A PROPOSED NONPARAMETRIC TEST FOR SIMPLE TREE ALTERNATIVE IN A BIBD DESIGN

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ABSTRACT

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A nonparametric test is proposed to test for the simple tree alternative in a Balanced Incomplete Block Design (BIBD). The details of the test statistic when the null hypothesis is true are given. The paper also introduces the calculations of the means and variances under a variety of situations.

A Monte Carlo simulation study based on SAS is conducted to compare the powers of the new proposed test and the Durbin test. The simulation study is used to generate the BIBD data from three distributions: the normal distribution, the exponential distribution, and the Student's t distribution with three degrees of freedom. The powers of the proposed test and the Durbin test are both estimated based on 10,000 iterations for three, four, and five treatments, and for different location shifts. According to the results of simulation study, the Durbin test is better when at least one treatment mean is close to or equal to the control mean; otherwise, the proposed test is better.

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CHAPTER 1. INTRODUCTION

The nonparametric methods are widely used in many fields, such as biostatistics, pharmaceutical statistics, business, psychology, and social sciences. The nonparametric tests are extremely useful because they have weaker assumptions about the underlying populations and the requirements for the measurement scales.

Assume that there are t populations where the first population is the control and the other t-1 populations are the treatments. We want to test whether it is true that at least one of the treatment means is larger than the control mean. The null hypothesis and alternative hypothesis is:

$$H_o: \mu_1 = \mu_2 = \cdots = \mu_t;$$

$$H_a: \mu_1 \leq [\mu_2, \mu_3, \cdots, \mu_t] \text{ (At least one is different).}$$

The above alternative hypothesis is called the simple tree alternative.

When blocks of homogeneous units are formed, and we cannot apply all treatments to each block, the Balanced Incomplete Block Design (BIBD) may be used. The BIBD means that every block contains an equal number of treatments, each treatment appears the same number of times, and each pair of treatments appears equal times.

Durbin (1951) presented a nonparametric rank test to test the null hypothesis that the treatments have identical effects in a Balanced Incomplete Block Design, which is for the general alternative. There is no existing nonparametric test for the simple tree alternative in the BIBD. Therefore, this paper proposes a new nonparametric test for this special alternative and compares the powers of the proposed test with the Durbin test. The powers will be estimated for a variety of distributions and location parameters using a Monte Carlo simulation study.

Chapter 2 gives the background information of some nonparametric tests; Chapter 3 proposes a new nonparametric test for the simple tree alternative in the BIBD; Chapter 4 gives the details of the simulation study for the comparison of the proposed test and the Durbin test; Chapter 5 introduces the results from the simulation study; and Chapter 6 draws the conclusions from the results.

CHAPTER 2. REVIEW OF LITERATURE

This chapter discuses some existing nonparametric tests for both the randomized complete block design (RCBD) and the balanced incomplete block design (BIBD). The tests in this chapter are either for the general alternative or for the non-decreasing ordered alternative.

2.1. Nonparametric Tests for RCBD

The Friedman test (Friedman, 1937, 1940) is a nonparametric test for testing the differences among treatment effects in a RCBD. Under the assumptions that the blocks are independent of each other; the variables are continuous; no interaction between blocks and treatments and the observations may be ranked in order of magnitude, the null and the alternative hypotheses for the Friedman test are:

$H_o: M_1 = M_2 = \cdots = M_t;$

H_a: At least one equality is violated.

The test statistic is

$$F = \frac{12}{bk(k+1)} \sum_{j=1}^{k} [R_j - \frac{b(k+1)}{2}]^2 \qquad (2.1)$$

where b is the number of blocks, k is the number of treatments and R_j is the sum of the ranks for the jth treatment.

The asymptotic distribution of F under the null hypothesis is chi-square with k-1 degrees of freedom. Reject H_o if the computed value of the test statistic F exceeds the tabulated value of chi-square for 1-α and k-1 degrees of freedom.

The Page's test (Page, 1963) is a nonparametric test for testing the ordered alternatives in a RCBD or a repeated measures design. Under the same assumptions, the null and the alternative hypotheses for the Page's test are :

$H_o: \tau_1 = \tau_2 = \cdots = \tau_t$;

$H_a: \tau_1 \le \tau_2 \le \tau_3 \le \cdots \le \tau_t$ (At least one inequality is strict).

The test statistic is

$$L = \sum_{j=1}^{t} jR_j = R_1 + 2R_2 + \cdots + tR_t \qquad (2.2)$$

where t is the number of treatments and R_j is the sum of the ranks for the jth treatment. It is noted that treatments are ranked only within each block. The ranks assigned within each block range from 1 to k, where k is the number of subjects per block (k < t). The standardized value of L is given by

$$Z_P = \frac{L-E(L)}{\sqrt{V(L)}} = \frac{L-[bt(t+1)^2/4]}{\sqrt{b(t^3-t)^2/144(t-1)}} \qquad (2.3)$$

where b is the number of blocks. Z_P has an asymptotic standard normal distribution under the null hypotheses and H_o is rejected if $Z_P \ge z_\alpha$.

Jiang (2009) compared the powers of the two-sample classical t-test and the Fligner-Wolfe test (Fligner and Wolfe, 1999) which can be viewed as an extension of the Wilcoxon rank-sum test to the simple tree alternative. He found that under the normal distribution, for equal variances, the t-test has higher powers; otherwise, the Fligner-Wolfe test has higher powers; under the exponential distribution, the Fligner-Wolfe test has higher powers regardless equal variances and unequal variance.

2.2. Nonparametric Tests for BIBD

The Durbin test (Durbin, 1951), is an extension from the Friedman test (Friedman, 1937, 1940), for a complete block design to a BIBD. Under the assumptions that the blocks are mutually independent of each other and the observations within each block may be ranked in order of magnitude, the null and the alternative hypotheses for the Durbin test are :

H_o: The treatments have equal effects;

H_a: The responses to at least one treatment tend to be larger than the responses to at least one other treatment.

The Durbin test statistic is

$$D = \frac{12(t-1)}{rt(k-1)(k+1)} \sum_{j=1}^{t} R_j^2 - \frac{3r(t-1)(k+1)}{k-1} \tag{2.4}$$

where t is the number of treatments, k is the number of subjects per block ($k < t$), r is the number of times each treatment occurs, and R_j is the sum of the ranks appearing under the jth treatment. The treatments are ranked only within each block.

The asymptotic distribution of D under the null hypothesis is chi-square with t-1 degrees of freedom. Reject H_0 if the computed value of the test statistic D exceeds the tabulated value of chi-square for α and t-1 degrees of freedom.

Ndungu and Magel (2011) proposed a nonparametric test for the non-decreasing ordered alternatives in a BIBD. The assumptions, null and alternative hypotheses are similar to the Page's test. The test statistic is

$$M = \sum_{j=1}^{t} j \times R_j \tag{2.5}$$

where R_j is the sum of the ranks for the jth treatment. Again, ranks are only assigned within each block. It is noted that not all treatments will appear in each block. Therefore, the ranks assigned in each block vary from 1 to k, where k is equal to the number of treatments appearing in each block with k being less than t. The standardized value of M is given by

$$M^* = \frac{M - E(M)}{\sqrt{V(M)}} \tag{2.6}$$

The E (M) and V (M) are given in the Table 1 for 3, 4 and 5 treatments based on the minimum number of blocks by Ndungu. The null hypotheses is rejected if $M^* \geq z_\alpha$, where z_α is the (1-α) percentile of a standard normal distribution.

Table 1. Mean and Variance for the Ndungu test

Cases	Minimum Number of Blocks	E (M)	V (M)
b = 3, k = 2	3	18	1.50
b = 4, k = 2	6	45	5.00
b = 4, k = 3	4	60	13.33
b = 5, k = 2	10	90	12.50
b = 5, k = 3	10	180	50.00
b = 5, k = 4	5	150	62.50

b = Number of treatments; k = Number of treatments per block

The number of blocks used for the Ndungu test must be a multiple of the minimum number of blocks. The mean and variance for the test statistic would be multiplied by this multiple number. Ndungu showed that the test was generally more powerful than the WSR test (Wilcoxon, 1945) and the Durbin test for the non-decreasing ordered alternatives in a BIBD regardless of the underlying distribution or sample size.

Cao (2010) compared the powers of the Durbin test and the Wilcoxon Signed-Rank test (WSR) which is a nonparametric test that tests for differences in location parameters when data are paired, with two observations per block in a BIBD. She found that the WSR test was more powerful for the ascending or descending ordered location parameters; but the Durbin test was more powerful for the random ordered location parameters.

A new nonparametric test will be proposed for the simple tree alternative in the BIBD in the Chapter 3. The estimated powers of this test will be compared with the estimated powers of the Durbin test.

CHAPTER 3. PROPOSED TEST

In this chapter, we propose a new nonparametric test for the simple tree alternative in a BIBD design. Assume that there are t populations where the first population is the control and the other t-1 populations are the treatments. Under the assumptions, the blocks are independent and the observations within each block may be ranked. We want to show that at least one treatment effect is larger than the control effect. Therefore, the null and alternative hypothesis is

$H_o: \tau_1 = \tau_2 = \cdots = \tau_t;$

$H_a: \tau_1 \leq [\tau_2, \tau_3, \cdots, \tau_t]$ (At least one inequality is strict).

The proposed test statistics is

$$T = R_1 + (t-1) * (R_2 + \cdots + R_t) \tag{3.1}$$

where t is the number of treatments, and R_j is the sum of the ranks for the jth treatment.

The exact null distribution of T could be found but would be based on sample sizes and number of treatments. It would change every time. This would be a time intensive process to consider the exact null distribution in every case. It would also become more time intensive as the sample size increases. Therefore, the asymptotic null distribution of T will be used.

T will have an asymptotic normal distribution under the null hypothesis. This is because T is based on rank statistics and is also similar to Page's test for a randomized complete block design. Page's test has an asymptotic normal distribution (Page, 1963). The standardized value of the test statistic, T^*, is given by

$$T^* = \frac{T - E(T)}{\sqrt{V(T)}} \tag{3.2}$$

The asymptotic null distribution of T^* is a standard normal distribution, and the null hypothesis is rejected when $T^* \geq z_\alpha$, where z_α is the $(1-\alpha)$ percentile of a standard normal distribution. The mean and variance of T will need to be calculated and these will vary based on the number of treatments as well as the number of blocks and the number of treatments in each block.

In order to have a balanced design of incomplete blocks, the number of blocks must be a multiple of a given number and this may be found in Table 2. For example, for 3 treatments with 2 treatments appearing in each block, the number of blocks must be of the form 3s, where s is an integer, and therefore must be a multiple of 3. Table 2 also gives the expected value and variance for the test statistic based on the null hypothesis for the minimum number of blocks in each case. The expected value and variance of the test statistic for the number of blocks used would be the expected value and variance of the minimum multiplied by the number of sets of minimum blocks, s.

Table 2. Mean and Variance for the proposed test

Sets	Number of Blocks	E (T)	V (T)
t = 3, k = 2	3s	15s	0.5s
t = 4, k = 2	6s	45s	3s
t = 4, k = 3	4s	60s	8s
t = 5, k = 2	10s	102s	9s
t = 5, k = 3	10s	204s	36s
t = 5, k = 4	5s	170s	45s

t = Number of treatments; k = Number of treatments per block (k < t)

In order to see how the expected value and variance is obtained for the test statistic based on the minimum number of blocks. The calculations are given for 4 treatments with 3 treatments appearing per block. In this case, the test statistic is given by

$$T = R_1 + 3 * (R_2 + R_3 + R_4) \tag{3.3}$$

The minimum number of blocks needed for a balanced design is $\binom{4}{3} = 4$. The treatments must appear in the blocks in the arrangement given below for the design to be balanced.

Cases	Treatment 1	Treatment 2	Treatment 3	Treatment 4
Case 1	×	×	×	.
Case 2	×	×	.	×
Case 3	×	.	×	×
Case 4	.	×	×	×

In Case 1, treatments 1, 2 and 3 appear in a block, but not treatment 4. The following arrangements of ranks are possible.

