UNDERSTANDING STUDENTS' PERCEPTIONS OF DIFFICULTY AND THE EFFECT DIFFICULTY HAS ON MATHEMATICAL ANXIETY

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Caleb Bridger Larson

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SUPERVISORY COMMITTEE:

Abraham Ayebo
Chair

Ben Duncan

Indranil Sengupta

James Nyachwaya

Approved:

April 10, 2018
Date

Ben Duncan
Department Chair

ABSTRACT

There exists a growing atmosphere surrounding mathematics that allows individuals to exclaim their belief they are deficient in math without any societal judgement. Compared to a state like being illiterate, we have reached a stage where it is acceptable to be math illiterate as well as hate math. To discover why so many people have this strong distaste towards math, we look towards the difficulty level of the subject. Students cite difficulty as one of the main reasons that they dislike math, so to fully understand the issue at hand, we must first understand students' perceptions of difficulty in mathematics. To this end, we use existing research to develop a survey targeting common issues in algebra that asks the students to complete the problems and describe to us what they think may be challenging about that problem. We then compare and contrast students' reactions to our own hypotheses.

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1. INTRODUCTION

Mathematics skills are essential for anyone to succeed in the society regardless of whether the person is in a technical or non-technical field. Mathematics skills are needed for routine activities such as balancing checkbooks, filing income taxes, interpreting charts and graphs, budgeting one's income, and so forth. However, despite the importance of mathematics skills, many students perform poorly in math courses or dread taking math courses in high school and college.

Researchers have noted that one affective variable used to explain both mathematics course avoidance and poor mathematics performance is *mathematics anxiety*. In learning mathematics, many students may feel anxiety. When faced with math problems, most people panic. In this state, learning becomes almost impossible for them. Because these students do not succeed in mathematics, they tend to develop a negative attitude towards it.

Mathematicians across the globe are in a constant effort to fight the bias that has developed around mathematics. We hear about people who claim to have a strong distaste towards math, potentially a result of "poor" teachers, lack of success, or math being "hard". For our purpose, we will focus specifically on what students think is challenging. To this end, we will provide a literary context for how difficulty affects students' mathematical careers and supply a basis for how our assessment is best formed. A description of how this study is focused as well as how the survey was administered will be given in the "Methods, Participants, and Context" section, along with a description of where the students and location where the assessment was administered. Then, our section "Development, Results, and Discussion" will detail the reasons each individual item on our assessment tool was selected as well as indicate our expected outcome of the item, provide sample quotes from the students which will tell us

what they found to be difficult about that assessment item, and allow for discussion regarding results from the students versus what we hypothesized. Finally, we will exhibit ways that this research can be further developed and pursued.

2. LITERATURE REVIEW

Belbase (2013) defines mathematics anxiety as the feeling of self-esteem threatening anxiety that comes about as the result of a situation regarding mathematics. It is well-known that the more apt to inducing anxiety a situation is, the more likely it is for individuals to avoid that situation (2013). Also, research suggests the level of each individual's math anxiety is connected to that individual's experience of success in mathematics (Ma and Kishor, 1997). Therefore, the conclusion can be drawn that people who find mathematics especially difficult will be more apt to having high levels of math anxiety and will be more likely to avoid math in general (Zan and Di Martino, 2007). However, a significant portion of the educated population is required to take at least some level of mathematics course, even at the university level. It is here that we end up with a selection of people who have this distaste towards math but still need to participate in and succeed in mathematics. Therefore, to minimize society's negative view of mathematics, it is the task of educators to make sure math is more accessible to the general population. One important aspect of this is to understand where the students' levels of understanding are, so within this paper we will try to more thoroughly understand students' perception of difficulty in introductory-level algebra. We define difficulty operationally as an anxiety and confusion-inducing construct. Difficulty will vary based upon the subject area. For example, a student who finds tedious calculations challenging may not obviously fit our definition of difficulty; however, carrying out the tedious calculations caused that student anxiety, hence fitting our definition. For our purposes, we are interested in the mathematics that introductory-level algebra students at the university level have the ability to understand and work with rather than the mathematics that is inaccessible because of student level. This excludes using material out of the students' reaches as material they could deem "difficult". Research shows that while doing well in mathematics

does little to increase enjoyment of learning, high levels of difficulty dissuades students desire to work on math (Ma, 1997). Ma goes on to say that difficulty is the number one characteristic that shapes enjoyment, and that if we can present higher difficulty mathematics at a more accessible and enjoyable level, students will be more likely to do well with it. This will lead, as Jansen, Louwerse, Straatemeier, Van der Ven, and Klinkenberg say, to a higher success rate which positively correlates with overall math performance (2013).

Previous studies have shown negative associations between mathematics performance and math anxiety (Ashcraft, 1994; Hembree, 1990; Khatoon & Mahmood, 2010; Ma, 1999; Miller & Bichsel, 2004). In addition, gender differences have been found although some findings are contradictory. While some studies have shown that mathematics anxiety is higher in male students than in females (Abed & Alkhateeb, 2001; Reavis, 1989; Sandman, 1979), other studies have shown that math anxiety negatively impacts mathematics performance and that female students may report higher levels of math anxiety than their male counterparts (Ashcraft & Faust, 1994; Betz, 1978; Hembree, 1990; Ho et al., 2000; Yuksel-Şahin, 2008). Another body of research have found no gender differences in math anxiety (Birgin, Baloglu, Çatlıoglu, & Gurbuz, 2010; Chinn, 2009).

To understand what students find difficult in math, we turn to the inner workings of students' memories. Working memory, by definition, is the aspect of our memory that deals with immediately processing and manipulating information, with a limited capacity. It is this limited capacity that is so heavily taxed in mathematics by the influence of the introduction of new symbols, processes, and applications being shown to students on top of any preexisting math anxiety (Ali and Reid, 2012). Once the capacity of the working memory is depleted, students will rely on memorization and, as a result, students will not internalize many of the key concepts.

It is in this result that math has received the common reputation of being a "roadmap" type subject, where coursework can be linear in fashion. Ali and Reid go on to say that if we create a stronger sense of confidence, students will feel more comfortable with new data and therefore be less inclined to succumb to high levels of math anxiety. To create this sense of confidence, it would be to educators' advantage to completely understand how students view mathematics, because it is the student's perception of math that most heavily influences the effect of education on that student (Belbase, 2013). As educators then, we must look more deeply into the misunderstandings of students and try to see why we commonly think that the students should simply see what we are getting at.

