RISK MANAGEMENT STRATEGIES FOR COMMODITY PROCESSORS

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Songjiao Chen

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Risk Management Strategies for Commodity Processors

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Songjiao Chen

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MASTER OF SCIENCE

SUPERVISORY COMMITTEE:

William Wilson
Chair

Ryan Larsen

Dragan Miljkovic

Ruilin Tian

Approved:

6/21/2013	William Nganje
Date	Department Chair

ABSTRACT

Recent years have witnessed an increase in agricultural commodity price volatilities. This thesis analyzes different models to derive optimal hedge strategies for commodity processors, with two components addressed. One is the dependence structure and joint distribution among inputs, outputs, and hedging instruments that impact hedging effectiveness. The second refers to different procurement and sales scenarios a processor may encounter. A domestic flour mill company is used to demonstrate alternative hedging strategies under different processing scenarios.

Copula is a relatively new method used to capture flexible dependence structure and joint distribution among assets. The applications of copulas in the agricultural literature are recent. This thesis integrates the concept of copula and widely studied risk measurement Value at Risk (VaR) to derive the optimal risk management strategy. Mean-VaR with copula calculation is shown to be an efficient and confident approach to analyze empirical studies.

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CHAPTER I. INTRODUCTION

1.1. Introduction

Commodity risk management is nothing new, despite the increased attention given to the subject over the past two decades. Farmers have engaged in risk management for well over a century, mitigating their risk against unexpected commodity price fluctuation. Corporations wish to reduce or eliminate the trading, procurement or sales risks by holding a diversified assets portfolio. End users actively seek ways to avoid buying risk by signing contracts. A growing body of literature derived and suggested the strategies to reduce volatility of income stream and to manage risk, but what are the elements of commodity risk?

According to Poitras (2013), the modern approach to risk management classifies risks into the following categories: business or commercial risks, market risks, credit risks, liquidity risks, operational risks, and legal risks. Commodity risk management is mainly concerned with the interaction between commercial and commodity price risk among all the above categories. These two types of risk are the central concern for non-financial firms who are involved in the production and consumptions of commodities.

Agribusiness firms are typical non-financial firms and form business supply chains. Optimal agribusiness supply chain performance requires a set of precise actions by each organization along the chain. It is hard to achieve the optimum because each supply chain participant seeks their own best strategy to reach the interest. This self-interest focus often results in poor risk management or inefficient performance for the entire chain. Fortunately, optimal performance can be achieved if the members coordinate by contracting so that each firm's objective becomes aligned with the supply chain's objective (Cachon & Lariviere, 2001). Forwards, futures and options contracts are the ones that play an important role in the coordination strategies and risk management.

Processors of agricultural commodities are key components in the agribusiness supply chain. Their planning and cooperation have a big impact on the growers and consumers. There are two important decisions for traditional processors to make: raw material procurement strategy and end products sales strategy. Price risk management is a crucial component in the success of processing business. Not only commodity ingredients price risk should be studied, but also the output price fluctuation. In addition, the relationships between inputs and outputs have a significant impact on the decision making. This thesis will not consider agribusiness supply chain as a whole, but only focuses on the commodity processing business. Studying this key supply chain component will help understand the objective of a processor, different hedging strategies, as well as the characteristics and relationships of multiple hedging instruments.

This paper takes wheat flour milling industry as an example of processors and provides a detailed analysis on how they manage their procurement and sales risk under different scenarios. Futures contract is the key study instrument in this paper that serves for procurement hedging and sales hedging. The optimal hedge ratio combination of multiple hedging instruments is the key decision variable that helps to determine the overall business strategy. Numerous techniques are available for estimating the optimal hedge ratios from past research, but Mean-VaR with copula is the main specification for this thesis. In addition, several business scenarios, covering procurement and sales stages a processor may encounter, are developed and studied to demonstrate the impact of market decisions on risk management strategies.

1.2. Problem Statement

A wide array of production processors includes flour miller, oilseed crusher, wet and dry corn miller, and etc. Each of these processors faces varying input and output demand and prices. Most of them are hedgers, who use futures contracts to mitigate input flat price risk and utilize forward contract to lock in sales price well ahead. However, effective procurement and sales strategies can give grain processors a competitive advantage over rivals and reach their goal of optimizing profit. Key components of determining an effective strategy include risk factor recognition, an accurate analysis of the relationship between input, output, and hedge instrument, and a better forecasting methodology. Due to the importance of determining strategy, much research effort has been devoted into those key components. This thesis focuses on grain processors, considering both input and output prices as two sources of risk under different business scenarios.

Procurement is as a major driver in a company's bottom line and its strategy plays an important role to determine the firm's competency in the global marketplace. Even though more advanced forecasting techniques have been developed, increasing volatility of price and demand still makes it extremely important to derive hedging strategy to mitigate procurement risk. For agricultural and energy commodities, processors make extensive use of futures and options for hedging purposes of their raw material procurement. Long futures or options positions of varying sizes, durations, and strike prices are adopted to offset volatile cash positions of ingredients. These traditional futures and options of typical crushing and milling ingredients have been traded in the electronic exchange market for a while. Because of the low transaction, a number of contract types associated with different strategies have become more and more favorable for procurement risk management purpose.

Another major source of uncertainty is the market price at which outputs are sold after processing takes place. Commodity processors often sign contracts, such as forward contracts, to lock in end-product prices. Without binding commitment from buyers for quantities or prices, processors will experience exposure to large price risk (Fu, Lee, & Teo, 2010). It may incur great loss if the sales price drops dramatically. In addition, even processing firms seem to produce only one major refined output, several by-products may also be generated to be sold in competitive markets. In some processing industries, futures or options exist on outputs, allowing simultaneous hedging. In numerous other commodity processing industries, hedging decisions become further important and complicated when futures or options do not exist on end products. Processors most often hedge major products with ingredient futures contract, while utilizing cross hedging to mitigate other by-products' price risk. Hedging instruments and decisions vary depending on interrelationships among assets.

Determining how far the hedge strategy should be implemented is known as the hedge horizon. Simply put, it is how many months ahead of the actual physical input buying or output selling the firm should hold futures as a hedge position. The longer the hedging strategy will cover, the higher the uncertainty is for the processor; the shorter the hedge horizon is, the less room there is for other strategic planning. Therefore, firms need to derive a decision whether to hedge for risks over the next month, three months, or even a longer time of period. It is important to note that the optimal hedge horizon depends on the seasonality factors, assets relationships, the potential market movements, and other important factors, hence it varies according to specific situations.

The dependence structure for inputs, outputs, and hedge instruments also has a significant impact on hedging decisions. Intuitively, when input and output prices move together overtime, the processing risk is naturally reduced, and the demand for protection from hedging instrument is mitigated. When risky assets are highly positive correlated with hedge instrument, it is likely that a hedge ratio close to one will play a better role in reducing the risk. Many studies have also addressed the reason and importance of analyzing dependence structure among assets (Jorion, 2007; Rachev, Menn, & Fabozzi, 2005; Wilson, Nganje, & Wagner, 2006; among others). Interestingly, among those literatures, the linear correlation has long been the only consideration for analyzing dependency, but it results in poor performance when the actual relationships are not linear. Therefore, it not only needs to learn the measure of association between assets, but also requires an accurate dependence measurement.

There are other factors that influence processor's hedging strategies, including the competitiveness of the market, transportation and energy cost, output demand uncertainty, and different hedging instrument involvement. However, these factors will not be particularly addressed in this thesis.

There are two main approaches taken by researchers when working with complicated hedging models, namely minimizing the risk of a cash position and maximizing the expected return of a portfolio. Though risk management is a very important task for an agribusiness firm, the goal of a firm is not simply to minimize risk but to find a balance in the tradeoff between expected return and risk. To achieve this goal, a utility function of a processing firm is derived according to firm's risk preference. An improper hedging strategy may narrow profit margins and increase the financial burden to a firm; on the contrast, an optimal hedging strategy will reduce processing risk and achieve processor's business running objective.

This thesis derives a risk management application to the wheat flour milling industry. Wheat price in the market becomes the source of procurement risk. The prices of major product flour and by-product mill feeds generate the sales risk to the flour processing industry. This thesis considers the problems stated above and tries to provide optimal solutions with a better analysis technique.

1.2.1. Brief discussion on price risk

If there is no commodity price risk, there is no need to derive hedging strategies. However, the reality is commodity prices are important sources of risk for agricultural processors. Such fluctuations of prices are mainly caused by supply and demand imbalances, political events, and unexpected weather patterns. Other than these traditional causing factors, price volatility can also stem from the behavior of speculations by some market participants (Giot & Laurent, 2003). Though there still exists a debate as to whether speculators play a vital role in increasing volatility to the commodity market, many primary commodities are experiencing extremely high volatility. Nissanke (2010) says, "after two decades of low, and at times dwindling, prices in the 1980s and 1990s, many primary commodities had registered a steep price increase since 2002, reaching an all-time high in the spring and summer of 2008 with extremely high volatility."

Grain and oil seeds exhibit the most seasonality due to harvest cycles. Figure 1 display historical futures prices in major commodity markets, including wheat, corn, and soybean. The volatile price behavior of agricultural commodities shown in Figure 1 & 2 suggests significant risk management issues. As presented in Figure 1, the anomaly raised the price of soybeans into $16 range in 2008, when it should fall into the range of $8 to $12 a bushel according to historical stock/consumption levels. The same situation happened to wheat prices, where the abnormal price raised into the $11 range, $4 or more than its normal price range. After that peak, wheat fell back to the normal price range in only a couple of months. Corn seems have experienced a similar price trend. All these increasing supply/demand uncertainties related to government policy, weather, and financial crisis, call for rigorous risk management to protect business from increasing price volatility.

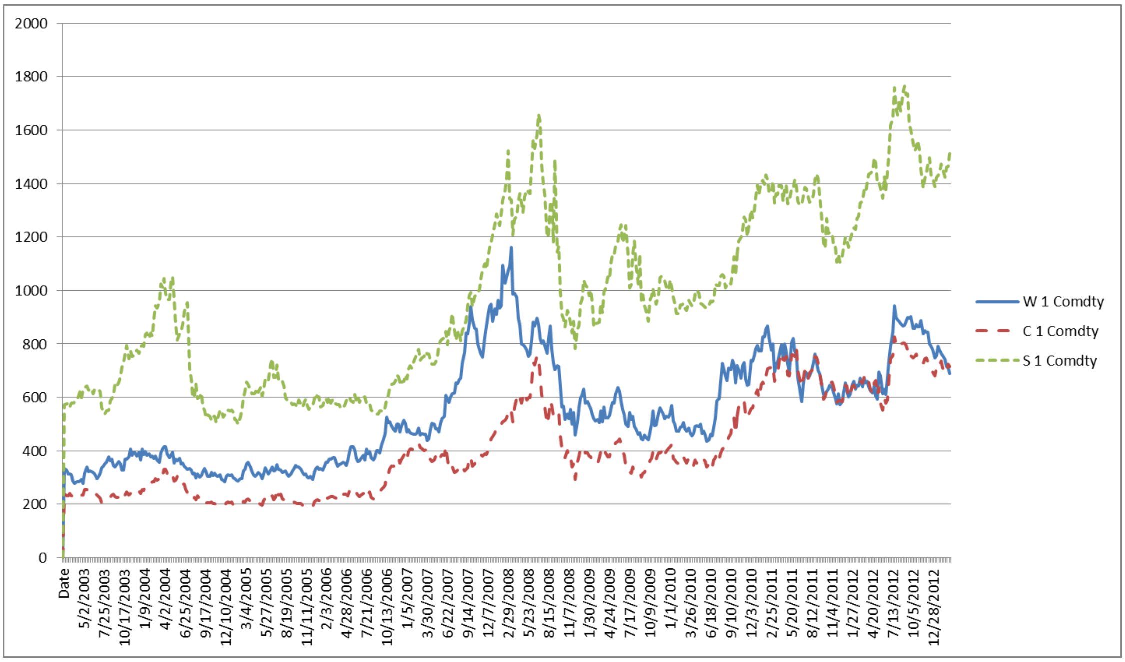

Figure 1. Price trends for major field crops 2003-2013

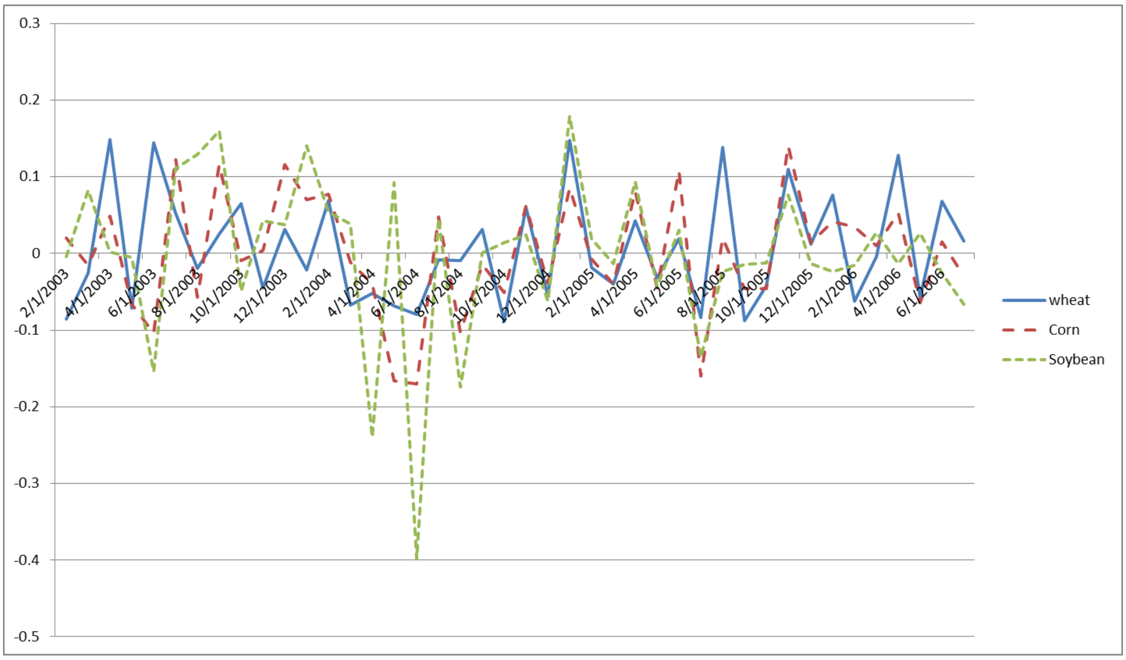

Figure 2. Monthly percentage price changes for corn, wheat and soybeans

Failure to control the cost of inputs and revenue from outputs has severe consequences and highlights the importance of effective hedging strategies. VeraSun Energy Corp., a leading producer of renewable fuel, filed for Chapter 11 bankruptcy due to the firm’s corn procurement and hedging strategy (Hannon, 2008). Another example is that General Mills Inc. reported $111 million total hedging loss (Wetzel, 2008). The number of companies that experienced loses were numerous in 2008/2009, including many ethanol producers, flour processor etc., because of their non-hedging action or misappropriates hedging strategies. According to Farrell and Blas (2010), roughly a third of the world’s largest food companies have established new hedging programs. Most large fuel-consuming companies, such as airline companies, choose hedging to mitigate risk exposure when facing volatile fuel costs, as well as natural gas and electricity costs. Many large grain companies have also established risk management department to evaluate potential commodity price risks. All the problems in the problem statement section lead to the need for this study.

1.3. Need for the Study

Numerous attempts have been made to derive hedging models using a variety of risk management tools for the best interests of processors. Recent studies with the traditional mean-variance approach to manage commodity risk include bakery industry study by Wilson et al. (2006), soybean crushing industry study by Dahlgran (2005), canola and western barley market price risk control by Mann (2010) and cross-hedging Distillers Dried Grains (DDG) study by Brinker, Parcell, and Dhuyvetter (2007). The two theses by Wagner (2001) and Oberholtzer (2011) both have a big portion focusing on flour milling industry that form the base theme of this thesis. Much of this literature has applied futures as hedging instruments and mean-variance as the main analysis methodology.

Futures contracts have most often been the instrument to complete the hedging task. The most important issue is the hedge ratio, which relates to the choice of the appropriate hedge instrument, hedging duration, and the interrelationship among risky assets and selected instrument. Normality assumption of the marginal and joint distribution and the assumption of linear correlation are frequently used. However, researchers have not paid much attention to the accurate dependence structure and joint distribution structure between ingredients, products, and hedging instruments. Concluding the optimal hedging ratio based solely on subjective assumption about the linear correlation and multi-normal distribution does not provide processors sufficient confidence to execute hedging decisions. For these reasons, this study will choose futures contracts to provide a processing hedging model, with better techniques to capture the accurate joint distribution, dependence structure, and convincing risk measurement.

There is by now a rich body of literature in the finance research area, focusing on Copula's application in Mean-VaR models, but its application in the agricultural industry is limited. This thesis incorporates copula using Mean-VaR method in agribusiness sectors to yield a better model and a more confident risk management strategy construction. Another issue is that it becomes very hard for processors to protect themselves when futures do not exist on outputs. This paper also addresses how to derive cross hedging strategy for processors to minimize the end products' price risk.

1.4. Study Objective and Contribution

The primary objective of this thesis is to develop utility maximizing models that balance between expected return and associated risk. It incorporates advanced techniques to derive more confident hedging strategies for different business scenarios.

The commodity processor's risk mainly concerns the ingredient price risk, the end product risk, the dependence between assets, and the hedging portfolio. The models illustrate the use of VaR and Copula in the context of agricultural processors.

There are several contributions to this study. First, it distinguishes several possible business scenarios a flour miller may encounter and derive different models to address on risk management strategies under each specific scenario. Second, it uses a flexible distribution Copula to capture the flexible interrelationship and joint distributions among input, output, and hedging instrument. Finally, it quantifies the risk measurement with VaR instead of traditional variance measurement.

The goal is to help processors effectively manage raw material procurement risk and product sales risk, and maximize the firm's utility with specifying an efficient and confident model.

1.5. Thesis Outline

The remainder of this thesis is organized as follows. In Chapter II, it provides a discussion about previous related studies. In Chapter III, it provides the theoretical model and a formal explanation for techniques and methodologies that applied in this thesis. This chapter provides a detailed description of the copula and its related theories and describes how it can be applied in the modeling process. Chapter IV discusses the empirical model for the flour milling industry and it also presents statistically valid fitting and fast sampling algorithms for the resulting copula-based multivariate model under different business scenarios. Chapter V discusses the results and Chapter VI summaries the work and presents future study possibilities.

CHAPTER II. LITERATURE REVIEW

2.1. Introduction

Firms that deal with agricultural commodities can be especially vulnerable to price risk. Agricultural commodity processors, such as flour millers and oilseed crushers, are mostly concerned with both ingredient procurement risk and product sales risk. Commodity prices nowadays become more volatile than ever before, which are caused by supply and demand imbalances, political events, unexpected weather patterns, and funds intervention. Price volatility significantly threats the profitability of a processing firm. Therefore, effective risk management strategy proves to be a critical function for these businesses.

This chapter begins with a discussion of commodity risk management and related studies. It also includes a hedging strategy discussion for agribusiness firms as a subsection. An introduction of financial hedging follows. The financial hedging section is divided into a detailed review of hedging models, the definition of optimal hedge ratio, and the portfolio theory. Next, it presents a literature review of copula applications in both finance and agricultural industry. A detailed review of traditional mean-variance and the recent mean-VaR modeling approach follows the copula section. Finally, it presents a brief literature discussion on an alternative sales strategy through physical options that could be considered in the future exploration. The final section is a summary of this literature review chapter.

2.2. Commodity Risk Management

Commodity risk management draws increasing attention from academic, government, and industry studies over the past decade or two. The volume and breath of the literature is immense, defying an inclusive and a brief description. Farmers, corporations, consumers, and institutions are all engaged in risk management, trying to mitigate their risk exposure against unexpected commodity price fluctuation. Nevertheless, what are the elements of commodity risk and what types of methods have been discussed to manage such risks?

2.2.1. Commodity risk

A commodity is legally defined by the Commodity Exchange Act (7 U.S.C. 1a) as the following,

> COMMODITY. —The term ‘‘commodity’’ means wheat, cotton, rice, corn, oats, barley, rye, flaxseed, grain sorghums, mill feeds, butter, eggs, Solanum tuberosum (Irish potatoes), wool, wool tops, fats and oils (including lard, tallow, cottonseed oil, peanut oil, soybean oil, and all other fats and oils), cottonseed meal, cottonseed, peanuts, soybeans, soybean meal, livestock, livestock products, and frozen concentrated orange juice, and all other goods and articles, except onions (as provided by the first section of Public Law 85–839 (7 U.S.C. 13–1)) and motion picture box office receipts (or any index, measure, value, or data related to such receipts), and all services, rights, and interests (except motion picture box office receipts, or any index, measure, value or data related to such receipts) in which contracts for future delivery are presently or in the future dealt in.

Poitras (2013) distinguishes the definition of a commodity between financial securities and physical commodities. He believes that a physical commodity should be involved in the costly storage and production while financial “commodities” such as stock indexes are apparently excluded from this definition. He states, “The dividing line between financial and physical commodities is provided by the physical commodity with the greatest financial use”. This thesis will follow his definition of “commodity” that is used as a shortened form of “physical commodity”.

Generally, risks have been categorized into the following ones: business or commercial risks, market risks, credit risks, liquidity risks, operational risks, and legal risks. While, commodity risk management is mainly concerned with the interaction between commercial risk and commodity market price risk. These two types of risk are the central concern for non-financial firms, like agricultural commodity processors, who are largely involved in the storage and consumptions of commodities.

The interest in the commodities markets has been gigantic for recent years, particularly in energy and agriculture. Commodity prices have been experiencing an unexpected rise in the last few years and have been very hard to predict. Pirrong (2012) states: “Structural models of commodity price behavior have improved our understanding of commodity price dynamics, but for storable commodities there is still a yawning gap between theory and evidence.” Agricultural planting or other commodity production decisions depend on expected prices and not price realization (Dana & Gilbert, 2008). If people don’t have proper structural models to forecast commodity price movements, production decisions will appear to be poor. Agricultural commodity prices, in particular, are volatile because short term production and consumption elasticity are low (Dana & Gilbert, 2008). Because of the volatile market and uncertainties, agents in the agricultural value chain are motivated to understand commodity risk and to manage commodity risk actively.