Treatment 1	Treatment 2	Treatment 3	Treatment 4	T
1	2	3	.	1+3*5=16
1	3	2	.	1+3*5=16
2	1	3	.	2+3*4=14
2	3	1	.	2+3*4=14
3	1	2	.	3+3*3=12
3	2	1	.	3+3*3=12

$$E(\text{case } 1) = \frac{16 * 2 + 14 * 2 + 12 * 2}{6} = 14$$

$$V(\text{case } 1) = \frac{(16 - 14)^2 * 2 + (14 - 14)^2 * 2 + (12 - 14)^2 * 2}{6} = \frac{8}{3}$$

In Case 2, treatments 1, 2 and 4 appear in a block, but not treatment 3. The following arrangements of ranks are possible.

Treatment 1	Treatment 2	Treatment 3	Treatment 4	T
1	2	.	3	1+3*5=16
1	3	.	2	1+3*5=16
2	1	.	3	2+3*4=14
2	3	.	1	2+3*4=14
3	1	.	2	3+3*3=12
3	2	.	1	3+3*3=12

$$E(\text{case } 2) = \frac{16 * 2 + 14 * 2 + 12 * 2}{6} = 14$$

$$V(\text{case } 2) = \frac{(16 - 14)^2 * 2 + (14 - 14)^2 * 2 + (12 - 14)^2 * 2}{6} = \frac{8}{3}$$

In Case 3, treatments 1, 3 and 4 appear in a block, but not treatment 2. The following arrangements of ranks are possible.

Treatment 1	Treatment 2	Treatment 3	Treatment 4	T
1	.	2	3	1+3*5=16
1	.	3	2	1+3*5=16
2	.	1	3	2+3*4=14
2	.	3	1	2+3*4=14
3	.	1	2	3+3*3=12
3	.	2	1	3+3*3=12

$$E(\text{case } 3) = \frac{16 * 2 + 14 * 2 + 12 * 2}{6} = 14$$

$$V(\text{case } 3) = \frac{(16 - 14)^2 * 2 + (14 - 14)^2 * 2 + (12 - 14)^2 * 2}{6} = \frac{8}{3}$$

In Case 4, treatments 2, 3 and 4 appear in a block, but not treatment 1. The following arrangements of ranks are possible.

Treatment 1	Treatment 2	Treatment 3	Treatment 4	T
.	1	2	3	0+3*6=18
.	1	3	2	0+3*6=18
.	2	1	3	0+3*6=18
.	2	3	1	0+3*6=18
.	3	1	2	0+3*6=18
.	3	2	1	0+3*6=18

$$E(\text{case } 4) = \frac{18 * 6}{6} = 18$$

$$V(\text{case } 4) = \frac{(18 - 18)^2 * 2 + (18 - 18)^2 * 2 + (18 - 18)^2 * 2}{6} = 0$$

Therefore,

$$E(\text{set}) = 14 + 14 + 14 + 18 = 60$$

$$V(\text{set}) = \frac{8}{3} + \frac{8}{3} + \frac{8}{3} + 0 = 8$$

The expected values and variances of the test statistics for other cases are worked out similarly. We are particularly interested in estimating the power of the proposed test. Details of a simulation study for the power comparison between the proposed test and the Durbin test are described in Chapter 4.

CHAPTER 4. SIMULATION STUDY

A simulation study is conducted comparing estimated powers of the proposed test to the Durbin test. Random samples from a balanced incomplete block design are generated under a variety of conditions. In order to estimate powers, 10,000 sets of samples are generated under the same conditions. The test statistics are calculated for each sample and it is determined whether or not the hypothesis is rejected in each case. The estimated power of a test is found by counting the number of times the null hypothesis is rejected by the test divided by 10,000. The level of significance is estimated for each test by generating 10,000 sets of samples from the populations when the null hypothesis is true, and counting the number of times the null hypothesis is rejected and dividing by 10,000. In this case, the populations are all the same.

Three distributions, including normal distribution, exponential distribution, and Student's t distribution with three degrees of freedom, are employed to generate the samples. Different numbers of treatments and replications are considered, and various location parameter arrangements are used in the simulation. Each combination of the distributions, locations, and replications, are simulated 10,000 times. The estimated power is defined by counting the numbers of times that the null hypothesis is rejected and then dividing by 10,000. All simulations are based on SAS 9.2.

4.1. Distributions

Three underlying distributions are considered in the simulation study. The block effects are always assumed to have a normal distribution.

The Ranuni routine (call Ranuni (seed, x)) is used to generate numbers from a uniform distribution to use as seeds in generating samples from various populations. The routines

Rannor and Ranexp are used to generate samples from normal and exponential populations, respectively.

For the t (3) distribution, the subroutine TINV (x, 3) is used to return the xth quantile from the Student's t distribution with degree freedom 3. In this case, x is the value returned from the subroutine Ranuni. Under these three distributions, different location parameters are used for different samples, but the variances were always equal to 1.

4.2. Number of Treatments and Replications

The numbers of populations considered in the simulation study are 3, 4, and 5. For all simulations, the population 1 is the control population, and the others are the treatment populations. The following cases are considered:

(1) t = 3; k = 2; r = 20; sigma = 1; b = 30;

(2) t = 4; k = 2; r = 15; sigma = 1; b = 30;

(3) t = 4; k = 3; r = 15; sigma = 1; b = 20;

(4) t = 5; k = 2; r = 32; sigma = 1; b = 80;

(5) t = 5; k = 3; r = 18; sigma = 1; b = 30;

(6) t = 5; k = 4; r = 12; sigma = 1; b = 15;

where

t = the number of treatments;

k = the number of subjects per block ($k < t$);

r = the number of replications of a treatment;

b = the number of blocks.

4.3. Location Parameters

The simulation study considers the simple tree alternative. The power is estimated for different parameter arrangements. For t = 3, 16 different location parameter arrangements are considered, which include the following types:

(1) The parameters are equally spaced, for example (0, 0.4, 0.8). It is noted that changing the ordering of the location parameters for any treatment situation besides the control population will not change the estimated powers. (0, 0.8, 0.4) will have same estimated powers;

(2) The space between the last two parameters is twice the space between the first two parameters, for example (0, 0.25, 0.75). Again, (0, 0.75, 0.25) will have same estimated powers;

(3) The space between the last two parameters is triple the space between the first two parameters, for example (0, 0.25, 1);

(4) The first two parameters are equal, while the others are different, for example (0.5, 0.5,1.5);

(5) The last two parameters are equal, while the others are different, for example (0.5, 1, 1);

(6) The spaces between the parameters are unequal, for example (0, 0.4, 1).

For t = 4, 26 different location parameter arrangements are considered, which include the following types:

(1) The parameters are equally spaced, for example (0, 0.4, 0.8, 1.2). Again, (0, 0.8, 1.2, 0.4) will have same estimated powers;

(2) The space between the last two parameters is twice the space between the third and second parameters; while the space between the third and second parameters is twice the space between the first two parameters, for example (0, 0.2, 0.6, 1.4);

(3) The space between the last two parameters is triple the space between the third and second parameters; while the space between the third and second parameters is triple the space between the first two parameters, for example (0, 0.1, 0.4, 1.3);

(4) The last three parameters are equal, while are different from the control, for example (0, 0.8, 0.8, 0.8);

(5) The two treatments are equal, while are different from the others, for example (0, 0.5, 1, 1);

(6) One treatment is different from the control, while others are equal to the control, for example (0, 0, 0, 1);

(7) The first two parameters are equal, while the last two parameters are equal, for example (0, 0, 1.5, 1.5);

(8) The spaces between the parameters are unequal, for example (0, 0.5, 0.75, 0.85).

For $t = 5$, 28 different location parameter arrangements are considered, which include the following types:

(1) The parameters are equally spaced, for example (0, 0.2, 0.4, 0.6, 0.8);

(2) The space between the last two parameters is twice the space between the fourth and third parameters; while the space between the fourth and third parameters is twice the space between the third and second parameters, and the space between the third and second parameters is twice the space between the first two parameters, for example (0, 0.05, 0.15, 0.35, 0.75);

(3) The space between the last two parameters is triple the space between the fourth and third parameters; while the space between the fourth and third parameters is triple the space between the third and second parameters, and the space between the third and second parameters is triple the space between the first two parameters, for example (0, 0.025, 0.1, 0.325, 1);

(4) The spaces between the parameters are unequal, for example (0, 0.5, 0.7, 0.9, 1);

(5) The first four parameters are equal, while are different from the last one, for example (0, 0, 0, 0, 1);

(6) The first three parameters are equal, while are different from the others, for example (0, 0, 0, 0.5, 1);

(7) The middle three parameters are equal, while are different from the others, for example (0, 0.5, 0.5, 0.5, 1);

(8) The last two parameters are equal, while are different from the others, for example (0, 0.25, 0.5, 0.75, 0.75);

(9) The first two treatments are equal, while the last treatments are equal, for example (0, 0.5, 0.5, 1, 1);

(10) The four treatments are equal, while are different from the control, for example (0, 0.5, 0.5, 0.5, 0.5);

(11) Only one treatment is different from the control, while others are equal to the control, for example (0, 1, 0, 0, 0).

The results from the simulation study may be found in Chapter 5. The conclusions are given in Chapter 6.

CHAPTER 5. RESULTS

This chapter presents the results of the simulation study described in Chapter 4. The powers of the proposed test and the Durbin test are estimated and compared on a Balanced Incomplete Block Design data for a variety of distributions and treatment means.

Before estimating the powers, the two tests are checked to see if the significance levels hold. The significance level $\alpha = 0.05$ is used in the simulation study. In all the following 18 tables, t is the number of treatments, k is the number of subjects in each block, and r is the number of times each treatment appears.

In all cases considered, the estimated significance levels of the Durbin test range from 3.69% - 4.93%. The significance levels of the proposed test range from3.63% - 5.91%. Overall, the estimated significance levels of the two tests are around 5%, which is the stated significance level. The estimated powers of the tests could be compared.

5.1. For t = 3 and k = 2

Tables 3 - 5 give the results of simulation study for t = 3 and k = 2, under the normal distribution, exponential distribution, and t - distribution with three degrees of freedom, respectively. The Durbin test has higher estimated powers when one treatment mean is equal to the control mean; otherwise, the estimated powers of the proposed test are higher. The powers of the two tests are highest under the exponential distribution. The powers of proposed test are almost twice as high as the powers of the Durbin test when all the two treatment means are equal, but different from the control mean.

5.2. For t = 4 and k = 2

Tables 6 - 8 give the results of simulation study for t = 4 and k = 2, under the normal distribution, exponential distribution, and t - distribution with three degrees of freedom, respectively. The Durbin test has high powers when at least one treatment mean is equal to or close to the control mean; otherwise, the proposed test has higher powers. The powers of the two tests are highest under the exponential distribution. The powers of proposed test are almost twice as high as the powers of the Durbin test when all the three treatment means are equal, but different from the control mean.

5.3. For t = 4 and k = 3

Tables 9 - 11 give the results of simulation study for t = 4 and k = 3, under the normal distribution, exponential distribution, and t - distribution with three degrees of freedom, respectively. The Durbin test has higher powers when at least one treatment mean is equal to or close to the control mean; otherwise, the proposed test has higher powers. The powers of the two tests are highest under the exponential distribution. The powers of proposed test are almost twice as high as the powers of the Durbin test when all the three treatment means are equal, but different from the control mean.

5.4. For t = 5 and k = 2

Tables 12 - 14 give the results of simulation study for t = 5 and k = 2, under the normal distribution, exponential distribution, and t - distribution with three degrees of freedom, respectively. The Durbin test has higher powers when at least one treatment mean is equal to or close to the control mean; otherwise, the proposed test has higher powers. The powers of the two tests are highest under the exponential. The powers of proposed test are almost twice as high as the powers of the Durbin test when all the four treatment means are equal, but different from the control mean.

5.5. For t = 5 and k = 3

Tables 15 - 17 give the results of simulation study for t = 5 and k = 3, under the normal distribution, exponential distribution, and t - distribution with three degrees of freedom, respectively. The Durbin test has higher powers when at least one treatment mean is equal to or close to the control mean; otherwise, the proposed test has higher powers. The powers of the two tests are highest under the exponential distribution. The powers of proposed test are almost twice larger than the Durbin test when all the four treatment means are equal.

5.6. For t = 5 and k = 4

Tables 18 - 20 give the results of simulation study for t = 5 and k = 4, under the normal distribution, exponential distribution, and t - distribution with three degrees of freedom, respectively. The Durbin test has higher powers when at least one treatment mean is equal to or close to the control mean; otherwise, the proposed test has higher powers. The powers of the two tests are highest under the exponential distribution. The powers of proposed test are almost twice larger than the Durbin test when all the four treatment means are equal.

