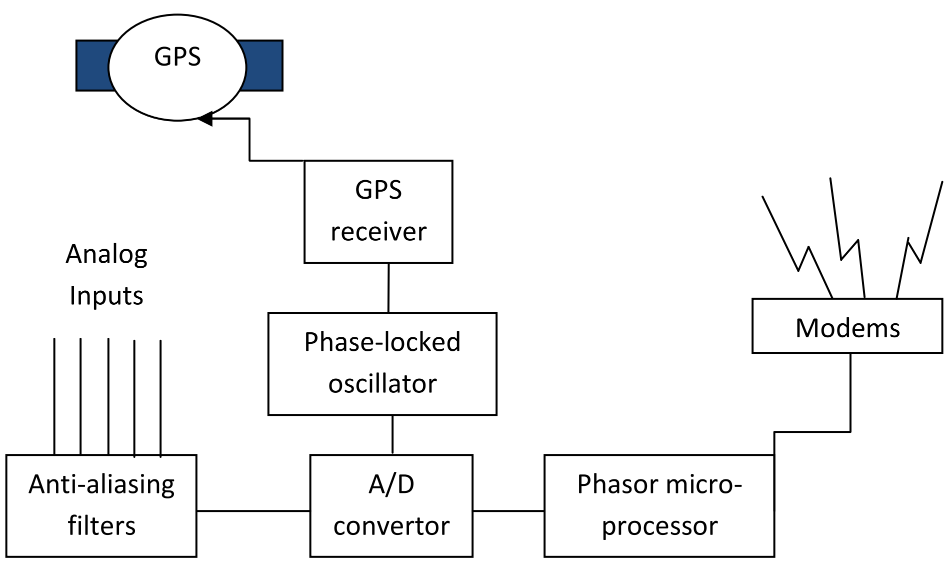

Figure 2.1. PMU block diagram.

The injected currents at the nodes of an interconnected network are related to the voltage at the nodes via admittance representation which finds wide spread application in determining the network solution and forms an integral part of most modern-day power analysis. The network may then be solved to find the node voltages. Bus admittance representation is obtained in terms of primitive representation, which characterizes the electrical behavior of the various network components (Bergen and Vittal, 2000). The binary connection matrix A of system can be directly obtained by transforming the bus admittance matrix's entries into binary form defined by:

$$A_{i,j} = \begin{cases} 1 & \textit{if either } i = j \textit{ or } i \textit{ is adjacent to } j \\ 0 & \textit{otherwise} \end{cases} \tag{9}$$

Building the A matrix for IEEE 14 bus system yields:

$$A = \begin{bmatrix} 11001000000000 \\ 11111000000000 \\ 01110000000000 \\ 01111010100000 \\ 11011000000000 \\ 00000100001110 \\ 00010011100000 \\ 00000011000000 \\ 00010010110001 \\ 00000000111000 \\ 00000100011000 \\ 00000100000110 \\ 00000100000111 \\ 00000000100011 \end{bmatrix} \tag{10}$$

System reliability is the probability that a system will perform its intended function for a given period of time under pre-specified operating conditions. The series system, parallel system, and K-out-of-N system are most used basic system configurations. In a series system configuration, the failure of any one item results in the failure of the system. In other words, for the functional success of a series system, all of its items (blocks) must successfully function during the intended mission time of the system. Figure 2.2 depicts the reliability block diagram of a series system consisting of N blocks. The reliability of the series system with N items is the probability that all N units succeed during its intended mission time t. Therefore, for the set of N independent units the system reliability $R_s(t)$ is calculated as follows:

$$R_s(t) = \prod_{i=1}^{N} R_i(t) \tag{11}$$

where $R_i(t)$ represent the reliability of the i^{th} item.

Figure 2.2. Block diagram of the series system reliability.

A reliability block diagram is a parallel configuration in which the failure of all units results in a system failure. Therefore success of only one unit would be sufficient to guarantee the success of the system. Figure 2.3 depicts the reliability block diagram of a parallel system consisting of N blocks. For the set of N independent units the system reliability $R_s(t)$ is calculated as follows:

$$R_s(t) = 1 - \prod_{i=1}^{N} [1 - R_i(t)] \tag{12}$$

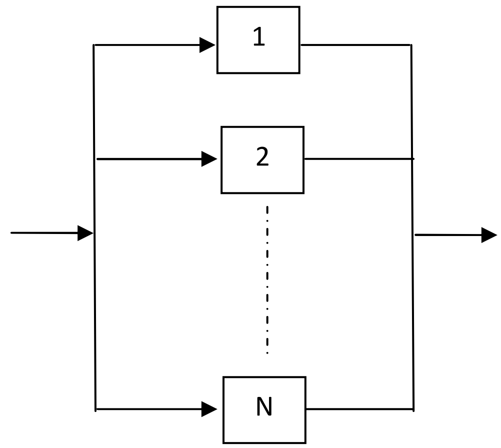

Figure 2.3. Block diagram of the parallel system reliability.

K-out-of-N system is a more general structure of series and parallel systems, in which if any combination of K units out of N independent units works, it guarantees the success of the system. By assuming that all units are identical, the binomial distribution can easily represent the probability that the system functions (Modarres et al., 2010):

$$R_s(t) = \sum_{r=k}^{N} \binom{N}{r} [R(t)]^r [1 - R(t)]^{N-r} \tag{13}$$

2.2. Literature Review

The PMU placement models in the literature mostly are related to cost minimization maintaining partial or full system observability. The synchronization in PMUs is obtained via signals available from the GPS (Phadke and Thorp, 2008). This characteristic makes PMU an important aid for several applications such as state estimation, protection, and control of power systems in future (Novosel et al., 2008 and Dua et al., 2008). Kamwa and Grondin (2002) proposed two numerical algorithms to minimize the overall sensor response of the signals captured via PMUs while minimizing the correlation among sensor outputs and to minimize the redundant information provided by multiple sensors. A PMU placed on a bus yields the voltage phasor at that bus and current phasors of all branches that are incident on the bus. Therefore, the presence of a PMU on a bus makes that particular bus and all of its immediate neighboring buses observable (Dongjie et al., 2004 and Denegri et al., 2002). Baldwin et al. (1993) reported a graph theoretic procedure to find a minimal (not necessarily the minimum) PMU placement based on topological observability theory. They used a modified bisecting search and simulated annealing based method to find the measurement set. In Marin et al. (2003) a genetic algorithm was developed to solve the optimal PMUs placement problem maintaining the network observability. Milosevic and Begovic (2003) proposed a non-dominated sorting genetic algorithm for the PMU placement problem. To reduce the initial number of PMU's candidate locations, they considered the conflicting objectives of minimization of the number of PMUs and maximization of the measurement redundancy by estimating the individual optimal solution for these conflicting objectives using the graph theoretical procedure and a simple genetic algorithm. Then using the non-dominated genetic algorithm they searched for the best tradeoff. An integer linear programming (ILP) approach to solve the minimal PMU placement problem based on network observability was proposed in Xu and Abur (2004). To further reduce the number of PMUs, this approach was extendable to include the pre-existing conventional measurements in the system. Gou (2008) extended the ILP approach to address the cases of redundancy, partial observability, and pre-existing conventional measurements. Also cases with and without zero injection and conventional measurements were considered. Nuqui and Phadke (2005) presented techniques for identifying placement sites for PMUs in power systems based on complete and incomplete observability. They introduced the novel concept of depth of unobservability and explained its impact on the number of PMU placements. Initially, they make use of spanning trees of the power system graph and a tree search technique to find the optimal locations of PMUs. Then extended the modeling to recognize limitations in the availability of communication facilities around the network and pose the constrained placement problem within the framework of Simulated Annealing. Brueni and Heath (2005) show that the optimal PMU placement problem to be NP-complete. They introduced a new simpler definition of graph observability and several complexity results for PMU placement problem proving that minimum PMU placement requires no more than 1/3 of the nodes in a connected graph of at least 3 nodes. Peng et al. (2006) presented an optimal PMU placement method for full network observability using Tabu search algorithm based on the linearized power system state estimator model and using augmented incidence matrix. A transmission network fault location observability with minimal PMU placement was presented in Lien et al. (2006). The scheme combines the fault-location algorithm and the fault-side selector. Chawasak et al. (2007) proposed a new method for an optimal PMU placement problem considering both single measurement loss and single-branch outage in order to obtain a reliable measurement system. An exhaustive binary search algorithm for PMU placement is presented in Chakrabarti and Kyriakides (2008). They considered single branch outages and proposed a strategy to select the solution, in case of more than one solution, resulting in the most preferred pattern of measurement redundancy. Dua et al. (2008) developed a systematic ILP approach for phasing of PMU placement considering failure of single PMU and modeling zero-injection buses. They showed that zero-injection constraints can also be modeled as linear constraints in an ILP framework. Nabil and Hanafy (2009) proposed a unified approach for the optimal PMU locations integrating the impact of both existing conventional flow measurements and the possibility of single or multiple PMU loss into the decision strategy of the optimal PMU allocation. Chakrabarti et al. (2010) investigated three different methods of inclusion of PMU current phasor measurements in a power system state estimator. They presented a comprehensive formulation of a hybrid state estimator incorporating conventional flow and PMU measurements. Aminifar et al. (2009) investigated the application of immunity genetic algorithm for the optimal PMU placement problem. Sodhi et al. (2011) proposed a multi-criteria decision-making scheme for placing PMUs in multiple stages over a given time period that ensures complete power system observability even under a PMU or a branch outage. Wang et al. (2012) developed an improved particle swarm optimization approach for optimal PMU placement problem to avoid long-run time and trapping local optimal. The point of genetic algorithm and simulated annealing process is involved in basic particle swarm optimization to develop the improved particle swarm optimization model.

Aminifar et al. (2010) presented contingency-constrained model such as measurement losses and line outages for the optimal placement of PMUs in electric power networks. In their proposed approach, optimal placement of PMUs are robust against line (branch) or bus (node) failures. Peng et al. (2010) developed a multi-objective optimal model of PMU placement using a non-dominated sorting differential evolution algorithm. Hurtgen and Maun (2010) used iterated local search metaheuristic for optimal PMU placement problem. Pai et al. (2010) presented algorithms to solve the restricted type of power domination on grids and provide approximation algorithms to deal with the fault-tolerant measurement placement when the number of faulty PMUs does not exceed three. Emami and Abur (2010) extended the PMU placement problem to those PMUs which are designed to monitor a single branch by measuring the voltage and current phasors at one end of the monitored branch, and also addressed the reliability of the resulting measurement design by considering PMUs, line, and transformer outages. Kavasseri and Srinivasan (2010) considered reducing the number of PMUs needed for system observability through judicious placement of the power-flow measurements. Vanfretti et al. (2011) presented a new approach for PMU based state estimation incorporating phase bias correction. In Korres (2011) an efficient integer-arithmetic algorithm for observability analysis of systems with both conventional measurements and PMUs is presented. Jamuna and Swarup (2011) presented an optimal placement of PMU and supervisory control and data acquisition (SCADA) measurements for security constrained state estimation using integer programming and a genetic algorithm approach. Hajian et al. (2011) developed a modified binary particle swarm optimization algorithm for optimal PMU placement problem maintaining network observability. They started with an optimal placement set to achieve full network observability during no shortages then modified it in order to consider contingency conditions such as PMU loss or a single transmission line outage maintaining network observability. Ahmadi et al. (2011) studied a binary swarm optimization based methodology for the optimal PMU placement problem considering measurement redundancy. They used a topology-based algorithm in order to ensure full network observability. The results were compared with some newly reported methods and showed that the whole system can be observable with PMU installation on less than 25% of the system buses. Sodhi et al. (2010) presented a two-stage optimal PMU placement method for complete topological and numerical observability of power system. The first stage seeks for the minimum number of the PMUs needed to make the power system topologically observable, and the second stage checks if the solution resulted from first stage leads to full ranked measurement Jacobean. Jiang et al. (2012) proposed a two-stage fault-location optimization model along with defining a matching degree index for large transmission networks which use PMUs. Cepeda et al. (2012) presented a probabilistic approach to addresses the problem of PMU placement with the aim of achieving high observability of system dynamics that are associated to transient and other short-term phenomena, in order to perform reliable real-time dynamic vulnerability assessment.Koutsoukis et al. (2013) introduced a recursive Tabu search approach for optimal

PMU placement problem. Unlike most existing metaheuristic optimal PMU placement approaches, which are based on topological observability methods, they proposed a numerical approach to check the network observability. Shahraeini et al. (2012) formulated a genetic algorithm approach for co-optimal simultaneous meters placement and their required communication infrastructure for state estimation in WAMS.

Aminifar et al. (2011) presented a probabilistic multistage PMU placement in electric power systems using mixed-integer programming approach. The problem constraint was a predefined probability of observability associated with each bus. Kavasseri and Sirnivasan (2011) considered the problem of joint optimal PMU and conventional power flow measurements for fault observability of power systems. The placement results require fewer PMUs for fault observability compared to systems with fixed locations of conventional power flow measurements. Mahaei and Tarafdar Hagh (2012) presented a new method for minimizing the number of PMUs for the optimal PMU placement problem in power systems considering existing conventional measurements. The method provides suitable constraint for power systems with two adjacent injection measurements and constraints for considering the connection of two buses to each other and to an injection bus. This results in a reduction in the required number of PMUs maintaining the full system observability. Mahaei and Tarafdar Hagh (2012) compared the results to recently published papers, and they found that number of PMUs is equal or even decreased and the measurement redundancy increased. Uddin et al. (2013) formulated the PMU placement problem as an integer programming problem using a linear minimum mean squared error estimator as the state estimator. They looked for suboptimal algorithms and bounds on the optimal performance because of the prohibitively complexness of the placement of PMU’s in a large network. Anderson and Chakrabortty (2012) developed a graph-theoretic based minimum cover algorithm for PMU placement problem for multi-area power system networks with the objective of identifying a dynamic equivalent model for the system. Wang et al. (2012) presented an optimal incremental PMU placement framework based on dynamic programming. Miljanic et al. (2013) developed the cellular genetic algorithm with an evolutional rule for PMU placement problem considering PMUs with different number of employed channels and communication constraints that may influence the optimal placement strategy. They take into account the robustness of a metering scheme and evaluated the ability of a metering scheme to maintain full observability even in case of contingencies such as single measurement or branch outage. An approach using topological charasteristics of the network was employed for reducing computational burden of the observability testing and for narrowing solution search space. Mosavi et al. (2012) presented an Ant Colony Optimization approach for optimal PMU placement problem using Global Positioning System. Huang and Wu (2013) presented a scalable solution for PMU placement problem under long-run data availabilities using Markov chains. Mahari and Seyedi (2013) developed a based on Binary Imperialistic Competition Algorithm method for optimal PMU placement in power systems considering different operating conditions. They considered both observability and maximum redundancy in the model. Venkateswaran and Kala (2012) presented a Differential Evolution algorithm based PMU placement problem considering the single PMU outage cases. Gao (2013) proposed a method incorporating both bus weight and voltage stability in order to further improve the accuracy and efficiency of PMU placement problem. Gomez and Rios (2013) developed a multistage optimal PMUs placement based on graph theory. The available budget, the power system expansion, redundancy in the PMU placement against the failure of a PMU or its communication links, user defined time constraints for PMU allocation, and the zero-injection effect has been taken account into the model. They also considered inter-area observability and intra-area observability criteria for dynamic stability monitoring. Huang and Wu (2012) formulated a fault-tolerant PMU placement method. They employed particle swarm optimization algorithm to minimize the placement of additional PMUs without violating the required control reconfigurability. Abdelaziz et al. (2013) developed an observability assessment based on topological analysis for PMU placement problem for both normal operating conditions and single branch outages. Ketabi et al. (2012) formulated a multi-objective optimization model based on Pareto optimum method for optimal placement of PMUs in state estimation considering uncertainty. Tai et al. (2013) considered both static and dynamic state estimation for optimal PMU placement in power systems to minimize the state estimation error covariance. Miljanić et al. (2012) proposed a cellular genetic algorithm approach for optimal PMU placement considering the availability of PMU measuring channels, and single measurement or branch outages. Jamuna et al. (2012) developed a multi-objective biogeography based optimization algorithm for PMU placement problem with two objectives, minimization of the number of PMUs and maximization of measurement redundancy. Kekatos et al. (2012) developed a convex relaxation approach for optimal PMU placement problem. A Multi-Stage simulated annealing algorithm for the joint placement of PMUs with the existing conventional measurement units in the power system proposed in Gopakumar et al. (2013). Aminifar et al. (2013) proposed an analytic technique for optimal PMU placement problem considering both long-term economic aspects and existing technical issues. Azizi et al. (2012) developed an equivalent integer linear programming method for the exhaustive search-based PMU placement.

Saha et al. (2012) presented a three stage optimal PMU placement method using network connectivity information. Gyllstrom et al. (2012) investigated the performance of a suitable greedy approximation algorithm for PMU placement and proved the NP-Completeness of

FullObserve, MaxObserve, FullObserve-XV, and MaxObserve-XV PMU placement problems. Xu et al. (2013) proposed a simplified version of chemical reaction optimization approach, a metaheuristic technique, for optimal PMU placement problem both with and without considering the zero-injection buses. A PMU placement approach ensuring minimum number of channels associated with the given placement and considering both topological and numerical observability for complete observability of power system proposed in Gupta et al. (2012). A comprehensive literature review on state of the art optimization methods on the optimal PMU placement problem and the solution methodologies are presented in Manousakis et al. (2012). A hybrid discrete particle swarm optimization technique for the solution of optimal placement of PMU in smart grids presented in Alinejad-Beromi et al. (2011).

In the foreseeable future, the power system operations will be increasingly dependent on PMUs. It is thus natural to consider and assess the reliability of a group of networked PMUs when it is a part of a WAMS , especially when several critical functions are entrusted to PMUs. The reliability of WAMS systems can be determined through the identification of its components such as PMUs, communication systems, and the central computation unit and then by estimating reliability of each of these components. The overall WAMS reliability can be increased by using a backup of its subsystems and/or components Marek et al. (2005). Li (2005) established a repairable reliability model of the power system based on a series parallel structure model and the Markov state-space method. Liu et al. (2009) proposed reliability modeling of WAMS using the Markov process. The analysis of data acquisition system takes the PMU and PDC as the study objects, and gets the reliability evaluation of data acquisition system by series system model, the parallel system model and k/n judgment system model of those facility units. However in this model the number of buses would not have an effect on the reliability evaluation. Yang et al. (2009a) proposed a reliability estimation model for a single PMU. The model develops a series-parallel structure for a single PMU viewed as a collection of seven subcomponents and identifies the most critical component within a PMU. Yang et al. (2009b) presented a hierarchical structure of WAMS to satisfy requirements of reliable real-time data transfer. They developed methods of evaluating multiple reliability indices of regional networks in WAMS. A reliability modeling of PMUs using fuzzy sets was proposed and extended to consider options for the PMU hardware in Aminifar et al. (2010). They employed the Markov process to analyze the proposed model and to present an equivalent two-state, up-and-down model of PMUs. Yang et al. (2010) presented a quantified reliability analysis for the backbone communication network in WAMS and the overall WAMS from a hardware reliability viewpoint using combined Markov modeling and state enumeration techniques. Singh et al. (2013) proposed a reliability analysis of PMU incorporating standby redundancy as well as switching failure probability. They formulated a Markov model of each individual module. Ghosh et al. (2013) presented a reliability analysis of geographic information system aided optimal PMU placement taking pragmatic spatial aspects into account for smart grid operation on eastern India. They investigated the impact of topological attributes on commissioning PMU to ensure reliability through different phasor measurement unit connectivity configurations.