This thesis will be concerned predominantly with price risk for commodity processors. Though, this is not the only or necessarily the most important risk factor. Other factors that need to be taken into account include demand and supply risk, weather-related quantity risk, political risk, currency exchange risk (Borovkova & Geman, 2012). Agents are urgent to adopt different strategies to manage different types of risk.

2.2.2. Commodity risk management studies

There are some questions that need to be answered in order to build risk management strategies. As Mackay and Moeller (2010) observe: “What risks do firm hedge? How much do they hedge? How far ahead do they hedge? What determines corporate hedging policy? Should firms hedge at all? Can corporate risk management create value?” Only if providing clear answers to these straightforward but important questions can we make further valuable discussions.

What risks to hedge and why they hedge have been addressed by a number of literatures (Froot, Scharfstein, & Stein, 2012; Hentschel & Kothari, 2001; Wilson et al., 2006; among others). Leland (1980), Carr, Jin, and Madan (2001), and Brown and Toft (2002) have provided good answers to how to manage risk. A classic example of the traditional approach to managing commodity risk is Hieronymus (1977). Hieronymous provided a detailed analysis of the agricultural commodities’ risk management motivations by producers, merchandisers, and consumers. There are many recent studies with the elements of traditional approach to manage commodity risk, such as Wilson, Nganje, and Hawes (2007) for bakeries, Mohapatra, Goodhue, Carter, and Chalfant (2010) for strawberries, Buguk and Brorsen (2005) for Turkish cotton, and so on.

Poitras (p.50-51, 2012) states that government reports and programs are also important approaches to commodity risk management. Such programs and reports provide background information and examine specific events, and are trusted to retrieve invaluable information. For example, CFTC report, risk management in agricultural production report by USDA, environmental hazard report by National Research Council, and many others provide risk management information and opinions in a professional way. In addition, many commodities sectors including investment banks, global firms, and commodity associations have websites or host seminars to provide information about “real time” commodity characteristics, practical guides, or policy positions concerning risk management practices (p.52-53).

Risk management strategy differs. Tomek and Peterson (2001) state the fact that the literature contains numerous models of optimal marketing portfolios because people's objective functions differ. These diverse models and results have not been unified to provide an "optimal" or useful generalization for decision makers. Dana and Gilbert (2008) further argued that different agents would take different responses to control volatility to their interest. Intermediaries aim to hold the commodity for as short a time as possible to avoid much exposure to price change; processors choose to offset commodity price risk using contracts; exporters and importers often aim to market on a back-to-back basis.

2.2.3. Previous studies in agribusiness risk management

Risk is a pervasive characteristic of business in the agriculture industry. Yield risk depends on weather conditions and frequent weather hazards, such as drought, floods, and windstorms. Grain price risk, on the other hand, mainly depends on the supply and demand relationship. Agribusinesses also face demand risk, refinery product risk, and business operational risk. All these different types of uncertainties and threats stimulate research and analysis of agribusiness risk management in the academia.

Hess, Richter, and Stoppa (2002) discuss how new weather risk management (WRM) can have a positive impact on the commodity production control and its advantages over the traditional WRM. They argue that weather risks cause substantial inefficiencies to rural areas, as well as threat agribusinesses in the market. Traditional under-developed WRM can cause underinvestment and over-diversification while the new WRM overcomes pitfalls and creates operational benefits. They apply the study in the case of WRM for cereals in Morocco.

Manfredo and Leuthold (2001) develop models to analyze cattle feeding margin and apply VaR to capture downside risk. Manfredo, Garcia, and Leuthold (2000) also develop optimal hedge strategies to account for the soybean processing margin. They consider a time-varying covariance matrix of soybean complex and examine the hedging effectiveness of the *Risk Metrics* method in estimating the margin and risk. The exponentially weighted moving average (EWMA) method is a main tool used in *Risk Metrics* for estimating variances and covariance for a multi-asset portfolio. They found that the complex hedging strategy derived from the *Risk Metrics* dominated other procedures in a mean-variance context. However, a minimum variance hedging (MVH) becomes the only framework to derive the optimal soybean complex hedge ratio.

Wilson et al. (2006) address on hedging model for food processors. The baking industry is used to illustrate hedging decisions by processors, with bread to be output and flour to be input. They acknowledge the risks confronting processors are ingredients price risk and products demand and price risk. They use wheat futures contract as the hedging instrument. Their goal is to develop a hedging strategy to maximize the difference between expected return and associated risk. They conclude that the hedge ratio calculation has been the core of the model and is highly dependent on a complicated set of interrelationship and hedge duration.

Boyabatlı, Kleindorfer, and Koontz (2011) provide insights on optimal procurement and selling decisions in the beef processing industry. The central question analyzed has been a contract-spot sourcing portfolio, where contract type is taken to be of a general window contract. The paper shows that a lower correlation between the spot price and product market variability benefits the firm. In addition, the results of the paper address the significant benefits of integrating input risk management and output risk management (Boyabatlı et al., 2011). Their theme and concept stimulate the work of this thesis.

There are still many other studies related to risk management in the agribusiness industry. Some of them are based on but have expansions from the mean-variance method (Dahlgran, 2005); more and more have been using VaR methodology to assess different risk management strategies (Baker & Gloy, 2000; Pritchett, Patrick, Collins, & Rios, 2004; Sanders & Manfredo, 2002; Wilson et al., 2007); some of them incorporate expected tail loss (ETL) into the optimal hedge ratio model (Zylstra, Kilmer, & Uryasev, 2003).

The risk management problem for producers of agricultural commodities is quite different from non-financial firms involved in metals and energy production. Agricultural production is still concentrated. Barnett and Coble (2009) describe that less than 6% of the farms in the United States are producing 75% of the value of production and they still produce undifferentiated commodities for markets. They also conclude that farming is still a risky business. Fortunately, agribusiness features a variety of risk management tools including crop insurance, contracts, and cash on hand. Keeping cash on hand for emergencies was explored to be number one strategy for most agribusiness (Harwood, Heifner, Coble, Perry, & Somwaru, 1999).

The theme of this thesis is also about risk management for an agribusiness agent and is similar to Wilson et al. (2006). This thesis develops an optimal hedging model for flour milling industry. The optimal hedge ratio becomes the goal of this analysis. However, the methodologies are different from others. VaR becomes the primary risk measurement to evaluate downside risk, and instead of traditional linear correlation and multivariate normal assumption the model is incorporating state of the art copula functions.

Dickie and Scott (2003) present an analysis of market volatility factors, such as natural variation and market-oriented factors, in the major segments of the agricultural industry. They conclude that business planning is required when making decisions in a volatile environment; however, predicting price expectations or making such price assumptions is deeply flawed. Rather, an understanding of the underlying causal forces is severely important. Furthermore, they conclude “Modern businesses need modern tools. Derivatives are available, the best business is using them sensibly and they are the only way both to protect against and to exploit the natural volatility of the agricultural and food industry operating environment in order to optimize revenue streams and cost profiles.”

The most important method to manage risk, in this thesis, is through financial hedging. The following section discusses previous literatures regarding hedging and portfolio theory.

2.3. Financial Hedging

Financial hedging has been a very important tool to conduct commodity risk management. It has become the focus in procurement and sales strategies for the past six decades. A financial hedge refers to utilizing a financial instrument position to offset potential losses that may incur by a companion investment. For processors, hedging with financial instruments reduces the risk of loss from fluctuations of input and output prices. Agribusiness agents who are interested in reducing their risk can engage in risk transference to speculators via futures or options markets (Rolfo, 1980). The most frequently used hedging instruments include forwards, future, and option contract.

Futures market ioriginated in Japan, to ensure the stability of exchanging rice to coins (Schaede, 1989). Futures contracts were first standardized from forward contract in 1984 by the Chicago Board of Trade (CBOT) and were solely used in the agricultural industry prior to 1970’s. The purpose of the contract is for grain trading;

it starts to observe more contracts created on a wide variety of commodities as well as futures exchange markets. Chance (1995) explains that currency futures contract, equity futures and options, interest rate futures contracts, treasury-bill futures contracts, treasury-bond futures contracts, and stock index futures contracts were then created and traded. For processors, the assurance of a profitable price for risky, costly and time-consuming production can provide strong cash market incentives to engage in the use of such contracting procedures (Poitras, 2013).

Lim and Wang (2007) argue that financial hedging and corporate diversification are often complementary rather than substitute means of risk management. This implies that the development of financial hedging markets will yield more need to manage risk through diversification for firms. Nevertheless, better ways to diversify and hedge request a proper selection of hedging instruments as well as a proper model to describe the relationship among assets. The following subsections provide previous discussion of hedging models, optimal hedge ratio, and portfolio theory.

2.3.1. Hedging models

Hedging was viewed as an activity used for the purpose of reducing price risk exposure, prior to the 1950s (Blank, Carter, & Schmiesing, 1991). This behavior was to take an exact opposite and equal position of contracts to the exposed position in the cash market. Futures were considered the major hedging tool since it is easy to follow and reduce a significant amount of risk. In the 1950s and early 1960s, Working (1962) separated hedging activities into three categories, and took place of traditional hedging concept. The three categories are namely arbitrage, operational, and anticipatory hedging, where operational hedging has been widely studied and used in a variety of fields-operations management, finance, strategy and international business (Boyabatlı & Toktay, 2004). The processors, such as flour millers, apply operational hedging frequently, using futures contracts to reduce procurement price risk.

Researchers constantly develop hedging models to contribute towards procurement decisions. Some early articles point out the importance of financial hedging. Hull (2009) indicates one reason for hedging is that the hedger "requires short-term protection in an uncertain market situation", which is true for processors that need short-term protection against the volatile input market. Manfredo et al. (2000) also illustrates the importance of time-varying hedging model for the soybean processors. Other examples deal with hedging models include Peck and Nahmias (1989), Lapan and Moschini (1994), Lence and Hayes (1994), and Koppenhaver and Swidler (1996). Some more recent articles have been developed to focus on processors procurement hedging strategies.

Bullock, Wilson, and Dahl (2007) introduce the strategic use of futures and European options by commodity processors. In addition to the futures and applied mean-variance (M-V) methodology, they included a call option (European) into the portfolio. They conclude "adding call options to the portfolio does not provide a hedging demand for options" (Bullock et al., 2007). For simplicity, this thesis will not consider options contract.

Wilson et al. (2006) address on hedging model by food processors. They use input futures contract as the hedging instrument. Their goal is to incorporate the hedge horizon and the input-output correlation into the hedging model when contracts for outputs do not exist. They conclude that the hedge ratio calculation has been the core of the model and highly dependent on a complicated set of statistical parameters. This thesis is most similar to their study while we apply mean-VaR with copula methodology to capture better statistical parameters, the dependence structure, marginal distribution, and joint distribution. In addition, this thesis adds VaR as downside risk measurement.

As Tomek and Peterson (2001) mentioned, there are numerous optimal hedging models because of numerous firms' objective functions of different commodities. These diverse models and results have not provided an "optimal" or useful generalization for decision makers. Dana and Gilbert (2008) also argued that different agents would take different strategies to respond to risk hedging. Optimal hedging model depends on the commodity and one's objective.

2.3.2. Optimal hedging ratio

One of the important questions in commodity procurement risk hedging relates to estimating the optimal hedge ratio, which is defined as the proportion of the underlying cash position hedged by financial instruments. Optimal hedge ratios depend on the correlation implied in cash and futures markets and one's objective function.

Risk management involves the identification, evaluation, and implementation of strategies to reduce uncertainty in the revenue flow, according to Baker and Gloy (2000). This does not necessarily imply to set the hedge ratio equal to 1.0 always, by taking exactly opposite positions. Spahr and Sawaya (1981) point out that hedging also often significantly reduces profits. Therefore, it is important to be clear with the hedging incentive, whether to minimize risk or to maximize utility, in order to derive an optimal hedge ratio when facing the tradeoff between risk and return.

Optimal hedge ratios can differ significantly, depending on the model specifications that are used to estimate the hedge ratio (Ghosh, 1993). Utility maximizing models are frequently discussed (Collins, 1997; Haigh & Holt, 1999;

Rolfo, 1980; Sakong, Hayes, & Hallam, 1993), because they are believed to have advantages over risk minimizing models. An agribusiness firm's incentive is never to minimize risk, but to maximize their utility or satisfaction. Therefore, it will consider utility maximizing model in this study, in order to reach a balance between expected return and associated risk.

Traditionally, the hedge ratio is calculated in terms of the variance of hedging portfolio, where the formula is listed below:

$$H = -(\frac{\sigma_{sf}}{\sigma_f^2}). \qquad (2.3.1.)$$

where H stands for hedge ratio, σ_{sf} is the covariance between hedge instrument and underlying asset and σ_f^2 is the variance of the underlying asset. The derivation can be found in Blank et al. (1991) and Rolfo (1980). This thesis will derive VaR, instead of traditional variance-covariance, to capture the risk. The optimal hedging ratio calculation will be presented in the empirical model chapter.

2.3.3. Portfolio theory

The hedging model is based on the portfolio theory; therefore, it is necessary to provide some background here. Portfolio theory has provided individuals with a means to measure and manage risk. It is first introduced by Markowitz in 1952. The risk measure used in the traditional portfolio problems has been variance. Diversification provides a method to manage risk based on minimizing variance by assigning different weight to each asset.

However, the crux of the portfolio has never been to minimize risk, but shows that expected return is desirable, and variance is undesirable. An efficient portfolio should reach a balance between expected return and undesired risk. There have been numerous studies that are based on portfolio theory. Robison and Brake (1979) reviews its application to the farmer and lender behavior, considers its limitation, and suggests some portfolio adjustments. Barkley and Peterson (2008) show that a portfolio of wheat varieties can enhance profitability and reduce the risk over the selection of a single variety, which represents another application of portfolio theory to improve returns. G. Alexander, Baptista, and Yan (2007) add VaR or CVaR constraint to the traditional mean-variance model for a comparison. Portfolio theory has been widely applied in the financial and commodity industries.

Szegö (2005) indicates that the variance is a valid risk measure, and linear correlation is the appropriate measure of dependence when the returns of the assets in the portfolio are normally distributed. Taking all possible combinations of assets and their correlation into account, an individual can identify an optimal portfolio returning the minimum variance. However, in reality, it has been shown that the existence of normally distributed assets and linear correlation is limited (Just & Weninger, 1999; W. Sun, S. Rachev, F. J. Fabozzi, & P. S. Kalev, 2009). Hence, alternative risk measures and dependency measures are required to account for non-normal data, to assess the joint distribution and correlation better(Roger B Nelsen, 2006; Stoica, 2006).

When variance is used as the risk measure, upside risk is penalized the same as downside risk. Upside risk is often considered being favorable since it is the riskless opportunities for unexpectedly high returns. Individuals only have incentives to measure the downside risk as the volatility below the target return. More recently, the use of downside risk measures in portfolio settings has been embraced by the corporate finance and banking industry (G. Alexander & Baptista, 2002; Artzner, Delbaen, Eber, & Heath, 1999; Buch & Dorfleitner, 2008). They use the downside risk measure estimated by value at risk (VaR), which can be well applied into the portfolio theory. Downside risk measures such as VaR can also be applied into agribusiness management. In addition, copula has been a more recent technology to describe dependence structure and joint distribution, which is of interest to this study.

2.4. Mean-Variance and Mean-VaR Literature

2.4.1. Mean-variance studies

The mean-variance (MV) approach is one of the earliest methods to solve the portfolio selection problems. Its framework is often used to evaluate portfolio return and risk, and it still has wide application in risk management. MV is based on the principle of diversification (Markowitz, 1952). The variance of the portfolio return is the only risk measure of this method.

In early days, Anderson and Danthine (1981) use the MV utility to specify the optimal hedging strategy. They recognize that the positions of cash and futures can be determined simultaneously by the hedging individual. Garcia, Adam, and Hauser (1994) found the mean-variance framework to be the most appropriate method for identifying risk management strategies when applying options contracts into the hedging decision.

Dahlgran (2005) uses MV method to examine the effect of transaction frequency on profits and cash flow risk for firms. He applies the analysis into the soybean crushing industry and compares MV-based hedging effectiveness with unhedged and direct-hedged strategies. Zhou and Yin (2003) propose a model to study the continuous-time version of the Markowitz mean-variance portfolio selection. There are many other recent studies that still utilize MV method to analyze problems, whether they are in the finance industry or commodity industry.

Further literature review of MV method will not be provided here as it is not the main focus in this thesis. On the other hand, VaR has become a new benchmark for managing and control risk; therefore it is necessary to present some literatures about the mean-VaR approach.

2.4.2. Mean-VaR studies

This thesis analyzes the agribusiness hedging problem incorporating mean-VaR method. Instead of traditional mean-variance model, mean-VaR imposes a Value-at-Risk (VaR) constraint on the hedging portfolio.

Risk to a commodity processor is associated with the possibility that the actual processing margin negatively differ from expectation. Variance and standard deviation are typical measures of risk; nevertheless, these two measures include both positive and negative outcomes. Few hedgers oppose positive outcomes; therefore, VaR is used to evaluate the left tail of the probability distribution.

Wilmott (1998) provides a useful definition for VaR:

> Value at risk is an estimate, with a given degree of confidence, of how much one can lose from one's portfolio over a given time horizon.

David Viniar, Chief Financial Officer for Goldman Sachs, provided his comment on VaR, "VaR is a useful tool. The more liquid the asset, the better the tool. The more history, the better the tool. The less of both, the worse it is. It helps you understand what you should expect to happen on a daily basis in an environment that is roughly the same (Nocera, 2009, January 4)."

Manfredo and Leuthold (2001) provide a review of agricultural applications of VaR prior to 2001 and explain a great potential of future use of VaR in agribusiness. He comments that VaR could provide elevators and agricultural producers with a great measure of downside market risk when using different contract types. Füss, Adams, and Kaiser (2010) demonstrate that unconditional VaR estimates for commodity price changes based on normal distributions can be improved by allowing non-normality assumption technics such as the GARCH-VaR. Though VaR has also been criticized from many sides, such as non-subadditive and ineffective measure, many agents in the industry have their own perspective and reasons to adopt it.

VaR has been profound for financial firms, even “the introduction of the US accounting standard FAS 133 has inspired financial firms to include VaR calculations in annual reports and other financial statements” (Poitras, 2013). Yet, VaR can be of importance for non-financial firms, firms associated with activities such as commodity trading, exporting/importing, and interest rate management.

Mean-VaR method has been studied and applied in numerous articles. G. Alexander and Baptista (2002) analyze the portfolio selection problem with both mean-VaR and mean-CVaR methodologies. When it relates VaR to mean-variance (mv) analysis, it is called mean-VaR method; on the other hand, instead of VaR, when related Conditional VaR to the mv analysis, people call it mean-CVaR framework. They discuss implications raised from imposing a VaR constraint as a risk measurement tool on an agent’s portfolio selection and also differentiate from implications raised from CVaR constraint. They conclude that a CVaR constraint is more effective than VaR or variance as risk measurement tools to slightly risk-averse individuals, but has a reverse effect to highly risk-averse agents. Wang (2000) provides a detailed discussion and comparison between the mean-variance approach and the mean-VaR approach. He also approaches from the mean-variance-VaR method, which uses variance and VaR as a double-risk measure simultaneously.

This thesis will incorporate mean-VaR with copulas methodologies to derive the optimal hedge ratio for agribusiness agents.

2.5. Copulas

A growing method of capturing relationships between assets is the copula. Copula functions enable people to describe a joint distribution better by allowing dealing separately with the needs of marginal univariate distributions and dependence structure. Copula offers a far more flexible method to describe multivariate joint distributions and capture the accurate dependence structure.

Copula has been gaining popularity in the financial literature. Bai and Sun (2007) apply copula function and copula-CVaR into the analysis of three important stock indexes (HS index, DJ industry index, and Nikkei index) in the global market and the design of portfolio optimization model. Rodriguez (2007) studies financial contagion using switching-parameter copulas to model dependence. He states "structural breaks in tail dependence are a dimension of the contagion phenomenon" and suggests taking into account tail dependence changes in the design of any sound asset allocation strategy. Patton (2009) provides a brief review of many applications of copulas in finance and economics but focuses on the literature of copulas application in the modeling of financial time-series problems. He points out that copulas stand out for characterizing the dependence between sequences of observations of a scalar time series process. Though, he also has concerns that more extension of copula-based multivariate time series models should be developed into higher dimensions. More recent studies include Wei, Zhang, and Guo (2004), Fernandez (2008), and Chollete, Heinen, and Valdesogo (2009) among others.

While copulas have been used in finance for some time, the applications of copulas in the agricultural literature are recent. Lee (2009) develops a copula GARCH model for optimal futures hedging. He sets the empirical investigation in agricultural commodity markets and models the dependence structure of spot and futures prices. Instead of typical bivariate normality assumption, he uses switching Gumbel-Clayton copula, which is considered to provide an out-of-sample hedging effectiveness. Zhu, Ghosh, and Goodwin (2008) provide a detailed copula modeling to describe the joint yield and price risk of corn and soybeans. Multivariate risk factors and their interrelations are hard but important to capture. They propose a copula approach to complete this hard task and calculate the premium rate of the whole farm insurance. In addition, Vedenov (2008a) goes beyond joint normality and mean-variance criteria and applies copula to model joint yield distributions among crops. He states “many applied problems call for joint distributions of various yields” other than a single yield distribution, and copula tackles the problem directly.

In addition to the flexibility when specifying the dependency among the distributions of returns, the other main advantage of copulas is the asset’s distributions can be specified as non-normal. A brief history and a detailed specification of copula are presented in Chapter III.

2.6. Alternative Sales Strategy

A concept of an alternative contracting for agricultural outputs is specified in Chapter VI. This concept should call for attention and needs future exploration in the agriculture industry. When futures do not exist on outputs, it brings significant risk to processors and buyers. Therefore, over-the-counter contract can be used to hedge the risk. The primary contract type is physical options contract, and the concept is to deliver whatever is contracted and to sell leftovers in the spot market. Option contract has been explored by many practitioners and studied by many scholars in supply chain management, because of its flexibility.

Flexible contracting strategies have drawn strong attention in the literature concerning output sales strategies. It appears that Wu, Kleindorfer, and Zhang (2002) are the first publication on integrating contract-spot purchases, in the stream of literature. They studied the coordination case of a single seller with one or more buyers. They divide the contract fees into two parts: a cost to take transaction and a cost to reserve capacity. However, it was a basic model, only incorporating the market risk.

Later Wu and Kleindorfer (2005) extend their previous model into multiple suppliers that lead to a significant new contributions and insights. The model combines several components (options, diverse technology, and contract-spot market) that have been frequently dealt with in finance, economics and supply chain management. Nevertheless, the model still assumes a price dependent demand function.