Table 3. Estimated Powers; t = 3, k = 2; r = 20; sigma = 1; Normal distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	Durbin (%)	T (%)
Type I error	0	0	0	4.14	5.39
Equal space	0	0.4	0.8	27.70	**46.73**
	0	0.5	1	41.26	**60.11**
	0	0.6	1.2	55.31	**72.50**
Double space	0	0.25	0.75	25.36	**36.31**
	0	0.3	0.9	34.97	**45.22**
	0	0.35	1.05	46.95	**54.54**
Triple space	0	0.2	0.8	29.55	**36.15**
	0	0.25	1	43.78	**46.87**
First 2 parameters are equal	0	0	1	**53.04**	34.37
	0.5	0.5	1.5	**52.46**	33.31
Last 2 parameters are equal	0	1	1	52.84	**81.89**
	0.5	1	1	15.68	**36.82**
Unequal space	0	0.4	1	41.96	**55.09**
	0	0.25	0.85	32.29	**40.86**
	0	0.8	1.1	52.18	**77.68**

Proposed Test: T = R1+2(R2+R3)

Table 4. Estimated Powers; t = 3, k = 2; r = 20; sigma = 1; Exponential distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	Durbin (%)	T (%)
Type I error	0	0	0	4.09	5.80
Equal space	0	0.4	0.8	49.58	**68.15**
	0	0.5	1	66.83	**80.53**
	0	0.6	1.2	79.87	**88.59**
Double space	0	0.25	0.75	46.90	**55.32**
	0	0.3	0.9	60.23	**65.73**
	0	0.35	1.05	71.68	**75.13**
Triple space	0	0.2	0.8	**54.08**	52.67
	0	0.25	1	**69.14**	66.53
First 2 parameters are equal	0	0	1	**74.23**	43.28
	0.5	0.5	1.5	**74.96**	44.30
Last 2 parameters are equal	0	1	1	75.20	**94.22**
	0.5	1	1	29.55	**59.23**
Unequal space	0	0.4	1	67.26	**75.85**
	0	0.25	0.85	55.59	**60.39**
	0	0.8	1.1	75.02	**92.58**

Proposed Test: T = R1+2(R2+R3)

Table 5. Estimated Powers; t = 3, k = 2; r = 20; t (3) distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	Durbin (%)	T (%)
Type I error	0	0	0	3.69	5.42
Equal space	0	0.4	0.8	18.78	35.56
	0	0.5	1	26.59	46.12
	0	0.6	1.2	37.89	57.93
Double space	0	0.25	0.75	17.86	28.27
	0	0.3	0.9	23.88	34.88
	0	0.35	1.05	32.18	42.83
Triple space	0	0.2	0.8	19.83	27.77
	0	0.25	1	29.32	35.50
First 2 parameters are equal	0	0	1	36.04	26.79
	0.5	0.5	1.5	35.64	26.34
Last 2 parameters are equal	0	1	1	36.12	66.02
	0.5	1	1	11.23	28.23
Unequal space	0	0.4	1	27.72	42.39
	0	0.25	0.85	22.49	32.06
	0	0.8	1.1	34.15	61.84

Proposed Test: T = R1+2(R2+R3)

Table 6. Estimated Powers; t = 4, k = 2; r = 15; sigma = 1; Normal distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	Durbin (%)	T (%)
Type I error	0	0	0	0	4.75	**5.88**
Equal space	0	0.4	0.8	1.2	38.32	**53.98**
	0	0.5	1	1.5	56.90	**69.45**
	0	0.6	1.2	1.8	73.02	**80.56**
Double space	0	0.2	0.6	1.4	**53.73**	45.91
	0	0.25	0.75	1.75	**73.67**	58.65
	0	0.3	0.9	2.1	**87.71**	69.58
Triple space	0	0.1	0.4	1.3	**73.71**	35.29
	0	0.15	0.6	1.95	**85.16**	54.13
	0	0.2	0.8	2.6	**97.92**	67.63
Last 3 treatments are equal	0	0.8	0.8	0.8	24.40	**55.90**
	0	1	1	1	36.80	**72.02**
	0.5	1.5	1.5	1.5	37.29	**72.30**
2 treatments are equal, others are different	0	0.5	1	1	34.94	**59.21**
	0	0.75	1.5	1.5	67.08	**85.56**
	0	0.75	0.75	1.5	52.53	**70.28**
	0	1	1	1.2	42.20	**75.97**
1 treatment is different from the control	0	0	0	1	**37.45**	18.26
	0	0	1.5	0	**69.83**	24.80
	0	2	0	0	**92.00**	30.88
First 2 parameters are equal, last 2 parameters are equal	0	0	1.5	1.5	**82.92**	64.94
	0	0	1	1	**47.72**	41.17
	0.5	0.5	2	2	**82.50**	65.04
Unequal space	0	0.5	0.75	0.85	22.62	**47.79**
	0	0.7	0.6	1	30.87	**53.11**
	0	0.8	1	1.3	44.69	**73.55**

Proposed Test: T = R1+3(R2+R3+R4)

Table 7. Estimated Powers; t = 4, k = 2; r = 15; sigma = 1; Exponential distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	Durbin (%)	T (%)
Type I error	0	0	0	0	4.48	5.91
Equal space	0	0.4	0.8	1.2	61.67	**73.31**
	0	0.5	1	1.5	78.09	**84.47**
	0	0.6	1.2	1.8	88.61	**91.72**
Double space	0	0.2	0.6	1.4	**73.44**	63.25
	0	0.25	0.75	1.75	**87.33**	73.56
	0	0.3	0.9	2.1	**94.84**	81.25
Triple space	0	0.1	0.4	1.3	**69.47**	48.85
	0	0.15	0.6	1.95	**92.18**	64.90
	0	0.2	0.8	2.6	**98.52**	77.98
Last 3 treatments are equal	0	0.8	0.8	0.8	41.72	**77.05**
	0	1	1	1	55.86	**86.94**
	0.5	1.5	1.5	1.5	55.83	**87.50**
Two treatments are equal, others are different	0	0.5	1	1	55.52	**76.55**
	0	0.75	1.5	1.5	82.63	**93.38**
	0	0.75	0.75	1.5	73.48	**85.36**
	0	1	1	1.2	62.56	**90.15**
1 treatment is different from the control	0	0	0	1	**55.08**	21.31
	0	0	1.5	0	**81.22**	26.50
	0	2	0	0	**93.54**	31.64
First 2 parameters are equal, last 2 parameters are equal	0	0	1.5	1.5	**91.66**	73.42
	0	0	1	1	**68.68**	56.08
	0.5	0.5	2	2	**91.62**	72.52
Unequal space	0	0.5	0.75	0.85	40.62	**68.91**
	0	0.7	0.6	1	46.88	**72.79**
	0	0.8	1	1.3	66.25	**88.30**

Proposed Test: T = R1+3(R2+R3+R4)

Table 8. Estimated Powers; t = 4, k = 2; r = 15; t (3) distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	Durbin (%)	T (%)
Type I error	0	0	0	0	4.93	5.89
Equal space	0	0.4	0.8	1.2	26.11	42.07
	0	0.5	1	1.5	38.60	53.96
	0	0.6	1.2	1.8	52.18	64.99
Double space	0	0.2	0.6	1.4	35.90	35.55
	0	0.25	0.75	1.75	51.23	45.91
	0	0.3	0.9	2.1	66.66	54.99
Triple space	0	0.1	0.4	1.3	33.42	27.41
	0	0.15	0.6	1.95	61.45	42.41
	0	0.2	0.8	2.6	83.19	55.49
Last 3 treatments are equal	0	0.8	0.8	0.8	16.72	42.99
	0	1	1	1	24.58	56.75
	0.5	1.5	1.5	1.5	24.54	56.38
Two treatments are equal, others are different	0	0.5	1	1	23.09	43.99
	0	0.75	1.5	1.5	45.56	68.93
	0	0.75	0.75	1.5	35.08	54.04
	0	1	1	1.2	28.01	59.42
1 treatment is different from the control	0	0	0	1	24.20	15.07
	0	0	1.5	0	49.12	19.79
	0	2	0	0	71.72	24.30
First 2 parameters are equal, last 2 parameters are equal	0	0	1.5	1.5	60.85	50.71
	0	0	1	1	33.28	32.41
	0.5	0.5	2	2	60.59	49.69
Unequal space	0	0.5	0.75	0.85	15.41	37.17
	0	0.7	0.6	1	18.37	40.21
	0	0.8	1	1.3	29.27	58.59

Proposed Test: T = R1+3(R2+R3+R4)

Table 9. Estimated Powers; t = 4, k = 3; r = 15; sigma = 1; Normal distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	Durbin (%)	T (%)
Type I error	0	0	0	0	4.24	4.01
Equal space	0	0.4	0.8	1.2	56.07	62.13
	0	0.5	1	1.5	77.42	**78.08**
	0	0.6	1.2	1.8	89.40	**90.79**
Double space	0	0.2	0.6	1.4	**73.82**	53.29
	0	0.25	0.75	1.75	**91.11**	68.52
	0	0.3	0.9	2.1	**98.11**	79.67
Triple space	0	0.1	0.4	1.3	**69.64**	38.79
	0	0.15	0.6	1.95	**96.66**	61.59
	0	0.2	0.8	2.6	**99.92**	77.46
Last 3 treatments are equal	0	0.8	0.8	0.8	35.56	**63.27**
	0	1	1	1	51.69	**79.89**
	0.5	1.5	1.5	1.5	53.31	**81.16**
2 treatments are equal, others are different	0	0.5	1	1	50.37	**67.16**
	0	0.75	1.5	1.5	86.40	**92.67**
	0	0.75	0.75	1.5	72.35	**78.76**
	0	1	1	1.2	60.46	**84.90**
1 treatment is different from the control	0	0	0	1	**53.59**	16.56
	0	0	1.5	0	**88.68**	25.02
	0	2	0	0	**98.97**	31.02
First 2 parameters are equal, last 2 parameters are equal	0	0	1.5	1.5	**96.31**	74.67
	0	0	1	1	**67.29**	46.37
	0.5	0.5	2	2	**96.45**	75.02
Unequal space	0	0.5	0.75	0.85	33.05	**53.73**
	0	0.7	0.6	1	38.74	**59.71**
	0	0.8	1	1.3	63.63	**83.35**

Proposed Test: T = R1+3(R2+R3+R4)

Table 10. Estimated Powers; t = 4, k = 3; r = 15; sigma = 1; Exponential distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	Durbin (%)	T (%)
Type I error	0	0	0	0	4.03	3.77
Equal space	0	0.4	0.8	1.2	82.28	81.05
	0	0.5	1	1.5	92.80	90.31
	0	0.6	1.2	1.8	98.03	95.40
Double space	0	0.2	0.6	1.4	92.47	70.50
	0	0.25	0.75	1.75	98.00	81.53
	0	0.3	0.9	2.1	99.68	89.10
Triple space	0	0.1	0.4	1.3	89.61	54.19
	0	0.15	0.6	1.95	99.38	74.90
	0	0.2	0.8	2.6	99.89	87.02
Last 3 treatments are equal	0	0.8	0.8	0.8	58.30	83.25
	0	1	1	1	73.53	92.15
	0.5	1.5	1.5	1.5	73.21	92.37
2 treatments are equal, others are different	0	0.5	1	1	75.19	84.37
	0	0.75	1.5	1.5	95.43	96.95
	0	0.75	0.75	1.5	90.76	90.80
	0	1	1	1.2	79.70	93.98
1 treatment is different from the control	0	0	0	1	78.46	22.50
	0	0	1.5	0	96.98	29.04
	0	2	0	0	97.80	33.83
First 2 parameters are equal, last 2 parameters are equal	0	0	1.5	1.5	98.91	81.67
	0	0	1	1	87.78	62.83
	0.5	0.5	2	2	98.90	82.42
Unequal space	0	0.5	0.75	0.85	56.98	75.45
	0	0.7	0.6	1	65.38	80.60
	0	0.8	1	1.3	83.61	92.49

Proposed Test: T = R1+3(R2+R3+R4)