CHAPTER 3. MODELING

As mentioned earlier Liu et al. (2009) proposed reliability modeling of WAMS using the Markov process. The analysis of data acquisition system considers the PMU and PDC as the study objectives. While considering these objectives it determines the reliability evaluation of data acquisition system by a model in series structure, or a model in parallel structure along with the k/n judgment model of those facility units. Some of the weaknesses and drawbacks of their model are as follows:

- The number of buses would not have any effect on the reliability evaluation.
- The division of the power grid into some areas is optional and is based on geographical scope only.
- The model is for reliability evaluation after PMUs are placed in the power system.

Therefore, the literature on analyzing the reliability of a network of PMUs which are placed to achieve a certain objective, such as observability, is scant. The existing PMU placement models consider cost and observability of buses ignoring reliability aspect of both WAMS and PMUs. However, such placement solutions call for additional or redundant PMUs to cover each bus or line failures. The existing PMU placement models require redundancy factor as an input to solve the placement problem. However, the redundancy factor is provided by the experts based on purely subjective assessment and, hence, there exists no means to assess or estimate the redundancy factor objectively. With traditional placement models, it is difficult to quantify the additional benefits from a reliability standpoint.

On the other hand, reliability-based models essentially focus on achieving a desired reliability of any given system while satisfying other requirements such as cost and observability. Reliability-based models have been used widely in system design optimization to improve system functionality and reliability. However, reliability models have not been exploited in existing optimal PMU placement based algorithms.

This motivates to consider the optimal placement problem from a reliability standpoint. In the proposed models, the reliability of an individual PMU and the desired system level reliability are factored as inputs into the model. The redundancy factors (which call for additional PMUs) are thus dictated by these inputs subject to the system topology. The solution thus achieves complete observability while meeting or exceeding a specified level of reliability. The problem is formulated as a multi-objective optimization formulation that maintains full system observability with minimum cost while exceeding a pre-specified level of reliability.

3.1. System Reliability Estimation

System reliability is the probability that a system will perform its intended function for a given period of time under pre-specified operating conditions. Moreover, for a system to perform its intended functions, it is important that all components and sub-systems contained in the system are highly reliable and able to perform specified functions within given requirements. The placement of a PMU at any given bus allows direct measurement of voltage phasor at that bus and computation of the voltage phasors at neighboring buses. Further, the reliability of power system observability depends on the reliability of PMUs covering each bus. A bus is said to be observable if the voltage phasor at that bus is known. Therefore, the power system will be fully observable if all buses are covered with one or more PMUs. On the other hand, if none of the PMUs were redundant, the failure of any one PMU would result in the complete loss of system observability, which could result in system failure. This understanding allows us to consider that from reliability point of view buses are connected in series structure. Further, in case of redundancy, each bus is covered by more than one PMU and these redundant PMUs will be treated as parallel connected. Thus a parallel structure model can be used to estimate the reliability of bus observability.

For example, consider the IEEE bus 14 in Figure 3.1 with PMUs placed on buses 1, 2, 6, 7 and 9. Assuming PMU reliability of 0.90, the reliability of observability of the bus 3 will be 0.90 because it is observed by one PMU only. On the other hand, the bus 1 is covered by two PMUs (the one PMU placed at bus 2 also covers bus 1), which act as parallel (or redundant). Therefore, the reliability of observability of bus 1 is given as:

$$r_1 = 1 - [(1-0.90)*(1-0.90)] = 0.99 \tag{14}$$

This clearly explains that if any given bus is observed by more than one PMU then for reliability estimation purpose all PMUs covering that bus are treated as parallelly connected. In that case, the reliability of observability of the i^{th} bus can be given as:

$$r_i = 1 - \prod_{j=1}^{f_i} q_j \tag{15}$$

where r_i represents reliability of the i^{th} bus, q_j is probability of failure of j^{th} PMU and f_i denotes the total number of PMUs covering i^{th} bus. This equation indicates that observability of i^{th} bus will fail only if all PMUs covering that bus have failed together.

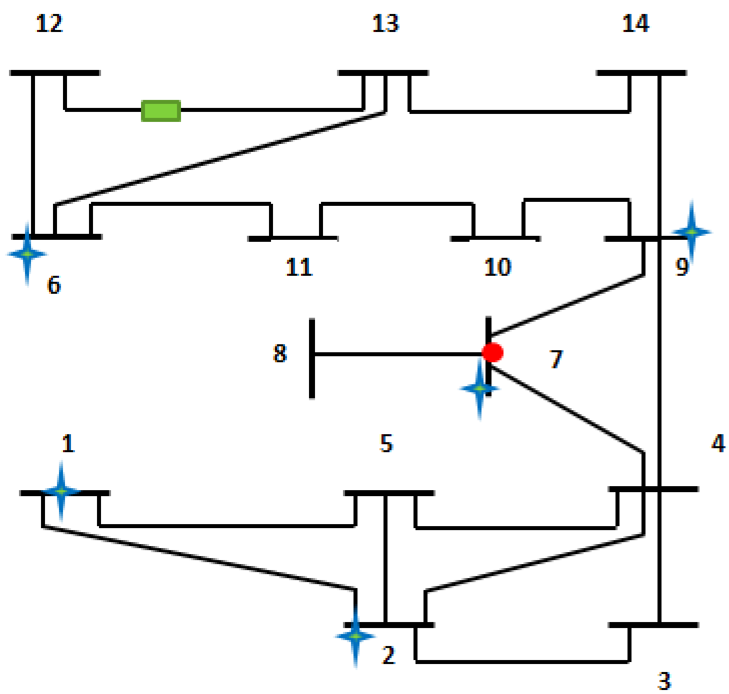

Figure 3.1. IEEE 14 bus system.

Now to develop a system reliability estimation model, let R denote the system wide level of reliability for whole network and r_i denote the reliability level for i^{th} bus. As discussed earlier, considering that from reliability point of view buses are connected in series structure, the reliability of a network can be estimated as:

$$R = \prod_{i=1}^{n} r_i \tag{16}$$

where n is the number of buses in the network. The equation (15) can be modified as:

$$r_i = 1 - P[B] = 1 - q_j^{f_i} \tag{17}$$

where P [B] represents probability of failure of all PMUs observing i^{th} bus and f_i indicates the number of PMUs covering i^{th} bus.

As discussed earlier, equation (17) is the equivalent form of the parallel system reliability model, which assumes that all PMUs covering i^{th} bus are parallelly connected for the purpose of reliability evaluation. The overall system reliability estimation equation (16) is now modified as:

$$R = \prod_{i=1}^{n} (1 - q_j^{\ f_i}) \tag{18}$$

The equation (18) provides more realistic estimation of system reliability by considering the number of buses in the power system network and PMU placement. Now we incorporate the reliability modeling in the network placement model to formulate a binary integer programming model in the next section. The objective would be to reach full system observability and to minimize total cost sustaining a minimum system wide reliability level.

3.2. Reliability-Based Placement

Reliability-based models have been used widely in system design optimization to improve system functionality and reliability. There are few reliability evaluation models for the WAMS and PMU in the literature. The reliability of WAMS can be determined through the identification of its components such as PMUs, communication systems, and the central computation unit and then by estimating reliability of each of these components. Marek et al. (2005) discussed the basic design and special applications of WAMS. They showed that the overall WAMS reliability can be increased by using back up of its subsystems and/or components. Yang et al. (2009) proposed a reliability estimation model for a single PMU. The model develops a series-parallel structure for a single PMU viewed as a collection of seven subcomponents and identifies the most critical component within a PMU. Li (2005) established a repairable reliability model of the power system based on a series parallel structure model and the Markov state-space method. Yang et al. (2009b) presented a hierarchical structure of WAMS to satisfy requirements of reliable real-time data transfer. They developed methods of evaluating multiple reliability indices of regional networks in WAMS. Yang et al. (2010) presented a quantified reliability analysis for the backbone communication network in WAMS and the overall WAMS from a hardware reliability viewpoint using combined Markov modeling and state enumeration techniques. A reliability modeling of PMUs using fuzzy sets proposed and extended to consider options for the PMU hardware in Aminifar et al. (2010). They employed the Markov process to analyze the proposed model and to present an equivalent two-state, up and down, model of PMUs. Liu et al. (2009) proposed reliability modeling of WAMS using the Markov process. The analysis of data acquisition system takes the PMU and PDC as the study objects, and gets the reliability evaluation of data acquisition system by series system model, the parallel system model and k/n judgment system model of those facility units.

The only study to attempt in evaluation of the PMU placement models incorporating reliability theory is proposed by Liu et al. (2009). However the proposed reliability evaluation by Liu et al. (2009) can be made after the placement of PMUs in the system. In addition, the reliability model does not take the number of buses into consideration for the reliability evaluation. Furthermore, the existing PMU placement models consider optimal placement of PMU where the main objective is to minimize cost and ensure system observability ignoring the reliability aspect of both WAMS and PMUs. However, neither the PMU placement nor a reliability evaluation model will ensure cost minimization and reliability simultaneously in the system, if used individually. Therefore both approaches need to be integrated into a single model in order to ensure that the placement is optimal and reliable. This motivated the development of a systematic approach to address both cost minimization and reliability simultaneously using this study. Given the twin and conflicting objectives of cost and reliability, in this section we modeled and solved a multi-objective formulation that maintains full system observability with minimum cost while exceeding a pre-specified level of reliability of observability. To do this a more thorough reliability evaluation approach coupled with PMU placement optimization will result in a more precise and effective solution than the solution presented by the model developed by Liu et al. (2009).

3.2.1. Reliability-Based Placement Model

This section proposes a reliability-based PMU placement model. The model attempts to decide on PMU placement to minimize total cost and ensure the minimum level of system wide reliability of observability for given PMU reliability. Let *R* be the desired system wide reliability level and p_j be reliability of each PMU to be known and are factored as inputs to our model. The objective is to place a minimum number of PMUs in the system subjected to minimum system level reliability of *R*. This objective function also takes care of the cost minimization goal since minimizing the number of PMUs means reducing cost. The proposed model is given as:

$$
\begin{aligned}
& Min \sum_{i=1}^{n} x_i \\
& st. \\
& R = \prod_{i=1}^{n} (1 - q_j^{\,f_i}) \qquad (19) \\
& R \geq R_{\min}
\end{aligned}
$$

where x_i is a binary decision variable, which will acquire value one if a PMU is installed on the i^{th} bus and zero otherwise. R_{min} is the predefined level of reliability of observability. Interestingly, f_i is no longer a decision variable for our model and is defined as:

$$f_i = \sum_{j \in N(i)} x_j \tag{20}$$

where $N(i)$ is the set of PMUs placed on buses j that are adjacent to i^{th} bus. Now to ensure that the proposed reliability-based placement model is an integer linear programming (ILP) model, we modify the reliability constraint in the proposed model as follows:

$$\prod_{i=1}^{n} r_i \geq R_{\min} \tag{21}$$

We further define r as:

$$r = \min_{1,2,\ldots,n}(r_i) \tag{22}$$

We then modify our model as given below:

$$Min \sum_{i=1}^{n} x_i \tag{23}$$

Subject to

$$r_i \geq r \qquad \textit{for all i} \tag{24}$$

$$r^n \geq R_{\min} \tag{25}$$

where r is the minimum reliability requirement of each bus required to meet system wide reliability level. Now assuming that all PMUs are identical and hence having the same level of PMU reliability, the constraint (24) can be written as:

$$1 - q^{f_i} \geq r \tag{26}$$

The Equation (26) ensures a minimum level of system reliability *R*. Further, the minimum number of PMUs required to cover i^{th} bus and to ensure given reliability target is given as:

$$f_i \geq \left\lceil \frac{\log(1-r)}{\log q} \right\rceil \tag{27}$$

$\lceil k \rceil$ is the smallest integer greater than or equal to *k*.

Finally denoting the right hand side of the equation (27) by b, the final model can be expressed as:

$$Min \sum_{i=1}^{n} x_i \tag{28}$$

Subject to

$$f_i \geq b \qquad for\ all\ i \tag{29}$$

$$r^n \geq R_{\min} \tag{30}$$

The value of *b*, which represents redundancy level for each bus, is derived based on system reliability target and factored as input into the model. The proposed ILP model may have several alternative optimal solutions, meaning there are multiple placement solutions meeting the minimum desired system reliability level.

In the second stage, the optimal number of PMUs obtained from the ILP model is considered as a constraint to identify the solution that maximizes system reliability. This is done in Equations 31-33. Let us denote the resulting total number of PMUs from minimization problem as *c* then maximization model would turn out to be:

$$Max\, R \tag{31}$$

Subject to

$$f_i \geq b \tag{32}$$

$$\sum_{i=1}^{n} x_i \leq c \tag{33}$$

Note that the objective function of the second stage Equation 31 is nonlinear. Also note that nonlinear optimization is the process of solving a system of equalities or inequalities, collectively termed as constraints, over a set of unknown variables along with an objective function to be maximized or minimized, where some of the constraints or objective functions are nonlinear. In our purposed model, the objective function, reliability maximization, turns out to be nonlinear. The pseudo-code for the proposed two-stage optimization model is shown in Figure 3.2.

Read Inputs R and q

$r = \sqrt[n]{R}$

//where n is the total number of nodes

$b = \left\lceil \frac{\log(1-r)}{\log q} \right\rceil$

Min $\sum_{i=1}^{n} x_i$

$Subject\ to$

$f_i \geq b \qquad\qquad for\ all\ i$

$r^n \geq R$

If (no solution exists)

end

else

//next stage is to find the best alternative (if needed)

c = resulting total number of PMUs from minimization problem

Max R

$Subject\ to$

$f_i \geq b \qquad\qquad for\ all\ i$

$\sum_{i=1}^{n} x_i \leq c$

end.

Figure 3.2. Pseudo-code for the two stage optimization model.

We demonstrate the applicability of the proposed approach by considering several examples of power systems. Consider the IEEE 14 bus system as shown in Figure 3.1. Suppose the system wide reliability level R is given as 0.75, and probability of failure of each PMU q is given as 0.1. Then minimum reliability of observability of each bus (r) would turn out to be $\sqrt[14]{R}$ $\approx$ 0.98. Now for given minimum reliability target of 0.98 for each bus, the minimum redundancy level b is calculated using Equation (27), which is equal to 2. Thus the ILP model for IEEE 14 bus system is given below:

$$\text{Min} \sum_{i=1}^{14} x_i \tag{34}$$

Subject to

$$x_1 + x_2 + x_5 \geq 2 \tag{35}$$

$$x_1 + x_2 + x_3 + x_4 + x_5 \geq 2 \tag{36}$$

$$x_2 + x_3 + x_4 \geq 2 \tag{37}$$

$$x_2 + x_3 + x_4 + x_5 + x_7 + x_9 \geq 2 \tag{38}$$

$$x_1 + x_2 + x_4 + x_5 \geq 2 \tag{39}$$

$$x_6 + x_{11} + x_{12} + x_{13} \geq 2 \tag{40}$$

$$x_4 + x_7 + x_8 + x_9 \geq 2 \tag{41}$$

$$x_7 + x_8 \geq 2 \tag{42}$$

$$x_4 + x_7 + x_9 + x_{10} + x_{14} \geq 2 \tag{43}$$

$$x_9 + x_{10} + x_{11} \geq 2 \tag{44}$$

$$x_6 + x_{10} + x_{11} \geq 2 \tag{45}$$

$$x_6 + x_{12} + x_{13} \geq 2 \tag{46}$$

$$x_6 + x_{12} + x_{13} + x_{14} \geq 2 \tag{47}$$

$$x_9 + x_{13} + x_{14} \geq 2 \tag{48}$$

The solution shows that at least 9 PMUs are required to maintain complete system observability and to achieve system wide reliability level of 0.75. These PMUs are placed at buses 1, 2, 3, 6, 7, 8, 9, 10 and 13 to ensure the redundancy level of 2 for each bus that gives system wide reliability as 0.9097. The results are comparable to single PMU outage case in Dua et al. (2008). As mentioned earlier, the ILP problems provide several alternative solutions. We therefore set out to search for a better alternative solution using the second stage that maximizes system reliability.

$$\text{Max } \boldsymbol{R} \tag{49}$$

$$\textit{Subject to}$$

$$f_i \geq 2 \qquad \textit{for all } i \tag{50}$$

$$\sum_{i=1}^{n} x_i \leq 9 \tag{51}$$

In the second stage, by changing the objective function to a system reliability maximization and adding additional constraint (51) to our initial model of IEEE 14 bus system, the better alternative solution also places 9 PMUs but on a different set of buses (2, 4, 5, 6, 7, 8, 9, 11 and 13) and maximizes system wide reliability to 0.9189, which is greater than the previous solution.

The proposed model is extendable for both considering zero injections and flow measurement cases. Having a flow measurement along a given branch allows the calculation of one of the terminal bus voltage phasors when the other one is known. Hence, the constraint equations associated with the terminal buses of the measured branch can be merged into a single constraint. Considering the power flow measurement on 12-13 line in Figure 3.1, the model can be updated as follows:

$$\left.\begin{aligned} f_{12} &= x_6 + x_{12} + x_{13} \\ f_{13} &= x_6 + x_{12} + x_{13} + x_{14} \end{aligned}\right\} f_{12new} = f_{12} + f_{13} = x_6 + x_{12} + x_{13} + x_{14} \tag{52}$$

In the case of zero-injection bus, considering bus 7 as zero-injection bus, if the phasor voltages at any three out of the four buses 4, 7, 8 and 9 are known, then the fourth one can be calculated using the Kirchhoff's Current Law applied at bus 7 where the net injected current is known. The three most used methods in the literature are as follows:

- Nonlinear constraint
- ILP approach presented in Dua et.al (2008)
- Topology transformation

The Nonlinear constraint method can be applied to the model by eliminating f_7 and following updates:

$$f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + f_7 . f_8 . f_9 \tag{53}$$

$$f_8 = x_7 + x_8 + f_4 . f_7 . f_9 \tag{54}$$

$$f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} + f_4 . f_7 . f_8 \tag{55}$$

By applying the following properties of logical AND as well as OR operators:

$$A \subset B \rightarrow A + B = B \;\; \& \;\; A.B = A \tag{56}$$

We can further simplify the equations (53), (54) and (55) as follows:

$$f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 + x_8.x_{10} + x_8.x_{14} \tag{57}$$

$$f_8 = x_4 + x_7 + x_8 + x_9 \tag{58}$$

$$f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} + x_2.x_8 + x_3.x_8 + x_5.x_8 \tag{59}$$

However the best approach for applying the zero injection-buses to our model would be the ILP approach presented in Dua et.al (2008). For instance in case of zero injection, we can update constraints (38), (41), (42) and (43) plus adding a new constraint (64) as follows:

$$f_4 = x_2 + x_3 + x_4 + x_5 + x_7 + x_9 \geq b_4 \tag{60}$$

$$f_7 = x_4 + x_7 + x_8 + x_9 \geq b_7 \tag{61}$$

$$f_8 = x_7 + x_8 \geq b_8 \tag*{(61)}$$

$$f_9 = x_4 + x_7 + x_9 + x_{10} + x_{14} \geq b_9 \tag{63}$$

$$b_4 + b_7 + b_8 + b_9 \geq 3b \tag{64}$$

This will yield a reduction in the number of PMUs required for desired leve of reliability of observability. In the case of topology transformation method, the main idea is to merge the bus which has the injection measurement, with any one of its neighbors (Eliminate f_7):

$$\left.\begin{aligned} f_7 &= x_4 + x_7 + x_8 + x_9 \\ f_8 &= x_7 + x_8 \end{aligned}\right\} \left\{\begin{aligned} f_4 &= x_2 + x_3 + x_4 + x_5 + x_{8n} + x_9 \\ f_{8n} &= x_4 + x_{8n} + x_9 \\ f_9 &= x_4 + x_{8n} + x_9 + x_{10} + x_{14} \end{aligned}\right. \tag{65}$$

Further, in order to deal with criticality of certain buses or lines in the network having a greater importance such as a generator, HV buses, or buses at intertie locations, the PMU placement at those critical buses can be made mandatory. This can be ensured by appending an additional constraint:

$$X_k = 1 \qquad \text{For all k} \tag{66}$$

where k represents critical buses. Since only 1 PMU can be placed at a bus, the addition of constraint (66) takes care of criticality of buses in a better way rather than assigning weights to those buses.