Misunderstandings are especially prevalent in the transition from arithmetic to basic algebra. Research often highlights reasons that algebra is particularly difficult for students at first. For example, some reasons lending to misconceptions are: misinterpretation of symbols, notational precision, letters and variables, and overall informal methods (Booth, 1988). Assumedly, in most educator's experience, word problems are the most anxiety-inducing problems experience in the algebra context. However, there is apparently limited research on what makes a math problem hard to students. Research on problem perception in mathematics is limited, but we have results regarding perception in novices versus experts in the field. Research concludes that as far as novices go, for example in beginning algebra students, students with similar backgrounds perceive problems in a similar fashion (Schoenfeld and Herrmann, 1982). Schoenfeld and Herrmann suggest that these novice students consider problems based on an aesthetic view, basing difficulty on if the problem looks "hard" after first read. On the other hand, these researchers conclude that the expert's perception of the problem is based on a much deeper understanding of the field, where they can rely on certain experience and conceptual

understanding to come to terms with the problem. Insofar as this paper goes, we are aiming to eventually take Schoenfeld and Herrmann's closing suggesting, first starting with a focus on the students' perception of difficulty. An important bit to consider when developing an assessment tool to analyze our research questions is what common issues are there in algebra. Based upon personal experience as an educator as well as research by Gunawardena Egodawatte from the University of Toronto, it is seen that computational procedures such as fraction and symbol manipulation are commonly difficult for algebra students (2014). In addition, there are difficulties with the conceptual understanding of certain items that students may have, such as understanding how to solve a word problem or understanding nuances within problems. As such, we will focus on targeting these items within our assessment tool.

In an early attempt at discovering the reasons that people have a distaste towards math, we are first going to focus on what students think is difficult. In future research we will be able to look at faculty members' perception of difficulty, and eventually work on what the gap between the two perceptions is. Therefore, our main research question we are going to focus on is: "What is the perception of difficulty among students in introductory-level university algebra courses?" In addition, we look to consider: "How does mathematical anxiety affect the students' perceptions of difficulty?"

3. METHODS, PARTICPANTS, AND CONTEXT

3.1. Methods

We employed a mixed-method approach for this study, using both quantitative and qualitative analysis techniques. This report, however, will focus on the qualitative approach, where future papers will focus on the quantitative data. A mixed-methods approach allows us to get a better understanding of the data than either a strictly quantitative or strictly qualitative study would. The qualitative approach of analyzing participants' essay responses to the survey questions will allow us to gain an understanding of the participants' feelings towards the survey items we are trying to analyze.

Cresswell (2003) explains qualitative researchers are "involved in the actual experiences of the participants" (p. 181). According to Creswell (2003), the inductive logic of research in a qualitative study includes the researcher gathering information in the form of interviews or observations, asking open-ended questions of participants, analyzing the data to form themes, looking for broad patterns or theories from themes, and linking the patterns to past experience and literature (p. 132).

Phenomenology is a qualitative research methodology in which "the researcher identifies the 'essence' of the human experiences concerning a phenomenon, as described by the participants in the study" (Cresswell, 2003, p. 15). Van Manen (2016), posits that phenomenological research investigates the phenomenon as it is lived, rather than how we think it is lived (p. 30). The primary focus of phenomenological inquiry is what people experience with regard to some phenomenon and how they interpret those experiences. The phenomenological researcher explores the 'essence' of the phenomenon as experienced by the people affected by it (van Manen, 2016). Through either one-on-one interviews or open-ended

questions on survey instruments, the researcher can gain a rich description of the phenomenon and details from those directly affected by it. In this study, open-ended questions on survey were used to obtain information about the students' perceptions of mathematics problem difficulty and how it relates to mathematics anxiety. Results from this study may provide educators with information about barriers or issues that keep students from successfully solving mathematical problems and / or give educators a glimpse into how to address students' mathematics anxiety.

3.2. Participants and Context

This study involved more than 300 students from the introductory-level algebra course at a research university in the Midwest United States. The course is titled "Intermediate Algebra" and is used as a bridge course between what the students would have learned in high school and the material taught at the college algebra level. At the time of the survey, the course had progressed far enough into the semester where the survey material had been covered and tested upon. To administer the survey, we blocked out a class period from the algebra course at the university and had each instructor of their individual course give the students the survey. The students had the entire class period to complete the survey.

3.3. Protection of the Rights of Human Subjects / Ethical Considerations

Prior to any data collection, approval was obtained from the university's Institutional Review Board (IRB). Informed consent for participation in the study was obtained at the time of testing and participants were reminded of the informed consent during the interview process. The consent forms and data have been maintained in a locked file at all times and are accessible only by the researcher. Participants were informed that there are no risks associated with participation in the study and that they may withdraw from the study at any time without risk of

penalty. There was no cost to the participants, and the risk to direct benefit to the participants was low.

4. DEVELOPMENT, RESULTS, AND DISCUSSION

Most of the assessment items were borrowed either directly from or influenced by the students' textbooks required by the university: *Intermediate Algebra* by Trigsted, Gallaher, and Bodden (2014). For each section, the table given will be responses to the part d) question of the associated problem on our developed survey. The survey can be found in Appendix A, and it would be useful for the reader to refer to the survey as he/she reads through each assessment item's development.

4.1. Assessment Item #1

4.1.1. Development

Based upon the research in common difficulties for students in courses containing basic algebra, one of the most difficult concepts at this stage is fractions (Egodawatte, 2014). Problems including fractions require students to not only know the material that they are working with but also have a firm grasp of how to work with fractions in the situation. Assessment item #1 was chosen to see how difficult participants think fractions are to work with, and how having fractions in a problem affects their anxiety level. In the development stage, on a scale from 1 to 5, with 1 being "very easy" to 5 being "very difficult", we ranked this item as a 1. We felt that after being over halfway through the algebra course, students should find this type of equation to be trivial to solve, and that the fractions would provide only an annoyance to the students. This leads us to our hypothesis for what students think is challenging about item #1: Students may struggle with item #1 because of the fractions involved in the problem.

4.1.2. Results

Table 1 consists of sample quotes from students responding to item #1. After scouring the survey responses, we developed five main codes that most of the students' responses could

be categorized under. The five codes were these common themes that answers represented: fractions are difficult, math anxiety, familiarity, common denominators, don't know how to start. The responses such as "I don't know" were not categorized, since there is no way to know what the students were thinking, and whether they thought the problem was anxiety-inducing or difficult. Nevertheless, there were valid responses that answered the question asked in 1d yet explained why the students did not know, so we have a category for that.