Table 11. Estimated Powers; t = 4, k = 3; r = 15; t (3) distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	Durbin (%)	T (%)
Type I error	0	0	0	0	4.40	3.62
Equal space	0	0.4	0.8	1.2	38.65	46.71
	0	0.5	1	1.5	54.91	61.42
	0	0.6	1.2	1.8	70.81	73.44
Double space	0	0.2	0.6	1.4	51.29	38.24
	0	0.25	0.75	1.75	71.31	51.51
	0	0.3	0.9	2.1	84.56	62.97
Triple space	0	0.1	0.4	1.3	48.42	28.40
	0	0.15	0.6	1.95	81.13	46.61
	0	0.2	0.8	2.6	95.81	61.57
Last 3 treatments are equal	0	0.8	0.8	0.8	24.41	48.66
	0	1	1	1	35.06	63.20
	0.5	1.5	1.5	1.5	34.94	63.23
2 treatments are equal, others are different	0	0.5	1	1	33.04	49.82
	0	0.75	1.5	1.5	63.82	78.20
	0	0.75	0.75	1.5	49.74	61.29
	0	1	1	1.2	40.72	67.83
1 treatment is different from the control	0	0	0	1	35.53	13.53
	0	0	1.5	0	66.79	19.17
	0	2	0	0	88.65	24.69
First 2 parameters are equal, last 2 parameters are equal	0	0	1.5	1.5	81.55	57.17
	0	0	1	1	46.95	34.31
	0.5	0.5	2	2	81.21	56.86
Unequal space	0	0.5	0.75	0.85	21.49	39.24
	0	0.7	0.6	1	25.61	43.40
	0	0.8	1	1.3	42.15	64.91

Proposed Test: T = R1+3(R2+R3+R4)

Table 12. Estimated Powers; t = 5, k = 2; r = 32; sigma = 1; Normal distribution

Location Parameter	$\mu1$	$\mu2$	$\mu3$	$\mu4$	$\mu5$	Durbin (%)	T (%)
Type I error	0	0	0	0	0	4.27	**5.64**
Equal space	0	0.2	0.4	0.6	0.8	36.40	**48.75**
	0	0.25	0.5	0.75	1	55.07	**62.91**
	0	0.3	0.6	0.9	1.2	73.26	**76.31**
Double space	0	0.05	0.15	0.35	0.75	**34.13**	27.96
	0	0.07	0.21	0.49	1.05	**92.09**	62.39
	0	0.1	0.3	0.7	1.5	**61.79**	42.07
Triple space	0	0.025	0.1	0.325	1	**59.48**	30.54
	0	0.04	0.16	0.52	1.6	**96.00**	52.07
	0	0.05	0.2	0.65	2	**99.73**	64.70
Unequal space	0	0.5	0.7	0.9	1	55.15	**79.54**
	0	0.8	0.6	0.7	1	49.67	**79.85**
	0	1	0.6	0.7	0.8	49.81	**80.02**
First 4 parameters are equal	0	0	0	0	1	**65.08**	18.36
	0.5	0.5	0.5	0.5	1.5	**65.49**	19.20
First 3 parameters are equal	0	0	0	0.5	1	**67.31**	32.08
	0	0	0	0.75	1.5	**96.53**	50.11
	0	0	0	1	1	**85.77**	45.72
Middle 3 treatments are equal	0	0.5	0.5	0.5	1	44.34	**63.51**
	0	0.75	0.75	0.75	1.5	82.26	**90.30**
Last 2 treatments are equal, others are different	0	0.25	0.5	0.75	0.75	37.67	**56.80**
	0	0.5	0.7	1.2	1.2	78.80	**88.49**
First 2 treatments are equal, last 2 treatments are equal	0	0.5	0.5	1	1	59.71	**76.49**
	0	0.6	0.6	1.3	1.3	85.67	**90.23**
4 treatments are equal	0	1	1	1	1	65.61	**94.27**
	0	0.5	0.5	0.5	0.5	18.42	**49.28**
1 treatment is different from the control	0	0	0	0	1	**66.36**	19.38
	0	1	0	0	0	**65.71**	19.21

Proposed Test: T = R1+4(R2+R3+R4+R5)

Table 13. Estimated Powers; t = 5, k = 2; r = 32; sigma = 1; Exponential distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	$\mu 5$	Durbin (%)	T (%)
Type I error	0	0	0	0	0	4.41	5.75
Equal space	0	0.2	0.4	0.6	0.8	69.06	**73.25**
	0	0.25	0.5	0.75	1	85.18	**85.43**
	0	0.3	0.6	0.9	1.2	92.38	**94.37**
Double space	0	0.05	0.15	0.35	0.75	**64.16**	44.83
	0	0.07	0.21	0.49	1.05	**98.95**	79.91
	0	0.1	0.3	0.7	1.5	**88.53**	60.82
Triple space	0	0.025	0.1	0.325	1	**85.37**	45.03
	0	0.04	0.16	0.52	1.6	**99.33**	67.26
	0	0.05	0.2	0.65	2	**99.95**	76.74
Unequal space	0	0.5	0.7	0.9	1	84.47	**94.59**
	0	0.8	0.6	0.7	1	79.78	**94.94**
	0	1	0.6	0.7	0.8	78.97	**95.01**
First 4 parameters are equal	0	0	0	0	1	**86.92**	23.32
	0.5	0.5	0.5	0.5	1.5	**86.72**	22.88
First 3 parameters are equal	0	0	0	0.5	1	**90.78**	44.26
	0	0	0	0.75	1.5	**99.59**	61.47
	0	0	0	1	1	**97.96**	59.75
Middle 3 treatments are equal	0	0.5	0.5	0.5	1	75.17	**86.46**
	0	0.75	0.75	0.75	1.5	96.95	**98.12**
Last 2 treatments are equal, others are different	0	0.25	0.5	0.75	0.75	70.31	**80.34**
	0	0.5	0.7	1.2	1.2	96.31	**97.55**
First 2 treatments are equal, last 2 treatments are equal	0	0.5	0.5	1	1	88.60	**93.67**
	0	0.6	0.6	1.3	1.3	97.89	**98.05**
4 treatments are equal	0	1	1	1	1	87.36	**99.20**
	0	0.5	0.5	0.5	0.5	37.72	**76.62**
1 treatment is different from the control	0	0	0	0	1	**86.70**	22.68
	0	1	0	0	0	**87.35**	23.02

Proposed Test: T = R1+4(R2+R3+R4+R5)

Table 14. Estimated Powers; t = 5, k = 2; r = 32; t (3) distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	$\mu 5$	Durbin (%)	T (%)
Type I error	0	0	0	0	0	4.29	5.51
Equal space	0	0.2	0.4	0.6	0.8	23.83	**37.02**
	0	0.25	0.5	0.75	1	36.66	**48.39**
	0	0.3	0.6	0.9	1.2	52.48	**59.79**
Double space	0	0.05	0.15	0.35	0.75	**22.73**	21.94
	0	0.07	0.21	0.49	1.05	**72.62**	48.44
	0	0.1	0.3	0.7	1.5	**41.59**	31.92
Triple space	0	0.025	0.1	0.325	1	**39.26**	23.56
	0	0.04	0.16	0.52	1.6	**80.39**	39.65
	0	0.05	0.2	0.65	2	**94.40**	49.65
Unequal space	0	0.5	0.7	0.9	1	36.97	**64.03**
	0	0.8	0.6	0.7	1	32.61	**64.22**
	0	1	0.6	0.7	0.8	34.06	**65.27**
First 4 parameters are equal	0	0	0	0	1	**44.49**	15.13
	0.5	0.5	0.5	0.5	1.5	**44.65**	15.64
First 3 parameters are equal	0	0	0	0.5	1	**44.99**	24.18
	0	0	0	0.75	1.5	**82.72**	39.01
	0	0	0	1	1	**65.68**	35.54
Middle 3 treatments are equal	0	0.5	0.5	0.5	1	29.19	**48.87**
	0	0.75	0.75	0.75	1.5	60.40	**76.75**
Last 2 treatments are equal, others are different	0	0.25	0.5	0.75	0.75	25.75	**43.49**
	0	0.5	0.7	1.2	1.2	56.86	**73.66**
First 2 treatments are equal, last 2 treatments are equal	0	0.5	0.5	1	1	40.37	**60.69**
	0	0.6	0.6	1.3	1.3	64.79	**76.78**
4 treatments are equal	0	1	1	1	1	44.83	**81.30**
	0	0.5	0.5	0.5	0.5	13.51	**37.75**
1 treatment is different from the control	0	0	0	0	1	**43.56**	15.36
	0	1	0	0	0	**44.30**	15.92

Proposed Test: T = R1+4(R2+R3+R4+R5)

Table 15. Estimated Powers; t = 5, k = 3; r = 18; sigma = 1; Normal distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	$\mu 5$	Durbin (%)	T (%)
Type I error	0	0	0	0	0	4.46	**5.71**
Equal space	0	0.2	0.4	0.6	0.8	29.58	**44.04**
	0	0.25	0.5	0.75	1	46.55	**57.98**
	0	0.3	0.6	0.9	1.2	64.42	**71.13**
Double space	0	0.05	0.15	0.35	0.75	**27.69**	25.40
	0	0.07	0.21	0.49	1.05	**52.80**	36.95
	0	0.1	0.3	0.7	1.5	**68.74**	56.24
Triple space	0	0.025	0.1	0.325	1	**50.43**	27.14
	0	0.04	0.16	0.52	1.6	**91.88**	46.94
	0	0.05	0.2	0.65	2	**98.80**	56.61
Unequal space	0	0.5	0.7	0.9	1	46.41	**72.65**
	0	0.8	0.6	0.7	1	41.38	**73.35**
	0	1	0.6	0.7	0.8	43.52	**71.36**
First 4 parameters are equal	0	0	0	0	1	**56.68**	17.78
	0.5	0.5	0.5	0.5	1.5	**56.75**	17.04
First 3 parameters are equal	0	0	0	0.5	1	**57.62**	28.92
	0	0	0	0.75	1.5	**92.40**	44.22
	0	0	0	1	1	**78.60**	40.92
Middle 3 treatments are equal	0	0.5	0.5	0.5	1	37.49	**58.01**
	0	0.75	0.75	0.75	1.5	75.07	**85.68**
Last 2 treatments are equal, others are different	0	0.25	0.5	0.75	0.75	32.83	**51.58**
	0	0.5	0.7	1.2	1.2	70.41	**82.73**
First 2 treatments are equal, last 2 treatments are equal	0	0.5	0.5	1	1	51.76	**71.11**
	0	0.6	0.6	1.3	1.3	78.14	**86.38**
4 treatments are equal	0	1	1	1	1	56.07	**89.47**
	0	0.5	0.5	0.5	0.5	15.49	**44.84**
1 treatment is different from the control	0	0	0	0	1	**57.35**	17.15
	0	1	0	0	0	**56.22**	16.48

Proposed Test: T = R1+4(R2+R3+R4+R5)

Table 16. Estimated Powers; t = 5, k = 3; r = 18; sigma = 1; Exponential distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	$\mu 5$	Durbin (%)	T (%)
Type I error	0	0	0	0	0	4.51	5.63
Equal space	0	0.2	0.4	0.6	0.8	59.70	**66.42**
	0	0.25	0.5	0.75	1	77.66	**77.82**
	0	0.3	0.6	0.9	1.2	86.65	**89.03**
Double space	0	0.05	0.15	0.35	0.75	**54.69**	39.31
	0	0.07	0.21	0.49	1.05	**82.57**	55.31
	0	0.1	0.3	0.7	1.5	**97.89**	73.76
Triple space	0	0.025	0.1	0.325	1	**79.19**	40.12
	0	0.04	0.16	0.52	1.6	**98.88**	59.84
	0	0.05	0.2	0.65	2	**99.96**	70.39
Unequal space	0	0.5	0.7	0.9	1	75.61	**89.90**
	0	0.8	0.6	0.7	1	70.69	**90.17**
	0	1	0.6	0.7	0.8	69.66	**90.06**
First 4 parameters are equal	0	0	0	0	1	**81.42**	21.90
	0.5	0.5	0.5	0.5	1.5	**81.91**	20.77
First 3 parameters are equal	0	0	0	0.5	1	**85.32**	39.66
	0	0	0	0.75	1.5	**99.06**	55.12
	0	0	0	1	1	**95.35**	53.20
Middle 3 treatments are equal	0	0.5	0.5	0.5	1	67.40	**78.88**
	0	0.75	0.75	0.75	1.5	93.10	**94.88**
Last 2 treatments are equal, others are different	0	0.25	0.5	0.75	0.75	60.66	**73.29**
	0	0.5	0.7	1.2	1.2	91.31	**93.60**
First 2 treatments are equal, last 2 treatments are equal	0	0.5	0.5	1	1	79.87	**88.10**
	0	0.6	0.6	1.3	1.3	94.31	**95.15**
4 treatments are equal	0	1	1	1	1	75.95	**96.78**
	0	0.5	0.5	0.5	0.5	32.31	**68.51**
1 treatment is different from the control	0	0	0	0	1	**82.04**	20.99
	0	1	0	0	0	**81.66**	21.79

