# VULNERABILITY ASSESSMENT OF INTERDEPENDENT POWER AND COMMUNICATIONS NETWORKS UNDER VARYING LEVEL OF INTERDEPENDENCY

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Rahul Banothu

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**Title**

VULNERABILITY ASSESSMENT OF INTERDEPENDENT POWER AND COMMUNICATIONS NETWORKS UNDER VARYING LEVEL OF INTERDEPENDENCY

**By**

Rahul Banothu

The Supervisory Committee certifies that this ***disquisition*** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

**MASTER OF SCIENCE**

SUPERVISORY COMMITTEE:

Dr. Chrysafis Vogiatzis

Chair

Dr. Om Prakash Yadav

Dr. Simone Ludwig

Approved:

| 11/17/2017 | Dr. Om Prakash Yadav |
|---|---|
| Date | Department Chair |

**ABSTRACT**

Maintaining a continuous and robust supply of power could be challenging task, because power networks depend on proper communication to coordinate and schedule supply, as well as recognize and mitigate failures; communication networks depend on power to function. This interdependency is a cause for greater failure risks due to the rapid cascading of failures from one network to the other.

The objective of this work is to investigate the vulnerability of interdependent networks under various scenarios and coupling assumptions. To do so, we employ heuristic techniques to detect critical nodes in either network which lead to the maximum number of failed nodes in the interdependent networks. We put to the test a series of topographical importance metrics to heuristically identify said important nodes and compare our results with the literature. Furthermore, we test different coupling methods for how interdependency works and compare the results under different failure assumptions.

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## LIST OF ABBREVIATIONS

BA ........................................................................ Barabási–Albert

CNP .................................................................... Critical Node Problem

CCNP ................................................................ Cardinality Constrained Critical Node Problem

ER......................................................................... Erdős–Rényi

GCC ................................................................... Giant Connected Component

IIC ....................................................................... Iterative Interdependent Centrality

LCC .................................................................... Largest Connected Component

## LIST OF SYMBOLS

$\gamma$ ............................................................... Scaling exponent or exponential factor

$G$ ............................................................... Graph or network

$V$ ............................................................... Vertex

$E$ ............................................................... Edge

$\forall$ ............................................................... For all

$\in$ ............................................................... Belongs to

$\exists$ ............................................................... There exists

$O$ ............................................................... Limiting behavior of a function when argument tends towards a particular value or infinity

$\Sigma$ ............................................................... Summation

# CHAPTER 1. INTRODUCTION

Societal welfare and well-being are intertwined with access to a fully functioning power system. This dependence of our lives on electricity has increased the necessity for power networks that are flexible and robust, and hence can be there for continuous support of human activity without any outage. To ensure the continuous and uninterrupted flow of power, modern power stations and substations depend on sophisticated communication systems for their control and coordination; similarly, communication systems depend on the power network for their support. This necessary interdependency renders both networks more vulnerable, as a failure in one of the networks could cascade to the other with catastrophic consequences. An example of such a failure comes from 2003, where cascading failures in the Northeast American power network affected 45 million people in 8 US states, and 10 million people in Canada. Moreover, power was not fully restored until one week after the event. From investigations, it was found that the sequence of events leading to the blackout was a different, seemingly unrelated failure in northern Ohio. The above situation was not an exception, as shown from more power blackouts observed in Italy in 2003, in Japan in 2011, and in India in 2012 (Feltes & Grande-Moran, 2014; Liu et al., 2014; Corsi & Sabelli, 2004; Mimura, Yasuhara, Kawagoe, Yokoki, & Kazama, 2011; Loi Lei Lai, Hao Tian Zhang, Chun Sing Lai, Fang Yuan Xu, & Mishra, 2013; Ramasubramanian et al., 2012). This phenomenon is attributed to an aging infrastructure, along with the deregularization of the power industry worldwide.

Such interdependency between modern infrastructures is not limited to power networks though. Instead, such coupled systems include water distribution, telecommunications, transportation, and social networks. These large socio-technical systems and the problem of random and targeted failures has attracted significant scientific interest recently.

To keep the flow continuous and uninterrupted, power stations depend on the communication network for control and management; the communication network also depends on a fully functional power network for electrical support and continuous operation. The interdependency of these networks renders the power network more vulnerable, as the overall scale of failure could be significantly increased due to cascading effect induced by communication network (Parandehgheibi & Modiano, 2013**;** Bashan, Berezin, Buldyrev, & Havlin, 2013). Should a perpetrator be interested in breaking down the power network, it would take only a targeted select set of nodes to significantly disrupt operations (Yilin Shen, Nguyen, Ying Xuan, & Thai, 2013). Seeing as a failure of certain nodes in the communication network can cascade and cause failure in the power network, and vice versa, protection from such attacks (or random failures) is a hard task. It is, hence, important to detect these nodes, as well as study the interdependency of these networks, in advance so that they can maintain the interdependency and at the same time mitigate the risk from targeted attacks.

Studying the importance of an entity in a network of operations is a topic that has attracted significant interest from a wide variety of scientific and practice fields. A brief literature review on this topic with an emphasis on power networks is provided in Chapter 2. In the general literature, some studies capture the importance of a node in a multi-layered network while treating each network as independent (Estrada, Estrada, Prof, & Knight, 2015;Freeman, 1978;Freeman, 1977): as an example, the *degree* of a node, which states the number of nodes directly connected to it, the *number of shortest paths* passing through a node, or the *sum of length of shortest paths* between nodes. As these approaches do not consider any underlying interdependencies, they are typically outperformed on interdependent networks by more specialized metrics and are inaccurate estimates of importance.

While assessing the vulnerability of interdependent networks, it became necessary to also investigate different interdependency models between the two networks, and how these could affect the existing approaches of finding interdependent network centrality. An interesting approach in literature called *Iterative Interdependent Centrality* (Nguyen, Shen, & Thai, 2013) aims to calculate the local intra-centrality (the centrality of a node within its network) of nodes using traditional centrality metrics (e.g., degree) and then iteratively update that value based on its interdependencies. The initial objective of this work then was to see how the IIC of a node varies when changing the means of calculating intra-centrality and how this would affect the efficiency and accuracy of finding such critical nodes. Then, a hybrid approach to measure interdependent network centrality is introduced, which includes combining a novel centrality metric with a modified version of IIC. Last, considering the fact that real world power networks usually receive information from several nodes in the communication network (Amin, 2001), it became necessary to also study different coupling models to assess the vulnerability of the power network under different scenarios. Our results from the mentioned models and approaches should provide more information regarding vulnerabilities of interdependent power networks and help making better informed decisions to protect power networks and render them less susceptible to targeted attacks and random failures.

# CHAPTER 2. BACKGROUND LITERATURE

The following section introduces some related literature on interdependent networks, the specific problem we are trying to address on coupled networks, and the models and algorithms used to study, analyze, and solve it.

## 2.1. Introduction to the power grid and network models

The United States power grid has faced 4 major large-scale blackouts due to cascading failures, starting from the first one in 1965 (Vassell, 1991) and reaching out to the latest on in 2003 which affected more than 45 million people (Farmer & Allen, 2006). Since 2003, scientific and practitioner interest in investigating the root causes of such network failures has peaked. That said, graph theoretic analyses of the underlying network were popular even in the early 1970s and 1980s. Networks were mined for their topological properties, and several metrics were proposed to explain and predict network behaviors. However, the inherent computational intractability of many of those metrics made progress slow due, in part, to the lack of the necessary computational resources, especially for studying real-life, large-scale networks, such as the power distribution network. Much has changed in the last decade, leading to the development of several complex models for network analysis, which albeit harder to solve, require less time to execute (Nardelli et al., 2014). These models have been helpful to better assess important real-world networks, like the power grid. Topological models have also been used to study other real-world networks, e.g., transportation networks (Yingfei, Chao, & Xiaohong, 2010), climate networks (Yamasaki, Gozolchiani, & Havlin, 2008), neural networks (Torres, Muñoz, Marro, & Garrido, 2004), among others.

## 2.2. Individual network models & scale free network

The introduction of the Erdős–Rényi (ER) random graph model (P Erdös & Rényi, 1959) gave rise to the development of multiple network models, including the Barabási–Albert (BA) random scale-free model (A. Barabási, 2013), and the Watts–Strogatz (WS) model (Watts & Strogatz, 1998), which can be utilized to explain most of the smaller and larger scale real-life complex networks. The ER model states that the probability of a vertex having an edge is independent of the other vertices present in the graph (P Erdös & Rényi, 1959; Gilbert, 1959). Using BA models, it was shown that most real world networks possess similar characteristics, like nodal degree or clustering coefficient distributions (Amaral, Scala, Barthelemy, & Stanley, 2000; A. L. Barabási, Albert, & Jeong, 1999), which classifies them as "scale-free networks". The BA model also reveals that networks consist of a small number of nodes, referred to as "*hubs*", which have a significantly higher degree than the rest of the nodes. Such networks are shown to have a degree distribution which follows the power law. This implies that the probability of a fraction of nodes $P_{deg}(k)$ having $k$ connections (a degree of $k$) is proportional to $\frac{1}{k^{\gamma}}$, where $\gamma$ is the scaling exponent (Barabasi, 2009). This scaling exponent is typically between 2 and 3 for most large-scale, real-world networks. The introduction of such models also provided us with insight on the *universality* of network topology in many real networks and the realization that such networks, independent of size or function, tend to converge to similar architectures. Examples of a well-known studied networks possessing the scale-free property include biological networks (see, e.g., Han et al., 2004), the world wide web (see, e.g., A.-L. Barabási & Albert, 1999a), collaborations in Hollywood (A.-L. Barabási & Albert, 1999b), research collaborations in neuroscience and mathematics (A. L. Barabási et al., 2002), the E. coli metabolism network (Oltvai, Barabási, Jeong, Tombor, & Albert, 2000), the S. cerevisiae protein interactions (Jeong, Mason, Barabási, & Oltvai, 2001), citation networks (Redner, 1998), phone call networks (Aiello, Chung, & Lu, 2001), as well as the co-occurrence of words (Cancho & Solé, 2001) and synonyms (Yook, Jeong, Barabási, & Tu, 2001).

An important parameter for the structural properties of a scale-free network is the power law exponent, with research revealing that the lower the exponent, the higher the number of hubs in the network (Reka Albert & Barabasi, 2002; Newman, 2003). Another important graph theoretic perspective comes from *percolation theory* (Stauffer & Aharony, 1994), which has been employed to evaluate network robustness. This is done through proper analysis of the structural properties of the *giant connected component*, which is qualitatively defined as the connected component of the network containing the majority of its nodes. Usually, the term $P_\infty$ is reserved to represent the probability of the existence of a giant connected component of a network. $P_\infty \sim 1$, then, represents the existence of a giant connected component almost surely, while $P_\infty \sim 0$ reveals the absence of a giant connected component. Now, randomly selecting and failing (removing them and its connections) a fraction of nodes equal to $1 - p$ gives us the *largest connected component* of the remaining network represented by $P_\infty$(p). There exists a critical threshold, or percolation threshold, $p_c \in [0,1]$ which determines the critical point where the network goes through a second order phase transition, also called a percolation phase transition. When $p > p_c$, the network converges into one giant connected component and goes into a super critical state. However, when $p < p_c$, the probability $P_\infty(p)$ is always 0 (i.e., $p < p_c$, $P_\infty(p) \equiv 0$) (Bollobás & Riordan, 2006; Gilbert, 1961; Wierman, 1990). Almost all scale-free networks with long tailed degree distributions have a threshold of $P_c \equiv 0$, which, in turn, explains the robustness of these networks to random failures (Cohen, Erez, Ben-Avraham, & Havlin, 2000). The ER model has a percolation threshold of $p_c = \frac{1}{k}$ where $k$ is the average nodal degree in the network (Bollobas & Erdös, 1976; P Erdös & Rényi, 1959; Paul Erdös & Rényi, 1960). Some examples of real-world networks, with their power law exponents and average path lengths can be found in Table 1.

Table 1

*Properties and data from several real-world networks.*

| Network | Size | k | $\gamma_{out}$ | $\gamma_{in}$ | $l_{real}$ | $l_{pow}$ | Reference |
|---|---|---|---|---|---|---|---|
| Internet | 325729 | 4.51 | 2.45 | 2.1 | 11.2 | 4.77 | Réka Albert, Jeong, & Barabási, 1999 |
| Internet | $4 * 10^7$ | 7 | 2.38 | 2.1 | | | Kleinberg, Kumar, Raghavan, Rajagopalan, & Tomkins, 1999 |
| Internet | $2*10^8$ | 7.5 | 2.72 | 2.1 | 16 | 7.61 | Broder et al., 2000 |
| Internet-Domains | 3015~4389 | 3.42~3.76 | 2.1~2.2 | 2.1~2.2 | 4 | 5.2 | Faloutsos, Faloutsos, & Faloutsos, 1999 |
| Internet-routers | 3888 | 2.57 | 2.48 | 2.48 | 12.15 | 7.67 | Faloutsos et al., 1999 |
| Internet-routers | 150000 | 2.66 | 2.4 | 2.4 | 11 | | Govindan & Tangmunarunkit, 2000 |
| Movie actors co-stardom network | 212250 | 28.78 | 2.3 | 2.3 | 4.54 | | A.-L. Barabási & Albert, 1999b |
| Co-authors in neuroscience | 209293 | 11.54 | 2.1 | 2.1 | 6 | | A. L. Barabási et al., 2002 |
| Co-authors in mathematics | 70975 | 3.9 | 2.5 | 2.5 | 9.5 | | A. L. Barabási et al., 2002 |

Table 1. *Properties and data from several real-world networks (continued)*

| Network | Size | k | $\gamma_{out}$ | $\gamma_{in}$ | $l_{real}$ | $l_{pow}$ | Reference |
|---|---|---|---|---|---|---|---|
| Metabolism E. coli | 778 | 7.4 | 2.2 | 2.2 | 3.2 | | Oltvai et al., 2000 |
| Protein S. cerevisiae | 1870 | 2.39 | 2.4 | 2.4 | | | Jeong et al., 2001 |
| Citation network | 783339 | 8.57 | | 3 | | | Redner, 1998 |
| Phone call | $53*10^6$ | 3.16 | 2.1 | 2.1 | | | Aiello et al., 2001 |
| Words, co-occurrence | 460902 | 70.13 | 2.7 | 2.7 | | | Cancho & Solé, 2001 |
| Words, synonyms | 22311 | 13.48 | 2.8 | 2.8 | | | Yook et al., 2001 |

All network sizes (total vertices), the average degree (k), the power law exponents for both in and out degrees ($\gamma$),the real network average path length ($l_{real}$) and the average path length for the power law degree distribution ($l_{pow}$) are provided. Note that most networks shown here have power law exponents between 2 and 3.

### 2.3. Interdependence and cascading

Most complex real-world networks do not function independently; instead they rely on information or resources from other networks. This phenomenon is common for several types of applications; for example, consider networks such as the communication network which is used for both voice and data by more than 90% of population (Poushter, 2016). Telecommunication networks today function correctly due to the electrical support of their operations from a functioning power grid. Similarly, power networks utilize communication networks for monitoring and control purposes (Hu, Yu, Cao, Ni, & Yu, 2014; Rinaldi, Peerenboom, & Kelly, 2001). Researchers study these networks to keep them robust (Parandehgheibi & Modiano, 2013; Zhang & Tse, 2015), well-connected (Bairey & Stowell, 2014), and with increased accessibility

(Wheeler & O’Kelly, 1999). Then, there also exist networks in which researchers are interested in identifying key elements to decrease connectedness and reachability, as in the epidemic spreading problem in which diseases spread to different locations due to a moving population (Son, Bizhani, Christensen, Grassberger, & Paczuski, 2012; Wheeler & O’Kelly, 1999), or as in financial networks where the banking firms are interdependent entities that can be modeled to analyze the failure propagation in the economy (Huang, Vodenska, Havlin, & Stanley, 2013). Unlike simple, single, isolated network models which consist of simple, local node-to-node links called connectivity edges (intra-links), interdependent networks also have a set of links which serve to connect nodes from different networks to one another: these are called dependency links (inter-links). However, it is not necessary for every network to be dependent on every other network in an interdependent setting. Moreover, we have the general case in which dependency is asymmetric. As an example, power networks rely on a functioning transportation network for fuel and maintenance operations, whereas the transportation network, in general, does not require the power network to be operational (albeit electrical support does make it safer to use).

The introduction and research of such models has revealed the importance of interdependency when studying robustness. When an interdependent network is considered as isolated or single network it leads to overestimation of network robustness (Huang, Shao, et al., 2013). This is due to the fact that failures occurring in interdependent networks tend to cascade over the other networks using inter-links causing more failures (Bashan et al., 2013; Dong, Du, Tian, & Liu, 2015; Havlin et al., 2010). For this reason, a broader degree distribution in an isolated network protects it from random attacks and increases robustness (Yuan, Shao, Stanley, & Havlin, 2015); instead, in the case of interdependent networks higher degree renders it, potentially, more vulnerable. The well-connected hub nodes could be interdependent on a failed node, which ultimately leads to their failures, and with them, the failure of a large fraction of nodes, which is, of course, a major concern (Gao, Buldyrev, Havlin, & Stanley, 2011). Furthermore, percolation in interdependent networks is significantly different than in single networks. In a single, isolated network the percolation transition is a second order continuous transition, whereas in interdependent networks percolation transition occurs when there is a discontinuity in the giant connected component due to cascading failures. Consider two networks which are interconnected where one of the network is subject to failure of $1 - p$ fraction of nodes. If the failed number of nodes are lower than the critical threshold ie. $p > p_c$, the size of giant connected component is finite $P_\infty > 0$ and there remains a cluster of nodes connected to GCC and the cascading failures stop before whole network collapses. But if the fraction of failed nodes $1 - p$ is higher than critical threshold ie. $p < p_c$, then this leads complete failure of nodes in both networks. When $p$ decreases below $p_c$ from one, $P_\infty$ falls to zero instantly showing discontinuity as first order transition. This cascading of failures is also referred to as an avalanche (Bashan, Parshani, & Havlin, 2011; Baxter, Dorogovtsev, Goltsev, & Mendes, 2012; Dong et al., 2015; Dong, Tian, Du, Fu, & Stanley, 2014; Havlin, Stanley, Bashan, Gao, & Kenett, 2015; Leicht & D'Souza, 2009a).

A simple pictorial example of percolation and cascading failures in interdependent networks is provided in Figure 1. Initially, a fraction of nodes $1 - p$ is disabled from Network 1 along with all their connections. This initial failure then propagates to the interdependent Network 2. All the nodes in Network 2 with dependency links to any failed nodes in Network 1 will also fail as a result. Based on percolation theory all nodes separated from the giant connected component are now non-functional. This failure further cascades back to Network 1 and its interdependent nodes, and this process goes on until either there is a mutual giant connected component, or when the network is completed disconnected. Interdependent networks are, then, more vulnerable to random attacks and failure of even a small subset of nodes can cause large scale failure (Havlin et al., 2015).

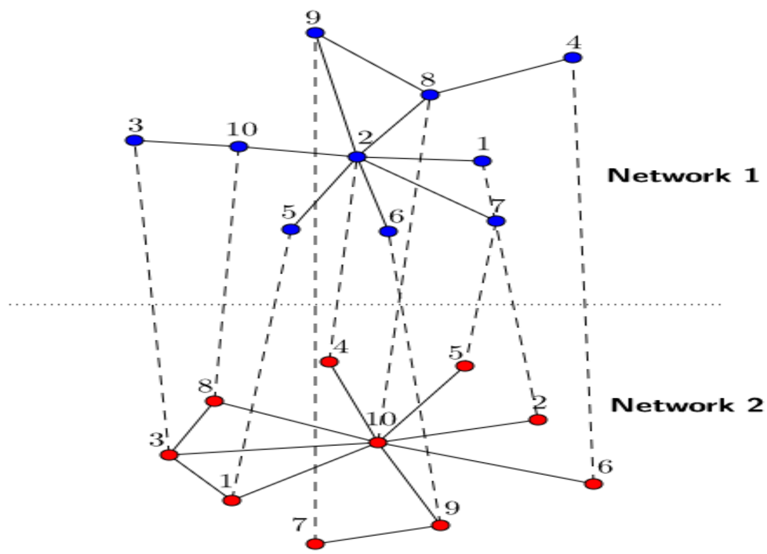


*Figure 1.* An example of percolation and cascading failures in interdependent networks.