3.2.2. Discussions and Computational Results

To further demonstrate the usefulness of the proposed approach, we considered several types of power systems such as IEEE 14, 30, 57, and 118 bus systems. We also looked into different alternative solutions of IEEE 30 bus system for the second stage optimization to select the better alternative with higher system reliability. For all power system types, it is assumed that all PMUs are identical and three different PMU reliability values (0.80, 0.90, and 0.95) are chosen to show how PMU reliability value influences decision on PMU placement. To get better understanding and further insight into the model, we selected system reliability target at three different levels: 0.70, 0.80, and 0.90. The computations were performed with Wolfram Mathematica 8.0. on a 2.66 GHz Intel(R) Core™ 2 Quad CPU with system memory of 2.96 GB.

For all power system types, results are summarized in Tables 3.1, 3.2, 3.3 and 3.4 giving number of PMUs required, level of redundancy (*b*), and actual system reliability achieved for all combinations of system reliability target and PMU reliabilities.

As shown in all Tables, for some combinations of system reliability target (R) and PMU reliability (p), there exists no feasible solution within given constraints or limitations. These scenarios are indicated as N/A (not applicable).

Table 3.1. IEEE 14 bus system analysis.

Desired R	p	#PMU	b	Achieved R
	0.8	N/A	N/A	N/A
0.9	0.9	N/A	N/A	N/A
	0.95	9	2	0.98
	0.8	N/A	N/A	N/A
0.8	0.9	9	2	0.91
	0.95	9	2	0.98
	0.8	N/A	N/A	N/A
0.7	0.9	9	2	0.91
	0.95	9	2	0.98

Table 3.2. IEEE 30 bus system analysis.

Desired R	p	#PMU	b	Achieved R
	0.8	N/A	N/A	N/A
0.9	0.9	N/A	N/A	N/A
	0.95	21	2	0.95
	0.8	N/A	N/A	N/A
0.8	0.9	N/A	N/A	N/A
	0.95	21	2	0.95
	0.8	N/A	N/A	N/A
0.7	0.9	21	2	0.83
	0.95	21	2	0.95

Table 3.3. IEEE 57 bus system analysis.

Desired R	p	#PMU	b	Achieved R
	0.8	N/A	N/A	N/A
0.9	0.9	57	3	0.96
	0.95	57	3	0.995
	0.8	N/A	N/A	N/A
0.8	0.9	57	3	0.96
	0.95	35	2	0.9
	0.8	N/A	N/A	N/A
0.7	0.9	57	3	0.96
	0.95	35	2	0.9

Table 3.4. IEEE 118 bus system analysis.

Desired R	p	#PMU	b	Achieved R
	0.8	N/A	N/A	N/A
0.9	0.9	N/A	N/A	N/A
	0.95	115	3	0.99
	0.8	N/A	N/A	N/A
0.8	0.9	115	3	0.93
	0.95	115	3	0.99
	0.8	N/A	N/A	N/A
0.7	0.9	115	3	0.93
	0.95	68	2	0.83

For example, consider IEEE 30 bus system. To achieve a system reliability target of 0.90 with having PMU reliabilities of 0.80 or 0.90, a redundancy level of 3 PMUs is needed for each bus system. However, this will require in total 33 PMUs assigned to 30 buses in the system violating our binary constraint of assigning one PMU to each bus. It is very clear from this analysis that to get a feasible solution and satisfy the binary constraint, the PMUs should be highly reliable. Further, the results also indicate that when individual PMU reliability is higher, the total number of PMUs and the redundancy level requirement decreases accordingly. For example, for IEEE 57 bus system when PMU reliability is changed from 0.90 to 0.95, the total number of PMUs and redundancy level values change from (57, 3) to (35, 2) for the given system wide reliability targets of 0.80 and 0.70. The same argument is applicable to other power systems as well.

The number of PMUs versus bus reliability (r) for all IEEE bus systems is shown in Figures 3.1 and 3.2. The figures clearly demonstrate that for higher system or bus reliability, the required number of PMUs increases initially and becomes constant after a certain level. This clearly puts a limit on the level of redundancy per bus and beyond which it does not influence system reliability but cost will certainly increase. The figures further indicate that as the system complexity increases (larger bus system), the required number of PMUs also increases.

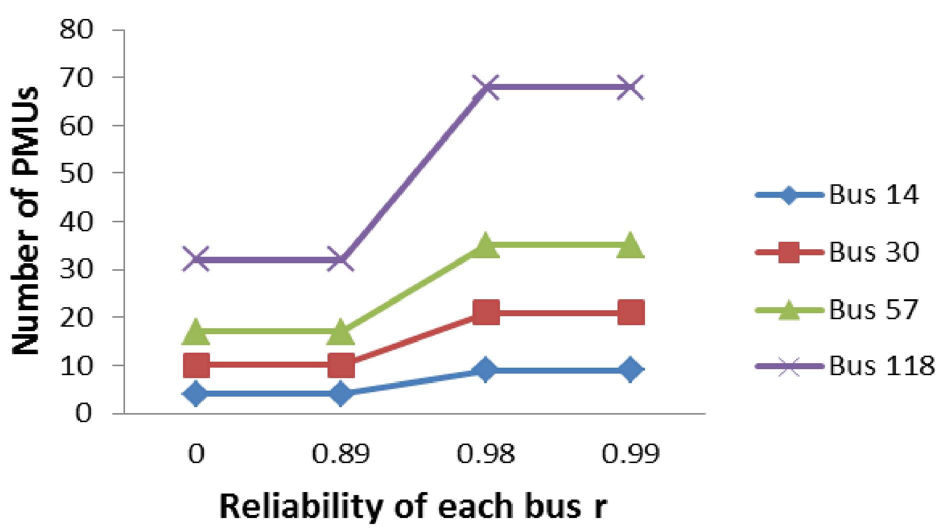

Figure 3.3. Number of PMU based on r.

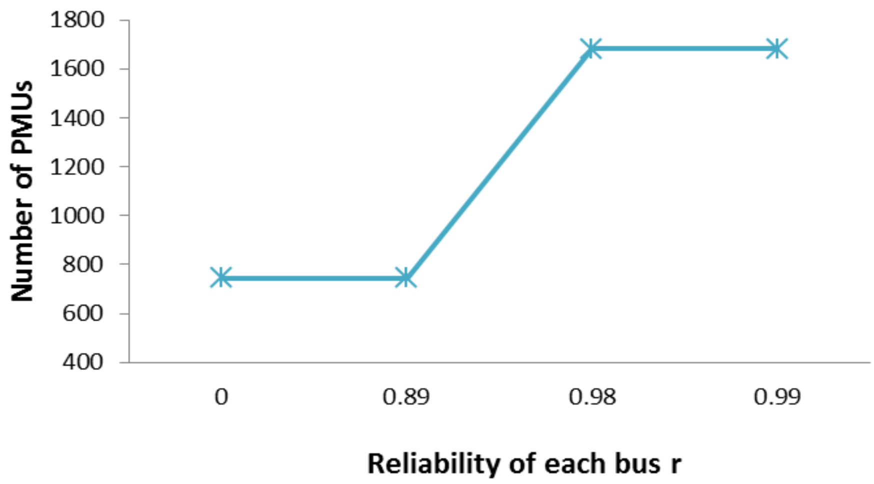

Figure 3.4. Number of PMU based on r for bus 2383 system.

To investigate the scalability of the proposed approach, a large 2383 bus system is also considered to demonstrate the model applicability. For given input values of system reliability targets and PMU reliability values, we were able to find an optimal solution for only one combination, i.e. having PMU reliability 0.99 and system reliability target of 0.70, which gave us actual reliability of 0.84 after finding the optimal solution. This clearly shows that for achieving higher system reliability for a large system, each PMU has to be highly reliable and hence demonstrates the criticality of PMU on power system network. The number of PMUs versus bus system reliability for IEEE 2383 bus system has the same shape but much larger number of PMUs as shown in Figure 3.4.

Comparing the results to the previous placement studies in the literature (Dua et al., 2008 and Kavasseri and Sirnivasan, 2010) although placement results (in terms of the number of PMUs) are very similar, the proposed approach allows a clear evaluation of reliability benefits gained by redundant PMUs, and provides further insight on system reliability and component criticality. Although, the inspection of results show that the trade-off between system wide reliability and the number of PMUs derived from optimization procedure result in more PMUs as compared to existing placement studies in the literature but this is a cost worth paying to achieve a certain reliability level. Moreover, this increase in total number of PMUs is dependent on individual PMU reliability as having highly reliable PMUs will require less number of PMUs to meet system wide reliability requirement as demonstrated earlier. Further, in existing placement models, the level of redundancy (b) was determined based on some expert knowledge without considering system reliability requirement and PMU reliabilities. However, the proposed approach determines level of redundancy based on system reliability requirement and individual PMU reliabilities. Such an approach may be more suitable and practical, especially as the electric grid grows in structural complexity.

The simple extension of the model, adding constraint (66), facilitates due consideration to critical buses or lines in power system and ensures higher reliability of observability for those buses of higher importance. The proposed model can also deal with situations of having different PMUs with different reliability. In that case, the proposed model considers lowest PMU reliability while solving the placement problem and ensures sort of lower bound of system reliability. Note that consideration of lowest PMU reliability in solving placement problem will provide the actual system reliability higher than the reliability suggested by the solution because of the higher PMU reliabilities. However, to consider the actual reliability of individual PMU's in the PMU placement model, one has to develop a different solution approach (or algorithm) to solve the placement model. Table 3.5 shows several alternative solutions for the IEEE 30 bus system with acheived reliability levels. It is clear from the Table that each alternative provides different system level reliability with the same number of 21 PMUs. The reliability maximization model in the second stage of our proposed approach searches for better solution from several alternative solutions. In this particular case, alternative 1 with 21 PMUs provides better system reliability values.

Table 3.5. Alternative PMU locations with corresponding reliabilities for IEEE 30 bus system.

Desired System	PMU(p)	Alternative solution	Placement node	Achieved R
0.9	0.8	N/A	N/A	N/A
	0.9	N/A	N/A	N/A
	0.95	1	1,2,3,5,6,9,10,11,12,13,15,16,18,19,22,24,25,26,27,28,29	0.959979
		2	1,3,5,6,7,8,9,10,11,12,13,15,17,18,19,22,24,25,26,29,30	0.957136
		3	2,3,4,6,7,9,10,11,12,13,15,17,19,20,22,24,25,26,27,28,30	0.955322
		4	1,2,3,5,6,9,10,11,12,13,15,16,18,19,21,23,25,26,27,28,30	0.955424
		5	1,2,3,5,6,9,10,11,12,13,15,16,18,19,21,23,25,26,27,28,29	0.955424
0.8	0.8	N/A	N/A	N/A
	0.9	N/A	N/A	N/A
	0.95	1	1,2,3,5,6,9,10,11,12,13,15,16,18,19,22,24,25,26,27,28,29	0.959979
		2	1,3,5,6,7,8,9,10,11,12,13,15,17,18,19,22,24,25,26,29,30	0.957136
		3	2,3,4,6,7,9,10,11,12,13,15,17,19,20,22,24,25,26,27,28,30	0.955322
		4	1,2,3,5,6,9,10,11,12,13,15,16,18,19,21,23,25,26,27,28,30	0.955424
		5	1,2,3,5,6,9,10,11,12,13,15,16,18,19,21,23,25,26,27,28,29	0.955424
0.7	0.8	N/A	N/A	N/A
	0.9	1	1,2,3,5,6,9,10,11,12,13,15,16,18,19,22,24,25,26,27,28,29	0.84570
		2	1,3,5,6,7,8,9,10,11,12,13,15,17,18,19,22,24,25,26,29,30	0.834451
		3	2,3,4,6,7,9,10,11,12,13,15,17,19,20,22,24,25,26,27,28,30	0.830008
		4	1,2,3,5,6,9,10,11,12,13,15,16,18,19,21,23,25,26,27,28,30	0.830591
		5	1,2,3,5,6,9,10,11,12,13,15,16,18,19,21,23,25,26,27,28,29	0.830591
	0.95	1	1,2,3,5,6,9,10,11,12,13,15,16,18,19,22,24,25,26,27,28,29	0.959979
		2	1,3,5,6,7,8,9,10,11,12,13,15,17,18,19,22,24,25,26,29,30	0.957136
		3	2,3,4,6,7,9,10,11,12,13,15,17,19,20,22,24,25,26,27,28,30	0.955322
		4	1,2,3,5,6,9,10,11,12,13,15,16,18,19,21,23,25,26,27,28,30	0.955424
		5	1,2,3,5,6,9,10,11,12,13,15,16,18,19,21,23,25,26,27,28,29	0.955424

3.3. Reliability-Based Multi-objective Placement

In the previous section we formulated the optimal PMU placement problem as a two-stage optimization model from a reliability standpoint where redundancy levels for all buses in the system are the same Khiabani et al. (2012a). However, maintaining separate objectives of minimizing the cost and maximizing the reliability results in infeasible solutions in some cases. This is because the formulation in (Khiabani et al., 2012a) overlooks combinations which could result in better solutions. This is because of the assumption of identical redundancy levels (bus reliabilities(r)) at all buses in the system. In this section we propose a multi-objective optimization model relaxing the identical redundancy level at all buses to reduce the number of PMUs required to reach a desired level of reliability of observability of overall system.

Given the twin and conflicting objectives of cost and reliability, in this section we present a multi-objective optimization formulation that maintains full system observability with minimum cost while exceeding a pre-specified level of reliability (Khiabani et al., 2012b). In other words, we present an optimization model to minimize cost per unit of reliability. This is achieved in the formulation by relaxing the assumption of identical redundancy levels (bus reliabilities(r)) at all buses in the system.

This multi-objective model solved for the IEEE 14, 30, 57 and 118 bus systems. The resulting formulation clearly dictates the placement of additional PMUs to achieve a specified level of overall reliability. It is to be noted that (i) the notion of reliability in this model is with respect to random failures in either the devices themselves or transmission line outages which affects state estimation and not that of the power system at large; and (ii) the accuracy of PMU measurements is not modeled in this formulation, meaning that a bus is considered observable as long as a non-trivial PMU measurement is available.

3.3.1. Reliability-Based Multi-Objective Model

To solve the multi-objective PMU placement problem, the following non-linear programming model is developed. The components of the model are developed based on Equations (14-18), binary decision variable vector and binary connection matrix definitions. The dual objectives of minimizing cost while maximizing unit reliability can be achieved with respect to exceeding a pre-specified level of reliability of observability and expressed as:

$$
\begin{aligned}
& Max\ J = \frac{\prod_{i=1}^{N} r_i}{\sum_{i=1}^{n} x_i} \\
& s.t. \\
& \prod_{i=1}^{N} r_i \geq R_{\min}
\end{aligned}
\qquad (67)
$$

where x_i is a binary decision variable, which acquires value one if a PMU is installed on the i^{th} bus, and zero otherwise. r_i is the reliability of the observability of the i^{th} bus and f_i denotes the total number of PMUs covering i^{th} bus which are given in Equations (15) and (20) respectively. Here, R_{min} is the desired system wide reliability of observability level. The objective function in (67) is to maximize unit reliability of observability of the system while minimizing the total number of PMUs required for complete system observability.

The model can be modified to incorporate both zero injection (Dua et al., 2008) and flow measurement cases (Kavasseri and Srinivasan, 2010) and to yield further reduction in the number of PMUs needed to achieve desired system reliability of observability.

3.3.2. Discussions and Computational Results

The proposed multi-objective PMU placement model is solved for the IEEE 14, 30, 57 and 118 test systems. A summary of results for all cases are shown in Table 3.6 for two combinations of the given desired reliability of observability level (R_{min}) and PMU reliabilities (p) , the achieved number of PMUs required and actual overall system reliability .

Table 3.6. Placement results with proposed formulation.

IEEE System	p=0.95		p=0.95	
	#PMUs	Achieved R	#PMUs	Achieved R
14	8	0.933	5	0.922
30	21	0.960	13	0.922
57	39	0.929	31	0.901
118	96	0.919	76	0.908

The levels of PMU reliability (p) were considered since PMU reliabilities are near 98%. Locations of PMU buses for each system for R_{min}= 0.90 and p = 0.99 are shown in Table 3.7. The robustness of the proposed placement model has been evaluated in Table 3.8 for R_{min}= 0.90 and p = 0.99 for the IEEE test systems given in Table 3.6.

Table 3.7. PMU locations for placement results.

IEEE System	PMU Location
14	2,4,6,7,9
30	1, 2, 6, 7, 9,10, 12, 15, 19, 24, 25, 27, 29
57	1, 3, 4, 6, 11, 12, 13, 15, 20, 21, 22, 24, 25, 27, 29, 30, 32, 33, 35, 36, 38, 39, 41, 45, 47, 48, 51, 52, 54, 56, 57
118	1, 3, 4, 5, 6, 9, 10, 12, 13, 15, 16, 18, 20, 21, 25, 27, 28, 29, 30, 32, 34, 35, 37, 39, 41, 42, 44, 45, 46, 47, 49, 52, 56, 57, 58, 59, 61, 62, 63, 64, 66, 67, 68, 70, 71, 72, 73, 75, 77, 78, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 100, 103, 105, 106,109, 110, 111, 112, 115, 116, 118

Table 3.8 shows the results for the two cases of PMU outages and line outages. The fraction of cases where the system is fully observable is reported for each system. When considering PMU outages, the lines are assumed to be intact. Starting with the initial configuration (i.e. placement results in Table 3.6), PMUs are disabled one at a time and after each PMU has been disabled the system is checked for observability. The number of placement scenarios for which the system is unobservable is noted in Table 3.8. For example, the IEEE 118 bus system is observable in all but 8 of 76 cases of PMU outage, meaning that the fraction of observable cases is 68/76 = 0.895. Similarly, when considering line outages, the PMUs are assumed to be intact. Starting with the initial configuration (i.e. placement results in Table 3.7), lines line are disabled one at a time and after each line has been disabled the system is checked for observability. For example, the IEEE 118 bus system is observable in all but 8 (out of 179) cases of lines outage, meaning that the fraction of observable cases is 171/179 = 0.995.

Table 3.8. Fraction of outage cases that preserve full system observability.