Table 1. Sample Quotes from Assessment Item #1

1. Fractions are difficult	2. Math anxiety	3. Familiarity	4. Common denominators	5. Don't know how to start
I get anxiety/very anxious/ worried whenever I see fractions. I did not learn them well and didn't master them when I was younger, and I still do not have them mastered to this day.	#1 is anxiety-inducing because of the current stress/anxiety from preexisting situations, like it is just more homework kind of. Also just the test room setting, and the fact that I'm blanking on how to divide fractions	It's hard for me to remember things from so long ago. I need to see an example before I can do it as a refresher.	Math just tends to make me freeze up, everything I know goes out the window. It's finding the same denominator that gets me, why are the numbers so big, etc. …	I am unsure of how to begin the problem therefore making me feel anxious and second guess myself.
The answer could look deceiving, most answers don't end in fractions like that.	I found it to be anxiety-inducing because after trying to solve, my numbers became larger, and that's why I'm not confident in it.	Seemed a little different than what I was taught.	Fractions can cause people to become anxious, especially ones that have different denominators.	If you don't know what step to take then it may be difficult and the fractions may make some feel like it was difficult
The answer was large and not a 'perfect' fraction/number.	I know how to do these problems, but I feel like my answer is way too long.	I think once I get review about how I do the problem I would be fine. I'm really good with doing an example then a problem.	It made me nervous because it involved adding/dividing fractions with different denominators.	I didn't know how to do it right when I looked at it.
As soon as I see a fraction in any problem I get anxious, it makes me have little [confidence] that my answer is correct.	Math just gives me anxiety.	I usually forget how to solve different math problems after having not worked with them for a while.	What made this "hard" is the number 2 in the problem. It's hard to find the same number that works with all 3 of them if 2 is even.	Not remembering how to solve this-panicking because I want to get it right but I don't know the steps well.
I found #1 to be anxiety-inducing because it involved fractions, which are my worst enemy. Fractions are difficult because we learn them in the 3rd grade and then never again.	I always forget how to do it during a test.	I feel we learn how to do the same thing so many ways that it's hard to remember how to do just one.	I wasn't sure if I should add the fractions or find an LCD for them. I know my answer is wrong.	When I looked at the question I felt anxiety right away because I wasn't positive on what to do right away.

4.1.3. Discussion

For assessment item #1, we expected that students would struggle with the fraction portion of the problem, but also expected that the students would not find the problem overly challenging. Our hypothesis was not far off, as most the students who failed to find the correct solution indicated having some trouble working with the fractions. An estimated 50% of the students correctly completed this problem, so students did as well as expected. Within the themes, we see contributing factors as to why the students may have struggled. We infer from the column 1 and column 4 of Table 1 that fractions not only cause students to struggle with the problem, but also intimidate students. We see comments such as "The answer could look deceiving; most answers don't end in fractions like that." This leads us to believe that the students potentially had not worked with fractions enough for them to become comfortable with them. We also infer from the first column that the one of the consequences of giving students problems with fractions is the lack of confidence in their answer, since most of the responses we received indicate that students were frightened by the lack of a "perfect" answer. Column 2, column 3, and column 5 are themes that recur through most of the assessment items. Regardless of each item's content, students responded with sentiments involving how afraid they are of both math and of tests. Some of the most interesting responses, which will be investigated as this study continues, were the responses such as "Seemed a little different than what I was taught." The material came directly from their textbooks and assignments that they had worked on, yet these comments were quite common throughout all assessment items. This allows us to conjecture that the students giving these responses had such a difficulty with this material that they did not recognize the material when given out of context.

4.2. Assessment Item #2

4.2.1. Development

For assessment item #2, we were trying to see how students reacted to problems ending differently than they expect. When selected, we expected that students would be able to successfully begin the problem using the elimination or substitution techniques for systems of equations, but as the students proceeded with the algorithm they are accustomed to using, they would arrive at a contradiction. This required the students to know that there is then no solution to the system. On our ranking scale when choosing this problem, we thought this would be a 2 out of 5 difficulty level, where students may find it difficult if they do not recall what the contradiction means, but overall students would do well on this problem. This led us to our hypothesis for item #2: "Students will struggle with the 'no solutions' ending to the problem."

4.2.2. Results

Table 2 includes quotes chosen from students' responses to assessment item #2. The horizontal headers contain the five themes that most accurately represented the students answers to the question. The codes we chose for this assessment item were: ending doesn't make sense, math anxiety, don't know how to start, two variables, and fractions.

Table 2. Sample Quotes from Assessment Item #2

1. Ending doesn't make sense	2. Math anxiety	3. Don't know how to start	4. Two variables	5. Fractions
I found it difficult because I feel like I solved it correctly but 0=8 doesn't make sense.	It seemed simple and almost too easy, which is why I'm not too confident in my answer.	This problem involves a lot of steps for solving, and then checking your work, and I only remember that we had to check the work from class.	It's hard to remember how to start the problem. You don't know if you should solve for x or y or use elimination, substitution, or what.	Once you solve for a variable, you'll end up with a fraction and that's what makes it difficult.
This question was anxiety-inducing because I got "no solution" because 0=4 as an answer which typically makes me question my work.	I don't find it hard or anxiety-inducing because we just learned this, it's just that I don't remember what to do by the end of it.	Can't remember the steps to solve other than elimination method, and that one didn't work out. Can't remember substitution.	May find it to be difficult or anxiety-inducing because of the two variables and because it is a system.	Some may not remember what to do first, and once you solve for one, the fractions may cause anxiety.
The difficult part of this problem would be the end. It's the "am I done?" question.	I always forget math when it shows up on exams so I need to review before I see it on an exam.	I remember learning this, but I couldn't remember how to do it, which was stressful.	Some might because the equations are set up (one on top of another) in an intimidating way.	Dealing with fractions answers make me feel like its wrong but when I solved I got 0=4 which would be no solution.
I know how to do this problem, I just don't know what to do because both of the x and y values cancelled out.	I'm not sure I can deal with math the next three years.	I can't remember, but know it's an easy problem. Math is hard because I forget formulas and problems after I haven't seen them in a while.	Because there are two equations with x and y in both.	I didn't know what I was supposed to do, thought it was elimination. With substitution the problem would have had fractions. I don't get why everything has to become a fraction.
Both variables cancel out which cause the problem to not work out.	When I take a test over material in which I feel inclined to do super well, I blank out on everything I learned.	I found it anxiety inducing because I panic when I can't find an answer to a problem.	It might take forever to figure out how many solutions there are.	Not getting right answer when checking work, weird fractions.