### 2.3.1. Interdependence of power and communication networks

One of the most studied pairs of interdependent networks are the power distribution and the communication network (Parandehgheibi & Modiano, 2013; Parandehgheibi, Modiano, & Hay, 2014). This is mainly because of how intertwined these networks are on one another in their current state, as well as the importance of maintaining the robustness of these networks seeing as they affect multiple and diverse facets of human activities. A series of probabilistic, deterministic, and heuristic methods have been developed to identify the network vulnerabilities and the risk of cascading failures in the power grid. An excellent overview of some of those methods has been curated by Papic et al. (2011). Nowadays, due to several load and tripping control measures, the power grid is more robust (NERC, 2017). Yet, there exist scenarios where failures occurring in specific substations, transmission lines, or power stations could render both the power distribution and the communication networks non-operational; as seen before the initial failure could also be on the communications side. It is indeed true that a common reason behind blackouts is often these types of cascading failures (Wei, Luo, & Zhang, 2012; Motter, 2004), and, when such failures occur, it is both a very expensive and long process to restore everything back to their normal state.

### 2.4. Critical node problem

The Critical Node Problem (CNP), introduced in (Borgatti, 2006) and (Arulselvan, Commander, Elefteriadou, & Pardalos, 2009), is described as an optimization problem of finding a set of *k* vertices, whose removal from the graph minimizes the pairwise connectivity (increased fragmentation) between nodes in the resulting subgraph. Another variation of CNP was later introduced in (Arulselvan, Commander, Shylo, & Pardalos, 2011), referred to as CC-CNP or Cardinality-Constrained Critical Node Detection Problem, with a different objective of finding the minimum set of vertices whose removal leads to a connectivity index below a specified limit. The CNP has many applications: as an example, in (Boginski & Commander, 2009) the authors use both CNP and CC-CNP to find a set of proteins which are responsible for the most important interactions in protein-protein interaction networks for drug design. In a different study by (Ventresca & Aleman, 2013) related to disease spread mitigation, critical nodes are considered as target nodes for vaccination to decrease the transmissibility of a disease.

Extensions of the critical node problem, like the Critical Node and Critical Link Disruptor problems, are studied in (Yilin Shen et al., 2013); therein, a linear programming based $O\left(\frac{n-k}{n^{\varepsilon}}\right)$-approximation rounding algorithm is proposed to help identify critical nodes and edges. Like in our work, studies have also been performed on finding critical nodes in interdependent networks: for example, the work by (Seo, Mishra, Li, & Thai, 2015) introduces and studies the

Cascading Critical Node Problem (CasCN) and proposes an $O(n^{1-\epsilon})$-approximation algorithm. This work employs the Load Redistribution model and weighted flow distribution model proposed in (Wu, Peng, Wang, Chan, & Wong, 2008) to find a set of critical nodes by failing nodes iteratively. This effectively captures the direct impact of a node and the mutual impact of a set of failed nodes. Other variations of CNP include node and edge disruptor problems like $\beta$ edge and vertex disruptor problem (Dinh, Xuan, Thai, Pardalos, & Znati, 2012) which admits an $O(\log n \log\log n)$ pseudo-approximation algorithm for node disruptor and an $O(\log^{1.5} n)$-approximation algorithm for the edge disruptor. This algorithm finds the minimum cardinality set of elements in a directed graph to cause a prespecified quantified level of degradation in its pairwise connectivity metric. When a level of degradation $\beta$ is given where $0 \leq \beta \leq 1$, the network overall pairwise connectivity is decreased to $\beta \binom{n}{2}$.

Last, a few studies have proposed strategies to reduce the risk of cascading failures in interdependent networks when subjected to targeted and random attacks. In their work, (Tang, Jing, He, & Stanley, 2016) study the interdependent supply chain network robustness to targeted attacks. Two networks, namely the physical supply chain network and the cyber layer network, each with the same number of nodes have one to one interdependence. Nodes are then assigned maximum capacity and failed nodes propagate their load onto neighboring nodes, based on the proposed priority redistribution model. Nodes are removed in ascending degree, descending degree, random single, random multiple order, and finally network robustness is measured in terms of a Comprehensive Effectiveness Index (CEI). In (Nguyen et al., 2013), the authors study the Interdependent Power Network Disruptor problem, a problem shown to be NP-Complete but that admits an approximation of $(2 - \varepsilon)$. The proposed algorithm, Iterative Interdependent Centrality uses weighted centrality from intra-links as well as inter-links providing the minimum cardinality set of critical nodes up to a given $k$ to decrease the initial LCC to the smallest possible size.

### 2.5. Interdependent network coupling models

In most studies that aim to quantify topological properties of or detect critical nodes in interdependent networks, coupling models to accurately model the interdependencies are used. This is based on several assumptions, and it needs to happen as a preprocessing step due to the lack of exact data on dependency links of real-world interdependent networks (Radicchi, 2015). As an example, whose paradigm we follow here, Nguyen, Shen, & Thai (2013) investigate coupling methods, such as the *random positive* and *random negative degree correlation coupling*, based on weighted permutations, reverse degree coupling, and same degree coupling. They then proceed to use the dependency links generated by the above coupling methods in order to determine the efficiency of their critical node detection algorithms. In their work, it is also assumed that the degree distributions for both intra- and inter- network connectivity follow a Poisson distribution (Leicht & D'Souza, 2009b).

# CHAPTER 3. METHODOLOGY

## 3.1. Power and communication network model

Many real-world networks are shown to belong to a class of networks called *scale-free networks*. These networks possess a small number of nodes with very high connectivity (*hubs*) and a big number of nodes with low connectivity. The degree distribution of scale-free networks is based on a *power* law. In a scale-free network with exponential factor γ, the fraction of nodes with degree k is proportional to $k^{-\gamma}$, that is P(k) ~ $k^{-\gamma}$. In practice, the exponential factor for the communications network is observed to be between 2 and 2.6, while the exponential factor for power networks is observed to be between 2.5 and 4. Due to the lack of exact graph data for the both networks, a synthetic network is generated using an exponential factor that varies from 2.2 to 3.0.

One method of generating a scale-free network is by using the Barabasi-Albert generator model. The BA model uses preferential attachment to form edges in the network based on the provided exponential factor γ. The insight is to form a network having degree distribution that follows a power law with the chosen scaling exponent.

## 3.2. Cascading failures model

Considering two network graphs $G_k = (V_k, E_k)$ and $G_l = (V_l, E_l)$ where $V_k$ are the vertices of graph $k$ and $E_k$ are the edges of graph $k$, whereas $V_l$ are the vertices of graph $l$ and $E_l$ are the edges of graph $l$, and $E_{kl} = \{(u, v): u \in V_k, v \in V_l\}$ represents the interdependency links between graphs $k\ and\ l$. Any node $u\ or\ v$ is only functional when they are connected to the giant connected component of their respective graph i.e.. $G_k$ or $G_l$.

The cascading failure model in this study (Havlin et al., 2010) has been used and evaluated in several studies before. Initially, a set of nodes in $G_k$ fail; nodes are then separated from the Giant Connected Component of $G_k$ because of the failures are also impacted and are considered failed. This new failure from these nodes propagates to the connected nodes in the interdependent network and causes failure in these nodes of $G_l$, which, in turn, are interdependent on failed nodes from $G_k$.

In this study, we consider three cascading effect scenarios. We opted for three scenarios so as to gather more detailed information on the cascading effects occurring due to the presence of interdependency links. The three failure scenarios are described as follows: (a) in the first scenario, a node fails when all of its interconnected nodes fail; (b) in the second scenario, a node only fails when at least 50% or more of its interconnected nodes fail; and (c) in the third scenario, a node fails when at least one of its interconnected nodes fail.

**3.3. Iterative interdependent centrality**

Iterative Interdependent Centrality, proposed in (Nguyen et al., 2013) is an algorithm to find critical nodes in interdependent networks. Considering an interdependent system $J(G_k, G_l, E_{kl})$ and $E_{kl} = \{(u, v): u \in V_k, v \in V_l\}$, IIC works on the phenomenon that if $u$ is critical then its coupled node $v$ should be treated as critical, too, and the neighbors of $u$ should also play a key role in determining the criticality of $u$. For this reason, IIC aims to capture both intra- and inter- centrality. Intra-centrality, being one of the traditional centrality measures like degree, closeness etc., gives the importance of a node within the network; these intra centrality scores are then updated on to the coupled nodes in the interdependent network to obtain new weighted centrality scores. The centrality vector $x^t$ of IIC at $t^{th}$ iteration is formed using

$$x^t = \frac{M^k_{u,v} M^l_{u,v} x^{t-2}}{C_k C_l} \text{ where } M^k_{u,v} \begin{cases} \alpha & if\ u = v \\ \frac{1}{d_v} & if\ (u,v) \in E_k \\ 0 & everything\ else \end{cases} \text{ and } M^l_{u,v} \begin{cases} \alpha & if\ u = v \\ \frac{1}{d_v} & if\ (u,v) \in E_l \\ 0 & everything\ else \end{cases}$$

are two matrices formed using networks $G_k\ and\ G_l$, whereas $C_k\ and\ C_l$ and two constants used for the convergence of the centrality vector. Our approach to finding critical nodes replaces the intra-centrality of IIC using a novel centrality algorithm explained below.

**3.4. Star degree and modified IIC**

Consider, like before, two interconnected networks $G_k$ and $G_l$ where $(V_k, E_k)$ represent nodes and edges of network $k$, similarly $(V_l, E_l)$ for network $l$, also $E_{kl} = \{(u, v): u \in V_k, v \in V_l\}$ represents interdependency links between nodes of $k$ and $l$. The calculation of the *star degree centrality score*, introduced by (Vogiatzis & Camur, 2017), is done by analyzing three levels of failure for each selected node. At the first level of failure the selected node and all its adjacent nodes in other interdependent networks stop working. These nodes are categorized as "center" nodes. Then the failures cascade further to all the nodes that are connected to "center" nodes but also do not have any inter- or intra-connections between these nodes. These nodes are categorized as "failed" nodes. After that the failure cascades further to all nodes that are connected to "failed" nodes, with these nodes being categorized as "affected" nodes. The main objective of Star Degree is to maximize the cardinality of the "affected" nodes set. Let us define the following three decision variables:

$$x_i^{(k)} \begin{cases} 1 \ if \ node \ i \in V_k \ is \ "center" \\ 0 \ otherwise \end{cases}$$

$$y_i^{(k)} \begin{cases} 1 \ if \ node \ i \in V_k \ is \ "failed" \\ 0 \ otherwise \end{cases}$$

$$z_i^{(k)} \begin{cases} 1 \ if \ node \ i \in V_k \ is \ "affected" \\ 0 \ otherwise \end{cases}$$

Moreover, let the following sets be defined as:

$$N^k(i): \ neighbors \ of \ i \in V_k$$

$$N^k[i]: \ neighbors \ of \ i \in V_k \ \& \ i$$

$$N^{kl}(i): \ neighbors \ of \ i \in V_k \ in \ V_l$$

$N^{kl}[i]: \ neighbors\ of\ i \in V_k\ in\ V_l\ \&\ i$

Then, the formulation can now be presented as:

$$\text{Maximize} \sum_{k} \sum_{i \in V_k} z_i^{(k)}$$

Subject to

$$z_i^{(k)} \leq \sum_{j \in N(i)} y_j^{(k)} + \sum_{l \neq k} \sum_{j \in N^{kl}(i)} y_j^{(l)} \qquad \forall i, \forall k$$

$$y_i^{(k)} \leq \sum_{j \epsilon N[i]} x_j^{(k)} + \sum_{l \neq k} \sum_{j \epsilon N^{kl}(i)} x_j^{(l)} \qquad \forall i, \forall k$$

$$y_i^{(k)} + y_j^{(k)} \leq 1 \qquad \forall (i,j) \in E_k, \forall k$$

$$y_i^{(k)} + y_j^{(l)} \leq 1 \qquad \forall (i,j) \in E_{kl}, \forall k, \forall l \neq k$$

$$y_i^{(k)} + z_i^{(k)} \leq 1 \qquad \forall\ i \in V_k, \forall k$$

$$x_i^{(k)}, y_i^{(k)}, z_i^{(k)} \in \{0,1\} \qquad \forall\ i \in V_k, \forall k$$

$$x_u^{(k)} = 1$$

$$x_j^{(l)} = x_i^{(k)} \qquad \forall \in V_l\ ; (u,j) \in E_{kl}, \forall k, \forall l \neq k$$

Larger and more well-connected networks require a higher number of critical nodes before the network breaks down, and this can consume a significant amount of computational time. It is for that reason that a modified version of IIC embeds the process of cascading into each iteration. Initially a converged centrality vector is calculated using the characteristic matrix obtained from the considered power and communication networks. The critical node is extracted from this centrality vector and then disconnected from the power network. This process causes a cascade of failures in both networks through inter- and intra-links, based on the cascading failure model explained earlier. Only one connected component exists in both graphs at the end of the cascading process implying that the failure cannot cascade any further because all the remaining nodes are connected to the largest connected component of their individual network and also have a functional interdependent node. Consider subgraphs of both power and communication network $G_k' = (V_k', E_k')$ and $G_l' = (V_l', E_l')$ where $(V_k', E_k')$ and $(V_l', E_l')$ are the vertices and edges of $LCC(G_k)$ and $LCC(G_l)$ respectively. These subgraphs are then used to find the new characteristic matrix $M_{u,v}^{k'} M_{u,v}^{l'}$ in the next iteration. The process continues until a number of $k$ critical nodes are found or the specified total level of disruption is reached. The size of the characteristic matrix decreases as the numbed of failed nodes increases.

## 3.5. Coupling models

### 3.5.1. One to one model

This coupling strategy uses the Random Positive Degree Correlation Coupling shown in (Nguyen et al., 2013). Two random weighted permutations are generated with nodes of graph $G_k$ and $G_l$ as elements of set and having the length equal to total number of vertices $n$ in each graph. The degree of the node is considered as the weight for the permutation. In both the generated sets $\{v_1^{k'}, v_2^{k'}, v_3^{k'}, \ldots v_n^{k'}\}$ and $\{v_1^{l'}, v_2^{l'}, v_3^{l'}, \ldots v_n^{l'}\}$, elements with higher degrees tend to have lower indices because of the considered weights. This results in positive degree correlation and $v_1^{k'}$ is coupled with $v_1^{l'}$, $v_2^{k'}$ with $v_2^{l'}$,…., $v_n^{k'}$ with $v_n^{l'}$.

### 3.5.2. One to multiple model

In the one to multiple coupling strategy the primary rule is that one node of the power network is allowed to be coupled with several nodes in the communications network; however, a node in the communications network can only be coupled with a single node from the power network $E_{kl} = \{(u, v): u \in V_k, v \in V_l\}$. The distribution of interdependent links follows a long-tailed distribution, like power law graphs. A small subset of nodes in $G_k$ have a high number of interdependent links per node whereas a large subset of nodes has fewer or no interdependent links. Initially, a random weighted permutation is generated for nodes of the power network where the weight of the nodes is their intra-centrality. Using this random weighted set, each node is assigned a certain number of interdependent links and from this permuted set, nodes which have lower indices are assigned a higher number of interdependent links. Further explanations on how this model is used are provided in Section 4.2.2 of the Computational Results.

### 3.5.3. Multiple to one model

The multiple to one model is similar to the previous case; in this one, though, nodes of the communications network ($G_l$) can have multiple interdependent links per node, but each node of the power network ($G_k$) can only have one interdependent link. Similar to one to multiple, a random weighted permutation is generated with the intra-centrality score of the nodes in the communications network as the weights. Nodes from the communications network are assigned interdependent links based on their indices in the permuted set. The total interdependent links per node for overall network follows a long tail distribution. The exact working is explained further in Section 4.2.3.

### 3.5.4. Multiple to multiple model

Finally, the multiple to multiple model is the combination of the above models. Nodes are selected on the similar basis such that the number of interdependent links per node follow a long tail distribution for both networks. Each node can have any number of interdependent links. Two random weighted permutations are generated for both networks with intra centrality score of nodes as their weight and nodes based on their indices in the permuted set, they are assigned total number of interdependent links. The exact working is explained further in Section 4.2.4.

# CHAPTER 4. COMPUTATIONAL RESULTS

In this section, we describe our experimental framework. We perform a series of experiments on two coupled (power and communications) networks, as described in Section 3.1. We compare the total breakdown of both networks for different values of $\gamma \in \{2.2, 2.6, 3.0\}$ and for different metrics for both networks, considering the cascading effects discussed in Section 3.2. More specifically, we compare the resulting Largest Connected Component (LCC) for Star Degree with Modified IIC (described in Section 3.4), Degree with the Original IIC (described in Section 3.3), Simple Star Degree (described in Section 3.4), Simple Degree, and Simple Betweenness centralities. We perform our experiments on a series of coupling models (see Section 3.5). Our results (per coupling model) follow in the remainder of this chapter.

## 4.1. One to one coupling model

The idea behind the one to one model is given in Section 3.5.1. This model is tested on synthetic scale free networks of sizes of a 150-nodes power network coupled with a 150-nodes communication network, 300-nodes power network coupled with 300-nodes communication network, 500-nodes power network coupled with 500-nodes communication network. The networks have degree distributions that follow the power law, and as the scaling exponent increases, the networks become denser having an increased number of intra-links. A total of 9 experiments are conducted which contain all possible combination of pairs with $\gamma$(2.2, 2.6, 3.0). The performance is evaluated across 5 independent runs and the average of the two outputs, namely the size of the LCC and the total number of critical nodes initially removed from the power network are used to construct the plots shown below in Figures 2-4.

