A NONPARAMETRIC TEST FOR THE NON-DECREASING

ALTERNATIVE IN AN INCOMPLETE BLOCK DESIGN

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The Supervisory Committee certifies that this **disquisition** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

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ABSTRACT

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The purpose of this paper is to present a new nonparametric test statistic for testing against ordered alternatives in a Balanced Incomplete Block Design (BIBD). This test will then be compared with the Durbin test which tests for differences between treatments in a BIBD but without regard to order. For the comparison, Monte Carlo simulations were used to generate the BIBD. Random samples were simulated from:

- Normal Distribution

- Exponential Distribution

- T distribution with three degrees of freedom

The number of treatments considered was three, four and five with all the possible combinations necessary for a BIBD. Small sample sizes were 20 or less and large sample sizes were 30 or more. The powers and alpha values were then estimated after 10,000 repetitions.

The results of the study show that the new test proposed is more powerful than the Durbin test. Regardless of the distribution, sample size or number of treatments, the new test tended to have higher powers than the Durbin test.

ACKNOWLEDGEMENTS

TABLE OF CONTENTS

LIST OF TABLES

1. INTRODUCTION

Nonparametric tests, compared to parametric tests, are ideal when the underlying assumptions necessary for the parametric test are not met; for instance, the distribution of the population violates a test assumption or when there is not enough information regarding the population. Nonparametric tests usually offer test legitimacy with very few assumptions. They can be used even when the distribution is unknown. There are several advantages to using nonparametric tests outlined in Applied Nonparametric Statistics by Wayne (1990) among which is the fact that the measurement scale required for some nonparametric tests can be as weak as the nominal.

One common area where nonparametric tests are employed is the testing of treatment effects. There are two basic assumptions when designing an experiment for testing the hypothesis – whether treatment effects differ from one another. One assumes the samples being tested are independent and the other that the samples are dependent on one another. This paper particularly looks at the latter.

The major problem when there is dependence is that the results of the test might be influenced by an external factor not of interest. A common term for this phenomenon is called confounding. For instance, suppose a researcher wants to know which diet plan is the best among several and scoring is based on the average weight loss. The Body Mass Index (BMI) of the subjects is a significant factor. Therefore, when randomly assigning subjects to the different diet plans, subjects with a high BMI may end up with one of the plans thereby showing that the particular plan is the best. In order to have a better design, one that eliminates confounding, a method called blocking is used. In blocking, subjects are divided into homogeneous subgroups called blocks (Wayne 1990) and the application

of treatments is done in random order within the blocks. As a result each subject within the block randomly receives a treatment. This design is called the Randomized Complete Block Design (RCBD). There are several tests for testing differences in treatment effects while blocking. However, we shall look at the two main ones: Friedman (Friedman, 1937, 1940) and Page's (Page, 1963).

The Friedman Test (Friedman, 1937, 1940) tests for differences in treatment effects using an RCBD. The treatment effects are ranked within the block thus comparisons are only within the subgroups (Wayne 1990). Suppose a researcher wants to know whether different over the counter medications have the same effect in reducing headaches. Subjects suffering from the headaches are randomly given one of the medications and they report their pain on a scale after a certain time. The next time they suffer another headache they are randomly given one of the remaining medication and again asked to report their pain after a certain time. Ranking is then done within the same person. The sum of the ranks of each medication is then obtained and the test statistic is computed using the sums.

Page's Test (Page, 1963) is used to test for differences in treatment effects also, but it has the additional assumption that if treatment effects are not equal, they are in a non-decreasing order. Thus, the alternative hypothesis is that the treatment effects are non-decreasing. For instance, a research company wants to find out if higher doses of a certain drug would cause a reduction in number of tumors. Testing is done at three levels 25mg, 50mg and 75mg. Therefore, it is believed that the number of tumors developed by subjects at 25mg will be more than at 50mg which in turn will be higher than at 75mg. Blocking is done based on gender and age level. Ranking is then done within the blocks as in the

Friedman case and the appropriate test statistic computed using the sum of the ranks. Page's, like Friedman, also uses the RCBD.

A Balanced Incomplete Block Design (BIBD) is a derivation of the RCBD. However, due to other factors such as limited resources, the RCBD may not be ideal thus a BIBD would be more practical. In the BIBD, only certain treatments would be applied within a block; not all treatments are applied within a block. Therefore, each block would have a different combination of treatments applied. However, the design is such that each block would have the same number of treatments applied, each treatment would be applied the same number of times, and each pair of treatments would appear the same number of times.

The Durbin Test (Durbin, 1951), like the Friedman, tests for differences in treatment effects. Ranking is done within a block also and the test statistic is computed based on the sum of the treatment ranks. However, unlike the Friedman and Page's Tests, the Durbin Test employs a Balanced Incomplete Block Design (BIBD).

Whereas the Page's statistic tests for ordered alternatives in an RCBD and the Durbin tests for differences in treatment effects in a BIBD, currently there is no test for ordered alternatives in a BIBD. The test statistic proposed in this paper thus aims to solve this problem. The test statistic assumes that the blocks are independent and that the data collected can be ranked in order of magnitude. The test extends the idea of Page's Test to a BIBD. The test also uses the ranking method used by Durbin when assigning ranks. Ranking, in the new test, is also done within each block and the sum of ranks of the treatments is obtained at the end. The test statistic is then computed using the sum of the treatment ranks. Chapter three looks at the proposed test in detail.

The next chapter goes over the survey of literature where the two tests – Page's and

Durbin – are further explored.

2. SURVEY OF LITERATURE

There are several nonparametric tests developed for testing for differences in treatment effects (or samples); each based on different sets of assumptions. The main assumption considered in this chapter is independence of samples tested. This chapter briefly looks at the development of these tests starting with the basic case where two samples are tested.

The Mann - Whitney Test (Mann and Whitney, 1947) proposed by Mann and Whitney (Daniel, 1990), tests for differences between two treatment effects. One assumption that must hold for validity of the test is that the samples are independent. Furthermore, Mann - Whitney uses ranks to compute its test statistic; therefore, the measurement scale must at least be ordinal. This test can be used for all three hypotheses:

- H_0: $\tau_1 = \tau_2$ vs. H_a: $\tau_1 \neq \tau_2$ (Two sided)

- H_0: $\tau_1 \geq \tau_2$ vs. H_a: $\tau_1 < \tau_2$ (One sided)

- H_0: $\tau_1 \leq \tau_2$ vs. H_a: $\tau_1 > \tau_2$ (One sided)

However, when the independence assumption does not hold and thus the two samples are related, a test called the Wilcoxon Signed Rank (WSR) Test (Wilcoxon, 1945) can be applied. It does not require the independence assumption. The WSR Test deals with pairs of observations; it is based on the differences between the matched pair of observations to compute its test statistic and the hypothesis is tested that M_D is zero where M_D is the median of the differences. Therefore, one requirement is that the measurement scale of the data is at least interval. The hypotheses tested that can be tested are:

- H_0: $M_D = 0$ vs. H_a: $M_D \neq 0$ (Two sided)

- H_0: $M_D \geq 0$ vs. H_a: $M_D < 0$ (One sided)

5

- H_0: $M_D \leq 0$ vs. H_a: $M_D > 0$ (One sided)

This test is further explored later on in the chapter.

When testing more than two treatments for differences in effect, one test that is used is the Kruskal – Wallis Test (Kruskal and Wallis, 1953). This test is equivalent to the Mann – Whitney Test in the case of two treatments (Wayne, 1990). Likewise, the assumption of independence of samples must hold for validity of the test and the measurement scale should at least be ordinal. The Kruskal – Wallis Test is based on the location of the medians and draws inferences on the population distribution of the treatment effects. Daniel (Daniel, 1990) gives the hypotheses tested as follows:

H_0: The k population distribution functions are identical;

H_a: The k populations do not all have the same medians.

Another test used for testing more than two treatments is the Jonckheere – Terpstra (JT) Test (Terpstra, 1952 & Jonckheere, 1954). Like the Kruskal – Wallis, this test also depends on the independence of the samples. However, the JT Test tests for differences in treatment effects when the effects are believed to be non-decreasing if different. This test provides more power when the non-decreasing assumption is true than just testing for inequality. The null and alternative are therefore:

H_0: $\tau_1 = \tau_2 = \ldots = \tau_k$

H_0: $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_k$ (at least one inequality is strict)

When testing several related samples, the independence assumption is violated and the three tests mentioned in Chapter One: Friedman (Friedman, 1937, 1940), Page's (Page, 1963) and the Durbin (Durbin 1951) tests can be used. These tests are based on a random block design where experimental units considered homogeneous are grouped together to

6

form a block. Both Page's test and the Friedman test use a complete block design in which every treatment appears in a block. The Durbin test uses an incomplete block design so not every treatment appears in every block. The following section looks at the WSR, Page's and the Durbin Tests in detail.

2.1. Wilcoxon Signed Rank (WSR) Test

As mentioned earlier, the WSR (Wilcoxon, 1945) tests for differences between two related samples. The design of the test is such that the samples form a paired set of data, say (X, Y), and the test statistic is based on the difference of X and Y. There are other assumptions that must hold for the validity of the test as outlined by Wayne (Wayne, 1990).

- The pair of observations must be taken on the same subject or similar subjects (a form of blocking);

- Differences, D_i's, are continuous random variables and independent of each other

- Distribution of difference must be symmetric around their median, M_D.

- The measurement scale is at least interval

The test statistic is calculated as follows:

- Calculate the difference absolute value of $|D_i| = |Y_i - X_i|$ and rank them

- Assign a + or − to the ranks based on the sign of the differences before taking the absolute value;

- Sum the + ranks T^+, or the − ranks T^-, (Wayne, 1990). These are the test statistics.

The null hypothesis is rejected for sufficiently small values of the test statistics (the test statistic used depends on the alternative hypothesis).

2.2. Page's Test

Page's Test (Page, 1963), as mentioned, considers non-decreasing ordered alternative in a Randomized Complete Block Design (RCBD). The assumptions necessary for Page's as listed by Daniel (Daniel, 1990) are:

- The blocks are independent
- Variable is continuous
- No interaction between blocks and treatments
- Observations may be ranked in order of magnitude

The hypotheses are

$H_0: \tau_1 = \tau_2 = \ldots = \tau_k$

$H_0: \tau_1 \leq \tau_2 \leq \ldots \leq \tau_k$ (at least one inequality is strict)

The test statistic is calculated as follows:

- Rank all observations within a block;
- Sum the ranks of each treatment, R_j – sum of the ranks of treatment j;
- The test statistic is,

$$L = \sum_{j=1}^{t} j \times R_j, \quad where \ t = total \ number \ of \ treatments$$

Note that Page's statistic assigns 'weights' to the sum of ranks with treatments of expected larger magnitude having larger 'weights'. Hence, if the alternative hypothesis can be sufficiently supported, L is expected to have larger values. Hence, reject the null for large L values.

A standardized value of Page's test statistic can be used and is given below as Z. The standardized version has an asymptotic standard normal distribution under the assumption that the null hypothesis is true

$$Z = \frac{L - [\frac{bt(t+1)^2}{4}]}{\sqrt{b(t^3 - t)^2/144(t-1)}} \quad \text{where } b = \text{total number of blocks.}$$

2.3. Durbin Test

The Durbin Test (Durbin, 1951) is used in a Balanced Incomplete Block Design (BIBD) to test for differences among treatment effects. There are two main assumptions with the Durbin:

- Blocks are mutually independent

- Observations may be ranked in order of magnitude

Ranking and summation of the treatment ranks is similar to Page's. Treatments not appearing in this design are assigned a rank of 0. However, the Durbin Test is based on the summation of the squared sum of Ranks, i.e. $\sum_{j=1}^{t} Rj^2$. The test statistic, as given is Daniel (Daniel, 1990), is

$$T = \frac{12(t-1)}{rt(k-1)(k+1)} \sum_{j=1}^{t} R_j^2 - \frac{3r(t-1)(k+1)}{k-1}$$

where

    t = the total number of treatments

    k = total number of subjects per block (k < t)

    r = the number of times each treatment occurs.

T has an asymptotically Chi Square distribution with $t - 1$ $d.f$ under H_0 for large values of r.

2.4. Cao and Magel's Comparison

Note that a BIBD with two subjects per block can be thought of as a paired observation within a block. In 2010, Cao conducted a simulation study comparing the

9

estimated powers of the Durbin test with the estimated powers of the WSR test when there were two treatments per block for the non-decreasing (or non-increasing alternative). Her findings were the following:

- If at least one treatment effect is different and the treatment effects following a non-decreasing (or non-increasing pattern), the WSR test is generally more powerful than the Durbin test.

- The Durbin test tends to be more powerful than the WSR test when the order is random

A new nonparametric test for the non-decreasing order alternative in a Balanced Incomplete Block Design (BIBD) is introduced in chapter 3. A simulation study is described in chapter 4 which compares the powers of the proposed test with existing tests. Results are given in chapter 5 and conclusions in chapter 6.

3. PROPOSED TEST STATISTIC

This chapter examines the development of the proposed test statistic. It will look at the assumptions underlying the test; the development of the expected value; the variance and the eventual large sample asymptotic distribution of the test statistic when the null hypothesis is true. The distribution of ranks is used to derive the expected value of the test statistic and its variance.

Throughout this chapter, the following are defined as such

t   = the total number of treatments in the experiment;

k   = the number of subjects within a block ($k < t$);

b   = the total number of blocks;

r   = the number of times a treatment appears ($r < b$);

R_j = the sum of the ranks of the jth treatment.

The hypotheses are

$H_0: \tau_1 = \tau_2 = \ldots = \tau_k$

$H_a: \tau_1 = \tau_2 = \ldots = \tau_k$

3.1. Assumptions

The assumptions underlying the test statistic are similar to those that the Page's and Durbin test require as the test is a derivation from the two. These are

- The *b* blocks are independent of each other

- The observations can be ranked in order of magnitude

- Variable of interest is continuous so theoretically there are no ties in the ranks

11

3.2. Procedure

The first step is to rank the observations within each block. Since it is a BIBD there are treatments that will not have an observation. For these a rank of 0 (zero) is assigned just like in the Durbin case. The ranks are then totaled for each of the j treatments in the experiment to get R_j. The test statistic is defined as follows:

$$M = \sum_{j=1}^{t} j \times R_j \tag{1}$$

	Treatment 1	Treatment 2	...	Treatment t
Block 1	r_{11}	r_{12}		r_{1t}
Block 2	r_{21}	r_{22}		r_{2t}
...				
Block b	r_{b1}	r_{b2}		r_{bt}
Total	R_1	R_2	...	R_t

$r_{ij} = 0$ if treatment is not applied.