4.2.3. Discussion

Our goal in selecting this problem was to determine how students react when they become forced to adjust the algorithmic path they choose to do common problems. We expected that the students would have completed enough of these "solve the system" type problems in their course that they would select one of the algorithms that they had learned (either elimination, substitution, or graphing) and proceed down that route. Then, when the students arrived at a point of contradiction (since there is no solution), we would be able to see how the students reacted. Instead, this problem provided some of the most interesting results out of the assessment items, and helps showcase the circumstances where the educator does not predict the issues the students may have. Instead of getting stuck at the ending, students commonly did not know where to begin. Most commonly, students' responses fell under column 3, where the correct route was unclear to the students from the start. A lot of the responses for column 3 mimic the response "I can't remember, but know it's an easy problem. Math is hard because I forget formulas and problems after I haven't seen them in a while." A lot of the students who answered this way tried to do some basic work but got stuck very quickly. Column 2 indicates that even after the first assessment item, students still were reacting to the test environment, enough to still write responses regarding it. This took us by surprise, as we assumed the anxiety of being given an exam would decrease following the first question. In support of our hypothesis that the students would struggle with the conclusion to the problem, column 1 shows us some sample quotes from students who struggled there. Column 1 was where most of the responses were from students who successfully began the problem, so we were correct in that assumption. Not only did the students find the assessment item difficult because the ending was different, the students also felt a lack of confidence when they reached the contradiction. Answers such as "I

found it difficult because I feel like I solved it correctly but 0=8 doesn't make sense" were quite common, especially when students did the work correctly. We infer from these responses that the "twist" ending proved to be difficult for the students, as hypothesized. Column 4 shows the same difficulty that we see in all the assessment item responses, where some students are confused and describe the difficulties with the problem instead of trying to describe their individual problems. While not exactly an answer to the question of what they found individually challenging, these quotes are valuable because the students are describing what they think may be difficult about the problem, which is what we are trying to get at. An interesting column of quotes is column 5, where we saw a popular response theme that we didn't expect to see. We did not consider that when you begin the problem using the elimination method, you quickly run into a problem involving fractions. As we saw in assessment item #1, the students are intimidated by fractions. This lead us to receiving answers such as "I didn't know what I was supposed to do, thought it was elimination. With substitution the problem would have had fractions. I don't get why everything has to become a fraction." That student seemed to be frustrated that our way to make these problems more "difficult" was to make them all involve fractions, when this was not our aim.

4.3. Assessment Item #3

4.3.1. Development

For item #3, we were aiming to view reactions to two different common algebraic issues that students have at this level, factoring and finding domains (Egodawatte, 2014). First, this problem requires the students to know how to factor quadratic expressions, something that is commonly difficult for students at this level. Research shows that students tend to panic when they do not see the factors of the expressions right away, so we wanted to see what portion of the

students worked through that. In addition, when students are finding the intercepts, one of them does not fall within the domain of the function. This part of the problem requires students to have an in-depth understanding of what is happening in the problem, not just run the "program" that they are used to. For our rank of this problem in development, we thought that this problem would be a 4 out of 5 on the difficulty scale for this level of student. There are a lot of different pieces to this problem, leaving open a lot of avenues for students to struggle or become confused. These factors led us to our hypothesis for this problem: "Students will struggle a bit with the factoring but even more coming to terms with the 'intercept' that doesn't fall in the domain of the function."

4.3.2. Results

Table 3 includes the selected quotes from assessment item #3. We chose four themes that the students' responses most commonly fell under: factoring, math anxiety, domain, don't know how.

Table 3. Sample Quotes from Assessment Item #3

1. Factoring	2. Math anxiety	3. Domain	4. Don't know how
When people see the first part they start to panic because they don't understand to factor it out first.	Number 3 makes me feel anxiety because it looks like it is supposed to be long and hard	I do not remember how to do this. I know "domain" means x-values. I don't feel anxiety and don't know if others do or don't.	I have no clue. I knew when we learned the material but I forgot because we haven't done any review.
Two problems both squared, one with a number in front of it makes it confusing.	I think that when you don't know how to do something it raises your anxiety level because you're panicking on how to figure out how to do the problem	I know we have gone over domain in class, but I don't remember how to do it. That is why this problem gave me anxiety.	I feel like I am missing key steps to help me solve the problem that I don't remember.
People might find questions about factoring anxiety-inducing	I have a hard time testing and remembering math material in a testing environment so for this problem I also can't recall the material or steps in order to solve it completely. The material is vague.	I got anxiety when I saw this because finding the domain is hard for me and I know exactly what the question is asking.	Some may find this to be difficult based of the fact that the question is a rational number, or if they do not understand what the question is looking for
I felt confident going into the problem, but when I couldn't factor the numerator it confused me.	I just needed to take a couple of minutes to calm myself and remind myself I know how to do the problem	If you didn't know to put the denominator equal to zero to find the domain it might give some people anxiety.	I remember seeing problems like this multiple times but I'm not remembering the steps on how to even start this problem.
I remember how to do the bottom but the 4 in front at the top is throwing me off. I don't feel confident on this because I know the top doesn't equal.		This problem was somewhat difficult because it was hard for me to remember what the domain was.	

4.3.3. Discussion

When reviewing the data and making sample quotes, assessment item #3 caused the most fear that our study would fail. Table 3 doesn't show the full picture of what happened in this

assessment item. The most common answer, by a large margin, was simply the answer "I don't know," with little-to-no work shown. There was also a large increase of students providing no written response after a failed attempt at the problem. When selecting item #3, the factoring was of major interest, and we were correct in hypothesizing that students would struggle with factoring. Column 1 provides quotes indicating that students thought people might struggle with the factoring in the problem, which we guessed correctly. What we did not anticipate, however, is that a total of one student in the study would correctly complete this problem. We gave this problem a 4 out of 5 on the difficulty scale, but did not expect only a single student to complete it correctly. We received very little data on the "intercept" that was not in the domain since most students failed to find their way that far into the problem. Another interesting pattern that the responses showed, but could not be indicated on a table, was that a lot of students seemed to be almost embarrassed to indicate what their individual struggles were with item #3. Instead of receiving responses such as "I had trouble with factoring", we received responses, usually after incorrect work, such as "Students may struggle with…" This indicates to us that the assessment item may have been challenging enough to the students that they began viewing the problem holistically rather than trying to solve it. Column 3 has responses from students regarding domain, which we expected would be an issue for students. What we did not foresee, however, was that a lot of students would not know what domain meant. We can infer from column 3 that recollection of definitions is much more challenging to students than we anticipated.

