OPTIMAL INVENTORY STRATEGY UNDER RISK: A CONTINGENT CLAIMS APPROACH

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Optimal Inventory Strategy Under Risk: A Contingent Claims Approach

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ABSTRACT

Inventory management in the agriculture industry involves many sources of risk in terms of demand uncertainty as well as uncertain margins. Divulging an optimal inventory strategy can prove cumbersome to logistics managers. In this thesis, inventory is viewed as a real option on the ability to operate. Contingent claims inventory (CCI) analysis, paired with stochastic binomial real option valuation, provides a model which values the real option embedded in holding inventory and iterates the purchasing strategy until expected profit is maximized. This framework is applied to three industry cases pertaining to: wheat flour milling, fertilizer merchandising, and bulk shipments via primary rail contracts.

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" 11 For no one can lay a foundation other than that which is laid, which is Jesus Christ. 12 Now if
anyone builds on the foundation with gold, silver, precious stones, wood, hay, straw— 13 each
one's work will become manifest, for the Day will disclose it, because it will be revealed by fire,
and the fire will test what sort of work each one has done. 14 If the work that anyone has built on
the foundation survives, he will receive a reward. 15 If anyone's work is burned up, he will suffer
loss, though he himself will be saved, but only as through fire."

- 1 Corinthians 3: 11-15

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CHAPTER 1. INTRODUCTION

1.1. Overview

Uncertainties in supply and demand have plagued logistics managers throughout time. Knowing what level of inventory to hold, and the risks associated with that strategy, can prove challenging to industry operatives. As volatility of supply and demand increase, the optimal quantities of stock tend to increase. This increase in buffer stock is credited to the real options embedded in holding inventory (Stowe and Su 1997). Unlike financial options, real options do not always hold a monetary value which can be easily traded (Trigeorgis [1996] 1999). However, they do process a premium which gauges how valuable holding that option is.

Uncertainties in supply and demand as well as margin create managerial challenges in the agricultural industry. Whether it is ordering wheat to mill flour, purchasing urea to meet fertilizer demand, or managing inventories to meet rail car supply; uncertainties in supply and demand as well as margin are ever present. This thesis applies an inventory management tool which utilizes real options, stochastic simulation, and contingent claims inventory analysis to optimize inventory strategy and maximize expected profit.

The agriculture industry is particularly different than conventional industries due to great uncertainties in price both intra and intra yearly as well as fluctuation in production. A severe drought throughout the country may lower the production of each commodity which greatly impacts both supply and cost. Alternatively, policy regarding genetically modified organisms (GMOs) may impact which commodity is demanded from year to the next. Conventional industries, such as car manufacturing, have a relatively stable demand and price; therefore, JIT concepts serve their effectiveness.

Real option premiums which are mapped onto uncertainties of supply and demand, combined with contingent claim inventory analysis (Stowe and Su 1997), help provide a mode to address optimal inventory strategy in the agriculture sector. This thesis builds a framework which uses real options to value inventory and applies this methodology to wheat flour milling, fertilizer merchandising, and soybean shuttle shipments.

1.2. Problem Statement

Issues in inventory management have been cited since before 600 B.C. (Kodukula and Papudesu 2006). Holding too much inventory ties up capital and accrues interest while not having enough may lead to company shutdown and foregone profits. In recent times, Just-In-Time manufacturing (JIT) concepts and lean production have moved industries towards inventory strategies which hold nearly zero buffer stocks (Ballou [1973] 1992; Jacobs and Chase [2008] 2017). Lean production concepts, if implemented without caution, may lead to stockout penalties such as demurrage and have a tenacity to cause congestion at key transfer points (Wilson and Dahl 2011).

Inventory management also has great implications at the industry level where the convenience of being able to operate holds value (Working 1949). Holding inventories creates a real option on the ability to sell a product. This is frequently called the convenience yield in commodity trading or processing and explains why firms may hold stocks even though prices may be cheaper in the future, simply for the convenience of not losing forgone profit from stockouts. This same option can be applied to a wheat mill needing stock to create flour, a fertilizer merchandiser who sells urea, or a shuttle loader who requires grain to fill rail cars. Empirical models which address these issues can be analyzed using the theoretical framework developed by Stowe and Su (1997) and stochastic binomial real option valuation (Churchill 2016; Landman 2017).

Holding inventory may be viewed as a portfolio of options which grant the owner the right to operate. Long call options establish a floor of value which encompasses net salvage value of unused inventory. A long call option gains value as demand rises until a point where they stockout. At the point of stockout, the owner's position is equivalent to a short call option which loses value as demand continues to increase. The loss of value represents foregone profit and any additional costs associated with stockout. This portfolio of options, combined with initial inventory and net salvage value, generate an expected profit from an inventory strategy. This net present value (NPV) can be maximized to determine the optimal level of inventories.

1.3. Objectives

The goal of this thesis is to develop a tool which measures the real option value created through holding inventory and identify key factors which affect optimal inventory strategy. The goals of this thesis may be subdivided as follows:

1. Build a framework which maps real option values onto demand. The real options in this study view option premiums as a relative likelihood of that option expiring in the money. Multiplying this premium by a number of options generate an expected value of the real option created through holding inventory.
2. Apply contingent claims inventory (CCI) analysis to three industry cases to determine the optimal purchasing strategy. These include: wheat flour milling, urea fertilizer merchandizing, and soybeans for bulk rail transportation.
3. Conduct sensitivities on the key variables to see how optimal strategy, expected profit, and risks associated with expected profit change.

1.4. Procedures

This thesis splits contingent claims inventory (CCI) analysis into module parts before it is applied to industry related issues. Not every situation utilizes CCI analysis in the same way so the procedure must be broken down and modified for efficient use. The framework was applied to three cases including: wheat flour milling, fertilizer, and rail shipments. Each application uses module which is closely related to the theoretical framework developed by Stowe and Su (1997) and solved using a stochastic binomial real option model (Shreve 2004) to value the real option; however, each application has properties which are unique to their specific application.

The first application is that of a wheat flour mill. The flour mill requires a quantity of bushels of wheat to be milled into flour each month. This quantity changes each month due to variability in the extraction rate, which is the percent of wheat which is milled into flour, and percent mill capacity utilization. Due to the nature of reoccurring demand, a material requirement planning (MRP) module is implemented in the overall CCI analysis (Ballou [1973] 1992).

The second application pertains to fertilizer merchandizing. Fertilizer is an uncommon agriculture industry because the United States relies heavily on imports to meet demand of certain fertilizers (Wilson, et al. 2014). Interior fertilizer distributors, referred to as country centroids, must prepare for fertilizer demand many months before demand occurs. The demand from each country centroid then relies heavily on spatial competition and market boundaries related to transportation costs. Given the nature of the fertilizer industry, a module which utilizes spatial arbitrage pricing is used to calculate both expected demand and competitive selling price for a representative country centroid (Tomek and Kaiser [1972] 2014).

The final application pertains to grain shuttle shipments. Bulk interior grain shippers primarily utilize the railroad when shipping grain (Wilson and Dahl 2011). There are many sources of risk when shipping grain including: secondary rail car prices, rail velocity, soybean futures spread, terminal basis spread, and changes in tariff rates. This application does not utilize any additional module parts, however the calculations within the CCI module are relatively complex and the real options must also be valued as American style options rather than European options (Guthrie 2009).

Each application combines the module parts into a dynamic iterative model. The model uses Monte Carlo simulation and RiskOptimizer™ to determine an optimal strategy which would maximize expected profits. Each application has key stochastic and structural variables which have uncertainties tied to supply and demand as well as price. Sensitives are conducted on these variables to see how optimal strategy and expected profit change with variable shifts.

1.5. Organization

The remainder of this thesis is divided into six additional chapters:

- Chapter 2 provides background and prior studies related to inventory management and real options.
- Chapter 3 develops the theoretical models used in this thesis.
- Chapter 4 applies theory in an empirical application related to a processor.
- Chapter 5 applies theory in an empirical application related to the fertilizer industry.
- Chapter 6 applies theory in an empirical application related to a bulk grain shipper.
- Chapter 7 discusses the findings in this thesis and its salient implications.

Chapter 2 provides the reader with the proper background in inventory management and real options. A review of related literature in inventory management discuss previous attempts to model inventory uncertainty, their findings, and their limitations. A review of real options literature presents its place in agriculture related industries, as well as papers which utilize stochastic binomial real option valuation in their procedures.

Chapter 3 presents the detailed theoretical framework of contingent claims inventory (CCI) analysis developed by Stowe and Su (1997). Chapter 3 also presents theoretical details of binomial real option valuation and its application in stochastic processes (Cox, et al. 1979; Shreve 2004; Hull [1995] 2008; Kodukula and Papudesu 2006).

Chapter 4 applies CCI analysis in the wheat flour milling industry. The application develops a representative Hard Red Winter (HRW) wheat mill based on 26 mills located in the upper Midwest. The application includes three module parts of material requirement planning (MRP), real option valuation, and contingent claims inventory (CCI). The MRP model is developed for four months of milling to capture the effects of markets spread on inventory strategy. The net present value (NPV) output by the CCI module represents the expected profit of four months of milling. The purchasing strategies at the end of each milling month are adjusted to maximized expected profit.

Chapter 5 applies CCI analysis in the urea merchandizing industry. The demand for urea is lumpy and therefore only occurs during certain times of the year, primarily during spring planting season. Urea imports from the US Gulf must be transported by barge, rail, or truck to state located in the upper Midwest (Wilson et al. 2014). This makes for an extended lead time during planting season. Demand must therefore be anticipated with relative certainty to not forego fertilizer sales while also limiting excess inventories which accrue great amounts of storage and interest if demand is overestimated. Competitive arbitrage pricing among country centroids makes demand highly volatile even if aggregate demand for urea is relatively stable. The urea purchasing strategy to meet uncertain demand is adjusted to maximize expected profit.

Chapter 6 apples CCI analysis to a representative bulk soybean shipper. A purchasing strategy is developed based on a soybean shipper which owns two primary BNSF one-year shuttle contracts. BNSF rail performance is measured in velocity, or total trips per month. Velocity has a large impact on rail car supply and thus the quantity of soybeans demanded to fill shuttle trains. Unused shuttle trains may be sold into the secondary rail market at either a premium or discount relative to the tariff rate which is recorded as daily car value (DCV). Trade West Brokerage Co. (2018) provides extensive data on velocity, daily car value, and terminal basis bids. A combination of multiple factors such as DCV, velocity, PNW terminal basis, tariff rate, and future market spread greatly impact net salvage value of unused inventory and stockout penalty if demand due to car supply is not met.

Chapter 7 summarizes the finding in chapters four, five, and six as well as their implications to the industry. Included in chapter 7 are also limitations in the applications and recommendations for further research.

CHAPTER 2. BACKGROUND AND RELEVANT STUDIES

2.1. Introduction

This chapter provides relevant background and literature on the research methods utilized in this thesis. The first portion of the chapter outlines conventional inventory management practices. This part concludes with a review of relevant literature in inventory management which utilizes financial theory. The second part of the chapter outlines the use of real options in managerial decisions and concludes with relevant studies which utilize stochastic binomial real option models.

2.2. Inventory Management Background and Relevant Studies

Over the past 50 years there has been a major movement in Operations and Supply Chain Management (OSCM) to reform the supply chain process and move towards lean production practices. The term lean production refers to generating high volume; high quality goods and services while also minimizing the use of inventories and raw materials. The basis of lean production originated in Tokyo, Japan when Toyota implemented just-in-time (JIT) manufacturing concepts. These concepts were meant to lower the inventory-to-sales ratio of the company. JIT concepts forecast the demand for certain parts of the supply chain and estimate lead times to minimize the number of components at each station (Ballou [1973] 1992; Jacobs and Chase [2008] 2017). Lean production and JIT concepts may help reduce the amount of capital being tied up in inventories.

In OSCM there is a major focus on managing inventories at each station in the supply chain. There are several factors that go into inventory management including, lead time uncertainty, convenience yield and future price uncertainty. These components all lead to the major problem in managing inventories which is affected largely by uncertainties in supply, demand and the logistical system. There is a certain level of risk associated with each of these variables. The level of risk in each variable is positively related with the amount of safety stocks (buffer stocks) that need to be readily available (Chang et al. 2015).

A buffer stock is inventory that is specifically ordered more than the projected requirement level (Coyle and Bardi [1976] 1984). Excess inventory must be well managed as it ties up capital and accrues carrying costs. However, not holding enough inventories in high levels of supply and demand uncertainty would leave a company susceptible to disruptions in OSCM. These disruptions in OSCM would therefore leave a company vulnerable to stock-out penalties. Seeking the balance between carrying costs and stock-out penalties has resulted in the evolution of several supply chain logistics strategy models.

Most supply chain and logistics models seek to manage inventory levels with the goal to either capture an advantage in the market or minimize risk while also maximizing expected profit. The reasons for inventory discussed in this section include: independence of operations, flexibility in production scheduling, achieving quantities of size, managing uncertainties of supply, and managing uncertainties of demand. The final two reason, managing uncertainties in supply and demand, are the focal point of the research conducted in this paper.

The first reason for holding inventories is to maintain an independence of operations. As raw materials move through the production phase it would pass many sites of operations until it is turned into the final product. Each of these phases would not take an identical amount of time so buffer stocks can be used to keep production moving fluidly. For example, corn must go through a series of phases before it is turned into ethanal. The ethanal plant would keep some level of inventory to serve as a "cushion" to keep the plant in operation.

The next reason for holding some level of inventory is to allow for flexibility in production scheduling. Depending on the firm's style of production, the same area of space in a production firm may be used to make different items or parts. If the same area of space is used, a set up phase must take place to convert equipment to make different products. These set up phases in the production process may be costly and take time. Therefore, it is advantageous to produce more of an item and store it while each phase in the process is set up.

Flexibility in production scheduling leads to the advantage of ordering in quantities of size. Companies generally prefer to deal in quantities of size to reduce the amount of management required in moving a product. This especially becomes the case when dealing with transportation of a good. Shipping costs tend to favor large orders, so greater the shipment order, the lower the per-unit cost (Jacobs and Chase [2008] 2017). For example, a grain buyer may offer a premium (or as referred to in the industry, an 'edge') to a farmer if he schedules a large quantity of bushels to be delivered during a specified period. The farmer is better off because he receives a premium, and the grain buyer is better off because he has relieved some uncertainty in supply by scheduling a large quantity.

The final two reasons covered for holding inventories, and the focus of this paper, is to deal with meeting variations in product supply and demand. First, there can be uncertainties in both the quantity and lead time of supply. The quantity of supply variation would typically originate from a raw material producer such as a farm, mine, or oil well. The raw material yielded by the producer would vary from period to period, especially if the raw material producer has onsite storage. When the producer has the ability to store raw materials on site, they may choose when and if to deliver depending on the level of price received for the good. If the price per barrel of oil is depressed, the oil producer may decide to turn down production until prices return to a favorable level. Along with uncertainty in quantity of supply, there may also be uncertainties in lead times. A lead time refers to the amount of time it takes for materials to arrive after an order is placed (Coyle and Bardi [1976] 1984). Sometimes, there are supply disruptions that prevent an order of material from arriving on schedule. The uncertainties in both quantity and lead time may lead to stock-out penalties. If stock-out penalties are high, then the level of inventories needed to serve as buffer stocks greatly increases with the increase in uncertainty.

Along with using inventories to deal with the uncertainties of supply, buffer stocks may also be used in dealing with the uncertainties of demand. If each gas station knew the exact amount of fuel which would be demanded from the pump each day there would be no need to hold an excess of fuel in bulk. However, the actual level of demand for a product is uncertain but can be predicted through forecasting methods to get close to what actual demand would be. In the case of the fuel pump, demand can be forecasted as function of both seasonality and the price of fuel. During certain times of the year most business would experience a surge in demand for a product. These surges can be predicted through the use of seasonality trends. Price also has a major influence in product demand. For instance, the underlying price of fuel would greatly influence how much people drive and thus how much fuel is demanded from the pump.

2.2.1. Cash, Futures, Basis, and Convenience Yield

The source of most OSCM disruptions in this paper link back to the underlying price of a certain state variable. Throughout this paper commodity prices are referred to as either cash, futures, or basis price. The producer, end-user, or other market participant would buy or sell the physical commodity at a cash price. The futures prices refer to the everchanging value of the underlying futures contract. There is generally a difference between what the cash price and what the futures price is; this difference in price is what is referred to in the market as basis (Kolb and Overdahl [1985] 2006).

$$Basis = Cash - Futures \tag{2.1}$$

where:

Basis	=	the difference between the cash price and futures price
Cash	=	the price received or paid for the physical commodity
Futures	=	the value of the futures contract.

This cash minus futures relationship is often referred to in the industry when quoting prices at local elevators and end-users. Many firms in the agriculture industry that are involved in handling commodities frequently refer to themselves as "basis traders," as most transactions occur on a level of basis rather than referring to a cash price.

Commodities with futures markets often have multiple months where the contract is traded. These "inter-temporal price relations" between deferred and nearby markets are called the *carry* of the market (Working 1949).

$$Carry = Deferred - Nearby \tag{2.2}$$

where:

Carry	=	intermonth spread between contract prices
Deferred	=	contract price of the deferred month
Nearby	=	contract price of the nearby month.

A normal market occurs when the intermonth spread between futures months is positive (Kolb and Overdahl [1985] 2006). A normal market encourages storage of a commodity to capture the carry in the market. There are different costs associated with carrying a commodity which include storage, insurance, transportation, and interest (Kolb and Overdahl [1985] 2006). There is a calculated cost of carry which considers all factors listed above and reports a value at which a storage facility would need to be compensated (Working 1949). A market is said to be in full carry if the intermonth spread is equal to the cost of carry. Figure 2.1 depicts the general interaction between the amount of wheat which is stored in relation to the intermonth spread of the commodity price of wheat.

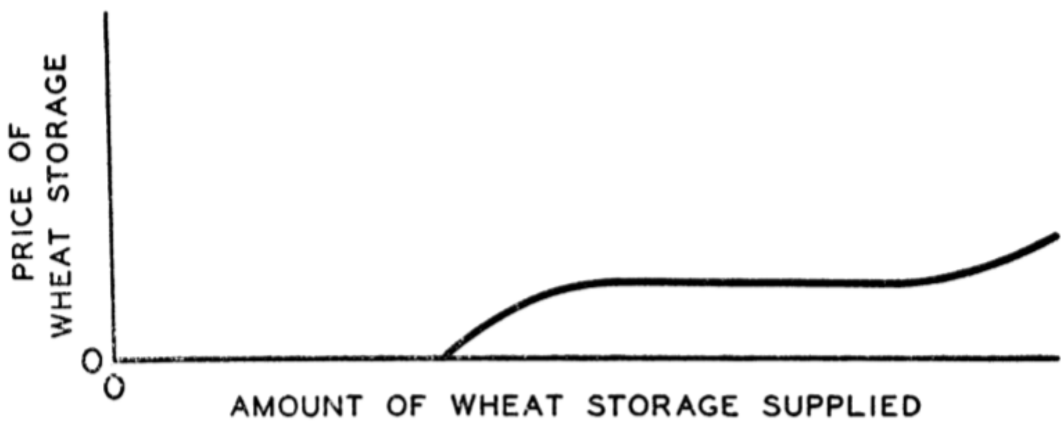

Figure 2.1. Partial Storage Supply Curve (Working 1949).

When the future price in the deferred month is less than the future price of the nearby month the market is said to be in backwardation (Kolb and Overdahl [1985] 2006). A market which is in backwardation discourages holding inventories of the physical commodity because it is worth more now than it would be in the future. However, sometimes a firm would store a commodity or good when the returns to storage is negative. This can occur for several reasons, but one explanation is the firm receives some level of convenience yield by holding the commodity or good.

When the market is in backwardation it would be assumed that an individual would sell all of their grain now instead of storing the commodity. The term convenience yield refers to the value the individual receives from of holding the physical commodity when the returns to storage are negative (Kaldor 1939). For example, a wheat flower mill needs to have a steady flow of high and low protein wheat to produce flour and wheat-midds. If the mill were to run out of wheat, it would shut down temporarily and the company would suffer losses. For this reason, there is a level of convenience yield the mill possess by holding some inventory, or buffer stocks, of high and low protein wheat.

Figure 2.2 shows how this relationship interacts with the amount of wheat stored in relation to the cost of storage. There is still a significant amount of wheat stored when the price of storage is negative or near zero. Working's conclusion was that for most potential suppliers of storage, the cost of storage was in adjunct with merchandising or processing (Working 1949).

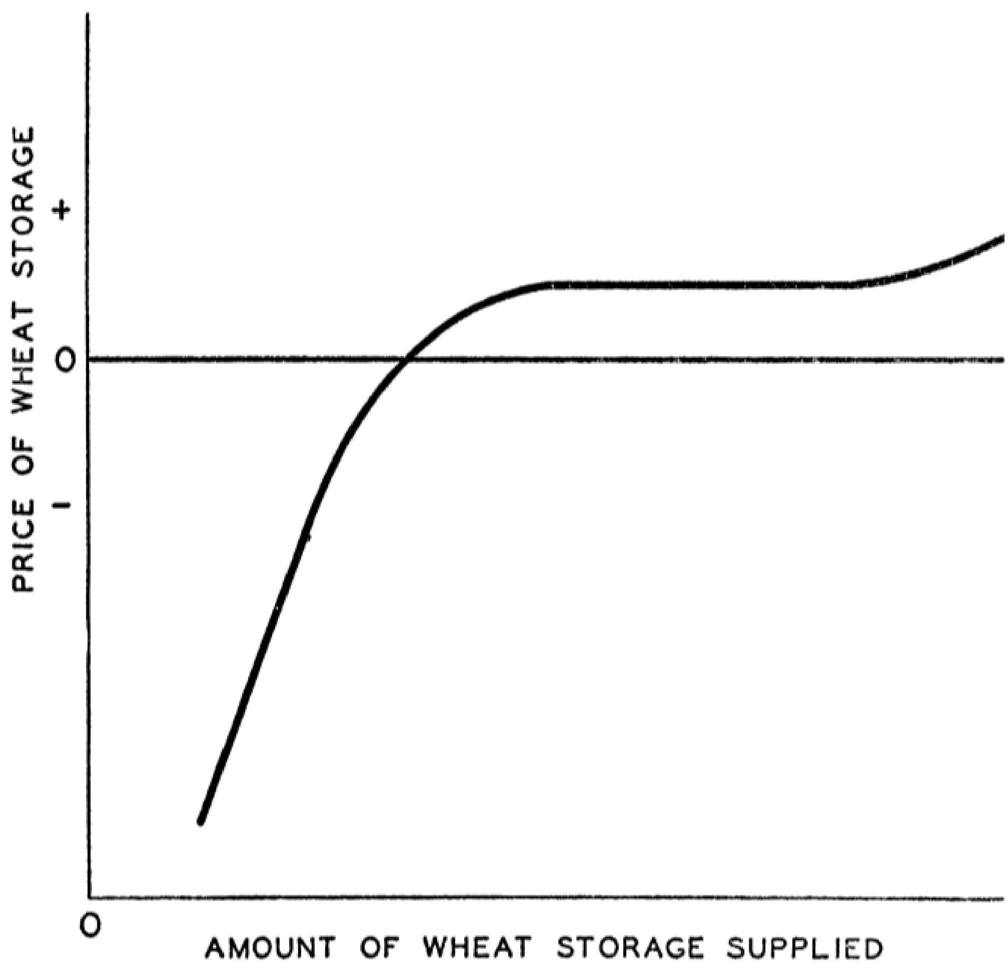

Figure 2.2. Complete Storage Supply Curve (Working 1949)

Gibson and Schwartz (1990) present a two-factor model which valued oil-linked assets under the assumption that the spot price of oil and the instantaneous net convenience yield of oil follow a joint stochastic process. They were then able to price one barrel of oil at any arbitrary future date and found the high importance convenience yield plays for a non-speculative commodity (Gibson and Schwartz 1990). Casassus and Collin-Dufresne (2005) furthered this study to also include interest rates and risk premia. They confirmed that spot prices follow a mean reverting process due to the significance of convenience yields (Casassus and Collin-Dufresne 2005).

2.2.2. Inventory Models in Operations and Supply Chain Management

In OSCM there several models that can be used when optimizing inventories when either supply, demand, or lead times are uncertain. The models discussed in the section include the single period model, economic order quantity (EOQ), and material requirement planning (MRP).

Jacobs and Chase depict a general rule of thumb in Figure 2.3 for where each step in the "make-to-stock environment" process coincides with the correct inventory model. However, it is also important to understand which type of demand structure is in play. There are two types of demand structures: dependent and independent demand.

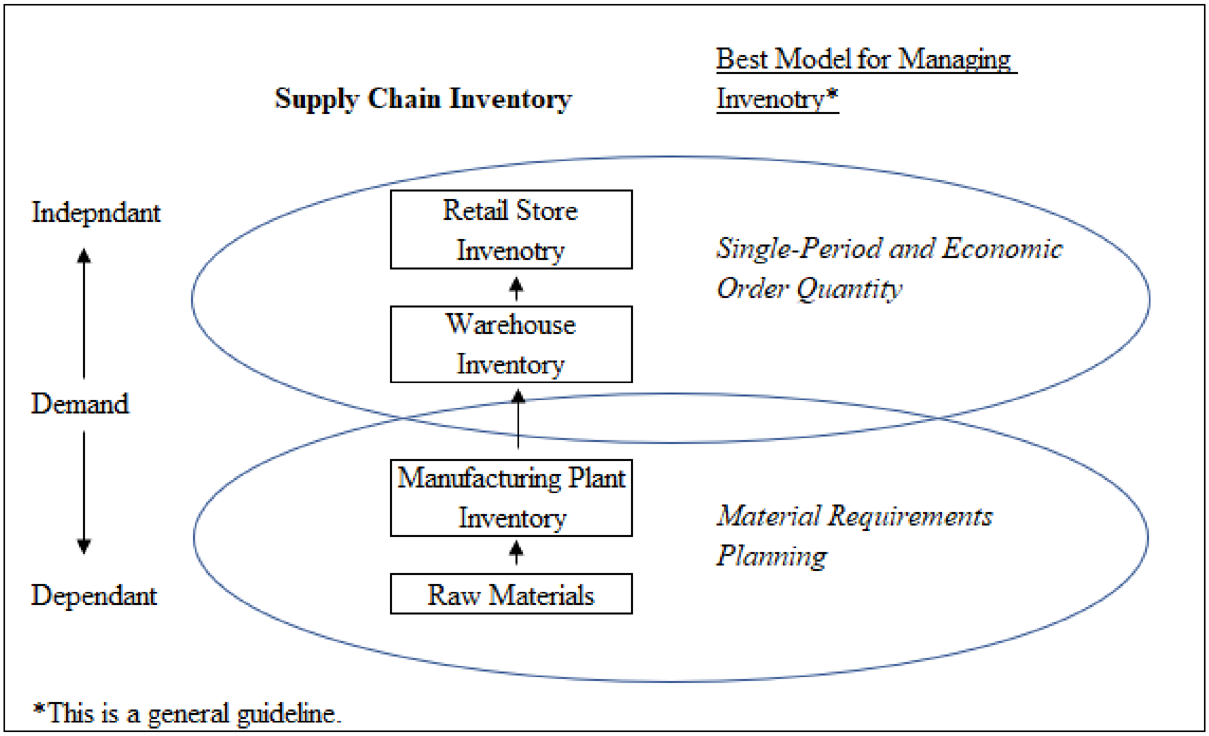

Figure 2.3. Recommended Inventory Model (Jacobs and Chase [2008] 2017)

With dependent demand, the need for the inventory item being stocked directly depends on the need for another item in the production process (Jacobs and Chase [2008] 2017). For example, there are two major components in the process of making malt: barley and water. If the malting plant knows how much barley it has on hand and how much malt it needs to generate, then the amount of water required is a simple calculation (Rosing 2018). This would be an example of the need for water having a dependent demand structure and the Material Requirements Planning (MRP) model would be appropriate.

Independent demand items differ from dependent demand items for obvious reasons, the demand for an independent good is unrelated to the demand of other goods in the production phase. The demand for beer on the other hand, has an independent demand structure since its demand is independent of the other production phases within the system. Knowing how much beer needs to be produced and held in stock is therefore an EOQ problem and requires some sort of forecast in demand (Jacobs and Chase [2008] 2017).

The single period model is used when the decision maker is going to submit a onetime order for a specified quantity. Usually this is due to the good becoming obsolete after a specified period. There are many practical situations where products are perishable, or the product demand is only available for a specific period of time. In situations like these, such as meeting car supply of the primary rail market, the single period inventory model is appropriate. In the case of the primary rail market, the supply of rail cars over a specified period of time is not known with absolute certainty. The theoretical model assumes that only one order may be placed, so how large the single order should be needs to be determined. To find the optimal stocking level (Q*) marginal economic analysis may be used (Ballou [1973] 1992). Using marginal analysis, the optimal stocking level occurs at a point where the marginal benefits of stocking one more unit becomes less than the expected costs of that unit. Jacobs and Chase (2017) refer to the classic example of the news vendor to illustrate how the single period model may be used to determine the optimal number of newspapers to print each morning to meet demand while also not stocking too many newspapers that have no salvage value if demand is overestimated.

The economic order quantity (EOQ) model is used when the demand for an item is continuous through time and occurring at a relatively constant rate. The EOQ model regulates inventory levels by specifying the order quantity and how frequently orders should be placed. This concept is a balancing act between two conflicting cost patterns: the cost of carry and procurement (Ballou [1973] 1992). This model was originally developed by Ford Harris (1913) when he recognized the problem factories where experiencing in ordering too large of quantities and tying up capital. Harris considered unit cost, set-up cost, interest and depreciation on stock, movement (demand), and manufacturing interval (lead time). Even in 1913, Harris recognized his model could not capture all aspects of the management process, however his model provides a guideline which managers could use to help minimize production cost (Harris 1913). Over the past century this EOQ model has served as the basis of many current management practices used today (Ballou [1973] 1992). The EOQ model is a derivative of the total cost equation which encompasses both procurement and carrying cost. Jacobs and Chase (2017) outline the conditions which must exist for the basic EOQ model to hold:

1. Demand for the product is constant and uniform throughout the period.
2. Lead time is constant.
3. Price per unit of product is constant.
4. Inventory holding cost is based on average inventory.
5. Ordering or setup costs are constant.
6. All demand for the product would be satisfied. (No backorders are allowed.)

Ballou (1992) illustrates an example of a parts manufacturing plant where continuous annual demand. Figure 2.4 shows how this example can be used to illustrate the “saw tooth” pattern of a standard inventory depletion and replacement cycle under the conditions laid out by Jacobs and Chase (2017).

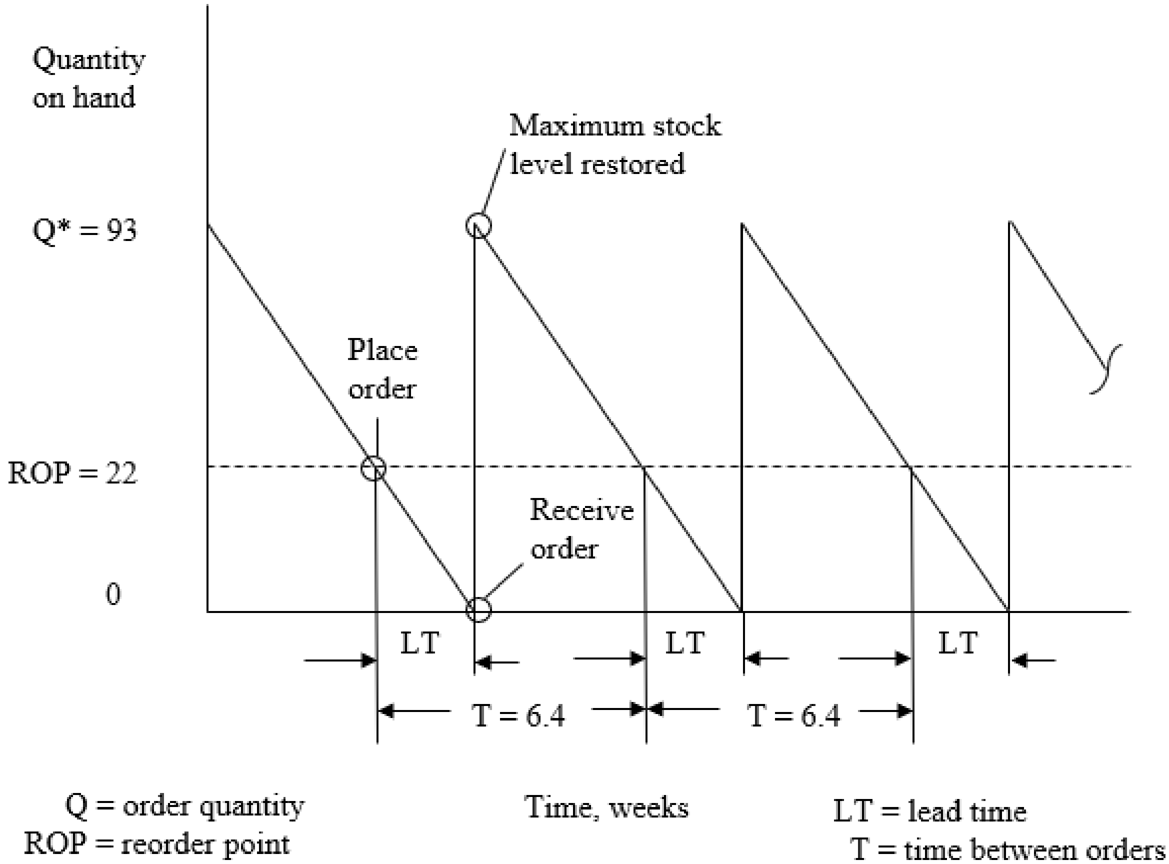

Figure 2.4. EOQ Representation with Reorder Point (Ballou [1973] 1992)

The basic EOQ model serves as guideline for inventory managers to pick an initial strategy. However, most conditions in the EOQ model do not hold in the real world. The greatest of these condition violations often being the uncertainty in supply or demand but may include uncertainty in logistical performance.

Unlike EOQ models, material requirement planning MRP is used when demand for an item is dependent on other items within the system. Landman (2016) uses MRP to estimate how many soybean shuttle trains were needed to meet shipping demand. The MRP model used by Landman (2016) accounted for several factors in the market including rail tariff rate, storage and interest, future market spreads, farmer deliveries, among several other variables to determine shipping demand.

2.2.3. Optimal Inventory Strategy Relevant Literature

The preceding section outlines the conventional approach to inventory management. This thesis uses real options, which is an extension of financial theory as it is applied to inventory decisions. The idea of applying financial theory to inventory management decisions started at the end of the 1980s when Kim and Chung (1989) applied a capital asset pricing model (CAPM) as an alternative to the profit maximization approach. Stowe and Su (1997) use the Black-Scholes (1973) model in a contingent claims approach. Goel and Gutierrez (2006) were one of the first to apply Monte Carlo Simulation and convenience yield. In more recent times, studies have become continuously more interested in the contingent claims approach and the different valuation methods in which it is accomplished (Shi et al. 2011; Chang et al. 2015; Li and Arreola-Risa 2017).

Kim and Chung (1989) use capital asset pricing (CAPM) theory to measure the effects of risk aversion and output market uncertainty on optimal inventory policy. They find that the optimal order quantity of the risk-adjusted maximizing firm is less than the expected-profit maximizing firm. They conclude that a risk averse firm would decrease their inventory policy in the presence of high uncertainty of demand (Kim and Chung 1989).

Stowe and Su (1997) provide examples of conventional inventory models which map uncertainty in demand onto discrete probabilities as well as a continuous demand framework. Stowe and Su then propose that payoffs to an inventory can be mapped onto an underlying state variable which is used as a proxy for either the discrete probability or a continues case. The value-maximizing NPV of the inventory payoff can then be found using an option-pricing model (Stowe and Su 1997).

Goel and Gutierrez (2006) determine optimal procurement policies of a stochastic inventory system. They consider marginal convenience yield between spot and futures prices to develop a procurement strategy which would minimize inventory costs. They use Monte Carlo simulation to generate 10,000 sample paths of unused inventories based on spot price and convenience yield as well as demand assumptions. However, they do not adequately account for stockout penalties because they assume additional spot purchases arrive instantaneously (Goel and Gutierrez 2006).

Shi, Wu, Chu, Sculli, and Xu (2011) use a portfolio approach to a multi-stage procurement process for a processor with a portfolio of long-term contracts, spot procurements, and option-based supply contracts. Their model is set up as a periodic review inventory policy for a material requirement planning schedule and solved using multi-stage stochastic programming. Their model accounts for the variability in input price as well as demand for products without a hedgeable futures price (Shi et al. 2011).

Ma, Yin, and Guan (2013) identify the optimal product order, component production and replenishment decisions in the presence of volatile spot prices and a random yield production process. They find that if inadequate buffer stocks are held in the presence of a random yield process that a “bullwhip” affect could occur and disrupt the supply chain; and, that these effects on profit are compounded when spot price is volatile (Ma et al. 2013).

Chang, Chang, and Shi (2015) develop optimal procurement and inventory policy by modeling the inventory as a portfolio of forward contracts. They use a real-asset martingale valuation methodology where the latent stochastic factor is the underlying trade arrival intensity. They find similar results to Stowe and Su (1997) in that a higher net salvage value would result in a greater initial inventory strategy (Chang et al. 2015).

Li and Arreola-Risa (2017) build on financial theory application models which use CAPM to aide in inventory management (Kim and Chung 1989). They extend the theory to view supplier capacity as a random variable with a minimum and maximum value. Li and Arreola-Risa find the optimal inventory quantity is independent of random supplier capacity while firm value is not (Li and Arreola-Risa 2017).

Of the above literature, the one most closely related to the methodology used in this thesis is by Stowe and Su (1997). Stowe and Su (1997) propose an alternative method to valuing inventory stocking decisions through option analysis. Using options, Stowe and Sus' methodology captures both volatility in supply and demand as well as the time to maturity. Osowski (2004) provided an extension of the model framework built by Stowe and Su (1997) to develop an optimal inventory strategy in the flour milling industry (2004). Both Stowe and Su (1997) and Osowski (2004) assume prices are known with some degree of certainty and that demand is the only random variable. Not all prices are forward contracted so there is generally some degree of uncertainty which needs to be accounted for in future price movements.

2.3. Real Options: Background and Relevant Studies

The use of real options has gained in popularity in recent decades for their use of managerial flexibility and ability to value opportunity through time. The research conducted in this paper relies heavily on the theory of real options and the payoff functions they represent. One of the first sited real options dates back to 600 B.C. when Thales, a famous Sophist philosopher, used real options to gain the right to rent olive presses. Thales paid a premium up front to gain the right, but no the obligation, to rent the olive presses a later date (Kodukla and Papudesu 2006).

A real option gives the owner the right, but not the obligation, to exercise that right at a later date at a negotiated price. Financial options theory defines a "call" option as the right to buy and a "put" option as the right to sell. In the example of Thales and the olive press, Thales had purchased a call option from the olive press owners for the right to rent, or "buy," their olive presses in the following year. The amount Thales paid the olive press owners up front is referred to as the option "premium." The option premium is a function of time to maturity, current price, strike price, interest, and the riskiness of the underlying asset (Trigeorgis [1996] 1999; Dixit and Pindyck 1994; Amram and Kulatilaka 1999).

Options can be classified into two broad categories: financial and real; based on the underlying asset. If the underlying asset is a financial instrument, such as a stock or bond, it is classified as a financial option and can generally be traded on an exchange such as the Chicago Board Options Exchange and the American Stock Exchange (Kodukula and Papudesu 2006). A real option refers to an option whose underlying asset is real. Real options are not generally traded on any form of exchange and therefore lack liquidity. However, real and financial options share many of the same properties so the same terminology is used. Table 2.1 illustrates the basic similarities and differences between real and financial options.

Table 2.1. Financial Options vs. Real Options (Trigeorgis [1996] 1999)

Component	Financial Options	Real Options
Underlying Variable:	Current value of stock	Gross present value of expected cash flows
Strike Value:	Exercise price	Investment cost
Time to Maturity:	Time to expiration	Time until opportunity disappears
Volatility:	Stock price uncertainty	Project value uncertainty
Risk-Free Rate:	Riskless interest rate	Riskless interest rate

Another key difference between financial and real options are the different classifications within real options. Each investment has different characteristics and should therefore be evaluated differently than other investment decisions. Trigeorgis (1999) breaks down real options into seven common categories which include: option to defer, timing option, option to alter operating scale, option to abandon, option to switch, growth options, and multiple interacting options.

2.3.1. ROA vs DCF

When investors are measuring the overall value of a project they tend to turn to one of the many Discounted Cash Flow (DCF) models to aid in the decision-making process. All these models stem from the net present value (NPV) model. The present value model assumes that a dollar is worth more tomorrow than it is today. For this reason, future values must be discounted adequately back to the present time by selecting an appropriate discount rate to reflect the riskiness of the investment (Trigeorgis [1996] 1999). However, most projects generate a stream of revenue over time while also requiring additional outlay which highlights the need for a DCF model.

A DCF model returns a Net Present Value (NPV) which helps decision makers value a project. Normally, if the NPV of a project is positive it is a good investment. Unfortunately, normal NPV analysis through DCF models have fixed assumptions on future cost and sale revenue which make it a static model (Kodukula and Papudesu 2006). The reality in most situations is that future cost and sales are dynamic. Real Option Analysis (ROA), offers supplemented methods to address dynamic properties. ROA is not a substitute for the DCF modeling, rather it is a compliment to the valuation process. ROA has since evolved in recent decades from the foundation of financial option valuation and has become the source of many state-of art decision models.

The model developed by Fisher Black and Myron Scholes (1973) laid the foundation from where real option valuation has grown. These individuals developed a formula which considers time to maturity, current price, strike price, discount interest, and the riskiness of the underlying asset to develop what has come to be known as the Nobel Prize-winning Black-Scholes model. The model calculates the premium for a European Call option which may only be exercised on the date of expiration.

Due to the nature of the model listed above, the value of a real option increases in value as either volatility or the value-to-cost metric increase (Luehrman 1998). The volatility metric has since been referred to as an options extrinsic value and the value-to-cost metric is an option's intrinsic value (Hull [1995] 2008). The extrinsic value is the time value of the option which increase as time to maturity and volatility increases. The intrinsic value is the amount the option is in-the-money, which has a maximum of zero and the value if the option where exercised immediately. Therefore, a call option value can be expressed as max(0, S-X) and put option as max(0, X-S). The calculations in the Black-Scholes model are mathematically complex and can only be evaluated in close form. Cox, Ross, and Rubinstein (1979) developed a method to value real options using a binomial tree.

The binomial tree has since been expanded to incorporate Monte Carlo Simulation to average future state variable forecasts and account for changing volatility. This method of calculating real options is keyed Stochastic Binomial Asset Pricing Model (Shreve 2004). Boyle (1977) uses Monte Carlo simulation in itself to calculate an option value. Churchill (2016) and Landman (2017) combine the Monte Carlo method binomial tree to calculate option premiums.

A study done by Churchill (2016) uses Monte Carlo simulation and provides a comprehensive framework on how different real options are embedded in biotech license agreements (Churchill 2016). Landman (2017) uses stochastic binomial real options to value primary rail contracts based on a one-year continuous contract (Landman 2017).

Real options do not always value assets that have explicit monetary value. Bhattacharya and Wright (2005) use real options to value human capital through assuming that human capital has value, and that the value changes over time. In their third proposition, Bhattacharya and Wright state that firms which have a greater uncertainty of volume, in their case workers demanded, should create HR options to alter operating scale (Bhattacharya and Wright 2005). The number of workers in this situation is viewed as an "inventory" of human resources in which a quantity option is created. However, Bhattacharya and Wright do note that valuing human resources is at best problematic, and at worst impossible (Bhattacharya and Wright 2005).

2.4. Conclusion

This chapter provides background and relevant literature related to inventory management and real options. This thesis pairs contingent claim inventory analysis (Stowe and Su 1997) with stochastic binomial real option valuation (Churchill 2016; Landman 2017) to develop an optimal inventory strategy. Real options on inventory is complementary to the site-based view in explaining the significance of maintaining a competitive advantage on the ability to operate (Leiblein 2003). This competitive advantage may be increased through maintaining adequate inventories and valued using real options.

CHAPTER 3. THEORETICAL MODELS

3.1. Introduction

Two theoretical methods are developed in this chapter. The first theoretical method is for contingent claims inventory analysis (Stowe and Su 1997). The methodology developed by Stowe and Su (1997) utilize the Black-Scholes (1973) option-pricing model; however, this thesis utilizes stochastic binomial real option valuation (Shreve 2004; Churchill 2016; Landman 2017). The second theoretical framework develops option valuation techniques as utilized in this thesis.

3.2. Theoretical Contingent Claims Inventory Model

The theoretical model used in applications of this thesis is a contingent claim inventory (CCI) model with its payoff function being mapped onto real options (Stowe and Su 1997). The methodology outputs a net present value (NPV) for an inventory strategy. CCI analysis entails a call option spread combined with valuation of discounted-net-salvage value and initial-inventory value. The call spread is comprised of a portfolio of long call options which represent the ability to utilize inventory as demand increases and short call options which represent forgone profits when stockout occurs. The NPV equation (5) in Stowe and Su (1997) is a difference between the summation of discounted net salvage value, long call value, and short call value minus the initial inventory value. Equation (3.1) breaks down Stowe and Sus' equation (5) into its four elements.

The CCI model which maximizes NPV through adjusting order quantities is shown in equation (3.1):

$$NPV = Q\Gamma e^{-r_f t} + Lf_L - Sf_S - IQ \tag{3.1}$$

where:

NPV	=	net present value of inventory stocking level
Q	=	order quantity
Γ	=	salvage value of unsold items
r_f	=	risk free interest rate
t	=	time to maturity
L	=	number of long call options
S	=	number of short call options
f_L	=	long call option premium
f_S	=	short call premium
I	=	investment per unit of inventory.

Equation (3.1) may be broken into four elements: vertical discount of net salvage value, gross revenues from satisfied demand, gross loss from unmet demand, and initial inventory value. The vertical discount, $Q\Gamma e^{-r_f t}$, considers the salvage value of all initial inventory stocked at time, t_0. The present value of the initial inventory's salvage value must be discounted at the risk-free interest rate, r_f, and time to maturity, t. Net salvage value is also a factor of the second element which is the gross revenues from satisfied demand.

Gross revenues from satisfied demand includes two components: the number of long call options, L, and the premium per long call option, f_L. The number of long calls, L, represent the additional revenue gained from a one unit increase in the underlying state variable. L can also be expressed as the slope of additional revenue w.r.t. a one unit change in the quantity demanded. Mathematically L can be expressed as:

$$L = \frac{\partial Q_D}{\partial \Psi} * (\Phi - \Gamma) \tag{3.2}$$

where:

Q_D	=	total quantity demanded for a given firm
Ψ	=	the underlying state variable which corresponds with change in demand
$\frac{\partial Q_D}{\partial \Psi}$	=	change in quantity demanded w.r.t. change in state variable level
Φ	=	price received per item sold.

Equation (3.2) subtracts the salvage value, Γ, from the price per item sold, Φ, before it is multiplied by the change in quantity demanded w.r.t. the change in the underlying state variable, $\frac{\partial Q_D}{\partial \Psi}$. The number of long calls thus has a negative relationship with salvage value. This results in a lower number of long call options as the salvage value increases.