The test statistic is then standardized by computing $M^* = \frac{M - E(M)}{\sqrt{var(M)}}$ which asymptotically follows a standard normal distribution when the null hypothesis is true. This follows from the asymptotic distribution of Page's Test (Page, 1963). Rejection rule: Reject H_0 if $M^* > Z_\alpha$.

3.3. Expected Value *E(M)* and Variance

The BIBD presents a challenge in finding a general formula for the expected values of the different cases. For each case of t treatments with k appearing at a time there are $\binom{t}{k}$ unique number of blocks. The $\binom{t}{k}$ is the minimum number of blocks required to form a BIBD. These are repeated to generate the required number of blocks in an experiment.

12

Therefore, b must be a multiple of $\binom{t}{k}$. Independence of the blocks allows computation to be done by looking at one block. The general steps involved are

- List the $\binom{t}{k}$ unique blocks [1]

- For each block, list the $k!$ combinations of ranks within that block given that the blanks remain fixed. [2]

- Calculate M_i, the test statistic for each combination i, $i = 1, \ldots, k!$ [3]

- Calculate the average, M_l, by $\frac{\sum_{i=1}^{k!} M_i}{k!}$ for the expected value of block l. [4]

- Compute the variance of M_i [5]

- The expected value and variance of the test statistic for the $\binom{t}{k}$ unique blocks is the sum of the M_l [6] and the sum of the variances of M_i respectively. [7]

- Finally, multiply the expected values in the above step appropriately for an experiment with b blocks. [8]

3.4. Example: Three Treatments with Two Treatments Per Block

For $t = 3$ and $k = 2$, there are $\binom{3}{2} = 3$ unique number of blocks that form a BIBD. The possible ranks to be assigned are 0, 1 and 2. The following table shows the step-by-step derivation of the expected value and variance.

The expected value and variance for 3 blocks are 18 and 1.5 respectively. Therefore, if an experiment is using 30 blocks, the expected values and variance are:

$Var(M) = 1.5 \times 10 = 15$ [8] & $E(M) = 18 \times 10 = 180$ [7].

Table 1. Expected Value and Variance when $t = 3$ and $k = 2$

	i^{th} combination	Treatment 1	Treatment 2	Treatment 3	M_i	M_l	$Var(M_i)$
(1)	(2)				(3)	(4)	(5)
Block 1	1	1	2	0	5	4.5	0.25
	2	2	1	0	4		
Block 2	1	1	0	2	7	6	1
	2	2	0	1	5		
Block 3	1	0	1	2	8	7.5	0.25
	2	0	2	1	7		
						(6)	(7)
						18	**1.5**

In this case Treatments 3, 2 and 1 are not applied in blocks 1, 2 and 3 respectively.

The following table lists computed values (Appendix A. tables A1 to A5 along with table 1 above).

Table 2. Number of Unique Blocks, Expected Values and Variances

Case	Number of unique blocks	E(M)	Var(M)
3 treatments 2 per block	3	18	1.5
4 treatments 2 per block	6	45	5
4 treatments 3 per block	4	60	13.33

14

Table 2. continued

5 treatments 2 per block	10	90	12.5
5 treatments 3 per block	10	180	50
5 treatments 4 per block	5	150	62.5

4. SIMULATION STUDY

This chapter discusses the details of a Monte Carlo Simulation study using SAS 9.1. This process provides random samples of data in a Balanced Incomplete Block Design (BIBD) to be used for the different cases discussed in this paper. The goal of the simulations is to determine the power of the proposed test statistic and compare it to that of Durbin. In selected cases, the estimated powers of the Wilcoxon Signed Ranks (WSR) (Wilcoxon, 1945) test as found by Cao (2010) will also be compared to the proposed test.

Samples are generated from three main distributions: Normal, Exponential and T (with three degrees of freedom). Power is then estimated for the different cases listed in Table 2 in chapter 3. The minimum number of blocks for the design to be balanced is first calculated. For example when there are 3 treatments with 2 treatments appearing per block, 3 blocks are needed as a minimum for the design to be balanced. When there are 5 treatments, with 3 treatments appearing per block, 10 blocks are needed for the design to be balanced. The minimum number of blocks with treatments appearing an equal number of times is first replicated to get the desired number of blocks. For instance, in the case of three treatments with two appearing per block, the unique blocks are replicated ten times resulting in a BIBD with thirty blocks. The data is simulated 10,000 times and then power is estimated by counting the number of times the test statistic rejects the null hypothesis divided by 10,000. This process is performed for both the Durbin and the proposed test. Also, several configurations of the location parameters are considered to see how power compares.

4.1. Distributions

SAS generates data from the three distributions using two main steps. The first step generates the seeds to be used in the second step. Seeds tell the SAS random number generator where to start generating values. For instance, if seed = 0 then SAS generates a seed based on time since midnight and each run of the code would therefore produce a different set of data (Bailer, 2010). The SAS routine for the first step is the RANUNI. This tells SAS to generate random numbers from a Uniform distribution. The second step uses the seeds to generate data from a specific distribution. In our case, there are three specific distributions used.

In order to generate values from a normal distribution, the Call RANNOR routine is used with the syntax

$$\text{call rannor(trt1, y)}.$$

The above subroutine generates a random sample of size one from a standard normal distribution. The symbol *Trt1* is the seed value and y is the name of the variable that stores the value that was generated. In order to generate a random sample of size n, the subroutine is called n times. It is noted that the seed value changes after each call. To create a random sample from a normal distribution with a different mean, say *mu*, the value *mu* is added to each value in the random sample from the standard normal.

The Exponential distribution random numbers are generated by the Call Ranexp routine. The syntax for the routine is

$$\text{call ranexp(trt1, y)}$$

where *trt1* is the seed and y stores the generated value. The mean and variance in this case are the same, 1. Again this routine would be called n times, the seed changing each time, to

17

create a random sample from a standard exponential distribution. If *mu* were then added to each of these values, this would simulate a random sample from an exponential with mean *mu* + 1 and variance still 1.

The T distribution, with 3 degrees of freedom, random numbers are generated by the RAND routine. This requires a different method of assigning the seed value so instead of using the Ranuni routine, seeds are generated by the function

$$Call\ streaminit\ (seed).$$

The syntax for generating the treatment values is

$$Y = rand\ ('T',\ 3) + mu,$$

where T is the name of the distribution, 3 is the degrees of freedom and *mu* is the constant added to create different location parameters.

4.2. Sample Sizes

This study examines the power of both the Durbin test and the proposed test for various sample sizes ranging from 10 to 30. Powers are estimated based on two sample sizes for each case. Note that the large sample approximation version of the Durbin test is always used. The large sample approximation was also always used for the proposed test. The following table shows the number of blocks considered for both sample sizes.

Table 3. Number of Blocks Considered

Case	Smaller Sample	Larger Sample
3 treatments 2 per block	12	30
4 treatments 2 per block	12	30
4 treatments 3 per block	12	32

Table 3. continued

Case	Smaller Sample	Larger Sample
5 treatments 2 per block	20[*]	30
5 treatments 3 per block	10	30
5 treatments 4 per block	10	30

* *A sample size of 10 yields dismal power hence 20 is used in order to get significant powers.*

4.3. Location Parameters

As mentioned before, different location parameters are used to compare the power of the proposed test to that of the Durbin. Shifting the location parameters in several ways does this. This section looks at the case-by-case methods categorized by the number of treatments being tested.

For the case with three treatments, there are five main ways used to shift the different location parameters, viz.:

    a.   The third parameter is different from the first two, e.g. (0, 0, 1);

    b.   The first parameter is different from the last two, e.g. (0, 1, 1);

    c.   The location parameters differ with equal spacing between them, e.g. (0, 1, 2);

    d.   The location parameters differ with unequal spacing between them, e.g. (0, 1, 3);

    e.   The location parameters are ordered randomly thus violating the non decreasing alternatives assumption, e.g. (1, 0, 1)

19

For the case with four treatments, there are six main ways used to shift the location parameters, viz.:

    a.   The fourth parameter is different from the others, e.g. (0, 0, 0, 1);

    b.   The first two parameters are equal but different from the last two which are also equal to each other, e.g. (0, 0, 1, 1);

    c.   The first parameter is different from the others, e.g. (0, 1, 1, 1);

    d.   The location parameters differ with equal spacing between them, e.g. (0, 1, 2, 3);

    e.   The location parameters differ with unequal spacing between them, e.g. (0, 0.5, 1.2, 1.5);

    f.   The location parameters are ordered randomly thus violating the non decreasing alternatives assumption, e.g. (1, 0, 1, 0.5);

For the case with five treatments, there are seven main ways used to shift the location parameters, viz.:

    a.   The fifth parameter is different from the others, e.g. (0, 0, 0, 0, 1);

    b.   The last two parameters are equal but different from the first three, e.g. (0, 0, 0, 1, 1);

    c.   The last three parameters are equal but different from the first two, e.g. (0, 0, 1, 1, 1);

    d.   The first parameter is different from the others, e.g. (0, 1, 1, 1, 1);

    e.   The parameters differ with equal spacing between them, e.g. (0, 1, 2, 3, 4);

    f.   The parameters differ with unequal spacing between them, e.g. (0, 1, 1.5, 3, 4);

g. The parameters are ordered randomly violating the non decreasing

alternatives assumption, e.g. (0, 1, 0, 0, 1);

5. RESULTS

This chapter discusses the findings of the Monte Carlo Simulations. The powers of the proposed (M), Wilcoxon Signed Rank (WSR) (Wilcoxon, 1945) and the Durbin (Durbin, 1951) tests are compared. The results are listed in Tables 4 to 39 at the end of this chapter. The results are grouped by the number of treatments being tested and the number of treatments appearing in each block.

5.1. Three Treatments with Two Appearing Per Block

5.1.1. Normal Distribution

In the large sample case, as shown in Table 4, when the location parameters are in a non-decreasing pattern, M and WSR have similar powers but both are more powerful than the Durbin Test. When the non-decreasing assumption is violated, the M test has powers lower than the Durbin and often less than WSR test. For example, when the parameters are ordered 1, 0, 0.5, the Durbin test rejects 40.11% of the time, the WSR test 17.88% of the time and the M test .24% of the time. When the assumption is violated we do not want to reject the null hypothesis. Therefore, we want a test with low power in this case.

The results in Table 5 show that when the sample size is small ($b = 12$), M tends to be more powerful than the Durbin Test provided the order is non-decreasing. However, when this assumption is violated, the Durbin Test tends be more powerful. We are looking for a test with this characteristic.

5.1.2. Exponential Distribution

When the sample size is large, Table 6 shows that the M tends to be the most powerful, followed by the WSR and the Durbin tests. This holds as long as the order is non-decreasing. When the order is no longer non-decreasing, the Durbin test becomes the

most powerful of the three; WSR and M suffer and have powers as low as around 2% compared to Durbin with around 75%.

In the small sample size comparison between the Durbin and M, the latter tends to be more powerful as shown in Table 7 when the assumption is correct. For example, when the location parameters are 0, 0.6, 0.9, the M test rejects 39.73% of the time while the Durbin test rejects 26.98% of the time. When the assumption is not correct, the M test has very low power in the situations we considered. We want this to be the case since the alternative hypothesis is not true.

5.1.3. T Distribution

In the T distribution case, Table 8 shows that when the order is non-decreasing, M tends to be the most powerful followed by WSR and Durbin being the least powerful. Generally, when the assumption of order is violated, the Durbin Test tends to be the most powerful. WSR and the proposed test tend to have similar power in most cases. When the assumed ordering is not correct, the M test generally has powers a lot lower than the Durbin and close to or less than the WSR test. The one exception is when the parameters are (0.25, 0, 0.75). The M test rejects 20.55% of the time while the Durbin test and WSR test reject 16.35% and 14.22% of the time, respectively.

When the sample size is small comparison between the proposed Test and the Durbin, Table 9 shows that the proposed Test is more powerful as long as the order is non-decreasing. The Durbin Test, however, is more powerful when the order is not non-decreasing.

5.2. Four Treatments with Two Appearing Per Block

5.2.1. Normal Distribution

Table 10 shows that when the sample size is large and the order is non-decreasing, M and WSR generally tend to have similar powers that are higher than the Durbin test. The M test does have estimated powers higher than the WSR test in more cases. However, the Durbin test tends to the most powerful when the order assumption is violated. In a few cases the estimated powers of the WSR test are quite a lot higher than the estimated powers of the M test when the assumption is violated.

Table 11 shows that when the sample size is small, M tends to have higher power than the Durbin test as long as the order assumption holds. The Durbin test has more power compared to M when the assumption is violated.

5.2.2. Exponential Distribution

The results in Table 12 show that M is the most powerful of the three tests as long as the order is non-decreasing; WSR in turn is generally more powerful than the Durbin test. The Durbin test, however, is the most powerful test when the order assumption is violated; WSR and M have small powers. When the assumption is violated, the M test never rejects more than 13.19% of the time in the cases considered. The WSR test has a higher rejection percentage than this in two cases.

Table 13 shows the results of the power comparison between M and the Durbin when the sample size is small. The former is more powerful when the order is non-decreasing but the latter has more power when the order assumption is violated.

5.2.3. T Distribution

M is the most powerful test when the order is non-decreasing and the sample size is large while WSR tends to be more powerful than the Durbin. The Durbin test becomes the most powerful test of the three when the order assumption is violated. This is shown in Table 14. The WSR test often has higher rejection percentages than the M test when the assumption is violated.

The comparison of power between M and the Durbin, when the sample size is small, is shown in Table 15 with the former being more powerful when the order is non-decreasing. When the order assumption is violated, the Durbin test becomes the more powerful one.

5.3. Four Treatments with Three Appearing Per Block

5.3.1. Normal Distribution

The results show that M is significantly more powerful than the Durbin test when the order is non-decreasing regardless of the sample size; as shown in both Tables 16 and 17. However, when the order assumption is violated, the Durbin test tends to be significantly more powerful.

5.3.2. Exponential Distribution

When the distribution is Exponential, M is still more powerful than the Durbin test when the order is non-decreasing regardless of the sample size. However, the Durbin test is significantly more powerful than M when the order is not non-decreasing. This is shown in Tables 18 and 19.

5.3.3. T Distribution

The results show that the conclusion is the same even when the distribution is T: M is significantly more powerful than the Durbin when the order is non-decreasing regardless of the sample size; the Durbin is significantly more powerful when the order is not non-decreasing. This is shown in Tables 20 and 21.