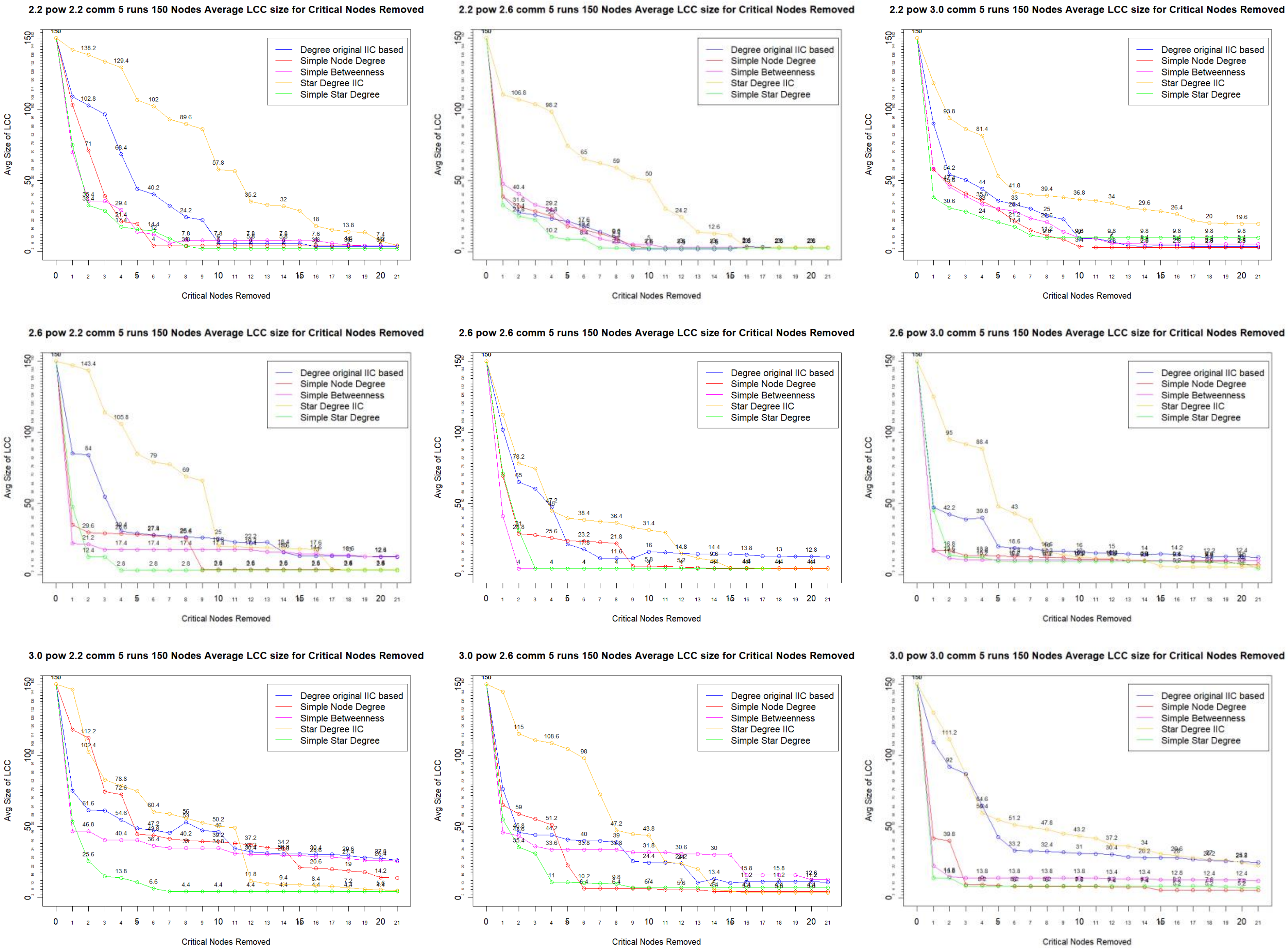


*Figure 2.* One to One model, Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

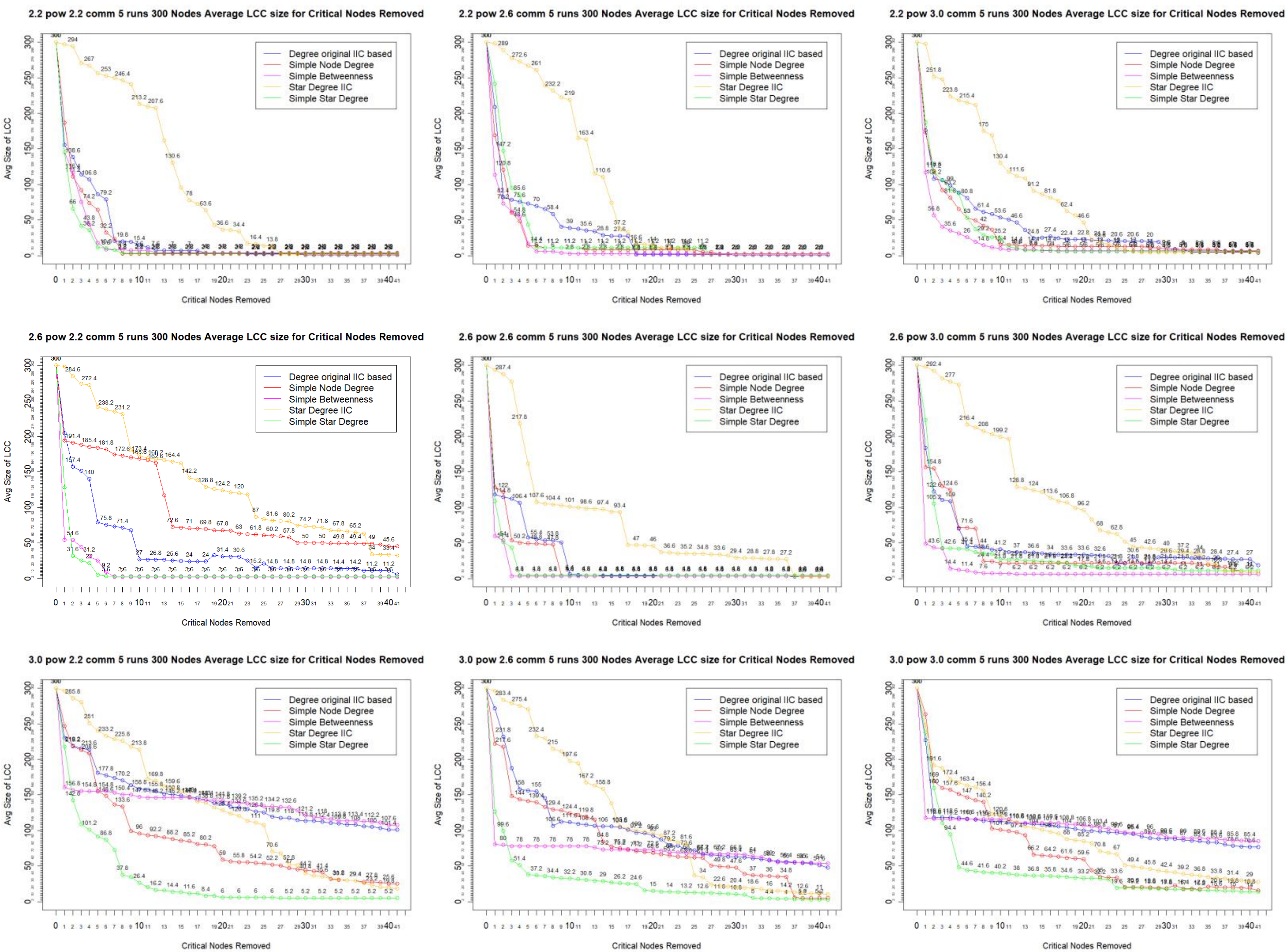


*Figure 3.* One to One model, Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

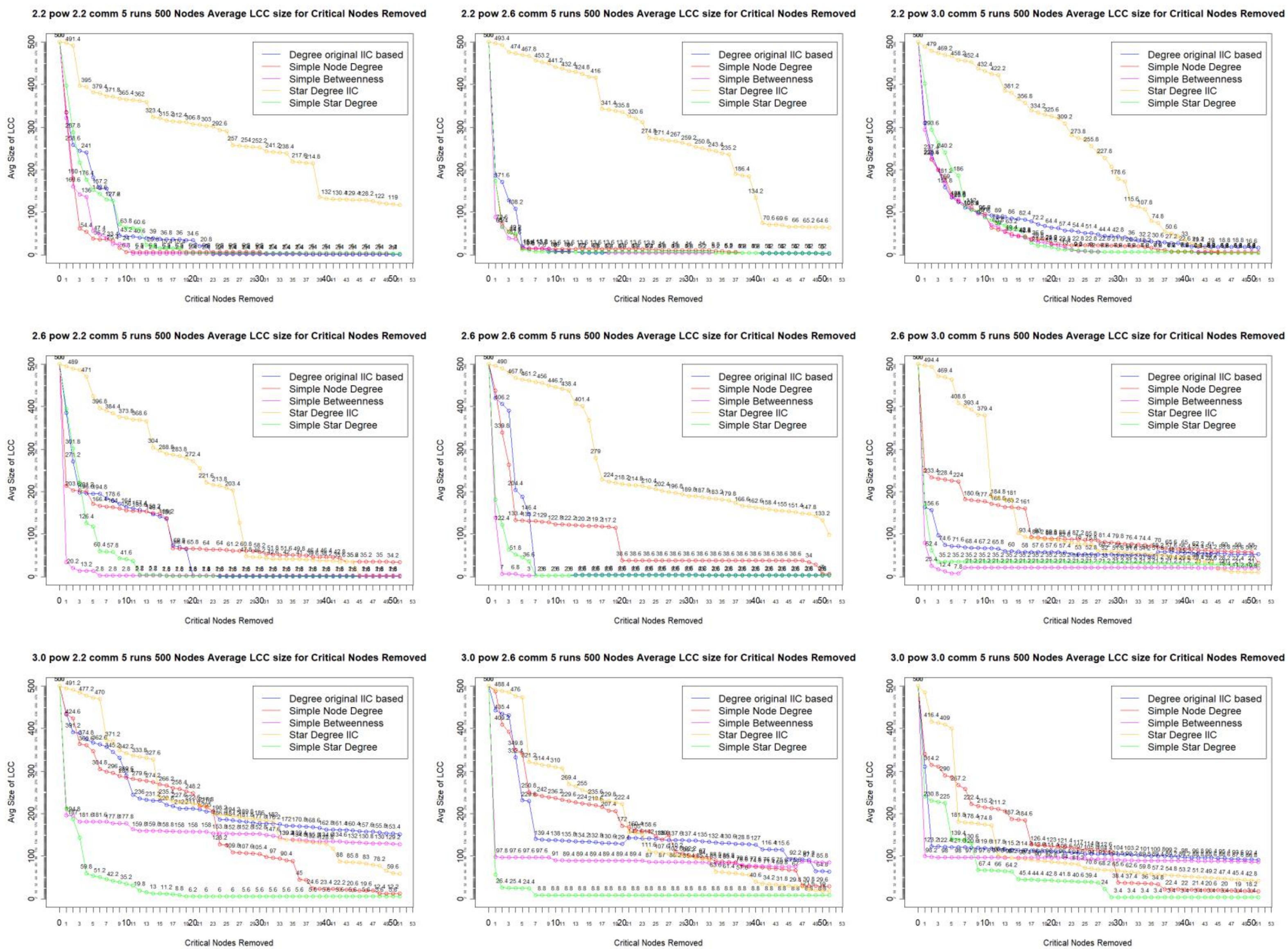


*Figure 4.* One to One model, Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

## 4.2. Other models

This subsection describes the experimental setup of the three other models used in this study. We first describe the different scenarios that arise for these models.

### 4.2.1. Scenarios

There are three main scenarios that arise in each of the models. In the first one, a node stops functioning either when all its interconnected nodes have failed or if the node has been disconnected from the largest connected component of its own network.

In the second scenario, a node stops functioning either when half or more of its interconnected nodes have failed or if the node has been disconnected from the largest connected component of its own network.

Last, in the final scenario under consideration, a node stops functioning when any of its interconnected nodes have failed or if the node has been disconnected from the largest connected component of its own network. We can now proceed to describe the remaining models.

### 4.2.2. One to multiple coupling model

For experimentation, a synthetic power and communications network are generated based on the selected combination of total number of nodes and scaling exponent $\gamma$ for both networks. A weighted random permutation is generated $\{d_1^{kl}, d_2^{kl}, \ldots, d_n^{kl}\}$ with a total length equal to the number of nodes in the power network where $d_1^{kl}$ is the the number of interdependent links for $v_1^k$. The permuted set contains one of these elements {1,2,3,4} and their weights are selected to be {0.6,0.2,0.15,0.05}. This implies that a power network node can be connected to 1, 2, 3, or 4 communications network nodes with probabilities 0.6, 0.2, 0.15, 0.05, respectively. Two more random weighted permutations are generated with vertices of the power and communications networks and now the degree of nodes are considered as weights $\{v_1^{k'}, v_2^{k'}, v_3^{k'}, \ldots v_n^{k'}\}$ and

$\{v_1^{l'}, v_2^{l'}, v_3^{l'}, \dots v_n^{l'}\}$. Both networks are coupled using the set $\{d_1^{kl}, d_2^{kl}, \dots, d_n^{kl}\}$, i.e. if $d_1^{kl}$ is 3 then $v_1^{k'}$ is coupled with $\{v_1^{l'}, v_2^{l'}, v_3^{l'}\}$. Results are plotted based on an average of 5 runs with power and communications network pairs of 150 nodes, 300 nodes, and 500 nodes (as was the case for the first model) for all 3 scenarios and all possible combinations of $\gamma$ from (2.2, 2.6. 3.0). This results again in a total of 9 experiments for each scenario under the one to multiple coupling model. The results are presented in Figures 5-13.

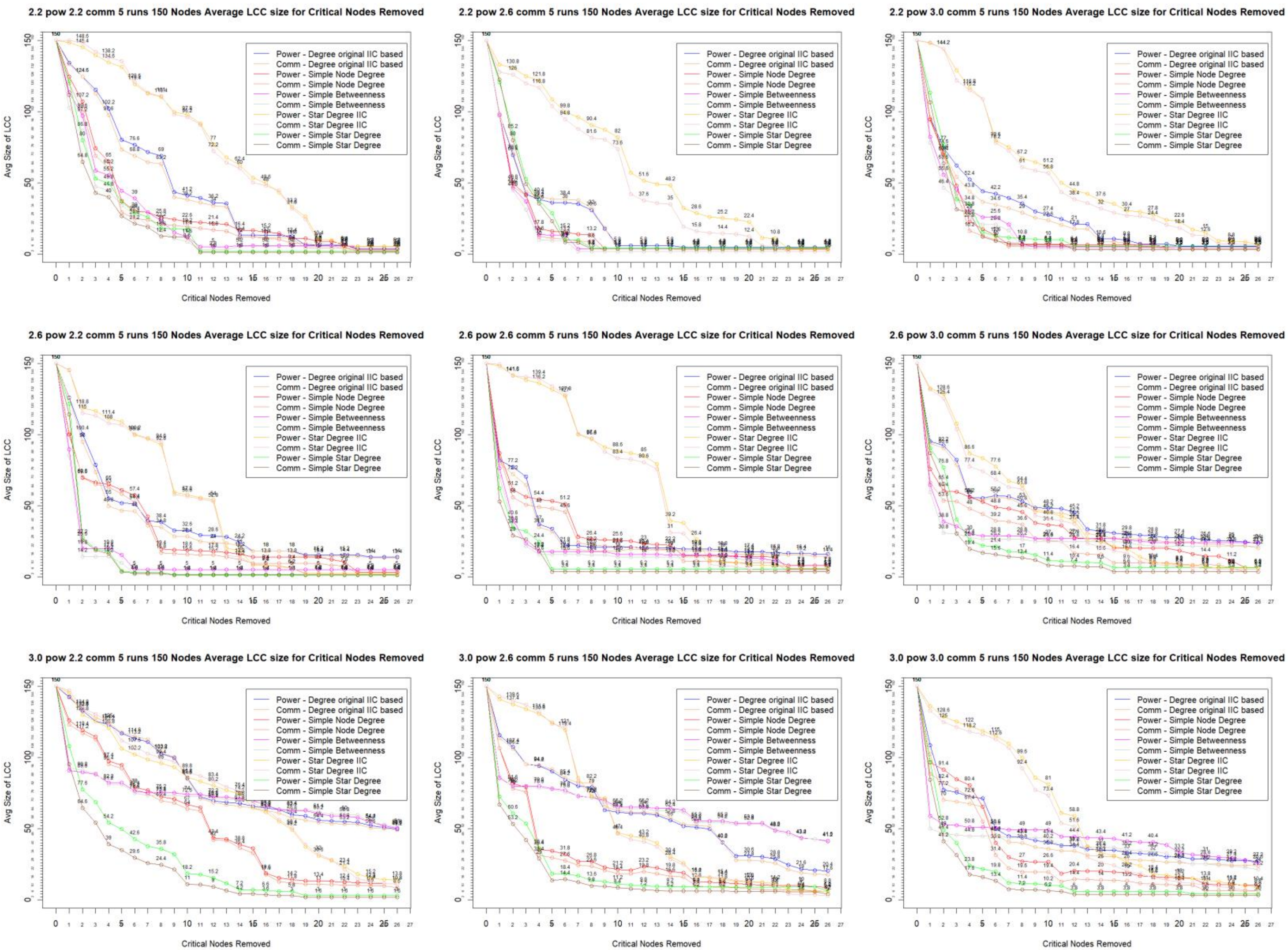


*Figure 5.* One to Multiple model, Scenario 1 Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

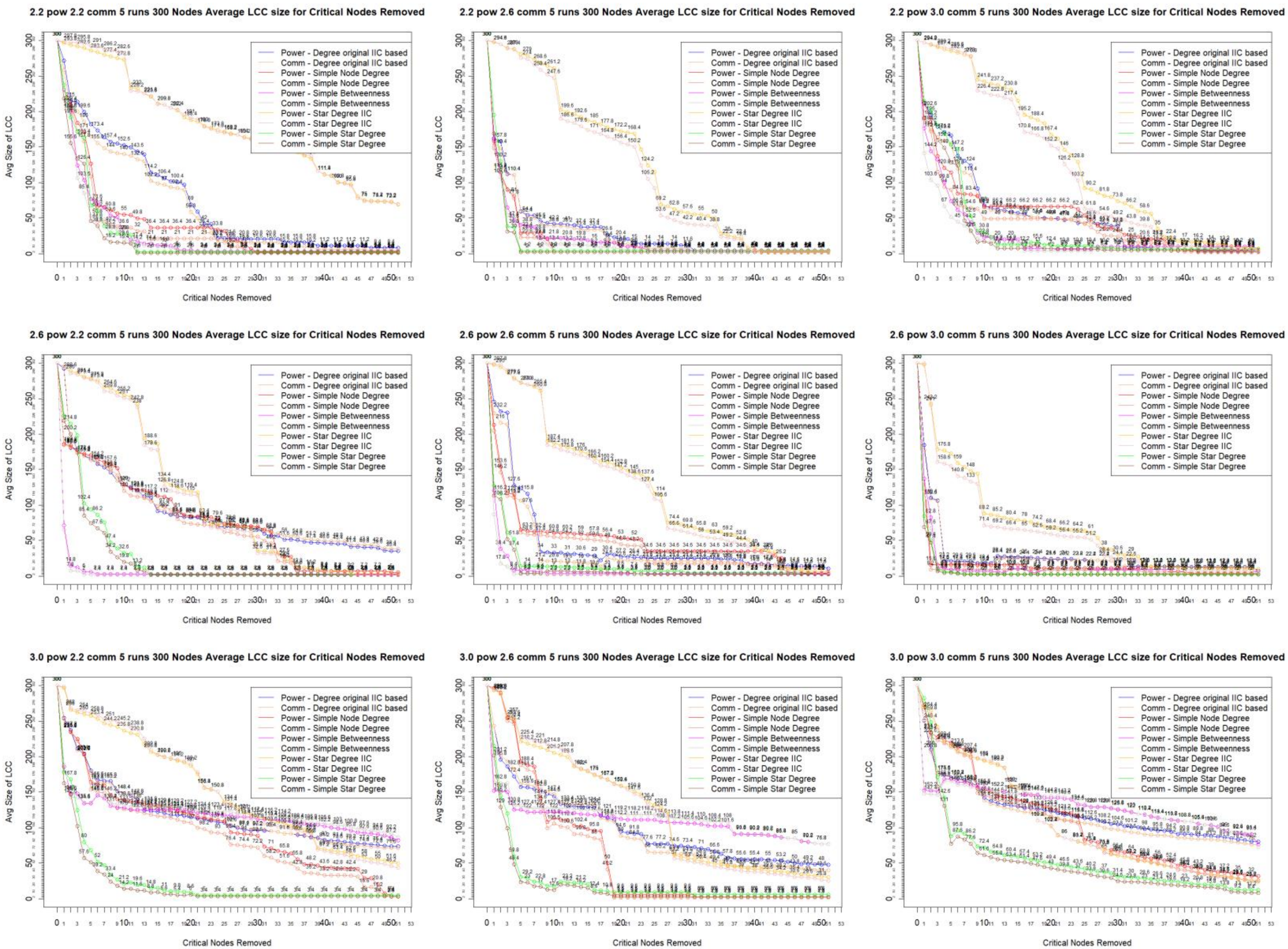


*Figure 6*. One to Multiple model, Scenario 1 Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