IEEE System	PMU outage		Line outage	
	#PMUs	Fraction	#Lines	Fraction
14	4	0.200	7	0.632
30	7	0.462	6	0.854
57	8	0.742	8	0.893
118	8	0.895	8	0.955

To investigate the usefulness of the proposed multi-objective optimization model, we compared the results obtained to the placement results in Kavasseri and Srinivasan (2011). Results for the case of p=0.99 are shown in Table 3.9 for IEEE 14, 30, 57 and 118 bus systems. Although the proposed model requires more PMUs, it achieves a higher system reliability level. The results show that with increasing system size, higher redundancy in terms of the number of PMUs is required to maintain the desired reliability levels. For instance in IEEE 14 bus system, the location of PMUs is the same for both models except that an additional PMU is placed on bus 4 in the proposed model. However in the case of IEEE 118 bus system, we need 44 additional PMUs compared with the traditional model, and recall that such placements are not fault tolerant.

Furthermore, the optimal solutions for the PMU placement problem results in alternative optimal solutions in which each alternative solution will result in a different reliability level. Therefore we need to investigate higher reliability level among the alternative solutions; however, the proposed model solves the placement model to reach the highest level of reliability using the least number of PMUs. Table 3.10 compares the results of the multi-objective PMU placement and reliability-based PMU placement presented in previous section (Khiabani et al., 2012a) for PMU reliability 0f 99% (p=0.99) and objective of predefined reliability of observability level of 90% (R_{min}=0.90).

Table 3.9. Comparison of placement results.

IEEE System	With R	#PMU	R	Without R	#PMU	R
14	2,4,6,7,9	5	0.92	2,6,7,9	4	0.89
30	1, 2, 6, 7, 9,10, 12, 15, 19, 24, 25, 27, 29	13	0.91	1,10,12, 15,18,2, 25, 27, 6, 9	10	0.84
57	1, 3, 4, 6, 11, 12, 13, 15, 20, 21, 22, 24, 25, 27, 29, 30, 32, 33, 35, 36, 38, 39, 41, 45, 47, 48, 51, 52, 54, 56, 57	31	0.90	1, 4, 9, 10, 19, 22, 25, 26, 29, 32, 36, 39, 41, 44, 46, 49, 53	17	0.62
118	1, 3, 4, 5, 6, 9, 10, 12, 13, 15, 16, 18, 20, 21, 25, 27, 28, 29, 30, 32, 34, 35, 37, 39, 41, 42, 44, 45, 46, 47, 49, 52, 56, 57, 58, 59, 61, 62, 63, 64, 66, 67, 68, 70, 71, 72, 73, 75, 77, 78, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 96, 100, 103, 105, 106,109, 110, 111, 112, 115, 116, 118	76	0.91	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 63, 68, 70, 71, 76, 77, 80, 85, 86, 90, 94, 101, 105,110,114	32	0.44

As shown in the Table 3.10, the proposed multi-objective model reaches the desired reliability level of 90% with fewer PMUs, which significantly decreases the cost. The cost saving occurs due to eliminating the need for the identical redundancy levels for all buses in the system, with the help of the multi-objective approach. Therefore, the proposed model solves the placement model to reach the highest level of reliability using the least number of PMUs. In the case of IEEE 118 bus system, the number of PMUs is reduced by ~34% compared to the reliability-based placement model (Khiabani et al., 2012a) with the proposed multi-objective model.

Table 3.10. Comparison of placement results considering system reliability of observability.

IEEE System	Multi-objective placement		Reliability-based placement	
	#PMUs	Achieved R	#PMUs	Achieved R
14	5	0.92	9	0.98
30	13	0.91	21	0.95
57	31	0.90	57	0.99
118	76	0.91	115	0.99

3.4. Goal Programming Approach for PMU Placement

In this section we developed a goal programming based approach with two objectives of maximizing the reliability and minimizing the placement cost of PMUs for full observability in power systems (Khiabani et al., 2013a). The model developed to investigate the possibility of existence of better trade-offs to be able to further optimize the multi-objective PMU placement model. The weighted sum goal programming formulation incorporates the reliability of individual PMUs and finds a placement to resolve the conflicting objectives of minimum number of PMUs cost-wise and maximum level of system-wide reliability. This multi-objective problem is formulated as a nonlinear goal programming model in which weights are associated with the objectives. A weight associated to a goal reflects the relative importance given to that goal. Therefore, a higher weight assigned to the overall system reliability of observability dictates the placement of additional PMUs as compared to the traditional PMU placement problems and eventually results in a higher cost. This extra cost is the cost of reliability, and is worth paying in order to achieve a higher level of reliability. Further cost minimization seeks for a zero-injection approach compatible with non-linear programming. Therefore, zero-injection buses were incorporated into the model to further optimize the model.

3.4.1. Goal Programming Model

The goal programming placement model has been formulated as a two-objective problem. The objectives are (1) to maximize the reliability of observability of the system while (2) minimizing the number of PMUs resulting in reduced cost. Thus the objectives are to seek the reliability of observability level R_{min} and to place minimum number of PMUs. It is assumed that the PMUs are identical; therefore minimization of the number of PMUs will result in cost minimization. The total cost (number of PMUs) must increase to reach higher redundancy in observability. Therefore, the objective of maximizing the reliability of observability and minimizing the cost are in conflict. To resolve the conflict, we developed a weighted sum goal programming model by assigning relative weight to each goal to combine the two conflicting objective functions into a single objective function. The goal programming model formulated as a weighted sum nonlinear programming is as follows:

$$
\begin{aligned}
&Max\ w\left(\frac{\prod_{i=1}^{n}(1-q_j{}^{f_i})-R_{\min}}{1-R_{\min}}\right)+(1-w)\left(\frac{Bus-\sum_{i=1}^{n}x_i}{Bus}\right)\\
&s.t\\
&x_i\in\{0,1\}\ i\in 1,2,3,\ldots n
\end{aligned}
\tag{68}
$$

where q_j represents the probability of failure of j^{th} PMU, x_i is a binary decision variable, which will acquire value one if a PMU is installed on the i^{th} bus and zero otherwise. f_i denotes the total number of PMUs of covering i^{th} bus which is given in Equation (20). Here, R_{min} is the desired system wide reliability of observability level, *Bus* is the number of buses in the system, and w defines the weight associated to each goal. The denominators are for normalizing the values of the objectives.

The objectives in equation (68) are to minimize the total number of PMUs required for maximum level of system reliability of observability. w defines the weight associated with the objectives and is a decision tool for the problem solver. If reliability maximization is more important, then w should be increased. However, if cost is more important than reliability, then a smaller value of w should be used.

To yield further reduction in the number of PMUs needed for system observability, the model can be modified to incorporate zero-injection buses. As explained in previous sections, a zero-injection bus is a bus where there is neither generation nor load; therefore, the sum of the flows on the all incident buses to the zero-injection bus is zero (Dua et al., 2008).

Hence if zero-injection buses are incorporated in the model, the total number of PMUs may further be reduced. If current phasors for each line incident to a zero-injection bus except for one are known, the current phasor of that line can be calculated using Kirchhoff's current law. Furthermore the voltage phasor of the bus at the other end of the line can be calculated using Kirchhoff's voltage law. Therefore the zero-injection models can be developed by updating f_i's of the adjacent buses of the zero-injection bus. Consider bus 7 in IEEE 14 bus system, shown in Figure 3.1, as zero-injection bus (in red), neighbored with the buses 4, 8, 9. With the zero-injection modification, a PMU at any of the buses 4, 7, 8 or 9 will make all four and other buses neighboring the bus with the PMU observable. A word of caution needs to be added here, if optimal solution chooses to place a PMU on bus 8, it would not make all four buses observable in case they are not covered by another PMU. However because of the nature of the reliability-based modeling this case would not happen. It means the algorithm will prevent placing a PMU on bus 8 to maximize reliability of observability. For instance with the placement of a PMU at bus 9 it will make buses 4, 7, 8, 9, 10, and 14 observable, of course with the consideration of bus 7 as a zero-injection bus.

3.4.2. Discussions and Computational Results

The proposed placement model is solved for the IEEE 14, 30, 57 and 118 bus standard test systems considering both with and without zero-injection buses. Results are reported with R_{min} = 0.9 for all system types.

A summary of results for the four standard IEEE types, not considering the zero-injection buses, are noted in Tables 3.11-3.14. In these tables the required number of PMUs and achieved actual overall system reliability are calculated for several combinations of the weight (w) and PMU reliabilities ($p=1-q$). In reality PMU reliabilities are near 98%, therefore we considered two levels of 95% and 99% for PMU reliability (p).

Table 3.11. GPB results for IEEE 14 test system.

w	p=0.95		p=0.99	
	#PMUs	Achieved R	#PMUs	Achieved R
0.1	7	0.8843	4	0.8952
0.2	9	0.9797	5	0.9315
0.3	9	0.9797	7	0.9793
0.4	9	0.9797	7	0.9793
0.5	9	0.9797	9	0.9992
0.6	11	0.9893	9	0.9992
0.7	11	0.9893	9	0.9992
0.8	14	0.9967	9	0.9992
0.9	14	0.9967	9	0.9992
0.99	14	0.9967	14	0.9999

Table 3.12. GPB results for IEEE 30 test system.

w	p=0.95		p=0.99	
	#PMUs	Achieved R	#PMUs	Achieved R
0.1	21	0.9598	10	0.8507
0.2	21	0.9599	14	0.9307
0.3	21	0.9599	21	0.9983
0.4	21	0.9600	21	0.9983
0.5	25	0.9784	21	0.9983
0.6	30	0.9906	21	0.9984
0.7	30	0.9906	21	0.9984
0.8	30	0.9906	21	0.9984
0.9	30	0.9906	21	0.9984
0.99	30	0.9906	30	0.9997

Table 3.13. GPB results for IEEE 57 test system.

w	p=0.95		p=0.99	
	#PMUs	Achieved R	#PMUs	Achieved R
0.1	38	0.9098	29	0.9188
0.2	40	0.9274	35	0.9861
0.3	46	0.9658	37	0.9963
0.4	47	0.9704	38	0.9964
0.5	55	0.9904	40	0.9968
0.6	55	0.9904	40	0.9968
0.7	55	0.9904	40	0.9968
0.8	55	0.9904	40	0.9974
0.9	57	0.9908	48	0.9991
0.99	57	0.9908	55	0.9998

Table 3.14. GPB results for IEEE 118 test system.

w	p=0.95		p=0.99	
	#PMUs	Achieved R	#PMUs	Achieved R
0.1	82	0.7584	71	0.9084
0.2	88	0.8061	79	0.9652
0.3	90	0.8066	86	0.9664
0.4	92	0.8161	89	0.9752
0.5	92	0.8161	89	0.9752
0.6	92	0.8161	91	0.9859
0.7	93	0.8512	91	0.9859
0.8	95	0.8718	91	0.9859
0.9	95	0.8718	92	0.9964
0.99	101	0.9220	101	0.9977

As mentioned earlier, incorporating zero-injection buses may result in reduced cost. Therefore, summary of results for all cases, considering the zero-injection buses, are shown in Tables 3.15-3.18. In these tables, the required number of PMUs and achieved overall system reliability of observability are calculated for several combinations of the weight (*w*) and PMU reliabilities *(p)*. The zero-injection buses for the IEEE standard bus systems are as follows (Dua et al., 2008):

14 bus :{ 7}

30 bus :{ 6, 9, 11, 25, 28}

57 bus :{ 4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48}

118 bus :{ 5, 9, 30, 37, 38, 63, 64, 68, 71, 81}

Table 3.15. GPB results for IEEE 14 considering zero-injection buses.

w	p=0.95		p=0.99	
	#PMUs	Achieved R	#PMUs	Achieved R
0.1	6	0.9240	3	0.8774
0.2	7	0.9749	5	0.9693
0.3	7	0.9749	6	0.9889
0.4	7	0.9749	6	0.9889
0.5	7	0.9749	7	0.9990
0.6	10	0.9915	7	0.9990
0.7	10	0.9915	7	0.9990
0.8	13	0.9990	7	0.9990
0.9	13	0.9990	7	0.9990
0.99	14	0.9992	13	1.0000

Table 3.16. GPB results for IEEE 30 considering zero-injection buses.

w	p=0.95		p=0.99	
	#PMUs	Achieved R	#PMUs	Achieved R
0.1	16	0.9625	8	0.9030
0.2	16	0.9625	10	0.9399
0.3	16	0.9625	16	0.9985
0.4	16	0.9625	16	0.9985
0.5	20	0.9811	16	0.9985
0.6	26	0.9955	16	0.9985
0.7	26	0.9955	16	0.9985
0.8	26	0.9955	16	0.9985
0.9	26	0.9955	16	0.9985
0.99	28	0.9959	26	0.9999

Table 3.17. GPB results for IEEE 57 considering zero-injection buses.

w	p=0.95		p=0.99	
	#PMUs	Achieved R	#PMUs	Achieved R
0.1	33	0.9324	27	0.9482
0.2	33	0.9368	31	0.9868
0.3	35	0.9589	32	0.9969
0.4	41	0.9788	32	0.9969
0.5	46	0.9913	32	0.9972
0.6	46	0.9931	33	0.9973
0.7	47	0.9937	34	0.9977
0.8	48	0.9939	34	0.9978
0.9	51	0.9948	38	0.9990
0.99	54	0.9951	47	0.9999

Table 3.18. GPB results for IEEE 118 considering zero-injection buses.

w	p=0.95		p=0.99	
	#PMUs	Achieved R	#PMUs	Achieved R
0.1	83	0.8227	65	0.8910
0.2	87	0.8809	76	0.9471
0.3	90	0.8777	83	0.9853
0.4	90	0.8777	85	0.9858
0.5	94	0.8901	85	0.9858
0.6	95	0.8923	85	0.9862
0.7	95	0.9342	85	0.9862
0.8	96	0.9395	85	0.9865
0.9	96	0.9395	90	0.9870
0.99	96	0.9395	90	0.9875

Comparing the results with zero-injection cases, when zero injection is incorporated, a certain reliability of observability is achievable with fewer PMUs.

The placement locations for the cases reached the minimum reliability level of 90% with the associated weights and tables not considering the zero-injection buses are shown in Table 3.19.

Table 3.19. PMU locations for GPB results with achieved minimum reliability of observability of 90%.

IEEE	PMU Location	w	Table
14	2, 6, 7, 9, 13	0.2	3.11
30	1, 2, 5, 6, 9, 10, 12, 15, 16, 19, 24, 25, 27, 29	0.2	3.12
57	1, 4, 6, 9, 12, 14, 15, 18, 20, 22, 24, 25, 27, 28, 29, 31, 32, 36, 37, 41, 45, 47, 48, 50, 51, 53, 54, 56, 57	0.1	3.12
118	2, 5, 7, 8, 9, 10, 11, 12, 15, 17, 20, 21, 23, 25, 26, 28, 29, 32, 33, 35, 37, 40, 41, 43, 44, 46, 49, 50, 52, 53, 54, 56, 57, 58, 59,61, 62, 63, 65, 67, 68, 70, 71, 72, 73, 76, 77, 78, 79, 80, 85, 86,87, 89, 91, 92, 93, 94, 96, 100, 102, 103, 105, 106, 108, 110, 111, 112, 114, 117, 118	0.1	3.14

The robustness of the proposed placement model has been evaluated for p = 0.99 for the IEEE test systems given in Table 3.19 and compared to the placement results in Kavasseri and Srinivasan (2011). Tables 3.20 and 3.21 show the results for p=0.99 for the two cases of PMU outages and line outages respectively. The fraction of cases where the system is fully observable is reported for each IEEE standard system. In the PMU outage scenario, it is assumed that there is no failure in the lines. Starting with the initial configuration (i.e. placement results in Table 3.19 and Table 3.22), a PMU is disabled then the system is checked for observability. Finally, the PMU is enabled and the same process is repeated for each PMU in sequence. The number of placement scenarios for which the system is unobservable is noted in Tables 3.20 and 3.21. For example, the IEEE 57 bus system is observable in all but 7 of 29 cases of PMU outage, meaning that the fraction of observable cases is 22/29 = 0.76. Similarly, in the line outage scenario, the PMUs are assumed to be fully functioning. Starting with the initial configuration (i.e. placement results in Table 3.19 and Table 3.22), a line is disabled then the system is checked for observability. Finally the line is enabled and the same process is repeated for each line in sequence. For example, the IEEE 57 bus system is observable in all but 7 of 75 cases of lines outages, meaning that the fraction of observable cases is 68/75 = 0.91. By comparing the results from both models from Tables 3.20 and 3.21 one can conclude that the proposed multi-objective goal programming model is more robust than the traditional cost-based method.

Table 3.20. Fraction of PMU outage cases that preserve full system observability for GPB approach.

IEEE System	Goal programming approach		Traditional cost-based approach	
	#PMUs	Fraction	#PMUs	Fraction
14	4	0.20	4	0.00
30	7	0.50	10	0.00
57	7	0.76	17	0.00
118	8	0.89	32	0.00

Table 3.21. Fraction of Line outage cases that preserve full system observability for GPB approach.

IEEE System	Goal programming approach		Traditional cost-based approach	
	#Lines	Fraction	#Lines	Fraction
14	6	0.68	9	0.53
30	4	0.90	15	0.63
57	7	0.91	34	0.55
118	9	0.95	64	0.64

To further investigate the usefulness of the proposed multi-objective goal programming model, we compared the results to the placement results in Khiabani et al. (2012a) and Kavasseri and Srinivasan (2011). Results for the case of p=0.99 with minimum desired system wide reliability of observability level of R_{min}=0.90 are shown in Table 3.22 and Table 3.23 for IEEE 14, 57, and 118 bus systems.

Based on the data from Table 3.22 it is clear that although the proposed model requires more PMUs, it achieves a higher system reliability level. The results show that, with increasing the system size, higher redundancy in terms of the number of PMUs is required to maintain the desired reliability level. For instance in IEEE 14 bus system, the location of PMUs is the same for both models, except that an additional PMU is placed on bus 13 in the proposed model. However, in the case of IEEE 118 bus system, we need 39 additional PMUs compared with the traditional model (Kavasseri and Srinivasan, 2011) to increase the reliability of observability by 47%. It should be noted that in the conventional PMU placement problems, loss of a PMU would result in loss of the observability of the majority of the neighboring buses. Therefore, loss of a single PMU will result in loss of observability of the system. Hence, such placements are not fault tolerant.

Table 3.22. Comparison of placement results.

IEEE System	With R	#PMU	R	Without R	#PMU	R
14	2, 6, 7, 9, 13	5	0.93	2,6,7,9	4	0.89
30	1, 2, 5, 6, 9, 10, 12, 15, 16, 19, 24, 25, 27, 29	14	0.93	1, 2, 6, 9, 10, 12, 15, 18, 25, 27	10	0.84
57	1, 4, 6, 9, 12, 14, 15, 18, 20, 22, 24, 25, 27, 28, 29, 31, 32, 36, 37, 41, 45, 47, 48, 50, 51, 53, 54, 56, 57	29	0.92	1, 4, 9, 10, 19, 22, 25, 26, 29, 32, 36, 39, 41, 44, 46, 49, 53	17	0.62
118	2, 5, 7, 8, 9, 10, 11, 12, 15, 17, 20, 21, 23, 25, 26, 28, 29, 32, 33, 35, 37, 40, 41, 43, 44, 46, 49, 50, 52, 53, 54, 56, 57, 58, 59,61, 62, 63, 65, 67, 68, 70, 71, 72, 73, 76, 77, 78, 79, 80, 85, 86,87, 89, 91, 92, 93, 94, 96, 100, 102, 103, 105, 106, 108, 110, 111, 112, 114, 117, 118	71	0.91	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56,62, 63,68, 70, 71, 76, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	32	0.44

Table 3.23. Comparison of placement results considering system reliability of observability.