5.4. Five Treatments with Two Appearing Per Block

5.4.1. Normal Distribution

The results in Table 22 show the same trend: M is the most powerful followed by WSR and the Durbin test when the order is non-decreasing and the sample size is large. However, when the order is not non-decreasing, the Durbin test tends to be the most powerful of all three. WSR and M tend to be equally powerful.

Table 23 shows the comparison between M and the Durbin test when the sample size is small. Here, M still is significantly more powerful than the Durbin provided that the order is non-decreasing. However, the reverse is true when the order is not non-decreasing.

5.4.2. Exponential Distribution

M again is significantly more powerful than WSR and the Durbin test given that the order is non-decreasing. WSR in turn is more powerful than the Durbin. When the order is not non-decreasing, however, the Durbin becomes the more powerful than the others. There are cases where M and WSR have high values of power albeit still smaller than Durbin's. This is shown in Table 24.

Table 25 shows that M is more powerful than the Durbin test when the order is non-decreasing and the sample size is small. The Durbin test tends to be more powerful when the order is not non-decreasing.

26

5.4.3. T Distribution

The results are the same for the T distribution with M being the most powerful when order is non-decreasing. When the order is not non-decreasing, the Durbin test tends to be the most powerful, regardless of sample size as seen in Tables 26 and 27.

5.5. Five Treatments with Three Appearing Per Block

The results show that M is the most powerful of the two tests given that the order is non-decreasing; and the Durbin test is the most powerful when order is not non-decreasing, regardless of the sample size or the distribution. The results are shown in Tables 28 to 33.

5.6. Five Treatments with Four Appearing Per Block

Tables 34 to 39 reflect the same trend that M is the more powerful than the Durbin test when the order is non-decreasing and vice versa when the order is not non-decreasing, regardless of the sample size.

Table 4. Power Comparison under Normal Distribution t = 3, k = 2 & b = 30

	μ1	μ2	μ3	M %	95% CI		Durbin %	95% CI		WSR	95% CI	
Non decreasing	0	0	0	4.57	4.16	4.98	3.81	3.43	4.19	5.33	4.8	5.8
	0	0.5	1	67.27	66.35	68.19	41.66	40.69	42.63	67.21	66.29	68.13
	0	0.25	0.25	47.65	46.67	48.63	25.64	24.78	26.5	43.81	42.84	44.78
	0	0.25	1	67.42	66.5	68.34	44.78	43.81	45.75	67.05	66.13	67.97
	0	0.8	1	66.9	65.98	67.82	44.87	43.9	45.84	66.83	65.91	67.75
	0	0.3	0.5	27.51	26.63	28.39	12.51	11.86	13.16	23.14	22.31	23.98
	0	0	1	65.94	65.01	66.87	52.79	51.81	53.77	64.32	63.38	65.26
	0	1	1	65.49	64.56	66.42	53.06	52.08	54.04	64.43	63.49	65.37
Randomly order	0	1	0	3.59	3.23	3.95	52.75	51.77	53.73	2.19	1.9	2.48
	1	0	1	4.03	3.64	4.42	53.87	52.89	54.85	2.2	1.91	2.49
	1	0	0.5	0.24	0.14	0.34	41.07	40.11	42.03	17.88	17.13	18.63
	0.3	0.5	0	0.95	0.76	1.14	12.52	11.87	13.17	10.39	9.79	10.99

Table 5. Power Comparison under Normal Distribution t = 3, k = 2 & b = 12

	μ1	μ2	μ3	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	2.92	2.59	3.25	5.13	4.70	5.56
	0	0	1	27.84	26.96	28.72	23.17	22.34	24.00
	0	0.5	1	29.89	28.99	30.79	18.82	18.05	19.59
	0	1	1	27.46	26.59	28.33	22.70	21.88	23.52
	0	0.5	1.5	51.99	51.01	52.97	36.37	35.43	37.31
Random order	0	1	0	2.68	2.36	3.00	23.30	22.47	24.13
	1	0	1	2.50	2.19	2.81	22.67	21.85	23.49

Table 6. Power Comparison under Exponential Distribution t = 3, k = 2 & b = 30

	μ1	μ2	μ3	M %	95% CI		Durbin %	95% CI		WSR	95% CI	
Non decreasing	0	0	0	4.65	4.24	5.06	4.17	3.78	4.56	4.99	4.56	5.42
	0	0.5	1	87.95	87.31	88.59	67.21	66.29	68.13	79.39	78.59	80.18
	0	0.25	0.75	71.32	70.43	72.21	46.44	45.46	47.42	57.3	56.33	58.27
	0	0.25	1	86.68	86.01	87.35	69.12	68.21	70.03	78.55	77.75	79.35
	0	0.8	1	85.71	85.02	86.40	69.78	68.88	70.68	77.72	76.9	78.54
	0	0.3	0.5	47.34	46.36	48.32	24.51	23.67	25.35	32.67	31.75	33.59
	0	0	1	82.83	82.09	83.57	75.02	74.17	75.87	74.84	73.99	75.69
	0	1	1	82.67	81.93	83.41	74.95	74.10	75.80	74.89	74.04	75.74
Random order	0	1	0	3.50	3.14	3.86	74.90	74.05	75.75	2.02	1.74	2.29
	1	0	1	3.61	3.24	3.98	74.26	73.40	75.12	1.56	1.32	1.8
	1	0	0.5	0.05	0.01	0.09	67.46	66.54	68.38	22.54	21.72	23.26
	0.8	0	1	11.66	11.03	12.29	69.45	68.55	70.35	4.76	4.34	5.18
	0.3	0.5	0	0.40	0.28	0.52	24.91	24.06	25.76	13.13	12.47	13.79
	0.25	0	0.75	39.55	38.59	40.51	47.01	46.03	47.99	27.05	26.18	27.92
	0.25	1	0	0.45	0.32	0.58	69.31	68.41	70.21	6.51	6.02	6.99

Table 7. Power Comparison under Exponential Distribution t = 3, k = 2 & b = 12

	μ1	μ2	μ3	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	3.20	2.86	3.54	4.99	4.56	5.42
	0	0	1	38.74	37.79	39.69	34.23	33.30	35.16
	0	1	1	39.41	38.45	40.37	34.18	33.25	35.11
	0	0.5	1	45.59	44.61	46.57	30.25	29.35	31.15
	0	0.6	0.9	39.73	38.77	40.69	26.98	26.11	27.85

Table 7. continued

	μ1	μ2	μ3	M%	95% CI		Durbin %	95% CI	
Random order	0	1	0	2.02	1.74	2.30	33.59	32.66	34.52
	1	0	1	2.03	1.75	2.31	34.97	34.04	35.90
	2	0	1	0.02	0.00	0.05	68.85	67.94	69.76
	0.25	1	0	0.81	0.63	0.99	30.40	29.50	31.30

Table 8. Power Comparison under the T Distribution t = 3, k = 2 & b = 30

	μ1	μ2	μ3	M %	95% CI		Durbin %	95% CI		WSR	95% CI	
Non decreasing	0	0	0	4.29	3.89	4.69	4.03	3.64	4.42	5.66	5.21	6.11
	0	0.5	1	51.12	50.14	52.10	26.77	25.90	27.64	42.25	41.28	43.22
	0	0.25	0.75	35.01	34.08	35.94	16.98	16.24	17.72	26.23	25.37	27.09
	0	0.25	1	50.77	49.79	51.75	29.11	28.22	30.00	41.47	40.50	42.44
	0	0.8	1	50.54	49.56	51.52	29.43	28.54	30.32	41.50	40.53	42.47
	0	0.3	0.5	21.34	20.54	22.14	9.14	8.58	9.70	14.85	14.15	15.55
	0	0	1	49.23	48.25	50.21	35.56	34.62	36.50	41.52	40.55	42.49
	0	1	1	49.48	48.50	50.46	34.33	33.40	35.26	40.46	39.50	41.42
Random order	0	1	0	4.00	3.62	4.38	35.38	34.44	36.32	3.93	3.55	4.31
	1	0	1	4.07	3.68	4.46	35.96	35.02	36.90	3.47	3.11	3.83
	1	0	0.5	0.49	0.35	0.63	27.89	27.01	28.77	12.33	11.69	12.97
	0.8	0	1	8.44	7.90	8.98	30.90	29.99	31.81	5.22	4.78	5.66
	0.3	0.5	0	1.23	1.01	1.45	9.09	8.53	9.65	8.36	7.81	8.90
	0.25	0	0.75	20.55	19.76	21.34	17.09	16.35	17.83	14.22	13.54	14.90
	0.25	1	0	1.48	1.24	1.72	29.23	28.34	30.12	5.05	4.62	5.48

30

Table 9. Power Comparison under the T Distribution t = 3, k = 2 & b = 12

	$\mu 1$	$\mu 2$	$\mu 3$	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	2.94	2.61	3.27	5.21	4.77	5.65
	0	0.6	1.5	37.00	36.05	37.95	24.94	24.09	25.79
	0	1	2	54.63	53.65	55.61	38.26	37.31	39.21
	0	2	2	48.31	47.33	49.29	43.70	42.73	44.67
	0	0	2	48.75	47.77	49.73	44.32	43.35	45.29
Random order	0	1	0	2.79	2.47	3.11	16.44	15.71	17.17
	1	0	1	2.45	2.15	2.75	16.77	16.04	17.50
	2.5	0	0	0.00	0.00	0.00	58.47	57.50	59.44
	0.25	1	0	1.47	1.23	1.71	14.85	14.15	15.55

Table 10. Power Comparison under Normal distribution t = 4, k = 2 & b = 30.

	$\mu 1$	$\mu 2$	$\mu 3$	$\mu 4$	M %	95% CI		Durbin %	95% CI		WSR %	95% CI	
Non decreasing	0	0	0	0	4.46	4.06	4.86	4.97	4.54	5.40	5.03	4.6	5.46
	0	0.5	1	1.5	84.65	83.94	85.36	56.95	55.98	57.92	84.81	84.11	85.51
	0	0.25	0.75	1.75	91.46	90.91	92.01	73.29	72.42	74.16	91.91	91.38	92.44
	0	0.1	0.4	1.3	70.85	69.96	71.74	49.89	48.91	50.87	68.61	67.7	69.52
	0	0.8	1	1.3	69.43	68.53	70.33	44.86	43.89	45.83	68.33	67.42	69.24
	0	0.3	0.5	0.6	27.23	26.36	28.10	12.99	12.33	13.65	23.56	22.73	24.39
	0	0	1	1	68.96	68.05	69.87	48.20	47.22	49.18	63.26	62.32	64.2
	0	0	0	1	44.96	43.98	45.94	36.94	35.99	37.89	39.72	38.76	40.67
Random order	1	0	0.5	0.8	3.68	3.31	4.05	28.41	27.53	29.29	3.23	2.88	3.58
	1	1.5	0.5	0.8	0.42	0.29	0.55	27.01	26.14	27.88	13.02	12.36	13.68
	0	1	0.1	0.5	7.60	7.08	8.12	30.36	29.46	31.26	4.41	4.01	4.81
	1	1	0	1	1.00	0.80	1.20	37.05	36.10	38.00	6.02	5.55	6.49

Table 11. Power Comparison under Normal distribution t = 4, k = 2 & b = 12.

	μ1	μ2	μ3	μ4	M %	CI		Durbin %	CI	
Non decreasing	0	0	0	0	5.04	3.65	4.43	2.79	2.47	3.11
	0	0	0	3	57.49	56.52	58.46	26.48	25.62	27.34
	0	0	1.5	1.5	56.89	55.92	57.86	31.12	30.21	32.03
	0	2	2	2	44.95	43.98	45.92	19.44	18.66	20.22
	0	0.7	1.4	2.1	69.62	68.72	70.52	27.13	26.26	28.00
	0	0.7	1.8	2	71.14	70.25	72.03	32.65	31.73	33.57
Random order	3	0	0.5	3	0.71	0.55	0.87	81.27	80.51	82.03
	3	0	0.5	0	0.00	0.00	0.00	30.13	29.23	31.03
	3	0	0	0	0.00	0.00	0.00	27.04	26.17	27.91
	3	0.75	0	0	0.00	0.00	0.00	35.00	34.07	35.93

Table 12. Power Comparison under Exponential distribution t = 4, k = 2 & b = 30

	μ1	μ2	μ3	μ4	M %	CI		Durbin %	CI		WSR %	CI	
Non decreasing	0	0	0	0	4.66	4.25	5.07	5.03	4.60	5.46	4.65	4.23	5.06
	0	0.5	1	1.5	94.97	94.54	95.40	76.80	75.97	77.63	91.15	90.59	91.77
	0	0.25	0.75	1.75	97.34	97.02	97.66	86.61	85.94	87.28	95.13	94.71	95.55
	0	0.1	0.4	1.3	86.64	85.97	87.31	69.75	68.85	70.65	78.13	77.32	78.94
	0	0.8	1	1.3	86.98	86.32	87.64	66.51	65.58	67.44	78.8	77.99	79.6
	0	0.3	0.5	0.6	46.56	45.58	47.54	24.03	23.19	24.87	31.6	30.69	32.51
	0	0	1	1	85.28	84.59	85.97	69.26	68.36	70.16	68.48	74.46	76.15
	0	0	0	1	60.42	59.46	61.38	55.55	54.58	56.52	49.86	48.88	50.84

Table 12. continued

	μ1	μ2	μ3	μ4	M%	95% CI		Durbin %	95% CI		WSR %	95% CI	
Random order	1	0	0.5	0.8	2.82	2.50	3.14	49.74	48.76	50.72	2.96	2.63	3.29
	1	1.5	0.5	0.8	0.16	0.08	0.24	47.61	46.63	48.59	15.13	14.42	15.83
	0	1	0.1	0.5	8.68	8.13	9.23	51.77	50.79	52.75	4.33	3.93	4.37
	1	0	0.5	0.1	0.03	0.00	0.06	51.63	50.65	52.61	28.47	27.59	29.35
	1	0	1	1	13.19	12.53	13.85	56.27	55.30	57.24	6.58	6.09	7.07
	1	1	0	1	0.75	0.58	0.92	54.86	53.88	55.84	6.66	6.17	7.15

Table 13. Power Comparison under Exponential distribution t = 4, k = 2 & b = 12

	μ1	μ2	μ3	μ4	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	3.86	3.48	4.24	3.02	2.68	3.36
	0	0	0	2	46.93	45.95	47.91	21.13	20.33	21.93
	0	0	1	1	47.03	46.05	48.01	22.46	21.64	23.28
	0	2	2	2	45.73	44.75	46.71	20.40	19.61	21.19
	0	1	2	3	93.96	93.49	94.43	59.17	58.21	60.13
	0	1	1.5	3	92.01	91.48	92.54	51.17	50.19	52.15
Random order	3	0	0.5	0	0.00	0.00	0.00	34.25	33.32	35.18
	3	0	0	0	0.00	0.00	0.00	24.89	24.04	25.74
	3	0.75	0	0	0.00	0.00	0.00	42.25	41.28	43.22
	3	0	0.5	3	0.90	0.71	1.09	78.02	77.21	78.83

Table 14. Power Comparison under the T distribution t = 4, k = 2 and b = 30.