Spinler, Huchzermeier, and Kleindorfer (2003) extend Wu et al. (2002) basic model to state-dependent spot price, production cost and demand and present a theoretical analysis of physical options contract for physical delivery. The paper scrutinizes the buyer and seller’s risk respectively and states that physical option contract achieves the goal of sharing risk between the buyer and the seller. However, this paper mentions only chemical and some capital-intensive industries, without indicating possible application for the agriculture industry.

Martinez-de-Albeniz and Simchi-Levi (2005) analyzes and optimizes a portfolio of supply contracts with establishing a general framework for supply contracts. They extend Wu et al. (2002) into a multi-period case and multi-supplier scenario, and add a critical assumption that that capacity is scalable at the time contracts are signed.

For recent studies, Haksöz and Kadam (2009) developed a supply-at-risk measure for a portfolio of long-term fixed price supply contracts. They focus on the loss distribution of a supply portfolio, which helps to study the supply value-at-risk. Fu et al. (2010) constructed an optimal single-period portfolio procurement strategy for the buyer, when both the product demand and spot price are random. They demonstrate that the value of option contracts increases with increased volatility of the spot price and correlation between the spot price and demand.

Alternative sales strategy could sell physical option contracts. Previous studies, mentioned above, have provided the buyer's optimal response. Taking their optimal response into consideration, it can help processor to maximize expected revenue, which needs future discussion.

2.6.1. Physical options

In the sales optimization model, the physical option contract is a contract type that performs the role to share risks between processors and buyers. Below is a brief introduction of physical options here.

An option contract offers the buyer the right to buy or sell an underlying asset at the strike price during a certain period of time before the exercise date (American options), but buyers do not have obligations to such actions. The seller incurs the corresponding obligation to meet the requirement of transaction. The buyer pays a premium or contracting fee to the seller for the right. A physical option is just an option that is based on a physical asset such as chemical products, semi-conductor, or beef, rather than stocks, futures, and indexes.

Processors or suppliers offer physical options to product buyers. Shi, Daniels, and Grey (2004) show how such physical options encourage risk sharing, enhance information flows, and improve supply chain efficiency. Spinler et al. (2003) and Fu et al. (2010) both demonstrate the efficiency to conduct physical delivery and risk sharing purpose via option contracts. Boyabatlı et al. (2011) analyzed on the beef supply chains problem via the long-term physical option contracts. Many industries have used such derivative to manage the risk associated with volatile prices of commodities, such as energy, agricultural products, and metals.

2.7. Summary

In the literature review, an overview of past studies in the areas of agribusiness risk control, overall commodity risk management, and analysis methodologies was presented. The literature develops and provides good background knowledge about this thesis, model, and methodologies. However, several key factors have not been addressed in the past. These factors include how to derive accurate joint distribution and dependence structure between underlying assets and hedging instrument, how to choose appropriate hedging instrument and duration, and how results differ from traditional methodology. In Chapter III, the theoretical model and methodologies will be introduces these additional questions.

CHAPTER III. THEORETICAL MODEL AND MODELING METHODS

3.1. Introduction

Traditionally, processors choose to hold the same amount but opposite futures position to hedge their raw material purchasing risk or end-product sales risk. However, the futures fluctuation does not have an ideal co-movement with the underlying asset. Though cash and futures prices for a given commodity closely move together over time, difference between the two is time-varying. In addition, the dependence between the output, input assets, and hedge instruments is ignored when exploiting the traditional hedging strategy with the hedging ratio being 1. All these deficiencies may result a poor hedging and put a limit on the profitability. Therefore, derivation of the optimal hedge ratio depends on the correlation implied in input and output commodities and their futures market. For grain processors, there is added complexity with the time lag between when to hedge, to procure input, and to sell output.

The theoretical model builds on the expected utility maximization model, and incorporates the copula to capture the dependence structure and joint distribution among assets, and VaR to measure the downside risk. Utility maximizing models (Collins, 1997; Garcia et al., 1994; Haigh & Holt, 1999; Wilson et al., 2006) include risk aversion coefficient to account for personal risk preference, and create a reasonable balance between the expected return and associated risk. The hedge ratio becomes our decision variable, which represents the hedging demand for futures. This model also considers the variability of hedging duration.

This theoretical model is a general form that can be applied to different processing industries, such as flour milling, corn milling, soybean crushing, and etc. It aims at solving the challenges in front of grain processors of how to mitigate buying and selling risks according to their individual preference. The utility function is comprised of expected return and expected risk with risk aversion joining the two. Different from utility maximizing models of previous papers, it uses portfolio VaR as risk measurement while they considered portfolio variance in their models.

3.2. Theoretical Model Specification

Expected return and risk functions are represented in this section. Below is the mathematical expression for the expected return:

$$E(\Pi) = Q_{O,t+n}\mathrm{E}(\dot{P}_{O,t+n}) - Q_{I,t}E(\dot{P}_{I,t}) + Q_{I,F,t-m}(E(\dot{P}_{I,F,t}) - \overline{P}_{I,F,t-m}) + Q_{O,F,t}(E(\dot{P}_{O,F,t+n}) - E(\dot{P}_{O,F,t})) - C \qquad (3.2.1.)$$

where $E(\Pi)$ is the expected return of the entire processing procedure, $Q_{O,t+n}$ is the number of units of output produced at time $t+n$, $Q_{I,t+n}$ is the quantity of inputs needed at period t, $Q_{I,F,t-m}$ is the quantity of futures hedged for inputs at time $t-m$, and $Q_{O,F,t}$ is the futures position hedged for outputs at time t. $\dot{P}_{O,t+n}$ is the price of processed products sold at time $t+n$, $\dot{P}_{I,t}$ is the price of inputs in the cash market at time t, $\dot{P}_{I,F,t}$ and $\overline{P}_{I,F,t-m}$ are the futures price hedged for inputs at time t and $t-m$ respectively, and $\dot{P}_{O,F,t+n}$ and $\dot{P}_{O,F,t}$ are the futures price hedged for outputs at time $t+n$ and t respectively. C represents the non-ingredient cost of the production process, comprising operational and hedging transaction cost, which are assumed to be constant. Dot ($\bullet$) above the prices means that they could be constants or stochastic variables, depending on the scenarios which processors may encounter. Different procurement and sales scenarios are elaborated Chapter IV.

The timing of hedging decisions, spot purchase, and product sales are illustrated in Figure 3. Hedging of ingredients *m* months ahead, when physical ingredients need to be acquired from the spot market at time *t*. Hedging of outputs occur at time *t* while outputs are expected to be sold in the market at time $t+n$. The first term of Equation 3.2.1 represents the revenue from selling products at $t+n$. The second term is the cost on ingredients, occurred at time *t* in the cash markets. The third and the fourth terms are the payoffs from hedging activities in the futures markets, one is hedging for inputs procurement, and the other is hedging for output sales. Payoffs generated from futures positions offset the risks of ingredient price increasing and output price decreasing. The duration of hedging time *m* and *n* depends on processing firms' practices and operation decisions.

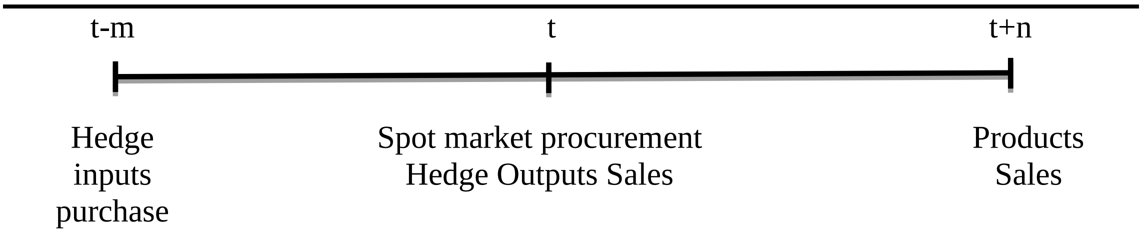

Figure 3. Timeline of hedging and procurement periods

VaR of the portfolio is defined as below:

$$VaR(\Pi,\alpha) = \min\{\zeta \in \mathbb{R} \mid \Psi(\Pi,\zeta) \geq \alpha\} \tag{3.2.2.}$$

where α is the confidence interval, ζ is the lowest possible value, and Ψ stands for the cumulative probability function (CDF) of Π. CDF Ψ is linked to the selected optimal multivariate copula, based on Sklar's Theorem specified in the following section. Prices of inputs, outputs, and hedging tools are estimated from multivariate copulas. Formulation of 3.2.2. indicates the most one can lose of their processing activity during the period $m+n$, at a confidence level α. Having defined the expected return and VaR measurement, it is now time to give the utility function. The traditional mean-variance utility model is the basic form to set up the utility maximization framework in this thesis, as the following:

$$\max U = E(\Pi) - \frac{\lambda}{2} VaR(\Pi, \alpha) \qquad (3.2.3.)$$

where λ is processor's risk aversion coefficient and U stands for the utility function. It shows a tradeoff between expected return and risk tolerance that can be evaluated and determined by an individual's risk preference, since high return is usually associated with high risk.

In practice, processors are interested in developing strategies at period t-m and t that significantly affects the processing return, whether or not to hedge, and how much to hedge. $Q_{I,F,t-m}$ and $Q_{O,F,t}$ can be evaluated by assessing the desired utility and different hedging durations. With iterative methods, it can find the optimal $Q_{I,F,t-m}{}^*$ and $Q_{O,F,t}{}^*$ that maximize the utility function. Dividing them by their hedging underlying asset would reveal the desired decision variable, the optimal hedge ratios $r_I{}^*$ and $r_O{}^*$, through the following equations

$$r_I{}^* = \frac{Q_{I,F,t-m}{}^*}{Q_{I,t}} \text{ and } r_O{}^* = \frac{Q_{O,F,t}{}^*}{Q_{O,t+n}}. \qquad (3.2.4.)$$

Hedging tools for ingredients and products are selected based on hedging effectiveness.

Whether to hedge or not depends on the questions of whether inputs have been bought and whether products have been sold. If inputs have been bought, there is no risk associated with ingredients procurement hence no reason to establish ingredient hedge positions. Likewise, if outputs have been sold through some type of contract like forward contract, there is no reason to set up output hedge positions. Different scenarios lead to different discussions regarding risk control strategies and optimal hedge ratio. A detailed scenario classification and corresponding models and solutions for flour milling industry are represented in the empirical model section.

3.3. Risk Measurement

3.3.1. Volatility

Agricultural processors are significantly exposed to input and output price risk. At various times, commodities markets have exhibited significant price volatility, which brought pains to processors. Speculation is part of the reason for market volatility. It is stated that speculation does not have a systematic impact, but is linked to short-lived volatility in commodity prices (Devlin, Woods, & Coates, 2011). The combination of inelastic demand and supply is also an important reason. Devlin et al. (2011) further states that "unanticipated changes in demand or supply can generate large price swings."

The fundamental driver of recent commodity volatility is the exceptionally strong demand from the emerging markets, particularly China. In addition, the response of supply to surging commodity prices after 2002 is sluggish, which results from the underinvestment in supply capacity globally and adverse weather conditions. Moreover, international trade generates multiple sources of price risk, such as exchange rates and freight rates risk, which helps to enlarge the global commodity volatility.

The fundamental driver and speculation activity both have a significant impact on the volatile commodities prices. Fortunately, there are various tools to measure the risk. It can be simply measured as the standard deviation of logarithmic returns (Dowd, 2003) while exponentially weighted moving average (EWMA) is an improved estimating methodology illustrated by Hull (2009). Value-at-Risk (VaR) has been largely adopted as a measure of downside risk. Its popularity as a risk measurement tool has risen dramatically to include firms from nearly every sector of the economy (Mina & Xiao, 2001). In addition, implied volatility provides a forwardlooking variance measure, instead of solely based upon historical data. VaR will be applied in this thesis; therefore it will provide a more detailed introduction to this measurement in the following subsection.

3.3.2. Value-at-Risk

The development of RiskMetrics™ (Metrics, 1997) stimulated the Value-at-Risk's (VaR) growth. Financial firms attempted to standardize the use of VaR throughout the industry (Linsmeier and Pearson, 2000) as it received increasing attention, both literarily and industrially. Jorion provided the definition of VaR, as "a single, summary statistic that measures the worst expected losses during a given time period, with a specified level of confidence, under normal market conditions." Mathematically, it is given:

$$VaR(x,\alpha) = \min\{\zeta \in \mathbb{R} \mid \Psi(\mathrm{x},\zeta) \geq \alpha\} \qquad (3.3.1.)$$

where x is the random variable, α is the confidence interval, ζ is the lowest possible value, and Ψ stands for the cumulative probability of x.

There are three widely used methods of VaR computation, namely parametric, historical, and Monte Carlo simulation. The parametric method is also referred to as variance/covariance approach, whose fundamental assumption is that the random variables are normally distributed. Historical simulation explores the VaR of a portfolio of assets over a historical period, which does not include any possible future information. Monte Carlo Simulation requires one to assign appropriate distributions to assets that can adequately approximate the portfolio possible changes (Linsmeier & Pearson, 1996). Since VaR is not the major focus of this study, its theory and mathematical derivation are not further explored.

Alternative to traditional risk measurement tools, VaR offers attractive features. When variance is used as the risk measure, upside risk is penalized the same as downside risk. Upside risk is an opportunity for unexpectedly high returns, hence is more favorable. Individuals only have incentives to measure the downside risk as the volatility below the target return. More recently, the use of downside risk measures in portfolio settings has been embraced by the corporate finance and banking industry (G. Alexander & Baptista, 2002; Artzner et al., 1999; Buch & Dorfleitner, 2008). It is receiving its popularity in the agribusiness industry for recent years, as well.

3.4. Copula Specification

Calculation of VaR requires knowledge of the joint distribution function of a portfolio. The traditional approach to this type of problem relies heavily on the multivariate normal distribution (Markowitz, 1952). However, the assumption of normality for agricultural commodities and output prices has been shown to be inconsistent (Goodwin & Ker, 2002; Just & Weninger, 1999). Copula has been gaining popularity in financial literatures as an alternative tool for modeling joint distributions and dependence structures (G. Alexander et al., 2007; S. Alexander, Coleman, & Li, 2006; Bai & Sun, 2007). Application of copulas to model multivariate distributions is described in numerous books (Cherubini, Luciano, & Vecchiato, 2004; Roger B Nelsen, 2006).

The term Copula originates from the Latin term which means to link, join, or connect. Copula functions enable us to tackle the problem of how to describe a joint distribution by letting people deal separately with the needs of marginal univariate distributions and market comovement and dependence. It offers a more flexible method for combining non-normal marginal distributions and real dependence structure into multivariate joint distributions. While copulas have been used in finance for quite some time, the applications of copulas in the agricultural literature are recent (Vedenov, 2008b; Y. Zhu, S. K. Ghosh, & B. K. Goodwin, 2008). Therefore, the rest of this section will provide adequate knowledge about copulas functions and application.

3.4.1. Definition and Sklar's theorem

This subsection provides the formal definitions of Copulas and Sklar's Theory. First of all, it is essential to start with defining subcopula as a certain class of grounded 2-increasing functions with margins.[1]

Definition 3.4.1.(a) A two-dimensional subcopula *C* is a real function defined on $A \times B$, where *A* and *B* are non-empty subsets of *I* = [0, 1], containing both 0 and 1:

$$C : A \times B \rightarrow \Re$$

(i) *C* is grounded ($C(u,0) = C(0,v) = 0$) and 2-increasing

(ii) for every (u,v) of $A \times B$

$$C(u,1) = u \,,\; C(1,v) = v \qquad (3.4.1.)$$

Note that for every (u,v) in Dom*C*, $0 \le C(u,v) \le 1$, so that Ran*C* is also a subset of *I*. Now is the time to define copulas, the subject of this section, as subcopulas with domain I^2 .

Definition 3.4.1.(b) (Sklar, 1959) A two-dimensional copula *C* is a two-dimensional subcopula with *A*=*B*=*I*.

Every copula is a subcopula, therefore, many of the important properties of copulas belong to the properties of subcopulas. The following theorem provides an inequality theorem for subcopulas, which also holds for copulas.

Theorem 3.4.1.(a)[2] (Nelsen, 2006) Let *C* be a subcopula. Then for every (u,v) in Dom*C*,

---

[1] For a detailed review of the definitions of grounding, 2-increasing, and a number of subcopula and copula properties, please refer to Nelsen 2006 and Cherubini et al. 2004.
[2]. If interested, proofs for these are in Nelsen (2006).

$$\max(u+v-1,0) \le C(u,v) \le \min(u,v). \qquad (3.4.2.)$$

It is important to point out that the bounds in inequality 3.4.2 are themselves copulas, called Fréchet-Hoeffding bounds. The upper and lower bound are commonly denoted by $M(u,v) = \min(u,v)$ and $W(u,v) = \max(u+v-1,0)$. People refer to *M* as the Fréchet-Hoeffding upper bound and *W* as the Fréchet-Hoeffding lower bound.

Sklar's Theorem is central and fundamental to the theory of copulas. It demonstrates the critical role that copula plays in the relationship between multivariate joint distribution and its corresponding univariate marginal distributions. The theorem for bivariates is stated as below:

Theorem 3.4.1.(b) (Sklar's Theorem) Let *H* be a joint distribution function with margins *F* and *G*. Then there exists a copula *C* such that for all x,y in *R*,

$$H(x,y) = C(F(x),G(y)). \qquad (3.4.3.)$$

If *F* and *G* are continuous, then *C* is unique; otherwise, *C* is uniquely determined on $RanF \times RanG$. Conversely, if *C* is a copula and *F* and *G* are distribution functions, then the function *H* defined by Equation 3.4.3 is a joint distribution function with margins *F* and *G*.

Equation 3.4.3 in the theorem provides an expression for the joint distribution functions in terms of a copula and two univariate distribution functions. Even though the definitions and theorems presented so far are for bivariate, they can also be extended to multi-dimensions. For the sake of length of this section, it only restates the n-dimensional Sklar's theorem as below:

Theorem 3.4.1.(c) (Sklar's Theorem in n-dimensions) Let *H* be an n-dimensional distribution function with margins $F_1, F_2, ..., F_n$. Then there exists an n-copula *C* such that for all $\vec{x}$ in R^n,

$$H(x_1, x_2, ..., x_n) = C(F_1(x_1), F_2(x_2), ..., F_n(x_n)). \quad (3.4.4.)$$

If $F_1, F_2, ..., F_n$ are all continuous, then C is unique; otherwise, C is uniquely determined on $RanF_1 \times RanF_2 \times \cdots \times RanF_n$. Conversely, if C is a n-copula and $F_1, F_2, ..., F_n$ are distribution functions, then the function H defined by Equation 3.4.4 is a joint distribution function with margins $F_1, F_2, ..., F_n$.

Generalization to the n-dimensional case of Sklar's theorem ensures that every copula is a joint distribution function if its arguments are marginal distribution functions. Another important fact tells that copulas are invariant with respect to increasing transformations. Also, from the fundamental probability statistical knowledge, it is able to transform random variables into uniformly distributed random variables by their respective cumulative distribution functions, i.e. $F_1(X_1), ..., F_n(X_n) \sim U_1, ..., U_n$. Therefore, multivariate copulas can be easily seen to be the cumulative distribution functions of multivariate uniform random variables $C(\vec{u}) = \Pr(U_1 \le u_1, ..., U_n \le u_n)$, and extends to the following remark.

Remark 3.4.1.(a) (Cherubini et al. 2004) The copula of the vector $\vec{X}$ is the joint distribution function of the probability-integral transforms of the functions F_i :

$$\Pr(F_1(X_1) \le u_1, ..., F_n(X_n) \le u_n)$$

$$= \Pr(X_1 \le F_1^{-1}(u_1), ..., X_n \le F_n^{-1}(u_n)) \quad (3.4.5.)$$

$$= C(F_1(F_1^{-1}(u_1)), ..., F_n(F_n^{-1}(u_n))) = C(u_1, ..., u_n)$$

Having given out the definitions of copula and Sklar's theorem, it is ready to look at some copula families and their functions. There are an infinite number of copula functions and thus an infinite number of joint distributions that may be generated for given marginals. Various copula families have been used in risk research. As examples, Gaussian, Archimedean copulas and etc. are discussed by Hennessy and Lapan (2002). Most frequently adopted ones are shown in the following subsections.

3.4.2. Bivariate copulas

This subsection introduces bivariate copulas. For each copula family, this subsection gives out their definitions, density functions, and cumulative functions, as well as their parameters. The next subsection will discuss the multivariate copulas. Bivariate copulas are applied to two variables while multivariate copulas can be applied several assets. In the n-dimensional case, $n>2$, the notions of copulas are very similar to the bivariate copulas; however, multivariate copulas obtain a broader applications in studying multi-assets.

Elliptical copulas are simply the copulas of elliptically contoured distributions. The advantage of it is that one can specify different levels of correlation between the marginal while it does not have closed form and is restricted to radial symmetry. Two most commonly used elliptical copulas, Gaussian copula and student-t copula, are introduced.

Definition 3.4.2.(a) (Bivariate Gaussian Copula) It is defined as follows:

$$C^{Ga}(u,v,\theta)=\Phi_{\theta}(\Phi^{-1}(u),\Phi^{-1}(v)) \qquad (3.4.6.)$$

where Φ_{θ} is the joint distribution function of a bivariate standard normal, with linear correlation coefficient θ, and Φ is the standard normal distribution function. Therefore, it can also be written as:

$$C^{Ga}(u,v;\theta)=\int_{-\infty}^{\Phi^{-1}(u)}\int_{-\infty}^{\Phi^{-1}(v)}\frac{1}{2\pi(1-\theta^2)^{1/2}}\exp\{-\frac{x^2-2\theta xy+y^2}{2(1-\theta^2)}\}dxdy \qquad (3.4.7.)$$

The density of the bivariate Gaussian copula is

$$c^{Ga}(u,v,\theta)=\frac{1}{\sqrt{1-\theta^2}}\exp(\frac{\Phi^{-1}(u)^2+\Phi^{-1}(v)^2}{2}$$

$$+\frac{2\theta\Phi^{-1}(u)\Phi^{-1}(v)-\Phi^{-1}(u)^2-\Phi^{-1}(v)^2}{2(1-\theta^2)}) \tag{3.4.8.}$$

Cherubini et al. (2004) also provide a proposition that the Gaussian copula generates the joint normal standard distribution functions, if and only if the margins are standard normal. For any other marginal choice, the Gaussian copula does not give a standard joint normal vector.