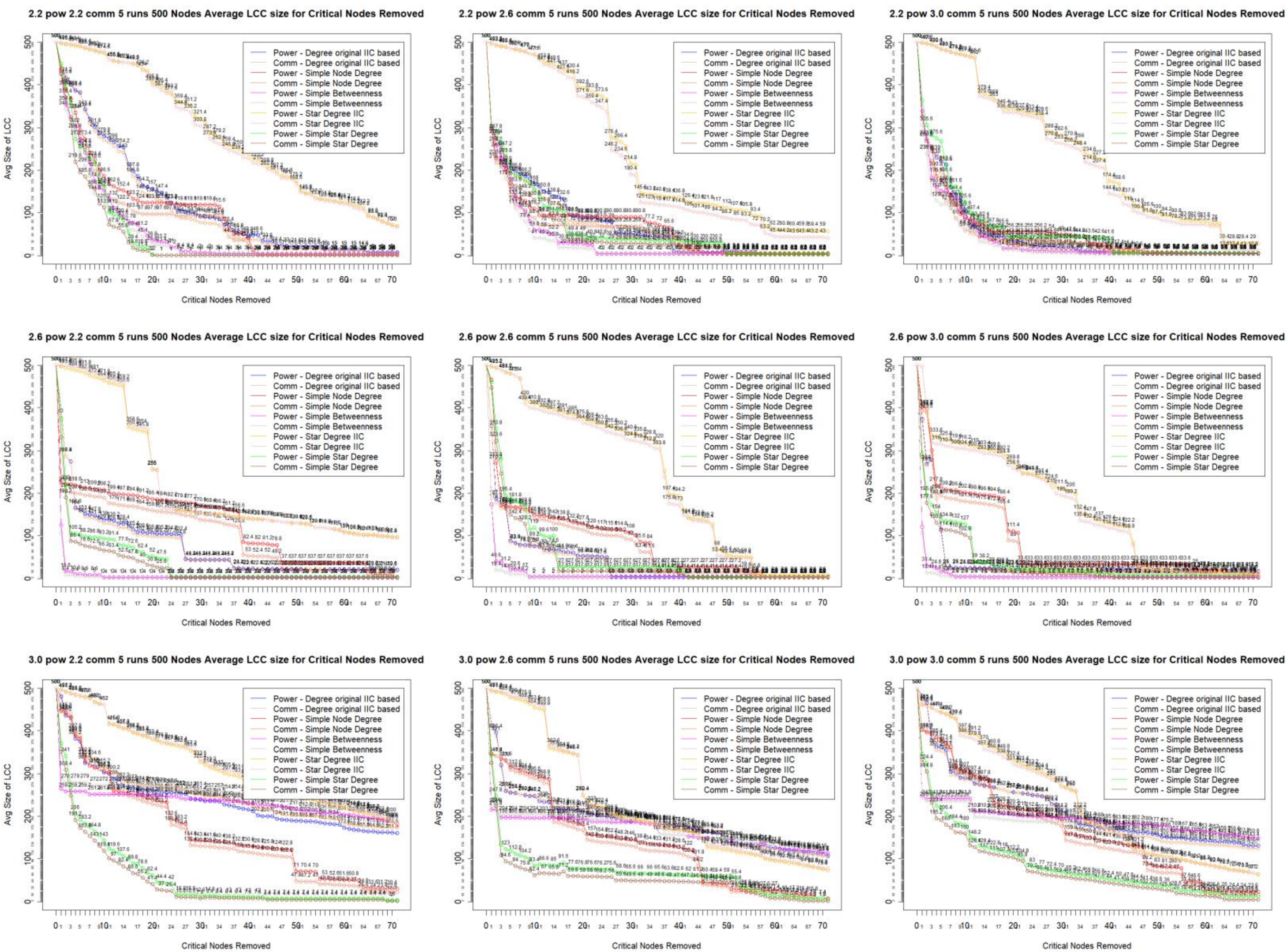


*Figure 7.* One to Multiple model, Scenario 1 Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

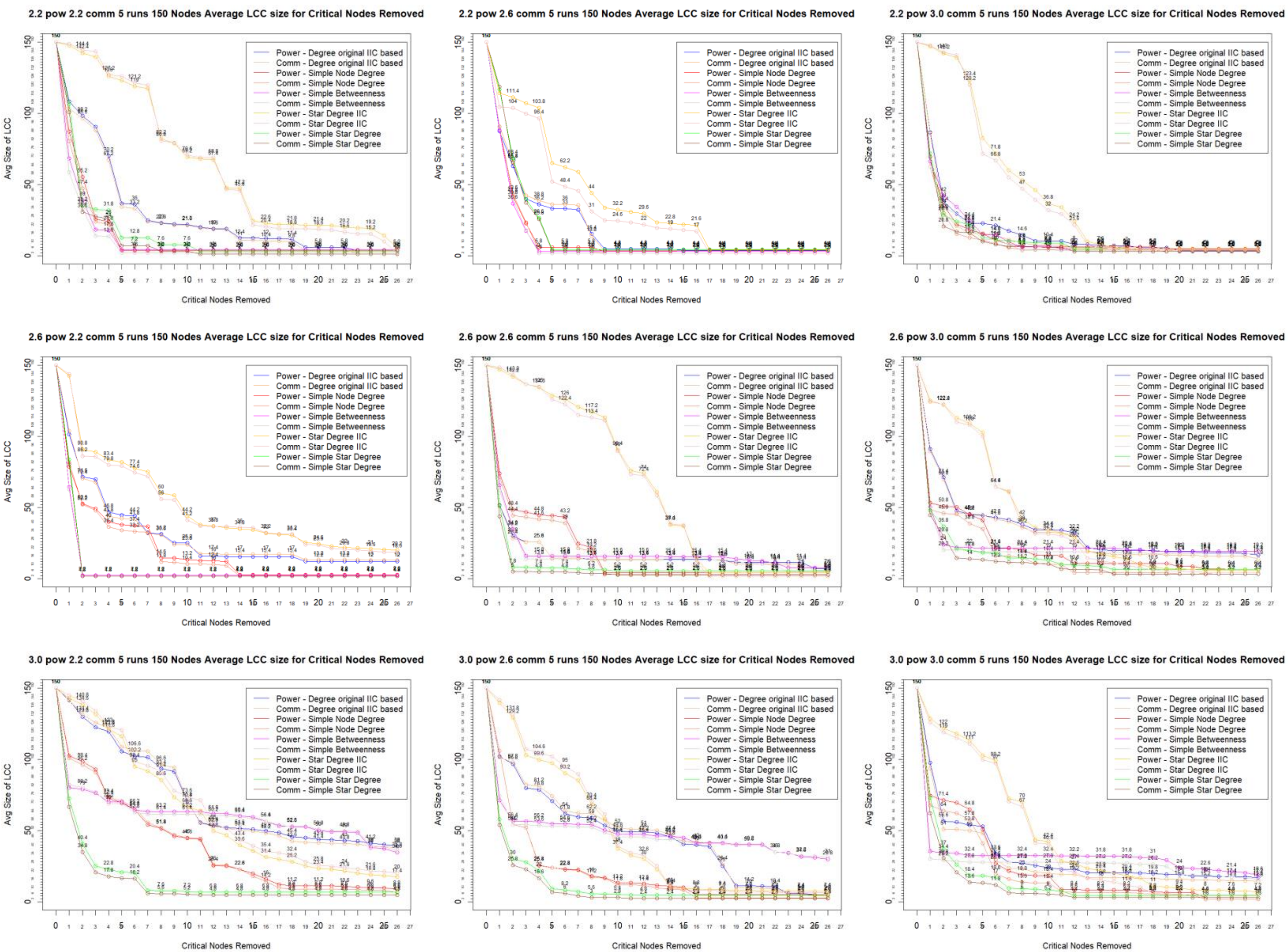


*Figure 8.* One to Multiple model, Scenario 2 Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

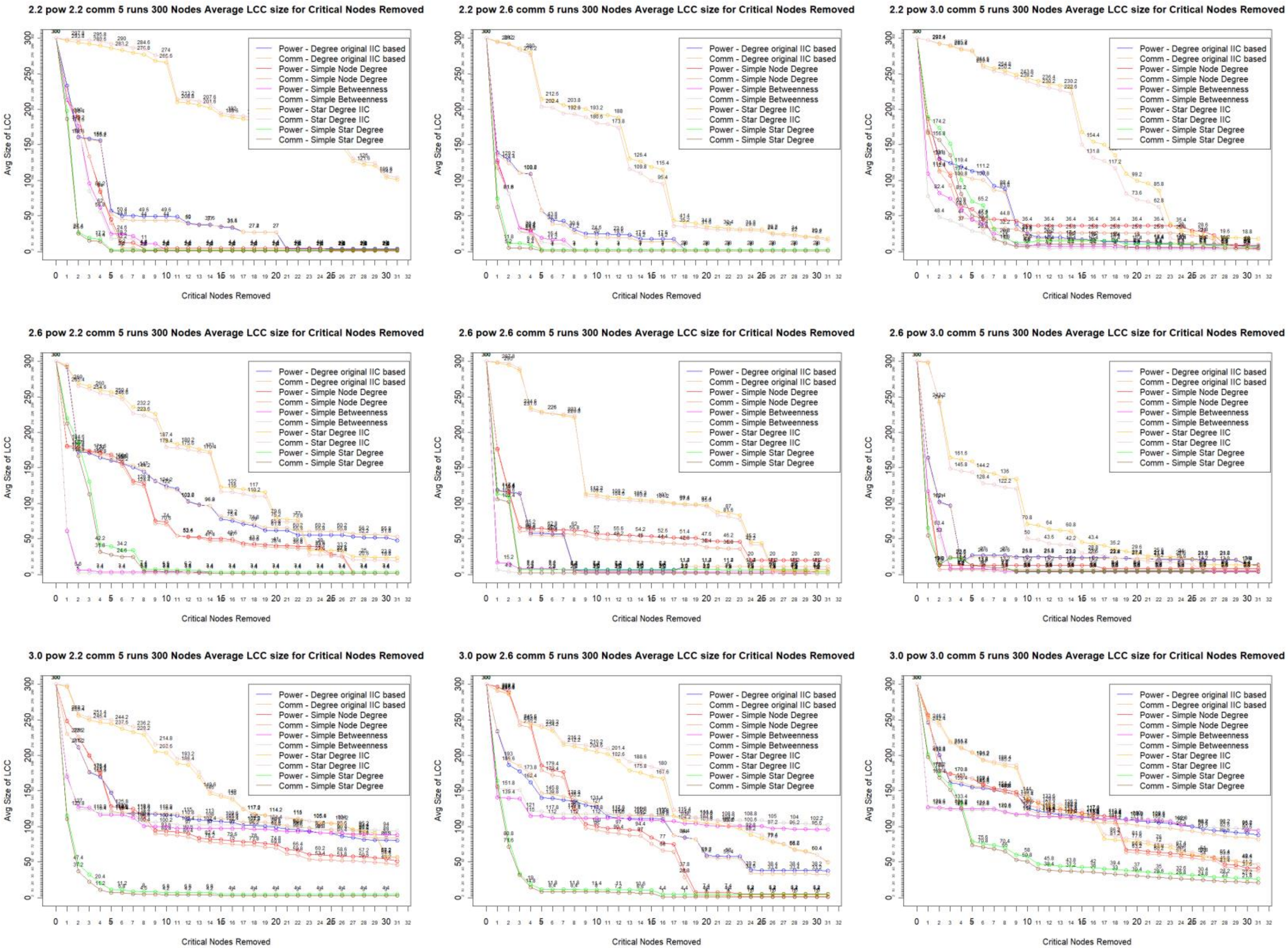


*Figure 9.* One to Multiple model, Scenario 2 Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

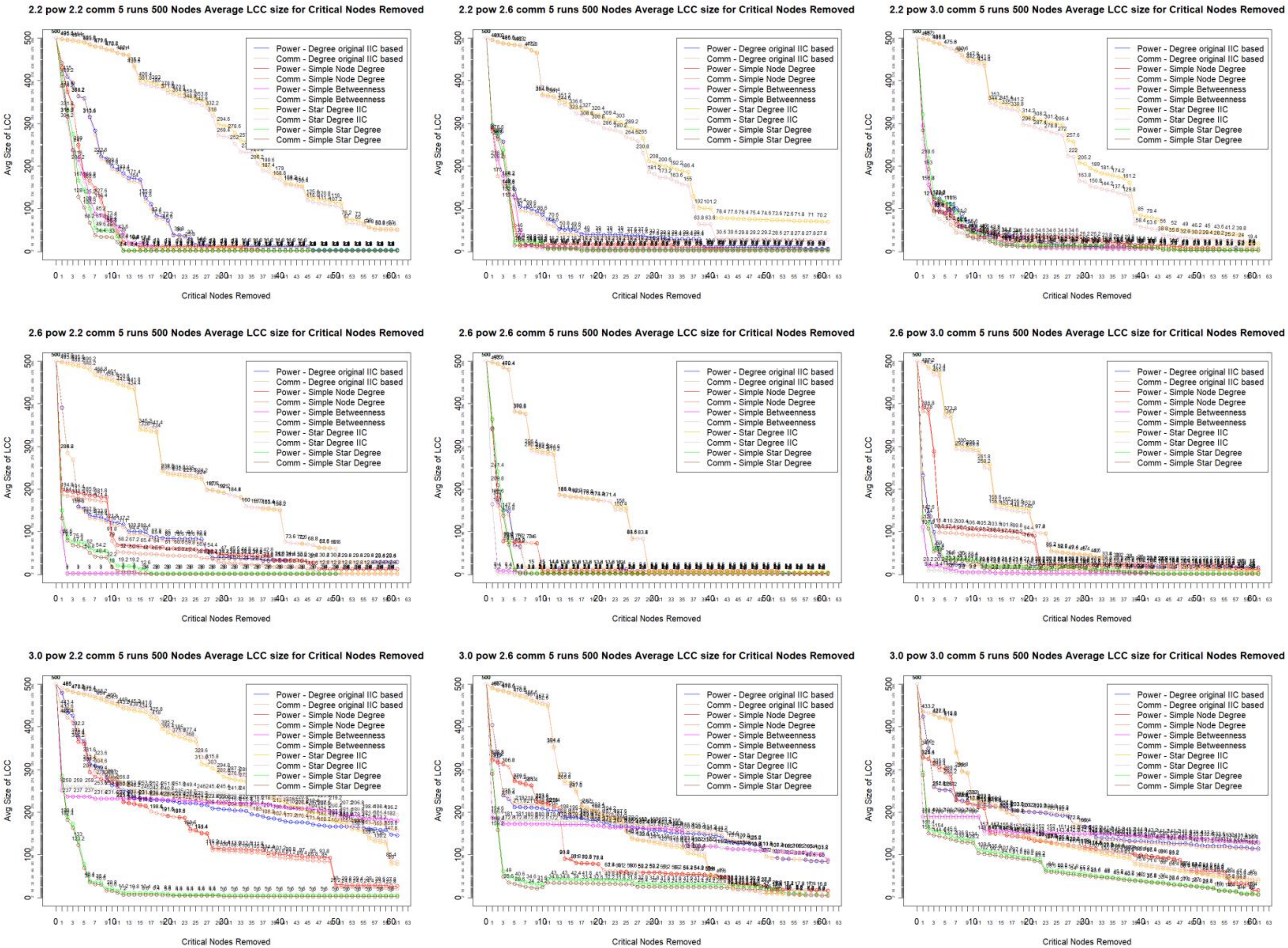


*Figure 10.* One to Multiple model, Scenario 2 Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

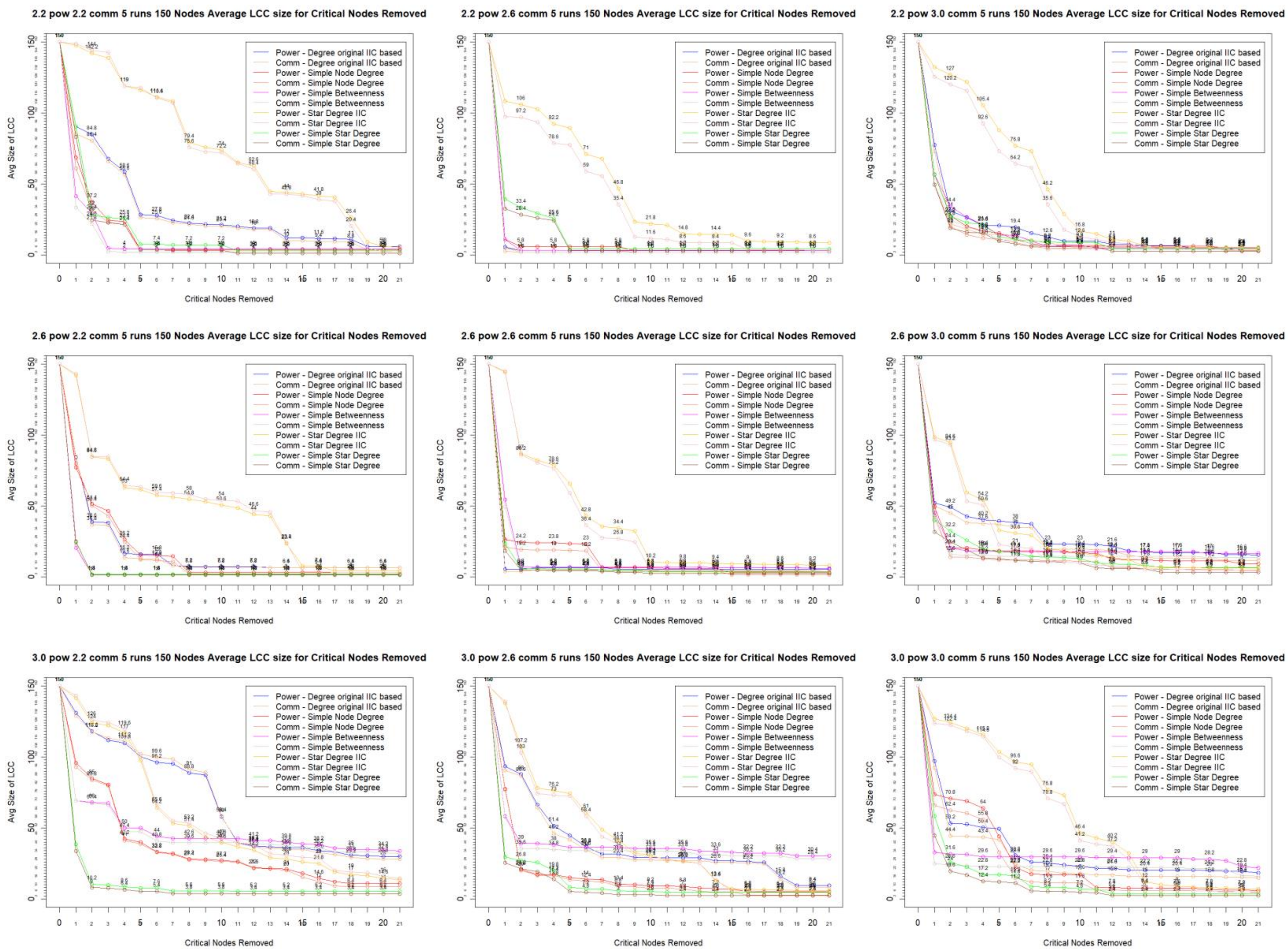


*Figure 11.* One to Multiple model, Scenario 3 Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

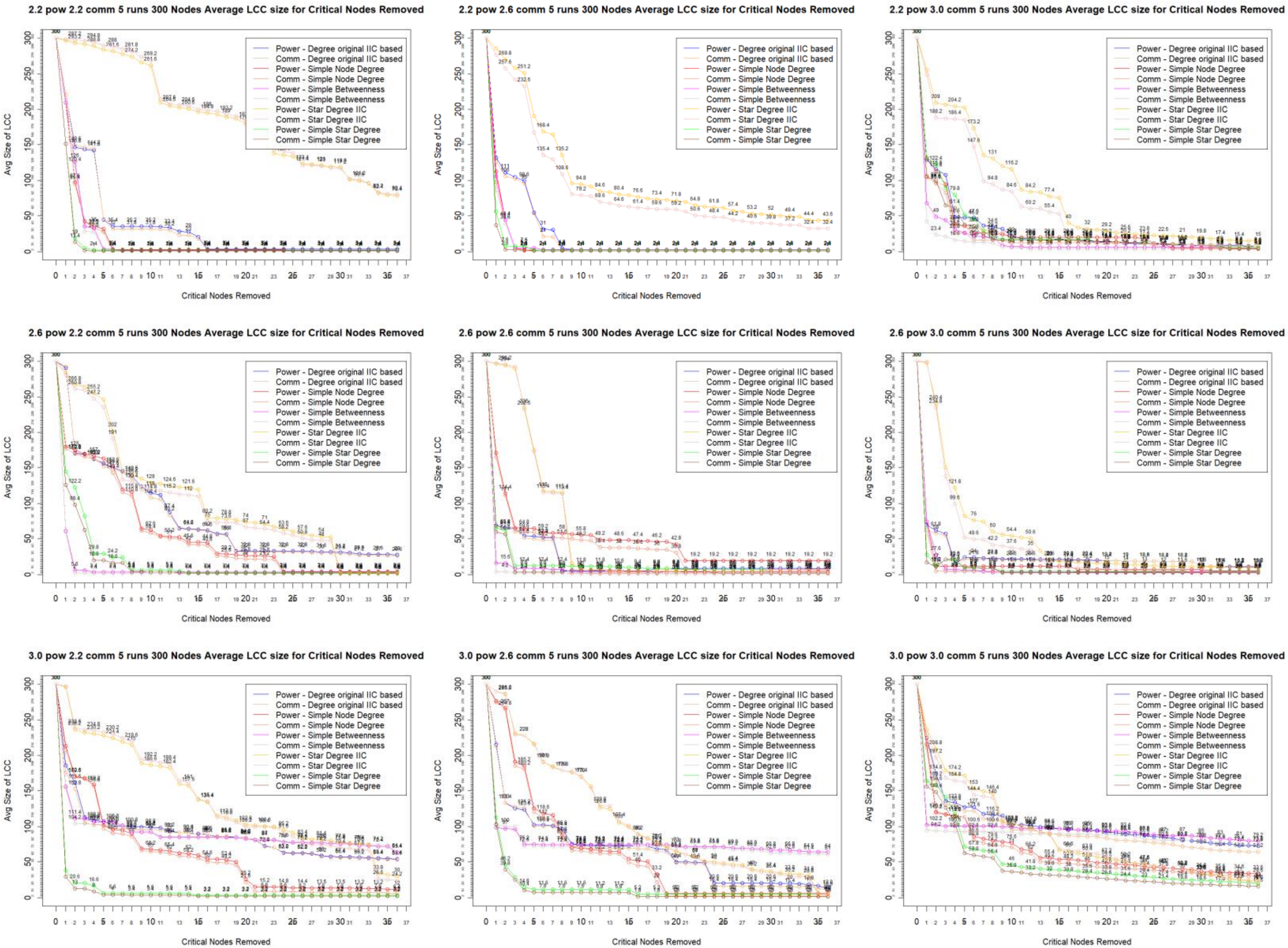


*Figure 12.* One to Multiple model, Scenario 3 Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