	μ1	μ2	μ3	μ4	M %	95% CI		Durbin %	95% CI		WSR %	95% CI	
Non decreasing	0	0	0	0	4.36	3.96	4.76	4.97	4.54	5.40	5.08	4.65	5.51
	0	0.5	1	1.5	68.34	67.43	69.25	38.26	37.31	39.21	58.03	57.06	58.99
	0	0.25	0.75	1.75	77.14	76.32	77.96	51.37	50.39	52.35	69.3	68.4	70.2
	0	0.1	0.4	1.3	54.26	53.28	55.24	32.82	31.90	33.74	44.01	43.04	44.98
	0	0.8	1	1.3	53.48	52.50	54.46	29.44	28.55	30.33	43	42.03	43.98
	0	0.3	0.5	0.6	21.50	20.69	22.31	10.13	9.54	10.72	14.83	14.13	15.53
	0	0	1	1	52.41	51.43	53.39	31.94	31.03	32.85	40.63	39.67	41.59
	0	0	0	1	34.07	33.14	35.00	24.29	23.45	25.13	25.32	24.47	26.17
Random order	1	0	0.5	0.8	3.99	3.61	4.37	18.98	18.21	19.75	4.08	3.69	4.47
	1	1.5	0.5	0.8	0.56	0.41	0.71	18.66	17.90	19.42	9.92	9.33	10.51
	0	1	0.1	0.5	6.87	6.37	7.37	21.20	20.40	22.00	4.77	4.35	5.19
	1	0	0.5	0.1	0.21	0.12	0.30	20.81	20.01	21.61	14.88	14.19	15.58

Table 15. Power Comparison under the T distribution t = 4, k = 2 and b = 12.

	μ1	μ2	μ3	μ4	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	3.88	3.50	4.26	2.81	2.49	3.13
	0	0.5	1	1.5	34.19	33.26	35.12	9.88	9.30	10.46
	0	0.25	0.75	1.75	39.68	38.72	40.64	12.22	11.58	12.86
	0	0.1	0.4	1.3	25.68	24.82	26.54	7.89	7.36	8.42
	0	0.8	1	1.3	25.63	24.77	26.49	7.25	6.74	7.76
	0	0	1	1	25.13	24.28	25.98	9.08	8.52	9.64
Random order	3	0.75	0	0	0.00	0.00	0.00	24.19	23.35	25.03
	3	0	0	0	0.00	0.00	0.00	19.91	19.13	20.69
	3	0	0.5	3	1.85	1.59	2.11	52.24	51.26	53.22

Table 16. Power Comparison under Normal distribution with t = 4, k = 3 & b = 32

	μ1	μ2	μ3	μ4	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	5.34	4.90	5.78	4.73	4.31	5.15
	0	0.25	0.5	0.75	71.21	70.32	72.10	38.39	37.44	39.34
	0	0.25	0.75	1	92.12	91.59	92.65	70.30	69.40	71.20
	0	0	0.75	0.75	83.49	82.76	84.22	65.58	64.65	66.51
	0	0	0	1	81.54	80.78	82.30	77.22	76.40	78.04
	0	0.75	0.75	0.75	62.03	61.08	62.98	50.41	49.43	51.39
Random order	1	0	0	0	0.00	0.00	0.00	78.48	77.67	79.29
	1	0.5	0.1	0	0.00	0.00	0.00	69.24	68.34	70.14
	0	1	0	1	51.03	50.05	52.01	89.92	89.33	90.51
	1	0.3	0.6	0.5	0.33	0.22	0.44	32.64	31.72	33.56

Table 17. Power Comparison under Normal distribution with t = 4, k = 3 & b = 12

	μ1	μ2	μ3	μ4	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	4.68	4.27	5.09	3.89	3.51	4.27
	0	0.25	0.75	1	56.24	55.27	57.21	24.76	23.91	25.61
	0	0	0	1	44.26	43.29	45.23	29.67	28.77	30.57
	0	0	0.75	0.75	45.06	44.08	46.04	23.38	22.55	24.21
	0	0.75	0.75	0.75	30.44	29.54	31.34	17.14	16.40	17.88
	0	0.25	0.5	0.75	37.03	36.08	37.98	13.30	12.63	13.97
Random order	1	0	0	0	0.04	0.00	0.08	29.68	28.78	30.58
	1	0.5	0.1	0	0.01	0.00	0.03	24.65	23.81	25.49
	0	1	0	1	23.66	22.83	24.49	41.01	40.05	41.97
	0	3	0	0.5	0.35	0.23	0.47	99.05	98.86	99.24

Table 18. Power Comparison under Exponential distribution with t = 4, k = 3 & b = 32

	μ1	μ2	μ3	μ4	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	5.61	5.16	6.06	4.84	4.42	5.26
	0	0.2	0.4	0.6	82.27	81.52	83.02	52.52	51.54	53.50
	0	0.2	0.45	0.5	75.37	74.53	76.21	45.38	44.40	46.36
	0	0	0.5	0.5	81.38	80.62	82.14	62.22	61.27	63.17
	0	0	0	0.5	60.43	59.47	61.39	49.53	48.55	50.51
	0	0.5	0.5	0.5	59.99	59.03	60.95	46.97	45.99	47.95
Random order	1	0	0	0	0.00	0.00	0.00	95.71	95.31	96.11
	1	0.5	0.1	0	0.00	0.00	0.00	93.51	93.03	93.99
	0	1	0	1	66.77	65.85	67.69	98.67	98.45	98.89
	1	0.3	0.6	0.5	0.08	0.02	0.14	63.68	62.74	64.62

Table 19. Power Comparison under Exponential distribution with t = 4, k = 3 & b = 12

	μ1	μ2	μ3	μ4	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	4.72	4.3	5.14	3.93	3.55	4.31
	0	0	0	1	58.93	57.97	59.89	47.18	46.20	48.16
	0	0	1	1	82.35	81.60	83.10	62.09	61.14	63.04
	0	1	1	1	58.40	57.43	59.37	46.53	45.55	47.51
	0	0.3	0.6	0.9	69.80	68.90	70.70	34.63	33.70	35.56
	0	0.3	0.5	1	72.73	71.86	73.60	38.68	37.73	39.63
Random order	1	0	0	0	0.00	0.00	0.00	46.43	45.45	47.41
	1	0.5	0.1	0	0.00	0.00	0.00	44.11	43.14	45.08
	0	1	0	1	31.02	30.11	31.93	61.97	61.02	62.92
	0	3	0	0.5	0.50	0.36	0.64	99.16	98.98	99.34

Table 20. Power Comparison under the T distribution with t = 4, k = 3 & b = 32

	μ1	μ2	μ3	μ4	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	5.38	4.94	5.82	4.51	4.10	4.92
	0	0	0	0.75	48.48	47.50	49.46	33.75	32.82	34.68
	0	0	0.75	0.75	68.96	68.05	69.87	45.07	44.09	46.05
	0	0.75	0.75	0.75	48.47	47.49	49.45	34.77	33.84	35.70
	0	0.25	0.5	0.75	56.96	55.99	57.93	25.91	25.05	26.77
	0	0.25	0.75	1	79.77	78.98	80.56	49.01	48.03	49.99
Random order	0	1	0	1	39.61	38.65	40.57	71.21	70.32	72.10
	1	0.3	0.6	0.5	0.67	0.51	0.83	21.51	20.70	22.32
	1	0	0	0	0.01	0.00	0.03	56.09	55.12	57.06
	1	0.5	0.1	0	0.01	0.00	0.03	48.98	48.00	49.96

Table 21. Power Comparison under the T distribution with t = 4, k = 3 & b = 12

	μ1	μ2	μ3	μ4	M %	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	4.74	4.32	5.16	4.00	3.62	4.38
	0	0	0	0.75	22.86	22.04	23.68	11.93	11.29	12.57
	0	0	0.75	0.75	34.07	33.14	35.00	15.58	14.87	16.29
	0	0.75	0.75	0.75	22.97	22.15	23.79	11.90	11.27	12.53
	0	0.25	0.5	0.75	26.97	26.10	27.84	9.70	9.12	10.28
	0	0.25	0.75	1	41.94	40.97	42.91	16.82	16.09	17.55
Random order	0	1	0	1	18.38	17.62	19.14	26.39	25.53	27.25
	1	0.3	0.6	0.5	1.21	1.00	1.42	8.69	8.14	9.24
	1	0	0	0	0.14	0.07	0.21	19.61	18.83	20.39
	0	3	0	0.5	0.58	0.43	0.73	86.29	85.62	86.96

Table 22. Power Comparison under Normal distribution t = 5, k = 2 & b = 30

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI		WSR %	95% CI	
Non	0	0	0	0	0	4.84	4.42	5.26	4.40	4.00	4.80	4.91	4.48	5.33
decreasing	0	0.25	0.5	0.75	1	47.87	46.89	48.85	18.50	17.74	19.26	43.3	42.32	44.27
	0	0.1	0.3	0.5	1.5	68.01	67.10	68.92	39.15	38.19	40.11	65.64	64.71	66.57
	0	0.05	0.2	0.65	2	87.12	86.46	87.78	66.51	65.58	67.44	86.52	85.85	87.19
	0	0.8	1	1.3	1.4	66.69	65.77	67.61	34.95	34.02	35.88	63.77	62.83	64.71
	0	0.3	0.5	0.6	1	43.03	42.06	44.00	16.07	15.35	16.79	38.17	37.22	39.12
	0	0	0	1	1	59.86	58.90	60.82	35.11	34.17	36.05	54.82	53.19	55.15
	0	1	1	1	1	32.05	31.14	32.96	22.04	21.23	22.85	26.94	26.07	27.81
	0	0	0	0	1	31.31	30.40	32.22	22.11	21.30	22.92	26.41	25.55	27.27
Random	1	0	0.5	0.1	0.6	1.02	0.82	1.22	19.58	18.80	20.36	5.86	5.4	6.32
order	1	1.5	0.5	0.8	0.2	0.06	0.01	0.11	28.46	27.58	29.34	33.24	32.32	34.16
	0	1	0.1	0.5	1	22.02	21.21	22.83	26.51	25.64	27.38	15.73	15.02	16.44
	1	0	1	1	1	12.79	12.14	13.44	22.40	21.58	23.22	8.42	7.88	8.96
	1	1	0	1	1	4.05	3.66	4.44	22.30	21.48	23.12	3.35	2.99	3.7

Table 23. Power Comparison under Normal distribution t = 5, k = 2 & b = 20.

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	4.54	4.13	4.95	4.51	4.10	4.92
decreasing	0	0.25	0.5	0.75	1	37.28	36.33	38.23	12.14	11.50	12.78
	0	0.1	0.3	0.5	1.5	52.42	51.44	53.40	23.20	22.37	24.03
	0	0.05	0.2	0.65	2	71.24	70.35	72.13	40.46	39.50	41.42
	0	0.8	1	1.3	1.4	51.03	50.05	52.01	21.42	20.62	22.22
	0	0.3	0.5	0.6	1	32.43	31.51	33.35	11.82	11.19	12.45
	0	0	0	1	1	46.08	45.10	47.06	22.65	21.83	23.47
	0	1	1	1	1	24.54	23.70	25.38	14.61	13.92	15.30
	0	0	0	0	1	24.79	23.94	25.64	14.20	13.52	14.88

Table 23. continued

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Random	0	2	0.5	0	2	15.56	14.85	16.27	65.17	64.24	66.10
order	1	1.5	0.5	0.8	0.2	0.05	0.01	0.09	17.63	16.88	18.38
	0	1	0.1	0.5	1	16.82	16.09	17.55	16.58	15.85	17.31
	1	0	1	1	1	10.97	10.36	11.58	13.96	13.28	14.64
	1	1	0	1	1	3.52	3.16	3.88	14.18	13.50	14.86

Table 24. Power Comparison under Exponential distribution t = 5, k = 2 & b = 30.

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI		WSR %	95% CI	
Non	0	0	0	0	0	4.30	3.90	4.70	4.24	3.85	4.63	5.11	4.68	5.54
decreasing	0	0.25	0.5	0.75	1	72.19	71.31	73.07	34.73	33.80	35.66	56.6	55.63	57.57
	0	0.1	0.3	0.5	1.5	83.71	82.99	84.43	56.26	55.29	57.23	80.29	79.51	81.07
	0	0.05	0.2	0.65	2	94.16	93.70	94.62	78.78	77.98	79.58	91.62	91.08	92.16
	0	0.8	1	1.3	1.4	85.24	84.54	85.94	53.29	52.31	54.27	75.04	74.19	75.89
	0	0.3	0.5	0.6	1	65.70	64.77	66.63	30.94	30.03	31.85	50.34	49.36	51.32
	0	0	0	1	1	77.72	76.90	78.54	54.30	53.32	55.28	65.25	64.32	66.18
	0	1	1	1	1	44.27	43.30	45.24	34.90	33.97	35.83	34.62	33.69	35.55
	0	0	0	0	1	43.01	42.04	43.98	34.51	33.58	35.44	33.47	32.55	34.39
Random	1	0	0.5	0.1	0.6	0.68	0.52	0.84	35.03	34.09	35.97	6.12	5.65	6.59
order	1	1.5	0.5	0.8	0.2	0.01	0.00	0.03	47.90	46.92	48.88	43.23	42.26	44.2
	0	1	0.1	0.5	1	30.21	29.31	31.11	45.97	44.99	46.95	19.33	18.56	21.1
	1	0	1	1	1	16.54	15.81	17.27	34.91	33.98	35.84	9.43	8.86	10
	1	1	0	1	1	3.48	3.12	3.84	35.10	34.16	36.04	2.74	2.42	3.06

Table 25. Power Comparison under Exponential distribution t = 5, k = 2 & b = 20.