IEEE System	Goal programming based placement		Multi-objective placement		Reliability-based placement	
	#PMUs	Achieved R	#PMUs	Achieved R	#PMUs	Achieved R
14	5	0.93	5	0.92	9	0.98
30	14	0.93	13	0.91	21	0.95
57	29	0.92	31	0.90	57	0.99
118	71	0.91	76	0.91	115	0.99

As shown in Table 3.23 the proposed model reaches the desired reliability of observability level of 90% with fewer PMUs compared to multi-objective (Khiabani et al., 2012b) and reliability-based (Khiabani et al., 2012a) models which significantly decreases the cost. The only exception is for IEEE 30 bus system in which the multi-objective model reaches the 90% system reliability of observability level with 13 PMUs where for the goal programming model it is 14 PMUs. The cost saving occurs due to eliminating the need for the identical redundancy levels for all buses in the system, with the help of the goal programming approach. Therefore, the proposed model solves the placement model to reach the highest level of reliability using the least number of PMUs for the given weight (w). In the case of IEEE 118 bus system, the number of PMUs is reduced by ~40% compared to the reliability-based placement model (Khiabani et al., 2012a) with the proposed goal programming approach. This means ~6% reduction in the number of PMUs as compared to multi-objective placement model. It is worth noting that the proposed goal programming approach could even reach better results by assigning different weights (w). For instance by assigning w=0.19 in the case of IEEE 30 bus system and running the model resulted in 13 PMUs required with the PMUs placed on buses 2, 3, 5, 6, 9, 10, 12, 15, 17, 19, 24, 25 and 27, and 91% achieved reliability of observability level.

3.5. Max Covering Approach for PMU Placement

The multi-objective optimization (Khiabani et al., 2012b) and goal programming multi-objective optimization (Khiabani et al., 2013a) models described earlier considered minimizing the number of PMUs to reach full system observability while maintaining a pre-specified level of reliability of observability. Both models relax the existence of limited number PMUs. However in practice the resources could be limited because of the high price of the purchasing and installing PMUs. In this case the decision maker will decide to allocate the limited recourses either to the strategic locations or to cover the maximum possible buses. Therefore we considered the PMU placement problem from a maximum covering standpoint (Khiabani et al., 2013b). In the proposed model, the number of existing PMUs is factored as inputs into the model. The maximum coverage thus dictated by this input is subject to the system topology.

In case that the number of the PMUs are sufficient for full system observability, the observability constraint was added to the model. The problem is formulated as an integer linear programming (ILP) model with the objective of maximizing the network coverage and reaching the full network observability when possible. The solution thus achieves maximum coverage with complete observability and incomplete observability depending on the availability of the recourses. Then the reliability evaluation method presented in Khiabani et al. (2012a) is used to evaluate the reliability of the resulting placement.

3.5.1. Max Covering Model

The maximum covering placement model has been formulated as an ILP problem. The main objective is to maximize the coverage of the buses in the power network through assigning the limited number of PMUs available to the strategic buses. Clearly the resource limitation would not allow reaching the complete observability of the power network. However in the case with the sufficient number of PMUs, the observability constraint will be added to the optimization model. The addition of an extra constraint may result in reduced coverage but will maintain the full system observability. The integer linear programming model formulated as a maximum covering is as follows:

$$
\begin{aligned}
& Max \sum_{i=1}^{n} f_i \\
& s.t \\
& \sum_{i=1}^{n} x_i \le c \\
& x_i \in \{0,1\} \;\; i \in 1,2,3,\ldots n
\end{aligned} \tag{69}
$$

where x_i is a binary decision variable, which will acquire value one if a PMU is installed on the i^{th} bus and zero otherwise. f_i denotes the total number PMUs of covering i^{th} bus which is given in Equation (20). Here, c is the number of PMUs available. The objective function in (69) is to maximize the coverage of the power system. In case a limited number of PMUs is sufficient to reach complete system observability the following constraint can be added to the model:

$$
\sum_{i=1}^{n} f_i \ge 1 \tag{70}
$$

Decision maker may need to cover some of the strategic buses in the system. To do this if the number of PMUs is not sufficient for full system observability, then only the i^{th} element of the constraint Equation (70) could be added to the optimization problem to make sure bus i is covered. The model can be modified to incorporate both zero-injection buses (Dua et al., 2008) and flow measurement cases (Kavasseri and Srinivasan, 2011) for further reduction in the total number of PMUs needed for full system observability. The model developed in Dua et al.(2008) has been modified for the proposed max covering problem to incorporate zero-injection buses in the system. The reliability evaluation portion of the reliability-based placement model presented in Khiabani et al. (2012a) is used to evaluate the reliability of observability of the power system.

3.5.2. Discussions and Computational Results

The proposed maximum covering placement model is solved for the IEEE 14, 30, 57, 118 and 2383 bus standard test systems. The observability constraint was added where complete power system observability was possible. The reliability of the placement solutions has been calculated. Results are reported with PMU reliabilities assumed to be 0.99 for all cases for both incorporating zero-injection buses and without zero-injection buses. The comparison plots have been done using Matlab. A Mathematica code using a 'For' loop has been applied for all sets of possible inputs for all IEEE standard bus systems.

The results for IEEE 14, 30, 57, 118 and 2383 standard bus systems are shown in Tables 3.24-3.28 for the number of PMUs given, the total coverage, and overall system reliability of observability achieved. The overall system reliability of observability has been calculated after and based on the optimization problem results and the PMU reliabilities was assumed to be 99%. The 99% level of PMU reliability assumed since PMU reliabilities are near 98%.

Table 3.24. Max covering placement results for IEEE 14 test system.

#PMU	Cover	R
1	6	0
2	11	0
3	16	0
4	18	0.89

Table 3.25. Max covering placement results for IEEE 30 test system.

#PMU	Cover	R
1	8	0
2	15	0
3	21	0
4	26	0
5	31	0
6	36	0
7	41	0
8	45	0
9	49	0
10	52	0.83

Table 3.26. Max covering placement results for IEEE 57 test system.

#PMU	Cover	R
1	7	0
2	14	0
3	20	0
4	26	0
5	31	0
6	36	0
7	41	0
8	46	0
9	51	0
10	56	0
11	60	0
12	64	0
13	68	0
14	72	0
15	76	0
16	80	0
17	69	0.62

Table 3.27. Max covering placement results for IEEE 118 test system.

#PMU	Cover	R	#PMU	Cover	R
1	10	0	17	119	0
2	19	0	18	125	0
3	27	0	19	131	0
4	35	0	20	137	0
5	42	0	21	143	0
6	49	0	22	148	0
7	56	0	23	153	0
8	63	0	24	158	0
9	70	0	25	163	0
10	77	0	26	168	0
11	83	0	27	173	0
12	89	0	28	178	0
13	95	0	29	183	0
14	101	0	30	188	0
15	107	0	31	193	0
16	113	0	32	164	0.45

Table 3.28. Max covering placement results for IEEE 2383 test system.

#PMU	Cover	R
1	10	0
2	20	0
3	30	0
4	40	0
5	50	0
6	60	0
7	69	0
8	78	0
9	87	0
10	96	0
11	105	0
12	114	0
13	123	0
14	132	0
⋮	⋮	⋮
⋮	⋮	⋮
745	3714	0
746	3288	3.90705*10^-8

The results for IEEE 14, 30, 57, 118 standard bus systems has been summarized and shown in Figures 3.5 and 3.6. The figures show the cover, number of buses with installed PMUs and the evaluated reliability. The results for IEEE 2383 standard bus system have been shown in Figure 3.7.

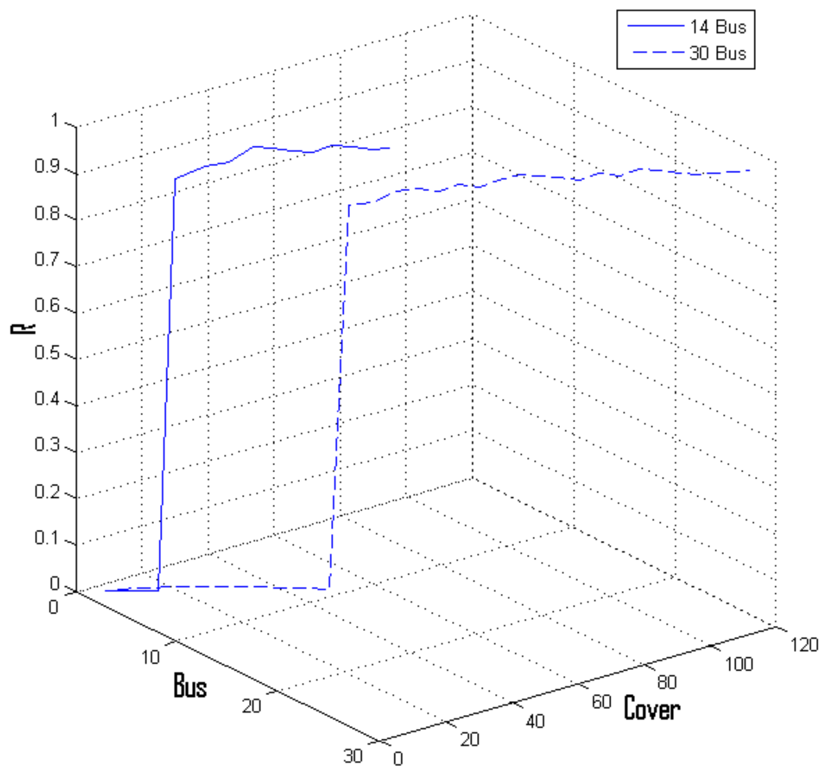

Figure 3.5. Comparison of coverage between IEEE 14 and 30 bus systems.

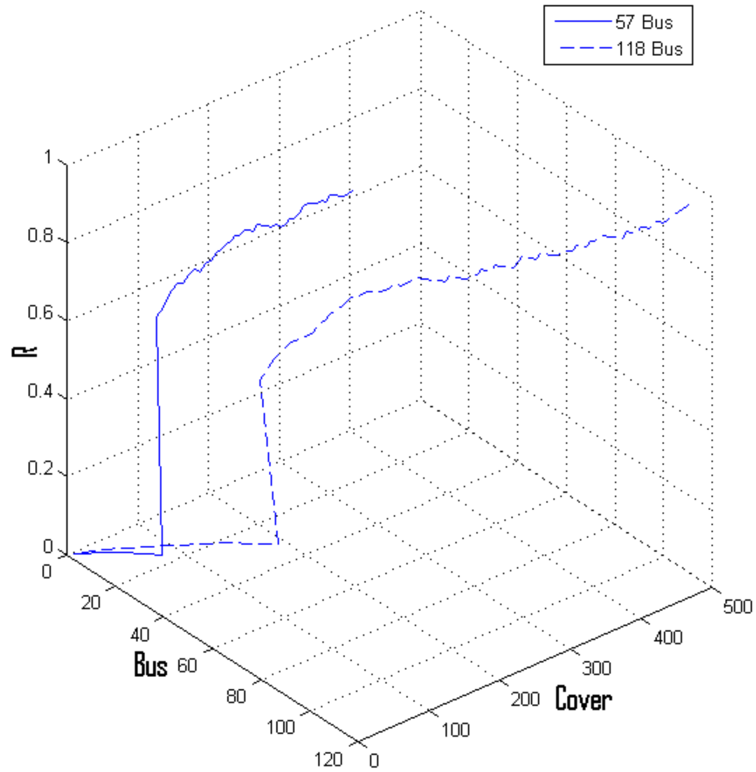

Figure 3.6. Comparison of coverage between IEEE 57 and 118 bus systems.

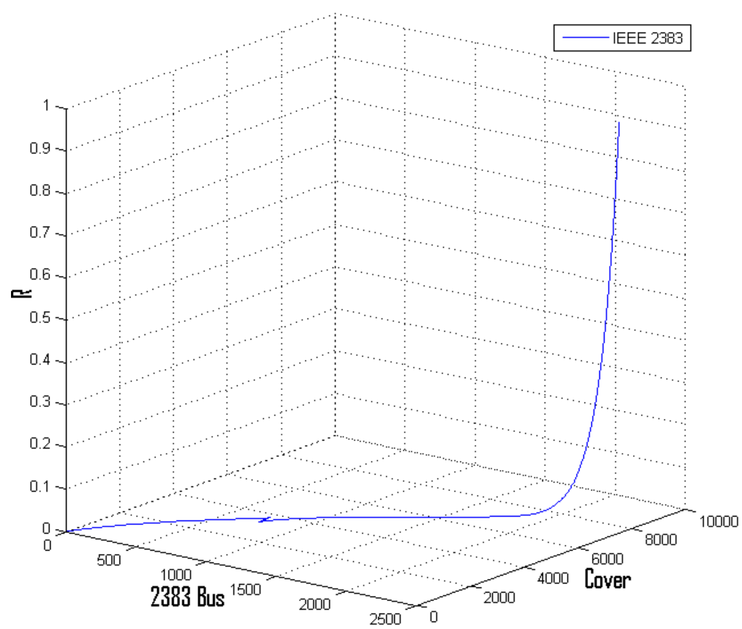

Figure 3.7. Coverage for IEEE 2383 bus system.

The results for IEEE 14, 30, 57, 118 standard bus systems considering the zero-injection buses are shown in Tables 3.29-3.32 for the number of PMUs given, the total coverage, and overall system reliability achieved. The overall system reliability has been calculated after and based on the optimization problem results, and the PMU reliabilities was assumed to be 99%.

Table 3.29. Max covering placement results for IEEE14 incorporating zero-injection buses.

#PMU	Cover	R
1	7	0
2	13	0
3	15	0.88

Table 3.30. Max covering placement results for IEEE30 incorporating zero-injection buses.

#PMU	Cover	R
1	13	0
2	23	0
3	33	0
4	43	0
5	52	0
6	61	0
7	57	0.86

Table 3.31. Max covering placement results for IEEE57 incorporating zero-injection buses.

#PMU	Cover	R
1	9	0
2	18	0
3	26	0
4	33	0
5	40	0
6	47	0
7	54	0
8	61	0
9	68	0
10	75	0
11	82	0
12	88	0
13	72	0.65

Table 3.32. Max covering placement results for IEEE118 incorporating zero-injection buses.

#PMU	Cover	R	#PMU	Cover	R
1	12	0	15	144	0
2	24	0	16	152	0
3	36	0	17	160	0
4	46	0	18	167	0
5	56	0	19	174	0
6	66	0	20	181	0
7	76	0	21	188	0
8	85	0	22	195	0
9	94	0	23	202	0
10	103	0	24	209	0
11	112	0	25	215	0
12	120	0	26	221	0
13	128	0	27	227	0
14	136	0	28	184	0.47

The results for IEEE 14, 30, 57, and 118 standard bus systems considering zero-injection buses have been summarized and shown in Figures 3.8 and 3.9. The figures show the cover, number of buses with installed PMUs, and the evaluated reliability.

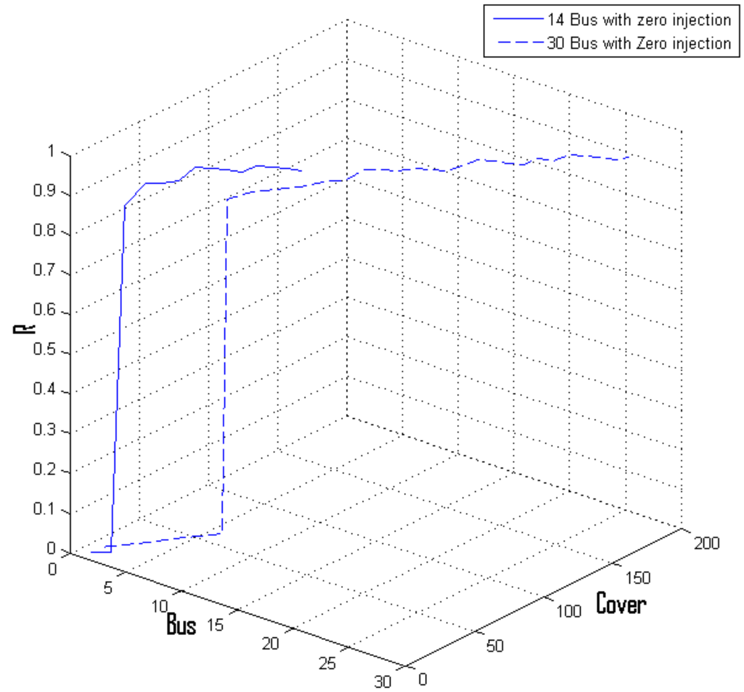

Figure 3.8. Comparison of coverage between IEEE 14 and 30 bus systems.

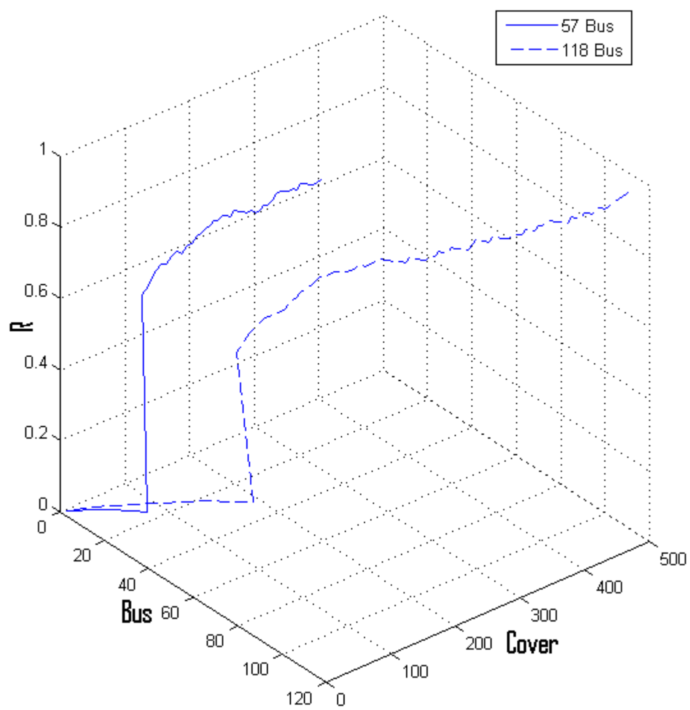

Figure 3.9. Comparison of coverage between IEEE 57 and 118 bus systems.

The usefulness of the proposed max covering optimization model was investigated, comparing the results to the PMU placement results in Kavasseri and Srinivasan (2011). Comparison results are shown in Table 3.33 for IEEE 14, 30, 57 and 118 bus systems. The comparison of the results shows that the models reach the same output with minor difference. However, the proposed model has less complexity and also can consider the cases where reaching the full observability is not feasible. This ability of the proposed model will empower the decision maker through availability of more options for the cases that involve limited resources. Almost each section in output is the same in Table 3.33 except the placement buses for the PMUs, this is trivial for the placement problems because of the existence of the alternative optimal solutions. However each alternative optimal solution will result in a different reliability level. This can be seen in Table 3.33 for the IEEE 30 bus system.

To further investigate the usefulness of the proposed model, we compared the results to the reliability-based placement results in Khiabani et al. (2012a). Since the results for the Khiabani et al. (2012a) analyzed for PMU reliability of 0.95 therefore for this comparison only we run the max covering model and evaluate the reliability with the PMU reliability of 0.95 with the selection of results reaching the overall system reliability of at least 0.90. Comparison results are shown in Table 3.34 reaching the minimum system wide reliability level of 0.90 and in Table 3.35 with the same amount of PMUs for IEEE 14, 30, 57 and 118 bus systems. The results show that the comparison between reliability-based placement model and the max cover model derived from optimization procedure result in more cover as compared to reliability-based placement in the literature. On the other hand the reliability-based placement model reached higher system wide reliability levels compared to the max covering problem.