The second component of gross revenue from satisfied demand is the long call premium. Premium per long call option, f_L, is obtained through option valuation. Option premium contains both the intrinsic and extrinsic value of the strike level with regards to the current value of the underlying state variable. This relationship also reflects the relative likelihood of the option expiring in-the-money, ITM. The strike value of the long call option, K_L, represents demand being equal to zero. Strike value of K_L is found outside the system, is specific to each situation, and remains static as other values in the model change. Multiplying the number of long calls and long call premium would give the net present value of gross revenues from satisfied demand. In contrast, unmet demand would result in a gross loss per unit increase in the underlying state variable.

Gross loss from unmet demand also includes two components: the number of short calls, S, and short call premium, f_S. S represents both the amount of additional revenue foregone by underestimating demand and stock-out penalties. If there is no additional stock-out penalty, the value of *S* is equal to the value of *L* in order to represent a constant level of revenue even if the underlying state variable continues to increase. However, if a stockout penalty is present, the value of *S* must represent the change in slope. Mathematically, *S* is equal to:

$$S = L + (\frac{\partial Q_D}{\partial \Psi} * \Lambda) \tag{3.3}$$

where:

Λ = shortage penalty.

In a most situations, gross profit is lost in the presence of a stock-out penalty. A positive stock-out penalty would increase the amount of revenue lost through underestimating demand and leads to a negative overall slope in the call spread. However; stock-out penalty can be negative and therefore result in overall revenue increasing if demand is underestimated.

Short call premium, f_S, coincides with the strike level of the short call. By the nature of the call spread used in this model, strike level of the short call would always be higher than strike level of the long call. The mathematical representation the short call strike value is:

$$K_S = K_L + (Q * \frac{1}{\frac{\partial Q_D}{\partial \Psi}}) \tag{3.4}$$

where:

K_S = strike level of the short call option

K_L = strike level of the long call option.

Strike level of the short call option, K_S, is found by adding K_L to the quantity stocked which is then multiplied by the inverse of the change in quantity demanded w.r.t. change in the state variable level, $\frac{\partial Q_D}{\partial \Psi}$. Given that a call option gives the owner the right to buy, the call option with a higher strike level, in this case the short call, would have less value than the call with a lower strike level. In (3.1), short call premium, f_S, reflects the relative likelihood of stockout; therefore, if Q is increased, strike call value, K_S, would increase and f_S would decrease which would decrease the probability of a stock-out.

The final component, initial inventory value, is the mark-to-market value of inventory. This value can be found by multiplying the mark-to-market investment per unit, I, and the order quantity, Q, during the initial period, t_0.

The first three components of the CCI model dictate the form of the payoff function. Figures 3.1 through 3.4 depict the examples used by Stowe and Su (1997). Figure 3.1 shows how the CCI payoff function looks without a salvage value or stockout penalty. In Figure 3.2 a salvage value for unused inventory is incorporated. Salvage value decreases the slope of additional revenue per increase in the underlying state variable; however, the minim payoff would increase. Figure 3.3 adds a positive stockout penalty when demand is greater than the underlying state variable. The addition of the stockout penalty increases the number of short call options which has a negative effect on overall payoff. Figure 3.4 compares each of these cases to show their differences in a convenient manor.

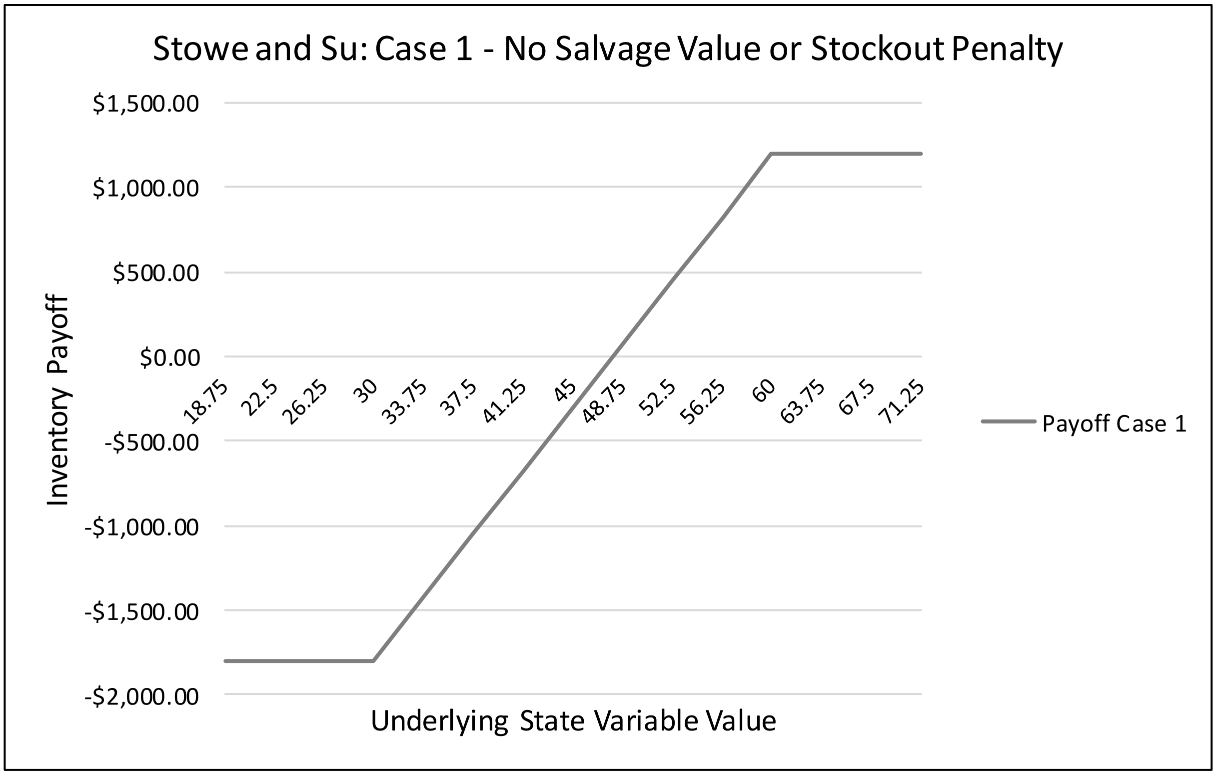

Figure 3.1. Case 1 – No Salvage Value or Stockout Penalty (Stowe and Su 1997)

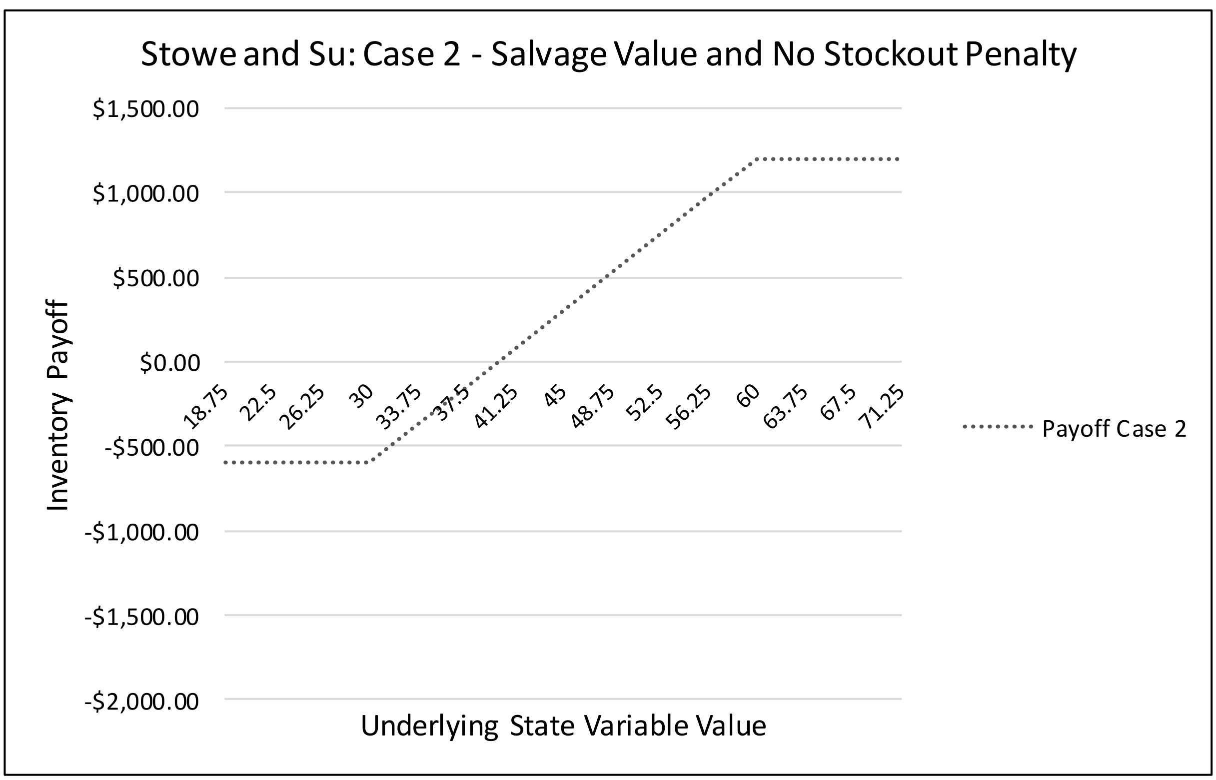

Figure 3.2. Case 2 – Salvage Value and No Stockout Penalty (Stowe and Su 1997)

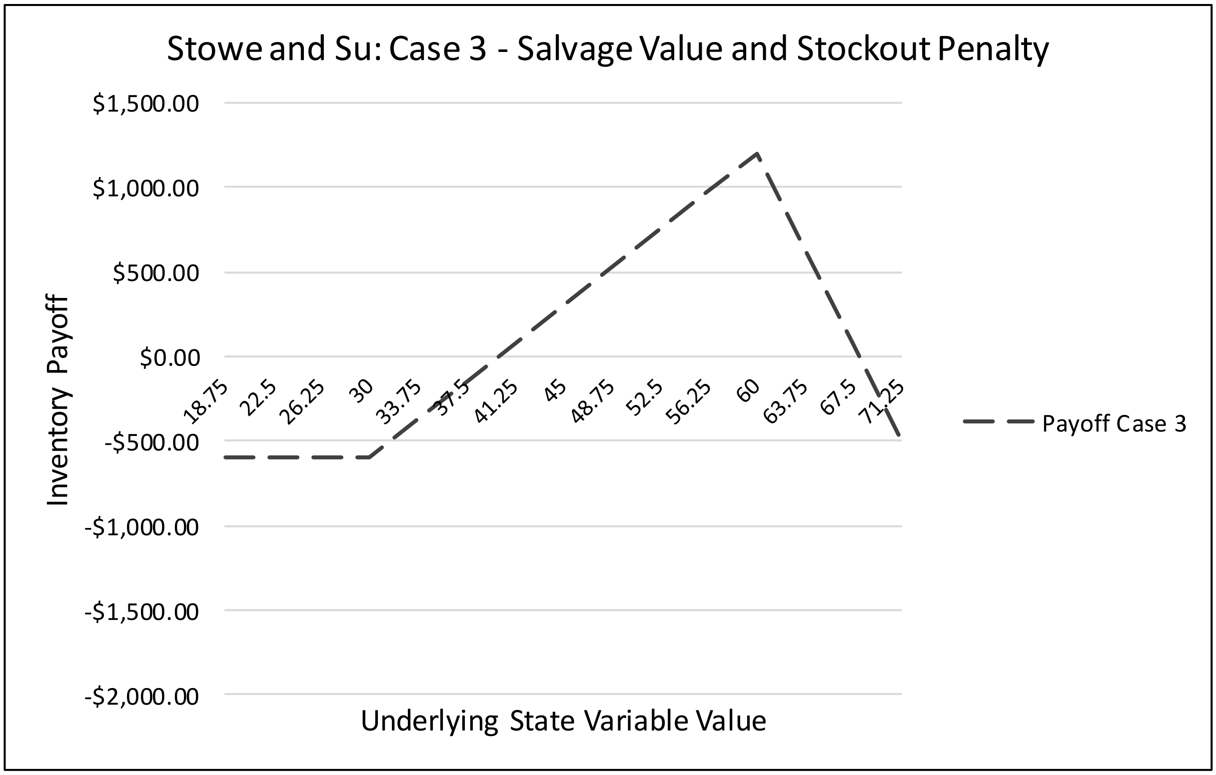

Figure 3.3. Case 3 – Salvage Value and Stockout Penalty (Stowe and Su 1997)

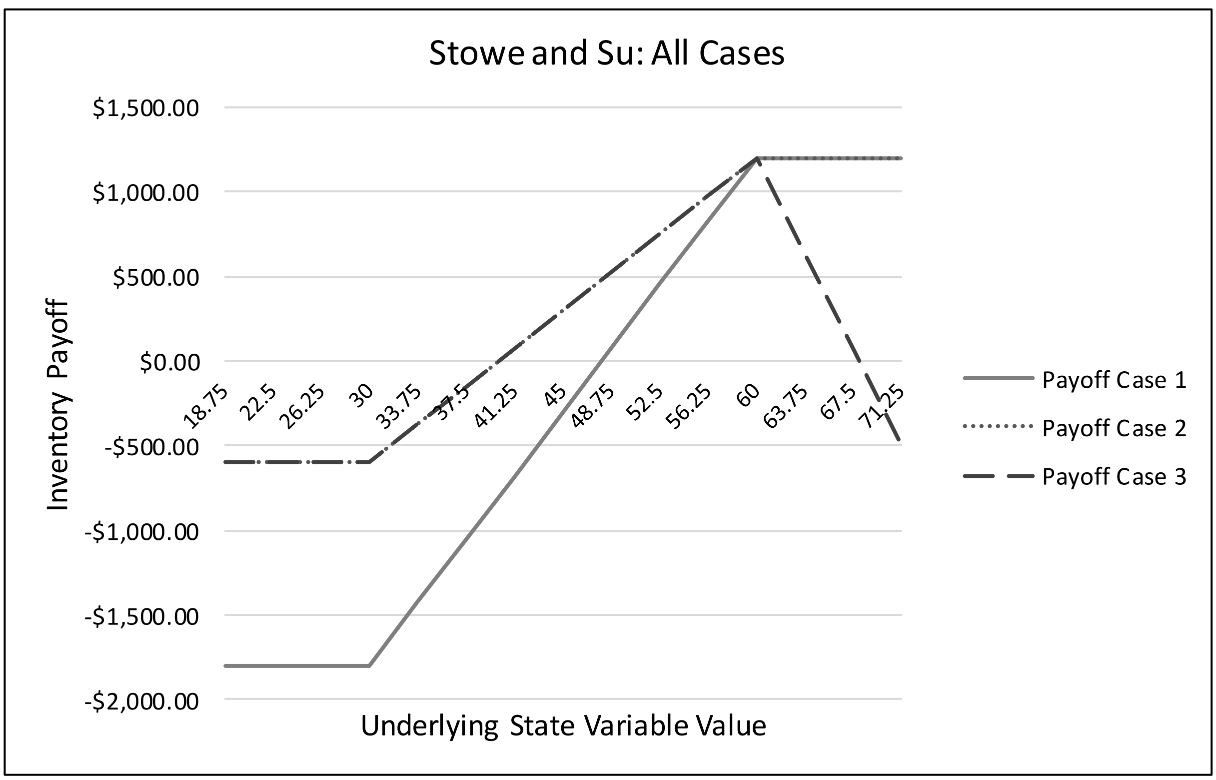

Figure 3.4. All Cases (Stowe and Su 1997)

3.3. Stochastic Binomial Real Option Valuation

Valuation of an option through use of a binomial tree was first published by Cox, Ross, and Rubinstein (1979). The method first divides total life on the option into smaller time periods, Δt. Binomial tree methodology assumes the underlying state variable moves from its original value, Ψ_0, to one of two new values, $\Psi_0 u$ or $\Psi_0 d$, during each period. Generally, the up factor, u, is greater than 1 and the down factor, d, is less than 1. If Ψ_0 moves to $\Psi_0 u$ it is described as an "up" move. If Ψ_0 moves to $\Psi_0 d$ it is described as a "down" move. Ψ_0 makes an up movement with a probability, p, and a down movemebent with a probability of $1 - p$ (Hull [1995] 2008). This basic one step binomial tree is shown in Figure 3.5.

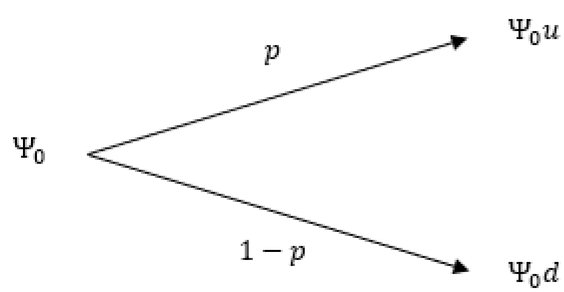

Figure 3.5. Underlying State Variable Movement Through Time, Δt

Figure 3.5. shows movement behavior of the underlying state variable in a risk-neutral world. Risk-neutral valuation is necessary to eliminate arbitrage opportunities for riskless profit (Cox et al. 1979). Therefore, the parameters *p, u,* and *d* must give correct values for the mean and variance of the underlying state-variable-return over time interval Δt. For this to happen three conditions by Cox, Ross, and Rubenstein must hold. These conditions deal with expected return, variance of return, as well as the "up" and "down" factors of the underlying state variable.

The first condition is that average return of the underlying state variable must be equal to expected return in a risk neutral world. In a risk neutral world, expected return on the underlying state variable is the risk-free interest rate, r_f. Expected return for the underlying state variable at end of Δt is then $\Psi e^{r_f \Delta t}$. Therefore, equation (3.5) must hold to match mean return of the underlying state variable.

$$\Psi e^{r_f \Delta t} = p\Psi u + (1-p)\Psi d \tag{3.5}$$

where:

Ψ	=	underlying state variable value at the beginning of the time interval
r_f	=	risk free rate of return
Δt	=	time interval
$e^{r_f \Delta t}$	=	growth factor
p	=	probability of an up move
u	=	up factor
d	=	down factor.

The second condition deals with variance of the underlying state variable. Variance of a variable Q is defined as $E(Q^2) - E(Q)^2$, where E denotes expected value (Hull [1995] 2008). Therefore, it follows that equation (3.6) would equal:

$$\sigma^2 \Delta t = pu^2 + (1-p)d^2 - [pu + (1-p)d]^2 \tag{3.6}$$

where:

σ^2	=	the variance of the underlying state variable.

The third condition is that "up" factor, *u*, must be the inverse of the "down" factor, *d*, as equation (3.7) shows (Cox et al. 1979):

$$u = \frac{1}{d}. \tag{3.7}$$

When Δt is small, the three conditions laid out in equations (3.5), (3.6), and (3.7) are satisfied by

$$p = \frac{a - d}{u - d} \tag{3.8}$$

$$u = e^{\sigma\sqrt{\Delta t}} \tag{3.9}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{3.10}$$

where:

a = $e^{r\Delta t}$.

Once appropriate values for *p*, *u*, and *d* have been derived, values for each node are calculated until the entire binomial tree is complete. At period Δt the underlying state variable has two possible values, $\Psi_0 u$ and $\Psi_0 d$. Due to the nature of the binomial tree, an "up" move followed by a "down" move would yield the same result as a "down" move followed by an "up" move, so that $\Psi_0 = \Psi_0 ud = \Psi_0 du$. Therefore, at period $2\Delta t$ the underlying state variable has three possible outcomes: $\Psi_0 u^2$, S_0, and $\Psi_0 d^2$. Figure 3.6 illustrates a binomial tree through four periods.

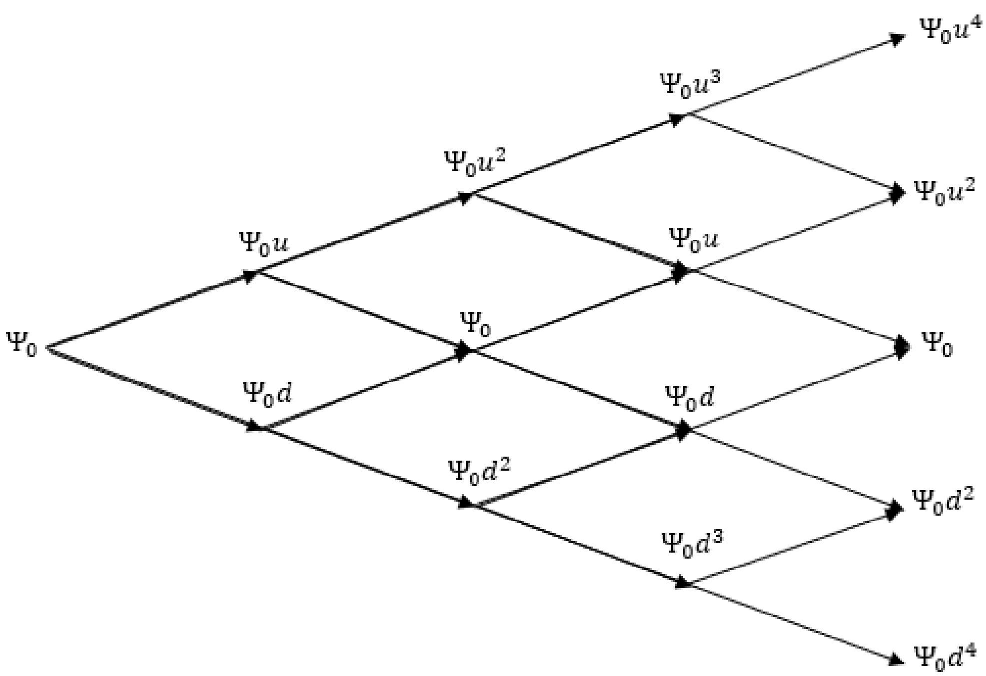

Figure 3.6. Binomial Tree Through $4\Delta t$.

Total number on *N* time periods, Δt, is determined by preference of the user and circumstances in the decision-making process. A greater number of decision nodes would result in a more robust valuation of the option (Hull [1995] 2008).

A process called backward induction is used when evaluating options which are modeled using a binomial tree. Backward induction first evaluates option value at each terminal node during time *T* and then works backward through time, discounting option value along the way.

Value of a call option at its terminal node is:

$$\max(\Psi_T - K, 0) \tag{3.11}$$

and value of a put option at its terminal node is:

$$\max(K - \Psi_T, 0) \tag{3.12}$$

where:

Ψ_T	=	the value of the underlying state variable at a terminal node during time T
T	=	the total life of the option
K	=	strike level.

Total life of the option, *T*, is broken into *N* subintervals of equal period length, Δt. The *j*th node at time $i\Delta t$ is referred to as the (i, j) node, where $0 \leq i \leq N$ is the number of time periods which have transpired, and $0 \leq j \leq i$ is the number of up movements (Hull [1995] 2008). Symbol expression $f_{i,j}$ would be defined as the option premium at the (i, j) node and $\Psi_0 u^j d^{i-j}$ would be defined as the underlying state variable level at the (i, j) node. Given equation (3.11), call option value at each terminal node after *N* time intervals is therefore

$$f_{N,j} = \max(\Psi_0 u^j d^{N-j} - K, 0). \tag{3.13}$$

Once each terminal node is evaluated using equation (3.13) the process of backward induction continues by evaluating each node during period $T - \Delta t$. As calculated in equation

(3.8), there is a probability, p, of the (i, j) node at time $T - \Delta t$ moving to the $(i + 1, j + 1)$ node at time T; and there is also a probity of $1 - p$ for the (i, j) node at time $T - \Delta t$ moving to the $(i + 1, j)$ node at time T (Hull [1995] 2008). When the option is European style, i.e., the option cannot be exercised before the expiration date, risk-neutral valuation gives

$$f_{i,j} = e^{-r_f \Delta t}\left[p f_{i+1,j+1} + (1 - p) f_{i+1,j}\right]. \tag{3.14}$$

This process works backward through the binomial tree for all nodes which $0 \leq i \leq N - 1$ and $0 \leq j \leq i$. The value at each $T - \Delta t$ node is calculated as the expected value at time T discounted at the risk-free rate, r_f. This process continues at each $T - i\Delta t$ node resulting in the entire tree being discounted appropriately. The option value of the initial node, $f_{0,0}$, at time $0\Delta t$ would consider all possible movements of the underlying state variable as defined in the binomial tree. Option premium, $f_{0,0}$, would represent the overall option premium required for the option in a risk-neutral world.

When the option is American style each node must also consider the possibility of early exercise. To accomplish this, option premium, $f_{i,j}$, in equation (3.14) must be compared with the intrinsic value of the option at each (i, j) node. Valuation of an American Call option at each node is

$$f_{i,j} = \max\left\{\Psi_0 u^j d^{i-j} - K,\ e^{-r_f \Delta t}\left[p f_{i+1,j+1} + (1 - p) f_{i+1,j}\right]\right\} \tag{3.15}$$

for $0 \leq i \leq N - 1$ and $0 \leq j \leq i$. The value for $f_{i,j}$ at $i\Delta t$ captures not only the possibility of early exercise at that node, but also the possibility of early exercise at every subsequent node in the binomial tree (Hull [1995] 2008).

The short call option demand premium, f_S, reflects the relative likelihood of running out of inventory. The relative likelihood may also be referred to as the strike-demand's Delta, or the probability of expiring in the money (ITM) (Hull [1995] 2008). The strike-demand delta is the absolute value of the calculation is equation (3.15):

$$\Delta_S = \left|\frac{\Delta f_S}{\Delta K_S}\right| \tag{3.15}$$

where:

Δ_S	=	short call option strike-demand delta
Δf_S	=	marginal change in short call strike demand premium
ΔK_S	=	marginal change in short call strike demand.

Monte Carlo simulation is combined with binomial tree valuation to generate a procedure referred to as stochastic binomial real option analysis as applied in Churchill (2016) and Landman (2017). The analysis used in this thesis fits a distribution based on data to forecast points into the future. Depending on the style of the option, either the average of the forecast or the end value is used as the current state level, Ψ_0. If the option is American style, the average is used in order to reflect the ability to exercise the option at any point between t_0 and T. If the option is European style, the last forecast value is the current state level (Churchill 2016; Landman 2017).

Volatility of the underlying state variable is found through taking the logarithmic first difference of the number of observations determined by the user (Kodukuka and Papudesu 2006). Volatility is not constant through time; therefore, in stochastic binomial real option analysis the logarithmic first differences of forecast values are also included in the volatility calculation. The standard deviation of the logarithmic first differences is then converted to annual volatility by multiplying the standard deviation by the square root of the number of time periods within one year (Kodukula and Papudesu 2006).

3.4. Summary

This chapter developed the theoretical methods which are used in this thesis. Section 3.2 provides the framework of contingent claims inventory (CCI) analysis (Stowe and Su 1997). Section 3.3 provides the framework of binomial tree valuation (Cox et al. 1979) as well as its extension into stochastic analysis (Shreve 2004). The chapter concludes with describing how the method used in this thesis calculates the underlying state variable of the real option (Churchill 2016; Landman 2017) as well as its stochastic volatility (Kodukula and Papudesu 2006).

CHAPTER 4. OPTIMAL FLOUR MILL PURCHASING STRATEGY UNDER RISK

4.1. Introduction

Processors in the agricultural industry all face some matter of contention in regard to transportation, logistics, and storage strategy. Weather it is a crushing facility turning soybeans into meal and oil, an ethanol plant turning corn into ethanol and DDGS, or a flour mill turning wheat into flour and wheat midds; each processor is subject to uncertainties in demand and margin. The optimal level of buffer stocks fluctuates month over month as demand for the main processing product changes. Just-in-time (JIT) manufacturing suggest a minimum of stock should be carried over (Ballou 1992; Jacobs and Chase [2008] 2017). However, this concept becomes potentially harmful to profits when demand contains a high level of volatility or margins are relatively high (Ptak and Smith [1975] 2011). Holding wheat in inventory at a flour mill is substantially different than holding its product, bread, in a grocery store. Wheat is a storable commodity with volatile prices where as its primary product, bread, is considered highly perishable with a low volatility in price. Stowe and Su (1997) view inventory, especially storable inventory, as a portfolio of real options on the ability to operate which also captures the effects of foregone profit from missed sales.

The value of the real option depends on numerous variables including: variability in processor capacity utilization, margin, price spreads, and raw material extraction rates. Raw material extraction rate is an example of a random yield in the production process where increased variability in extraction rate dictates increased volatility in required supply to meet the same milling demand (Ma et al. 2013). Wheat flour mills may view buffer stocks of wheat as a real option to mill flour. Stowe and Su (1997) use a continent claim inventory (CCI) model which values inventories as a call spread. The call spread gives the processor the right to operate until a certain level of demand is reached, i.e., long call options. When demand exceeds inventory the processor's short call options would take effect and limit profits. The portfolio of options outputs a net present value (NPV) which may be viewed as the expected profit given current market uncertainties. CCI analysis may be combined with material requirement planning (MRP) to establish a strategy which would maximize the expected profit of the processor.

This chapter applies material requirement planning (MRP), stochastic binomial real option valuation, and contingent claim inventory (CCI) analysis to a representative wheat flour mill. Output of flour is relatively stable; however, there are uncertainties in mill capacity utilization, extraction rate (the amount of flour extracted wheat), stockout penalties, and market spreads. Under these uncertainties, an optimal purchasing strategy to replenish inventory must be established which would maximize expected profit while also establishing adequate buffer stocks to hedge against uncertainty in demand.

This chapter first develops the conceptual model of a representative wheat flour mill. Next, the empirical section specifies each component of the MRP system, the stochastic binomial real option model, and the contingent claims inventory model. Then, data sources are defined with non-random and random inputs presented. Finally, the base case results are discussed with relevant sensitivities performed on key variables.

4.2. Conceptual Model

CCI analysis as it applies to a flour mill is split into three module parts. The three module parts include:

- Module 1: an MRP system
- Module 2: a stochastic binomial real option model
- Module 3: a contingent claim inventory (CCI) model

Module flow of CCI analysis as it applies to a wheat flour mill is displayed in Figure 4.1.

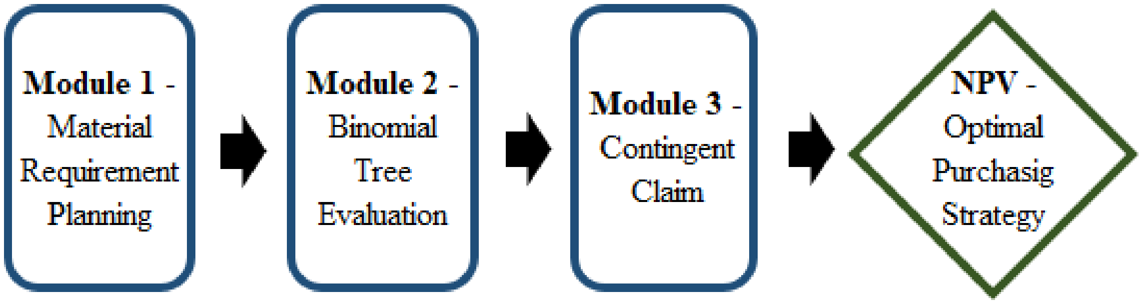

Figure 4.1. CCI Analysis Module Flow: Flour Mill Application

In this application, a representative flour mill replenishes its inventories at the end of each milling month. The purchasing strategy to replenish inventories is expressed as a percent of expected milling demand. Monthly milling demand is set up as material requirement planning (MRP) system for four months of milling. The MRP outputs four key variables which are evaluated using stochastic binomial real options and contingent claims inventory (CCI) analysis. The four key variables are total bushels demanded for milling, beginning inventory bushels expressed as flour cwt equivalents, cwts of flour milled, and cwts of byproducts milled. The logarithmic first differences of the quantity of bushels demanded is evaluated using real option analysis to generate “option-demand-premiums” which serve as proxies for the relative likelihood of sales and stockout quantity. The option-demand-premiums, combined with the other three MRP outputs, are evaluated using (CCI) analysis to generate an NPV which represents expected profit.

Module 1’s MRP system has a beginning inventory, expressed as bushels, which is converted to flour cwt equivalents based on a conversation factor which accounts for extraction rate. The flour cwt equivalents are then compared to milling demand. At the end of each month, inventories are evaluated and replenished using a purchasing strategy specified by the milling manager. The beginning inventory of each month is then evaluated as a real option to mill.

Inventory is viewed as a real option to meet uncertain demand (Stowe and Su 1997). In Stowe and Su's model, demand is tied to an underlying financial asset and the Black-Scholes (1973) model values the option. This application values demand directly as the underlying state variable of the real option. The "option-demand-premium" output from the stochastic-binomial-real-option model serves as a proxy for the likelihood of that option expiring in-the-money (ITM) which is discounted for time value with the risk-free interest rate. The option demand premium is then used in the contingent claim inventory (CCI) model of Module 3 to generate a net present value (NPV) of the purchasing strategy.

Inventory may be valued with real options as a call spread using contingent claim inventory (CCI) analysis (Stowe and Su 1997). Long calls with a minimum strike demand represent the ability to mill flour when demand is above the long call strike and below the short call strike demand. The short calls with a greater strike demand coincide with flour cwt equivalents of wheat bushels. The delta of the short calls represents the likelihood of some level of stockout occurring. If demand is greater than the short call strike demand, the mill forgoes additional margins and accrues any additional penalties associated with stocking out. The option demand premiums reflect the relative likelihood of both the long and short calls expiring in the money, and thus being exercised. The effect the premiums have on expected profit is tied to the number of long and short calls held by the flour mill. Adjusting the purchasing stagey would adjust the short call strike and thus the option demand premium.

4.3. Empirical Model

The empirical model for a representative flour mill builds on the methods developed in Chapter 3 pertaining to contingent claims inventory (CCI) analysis (Stowe and Su 1997) and stochastic binomial real option valuation (Churchill 2016; Landman 2017). The application for a flour mill also utilizes a material requirement planning (MRP) system which maps out wheat demand for flour milling over time. This section is divided into three subsections. First, the MRP system of Module 1 is empirically explained as it applies to a flour mill. Second, stochastic binomial real option valuation of Module 2 is developed for wheat. Finally, the elements of the CCI module is derived for the flour mill.

4.3.1. Module 1: Flour Mill Material Requirement Planning

Material Requirement Planning (MRP) is one of the key systems in a production plan when raw material is being processed in a consistent manner (Jacobs and Chase [2008] 2017). Wheat being milled into flour reoccurs monthly with slight milling demand fluctuations. Milling demand can fluctuate due to changes in extraction rate and mill capacity utilization. Extraction rate refers to the amount of wheat which is processed into flour, expressed as a percentage. Wheat which is not processed into flour may be sold as a byproduct. The amount of wheat required to meet milling demand is negatively related with extraction rate which may also affect mill capacity utilization. Each mill has a capacity of flour, measured in hundred weights (cwt), which it can process in one day. Mill capacity utilization may fluctuate from month to month from an array of reason including, but not limited to: extraction rate, end-user contract requirements, worker efficiency, and number of non-milling days.

The MRP developed in this application is based on the model from *Orlicky's Material Requirements Planning* (Ptak and Smith [1975] 2011, 100). Figure 4.2 shows the basic model setup.

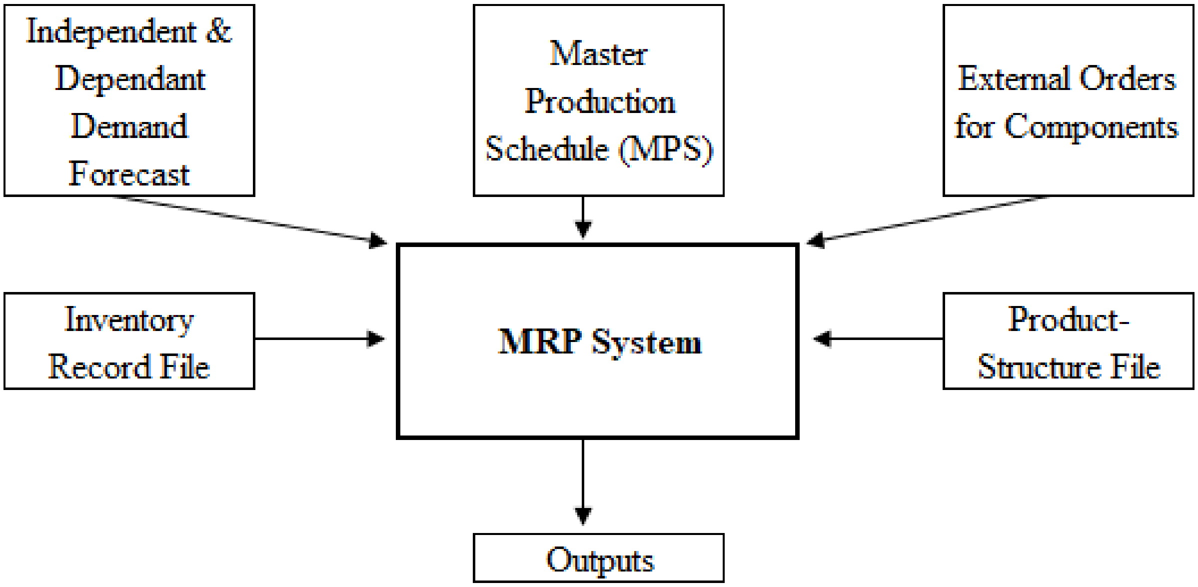

Figure 4.2. MRP System Setup (Ptak and Smith [1975] 2011, 100)

There are several inputs included in the MRP system as outlined in Table 4.1. Each parameter is defined along with its units, symbol, and which system it belongs to.

Table 4.1. MRP Module Parameters

MRP Module Parameter	Units	Symbol	Input System
Storage Capacity	Bushels	Z	MPS
Daily Milling Capacity	Hundred Weights (cwts)	C_M	MPS
Mill Capacity Utilization	Percent	C_U	External Orders for Components
Days Mill Operates Per Month	Days	Θ	External Orders for Components
Test Weight of Wheat	Pounds Per Bushel	ψ	Product-Structure File
Pounds Per Hundred Weight	Pounds	ω	Product-Structure File
Beginning Wheat Inventory	Bushels	Q_B	Inventory Record File; Output
Wheat Demanded to Meet Milling Demand	Bushels	Q_D	Dependent Demand Forecasts
Wheat Milled into Flour	Bushels	Q_M	Inventory Record File
Ending Wheat Inventory	Bushels	Q_E	Inventory Record File
Wheat Purchased Prior to Milling Month	Bushels	Q_P	MPS
Wheat Equivalent of Flour Prior to Milling	Hundred Weights (cwts)	F_B	Inventory Record; Output
Flour Demand	Hundred Weights (cwts)	F_D	Independent Demand Forecast
Flour Milled	Hundred Weights (cwts)	F_M	Output
Flour Byproducts Produced	Hundred Weights (cwts)	B_M	Output
Extraction Rate	Percent	ϕ	Product-Structure File
Ending Inventory Expressed as Percent of Expected Bushels Required	Percent	γ	Inventory Record
Storage Capacity Expressed as Percent of Expected Bushels Required	Percent	ζ	MPS
Purchasing Strategy Expressed as Percent of Expected Bushels Required	Percent	δ	MPS
Unit of Byproduct produced per Unit of Flour Produced	Ratio	χ	Product-Structure File
Conversion Factor of Bushels to One cwt of Flour	Factor	π	Product-Structure File

Storage capacity, Z, equals the total raw bushels wheat which can be stored at the mill. Storage capacity is also expressed as the percent of monthly wheat bushels required, ζ, as calculated in equation (4.1):

$$\zeta = \frac{\mathrm{Z}}{E(Q_D)} \tag{4.1}$$

where:

ζ = storage capacity expressed as a percent of expected bushels required

Z = storage capacity of raw wheat measured in bushels

$E(Q_D)$ = expected wheat required to meet monthly milling demand.

The expected wheat required to meet monthly milling demand, $E(Q_D)$, multiples the four variables of expected wheat bushels required per flour cwt, π, daily milling capacity, C_D, expected mill capacity utilization, $E(C_U)$, and days operating per month. Expected bushels per month is calculated using (4.2):

$$E(Q_D) = E(\pi) * C_M * E(C_U) * \Theta \tag{4.2}$$

where:

$E(\pi)$ = expected wheat bushels required per flour cwt, conversion factor

C_M = milling capacity per day, measured in cwt

$E(C_U)$ = expected mill capacity utilization

Θ = days operating per month.

The expected conversion factor, $E(\pi)$, equals the expected number of bushels required to one cwt of flour and is calculated using (4.3):

$$E(\pi) = \frac{\omega/\psi}{E(\phi)} \tag{4.3}$$

where:

ω = pounds per cwt

ψ = expected pounds per bushel of wheat

$E(\phi)$ = expected extraction rate.

Beginning inventory is a logic calculation which multiples the maximum between the previous months ending inventory expressed as a percent of expected milling demand, γ_{i-1}, and the current months purchasing strategy expressed as a percent of expected milling demand, δ_i. The maximum of the two variables is multiplied by the expected milling demand, $E(Q_D)$. Beginning inventory is calculated using (4.4):

$$Q_{B,i} = \max(\gamma_{i-1}, \delta_i) * E(Q_D) \tag{4.4}$$

where:

$Q_{B,i}$ = inventory at the beginning of *i* processing month

γ_{i-1} = inventory at the end of previous milling month expressed as a percent of expected milling demand

δ_i = purchasing strategy of the *i* processing month expressed as a percent of expected milling demand

The maximum logic function in equation (4.4), combined with a constraint which has purchasing strategy during month *i* less than or equal to the storage capacity, ζ, ensures that beginning inventory is always less than or equal to storage capacity.

The flour cwt equivalent of wheat, F_B, evaluates the beginning inventory of wheat, Q_B, with the current milling month's conversion factor, π_i, as shown in (4.5):

$$F_{B,i} = \frac{Q_{B,i}}{\pi_i} \tag{4.5}$$

where:

$F_{B,i}$ = beginning inventory of wheat expressed as flour cwt equivalents during milling month *i*

$Q_{B,i}$ = beginning inventory of wheat during milling month *i* measured in bushels

π_i = conversion factor during month *i*.

The amount of flour milled, F_M, evaluates the minimum between beginning cwt flour equivalents, $F_{B,i}$, and milling demand during month *i*, $F_{D,i}$. The amount of flour milled, F_M, is evaluated as (4.6):

$$F_{M,i} = \min(F_{B,i}, F_{D,i}) \tag{4.6}$$

where:

$F_{M,i}$ = the amount of flour produced in month *i* measured in cwts

$F_{D,i}$ = amount of flour demanded during month *i* measured in cwts.

The amount of flour demanded during month *i*, $F_{D,i}$, multiplies the three variables of milling capacity per day, C_M, milling capacity utilization during month *i*, $C_{U,i}$, and days milling during the month, Θ. The amount of flour demand during month *i* is calculated using (4.7):

$$F_{D,i} = C_M * C_{U,i} * \Theta \tag{4.7}$$

where:

$C_{U,i}$ = mill capacity utilization during month *i*.

To calculate the number of bushels milled into flour during month *i*, $Q_{M,i}$; the amount of flour produced in month *i*,$F_{M,i}$, is multiplied by month *i*'s conversion factor, π_i, as shown in (4.8):

$$Q_{M,i} = F_{M,i} * \pi_i \tag{4.8}$$

where:

$Q_{M,i}$ = wheat turned into flour during month *i* measured in bushels.

Inventory at the end of milling month i, $Q_{E,i}$, is found by subtracting wheat turned into flour, $Q_{M,i}$, from beginning wheat inventory, $Q_{B,i}$, as shown in (4.9):

$$Q_{E,i} = Q_{M,i} - Q_{B,i} \tag{4.9}$$

where:

$Q_{E,i}$ = ending inventory of wheat after the *i* milling month measured in bushels

The purchasing strategy is expressed as a percent of expected milling demand; therefore, ending inventory must also be expressed as a percent of expected milling demand as in (4.10):

$$\gamma_i = \frac{Q_{E,i}}{E(Q_D)} \tag{4.10}$$

where:

γ_i = ending inventory of milling month *i* expresses as a percent of expected milling demand.

The amount of wheat to purchase as the end of milling month i, $Q_{P,i}$ is then calculated using (4.11):

$$Q_{P,i} = \max\{[\max(\gamma_i, \delta_i) * E(Q_D)] - Q_{E,i}, 0\} \tag{4.11}$$

where:

$Q_{P,i}$ = wheat purchased at the end of milling month *i* measured in bushels.

The amount of flour byproducts produced, $B_{M,i}$, is a function of both flour milled, $F_{M,i}$, and the quantity ratio of byproduct produced per unit of flour produced, χ_i, as in (4.12):

$$B_{M,i} = F_{M,i} * \chi_i \tag{4.12}$$

where:

$B_{M,i}$ = flour byproducts produced measured in cwts

χ_i = quantity ratio of unit byproduct produced per unit of flour produced

and χ_i is calculated using (4.13):

$$\chi_i = \frac{(1 - \pi_i)}{\pi_i} \tag{4.13}$$

Figure 4.3 shows how equations (4.1) through (4.13) were used to set up an MRP system. The four key outputs of the MRP system are in the columns highlighted in green. Columns highlighted in grey are part of the production-structure file. Columns highlighted in blue directly impact the inventory record file. Columns in orange are static and specific to each mill.

Month Number	Month	Beginning Inventory	Flour CWT Equivalents	Mill Capacity Utilization	Flour Demand CWT	Wheat Demanded to Meet Milling Demand	Bushels Turned Into Flour	Ending Inventory Bushels	Ending Inventory in Terms Percent Expected Demand	Purchasing Strategy for i milling month	End of Month Purchased Bushels
i		Q_B	F_B	C_U	F_D	Q_D	Q_M	Q_E	γ	δ	Q_P
	November							0	0.0%		532,113
1	December	532,113	243,683	94%	224,623	490,493	490,493	41,620	8.9%	114.0%	518,499
2	January	560,119	259,420	91%	217,781	470,216	470,216	89,904	19.3%	120.0%	479,551
3	February	569,455	260,730	85%	203,929	445,397	445,397	124,058	26.6%	122.0%	454,732
4	March	578,790	269,351	93%	203,929	438,210	479,027	99,763	21.4%	124.0%	

Month Number	Month	CWT Flour Milled	CWT Wheat Byproducts	Quantity Ratio	Factor of Wheat to Flour	Extraction Rate	Days Milling	Daily Milling Capacity	Expected Bushels Required	Storage Capacity	Stock of Bushels Required
i		F_M	B_M	χ	π	ϕ	Θ	C_M	$E(Q_D)$	Z	ζ
1	December	224,623	69,673	0.310	2.184	76.3%	27	8,890	466,766	1,520,000	326%
2	January	217,781	64,348	0.295	2.159	77.2%	27	8,890	466,766	1,520,000	326%
3	February	203,929	63,309	0.310	2.184	76.3%	27	8,890	466,766	1,520,000	326%
4	March	222,924	64,492	0.289	2.149	77.6%	27	8,890	466,766	1,520,000	326%

Figure 4.3. MRP System for Wheat Flour Mill

4.3.2. Module 2: Stochastic Binomial Real Option Valuation

When a flour mill purchases wheat, they create a real option on the ability to mill flour. This real option can be viewed as a long call option which gains value as milling demand increases. Purchasing a set quantity of wheat also creates a short call option. The short call strike demand coincides with the quantity of wheat purchased and caps the ability to mill flour.

Long and short call options have an option demand premium which represents the relative likelihood of expiring "in the money." Generally, options are quoted in a monetary value. However, option demand premiums are simply a proxy value which reflects the riskiness of demand given time to maturity, stocking level, forecast demand, and risk-free interest. Module 2 uses stochastic binomial trees to value the premium for a real option using backward induction (Cox, et al., 1979).