	$\mu1$	$\mu2$	$\mu3$	$\mu4$	$\mu5$	M%	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	0	4.57	4.16	4.98	4.37	3.97	4.77
	0	0.25	0.5	0.75	1	56.15	55.18	57.12	21.61	20.80	22.42
	0	0.1	0.3	0.5	1.5	69.01	68.10	69.92	34.22	33.29	35.15
	0	0.05	0.2	0.65	2	81.96	81.21	82.71	51.53	50.55	52.51
	0	0.8	1	1.3	1.4	69.06	68.15	69.97	33.37	32.45	34.29
	0	0.3	0.5	0.6	1	50.26	49.28	51.24	18.89	18.12	19.66
	0	0	0	1	1	62.11	61.16	63.06	34.89	33.96	35.82
	0	1	1	1	1	32.41	31.49	33.33	20.71	19.92	21.50
	0	0	0	0	1	32.46	31.54	33.38	21.25	20.45	22.05
Random order	0	2	0.5	0	2	16.59	15.86	17.32	73.14	72.27	74.01
	1	1.5	0.5	0.8	0.2	0.04	0.00	0.08	29.52	28.63	30.41
	0	1	0.1	0.5	1	23.30	22.47	24.13	29.14	28.25	30.03
	1	0	1	1	1	12.88	12.22	13.54	20.42	19.63	21.21
	1	1	0	1	1	3.91	3.53	4.29	19.70	18.92	20.48

Table 26. Power Comparison under the T distribution t = 5, k = 2 & b = 30.

	$\mu1$	$\mu2$	$\mu3$	$\mu4$	$\mu5$	M%	95% CI		Durbin %	95% CI		WSR %	95% CI	
Non decreasing	0	0	0	0	0	4.28	3.88	4.68	4.21	3.82	4.60	4.94	4.52	5.36
	0	0.25	0.5	0.75	1	36.56	35.62	37.50	13.19	12.53	13.85	26.35	25.49	27.21
	0	0.1	0.3	0.5	1.5	52.48	51.50	53.46	25.53	24.68	26.38	41.14	40.18	42.1
	0	0.05	0.2	0.65	2	71.09	70.20	71.98	44.15	43.18	45.12	61.65	60.7	62.6
	0	0.8	1	1.3	1.4	51.32	50.34	52.30	22.82	22.00	23.64	39.94	38.98	40.9
	0	0.3	0.5	0.6	1	32.84	31.92	33.76	11.99	11.35	12.63	23.42	22.59	24.25
	0	0	0	1	1	45.85	44.87	46.83	22.52	21.70	23.34	34.42	33.49	35.35
	0	1	1	1	1	24.23	23.39	25.07	15.46	14.75	16.17	17.53	16.78	18.28
	0	0	0	0	1	24.56	23.72	25.40	15.32	14.61	16.03	16.86	16.13	17.59

40

Table 26. continued

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI		WSR %	95% CI	
Random	1	0	0.5	0.1	0.6	1.59	1.34	1.84	13.49	12.82	14.16	5.55	5.1	6
order	1	1.5	0.5	0.8	0.2	0.14	0.07	0.21	18.09	17.34	18.84	21.67	20.86	22.48
	0	1	0.1	0.5	1	16.87	16.14	17.60	17.82	17.07	18.57	10.98	10.37	11.59
	1	0	1	1	1	11.05	10.44	11.66	15.22	14.52	15.92	7	6.5	7.5
	1	1	0	1	1	3.85	3.47	4.23	14.94	14.24	15.64	3.93	3.55	4.31

Table 27. Power Comparison under the T distribution t = 5, k = 2 & b = 20.

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	4.60	4.19	5.01	4.31	3.91	4.71
decreasing	0	0.25	0.5	0.75	1	27.61	26.73	28.49	9.96	9.37	10.55
	0	0.1	0.3	0.5	1.5	39.22	38.26	40.18	16.15	15.43	16.87
	0	0.05	0.2	0.65	2	55.15	54.18	56.12	26.49	25.63	27.35
	0	0.8	1	1.3	1.4	38.40	37.45	39.35	14.90	14.20	15.60
	0	0.3	0.5	0.6	1	24.96	24.11	25.81	9.07	8.51	9.63
	0	0	0	1	1	34.44	33.51	35.37	15.41	14.70	16.12
	0	1	1	1	1	19.15	18.38	19.92	10.71	10.10	11.32
	0	0	0	0	1	19.24	18.47	20.01	10.80	10.19	11.41
Radom	0	2	0.5	0	2	12.95	12.29	13.61	42.61	41.64	43.58
order	1	1.5	0.5	0.8	0.2	0.32	0.21	0.43	12.95	12.29	13.61
	0	1	0.1	0.5	1	13.98	13.30	14.66	12.85	12.19	13.51
	1	0	1	1	1	10.09	9.50	10.68	10.85	10.24	11.46
	1	1	0	1	1	4.28	3.88	4.68	10.78	10.17	11.39

Table 28. Power Comparison under Normal Distribution t = 5, k = 3 & b = 30

	$\mu1$	$\mu2$	$\mu3$	$\mu4$	$\mu5$	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	4.72	4.30	5.14	4.51	4.10	4.92
decreasing	0	0	0	0	1	60.68	59.72	61.64	55.80	54.83	56.77
	0	0	0	1	1	90.92	90.36	91.48	78.88	78.08	79.68
	0	0	1	1	1	91.17	90.61	91.73	78.60	77.80	79.40
	0	1	1	1	1	61.01	60.05	61.97	56.08	55.11	57.05
	0	0.25	0.5	0.75	1	81.63	80.87	82.39	47.82	46.84	48.80
	0	0.25	0.6	0.9	1	84.13	83.41	84.85	53.88	52.90	54.86
Random order	0.5	0	0	0	0	0.29	0.18	0.40	15.96	15.24	16.68
	0.5	0.6	0	1	0	0.71	0.55	0.87	53.12	52.14	54.10
	0.6	0.5	0.4	0.3	0.2	0.34	0.23	0.45	9.60	9.02	10.18
	0	0	0.5	0	0	4.53	4.12	4.94	15.56	14.85	16.27

Table 29. Power Comparison under Normal Distribution t = 5, k = 3 & b = 10

	$\mu1$	$\mu2$	$\mu3$	$\mu4$	$\mu5$	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	5.39	4.95	5.83	3.33	2.98	3.68
decreasing	0	0	0	0	1	29.45	28.56	30.34	13.63	12.96	14.30
	0	0	0	1	1	53.13	52.15	54.11	21.98	21.17	22.79
	0	0	1	1	1	53.48	52.50	54.46	21.45	20.65	22.25
	0	1	1	1	1	30.07	29.17	30.97	14.50	13.81	15.19
	0	0.25	0.5	0.75	1	42.48	41.51	43.45	11.54	10.91	12.17
	0	0.25	0.6	0.9	1	46.56	45.58	47.54	13.16	12.50	13.82
Random order	1	2	0	0	0	0.00	0.00	0.00	53.64	52.66	54.62
	0.5	0.6	0	1	0	1.94	1.67	2.21	13.57	12.90	14.24
	0.6	0.5	0.4	0.3	0.2	1.28	1.06	1.50	4.18	3.79	4.57
	0	0	0.5	0	0	4.96	4.53	5.39	5.63	5.18	6.08

Table 30. Power Comparison under Exponential Distribution t = 5, k = 3 & b = 30

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	4.75	4.33	5.17	4.64	4.23	5.05
decreasing	0	0	0	0	0.5	40.79	39.83	41.75	31.47	30.56	32.38
	0	0	0	0.5	0.5	70.81	69.92	71.70	49.06	48.08	50.04
	0	0	0.5	0.5	0.5	69.81	68.91	70.71	47.35	46.37	48.33
	0	0.5	0.5	0.5	0.5	41.35	40.38	42.32	32.09	31.18	33.00
	0	0.1	0.2	0.3	0.4	48.01	47.03	48.99	19.81	19.03	20.59
	0	0.1	0.22	0.3	0.6	68.02	67.11	68.93	35.65	34.71	36.59
Random	0.5	0	0	0	0	0.03	0.00	0.06	31.10	30.19	32.01
order	0.5	0.6	0	1	0	0.39	0.27	0.51	82.77	82.03	83.51
	0.6	0.5	0.4	0.3	0.2	0.02	0.00	0.05	19.75	18.97	20.53
	0	0	0.5	0	0	4.37	3.97	4.77	30.29	29.39	31.19

43

Table 31. Power Comparison under Exponential Distribution t = 5, k = 3 & b = 10

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	5.12	4.69	5.55	3.24	2.89	3.59
decreasing	0	0	0	0	1	38.56	37.61	39.51	20.09	19.30	20.88
	0	0	0	1	1	70.15	69.25	71.05	35.59	34.65	36.53
	0	0	1	1	1	69.81	68.91	70.71	36.67	35.73	37.61
	0	1	1	1	1	39.48	38.52	40.44	24.58	23.74	25.42
	0	0.25	0.5	0.75	1	64.52	63.58	65.46	21.22	20.42	22.02
	0	0.25	0.6	0.9	1	66.53	65.61	67.45	24.81	23.96	25.66
Random	1	2	0	0	0	0.00	0.00	0.00	65.94	65.01	66.87
order	0.5	0.6	0	1	0	1.18	0.97	1.39	24.94	24.09	25.79
	0.6	0.5	0.4	0.3	0.2	0.58	0.43	0.73	6.31	5.83	6.79
	0	0	0.5	0	0	5.05	4.62	5.48	8.26	7.72	8.80

Table 32. Power Comparison under the T Distribution t = 5, k = 3 & b = 30

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	4.45	4.05	4.85	4.80	4.38	5.22
decreasing	0	0	0	0	0.5	46.47	45.49	47.45	36.93	35.98	37.88
	0	0	0	0.5	0.5	77.17	76.35	77.99	56.58	55.61	57.55
	0	0	0.5	0.5	0.5	76.96	76.13	77.79	56.28	55.31	57.25
	0	0.5	0.5	0.5	0.5	45.67	44.69	46.65	37.24	36.29	38.19
	0	0.1	0.2	0.3	0.4	64.62	63.68	65.56	30.92	30.01	31.83
	0	0.1	0.22	0.3	0.6	69.08	68.17	69.99	36.73	35.79	37.67
Random	0.5	0	0	0	0	0.04	0.00	0.08	36.54	35.60	37.48
order	0.5	0.6	0	1	0	0.74	0.57	0.91	42.60	41.63	43.57
	0.6	0.5	0.4	0.3	0.2	0.01	0.00	0.03	31.04	30.13	31.95
	0	0	0.5	0	0	4.23	3.84	4.62	37.11	36.16	38.06

Table 33. Power Comparison under the T Distribution t = 5, k = 3 & b = 10

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	5.12	4.69	5.55	3.50	3.14	3.86
decreasing	0	0	0	0	1	22.82	22.00	23.64	9.80	9.22	10.38
	0	0	0	1	1	40.08	39.12	41.04	14.55	13.86	15.24
	0	0	1	1	1	40.05	39.09	41.01	14.40	13.71	15.09
	0	1	1	1	1	22.66	21.84	23.48	9.93	9.34	10.52
	0	0.25	0.5	0.75	1	32.27	31.35	33.19	8.33	7.79	8.87
	0	0.25	0.75	0.9	1	34.42	33.49	35.35	9.84	9.26	10.42
Random	1	3	0	0	0	0.00	0.00	0.00	56.94	55.97	57.91
order	1	2	0	0	0	0.01	0.00	0.03	33.70	32.77	34.63
	1	0	2	0	0	0.37	0.25	0.49	32.83	31.91	33.75
	0	0	1	0	0	4.56	4.15	4.97	9.68	9.10	10.26

Table 34. Power Comparison under Normal distribution t = 5, k = 4 & b = 30

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	0	5.50	5.05	5.95	5.00	4.57	5.43
	0	0	0	0	1	80.40	79.62	81.18	81.86	81.10	82.62
	0	0	0	0.5	0.5	62.19	61.24	63.14	38.21	37.26	39.16
	0	0	0.5	0.5	0.5	62.59	61.64	63.54	37.98	37.03	38.93
	0	0.5	0.5	0.5	0.5	36.47	35.53	37.41	26.13	25.27	26.99
	0	0.2	0.4	0.6	0.8	83.83	83.11	84.55	50.69	49.71	51.67
	0	0.2	0.5	0.6	0.9	88.70	88.08	89.32	60.15	59.19	61.11
Random order	0.5	0	0	0	0	0.18	0.10	0.26	25.87	25.01	26.73
	0.5	0	1	0	0.2	0.62	0.47	0.77	78.31	77.50	79.12
	0.8	0.4	0.3	0.1	0	0.00	0.00	0.00	48.41	47.43	49.39
	0	0	0.3	0	1	82.26	81.51	83.01	80.39	79.61	81.17

Table 35. Power Comparison under Normal distribution t = 5, k = 4 & b = 10

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non decreasing	0	0	0	0	0	5.09	4.66	5.52	3.91	3.53	4.29
	0	0	0	0	1	40.23	39.27	41.19	26.74	25.87	27.61
	0	0	0	0.5	0.5	30.20	29.30	31.10	11.42	10.80	12.04
	0	0	0.5	0.5	0.5	29.49	28.60	30.38	12.34	11.70	12.98
	0	0.5	0.5	0.5	0.5	18.12	17.37	18.87	8.78	8.23	9.33
	0	0.2	0.4	0.6	0.8	43.25	42.28	44.22	15.23	14.53	15.93
	0	0.2	0.5	0.6	0.9	49.60	48.62	50.58	17.51	16.77	18.25
Random order	2	1	0	0	0	0.00	0.00	0.00	85.89	85.21	86.57
	0.5	0	1	0	0.2	1.44	1.21	1.67	25.25	24.40	26.10
	0.8	0.4	0.3	0.1	0	0.09	0.03	0.15	14.26	13.57	14.95
	0	0	0.3	0	1	40.90	39.94	41.86	25.30	24.45	26.15

Table 36. Power Comparison under Exponential distribution t = 5, k = 4 & b = 30

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	5.22	4.78	5.66	4.66	4.25	5.07
decreasing	0	0	0	0	0.25	27.16	26.29	28.03	16.74	16.01	17.47
	0	0	0	0.25	0.25	46.33	45.35	47.31	24.44	23.60	25.28
	0	0	0.25	0.25	0.25	47.47	46.49	48.45	24.85	24.00	25.70
	0	0.5	0.5	0.5	0.5	58.86	57.90	59.82	50.76	49.78	51.74
	0	0.1	0.2	0.3	0.4	65.81	64.88	66.74	31.32	30.41	32.23
	0	0.1	0.15	0.4	0.55	87.16	86.50	87.82	58.40	57.43	59.37
Random	0.5	0	0	0	0	0.01	0.00	0.03	54.28	53.30	55.26
order	0.5	0	1	0	0.2	0.22	0.13	0.31	97.67	97.37	97.97
	0.8	0.4	0.3	0.1	0	0.00	0.00	0.00	83.01	82.27	83.75
	0	0	0.3	0	1	95.09	94.67	95.51	97.52	97.22	97.82