Definition 3.4.2.(b) (Bivariate Student t Copula) It is defined as follows:

$$C^T(u,v,\theta,\varsigma)=t_{\theta,\varsigma}(t_\varsigma^{-1}(u),t_\varsigma^{-1}(v)) \tag{3.4.9.}$$

where $t_{\theta,\varsigma}$ is defined as the standardized bivariate Student-t distribution function, θ is the correlation coefficient, and ς is the degrees of freedom. $t_\varsigma^{-1}(u),t_\varsigma^{-1}(v)$ are used to denote the inverse of the Student's t cdf functions. In the two dimensional case, the T copula density can be written as

$$c^T(u,v,\theta,\varsigma)=\theta^{-\frac{1}{2}}\frac{\Gamma(\frac{\varsigma+2}{2})\Gamma(\frac{\varsigma}{2})}{\Gamma(\frac{\varsigma+2}{2})^2}$$

$$\cdot\frac{(1+\frac{t_\varsigma^{-1}(u)^2+t_\varsigma^{-1}(v)^2-2\theta t_\varsigma^{-1}(u)t_\varsigma^{-1}(v)}{\varsigma(1-\theta^2)})^{-(\varsigma+2)/2}}{((1+\frac{t_\varsigma^{-1}(u)^2}{\varsigma})(1+\frac{t_\varsigma^{-1}(v)^2}{\varsigma}))^{-(\varsigma+2)/2}} \tag{3.4.10.}$$

As its degrees of freedom get larger, the student t-copula converges to the Gaussian copula. However, for a limited number of degrees of freedom, the behavior of t copula is quite different from Gaussian. It is also noticeable that the student-t copula presents more observations in the tails than Gaussian copula.

Archimedean copula is an important class of copulas that has a wide range of applications because of many nice unique properties it possesses. The class originates from the study of probabilistic metric spaces. Please refer to (Schweizer, 1991) for an account of its history. Here provides its general definitions and three specific Archimedean copulas named Clayton, Frank, and Gumbel.

Definition 3.4.2.(c) Given a generator ϕ and its pseudo-inverse $\phi^{[-1]}$ [3], an Archimedean copula C^A is generated as follows:

$$C^A(u,v) = \phi^{[-1]}(\phi(u) + \phi(v)) \tag{3.4.11.}$$

Three specific Archimedean copulas used in this thesis are called Clayton, Frank, and Gumbel. The Clayton copula is asymmetric and exhibits greater dependence in the lower tail. The Frank copula, on the other hand, is a symmetric copula and weights the tails of the data equally. The Gumbel copula is an asymmetric copula and exhibits greater dependence in the upper tail. They are defined individually in the following.

Definition 3.4.2.(d) (Clayton Copula) The generator is given by $\varphi(u) = u^{-\alpha} - 1$, hence $\varphi^{-1}(t) = (t+1)^{-\frac{1}{\alpha}}$

$$C(u,v;\alpha) = (u^{-\alpha} + v^{-\alpha} - 1)^{-1/\alpha} \tag{3.4.12.}$$

Definition 3.4.2.(e) (Gumbel Copula) The generator is given by $\varphi(u) = (-\ln(u))^{\alpha}$, hence $\varphi^{-1}(t) = \exp(-t^{\frac{1}{\alpha}})$

$$C(u,v;\alpha) = \exp\{-[(-\ln u)^{\alpha} + (-\ln v)^{\alpha}]^{1/\alpha}\} \tag{3.4.13.}$$

Definition 3.4.2.(f) (Frank Copula) The generator is given by $\varphi(u) = \ln(\frac{\exp(-\alpha u) - 1}{\exp(-\alpha) - 1})$, hence $\varphi^{-1}(u) = -\frac{1}{\alpha}\ln(1 + e^{t}(e^{-\alpha} - 1))$

[3] For the complete definition of generator and pseudo-inverse, see Nelsen2006.

$$C(u,v;\alpha) = -\frac{1}{\alpha}\ln(1+\frac{(e^{-\alpha u}-1)(e^{-\alpha v}-1)}{e^{-\alpha}-1}) \qquad (3.4.14.)$$

This subsection has shown several fundamental and frequently used bivariate copulas in this subsection. It will briefly extend to their multivariate form in the following subsection.

In the n-dimensional case, n>2, the notions of copulas are very similar to the bivariate copulas presented previously. However, multivariate copulas obtain more practical applications. For an n-variate function H with in total n univariate marginal distributions $F_1,...,F_n$, there exists a copula function C such that:

$$H(x_1,...,x_n) = C(F_1(x_1),...,F_n(x_n),\theta), \qquad (3.4.15.)$$

Where θ is a scalar based dependence parameter, and $F_1(x_1) = u_1,...,F_m(x_m) = u_m$, by probability integral transform, $F_1,...,F_n \sim U_1,...,U_n$. If $F_1,...,F_n$ are all continuous, then C is unique as defined previously.

3.4.3. Multivariate copulas

The definitions of Elliptical and Archimedean copula families in the n-dimensional form are provided.

Definition 3.4.3.(a) (Multivariate Gaussian Copula (MGC)) Let R be a symmetric, positive definite matrix with $\text{diag}(R) = (1,\ldots,1)^T$ and Φ_R the standardized multivariate normal distribution with correlation matrix R. MGC is defined as follows:

$$C^{Ga}(\vec{u}) = \Phi_R(\Phi^{-1}(u_1),...,\Phi^{-1}(u_n)), \qquad (3.4.16.)$$

Similar to the bivariate case, MGC presents a standard Gaussian joint distribution function, if and only if the margins are standard normal. It will not be standard Gaussian when other marginal distributions are chosen.

Definition 3.4.3.(b) (Multivariate Student-t Copula (MTC)) Let R be a symmetric, positive definite matrix with $\mathrm{diag}(R) = (1, \ldots, 1)^T$ and $t_{R,\varsigma}$ the standardized multivariate student-t distribution with correlation matrix R and ς degree of freedom. Then the MTC is defined as follows:

$$C^T(\vec{u}, R, \varsigma) = t_{R,\varsigma}(t_\varsigma^{-1}(u_1), ..., t_\varsigma^{-1}(u_n)), \tag{3.4.17.}$$

Similar to the bivariate case, MTC converges to the MGC as degrees of freedom getting larger.

Multivariate Archimedean copulas are an associative class of copulas and have a wide application in different areas. They are defined as:

Definition 3.4.3.(c) Let φ be a strict generator, with φ^{-1} completely monotonic on $[0, \infty]$. Then an n-variate Archimedean copula is the function

$$C(u_1, ..., u_n) = \varphi^{-1}(\varphi(u_1) + ... + \varphi(u_n)), \tag{3.4.18.}$$

With the definition of Archimedean family, it is able to give out the three most frequently used Archimedean copula functions, namely Clayton, Gumbel, and Frank.

Definition 3.4.3.(d) (Multivariate Clayton Copula) The generator is given by $\varphi(u) = u^{-\alpha} - 1$, hence $\varphi^{-1}(t) = (t+1)^{-\frac{1}{\alpha}}$. It is completely monotonic if $\alpha > 0$. The Clayton n-copula is

$$C(u_1, ..., u_n) = (\sum_{i=1}^{n} u_i^{-\alpha} - n + 1)^{-1/\alpha} \tag{3.4.19.}$$

Definition 3.4.3.(e) (Multivariate Gumbel Copula) The generator is given by $\varphi(u) = (-\ln(u))^\alpha$, hence $\varphi^{-1}(t) = \exp(-t^{\frac{1}{\alpha}})$. It is completely monotonic if $\alpha > 1$. The Gumbel n-copula is

$$C(u_1, ..., u_n) = \exp\{-[\sum_{i=1}^{n} (-\ln u_i)^\alpha]^{1/\alpha}\} \tag{3.4.20.}$$

Definition 3.4.3.(f) (Multivariate Frank Copula) The generator is given by

$\varphi(u) = \ln(\frac{\exp(-\alpha u)-1}{\exp(-\alpha)-1})$, hence $\varphi^{-1}(u) = -\frac{1}{\alpha}\ln(1+e^{t}(e^{-\alpha}-1))$. It is completely

monotonic if $\alpha > 0$. The Gumbel n-copula is

$$C(u_1,...,u_n) = -\frac{1}{\alpha}\ln(1+\frac{\prod_{i=1}^{n}(e^{-\alpha u_i}-1)}{(e^{-\alpha}-1)^{n-1}}) \qquad (3.4.21.)$$

It follows from Cherubini et al. (2004) that “the Gumbel family can represent independence and “positive” dependence only, since the lower and upper bound for its parameter correspond to the product copula and the upper Fréchet bound. On the other hand, the Frank and Clayton family both cover the whole range of dependence.”

Archimedean copulas are related to measures of dependence in mathematical forms easily. They are also famous for tail dependency measurement. It can be shown that the Clayton copula has lower tail dependence for $\alpha > 0$, since $\lambda_L = 2-1/\alpha$, while Gumbel has upper tail dependence with $\lambda_U = 2-2^{1/\alpha}$, where λ stands for tail dependence measurement. However, the Frank copula shows neither lower nor upper tail dependency. So far, copula definitions, joint and marginal distributions and different families of bivariate and multivariate copula families are presented. Yet have mentioned dependence structure, another important feature of copulas. The following subsection briefly presents different measurement of dependence and their relationships with copula functions.

3.4.4. Dependence structure

Copula functions provide a way to preserve the specification of the marginal distributions of each asset, as well as to capture an accurate dependence structure between assets. Linear correlation coefficient is a widely adopted traditional tool to describe the co-movements between different assets and different markets. However, the weakness of linear correlation concept is also very clear: assets must preserve a linear relationship. Otherwise, the coefficient may not provide much useful information. A typical example to show it is a flawed instrument measuring correlation coefficient in the presence of non-linear links is Z and Z^2, where Z is a random variable that follows a standard normal distribution. It is easy to tell by observing that these two random variables are perfectly correlated, but their linear correlation coefficient is zero from simple calculation.

The concept of dependence embedded in copula function is much more general than the standard linear correlation concept, which urges to present definitions of some other dependence measurements.

There are three concepts that can reflect the association or dependency between random variables, i.e. concordance, linear correlation, and tail dependence. Kendall's *tau* and Spearman's *rho* are measurements for concordance; the linear correlation coefficient is the measurement for linear correlation as have already known; the indices of tail dependency measure the tail dependence.

The discussion about linear correlation is omitted here. Concordance concepts, on the other hand, aim at capturing the probability of having large (small) values of both *X* and *Y* simultaneously. Kendall's *tau* and Spearman's *rho*, the most widely known scale-invariant measures of association, both measure such form of dependence-concordance. These two coefficients are defined differently, but are interchangeable, and both are related to copulas.

Kendall's coefficient measures the difference between the probabilities of concordance and discordance for two independent random variables, (X_1, Y_1) and (X_2, Y_2), each with the same joint distribution function *F* and copula *C*. Mathematically, it is defined as below

$$\tau = \tau_{X,Y} = \Pr[(X_1 - X_2)(Y_1 - Y_2) > 0] - \Pr[(X_1 - X_2)(Y_1 - Y_2) < 0] \quad (3.4.22.)$$

Thereafter, it can be shown that Kendall's *tau* depends on the distributions of (X_1, Y_1) and (X_2, Y_2) only through their copulas, in the following theorem.

Theorem 3.4.4.(a) (Nelsen, 2006) Let (X_1, Y_1) and (X_2, Y_2) be independent vectors of continuous random variables with joint distribution functions H_1 and H_2, respectively, with common margins *F* (of X_1 and X_2) and G (of Y_1 and Y_2). Let C_1 and C_2 denote the copulas of (X_1, Y_1) and (X_2, Y_2), respectively, so that $H_1(x, y) = C_1(F(x), G(y))$ and $H_2(x, y) = C_2(F(x), G(y))$. Let τ denote the difference between the probabilities of concordance and discordance of (X_1, Y_1) and (X_2, Y_2), then

$$\tau = \tau(C_1, C_2) = 4\iint_{I^2} C_2(u, v)dC_1(u, v) - 1. \quad (3.4.23.)$$

If X and Y are continuous random variables whose copulas is C, then the population version of Kendall's tau for X and Y is given below.

$$\tau_C = \tau(C, C) = 4\iint_{I^2} C(u, v)dC(u, v) - 1. \quad (3.4.24.)$$

For Spearman's coefficient, in contrast to Kendall's *tau* coupled random vectors, it starts from three couples of i.i.d. random variables (X_1, Y_1), (X_2, Y_2), and (X_3, Y_3) with copula *C*. it will provide its mathematical definition and relationship to copula in one theorem.

Theorem 3.4.4.(b) Given (X_1, Y_1), (X_2, Y_2), and (X_3, Y_3) i.i.d. with copula *C*, then

$$\lambda_U = 0 \quad (3.4.25.)$$

The relationship with copulas is given

$$\rho_s = 12\iint_{I^2} [C(u, v) - uv]dudv \quad (3.4.26.)$$

It is important to note that Spearman's *rho* is the rank correlation, the correlation of the integral transforms of *X* and *Y*.

Another important dependence concept associated with copula is the tail dependence. Verbally defined, tail dependence is the concordance in the tail or extreme values of random variables. In other words, it is observable of a concentration on either upper or lower quadrant of the joint distribution function. Below is the formal definition of tail dependence.

Definition 3.4.4.(a) Recall that $C(v,v) = \Pr(U_1 \le v, U_2 \le v)$ and $\bar{C}(v,v) = \Pr(U_1 > v, U_2 > v)$. Let

$$\lim_{v \to 1^-} \frac{\bar{C}(v,v)}{1-v} = \lambda_U \qquad (3.4.27.)$$

exist finite. *C* is said to have upper tail dependence iff $\lambda_U \in (0,1]$, no upper tail dependence iff $\lambda_U = 0$. Similarly, let

$$\lim_{v \to 0^+} \frac{C(v,v)}{v} = \lambda_L \qquad (3.4.28.)$$

exist finite. *C* is said to have lower tail dependence iff $\lambda_L \in (0,1]$, no lower tail dependence iff $\lambda_L = 0$.

As shown previously, Archimedean copulas are famous for tail dependency measurement. Clayton copula has lower tail dependence for $\alpha > 0$, since $\lambda_L = 2 - 1/\alpha$, while Gumbel has upper tail dependence with $\lambda_U = 2 - 2^{1/\alpha}$. However, the Frank copula has neither lower nor upper tail dependency. Archimedean copulas are constructed with only one parameter α through their individual generators. The relationships between the parameter of Archimedean copulas and measures of concordance are presented in Table 1. Once properly estimated the parameter values, people can compute the joint probability as well as the dependence structure among assets.

Table 1. Relationship between the parameter of Archimedean copulas and the measures of concordance

Family	Kendall's tau	Spearman's rho
Clayton	$1-\alpha^{-1}$	No closed form
Gumbel	$\alpha/(\alpha+2)$	Complicated expression
Frank	$1+4[D_1(\alpha)-1]/\alpha$	$1-12[D_2(-\alpha)-D_1(-\alpha)]/\alpha$

Source: Cherubini et al., (2004)

3.4.5. Copula summary

Implementation of copulas involves three steps including 1) select and construct a copula, 2) estimate the parameters associated with the copula, and 3) sample from the parameterized copula. Copula parameters are estimated through a maximum likelihood estimation method of the form of

$$\hat{\delta}_2 = argmax_{\hat{\delta}_2} \sum_{t=1}^{T} \ln c\left(\hat{G}_x(x_t), \hat{H}_y(y_t), \delta_2\right), \tag{3.4.29.}$$

where $\hat{\delta}_2$ is the estimated copula parameter, argmax is the mathematical functions that provides the argument associated with the maximum, ln is the natural logarithm, and $\hat{G}_x(x_t), \hat{H}_y(y_t)$ are the estimated marginal distributions for x and y. To avoid distributional assumptions, a non-parametric distribution is used for the marginal distributions. The Schwarz Information Criteria (SIC) and Akaike Information Criteria (AIC) were utilized for selecting the most appropriate multivariate copula. AIC and SIC are goodness of fit statistics that are superior to other fit ranking criteria (e.g. chi-squared).

Copula represents a powerful tool for decomposing the joint distribution problem into marginal distribution and dependence structure problems that can be dealt with separately. One can choose the marginal distribution that best fits each data asset, and afterwards integrate everything using some desirable properties of a copula function.

As described previously, Copulas have been applied to the measurement of credit and market risk, in particular to the assessment of the VaR of a portfolio. It allows people to compute VaR while avoiding the usual assumption of marginal and joint normality and linear correlation structure. Copula and VaR are associated in the empirical model section and call it mean-VaR with copula method.

3.5. Summary

This chapter introduces the theoretical model for general commodity processor's risk management strategies. It also provides detailed descriptions of risk measurement methods and copula method. Chapter IV applies theoretical framework to the real-world situations and develops an empirical model that focuses on the flour milling industry.

CHAPTER V. EMPIRICAL MDOELS

4.1. Introduction

Chapter III developed the general theoretical model that can be applied to different processing industries, such as corn milling, oilseeds crushing, flour milling etc. The empirical model, in this chapter, only applies theoretical framework to the flour milling industry.

Flour milling is an important agent in the wheat-product supply chain where wheat is processed into value-added products. This industry is a mature, low margin industry with overcapacity. Managing ingredient procurement risk and product sales risk become primary tasks for flour millers since it has a significant impact on the margin of the business. How to develop strategies and advanced tools to build the model is the main topic of this chapter. Another purpose of the empirical analysis is to examine the effectiveness of the theoretical model, by evaluating its ability to explain the business behaviors of firms. Certainly, specific assumptions or adjustments will be adopted, according to the business requirement and characteristic of the flour milling industry.

Chapter IV is organized as the following. Section 4.2 introduces the business background of a typical flour milling firm. Section 4.3 spends a lot effort on specifying models for different business scenarios. It focuses on three major business scenarios, under each associated with three model specifications. It is followed by data source and analysis section, which explains the data origins and presents results of data manipulation and analysis. This chapter ends with a brief summary.

4.2. Flour Milling Business Description

Wheat flour is a powder made from the grinding of wheat, and it is the most common flour for human consumption. There are different types of wheat flour, which are distinguished by the amount of gluten they contain. Gluten, simply speaking, is a protein composition that gives baked goods their structure. Hard flour, alternatively called bread flour, contains high gluten content, about 12% to 14%. Such flour is usually made from hard red spring wheat (HRS Wheat). HRS Wheat has relatively high protein content, making it ideal for yeast bread baking.

In this thesis, the representative flour mill is located in Minneapolis, Minnesota. It is assumed that the chosen flour miller produces only one type of flour, 13.5% Protein Baker's Standard Patent Flour. When considering typical 1% of protein loss, the miller need to procure a combination of 14% Protein #1 Dark Northern Hard Red Spring Wheat and 15% Protein #1 Dark Northern Hard Red Spring Wheat in equal amounts. The spot market to purchase these ingredients will occur in Minneapolis terminal market.

In addition, some technical assumptions are made. To produce from inputs into outputs, a flour extraction rate is of 72%, the industry standard. The 28% production residual is mill feeds that can also be sold in the market. The daily milling capacity is assumed to be 15,000 hundred weights of flour a day. If running for 21 days per month at full capacity, the representative mill produces 315,000 hundredweight of flour and 122,500 hundredweight of mill feeds per month. It requires him to buy 364,584 bushels of 14% HRS Wheat and 364,584 bushels of 15% HRS Wheat. This flour mill firm is in a perfect competitive market environment with no government intervention. Table 2 specifies the input and output quantities of the milling process.

Table 2. Milling input and output quantities

	Input		Output	
Assets	14% HRS Wheat (bu)	15% HRS Wheat (bu)	13.5% Flour (cwt)	Mill feeds (cwt)
Quantity	364,583	364,583	315,000	122,500

Flour milling is a low margin industry in a competitive market environment. Therefore, managing procurement risk and forecasting output price and demand become primary tasks for flour millers. The rest of this chapter discusses different risk management models for the above prototypical flour mill, according to different procurement and sales scenarios.

4.3. Procurement and Sales Scenarios

The following four scenarios are considered in this thesis: 1.) wheat has been purchased, products have been sold in advance; 2.) Products sold, wheat not purchased; 3.) wheat purchased, products not sold; 4.) wheat not purchased, products not sold. Mill feeds are managed with the same sales strategy as flour, simultaneously.

The four scenarios can intuitively be explained in a position report (Table 3). For the first case, if products are sold in advance through forward contracts and wheat is already bought from the spot market, the miller faces no potential price risk. There is no incentive to hedge with futures market, which is shown in the December case, net cash position equals 0. January transaction corresponds to the second case where flour is sold with no wheat bought yet. The net cash wheat position of -1000 requires the miller to long MGE March Futures in order to hedge the wheat price fluctuation. If the futures position is 1000, then the hedge ratio is 1. February transaction represents the third case that cash wheat is bought, but flour has not been sold under forward contract. The net position is +1000, which suggests shorting March futures to mitigate the price risk of end products. The last scenario is indicated by March transaction, where none flour is sold nor is wheat bought. The miller then faces both input and output price risk, which requires it to find an appropriate balance between long and short futures position to reduce the total risk. April and May examples show cases when hedge ratio differs from 1.

Table 3. Position report

	Flour Sales	**Cash Wheat Position**	**Net Cash Position**	**Futures**						
				March	May	July	…	Dec	Total Futures	Hedge ratio
December	-1000	1000	0	0					0	#DIV/0!
January	-1000	0	-1000	1000					1000	1
February	0	1000	1000	-1000	0				-1000	1
March	0	0	0		y				y	z
April	-500	1000	500		-200				-200	0.4
May	-1000	500	-500			300			300	0.6
…	…	…	…						…	…
November	0	0	0					0	0	#DIV/0!

The position report presents a straightforward way to look at risk management, by taking an opposite position of the hedging instrument to offset underlying asset risk. Detailed model specification for each scenario is introduced in the following sub-sections. The last three scenarios are our main focus. Under each scenario, there are three risk management model specifications. The first one is the traditional risk hedging strategy, by which the miller always sets the hedge ratio equals 1.0. However, HR=1 may reduce the possibility of making larger profits and it does not show flexibility to meet our utility expectation. Therefore, it requires two other approaches to compute the optimal hedge ratio that meets the requirements. The second approach is Mean-VaR optimization, under multi-normal distribution and linear correlation assumptions. The third is Mean-VaR that incorporates Copula to specify more accurate distribution and dependence structure. Construction of each model specification in detail for these three scenarios respectively is shown in the following sub-sections.