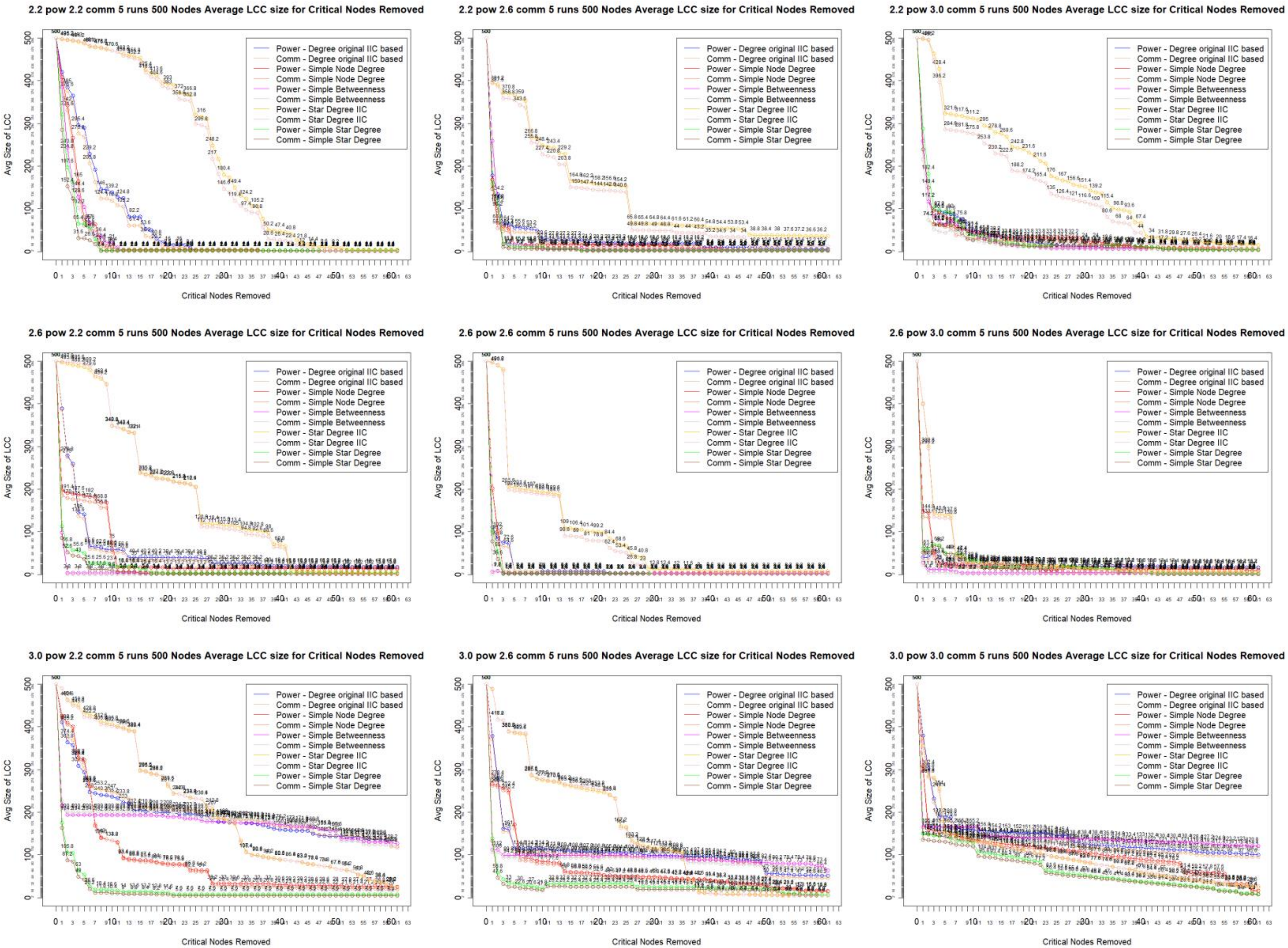


*Figure 13.* One to Multiple model, Scenario 3 Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

### 4.2.3. Multiple to one coupling model

The opposite model is also designed for experimentation. Now, a weighted random permutation is generated $\{d_1^{lk}, d_2^{lk}, \dots, d_n^{lk}\}$ with total length as the number of nodes in the communications network where $d_1^{lk}$ is the the number of interdependent links for $v_1^l$ and $d_1^{lk}$ is the the number of interdependent links for $v_1^l$. Following the same setup as before, but starting the discussion from the communications network, we obtain the results again a total of 9 experiments for each scenario under the multiple to one coupling model. The results of all three scenarios are shown in Figures 14-22.

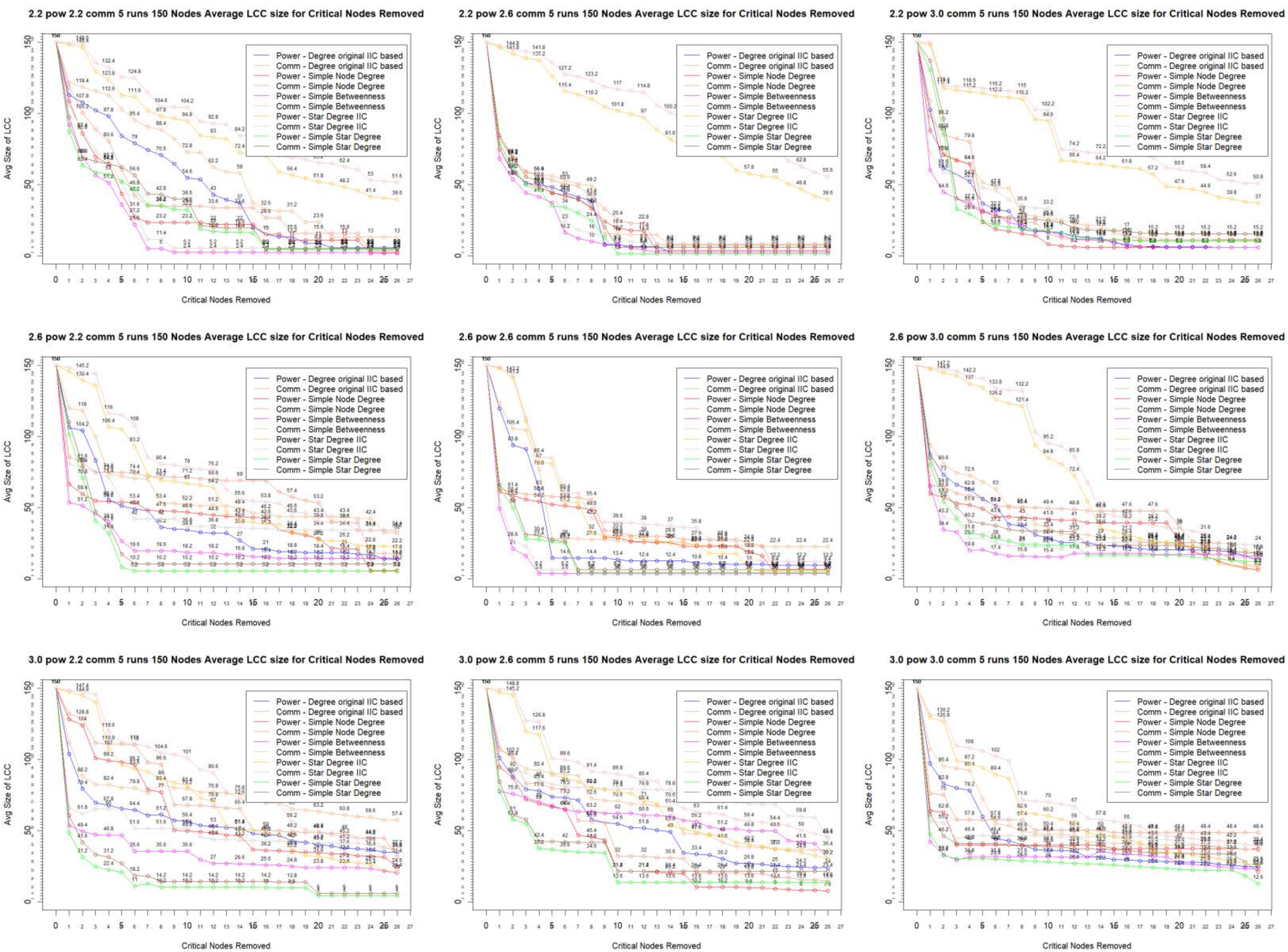


*Figure 14.* Multiple to One model, Scenario 1 Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

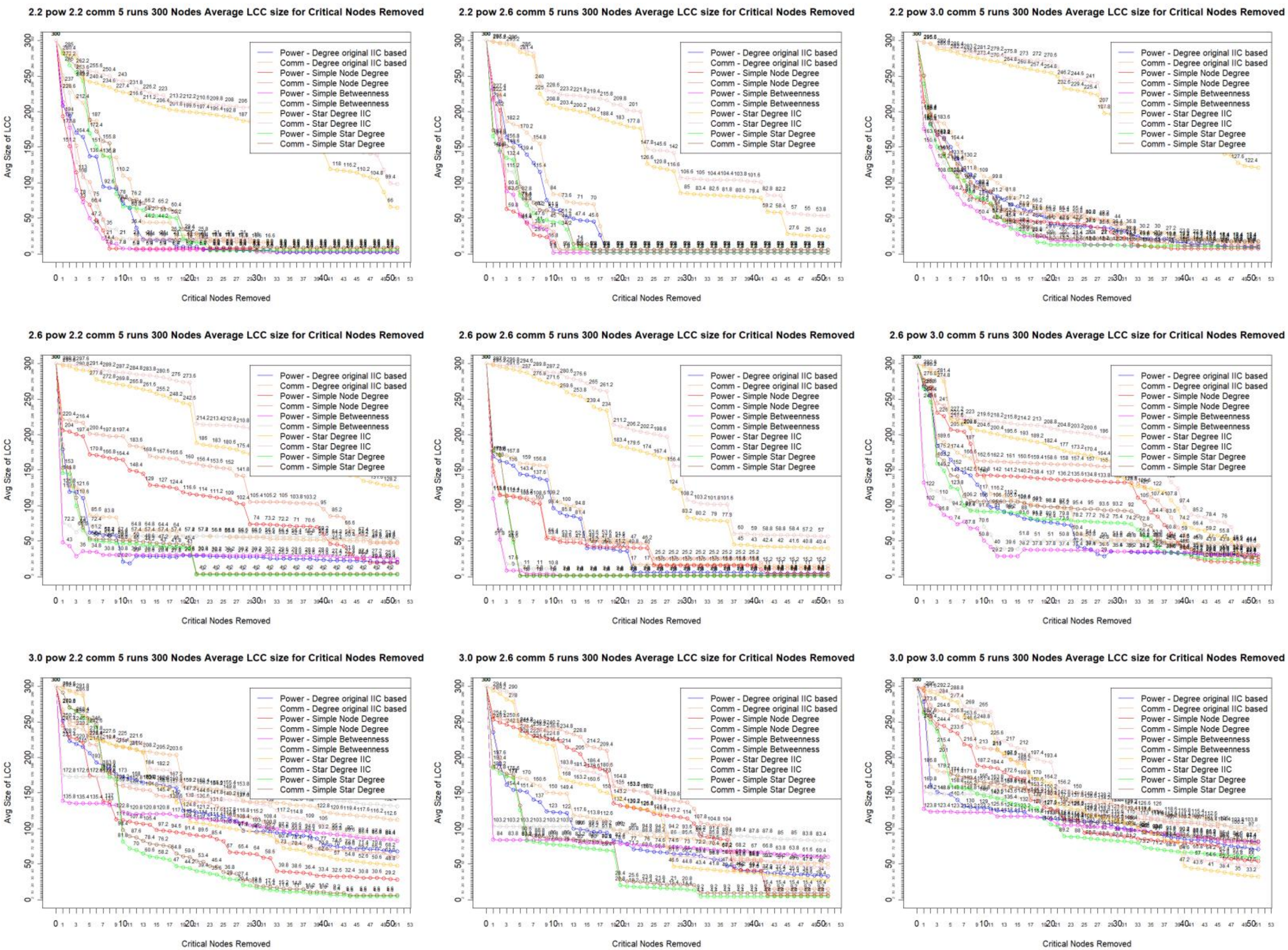


*Figure 15.* Multiple to One model, Scenario 1 Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

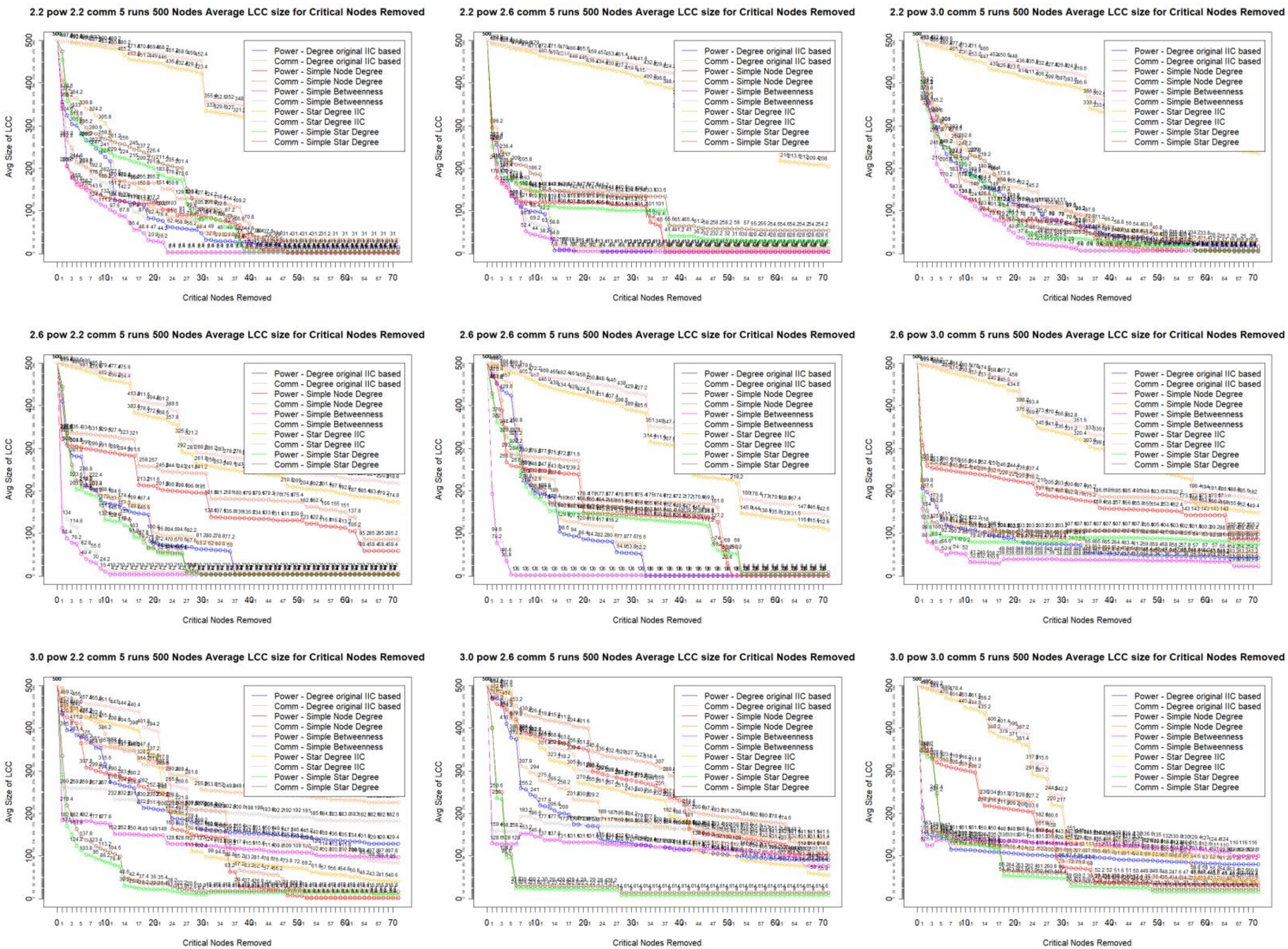


*Figure 16.* Multiple to One model, Scenario 1 Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

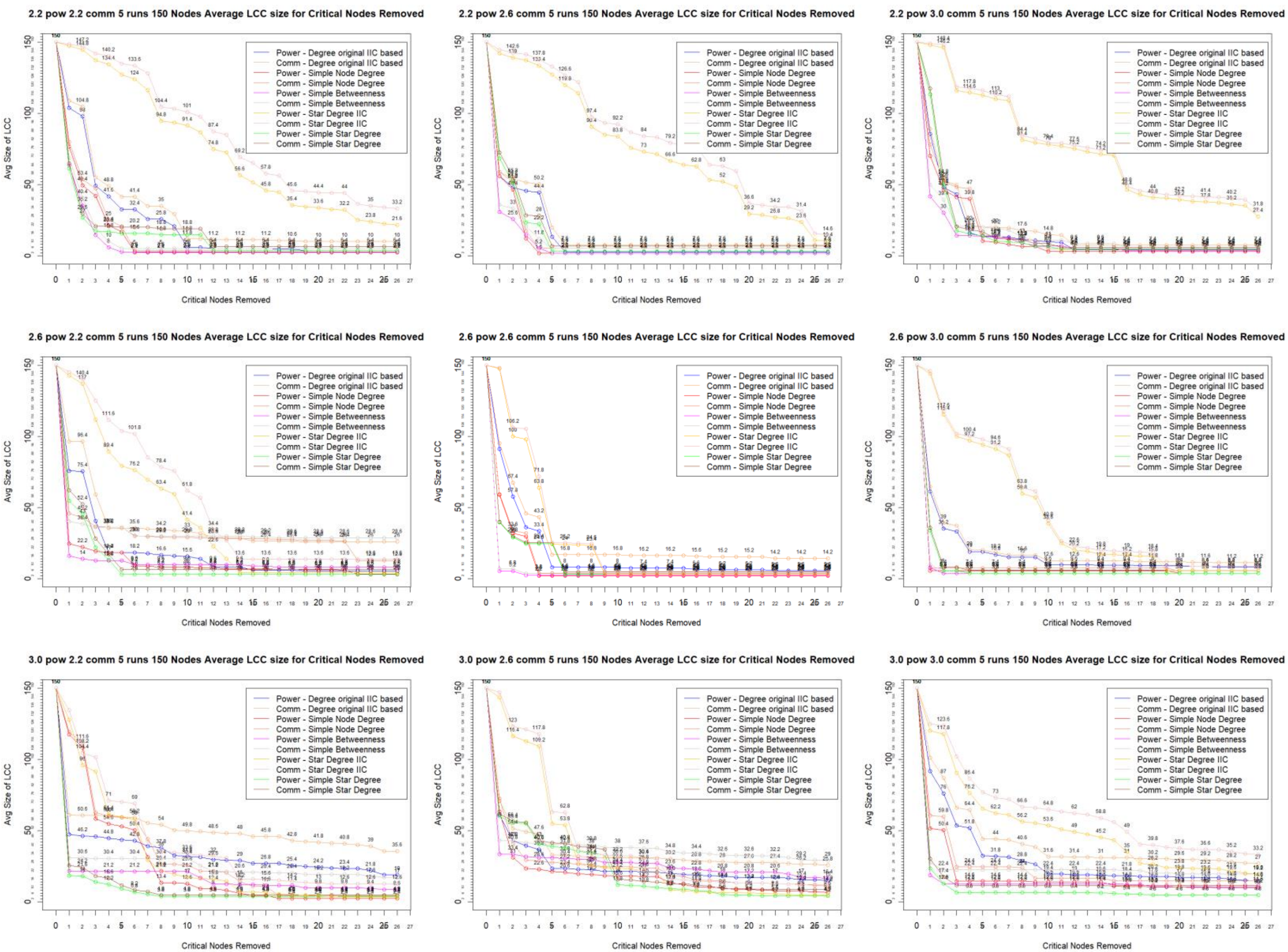


*Figure 17.* Multiple to One model, Scenario 2 Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