Table 37. Power Comparison under Exponential distribution t = 5, k = 4 & b = 10

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	4.83	4.41	5.25	3.53	3.17	3.89
decreasing	0	0	0	0	1	52.72	51.74	53.70	41.87	40.90	42.84
	0	0	0	0.5	0.5	48.37	47.39	49.35	22.48	21.66	23.30
	0	0	0.5	0.5	0.5	47.37	46.39	48.35	23.11	22.28	23.94
	0	0.5	0.5	0.5	0.5	27.54	26.66	28.42	16.42	15.69	17.15
	0	0.2	0.4	0.6	0.8	67.65	66.73	68.57	29.28	28.39	30.17
	0	0.2	0.5	0.6	0.9	73.37	72.50	74.24	34.43	33.50	35.36
Random	0.5	0	0	0	0	0.28	0.18	0.38	14.50	13.81	15.19
order	0.5	0	1	0	0.2	0.80	0.63	0.97	44.12	43.15	45.09
	0.8	0.4	0.3	0.1	0	0.00	0.00	0.00	27.21	26.34	28.08
	2	1	0	0	0	0.00	0.00	0.00	94.81	94.38	95.24

Table 38. Power Comparison under the T distribution t = 5, k = 4 & b = 30

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	5.27	4.83	5.71	4.77	4.35	5.19
decreasing	0	0	0	0	1	65.17	64.24	66.10	60.91	59.95	61.87
	0	0	0	1	1	92.71	92.20	93.22	81.32	80.56	82.08
	0	0	1	1	1	92.77	92.26	93.28	81.54	80.78	82.30
	0	1	1	1	1	64.88	63.94	65.82	60.76	59.80	61.72
	0	0.25	0.5	0.75	1	83.75	83.03	84.47	50.57	49.59	51.55
	0	0.25	0.75	0.9	1	87.35	86.70	88.00	59.10	58.14	60.06
Random order	1	0	0	0	0	0.02	0	0.05	60.02	59.06	60.98
	0.8	0.2	0	1	0	0.56	0.41	0.71	66.49	65.56	67.42
	1	0.5	0.25	0.1	0	0.01	0	0.03	50.41	49.43	51.39
	0	0	1	0	0	4.79	4.37	5.21	60.77	59.81	61.73

Table 39. Power Comparison under the T distribution t = 5, k = 4 & b = 10

	μ1	μ2	μ3	μ4	μ5	M%	95% CI		Durbin %	95% CI	
Non	0	0	0	0	0	4.85	4.43	5.27	4.01	3.63	4.39
decreasing	0	0	0	0	1	30.21	29.31	31.11	17.56	16.81	18.31
	0	0	0	1	1	53.97	52.99	54.95	27.39	26.52	28.26
	0	0	1	1	1	54.35	53.37	55.33	27.73	26.85	28.61
	0	1	1	1	1	30.29	29.39	31.19	18.23	17.47	18.99
	0	0.25	0.5	0.75	1	44.04	43.07	45.01	15.06	14.36	15.76
	0	0.25	0.75	0.9	1	47.14	46.16	48.12	18.10	17.35	18.85
Random order	1	0	0	0	0	0.21	0.12	0.30	17.69	16.94	18.44
	0.8	0.2	0	1	0	1.38	1.15	1.61	19.86	19.08	20.64
	2	1	0	0	0	0.00	0.00	0.00	61.61	60.66	62.56
	0	0	1	0	0	4.44	4.04	4.84	18.07	17.32	18.82

6. CONCLUSION

The study shows that, overall, the proposed test (M) can be used to test for ordered alternatives in a Balanced Incomplete Block Design (BIBD). The results, however, are limited to the three main cases looked at: three, four and five treatments. The power of the test is generally good while the probability of type I error is comparable to other established tests at around 0.05.

The study also compared M to the Durbin (Durbin, 1951) test. The study shows that M is significantly more powerful when testing for ordered alternatives regardless of the underlying distribution or sample size. Furthermore, M was compared to the Durbin and Wilcoxon Signed Rank (WSR) (Wilcoxon, 1945) tests when testing two non-decreasing treatment effects in a BIBD. The study shows that M is generally more powerful than both WSR and the Durbin regardless of the underlying distribution; comparison on a small sample was not done.

The study also shows that M is heavily dependent on the assumption that the order of treatment effects is non-decreasing. When this assumption is violated, M tends to have very low power regardless of the underlying distribution or the sample size. This is a good characteristic to have because we do not want a test to reject if H_a is not true and if the treatment effects are not non-decreasing, H_a is not true.

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APPENDIX A. EXPECTED VALUES AND VARIANCES

Table A1. Four Treatments with Two Treatments Per Block

For $t = 4$ and $k = 2$, there are $\binom{4}{2} = 6$ unique of blocks. The possible ranks are 0, 1 and 2. The following table illustrates the calculation.

	i^{th} combination	t1	t2	t3	t4	M_i	M_l	Var(M_i)
Block 1	1	1	2	0	0	5	4.5	0.25
	2	2	1	0	0	4		
Block 2	1	1	0	2	0	7	6	1
	2	2	0	1	0	5		
Block 3	1	0	1	2	0	8	7.5	0.25
	2	0	2	1	0	7		
Block 4	1	0	1	0	2	10	9	1
	2	0	2	0	1	8		
Block 5	1	0	0	1	2	11	10.5	0.25
	2	0	0	2	1	10		
Block 6	1	1	0	0	2	9	7.5	2.25
	2	2	0	0	1	6		
							45	5

$$E(M) = 45 \ and \ Var(M) = 5$$

Table A2. Four Treatments with Three Treatments Per Block

For $t = 4$ and $k = 3$, there are $\binom{4}{3} = 4$ unique blocks. The possible ranks are

0, 1, 2 and 3. The following table illustrates the calculation.

	i^{th} combination	t1	t2	t3	t4	M_i	M_l	$Var(M_l)$
Block 1	1	1	2	3	0	14	12	2.000
	2	1	3	2	0	13		
	3	2	1	3	0	13		
	4	2	3	1	0	11		
	5	3	1	2	0	11		
	6	3	2	1	0	10		
Block 2	1	1	2	0	3	17	14	4.667
	2	1	3	0	2	15		
	3	2	1	0	3	16		
	4	2	3	0	1	12		
	5	3	1	0	2	13		
	6	3	2	0	1	11		
Block 3	1	1	0	2	3	19	16	4.667
	2	1	0	3	2	18		
	3	2	0	1	3	17		
	4	2	0	3	1	15		
	5	3	0	1	2	14		
	6	3	0	2	1	13		
Block 4	1	0	1	2	3	20	18	2.000
	2	0	1	3	2	19		
	3	0	2	1	3	19		
	4	0	2	3	1	17		
	5	0	3	1	2	17		
	6	0	3	2	1	16		
							60	13.3333

$$E(M) = 60 \text{ and } Var(M) = 13.3333$$

51

Table A3. Five Treatments with Two Treatments Per Block

For $t = 5$ and $k = 2$, there are $\binom{5}{2} = 10$ unique blocks. The possible ranks are

0, 1 and 2. The following table illustrates the calculation.

	i^{th} combination	t1	t2	t3	t4	t5	M_i	M_l	$Var(M_i)$
Block 1	1	1	2	0	0	0	5	4.5	0.25
	2	2	1	0	0	0	4		
Block 2	1	1	0	0	0	2	11	9	4
	2	2	0	0	0	1	7		
Block 3	1	0	0	0	1	2	14	13.5	0.25
	2	0	0	0	2	1	13		
Block 4	1	1	0	0	2	0	9	7.5	2.25
	2	2	0	0	1	0	6		
Block 5	1	0	0	1	2	0	11	10.5	0.25
	2	0	0	2	1	0	10		
Block 6	1	0	0	1	0	2	13	12	1
	2	0	0	2	0	1	11		
Block 7	1	0	1	0	0	2	12	10.5	2.25
	2	0	2	0	0	1	9		
Block 8	1	0	1	2	0	0	8	7.5	0.25
	2	0	2	1	0	0	7		
Block 9	1	1	0	2	0	0	7	6	1
	2	2	0	1	0	0	5		
Block 10	1	0	1	0	2	0	10	9	1
	2	0	2	0	1	0	8		
								90	12.5

$$E(M) = 90 \ and \ Var(M) = 12.5$$

52

Table A4. Five Treatments with Three Per Block

For $t = 5$ and $k = 3$, there are $\binom{5}{3} = 10$ unique blocks. The possible ranks are

0, 1, 2 and 3. The following table illustrates the calculation.

	i^{th} combination	t1	t2	t3	t4	t5	M_i	M_l	Var(Mi)
Block 1	1	1	2	3	0	0	14	12	2.000
	2	1	3	2	0	0	13		
	3	2	1	3	0	0	13		
	4	2	3	1	0	0	11		
	5	3	1	2	0	0	11		
	6	3	2	1	0	0	10		
Block 2	1	1	2	0	3	0	17	14	4.667
	2	1	3	0	2	0	15		
	3	2	1	0	3	0	16		
	4	2	3	0	1	0	12		
	5	3	1	0	2	0	13		
	6	3	2	0	1	0	11		
Block 3	1	1	0	2	3	0	19	16	4.667
	2	1	0	3	2	0	18		
	3	2	0	1	3	0	17		
	4	2	0	3	1	0	15		
	5	3	0	1	2	0	14		
	6	3	0	2	1	0	13		
Block 4	1	0	1	2	3	0	20	18	2.000
	2	0	1	3	2	0	19		
	3	0	2	1	3	0	19		
	4	0	2	3	1	0	17		
	5	0	3	1	2	0	17		
	6	0	3	2	1	0	16		
Block 5	1	0	1	2	0	3	23	20	4.667
	2	0	1	3	0	2	21		
	3	0	2	1	0	3	22		
	4	0	2	3	0	1	18		
	5	0	3	1	0	2	19		
	6	0	3	2	0	1	17		

Block 6	1	0	1	0	2	3	25	22	4.667
	2	0	1	0	3	2	24		
	3	0	2	0	1	3	23		
	4	0	2	0	3	1	21		
	5	0	3	0	1	2	20		
	6	0	3	0	2	1	19		
Block 7	1	0	0	1	2	3	26	24	2.000
	2	0	0	1	3	2	25		
	3	0	0	2	1	3	25		
	4	0	0	2	3	1	23		
	5	0	0	3	1	2	23		
	6	0	0	3	2	1	22		
Block 8	1	1	0	2	0	3	22	18	8.000
	2	1	0	3	0	2	20		
	3	2	0	1	0	3	20		
	4	2	0	3	0	1	16		
	5	3	0	1	0	2	16		
	6	3	0	2	0	1	14		
Block 9	1	1	0	0	2	3	24	20	8.667
	2	1	0	0	3	2	23		
	3	2	0	0	1	3	21		
	4	2	0	0	3	1	19		
	5	3	0	0	1	2	17		
	6	3	0	0	2	1	16		
Block 10	1	1	2	0	0	3	20	16	8.667
	2	1	3	0	0	2	17		
	3	2	1	0	0	3	19		
	4	2	3	0	0	1	13		
	5	3	1	0	0	2	15		
	6	3	2	0	0	1	12		
								180	50

$$E(M) = 180 \; and \; var(M) = 50$$

Table A5. Five Treatments with Four Per Block

For $t = 5$ and $k = 4$, there are $\binom{5}{4} = 5$ unique blocks. The possible ranks are

0, 1, 2, 3 and 4. The following table illustrates the calculation.

	i^{th} combination	t1	t2	t3	t4	t5	M_i	M_l	var(Mi
Block 1	1	1	2	3	4	0	30	25	8.333
	2	1	2	4	3	0	29		
	3	1	3	2	4	0	29		
	4	1	3	4	2	0	27		
	5	1	4	2	3	0	27		
	6	1	4	3	2	0	26		
	7	2	1	3	4	0	29		
	8	2	1	4	3	0	28		
	9	2	3	1	4	0	27		
	10	2	3	4	1	0	24		
	11	2	4	1	3	0	25		
	12	2	4	3	1	0	23		
	13	3	1	2	4	0	27		
	14	3	1	4	2	0	25		
	15	3	2	1	4	0	26		
	16	3	2	4	1	0	23		
	17	3	4	1	2	0	22		
	18	3	4	2	1	0	21		
	19	4	1	2	3	0	24		
	20	4	1	3	2	0	23		
	21	4	2	1	3	0	23		
	22	4	2	3	1	0	21		
	23	4	3	1	2	0	21		
	24	4	3	2	1	0	20		
Block 2	1	1	2	3	0	4	34	27.5	14.583
	2	1	2	4	0	3	32		
	3	1	3	2	0	4	33		
	4	1	3	4	0	2	29		
	5	1	4	2	0	3	30		
	6	1	4	3	0	2	28		
	7	2	1	3	0	4	33		
	8	2	1	4	0	3	31		
	9	2	3	1	0	4	31		
	10	2	3	4	0	1	25		

	11	2	4	1	0	3	28	
	12	2	4	3	0	1	24	
	13	3	1	2	0	4	31	
	14	3	1	4	0	2	27	
	15	3	2	1	0	4	30	
	16	3	2	4	0	1	24	
	17	3	4	1	0	2	24	
	18	3	4	2	0	1	22	
	19	4	1	2	0	3	27	
	20	4	1	3	0	2	25	
	21	4	2	1	0	3	26	
	22	4	2	3	0	1	22	
	23	4	3	1	0	2	23	
	24	4	3	2	0	1	21	
Block 3	1	1	2	0	3	4	37	30	16.667
	2	1	2	0	4	3	36	
	3	1	3	0	2	4	35	
	4	1	3	0	4	2	33	
	5	1	4	0	2	3	32	
	6	1	4	0	3	2	31	
	7	2	1	0	3	4	36	
	8	2	1	0	4	3	35	
	9	2	3	0	1	4	32	
	10	2	3	0	4	1	29	
	11	2	4	0	1	3	29	
	12	2	4	0	3	1	27	
	13	3	1	0	2	4	33	
	14	3	1	0	4	2	31	
	15	3	2	0	1	4	31	
	16	3	2	0	4	1	28	
	17	3	4	0	1	2	25	
	18	3	4	0	2	1	24	
	19	4	1	0	2	3	29	
	20	4	1	0	3	2	28	
	21	4	2	0	1	3	27	
	22	4	2	0	3	1	25	
	23	4	3	0	1	2	24	
	24	4	3	0	2	1	23	