4.4. Assessment Item #4

4.4.1. Development

Since factoring is such a common problem for students to struggle with at this level of algebra, we decided a second problem was necessary to understand students' reactions to

factoring. Item #4 assesses a specific technique for factoring rather than a general problem including factoring. During selection, we expected students to use the factoring method "difference of squares" when working through this problem. One of the common aspects of algebra that students struggle with is remembering "formulas" such as the difference of squares, so this item was specifically supposed to check their reactions to being asked to use such a formula. Another aspect that we looked at with this problem was challenging students' initial reactions to large exponents. At the time of development, we thought that this problem would be a 3 out of 5 on the difficulty scale, earning a higher difficulty rating because the problem becomes much more difficult if we consider that a student may not know the formula. Hence, our hypothesis for item #4 was: Students may struggle because of the large exponent initially and will have a difficult time remembering the formula for the difference of squares.

4.4.2. Results

Table 4 includes sample quotes from assessment item #4. We chose 4 themes to categorize most of the responses: special case, numbers, large exponent, math anxiety.

Table 4. Sample Quotes from Assessment Item #4

1. Special case	2. Math anxiety	3. Numbers	4. Large exponent
It's difficult because its missing a variable and it's not in the typical form of ax^2+bx+c.	It can be scary doing problems on your own	Thought it was going to be easy but 81 and 16 don't have anything in common at all so it was hard for me at first to factor it.	The 6 threw me off. I usually know how to factor.
I forgot the formula for a difference of squares.	I look at the equation and have somewhat of an idea of how to solve it, but honestly, I always come up blank and don't even remember what a polynomial is. These tests make me feel like an idiot.	I could not figure out a number that goes into both numbers, so I can't remember how to correctly solve it	Seems easy enough but having such a high exponent throws people off.
Was not sure what method I would use to factor of if I had fully factored.	I love factoring, but I feel like even the simplest of problems for me can be the most challenging and I am forgetting all of what I learned.	I have no idea. 81 doesn't go into anything that 16 does, so you can't take anything out. They are both perfect squares, but I don't know what to do for the exponent.	Working out the problem made it easier to understand, but seeing the large exponent made me panic because I dislike large exponents because I know there will be multiple steps.
Without it saying what method to use I was confused on how to do it.	Yeah, I just really am not good at math, so sorry for wasting your time.	I looked at all the different factors and there isn't a GCF so it would already be prime.	I think this brings anxiety because of the 6th power. The 6th power is pretty high and we don't have an equals sign which throws students off as well.
I am not remembering whether or not it is an easy question. I don't know when to use the AC method.			

4.4.3. Discussion

Judging by the results of assessment item #3, it seemed that a lot of the students had all but given up at this point, so we were nervous that we would not receive any usable data regarding item #4. During development, we hypothesized that students would have trouble with

remembering the “special” case for factoring (i.e. difference of squares), and that the students may also be intimidated by the magnitude of the exponent. The students impressed us, however, by doing quite well on this problem. In fact, most of the students did this problem completely correctly, even wrote “easy” as a response. As a result, there was a sizable column with responses of this sort. We chose not to include these responses in the table since they do not actually provide insight to what the students found difficult or what others could have found difficult, but, nonetheless, it is important to note that a lot of students succeeded with this problem. On the other hand, some students who had been doing poorly on the previous three questions showed signs of giving up or losing confidence with responses such as “again, I don’t know how to do this.” Column 1 gives us the responses related to factoring, with answers such as “It's difficult because its missing a variable and it’s not in the typical form of ax^2+bx+c” or “Without it saying what method to use I was confused on how to do it.” The first answer here was a common theme echoed by students. They seemed not to know how to factor the polynomial when there were only two terms and not three. This indicates that these students not only forgot the formulas but also were unaware of the fact that they should have even recalled a formula. The second quote was also quite common. Students are very used to being told when a special case will be required to do a problem. For example, on a class exam we may say “Use the method of the difference of squares…” or “Use the squeeze theorem to…,” where as in this assessment the students went in “blind”. This proved to be challenging for some students. Column 2 shows the insistence of students to respond about their math anxiety rather than addressing the difficulties of item #4, but as the study goes on this information is becoming quite interesting. Instead of answers such as: “Tests really scare me,” we are receiving answers such as: “Yeah, I just really am not good at math, so sorry for wasting your time.” Obviously being

only a sample quote, it is hard to quantify how many students felt this way, but analyzing the surveys resulted in us inferring that a lot of the students that referenced test anxiety and math anxiety in the first couple of problems continued to reference it at this point, but became apologetic. Instead of stating that they have math anxiety or that tests scare them, they are apologizing for being wrong or being confused. Column 3 indicates that a lot of students were trying to employ the "AC" method that they had learned in class, which is one approach (although most likely unsuccessful here), but showed that the students were at least still trying to make progress. This is of interest to us since one of the most common, glaring issues that these introductory-level algebra students face will come in items #5 and #6: word problems. It's encouraging to us that the students are still motivated and are attempting answers this deeply into the survey. Column 4 holds responses from the students who either knew they needed to use the difference of squares, or had no idea where to begin. The six in the exponents scared both of these groups, which was expected. Everything considered, the students seemed to agree with our hypothesis in item #4.

4.5. Assessment Item #5

4.5.1. Development

Item #5 is arguably the most interesting assessment item on the list (by design). The most common complaint from students beginning algebra is that word problems are incredibly difficult and overly complicated. This assessment item was chosen to assess the students' reactions to word problems specifically. The problem, being relatively easy, is supposed to look intimating based only upon the wall of text that is presented to the student as well as the large blank graph presented. On our rating scale, this problem is a 1 out of 5, being very easy once the

student realizes what's going on. This leads us to the hypothesis: "Students will struggle with the intimidation of the paragraph of text, all of the instructions, and the graphing required."

4.5.2. Results

Table 5 shows sample quotes from students for assessment item #5. The students' responses fell under four themes: directions/labels, word problems, graphing, and easy.