Table 4.2 shows the five components of an option to mill flour and presents the relationship between three types of options, which builds on Table 2.1.

Table 4.2. Five Components of Option to Sell Fertilizer

Component	Financial Option	Real Option	Option to Mill Flour
Underlying Variable:	Current value of stock	Gross present value of expected cash flows	Forecast milling demand (cwts)
Strike Value:	Exercise price	Investment cost	Milling demand which is supported by purchased quantity of wheat
Time to Maturity:	Time to expiration	Time until opportunity disappears	Time from wheat purchase until end of milling month
Volatility:	Stock price uncertainty	Project value uncertainty	Milling demand volatility
Risk-Free Rate:	Riskless interest rate	Riskless interest rate	52 Week T-Bill rate

Module 2 requires the five inputs outlined in Table 4.2. Once inputs are known, equations (4.14), (4.15), and (4.16) are used to set up the binomial option tree (Hull [1995] 2008):

$$p = \frac{\alpha - d}{u - d} \tag{4.14}$$

$$u = e^{\sigma\sqrt{\Delta t}} \tag{4.15}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{4.16}$$

where:

p	=	probability of an up move
α	=	growth factor
u	=	multiplicative up factor
d	=	multiplicative down factor
σ	=	milling demand volatility; standard deviation of log first differences
t	=	life of option in terms of a fraction of a year
Δt	=	length of one option move; fraction of total moves to t.

The growth factor, α, represents the expected annual growth rate of extraction rate (4.17):

$$\alpha = e^{r_d \Delta t} \tag{4.17}$$

where:

r_d	=	average historical logarithmic first differences of extraction rate.

Values at terminal nodes are evaluated as a call option using (4.18):

$$\max(\Psi_{t,j} - K, 0) \tag{4.18}$$

where:

$\Psi_{t,j}$	=	milling demand values at terminal nodes t with j up moves

j = number of up moves which have occurred since time zero

K = strike milling demand.

Option demand premiums work backward through the binomial tree from right to left. Premiums are evaluated as American style options using equation (4.19) at each node until the final option value is derived at the initial node:

$$f_{i,j} = \max\{\Psi_{i,j} u^j d^{i-j} - K,\ e^{-r\Delta t}[p f_{i+1,j+1} + (1-p) f_{i+1,j}]\} \qquad (4.19)$$

where:

$f_{i,j}$ = option demand premium at node i, j

i = number of milling demand moves which have occurred since time zero

r = risk free interest rate.

Information from Module 1 is used to forecast milling demand four months forward for the months of December, January, February, and March. This application utilizes an MRP system for four months of milling; therefore, individual stochastic binomial trees are set up for each milling month. Forecast milling demand for each month is used as the current state variable for each *i* binomial tree. A previous month milling demand is used as starting point to calculate the logarithmic first difference for each *i* milling month. The standard deviation of the four logarithmic first differences is multiplied by the square root of 12 to generate annualized forecast volatility of milling demand (Kodukula and Papudesu 2006).

Table 4.3 shows an iteration example of forecast milling demand, the logarithmic first differences, and annualized flour demand volatility. Table 4.4 provides an example of all inputs used in the Module 2 example calculation for March milling. Figure 4.4 shows how this iteration example is input to the stochastic binomial option tree for March to return an option demand premium of 5,307 bushels for a short call strike demand **of** 513,443 bushels of wheat**.**

Table 4.3. Logarithmic First Differences of Milling Demand Calculation

i	**Month**	**Milling Demand**	**Logarithmic First Difference**
0	Previous Month	464,819	
1	December	500,554	0.074
2	January	486,030	-0.029
3	February	461,175	-0.052
4	March	465,478	0.009
	Volatility Monthly	0.055	
	Volatility Annualized	0.192	

Table 4.4. Module 2 Iteration Example Inputs for March Milling

Parameter	**Derivation**	**Value**
Forecast Milling Demand	Ψ_T	465,478
Strike Demand	K	513,443
Interest Rate	r	2.6%
Volatility	σ	0.192
Time Until Expiration	t	4/12
Period Length	Δt	0.083
Up Factor	$e^{\sigma\sqrt{\Delta t}}$	1.057
Down Factor	$e^{-\sigma\sqrt{\Delta t}}$	0.946
Probability of Up Move	$p = \frac{1-d}{u-d}$	0.487
Probability of Down Move	$1 - p$	0.513

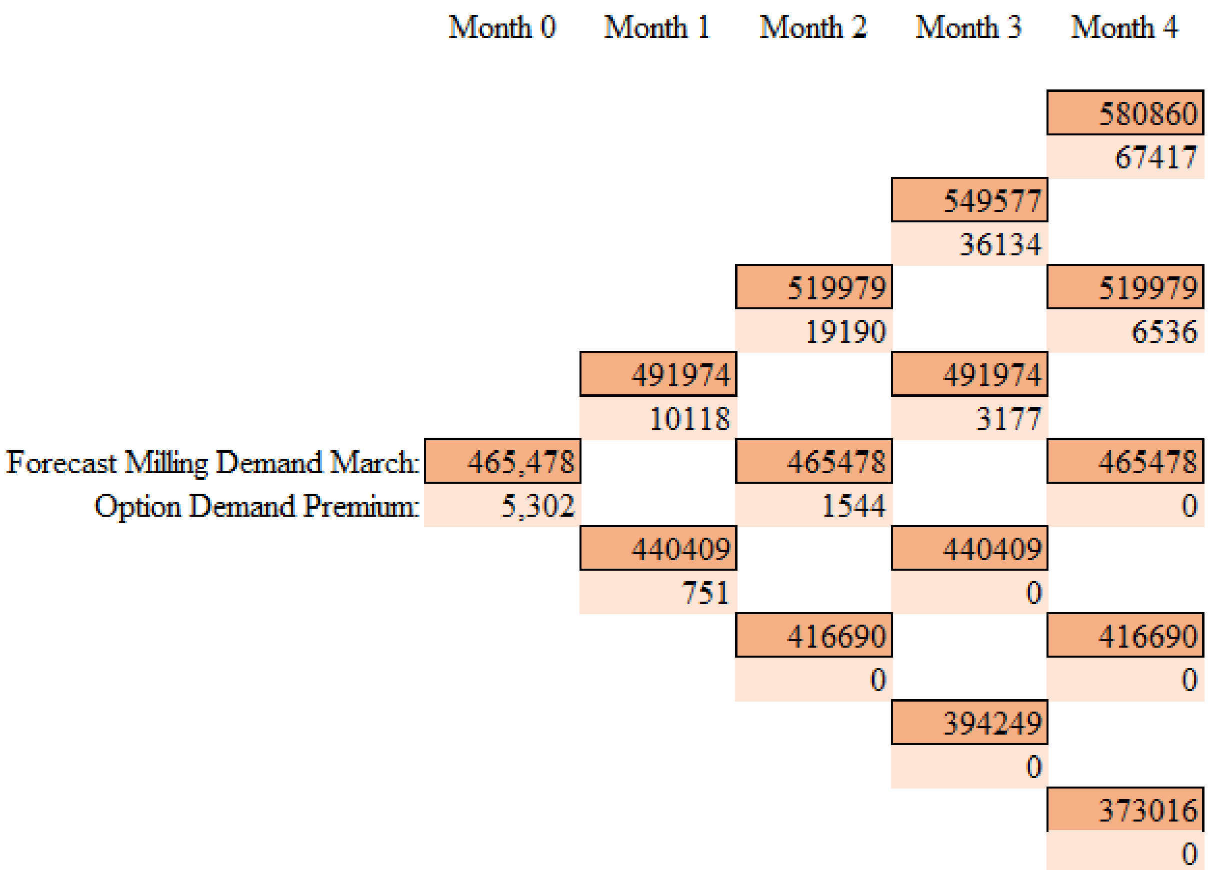

Figure 4.4. Stochastic Binomial Tree (Iteration Example for March Milling)

4.3.3. Module 3: Flour Mill Inventory Contingent Claim

Stowe and Su's contingent claim model is made up of four elements to generate a net present value (NPV) of inventory. The four elements include: salvage value of unused inventory, value of long call options, value of short call options, and the initial inventory value. The first three elements of the contingent claim module make up potential sources of revenue for the purchasing strategy. In this application, salvage value includes one-month worth of storage and interest plus any carry in the market if the milling month is before the delivery month of the futures contract. The long call options represent expected revenue gained per cwt of flour milled plus an adjustment for byproducts. The short call options represent expected revenue foregone per cwt milling demand missed. Combining the first three elements generate expected revenues of the purchasing strategy which are discounted at the risk-free interest rate. The fourth element, which values the initial total inventory value, is then subtracted which outputs an overall net present value of the purchasing strategy. The CCI formula of Module 3 results in an NPV and is shown in (4.20):

$$NPV = \sum_{i=1}^{n} F_{B,i}\Gamma_{B,i}e^{-r_f t} + L_i f_{L,i} - S_i f_{S,i} - I_{B,i}F_{B,\boldsymbol{i}} \tag{4.20}$$

where:

NPV	=	net present value for *i* months of milling
$F_{B,i}$	=	purchasing strategy of the wheat cwt equivalents of flour during month *i*
$\Gamma_{E,i}$	=	salvage value for unsold wheat at the end of the *i* milling month
r_f	=	risk free interest rate
t	=	duration of purchasing strategy
L_i	=	number of long call options during the *i* milling month
S_i	=	number of short call options during the *i* milling month
$f_{L,i}$	=	long call option demand premium during the *i* milling month
$f_{S,i}$	=	short call option demand premium during the *i* milling month
$I_{B,i}$	=	investment per wheat cwt equivalents of flour during month *i*.

Each of the four elements are explained in further detail in the following subsection. The first element calculates the discounted salvage value of any unsold wheat at the end of the milling month. Salvage value of any unsold wheat, $\Gamma_{B,i}$, considers the investment of wheat cwt equivalents of flour, $I_{B,i}$, storage and interest costs, and the carry in the market if applicable. Equation (4.21) shows the salvage value calculation:

$$\Gamma_{B,i} = I_{B,i} - \left[(\pi_i * \tau) + \left(r_L * I_{B,i}\right)\right] + \Omega_i \tag{4.21}$$

where:

π_i	=	conversion rate during milling month *i*
τ	=	storage rate per bushel per month
r_L	=	loan rate
Ω_i	=	futures spread if applicable in milling month *i*.

Equation (4.22) shows how investment per wheat cwt flour equivalents is calculated:

$$I_{B,i} = P_{W,i} * \pi_i \tag{4.22}$$

where:

$P_{W,i}$	=	spot price of wheat during milling month *i*.

The second element of the CCI Module calculates the expected revenue gained each cwt of flour milled using long call options. The number of long calls equal the slope of additional revenue gained per cwt of flour sold. The number of long call options subtracts salvage value from selling price and multiplies it by the demand increase per unit change in the underlying state variable as shown in (4.23):

$$L_i = \frac{\partial F_D}{\partial \Psi} * (\Phi_{F,i} - \Gamma_{B,i}) \tag{4.23}$$

where:

$\frac{\partial F_D}{\partial \Psi}$	=	increase in milling demand per increase in underlying state variable
$\Phi_{F,i}$	=	net price received per cwt of flour sold during milling month *i*.

The net price received per cwt of flour sold, $\Phi_{F,i}$ adds in revenues from selling the byproducts of milling and subtracts the processing cost of wheat into flour. The processing costs included, T_M, include: freight, packaging, enrichments, commission fees, and overhead milling costs (Wilson 2019). Net price received per cwt of flour sold is calculated in (4.24):

$$\Phi_{F,i} = P_{F,i} + (\chi_i * P_{B,i}) - T_M \tag{4.24}$$

where:

$P_{F,i}$ = price received per cwt flour sold during milling month *i*

χ_i = quantity ratio of byproducts produced per unit flour produced during milling month *i*

$P_{B,i}$ = price received per cwt milling byproducts sold during milling month *i*

T_M = total cost of milling.

The number of long calls is multiplied by the long call option demand premium calculated in Module 2 which reflects expected milling demand based on the purchasing strategy. The Module 2 calculation of $f_{L,i}$ is in terms of bushels; therefore, $f_{L,i}$ must be multiplied by the conversion factor, π_i, to convert to cwts. Multiplying $f_{L,i}$ and L_i together generates discounted expected revenue gained from selling flour and byproducts. Expected revenue foregone is then calculated in the third element using short calls.

The short call reduces NPV through the subtraction of expected revenue foregone from expected profits. The number of short calls equals the number of long calls plus the product of shortage penalty and milling demand increase. This application assumes zero shortage penalty in the base case, so the number of short calls would equal the number of long calls as shown in (4.25):

$$S_i = L_i + (\frac{\partial F_D}{\partial \Psi} * \Lambda_{F,i}) \tag{4.25}$$

where:

$\Lambda_{F,i}$ = shortage penalty per unmet milling demand, assumed zero in the base case.

Short call strike demand coincides with the wheat purchasing strategy. As discussed in Chapter 3, this value is found using (4.26):

$$K_{S,i} = K_L + (F_{B,i} * \frac{1}{\frac{\partial F_D}{\partial \Psi}}). \tag{4.26}$$

where:

$K_{S,i}$ = short call strike demand during milling month *i*

K_L = long call strike demand which is found outside the system of equations and assumed to be constant at zero.

The short call option demand premium, f_S, reflects the relative likelihood of running out of wheat and is calculated in Module 2. The relative likelihood may also be referred to as the strike-demand's delta, or the probability of expiring in the money (ITM), as calculated in Chapter 3 (Hull [1995] 2008). The strike-demand delta for wheat is the absolute value of the calculation is (4.27):

$$\Delta_{S,i} = \left|\frac{\Delta f_{S,i}}{\Delta K_{S,i}}\right| \tag{4.27}$$

where:

$\Delta_{S,i}$ = short call option strike-demand delta during *i* milling month

$\Delta f_{S,i}$ = marginal change in short call strike demand premium during *i* milling month

$\Delta K_{S,i}$ = marginal change in short call strike demand during *i* milling month.

The Module 2 calculation of $f_{S,i}$ is in terms of bushels; therefore, $f_{S,i}$ must first be multiplied by the conversion factor, π_i, to convert to cwts. Multiplying $f_{S,i}$ and S_i together generates the discounted expected foregone margin. The values from the first three elements combine and represent total expected revenue gained from the purchasing strategy. Finally, the fourth element, initial inventory value, is subtracted to generate an NPV of the purchasing strategy.

Figure 4.5 shows how Module 3 calculate a four-month MRP contingent claim. @Risk™ is used to simulate the overall module flow using Monte Carlo Simulation on random inputs. RiskOptimizer™ iterates the purchasing strategy until expected profit of the MRP is maximized. The process of changing the purchasing strategy and maximizing NPV represents a dynamic iterative model.

Minimal Purchase Quantity		0
Maximum Purchase Quanity	i	1,520,000
December Inventory Replenishment Strategy	1	114%
January Inventory Replenishment Strategy	2	120%
February Inventory Replenishment Strategy	3	122%
March Inventory Replenishment Strategy	4	124%

December	
Dec Flour CWT Prepared For	249,484
Dec Inv Per CWT	$12.99
Dec Selling Price Per CWT	$16.44
Dec Profit Per CWT Sold	$3.45
Dec Salvage Value	$12.79
Dec Shortage Penalty	$0.00
Dec Vertical Discount	$3,233,445.79
Dec Long Calls	3.66
Dec Long Call Premium:	218,589
Dec Long Exercise Demand:	0
Dec Short Calls	3.66
Dec Short Call Premium:	0
Dec Short Exercise Demand:	249,484
Dec NPV:	$792,282.17

January	
Jan Flour CWT Prepared For	262,870
Jan Inv Per CWT	$12.98
Jan Selling Price Per CWT	$16.43
Jan Profit Per CWT Sold	$3.46
Jan Salvage Value	$12.77
Jan Shortage Penalty	$0.00
Jan Vertical Discount	$3,403,627.15
Jan Long Calls	3.66
Jan Long Call Premium:	220,705
Jan Long Exercise Demand:	0
Jan Short Calls	3.66
Jan Short Call Premium:	0
Jan Short Exercise Demand:	262,870
Jan NPV:	$800,320.98

February	
Feb Flour CWT Prepared For	262,307
Feb Inv Per CWT	$13.22
Feb Selling Price Per CWT	$16.61
Feb Profit Per CWT Sold	$3.39
Feb Salvage Value	$13.08
Feb Shortage Penalty	$0.00
Feb Vertical Discount	$3,460,354.27
Feb Long Calls	3.53
Feb Long Call Premium:	213,872
Feb Long Exercise Demand:	0
Feb Short Calls	3.53
Feb Short Call Premium:	0
Feb Short Exercise Demand:	262,307
Feb NPV:	$747,607.54

March	
Mar Flour CWT Prepared For	264,708
Mar Inv Per CWT	$14.18
Mar Selling Price Per CWT	$18.22
Mar Profit Per CWT Sold	$4.03
Mar Salvage Value	$13.97
Mar Shortage Penalty	$0.00
Mar Vertical Discount	$3,745,798.59
Mar Long Calls	4.03
Mar Long Call Premium:	219,954
Mar Long Exercise Demand:	0
Mar Short Calls	4.03
Mar Short Call Premium:	0
Mar Short Exercise Demand:	264,712
Mar NPV:	$879,141.49
Total Four Month NPV:	$3,219,352.19

***Rolling Hedges at the end of February and April causes Salvage Value Calculation to Change

Figure 4.5. Flour Mill Contingent Claim

4.4. Data

Data analyzed in this application is monthly, quarterly, and annually from January 1998 through November 2018. Data is retrieved to form the representative mill parameters, mill prices, spreads, and extraction rates.

Data sources used in this application are found in Table 4.5.

Table 4.5. Data Sources: Four Mill Application

Data	Source
Representative Mill Parameters	Grain & Milling Annual (2015); Wilson (2019)
No. 1 Hard Red Winter Wheat, Wholesale Baker Flour, and Byproduct Prices	USDA-AMS; Monthly Feedstuffs Prices; Milling and Baking News; Market Fax; USDA-ERS (2019)
Kansas City Wheat Futures Spread	Chicago Mercantile Exchange; Data Retrieved from DTN (2019)
Extraction Rate	USDC-Bureau of the Census' Flour Milling Products; North American Millers Association; USDA-NASS (2019)

The following four subsections discuss data used in the flour mill application along with distributions. The final subsection provides non-random and random inputs of the base case.

4.4.1. Representative Mill Parameters

Representative mill parameters are set up from guidance of industry source contributions as well as data from the Grain & Milling Annual (2015). Industry sources provided guidance in forming assumptions for the total cost of milling. Total cost of milling flour include freight, packaging, enrichments, commission fees, and overhead milling costs (Wilson 2019).

The representative mill's storage and daily milling capacity are an average of 24 mills located in Kansas, Nebraska, and North Dakota (Grain & Milling Annual 2015). The maximum storage represents the total available space for wheat bushels waiting to be milled. Milling capacity equals the total hundred weights (cwts) of flour which is able to be processed during one day of milling.

Data on the 24 mills used in the application are shown in Figure 4.6. Mill names and locations are on the X-axis. The primary Y-axis measures storage capacity of each mill in bushels and the secondary Y-axis measures daily milling capacity in hundred weights.

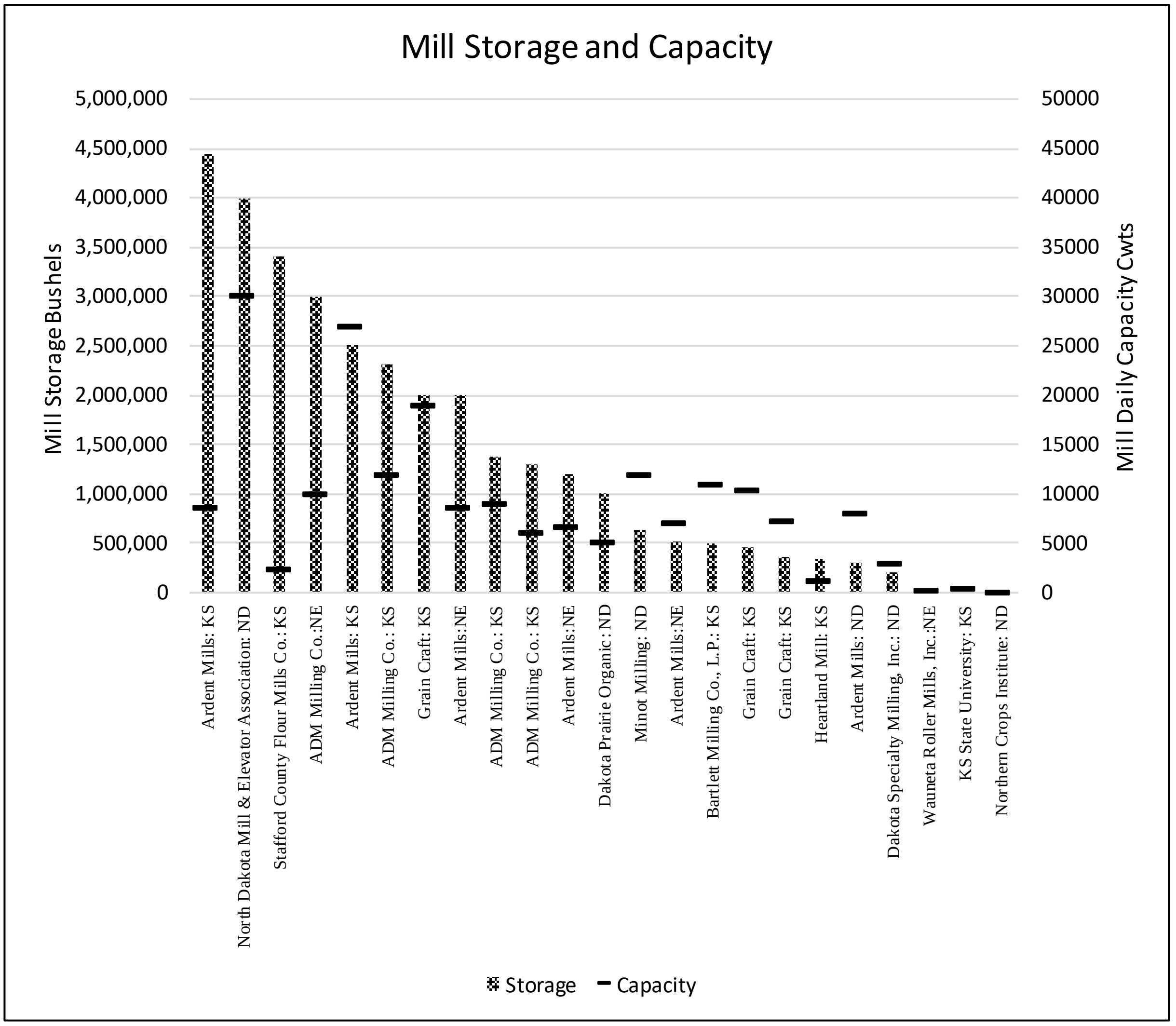

Figure 4.6. Mill Storage and Capacity: KS, NE, and ND (Grain & Milling Annual 2015)

4.4.2. Wheat, Flour, and Byproduct Prices

Data for No. 1 hard red winter (HRW) wheat, whole sale price of bakery flour, and product prices are quarterly from 1988 through 2018 (USDA-ERS 2019a).

Raw data for No. 1 HRW wheat is converted from dollars per hundred weight (cwt) to dollars per bushel using a standard conversion factor of 2.28 (USDA-ERS 2019a).

The wholesale price of bakery flour is recorded in dollars per cwt and is quoted as mid-month bakers' standard patent, bulk press (USDA-ERS 2019a).

The price of flour byproducts is recorded as a premium received by the flour mill per cwt of flour sold. The premium is divided by a quantity ratio of .37 to convert to a standard dollars per cwt of byproduct (USDA-ERS 2019a).

Data for No. 1 HRW wheat, Flour, and Byproducts are graphed in Figure 4.7.

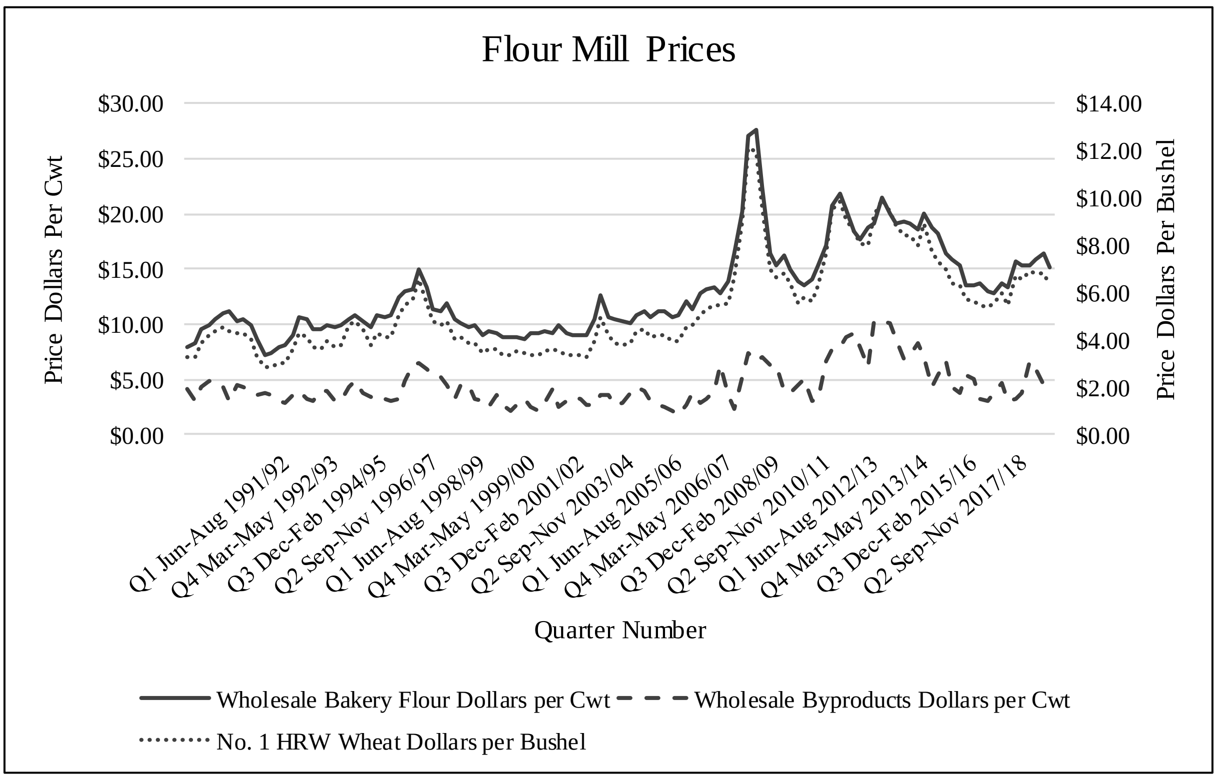

Figure 4.7. Flour Mill Prices (USDA-ERS 2019a)

Figure 4.8 shows how wholesale flour margins have changed through time. The milling margin assumes a constant extraction rate of 73% through time (USDA-ERS 2019a).

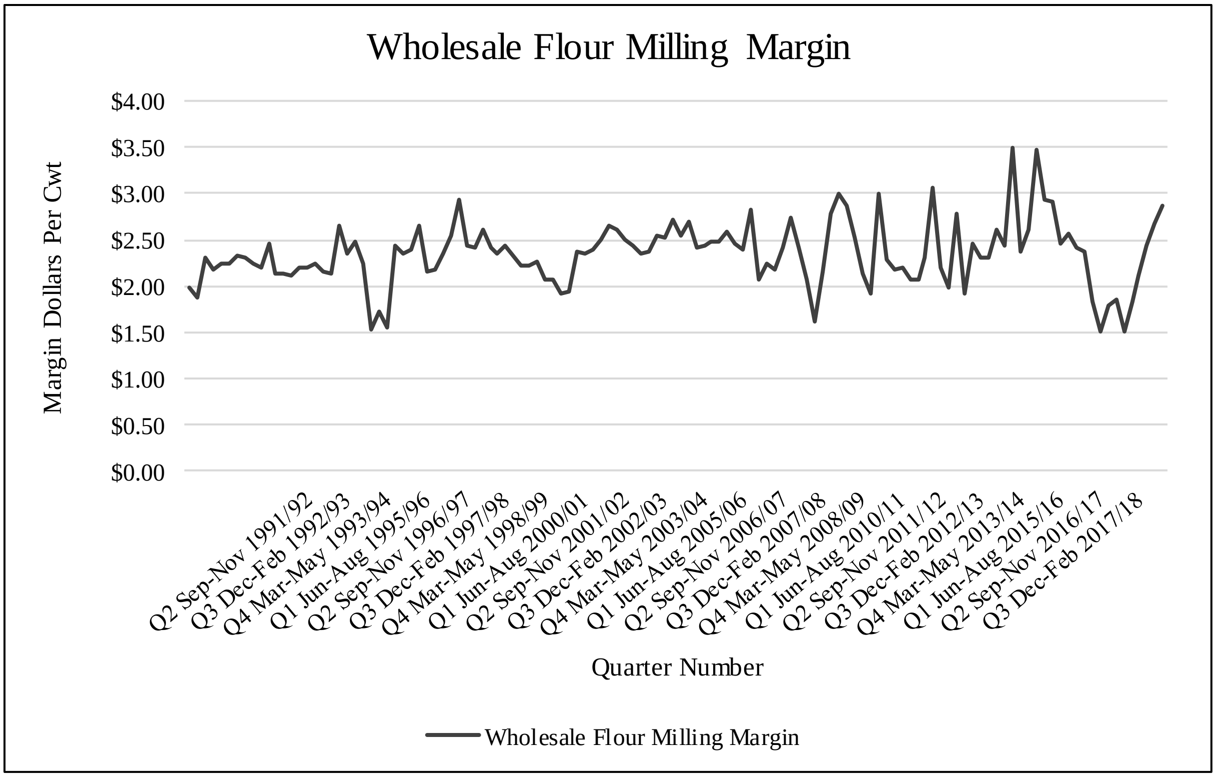

Figure 4.8. Flour Mill Margin (USDA-ERS 2019a)

Bestfit™ in @Risk™ is used to fit time series distributions which are used for forecasting based on the Akaike information criterion (AIC). Bestfit™ compares variations of autoregressive, moving average, Brownian motion, auto regressive conditional heteroscedasticity (ARCH), and generalized auto regressive conditional heteroscedasticity (GARCH) models when fitting time series data. Appendix G shows a complete description of the times series distributions compared by Bestfit™. Bestfit™ detects seasonality, trend, and stationarity to make proper transformations before fitting data. The time series functions use Spearman Rank Order correlation of the error terms when fitting the time series distributions (Palisade 2016). After the proper time series models have been fit, @Risk™ formulates a forecast based on specifications of the user.

Tables 4.6 and 4.7 report the time series functions and correlations of flour mill prices.

Table 4.6. Flour Mill Price Time Series Functions (@Risk™)

Parameter	No. 1 HRW Wheat	Wholesale Flour	Flour Byproducts
Distribution:	Moving Average I	Moving Average I	Autor Regressive II
Function:	RiskMA1(0.005243, 0.09682,0.35961, -0.083936)	RiskMA1(0.0052221, 0.083184,0.32084, -0.095639)	RiskAR2(0.00225, 0.21546, -0.040227, -0.54279,0.18894, -0.26134)
AIC Score	-225.4134	-261.5614	-22.101
Transformation	Logarithmic First Difference	Logarithmic First Difference	Logarithmic First Difference

Table 4.7. Flour Mill Price Correlation Matrix (@Risk™)

Correlation	No. 1 HRW Wheat	Wholesale Flour	Flour Byproducts
No. 1 HRW Wheat	1.000		
Wholesale Flour	0.865	1.000	
Flour Byproducts	0.380	0.029	1.000

Figures 4.9 through 4.11 show the @Risk™ time series function forecasts. Negative values of the X-axis represent historical data. Observations greater than zero on the X-axis are forecast rail freight values. The dark line represents the mean of forecast prices, the gray areas above and below the mean represent confidence intervals, and the red line is a sample path.

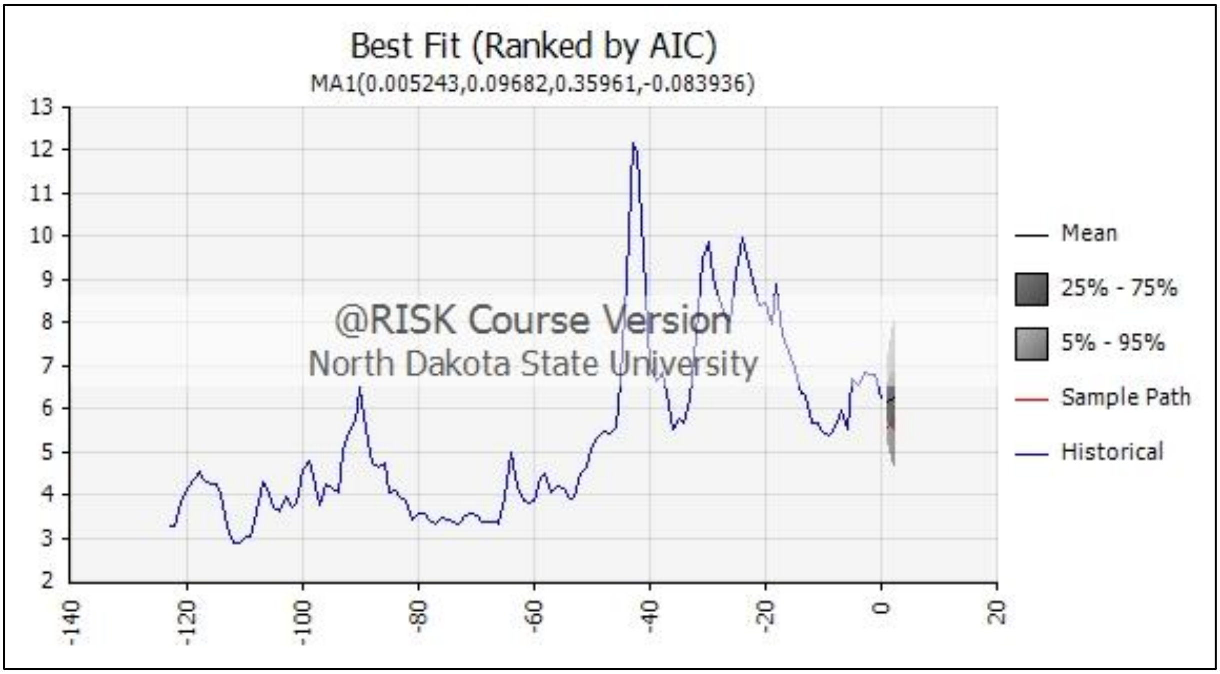

Figure 4.9. Time Series Forecast of HRW Wheat Price (@Risk™)

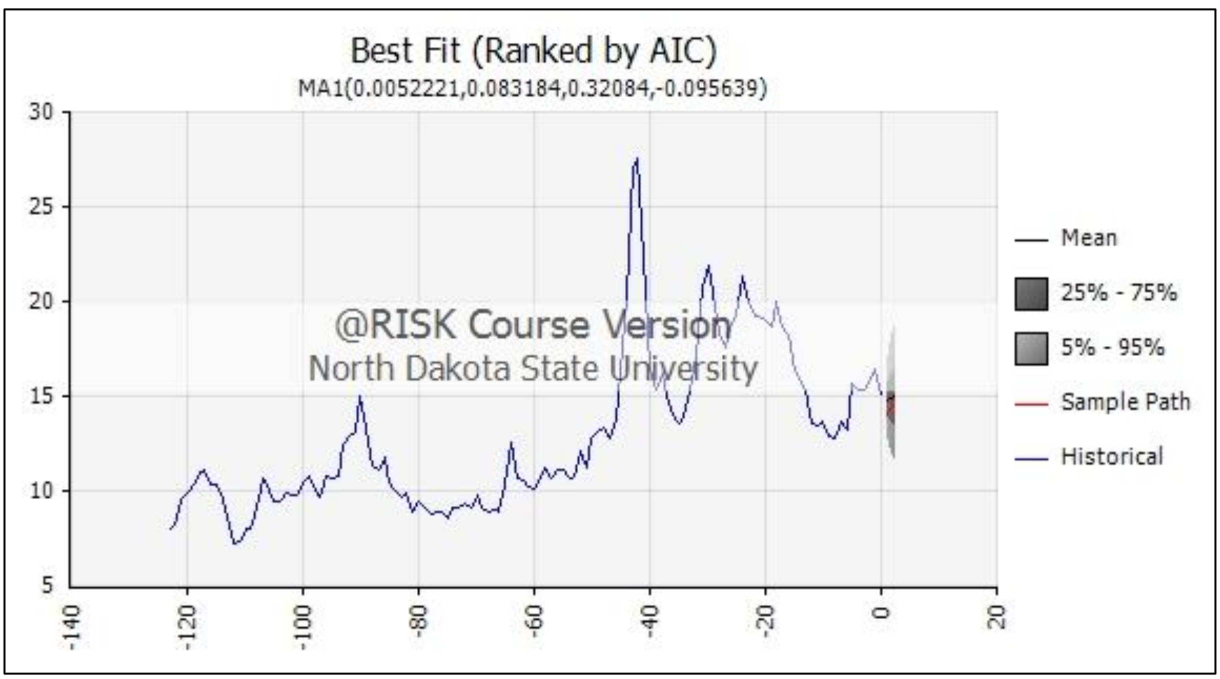

Figure 4.10. Time Series Forecast of Wholesale Flour Price (@Risk™)

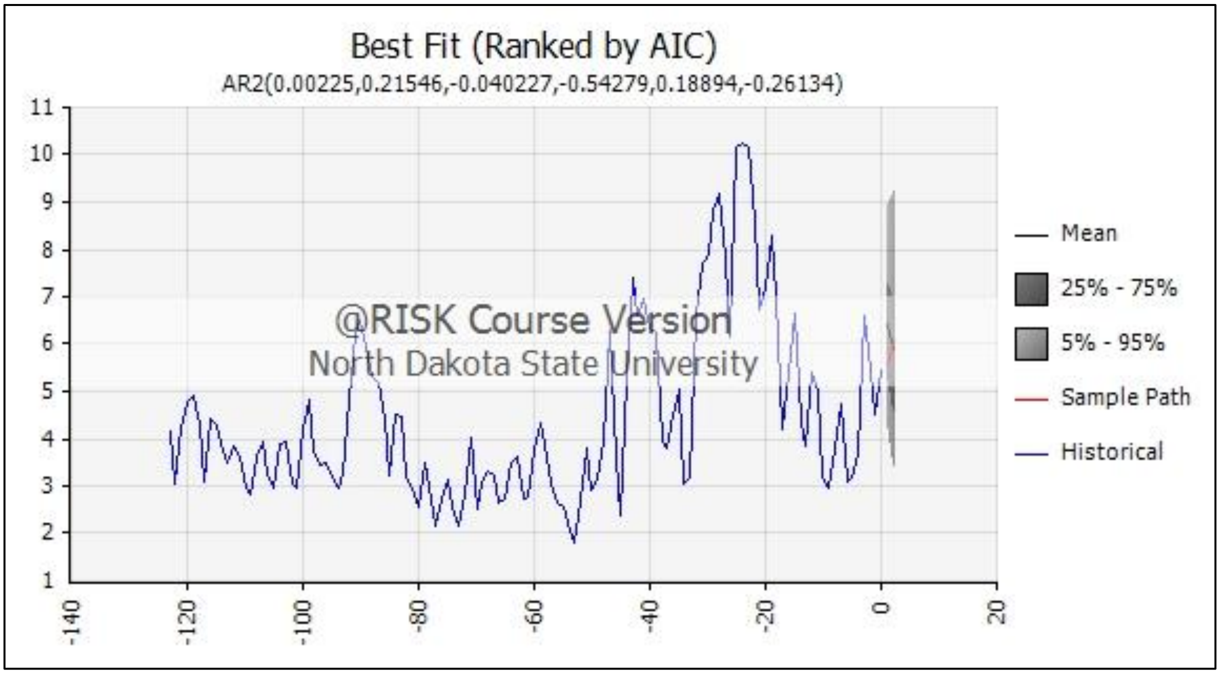

Figure 4.11. Time Series Forecast of Flour Byproduct Price (@Risk™)

4.4.3. Future Price Spreads

HRW futures spread data is observed monthly from January 1988 through November 2018 for a total of 371 observations (CME 2019). Data is recorded by subtracting the nearby futures contract from the deferred futures contract. Prices are record as the closing price of each month and rolled over the month prior to delivery.

The application assumes any excess HRW wheat inventory is hedged using the futures price of Kansas City HRW wheat. If the representative mill hedges in the nearby futures moth, any open contracts must be rolled into the deferred futures month to avoid initiation of the delivery process.

Figure 4.12 shows the behavior of HRW wheat price spread through time.

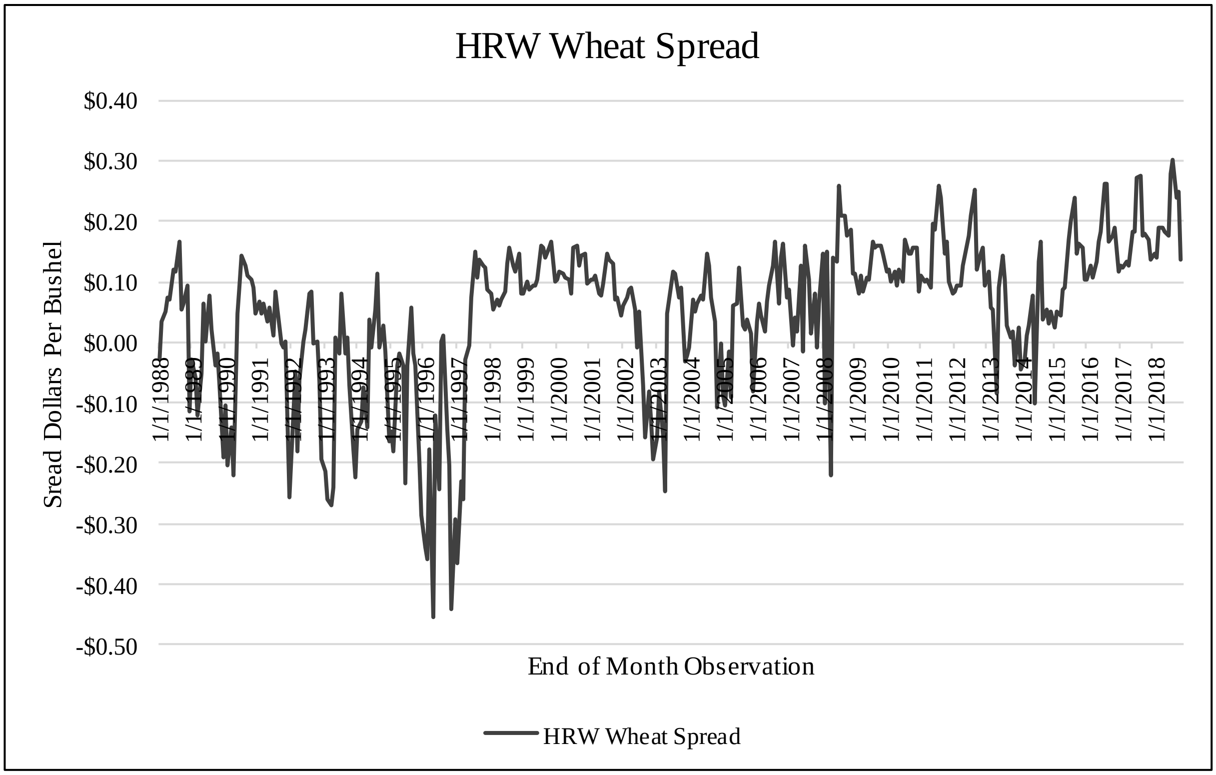

Figure 4.12. HRW Futures Price Spread Behavior (CME 2019)

A time series forecast using BestFit™ in @Risk™ fits HRW wheat spread as shown in Table 4.8.

Table 4.8. HRW Wheat Futures Spread Time Series Function (@Risk™)

Variable	Distribution	Function	AIC Score	Transformation
HRW Wheat Spread	Moving Average I	RiskMA1(0.0003 7391,0.06319, -0.46683, -0.092252)	-999.7492	First Difference Additive Deseasonalize

Figure 4.13 shows the @Risk™ time series forecast for HRW wheat futures spread.

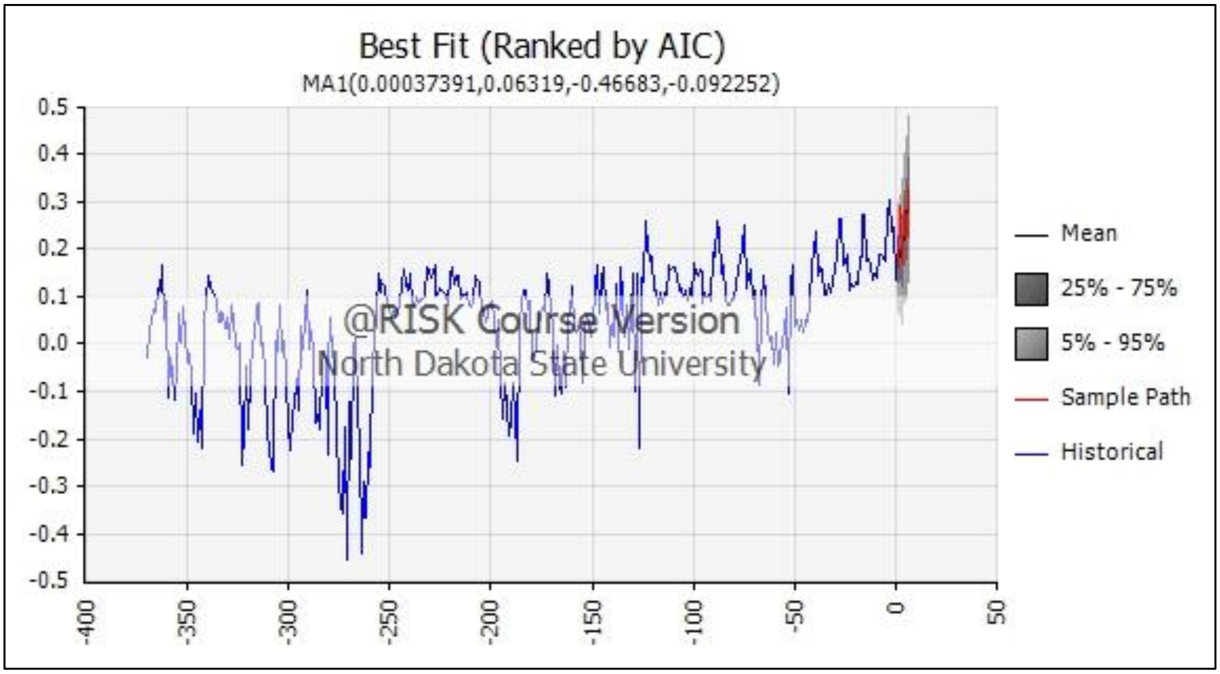

Figure 4.13. Time Series Forecast of HRW Wheat Spread (@Risk™)

4.4.4. Extraction Rate

Data on extraction rate (the efficiency rate at which wheat is converted to flour) is yearly from 1988 through 2018 (USDA-ERS 2019b). The extraction rate data is trending due to improvements in technology so a regression via ordinary least squares (OLS) regression is used to make the data stationary, see Appendix A for OLS regression results.

Figure 4.14 shows the raw and detrended data set.