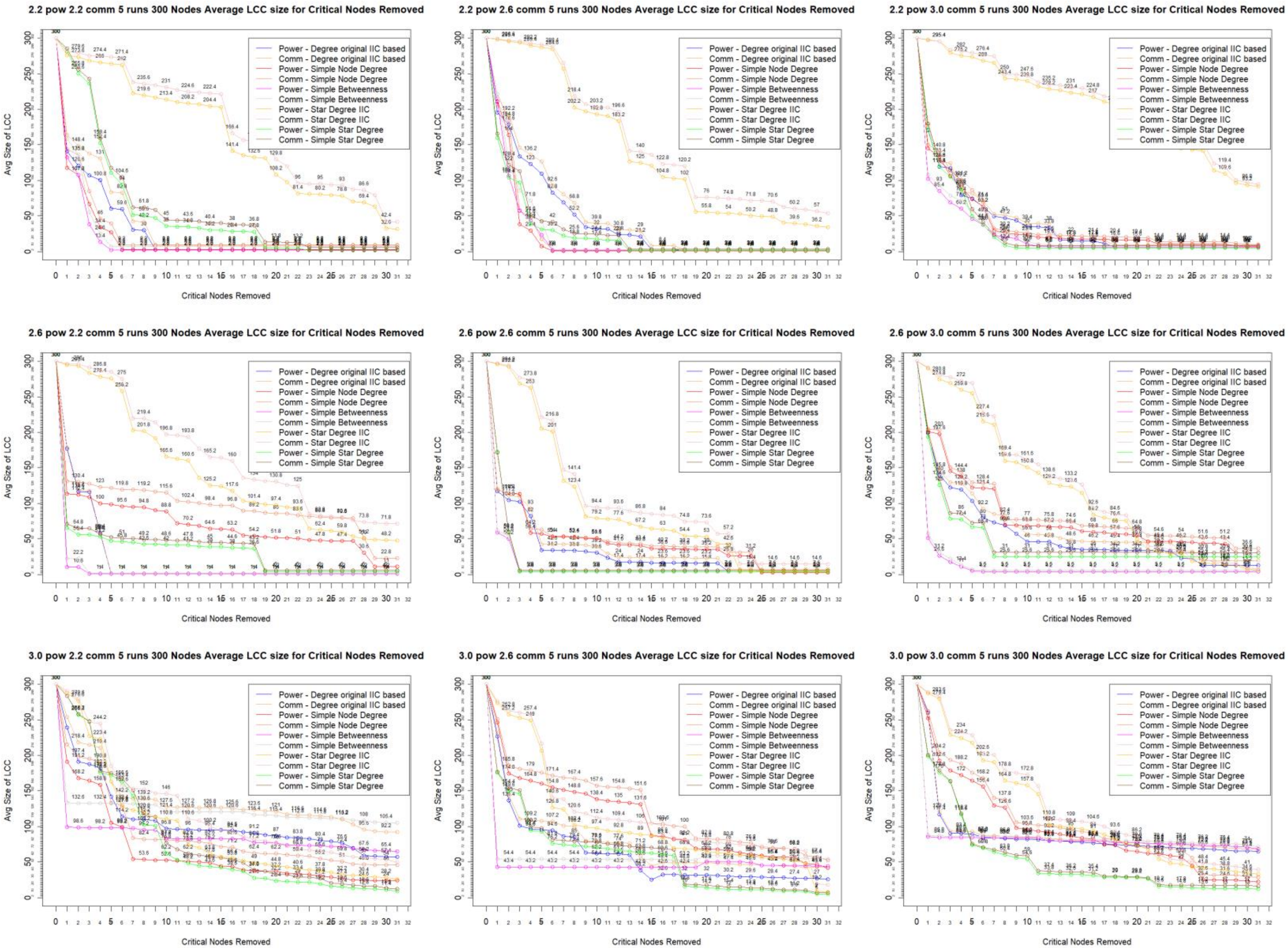


*Figure 18.* Multiple to One model, Scenario 2 Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

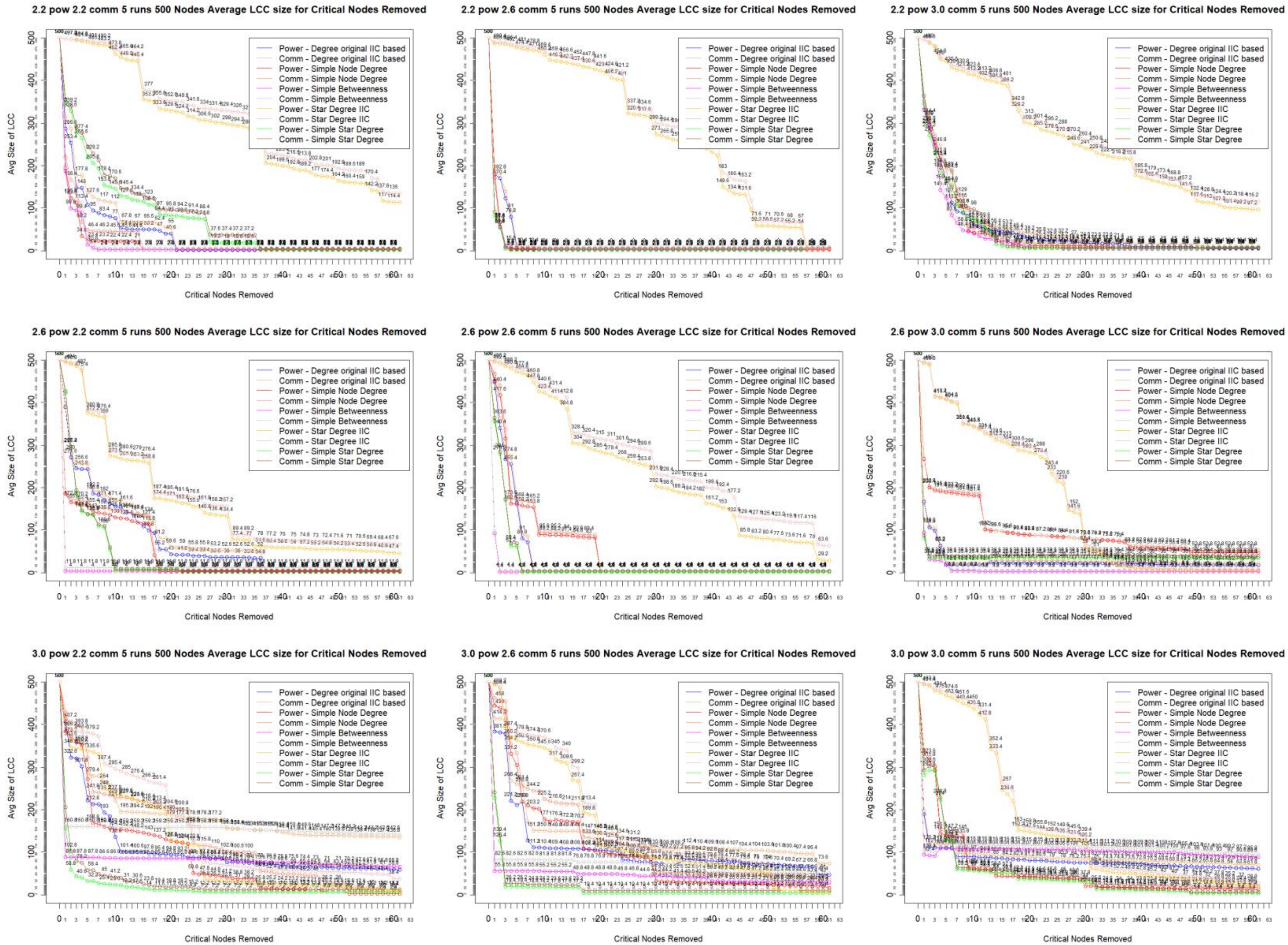


*Figure 19.* Multiple to One model, Scenario 2 Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

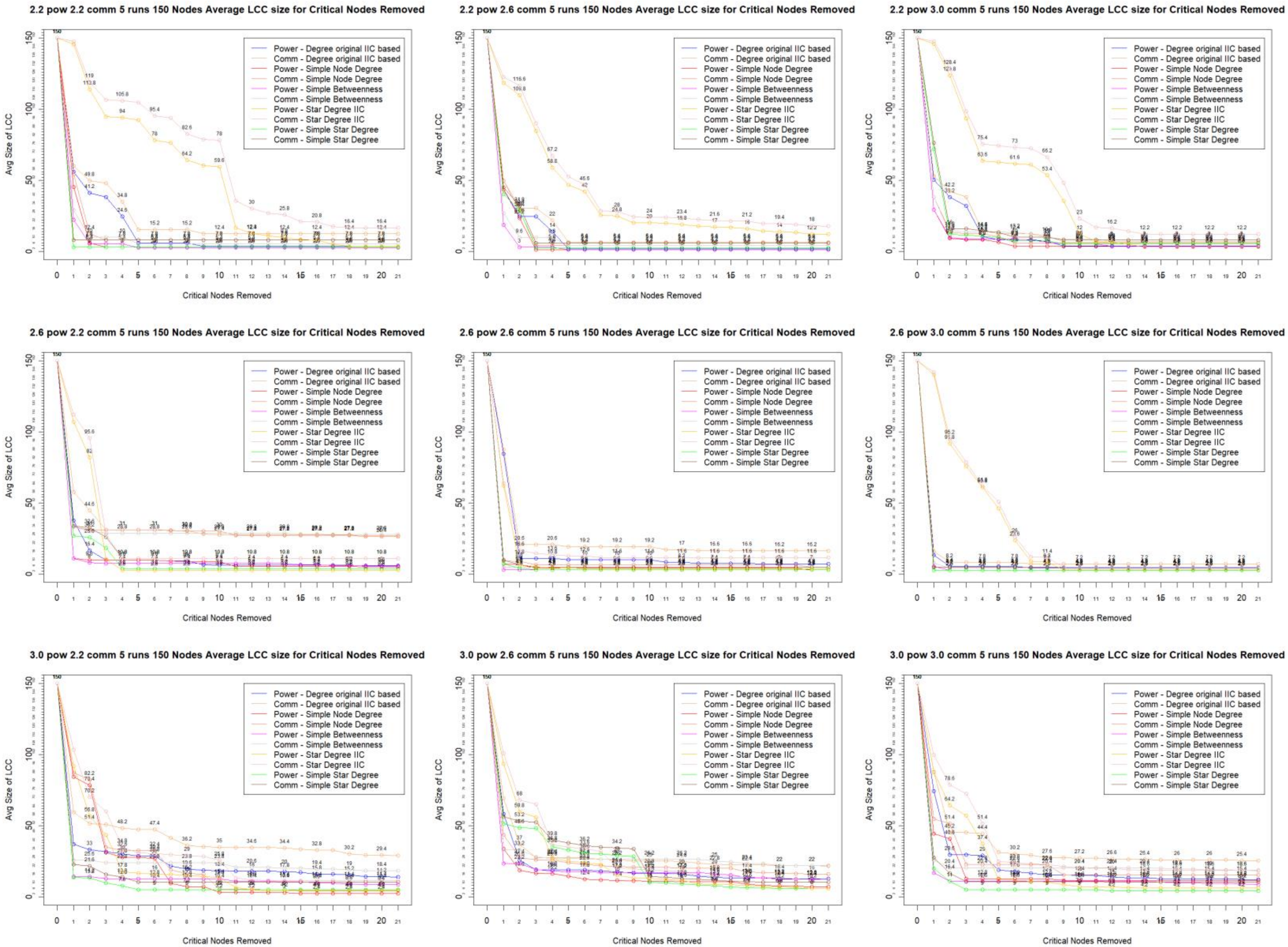


*Figure 20.* Multiple to One model, Scenario 3 Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

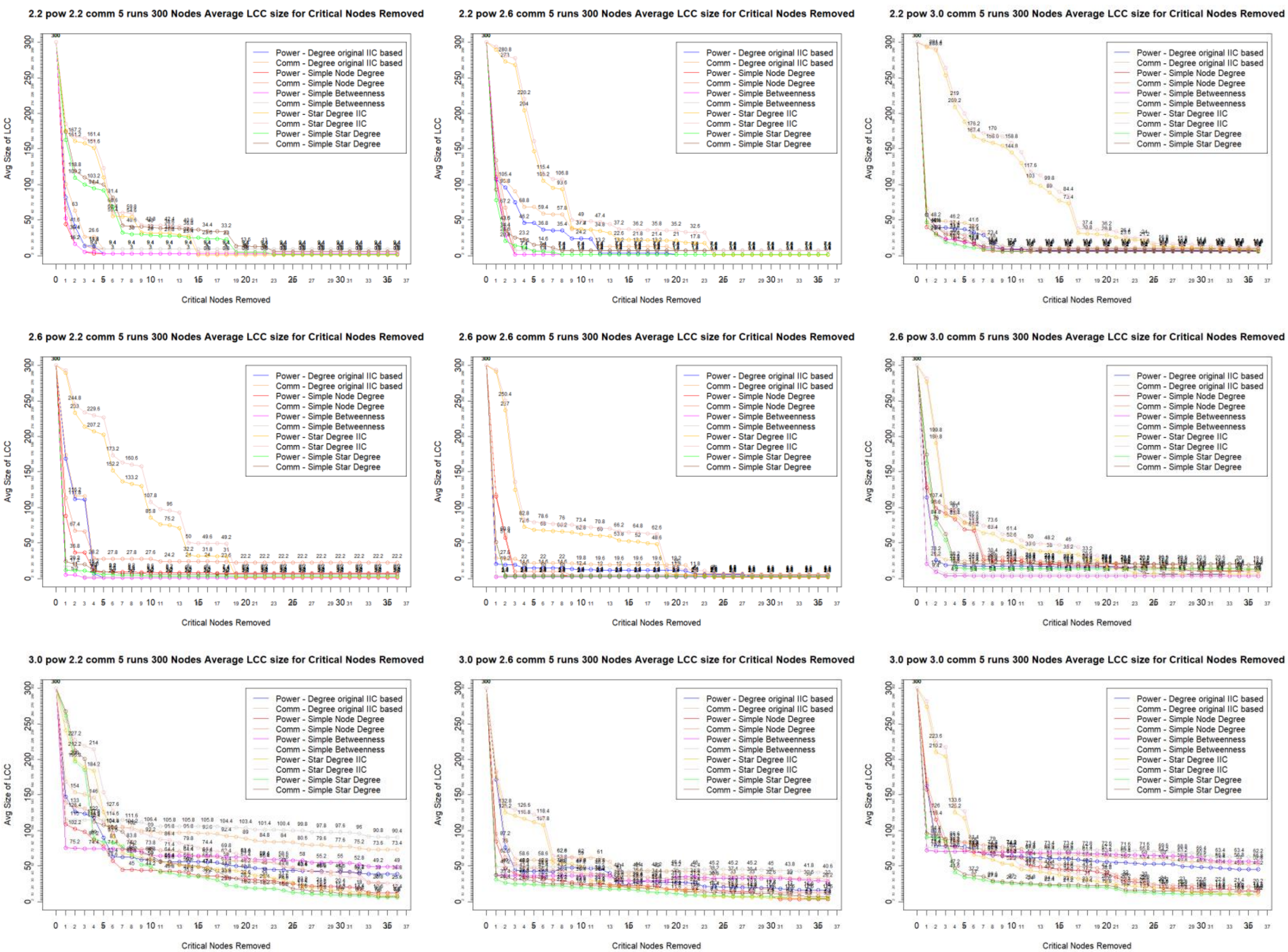


*Figure 21*. Multiple to One model, Scenario 3 Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

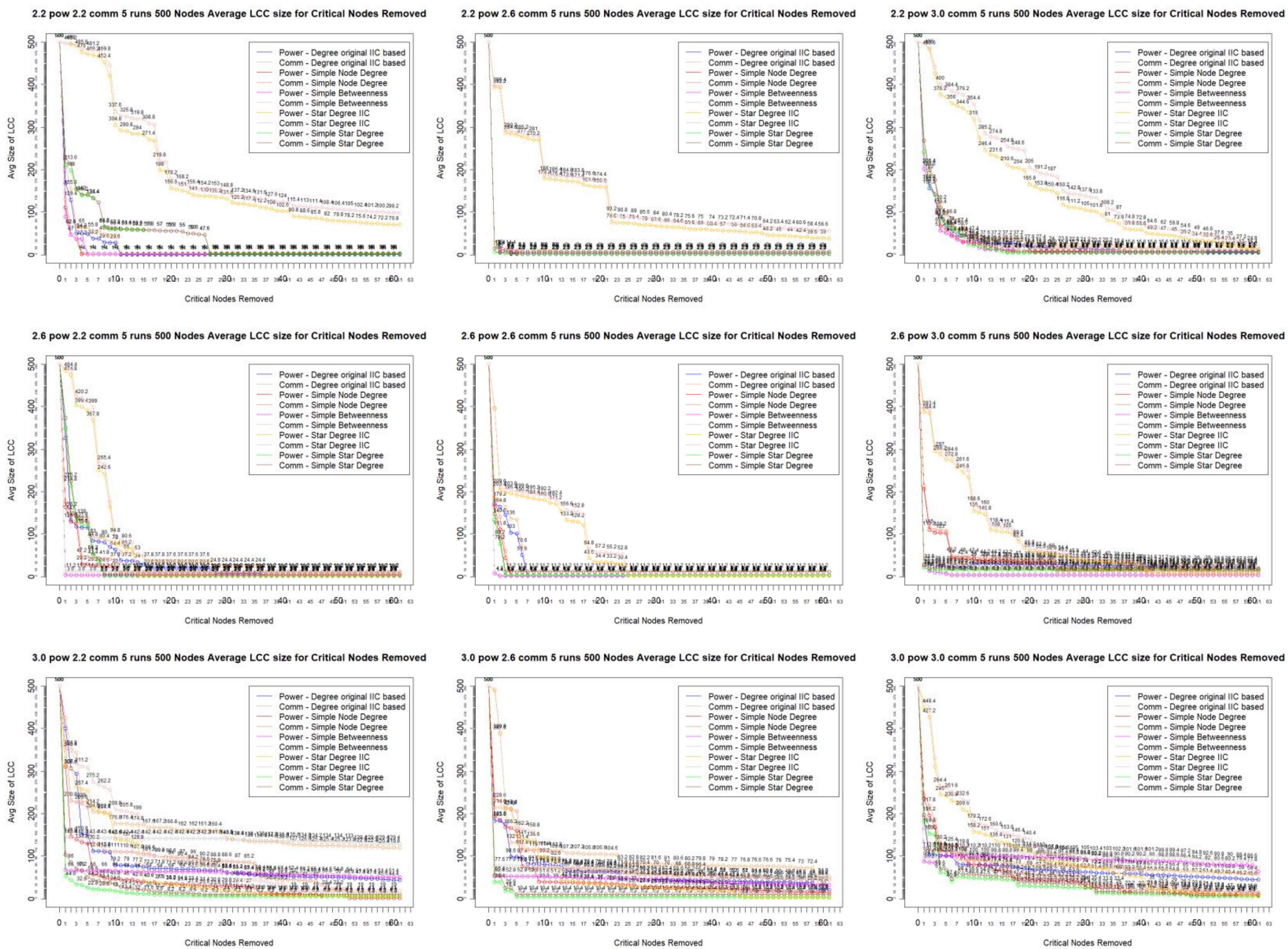


*Figure 22.* Multiple to One model, Scenario 3 Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

### 4.2.4. Multiple to multiple coupling model

Last, we investigate a multiple to multiple coupling model. In this model, two weighted random permutations are generated $\{d_1^{kl}, d_2^{kl}, \dots, d_n^{kl}\}$ and $\{d_1^{lk}, d_2^{lk}, \dots, d_n^{lk}\}$ with total length equal to the number of nodes in the power network and the communications network, respectively, where $d_1^{kl}$ is the the number of interdependent links for $v_1^k$ and $d_1^{lk}$ is the number of interdependent links for $v_1^l$. The permuted sets contain one of these elements $\{1,2,3,4\}$ and their weights are (as earlier) $\{0.6,0.2,0.15,0.05\}$. Two more random weighted permutations are generated with elements as vertices of power and communications network and the degree of the nodes in their network are considered as weights $\{v_1^{k'}, v_2^{k'}, v_3^{k'}, \dots v_n^{k'}\}$ and $\{v_1^{l'}, v_2^{l'}, v_3^{l'}, \dots v_n^{l'}\}$, respectively. Both networks are coupled using the set $\{d_1^{kl}, d_2^{kl}, \dots, d_n^{kl}\}$, i.e. if $d_1^{kl}$ is 3 then $v_1^{k'}$ is coupled with $\{v_1^{l'}, v_2^{l'}, v_3^{l'}\}$; similarly, if $d_1^{lk}$ is 3 then $v_1^{l'}$ is coupled with $\{v_1^{k'}, v_2^{k'}, v_3^{k'}\}$. Using the same setup as before for sizes and $\gamma$ results in a total of 9 experiments per scenario under the multiple to multiple coupling model. The results are shown in Figures 23-31.