Table 5. Sample Quotes from Assessment Item #5

1. Directions/labels	2. Word problems	3. Graphing	4. Easy
Hard to follow some of the directions.	Word problems always confuse me and tend to be more difficult.	Don't know how to graph.	The explanation was very clear and easy to follow. It was more fun than algebraic equations.
It could cause anxiety if you don't understand some of the directions.	Word problems are not my strong suit. There are way too many moving parts for me in this equation. I find it difficult to do.	Graphs are always tricky and always cause me anxiety.	The directions were simple but knowing me I probably did something wrong.
The directions were very vague and made it really hard to understand what was being asked, which made me uncertain of whether I was doing it right or if I was doing it very wrong.	Word problems scare a lot of people and with them being big, it makes it even harder to solve sometimes.	I actually skipped this one and came back to it because I am terrible at graphs. I don't know how to do it.	The problem looks difficult, but upon reading it/ analyzing it, one realizes it is not.
The labels were hard to place on the graph (x and y axis).	The paragraph was worded long and confusing. I don't know how to graph the decrease and increase in shares. This is because I can't tell if it lost 6 dollars on each share or if it lost 6 dollars in revenue. The wording is confusing overall.	Graphs are difficult and always seem to cause some sort of problem for me. They always have been difficult.	It was anxiety inducing because it was intimidating with the long paragraph. After reading it and understanding it I realized it was pretty easy.
Not knowing what label to put on each axis.	It was a wordy problem so I don't have confidence and I got anxiety.	There are so many words and its hard to comprehend how we are supposed to draw the graph.	I found it a little easier than the others. It was better because it used simple English and not all those foofy math words. It was a simple/easy to comprehend instructions that was easier to follow.

4.5.3. Discussion

Assessment item #5 was the first of the two questions to target the students' reactions towards word problems. The literature, prior to beginning the research, indicated that word problems are the most difficult kind of problems for students of an introductory level to complete

(Egodawatte, 2014). We hypothesized that students would struggle with intimidation immediately after reading the problem. For the most part, we were completely correct in our hypothesis. A lot of students, as indicated in columns 1, 2, and 3, were intimidated by the wall of text along with the blank graph. A significant portion of students that struggled with this problem had responses echoing the following:

> "The paragraph was worded long and confusing. I don't know how to graph the decrease and increase in shares. This is because I can't tell if it lost 6 dollars on each share or if it lost 6 dollars in revenue. The wording is confusing overall."

These responses allow us to infer that these students have a difficult time parsing through the important details of the problem. These students could be grouped with the students in column 1 who struggled with the directions and the labels (common issues at this level) and column 3 who struggled with the graphing. We did not foresee students struggling with the labeling, although that is more likely to our own error than to our students, lending again to the data of the difference in perception of difficulty between the educator and the student. It is common that students struggle with labeling and knowing which axis represents what, so looking back that should have been a portion of our hypothesis. It was intriguing to see how many students responded with answers such as "Don't know how to graph," since the problem had very little to do with graphing. Often that response was accompanied by very little work or attempt at a solution, indicating that the student perhaps was intimidated by the problem and didn't dive in and try to solve it. Column 4 provided us with responses entailing the reactions of the students who succeeding in seeing past the "tricky" problem. These students noticed that once the problem is understood, it was quite easy.

4.6. Assessment Item #6

4.6.1. Development

Item #6 assesses many of the same elements as item #5 and #4, putting common difficulties at this introductory level together into one problem. The students must first work past the intimidation of a word problem, then apply a formula to solve. Added to the difficulty of this question is that fact that it is the last question in a survey of 6 that would be an hour long, so we expected that a lot of students would be rushed or frustrated at this point, leading to hasty, incomplete answers. During selection, we deemed the formula for area not too difficult to remember and the paragraph of text is not overly intimidating, so we gave this item a difficulty of 2 out of 5. The ease of this question also lies in the fact that this was a question on one of the students' previous in-class exams. This led us to the hypothesis: "Students will be intimidated by the 'application problem' structure as well as the recollection of formulas necessary to complete the problem."

4.6.2. Results

Table 6 shows students' responses to assessment item #6. There were, likely as the result of being the end of a lengthy survey, less responses to this question in general, but the responses fell under 3 themes: word problems, formulas, and couldn't do it.

Table 6. Sample Quotes from Assessment Item #6

1. Word problems	2. Formulas	3. Couldn't do it
Word problems with various formulas can be difficult, especially when trying to use the right formula to solve the equation.	Couldn't remember the formulas, so wasn't sure how to finish.	Did not finish. I find the problem with math is that you learn it and then forget it right away because it is never used in the work force or outside world unless your career depends upon it.
I don't like all word problems, and I don't ever want to do any of them.	I forget equation so that makes this question more difficult.	I have no idea so I pretty much gave up.
I've always been bad at word problems.	I found it to be a little difficult because I'm not sure on the formula.	I've done this problem multiple times but I can't figure it out. It has been a while since we have done these, so I don't remember.
Word problems! Don't have patience for them and take up too much time, and have too much extra unneeded information.	This was difficult, even with a picture I can't seem to follow this problem, word problems are difficult and even more difficult when trying to find dimensions.	You know what I have trouble with? When will I actually use these equations? Never. I'm going to move cattle across Montana and hunt my own food, not measure some yuppee's tennis court.
	Perimeter and area is something I have struggled with. The concept itself is confusing and I've never really caught on.	Running out of time so when I got stuck I couldn't finish.

4.6.3. Discussion

Assessment item #6 finishes out the survey. Six problems in under an hour along with a survey following each question is quite a bit of material for students of any level, especially introductory course students, so we expected to see less answers for this assessment item. This was a problem that had been given on one of the students' previous exams, so along with assessing their response to needing to recall formulas and tackle a word problem, this problem

also allowed us to look at their problem recognition. No student acknowledged that they had so recently done this problem on an exam, although we did see a response in column 3 that references having done this problem prior to the survey. Students responded similarly to how we expected them to, responding with word problems and formulas as the main difficulty for this problem. Column 1 shows some frustrations were beginning to form with the survey with responses such as "Word problems! Don't have patience for them and take up too much time, and have too much extra unneeded information." That response, along with responses like "I don't like all word problems, and I don't ever want to do any of them" were generally accompanied by a lack of work for this problem. It seemed some students were so frustrated with the work by this point that the word problem formatting was enough to steer them away from attempting the problem. Column 2 shows us the lack of the ability to recollect formulas, which is to be expected. It was a bit surprising, however, that the majority of the students recalled the formula for the difference of squares, yet completely forgot how to find perimeter and area, which we would consider to be a much easier task. Column 3 shows that some of the students were getting quite upset, with mentions of giving up or lamentations about never using this math. Perhaps one of the most humorous answers (not listed on the table for good reason) was "I should be able to do this- I blame the weather." Most likely the frustrated attitude and overall lack of answers can be attributed to the lack of time the students had to finish this last problem, but perhaps these answers are the most honest since they are the raw reactions students had to the problem.