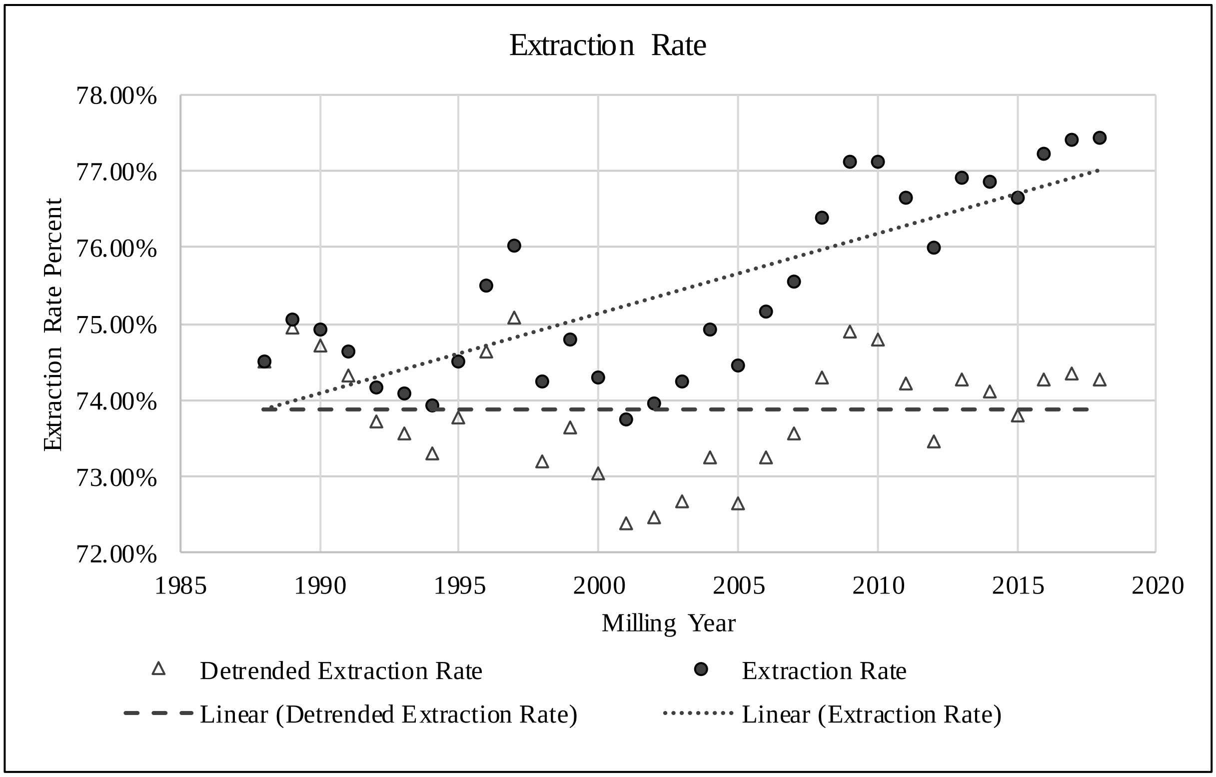

Figure 4.14. Extraction Rate Data (USDA-ERS 2019b)

Extraction rate is forecast for 2019 for each individual month in the MRP system. A probability density function is fit by BestFit™ to generate a detrended extraction rate. A trended addition of 3.27% is then added to the detrended forecast to generate an extraction rate for 2019. Each month's extraction rate is independently forecast to reflect a realistic variability in extraction rate.

Table 4.9 shows the distribution properties of extraction rate. The best fit distribution is chosen using Anderson-Darling criterion. A full set of distribution functions evaluated by BestFit™ is shown in Appendix F.

Table 4.9. Extraction Rate Distribution Function (@Risk™)

Variable	Distribution	Function	A-D Statistic	Mean	Standard Deviation
Extraction Rate	Normal	RiskNormal(0.7386325,0.0076086)	0.38	0.739	0.008

The best fit distribution was a Weibull distribution; however, a normal distribution was the third best distribution and chosen for its ease in sensitivity analysis. Figure 4.15 shows how the Weibel and Normal distribution compare with the raw data distribution.

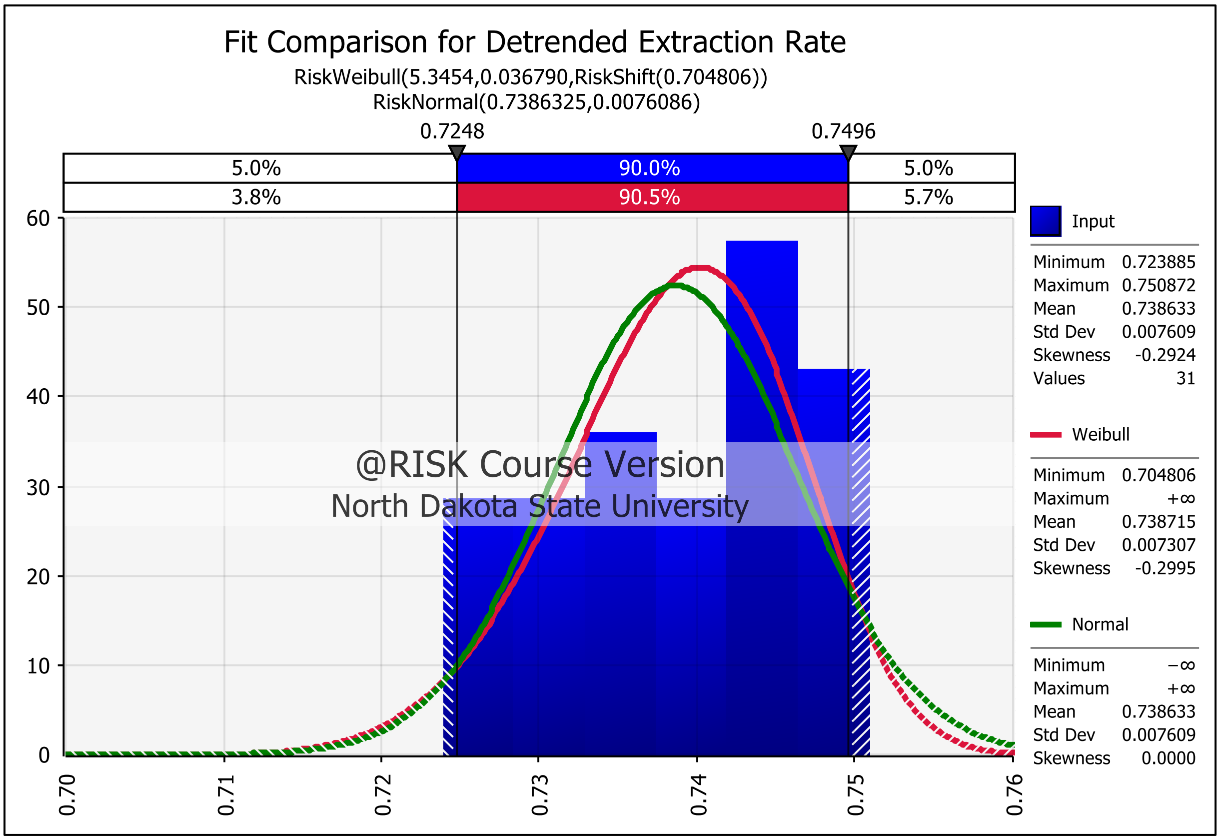

Figure 4.15. Detrended Extraction Rate Distribution Fit (@Risk™)

4.4.5. Random and Non-Random Inputs

Input parameters are split into two groups: random and non-random inputs. Random input parameters are linked, or have calculations linked, to distributions in @Risk™. Non-random inputs are static and do not change during Monte Carlo simulation. Random input parameters are summarized in Table 4.10 and non-random inputs are summarized in Table 4.11.

Table 4.10. Random Input Means

HRW Wheat Mill Model Inputs	Value	Units	Source
Dec HRW Price	$6.14	$$/bu	USDA-ERS 2019a; Calculation
Jan HRW Price	$6.14	$$/bu	USDA-ERS 2019a; Calculation
Feb HRW Price	$6.14	$$/bu	USDA-ERS 2019a; Calculation
Mar HRW Price	$6.23	$$/bu	USDA-ERS 2019a; Calculation
Dec HRW Conversion	2.161	Factor	USDA-ERS 2019b; Calculation
Jan HRW Conversion	2.161	Factor	USDA-ERS 2019b; Calculation
Feb HRW Conversion	2.161	Factor	USDA-ERS 2019b; Calculation
Mar HRW Conversion	2.161	Factor	USDA-ERS 2019b; Calculation
Feb Spread	$0.17	$$/bu	CME 2019; Calculation
Dec Flour Price	$14.82	$$/cwt	USDA-ERS 2019a
Jan Flour Price	$14.82	$$/cwt	USDA-ERS 2019a
Feb Flour Price	$14.82	$$/cwt	USDA-ERS 2019a
Mar Flour Price	$14.99	$$/cwt	USDA-ERS 2019a
Dec Byproduct Price	$6.41	$$/cwt	USDA-ERS 2019a; Calculation
Jan Byproduct Price	$6.41	$$/cwt	USDA-ERS 2019a; Calculation
Feb Byproduct Price	$6.41	$$/cwt	USDA-ERS 2019a; Calculation
Mar Byproduct Price	$5.90	$$/cwt	USDA-ERS 2019a; Calculation
Dec Storage & Int.	$0.21	$$/cwt	Calculation
Jan Storage & Int.	$0.21	$$/cwt	Calculation
Feb Storage & Int.	$0.21	$$/cwt	Calculation
Mar Storage & Int.	$0.21	$$/cwt	Calculation
Dec Capacity Utilization	90.0%	Percent	Assumption
Jan Capacity Utilization	90.0%	Percent	Assumption
Feb Capacity Utilization	90.0%	Percent	Assumption
Mar Capacity Utilization	90.0%	Percent	Assumption
Dec Milling Demand	215,942	Cwt	Calculation
Jan Milling Demand	215,942	Cwt	Calculation
Feb Milling Demand	215,942	Cwt	Calculation
Mar Milling Demand	215,942	Cwt	Calculation
Dec Byproduct Ratio	0.296	Quantity Ratio	USDA-ERS 2019b; Calculation
Jan Byproduct Ratio	0.296	Quantity Ratio	USDA-ERS 2019b; Calculation
Feb Byproduct Ratio	0.296	Quantity Ratio	USDA-ERS 2019b; Calculation
Mar Byproduct Ratio	0.296	Quantity Ratio	USDA-ERS 2019b; Calculation

Table 4.11. Non-Random Inputs

HRW Wheat Mill Model Inputs	Value	Units	Source
Milling Capacity Daily	8,890	cwt	Grain & Milling Annual 2015
Storage Capacity	1,520,000	Bushels	Grain & Milling Annual 2015
Mill Utilization	90%	Percent	Assumption
Mill Utilization Stdev	3.5%	Percent	Assumption
Maximum Capacity Utilization	100%	Percent	Wilson 2019
Risk-Free Interest Rate	2.62%	APY	USDT 2018
Day Operating Per Month	27	Days	Wilson 2019
HRW Pounds Per Bushel	60	lbs.	USDA-ERS 2019a
Pounds Per CWT	100	lbs.	USDA-ERS 2019a
Shortage Penalty	$0.00	$$/cwt	Assumption
Demand Increase	1	CCI Factor	CCI Calculation
Exp(-r*t)	0.998	Discount Factor	Calculation
Expected Extraction Rate	77.1%	Mill Extraction Rate	USDA-ERS 2019b; Calculation
Expected Bushels per CWT	2.16	Conversion Factor	USDA-ERS 2019b: Calculation
Expected Bushels Per Month	466,766	Bushels	Calculation
Max Buffer Stock	326%	Percent	Gain & Milling Annual 2015; Calculation
Total Cost of Milling	$2.53	$$/cwt	Wilson 2019
Dec Maturity	0.08	Years	Calculation
Jan Maturity	0.17	Years	Calculation
Feb Maturity	0.25	Years	Calculation
Mar Maturity	0.33	Years	Calculation
Order Interval	Monthly	Interval	Assumption
Storage Per Bushel	0.07	Monthly	Wilson 2019
Credit Rate	5%	APR	Assumption

4.5. Base Case Results

The empirical modules are applied to a representative mill to develop an optimal purchasing strategy for four months of milling where wheat inventories are replenished at the end of each milling moth. Monte Carlo simulation is implemented using @Risk™ to run 10,000 iterations of the model based on structural and stochastic variables. RiskOptimizer™ iterates the purchasing strategy of each month until mean expected profit is maximized. Specific @Risk™ settings are shown in Table 4.12.

Table 4.12. Flour Mill @Risk™ Settings

@Risk™ Specification	@Risk™ Setting
Sampling Type	Latin Hypercube
Generator	Mersenne Twister
Initial Seed Value	1
Macros	VBA

Results of the base case, and subsequent sensitivities in Section 4.6, reflect the mean values of Monte Carlo simulation for the optimal purchasing strategy. RiskOptimzer™ maximizes equation (4.20) by changing the purchasing strategy expressed as a percent of expected milling demand. Purchasing strategies are changed in step sizes of 1% with constraints imposed on RiskOptimzer™ which prevent purchasing strategies less than zero or greater than maximum buffer stock. The base case results are formulated using data and distributions discussed in Section 4.4.

The optimal average buffer stock of the representative mill is 120% of expected milling demand. The expected profit of four months of milling is $843,564 with a standard deviation of $1,623,748. The deltas for December, January, February and March are 3.7%, 3.0%, 3.6%, and 4.0% respectively. This would infer that the mill would use a purchasing strategy which almost eliminates any probability of stockouts. This is attributed to the high convenience yield which can be measured using the number of short calls, 1.04. This means the mill would forgo $1.04 in additional revenue for each cwt of milling demand missed. The expected storage and interest is $0.21 per cwt which is relatively small compared to the $1.04 per cwt of forgone profit (Table

4.10). The low storage costs relative to forgone profit is a choice example of convenience yield in agriculture processing.

Table 4.13 shows the complete results of the optimal base case.

Table 4.13. Flour Mill Base Case Results

Observation	Value
Expected Profit	$843,565
Standard Deviation of Expected Profit	$1,623,748
Average Purchasing Strategy	120.0%
Average Number of Long Calls	1.04
Average Long Call Demand Premium	172,753
Average Number of Short Calls	1.04
Average Short Call Demand Premium	500
Total Bushels Demanded	1,866,512
Standard Deviation of Total Bushels Demanded	36,740
December Purchasing Strategy	114%
January Purchasing Strategy	120%
February Purchasing Strategy	122%
March Purchasing Strategy	124%
December Delta	3.7%
January Delta	3.0%
February Delta	3.6%
March Delta	4.0%

Figure 4.16 shows the payoff function for each milling month. In the month of January, the long call options establish a price floor of a minimum expected profit of -$50,000 which represents the expected net salvage value of inventory. When demand surpasses long call strike demand, the payoff increases with a slope equal to the number of long call options. The number of long call options hold a monetary value indicating the amount of revenue gained per cwt of flour sold. The number of long call options is greater than the milling margin because of net salvage value is incorporated into additional revenue gained.

Net profit continues to increase as milling demand increases until the short call strike demand is reached. In the month of January, this point occurs at a maximum expected profit of $240,000. At this point, inventory would run out and the short strike demand takes effect. The number of short calls represent the amount of additional profits foregone from missed milling demand. If the stockout penalty is zero, the number of short calls is equal to the number of long calls (equation 4.23). An equal number of long and short calls would result in a horizontal slope in net profits, as seen in Figure 4.16, when milling demand continues to increase past the short call strike-demand.

The payoff function for the month of February has a higher net salvage value due to the futures spread. The base case has an expected spread of a positive $0.17 per bushel or equivalently $0.37 per cwt. Any excess bushels are hedged through selling futures contracts. The short position in HRW wheat futures contracts is rolled into the next futures month at the end of February. A gain in net salvage value occurs when spread is positive; therefore, a $0.37 increase in net salvage value raised the minimum expected profit to nearly $0.

The March payoff function is considerably different from the other three months because the margin decreases during the next milling quarter. A decreased margin lowers the number of long and short calls. This lowers the slope of additional payoff and thus the convenience yield of holding inventory. The purchasing strategy is still higher in March due the added time value of the real option. The added time value in the option increases the uncertainty in expected demand. The increase in expected demand means there is a higher possibility for stockout. Even though the convenience yield is lower in March, there is still enough forgone profit for the mill to utilize a strategy which further lowers the risk of a stockout occurring.

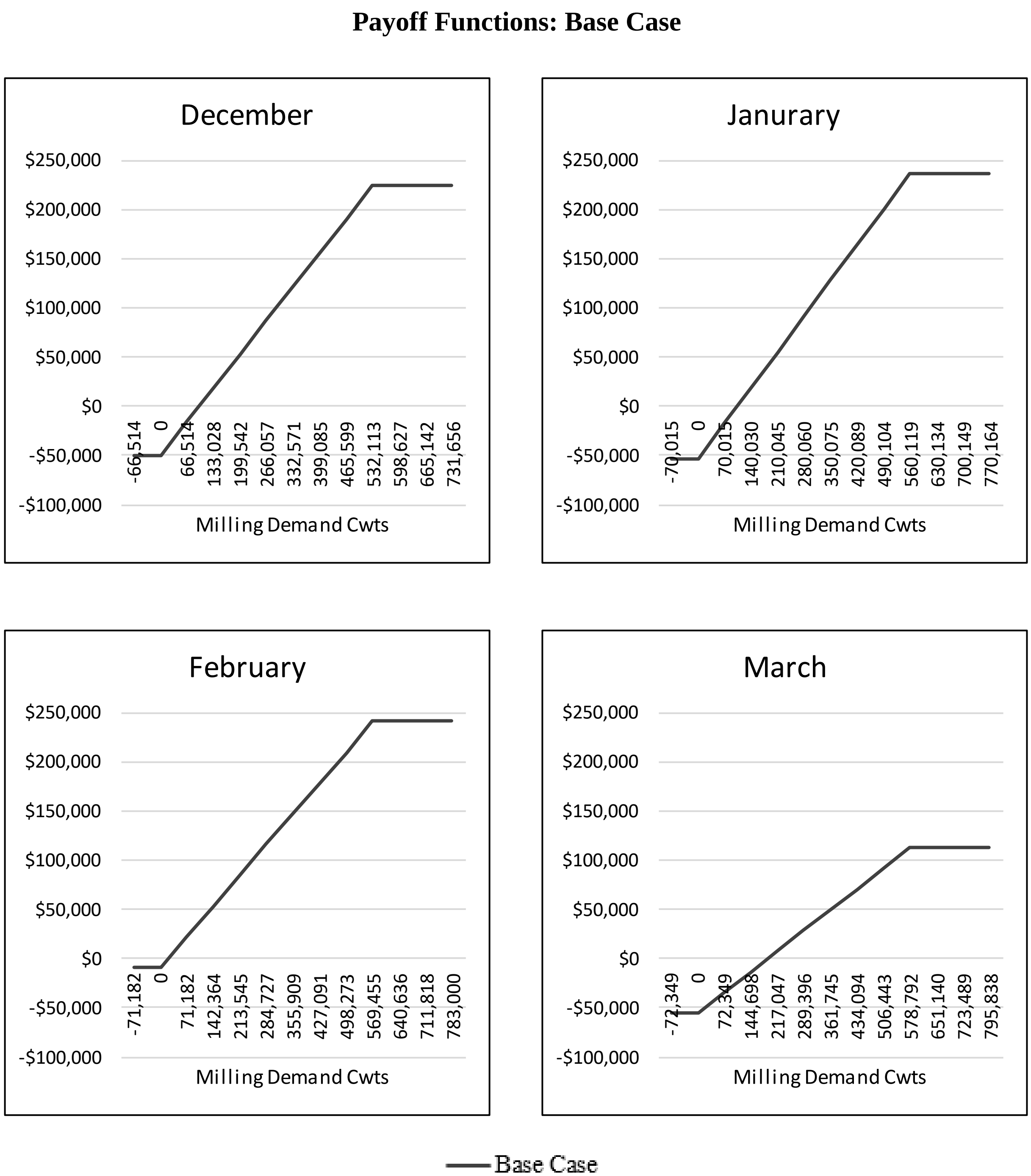

Figure 4.16. Payoff Function Base Case

Total expected milling demand follows a normal distribution because both capacity utilization and extraction rate follow a normal distribution function. Expected total milling demand is 1,866,510 bushels with a minimum of 1,741,097 bushels and a maximum of 2,011,784 bushels.

Figure 4.17 shows the distribution of total milling demand.

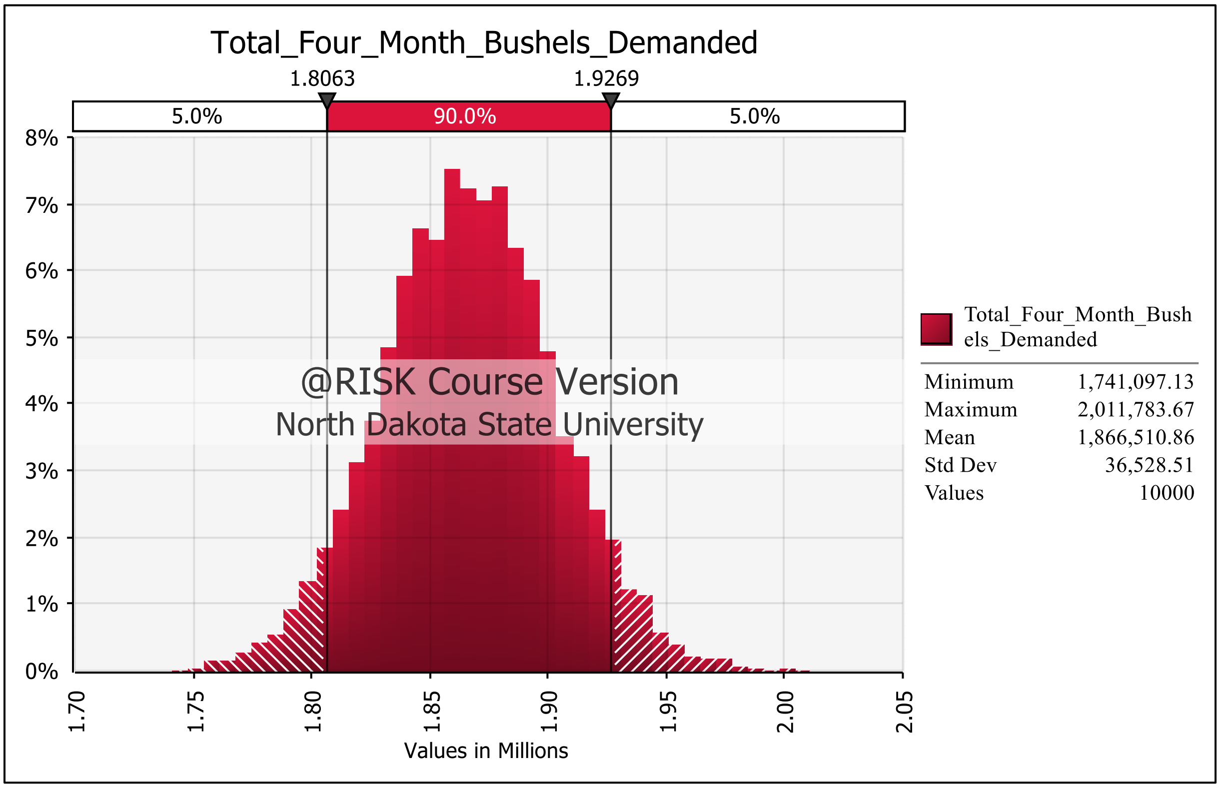

Figure 4.17. Base Case Total Milling Demand (@Risk™)

The net present value of equation (4.20) represents the expected profit of four months of milling. Expected profit may change due to changes in margins as well as changes in milling demand. Expected profit of the representative flour mill is $843,564 with a standard deviation of $1,623,748. Minimum expected profit occurs at -$5,8111,756.45 and a maximum of $6,966,355.00. The distribution of profit follows a normal distribution because demand, as well as the error in price forecasts are normally distributed.

Figure 4.18 shows the distribution of expected profit.

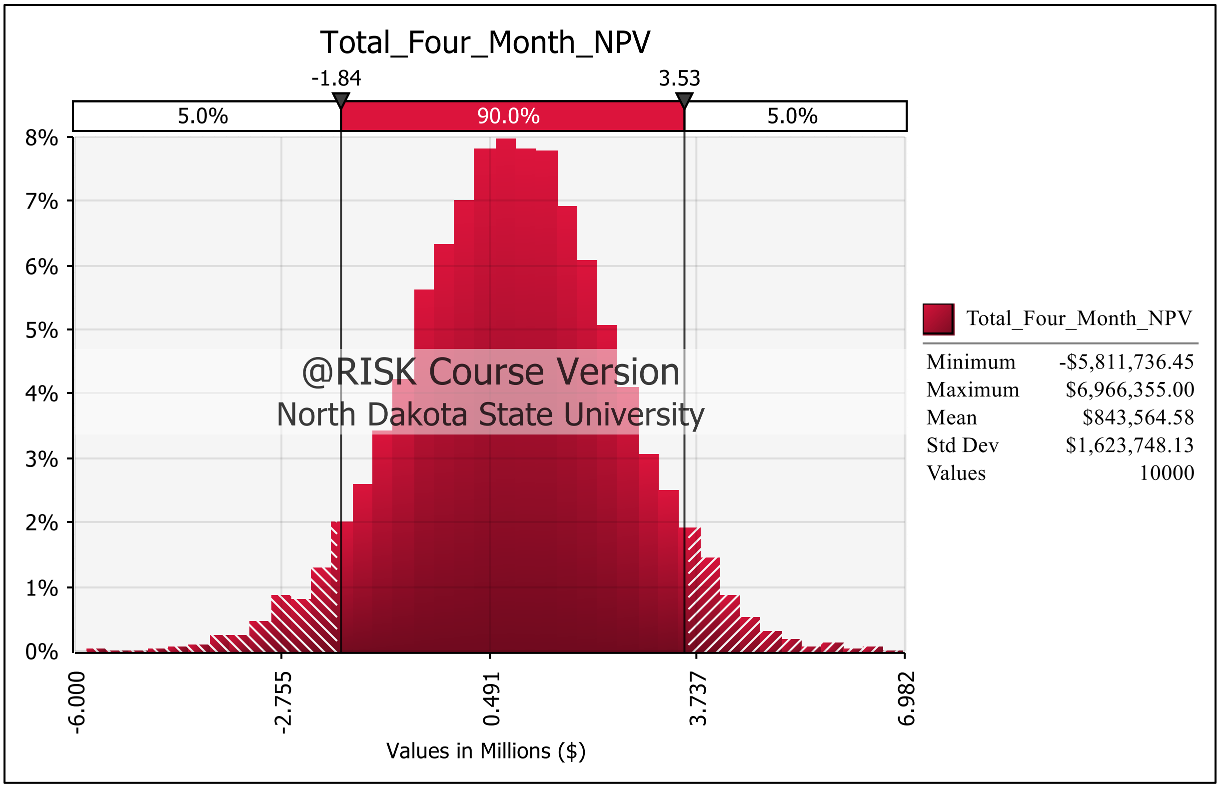

Figure 4.18. Base Case Distribution of Expected Profit (@Risk™)

Figure 4.19 shows the cumulative ascending probability distribution of short call premiums. The short call option premium increases in value the further out the milling month would occur. This relationship is explained by the increase in option value as the time value metric increases (Luehrman 1998). Each option premium has a minimum value of zero, which is attributed to the call premium calculations of equation (4.18) and (4.19). The option demand premium of December has a maximum of 16,122. This means the maximum stockout which could occur in December is 16,122 bushels of missed milling demand under the optimal purchasing strategy. The shape of the CDF in March is different because of Marche's relatively higher time value and delta compared to the other milling months.

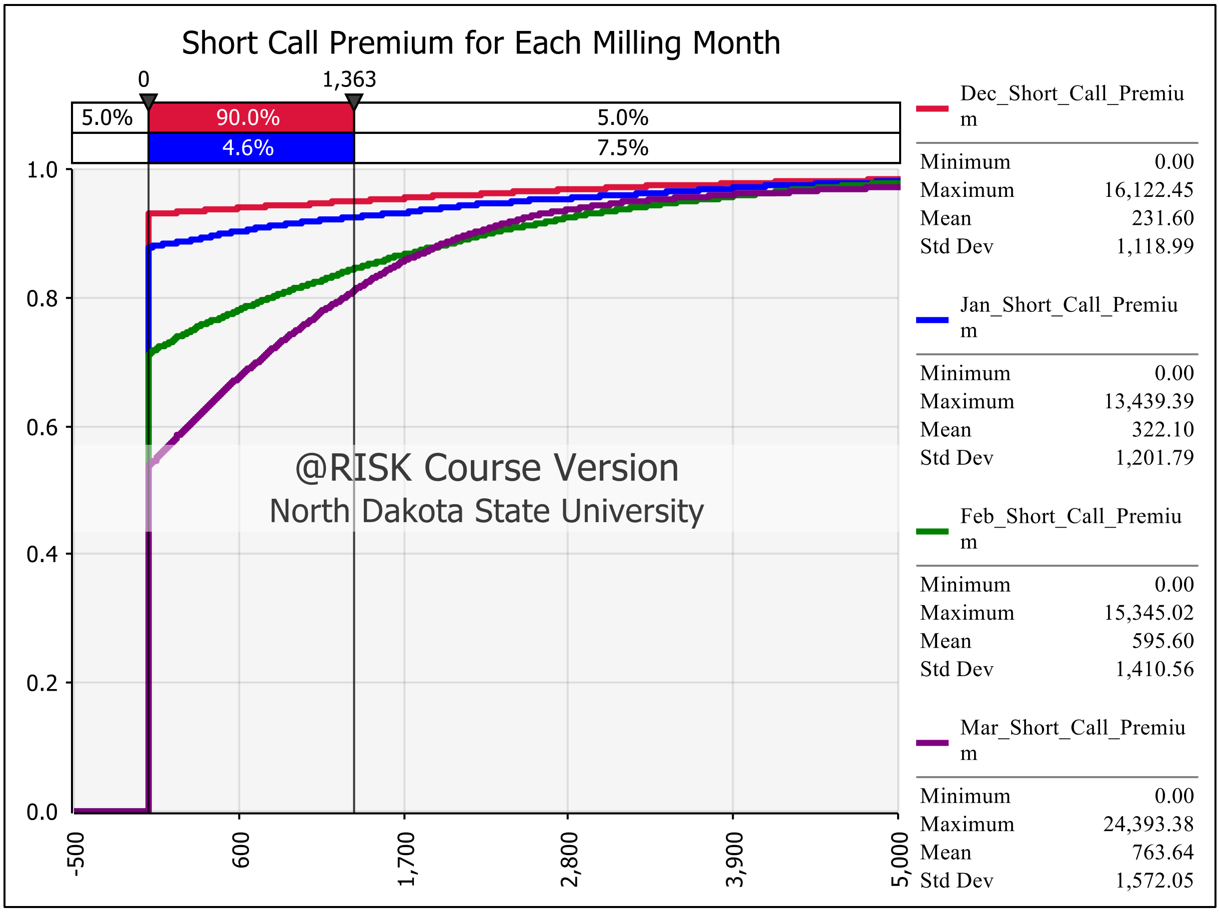

Figure 4.19. CDF: Short Call Option Demand Premiums (@Risk™)

The tornado graph in Figure 4.20 ranks random inputs based on their effects on expected profit under the optimal purchasing strategy. The top six effects are all price variables of flour inputs and products. The next three inputs are extraction rate, capacity utilization, and spread. It is evident that these effects are marginal compared to price effects. This would infer that using inventory as a real option serves as an effective hedging mechanism against demand uncertainties. The high level of risk in price may be dealt with forward contracting or some other hedging mechanism which is not discussed in this application.

Figure 4.20 shows the tornado graph for expected profit.

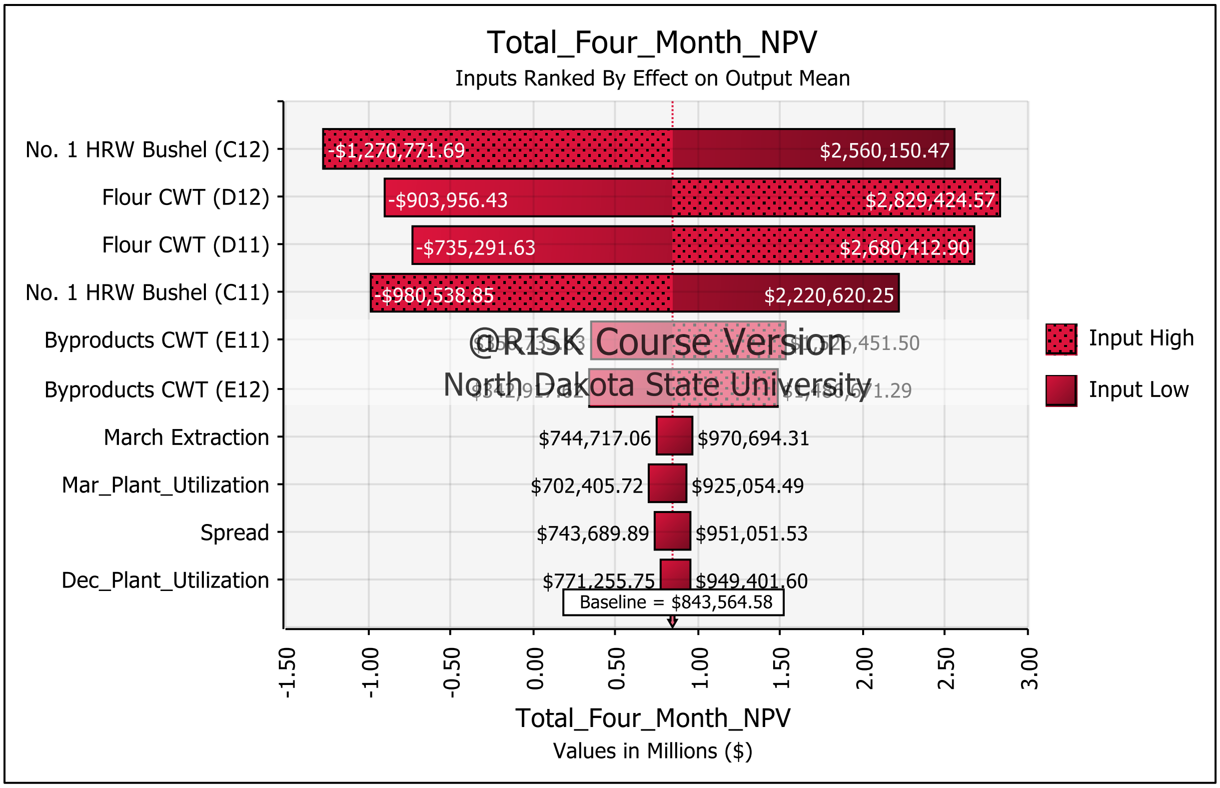

Figure 4.20. Tornado Graph of Expected Profit: Flour Mill Application (@Risk™)

4.6. Sensitivities

The factors with the most influence on expected profit are margin and demand related. These factors include extraction rate, capacity utilization, and margin. Table 4.14 provides a summary of the five sensitive analysis conducted on key structural variables.

Table 4.14. Flour Mill Sensitivity Analysis

Sensitivity	Base Case	Sensitivity Low	Sensitivity High
Extraction Rate Standard Deviation	Standard Deviation of 0.8%	0.0%	3.0%
Capacity Utilization Standard Deviation	Standard Deviation of 3.5%	2.0%	5.0%
Market Spread	$0.17	-$0.03	$0.37
Stockout Penalty	$0.00	$1.00	$2.00
Milling Cost	$2.53	$2.78 per cwt	$3.53

The first two sensitivities of extraction rate and capacity utilization pertain to demand uncertainty and do not affect the payoff function. The other three sensitivities of market spread, stockout penalty, and milling cost impact margin and the payoff function but do not affect demand.

4.6.1. Sensitivity: Extraction Rate

The first sensitivity of extraction rate effects how efficiently flour is extracted from wheat. A higher extraction rate would lower milling demand and a lower extraction rate would increase milling demand. Increasing the standard deviation of extraction rate increases demand uncertainty and thus the value of the real option on the ability to mill. The effects of extraction rate on the demand for bushels required to mill flower represents a random yield in the production process (Ma et al. 2013). Increasing the extraction rate from 0.8% to 3% increases the average buffer stock from 120% to 125.8% of expected milling demand. Alternatively, decreasing the extraction rate to 0.8% lowers the average buffer stock by .5% indicating current extraction rate has little effect on optimal purchasing strategy.

Table 4.15 contains the complete set of results on the sensitivity on extraction rate.

Table 4.15. Sensitivity Results to Extraction Rate

Observation	Base Case	Decrease	Increase
Standard Deviation of Extraction Rate	0.8%	0.0%	3.0%
Expected Profit	$843,564	$844,298	$835,126
Standard Deviation of Expected Profit	$1,587,776	$1,587,867	$1,581,965
Average Purchasing Strategy	120.0%	119.5%	125.8%
Average Number of Long Calls	1.04	1.04	1.03
Average Long Call Demand Premium	172,753	172,753	172,753
Average Number of Short Calls	1.04	1.04	1.03
Average Short Call Demand Premium	500	476	958
Total Bushels Demanded	1,866,512	1,866,325	1,869,177
Standard Deviation of Total Bushels Demanded	36,740	35,486	50,152
December Purchasing Strategy	114%	114%	119%
January Purchasing Strategy	120%	120%	126%
February Purchasing Strategy	122%	122%	130%
March Purchasing Strategy	124%	122%	128%
December Delta	3.7%	3.3%	3.7%
January Delta	3.0%	2.9%	3.9%
February Delta	3.6%	3.3%	3.9%
March Delta	4.0%	4.5%	6.2%

The demand of extraction rate is very sensitive to an increase in extraction rate standard deviation. Increasing the standard deviation of extraction rate increases the standard deviation of total milling demand from 36,981 bushels to 51,247 bushels. Decreasing the standard deviation has a marginal effect on standard deviation of demand.

Figure 4.21 shows the change in total milling demand.

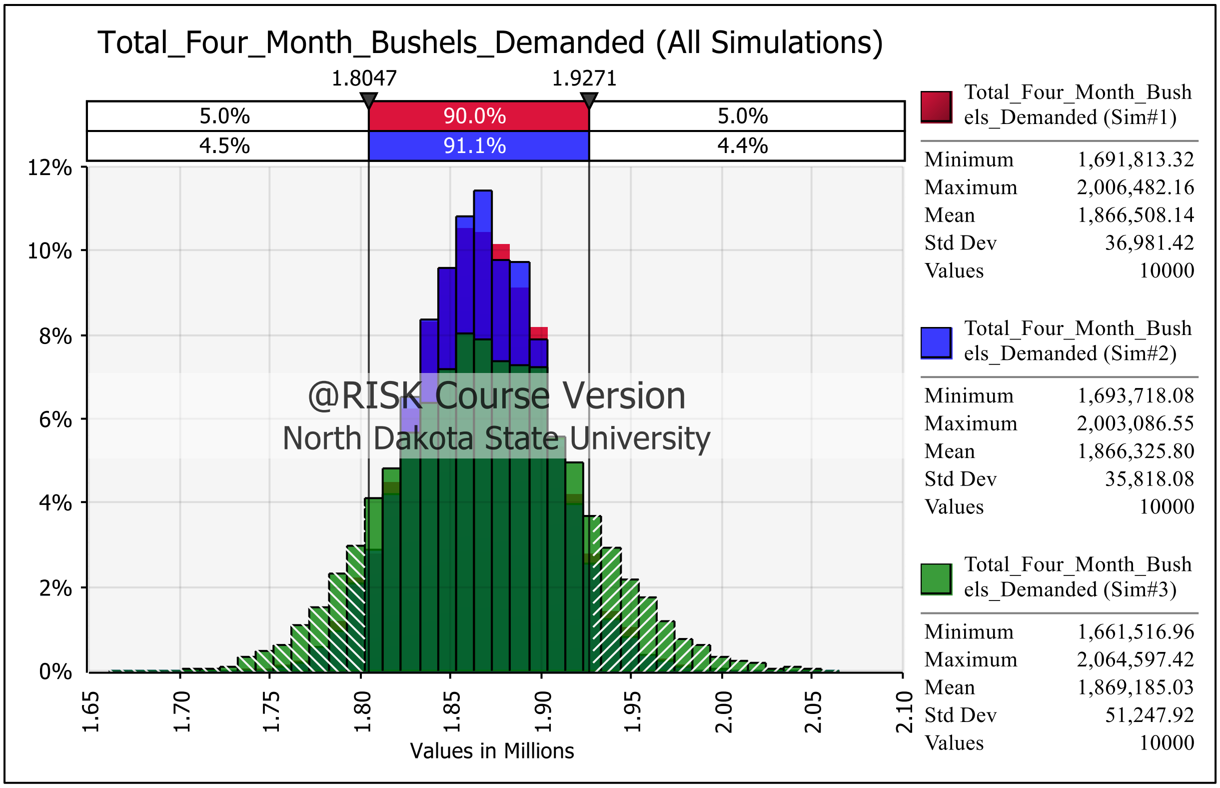

Figure 4.21. Change in Total Demand: Sensitivity to Extraction Rate (@Risk™)

In each of the sensitivities on demand there is little effect on the distribution of expected profit. This is because the mill is already utilizing an optimal purchasing strategy which all but eliminates stockouts. Figure 4.22 show how the sensitivity on extraction rate slightly shifts the distribution to the left when extraction rate standard deviation increases.

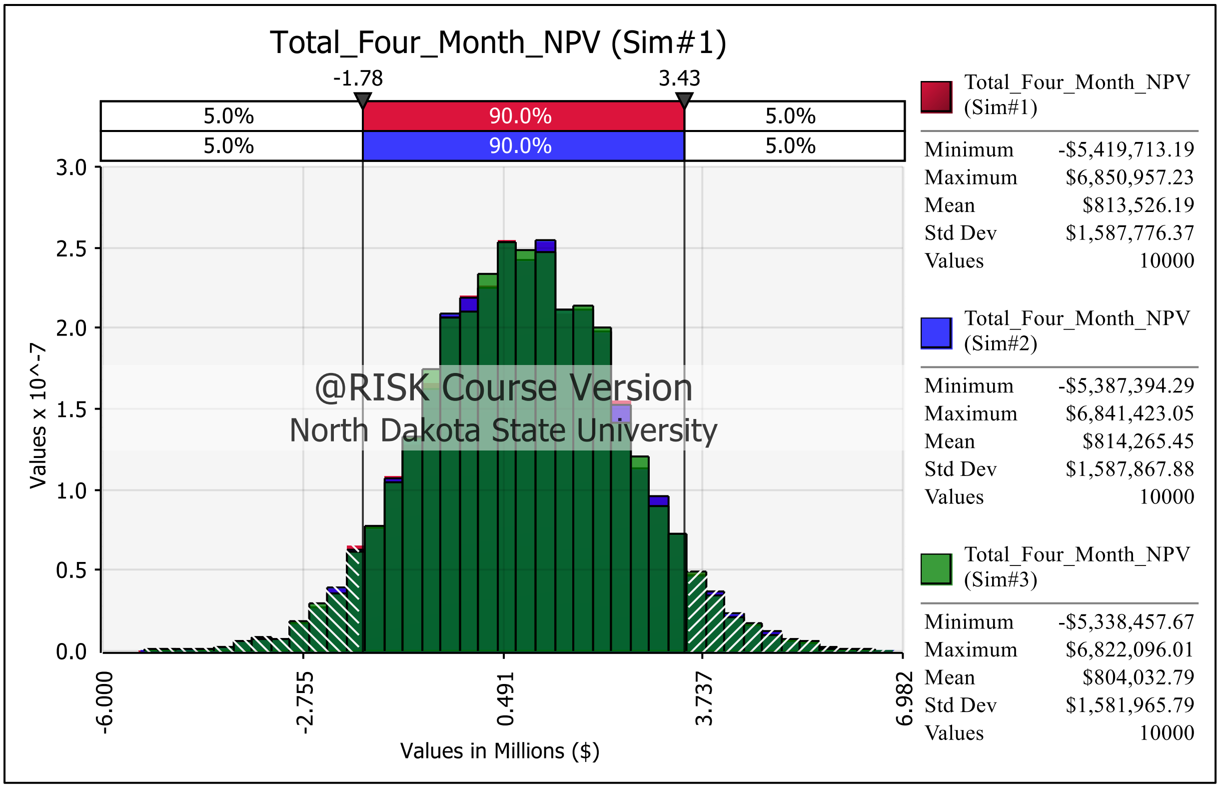

Figure 4.22. Change in Expected Profit: Sensitivity to Extraction Rate (@Risk™)

4.6.2. Sensitivity: Capacity Utilization

The sensitivity to capacity utilization affects the uncertainty of how much flour is demanded each milling month. An increase in capacity utilization standard deviation may result from an increase in spot flour demand, logistical issues due to weather or some other external factor.

Decreasing the volatility of capacity utilization standard deviation decreases average buffer stock by 7.7%. The large decrease in purchasing strategy is attributed to the decreased risk in demand and thus less need to use real options as a hedging instrument. Increasing the standard deviation increase the purchasing strategy and decreases expected profit. Expected profit decreases because of the effect from short calls. The short call option demand premium is increased by over 200 bushels and thus increases the effect of the short calls relative to the long call option which have a relatively unaffected option demand premium.

Table 4.16 shows the full results of the sensitivity.

Table 4.16. Sensitivity Results of Capacity Utilization

Observation	Base Case	Decrease	Increase
Utilization Standard Deviation	3.5%	2.0%	5.0%
Expected Profit	$843,564.58	$865,568.59	$856,735.09
Standard Deviation of Expected Profit	$1,587,776	$1,625,896.39	$1,613,991.16
Average Purchasing Strategy	120.0%	112.3%	126.0%
Average Number of Long Calls	1.04	1.04	1.04
Average Long Call Demand Premium	172,753	172,822	172,172
Average Number of Short Calls	1.04	1.04	1.04
Average Short Call Demand Premium	500	276	725
Total Bushels Demanded	1,866,512	1,867,249	1,860,230
Standard Deviation of Total Bushels Demanded	36,740	22,663	48,607
December Purchasing Strategy	114%	109%	118%
January Purchasing Strategy	120%	112%	127%
February Purchasing Strategy	122%	113%	130%
March Purchasing Strategy	124%	115%	129%
December Delta	3.7%	3.3%	3.5%
January Delta	3.0%	3.2%	2.9%
February Delta	3.6%	4.0%	3.3%
March Delta	4.0%	3.6%	5.1%

Changing capacity utilization has a large impact on the distribution of demand. When standard deviation increases by 1.5%, the standard deviation of total demand decreases by 13,000 bushels. When standard deviation increases by 1.5% the standard deviation of total demand increases by the same amount.