Table 3.33. Comparison of placement results with traditional PMU placement results.

IEEE System	Max cover			PMU placement		
	Placement	#PMU	R	Placement	#PMU	R
14	2,6,7,9	4	0.89	2,6,7,9	4	0.89
30	2,4,6,9,10,12,15,18,25,27	10	0.83	1, 2, 6, 9, 10, 12, 15, 18, 25, 27	10	0.84
57	1,4,9,13,19,22,25,26,29,32,36,39,41,45,47,50,53	17	0.62	1, 4, 9, 10, 19, 22, 25, 26, 29, 32, 36, 39, 41, 44, 46, 49, 53	17	0.62
118	3,5,9,12,15,17,20,23,28,30,34,37,40,45,49,52,56,62,64,68,71,75,77,80,85,86,90,94,101,105,110,114	32	0.45	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56,62, 63,68, 70, 71, 76, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114	32	0.44

Table 3.34. Comparison of placement results with RBP with R=0.90.

IEEE System	Max cover			Reliability-based placement		
	#PMU	Cover	R	#PMU	Cover	R
14	11	44	0.94	9	37	0.98
30	28	108	0.90	21	85	0.95
57	55	203	0.90	57	207	0.99
118	117	474	0.93	115	470	0.99

Table 3.35. Comparison of placement results with RBP with same number of PMUs.

IEEE System	Max cover			Reliability-based placement		
	#PMU	Cover	R	#PMU	Cover	R
14	9	38	0.85	9	37	0.98
30	21	88	0.69	21	85	0.95
57	57	207	0.99	57	207	0.99
118	115	470	0.84	115	470	0.99

3.6. Genetic Algorithm Approach for PMU Placement

The reliability-based PMU placement model is able to solve small size problems such as IEEE 14, 30, 57 and 118. However, it can also solve some large size problems but not in a timely manner. As the problem size increases the complexity of the system increases exponentially rendering the problem mathematically unsolvable. As noted earlier, the PMU placement model is NP-hard and cannot be solved using exact algorithm for larger size problems. Furthermore, the addition of the second objective, maximization of reliability of observability makes it even more complex as it brings non-linearity into the mathematical model. Therefore, in this section we developed a genetic algorithm for multi-objective optimal PMU placement problem in order to increase the scalability of the model and solve the large size problems. The genetic algorithm approach is based on binary encoding and consists of two main objectives to tackle large scale problems (Khiabani et al., 2013c).

3.6.1. Genetic Algorithm Model

We proposed a multi-objective optimal PMU placement model using a genetic algorithm based on binary encoding. The model consists of two main and contradicting objectives--that is maximization of reliability of observability of system and minimization of the number of PMUs and is designed to be able to tackle large scale problem instances. The model ensures full system observability and aims to reach a pre-specified level of reliability of observability while placing the minimal number of PMUs. The model is solved for the IEEE 14, 30, 57, 118, and 2383 bus systems. The genetic algorithm consists of four objectives: 1) Ensuring the overall system observability; 2) Placing the minimum number of PMUs in the system; 3) Reaching the pre-specified system level of reliability of observability; and 4) Maximizing of the overall system reliability of observability. The weights associated with the objectives are derived from both the relative importance given to the goals and computation runs. Clearly, incorporating the reliability into the model will dictate placement of additional PMUs as compared to traditional PMU placement problems and, eventually, results in more costlier solution. This extra cost is the cost of achieving a higher reliability level.

The objectives of PMU minimization and reliability of observability maximization are in conflict since the number of PMUs must increase to reach higher levels of reliability of observability. To resolve the conflict, relative weights should be assigned to each objective to combine two conflicting objective functions into a single objective function. The multi-objective programming model can be formulated as follows:

$$Max\ w_1\left(R - R_{desired}\right) + w_2\left(\frac{\sum_{i=1}^{N}(1 - x_i)}{N}\right) \tag{70}$$

where R is given in (18), $R_{desired}$ is the desired system wide reliability of observability level, x_i is the binary decision variable indicating whether or not a PMU placed on bus i, and w_i defines the weight associated to each objective. Note that the sum of w_i is equal to one, and N is the total number of buses in the system. The weights in (70) are derived both from the relative importance given to the objectives and pilot runs. The scales are different for R and x, therefore the model in (70) has been standardized by dividing the total number of buses without PMUs by the total number of buses in the system for the second objective to assign the value between 0 and 1, so that the two objectives can be represented by a compatible scale.

In the multi-objective PMU placement problem, we develop a genetic algorithm based approach. The algorithm basically mimics the process of natural evolution using the inheritance and adaptive processes. In addition to those, the mutation and crossover operators are also used.

In this approach, a binary encoding is implemented for the genetic algorithm. The presence of the PMU at a particular bus is represented with a binary number. If the PMU is placed on that particular bus, then the corresponding value of the bus is set to 1 indicating that a PMU is placed. Otherwise, it takes the value of 0. Below is an example representation of a particular solution for the genetic algorithm in a 10-node system.

1 0 1 0 1 0 0 0 1 0

Based on this particular representation, the PMUs are placed on the nodes 1, 3, 5, and 9. The genetic algorithm begins with the generation of the initial population.

For creating the solutions that constitute the initial population, the first step is creating random numbers that are uniformly distributed between 0 and the bus size. Based on this number, a reference threshold value is calculated by dividing this randomly generated number by the number of buses. For each bus, a random number that is uniformly distributed between 0 and 1 is generated to decide whether a PMU is placed on a particular bus. If the generated number is less than the threshold value, then a PMU is placed in the corresponding bus, otherwise no PMU is placed. For example, in a 30 bus system, suppose that initially, number 18 is generated. That corresponds to the threshold value of 0.6. Suppose that for the first bus the generated random number turns out to be 0.65. Since the number is larger than 0.6, a PMU is not placed on that bus. This will be repeated for each bus in the system to obtain the first randomly generated solution. This procedure is repeated for each solution in the initial pool. Table 3.36 provides the parameters associated with the genetic algorithm. Note that some values are divided by the separator (i.e., |), which means that for the 14, 30, 57, and 118 bus systems, the first value applies, whereas for 2383 bus system, the second value is utilized. To cite an instance, for the 2383 bus system, a population size of 500 is used, whereas for the others, a population size of 60 is used.

Table 3.36. Genetic algorithm parameters.

Parameter	Values
Population Size	60|500
Number of offsprings created in each generation	30|74
Number of population members selected by the elitist selection rule	10|50
Number of population members selected by the roulette wheel selection	50|450
Mutation probability	0.01
Generation Limit	5000|15000
ω1	4/9
ω2	1/10
ω3	2/5
ω4	1/18

After the initial population generation, a corresponding fitness function is calculated. The fitness function is calculated based on several factors:

- Number of covered buses divided by total number of buses (i.e., θ_1)
- Number of buses that no PMU is placed divided by total number of buses (i.e., θ_2)
- Whether the system threshold reliability level is exceeded or not (i.e., θ_3-a binary value, 0 if not exceeded, 1 otherwise)
- Overall system reliability level (θ_4)

Note that all θ values are between 0 and 1. This standardization helps us develop the corresponding weights for each factor. Based on these criteria, the following fitness function is devised:

$$\mu = \sum_{i=1} \omega_i \theta_i \qquad (71)$$

Where μ is the corresponding fitness function value of the particular solution and ω_i is the corresponding weights associated with the particular criterion which is listed above.

After the fitness value for each solution is calculated, the solutions are ranked according to the descending order of fitness values. Based on these values, using roulette wheel selection scheme, the chromosomes that will undergo reproduction will be selected. Based on the roulette wheel selection scheme, the chromosomes having higher fitness function value, have higher probability of being selected for producing offsprings. For producing offspring, two different approaches are followed depending on the length of the chromosome (i.e., number of buses). For

14, 30, and 57bus systems, a traditional two-point crossover operator is applied. For the 118 and 2383 bus systems, a four-point traditional crossover operator is utilized.

In a two-point traditional crossover operator, two crossover sites are randomly selected and the part of the chromosome between those sites is exchanged among the parents. An example of the traditional two-point crossover is as follows:

Parent 1: 1 0 0 0 | 1 0 0 | 0 0 0 1 0 0 0

Parent 2: 0 0 1 1 | 0 1 0 | 1 0 1 0 1 0 1

Offspring 1: 1 0 0 0 | 0 1 0 | 0 0 0 1 0 0 0

Offspring 2: 0 0 1 1 | 1 0 0 | 1 0 1 0 1 0 1

For 118 and 2383 bus systems, four-point crossover operator is applied. In the four-point crossover operator, four crossover sites are randomly selected. In that scheme, the bits between the first and the second, and the third and fourth sites are exchanged among the parents to produce offsprings. In addition to two-point crossoveroperator (Weile and Michielssen, 1997), the efficiency of multi-point crossover operator especially for the chromosome representations involving long strings has been analyzed in the literature as well (De Jong and Spears, 1992). An example of the four-point crossover is as follows:

Parent 1: 1 0 0 | 0 1 0 0 | 0 0 0 | 1 0 0 0

Parent 2: 0 0 1 | 1 0 1 0 | 1 0 1 | 0 1 0 1

Offspring 1: 1 0 0 | 1 0 1 0 | 0 0 0 | 0 1 0 1

Offspring 2: 0 0 1 | 0 1 0 0 | 1 0 1 | 1 0 0 0

After offsprings are created, the mutation operator is applied for the offpsrings. Bit by bit consideration is provided for the mutation. A random number uniformly distributed between 0 and 1 is generated for each bus in the chromosome representation. If the generated number is smaller than the mutation probability, then the corresponding bit is switched from 0 to 1 or 1 to 0, thus placing or removing the PMU on the corresponding bus.

After all the offpsprings are created using the crossover operator and modified using mutation operator, the existing population and created offsprings are collected in a single pool and ranked based on the descending order of the fitness function which is presented in Equation(71). A combination of the elitist and roulette wheel selection is applied for forming the new generation. Again a distinction is made based on the problem size. For 14, 30, 57, and 118 bus systems, the top 10 chromosomes are selected and included directly in the new generation using the elitist generation scheme. For the 2383 bus system, this number is set to be 50. The remaining chromosomes are selected based on the roulette wheel selection rule. After forming the new generation, the same sequence of procedures are applied (i.e., selection for producing offsprings, crossover, mutation, and the selection for the new generation) on the new generation, and this is repeated until the generation limit is reached (i.e., 15,000 for 2383 bus systems, 5000 for the rest).

3.6.2. Discussions and Computational Results

The proposed genetic algorithm is tested for the IEEE 14, 30, 57, 118, and 2383 bus standard test systems. The code is developed on Matlab 2010a platform and run on a computer having 2.66 GHz Intel(R) Core™ 2 Quad CPU with memory of 2.96 GB. The results are reported with $R_{desired}$ parameter set to be 0.90 for all system types.

A summary of the results for the all standard IEEE types is presented in Tables 3.37, 3.38, and 3.39 for individual PMU reliability of 0.95, 0.99, and 0.99833 respectively. The IEEE 2383 test system is missing in Table 3.37 since the test system is not able to reach the desired reliability of observability of 0.90 with the PMU reliability of 0.95. The PMU reliabilities of 0.99 and 0.99833 are achieved for the reliability of observability of 0.90 in IEEE 2383 standard bus system. The required number of PMUs and achieved overall system reliability of observability are calculated for PMU reliabilities of 0.95, 0.99, and 0.99833. In practice, PMU reliabilities are reported around the value of 0.99 (Yang Wang et al., 2009b).

Table 3.37. Genetic algorithm placement results for PMU reliability of 0.95.

IEEE System	#PMU	Reliability Achieved
14	8	0.9329
30	20	0.9142
57	35	0.901
118	82	0.9009

Table 3.38. Genetic algorithm placement results for PMU reliability of 0.99.

IEEE System	#PMU	Reliability Achieved
14	5	0.9315
30	13	0.9123
57	27	0.9004
118	59	0.907
2383	2250	0.9003

Table 3.39. Genetic algorithm placement results for PMU reliability of 0.99833.

IEEE System	#PMU	Reliability Achieved
14	4	0.9818
30	10	0.9736
57	17	0.9244
118	35	0.9045
2383	1993	0.9004

The PMU locations for the standard IEEE test systems for PMU reliability of 0.95, 0.99, and 0.99833 are shown in Tables 3.40, 3.41, and 3.42 respectively. It should be noted that for the IEEE 2383 test system, non-PMU buses rather than PMU-buses are presented for the purpose of brevity.

Based on the results shown in Tables 3.37, 3.38 and 3.39, it is clear that with the increase in the bus size, higher redundancy level in terms of the number of PMUs is required to maintain the desired reliability of observability levels since overall reliability is calculated by multiplication of the individual bus reliabilities.

Table 3.40. Genetic algorithm locations for PMU reliability of 0.95.

IEEE System	PMU Locations
14	2,4,5,6,7,9,11,13
30	1, 2, 3, 5, 6, 9, 10, 11, 12, 13, 15, 16, 18, 19, 22, 24, 25, 27, 28
57	1, 2, 4, 6, 9, 10, 11, 12, 15, 18, 19, 21, 22, 24, 25, 27, 28, 29, 30, 32, 33, 34, 36, 37, 39, 41, 44, 45, 46, 47, 49, 50, 53, 54, 56
118	1, 2, 5, 6, 7, 9, 10, 11, 12, 15, 17, 19, 20, 21, 22, 23, 24, 26, 27, 28, 29, 30, 32, 34, 35, 36, 37, 40, 42, 43, 44, 45, 46, 47, 49, 51, 52, 53, 54, 56, 57, 59, 61, 62, 64, 65, 66, 68, 70, 71, 73, 75, 76, 77, 78, 79, 80, 83, 84, 85, 86, 87, 89, 90, 91, 92, 94, 96, 100, 101, 105, 106, 108, 109, 110, 111, 112, 114, 115, 116, 117, 118

Table 3.41. Genetic algorithm locations for PMU reliability of 0.99.

IEEE System	PMU Locations
14	2,6,7,9,13
30	1, 2, 6, 9, 10, 12, 15, 16, 19, 24, 25, 27, 30
57	1, 4, 6, 9, 12, 15, 19, 21, 22, 24, 26, 27, 29, 30, 32, 34, 36, 37, 41, 45, 46, 47, 49, 50, 52, 54, 56
118	1, 5, 7, 9, 10, 11, 12, 15, 17, 19, 21, 22, 24, 26, 27, 28, 30, 32, 34, 36, 37, 40, 44, 45, 46, 49, 51, 52, 54, 56, 57, 59, 62, 64, 65, 66, 68, 70, 71, 75, 77, 78, 80, 83, 85, 86, 89, 90, 92, 94, 96, 100, 101, 105, 106, 109, 110, 114, 118
2383	All buses except {17, 25, 26, 27, 31, 36, 52, 54, 59, 69, 79, 95, 98, 115, 120, 129, 160, 165, 166, 199, 203, 208, 221, 234, 283, 286, 318, 323, 347, 349, 376, 378, 413, 417, 431, 439, 443, 465, 497, 503, 549, 561, 565, 570, 590, 596, 598, 604, 610, 618, 621, 643, 653, 702, 725, 770, 771, 772, 775, 785, 804, 808, 838, 890, 893, 918, 921, 926, 947, 1055, 1058, 1066, 1088, 1089, 1130, 1143, 1169, 1193, 1196, 1215, 1220, 1223, 1266, 1344, 1372, 1380, 1398, 1411, 1445, 1479, 1500, 1501, 1527, 1536, 1552, 1566, 1579, 1582, 1638, 1658, 1663, 1674, 1702, 1704, 1724, 1742, 1752, 1826, 1833, 1838, 1863, 1881, 1902, 1950, 1960, 1962, 1965, 1971, 2014, 2020, 2037, 2038, 2097, 2138, 2155, 2156, 2194, 2249, 2321, 2344, 2352, 2357, 2380}

Table 3.42. Genetic algorithm locations for PMU reliability of 0.99833.

IEEE System	PMU Locations
14	2,6,7,9
30	1, 2, 6, 9, 10, 12, 15, 19, 25, 27
57	1, 4, 9, 10, 20, 22, 25, 27, 29, 32, 36, 39, 41, 45, 46, 49, 54
118	3, 5, 9, 12, 15, 17, 21, 23, 27, 29, 30, 32, 34, 37, 40, 45, 49, 51, 54, 56, 62, 64, 68, 71, 75, 77, 80, 85, 86, 89, 92, 96, 100. 105, 110
2383	All buses except {5, 10, 11, 14, 15, 20, 21, 24, 26, 27, 29, 35, 36, 41, 44, 46, 47, 59, 60, 66, 70, 75, 76, 80, 83, 87, 88, 91, 98, 101, 110, 115, 117, 123, 126, 131, 143, 144, 150, 154, 159, 162, 163, 166, 167, 169, 170, 172, 182, 187, 194, 195, 210, 211, 212, 220, 222, 226, 234, 237, 238, 244, 253, 254, 256, 269, 270, 272, 281, 282, 283, 290, 294, 296, 298, 303, 304, 307, 308, 317, 324, 333, 340, 342, 349, 363, 370, 372, 381, 389, 400, 410, 412, 417, 420, 426, 427, 430, 431, 432, 439, 449, 451, 452, 457, 478, 484, 487, 489, 491, 506, 523, 532, 534, 536, 537, 544, 547, 553, 559, 564, 567, 570, 572, 575, 579, 581, 584, 595, 596, 603, 605, 607, 609, 616, 617, 627, 634, 636, 637, 640, 641, 651, 655, 665, 668, 670, 684, 687, 704, 705, 709, 714, 729, 731, 732, 735, 739, 746, 748, 749, 757, 773, 779, 782, 789, 793, 794, 813, 823, 830, 834, 840, 842, 865, 866, 888, 890, 899, 900, 917, 924, 932, 960, 963, 964, 975, 982, 994, 1000, 1001, 1003, 1016, 1021, 1042, 1045, 1062, 1074, 1077, 1079, 1098, 1105, 1117, 1134, 1142, 1144, 1155, 1164, 1169, 1173, 1189, 1194, 1195, 1204, 1207, 1210, 1223, 1227, 1235, 1236, 1252, 1262, 1264, 1276, 1291, 1292, 1312, 1320, 1326, 1328, 1329, 1339, 1342, 1343, 1344, 1363, 1372, 1373, 1374, 1375, 1377, 1390, 1394, 1395, 1401, 1403, 1411, 1417, 1420, 1421, 1427, 1444, 1450, 1459, 1466, 1471, 1478, 1491, 1492, 1495, 1501, 1515, 1517, 1526, 1549, 1553, 1557, 1560, 1563, 1565, 1566, 1567, 1577, 1583, 1586, 1591, 1598, 1606, 1612, 1613, 1634, 1644, 1646, 1650, 1659, 1670, 1683, 1700, 1702, 1705, 1709, 1715, 1718, 1720, 1737, 1743, 1744, 1745, 1752, 1759, 1762, 1775, 1777, 1778, 1788, 1791, 1795, 1801, 1815, 1819, 1831, 1838, 1847, 1848, 1853, 1869, 1891, 1897, 1911, 1916, 1924, 1932, 1935, 1942, 1945, 1947, 1956, 1959, 1960, 1965, 1966, 1967, 1983, 1988, 1989, 1990, 1992, 1995, 2000, 2014, 2039, 2073, 2087, 2088, 2090, 2093, 2097, 2107, 2123, 2126, 2127, 2128, 2136, 2147, 2165, 2171, 2180, 2188, 2193, 2198, 2220, 2227, 2229, 2238, 2242, 2250, 2254, 2264, 2265, 2267, 2285, 2292, 2304, 2309, 2316, 2319, 2322, 2326, 2328, 2331, 2335, 2336, 2341, 2344, 2353, 2359, 2362, 2368, 2375, 2376}

Figure 3.10 provides the evolution of the quality of the best solution found during the computation with respect to generations based on individual PMU reliability of 95% for GA. The left y-axis indicates the number of PMUs, whereas the right y-axis indicates the overall score and the total system reliability. The x-axis indicates the generation number. Note that throughout the generations, the number of PMUs is decreasing, whereas the overall score that is provided in Equation(71) is increasing. In terms of the total system reliability, there is a fluctuation. Initially, the system reliability at some generations exceeds the level of 0.96, but throughout the generations, it converges to the target level of 0.9, whereas the number of PMUs is decreasing initially from the 114 to 82. Better PMU placement, results in fewer PMUs which leads to less costly PMU placement strategies, can retain relatively the same level of reliability of observability. Another interesting point to note is that after approximately a generation number of 2750, the population converges and no other changes are observed afterwards.