Proposed Test: T = R1+4(R2+R3+R4+R5)

Table 17. Estimated Powers; t = 5, k = 3; r = 18; t (3) distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	$\mu 5$	Durbin (%)	T (%)
Type I error	0	0	0	0	0	4.35	5.70
Equal space	0	0.2	0.4	0.6	0.8	20.20	**33.71**
	0	0.25	0.5	0.75	1	29.79	**43.54**
	0	0.3	0.6	0.9	1.2	43.15	**54.57**
Double space	0	0.05	0.15	0.35	0.75	**20.32**	19.57
	0	0.07	0.21	0.49	1.05	**63.93**	42.80
	0	0.1	0.3	0.7	1.5	**35.14**	29.15
Triple space	0	0.025	0.1	0.325	1	**33.77**	21.54
	0	0.04	0.16	0.52	1.6	**70.91**	35.20
	0	0.05	0.2	0.65	2	**88.73**	45.11
Unequal space	0	0.5	0.7	0.9	1	31.69	**58.57**
	0	0.8	0.6	0.7	1	27.90	**57.93**
	0	1	0.6	0.7	0.8	27.84	**58.65**
First 4 parameters are equal	0	0	0	0	1	**38.02**	14.36
	0.5	0.5	0.5	0.5	1.5	**37.58**	14.27
First 3 parameters are equal	0	0	0	0.5	1	**37.77**	22.44
	0	0	0	0.75	1.5	**73.19**	34.70
	0	0	0	1	1	**56.61**	31.23
Middle 3 treatments are equal	0	0.5	0.5	0.5	1	24.90	**44.51**
	0	0.75	0.75	0.75	1.5	51.07	**70.17**
Last 2 treatments are equal, others are different	0	0.25	0.5	0.75	0.75	21.81	**39.17**
	0	0.5	0.7	1.2	1.2	48.61	**68.20**
First 2 treatments are equal, last 2 treatments are equal	0	0.5	0.5	1	1	33.70	**55.14**
	0	0.6	0.6	1.3	1.3	55.83	**71.41**
4 treatments are equal	0	1	1	1	1	37.38	**76.17**
	0	0.5	0.5	0.5	0.5	11.55	**34.18**
1 treatment is different from the control	0	0	0	0	1	**38.57**	14.28
	0	1	0	0	0	**37.86**	14.37

Proposed Test: T = R1+4(R2+R3+R4+R5)

Table 18. Estimated Powers; t = 5, k = 4; r = 12; sigma = 1; Normal distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	$\mu 5$	Durbin (%)	T (%)
Type I error	0	0	0	0	0	4.48	4.47
Equal space	0	0.2	0.4	0.6	0.8	24.27	34.87
	0	0.25	0.5	0.75	1	37.02	46.52
	0	0.3	0.6	0.9	1.2	52.01	58.26
Double space	0	0.05	0.15	0.35	0.75	22.47	19.38
	0	0.07	0.21	0.49	1.05	75.19	43.99
	0	0.1	0.3	0.7	1.5	42.66	28.68
Triple space	0	0.025	0.1	0.325	1	39.93	20.92
	0	0.04	0.16	0.52	1.6	82.84	35.87
	0	0.05	0.2	0.65	2	96.21	45.51
Unequal space	0	0.5	0.7	0.9	1	35.72	62.03
	0	0.8	0.6	0.7	1	33.67	61.59
	0	1	0.6	0.7	0.8	33.06	62.28
First 4 parameters are equal	0	0	0	0	1	45.40	13.28
	0.5	0.5	0.5	0.5	1.5	45.36	13.20
First 3 parameters are equal	0	0	0	0.5	1	46.58	21.71
	0	0	0	0.75	1.5	84.17	34.64
	0	0	0	1	1	66.55	31.41
Middle 3 treatments are equal	0	0.5	0.5	0.5	1	28.79	45.95
	0	0.75	0.75	0.75	1.5	60.78	74.51
Last 2 treatments are equal, others are different	0	0.25	0.5	0.75	0.75	24.98	39.75
	0	0.5	0.7	1.2	1.2	58.36	71.87
First 2 treatments are equal, last 2 treatments are equal	0	0.5	0.5	1	1	41.00	58.96
	0	0.6	0.6	1.3	1.3	67.72	76.61
4 treatments are equal	0	1	1	1	1	45.47	81.33
	0	0.5	0.5	0.5	0.5	12.84	35.25
1 treatment is different from the control	0	0	0	0	1	45.49	13.09
	0	1	0	0	0	45.99	12.55

Proposed Test: T = R1+4(R2+R3+R4+R5)

Table 19. Estimated Powers; t = 5, k = 4; r = 12; sigma = 1; Exponential distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	$\mu 5$	Durbin (%)	T (%)
Type I error	0	0	0	0	0	4.45	5.05
Equal space	0	0.2	0.4	0.6	0.8	47.57	**54.68**
	0	0.25	0.5	0.75	1	64.97	**66.27**
	0	0.3	0.6	0.9	1.2	76.44	**79.74**
Double space	0	0.05	0.15	0.35	0.75	**44.43**	31.89
	0	0.07	0.21	0.49	1.05	**93.74**	60.51
	0	0.1	0.3	0.7	1.5	**70.99**	44.68
Triple space	0	0.025	0.1	0.325	1	**67.06**	32.30
	0	0.04	0.16	0.52	1.6	**95.96**	49.28
	0	0.05	0.2	0.65	2	**99.48**	57.52
Unequal space	0	0.5	0.7	0.9	1	62.51	**80.02**
	0	0.8	0.6	0.7	1	57.88	**80.32**
	0	1	0.6	0.7	0.8	57.86	**80.52**
First 4 parameters are equal	0	0	0	0	1	**70.11**	15.97
	0.5	0.5	0.5	0.5	1.5	**70.70**	16.14
First 3 parameters are equal	0	0	0	0.5	1	**74.13**	31.74
	0	0	0	0.75	1.5	**96.72**	44.02
	0	0	0	1	1	**88.86**	41.25
Middle 3 treatments are equal	0	0.5	0.5	0.5	1	54.08	**68.13**
	0	0.75	0.75	0.75	1.5	84.51	**87.55**
Last 2 treatments are equal, others are different	0	0.25	0.5	0.75	0.75	49.80	**61.02**
	0	0.5	0.7	1.2	1.2	82.77	**86.10**
First 2 treatments are equal, last 2 treatments are equal	0	0.5	0.5	1	1	68.83	**77.39**
	0	0.6	0.6	1.3	1.3	88.03	**88.05**
4 treatments are equal	0	1	1	1	1	64.92	**91.16**
	0	0.5	0.5	0.5	0.5	25.41	**58.04**
1 treatment is different from the control	0	0	0	0	1	**71.03**	16.30
	0	1	0	0	0	**70.42**	17.26

Proposed Test: T = R1+4(R2+R3+R4+R5)

Table 20. Estimated Powers; t = 5, k = 4; r = 12; t (3) distribution

Location Parameter	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	$\mu 5$	Durbin (%)	T (%)
Type I error	0	0	0	0	0	4.82	4.44
Equal space	0	0.2	0.4	0.6	0.8	17.17	26.21
	0	0.25	0.5	0.75	1	23.18	34.15
	0	0.3	0.6	0.9	1.2	34.33	44.24
Double space	0	0.05	0.15	0.35	0.75	15.94	15.44
	0	0.07	0.21	0.49	1.05	52.91	32.52
	0	0.1	0.3	0.7	1.5	27.53	22.50
Triple space	0	0.025	0.1	0.325	1	26.48	16.70
	0	0.04	0.16	0.52	1.6	60.09	27.16
	0	0.05	0.2	0.65	2	79.18	35.84
Unequal space	0	0.5	0.7	0.9	1	25.13	47.36
	0	0.8	0.6	0.7	1	21.24	45.76
	0	1	0.6	0.7	0.8	20.99	46.65
First 4 parameters are equal	0	0	0	0	1	29.46	10.87
	0.5	0.5	0.5	0.5	1.5	28.89	11.19
First 3 parameters are equal	0	0	0	0.5	1	30.12	17.48
	0	0	0	0.75	1.5	61.40	26.49
	0	0	0	1	1	45.25	24.37
Middle 3 treatments are equal	0	0.5	0.5	0.5	1	19.87	34.57
	0	0.75	0.75	0.75	1.5	41.99	59.38
Last 2 treatments are equal, others are different	0	0.25	0.5	0.75	0.75	17.46	30.32
	0	0.5	0.7	1.2	1.2	38.44	55.65
First 2 treatments are equal, last 2 treatments are equal	0	0.5	0.5	1	1	26.67	44.07
	0	0.6	0.6	1.3	1.3	45.01	59.02
4 treatments are equal	0	1	1	1	1	29.78	65.42
	0	0.5	0.5	0.5	0.5	9.78	26.40
1 treatment is different from the control	0	0	0	0	1	28.79	11.45
	0	1	0	0	0	29.34	11.32

Proposed Test: T = R1+4(R2+R3+R4+R5)

CHAPTER 6. CONCLUSIONS

This paper is aimed to compare the powers of the new proposed test and the Durbin test (Durbin, 1951) for the simple tree alternative in the Balanced Incomplete Block Design under three distributions – normal distribution, exponential distribution, and t (3) distribution.

The powers of the two tests are not changed if we change the order of the treatment mean. From the results of simulation study, the significance levels of the Durbin test range from 3.69% to 4.93%, and the estimated significance levels of the proposed test range from 3.62% to 5.91%. The estimated powers of the tests could be compared.

The tests are compared on the basis of estimated power. The test with the higher power is considered the best test in that situation. From the results of simulation study, overall, the Durbin test is better when at least one treatment mean is close to or equal to the control mean; otherwise, the proposed test is better. This is true for all population distributions studied. It is also for $t = 3, 4$ or 5. Generally, the powers of the two tests are highest under exponential distribution than normal distribution and t - distribution with three degrees of freedom.