Figure 4.23 shows how the distribution of total demand changes with a change in capacity utilization standard deviation and Figure 4.24 shows how the distribution of expected profits changes

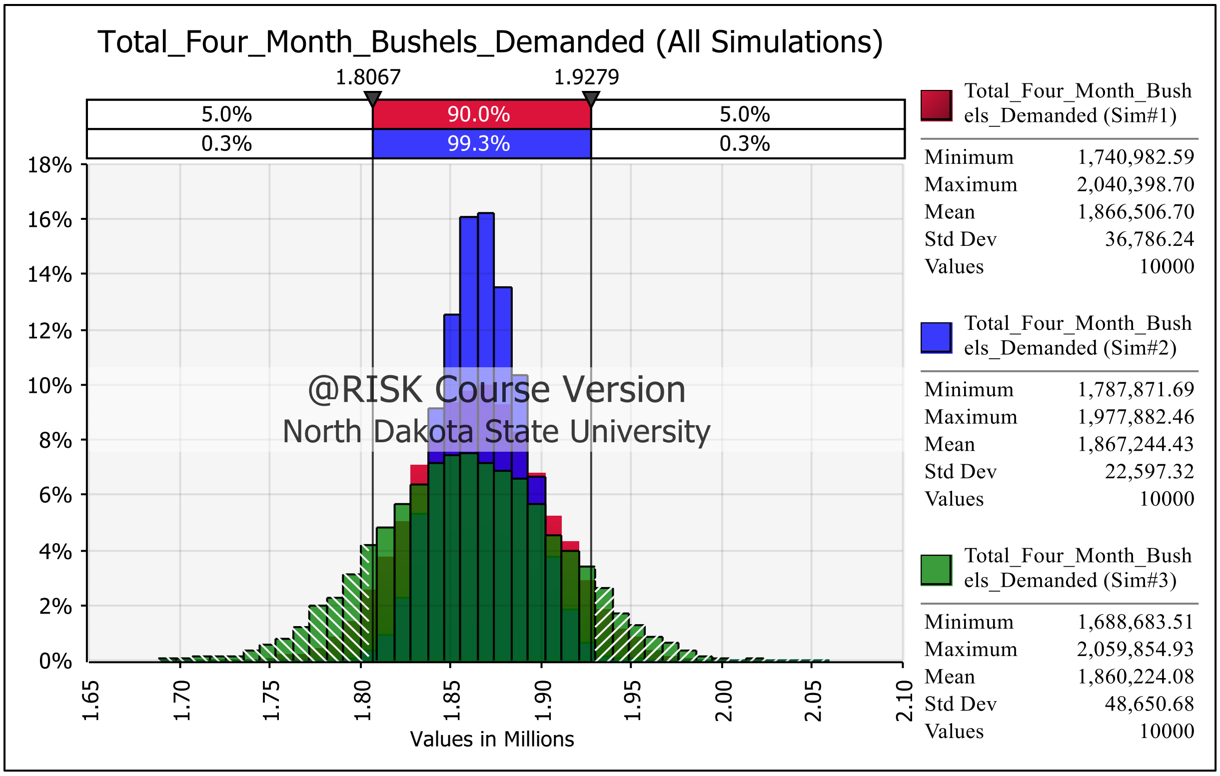

Figure 4.23. Change in Total Demand: Sensitivity to Capacity Utilization (@Risk™)

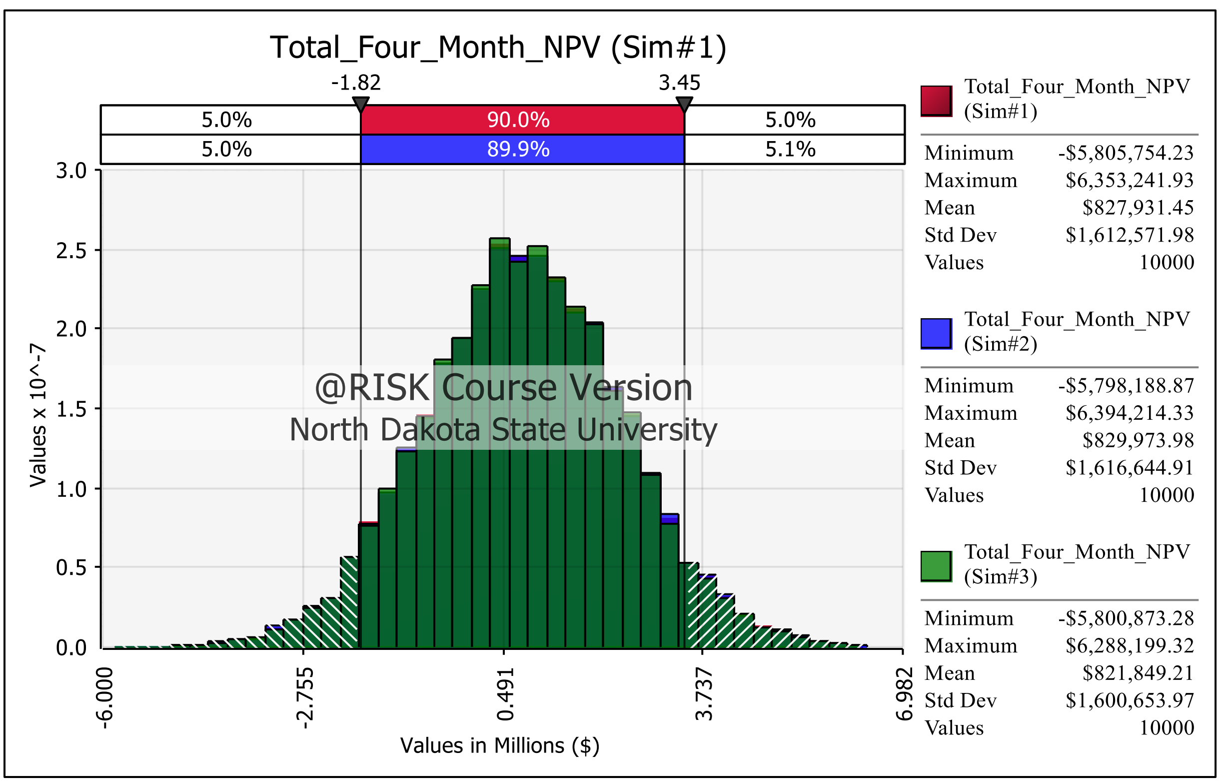

Figure 4.24. Change in Expected Profit: Sensitivity to Capacity Utilization (@Risk™)

4.6.3. Sensitivity: Market Spread

Changes in market spread affects both the futures market and the expected selling price of flour. Changing the market spread affects margin in the future and thus the payoff functions of deferred milling months.

When the future market is in an inverse, it is likely the flour market would be in an inverse as well because the price of wheat and the price of flour are 86% correlated (Table 4.7) . Decreasing the spread lowers average buffer stock to 116.8%; however, the optimal strategy is still to carry 16.8% more stocks than expected milling demand. This result is of interest and relates to the conventional interpretation of the convenience yield (Working 1949). The result shows that even though the market is in an inverse (ie negative price of storage), processors would still hold excess inventories, although reduced, due to the convenience of doing so. Here it means that even though there is an inverse, the loss associated with storage is less than the margin earned from processing.

Table 4.17 shows the complete results of the sensitivity on market spread.

Table 4.17. Sensitivity Results of Market Spread

Market Inverse	Base Case	Market Inverse	Increase Carry
HRW Spread $$/cwt	$0.17	-$0.03	$0.37
Monthly Flour Spread $$/cwt	$0.00	-$0.20	$0.20
Expected Profit	$843,564	$628,667	$1,058,881
Standard Deviation of Expected Profit	$1,587,776	$1,587,715	$1,587,789
Average Purchasing Strategy	120.0%	116.8%	123.8%
Average Number of Long Calls	1.04	0.79	1.29
Average Long Call Demand Premium	172,753	172,753	172,753
Average Number of Short Calls	1.04	0.79	1.29
Average Short Call Demand Premium	500	980	301
Total Bushels Demanded	1,866,512	1,866,512	1,866,512
Standard Deviation of Total Bushels Demanded	36,740	36,740	36,740
December Purchasing Strategy	114%	114%	119%
January Purchasing Strategy	120%	119%	122%
February Purchasing Strategy	122%	121%	124%
March Purchasing Strategy	124%	113%	130%
December Delta	3.7%	3.7%	1.0%
January Delta	3.0%	3.6%	2.3%
February Delta	3.6%	4.0%	3.0%
March Delta	4.0%	13.6%	2.0%

The inversion of flour price occurs each month while the inversion of wheat only occurs in the month of contract roll over. This causes January and March milling months to be affected differently than February. In January and February, only the selling price is affected relative to the base case which only changes the number of long and short calls but not net salvage value.

In the milling month of February both the net salvage value and the number of options changes. A change in net salvage value shifts the floor of expected profit. Increasing the spread raises the floor and causes a higher purchasing strategy to increase during that month. This is because the mill is making money whether wheat is stored to collect the carry or if it is milled into wheat.

Figure 4.25 shows the change in payoff function with a change in market spread for each of the four months in the MRP system.

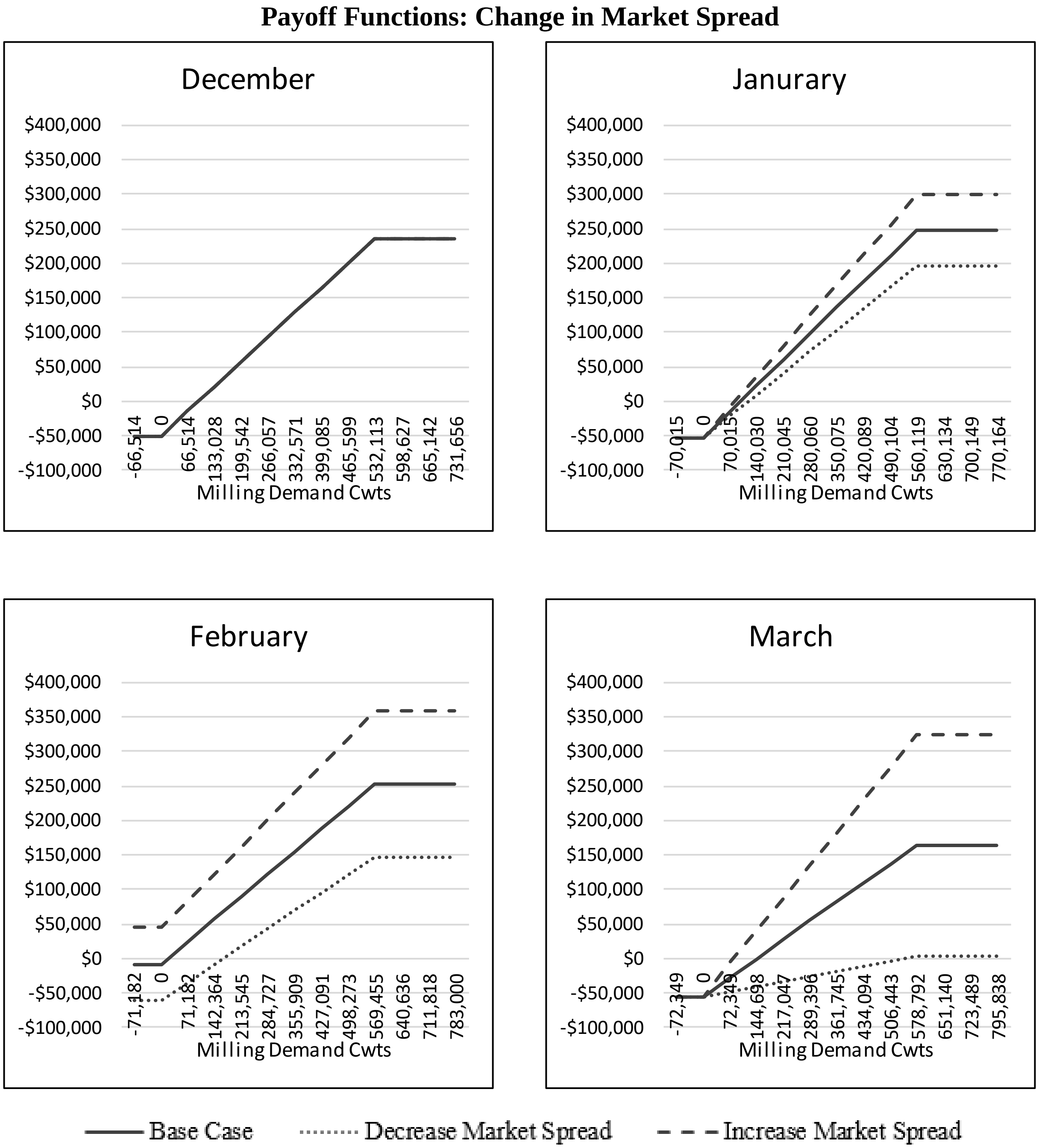

Figure 4.25. Payoff Functions with Change in Market Spreads

Figure 4.26 shows how increasing the market spread would shift the distribution of expected profit to the right while decreasing the spread would shift the distribution to the left. In both cases the standard deviation is relatively unchanged because there are not change in demand uncertainty.

Figure 4.26 shows how the distribution of expected profit changes with changes in market spread.

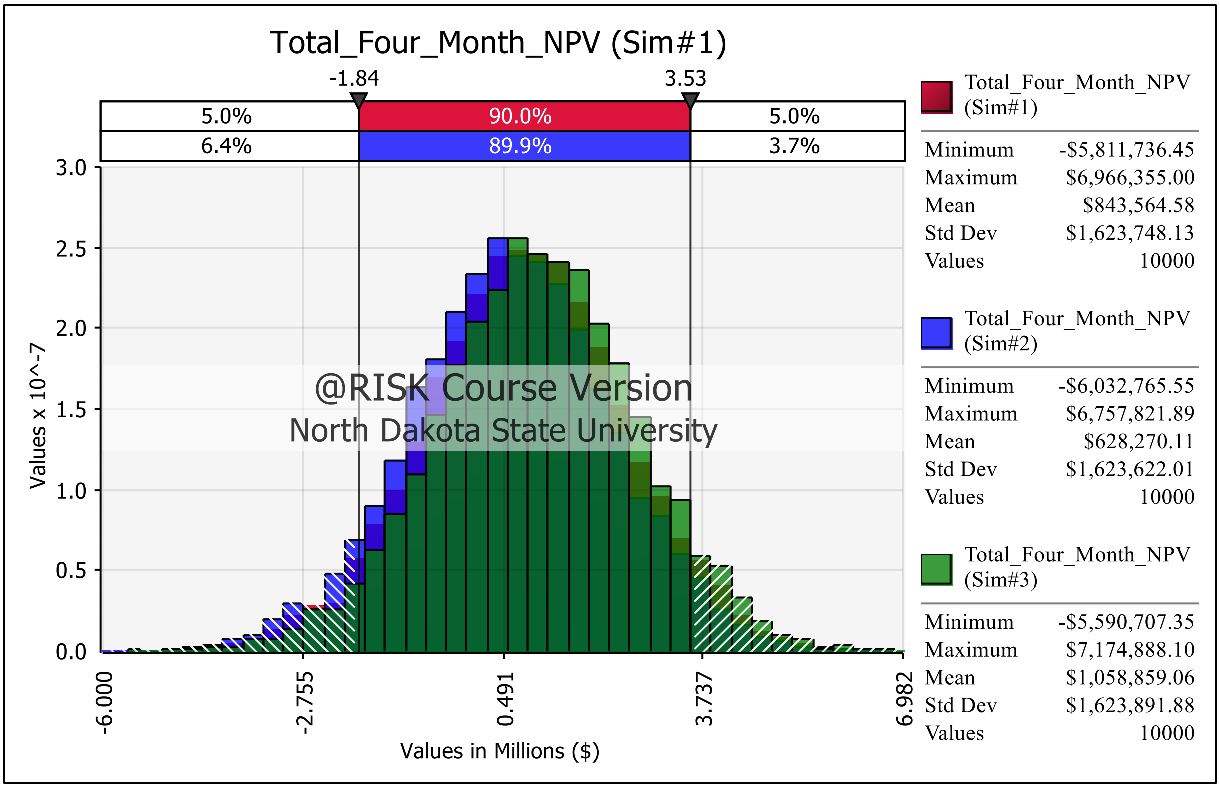

Figure 4.26. Change in Expected Profit: Sensitivity to Market Spread (@Risk™)

4.6.4. Sensitivity: Stockout Penalty

In the base case it is assumed there is no stockout penalty for unmet milling demand. In reality there are stockout penalties which may result from a variety of reasons, such as having to redirect demand to fulfill a flour contract (Wilson 2019).

Increasing the stockout penalty directly affects the number of short calls options. The increase in short call options increases the effect of the option demand premium on the net present value (NPV). The increased effect causes the optimal purchasing strategy to increase which in turn increases the short call demand strike and lowers the delta.

Table 4.18 shows the complete results of the sensitivity on stockout penalty.

Table 4.18. Sensitivity Results of Stockout Penalty

Observation	Base Case	Increase	Increase
Stockout Penalty	$0.00	$1.00	$2.00
Expected Profit	$843,564.58	$842,343.72	$841,667.28
Standard Deviation of Expected Profit	$1,587,776	$1,587,715	$1,587,785
Average Purchasing Strategy	120.0%	125.8%	128.5%
Average Number of Long Calls	1.04	1.04	1.04
Average Long Call Demand Premium	172,753	172,753	172,753
Average Number of Short Calls	1.04	2.04	3.04
Average Short Call Demand Premium	500	210	136
Total Bushels Demanded	1,866,512	1,866,512	1,866,512
Standard Deviation of Total Bushels Demanded	36,740	36,740	36,740
December Purchasing Strategy	114%	117%	119%
January Purchasing Strategy	120%	125%	127%
February Purchasing Strategy	122%	129%	133%
March Purchasing Strategy	124%	132%	135%
December Delta	3.7%	1.7%	1.0%
January Delta	3.0%	1.5%	1.0%
February Delta	3.6%	1.7%	1.0%
March Delta	4.0%	1.7%	1.2%

The addition of a stockout penalty increases the number of short calls which has a direct effect on the shape of the payoff function. Increasing the number of short calls increases the slope change when stockout occurs. The increase in slope change causes the slope in profit to become negative during a stockout. Figure 4.27 shows how the shape of the payoff function changes in response to the addition of a stockout penalty.

Payoff Functions: Addition of Stockout Penalty

Figure 4.27. Payoff Functions with Addition of Stockout Penalty

The addition of a stockout penalty has relatively little effect on expected profit in this application. This is because there is a high margin in milling wheat and the optimal strategy already had a low likelihood of stockout occurring. The effects of the stockout penalty would be more abrasive if the milling margin were lower and the base case optimal strategy was closer to 100% of expected milling demand.

Figure 4.28 shows how the distribution of expected profit changes with the addition of a stockout penalty.

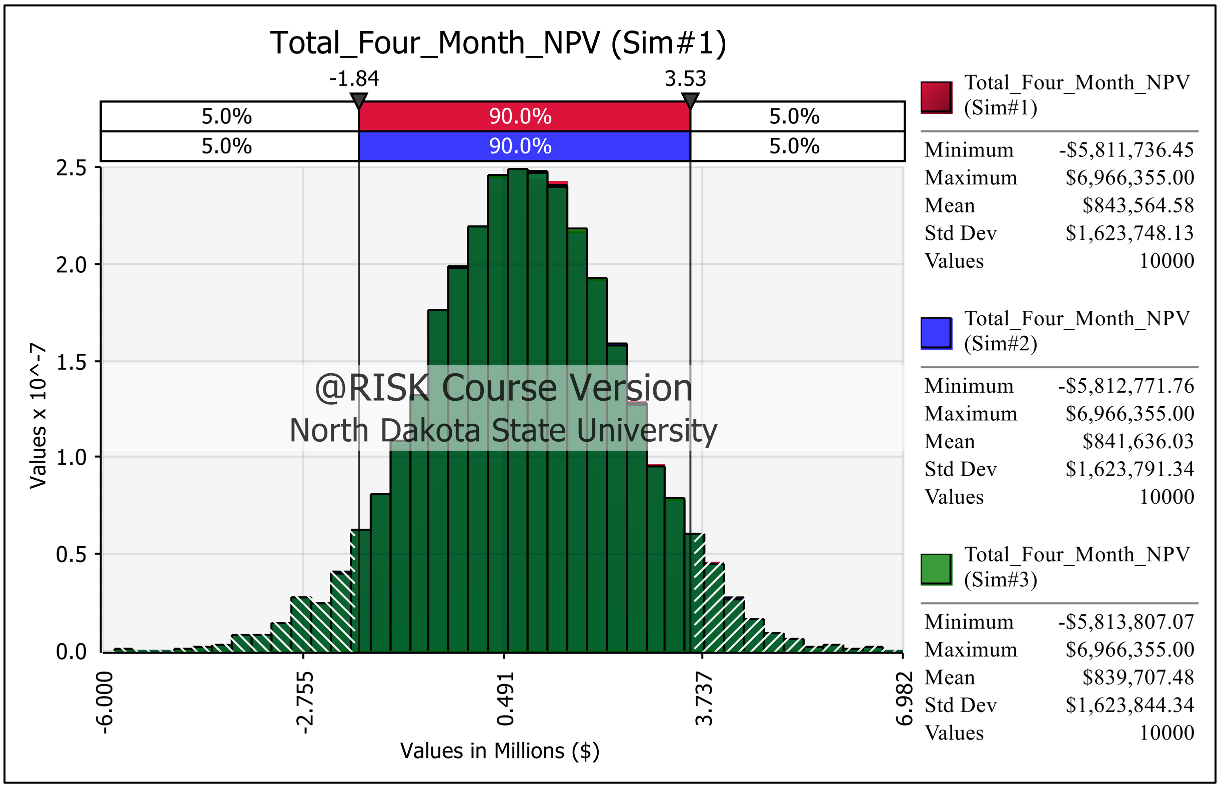

Figure 4.28. Change in Expected Profit: Sensitivity to Stockout Penalty (@Risk™)

4.6.5. Sensitivity: Cost of Milling

Total cost of milling includes all costs associated with turning wheat into flour. These costs include: freight, packaging, enrichments, commission fees, and overhead milling costs (Wilson 2019). The base case assumes total coast of milling equals \$2.53 per cwt. Sensitivity analysis conducted on total cost of milling increases the total cost by \$0.25 increments. There is little change in purchasing strategy with the first incremental increase; however, the optimal purchasing strategy decreases at an increasing rate until margin is reduced to a point where the optimal purchasing strategy changes drastically.

The full results of the sensitivity analysis are shown in Table 4.19.

Table 4.19. Sensitivity Results of Total Cost of Milling

Observation	Base Case	Increase $0.25 Per Cwt	Increase $0.50 Per Cwt	Increase $0.75 Per Cwt	Increase $1.00 Per Cwt
Total Milling Cost Dollars Per Cwt	$2.53	$2.78	$3.03	$3.28	$3.53
Expected Profit	$843,564.58	$646,823	$431,535	$216,717	$55,706
Standard Deviation of Expected Profit	$1,587,776	$1,521,769	$1,519,644	$1,514,032	$748,519
Average Purchasing Strategy	120.0%	119.3%	117.3%	113.0%	56.8%
Average Number of Long Calls	1.04	0.79	0.54	0.29	0.06
Average Long Call Demand Premium	172,753	172,753	172,753	172,753	172,754
Average Number of Short Calls	1.04	0.79	0.54	0.29	0.06
Average Short Call Demand Premium	500	565	776	1,439	98,833
Total Bushels Demanded	1,866,512	1,866,512	1,866,512	1,866,512	1,866,519
Standard Deviation of Total Bushels Demanded	36,740	36,740	36,740	36,740	36,936
December Purchasing Strategy	114%	114%	113%	112%	106%
January Purchasing Strategy	120%	120%	119%	114%	104%
February Purchasing Strategy	122%	121%	119%	113%	2%
March Purchasing Strategy	124%	122%	118%	113%	0%
December Delta	3.7%	3.7%	4.8%	5.8%	13.9%
January Delta	3.0%	3.0%	3.6%	7.4%	28.0%
February Delta	3.6%	4.0%	5.0%	11.0%	100.0%
March Delta	4.0%	5.1%	7.9%	13.6%	100.0%

Of interest is the milling month of March where the optimal purchasing strategy drops from 113% of forecast demand to 0% forecast demand in one incremental increase. The drastic drop occurs because the number of long calls becomes negative. A negative amount of long calls would mean addition profit decreases with each cwt of flour processed and sold. Therefore, the mill would be better off to cease operations then operate with an expected deficit. Of course, other factors must be considered before a manger shuts down the flour mill.

Figure 4.29 shows the relationship between an increase in cost of milling and the short strike delta as well as the optimal purchasing strategy of each month. The percentages depicted in the graph are for the milling month of March.

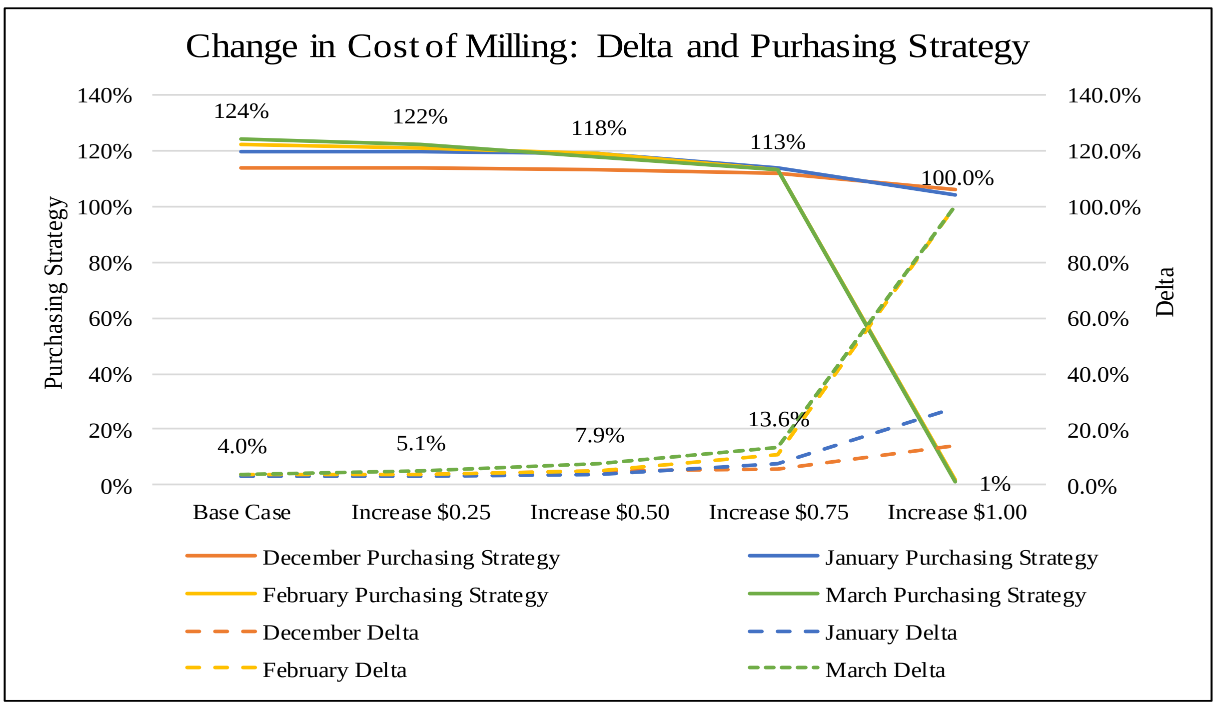

Figure 4.29. Change in Cost of Milling: Delta and Optimal Purchasing Strategy

An increase in total cost of milling decrease the margin and thus decrease the number of long and short calls. Decreasing the slope of long and short calls decreases the amount of revenue gained per cwt of flour milled as well as decreases the impact of stockouts on expected profit. Figure 4.30 shows how the sensitivity impacts the base case purchasing strategy for each milling month. In March, an increase of $1.00 in total cost means the number of long calls would be less than zero, ie., profit decreases as milling demand increases because the mill is operating at a loss. In this case, the previous minimum of -$50,000 of expected profit in March effectively becomes the expected maximum profit.

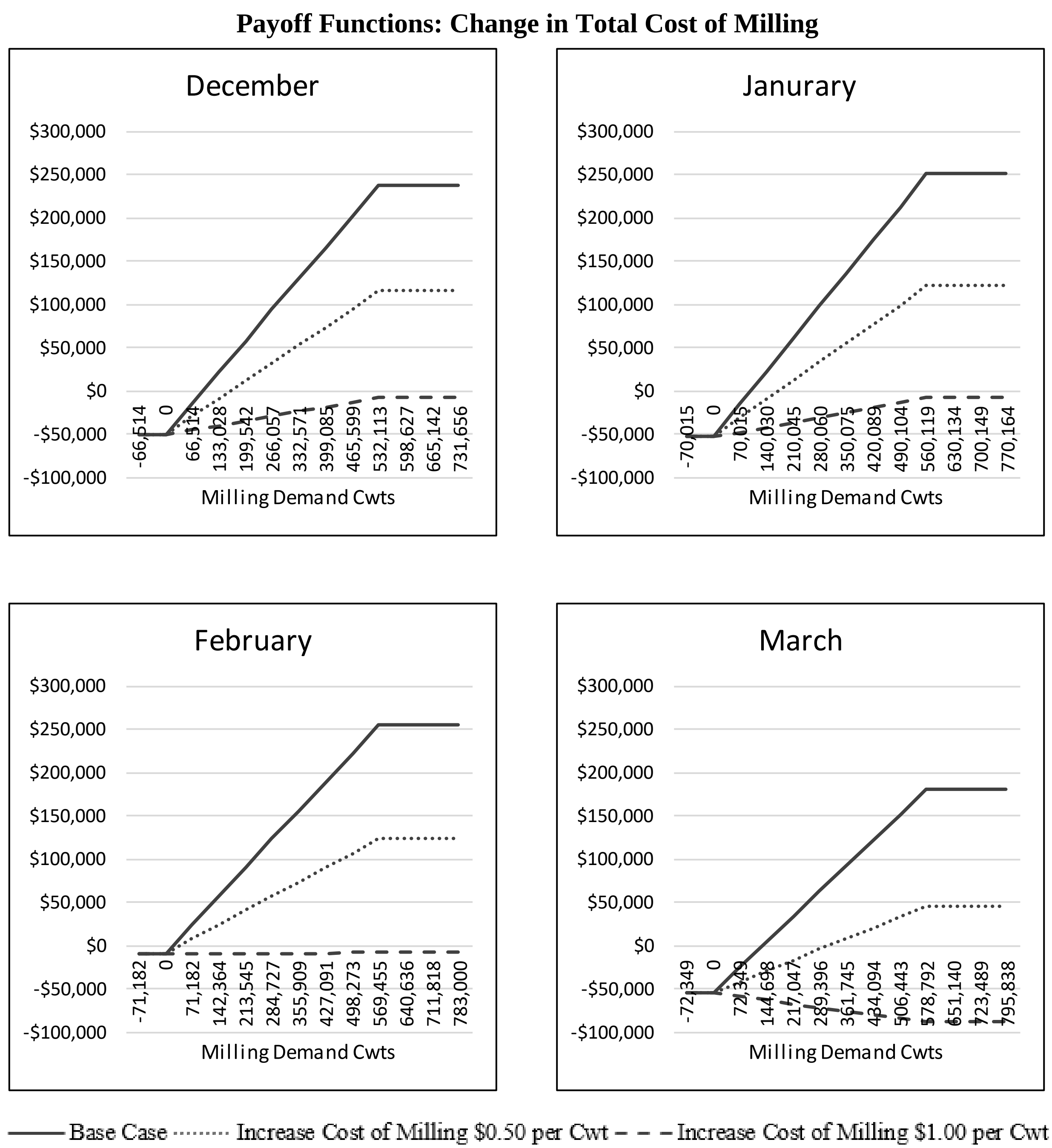

Figure 4.30. Payoff Functions with Change in Total Cost of Milling

Figure 4.31 shows how the CDFs of each sensitivity analysis on total milling cost would affect the cumulative distribution of expected profits for the base case strategy. In the base case, positive expected profit occurs between 60% and 80% of the time. When total milling cost increases to $3.53 per cwt positive expect profit only occurs 40%-60% of the time.

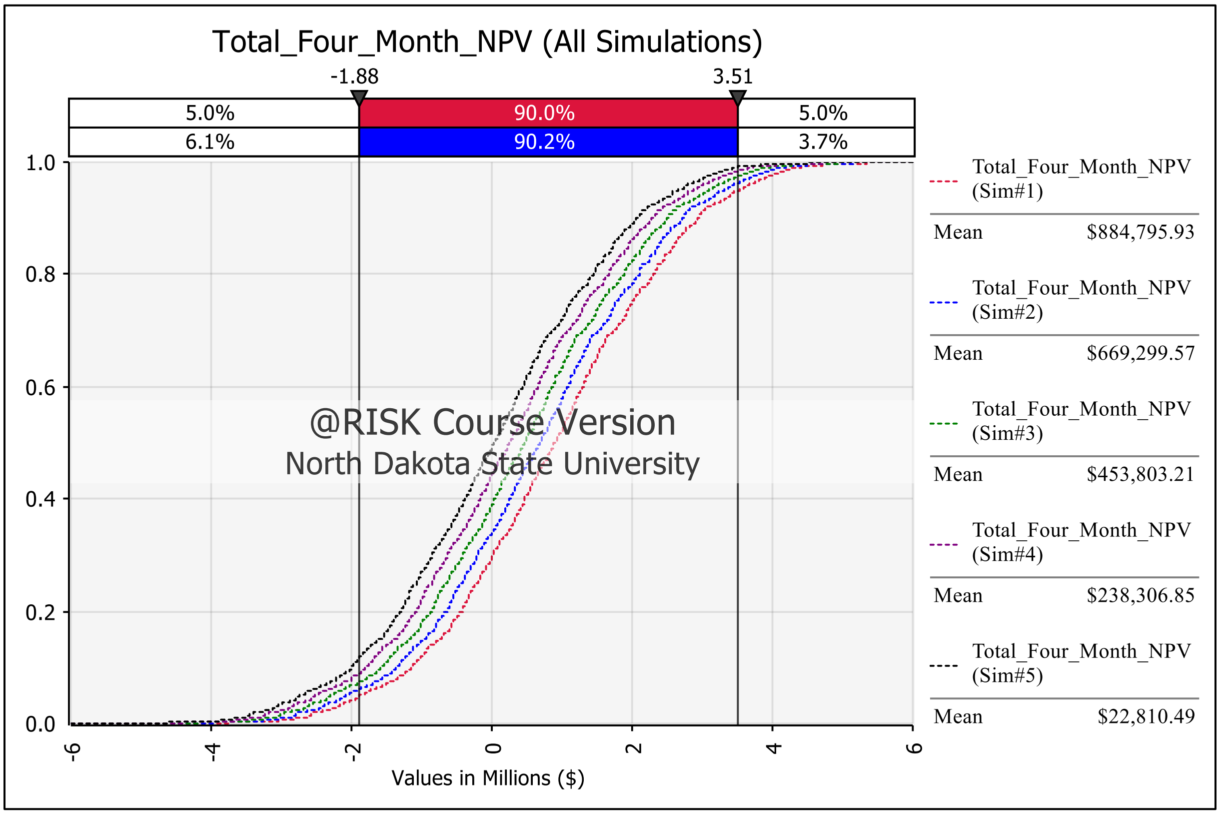

Figure 4.31. CDF: Sensitivity on Total Milling Cost (@Risk™)

4.7. Conclusion

Wheat Mills are exposed to many areas of risk and uncertainty from both milling demand and margin. Just-in-time (JIT) manufacturing concepts suggest that processors should gravitate to a minimal level of buffer stocks (Ballou [1973] 1992; Jacobs and Chase [2008] 2017). This concept give way to the issue of optimal inventory strategy. Recent research has looked into the optimal purchasing strategy and different ways of valuing inventory (Shi et al. 2011; Chang et al. 2015; Li and Arreola-Risa 2017).

Stowe and Su (1997) view inventory as a real option to operate. This chapter is an application of their methodology to a flour mill processor which views inventory as a real option to mill. This methodology, combined with an MRP system and Stochastic Binomial Real Option Valuation, gives logistics managers a mode to value the embedded real option in holding inventory. The methodology of real option valuation views option premiums not having monetary meaning, but rather a relative likelihood of the real option expiring in the money. In the case of flour milling, or virtually any agricultural processing industry, these relationships are important. In this case there are several sources of uncertainty including demand, extraction rates, and price spreads, making inventory decisions critical. Viewing inventories as a real option means that a firm may choose a non-nil level of inventories to have the option of processing and earning margin. The model was developed to determine that level of inventories and how factors have an impact on those values.

This methodology uses Monte Carlo simulation and RiskOptimizer™ to iterate the purchasing strategy until expected profit is maximized. It is found that the optimal purchasing strategy of a representative flour mill would be to order buffer stocks in excess of 20% more than expected demand. The high level of buffer stocks represents the high convenience yield in holding the real option to mill flour (Working 1949).

Sensitivity analysis on key variables have relatively predictable results on the optimal purchasing strategy. However; the change in expected profit does not always change in proportion to the changes in optimal inventory. This may indicate that back testing or a statistics test should be developed to check how robust the purchasing strategy is.

CHAPTER 5. OPTIMAL FERTILIZER PURCHASING STRATEGY UNDER RISK

5.1. Introduction

Fertilizer handlers experience high levels of uncertainty when it comes to fertilizer price and demand. Demand for fertilizer, as it applies to planting, only occurs a few times per year and generally in mass quantities. County centroids located in the upper Midwest must buy quantities of fertilizer in advance in anticipation of demand during the spring planting season. For the fertilizer handler, two prominent sources of risk are uncertainty in demand due to shifting crops and market boundaries as well as uncertainty in margin. County centroids must evaluate these sources of risk and choose a purchasing strategy which maximizes expected profit. Stowe and Su (1997) view inventory as an option on future sales modeled as a call spread. Similarly, an inventory of fertilizer can be viewed as a real option on future sales. Methodology developed by Stowe and Su (1997) combined with competitive arbitrage pricing and real option valuation can help fertilizer managers select a purchasing strategy which will maximize their expected profit.

This chapter applies competitive arbitrage pricing, real option valuation, and contingent claim inventory (CCI) to address optimal purchasing strategies for a representative fertilizer trading firm. First, a conceptual section outlines the application structure and module flow. Next, each component of the fertilizer application is specified in the empirical section of the chapter. Data sources and transformations are defined with random and non-random input parameters presented. Finally, base case results are discussed with relevant sensitives performed.

5.2. Conceptual Model

Urea merchandisers buy fertilizer from an array of sources in anticipation of farmer demand in the spring. Merchants are exposed to two main areas of uncertainty: fertilizer demand and future fertilizer margin. This application develops a model which optimizes the net present value (NPV) of a urea purchasing strategy. NPV, which represents expected profit of the county centroid, can be maximized by changing the quantity of fertilizer purchased. County centroids are locations which distribute fertilizer to farmers and other elevators to meet urea demand. County centroids located in United States may purchase urea from a variety of sources including: Canada-USA boarder points, inland transshipment points, USA fertilizer plants, or imports from the United States Gulf (Wilson et al. 2014). Figure 5.1 demonstrates the flow of fertilizer via barge, rail, and truck to county centroids.

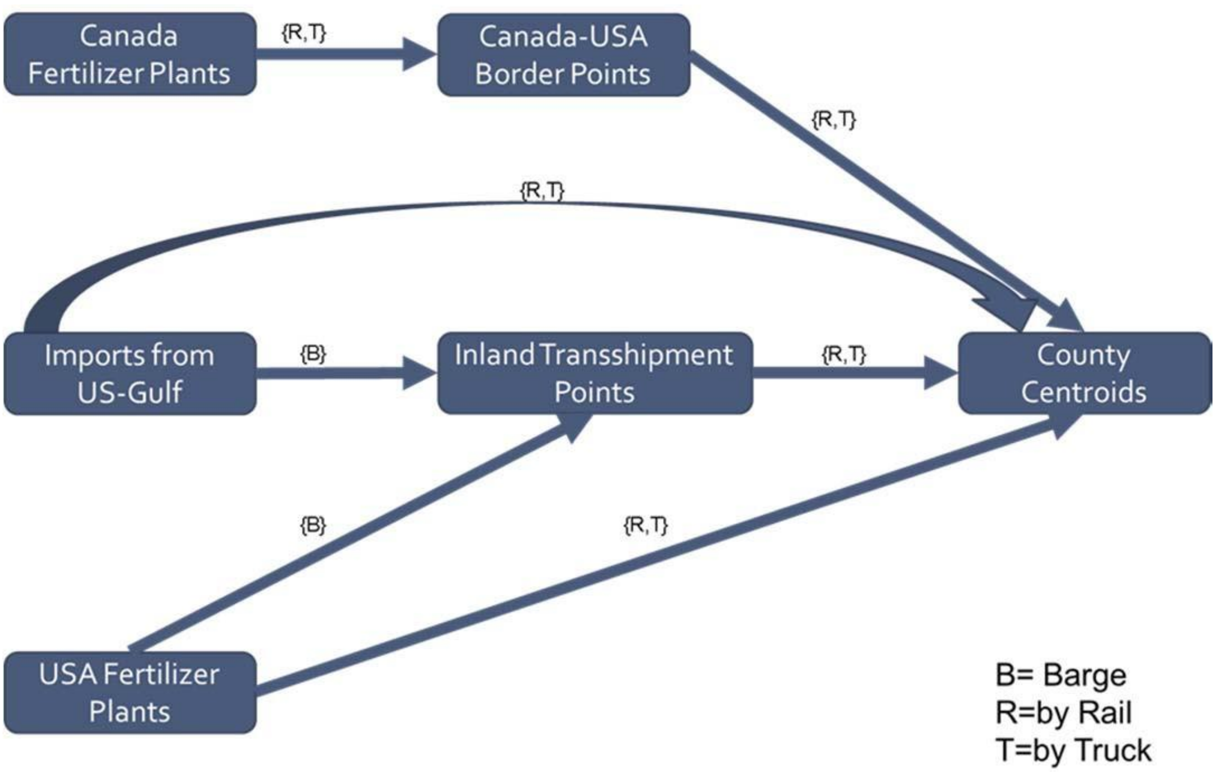

Figure 5.1. United States Fertilizer Flow by Barge, Rail, and Truck (Wilson et al. 2014)

Investment per short ton of urea at county centroids accrue three input costs: fertilizer price at inland transshipment points, transportation cost, and accrued interest. This application assumes a county centroid located in the Upper Midwest would purchase all urea from inland transshipment points located on the Mississippi River. Price of urea at inland transshipment points are generally priced off imports from the US Gulf plus barge transportation costs and margin. An efficient way to transport bulk fertilizer from inland transshipment points to Upper Midwest county centroids is by rail, so rail freight is the only mode of transportation considered to move fertilizer from inland transshipment points to county centroids (Rolf 2019). County centroids tend to purchase fertilizer at least five months prior to the planting season. This application assumes county centroids develop their purchasing strategy in November in anticipation of farmer demand in April. Purchasing fertilizer this far in advance requires county centroids to accrue interest costs from the time of purchase until farmer demand is met.

This application has three module parts which make up the overall model. The three module parts include: competitive arbitrage pricing, real option valuation, and contingent claim inventory. Module 1 uses competitive arbitrage pricing to determine both the selling price and the total demand of urea based on competitive market boundaries. Forecast demand from Module 1 is input to real option valuation in Module 2. Module 2 generates a theoretical option premium which represent the likelihood of the coinciding call options expiring in the money. Option premiums from Module 2 are used in the contingent claim inventory evaluation of Module 3. Module 3 is comprised of a portfolio of long and short call options ie., a call spread, which generates a NPV of the chosen purchasing strategy. The overall module flow is shown in Figure 5.2.

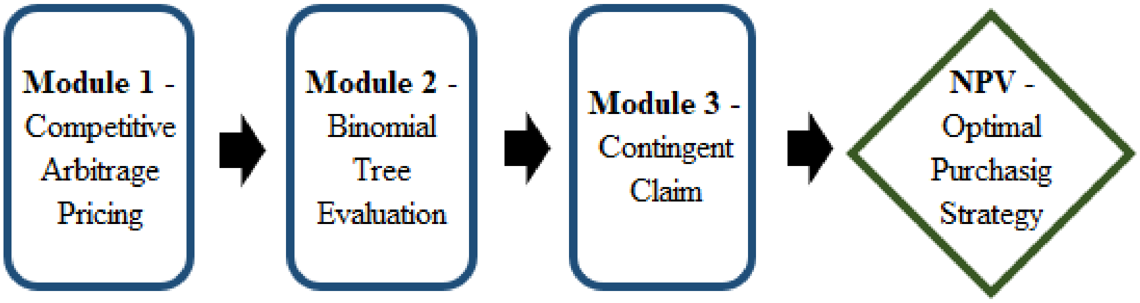

Figure 5.2. Fertilizer Module Flow

@Risk™ is used to simulate the overall model using Monte Carlo simulation on random input parameters. RiskOptimier™ changes the quantity of urea to purchase until mean NPV is maximized. This process demonstrates a dynamic iterative model.

5.3. Empirical Model

The empirical model builds on methods developed in chapter three pertaining to contingent claims inventory (CCI) methodology, developed by Stowe and Su (1997), and real options valuation. This application also uses competitive arbitrage pricing to determine both the selling price and the demand in a given market region for a county centroid. This section first explains how the competitive arbitrage pricing from Module 1 applies to fertilizer. Then, real option valuation in Module 2 is empirically applied. Finally, parameters of CCI evaluation in Module 3 are explained in how they pertain to fertilizer.

5.3.1. Competitive Arbitrage Pricing: Price and Demand Calculation

Competitive arbitrage pricing assumes locations in a regional draw area follow the law of one price; therefore, difference in pricing is attributed to transportation costs and spatial differences (Tomek and Kaiser [1972] 2014). Locations competing in this way would set their prices to eliminate any riskless arbitrage opportunities. This application assumes location, *j*, would set their price to acquire a minimum market share from any competitive *i* locations. The price at location *j*, P_j, is found using equation (5.1):

$$P_j = Min\{P_i + [(1 - S_{j,i}) * M_{j,i} * T] - [S_{j,i} * M_{j,i} * T]\} \tag{5.1}$$

where:

P_j = competitive price at location *j*

P_i = price at location *i*

$S_{j,i}$ = minimum percent market share of location *j* from location *i*

$M_{j,i}$ = miles between location *j* and location *i*

T = trucking cost per mile per short ton of fertilizer.

Once the competitive price at location *j* is found, market share of location *j* from all other *i* locations needs to be calculated. Rearranging equation (5.1) allows location *j* to find its market share, $S_{j,i}$, from all remaining *i* locations using equation (5.2):

$$S_{j,i} = \frac{P_i + M_{j,i}T - P_j}{2M_{j,i}T} \tag{5.2}$$

where:

$S_{j,i}$ = location *j*'s percent market share from location *i*.

Total demand at location *j* is found through summing all $S_{j,i}$ percent market share of each $R_{j,i}$ market region as in equation (5.3):

$$D_j = \sum_{i=1}^{n} (S_{j,i} * R_{j,i}) \tag{5.3}$$

where:

D_j = total demand at location *j*

$R_{j,i}$ = aggregate demand in the *j*,*i* market region.

Using equations (5.1), (5.2), and (5.3), location *j* may find its expected demand and competitive price based on forecast prices from each *i* location. The forecast demand level is then input to the real option valuation of Module 2.

5.3.2. Real Option Valuation

When merchandisers purchase fertilizer, they create a real option on future sales. This real option can be viewed as a long call option which gains value as demand increases. Purchasing a set quantity of fertilizer also creates a short call option. Short call strike demand coincides with the quantity purchased and caps the ability to meet demand.

Long and short call options have an option demand premium which represents the relative likelihood of expiring "in the money." Generally, options are quoted in a monetary value. However, option demand premiums are simply a proxy value which reflects the riskiness of demand given time to maturity, stocking level, forecast demand, and risk-free interest. Module 2 uses stochastic binomial trees to value the premium for a real option using backward induction (Cox et al. 1979).

Table 5.1 shows the five components of an option to sell fertilizer and presents the relationship between three types of options, which builds on Table 2.1.