It is important to distinguish between mean-variance and mean-VaR approaches. Numerous studies in the literature have used portfolio optimization (Anderson & Danthine, 1981; Dahlgran, 2005; Garcia et al., 1994; among others) that are utilizing mean-variance approach. In conventional single period mean-variance optimization, processors aim at maximizing expected return subject to a selected level of risk, and make portfolio allocation for a single upcoming period. Processors also develop utility function to demonstrate the tradeoff between the expected return and risk, and then optimize the utility. In either way, the risk measurement is portfolio variance, which accounts for the variation on both sides of the portfolio distribution. Alternatively, instead of penalizing both tails, the VaR method can be applied to penalize downside risk only. The procedure of mean-VaR is to capture the portfolio expected return and the maximal portfolio loss at a level of confidence. The difference is the risk measurement, minor but important.

4.3.1. Scenario #1: products sold, wheat bought

This is a trivial scenario. Flour miller has paid for ingredients and sold products under contract; therefore, he faces no price or market risk. He does not need hedging protections from futures market.

4.3.2. Scenario #2: products sold, wheat not purchased

The timeline of the hedging decision and spot purchase is illustrated in Figure 4. Under this scenario, flour and mill feeds sales price and quantity demanded have been determined in advance, which leaves millers plenty of room to determine their procurement strategy. The only risk in this scenario is ingredient price risk. This thesis assumes that the hedging decision occurs *n* months ahead of the spot purchase, say *t-n*, where *n*=1,2,3 and *t* is month to acquire the ingredient. Then the expected return Π formula of this scenario is presented as below:

$$
\begin{aligned}
E(\Pi) = {} & \bar{Q}_f \bar{P}_f + \bar{Q}_{mf} \bar{P}_{mf} - \left(\bar{P}_{14,8} \prod_{j=9}^{8+n} (1+\tilde{R}_{14,j}) + \bar{P}_{15,8} \prod_{j=9}^{8+n} (1+\tilde{R}_{15,j})\right) \cdot \bar{Q}_w / 2 \\
& + \left(\bar{P}_{wf,8} \prod_{j=9}^{8+n} (1+\tilde{R}_{wf,j}) - \bar{P}_{wf,8}\right) \cdot \hat{Q}_{wf}
\end{aligned}
\qquad (4.3.1.)
$$

where $\bar{Q}_f$ and $\bar{Q}_{mf}$ are contracted quantify of flour and mill feeds, $\bar{P}_f$ and $\bar{P}_{mf}$ are flour and mill feeds prices, $\bar{P}_{14,8}$ and $\bar{P}_{15,8}$ are 14% and 15% protein wheat spot prices in August, $\bar{P}_{wf,8}$ stands for wheat futures price in August, and all of them are acknowledged constants. $\tilde{R}_{14,j}$, $\tilde{R}_{15,j}$, and $\tilde{R}_{wf,j}$ represent monthly rate of return of 14%, 15% cash wheat and wheat futures in month j, and they are random variables. Then, $\bar{P}_{i,8} \prod_{j=9}^{8+n} (1+\tilde{R}_{i,j})$ is the projected price after n months for asset i. $\hat{Q}_{wf}$ is an important decision variable, wheat futures position. Hence, (Q_{wf} / Q_w) stands for the hedge ratio. The first two terms in the equation are revenues generated from selling products. The third term is the total cost on procuring inputs. The final term is the payoff from wheat futures contract. It assumes the contract transaction cost and firm's constant operational cost being zeros in the empirical model.

In the formulation, output price and quantity demanded are fixed while wheat spot and futures price at time *t* are random. Payoffs from futures position offset price fluctuations in cash markets. Month *t* and *t-n* depend on flour mill industry's practice and the overall wheat market movements. For simplicity, it assumes the month to hedge always happens in August. Then the spot purchase of wheat happens in September, October, and November for n=1,2,3 correspondingly. Hedge ratio can be evaluated from different methodologies.

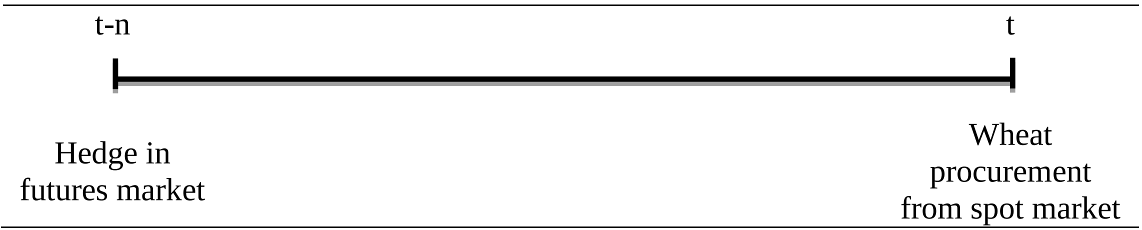

Figure 4. Timeline for Scenario #2

4.3.2.1. Specification 1: mean-VaR with HR=1

Hedge ratio sets equal to 1 need to long the same amount of futures as desired wheat. Therefore, $\hat{Q}_{wf}$ equals $\bar{Q}_w$ in Equation 4.3.1.

This approach follows traditional assumptions, which are multi-normal joint distribution and linear dependence structure. Each asset return $\tilde{R}_{14,j}$, $\tilde{R}_{15,j}$, and $\tilde{R}_{wf,j}$ is following a normal distributions, as well. Therefore, the standard deviation of the portfolio is as below:

$$\sigma(\Pi) = \sqrt{(\sigma_{14}Q_{14}, \sigma_{15}Q_{15}, \sigma_{wf}Q_{wf})(\Sigma)(\sigma_{14}Q_{14}, \sigma_{15}Q_{15}, \sigma_{wf}Q_{wf})^T} \qquad (4.3.2.)$$

where σ_i is the price standard deviation of the asset i, $2Q_{14} = 2Q_{15} = Q_w = Q_{wf}$, and Σ stands for the linear correlation matrix. VaR is used as the risk measurement. Based on the normal distribution assumption for Π, 5% VaR formula is presented as following:

$$VaR(\Pi) = E(\Pi) - 1.64\sigma(\Pi) \qquad (4.3.3.)$$

Individual's utility can be easily calculated from Equation 4.3.4. The risk-aversion of the firm is characterized by the parameter λ, where λ is always non-negative. The firm appears to be risk-neutral when the parameter equals zero; while the firm becomes more and more risk averse as the coefficient increases. It is difficult to measure a realistic value of / for a particular company, but through a variety of values of / it is able to see different hedging strategies for firms with different risk tolerance. This thesis chooses the risk aversion coefficient to be 2 in our base case, and then conduct a sensitivity analysis on / . $\lambda = 2$ ensures a one-to-one tradeoff between expected return and VaR, which makes it standard initial case.

$$U = E(\Pi) - \frac{\lambda}{2} VaR(\Pi) \qquad (4.3.4.)$$

4.3.2.2. Specification 2: non-copula-based mean-VaR

Instead of pre-determining HR=1, hedgers can maximize utility by determining the optimal hedge ratio. All the assumptions and formulations are the same as specification 1 but $\hat{Q}_{wf}$ is no longer necessarily equal to $\overline{Q}_w$. The objective is to maximize the utility function 4.3.4.

Since large speculative position is not considered, the hedge ratio is chosen to lie between -2 and 2. Through iterative method, we determine the optimal hedge ratio so that 4.3.4 is maximized.

4.3.2.3. Specification 3: copula-based mean-VaR

Linear correlation is the measure of dependence when returns to assets in the portfolio are normally distributed (Szegö, 2005). However, the normality assumption for asset returns has been shown to be limited (Just & Weninger, 1999; W. Sun, S. Rachev, F. Fabozzi, & P. Kalev, 2009). Alternative risk measures and dependency measures have been developed to account for non-normal data (Roger B. Nelsen, 2006; Stoica, 2006). Here, it uses Copula to capture the flexible non-Gaussian joint distribution and non-linear dependence structure among assets' monthly returns, and VaR to measure the portfolio downside risk.

Variables $\tilde{R}_{14,j}$, $\tilde{R}_{15,j}$, and $\tilde{R}_{wf,j}$ in Equation 4.3.1 are simulated from the best-fit copula joint distribution, with the specification of marginal distributions and dependence structure. This change from previous methodologies impacts the expected return in the hedging model. Then from copula-based portfolio return distribution, it is capable to capture the lowest 5% value to be our portfolio VaR. The objective can be mathematically interpreted as

$$\text{Max}U = E(\Pi_C) - \frac{\lambda}{2} VaR(\Pi_C) \qquad (4.3.5.)$$

Iterating hedge ratio $(\hat{Q}_{wf} / Q_w)$ from -2 and 2 allows us to find the maximum utility of Equation 4.3.5. Specification 3 allows for non-Gaussian joint distribution and non-linear dependence structure among assets, and is considered a confident tool to determine hedging strategy.

4.3.3. Scenario #3: wheat bought, products not sold

Under this scenario, flour and mill feeds sales prices become the only risk. There is no concern with respect to the ingredient price since people have already paid for it. The hedging decision against the product price fluctuation can occur *m* months ahead of the sales in the market, say *t+m*, where m=1,2,3 and *t* is the month to obtain hedging position. The timeline of the hedging decision and sales in the spot market is illustrated in Figure 5. Then the expected return formula of this scenario is presented as below:

$$E(\Pi) = (\bar{P}_{f,9}\prod_{j=10}^{9+n}(1+\tilde{R}_{f,j}))\cdot\bar{Q}_f + (\bar{P}_{mf,9}\prod_{j=10}^{9+n}(1+\tilde{R}_{mf,j}))\cdot\bar{Q}_{mf}$$

$$-(\bar{P}_{14,9}+\bar{P}_{15,9})\cdot\bar{Q}_w/2+(\bar{P}_{wf,9}\prod_{j=10}^{10+n}(1+\tilde{R}_{wf,j})-\bar{P}_{wf,9})\cdot\hat{Q}_{wf} \qquad (4.3.6.)$$

$$+(\bar{P}_{cf,9}\prod_{j=10}^{9+n}(1+\tilde{R}_{cf,j})-\bar{P}_{cf,9})\cdot\hat{Q}_{cf}$$

where all notations are with the same meanings as in Scenario #2. The difference in the formulation is to add another hedging component, corn futures, to cross hedge risk from mill feeds price. $\hat{Q}_{wf}$ and $\hat{Q}_{cf}$ are our important decision variables, wheat and corn futures positions. In this scenario, (Q_{wf}/Q_f) and (Q_{cf}/Q_{mf}) stand for the hedge ratios of flour and mill feeds. The first two terms in the equation are anticipated revenue from selling products. The third term is the known cost on procuring inputs. The final two terms are the payoffs from hedging instruments.

In the formulation, output quantities demanded are assumed fixed, which follow the extraction rate from obtained wheat. Payoffs from wheat and corn futures positions offset flour and mill feed price fluctuations in the spot markets. This case assumes the month to hedge happens in September. Then the sales may happen in October, November, December for *m*=1,2,3 correspondingly. Similar to Scenario #2, it follows three specifications to evaluate hedge ratios in order to balance between expected return and associated risks.

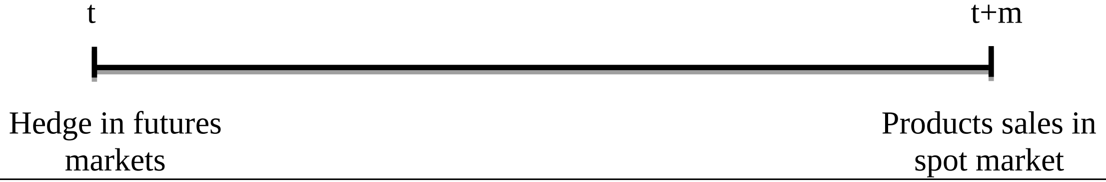

Figure 5. Timeline for Scenario #3

4.3.3.1. Specification 1: HR=1

When hedging ratios set equal to 1, it needs to short both wheat and corn futures, where $\hat{Q}_{wf} = -\bar{Q}_f$ and $\hat{Q}_{cf} = -\bar{Q}_{mf}$ in Equation 4.3.6.

Normal distribution and linear correlation for assets are still the primary assumptions, i.e. $\tilde{R}_{i,j} \sim N(\mu_i, \sigma)$, where *i*=flour, mill feeds, flour futures, and corn futures. Therefore, the standard deviation of the portfolio is as below:

$$\sigma(\Pi) = \sqrt{\begin{array}{l}(\sigma_f \bar{Q}_f, \sigma_{mf} \bar{Q}_{mf}, \sigma_{wf} \hat{Q}_{wf}, \sigma_{cf} \hat{Q}_{cf})(\Sigma) \\ \\ (\sigma_f \bar{Q}_f, \sigma_{mf} \bar{Q}_{mf}, \sigma_{wf} \hat{Q}_{wf}, \sigma_{cf} \hat{Q}_{cf})^T\end{array}} \qquad (4.3.7.)$$

where σ_i is the price standard deviation of the asset i and Σ stands for the linear correlation matrix. We compute 5% VaR based on the following formula:

$$VaR(\Pi) = E(\Pi) - 1.64\sigma(\Pi) \qquad (4.3.8.)$$

Choosing the risk aversion coefficient to be 2, it can calculate an individual's utility from Equation 4.3.9. Again, $/ = 2$is our base case, followed by sensitivity analysis.

$$U = E(\Pi) - \frac{\lambda}{2} VaR(\Pi) \qquad (4.3.9.)$$

4.3.3.2. Specification 2: non-copula-based mean-VaR

Instead of setting hedge ratios to be 1 people can select the optimal hedge ratio to maximize an individual's utility. Under this scenario, all the assets are following normal distribution assumption, and their correlation structure is linear. The goal is to find optimal Q_{wf} and Q_{cf}.

Since great speculative position is not considered, the hedge ratios lie between -2 and 2. Through iterative method, it is capable to find the optimal hedge ratio so that 4.3.9 is maximized.

4.3.3.3. Specification 3: copula-based mean-VaR

Similarly to Scenario #1, it uses Copula to capture the flexible non-Gaussian joint distribution and dependence structure between assets, and VaR to measure the portfolio downside risk.

Variables $\tilde{R}_{i,j}$ (i=flour, mill feeds, wheat futures, and corn futures) in Equation 4.3.6 are simulated from best-fit copula joint distribution. Multivariate copula, instead of traditional normal distribution and linear correlation assumption, has a significant impact on the expected return and downside risk of our hedging model. From copula-based portfolio return distribution, VaR is chosen to be the lowest 5% value of the portfolio. The objective, maximizing utility function, shares the same mathematical form with Equation 4.3.5.

Iterating hedge ratios (Q_{wf} / Q_f) and (Q_{cf} / Q_{mf}) from -2 and 2 enables an individual to maximize his utility. The hedge ratios provide us the best hedging strategy by indicating the optimal futures positions in wheat and corn markets. Scenario #3 only involves product sales risk and specification 3, allowing for non-Gaussian joint distribution and non-linear dependence structure among assets, provides a confident tool to determine hedging strategy.

4.3.4. Scenario #4: wheat not bought, products not sold

This is the most difficult scenario among the four since it needs to consider both ingredients and products price risk. Under this scenario, none of the wheat has been bought nor have products been sold. Therefore, hedgers need to include instruments to hedge procurement and sales risk. The hedging decision against ingredients price fluctuation can occur n months ahead of the wheat procurement, say $t-n$, where n=1,2,3 and t is the month to acquire physical wheat. The hedger lifts the wheat futures position once he bought physical wheat from cash market, and then open corn and wheat futures position to hedge against products sales risk. In this scenario, for simplicity, it only considers m =1. The timeline of hedging decision, procurement, and sales time point is illustrated in Figure 6. Then the expected return formula of this scenario is presented as below:

$$
\begin{aligned}
E(\Pi) = {} & (\bar{P}_{f,8}\prod_{j=9}^{8+n+1}(1+\tilde{R}_{f,j}))\cdot\bar{Q}_f + (\bar{P}_{mf,8}\prod_{j=9}^{8+n+1}(1+\tilde{R}_{mf,j}))\cdot\bar{Q}_{mf} \\
& -(\bar{P}_{14,8}\prod_{j=9}^{8+n}(1+\tilde{R}_{14,j}) + \bar{P}_{15,8}\prod_{j=9}^{8+n}(1+\tilde{R}_{15,j}))\cdot\bar{Q}_w / 2 \\
& +(\bar{P}_{wf,8}\prod_{j=9}^{8+n}(1+\tilde{R}_{wf,j}) - \bar{P}_{wf,9})\cdot\hat{Q}_{wf,p} + (\tilde{P}_{wf,8+n}\prod_{j=9+n}(1+\tilde{R}_{wf,j}) \\
& -\tilde{P}_{wf,8+n})\cdot\hat{Q}_{wf,s} + (\tilde{P}_{cf,8+n}\prod_{j=9+n}(1+\tilde{R}_{cf,j}) - \tilde{P}_{cf,8+n})\cdot\hat{Q}_{cf}
\end{aligned} \tag{4.3.10.}
$$

where all notations are with the same meanings as in Scenarios #2 and #3. The difference in the formulation is to predict prices of flour, mill feeds, physical wheat, wheat futures, and corn futures at different time points. Still, $\hat{Q}_{wf}$ and $\hat{Q}_{cf}$ are our decision variables, wheat and corn futures positions. While, $\hat{Q}_{wf,p}$ stands for the procurement hedge position with wheat futures and $\hat{Q}_{wf,s}$ is the sales hedge position with wheat futures. In this scenario, $(\hat{Q}_{wf,p}/Q_w), (\hat{Q}_{wf,s}/Q_f)$, and $(\hat{Q}_{cf}/Q_{mf})$ stand for the hedge ratios for physical wheat, flour and mill feeds. It is assumed the month to hedge procurement risk happens in August. Physical ingredient procurement may happen in October, November, December for *n*=1,2,3 correspondingly. Then it allows one month processing and selling all the products in the spot market at time t + 1.

The first two terms in the equation are revenues from selling products. The third term is the projected cost for procuring inputs. The fourth term is the payoff from lifting wheat futures position against procurement risk. The final two terms are the payoffs from hedging instruments against sales risk. Payoffs from wheat and corn futures positions offset physical wheat, flour and mill feeds price fluctuations in the spot markets. Similar to previous two scenarios, it is following three model specifications to evaluate hedge ratios.

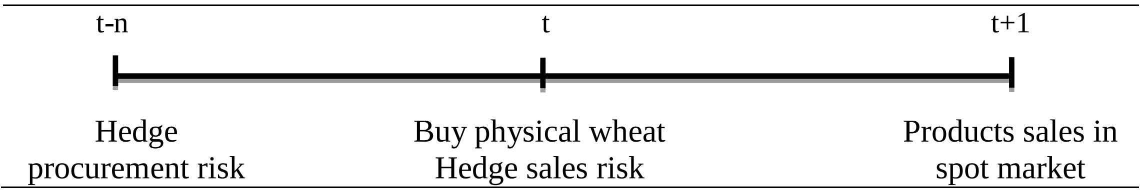

Figure 6. Timeline for Scenario #4

4.3.4.1. Specification 1: HR=1

When hedging ratios set equal to 1, hegers long wheat futures to hedge ingredient risk and short wheat and corn futures at time *t* to hedge against product price risk, where $\hat{Q}_{wf,p} = \bar{Q}_w$, $\hat{Q}_{wf,s} = -\bar{Q}_f$, and $\hat{Q}_{cf} = -\bar{Q}_{mf}$ in Equation 4.3.10.

Normal distribution and linear correlation for assets are still our primary assumptions, i.e. $\tilde{R}_{i,j} \sim N(\mu_i, \sigma)$, where *i*=flour, mill feeds, 14% cash wheat, 15% cash wheat, wheat futures for both purchase and sales hedge purpose, and corn futures. Therefore, the standard deviation of our portfolio is as below:

$$\sigma(\Pi) = \sqrt{\begin{array}{l}(\sigma_f \bar{Q}_f, \sigma_{mf} \bar{Q}_{mf}, \sigma_{14} \bar{Q}_{14}, \sigma_{15} \bar{Q}_{15}, \sigma_{wf,p} \hat{Q}_{wf,p}, \sigma_{wf,s} \hat{Q}_{wf,s}, \sigma_{cf} \hat{Q}_{cf})(\Sigma) \\ (\sigma_f \bar{Q}_f, \sigma_{mf} \bar{Q}_{mf}, \sigma_{14} \bar{Q}_{14}, \sigma_{15} \bar{Q}_{15}, \sigma_{wf,p} \hat{Q}_{wf,p}, \sigma_{wf,s} \hat{Q}_{wf,s}, \sigma_{cf} \hat{Q}_{cf})^T\end{array}} \quad (4.3.11.)$$

where σ_i is the price standard deviation of the asset i and Σ stands for the linear correlation matrix. Then the following formula computes 5% VaR:

$$VaR(\Pi) = E(\Pi) - 1.64\sigma(\Pi) \quad (4.3.12.)$$

Choosing the risk aversion coefficient to be 2, it can calculate an individual's utility from Equation 4.3.13.

$$U = E(\Pi) - \frac{\lambda}{2} VaR(\Pi) \qquad (4.3.13.)$$

4.3.4.2. Specification 2: non-copula based mean-VaR

Similar to previous scenarios, hedgers can select the optimal hedge ratio to maximize the utility. Still, all the assets follow normal distributions assumption and linear correlation. The objective is to find optimal $Q_{wf,p}, Q_{wf,s}$, and Q_{cf} to maximize utility function. Through iterative method, it is capable to find the optimal hedge ratio so that 4.3.13 is maximized.

4.3.4.3. Specification 3: copula based mean-VaR

Copula allows flexible marginal distributions, non-Gaussian joint distribution, and dependence structure between assets.

Variables $\tilde{R}_{i,j}$ (*i*=flour, mill feeds, 14% and 15% cash wheat, wheat futures, and corn futures) in Equation 4.3.10 are simulated from best-fit copula joint distribution. It strictly follows the specifications with copula under previous scenarios.

4.4. Data Sources

There are in total six assets studied in this empirical analysis. The database was aggregated from several different sources. The products, 13.5% protein Baker's Standard Patent and mill feeds in the Minneapolis market, were retrieved from *Milling and Baking News*, Ingredient Market Trends sections from a number of issues for many years. These two assets are weekly prices. Hard red spring 14% and 15% protein wheat daily basis were taken from the Minneapolis Grain Exchange website. HRS wheat futures daily price is also retrieved from MGE while corn futures daily price is taken from CME. Since the duration of product prices is weekly, all other daily prices are converted into weekly averages. The time period of the data is from January 2005 to December 2012, 418 observations in total.