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APPENDIX. SAS CODE FOR THE SIMULATION STUDY

```
/*Normal distribution t = 3, k = 2*/
%macro generate(sim,rep,r,mu1,mu2,mu3,sigma);

%let t=3;
%let k=2;
data raw(keep=sim rep block y1-y3);
  array seeds{3}seed1-seed3;
  do i=1 to 3;
  seeds{i}=int(ranuni(0)*1e6);
  end;
  put seed1-seed3;
do sim=1 to &sim;
    do rep=1 to &rep;
       do block=1 to 3;
              call rannor(seed1,ya);
                  y1=(&mu1)+&sigma*ya;
              call rannor(seed2,yb);
                  y2=(&mu2)+&sigma*yb;
      call rannor(seed3,yc);
                  y3=(&mu3)+&sigma*yc;
     if block=1 then do; y1=.; end;
       else if block=2 then do; y2=.; end;
         else if block=3 then do; y3=.; end;
      output;
      end;
    end;
 end;
run;

/*Durbin Test*/
data one(drop=cctr i);
  set raw end=eof;
  by sim;
    array orgs{3}y1 y2 y3;
    array ab{2}A B;
    array trt{2}Ai Bi;
    array r{3}r1-r3;
    array sumr{3}sumr1-sumr3;
    array sumrsq{3}sumrsq1-sumrsq3;
      cctr=0;
      do i=1 to 3;
        if orgs{i}>. then do;
         cctr+1;
         ab{cctr}=orgs{i};
```

```
      trt{cctr}=i;
    end;
  end;
  if (A<B) then do; r{Ai}=1;r{Bi}=2;end;
  else if (A>B) then do; r{Ai}=2; r{Bi}=1; end;
  do i=1 to 3;
   if first.sim then do;
    sumr{i}=0;
    sumrsq{i}=0;
    end;
   sumr{i}+r{i};
   if last.sim then do;
    sumrsq{i}=sumr{i}**2;
    end;
  end;
  totRsqsum=sum(of sumrsq:);
  if last.sim then do;
   durbin=(12*(&t-1)*totRsqsum/(&r*&t*(&k-1)*(&k+1)))-(3*&r*(&t-1)*(&k+1))/(&k-
1);
     if durbin>5.991 then pow_durbin+1;
       output;
       end;
  if eof;
  run;

  proc print;
  var durbin pow_durbin;
  run;

  /*Proposed Test*/
  data two;
  set one;
   T=((sumr1+2*(sumr2+sumr3))-15*(&r/2))/sqrt(0.5*(&r/2));
  if T>1.645 then pow_T+1;
  run;

  proc print;
  var T pow_T;
  run;

  %mend generate;
  %generate (10000,10,20,0,0,0,1);
  %generate (10000,10,20,0,0.4,0.8,1);
  %generate (10000,10,20,0,0.5,1,1);
  %generate (10000,10,20,0,0.6,1.2,1);
  %generate (10000,10,20,0,0.25,0.75,1);
```

```
%generate (10000.10.20.0.0.3.0.9.1):
%generate (10000.10.20.0.0.35.1.05.1);
%generate (10000.10.20.0.0.2.0.8.1):
%generate (10000.10.20.0.0.25.1.1):
%generate (10000.10.20.0.0.1.1):
%generate (10000.10.20.0.5.0.5.1.5.1):
%generate (10000.10.20.0.1.1.1):
%generate (10000.10.20.0.5.1.1.1):
%generate (10000.10.20.0.0.4.1.1):
%generate (10000.10.20.0.0.25.0.85.1);
%generate (10000.10.20.0.0.8.1.1.1):
```

```
/*Exponential distribution t = 3, k = 2*/
%macro generate(sim,rep,r,mu1,mu2,mu3,sigma);

%let t=3;
%let k=2;
data raw(keep=sim rep block y1-y3);
  array seeds{3}seed1-seed3;
  do i=1 to 3;
  seeds{i}=int(ranuni(0)*1e6);
  end;
  put seed1-seed3;
do sim=1 to &sim;
    do rep=1 to &rep;
       do block=1 to 3;
              call ranexp(seed1,ya);
                  y1=(&mu1)+&sigma*ya;
              call ranexp(seed2,yb);
                  y2=(&mu2)+&sigma*yb;
      call ranexp(seed3,yc);
                  y3=(&mu3)+&sigma*yc;
     if block=1 then do; y1=.; end;
       else if block=2 then do; y2=.; end;
         else if block=3 then do; y3=.; end;
      output;
      end;
    end;
 end;
run;

/*Durbin Test*/
data one(drop=cctr i);
 set raw end=eof;
 by sim;
   array orgs{3}y1 y2 y3;
   array ab{2}A B;
   array trt{2}Ai Bi;
   array r{3}r1-r3;
   array sumr{3}sumr1-sumr3;
   array sumrsq{3}sumrsq1-sumrsq3;
     cctr=0;
     do i=1 to 3;
       if orgs{i}>. then do;
        cctr+1;
        ab{cctr}=orgs{i};
        trt{cctr}=i;
     end;
```

```
end;
if (A<B) then do; r{Ai}=1;r{Bi}=2;end;
else if (A>B) then do; r{Ai}=2; r{Bi}=1; end;
do i=1 to 3;
 if first.sim then do;
  sumr{i}=0;
  sumrsq{i}=0;
  end;
 sumr{i}+r{i};
 if last.sim then do;
  sumrsq{i}=sumr{i}**2;
  end;
end;
totRsqsum=sum(of sumrsq:);
if last.sim then do;
 durbin=(12*(&t-1)*totRsqsum/(&r*&t*(&k-1)*(&k+1)))-(3*&r*(&t-1)*(&k+1))/(&k-
1);
   if durbin>5.991 then pow_durbin+1;
     output;
     end;
if eof;
run;

proc print;
var durbin pow_durbin;
run;

/*Proposed Test*/
data two;
set one;
 T=((sumr1+2*(sumr2+sumr3))-15*(&r/2))/sqrt(0.5*(&r/2));
if T>1.645 then pow_T+1;
run;

proc print;
var T pow_T;
run;

%mend generate;
%generate (10000,10,20,0,0,0,1);
%generate (10000,10,20,0,0.4,0.8,1);
%generate (10000,10,20,0,0.5,1,1);
%generate (10000,10,20,0,0.6,1.2,1);
%generate (10000,10,20,0,0.25,0.75,1);
%generate (10000,10,20,0,0.3,0.9,1);
%generate (10000,10,20,0,0.35,1.05,1);
```

```
%generate (10000,10,20,0,0.2,0.8,1);
%generate (10000,10,20,0,0.25,1,1);
%generate (10000,10,20,0,0,1,1);
%generate (10000,10,20,0.5,0.5,1.5,1);
%generate (10000,10,20,0,1,1,1);
%generate (10000,10,20,0.5,1,1,1);
%generate (10000,10,20,0,0.4,1,1);
%generate (10000,10,20,0,0.25,0.85,1);
%generate (10000,10,20,0,0.8,1,1,1);
```

```
/*t (3) distribution t = 3, k = 2*/
%macro generate(sim,rep,r,mu1,mu2,mu3,sigma);

%let t=3;
%let k=2;
data raw(keep=sim rep block y1-y3);
  array seeds{3}seed1-seed3;
  do i=1 to 3;
  seeds{i}=int(ranuni(0)*1e6);
  end;
  put seed1-seed3;
do sim=1 to &sim;
   do rep=1 to &rep;
      do block=1 to 3;
            call ranuni(seed1,ya);
    ya=tinv(ya,3);      y1=(&mu1)+&sigma*ya;
            call ranuni(seed2,yb);
    yb=tinv(yb,3);      y2=(&mu2)+&sigma*yb;
      call ranuni(seed3,yc);
    yc=tinv(yc,3);      y3=(&mu3)+&sigma*yc;
    if block=1 then do; y1=.; end;
      else if block=2 then do; y2=.; end;
        else if block=3 then do; y3=.; end;
      output;
      end;
   end;
 end;
run;

/*Durbin Test*/
data one(drop=cctr i);
 set raw end=eof;
 by sim;
  array orgs{3}y1 y2 y3;
  array ab{2}A B;
  array trt{2}Ai Bi;
  array r{3}r1-r3;
  array sumr{3}sumr1-sumr3;
  array sumrsq{3}sumrsq1-sumrsq3;
   cctr=0;
   do i=1 to 3;
     if orgs{i}>. then do;
      cctr+1;
      ab{cctr}=orgs{i};
      trt{cctr}=i;
   end;
```

```
end;
if (A<B) then do; r{Ai}=1;r{Bi}=2;end;
else if (A>B) then do; r{Ai}=2; r{Bi}=1; end;
do i=1 to 3;
 if first.sim then do;
  sumr{i}=0;
  sumrsq{i}=0;
  end;
 sumr{i}+r{i};
 if last.sim then do;
  sumrsq{i}=sumr{i}**2;
  end;
end;
totRsqsum=sum(of sumrsq:);
if last.sim then do;
 durbin=(12*(&t-1)*totRsqsum/(&r*&t*(&k-1)*(&k+1)))-(3*&r*(&t-1)*(&k+1))/(&k-
1);
   if durbin>5.991 then pow_durbin+1;
      output;
      end;
if eof;
run;

proc print;
var durbin pow_durbin;
run;

/*Proposed Test*/
data two;
set one;
 T=((sumr1+2*(sumr2+sumr3))-15*(&r/2))/sqrt(0.5*(&r/2));
if T>1.645 then pow_T+1;
run;

proc print;
var T pow_T;
run;

%mend generate;
%generate (10000,10,20,0,0,0,1);
%generate (10000,10,20,0,0.4,0.8,1);
%generate (10000,10,20,0,0.5,1,1);
%generate (10000,10,20,0,0.6,1.2,1);
%generate (10000,10,20,0,0.25,0.75,1);
%generate (10000,10,20,0,0.3,0.9,1);
%generate (10000,10,20,0,0.35,1.05,1);
```

```
%generate (10000.10.20.0.0.2.0.8.1);
%generate (10000.10.20.0.0.25.1.1);
%generate (10000.10.20.0.0.1.1);
%generate (10000.10.20.0.5.0.5.1.5.1);
%generate (10000.10.20.0.1.1.1);
%generate (10000.10.20.0.5.1.1.1);
%generate (10000.10.20.0.0.4.1.1);
%generate (10000.10.20.0.0.25.0.85.1);
%generate (10000.10.20.0.0.8.1.1.1);
```

```
/*Normal distribution t = 4, k = 2*/
%macro generate(sim,rep,r,mu1,mu2,mu3,mu4,sigma);

%let t=4;
%let k=2;
data raw(keep=sim rep block y1-y4);
  array seeds{4}seed1-seed4;
  do i=1 to 4;
  seeds{i}=int(ranuni(0)*1e6);
  end;
  put seed1-seed4;
do sim=1 to &sim;
   do rep=1 to &rep;
      do block=1 to 6;
            call rannor(seed1,ya);
               y1=(&mu1)+&sigma*ya;
            call rannor(seed2,yb);
               y2=(&mu2)+&sigma*yb;
      call rannor(seed3,yc);
               y3=(&mu3)+&sigma*yc;
            call rannor(seed4,yd);
               y4=(&mu4)+&sigma*yd;
     if block=1 then do; y3=.; y4=.; end;
       else if block=2 then do; y2=.; y4=.; end;
         else if block=3 then do; y2=.; y3=.; end;
                else if block=4 then do; y1=.; y4=.; end;
                else if block=5 then do; y1=.; y3=.; end;
                else if block=6 then do; y1=.; y2=.; end;
      output;
      end;
   end;
end;
run;

/*Durbin Test*/
data one(drop=cctr i);
 set raw end=eof;
 by sim;
  array orgs{4}y1 y2 y3 y4;
  array ab{2}A B;
  array trt{2}Ai Bi;
  array r{4}r1-r4;
  array sumr{4}sumr1-sumr4;
  array sumrsq{4}sumrsq1-sumrsq4;
   cctr=0;
   do i=1 to 4;
```

```
        if orgs{i}>. then do;
         cctr+1;
         ab{cctr}=orgs{i};
         trt{cctr}=i;
      end;
   end;
   if (A<B) then do; r{Ai}=1;r{Bi}=2;end;