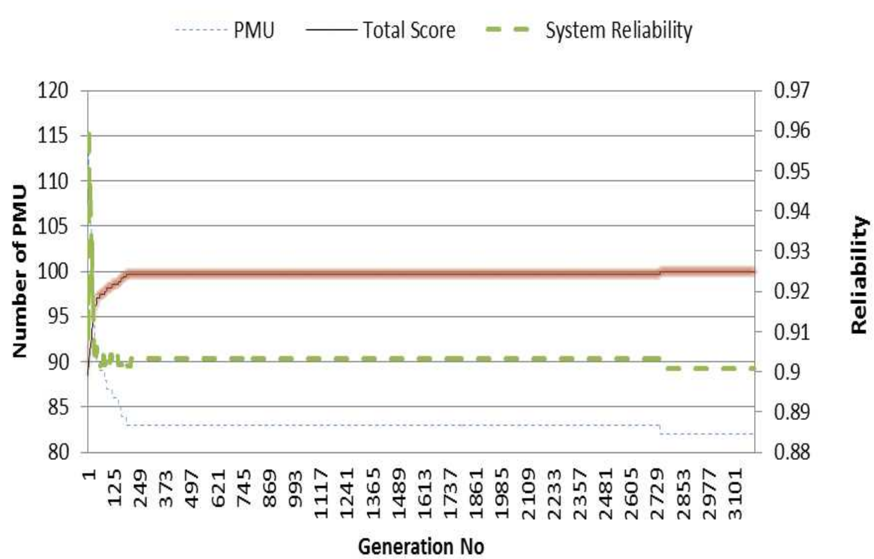

Figure 3.10. Evolution of the quality of the incumbent solution provided by the GA for individual PMU reliability of 95% for IEEE 118 bus system.

CHAPTER 4. DISSCUSSIONS

The effectiveness of the proposed approaches is further investigated by comparing the results of the Genetic Algorithm (GA) presented in Khiabani et al. (2013c), the Reliability-Based Placement (RBP) approach presented in Khiabani et al. (2012a), Goal Programming Based (GPB) approach in Khiabani et al. (2013a), and Optimal PMU Placement (OPP) results in Kavasseri and Srinivasan (2011). Results for the case of p=0.99 with minimum desired system wide reliability of observability level of 0.90 (i.e., $R_{desired}$) are presented in Table 4.1 for IEEE 14, 30, 57, 118, and 2383 standard bus systems.

Table 4.1. Comparison of results for PMU reliability of 0.99.

	#PMU				R			
IEEE System	GA	GPB	OPP	RBP	GA	GPB	OPP	RBP
14	5	5	4	9	0.93	0.93	0.89	0.98
30	13	14	10	21	0.91	0.93	0.84	0.95
57	27	29	17	57	0.90	0.92	0.62	0.99
118	59	71	32	115	0.90	0.91	0.44	0.99
2383	2250	N/A	N/A	N/A	0.90	N/A	N/A	N/A

The mentioned approaches fail to solve IEEE 2383 bus system with reliability considerations (Kavasseri and Srinivasan, 2011, Khiabani et al., 2012a, and Khiabani et al., 2013a). Not only the GA approach is able to solve the 2383 bus-system problems but also performs better in terms of solution quality as compared to other approaches for solving large scale systems.

Comparison of GA and OPP based on Table 4.1 indicates that although the GA approach presents the solutions with more PMUs as compared to OPP, the solution achieves a higher system wide reliability of observability level. Only for the case of IEEE 14 bus system, the OPP performs slightly worse than the GA based-solution by placing 4 PMUs and almost reaching the minimum required reliability of 0.9 as compared to GA based-solution. However, in the case of IEEE 118 bus system, the system requires 27 additional PMUs to increase the reliability of observability from 44% to the desired target value of 90%. It should be noted that in the conventional PMU placement problems, loss of a PMU would result in loss of the observability of the majority of the neighboring buses and, therefore, loss of observability of the system. Hence, such placements are not fault tolerant.

Also comparing the GA approach to RBP and GPB, it is clear that the former outperforms the latter ones. Since the proposed GA model reaches the desired reliability of observability level of 90% with approximately half the number of PMUs that would be required for the RBP based solution, using the GA based approach might lead to significant cost savings. To cite an instance, GA based solution for the 118 bus system specifies 12 fewer PMUs as compared to GBP. For the case of comparison of GA with the RBP, the difference is much more significant, i.e., 56 PMUs. Although RBP and GPB based approaches provide higher level of reliability of observability at the expense of higher number of PMUs, they are considered to be overkill especially when the target level is set to be 0.9. GA based approach reaches the desired level of reliability with fewer PMUs. Hence, the proposed GA model not only is able to solve the large scale problems but also gives a better solution for the majority of the small size problems as compared to the two other reliability-based approaches by using the least number of PMUs given the desired level of reliability of observability. The GA based approach provides the solution with a closer value of system reliability to the target level as compared to the other approaches.

The comparison of the effect of PMU reliability on the multi-objective placement has been shown in Figure 4.1. From the figure, as expected, it can be seen that higher level of individual PMU reliability results in fewer PMUs required to reach the desired overall system reliability of observability and the effect increases as the size of the system grows. The secondary y-axis indicates the IEEE 2383 bus system values only.

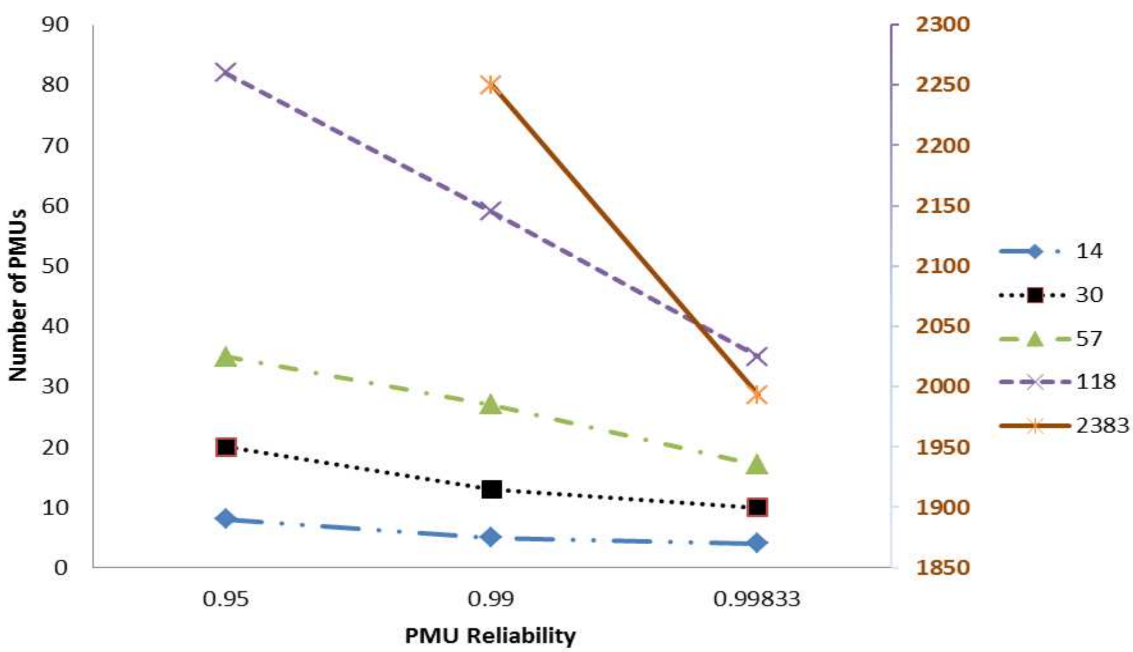

Figure 4.1. Effect of PMU reliability on the different IEEE standard test systems.

These findings are helpful to understand the effectiveness of the proposed models. However to further investigate the effectiveness of the presented approaches, lets analyze the results cost wise. Damir Novosel (2007) reported the cost of the blackouts for the customers and society in general as well as for the power companies and emphasized on the importance of the reliability of the power grid. Although large-scale blackouts are rare, they carry enormous costs and consequences for the customers, society, and power companies. The research reported an estimated society cost of six billion dollars for August 14, 2003 blackout in the US and Canada. They reported typical PMU deployment, High-end hardware, Engineering and Training and installation costs of $47,000, $30,000 and $25,000 respectively.

Abbasy and Ismail (2009) reported the cost of PMUs to be approximately between $30,000 and $40,000. They described that the prices would vary based on many factors such as number of channels (terminals), GPS antenna connection, power connection, station ground connection, current transformers (CT) and potential transformers (PT) connections. Miller (2010) reported reduction in the PMU prices, with the new cost of $14,000 on average with the installation costs which typically exceed $20,000. Because of the trend of decreasingcost of PMUs, a cost of $30,000 in total for a PMU and installation was assumed. Table 4.2 shows the costs associated with the placement results in Table 4.1. Based on the results, OPP approach is cost effective, yet does not consider reliability of power systems.

By not considering the grids reliability, OPP may result in immense blackout costs in the long term. Therefore considering both the reliability aspect and cost of the power system, GA is the best scenario. Figures 4.2 and 4.3 show the effect of the OPP, GA, GPB and RBP approaches on the PMU placement costs for the IEEE 14, 30, 57 and 118 bus systems.

Table 4.2. Comparison of placement prices for IEEE systems.

IEEE System	GA	GPB	OPP	RBP
14	$150,000	$150,000	$120,000	$270,000
30	$390,000	$420,000	$300,000	$630,000
57	$810,000	$870,000	$510,000	$1,710,000
118	$1,770,000	$2,130,000	$960,000	$3,450,000

As mentioned before, prior studies that have noted the importance of the reliability of the smart grid emphasized on the long term cost effectiveness associated with the reliability of the smart grid. Therefore the models developed in this dissertation will be beneficial and useful for the power customers, power producers, transmission companies, distributed energy resources, and electric utility companies. The avoidance of the cost of blackouts on society and the economy would far outweigh the costs invested on the reliability of the grid.

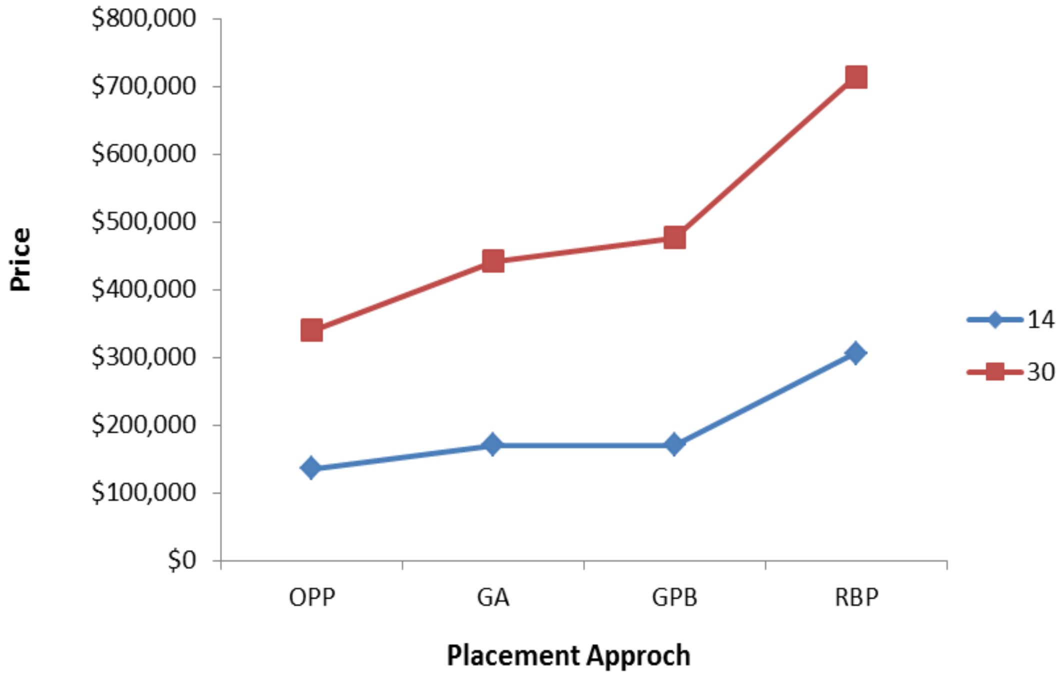

Figure 4.2. Effect of the placement approach on the price of IEEE 14 & 30 test systems.

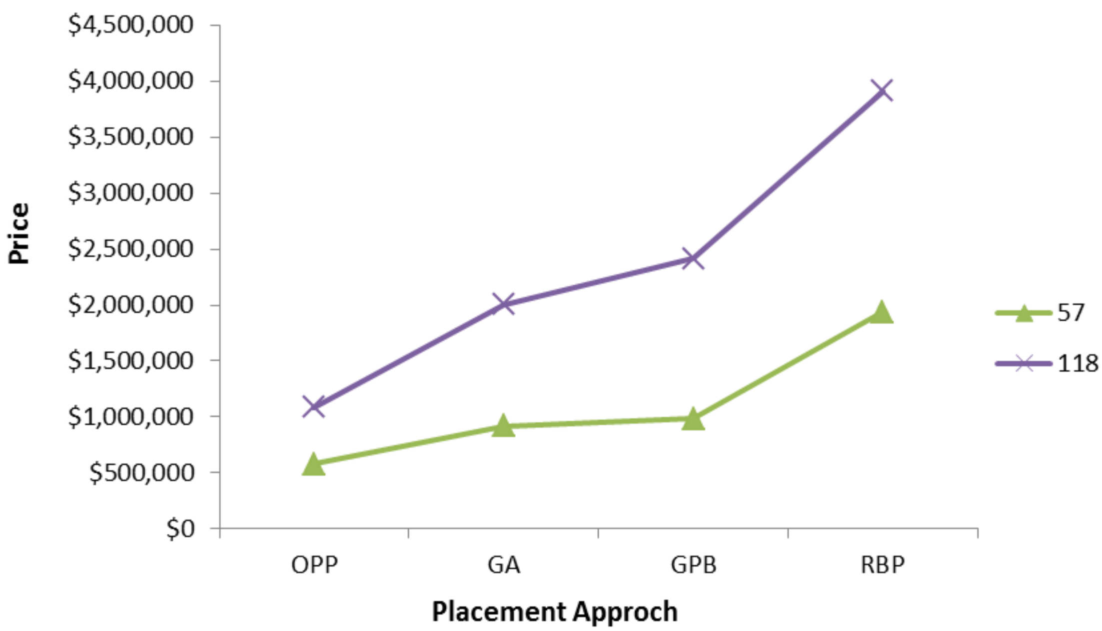

Figure 4.3. Effect of the placement approach on the price of IEEE 57 & 118 test systems.

CHAPTER 5. CONCLUSIONS

This dissertation presents modeling and solving multi-objective PMU placement approach for power system observability. Efficient and reliable WAMS is crucial to preventing outages and cascading failures in the smart grid. Since PMUs are the critical parts of the WAMS, the questions of the arrangement and number of PMUs to use and place in order to assess risk must be addressed.

The key idea is in the consideration of system and PMU reliabilities in order to determine different redundancy levels at different buses. The main contribution of this dissertation was to bring reliability considerations into PMU placement in large and complex power systems. The consideration of targeting overall system reliability along with individual PMU reliability provides much better understanding and insight while determining redundancy level at each node or bus. The concept of reliability of observability was introduced to incorporate and connect PMU reliabilities to power network observability.

In this study a reliability-based two-stage optimization model was proposed (Khiabani et al., 2012a). The model was then improved by developing a multi-objective optimization approach (Khiabani et al., 2012b) and a goal programming multi-objective optimization formulation (Khiabani et al., 2013a) to fix the identical level of the redundancy levels for all buses in Khiabani et al. (2012a) relaxing the existence of the limited number of PMUs. In Khiabani et al. (2013b) the PMU placement problem was considered from a maximum coverage standpoint, since in practice the resources are limited due to the high cost of PMUs and their installation.

Previous reliability-based approaches fail to provide a solution for IEEE-2383 bus system; therefore, a genetic algorithm approach was proposed. In this approach, two conflicting objectives of exceeding a target reliability of system observability and minimizing the number of PMUs were tackled. The proposed algorithm is compared to other approaches using the IEEE-14, 30, 57, 118, and 2383 bus power systems with different individual PMU reliabilities. Compared to the traditional optimal PMU placement methods, the proposed approaches are superior in terms of reliability of system observability. As compared to OPP based approach, the GA approach significantly improved the system reliability of observability from ~45% to more than ~90% for IEEE 118 bus system.

In short, the proposed GA based solution methodology provides a balanced approach for providing the desired level of system reliability of observability with the optimal or near-optimal number of PMUs as compared to other approaches. The proposed approach is the most balanced approach in satisfying both objectives of reaching the level of the desired reliability of system observability and minimizing the total number of PMUs placed, as compared to other reliability-based approaches. At the same time, it also considers the reliability perspective of the system that is neglected by traditional PMU placement approaches by placing the minimum additional number of PMUs at the expense of reduced reliability where the failure of one PMU might result in the total loss of system observability.

Although the inclusion of system reliability constraint results in the placement of more PMUs, this improves reliability of observability significantly. However, this increase in the number of PMUs can be managed by ensuring the usage of highly reliable PMUs. These models are very effective in computing placement solutions with the desired levels of system reliability, given the reliability of individual PMUs. This connection between observability and system reliability could be potentially useful in evaluating the reliability of placement scenarios in large and complex electric grids.

This dissertation presents and highlights the reliability-based PMU placement. However in the future the reliability of the WAMS can further be investigated with the inclusion of the real time PMU failure rates, PDC failure rates, etc. in the multi-objective model and investigating the placement scenario considering the reliability of observability target of the WAMS. Further studies with the focus on economic analysis of the WAMS could also be considered. A comprehensive model considering cost of reliability, blackouts, PMUs of different channel types, PDCs, etc. could be developed and the number and locations of the PMUs could be determined incorporating the long run blackout costs. In this model the goal will be to maximize the overall power system reliability with the limited number of PMUs dictated by the economic factors. In addition incorporating the conventional flow measurements into the multi-objective model is suggested. This model will be helpful in reducing the investment needed to reach the predefined level of reliability of observability. The conventional flow measurements incorporation would be helpful since fewer number of PMUs will be used, given that the flow measurement costs are 1/3 of the PMU cost. In that scenario, the PMU and flow measurement reliabilities and the desired system wide reliability of observability would be factored as inputs into the model. In the case of existing power systems with the conventional flow measurements already installed, the model could be used to find the required number of PMUs and their arrangement.