Table A5. continued

Block 4	1	1	0	2	3	4	39	32.5	14.583
	2	1	0	2	4	3	38		
	3	1	0	3	2	4	38		
	4	1	0	3	4	2	36		
	5	1	0	4	2	3	36		
	6	1	0	4	3	2	35		
	7	2	0	1	3	4	37		
	8	2	0	1	4	3	36		
	9	2	0	3	1	4	35		
	10	2	0	3	4	1	32		
	11	2	0	4	1	3	33		
	12	2	0	4	3	1	31		
	13	3	0	1	2	4	34		
	14	3	0	1	4	2	32		
	15	3	0	2	1	4	33		
	16	3	0	2	4	1	30		
	17	3	0	4	1	2	29		
	18	3	0	4	2	1	28		
	19	4	0	1	2	3	30		
	20	4	0	1	3	2	29		
	21	4	0	2	1	3	29		
	22	4	0	2	3	1	27		
	23	4	0	3	1	2	27		
	24	4	0	3	2	1	26		
Block 5	1	0	1	2	3	4	40	35	8.333
	2	0	1	2	4	3	39		
	3	0	1	3	2	4	39		
	4	0	1	3	4	2	37		
	5	0	1	4	2	3	37		
	6	0	1	4	3	2	36		
	7	0	2	1	3	4	39		
	8	0	2	1	4	3	38		
	9	0	2	3	1	4	37		
	10	0	2	3	4	1	34		
	11	0	2	4	1	3	35		
	12	0	2	4	3	1	33		
	13	0	3	1	2	4	37		
	14	0	3	1	4	2	35		

Table A5. continued

15	0	3	2	1	4	36		
16	0	3	2	4	1	33		
17	0	3	4	1	2	32		
18	0	3	4	2	1	31		
19	0	4	1	2	3	34		
20	0	4	1	3	2	33		
21	0	4	2	1	3	33		
22	0	4	2	3	1	31		
23	0	4	3	1	2	31		
24	0	4	3	2	1	30		
							150	62.5

$$E(M) = 150 \ and \ var(M) = 62.5$$

APPENDIX B. SAS CODE FOR RANKING DATA

```
************************************************************************
```

This macro ranks the data for all the cases considered. It is called by each of the other codes listed from Appendix III to V. The name of the macro is rankvar.

```
************************************************************************
```

```
%macro rankvar(dta, varn, newdta);

  data tmp1;
    set &dta;

    idn+1;
  run;

  data temp(keep=idn y grp);
    set tmp1;


    array dummy{*} y1-y&varn;

    do i=1 to &varn;

      y=dummy{i};
      grp=i;
      output;
    end;

  run;

  proc sort data=temp;
    by idn;
  run;


  proc rank data=temp out=rankpair;
    by idn;
    var y;
  run;


  data rankpair;
    set rankpair;

    if y=. then y=0;
  run;
```

```
proc sort data=rankpair;
  by idn;
run;


proc transpose data=rankpair out=tmp prefix=r;
  by idn ;
  var y;
run;


data &newdta(drop=idn _NAME_ _LABEL_ );
  merge tmp1 tmp;
  by idn;
run;

%mend rankvar;
```

APPENDIX C. SAS CODE FOR THREE POPULATIONS

****

This code generates data for 3 treatments with 2 appearing per block in a Balanced Incomplete Block Design for the three distributions Normal, Exponential and T (3 d.f). The Normal and Exponential distributions here have a variance of 1. The powers of the proposed test (M) and the Durbin test are calculated using a 5% significance level.
****

```
%macro generate32(sim,dist,t,k,r,reps,mu1,mu2,mu3,sigma);

        data raw32 (keep=sim rep block y1-y3);
        array trt{3} trt1-trt3;
        do i=1 to 3;
                trt{i}=int(ranuni(0)*1e6);
        end;
        put trt1-trt3;

                        call streaminit(321);

                do sim=1 to &sim;
                        do rep=1 to &reps;
                                do block=1 to 3;
                                        if &dist = 'normal' then do;
                                                call rannor(trt1 ,y1 );
        y1=y1+&mu1;
                call rannor(trt2 ,y2 );
        y2=y2+&mu2;
                call rannor(trt3 ,y3 );
        y3=y3+&mu3;
                                        end;
                                        else if &dist = 'exp' then do;
                                                call ranexp(trt1 ,y1 );
                                                        y1=(&mu1)+y1;
                                                call ranexp(trt2 ,y2 );
                                                        y2=(&mu2)+y2;
                                                call ranexp(trt3 ,y3 );
                                                        y3=(&mu3)+y3;
                                        end;
                                        else if &dist = 't' then do;
                                                y1=rand('T', 3) + &mu1;
        y2=rand('T', 3) + &mu2;
        y3=rand('T', 3) + &mu3;
                                        end;


                                if block=1 then do; y1=.; end;
```

61

```
                                        else if block=2 then do; y2=.; end;
                                        else if block=3 then do; y3=.; end;


                                output;
                        end;
                end;
            end;
    run;

    %rankvar(raw32, 3, new32);

    data n32;
    set new32 end=eof;
    by sim;

            array   r{3} r1-r3;
            array sumr{3} sumr1-sumr3;
    array sumrtn{3} sumrtn1-sumrtn3;
            array sumrtd{3} sumrtd1-sumrtd3;

        do i=1 to 3;
     if first.sim then
       do;
         sumr{i}=0;
       end;

      sumr{i}+r{i};
    if last.sim then
        do;
          sumrtn{i}=sumr{i}*i;
                    sumrtd{i}=sumr{i}**2;
        end;
      end;

      totRsumn=(sumrtn1 + sumrtn2 + sumrtn3);
          totRsumd=(sumrtd1 + sumrtd2 + sumrtd3);

          if last.sim then do;

                ar = (totRsumn - (18*&reps))/((1.5*&reps)**(1/2));
                if ar>1.645 then pow_ar+1;

                durbin = (12*(&t - 1) * totRsumd /((&r*&reps)*&t*(&k - 1)*(&k + 1))) -
    (3*(&r*&reps)*(&t -1)*(&k+1))/(&k-1);
                if durbin > 5.991 then pow_durbin+1;
```

62

```
                output;
end;

    if eof then do;
            p_ar=pow_ar/&sim;
            p_durbin=pow_durbin/&sim;
            file 'C:\Documents and Settings\Alfred.Ndungu\Desktop\power32.txt' mod;
                put @1 "&dist,&mu1,&mu2,&mu3," p_ar "," p_durbin ;



                end;
        run;
%mend generate32;
```
****
This section illustrates an example of the command used to call the above macro
generate32.
****
%generate32(10000,'normal',3,2,2,4,0,0,0,1);

%generate32(10000,'exp',3,2,2,4,0.25,1,0,1);

%generate32(10000,'t',3,2,2,4,0.25,1,0,1);

APPENDIX D. SAS CODE FOR FOUR POPULATIONS
****

This code generates data for 4 treatments with 2 appearing per block in a Balanced Incomplete Block Design for the three distributions Normal, Exponential and T (3 d.f). The Normal and Exponential distributions here have a variance of 1.The powers of the proposed test (M) and the Durbin test are calculated using a 5% significance level.
****


```
%macro generate42(sim,dist,t,k,r,reps,mu1,mu2,mu3,mu4,sigma);

        data raw42 (keep=sim rep block y1-y4);
                        array trt{4} trt1-trt4;
        do i=1 to 4;
           trt{i}=int(ranuni(0)*1e6);
        end;
        put trt1-trt4;


                        call streaminit(421);


        do sim=1 to &sim;
                do rep=1 to &reps;
                        do block=1 to 6;
                                if &dist = 'normal' then do;
                                        call rannor(trt1 ,y1 );
                                                y1=(&mu1)+&sigma*y1;
                                        call rannor(trt2 ,y2 );
                                                y2=(&mu2)+&sigma*y2;
                                        call rannor(trt3 ,y3 );
                                                y3=(&mu3)+&sigma*y3;
                                        call rannor(trt4 ,y4 );
                                                y4=(&mu4)+&sigma*y4;
                                end;
                                else if &dist = 'exp' then do;
                                        call ranexp(trt1 ,y1 );
                                                y1=(&mu1)+y1;
                                        call ranexp(trt2 ,y2 );
                                                y2=(&mu2)+y2;
                                        call ranexp(trt3 ,y3 );
                                                y3=(&mu3)+y3;
                                        call ranexp(trt4 ,y4 );
                                                y4=(&mu4)+y4;
                                end;
                                else if &dist = 't' then do;
                                        y1=rand('T', 3) + &mu1;
                        y2=rand('T', 3) + &mu2;
```

```
                              y3=rand('T', 3) + &mu3;
                              y4=rand('T', 3) + &mu4;
                                            end;


                                     if block=1  then do; y1=.; y2=.; end;
            else if block=2  then do; y1=.; y3=.; end;
            else if block=3  then do; y1=.; y4=.; end;
            else if block=4  then do; y2=.; y3=.; end;
            else if block=5  then do; y2=.; y4=.; end;
            else if block=6  then do; y3=.; y4=.; end;



                                     output;
                              end;
                       end;
                 end;
            run;

            %rankvar(raw42, 4, new42);

            data n42;
            set new42 end=eof;
            by sim;

                array   r{4} r1-r4;
                      array sumr{4} sumr1-sumr4;
            array sumrtn{4} sumrtn1-sumrtn4;
                      array sumrtd{4} sumrtd1-sumrtd4;

            do i=1 to 4;
          if first.sim then
            do;
              sumr{i}=0;
            end;

          sumr{i}+r{i};
        if last.sim then
            do;
              sumrtn{i}=sumr{i}*i;
                      sumrtd{i}=sumr{i}**2;
            end;
          end;

        totRsumn=sum(of sumrtn:);
```
65

```
        totRsumd=sum(of sumrtd:);

    if last.sim then do;

            ar = (totRsumn - (45*&reps))/((5*&reps)**(1/2));
            if ar>1.645 then pow_ar+1;

            durbin = (12*(&t - 1) * totRsumd /((&r*&reps)*&t*(&k - 1)*(&k + 1))) -
(3*(&r*&reps)*(&t -1)*(&k+1))/(&k-1);
        if durbin > 7.815 then pow_durbin+1;


            output;
    end;

    if eof then do;

            p_ar=pow_ar/&sim;
            p_durbin=pow_durbin/&sim;
            file 'C:\Documents and Settings\Alfred.Ndungu\Desktop\power42.txt' mod;
                put @1 "&dist,&mu1,&mu2,&mu3,&mu4," p_ar "," p_durbin ;



            end;
        run;

%mend generate42;

**************************************************************************
This section illustrates the command used to call the above macro generate42.
**************************************************************************

%generate42(10000,'normal',4,2,3,2,0,0,0,0,1);
%generate42(10000,'exp',4,2,3,2,0,0,0,0,1);
%generate42(10000,'t',4,2,3,2,0,0,0,0,1);
```

****

This code generates data for 4 treatments with 3 appearing per block in a Balanced Incomplete Block Design for the three distributions Normal, Exponential and T (3 d.f). The Normal and Exponential distributions have a variance of 1. The powers of the proposed test (M) and the Durbin test are calculated using a 5% significance level.
**;

```
%macro generate43(sim,dist,t,k,r,reps,mu1,mu2,mu3,mu4,sigma);


        data raw43 (keep=sim rep block y1-y4);
                        array trt{4} trt1-trt4;
        do i=1 to 4;
          trt{i}=int(ranuni(0)*1e6);
        end;
        put trt1-trt4;
                                call streaminit(431);


        do sim=1 to &sim;
                do rep=1 to &reps;
                        do block=1 to 4;
                                if &dist = 'normal' then do;
                                        call rannor(trt1 ,y1 );
                                                y1=(&mu1)+&sigma*y1;
                                        call rannor(trt2 ,y2 );
                                                y2=(&mu2)+&sigma*y2;
                                        call rannor(trt3 ,y3 );
                                                y3=(&mu3)+&sigma*y3;
                                        call rannor(trt4 ,y4 );
                                                y4=(&mu4)+&sigma*y4;
                                end;
                                else if &dist = 'exp' then do;
                                        call ranexp(trt1 ,y1 );
                                                y1=(&mu1)+y1;
                                        call ranexp(trt2 ,y2 );
                                                y2=(&mu2)+y2;
                                        call ranexp(trt3 ,y3 );
                                                y3=(&mu3)+y3;
                                        call ranexp(trt4 ,y4 );
                                                y4=(&mu4)+y4;
                                end;
                                else if &dist = 't' then do;
                                                y1=rand('T', 3) + &mu1;
                        y2=rand('T', 3) + &mu2;
```

67

```
                              y3=rand('T', 3) + &mu3;
                              y4=rand('T', 3) + &mu4;
                                      end;


                                 if block=1  then do; y1=.; end;
          else if block=2  then do; y2=.; end;
          else if block=3  then do; y3=.; end;
          else if block=4  then do; y4=.; end;

                              output;
                        end;
                  end;
            end;
      run;

      %rankvar(raw43, 4, new43);

      data n43;
      set new43 end=eof;
      by sim;

        array   r{4} r1-r4;
            array sumr{4} sumr1-sumr4;
      array sumrtn{4} sumrtn1-sumrtn4;
            array sumrtd{4} sumrtd1-sumrtd4;

        do i=1 to 4;
      if first.sim then
        do;
          sumr{i}=0;
        end;

        sumr{i}+r{i};
      if last.sim then
          do;
            sumrtn{i}=sumr{i}*i;
                  sumrtd{i}=sumr{i}**2;
          end;
        end;

      totRsumn=sum(of sumrtn:);
          totRsumd=sum(of sumrtd:);

          if last.sim then do;
```