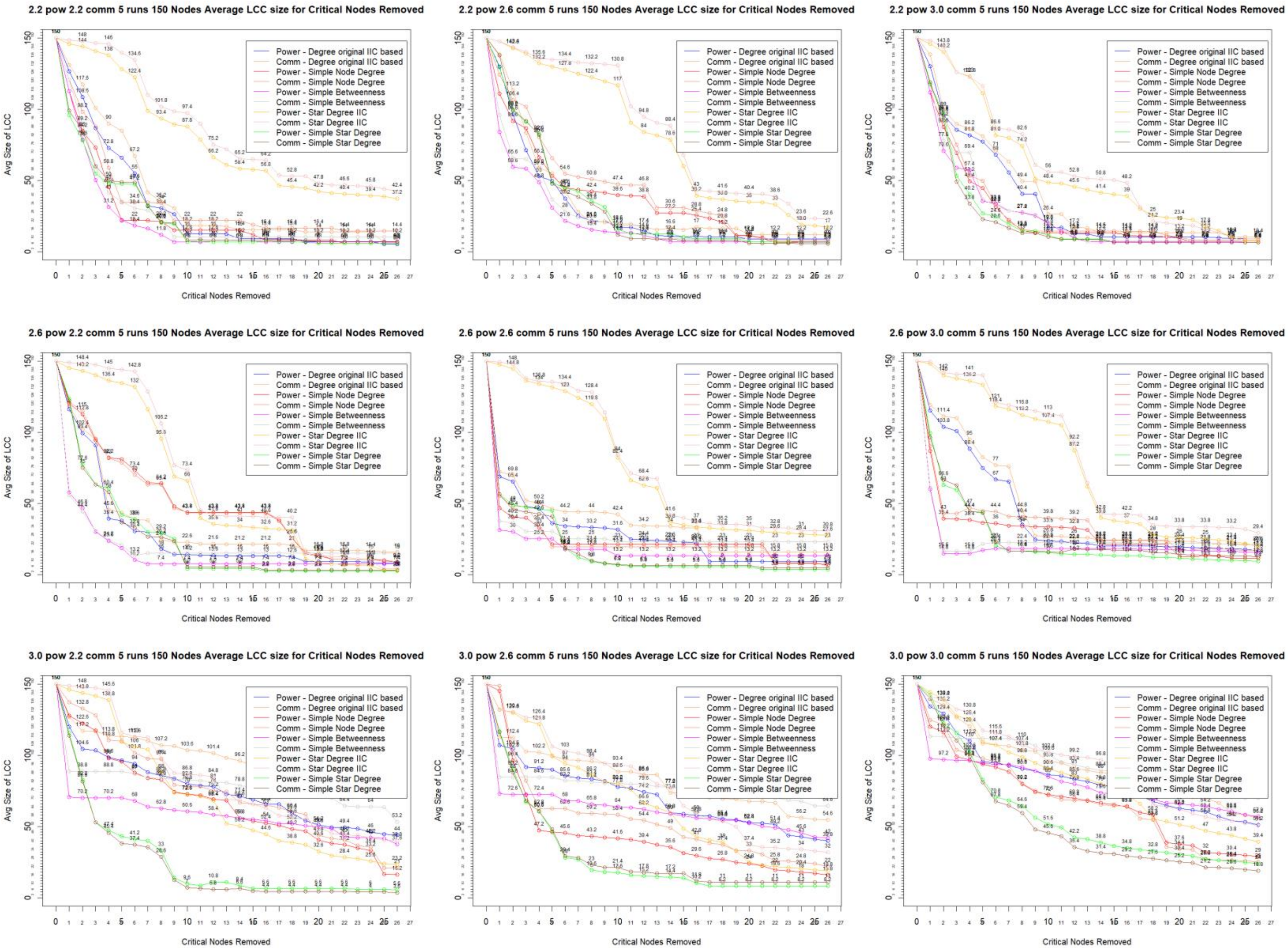


*Figure 23.* Multiple to Multiple model, Scenario 1 Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

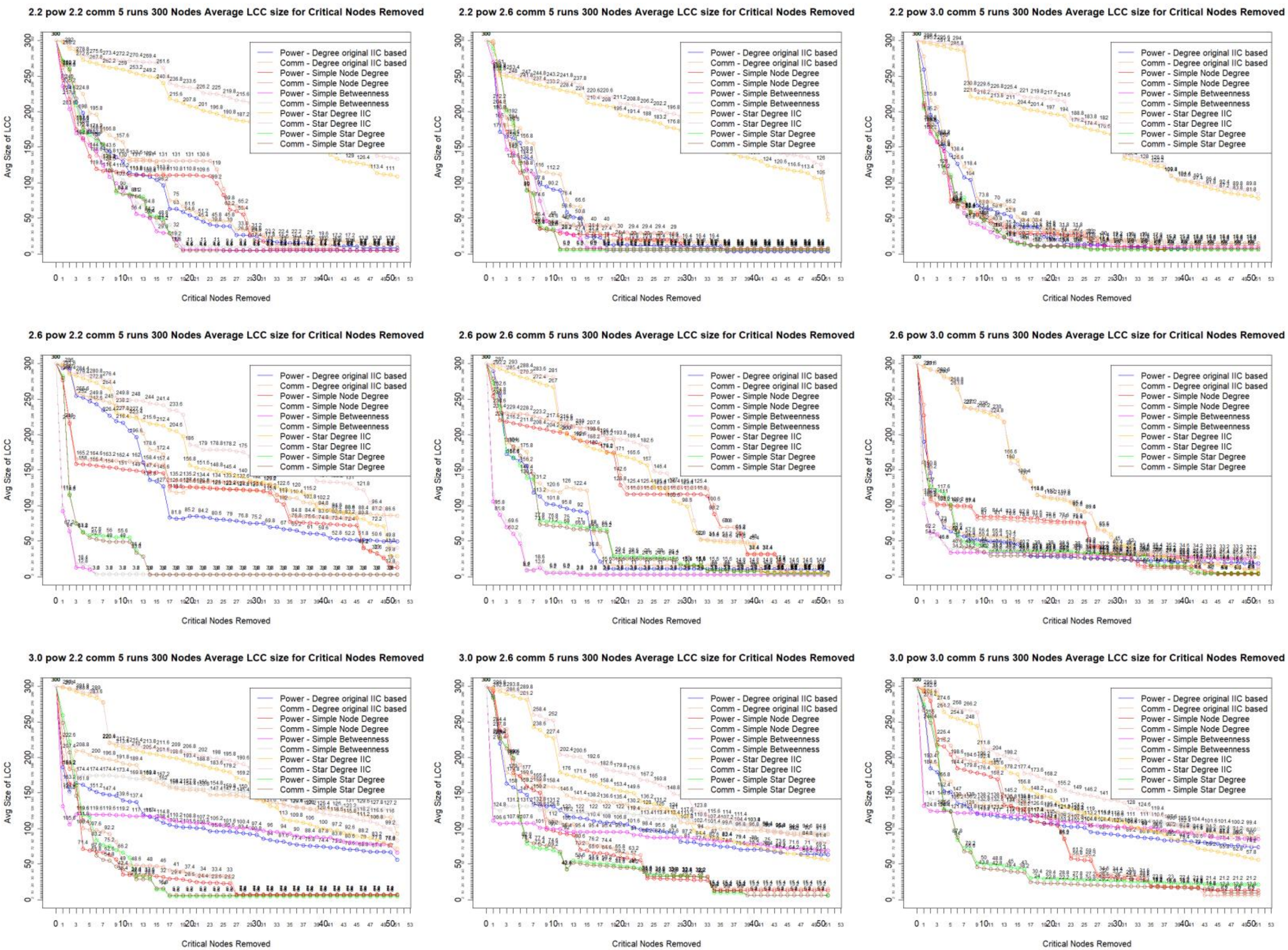


*Figure 24.* Multiple to Multiple model, Scenario 1 Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

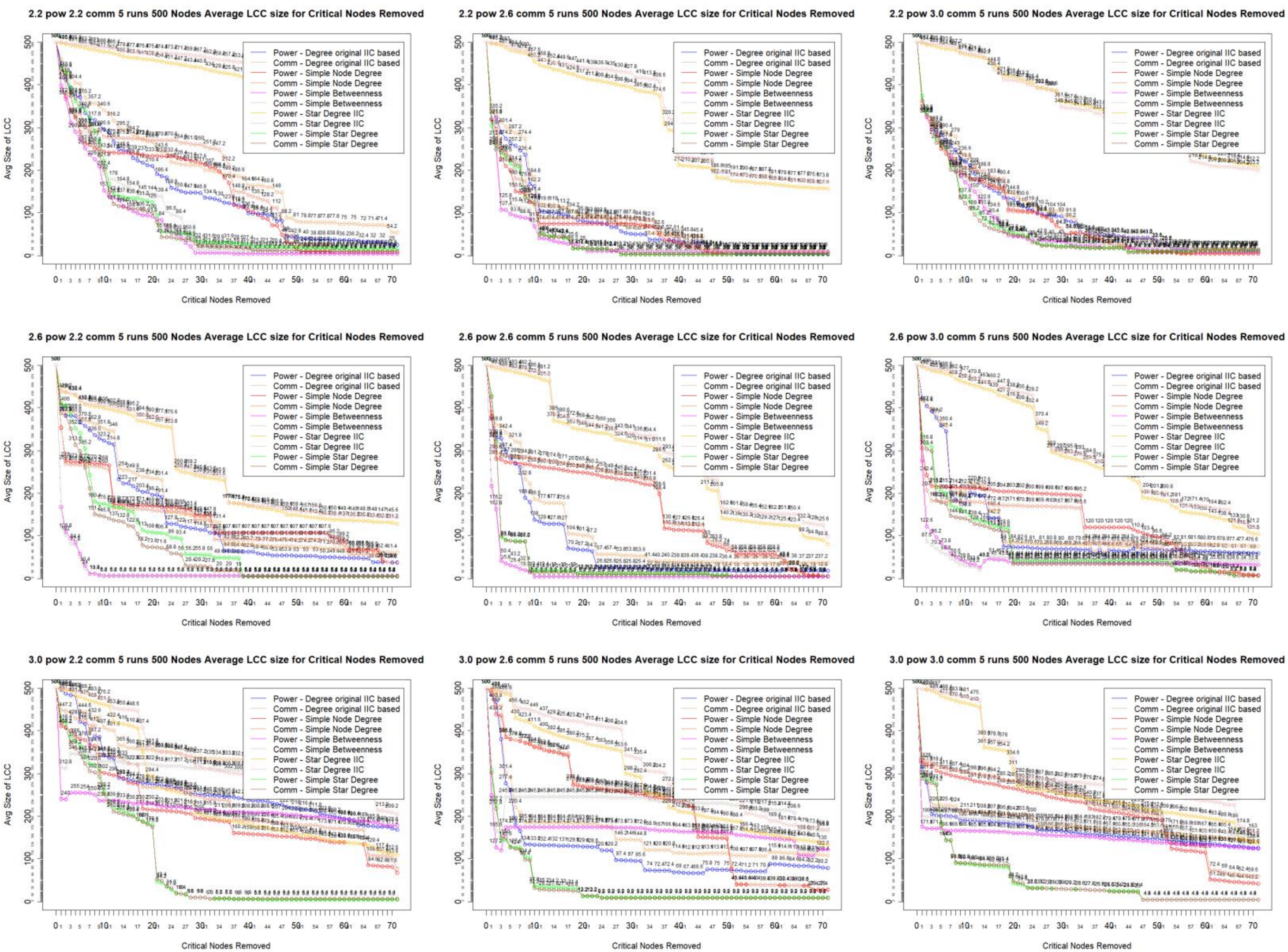


*Figure 25.* Multiple to Multiple model, Scenario 1 Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

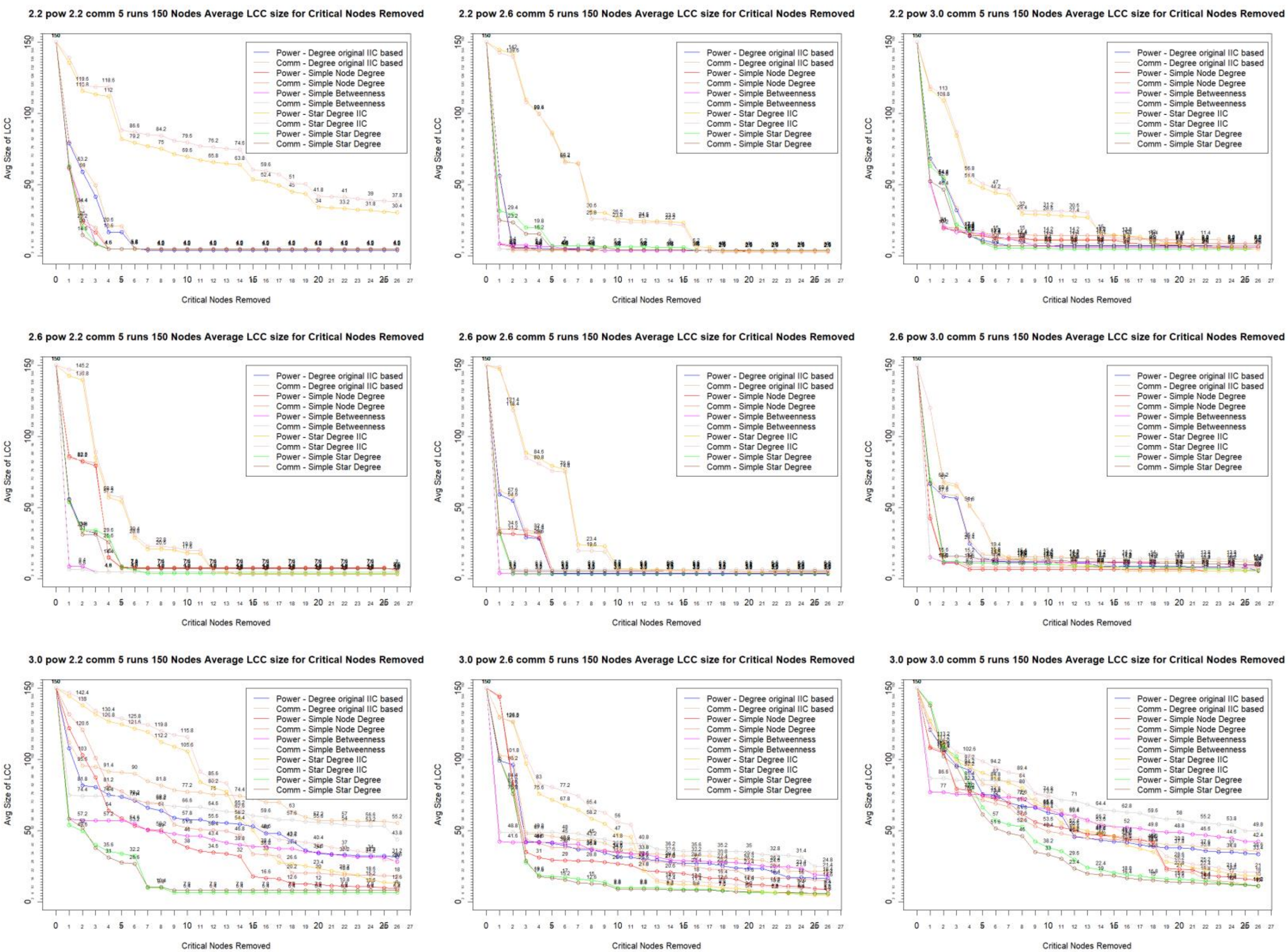


*Figure 26.* Multiple to Multiple model, Scenario 2 Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

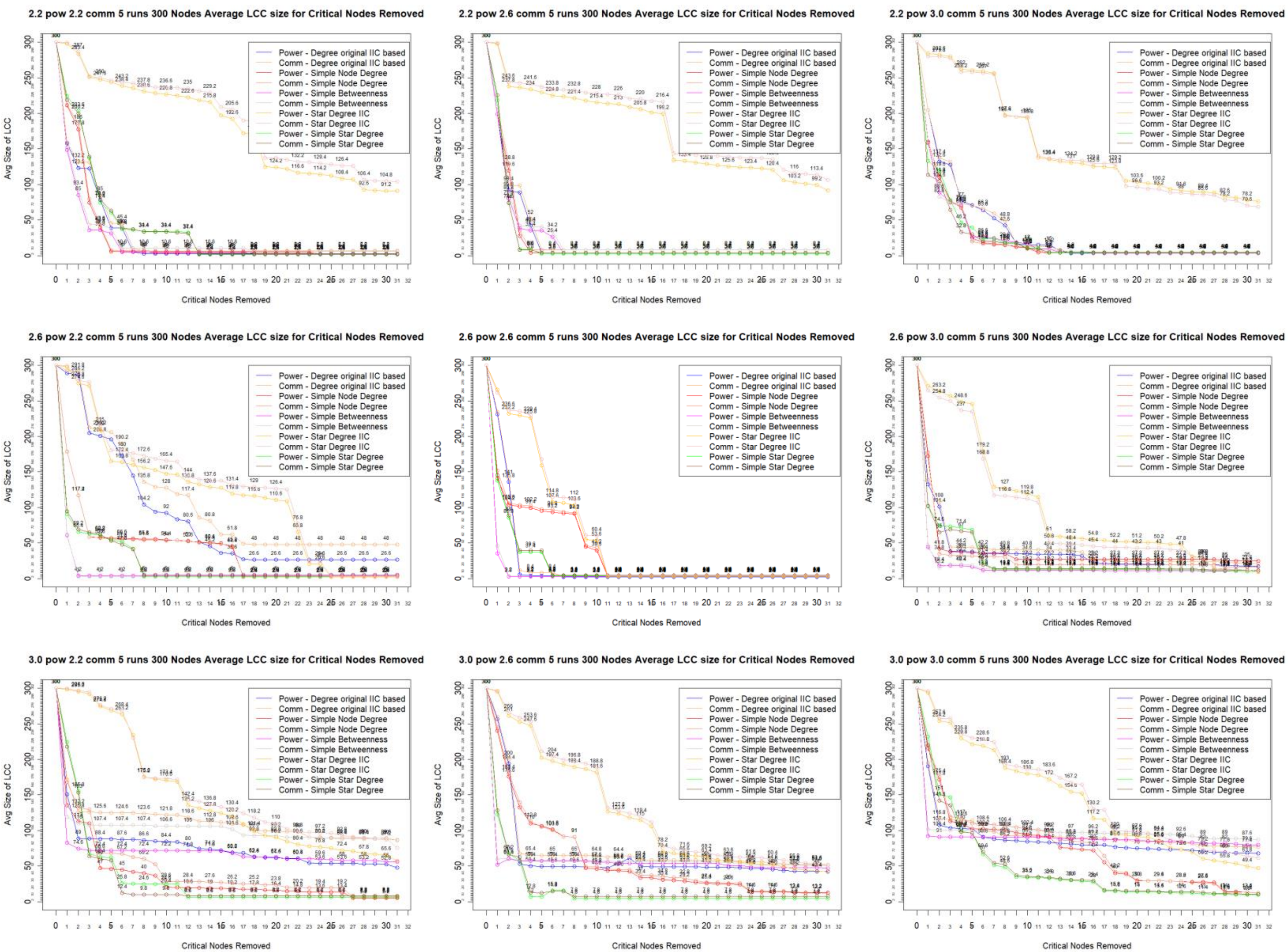


*Figure 27.* Multiple to Multiple model, Scenario 2 Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