In future investigations, it might be possible to incorporate the lines and their reliabilities into the model to develop a comprehensive multi objective model.

REFERENCES

Abur Ali and Exposito Antonio. Gomez (2004). *Power System State Estimation Theory and Implementation.* CRC Press.

Abbasy, N. H., & Ismail, H. M. (2009). A unified approach for the optimal PMU location for power system state estimation. *IEEE Transactions on Power Systems, 24*(2), 806-813.

Abdelaziz, A. Y., Ibrahim, A. M., & Salem, R. H (2013). Power system observability with minimum phasor measurement units placement. *International Journal of Engineering, Science and Technology, 5*(3), 1-18.

Ahmadi A, Alinejad-Beromi Y and Moradi M (2011). Optimal PMU placement for power system observability using binary particle swarm optimization and considering measurement redundancy. *Expert Systems with Applications, 38*, 7263-7269.

Alinejad-Beromi Y, Ahmadi A, Rezai. Soleymanpour H (2011). Optimal PMU Placement Considering Contingencies by Using Hybrid Discrete Particle Swarm Optimization Technique. *International Review of Electrical Engineering-IREE, 6*(4), 1927-1938.

Aminifar F, Khodaei A, MF-Firuzabad and Mohammad Shahidehpour (2010). Contingency-Constrained PMU Placement in Power Networks. *IEEE Trans. Power Systems, 25*(1), 516-523.

Aminifar F, Lucas C, Khodaei A and Fotuhi-Firuzabad M (2009). Optimal Placement of Phasor Measurement Units Using Immunity Genetic Algorithm. *IEEE Trans on Power Delivery, 24*(3), 1014-1020.

Aminifar Farrokh, Fotuhi-Firuzabad Mahmud, Shahidehpour Mohammad and Khodaei Amin (2011). Probabilistic Multistage PMU Placement in Electric Power Systems. *IEEE Trans on Power Delivery*, *26*(2), 841-849.

Aminifar, F., Fotuhi-Firuzabad, M., & Safdarian, A (2013). Optimal PMU Placement Based on Probabilistic Cost/Benefit Analysis. *IEEE Transactions on Power Systems*, *28*(1), 566-567.

Anderson, J. E., & Chakrabortty, A (2012). A minimum cover algorithm for PMU placement in power system networks under line observability constraints. *IEEE Power and Energy Society General Meeting,* 1-7.

Azizi, S., Dobakhshari, A. S., Nezam Sarmadi, S. A., & Ranjbar, A. M (2012). Optimal PMU placement by an equivalent linear formulation for exhaustive search. *IEEE Transactions on Smart Grid*, *3*(1), 174-182.

Baldwin T. L, Mili L, Boisen M. B and Adapa R (1993). Power system observability with minimal phasor measurement placement. *IEEE Trans. Power Systems, 8*(2), 707-715.

Bergen Arthur. R and Vittal Vijay (2000). *Power system analysis.* New Jersey: Prentice-Hall.

Brueni D.J and Heath L.S. The PMU placement problem (2005). *SIAM J. Discrete Math*, *19*(3), 744-761.

Cepeda, J. C., Rueda, J. L., Erlich, I., & Colome, D. G (2012). Probabilistic approach-based PMU placement for real-time power system vulnerability assessment. *IEEE PES International Conference and Exhibition on Innovative Smart Grid Technologies (ISGT Europe),* 1-8.

Chakrabarti S and Kyriakides E (2008). Optimal placement of phasor measurement units for power system observability. *IEEE Trans. Power Systems*, *23*(3), 1433-1440.

Chakrabarti S, Kyriakides E, Ledwich G and Ghosh A (2010). Inclusion of PMU current phasor measurements in a power system state estimator. *IET Generation, Transmission & Distribution, 4*(10), 1104-1115.

Chawasak R, Suttichai P, Sermsak U and Neville R. W (2007). An Optimal PMU Placement Method against Measurement Loss and Branch Outage. *IEEE Trans. On Power Delivery, 22*(1), 101-107.

Damir Novosel (2007). PHASOR MEASUREMENT APPLICATION STUDY. FINAL PROJECT REPORT Prepared for CIEE By: KEMA, Inc. Project Manager: Damir Novosel.

De Jong, K. A., & Spears, W. M. (1992). A formal analysis of the role of multi-point crossover in genetic algorithms. *Annals of Mathematics and Artificial Intelligence*, *5*(1), 1-26.

Denegri G.B, Invernizzi M and Milano F (2002). A security oriented approach to PMU positioning for advanced monitoring of a transmission grid. *In Proc. IEEE Int. Conf. Power Syst*, *2,* 798–803.

Dongjie X, Renmu H, Peng W and Tao X (2004). Comparison of several PMU placement algorithms for state estimation. *In Proc. Inst. Elect. Eng. Int. Conf. Develop. Power Syst. Protection,* 32–35.

Dua D, Dhambhare S, Gajbhiye R.K and Soman S.A (2008). Optimal multistage scheduling of PMU placement: An ILP approach. *IEEE Trans. Power Delivery, 23*(4), 1812-1820.

Emami R and Abur A (2010). Robust Measurement Design by Placing Synchronized Phasor Measurements on Network Branches. *IEEE Trans on Power Systems*, *25*(1), 38-43.

Gao, X (2013). An optimal PMU placement method considering bus weight and voltage stability. *IEEE International Conference on Environment and Electrical Engineering (EEEIC),* 124-129.

Ghosh, D., Ghose, T., & Mohanta, D. K (2013). Reliability Analysis of Geographic Information System (GIS) Aided Optimal Phasor Measurement Unit Location for Smart Grid Operation. *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*.

Gou B (2008). Generalized integer linear programming formulation for optimal PMU placement. *IEEE Trans. on Power Systems*, *23*(3), 1099-1104.

Gou B (2008). Optimal placement of PMUs by integer linear programming. *IEEE Trans. on Power Systems*, *23*(3), 1525-1526.

Gómez, O., & Ríos, M. A (2013). ILP-based multistage placement of PMUs with dynamic monitoring constraints. *International Journal of Electrical Power & Energy Systems*, *53*, 95-105.

Gopakumar, P., Reddy, M. J. B., & Mohanta, D. K (2013). Novel multi-stage simulated annealing for optimal placement of PMUs in conjunction with conventional measurements. *IEEE International Conference on Environment and Electrical Engineering (EEEIC),* 248-252.

Gupta, N., Goyal, M., & Tripathy, P (2012). A novel approach for optimal placement of PMUs with minimum measurement channels. *IEEE International Conference on Power and Energy (PECon)*, 505-509.

Gyllstrom, D., Rosensweig, E., & Kurose, J (2012). On the impact of PMU placement on observability and cross-validation. *In Proceedings of the 3rd International Conference on Future Energy Systems: Where Energy, Computing and Communication Meet*, 20.

Hajian Mahdi, Ranjbar Ali Mohammad, Amraee Turaj and Mozafari Babak (2011). Optimal placement of PMUs to maintain network observability using a modified BPSO algorithm. *Electrical Power and Energy Systems, 33*, 28-34.

Huang, J., & Wu, N. E (2012). Fault-tolerant placement of phasor measurement units based on control reconfigurability. *Control Engineering Practice, 21*(1), 1-11.

Huang, J., & Wu, N. E (2013). A new scalable solution to optimal PMU placement under a long-run data availability criterion. *American Control Conference (ACC),* 5068-5073.

Hurtgen M and Maun J.-C (2010). Optimal PMU placement using Iterated Local Search. *Electrical and Energy Systems, 32,* 857-860.

IEEE Working Group H-8 (1998). IEEE Standard for Synchrophasors for Power Systems. *IEEE Transactions on Power Delivery*, *13*(1).

Jamuna K and Swarup K.S (2011). Optimal placement of PMU and SCADA measurements for security constrained state estimation. *International Journal of Electrical Power & Energy Systems.*

Jamuna, K., & Swarup, K. S (2012). Multi-objective biogeography based optimization for optimal PMU placement. *Applied Soft Computing*, *12*(5), 1503-1510.

Jiang, Q., Li, X., Wang, B., & Wang, H (2012). PMU-Based Fault Location Using Voltage Measurements in Large Transmission Networks. *IEEE Transactions on Power Delivery, 27*(3), 1644-1652.

Kamwa Innocent and Grondin Robert (2002). PMU Configuration for System Dynamic Performance Measurement in Large Multiarea Power Systems. *IEEE Trans. Power Systems, 17*(2), 385-394.

Kavasseri R and Srinivasan S.K (2010). Joint optimal placement of PMU and conventional measurements in power systems. *Circuits and Systems (ISCAS), Proceedings of 2010 IEEE International Symposium*, 3449-3452.

Kavasseri R and Srinivasan S.K (2011). Joint Placement of Phasor and Conventional Power Flow Measurements for Fault Observability of Power Systems. *IET Generation, Transmission & Distribution, 5*(10), 1019-1024.

Kekatos, V., Giannakis, G. B., & Wollenberg, B (2012). Optimal placement of phasor measurement units via convex relaxation. *IEEE Transactions on Power Systems*, *27*(3), 1521-1530.

Ketabi, A., Nosratabadi, S. M., & Sheibani, M. R (2012). Optimal PMU Placement with Uncertainty Using Pareto Method. *Mathematical Problems in Engineering*, 2012.

Khiabani, V., Yadav, O. P., & Kavasseri, R (2012a). Reliability-based placement of Phasor Measurement Unit in Power Systems. *Proc. Institution of Mechanical Engineers (IMechE), Part O: Journal of Risk and Reliability, 226*(1), 109-117.

Khiabani, V., R. Kavasseri and K. Farahmand (2012b). A Reliability Based Multi-Objective Formulation for Optimal PMU Placement, *Journal of International Review on Modelling and Simulations (I.RE.MO.S.), 5*(4), 1640-1644.

Khiabani, V., M. Hamidi, K. Farahmand and R. Aghatehrani (2013a). A Goal Programming Approach for Optimal PMU placement. *Journal of International Review on Modelling and Simulations (I.RE.MO.S.), 6*(2), 490-497.

Khiabani, V., and K. Farahmand (2013b). Max Covering Phasor Measurement Units Placement for Partial Power system Observabilitity. *Engineering Management Research, 2*(1), 43-54. http://dx.doi.org/10.5539/emr.v2n1p43.

Khiabani, V., E. Erdem, K. Farahmand K. Nygard (2013c). Genetic Algorithm for Instrument Placement in Smart Grid", *Fifth World Congress on Nature and Biologically Inspired Computing (NaBIC), Fargo, USA, August 12-14*, 231-218.

Korres G.N (2011). An integer-arithmetic algorithm for observability analysis of systems with SCADA and PMU measurments. *Electric Power Systems Research, 81*, 1388-1402.

Koutsoukis, N. C., Manousakis, N. M., Georgilakis, P. S., & Korres, G. N (2013). Numerical observability method for optimal phasor measurement units placement using recursive Tabu search method. *IET Generation, Transmission & Distribution, 7*(4), 347-356.

Li Wenyuan (2005). *Risk Assessment of Power Systems: Models, Methods, and Applications. 21*, Wiley-IEEE Press.

Lien K.L, Liu C.W, Yu C.S and Jiang J.A (2006). Transmission Network Fault Location Observability with Minimal PMU Placement. *IEEE Trans on Power Delivery, 21*(3), 1128-1136.

Liu Wen-xia, Liu Nian, Fan Yong-feng, Zhang Li-xin and Zhang Xin (2009). Reliability analysis of Wide Area Measurement System based on the Centralized Distributed model. *Power Systems Conference and Exposition IEEE/PES*, 1-6.

Mahaei S. Mehdi and Tarafdar Hagh M (2012). Minimizing the number of PMUs and their optimal placement in power systems. *Electric Power Systems Research, 83*, 66-72.

Marek Zima, Mats Larsson, Peter Korba, Christian Rehtanaz and Goran Andersson (2005). Design Aspects for Wide-Area Monitoring and Control Systems. *Proceedings of the IEEE, 93*(5), 980-996.

Marin F.J, Garcia-Lagos F, Joya G and Sandoval F (2003). Genetic algorithms for optimal placement of phasor measurement units in electrical networks. *Electronics Letters, 39*(19), 1403-1405.

Mahari, A., & Seyedi, H (2013). Optimal PMU placement for power system observability using BICA, considering measurement redundancy. *Electric Power Systems Research, 103*, 78-85.

Manousakis, N. M., Korres, G. N., & Georgilakis, P. S (2012). Taxonomy of PMU placement methodologies. *IEEE Transactions on Power Systems, 27*(2), 1070-1077.

Miller, B. R. (2010). Concept for Next Generation Phasor Measurement: A Low-Cost, Self-Contained, and Wireless Design. Master's thesis. University of Tennessee – Knoxville.

Miljanić, Z., Djurović, I., & Vujošević, I (2012). Optimal placement of PMUs with limited number of channels. *Electric Power Systems Research, 90*, 93-98.

Miljanić, Z., Djurović, I., & Vujošević, I (2013). Multiple channel PMU placement considering communication constraints. *Energy Systems*, 4(2), 125-135.

Milosevic B and Begovic M (2003). Nondominated sorting genetic algorithm for optimal phasor measurement placement. *IEEE Trans. on Power Syst*, *18*(1), 69–75.

Modarres Mohammad, Kaminskiy Mark and Krivtsov Vasiliy (2010). *Reliability Engineering and Risk Analysis.* CRC Press.

Mosavi, M. R., Akhyani, A. A., & Rahmati, A (2012). A PMU Placement Optimal Method in Power Systems using Modified ACO Algorithm and GPS Timing. *PRZEGLAD ELEKTROTECHNICZNY, 88*(8), 346-349.

Nabil H. Abbasy, and Hanafy M. I (2009). A Unified Approach for the Optimal PMU Location for Power System State Estimation. *IEEE Trans on Power Systems, 24*(2), 806-813.

Novosel D, Madani V, Bhargava B, Vu K and Cole J (2008). Dawn of the Grid Synchronization. *IEEE Power and Energy Magazine*, *6*(1), 49-60.

Nuqui Reynaldo F and Phadke Arun G (2005). Phasor Measurement Unit Placement Techniques for Complete and Incomplete Observability. *IEEE Trans. Power Systems, 20*(4), 2381-2388.

Pai Kung-Jui, Chang Jou-Ming and Wang Yue-Li (2010). Restricted power domination and fault-tolerant power domination on grids. *Discrete Applied Mathematics, 158*(10), 1079-1089.

Peng Chunhua, Sun Huijuan and Guo Jianfeng (2010). Multi-objective optimal PMU placement using a non-dominated sorting differential evolution algorithm. *Electrical and Energy Systems, 32*(8), 886-892.

Peng J, Sun Y and Wang H.F (2006). Optimal PMU placement for full network observability using Tabu search algorithm. *Elec. Power Syst*, *28*(4), 223–231.

Phadke A.G (1993). Synchronized phasor measurements in power systems. *IEEE Comp. Appl. in Power Systems, 6*(2), 10-15.

Phadke A.G and Thorp J.S (2008). *Synchronized Phasor Measurements and Their Applications*. New York: Springer.

Phadke, A.G., & de Moraes, R.M. (2008). The wide world of wide-area measurement. *IEEE Power Energy*, *6*(5), 52–65.

Saha Roy, B. K., Sinha, A. K., & Pradhan, A. K (2012). An optimal PMU placement technique for power system observability. *International Journal of Electrical Power & Energy Systems*, *42*(1), 71-77.

Shahraeini, M., Ghazizadeh, M. S., & Javidi, M. H (2012). Co-Optimal Placement of Measurement Devices and Their Related Communication Infrastructure in Wide Area Measurement Systems. *IEEE Transactions on Smart Grid*, *3*(2), 684-691.

Singh, M. S., Mohanta, D. K., Murthy, C., & Roy, D. S (2013). Effect of switching on standby redundancy operating mode of phasor measurement unit. *IEEE International Conference on In Environment and Electrical Engineering (EEEIC)*, 287-291.

Sodhi R, Srivastava S.C and Singh S.N (2011). Multi-criteria decision-making approach for multi-stage optimal placement of phasor measurement units. *IET Generation, Transmission & Distribution*, *5*(2), 181-190.

Sodhi Ranjana, Srivastava S.C and Singh S.N (2010). Optimal PMU placement method for complete topological and numerical observability of power system. *Electric Power Systems Research*, *80*(9), 1154-1159.

Tai, X., Marelli, D., Rohr, E., & Fu, M (2013). Optimal PMU placement for power system state estimation with random component outages. *International Journal of Electrical Power & Energy Systems*, *51*, 35-42.

Uddin, M., Kuh, A., Kavcic, A., Tanaka, T., & Mandic, D. P (2013). Grid Monitoring: Bounds on Performances of Sensor Placement Algorithms. *The Third International Conference on Smart Grids, Green Communications and IT Energy-aware Technologies*, 89-95.

Vanfretti Luigi, Chow Joe H, Sarawgi Sanjoy and Fardanesh Behruz(Bruce) (2011). A Phasor-Data-Based State Estimator Incorporating Phase Bias Correction. *IEEE Trans on Power Systems, 26*(1), 111-119.

Venkateswaran, V. B., & Kala, V (2012). Observability analysis and optimal placement of PMU using Differential Evolution algorithm. *IEEE International Conference on Emerging Trends in Electrical Engineering and Energy Management (ICETEEEM)*, 205-209.

Wang, F., Zhang, W., & Li, P (2012). Optimal incremental placement of PMUs for power system observability. *IEEE Power and Energy Society General Meeting*, 1-7.

Wang, J. M., Li, C., & Zhang, J (2012). Optimal Phasor Measurement Unit Placement by an Improved PSO Algorithm. *Asia-Pacific Power and Energy Engineering Conference (APPEEC)*, 1-4.

Weile, D. S., & Michielssen, E. (1997). Genetic algorithm optimization applied to electromagnetics: A review. *IEEE Transactions on Antennas and Propagation*, *45*(3), 343-353.

Xu B and Abur A (2004). Observability analysis and measurement placement for systems with PMUs. *IEEE Power Systems Conf. Expo, 2*, 943-946.

Xu, J., Wen, M. H., Li, V. O., & Leung, K. C (2013). Optimal PMU placement for wide-area monitoring using chemical reaction optimization. *IEEE Innovative Smart Grid Technologies (ISGT)*, 1-6.

Yang Wang, Wenyuan Li and Jiping Lu (2009b). Reliability Analysis of Phasor Measurement Unit Using Hierarchical Markov Modeling. *Electric Power Components and Systems. Taylor & Francis, 37*(5), 517-532.

Yang Wang, Wenyuan Li and Jiping Lu (2010). Reliability Analysis of Wide-Area Measurement System. *IEEE Transactions on Power Delivery, 25*(3), 1483-1491.

Yang Wang, Wenyuan Li, Jiping Lu and Honghong Liu (2009a). Evaluating multiple reliability indices of regional networks in wide area measurement system. *Electric Power Systems Research*, *79*(10), 1353-1359.