   else if (A>B) then do; r{Ai}=2; r{Bi}=1; end;
   do i=1 to 4;
    if first.sim then do;
     sumr{i}=0;
     sumrsq{i}=0;
     end;
    sumr{i}+r{i};
    if last.sim then do;
     sumrsq{i}=sumr{i}**2;
     end;
   end;
   totRsqsum=sum(of sumrsq:);
   if last.sim then do;
    durbin=(12*(&t-1)*totRsqsum/(&r*&t*(&k-1)*(&k+1)))-(3*&r*(&t-1)*(&k+1))/(&k-
1);
      if durbin>7.815 then pow_durbin+1;
        output;
        end;
   if eof;
   run;

   proc print;
   var durbin pow_durbin;
   run;

   /*Proposed Test*/
   data two;
   set one;
    T=((sumr1+3*(sumr2+sumr3+sumr4))-45*(&r/3))/sqrt(3*(&r/3));
   if T>1.645 then pow_T+1;
   run;

   proc print;
   var T pow_T;
   run;

   %mend generate;
   %generate (10000,5,15,0,0,0,0,1);
   %generate (10000,5,15,0,0.4,0.8,1.2,1);
```

```
%generate (10000.5.15.0.0.5.1.1.5.1):
%generate (10000.5.15.0.0.6.1.2.1.8.1):
%generate (10000.5.15.0.0.2.0.6.1.4.1):
%generate (10000.5.15.0.0.25.0.75.1.75.1):
%generate (10000.5.15.0.0.3.0.9.2.1.1):
%generate (10000.5.15.0.0.1.0.4.1.3.1):
%generate (10000.5.15.0.0.15.0.6.1.95.1):
%generate (10000.5.15.0.0.2.0.8.2.6.1):
%generate (10000.5.15.0.0.8.0.8.0.8.1):
%generate (10000.5.15.0.1.1.1.1):
%generate (10000.5.15.0.5.1.5.1.5.1.5.1):
%generate (10000.5.15.0.0.5.1.1.1):
%generate (10000.5.15.0.0.75.1.5.1.5.1):
%generate (10000.5.15.0.0.75.0.75.1.5.1):
%generate (10000.5.15.0.1.1.1.2.1):
%generate (10000.5.15.0.0.0.1.1):
%generate (10000.5.15.0.0.1.5.0.1):
%generate (10000.5.15.0.2.0.0.1):
%generate (10000.5.15.0.0.1.5.1.5.1):
%generate (10000.5.15.0.0.1.1.1):
%generate (10000.5.15.0.5.0.5.2.2.1):
%generate (10000.5.15.0.0.5.0.75.0.85.1):
%generate (10000.5.15.0.0.7.0.6.1.1):
%generate (10000.5.15.0.0.8.1.1.3.1):
```

```
/*Normal distribution t = 4, k = 3*/
%macro generate(sim,rep,r,mu1,mu2,mu3,mu4,sigma);

%let t=4;
%let k=3;
data raw(keep=sim rep block y1-y4);
  array seeds{4}seed1-seed4;
  do i=1 to 4;
  seeds{i}=int(ranuni(0)*1e6);
  end;
  put seed1-seed4;
do sim=1 to &sim;
   do rep=1 to &rep;
      do block=1 to 4;
            call rannor(seed1,ya);
               y1=(&mu1)+&sigma*ya;
            call rannor(seed2,yb);
               y2=(&mu2)+&sigma*yb;
      call rannor(seed3,yc);
               y3=(&mu3)+&sigma*yc;
            call rannor(seed4,yd);
               y4=(&mu4)+&sigma*yd;
    if block=1 then do; y4=.; end;
      else if block=2 then do; y3=.; end;
        else if block=3 then do; y2=.; end;
              else if block=4 then do; y1=.; end;
      output;
      end;
   end;
 end;
run;

/*Durbin Test*/
data one(drop=cctr i);
 set raw end=eof;
 by sim;
   array orgs{4}y1 y2 y3 y4;
   array abc{3}A B C;
   array trt{3}Ai Bi Ci;
   array r{4}r1-r4;
   array sumr{4}sumr1-sumr4;
   array sumrsq{4}sumrsq1-sumrsq4;
    cctr=0;
    do i=1 to 4;
      if orgs{i}>. then do;
       cctr+1;
```

```
        abc{cctr}=orgs{i};
        trt{cctr}=i;
      end;
   end;
   if (A<B<C) then do; r{Ai}=1;r{Bi}=2;r{Ci}=3; end;
   else if (A<C<B) then do; r{Ai}=1; r{Bi}=3; r{Ci}=2; end;
   else if (B<A<C) then do; r{Ai}=2; r{Bi}=1; r{Ci}=3; end;
   else if (C<A<B) then do; r{Ai}=2; r{Bi}=3; r{Ci}=1; end;
   else if (B<C<A) then do; r{Ai}=3; r{Bi}=1; r{Ci}=2; end;
   else if (C<B<A) then do; r{Ai}=3; r{Bi}=2; r{Ci}=1; end;
   do i=1 to 4;
    if first.sim then do;
     sumr{i}=0;
     sumrsq{i}=0;
     end;
    sumr{i}+r{i};
    if last.sim then do;
     sumrsq{i}=sumr{i}**2;
     end;
   end;
   totRsqsum=sum(of sumrsq:);
   if last.sim then do;
    durbin=(12*(&t-1)*totRsqsum/(&r*&t*(&k-1)*(&k+1)))-(3*&r*(&t-1)*(&k+1))/(&k-
1);
      if durbin>7.815 then pow_durbin+1;
        output;
        end;
   if eof;
   run;

   proc print;
   var durbin pow_durbin;
   run;

   /*Proposed Test*/
   data two;
   set one;
    T=((sumr1+3*(sumr2+sumr3+sumr4))-60*(&r/3))/sqrt(8*(&r/3));
   if T>1.645 then pow_T+1;
   run;

   proc print;
   var T pow_T;
   run;

   %mend generate;
```

```
%generate (10000.5.15.0.0.0.0.1):
%generate (10000.5.15.0.0.4.0.8.1.2.1):
%generate (10000.5.15.0.0.5.1.1.5.1):
%generate (10000.5.15.0.0.6.1.2.1.8.1):
%generate (10000.5.15.0.0.2.0.6.1.4.1):
%generate (10000.5.15.0.0.25.0.75.1.75.1):
%generate (10000.5.15.0.0.3.0.9.2.1.1):
%generate (10000.5.15.0.0.1.0.4.1.3.1):
%generate (10000.5.15.0.0.15.0.6.1.95.1):
%generate (10000.5.15.0.0.2.0.8.2.6.1):
%generate (10000.5.15.0.0.8.0.8.0.8.1):
%generate (10000.5.15.0.1.1.1.1):
%generate (10000.5.15.0.5.1.5.1.5.1.5.1):
%generate (10000.5.15.0.0.5.1.1.1):
%generate (10000.5.15.0.0.75.1.5.1.5.1):
%generate (10000.5.15.0.0.75.0.75.1.5.1):
%generate (10000.5.15.0.1.1.1.2.1):
%generate (10000.5.15.0.0.0.1.1):
%generate (10000.5.15.0.0.1.5.0.1):
%generate (10000.5.15.0.2.0.0.1):
%generate (10000.5.15.0.0.1.5.1.5.1):
%generate (10000.5.15.0.0.1.1.1):
%generate (10000.5.15.0.5.0.5.2.2.1):
%generate (10000.5.15.0.0.5.0.75.0.85.1):
%generate (10000.5.15.0.0.7.0.6.1.1):
%generate (10000.5.15.0.0.8.1.1.3.1):
```

```
/*Normal distribution t = 5, k = 2*/
%macro generate(sim,rep,r,mu1,mu2,mu3,mu4,mu5,sigma);

%let t=5;
%let k=2;
data raw(keep=sim rep block y1-y5);
  array seeds{5}seed1-seed5;
  do i=1 to 5;
  seeds{i}=int(ranuni(0)*1e6);
  end;
  put seed1-seed5;
do sim=1 to &sim;
    do rep=1 to &rep;
       do block=1 to 10;
              call rannor(seed1,ya);
                 y1=(&mu1)+&sigma*ya;
              call rannor(seed2,yb);
                 y2=(&mu2)+&sigma*yb;
      call rannor(seed3,yc);
                 y3=(&mu3)+&sigma*yc;
              call rannor(seed4,yd);
                 y4=(&mu4)+&sigma*yd;
              call rannor(seed5,ye);
                 y5=(&mu5)+&sigma*ye;
     if block=1 then do; y3=.; y4=.;y5=.; end;
       else if block=2 then do; y2=.;y4=.; y5=.; end;
        else if block=3 then do; y2=.;y3=.; y5=.; end;
              else if block=4 then do; y2=.; y3=.;y4=.; end;
              else if block=5 then do; y1=.; y4=.;y5=.; end;
              else if block=6 then do; y1=.; y3=.;y5=.; end;
              else if block=7 then do; y1=.; y3=.;y4=.; end;
              else if block=8 then do; y1=.; y2=.;y5=.; end;
              else if block=9 then do; y1=.; y2=.;y4=.; end;
              else if block=10 then do; y1=.; y2=.;y3=.; end;
      output;
      end;
    end;
 end;
run;

/*Durbin Test*/
data one(drop=cctr i);
 set raw end=eof;
 by sim;
  array orgs{5}y1 y2 y3 y4 y5;
  array ab{2}A B;
```

```
      array trt{2}Ai Bi;
      array r{5}r1-r5;
      array sumr{5}sumr1-sumr5;
      array sumrsq{5}sumrsq1-sumrsq5;
       cctr=0;
       do i=1 to 5;
         if orgs{i}>. then do;
          cctr+1;
          ab{cctr}=orgs{i};
          trt{cctr}=i;
       end;
   end;
   if (A<B) then do; r{Ai}=1;r{Bi}=2; end;
   else if (A>B) then do; r{Ai}=2;r{Bi}=1; end;
   do i=1 to 5;
    if first.sim then do;
     sumr{i}=0;
     sumrsq{i}=0;
     end;
    sumr{i}+r{i};
    if last.sim then do;
     sumrsq{i}=sumr{i}**2;
     end;
   end;
   totRsqsum=sum(of sumrsq:);
   if last.sim then do;
    durbin=(12*(&t-1)*totRsqsum/(&r*&t*(&k-1)*(&k+1)))-(3*&r*(&t-1)*(&k+1))/(&k-
1);
      if durbin>9.488 then pow_durbin+1;
         output;
         end;
   if eof;
   run;

   proc print;
   var durbin pow_durbin;
   run;

   /*Proposed Test*/
   data two;
   set one;
    T=((sumr1+4*(sumr2+sumr3+sumr4+sumr5))-102*(&r/4))/sqrt(9*(&r/4));
   if T>1.645 then pow_T+1;
   run;

   proc print;
```

```
var T pow_T;
run;

%mend generate;
%generate (10000,8,32,0,0,0,0,0,1);
%generate (10000,8,32,0,0.2,0.4,0.6,0.8,1);
%generate (10000,8,32,0,0.25,0.5,0.75,1,1);
%generate (10000,8,32,0,0.3,0.6,0.9,1.2,1);
%generate (10000,8,32,0,0.05,0.15,0.35,0.75,1);
%generate (10000,8,32,0,0.1,0.3,0.7,1.5,1);
%generate (10000,8,32,0,0.07,0.21,0.49,1.05,1);
%generate (10000,8,32,0,0.025,0.1,0.325,1,1);
%generate (10000,8,32,0,0.04,0.16,0.52,1.6,1);
%generate (10000,8,32,0,0.05,0.2,0.65,2,1);
%generate (10000,8,32,0,0.5,0.7,0.9,1,1);
%generate (10000,8,32,0,0.8,0.6,0.7,1,1);
%generate (10000,8,32,0,1,0.6,0.7,0.8,1);
%generate (10000,8,32,0,0,0,0,1,1);
%generate (10000,8,32,0.5,0.5,0.5,0.5,1.5,1);
%generate (10000,8,32,0,0,0,0.5,1,1);
%generate (10000,8,32,0,0,0,0.75,1.5,1);
%generate (10000,8,32,0,0,0,1,1,1);
%generate (10000,8,32,0,0.5,0.5,0.5,1,1);
%generate (10000,8,32,0,0.75,0.75,0.75,1.5,1);
%generate (10000,8,32,0,0.25,0.5,0.75,0.75,1);
%generate (10000,8,32,0,0.5,0.7,1.2,1.2,1);
%generate (10000,8,32,0,0.5,0.5,1,1,1);
%generate (10000,8,32,0,0.6,0.6,1.3,1.3,1);
%generate (10000,8,32,0,1,1,1,1,1);
%generate (10000,8,32,0,0.5,0.5,0.5,0.5,1);