Table 5.1. Five Components of Option to Sell Fertilizer

Component	Financial Option	Real Option	Option to Sell Fertilizer
Underlying Variable:	Current value of stock	Gross present value of expected cash flows	Forecast fertilizer demand (short tons)
Strike Value:	Exercise price	Investment cost	Fertilizer demand supported by purchased quantity
Time to Maturity:	Time to expiration	Time until opportunity disappears	Time of fertilizer purchase until end of application season
Volatility:	Stock price uncertainty	Project value uncertainty	Fertilizer demand volatility
Risk-Free Rate:	Riskless interest rate	Riskless interest rate	52 Week T-Bill rate

Module 2 requires the five inputs outlined in Table 5.1. Once inputs are known, equations (5.4), (5.5), and (5.6) are used to set up the binomial option tree (Hull [1995] 2008):

$$p = \frac{\alpha - d}{u - d} \tag{5.4}$$

$$u = e^{\sigma\sqrt{\Delta t}} \tag{5.5}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{5.6}$$

where:

p = probability of an up move

α = growth factor

u = multiplicative up factor

d = multiplicative down factor

σ = fertilizer demand volatility; standard deviation of log first differences

t = life of option in terms of a fraction of a year

Δt = length of one option move; fraction of total moves to t.

The growth factor, α, represents the expected annual growth rate of aggregate fertilizer demand in the market region. Equation (5.7) calculates α as:

$$\alpha = e^{r_d \Delta t} \tag{5.7}$$

where:

r_d = average historical logarithmic first differences of aggregate demand

Option demand premiums at binomial tree terminal nodes are valued as call options using equation (5.8):

$$\max(\Psi_{t,j} - K, 0) \tag{5.8}$$

where:

$\Psi_{t,j}$ = fertilizer demand at terminal nodes t with j up moves

j = number of up moves which have occurred since time zero

K = strike fertilizer demand.

Option demand premiums work backward through the binomial tree from right to left. Premiums are evaluated as European style options using equation (5.9) at each node until the final option value is derived at the initial node:

$$f_{i,j} = e^{-r\Delta t}\left[pf_{i+1,j+1} + (1-p)f_{i+1,j}\right] \tag{5.9}$$

where:

$f_{i,j}$ = option demand premium at node i, j

i = number of fertilizer demand moves which have occurred since time zero

r = risk free interest rate.

Information gained from Module 1 is used to generate a fertilizer demand forecast for location j. Forecast demand is the current state value used in the binomial tree. Volatility is calculated by taking logarithmic first differences of the preceding five years and forecast fertilizer demand. The standard deviation of these six demand levels equals fertilizer demand volatility (Kodukula and Papudesu 2006).

Table 5.2 shows an iteration example of logarithmic first differences for historical fertilizer demand, forecast fertilizer demand, and fertilizer demand volatility. Table 5.3 provides an example of all inputs used in the Module 2 example calculation. Figure 5.3 shows how this iteration example is input to the stochastic binomial option tree to return an option demand premium of 30,692 for a short call strike demand of 96,000 short tons**.**

Table 5.2. Logarithmic First Differences of Fertilizer Demand Calculation

Year	**Verona Demand Short Tons**	**Logarithmic First Difference**
2013	85,957	-
2014	69,102	-0.218
2015	91,586	0.282
2016	104,700	0.134
2017	71,976	-0.375
2018	85,880	0.177
2019(Forecast)	125,979	0.383
Forecast Demand:		125,979
Demand Volatility:		0.296

Table 5.3. Module 2 Iteration Example Inputs

Parameter	**Derivation**	**Value**
Forecast Urea Demand	Ψ_T	125,979
Strike Demand	K	96,000
Interest Rate	r	2.7%
Volatility	σ	0.296
Time Until Expiration	t	5/12
Period Length	Δt	0.08
Up Factor	$e^{\sigma\sqrt{\Delta t}}$	1.089
Down Factor	$e^{-\sigma\sqrt{\Delta t}}$	0.918
Probability of Up Move	$p = \frac{1-d}{u-d}$	0.484
Probability of Down Move	$1-p$	0.516

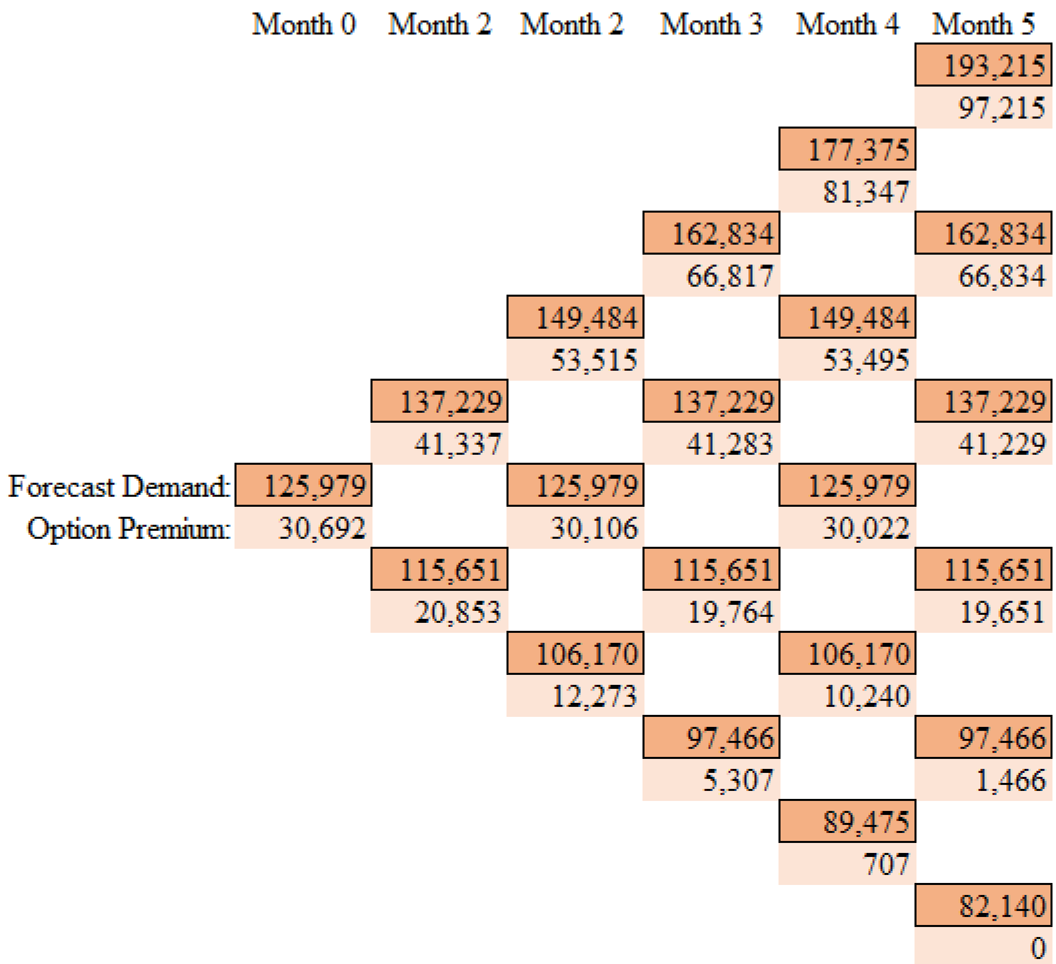

Figure 5.3. Stochastic Binomial Tree (Iteration Example)

5.3.3. Inventory Contingent Claim

Stowe and Su's contingent claim model is made up of four elements to generate a net present value (NPV) of inventory. The four elements include: salvage value of unused inventory, value of long call options, value of short call options, and the initial inventory value. The first three elements of the contingent claim module make up potential sources of revenue for the purchasing strategy. In this application, salvage value includes initial investment per short ton of urea minus seven months of storage to hold unsold fertilizer until the next purchasing period. The long call options represent expected revenue gained per ton of urea sold. The short call options represent expected revenue foregone per short ton of urea demand missed.

Combining the first three elements generate expected revenues of the purchasing strategy which are discounted at the risk-free interest rate. The fourth element, which values the initial total inventory value, is then subtracted which outputs an overall net present value of the purchasing strategy. The CCI formula of Module 3 is shown in equation (5.10):

$$NPV = Q_u \Gamma_u e^{-r_f t_1} + L f_L - S f_S - I_u Q_u \tag{5.10}$$

where:

NPV	=	net present value of urea purchasing strategy
Q_u	=	purchasing strategy quantified in short tons of urea
Γ_u	=	salvage value for unsold tons of urea
r_f	=	risk free interest rate
t_1	=	duration of purchasing strategy
L	=	number of long call options
S	=	number of short call options
f_L	=	long call option demand premium
f_S	=	short call option demand premium
I_u	=	overall investment per short ton of urea at county centroid.

Each of the four elements are further explained in the remainder of this subsection. The first element calculates the discounted salvage value of any unsold short tons of urea. Before salvage value can be calculated, investment per short ton of urea, I_u, sums the price of urea at inland transshipment points, transportation cost to county centroid, and accrued interest from time of purchase until urea is sold. Equation (5.11) shows the calculation of I_u:

$$I_u = P_s + T_R + (r_l * t_1 * P_S) \tag{5.11}$$

where:

P_s = urea price per short ton at inland transshipment point

T_R = transportation cost of rail to county centroid

r_l = loan interest rate.

Salvage value per short of urea, Γ_u, considers interest accrued by urea until the next purchasing period. Γ_u assumes accrued interest is payed before the next purchasing period as in equation (5.12):

$$\Gamma_u = 2I_u - I_u e^{r_l t_2} \tag{5.12}$$

where

t_2 = time until next purchasing period from sale period.

The first element is discounted at the risk-free interest rate to generate a present value of unsold urea short tons. The second element of the CCI Module calculates the expected revenue gained for urea sold using long call options. The number of long calls equal slope of additional revenue gained per short ton sold. Number of long call options subtracts salvage value from selling price, calculated in Module 1, and multiplies it by the demand increase per unit change in the underlying state variable as shown in equation (5.13):

$$L = \frac{\partial Q_D}{\partial \Psi} * (\Phi_u - \Gamma_u) \tag{5.13}$$

where:

$\frac{\partial Q_D}{\partial \Psi}$ = increase in fertilizer demand per increase in underlying state variable

Φ_u = price received per short ton of urea sold, calculated in Module 1.

The number of long calls is multiplied by the long call option demand premium calculated in Module 2 which reflects expected short tons of urea sold based on the purchasing strategy. Multiplying f_L and L together generates discounted expected revenue gained from selling urea. Expected revenue foregone is then calculated in the third element using short calls.

The short calls lessons NPV through the subtraction of expected revenue foregone from expected profits. Number of short calls equals number of long calls plus the product of shortage penalty and fertilizer demand increase. This application assumes zero shortage penalty, so the number of short calls would equal the number of long calls as shown in equation (5.14):

$$S = L + (\frac{\partial Q_D}{\partial \Psi} * \Lambda_u) \qquad (5.14)$$

where:

Λ_u = shortage penalty per unmet urea demand, assumed zero in this application.

Short call strike demand coincides with the fertilizer purchasing strategy. As discussed in Chapter 3, this value is found in equation (5.15):

$$K_S = K_L + (Q_u * \frac{1}{\frac{\partial Q_D}{\partial \Psi}}). \qquad (5.15)$$

where:

K_S = short call strike demand

K_L = long call strike demand which is found outside the system of equations and assumed to be constant at zero.

The short call option demand premium, f_S, reflects the relative likelihood of running out of fertilizer and is calculated in Module 2. The delta of the short call strike may also be found using equation (3.15) to express the probability of a stockout occurring. The premium itself represents the magnitude of stockout expected to occur if the firm experiences a shortage of inventory. Multiplying f_S and S together generates the discounted expected revenue foregone from missed fertilizer sales. The values from the first three elements combine and represent total expected revenue gained from the purchasing strategy. Finally, the fourth element, initial inventory value, is subtracted to generate an NPV of the purchasing strategy.

5.4. Data

Data analyzed in this application is monthly from November 30, 2012 through November 30, 2018 for a total of 73 observations. The data is gathered for the county level demand of urea, urea price per short ton from three county centroids, SWAP Price, urea price in St. Louis MO, SWAP spread, and rail freight.

Data sources can be found in Table 5.4.

Table 5.4. Urea Application Data Sources

Data	Source
Demand ND County Level	USDA-ERS (2013); USDA-NASS (2013); AAPFCO (2011)
County Centroid Prices	Data Transmission Network (2018)
Inland Transshipment Prices	Green Markets (2018)
Swat Futures Prices	Chicago Board of Trade (2018)
Repetitive Location Parameters	Rolf (2019)
Rail Prices	BNSF Distance Calculator (2018); BNSF Rate Item Price List (2018a); Rail Cost Adjustment Factor (2018)

The following four subsections discuss the data used in this application along with distributions. The final subsection provides random and non-random input parameters.

5.4.1. Urea Demand Verona

County level urea demand was derived using the same data compiled by Wilson et al., 2014. County level demand for urea was derived using data on nitrogen use by crop type and acres planted. Acres planted were for barley, canola, corn, cotton, peanuts, rice, sorghum, soybeans, wheat and potatoes for 2010-2012 (USDA-NASS 2013a). Nitrogen use by crop type was obtained from USDA-ERS (2013) and USDA-NASS (2013b) on a state level basis and applied to all counties within the state. Urea use per county was found by multiplying total nitrogen use per county by the state level proportion of urea to total nitrogen (AAPFCO 2011). The forecast demand for 2018 was estimated by assuming planted acres by crop within a county increase by the average annual rate of change for planted acres from 2000-2012.

Urea demand is based on data from 2013 and an estimation for 2018. Aggregate county data for 2014-2017 and forecast for 2019 are estimated assuming aggregate demand follows trend line consumption between 2013 and 2018. Urea consumption in the United States has followed a relatively stable trendline since 1973 with an adjusted R^2 of 0.974 as shown in Figure 5.4 (USDA-ERS 2018); therefore, it is reasonable to assume county level demand also follows a consumption trendline.

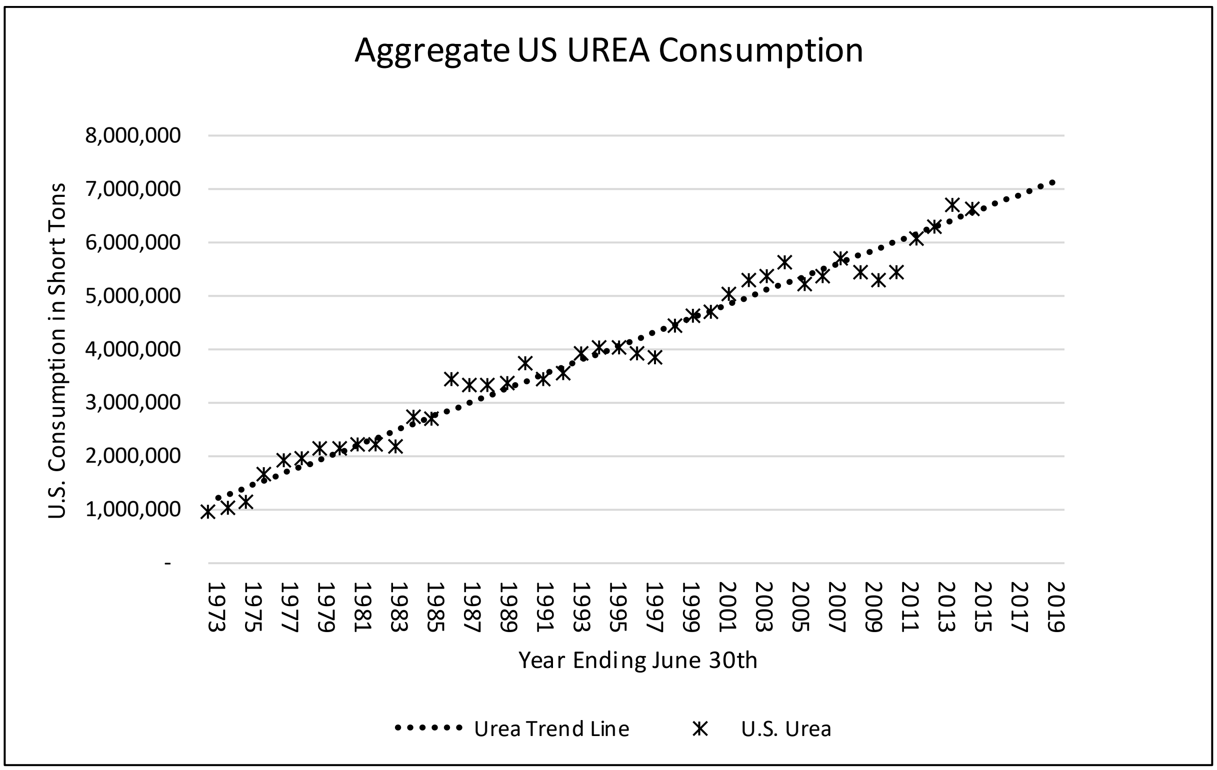

Figure 5.4. Aggregate U.S. Urea Consumption (USDA-ERS 2018)

The representative location for this application is Verona, ND. Verona is strategically located between three county centroids of Cargill Milling Inc., Woodworth Farmers Grain, and

Northern Plains Cooperative. The draw region of Verona is separated into three competitive market boundaries. Urea demand from each draw area depends on competitive arbitrage pricing which assumes the market areas are efficient and follow the law of one price (Tomek and Kaiser 2014). Thus, urea demand from Verona largely depends on competitor prices and transportation costs between markets. Under this assumption, historical draw areas between Verona and competitive county centroids are calculated based off data used by Wilson et al. (2014).

Transportation costs via truck are built on industry level conversations. In the base case, historical trucking is assumed to be constant at $0.60-per-short-ton-per-mile for a commercial trucker transporting urea between rural locations. Verona will also competitively price at the minimum of three 30% Verona market-share-prices from each competitive county centroid. Verona's market share from each of the other two draw areas is calculated based on Verona's competitive price in Module 1.

Table 5.5 shows counties associated within each draw area, historical trucking cost-per-short-ton-per-mile, and miles between each county centroid and Verona.

Table 5.5. Competitive Arbitrage Pricing Specifications

County Centroid	Historical Trucking Cost per Mile	Miles from Verona	Competitive Market Share	Draw Area Color/Pattern	Counties in Draw Area
Cargill Milling Inc.	$0.60	87	30% (Variable)	Green Dots & 1/3 Red	Ransom, Richland, Sargent, 1/3 La Moure
Woodworth Farmers Grain	$0.60	110	30% (Variable)	Orange Diagonal Lines & 1/3 Red	Stutsman, Barnes, 1/3 La Moure
Northern Plains Cooperative	$0.60	83	30% (Variable)	Blue Cross Hatch & 1/3 Red	Logan, McIntosh, Dickey, 1/3 La Moure

Draw areas and counties are displayed in Figure 5.5. Counties with green dots represent counties in the Cargill Milling Inc. draw area, counties with orange diagonal lines represent counties in the Woodworth Farmers Grain draw area, counties with blue cross hatch represent counties in the Northern Plains Cooperative draw area, and all county centroids compete with Verona for 1/3 of La Moure county which is in red. Figure 5.6 shows historical demand for Verona based on competitive arbitrage pricing assumptions.

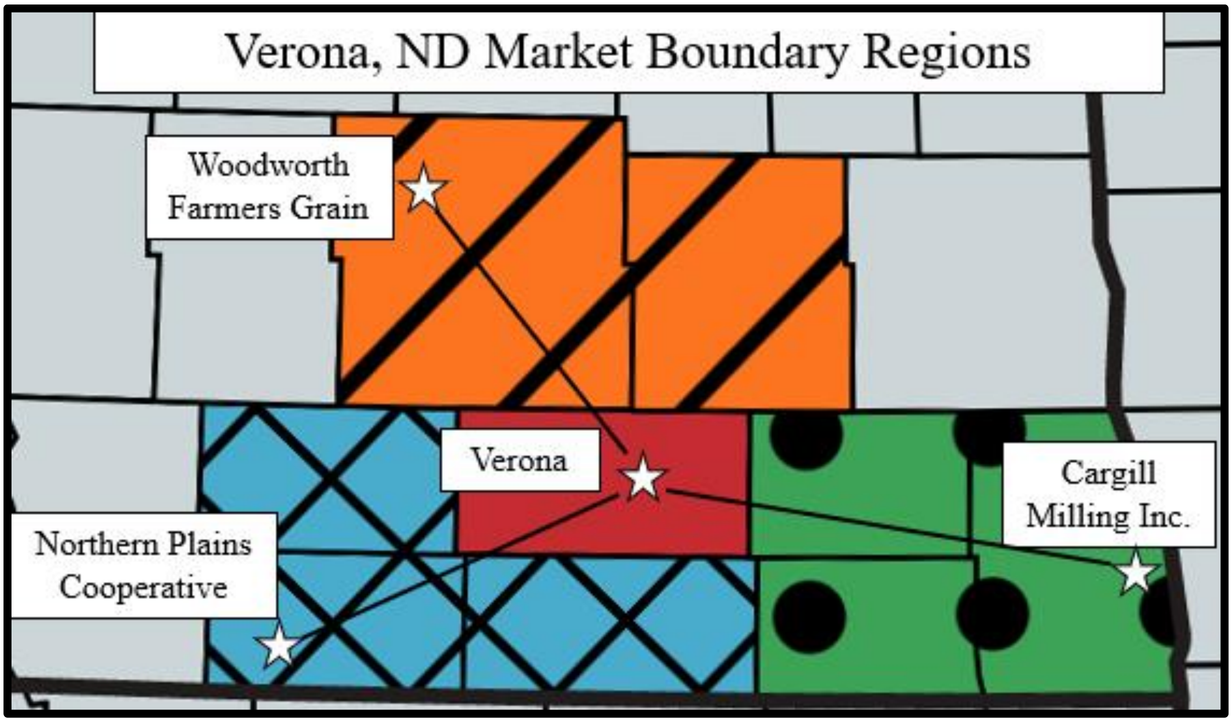

Figure 5.5. Verona, ND Market Boundary Regions (MapChart™ 2019)

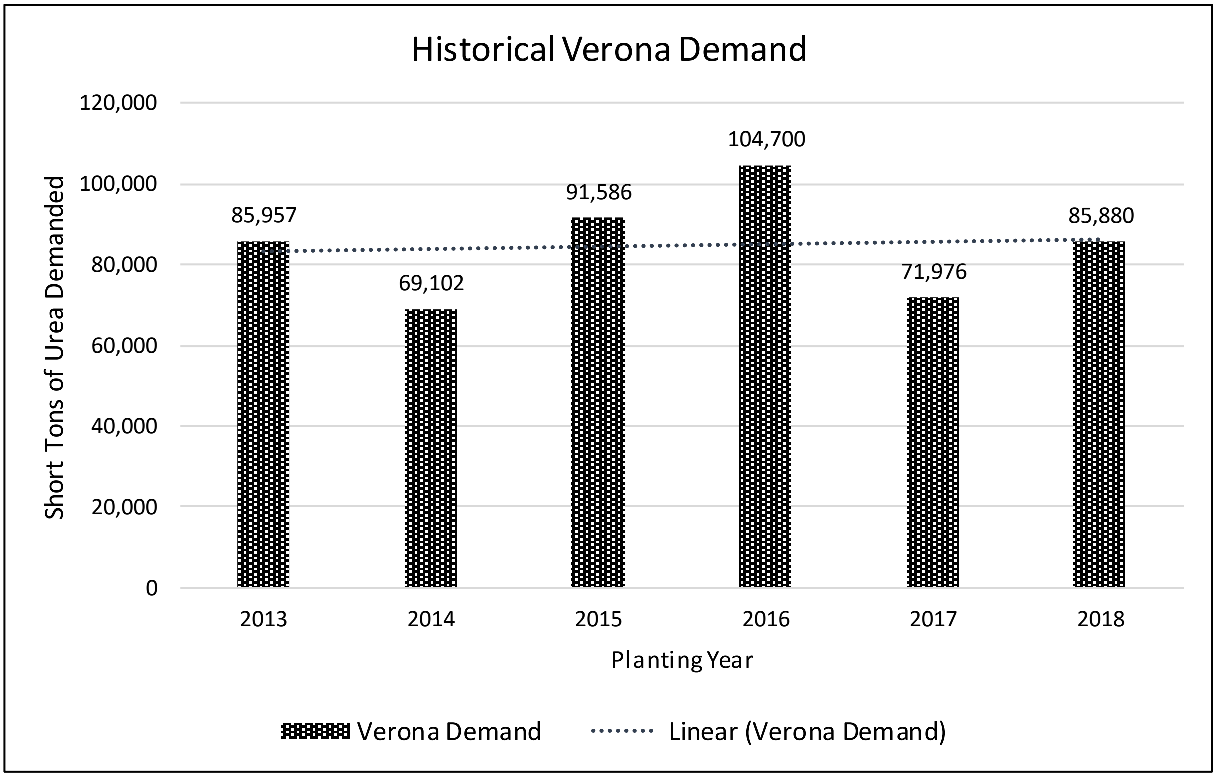

Figure 5.6. Historical Verona Demand (Representative Location Based Module 1)

5.4.2. Urea Prices

Urea prices in terms of dollars per short ton were gathered from three county centroids, urea Swap Futures, and Western Corn Belt. Data was extracted from Bloomberg using symbols: AMURUSND AFTX DTN Index, AMURUSND AFUI DTN Index, AMURUSND AFPP DTN Index, URE1 Comdty, and GCFPURWS Index for Cargill Milling Inc., Woodworth Farmers Grain, Northern Plains Cooperative, urea Swap Futures, and St. Louis, respectively; as well as instrument variables: GCFPURGB Index and GCFPURSE Index for New Orleans and South East spot price. Data was extracted on a monthly interval using average price of observed data from that month.

There were seven missing data points from Northern Plains Cooperative and three missing points from St. Louis. An instrument variable from New Orleans spot price was used to fill in missing data for Northern Plains Cooperative and an instrument variable from South East

United States was used to fill in St. Louis. Regressions used to fit missing data can be found in Appendix B and Appendix C.

The data for urea prices is displayed in Figure 5.7.

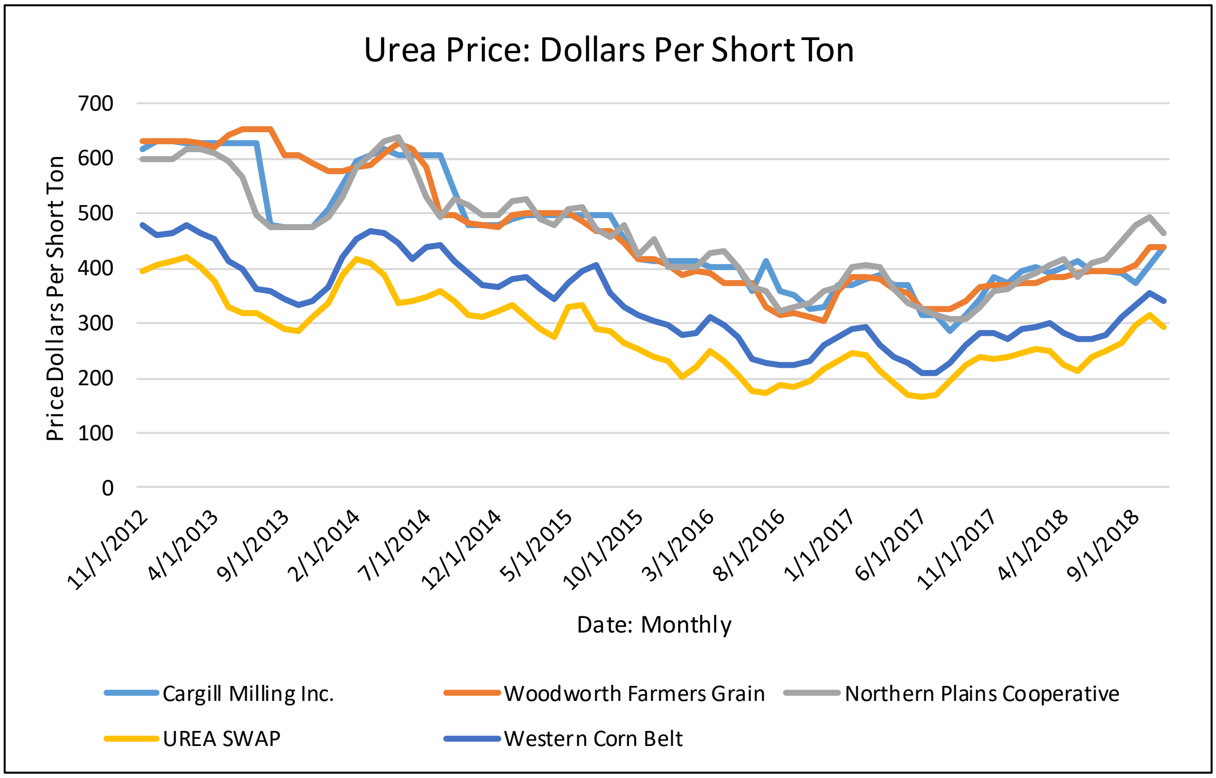

Figure 5.7. Urea Price in Dollars per Short Ton (DTN 2018; Green Markets 2018)

Price at the county centroids of: Cargill Milling Inc., Woodworth Farmers Grain, and Northern Plains Cooperative; the inland transshipment point of the Western Corn Belt; and the urea Swap Futures price are all highly correlated. However, it can be seen in Figure 5.7 that data at county centroids are slightly lagged the price at St. Louis which is lagged slightly behind urea Swap Futures price. This results from lagged transportation time up the Mississippi river via Barge and from rail transportation time from St. Louis to county centroids.

Correlation matrix at price levels can be found in Table 5.6.

Table 5.6. Urea Price Level Correlation Matrix (@Risk™)

	Cargill Milling Inc.	Woodworth Farmers Grain	Northern Plains Cooperative	Urea SWAP	Western Corn Belt
Cargill Milling Inc.	1.000				
Woodworth Farmers Grain	0.929	1.000			
Northern Plains Cooperative	0.926	0.923	1.000		
Urea SWAP	0.879	0.883	0.942	1.000	
Western Corn Belt	0.921	0.902	0.962	0.974	1.000

The first differences of urea price were used to forecast price movements from December to April. @Risk™ uses Bestfit™ to fit distributions of the first differences automatically. @Risk™ compares and chooses the best fit distribution based on Akaike Information Criteria (AIC). Appendix F shows the distributions used by Bestfit™ with descriptions. @Risk™ uses Spearman Rank-Order Correlations to fit a correlation matrix to the distributions.

Forecast price movements are assumed to be spatially correlated but temporally independent. Therefore, the correlation matrix in Table 5.7 is used instead of the correlation matrix chosen by @Risk™. The result is a stochastic price movement which follows the individual price distributions but returns a data set which has a correlation similar to price level correlations. Each distribution is truncated to the minimum and maximum price movement of the individual dataset to eliminate any nonsense results.

First difference distributions for the county centroids and urea SWAP price are found in Table 5.7.

Table 5.7. Urea Price Distributions (@Risk™)

Variable:	Cargill Milling Inc.	Woodworth Farmers Grain	Northern Plains Cooperative	Urea SWAP
Distribution:	Laplace	Laplace	Triangular	Normal
Function:	RiskLaplace (0,23.5463, RiskTruncate (-148.81,55.12))	RiskLaplace (0,16.3903, RiskTruncate (-88.18,55.11))	RiskTriang (-72.814,8.27,55.745, RiskTruncate (-66.69,52.35))	RiskNormal (-1.4208,21.13, RiskTruncate (-49.14,51.71))
AIC Score:	652.97	600.80	674.43	646.80
Mean:	0.00	0.00	-2.93	-1.42
Standard Deviation:	23.55	16.39	26.54	21.13

The distribution fits for Cargill Milling Inc., Woodworth Farmers Grain, Northern Plains Cooperative, and urea Swap Futures are found in Figures 5.8 through 5.11.

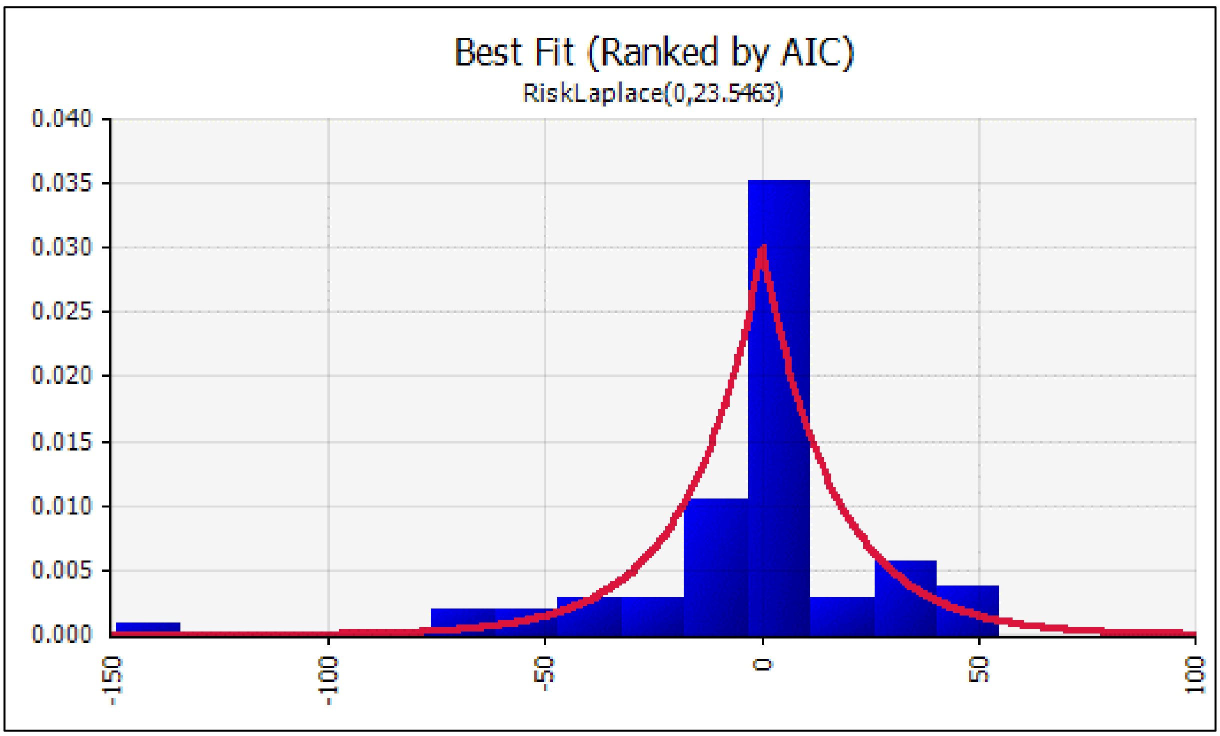

Figure 5.8. Cargill Milling Inc. First Difference Distribution Fit (@Risk™)

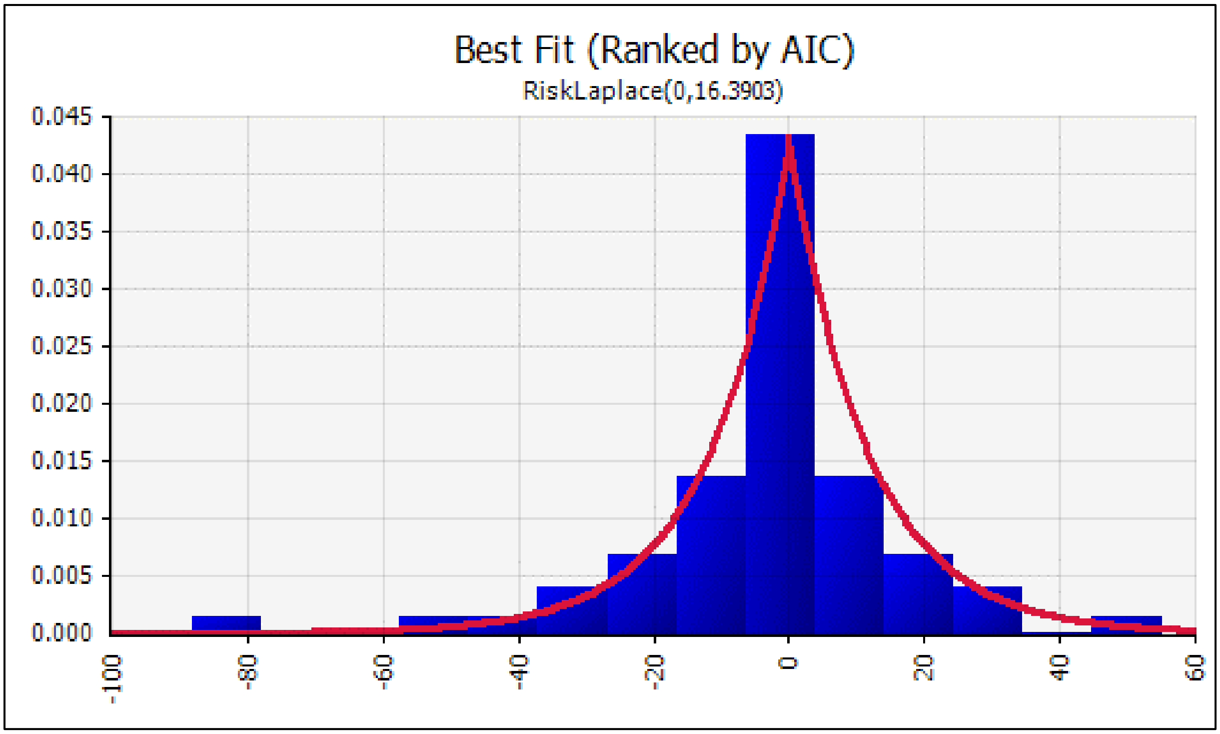

Figure 5.9. Woodworth Farmers Grain First Difference Distribution Fit (@Risk™)

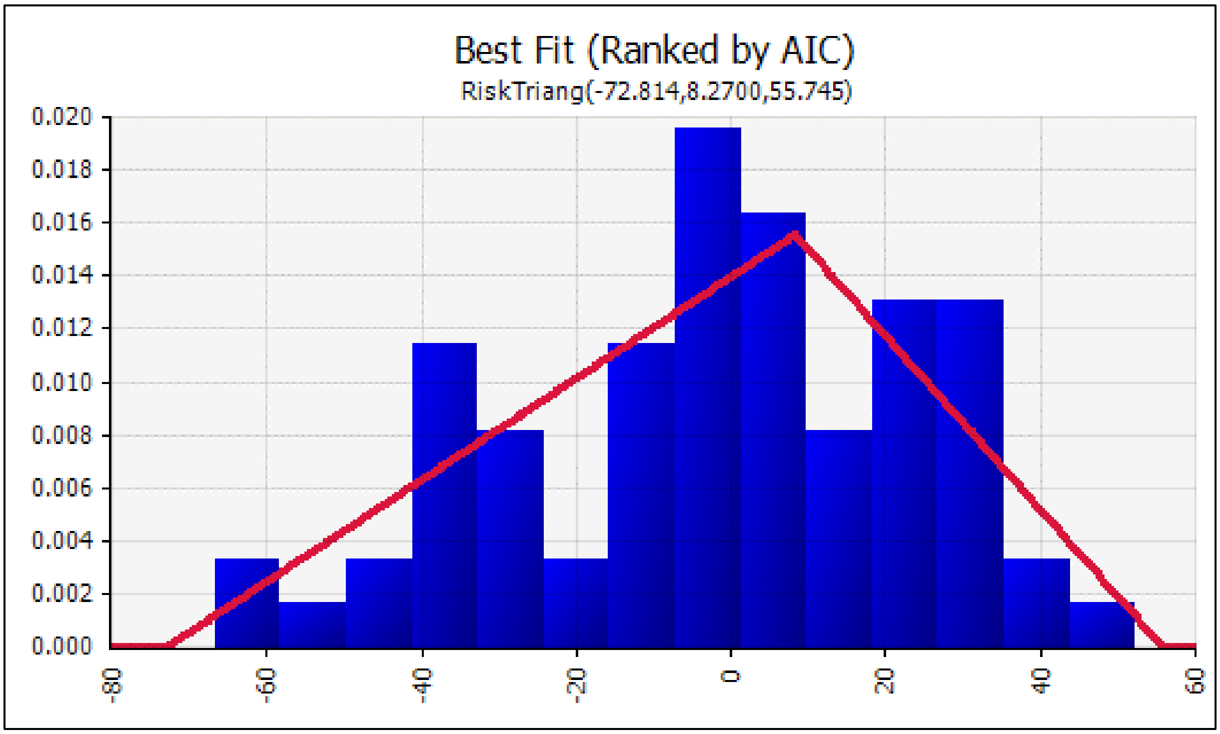

Figure 5.10. Northern Plain Cooperative First Difference Distribution Fit (@Risk™)

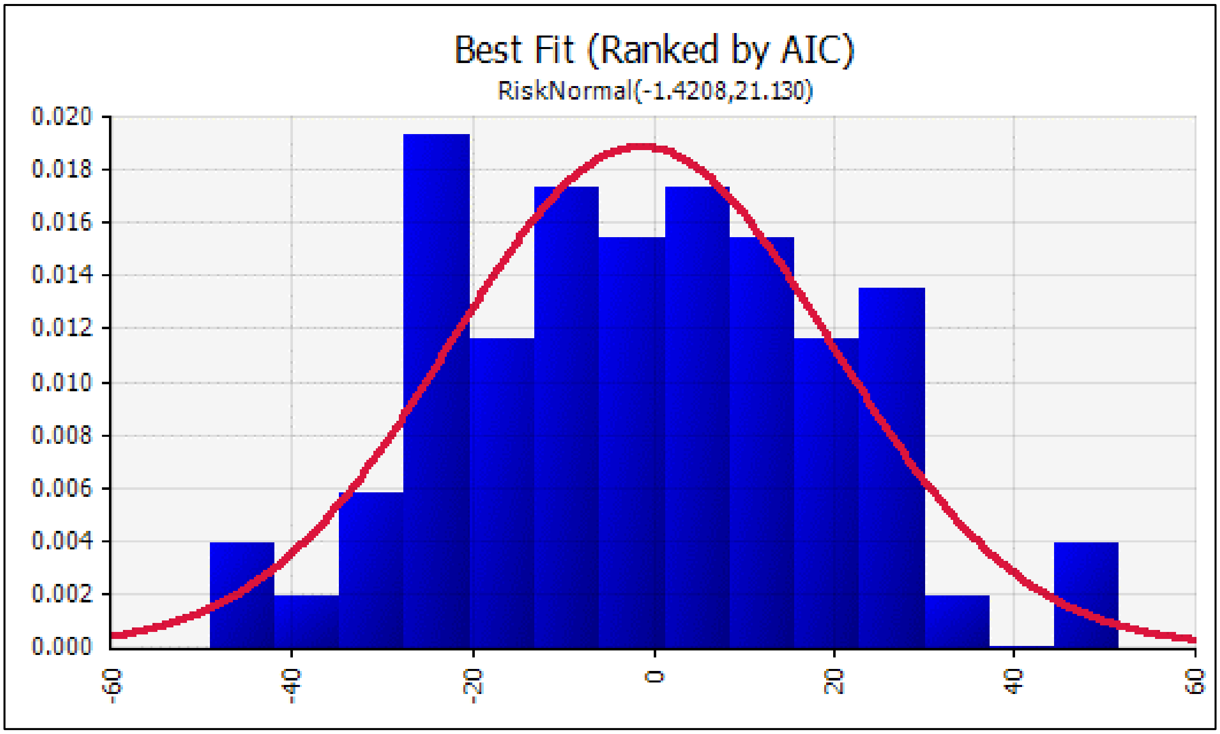

Figure 5.11. Urea Swap First Difference Distribution Fit (@Risk™)

The five-month forecast uses the same distribution for each month with its own correlation matrix identical to Table 5.6. An example of one iteration of forecast price movements for the months of December through April are shown in Figure 5.11.

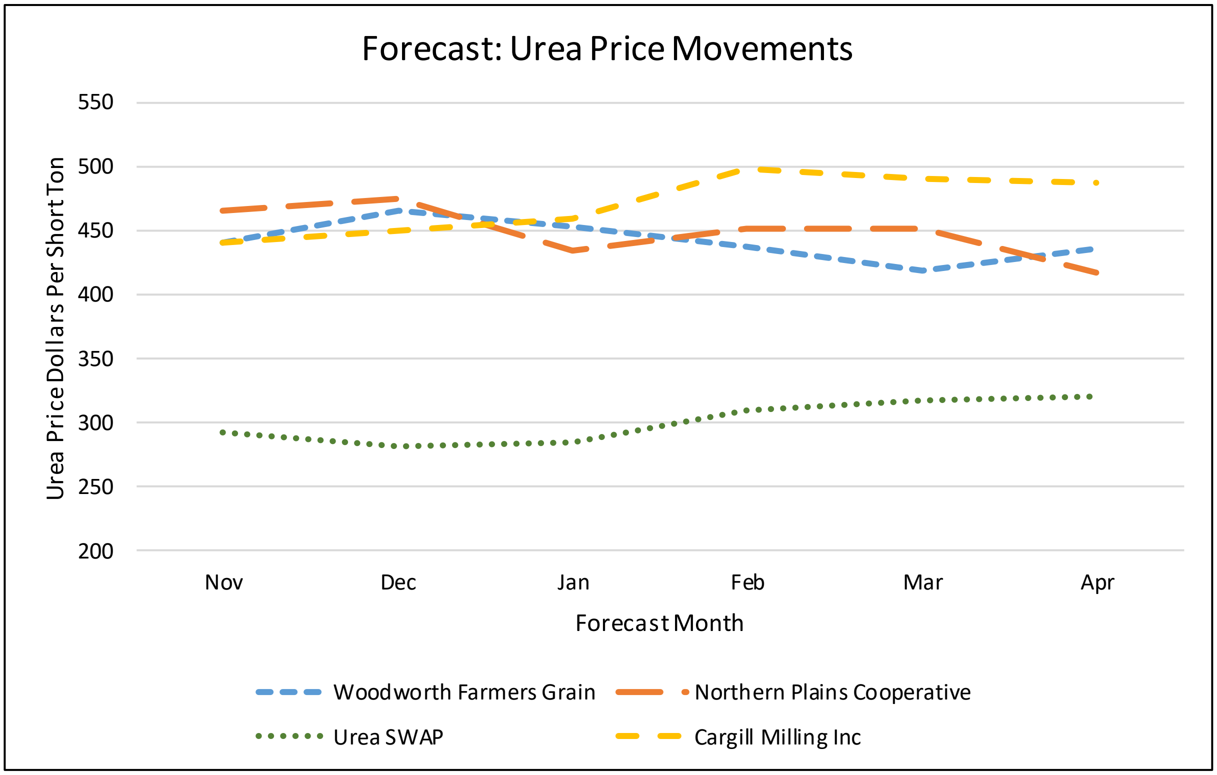

Figure 5.12. Example of Five-Month Price Forecast

5.4.3. Urea Swap Futures Spread

The urea Swap Futures spread is found by subtracting nearby Swap Futures from the deferred Swap Futures contract. Urea Swap Futures contracts are cash settled so the spread may be observed up until contract close (CBOT 2018). Swap Futures prices were gathered from DTN ProphetX on monthly intervals using the monthly closing price. Swap Futures spread follows a seasonal pattern, generally, resulting in an inverse during the spring months of the year. An inverse occurs because after spring planting there is very little demand for urea until the next application season.

Urea Swap Futures spread data is found in Figure 5.13.