5. FUTURE RESEARCH AND CONCLUSION

5.1. Future Research

While the qualitative approach to the analysis of the assessment items allowed us to gain a feel for the students' perceptions of difficulty, quantitative analysis will permit us to have an even deeper understanding of what students found to be challenging. The survey was designed in such a way that the quantitative analysis will be reasonable to accomplish, since students were asked to indicate their reactions on a scale from 1 to 5. As well as further developing the quantitative side, we began development of an interview assessment that would ask students to explain their survey answers as well as have students complete some math problems on a board in front of the interviewers. This process would allow us to assess body language as well as tone in their voice as a better avenue to understanding the anxiety students feel under pressure.

After an understanding of students' perceptions of difficulty has been achieved, the clear path for us to continue this research would be to attempt to understand faculty members' perceptions of difficulty. As a beginning to this research, we gave the same assessment item to about 10 faculty members at the same university that we gave the survey to students at, and have collected some sample quotes from their surveys.

Table 7. Sample Quotes from Faculty Members

Item #1	Item #2	Item #3	Item #4	Item #5	Item #6
It seems as though students have a difficult time working with fractions, particularly adding fractions with different denominators.	Students always have issues with no/all solutions. It isn't " normal" so they have trouble.	Students forgot how to find "issues" with a function.	It is a special case with a large exponent.	Students have a hard time with word problems, particularly isolating the pieces.	Word problems, forgetting formulas, and big numbers.
There are many steps involving fraction manipulation.	Usually, for these problems, students will get a solution. Since this will not happen, someone might think they made a mistake.	They forget "hole", how to factor which is even worse by an ax^2, and they hate domain.	Factoring higher dimensional polynomials is challenging. I feel decent that I can't go further.	Lots of information to take in. Its every easy to start a line or make a small error.	People forget to consider what we know and what we want.
The appearance of fractions immediately is frightening; also 2,5,7 are all prime to each other making it more difficult to simplify these fractions. Adding fractions with different denominators is challenging.	Anxiety could be caused by being intimidated at "having to remember all of the steps" to solve the system. Could also be caused by being confused by the conclusion to the problem.	Factoring could be challenging. Also, trying to remember what the words "domain" and "x-intercept" mean in terms of what they look like while manipulating the function.	It is difficulty firstly because it is not clear whether we are supposed to factor over the reals, complex numbers, or irrationals. It requires the knowledge of difference of squares and cubes as well as special factorings. Also it is of degree 6, so there are many factors and much more chance for errors.	The length of the question is intimidating, I believe some would see the paragraph and immediately be intimidated.	At a loss for how to set up the problem0- you need two equations from just words.

Table 7. Sample Quotes from Faculty Members (continued)

Item #1	Item #2	Item #3	Item #4	Item #5	Item #6
Fractions, need I say more?:)	Remembering the difference between zero, one, and infinitely many solutions.	Some could find it difficult because of the factorization and I have a feeling that the math 98 students would forget to exclude 1 as a possible x intercept.	Don't know where to stop where it is factored completely!	It is a word problem so it has great length, and there are many extraneous details- all we care about is the value of the stock over time, so it doesn't matter that Bill bought or sold it for example. Also the information of the units for the graph and what is asked at the very end of the question.	The numbers are messy to work with, not nice small values so they have a chance for an arithmetic error.
The fraction 58/175 feels uncomfortable because it cannot be simplified even though the numbers are "large".	The verbiage in the phrase "system of linear equations" may be intimidating. Also it has two parts (to determine the number of solutions and the solutions themselves) plus, working with two equations at the same time is harder than one.	It is a multiple part problem with fractions and quadratics on top and bottom, requires knowledge of factoring, and one needs to know the meaning of terms like "domain" and "x intercept" in order to even have a chance.	Students will not recognize that x^6 is a perfect square.	It looks long!	Fractions usually make lower level students nervous.

For future research, it would be interesting to get a better understanding of what faculty members assume would be difficult for students, so we would want to administer the survey to

significantly more faculty members. Then, after analyzing the responses from faculty members, it would be great to develop an understanding of the difference in the perception of difficulty between students and faculty. Doing this would provide educators a potential understanding of the gap between student and instructor, and hopefully allow us to make mathematics more approachable for the student.

5.2. Conclusion

The elements of a problem that students think are difficult is one of the most interesting topics for an educator to research. As educators, it is our goal to completely understand our students' needs. To try to understand this perception of difficulty, we attempted to answer two main research questions: "What is the perception of difficulty among students in introductory-level university algebra courses?" and "How does mathematical anxiety affect the students' perceptions of difficulty?" To answer these questions, using the students' textbook, we developed a survey targeting common issues in introductory-level algebra. We then analyzed the students' responses to what would make each question difficult for a student. With further investigation into the perception of difficulty of both students and faculty, we may be able to develop the tools to make mathematics more approachable to students, hopefully allowing a more accessible field of study and lowering the negative stigma that mathematics has.

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APPENDIX. PERCEPTION OF DIFFICULTY SURVEY

The following optional, anonymous survey assessment has been developed by Caleb Larson and Dr. Abraham Ayebo to evaluate different perceptions of difficulty as well as the causes of mathematical anxiety. Please answer all questions to the best of your ability and explain all answers where possible. There are four questions about each problem immediately following the problem. (Some of the problems were adapted from *Intermediate Algebra*: Trigsted, Gallaher, & Bodden.)

1. **Solve the following equation for x . Please show all steps and write legibly.**

$$\frac{5}{2}x - \frac{3}{7} = \frac{2}{5}$$

a) Circle the difficulty level of problem #1, with 1 being "very easy" and 5 being "very difficult".

1 2 3 4 5

b) Circle the level of anxiety you experienced upon seeing problem # 1, with 1 being "very calm" and 5 being "very anxious".

1 2 3 4 5

c) Circle the level of confidence you have in your answer being correct, with 1 being "very doubtful" and 5 being "very confident".

1 2 3 4 5

d) If you found problem #1 to be difficult or anxiety-inducing, what makes the problem that way? If you did not find it to be difficult or anxiety-inducing, why do you think some would?

2. Determine the number of solutions of the following system of linear equations and find the solutions if possible.

$$\begin{cases} 3x - 2y = 3 \\ 6x - 4y = 4 \end{cases}$$

a) Circle the difficulty level of problem #2, with 1 being "very easy" and 5 being "very difficult".

1 2 3 4 5

b) Circle the level of anxiety you experienced upon seeing problem # 2, with 1 being "very calm" and 5 being "very anxious".

1 2 3 4 5

c) Circle the level of confidence you have in your answer being correct, with 1 being "very doubtful" and 5 being "very confident".