The model computes 4-week logarithmic price returns for the assets and adjusts for seasonality. All returns are calculated as the percentage logarithmic price ratio with the formula $r_{t,t+4} = \ln(\frac{p_{t+4}}{p_t})$. Table 4 shows the seasonal factors extracted from the original 4-week return that will be adjusted back when calculating gross margin. Table 5 summarizes the statistics of 4-week price change of all six assets, including mean return, standard deviation, skewness, and kurtosis, note that the returns have been adjusted for seasonality. The skewness and kurtosis for assets suggest that normal distribution is a poor assumption. This encourages to utilize copula to include non-normal marginal distributions in the description of an accurate joint distribution. In the analysis, SAS is used to select the best-fit copula to conduct multivariate analysis under each scenario. It also estimates the selected copula's parameters and simulates 10,000 vector returns from the multivariate joint distribution. Important copula component, marginal distribution for each asset, is listed in Table 6. However, in this thesis, empirical marginal distributions are utilized as the copula component. Copula parameters are estimated through a maximum likelihood estimation method of the form of

$$\delta_2 = \operatorname{argmax}_{\delta_2} \sum_{t=1}^{T} \ln c\left(G_x\left(x_t\right), H_y\left(y_t\right), \delta_2\right), \qquad (4.4.1.)$$

where δ_2 is the estimated copula parameter, argmax is the mathematical function that provides the argument associated with the maximum, $\ln$ is the natural logarithm, and $G_x\left(x_t\right), H_y\left(y_t\right)$ are the estimated marginal distributions for x and y. To avoid distributional assumptions, a non-parametric distribution is used for the marginal distributions. Akaike Information Criteria (AIC) was utilized for selecting the most appropriate multivariate copula. AIC is considered a superior goodness of fit statistics to other fit ranking criteria (e.g. chi-squared).

Table 4. Monthly averages as the seasonal factors

	Seasonal Factor					
Month	**Flour**	**MF**	**Cash14**	**Cash15**	**wheat Futures**	**Corn Futures**
1	0.041	-0.068	0.022	0.022	0.021	0.062
2	0.060	-0.000	0.063	0.058	0.055	0.008
3	-0.012	-0.047	-0.010	-0.007	-0.015	0.014
4	-0.007	-0.101	-0.024	-0.018	-0.023	0.010
5	-0.004	-0.019	-0.001	-0.001	0.001	0.012
6	0.004	0.025	0.023	0.013	0.025	0.010
7	-0.038	0.131	-0.039	-0.042	0.002	0.001
8	-0.023	0.031	-0.012	-0.032	0.012	-0.027
9	-0.011	0.117	-0.002	0.022	0.008	0.019
10	0.015	-0.010	0.019	0.023	0.003	0.009
11	0.043	0.012	0.039	0.018	0.002	0.018
12	-0.003	0.091	0.007	0.014	0.019	0.010

Table 5. Descriptive statistics for assets

Assets price return	**Mean**	**Std.Dev**	**Skewness**	**Kurtosis**
Flour	0	0.094	0.137	1.521
Mill feeds	0	0.196	0.008	0.138
14% cash wheat	0	0.088	0.375	2.400
15% cash wheat	0	0.093	0.504	3.241
Wheat futures	0	0.096	0.114	2.950
Corn futures	0	0.090	-0.247	1.012

Table 6. Marginal distributions for assets

Asset	De-seasonalized return best-fit Distribution
Flour	Logistic(0.0000436938,0.051342)
Mill feeds	Normal(-1.6593E-18,0.19572)
14% wheat	Logistic(-0.0012344,0.047016)
15% wheat	Loglogistic(-1.3758,1.3724,27.939)
Wheat futures	Laplace(-0.00075447,0.097246)
Corn futures	Logistic(0.0015988,0.049722)

Interestingly, for all three scenarios, t-copula is selected to be the best fit copula. Table 7 provides an AIC ranking for major copula fit under each scenario, the more negative value, the better fit. Table 8-10 presents t-copula parameters for each scenario.

Table 7. AIC ranking for available copulas, the smaller number, the better fit

Scenario	t-copula	Gaussian	Gumbel	Clayton	Frank
2	-1183	-1088	-957	-726	-819
3	-484	-383	-112	-136	Invalid
4	-2004	-1800	-534	-585	Invalid

Table 8. Student t-copula parameter for Scenario #2

	14% wheat	15% wheat	Wheat futures
14% wheat	1	0.901	0.802
15% wheat	0.901	1	0.712
Wheat futures	0.802	0.712	1

Table 9. Student t-copula parameter for Scenario #3

	Flour	**Mill feeds**	**Wheat futures**	**Corn futures**
Flour	1	-0.108	0.727	0.254
Mill feeds	-0.108	1	0.084	0.236
Wheat futures	0.727	0.084	1	0.532
Corn futures	0.254	0.236	0.532	1

Table 10. Student t-copula parameter for Scenario #4

	Flour	**Mill feeds**	**14% wheat**	**15% wheat**	**Wheat futures**	**Corn futures**
Flour	1	-0.119	0.877	0.797	0.709	0.239
Mill feeds	-0.119	1	0.078	0.101	0.070	0.237
14% wheat	0.877	0.078	1	0.901	0.793	0.336
15% wheat	0.797	0.101	0.901	1	0.694	0.253
Wheat futures	0.709	0.070	0.793	0.694	1	0.530
Corn futures	0.239	0.237	0.336	0.253	0.530	1

Some bivariate copula relationships of interest are demonstrated in the following figures for illustration purposes (as the empirical analysis use multivariate copula). Notice that a good ellipse relationship of 14% HRS wheat/15% HRS wheat (Figure 7) and 14% HRS wheat/flour (Figure 8), which suggests that a bivariate normal copula may be the best fit for these two variables. However, a star shaped scatter plot in Figure 9 and 10, which implies a t-copula for the two variable pairs that capture some inverse dependence. The scatter plots in Figure 11 and 12 are more dispersed that not display a clear shape; it is hard to conclude a copula type for these two pairs. Each pair may have different copula dependence, but the multivariate copula fitted for the whole data set is more of interested. The flexibility of the copula modeling allows the shape of each marginal distribution to be maintained and in theory more accurately capturing the risk (VaR) that exists in the lower tails.

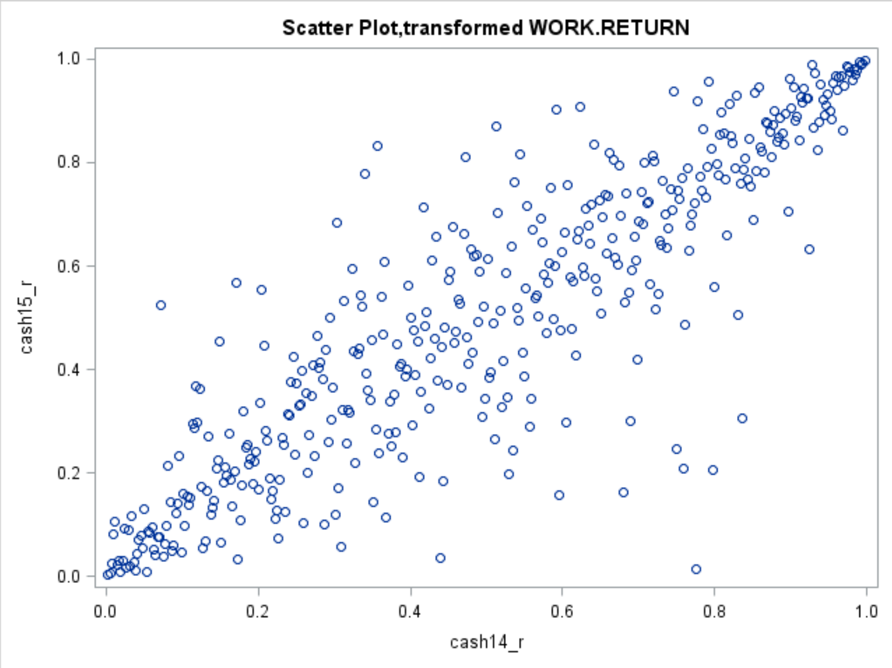

Figure 7. Scatter plot of uniformly transformed 14% and 15% Protein HRS wheat weekly return

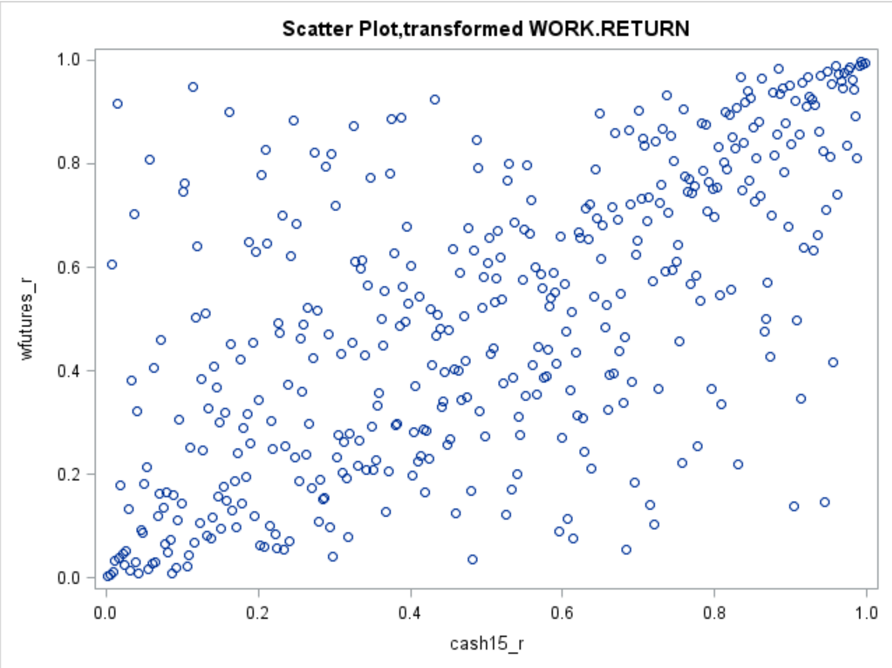

Figure 8. Scatter plot of uniformly transformed wheat futures and 15% Protein HRS wheat weekly return

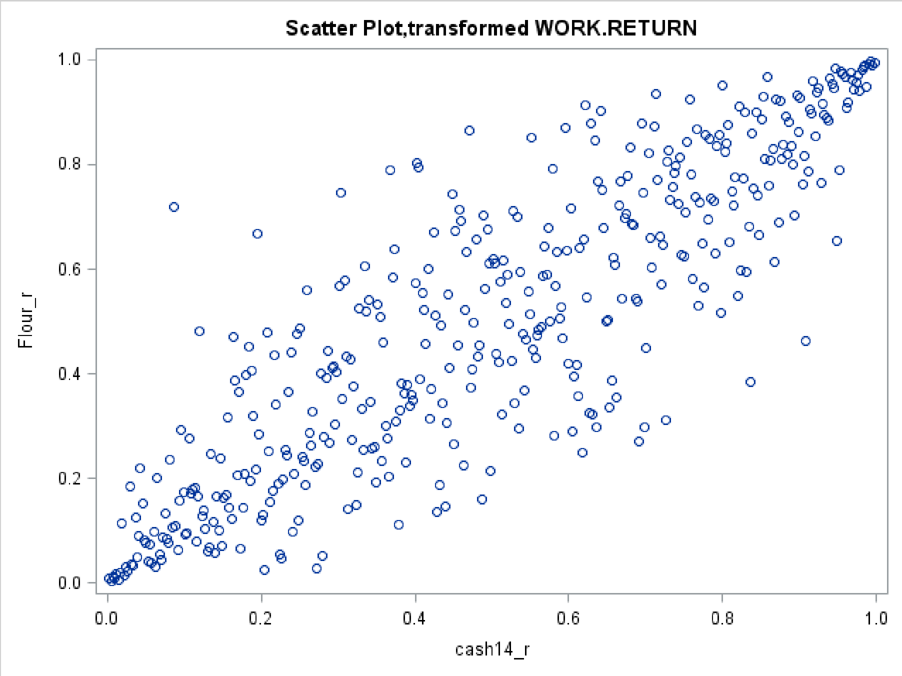

Figure 9. Scatter plot of uniformly transformed 15% Protein HRS wheat and flour weekly return

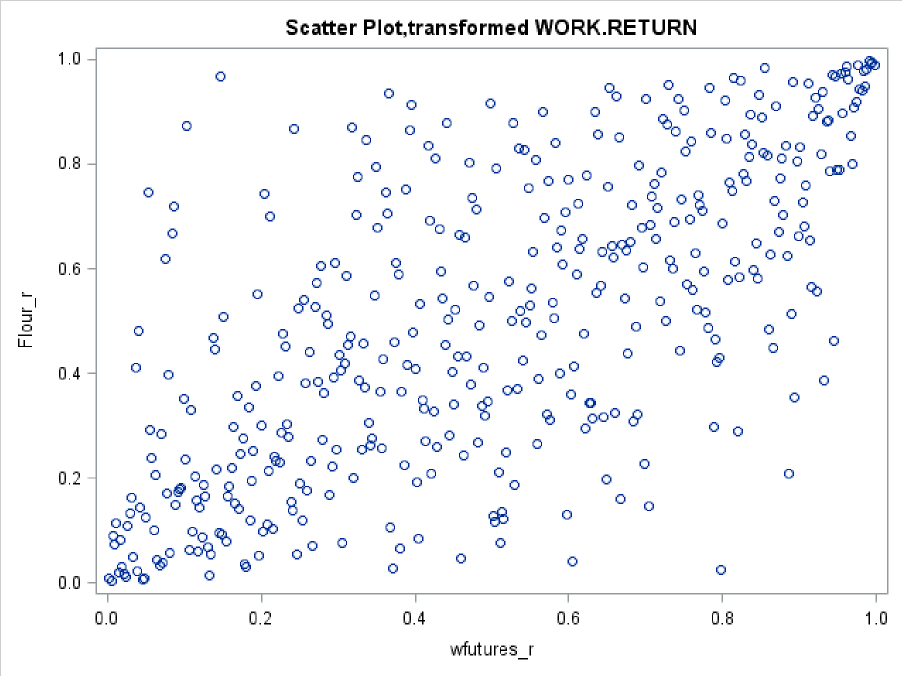

Figure 10. Scatter plot of uniformly transformed wheat futures and flour weekly return

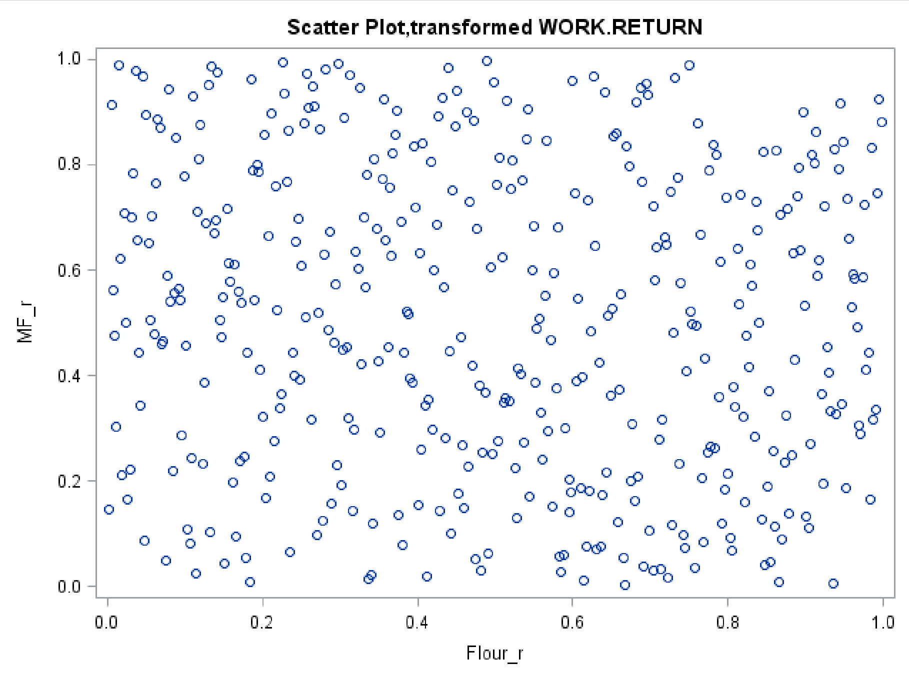

Figure 11. Scatter plot of uniformly transformed mill feeds and flour weekly return

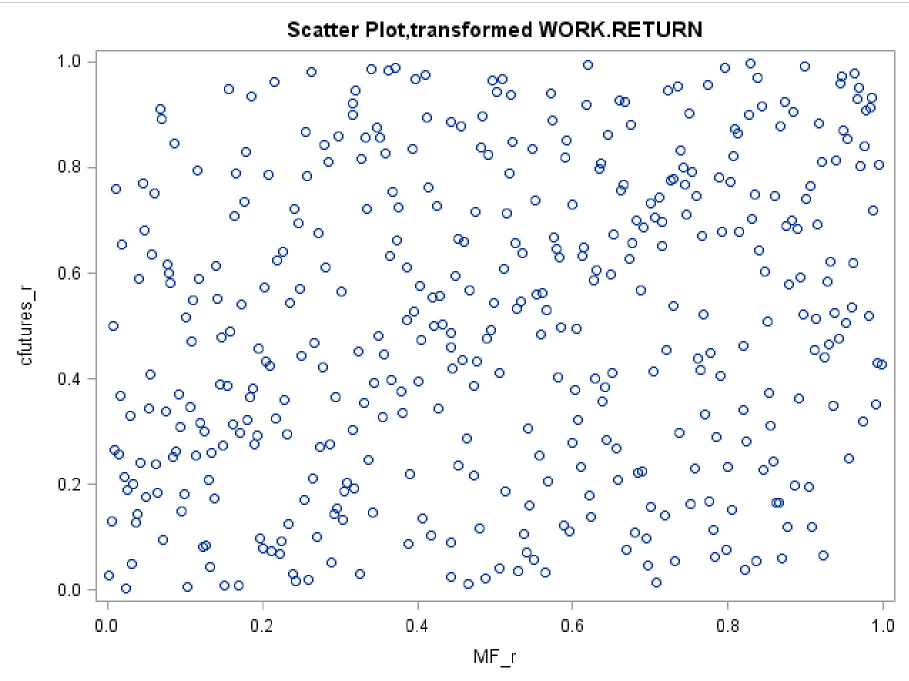

Figure 12. Scatter plot of uniformly transformed corn futures and mill feeds weekly return

The final step is to simulate values from the estimated copula. Monte Carlo simulation samples 10,000 monthly returns for each asset based on copula joint distribution. The simulations are 4-week price return for each asset, based on copula specified dependence structure. In this t-copula case, the dependence structure is still linear correlation matrix. The multivariate student-t copula converges to a Gaussian copula as degrees of freedom increases (Cherubini et al., 2004). However, the t-copula has more dependence in the tail for a smaller number of degrees of freedom.

4.5. Summary

This chapter developed three major scenario studies analyzed in this thesis. It provided detailed explanations of the model specifications used to resolve each scenario. The model setup and statistical techniques were also given. In addition, the various data sets used in the study and the sources of the data were shown. Price data were analyzed for statistical characteristics and to serve as inputs for the analytical models. Chapter V presents the results of the empirical analysis for the flour milling industry.

CHAPTER V. RESULTS

5.1. Introduction

Chapter IV derived the empirical model specifications on risk management strategies for flour milling processors. This chapter presents the empirical results for different flour milling scenarios along with three different analytical specifications. The results provide optimal hedge ratios and quantitative measures of the expected gross margin, the worst probable margin, and individual's favorable balance between the expected return and risk. Three modeling specifications are used to evaluate risk and returns, which were defined in detail in Chapter IV. The first specification is the traditional risk hedging strategy that sets the hedge ratio equals 1. The second approach is Mean-VaR methodology under multi-normal distribution and linear correlation assumptions. The third specification is Mean-VaR with copula that allows for flexible non-normal marginal and joint distribution and non-linear dependence structure. Utility is the measurement of tradeoff between return and risk, which provides a benchmark to evaluate the effectiveness of risk management specifications 1 and 2. Since specification 3 models return and risk differently, it is not directly comparable to the results from previous two specifications. Specification 3 provides a new approach, mean-VaR with copula, to quantify a firm's utility.

For the flour miller, there are four possible scenarios included in this thesis. Scenario #1 is where wheat has been purchased, and products have also been sold in advance. In this scenario, there is no price risk associated; hence it is a trivial case. Products being sold in advance but wheat not being purchased form the Scenario #2. This case only concerns ingredients price risk. Scenario #3 describes when wheat is purchased, but products have not been sold. Products' sales price risk becomes the major concern. Scenario #4 is the case when wheat is not purchased, and products are not sold; both ingredients and products price risk are the focus.

This thesis compares and contrasts three specifications used for each scenario. The first specification sets hedge ratios equal 1, which represents the traditional hedging strategy. In this specification, Mean-VaR with normal distribution and linear correlation assumptions helps to quantify gross margin while using VaR as risk measurement. However, HR=1 reduces the possibility of reaching a greater utility by restricting profit and risk. Therefore, in the second specification, based on previous Mean-VaR approach, the hedge ratios iterate from -2 to 2 in order to seek for the largest utility value. The third specification is Mean-VaR that incorporates Copula to specify joint distribution and dependence structure, in order to find optimal hedge ratios.

The following section (Section 5.2) discusses the results calculated from each specification under each scenario. Section 5.2 divides into four subsections according to four business scenarios and presents result respectively. A brief summary of this chapter is presented in Section 5.3.

5.2. Scenario Results

This section includes four subsections, each presenting the result and illustrating the analysis on the result under one particular business scenario. Charts and tables are provided to illustrate the results and analysis for each risk management strategies.

5.2.1. Scenario #1: products sold, wheat bought

The first scenario is when wheat has been bought from the market, and flour and mill feed have been sold in advance under forward contract. The flour miller then faces no potential market price risk. This is a trivial case with no need for further study.

5.2.2. Scenario #2: products sold, wheat not purchased

The second scenario evaluates when flour and mill feed sales price and quantity demanded have been determined in advance while the ingredients--two types of wheat--need to be purchased from the cash market. This case leaves processor some room to determine their procurement strategy. The only risk involved in this scenario is ingredient price risk. Three assets have been studied, 14% and 15% protein HRS wheat and MGE wheat futures. MGE wheat futures are considered as the hedging instrument. The firm's cash position risk can be offset by positions taken in futures contracts.