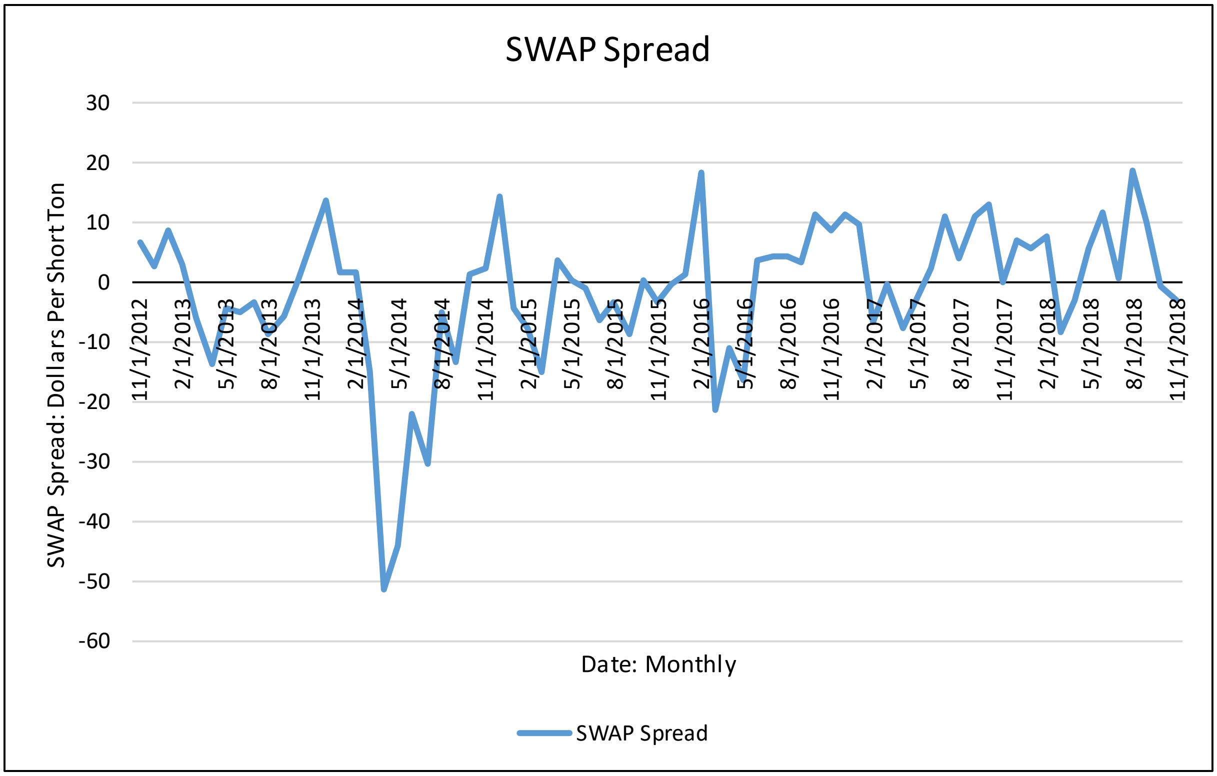

Figure 5.13. Urea Swap Futures Spread (DTN 2018)

The urea Swap Futures spread distribution is fit using BestFit™. The model is only concerned with the Swap Futures spread at the end of April, so only six data points were fit at levels. The distribution was truncated with the spread minimum and maximum of the entire spread data set to eliminate any nonsense results.

@Risk™ distribution specifications for SWAP spread are found in Table 5.8.

Table 5.8. SWAP Spread Distribution (@Risk™)

Variable	Distribution	Function	AIC Score	Mean	Standard Deviation
SWAP Spread	Gumble Minimum	RiskExtvalueMin (-6.5508,11.0655, RiskTruncate (-51.47,18.63))	56.8285	-12.938	14.1921

Urea SWAP spread distribution is shown if Figure 5.14.

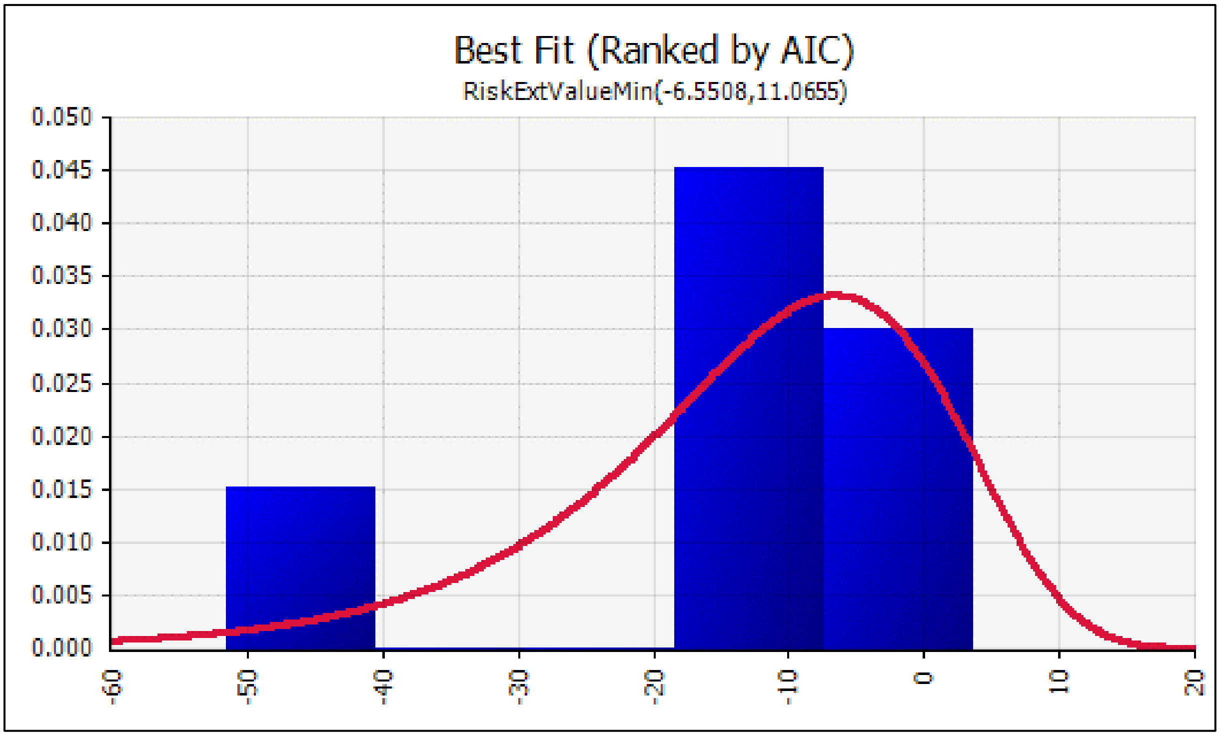

Figure 5.14. Urea SWAP Distribution Fit (@Risk™)

5.4.4. Rail Freight

Rail rates are generated on a quarterly basis using information from BNSF (2018a; 2019) and the Association of American Railroads (AAR 2018). Quarterly rates from the fourth quarter 2012 through the third quarter 2018 are generated using price indexes provided by the AAR. Rail rates are then forecast for each of the five months coinciding with the appropriate quarter. The average cost to ship fertilizer over the five months is the priced used in this application. The price to ship urea from St. Louis, MO to Verona, ND is observed from the end of fourth quarter 2018 through beginning of the second quarter 2019.

The representative county centroid location in Verona, ND purchases urea from St. Louis, MO and has it delivered via railroad. BNSF quotes freight rates from origin to destination based on transportation miles. Distance from Verona, ND to St. Louis, MO is 901 miles using the BNSF miles calculator (BNSF 2019). The price to ship urea from St. Louis, MO to Verona, ND during the third quarter of 2018 was $62.80 (BNSF 2018a). AAR publishes rail cost adjustment factors which are used to calculate freight costs for all remaining quarters from 2012 through 2018 with quarter four of 2017 as the base quarter (AAR 2018).

Rail cost from St. Louis, MO to Verona, ND is shown in Figure 5.15.

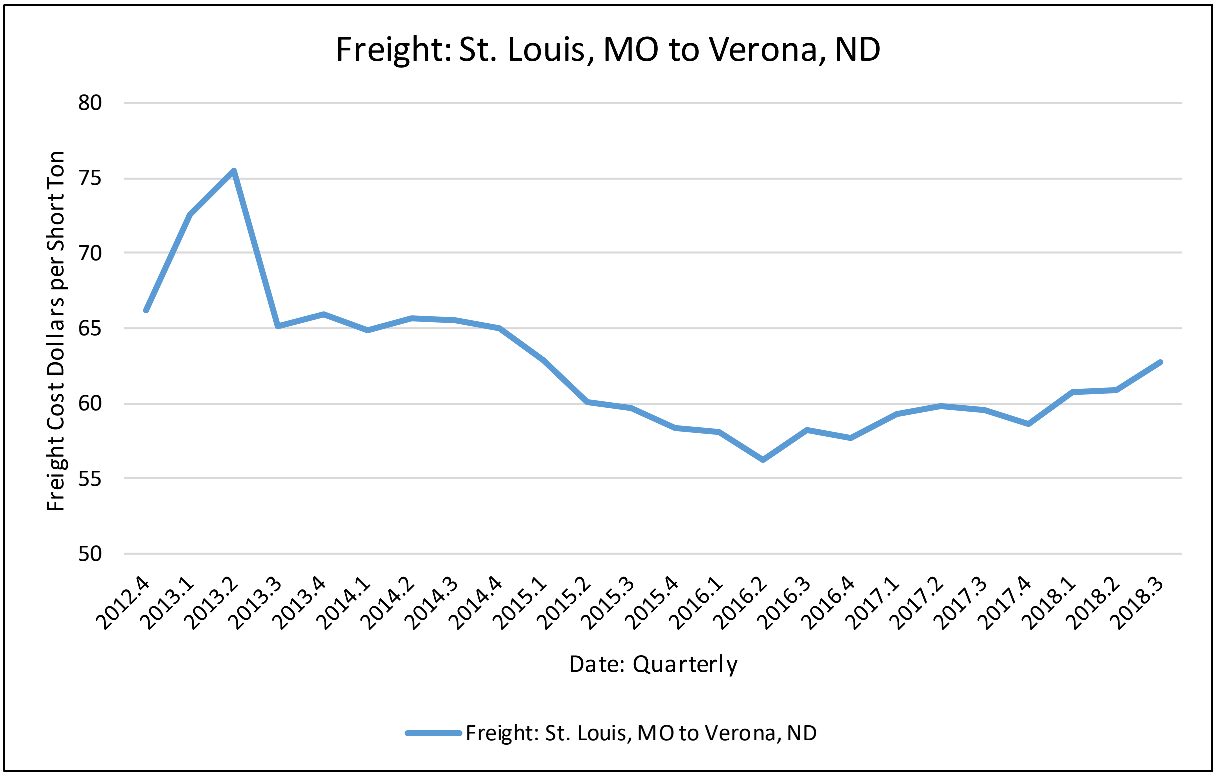

Figure 5.15. Freight from MO to ND (BNSF 2018a; BNSF 2019; AAR 2018)

@Risk™ has the ability to fit time series functions using Bestfit™. Bestfit™ compares variations of autoregressive, moving average, Brownian motion, auto regressive conditional heteroscedasticity (ARCH), and generalized auto regressive conditional heteroscedasticity (GARCH) models when fitting time series data. Appendix G shows a complete description of the times series distributions compared by Bestfit™. Bestfit™ detects seasonality, trend, and stationarity to make proper transformations before fitting data. After a proper time series function has been fit, @Risk™ formulates a forecast based on specifications of the user.

Rail freight is fit on a quarterly basis to forecast the quarter four 2018, quarter one 2019, and quarter two 2019 costs. Rail freight values for each month in those quarters is then extracted from the forecast.

BestFit™ time series distribution specifications for rail freight are found in Table 5.9.

Table 5.9. Time Series Function of Rail Freight (@Risk™)

Variable	Distribution	Function	AIC	Transformation
Rail Freight	Moving Average	RiskMA1 (-0.0023252,0.036907, -0.20906,0.035975)	-86.33	Logarithmic; First Difference

The time series function in Figure 5.16 provides a sample path of forecast rail freight. BestFit™ ranks the MA1 process on the logarithmic first difference transformation of rail freight as the best forecasting function. MA1 is a moving average of forecast errors with one lag and four @Risk™ parameters. The first parameter, -0.0023252, is the mean logarithmic first difference of the stochastic process. The second parameter, 0.036907, is the standard deviation of logarithmic first difference errors. The third parameter, -0.20906, is the β_1 coefficient which is multiplied by the lagged one period error term. The final parameter, -0.035975 is the initial error of the MA1 process (Palisade 2016). In Figure 5.16, the negative values of the X-axis represent historical data. Observations greater than zero on the X-axis are forecast rail freight values. The dark line represents the mean of forecast rail freight, the gray areas above and below the mean represent confidence intervals, and the red line is a sample path.

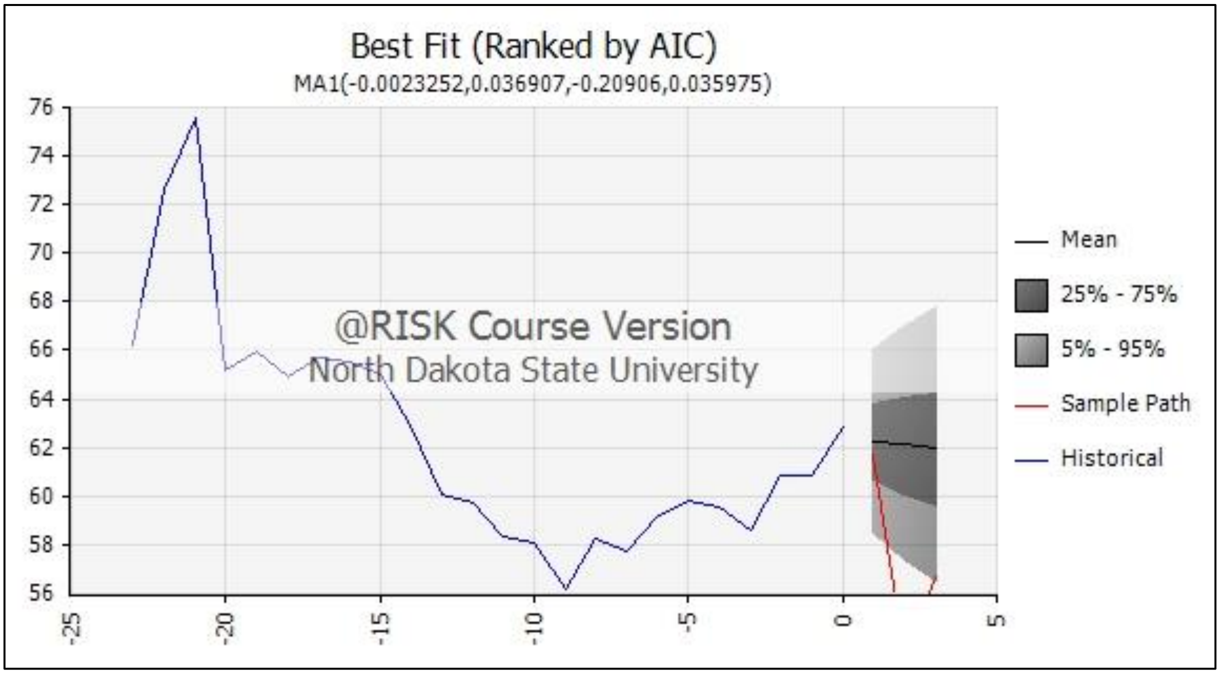

Figure 5.16. Rail Freight Time Series Function (@Risk™)

5.4.5. Random and Non-Random Inputs

Input parameters are split into two groups: non-random inputs and random inputs. Non-random inputs are static and do not change during sensitivity analysis. Random input parameters are either linked, or have calculations linked, to distributions in @Risk™. Non-random inputs are summarized in Table 5.10 and random input parameters are summarized in Table 5.11.

Table 5.10. Non-Random Inputs

Non-Random Inputs	Value	Units	Source
Purchase Price St. Louis	$340.00	Dollars Per Short Ton	Green Markets 2018
Rail Miles: St. Louis to Verona	901	Miles	BNSF 2019
Purchasing Period: Nov 1-April 1	5	Month	Rolf 2019
Time Held if Not Sold	7	Month	Assumption
Storage Capacity Minimum	0	Short Ton	Assumption
Storage Capacity Maximum	120,000	Short Ton	Assumption
Increase in Demand per Underlying State Variable Increase	1	Short Ton	Assumption
Risk Free Interest Rate	2.70%	APY	USDT 2018
Loan Interest Rate	5.00%	APR	Assumption

Table 5.11. Random Inputs

Random Input	Value Mean	Units	Source
Verona Competitive Price	$447.62	Dollars Per Short Ton	Module 1
t_1 Accrued Interest	$7.08	Dollars Per Short Ton	Calculation
Rail Cost	$62.11	Dollars Per Short Ton	BNSF 2018a
Total Investment Per Ton	$409.19	Dollars Per Short Ton	Calculation
Expected Margin	$38.43	Dollars Per Short Ton	Calculation
Stockout Penalty	$0.00	Dollars Per Short Ton	Assumption
Swap Change	-$5.57	Dollars Per Short Ton	DTN 2018
Swap Spread	-$12.08	Dollars Per Short Ton	DTN 2018
Verona Demand	93,300	Short Tons	USDA-ERS (2013); USDA-NASS (2013); AAPFCO (2011)

5.5. Base Case Results

Monte Carlo simulation is implemented using @Risk™ to run 1,000 iterations of the model based on structural and stochastic variables. Specific @Risk™ settings are shown in Table 5.12.

Table 5.12. @Risk™ Settings

@Risk™ Specification	@Risk™ Setting
Sampling Type	Latin Hypercube
Generator	Mersenne Twister
Initial Seed Value	500
Macros	VBA

Results of the base case, and subsequent sensitives, reflect mean values of stochastic simulation for a specific purchasing strategy. RiskOptimizer™ maximizes mean NPV in equation (5.10) by changing the quantity of short tons to purchase. The purchasing quantity is changed in discrete step sizes of 2,000 short tons. Constraints are set on RiskOptimizer™ to reflect a minimum purchasing strategy of zero short tons and a maximum strategy of 120,000 which is the storage capacity of the base case location. The base case results are formulated using distributions from monthly data collected from November 2012 through November 2018.

Base case results are in Table 5.13.

Table 5.13. Base Case Results

Observation	Value
Hedged:	No
Purchasing Quantity	96,000
Purchasing Strategy	102.9%
Inventory NPV: Expected Profit	$2,602,162
Standard Deviation	$4,200,135
Number of Short Calls	51
Short Call Demand Premium	7,881
Number of Long Calls	51
Forecast Demand	93,300
Selling Price	$447.62
Probability of Positive Profit	75.0%
Expected Margin	$38.43

The base case optimal purchasing quantity is 96,000 short tons which is a purchasing strategy 102.9% of forecast demand. Figure 5.17 shows the payoff function for the optimal purchasing strategy with the base case specifications.

Urea demand levels are on the X-axis and expected profit is on the Y-axis. The X-axis shows the possibility for demand to be negative; however, this is impossible but is included to depict the function the long call strike demand. Mean Expected profit has a minimum which occurs at -$1.1 million. This minimum coincides with the long call strike demand, which is 0, and shows the results of net salvage value on an inventory investment of 96,000 short tons.

The fertilizer merchant is long 51 long call options with a strike demand of zero and short 51 calls with a strike demand of 96,000. Strike demand coincides directly with the purchasing strategy because it is assumed the merchant does not have any initial inventory from the previous year. If there were carry over fertilizer, strike demand would have to be adjusted by adding the purchasing strategy to the initial inventory. The Number of long calls equals the slope of expected profit per one ton increase in demand. A base case value of 51 means the merchant gains $51.00 per ton in expected profit per one-ton increase of demand. This number is different than expected margin, which is $38.43, because net salvage value is less than investment costs. The merchant is also short 51 call options at a strike demand of 96,000 short tons. When demand exceeds this number, the merchant does not gain any additional profit from selling urea, i.e., number of short calls equals foregone profit from missed sales. Profit is therefore net zero per one ton increase in demand and the payoff function becomes horizontal.

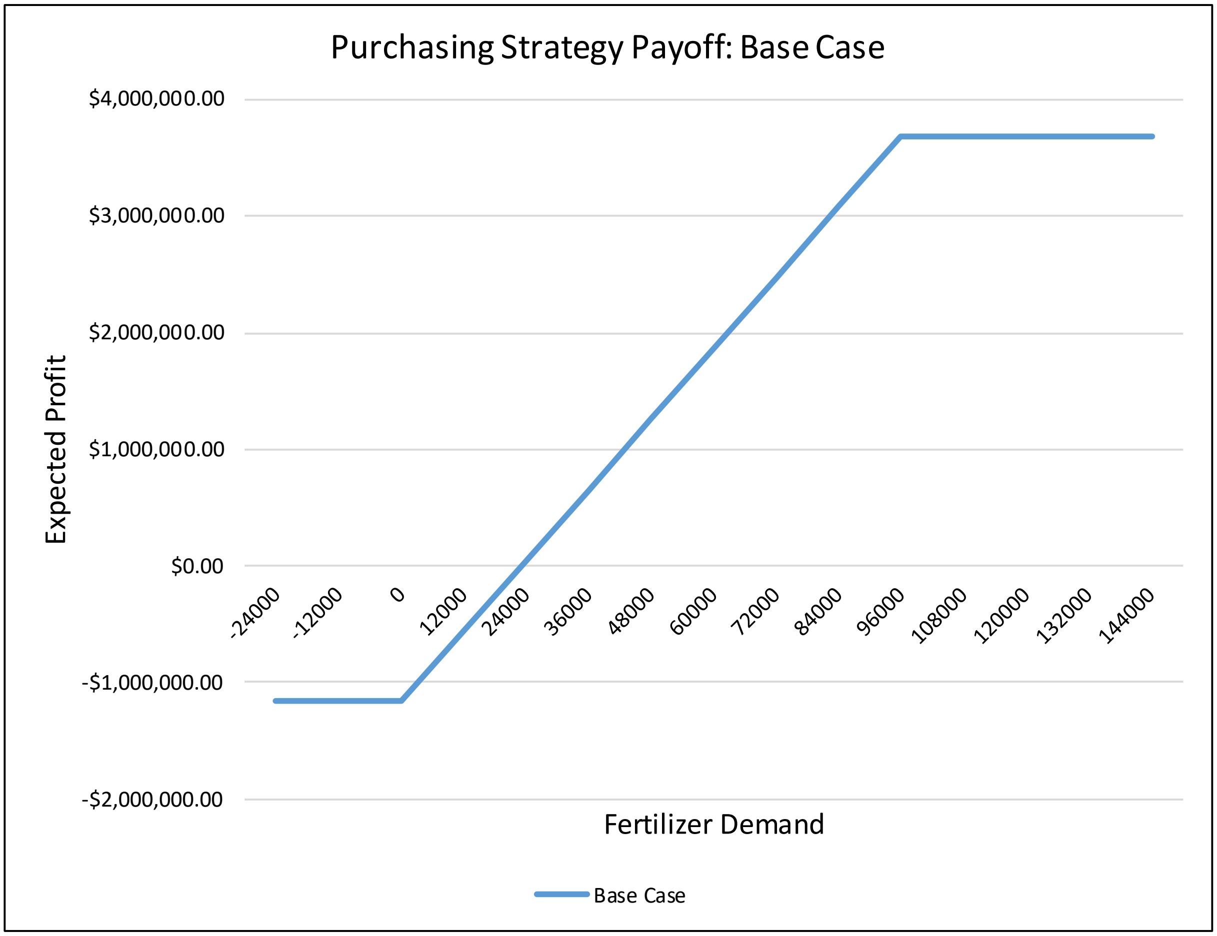

Figure 5.17. Base Case Payoff Function

Demand for fertilizer is skewed to the right. The mean demand occurs at 93,300 short tons with a standard deviation of 16,231 and a maximum demand of 163,726 short tons. The distribution of demand is shown in Figure 5.18.

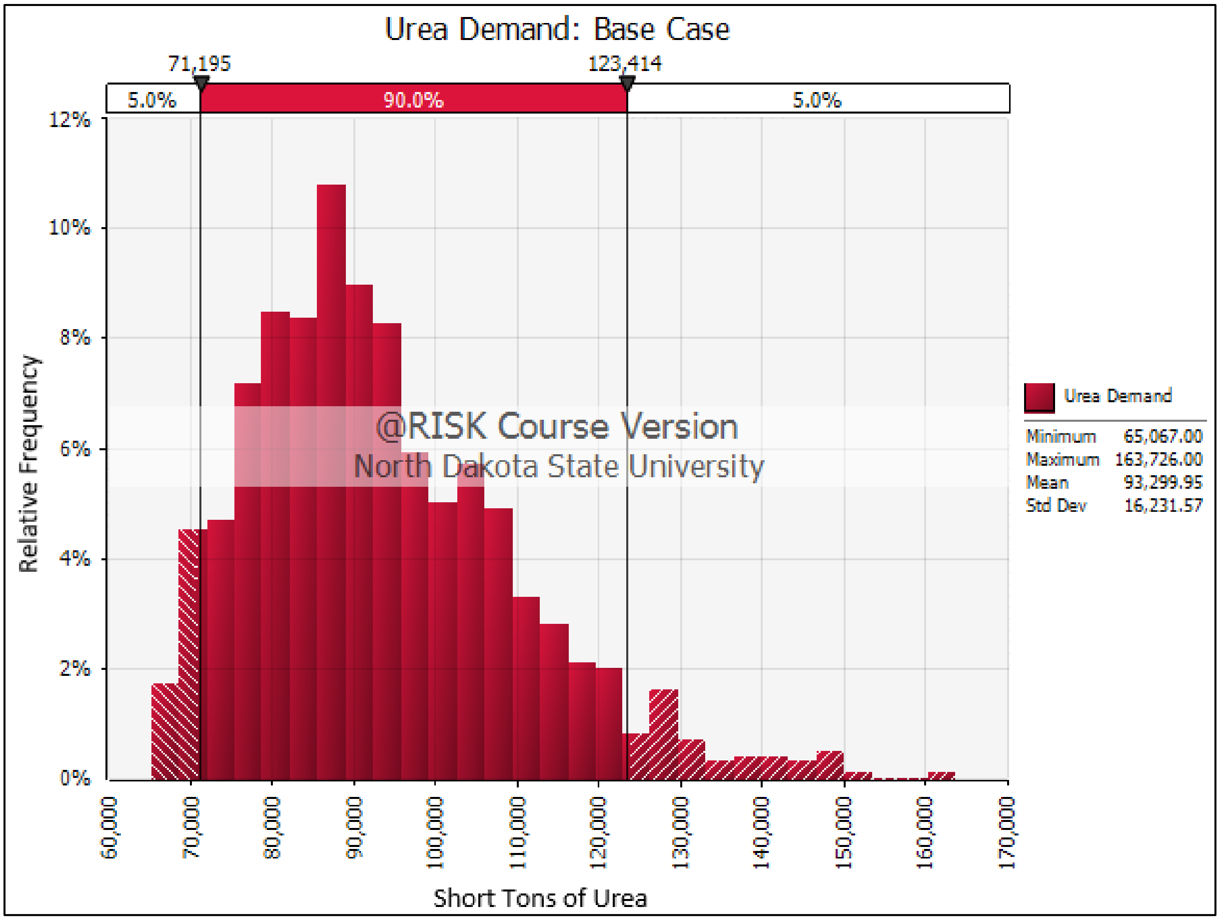

Figure 5.18. Base Case Distribution of Urea Demand (@Risk™)

Expected profit has a relatively wide distribution with a mean expected profit of $2,602,162 and a standard deviation of $4,200,135. Expected profit is above zero 75% of the time but has a minimum expected profit of -$16,342,337 because the distribution is negatively skewed.

Figure 5.19 shows the base case expected profit probability distribution.

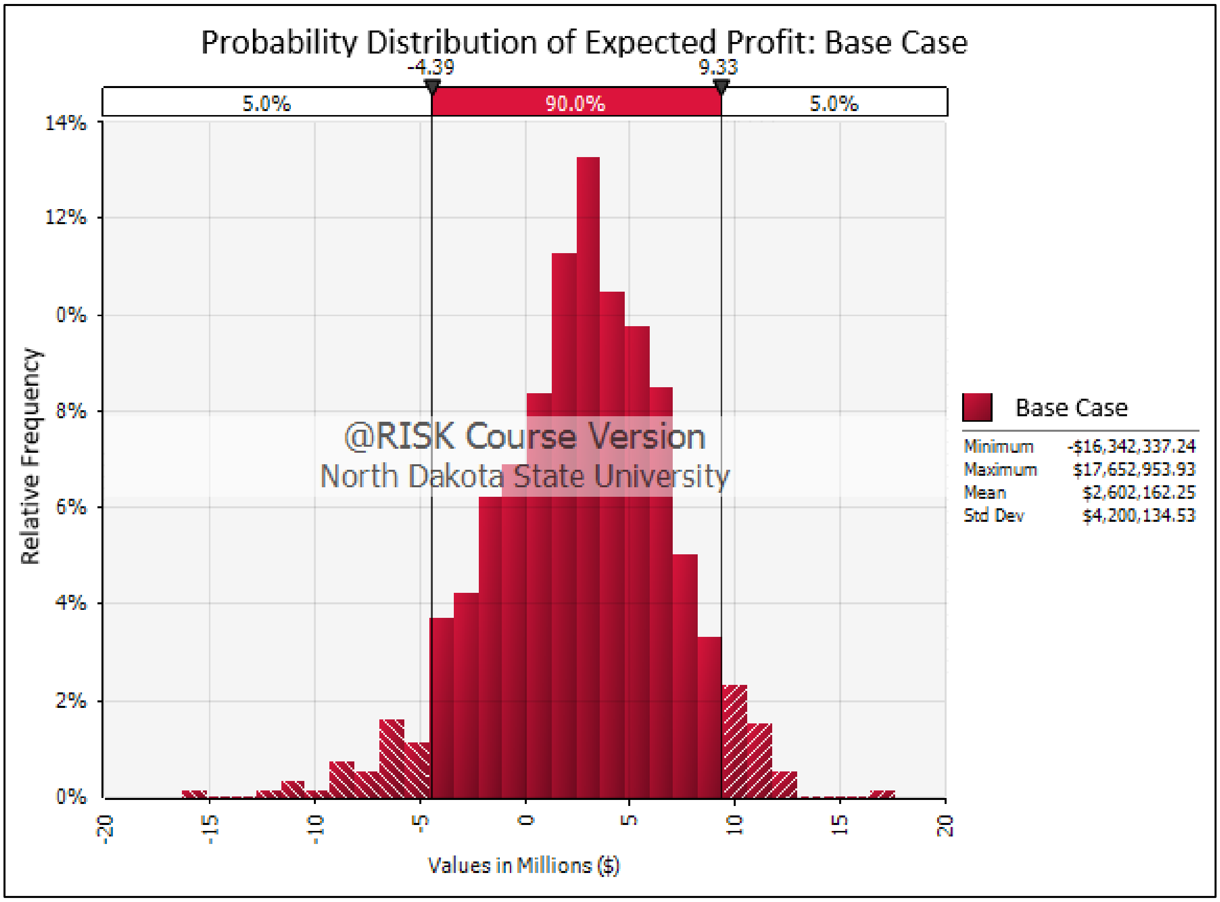

Figure 5.19. Base Case Probability Distribution (@Risk™)

Verona's market share from each region, which is divided into three regions, is shown in Figure 5.20. Verona draws the least from Cargill Milling Inc. because it is the area from which Verona prices its fertilizer off the most.

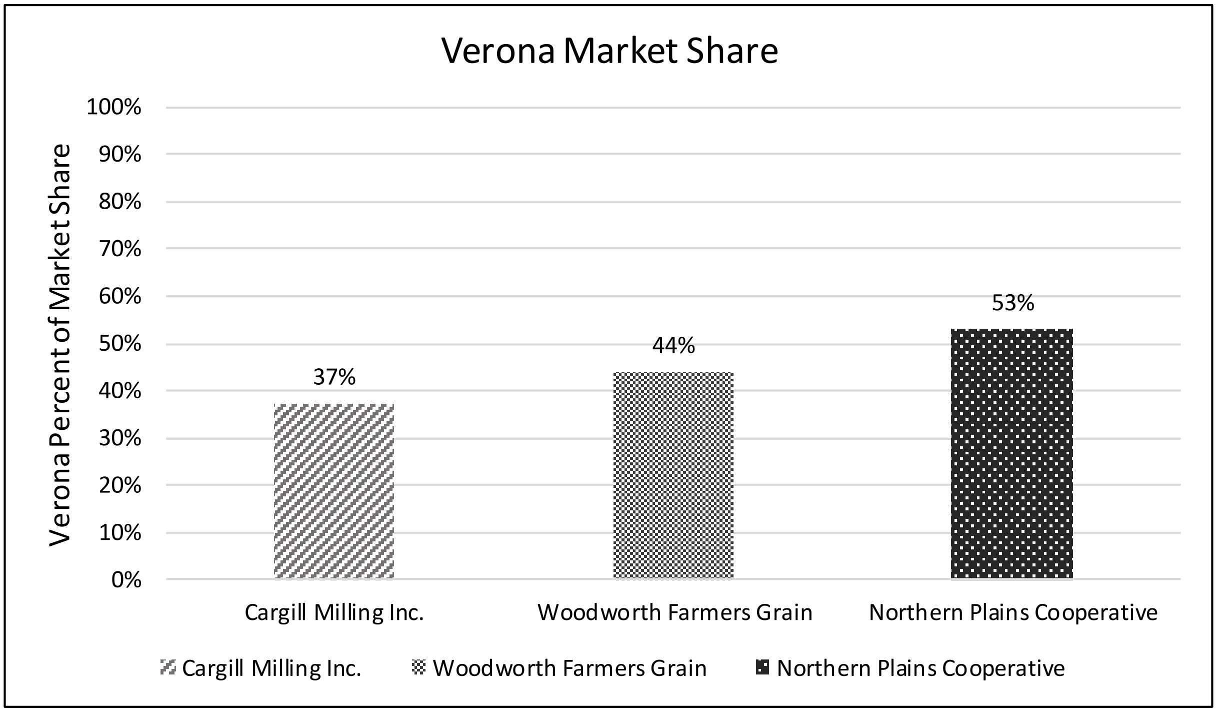

Figure 5.20. Base Case Verona Market Share

5.6. Sensitivities

The factors with the most influence on expected profit are the inputs of competitive arbitrage pricing which impact both margin and demand. The greatest factors in competitive arbitrage pricing are percentage draw area and transportation costs between locations; as these two parameters define market boundaries. Sensitivities in this section pertain to changes in trucking costs between the competitive locations and draw area; as wells as the introduction of hedging in urea Swap Futures contracts.

Sensitivity analysis conducted in this section is summarized in Table 5.14.

Table 5.14. Urea Sensitivity Analysis Summary

Sensitivity	Base Case	Sensitivity Analysis
Draw Area	30% (Minimum Price)	+/- 10%
Trucking Cost	$0.60/mile	+$0.20 / -$0.40
SWAP Hedging	No Hedge	100% Hedged in SWAP

5.6.1. Draw Area

Verona prices urea using competitive arbitrage pricing between Cargill Milling Inc., Woodworth Farmers Grain, and Northern Plains Cooperative. The base case assumes Verona prices to acquire a minimum of 30% market share from any one competitor. The market share that Verona acquires from the other two competitors is then adjusted due to spatial differences and differences in competitive price. Increasing market share makes Verona more competitive which would decrease margin and increases expected demand. Contrary, decreasing minimum market share gives Verona added flexibility in pricing which would increase margin but decreases expected fertilizer demand.

Table 5.15 shows the results of draw area sensitivity analysis.

Table 5.15. Draw Area Sensitivity Results

Observation	20% Minimum Draw Area	Base Case: 30% Minimum Draw Area	40% Minimum Draw Area
Purchasing Quantity	78,000	96,000	112,000
Purchasing Strategy	107.6%	102.9%	98.2%
Inventory NPV: Expected Profit	$2,629,908	$2,602,162	$2,109,796
Standard Deviation	$3,322,317	$4,200,135	$4,951,456
Number of Short Calls	61	51	39
Short Call Demand Premium	6,055	7,881	11,421
Number of Long Calls	61	51	39
Forecast Demand	72,489	93,300	114,084
Selling Price Per Ton	$458.56	$447.62	$436.56
Probability of Positive Profit	79.8%	75.0%	69.4%
Expected Margin	$49.37	$38.43	$27.37

Decreasing minimum draw area from 30% to 20% causes the purchasing quantity to decrease, purchasing strategy to increase, expected profit to increase, and standard deviation of expected profit to decrease. The optimal purchasing strategy increases to 107.6% of forecast demand. Due to the minimum draw area decreasing, the forecast demand also decreases so an increase in percentage purchasing strategy still results in 18,000 less short tones. The expected margin per short increases from $38.43 in the base case to $49.37, i.e., Verona can charge a higher price for their fertilizer because their market boundaries move in closer which also makes their probability of positive expected profit increase to 79.8%. Decreasing the minimum draw area has positive effects on all aspects of the business.

Increasing the minimum draw are to 40% decreases the margin to a point where Verona's optimal purchasing strategy is below forecast demand i.e., Verona would rather under purchase urea because the cost of storing excess inventories is much greater than margin foregone by missing fertilizer sales.

The payoff function changes because the price at which Verona can sell their fertilizer changes. Decreasing draw area increases slope of expected profit which is represented by the number of long calls. Increasing minimum draw area to 40% causes Verona to become more competitive in their pricing. A more competitive price means lower expected margins and therefore a decreased slope in marginal profit per short ton of urea sold. Each sensitivity has the same salvage value because initial investment costs, rail transportation, and interest costs remain unchanged. Thus, a minimum mean expected profit occurs at -$1.1 million which reflects the net salvage value if demand where zero in each sensitivity.

Figure 5.21 shows how the payoff function changes when the minimum draw area changes relative to the base case purchasing quantity.

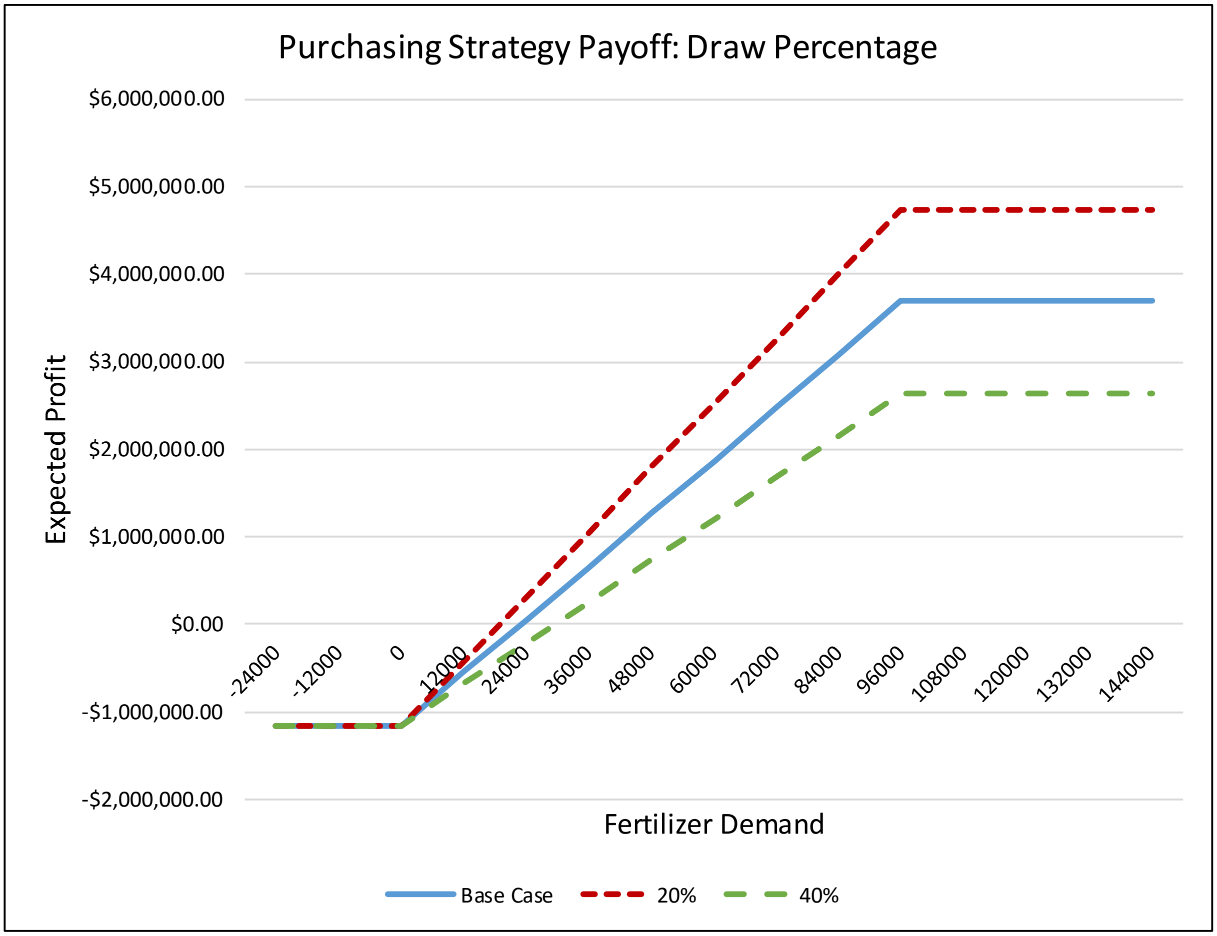

Figure 5.21. Sensitivity Minimum Draw Area Payoff Function

Changing percent draw area has a large impact on the distribution of demand. Increasing minimum draw area to 40% shifts the distribution to the right but decreases standard deviation of demand. This causes less risk in demand and for the optimal purchasing strategy to have a decreased option demand premium. A decreased premium on the number of short calls represents the relatively likelihood of stocking out, i.e., decreased premium means the elevator has higher certainty in their expected demand level.

Verona would purchase a greater percentage of forecast demand when minimum draw area decreases. This result is caused by two reasons: the margin increases and risk in total demand also increases. Increased risk in demand would cause an increased purchasing strategy because an increase in urea demand volatility causes option demand premiums to increase. An increase in premium causes the effect of short calls in the CCI model to increase which lowers expected profit; therefore, a merchandizer will purchase more to lower the option premium; thus, lowering this effect.

Figure 5.22 shows how distribution of demand changes with an increase and decrease in minimum percentage draw.

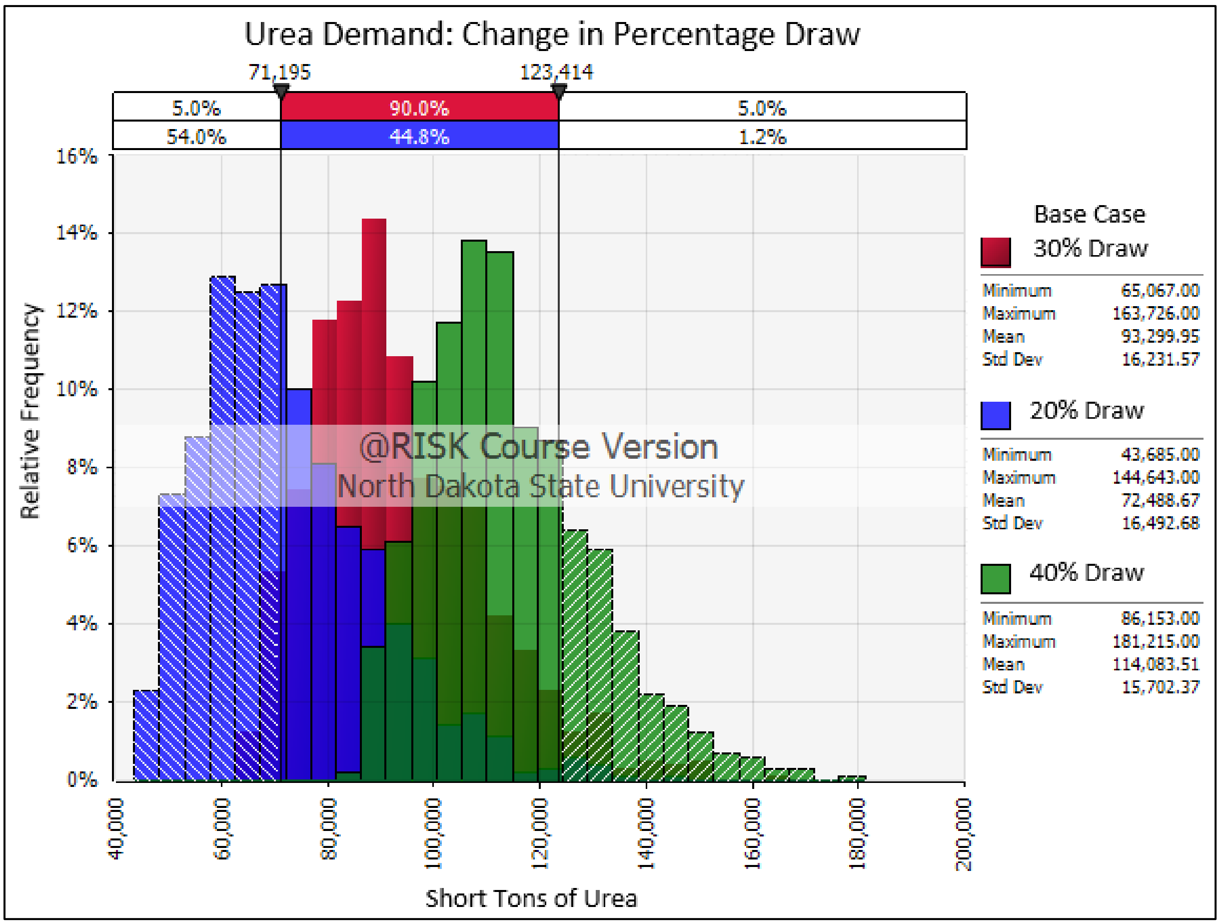

Figure 5.22. Urea Demand: Sensitivity to Minimum Draw Area (@Risk™)

The distribution of profits in Figure 5.23 shows how increasing draw area percentage widens the distribution of expected profit. Decreasing the percentage draw causes expected profit to cluster around the mean and narrow the distribution.