68

```
                ar = (totRsumn - (60*&reps))/((13.3333*&reps)**(1/2));
                if ar>1.645 then pow_ar+1;

                durbin = (12*(&t - 1) * totRsumd /((&r*&reps)*&t*(&k - 1)*(&k + 1))) -
(3*(&r*&reps)*(&t -1)*(&k+1))/(&k-1);
            if durbin > 7.815 then pow_durbin+1;


                output;
        end;

        if eof then do;

                p_ar=pow_ar/&sim;
                p_durbin=pow_durbin/&sim;
                file 'C:\Documents and Settings\Alfred.Ndungu\Desktop\power43.txt' mod;
                        put @1 "&dist,&mu1,&mu2,&mu3,&mu4," p_ar "," p_durbin ;


                end;
        run;

%mend generate43;

**********************************************************************
This section illustrates an example of the command used to call the above macro
generate43.
**********************************************************************

%generate43(10000,'normal',4,3,3,3,1,0,0,0,1);
%generate43(10000,'exp',4,3,3,3,1,0.3,0.6,0.5,1);
%generate43(10000,'t',4,3,3,3,0,1,0,1,1);
```

APPENDIX E. SAS CODE FOR FIVE POPULATIONS

****

This code generates data for 5 treatments with 2 appearing per block in a Balanced Incomplete Block Design for the three distributions Normal, Exponential and T (3 d.f). The Normal and Exponential distributions have a variance of 1. The powers of the proposed test (M) and the Durbin test are calculated using a 5% significance level.
** ;


```
%macro generate52(sim,dist,t,k,r,reps,mu1,mu2,mu3,mu4,mu5,sigma);

        data raw52 (keep=sim rep block y1-y5);
                    array trt{5} trt1-trt5;
        do i=1 to 5;
          trt{i}=int(ranuni(0)*1e6);
        end;
        put trt1-trt5;
                            call streaminit(521);


            do sim=1 to &sim;
                    do rep=1 to &reps;
                            do block=1 to 10;
                                if &dist = 'normal' then do;
                                        call rannor(trt1 ,y1 );
                                                y1=(&mu1)+&sigma*y1;
                                        call rannor(trt2 ,y2 );
                                                y2=(&mu2)+&sigma*y2;
                                        call rannor(trt3 ,y3 );
                                                y3=(&mu3)+&sigma*y3;
                                        call rannor(trt4 ,y4 );
                                                y4=(&mu4)+&sigma*y4;
                                        call rannor(trt5 ,y5 );
                                                y5=(&mu5)+&sigma*y5;
                                end;
                                else if &dist = 'exp' then do;
                                        call ranexp(trt1 ,y1 );
                                                y1=(&mu1)+y1;
                                        call ranexp(trt2 ,y2 );
                                                y2=(&mu2)+y2;
                                        call ranexp(trt3 ,y3 );
                                                y3=(&mu3)+y3;
                                        call ranexp(trt4 ,y4 );
                                                y4=(&mu4)+y4;
                                        call ranexp(trt5 ,y5 );
                                                y5=(&mu5)+y5;
```

70

```
                                          end;
                                          else if &dist = 't' then do;
                                                  y1=rand('T', 3) + &mu1;
                         y2=rand('T', 3) + &mu2;
                         y3=rand('T', 3) + &mu3;
                         y4=rand('T', 3) + &mu4;
                         y5=rand('T', 3) + &mu5;
                                          end;


                                 if block=1  then do; y1=.; y2=.; y3=.; end;
          else if block=2  then do; y2=.; y3=.; y4=.; end;
          else if block=3  then do; y3=.; y4=.; y5=.; end;
          else if block=4  then do; y1=.; y3=.; y4=.; end;
          else if block=5  then do; y1=.; y4=.; y5=.; end;
          else if block=6  then do; y2=.; y4=.; y5=.; end;
          else if block=7  then do; y1=.; y2=.; y4=.; end;
          else if block=8  then do; y1=.; y2=.; y5=.; end;
          else if block=9  then do; y2=.; y3=.; y5=.; end;
          else if block=10 then do; y1=.; y3=.; y5=.; end;



                                 output;
                         end;
                 end;
         end;
   run;

   %rankvar(raw52, 5, new52);

   data n52;
   set new52 end=eof;
   by sim;

      array   r{5} r1-r5;
            array sumr{5} sumr1-sumr5;
   array sumrtn{5} sumrtn1-sumrtn5;
            array sumrtd{5} sumrtd1-sumrtd5;

      do i=1 to 5;
   if first.sim then
    do;
      sumr{i}=0;
    end;

    sumr{i}+r{i};
   if last.sim then
```

71

```
        do;
          sumrtn{i}=sumr{i}*i;
                    sumrtd{i}=sumr{i}**2;
          end;
       end;

    totRsumn=sum(of sumrtn:);
        totRsumd=sum(of sumrtd:);

       if last.sim then do;

              ar = (totRsumn - (90*&reps))/((12.5*&reps)**(1/2));
              if ar>1.645 then pow_ar+1;

              durbin = (12*(&t - 1) * totRsumd /((&r*&reps)*&t*(&k - 1)*(&k + 1))) -
(3*(&r*&reps)*(&t -1)*(&k+1))/(&k-1);
         if durbin > 9.488 then pow_durbin+1;


              output;
         end;

       if eof then do;

              p_ar=pow_ar/&sim;
              p_durbin=pow_durbin/&sim;
              file 'C:\Documents and Settings\Alfred.Ndungu\Desktop\power52.txt' mod;
                    put @1 "&dist,&mu1,&mu2,&mu3,&mu4,&mu5," p_ar ","
p_durbin ;


              end;
          run;

%mend generate52;
**********************************************************************
This section illustrates an example the command used to call the above macro generate52.

**********************************************************************
%generate52(10000,'normal',5,2,4,3,0,0,0,0,0,1);
%generate52(10000,'exp',5,2,4,3,0,0.25,0.6,0.7,1,1);
%generate52(10000,'t',5,2,4,3,0,1,1,1,1,1);
```

```
************************************************************
This code generates data for 5 treatments with 3 appearing per block in a Balanced
Incomplete Block Design for the three distributions Normal, Exponential and T (3 d.f).
The Normal and Exponential distributions have a variance of 1.The powers of the
proposed test (M) and the Durbin test are calculated using a 5% significance level.
************************************************************;


%macro generate53(sim,dist,t,k,r,reps,mu1,mu2,mu3,mu4,mu5,sigma);


        data raw53 (keep=sim rep block y1-y5);
                    array trt{5} trt1-trt5;
        do i=1 to 5;
          trt{i}=int(ranuni(0)*1e6);
        end;
        put trt1-trt5;
                                call streaminit(531);



            do sim=1 to &sim;
                    do rep=1 to &reps;
                        do block=1 to 10;
                            if &dist = 'normal' then do;
                                call rannor(trt1 ,y1 );
                                        y1=(&mu1)+&sigma*y1;
                                call rannor(trt2 ,y2 );
                                        y2=(&mu2)+&sigma*y2;
                                call rannor(trt3 ,y3 );
                                        y3=(&mu3)+&sigma*y3;
                                call rannor(trt4 ,y4 );
                                        y4=(&mu4)+&sigma*y4;
                                call rannor(trt5 ,y5 );
                                        y5=(&mu5)+&sigma*y5;
                            end;
                            else if &dist = 'exp' then do;
                                call ranexp(trt1 ,y1 );
                                        y1=(&mu1)+y1;
                                call ranexp(trt2 ,y2 );
                                        y2=(&mu2)+y2;
                                call ranexp(trt3 ,y3 );
                                        y3=(&mu3)+y3;
                                call ranexp(trt4 ,y4 );
                                        y4=(&mu4)+y4;
                                call ranexp(trt5 ,y5 );
```

73

```
                                        y5=(&mu5)+y5;
                                  end;
                            else if &dist = 't' then do;
                                        y1=rand('T', 3) + &mu1;
                        y2=rand('T', 3) + &mu2;
                        y3=rand('T', 3) + &mu3;
                        y4=rand('T', 3) + &mu4;
                        y5=rand('T', 3) + &mu5;
                                  end;

                                  if block=1  then do; y1=.; y2=.; end;
            else if block=2  then do; y1=.; y3=.; end;
            else if block=3  then do; y1=.; y4=.; end;
            else if block=4  then do; y1=.; y5=.; end;
            else if block=5  then do; y2=.; y3=.; end;
            else if block=6  then do; y2=.; y5=.; end;
            else if block=7  then do; y2=.; y4=.; end;
            else if block=8  then do; y3=.; y4=.; end;
            else if block=9  then do; y3=.; y5=.; end;
            else if block=10 then do; y4=.; y5=.; end;

                              output;
                        end;
                  end;
            end;
      run;

      %rankvar(raw53, 5, new53);

      data n53;
      set new53 end=eof;
      by sim;

         array   r{5} r1-r5;
              array sumr{5} sumr1-sumr5;
      array sumrtn{5} sumrtn1-sumrtn5;
              array sumrtd{5} sumrtd1-sumrtd5;

         do i=1 to 5;
      if first.sim then
        do;
          sumr{i}=0;
        end;

        sumr{i}+r{i};
      if last.sim then
```

74

```
                do;
                  sumrtn{i}=sumr{i}*i;
                            sumrtd{i}=sumr{i}**2;
                end;
             end;

          totRsumn=sum(of sumrtn:);
               totRsumd=sum(of sumrtd:);

            if last.sim then do;

                    ar = (totRsumn - (180*&reps))/((50*&reps)**(1/2));
                    if ar>1.645 then pow_ar+1;

                    durbin = (12*(&t - 1) * totRsumd /((&r*&reps)*&t*(&k - 1)*(&k + 1))) -
(3*(&r*&reps)*(&t -1)*(&k+1))/(&k-1);
               if durbin > 9.488 then pow_durbin+1;


                    output;
               end;

            if eof then do;

                    p_ar=pow_ar/&sim;
                    p_durbin=pow_durbin/&sim;
                    file 'C:\Documents and Settings\Alfred.Ndungu\Desktop\power53.txt' mod;
                         put @1 "&dist,&mu1,&mu2,&mu3,&mu4,&mu5," p_ar ","
p_durbin ;


                    end;
               run;


%mend generate53;
**********************************************************************
This section illustrates an example of the command used to call the above macro
generate53.
**********************************************************************

%generate53(10000,'normal',5,3,6,1,0,0,0.5,0,0,1);
%generate53(10000,'exp',5,3,6,1,0.6,0.5,0.4,0.3,0.2,1);
%generate53(10000,'exp',5,3,6,1,0,0,0.5,0,0,1);
%generate53(10000,'t',5,3,6,1,0,0,1,0,0,1);
```

```
******************************************************************
This code generates data for 5 treatments with 4 appearing per block in a Balanced
Incomplete Block Design for the three distributions Normal, Exponential and T (3 d.f).
The Normal and Exponential distributions have a variance of 1.The powers of the
proposed test (M) and the Durbin test are calculated using a 5% significance level.
******************************************************************;

%macro generate54(sim,dist,t,k,r,reps,mu1,mu2,mu3,mu4,mu5,sigma);


        data raw54 (keep=sim rep block y1-y5);
                    array trt{5} trt1-trt5;
        do i=1 to 5;
          trt{i}=int(ranuni(0)*1e6);
        end;
        put trt1-trt5;
                            call streaminit(541);


            do sim=1 to &sim;
                do rep=1 to &reps;
                    do block=1 to 5;
                        if &dist = 'normal' then do;
                                call rannor(trt1 ,y1 );
                                        y1=(&mu1)+&sigma*y1;
                                call rannor(trt2 ,y2 );
                                        y2=(&mu2)+&sigma*y2;
                                call rannor(trt3 ,y3 );
                                        y3=(&mu3)+&sigma*y3;
                                call rannor(trt4 ,y4 );
                                        y4=(&mu4)+&sigma*y4;
                                call rannor(trt5 ,y5 );
                                        y5=(&mu5)+&sigma*y5;
                        end;
                        else if &dist = 'exp' then do;
                                call ranexp(trt1 ,y1 );
                                        y1=(&mu1)+y1;
                                call ranexp(trt2 ,y2 );
                                        y2=(&mu2)+y2;
                                call ranexp(trt3 ,y3 );
                                        y3=(&mu3)+y3;
                                call ranexp(trt4 ,y4 );
                                        y4=(&mu4)+y4;
                                call ranexp(trt5 ,y5 );
                                        y5=(&mu5)+y5;
                        end;
```

76

```
                                    else if &dist = 't' then do;
                                            y1=rand('T', 3) + &mu1;
                    y2=rand('T', 3) + &mu2;
                    y3=rand('T', 3) + &mu3;
                    y4=rand('T', 3) + &mu4;
                    y5=rand('T', 3) + &mu5;
                                            end;


                            if block=1 then do; y1=.; end;
                            else if block=2 then do; y2=.; end;
                            else if block=3 then do; y3=.; end;
                            else if block=4 then do; y4=.; end;
                            else if block=5 then do; y5=.; end;


                    output;
                end;
            end;
        end;
    run;

    %rankvar(raw54, 5, new54);

    data n54;
    set new54 end=eof;
    by sim;
            array   r{5} r1-r5;
            array sumr{5} sumr1-sumr5;
    array sumrtn{5} sumrtn1-sumrtn5;
            array sumrtd{5} sumrtd1-sumrtd5;

        do i=1 to 5;
    if first.sim then
      do;
        sumr{i}=0;
      end;

    sumr{i}+r{i};
    if last.sim then
        do;
        sumrtn{i}=sumr{i}*i;
                sumrtd{i}=sumr{i}**2;
        end;
    end;
```

```
        totRsumn=sum(of sumrtn:);
            totRsumd=sum(of sumrtd:);

        if last.sim then do;

                ar = (totRsumn - (150*&reps))/((62.5*&reps)**(1/2));
                if ar>1.645 then pow_ar+1;

                durbin = (12*(&t - 1) * totRsumd /((&r*&reps)*&t*(&k - 1)*(&k + 1))) -
(3*(&r*&reps)*(&t -1)*(&k+1))/(&k-1);
            if durbin > 9.488 then pow_durbin+1;


                output;
        end;

        if eof then do;

                p_ar=pow_ar/&sim;
                p_durbin=pow_durbin/&sim;
                file 'C:\Documents and Settings\Alfred.Ndungu\Desktop\power54.txt' mod;
                    put @1 "&dist,&mu1,&mu2,&mu3,&mu4,&mu5," p_ar ","
p_durbin ;


                end;
        run;


%mend generate54;
********************************************************************************
This section illustrates an example of the command used to call the above macro
generate54.

********************************************************************************
%generate54(10000,'normal',5,4,4,2,0,0,0.3,0,1,1);
%generate54(10000,'exp',5,4,4,2,0,0,0.3,0,1,1);
%generate54(10000,'t',5,4,4,2,0,0,1,0,0,1);
```