%generate (10000,8,32,0,0,0,0,1,1);
%generate (10000,8,32,0,1,0,0,0,1);
```

```
/*Normal distribution t = 5, k = 3*/
%macro generate(sim,rep,r,mu1,mu2,mu3,mu4,mu5,sigma);

%let t=5;
%let k=3;
data raw(keep=sim rep block y1-y5);
  array seeds{5}seed1-seed5;
  do i=1 to 5;
  seeds{i}=int(ranuni(0)*1e6);
  end;
  put seed1-seed5;
do sim=1 to &sim;
   do rep=1 to &rep;
      do block=1 to 10;
            call rannor(seed1,ya);
               y1=(&mu1)+&sigma*ya;
            call rannor(seed2,yb);
               y2=(&mu2)+&sigma*yb;
      call rannor(seed3,yc);
               y3=(&mu3)+&sigma*yc;
            call rannor(seed4,yd);
               y4=(&mu4)+&sigma*yd;
            call rannor(seed5,ye);
               y5=(&mu5)+&sigma*ye;
     if block=1 then do; y4=.; y5=.; end;
       else if block=2 then do; y3=.; y5=.; end;
         else if block=3 then do; y3=.; y4=.; end;
              else if block=4 then do; y2=.; y5=.; end;
              else if block=5 then do; y2=.; y4=.; end;
              else if block=6 then do; y2=.; y3=.; end;
              else if block=7 then do; y1=.; y5=.; end;
              else if block=8 then do; y1=.; y4=.; end;
              else if block=9 then do; y1=.; y3=.; end;
              else if block=10 then do; y1=.; y2=.; end;
      output;
      end;
   end;
 end;
run;

/*Durbin Test*/
data one(drop=cctr i);
 set raw end=eof;
 by sim;
   array orgs{5}y1 y2 y3 y4 y5;
   array abc{3}A B C;
```

```
    array trt{3}Ai Bi Ci;
    array r{5}r1-r5;
    array sumr{5}sumr1-sumr5;
    array sumrsq{5}sumrsq1-sumrsq5;
     cctr=0;
     do i=1 to 5;
       if orgs{i}>. then do;
        cctr+1;
        abc{cctr}=orgs{i};
        trt{cctr}=i;
     end;
end;
if (A<B<C) then do; r{Ai}=1;r{Bi}=2;r{Ci}=3; end;
else if (A<C<B) then do; r{Ai}=1; r{Bi}=3; r{Ci}=2; end;
else if (B<A<C) then do; r{Ai}=2; r{Bi}=1; r{Ci}=3; end;
else if (C<A<B) then do; r{Ai}=2; r{Bi}=3; r{Ci}=1; end;
else if (B<C<A) then do; r{Ai}=3; r{Bi}=1; r{Ci}=2; end;
else if (C<B<A) then do; r{Ai}=3; r{Bi}=2; r{Ci}=1; end;
do i=1 to 5;
 if first.sim then do;
  sumr{i}=0;
  sumrsq{i}=0;
  end;
 sumr{i}+r{i};
 if last.sim then do;
  sumrsq{i}=sumr{i}**2;
  end;
end;
totRsqsum=sum(of sumrsq:);
if last.sim then do;
 durbin=(12*(&t-1)*totRsqsum/(&r*&t*(&k-1)*(&k+1)))-(3*&r*(&t-1)*(&k+1))/(&k-
1);
    if durbin>9.488 then pow_durbin+1;
      output;
      end;
if eof;
run;

proc print;
var durbin pow_durbin;
run;

/*Proposed Test*/
data two;
set one;
 T=((sumr1+4*(sumr2+sumr3+sumr4+sumr5))-204*(&r/6))/sqrt(36*(&r/6));
```

```
if T>1.645 then pow_T+1;
run;

proc print;
var T pow_T;
run;

%mend generate;
%generate (10000,3,18,0,0,0,0,0,1);
%generate (10000,3,18,0,0.2,0.4,0.6,0.8,1);
%generate (10000,3,18,0,0.25,0.5,0.75,1,1);
%generate (10000,3,18,0,0.3,0.6,0.9,1.2,1);
%generate (10000,3,18,0,0.05,0.15,0.35,0.75,1);
%generate (10000,3,18,0,0.1,0.3,0.7,1.5,1);
%generate (10000,3,18,0,0.07,0.21,0.49,1.05,1);
%generate (10000,3,18,0,0.025,0.1,0.325,1,1);
%generate (10000,3,18,0,0.04,0.16,0.52,1.6,1);
%generate (10000,3,18,0,0.05,0.2,0.65,2,1);
%generate (10000,3,18,0,0,0,0,1,1);
%generate (10000,3,18,0.5,0.5,0.5,0.5,1.5,1);
%generate (10000,3,18,0,0,0,0.5,1,1);
%generate (10000,3,18,0,0,0,0.75,1.5,1);
%generate (10000,3,18,0,0,0,1,1,1);
%generate (10000,3,18,0,0.5,0.5,0.5,1,1);
%generate (10000,3,18,0,0.75,0.75,0.75,1.5,1);
%generate (10000,3,18,0,0.25,0.5,0.75,0.75,1);
%generate (10000,3,18,0,0.5,0.7,1.2,1.2,1);
%generate (10000,3,18,0,0.5,0.5,1,1,1);
%generate (10000,3,18,0,0.6,0.6,1.3,1.3,1);
%generate (10000,3,18,0,1,1,1,1,1);
%generate (10000,3,18,0,0.5,0.5,0.5,0.5,1);
%generate (10000,3,18,0,0,0,0,1,1);
%generate (10000,3,18,0,1,0,0,0,1);
```

```
/*Normal distribution t = 5, k = 4*/
%macro generate(sim,rep,r,mu1,mu2,mu3,mu4,mu5,sigma);

%let t=5;
%let k=4;
data raw(keep=sim rep block y1-y5);
  array seeds{5}seed1-seed5;
  do i=1 to 5;
  seeds{i}=int(ranuni(0)*1e6);
  end;
  put seed1-seed5;
do sim=1 to &sim;
   do rep=1 to &rep;
      do block=1 to 5;
             call rannor(seed1,ya);
                y1=(&mu1)+&sigma*ya;
             call rannor(seed2,yb);
                y2=(&mu2)+&sigma*yb;
     call rannor(seed3,yc);
                y3=(&mu3)+&sigma*yc;
             call rannor(seed4,yd);
                y4=(&mu4)+&sigma*yd;
             call rannor(seed5,ye);
                y5=(&mu5)+&sigma*ye;
    if block=1 then do; y5=.; end;
      else if block=2 then do; y4=.; end;
       else if block=3 then do; y3=.; end;
             else if block=4 then do; y2=.; end;
             else if block=5 then do; y1=.; end;
     output;
     end;
   end;
 end;
run;

/*Durbin Test*/
data one(drop=cctr i);
 set raw end=eof;
 by sim;
   array orgs{5}y1 y2 y3 y4 y5;
   array abcd{4}A B C D;
   array trt{4}Ai Bi Ci Di;
   array r{5}r1-r5;
   array sumr{5}sumr1-sumr5;
   array sumrsq{5}sumrsq1-sumrsq5;
    cctr=0;
```

```
      do i=1 to 5;
       if orgs{i}>. then do;
        cctr+1;
        abcd{cctr}=orgs{i};
        trt{cctr}=i;
      end;
   end;
   if (A<B<C<D) then do; r{Ai}=1;r{Bi}=2;r{Ci}=3; r{Di}=4;end;
   else if (A<B<D<C) then do; r{Ai}=1;r{Bi}=2;r{Ci}=4; r{Di}=3;end;
   else if (A<C<B<D) then do; r{Ai}=1;r{Bi}=3;r{Ci}=2; r{Di}=4;end;
   else if (A<D<B<C) then do; r{Ai}=1;r{Bi}=3;r{Ci}=4; r{Di}=2;end;
   else if (A<C<D<B) then do; r{Ai}=1;r{Bi}=4;r{Ci}=2; r{Di}=3;end;
   else if (A<D<C<B) then do; r{Ai}=1;r{Bi}=4;r{Ci}=3; r{Di}=2;end;
   else if (B<A<C<D) then do; r{Ai}=2;r{Bi}=1;r{Ci}=3; r{Di}=4;end;
   else if (B<A<D<C) then do; r{Ai}=2;r{Bi}=1;r{Ci}=4; r{Di}=3;end;
   else if (C<A<B<D) then do; r{Ai}=2;r{Bi}=3;r{Ci}=1; r{Di}=4;end;
   else if (D<A<B<C) then do; r{Ai}=2;r{Bi}=3;r{Ci}=4; r{Di}=1;end;
   else if (C<A<D<B) then do; r{Ai}=2;r{Bi}=4;r{Ci}=1; r{Di}=3;end;
   else if (D<A<C<B) then do; r{Ai}=2;r{Bi}=4;r{Ci}=3; r{Di}=1;end;
   else if (B<C<A<D) then do; r{Ai}=3;r{Bi}=1;r{Ci}=2; r{Di}=4;end;
   else if (B<D<A<C) then do; r{Ai}=3;r{Bi}=1;r{Ci}=4; r{Di}=2;end;
   else if (C<B<A<D) then do; r{Ai}=3;r{Bi}=2;r{Ci}=1; r{Di}=4;end;
   else if (D<B<A<C) then do; r{Ai}=3;r{Bi}=2;r{Ci}=4; r{Di}=1;end;
   else if (C<D<A<B) then do; r{Ai}=3;r{Bi}=4;r{Ci}=1; r{Di}=2;end;
   else if (D<C<A<B) then do; r{Ai}=3;r{Bi}=4;r{Ci}=2; r{Di}=1;end;
   else if (B<C<D<A) then do; r{Ai}=4;r{Bi}=1;r{Ci}=2; r{Di}=3;end;
   else if (B<D<C<A) then do; r{Ai}=4;r{Bi}=1;r{Ci}=3; r{Di}=2;end;
   else if (C<B<D<A) then do; r{Ai}=4;r{Bi}=2;r{Ci}=1; r{Di}=3;end;
   else if (D<B<C<A) then do; r{Ai}=4;r{Bi}=2;r{Ci}=3; r{Di}=1;end;
   else if (C<D<B<A) then do; r{Ai}=4;r{Bi}=3;r{Ci}=1; r{Di}=2;end;
   else if (D<C<B<A) then do; r{Ai}=4;r{Bi}=3;r{Ci}=2; r{Di}=1;end;
   do i=1 to 5;
    if first.sim then do;
     sumr{i}=0;
     sumrsq{i}=0;
     end;
    sumr{i}+r{i};
    if last.sim then do;
     sumrsq{i}=sumr{i}**2;
     end;
   end;
   totRsqsum=sum(of sumrsq:);
   if last.sim then do;
    durbin=(12*(&t-1)*totRsqsum/(&r*&t*(&k-1)*(&k+1)))-(3*&r*(&t-1)*(&k+1))/(&k-
1);
     if durbin>9.488 then pow_durbin+1;
```

```
    output:
    end:
if eof:
run:

proc print:
var durbin pow_durbin:
run:

/*Proposed Test*/
data two:
set one:
 T=((sumr1+4*(sumr2+sumr3+sumr4+sumr5))-170*(&r/4))/sqrt(45*(&r/4)):
if T>1.645 then pow_T+1:
run:

proc print:
var T pow_T:
run:

%mend generate:
%generate (10000.3.12.0.0.0.0.0.1):
%generate (10000.3.12.0.0.2.0.4.0.6.0.8.1):
%generate (10000.3.12.0.0.25.0.5.0.75.1.1):
%generate (10000.3.12.0.0.3.0.6.0.9.1.2.1):
%generate (10000.3.12.0.0.05.0.15.0.35.0.75.1):
%generate (10000.3.12.0.0.1.0.3.0.7.1.5.1):
%generate (10000.3.12.0.0.07.0.21.0.49.1.05.1):
%generate (10000.3.12.0.0.025.0.1.0.325.1.1):
%generate (10000.3.12.0.0.04.0.16.0.52.1.6.1):
%generate (10000.3.12.0.0.05.0.2.0.65.2.1):
%generate (10000.3.12.0.0.5.0.7.0.9.1.1):
%generate (10000.3.12.0.0.8.0.6.0.7.1.1):
%generate (10000.3.12.0.1.0.6.0.7.0.8.1):
%generate (10000.3.12.0.0.0.0.1.1):
%generate (10000.3.12.0.5.0.5.0.5.0.5.1.5.1):
%generate (10000.3.12.0.0.0.0.5.1.1):
%generate (10000.3.12.0.0.0.0.75.1.5.1):
%generate (10000.3.12.0.0.0.1.1.1):
%generate (10000.3.12.0.0.5.0.5.0.5.1.1):
%generate (10000.3.12.0.0.75.0.75.0.75.1.5.1):
%generate (10000.3.12.0.0.25.0.5.0.75.0.75.1):
%generate (10000.3.12.0.0.5.0.7.1.2.1.2.1):
%generate (10000.3.12.0.0.5.0.5.1.1.1):
%generate (10000.3.12.0.0.6.0.6.1.3.1.3.1):
%generate (10000.3.12.0.1.1.1.1.1):
```

```
%generate (10000,3,12,0,0.5,0.5,0.5,0.5,1);
%generate (10000,3,12,0,0,0,0,1,1);
%generate (10000,3,12,0,1,0,0,0,1);
```