Figures for mean versus VaR are presented below for different hedge durations. Each point on the efficient frontier describes a combination of expected return and VaR risk that is based on a series of hedge ratios. This is the base case analysis that takes place on risk aversion coefficient equals to two. There are two efficient frontiers on each chart, representing the results calculated from mean-VaR with non-copula and mean-VaR with copula specification respectively. Even though it is not directly comparable to conclude which specification is better because of distinct assumptions, it is still important to discuss the differences and characteristics implied from the frontiers.

Observing from these three charts, it is easy to find that expected margin shifts downward and VaR shifts rightward for both specifications as hedge duration increases from 1 month to 3 months. This is likely due to assets' prices experiencing greater uncertainty as hedging length extends and ingredients market price rising in a faster pace than wheat futures after the harvest season. Margin distribution is related to seasonality factor of ingredients and futures market since it is more reasonable for processors to purchase input at a low market price season. On the other hand, it tends more difficult to predict the price in a longer term since the price variation is larger, which becomes another reason for a greater VaR value in a longer hedging period.

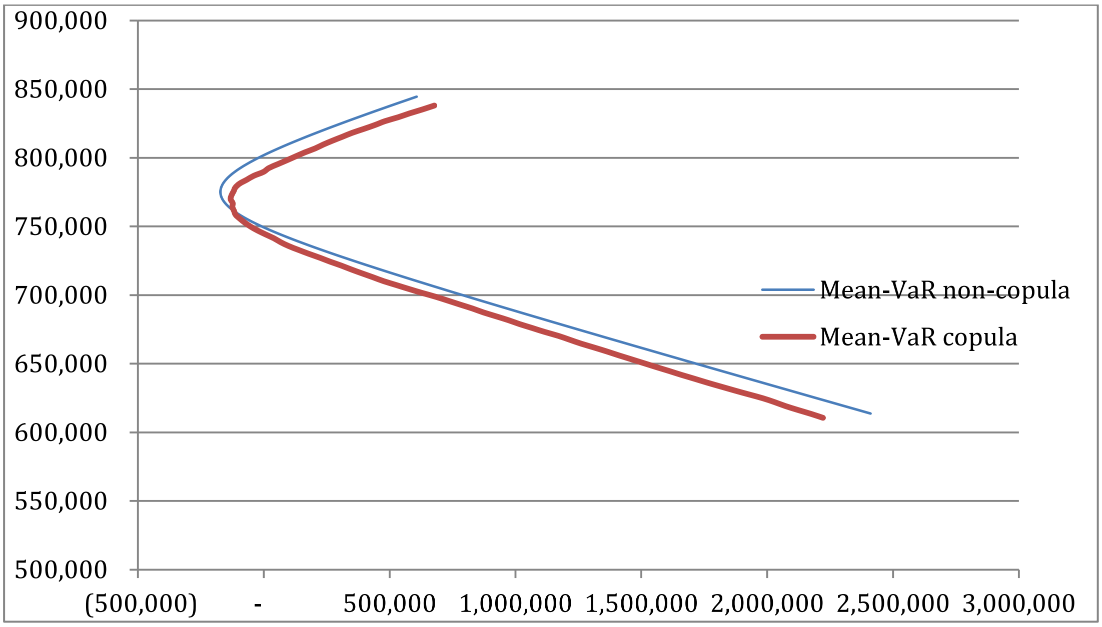

Figure 13. Hedge duration 1 month, Scenario #2

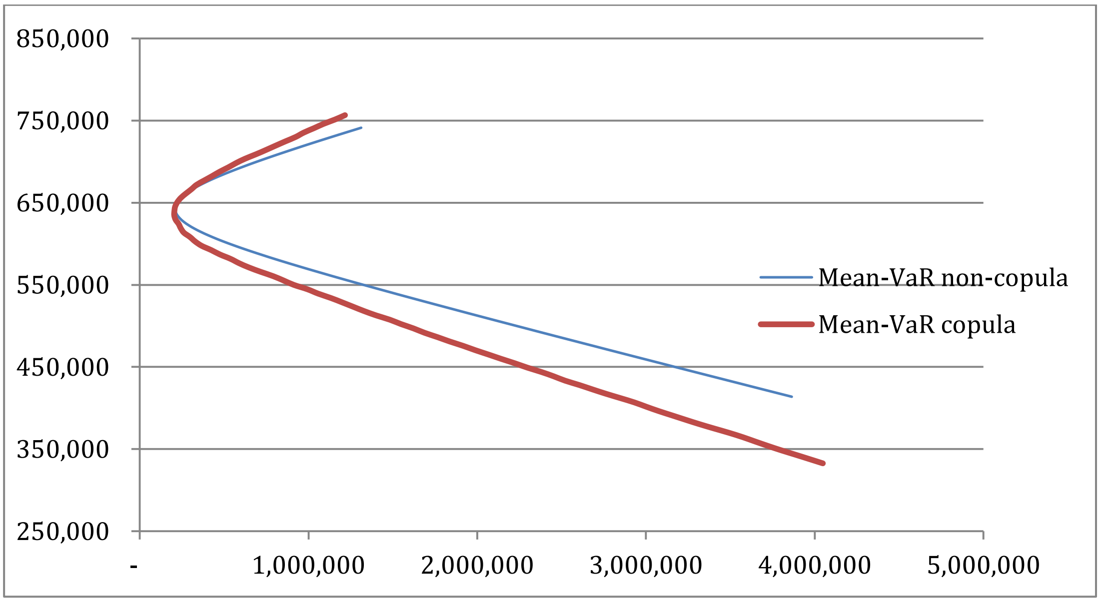

Figure 14. Hedge duration 2 month, Scenario #2

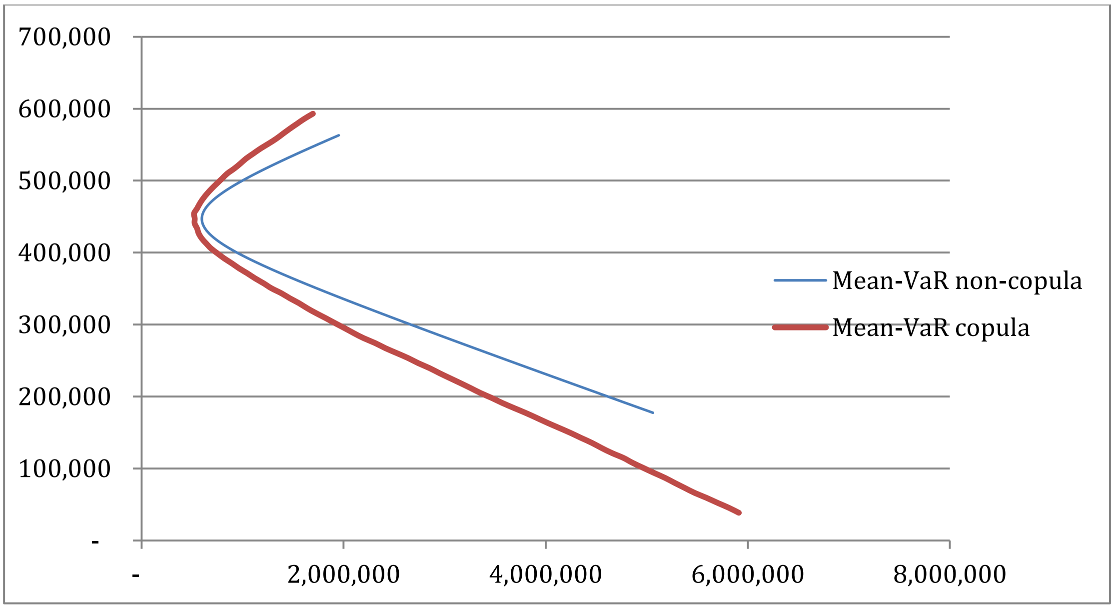

Figure 15. Hedge duration 3 month, Scenario #2

When the hedge duration is 1 month, there appears no remarkable difference between the results of two specifications. As duration is extended, expected margin becomes more sensitive to the change of VaR of mean-VaR copula specification than of mean-VaR non-copula specification. It is likely caused by the normal distribution assumption that under evaluate the variation of the portfolio value. On the contrast, t-copula connects the empirical marginal distribution of each asset to describe the joint distribution in a flexible way.

The difference between expected return at the same level of risk of two specifications is not significant when hedge ratio ranges between -1 and 1. If speculative position is not considered in the futures market, the decisions derived from two specifications appear similar or close.

Table 11 reports the results of the various hedging cases analyzed in Scenario #2. The first column simply indicates the hedge duration from 1 to 3 months. The next column repeats the applicable three specifications. Column three lists the optimal hedge ratios derived under each specific specification. The optimal hedge ratio is the decision variable that leads to the firm's risk management decision. Column four and five provide the corresponding expected gross margin and standard deviation. It reports the one-month VaR statistics at the 95% confidence level in column six, indicating the most value the firm could imagine to loss under normal market movements in one month. This method of reporting risk is in contrast to traditional variance or standard deviation statistics, which is shown in column five.

The last column reports the utility statistics. Utility values are not ranked since they are evaluated under distinct specifications and are not directly comparable. Higher utility of mean-VaR with non-copula specification than alternative mean-VaR with copula specification does not necessarily mean a better strategy to resolve the problem. Instead, it shows the value that is most likely to occur under this particular specification that the firm is the most comfortable to go. Nevertheless, hedgers can have a direct comparison between specification 1 and 2 since they specify the expected margin and risk in an identical way. In addition, individuals can compare statistics of the same specification for different hedge durations.

Specification 2 always yields a higher utility than specification one, with its hedge ratio about 0.17 lower than traditionally conceived hedge ratio 1, from Table 11. Though the expected gross margin from method 2 is not as high as method 1, its associated risk is also much lower, which reflects a higher utility statistic. Since everything applied in the two specifications are identical besides that one is pre-assigned the hedge ratio while the other is seeking the optimal, it can be concluded that specification 2 dominates specification 1. This implies that the processor can achieve a better balance between expected gross margin and risk compared to traditional hedging strategy.

Mean-VaR with copula is defined differently from specifications 1 and 2 by allowing flexible marginal and joint distributions and non-linear correlation. When hedge duration is 1 month, the optimal hedge ratio is a little lower than method 2 by approximately 0.02. However, as hedging length extends, the ratio approaches closer to 1 but 5%VaR significantly goes higher and utility goes lower. This suggests a processor to go with shorter hedge duration and fully protect themselves in the futures market if choosing a long hedging duration. Except duration 1, specification 3 gives out a slightly higher utility value than the other two which hints that the other two specifications may under estimate the possible optimal utility. It is likely reasonable to apply copula to allow for flexible marginal distribution for each asset and capture a better dependence structure.

Table 11. Results for Scenario #2

Duration (month)	**Specification**	**Optimal HR**	**E(Π) $1,000**	**σ(Π) $1,000**	**VaR(Π) $1,000**	**Optimal utility in 1,000**
1	Mean-variance	0.77	773	367	-172	-1.35E+8
	HR=1	1.00	786	397	-135	922
	Mean-VaR non-copula	0.82	777	369	-172	948
	Mean-VaR copula	0.83	771	419	-133	903
2	Mean-variance	0.77	640	519	211	-2.69E+8
	HR=1	1.00	659	562	262	397
	Mean-VaR non-copula	0.82	645	522	211	434
	Mean-VaR copula	0.95	646	551	206	439
3	Mean-variance	0.77	444	636	598	-4.04E+8
	HR=1	1.00	467	688	662	-195
	Mean-VaR non-copula	0.82	449	639	598	-148
	Mean-VaR copula	0.98	453	670	513	-60

Much of the previous literature use mean-variance approach to specify models and optimize portfolios. Though mean-variance is not the focus in this thesis, the results under this approach were evaluated in Table 5.1 for comparison. The result remains unchanged for three different hedge durations, which always equals 0.77. It is also interesting that the expected gross margin, standard deviation, and 5% VaR are nearly the same for mean-variance and mean-VaR non-copula specifications. These two specifications share normal distribution and linear correlation assumptions, while differentiate in risk measurements. Comparing mean-variance to mean-VaR with copula results, the standard deviation of the prior specification is always lower than the later while the VaR is always greater. This implies that mean-variance approach may underestimate the volatility, but overestimate the most possible loss. The optimal hedge ratios computed from mean-variance are always lower than that derived from mean-VaR with copula.

We provided a discussion on the results from the base case that takes place on risk aversion coefficient equals 2. The following presents sensitivity analysis on the risk aversion coefficient that ranges from -0.5 to 3.

Table 12 shows that 5% VaR decreases as λ increases, which conforms to the definition of risk aversion coefficient. The optimal hedge ratios for three hedge durations decrease as risk aversion coefficient increases, for both mean-VaR with and without copula specifications. This suggests processors to decrease their exposure to the futures market if they become opposed to risk. There appears not much indication in hedge ratios when hedge duration extends.

Table 12. Sensitivity analysis on risk aversion coefficient, Scenario #2

Duration (month)	**Specification**	**Optimal HR**	**E(Π) $1,000**	**σ(Π) $1,000**	**VaR(Π) $1,000**	**Optimal utility in 1,000**
1	HR=1	1.00	786	397	-135	922
	Mean-VaR non-copula	0.82	777	369	-172	948
	Mean-VaR copula	0.83	771	419	-133	903
2	HR=1	1.00	659	562	262	397
	Mean-VaR non-copula	0.82	645	522	211	434
	Mean-VaR copula	0.95	646	551	206	439
3	HR=1	1.00	467	688	662	-195
	Mean-VaR non-copula	0.82	449	639	598	-148
	Mean-VaR copula	0.98	453	670	513	-60

5.2.3. Scenario #3: wheat bought, products not sold

The third scenario evaluates when wheat ingredients have already been purchased from the market while processing products need to be sold into the market in the future. This case leaves the processor a question of deciding the sales strategy. The only risk involved in this scenario is products' price risk. Four assets have been studied, namely flour, mill feeds, MGE wheat futures, and CBOT corn futures. MGE wheat futures are selected to hedge the flour price fluctuation while CBOT corn futures are chosen to cross hedges mill feeds price risk, based on the relative high dependency. There does not exist futures market on the products, which enforces us to cross hedge with close related assets.

Unlike the previous scenario, this scenario has two decision variables, which are flour hedge ratio and mill feeds hedge ratio. It becomes difficult to present an intuitive result on the efficient frontier, as the number of iterative variables increases to two. However, the following truncated Table 13 shows the concept of how to find the optimal hedge ratios. Specification 2 and 3 both obtain such similar tables. Column 1 represents the wheat futures hedge ratio; correspondingly, corn hedge ratio is listed in column 2. The remaining four columns are expected gross margin, margin standard deviation, margin associated 5% VaR, and the firm's utility respectively. For each combination of the two hedge ratios, a corresponding utility is derived. Conversely, the largest utility statistic will return the optimal hedge ratio vector. It is interesting to note from the original tables[4] for both specifications 2 and 3, that when corn hedge ratio range from -2 to 2 on a determined wheat hedge ratio, the utility value increases then decreases. This shape forms an inverse parabola.

[4] For an original result table, please contact the author.

Table 13. Truncated table of hedge ratio combinations and relevant statistics

Wheat Hedge Ratio	Corn Hedge Ratio	E(Π) $1,000	σ(Π) $1,000	VaR(Π) $1,000	Optimal utility in 1,000
-2.00	-2.00	820	877	555	264
	-1.75	824	851	497	326
	⋮	⋮	⋮	⋮	⋮
	2.00	888	641	116	771
-1.75	-2.00	825	785	415	409
⋮	⋮	⋮	⋮	⋮	⋮
-1	0.25	877	459	-118	995
	0.5	882	462	-122	1004
	0.75	886	467	-115	1001
	1	890	476	-109	1,000
	1.25	894	488	-90	985
	1.5	898	503	-78	977
	1.75	903	520	-50	953
	2	907	540	-26	934
⋮	⋮	⋮	⋮	⋮	⋮
2	1.75	961	1,642	1,697	-736
	2	965	1,666	1,742	-777

Table 14 reports the results for Scenario #3. Column 1 is the hedge duration, column 2 indicates the specifications, column 3 lists the optimal hedge ratio vectors, and the rest are corresponding expected gross margin, standard deviation, 5% VaR, and optimal utility. The optimal hedge ratio vector is our decision variable that leads to the firm's risk management decision, with the first dimension being the wheat hedge ratio and the second being the corn hedge ratio.

The last column lists the utility statistics. For the same reason as previous scenario, the utilities are not ranked since they are not directly comparable. Nevertheless, people can have a direct comparison between specification 1 and 2.

Noted from the table below, specification 2 always yields a higher utility than specification one, accompany with a higher expected margin and much lower 5% VaR. It is capable to conclude that specification 2 dominates specification one in this scenario, which suggests the processor to adjust traditional hedge strategies to achieve a greater satisfaction.

Since corn futures and mill feed are positively correlated, the conventional hedging concept is to short corn futures to protect against the rise of mill feeds price. Though, it may challenge processing firm's decision group when holding wheat and corn futures position simultaneously. Traditional intuitive strategy is to 100% short wheat and corn futures, which is represented in specification one. However, through optimization method, we realize that long some portion of corn futures has a positive effect on enlarging profit while reducing risk. Under specification two for three different hedge durations, the wheat futures are all fairly close to -1 that suggests a processor to obtain nearly 100% short positions, which conforms to our traditional knowledge. On the contrast, the result recommends a long position in corn futures market. When hedge duration is one, the optimal mill feeds hedge ratio is +60%; while, it approaches 104~115% long position when hedge duration becomes 2 and 3 months. Such speculative ambition may result from the relatively low correlation between corn futures and mill feeds, as well as comprise to wheat futures position. Short wheat and long corn futures strategy provides a higher expected gross margin and a lower VaR value than traditional hedging strategy, which is considered superior. Exact hedge ratio combination is listed in the table, under mean-VaR non-copula specification.

Table 14. Results for Scenario #3

Duration (month)	**Specification**	**Optimal HR**	**E(Π) $1,000**	**σ(Π) $1,000**	**VaR(Π) $1,000**	**Optimal utility in 1,000**
1	Mean-variance	-1.01, 0.16	867	446	-135	-1.99E+8
	HR=1	-1, -1	848	480	-61	909
	Mean-VaR non-copula	-1.04, 0.59	874	450	-135	1,010
	Mean-VaR copula	-0.92, 0.53	883	464	-127	1,011
2	Mean-variance	-1.01, 0.16	1,159	631	-124	-3.99E+8
	HR=1	-1, -1	1,104	679	9	1,095
	Mean-VaR non-copula	-1.12, 1.15	1,204	659	-123	1,328
	Mean-VaR copula	-0.97, 0.36	1,171	607	-192	1,364
3	Mean-variance	-1.01, 0.16	1,198	773	69	-5.98E+8
	HR=1	-1, -1	1,121	831	241	880
	Mean-VaR non-copula	-0.97, 1.03	1,262	813	71	1,190
	Mean-VaR copula	-0.99, 0.13	1,174	722	-47	1,220

Mean-VaR with copula allows for flexible marginal and joint distributions and non-linear correlation. There are some significant differences between the results of mean-VaR with and without copula specifications. Similarly, the result suggests processors to short nearly 100% wheat futures. As hedge duration extends from 1 to 3 months, the flour hedge ratio rises from -0.92 to -0.99, getting very close to -1; on the other hand, the speculative position of corn futures shrinks from 53.1% to 13.0%. Unlike previous specification, the mill feed hedge ratio is not very large that keeps processors away from high speculative positions. It looks like the utilities derived from specification three are the highest among the three methods, even though they are not directly comparable. The expected gross margin, 5% VaR, and utility statistics under specification three provide firms a more confident way to evaluate their market return and risk.

As observed from Table 14, the expected margin increases as the hedge duration increases, however, the 5% VaR increases, as well. The increase in gross margin may result from a speculative position in corn futures. The utility values do not follow any increasing or decreasing pattern along with the hedge duration movements. It is likely reasonable to apply copula to allow for flexible marginal distribution for each asset and capture a better dependence structure.

The results of mean-variance approach are presented in Table 14. The result-vector remains unchanged for three different hedge durations, which always equals (-1.01, 0.16). The flour hedge ratio is fairly close to -1 while it appears a small amount of speculative position in the mill feeds hedging instrument. The expected gross margin, margin standard deviation, and margin 5% VaR are nearly the same for mean-variance and mean-VaR non-copula specifications. These two specifications share normal distribution and linear correlation assumptions, but differentiate in risk measurements. Comparing mean-variance to mean-VaR with copula results, the optimal hedge ratios of the two specifications converge as hedge duration extends. There appears no clear pattern when comparing gross margin, standard deviation, and VaR of these two specifications.

Table 15 presents a sensitivity analysis on the risk aversion coefficient that ranges from -0.5 to 3. 5%VaR decreases as λ increases, which conforms to the definition of risk aversion coefficient. The optimal hedge ratio for flour decreases in a mild pace while it is in a faster pace for mill feeds, under both specifications. One important thing to note here is that the change of λ doesn't influence the overall shorting perspective of wheat futures and longing perspective of corn futures. It is

Table 15. Sensitivity analysis on risk aversion coefficient, Scenario #3

Duration (month)	**Specification**	**Optimal HR**	**E(Π) $1,000**	**σ(Π) $1,000**	**VaR(Π) $1,000**	**Optimal utility in 1,000**
1	HR=1	-1, -1	848	480	-61	909
	Mean-VaR non-copula	-1.04, 0.59	874	450	-135	1,010
	Mean-VaR copula	-0.92, 0.53	883	464	-127	1,011
2	HR=1	-1, -1	1,104	679	9	1,095
	Mean-VaR non-copula	-1.12, 1.15	1,204	659	-123	1,328
	Mean-VaR copula	-0.97, 0.36	1,171	607	-192	1,364
3	HR=1	-1, -1	1,121	831	241	880
	Mean-VaR non-copula	-0.97, 1.03	1,262	813	71	1,190
	Mean-VaR copula	-0.99, 0.13	1,174	722	-47	1,220

hard to conclude the monotonic movements of utility with respect to the movement of λ, but different individual with different risk preference is associated with a particular utility level.

5.2.4. Scenario #4: wheat not bought, products not sold

Under this scenario, none of the wheat has been bought nor have products been sold. Therefore, we need to include instruments to hedge both procurement risk as well as sales risk. In total six assets have been studied, which are14% and 15% protein HRS wheat, MGE wheat futures, CBOT corn futures, flour, and mill feed price. Wheat futures contract has been selected to hedge physical wheat procurement risk and flour sales risk while CBOT corn futures only hedge mill feeds price variation.