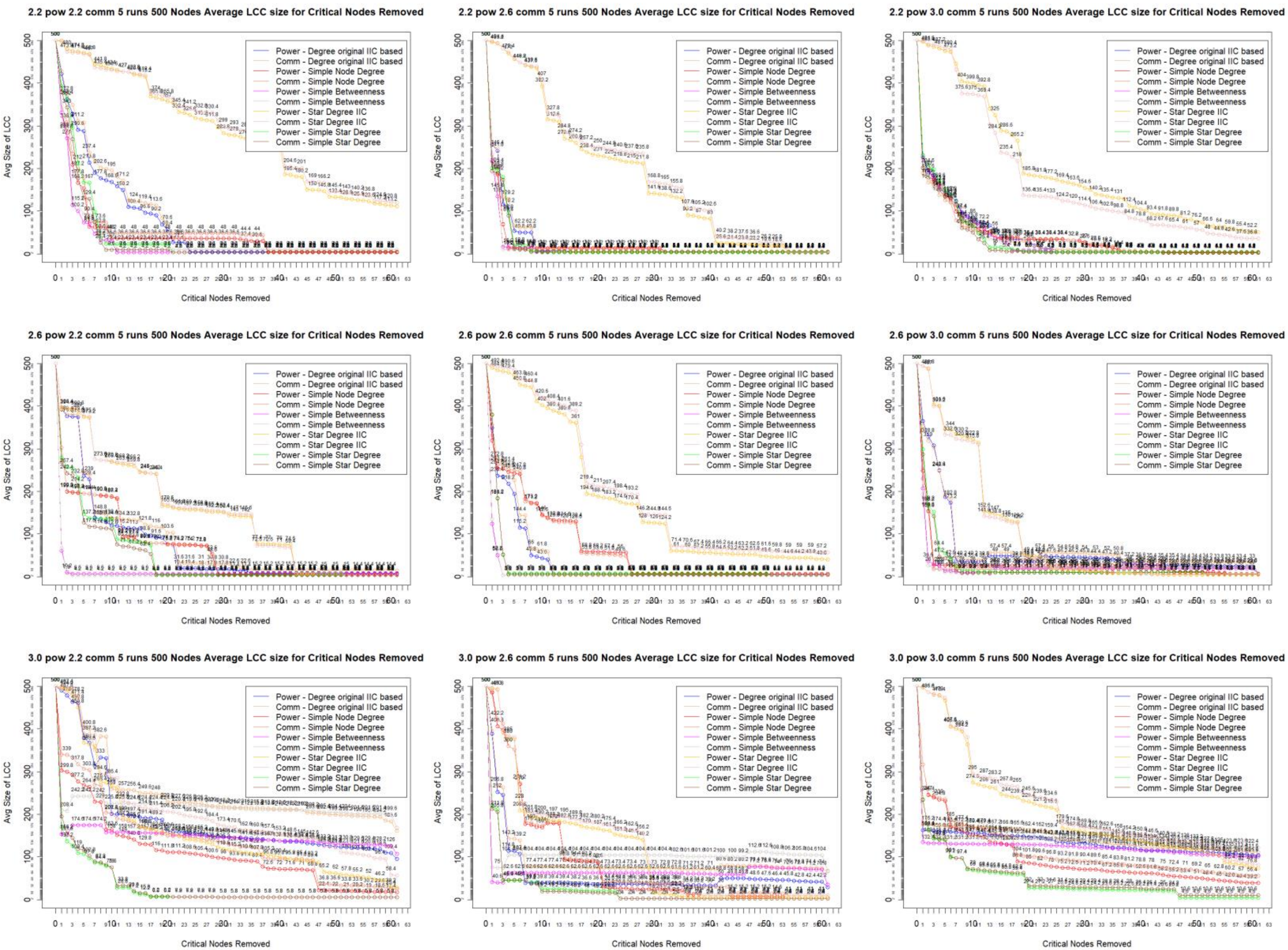


*Figure 28.* Multiple to Multiple model, Scenario 2 Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

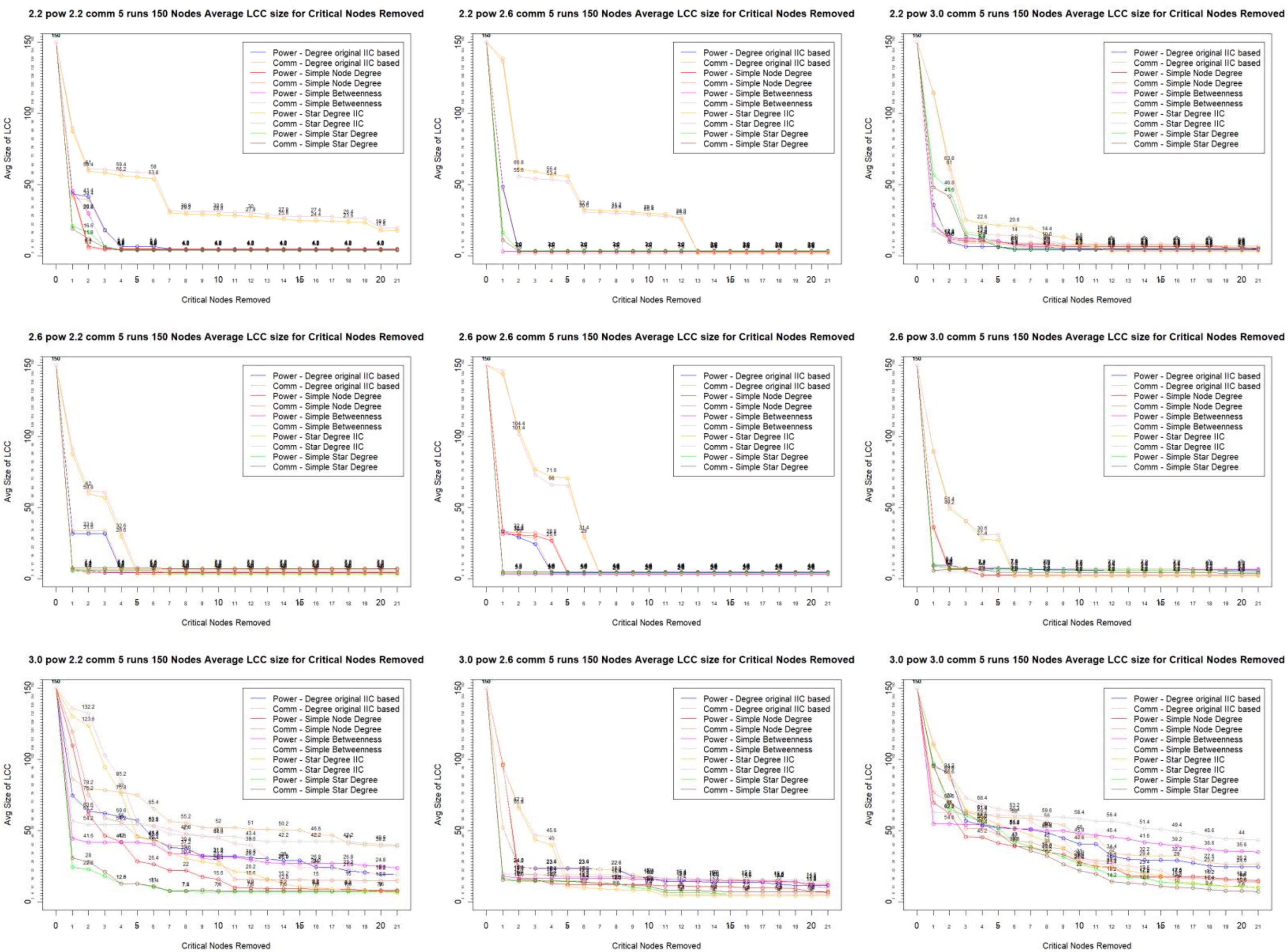


*Figure 29.* Multiple to Multiple model, Scenario 3 Average LCC vs Critical Nodes removed for 150-node networks across 5 runs

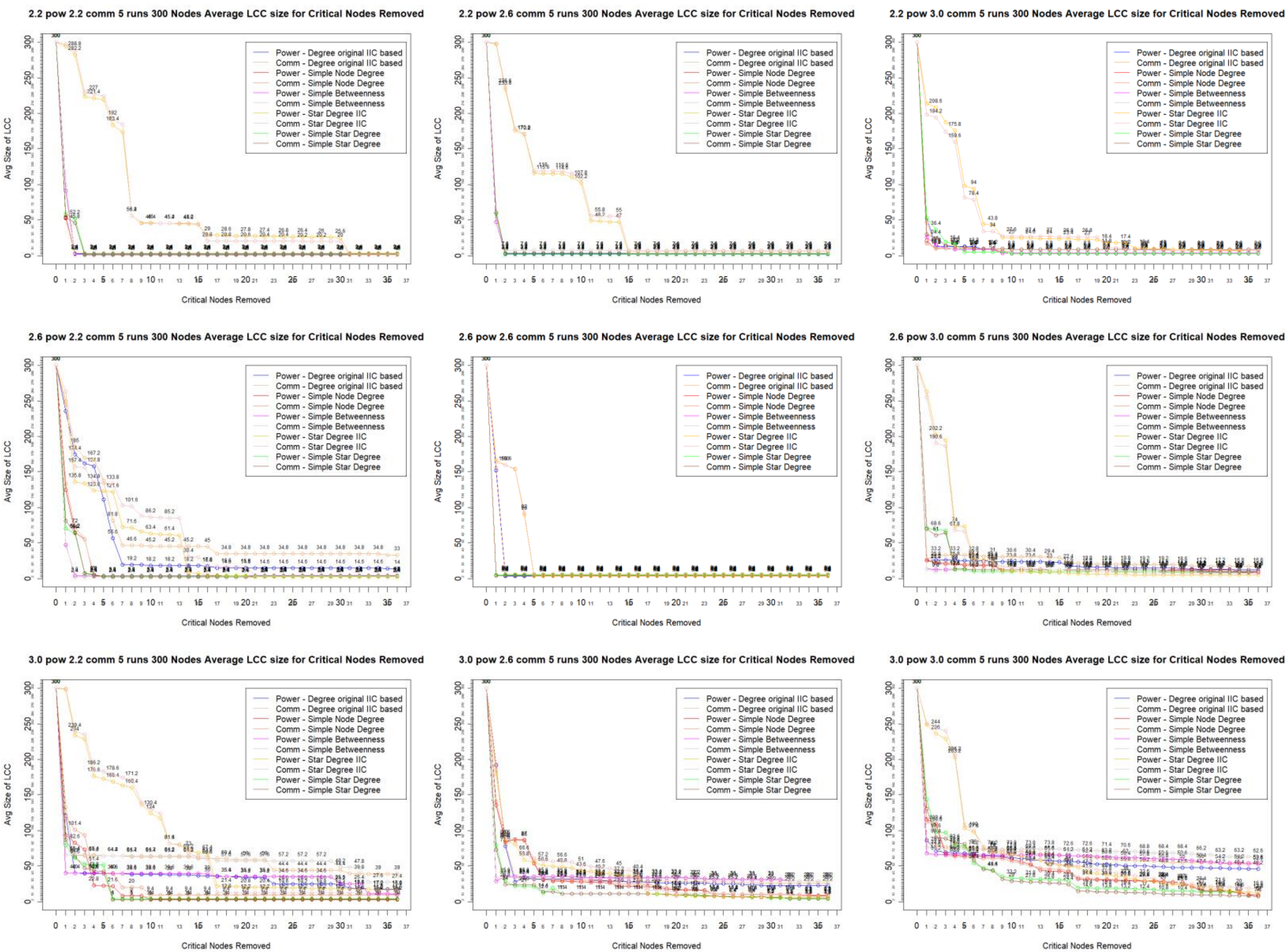


*Figure 30.* Multiple to Multiple model, Scenario 3 Average LCC vs Critical Nodes removed for 300-node networks across 5 runs

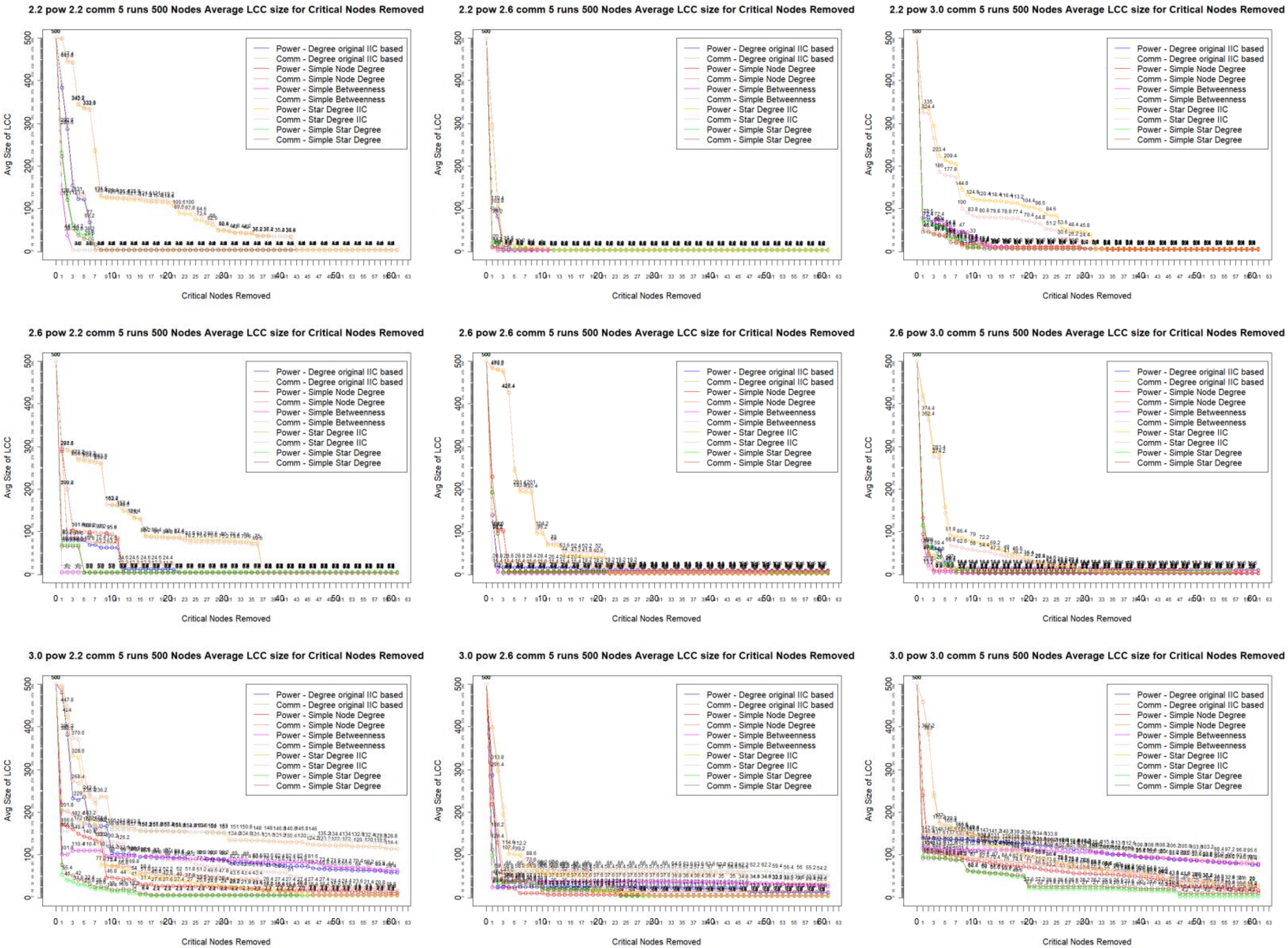


*Figure 31.* Multiple to Multiple model, Scenario 3 Average LCC vs Critical Nodes removed for 500-node networks across 5 runs

### 4.3. Analysis

Let us begin with the one to one model. As we observe, simple star degree shows a superior performance to all other metrics compared, and finds the smallest number of critical nodes that lead to maximum network breakdown. This behavior is the same for densely and loosely connected networks. Simple betweenness performs similarly well in loosely connected networks ($\gamma$=2.2, 2.6); however, it fails to find a small set of critical nodes in densely connected networks. Star degree IIC is outperformed in loosely connected networks, but reaches maximum breakdown faster in densely connected networks, even in cases where convergence is slower than other metrics for the first few critical nodes. Degree IIC performs slower than simple star degree in all network configurations tested, while in dense configurations it surpasses other conventional metrics, and surpasses node degree in loosely connected networks.

For the one to multiple model, simple star degree performs best and converges faster in all the tests conducted. Although simple betweenness shows good performance in loosely connected networks for all scenarios of this model, once the nodes are more densely connected the performance starts going down and the network stays connected even when a larger number of high betweenness nodes are disconnected from the power network. Simple node degree performance stays similar to betweenness but is more efficient in scenarios where the power network is densely connected. Degree with original IIC maintains its performance in loosely connected networks and combination of loosely connected power network with dense communication network. Star degree with modified IIC converges slower than other centrality scores in loosely connected power network. When it comes to densely connected power and communication network, though, the maximum network breakdown is achieved sooner with some scenarios where it converges significantly faster than degree IIC, betweenness, or simple degree. These patterns are reflected in networks of all tested sizes.

The results of multiple to one model reveal a similar pattern to the one to multiple model, where simple star degree shows a superior performance in all scenarios and star degree with modified IIC performs well in scenarios with a dense power network. Even though it converges slowly after the removal of the first few critical nodes, the maximum breakdown is reached faster than betweenness, simple degree, or degree IIC. Failure scenario 3 causes the most amount of disruption with the smallest number of critical nodes out of all 3 scenarios, due to its setup.

Last, the results of multiple to multiple model show that in scenarios 1 and 2, the network is much less vulnerable since for a node to fail all or at least half of the interdependent nodes from other networks need to fail as well. In this scenario, both networks have multiple interdependent links keeping them well-connected and, hence, the cascading of failures is stopped sooner than all other coupling models. However, in scenario 3, this effect is completely the opposite as the failure propagates much further for every critical node failed. As we can observe in the Figures, it takes only 5 to 15 critical nodes for the whole network to completely break apart in scenario 3. As seen previously in other models, simple star degree performs better here, as well with degree IIC performing better than simple node degree and betweenness in dense networks. Star degree IIC performs better in densely connected power networks and converges faster than other models in these dense network configurations.

# CHAPTER 5. CONCLUDING REMARKS

In this work, we investigated the coupling of two networks (namely a power and a communications network) from a graph theoretic perspective. Our objective was to identify metrics that can help us predict the importance of a node and the overall breakdown of the network should it fail. We proposed three new coupling models, based on the literature, and extended the one to one coupling model that is typically used. We also proposed three failure scenarios for the different coupling models.

More importantly though we developed a new modified IIC technique to identify critical nodes, as well as tried a new centrality metric (star centrality) in both its simple and its IIC versions. From our experimental setup, we were able to show that the newly proposed metrics are performing well in small and medium sized networks that are generated by a power law distribution.

The star centrality metric is also defined for more than 2 coupled networks. It is hence one of our goals to investigate how well its performance is in the presence of multiple interdependent networks. This would also have applications in real life, as it is usually many infrastructures that are coupled (pipelines, transportation networks, power, communications, etc.). Another important aspect of our work has to do with the study of networks that are scale-free: it would be interesting to investigate how the performance of the studied metrics is affected when different networks follow different distributions. Last, our cascading setup and metrics are computationally expensive, which makes them prohibitive to use in very large-scale networks. It is for that reason that we would like to propose new heuristic techniques to identify critical nodes in such intertwined networks.

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