1 2 3 4 5

d) If you found problem #2 to be difficult or anxiety-inducing, what makes the problem that way? If you did not find it to be difficult or anxiety-inducing, why do you think some would?

3. Indicate the domain of the following rational function and find the *x*-intercepts of the function.

$$h(x) = \frac{4x^2 - 5x + 1}{x^2 + x - 2}$$

a) Circle the difficulty level of problem #3, with 1 being "very easy" and 5 being "very difficult".

1 2 3 4 5

b) Circle the level of anxiety you experienced upon seeing problem # 3, with 1 being "very calm" and 5 being "very anxious".

1 2 3 4 5

c) Circle the level of confidence you have in your answer being correct, with 1 being "very doubtful" and 5 being "very confident".

1 2 3 4 5

d) If you found problem #3 to be difficult or anxiety-inducing, what makes the problem that way? If you did not find it to be difficult or anxiety-inducing, why do you think some would?

4. Factor the following polynomial using any method:

$$81x^6 - 16$$

a) Circle the difficulty level of problem #4, with 1 being "very easy" and 5 being "very difficult".

1 2 3 4 5

b) Circle the level of anxiety you experienced upon seeing problem # 4, with 1 being "very calm" and 5 being "very anxious".

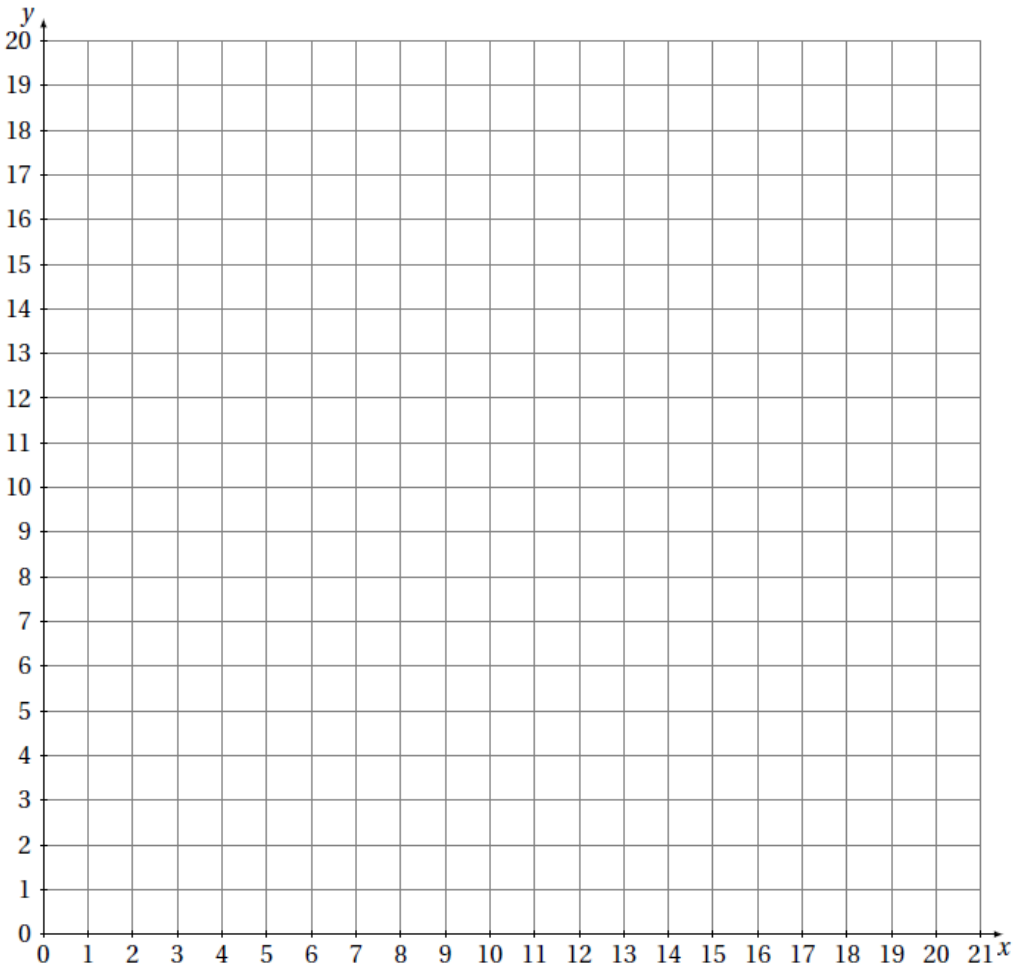
1 2 3 4 5

c) Circle the level of confidence you have in your answer being correct, with 1 being "very doubtful" and 5 being "very confident".

1 2 3 4 5

d) If you found problem #4 to be difficult or anxiety-inducing, what makes the problem that way? If you did not find it to be difficult or anxiety-inducing, why do you think some would?

5. Draw and label a graph of the function that describes the situation. Bill bought a stuck in a company for $10 per share. After his purchase, the price remained constant for two days. At that time, a negative report was released about the company, so the price declined steadily for three days until it lost a total of $6 per share. The price remained steady for four days. Then, a positive report was released about the company, so the price rose sharply for two days, gaining $8 per share. The price continued gaining gradually for four more days, ultimately rising $1 more per share. Then, Bill sold the stock. Draw a graph of the stock's value (in dollars per share) as a function of the time (in days).



a) Circle the difficulty level of problem #5, with 1 being “very easy” and 5 being “very difficult”.

1 2 3 4 5

b) Circle the level of anxiety you experienced upon seeing problem # 5, with 1 being “very calm” and 5 being “very anxious”.

1 2 3 4 5

c) Circle the level of confidence you have in your answer being correct, with 1 being “very doubtful” and 5 being “very confident”.

1 2 3 4 5

d) If you found problem #5 to be difficult or anxiety-inducing, what makes the problem that way? If you did not find it to be difficult or anxiety-inducing, why do you think some would?

6. A doubles tennis court a perimeter of 228 feet. If 6 times the length of the court equals 13 times the width, what are its dimensions?

a) Circle the difficulty level of problem #6, with 1 being "very easy" and 5 being "very difficult".

1 2 3 4 5

b) Circle the level of anxiety you experienced upon seeing problem # 6, with 1 being "very calm" and 5 being "very anxious".

1 2 3 4 5

c) Circle the level of confidence you have in your answer being correct, with 1 being "very doubtful" and 5 being "very confident".

1 2 3 4 5

d) If you found problem #6 to be difficult or anxiety-inducing, what makes the problem that way? If you did not find it to be difficult or anxiety-inducing, why do you think some would?