For this scenario, three decision variables considered are physical wheat, flour, and mill feeds hedge ratios. It is difficult to present and compare results in an efficient frontier or a data table, since the dimension of variables is three. However, we can demonstrate some extractable information from the result in Table 16. Table 16 is organized the same as previous scenarios, with column 3 being of our priority interest. The optimal hedge ratio vector consists of three components, wheat, flour, and corn hedge ratio in sequence.

Comparing the optimal utility statistics, specification 2 always returns a much higher utility than specification one. At the same time, it yields a higher expected margin and much lower standard deviation and 5% VaR. It is reasonable to draw the same conclusion as previous scenarios that specification 2 dominates specification 1. This suggests the processors to adjust traditional hedge strategies of restricting all hedge ratios to be unit; instead, they can achieve a greater satisfaction by flexibly accommodate assets' hedge ratios.

The conventional hedging strategy to processors is longing wheat futures to hedge procurement risk in the first period while shorting wheat futures and corn futures to hedge against products price risk in the second period. Such traditional intuitive strategy is represented in specification one, with long wheat HR being 1, short wheat HR and corn HR being -1. However, it is hard to see the validity of such intuitive thinking when hedging procurement and sales risk simultaneously. Through optimization in specification 2, it turns out to be very ambitious in longing wheat futures for all three hedge durations. The physical hedge ratios range from 1.775 to 1.966, indicating a large percentage of speculative positions. On the other hand, the flour hedge ratios lie between -1.686 and -1.980 that conform our intuitive short position perspective, but also they introduce a large speculative shorting position. Corn futures are considered the hedging instrument against mill feeds price risk. The hedge ratios for different durations are near -10% to -20%, which are considered small hedge positions. The corn futures conservative position may result from a relatively low correlation with mill feeds and its cross hedging characteristic. There seems a great risk when applying cross hedging; therefore, only a small amount of such position is preferred. The results generated from specification 2 seem more reasonable compared to restricting all hedge ratios to unit. This specification provides a higher expected gross margin and a lower VaR value than traditional hedging strategy, which is considered superior.

There are significant differences between the results of mean-VaR with and without copula specifications. The physical wheat hedge ratio is around 5% in specification three, dramatically less than 200% in specification 2. Flour hedge ratio is no smaller than -100%, which suggests non speculative position taken in shorting wheat futures. Lastly, instead of shorting corn futures to protect against mill feeds

Table 16. Results for Scenario #4

Duration (month)	Specification	Optimal HR	E(Π) $1,000	σ(Π) $1,000	VaR(Π) $1,000	Optimal utility in 1,000
1	Mean-variance	1.88, -2.00, -0.30	987	510	-150	-2.60E+8
	HR=1	1, -1, -1	959	528	-93	1,052
	Mean-VaR non-copula	1.94, -1.98, -0.18	992	513	-150	1,142
	Mean-VaR copula	0.04, -1.00, 0.50	895	570	-1	896
2	Mean-variance	1.74, -2.00,- 0.36	1,162	623	-139	-3.89E+8
	HR=1	1, -1, -1	1,091	662	-5	1,097
	Mean-VaR non-copula	1.77,-1.97,-0.09	1,174	630	-139	1,314
	Mean-VaR copula	0.06, -0.98, 0.53	1,051	652	-48	1,100
3	Mean-variance	2.00, -1.78, -0.19	1,038	747	186	-5.58E+8
	HR=1	1, -1, -1	999	783	284	715
	Mean-VaR non-copula	1.96, -1.68, -0.16	1,047	752	186	861
	Mean-VaR copula	0.06, -0.52, 0.24	904	725	213	690

price risk, the result of specification three indicts a long position. The mill feed hedge ratio ranges from 24.7% to 53.1%. It looks like the utilities derived under copula method is less than the ones without copula. The expected gross margin also appears less, while 5% VaR shows a larger value.

The much lower holding positions in all hedge instruments computed from specification three may be because that we consider procurement and sales hedging simultaneously. Economic market tells us that ingredients and products prices most often move closely together. If ingredient price increases, it is likely the price of the product will increase, in which situation the processor’s market price risk is self-reduced, and vice versa. It appears reasonable not to apply the risk management strategy with large hedge ratios, not even intensive speculative positions. Mean-VaR with copula approach provides a result that takes input-output co-movement into careful and accurate consideration, and keeps processors away from excessively exposed to derivative market. On the other hand, non-copula specification seems have undervalued the risk and overstated the expected gross margin, from its statistics shown in table 16.

It is likely reasonable to apply copula to allow for flexible marginal distribution for each asset and capture a better dependence structure, especially there are multiple variables to consider. There appears not much difference between specification 1 and 2 in Scenario #1 since there are only three variables. However, the difference becomes more significant across Scenario #2 and #3 since more variables come into the model for analysis. When more variables need consideration, it becomes severely important to capture an accurate dependence structure and to consider flexible marginal and joint distributions. The results illustrate that processors are facing much greater risk and lower expected margin with copula method than other traditional specifications. As hedge duration extends from 1 month to 3 months, the utility a processor will receive tends to decrease. The probable reason is that they face multiple uncertainties associated with multiple assets.

It is interesting to show the results of mean-variance approach in Table 16. Unlike the static behavior of last two scenarios' results, the results vector in this scenario varies a little for different hedge durations. It is important to note that the results are fairly close to those under mean-VaR non-copula specification. It appears large speculative positions in both physical wheat procurement and flour sales hedging instruments. Comparing mean-variance to mean-VaR with copula results, it seems that the standard deviation of the prior specification is lower than the latter, which suggests that mean-variance approach may underestimate the volatility. We do not observe clear patterns when comparing VaR of the two specifications. The optimal hedge ratios computed from mean-variance are much aggressive than those derived from mean-VaR with copula.

Sensitivity analysis on the risk aversion coefficient is represented in Table 17. λ ranges from -0.5 to 3. VaR statistic still decreases as λ increases, which conforms to intuition that the most risk an individual can tolerate decreases when he becomes more risk averse. The optimal hedge ratio for physical wheat decreases gradually while there appears no clear pattern for both flour and mill feed hedge ratios. It is likely that the flour hedge ratio remains unchanged and mill feeds ratio turned over from slightly longing into shorting position, under mean-VaR non-copula specification. On the other hand, both flour and mill feeds hedge ratios are nearly unchanged as risk aversion coefficient increases, under mean-VaR with copula specification. The monotonic trend of utility with respect to the movement of λ is not clear; different individual with distinct risk preference is associated with his own utility level.

Table 17. Sensitivity analysis on risk aversion coefficient, Scenario #4

Duration (month)	Specification	Optimal HR	E(Π) $1,000	σ(Π) $1,000	VaR(Π) $1,000	Optimal utility in 1,000
1	HR=1	1, -1, -1	959	528	-93	1,052
	Mean-VaR non-copula	1.94, -1.98, -0.18	992	513	-150	1,142
	Mean-VaR copula	0.04, -1.00, 0.50	895	570	-1	896
2	HR=1	1, -1, -1	1,091	662	-5	1,097
	Mean-VaR non-copula	1.77, -1.97, -0.09	1,174	630	-139	1,314
	Mean-VaR copula	0.06, -0.98, 0.53	1,051	652	-48	1,100
3	HR=1	1, -1, -1	999	783	284	715
	Mean-VaR non-copula	1.96, -1.68, -0.16	1,047	752	186	861
	Mean-VaR copula	0.06, -0.52, 0.24	904	725	213	690

5.3. Summary

This chapter presents the hedging results for four flour milling business scenarios. Scenario #1 discusses the situation when both procurement and sales prices are fixed, hence with no price risk associated. Scenario #2 only concerns ingredients price risk. Scenario #3 discusses the products sales risk. Scenario #4 takes both input procurement risk and output sales risk into account. We applied and compared three analysis specifications, namely traditional strategy when HR=1, mean-VaR without copula, and mean-VaR with copula, for each scenario.

The results conclude that specification 2 always dominates specification 1, which suggests processors to seek for optimal hedge ratio instead of applying conventional HR=1 strategy. They can achieve a better balance between expected gross margin and risk, compared to traditional hedging strategy.

Mean-VaR with copula allows for flexible marginal and joint distributions and non-linear correlation. This specification is more efficient in base assumptions of distribution and dependence structure, and it provides processors more confidence to assess returned statistics, such as expected gross margin, 5% VaR, and utility. When the number of variables in the model is only three, there is not great difference between specification two and three. Whereas the number increases to six, we observed significant differences of results between method two and three, which indicates copula's severe role in the model. It is likely reasonable to apply copula to allow for flexible marginal distribution for each asset and capture a better dependence structure, when there are multiple variables to consider. Hence, mean-VaR with copula is considered a more efficient, hence a more confident, specification in modeling. The final chapter draws a conclusion for this thesis.

CHAPTER VI. CONCLUSION

6.1. Introduction

Commodity risk management has drawn an increasing attention over the past several decades. Farmers try to mitigate the risk from selling agricultural commodities. Traders aim to maximize arbitrage profitability while minimize market risk in a certain amount of time. Processors typically hedge their ingredients and products' price risk in futures and options market to protect profit margins. Buyers seek ways to lower fluctuating procurement price risk through different types of contracts. Traditional approaches take a position in a closely related derivative market to offset the exposure in the underlying commodity. The idea within such an approach is to synchronize the decrease (increase) of the derivative asset with the increase (decrease) of the underlying asset, but depend on the dependence structure of the two or multiple assets.

Perfect protection results from perfect linear correlation between underlying commodity and corresponding financial asset. However, there are two major reasons for agribusiness agents to derive optimal hedge ratios actively other than taking perfect protection. The first is that few commodities have an associated derivative asset with an exact price correlation. Instead, hedgers are more likely to find a better co-movement or dependence structure between assets. Secondly, agents in the agribusiness supply chain would never set minimizing risk as their ultimate goal. Rather, the goal of maximizing profit margins or utility drives their business. Therefore, an optimal hedge ratio or a better size of the position in the derivative asset is selected to balance between the expected margin and associated risk, hence to meet the objectives of the agribusiness agent. A growing body of literature is built to derive the optimal hedge ratio and manage risk.

Agricultural commodity processor is a key component in the entire agribusiness supply chain. Past studies about processors mostly focused on managing the risk of price fluctuations in ingredients. Though, it is important to model strategies of a firm incorporating both ingredients and products markets at the same time. The decision-making process should closely depend on different scenarios the processor may face. Another drawback of past literatures is that they could not present an accurate measurement when taking the relationship between assets into consideration. Mean-VaR methods with linear correlation and multi-normal distribution assumptions are not enough to describe the relationship among assets, especially when the asset pool is large. If assets are not linearly dependent, then the parameter poorly describes the dependence structure. At the same time, involving more assets may result in more non-normal marginal distributions hence lower the chance of having multi-normal joint distribution. Fortunately, copula is a methodology to overcome the dependence and pre-assumed normal distribution drawbacks of traditional models. In addition, value-at-risk substitutes traditional variance measure of risk and provides firms a clear view of the most value they can lose.

The hedge horizon is known as a concept of determining how far the hedging strategy should be implemented. The amount of months ahead of the physical asset procurement and output sales to hold positions in derivative markets should also be addressed to processors. Longer hedge horizon results in higher uncertainty, while shorter hedge horizon leaves less room to make other strategic planning. Therefore, in addition to previous studies, this thesis discusses on the decision whether to hedge for risks over the next month, three months, or even a longer time of period.

The objective of this study is to develop utility maximizing models that balance between expected return and associated risk for commodity processors.

Considering different business scenarios a processor may encounter, this thesis incorporates advanced technics to derive more confident hedging strategies.

6.2. Methodologies

6.2.1. Value-at-Risk (VaR)

The development of RiskMetrics™ by the risk management group at J.P. Morgan in 1994 stimulated the Value-at-Risk's (VaR) growth. It received its popularity in both academia and industrial areas promptly. VaR is defined by Jorion (2007) as a single, summary statistic that measures the worst expected losses during a given time period, with a specified level of confidence, under normal market conditions.

The three widely used VaR-computation approaches are historical, parametric, and Monte Carlo simulation. Historical simulation explores the VaR of a portfolio solely based on historical dataset, which does not include any possible future information. Parametric method is referred as variance/covariance approach, whose fundamental assumption is that the random variables are normally distributed. Monte Carlo Simulation requires individuals to assign appropriate distributions to assets, and then simulate out portfolio distribution based on selected marginal distributions.

Both upside and downside tails of a distribution are penalized when variance is used as the conventional risk measure. However, upside tail is more favorable to an individual since it represents an unexpected profit. Individuals only have incentives to measure the downside loss. VaR is an attractive alternative tool to capture only downside risk. Not only in corporate finance and banking industry, it is drawing strong attention in the agribusiness industry, as well.

VaR addresses on an alternative risk measurement from variance. Copula is utilized to solve the crucial issues of how to account for dependence structure of multiple assets and joint multivariate distribution.

6.2.2. Copula

Copula functions enable one to describe a joint distribution by tackling separately with the needs of the marginal distribution and dependence structure. They are a far more flexible method to capture the real dependence structure and multivariate joint distribution rather than the traditional use of linear correlation matrix and multinormal assumption. Copula has been used as a state-of-the-art technology in this article and is compared with traditional methods.

Sklar’s Theorem is central and fundamental to the theory of copulas. It demonstrates the critical role that copula plays in the relationship between multivariate joint distribution and its corresponding univariate marginal distributions. The generalization to the n-dimensional case of Sklar’s theorem ensures that every copula is a joint distribution function. Multivariate copulas obtain more practical applications. Chapter III introduced multivariate Gaussian, student-t, Frank, Gumbel, and Clayton copulas; In Chapter IV empirical analysis, it picked the best fit one among these frequently used copulas.

Copula functions provide a way to capture an accurate dependence structure between assets. Linear correlation coefficient is a widely adopted tool to describe the relationship between among assets in different markets. However, when assets don’t present a linear relationship, the correlation coefficient doesn’t provide useful information. Fortunately, the concept of dependence embedded in copula function is much more general than the standard linear correlation concept. The dependence structure described by copula includes the concordance (Kendall's *tau* and Spearman's *rho*), linear correlation, and tail dependence.

Implementation of copulas involves three steps: 1) select and construct a copula 2) estimate the parameters associated with the copula and 3) sample from the parameterized copula. Following these three steps, we applied copula in the empirical section and contrast the results from traditional methods. The benefits of copula are that it allows for flexible joint distributions of returns rather than the more typical multivariate normal joint distribution assumption and a more general dependence structure.

Copula has been gaining popularity in financial literatures as an alternative tool for modeling joint distributions and dependence structures for quite some time, but the applications of copula in the agricultural industry are recent. This thesis applies copula in the agribusiness industry and addresses its benefits in studying multiple assets.

6.3. Summary of Results and Contributions

In order to demonstrate different risk management strategies for an agribusiness processor, this study developed the case of a hypothetical flour mill located in Minneapolis, Minnesota. Flour milling is a low margin industry in a perfect competitive market environment. Therefore, effectively managing risk becomes primary tasks for flour millers. This thesis broke down the flour milling business into four scenarios that concerns with distinct risks. Scenario #1 discusses the situation when both procurement and sales prices are settled, hence with no price risk associated. Scenario #2 only concerns ingredient procurement risk. Scenario #3 discusses the product sales risk. Scenario #4 takes both input procurement risk and output sales risk into account simultaneously.

There are in total six assets included and analyzed in the models, namely hard red spring 14% and 15% protein wheat, MGE wheat futures, CBOT corn futures, flour, and mill feed price. Physical wheat is our input; flour and mill feeds are outputs. Wheat futures contract plays a role to hedge physical wheat procurement and flour sales risk while CBOT corn futures only cross hedge mill feeds price variation. Wheat futures and corn futures are considered hedging instruments.

Under each scenario, there are three different modeling specifications. The first is the traditional risk hedging strategy, by which the miller always sets the hedge ratio equals 1.0. However, HR=1 lacks the flexibility to changes, reduces the possibility of making larger profits, and it does not adapt to meet the utility expectation. Therefore, it needs two other specifications to compute the optimal hedge ratio that meets requirements. The second approach is Mean-VaR methodology under multi-normal distribution and linear correlation assumptions. The third specification is Mean-VaR that incorporates Copula to specify more accurate joint distribution and dependence structure.

Scenario #1 is a trivial case with non-price risk associated. The last three scenarios are the main focus. Scenario #2 analyses when flour and mill feed sales price and quantity demanded have been determined in advance while ingredients need to be purchased from the market place. MGE wheat futures are considered the instrument to hedge against the physical wheat procurement risk. In this scenario, hedge ratios of specification two are about 0.17 lower than specification one. Specification 2 dominates one as it always yields a higher utility. On the other hand, the hedge ratios computed from mean-VaR with copula approach very close to 1 as hedge duration extends. This suggests processors to hedge in the futures market as much as possible when hedge duration is long.

Scenario #3 analyzes when wheat ingredients have already been purchased from the market while products need to be sold into the market in the future. MGE wheat futures are selected to hedge the flour price fluctuation while CBOT corn futures cross hedge mill feeds price risk. Still, specification 2 dominates one. In specification 2, the wheat futures are all fairly close to -1 while a long and speculative position in corn futures market is recommended from the result. Mean-VaR with copula shows a similar result in flour hedge ratio, but presents a much lower mill feeds hedge ratio. This keeps processors away from high speculative positions.

Scenario #4 analyses the situation when no wheat has been bought or products have been sold. Wheat futures contract has been selected to hedge physical wheat procurement risk and flour sales risk while CBOT corn futures only hedge mill feeds price variation. Tediously, specification 2 is superior to 1 as its utility is always higher. Interestingly, the result of specification three is quite different from previous two methods. In mean-VaR with copula method, the holding positions in all hedge instruments are much lower. It is likely because of the close co-movements between ingredients and products prices that processor's market price risk is self-reduced. However, the other two specifications may not have properly captured such characteristics.

Mean-VaR with copula allows for flexible non-normal marginal and joint distributions and non-linear correlation. This specification provides processors more confidence to assess these issues. When the number of variables in the model is only three, we don't see a great difference between specification two and three. Whereas the number increases to six, we observed significant differences of results between specification two and three, which indicates copula's severe role in the model. It is likely reasonable to apply copula when there are multiple variables to consider, because of its unique features. Hence, mean-VaR with copula is considered an efficient specification in the models that provide processors more confidence with the statistics of the anticipated margins and risks.

6.4. Contributions

There are several contributions of this study to the literature. First, it derives and compares different hedging strategies under distinct processing business scenarios. Second, it tests and compares different hedge horizons for commodity processors. Third, it uses an efficient technology, copula, to capture the flexible interrelationship and joint distributions among input, output, and hedging instrument accurately. Finally, this thesis quantifies the risk with VaR other than traditional variance measurement.

6.5. Limitations

This study provides a detailed explanation of what risk management strategies could be used by an agricultural processor based on distinct business scenarios. However, this thesis is limited by several factors. The first limitation is the availability of reliable data. The second one is the assumption of not including operational cost, other business-running cost, or contract transaction cost. The third limitation is the assumption about by-products sales strategy. The final limitation is that risk management instrument in the empirical model only includes futures contract. The following provides a brief discussion about each limitation.

The empirical analysis is limited by the availability of reliable price data. The frequency of observations varies from daily to weekly averages. The output price series are in weekly average; hence, all cash and futures prices had to be converted to weekly averages, which may reduce volatility. In addition, in order to incorporate seasonality, it computed four-week return to be monthly return which may differ from the actual monthly return a bit. In other words, the sample statistics measured in four-week return do not account for a shorter period fluctuation and may under evaluate the actual volatility.

The empirical model assumed business operating cost, business overhead cost, and equipment operating cost to be zero for simplification. All these cost factors other than ingredients procurement cost are considered by a single parameter called “all other costs” in the theoretical model but neglected in the empirical model specification. “All other costs” varies according to individual firms. One other important cost factor is transaction costs incurred for hedging instrument. For forward contracts, it comes in the form of negotiation costs; broker commissions are typically required for futures contracts. Compared to value change of assets in the portfolio, all these transaction cost are of a small amount and can be ignored but should be addressed in the study of real case.

The third limitation is the assumption of mill feed being sold with exactly the same contract type at the same time as flour. However, in reality, the quantity of mill feeds sold under contract and the proportion sold in the market vary according to firms and market condition. Flour customer may not contract both flour and mill feed in the production ratio, which brings flour miller the market demand risk.

The final limitation is that forward contracts and cross hedging futures contracts were the only derivative instruments considered as sales and risk management tools when futures market does not exist on refinery products. In reality, a variety of contract embedded, exchange traded, or over the counter (OTC) options may be viable risk-management alternatives. Physical options contract is a far more flexible tool that has been used by many commodity processors. It protects both processors from product sales risk and consumers from procurement risk more efficiently than futures and forward contracts and, therefore, calls for further exploration.

6.6. Further Study

Agricultural commodity processors are key components in agribusiness supply chain. Their planning and cooperation have a big impact on farmers and consumers. People can extend the risk management model into the entire flour business supply chain, such as inclusion of wheat growers and bakery industry. Instead of solely reaching the goal of flour millers, it can optimize the supply chain's performance by having agents coordinate through contracting so that each agent's objective becomes aligned with the supply chain's objective. Forwards, futures, options, and other types of contracts can play important roles in the coordination strategies.

In this thesis, commodity price risk has been the only risk factor studied and managed. An interesting extension of the analysis would be to derive models for multiple-risk hedging purposes. In reality, the additional sources of risks include demand uncertainty, transportation cost, resources cost during production, and credit risk. Among these, demand uncertainty is the most urgent one to be included and managed. In the scenario when products cannot be sold entirely under contracts, processors are facing a great risk of demand uncertainty from the market place. Future studies are able to apply Monte Carlo simulation to forecast quantity demanded to improve the margin specification.

One of the most important areas could be extended and studied is to incorporate physical options into the theoretical and empirical model. The benefits of using physical option contract to manage the product sales risk are flexible, limiting the credit risk by receiving or paying premiums, sharing risks between processors and buyers, and improving supply chain efficiency. Many studies of using such derivative to contract for products have been provided by scholars. Industries, such as chemical products, semi-conductor, and beef industry, have been using physical options for procurement and sales purpose. Their goal is to manage the risk associated with the prices of their desired commodities. A brief review of past studies using such contract to mitigate risk is presented in the second chapter. Future study can follow existing literatures to incorporate physical options and extend this thesis in the agricultural commodity processing industry.

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