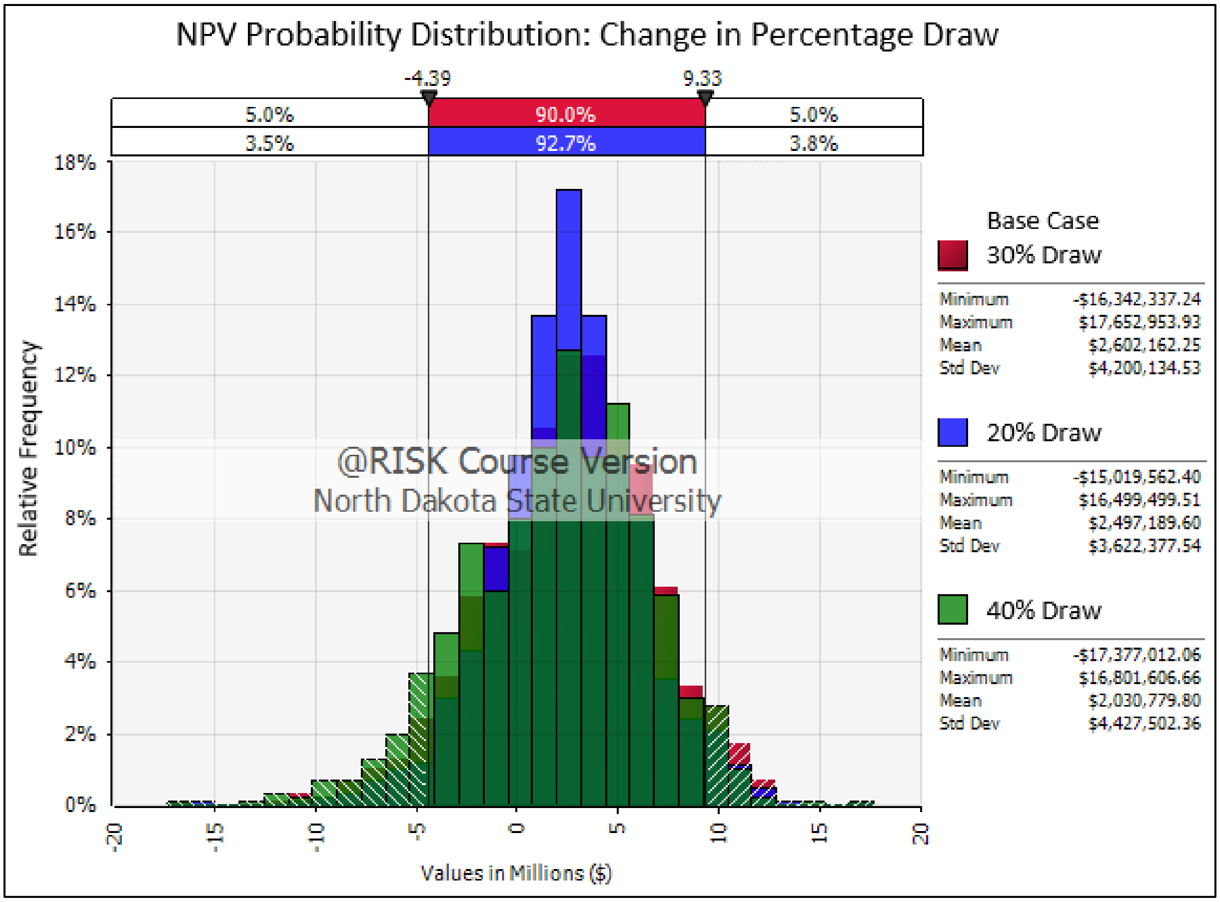

Figure 5.23. Sensitivity Minimum Draw Area Probability Distribution (@Risk™)

5.6.2. Trucking Cost

Transportation cost plays a major role in competitive arbitrage pricing as it is the multiplicative component with spatial distance. Increasing and decreasing trucking cost adds and takes away the flexibility of competitive markets. When transportation costs are zero, all prices should be equal under the law of one price to eliminated arbitrage opportunity (Tomek and Kaiser 2014). Therefore, high trucking costs create rigid market boundaries which is why it is a sensitivity analysis.

Base case trucking costs are assumed to be $0.60 per ton per mile for commercial truckers transporting urea between rural locations. However, this assumption is quite high according to industry sources (Rolf 2019). Sensitivity on transportation costs lowers trucking to $0.20 per ton per mile as well as increases to $0.80.

Results of sensitivity on trucking costs are shown in Table 5.16.

Table 5.16. Trucking Costs Sensitivity Results

Observation	$0.20 Trucking Cost	Base Case: $0.60 Trucking Cost	$0.80 Trucking Cost
Purchasing Quantity	114,000	96,000	92,000
Purchasing Strategy	89.3%	102.9%	106.4%
Inventory NPV: Expected Profit	$1,761,360	$2,602,162	$3,033,772
Standard Deviation	$5,067,804	$4,200,135	$3,989,694
Number of Short Calls	36	51	58
Short Call Demand Premium	21,929	7,881	5,412
Number of Long Calls	36	51	58
Forecast Demand	127,683	93,300	86,438
Selling Price Per Ton	$432.84	$447.62	$454.92
Probability of Positive Profit	67.1%	75.0%	78.0%
Expected Margin	$23.65	$38.43	$45.73

Decreasing trucking costs to an industry acceptable level increases purchasing quantity, decreases purchasing strategy, lowers probability of expected profit, and decreases expected margin. The purchasing quantity increases from 96,000 short tons to 114,000 short tons. The purchasing strategy is reduced to ordering only 89.3% of forecast demand. The expected margin falls by almost $15 per short ton of urea to only $23.65 per short ton. This reduction causes the slope of profit increase, which is the number of long calls, to be reduced to 36. Expected profit decreases to $1,761,360 and the standard deviation increases to $5,067,804. This also lowers the probability of experiencing a positive profit to only 67.1%.

Figure 5.24 shows how the payoff function changes when trucking cost changes relative to base case purchasing quantity. Decreasing trucking cost lowers maximum expected profit to

$2.2 million as shown by the red dotted line. The green dashed line shows how increasing trucking cost, which allows Verona to increase their selling price due to a more rigid market boundary, would increase maximum expected profit to $4.4 million. The minimum expected profit, which occurs at -$1.1 million, does not change because net salvage value does not change.

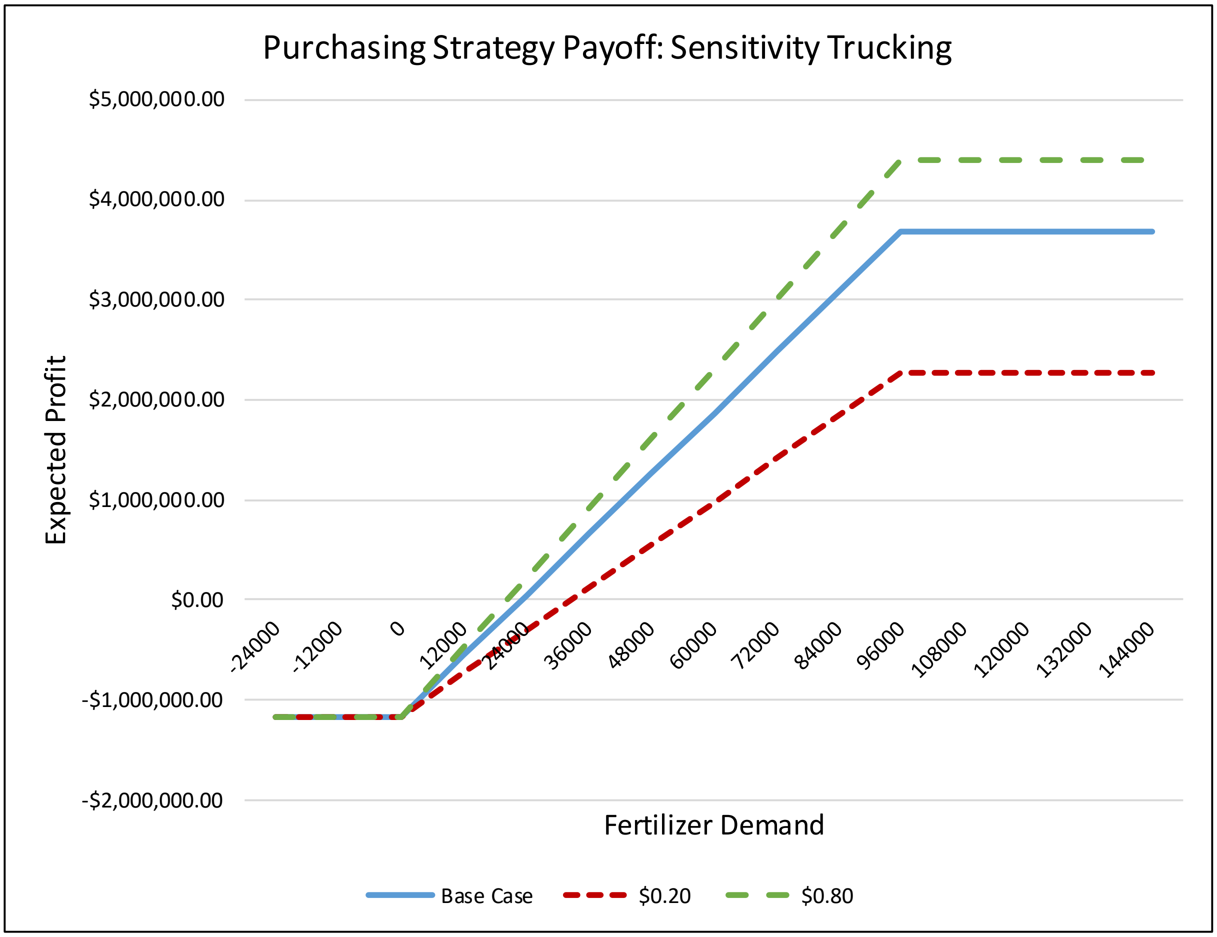

Figure 5.24. Sensitivity Trucking Cost Payoff Function

Increasing trucking cost to $0.80 per mile creates a more rigid market boundary for competitive arbitrage pricing which increases expected margin per short ton of urea. Increased expected margin also increases the number of long calls to 58. The increased slope is reflected in the green dashed line of Figure 5.24.

Changing trucking costs has a major impact on Verona's total demand distribution. Increasing trucking cost shifts the demand distribution to the left and decreases standard deviation because the market boundary has become more rigid.

When trucking cost is reduced to industry levels, the distribution of demand returns nonsense results. A minimum function is required to be added to the competitive arbitrage pricing formula to eliminate nonsense results of over 100% demand. These nonsense results insinuate that urea markets are not efficient at industry levels.

The distribution of demand under sensitivities to trucking cost are in Figure 5.25.

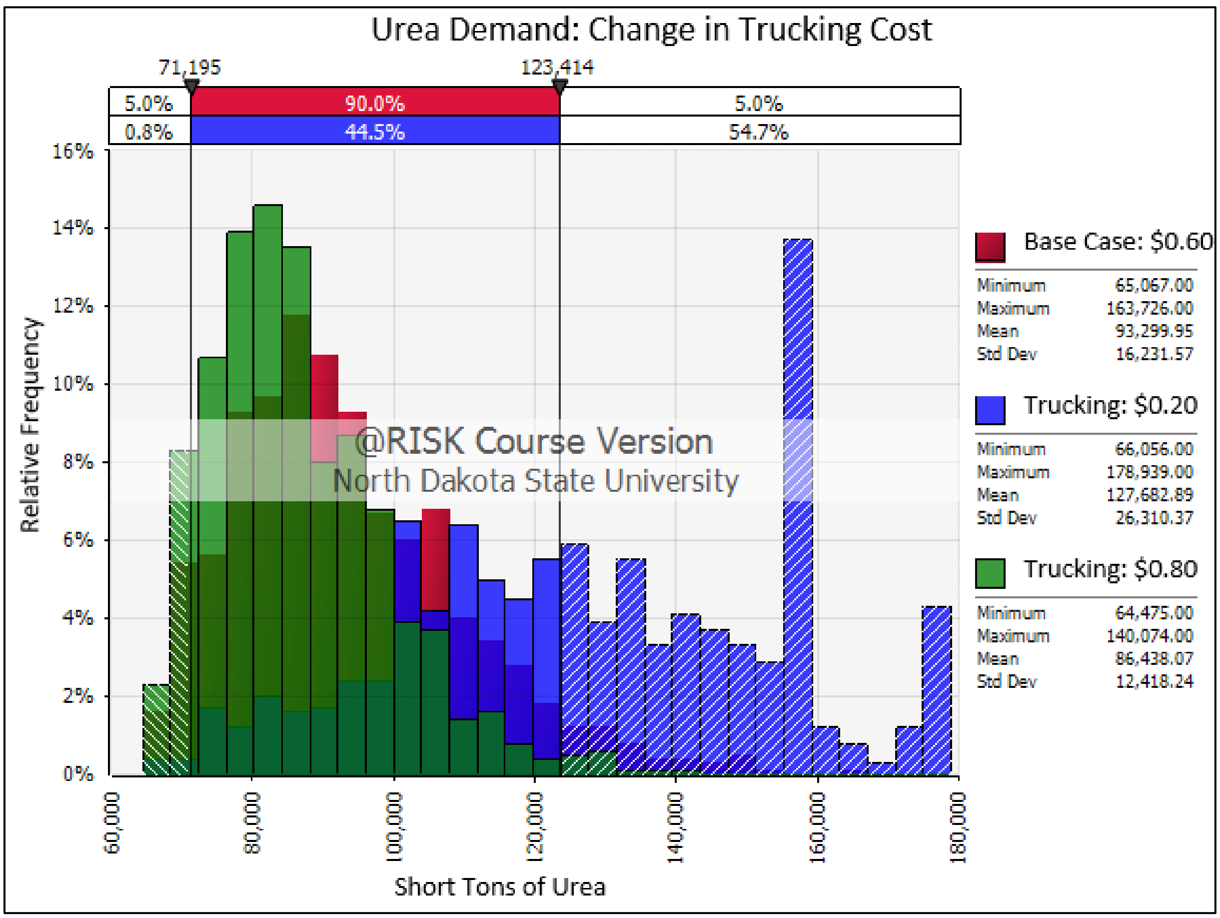

Figure 5.25. Distribution of Urea Demand: Sensitivity to Trucking Costs (@Risk™)

Changing trucking cost also greatly impacts Verona market share from each draw area. In Figure 5.26, increasing trucking cost causes Verona's market share to become narrow at each location and only range from 35% market share to 47% market share. Decreasing trucking costs increases the average market share form each location. It also causes average market share to have a low of 51% at Cargill Milling Inc.'s market and a high at 74% of Northern Plains Cooperative' market. The competitive arbitrage pricing criteria is still at 30%; therefore, these levels of market share and their range further infer inefficient markets at industry level trucking cost.

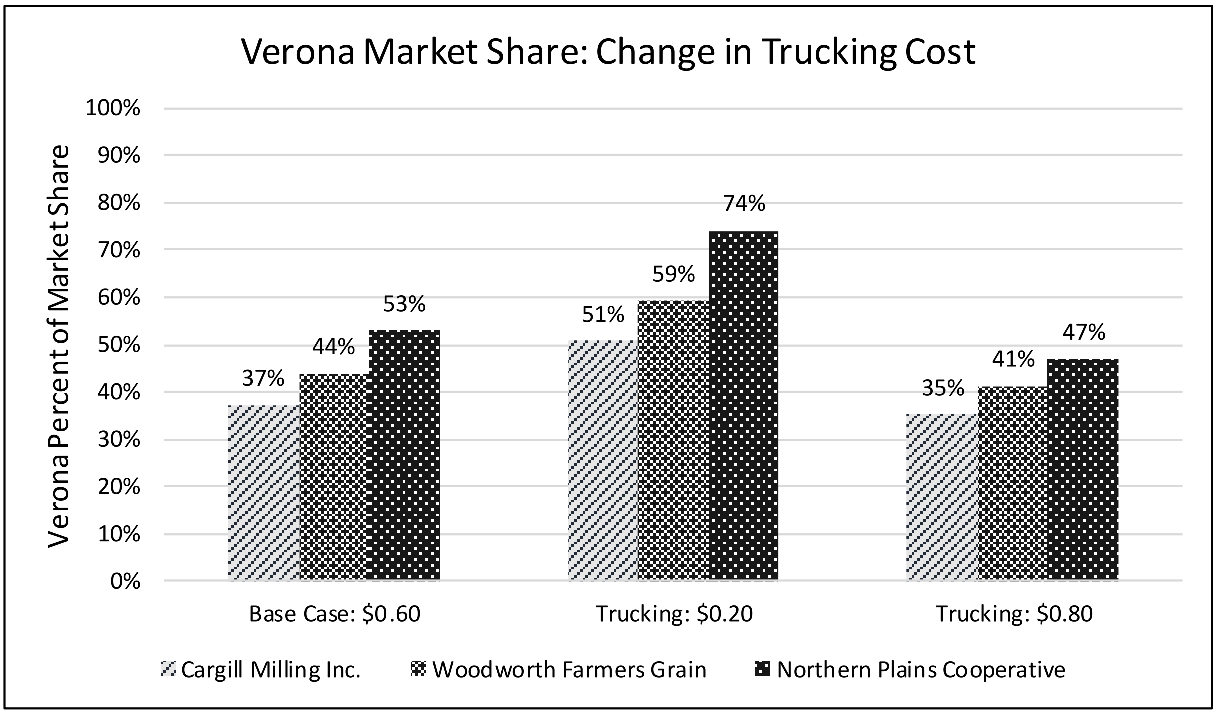

Figure 5.26. Verona's Market Share: Sensitivity to Trucking Cost

5.6.3. Urea Swap

Urea Swap futures contracts are offered by the Chicago Board of Trade (CBOT) based on freight on board (FOB) urea prices in the United State Gulf. A buyer of a Swap Futures contract gains money from the seller when the closing price of FOB US Gulf increases. Fertilizer merchandisers may effectively hedge their long cash position by selling Swap Futures contracts. The offsetting contract could gain money if prices fell while the cash position loses money.

Under competitive arbitrage pricing assumptions, urea price at Verona, ND has a correlation coefficient of 0.93 with Swap Futures at levels and a coefficient of 0.33 when first differenced. Swap futures would not result in a perfect hedge; but, because the hedge is for a duration of five months, the coefficient of 0.93 at levels is good enough to justify a hedge. If the duration of the hedge were short term, a correlation coefficient of 0.33 when first differenced would indicate a poor hedge (Blank et al. 1991).

Urea Swap sensitivity compares optimal purchasing strategies of the unhedged base case with being 100% hedged using Swap Futures. Adding the hedging component causes net selling price and salvage value to be calculated differently. Gains or losses from hedging must be added into the final calculation of net selling price. Verona is classified as a "short" hedger because they are long cash urea and would therefore short urea Swap Futures contracts. Their entire long urea position is hedged by taking an equal and opposite position in urea Swap Future because it is assumed no forward contracting takes place. If forward contracting took place, the calculation for unhedged urea would need to be different from hedged urea. Change in Swap Future price is subtracted from net selling price which adjusts the calculation of long call options (equation 5.13) as in equation (5.16):

$$L = \frac{\partial Q_D}{\partial \Psi} * (\Phi_u - \Delta P_{Swap} - \Gamma_u) \tag{5.16}$$

where:

L	=	number of long call options
$\frac{\partial Q_D}{\partial \Psi}$	=	increase in fertilizer demand per increase in underlying state variable
Φ_u	=	price received per short ton of urea sold, calculated in Module 1
ΔP_{Swap}	=	change in Swap Futures price
Γ_u	=	salvage value of unsold urea.

In addition to changing calculation of long call options, calculation of salvage value (equation 5.12) must also be adjusted to account for spreads in Swap Futures contracts. Swap Futures spreads are highly seasonal and are usually inverted at the end of planting season. An inverted Swap Futures spread would lower salvage value of unsold short tons of urea if the merchandiser wishes to continue to hedge their position until next purchasing season. Salvage value is now calculated as in equation (5.17):

$$\Gamma_u = 2I_u - I_u e^{r_l t_2} + \Omega_{Swap} \tag{5.17}$$

where

I_u = overall investment per short ton of urea at county centroid.

t_2 = time until next purchasing period from sale period

r_l = loan interest rate

Ω_{Swap} = Swap Futures spread.

Results of sensitivity on hedging using Swap Futures are shown in Table 5.17.

Table 5.17. Swap Futures Hedging Sensitivity Results

Observation	**Base Case**	**Hedge Using Swap Futures**
Hedged	No	Yes
Purchasing Quantity:	96,000	92,000
Purchasing Strategy:	102.9%	98.6%
Inventory NPV	\$2,602,162	\$2,969,081
Standard Deviation	\$4,200,135	\$1,689,750
Number of Short Calls	51	68
Short Call Demand Premium	7,881	9,600
Number of Long Calls	51	68
Forecast Demand:	93,300	93,300
Selling Price:	\$447.62	\$453.19
Probability of Positive Profit:	75.0%	95.2%
Expected Margin:	\$38.43	\$44.00

When hedging using Swap Futures, the optimal purchasing quantity falls 4,000 short tons and optimal purchasing strategy is reduced to 98.6% of forecast demand. The fall in purchasing strategy result is twofold. The net salvage value for unsold urea decreases because the urea Swap Futures spread during the spring months has a mean value of -$12.08 (Table 5.10). The negative spread further lowers net salvage value which further penalizes the urea merchant for overestimating urea demand and would lower the purchasing strategy (Stowe and Su 1997). The fall in purchasing strategy is also credited to no longer needing additional inventories to hedge against price movements, i.e., hedging using Swap Futures lessons the need for maintaining a real option. Expected profit increases to $2,969,081 and standard deviation of expected profit falls from $4,200,145 in base case to only $1,689,750 when hedged. Probability of positive expected profit also increases from 75% to 95% of the time.

The number of long and short calls increase largely due to salvage value being reduced. A lower salvage value means slope of additional profit per sold short ton would increase. Expected margin also increases nearly $6.00 per short ton when hedged. An increase in margin further infers that hedging using Swap Futures does not result in a perfect hedge.

The decreased salvage value plus an increased expected margin causes the payoff function to widen out as shown in Figure 5.27. The minimum expected profit lowers from -$1.1 million to -$2.2 million if urea demand where zero because urea Swap Futures spread is negative which lowers the net salvage value. Maximum expected profit also increases to $4.1 million because hedging using urea Swap Futures is not a perfect hedging mechanism. The net change is typically a negative $5.11 per short ton relative to cash price change which increases the net selling price due to expected gains from the short hedge (Table 5.10).

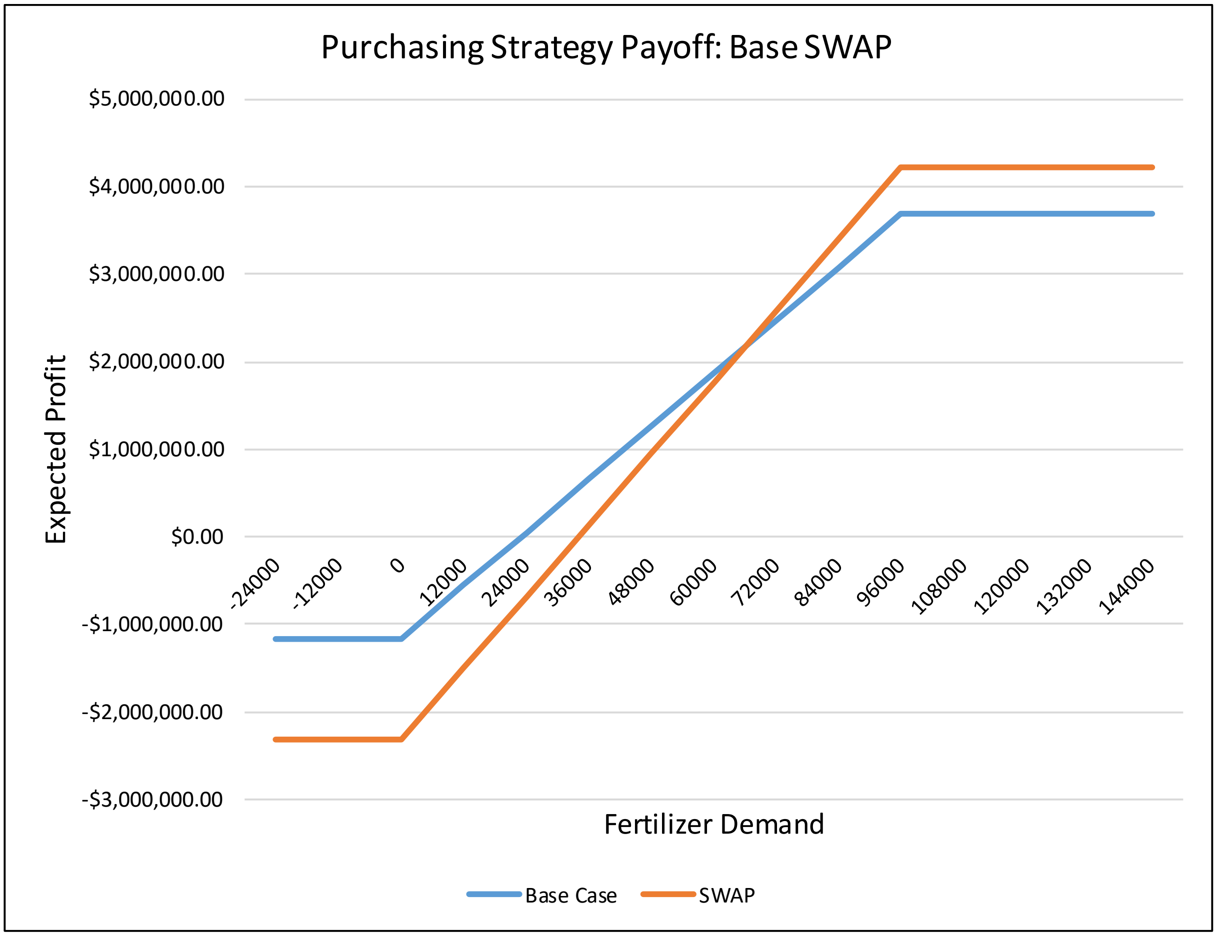

Figure 5.27. Sensitivity Hedging using Swap Futures Payoff Function

The distribution of expected profit moves closer to being normally distributed when hedged. Distribution of profits narrows greatly from a standard deviation of $4.2 million in the base case to $1.68 million when hedged using Swap futures. This major decreases in standard deviation of expected profit is good for fertilizer merchants who wish to mitigate their downside risk. However, hedging using Swap Futures also partially eliminates the opportunity to make large profits through competitive arbitrage pricing.

Distribution of being hedged versus being unhedged are shown in Figure 5.28.

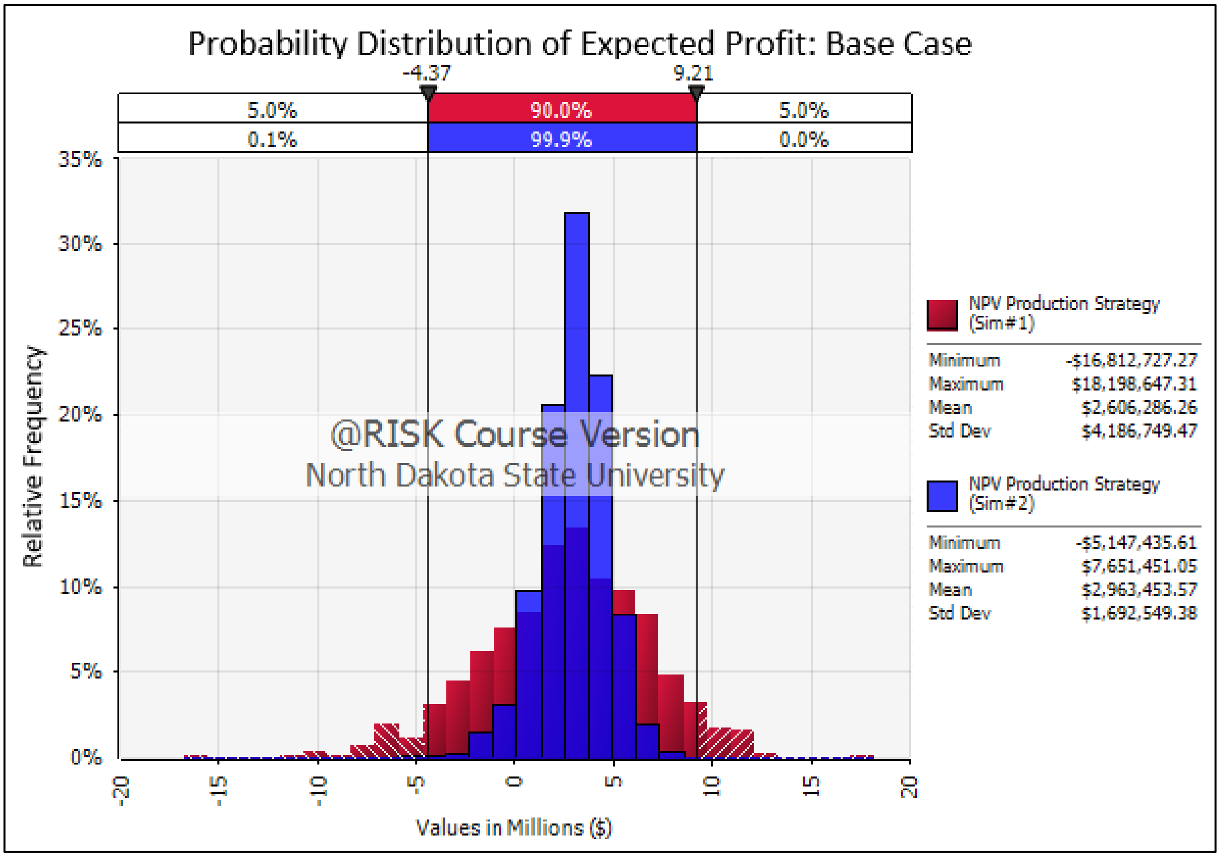

Figure 5.28. Sensitivity Hedged using Swap Futures: Probability Distribution (@Risk™)

5.7. Conclusion

Fertilizer merchandisers located in interior markets are exposed to large levels of risk from both demand and competitive pricing. Competitive arbitrage pricing uses competitor price levels and transportation costs to form artificial market boundaries. These market boundaries shift both inter and intra-yearly which causes great uncertainty in fertilizer demand. An excess purchasing strategy of forecast demand may be used as a real option on futures sales to hedge against price and demand uncertainties. Methodology used by Stowe and Su (1997) combined with real option valuation and competitive arbitrage pricing allows fertilizer merchants to develop an optimal purchasing strategy which would maximize expected profit.

This chapter used a representative urea merchant located strategically between three major competitors. Contingent claims inventory (CCI) analysis, using methodology from Stowe and Su (1997) along with real option valuation and competitive arbitrage pricing, developed a way to model uncertainty in fertilizer and maximize expected profit. Monte Carlo simulation and stochastic optimization produces a purchasing strategy which would maximize expected profit under base case assumptions and data distributions.

Stochastic simulation and RiskOptimizer™ allows fertilizer merchants to simulate multiple scenarios and choose the best strategy based on maximizing expected profit. Sensitivities on market parameters show how optimal quantities and strategy change with shifts in key model parameters such as: minimal market share, transportation costs, and hedging using urea Swap Futures:

- Increasing minimal market share, i.e., lowering urea selling price to acquire more sales, causes purchasing quantities to increases, margins to decreases, expected profit to decrease, and standard deviation of expected profit to increase.
- Base case transportation costs are considered too high by industry sources, however, lowering trucking cost to industry standards causes markets to become inefficient. This means one of two things: there is a large area of opportunity in trading fertilizer or urea is extremely risky and locations are forced to sell at prices regardless of competitor pricing to remain profitable.
- Hedging a long fertilizer position using Swap Futures contracts based on spot prices at New Orleans does not result in a perfect hedge, but it does lower risk substantially. Price levels at interior markets are highly correlated with urea Swap Futures but have a relatively low correlation coefficient when first differenced. This indicates an

alternative method such as Vector Auto Regression (VAR) or a similar multivariate method could be used when hedging for short durations of time.

There are many sources of risk when merchandising fertilizer. However, CCI analysis and real options provide a way to measure this risk. When risk is measured it can be managed. Merchandizers may then alter their purchasing strategy based on sensitives to market risk to make key inventory decisions.

CHAPTER 6. OPTIMAL GRAIN PURCHASING STRATEGY UNDER RISK

6.1. Introduction

Shippers are exposed to several areas of risk including: velocity of shuttle trains, market carry, and price of rail cars on the secondary market. Purchased grain can be viewed as a real option to ship grain. This builds on Stowe and Stu (1997) which views inventory as a real option on future sales using a contingent claim inventory (CCI) model. In this case, the CCI model is interpreted as a call spread. Long calls represent the option to sell and short calls represents forgone profit when there is a shortage of inventory. Shippers can apply this same model to the option to ship grain. A shipper that purchases primary rail contracts up to 12+ months forward has an uncertain supply of rail cars due to randomness in velocity. To accommodate uncertain car supply, among other market variables, the shipper determines an optimal grain purchasing strategy. The CCI model outputs a net present value (NPV), which represents expected profit, of the purchasing strategy which can be maximized by altering the quantity of bushels purchased over time.

The optimal purchasing strategy of a shipper depends on three main components: velocity of rail cars, market carry, and secondary-rail-market prices. These three components can be translated into car supply, salvage value, and stockout penalty. Velocity of rail cars effects how many shuttle trains arrive over a one-month period and thus shipping demand due to car supply. Market carry, which is comprised of terminal basis spread, futures spread, and tariff spread, has a great impact on unshipped bushels because shippers roll their positions into the next shipping period which affects net salvage value of the purchasing strategy. Underestimating car supply leads to excess shuttle trains being sold into secondary market. Shuttle trains sold into the secondary market are for either a premium or discount and thus either a negative or positive stockout penalty. Given these components, an optimal purchasing strategy can be found using Stowe and Su's contingent claims inventory (CCI) model (1997). The CCI model derives NPV which is interpreted as a shipper's expected profit given the current market characteristics. Shippers can maximize their NPV by obtaining an optimal purchasing strategy through stochastic optimization.

This chapter presents an empirical model for determining an optimal purchasing strategy for a bulk shipper. First, a conceptual model outlines the application structure and input parameters. Next, each component is derived along with data sources and distributions of stochastic variables. Finally, results of the base case are presented, followed by relevant sensitivities.

6.2. Conceptual Model

The model represents a typical shuttle elevator located in the great plains who ships soybeans using primary rail contracts. This model represents a single-elevator shipper but could be adapted to utilize multiple locations. The shipper procures soybeans from producers via forward contracts and resells soybeans to terminal markets located in the Pacific Northwest (PNW). The elevator ships soybeans by rail, using BNSF primary rail contracts. Current primary contracts offered by the BNSF are for a one-year duration of continuous shipments (TradeWest Brokerage Co. 2018). In the base case, the shuttle elevator only buys and sells soybeans and does not buy additional rail cars on the secondary market. However, the elevator can sell unused trains into the secondary market at either a premium or discount. This model represents a purchasing strategy for three months - or fourteen weeks - of soybean forward contracts. This timeline matches new-crop delivery of soybeans which starts in September and goes through November. This application assumes a shipper makes one purchasing strategy for three months which may be adjusted over time. However, as Stowe and Su (1997) state, this model can be expanded to a material requirement planning (MRP) model which would reflect weekly inflows and outflows of grain as well as account for randomness in spot deliveries.

There are two module components in the overall model: Module 1 which consists of a stochastic-binomial-option-valuation tree and Module 2 which is a purchasing strategy contingent claim. The module operates as follows:

- First, option strike velocities are derived from input parameters based on a chosen purchasing strategy.
- Second, option strike velocities are evaluated using stochastic binomial valuation from Module 1.
- Finally, derived premiums of Module 1 are used in the CCI module to generate an NPV of the purchasing strategy.

This process continues using adjusted purchasing strategies until NPV of the shipper is maximized. This is a dynamic iterative model and uses @Risk™ and RiskOptimizer™ which are products of Palisade Software (Winston 2008). RiskOptimizer™ changes the quantity of bushels in the purchasing strategy until NPV is maximized. Stowe and Su (1997) manually iterate their model until a closed form derivative is equated which coincides with maximizing NPV. Their model does not include distribution functions and does not need stochastic simulation; therefore, a closed form solution was possible.

Module flow is shown in Figure 6.1.

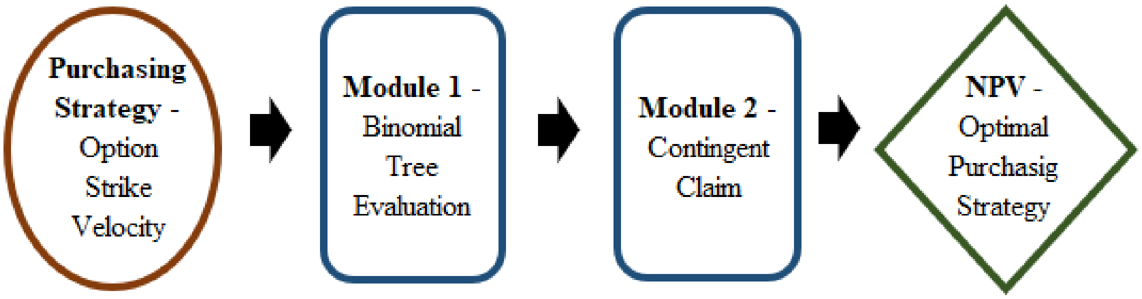

Figure 6.1. Module Flow

Stowe and Su (1997) map demand of a product onto the price level of an underlying state variable. For Stowe and Su, the underlying state variable is a financially traded asset so Black-Schole's model is used to value option premium. In this application, the underlying state variable is velocity of rail cars and binomial trees are used to value the real option

Stochastic binomial option pricing trees calculate option premiums for both long and short call options. The three-month purchasing strategy is divided into weekly intervals for a total of 14 moves. Note: if this were set up as an MRP model, it would need 14 individual binomial trees (Landman 2017). Weekly intervals demonstrate how often a shuttle train may arrive and thus represents the frequency of evaluation. Options are valued as American style because an option to ship may be exercised at any time during the option life. Table 6.1 shows the five components of the option to ship and presents the relationship between the three types of options, which builds on Table 2.1.

Table 6.1. Five Components of Option to Ship

Component	Financial Option	Real Option	Option to Ship
Underlying Variable:	Current value of stock	Gross present value of expected cash flows	Forecast velocity of shuttle trains
Strike Value:	Exercise price	Investment cost	Velocity coinciding with car supply
Time to Maturity:	Time to expiration	Time until opportunity disappears	Duration of purchasing strategy
Volatility:	Stock price uncertainty	Project value uncertainty	Velocity Volatility
Risk-Free Rate:	Riskless interest rate	Riskless interest rate	52 Week T-Bill rate

Stowe and Su developed a contingent claims model for a processing firm which treats inventory as a strategic variable. The firm can view their inventory as a real option because it may be used for futures sales, i.e., as an option on future sales. Model framework developed by Stowe and Su is used in this study. The problem for a shipper which orders rail cars via primary rail markets is to determine an optimal level of grain purchases to meet car supply. In concept, grain shippers would buy an excess amount of grain than forecast car supply which may be used as a real option. An excess purchasing strategy results in part to randomness of velocity which results in uncertainty of rail cars supplied. If velocity is greater than forecast, it would be in the shipper's interest to have a surplus grain held in inventory as an option to ship.

6.3. Empirical Model

This section has two parts. First, it explains calculations in binomial tree evaluation of Module 1 as it applies to shuttle velocity. Second, each component of the CCI model is developed in its application to a shuttle elevator.

6.3.1. Stochastic Binomial Real Option Module

Stowe and Su use Black-Schole's model to evaluate option premiums. This application uses backward induction via binomial tree from Cox, Ross, and Rubinstein (1979). The stochastic binomial option module for velocity requires five components: forecast velocity of shuttle trains, strike velocity, duration of option, velocity volatility, and risk-free interest rate. These components ultimately provide inputs into Module 1 (Figure 6.1). Once inputs are known, equations (6.1), (6.2), and (6.3) are used to set up the binomial option tree:

$$p = \frac{a - d}{u - d} \tag{6.1}$$

$$u = e^{\sigma\sqrt{\Delta t}} \tag{6.2}$$

$$d = e^{-\sigma\sqrt{\Delta t}} \tag{6.3}$$

where:

p	=	probability of an up move
a	=	growth factor
u	=	multiplicative up factor
d	=	multiplicative down factor
σ	=	velocity volatility; annualized standard deviation of log first differences
t	=	life of the option in terms of fraction of a year
Δt	=	length of one option move; fraction of total moves to t.

Velocity is the underlying state variable in this real option valuation module. Velocity volatility is the standard deviation of logarithmic first differences of forecast values. Growth factor, a, equals one in this application because the module is valuing a real option on an asset that does not experience a constant expected growth (Hull [1995] 2008).

Values at terminal nodes are evaluated as a call option using equation (6.4):

$$\max(\Psi_{t,j} - K, 0) \tag{6.4}$$

where:

$\Psi_{t,j}$ = velocity value at terminal nodes t with j up moves

j = number of up moves which have occurred since time zero

K = strike velocity.

Option premiums work backward through the tree from right to left. Premiums are evaluated as American style options using equation (6.5) at each node until the final option value is derived at the initial node:

$$f_{i,j} = \max\{\Psi_{i,j} u^j d^{i-j} - K,\ e^{-r\Delta t}[pf_{i+1,j+1} + (1-p)f_{i+1,j}]\} \tag{6.5}$$

where:

$f_{i,j}$ = option premium at node i, j

i = number of velocity moves which have occurred since time zero

r = risk free interest rate.

@Risk™ fits a time series forecast based on historical data. This forecast generates 14 weeks of expected velocity. Average velocity of this forecast is the current state value for the binomial tree. The average is used because car supply occurs each week, so simply taking the last value would give an inaccurate forecast of car supply over 14 weeks. To calculate volatility, logarithmic first differences for each week are derived as shown in equation (6.6):

$$Log\ Difference\ at\ Week_i = \ln(week_i) - \ln(week_{1-i}). \tag{6.6}$$

where:

$week_i$ = velocity forecast value at week i.

The standard deviation of logarithmic first differences over a 14-week time span is the weekly velocity volatility (Kodukula and Papudesu 2006). Weekly volatility is converted to annual through the multiplication by the square root of 52. Table 6.2, Table 6.3, and Figure 6.2 show the results of one iteration from the stochastic binomial real option model. Table 6.2 shows an example of logarithmic first differences for forecast velocity, velocity volatility, and the forecast velocity state variable. Table 6.3 provides an example of all values taken from the short call option calculation in Module 1. Figure 6.2 shows how the stochastic binomial option tree returns an option premium of 0.23 when strike velocity is 3.04 and the iteration forecast velocity is 3.16.

Table 6.2. Velocity Logarithmic First Differences

Forecast Week	Forecast Velocity	Logarithmic Difference
Week 0	2.90	-
Week 1	3.00	0.035
Week 2	3.03	0.008
Week 3	2.89	-0.048
Week 4	3.08	0.064
Week 5	3.09	0.003
Week 6	3.04	-0.015
Week 7	3.09	0.014
Week 8	3.14	0.017
Week 9	3.38	0.073
Week 10	3.38	0.001
Week 11	3.26	-0.035
Week 12	3.29	0.007
Week 13	3.26	-0.008
Week 14	3.35	0.027
	Velocity Volatility; Weekly:	0.033
	Velocity Volatility; Annualized:	0.239
	Forecast State Velocity:	3.16

Table 6.3. Binomial Tree Inputs Example

Parameter	Derivation	Value
Forecast State Velocity	Ψ_T	3.16
Strike Velocity	K	3.04
Interest Rate	r	2.7%
Volatility	σ	0.239
Time Until Expiration	t	14/52
Period Length	Δt	0.019
Up Factor	$e^{\sigma\sqrt{\Delta t}}$	1.034
Down Factor	$e^{-\sigma\sqrt{\Delta t}}$	0.967
Probability of Up Move	$p = \frac{1-d}{u-d}$	0.492
Probability of Down Move	$1 - p$	0.508

	Week 0	Week 1	Week 2	Week 3	Week 4	Week 5	Week 6	Week 7	Week 8	Week 9	Week 10	Week 11	Week 12	Week 13	Week 14
															5.03 / 1.99
														4.87 / 1.83	
													4.71 / 1.67		4.71 / 1.67
												4.55 / 1.52		4.55 / 1.52	
											4.40 / 1.37		4.40 / 1.37		4.40 / 1.37
										4.26 / 1.23		4.26 / 1.23		4.26 / 1.23	
									4.12 / 1.09		4.12 / 1.09		4.12 / 1.09		4.12 / 1.09
								3.99 / 0.95		3.99 / 0.95		3.99 / 0.95		3.99 / 0.95	
							3.86 / 0.82		3.86 / 0.82		3.86 / 0.82		3.86 / 0.82		3.86 / 0.82
						3.73 / 0.70		3.73 / 0.70		3.73 / 0.70		3.73 / 0.70		3.73 / 0.70	
					3.61 / 0.58		3.61 / 0.58		3.61 / 0.58		3.61 / 0.57		3.61 / 0.57		3.61 / 0.57
				3.49 / 0.47		3.49 / 0.47		3.49 / 0.46		3.49 / 0.46		3.49 / 0.46		3.49 / 0.46	
			3.38 / 0.38		3.38 / 0.37		3.38 / 0.36		3.38 / 0.35		3.38 / 0.35		3.38 / 0.34		3.38 / 0.34
		3.27 / 0.30		3.27 / 0.29		3.27 / 0.28		3.27 / 0.27		3.27 / 0.25		3.27 / 0.24		3.27 / 0.23	
Forecast State Velocity: / Option Premium:	3.16 / 0.23		3.16 / 0.22		3.16 / 0.20		3.16 / 0.19		3.16 / 0.18		3.16 / 0.16		3.16 / 0.15		3.16 / 0.13
		3.06 / 0.16		3.06 / 0.15		3.06 / 0.14		3.06 / 0.12		3.06 / 0.11		3.06 / 0.09		3.06 / 0.06	
			2.96 / 0.10		2.96 / 0.09		2.96 / 0.08		2.96 / 0.07		2.96 / 0.05		2.96 / 0.03		2.96 / 0.00
				2.86 / 0.06		2.86 / 0.05		2.86 / 0.04		2.86 / 0.03		2.86 / 0.02		2.86 / 0.00	
					2.77 / 0.03		2.77 / 0.02		2.77 / 0.02		2.77 / 0.01		2.77 / 0.00		2.77 / 0.00
						2.68 / 0.01		2.68 / 0.01		2.68 / 0.00		2.68 / 0.00		2.68 / 0.00	
							2.59 / 0.00		2.59 / 0.00		2.59 / 0.00		2.59 / 0.00		2.59 / 0.00
								2.51 / 0.00		2.51 / 0.00		2.51 / 0.00		2.51 / 0.00	
									2.43 / 0.00		2.43 / 0.00		2.43 / 0.00		2.43 / 0.00
										2.35 / 0.00		2.35 / 0.00		2.35 / 0.00	
											2.27 / 0.00		2.27 / 0.00		2.27 / 0.00
												2.20 / 0.00		2.20 / 0.00	
													2.12 / 0.00		2.12 / 0.00
														2.06 / 0.00	
															1.99 / 0.00

Figure 6.2. Short Call Binomial Tree: One Iteration

6.3.2. Contingent Claim Module

The contingent claim inventory (CCI) model developed by Stowe and Su (1997) is broke into four elements: salvage value of unused inventory, long call payoff, short call payoff, and initial inventory value. The CCI module evaluates NPV of the purchasing strategy using Stowe and Su's equation (5) as discussed in chapter 3. The CCI module objective function, as it applies to a grain purchasing strategy, is:

$$NPV = Q_b \Gamma_b e^{-rt} + Lf_L - Sf_S - I_b Q_b \qquad (6.7)$$

where:

NPV	=	net present value of inventory stocking level
Q_b	=	purchasing strategy quantified in bushels of soybeans
Γ_b	=	salvage value for unsold soybeans
L	=	number of long call options
S	=	number of short call options
f_L	=	long call option premium
f_S	=	short call premium
I_b	=	elevator cash price to producers for soybeans.

The first of four equation elements is a vertical discount. Vertical discount considers salvage value of the purchasing strategy discounted at the risk-free interest rate and time to maturity.

In this application, salvage value equals return to storage of unsold soybeans. Returns to storage is comprised of three parts: investment per bushel of soybeans, market carry, and the cost of carry. Investment per bushel is the shipper's cash price to producers. The shipper's cash price is derived by first adding nearby futures price to terminal basis at the Pacific Northwest

(PNW); then, subtracting rail tariff per bushel and the shipper's gross margin per bushel as shown in equation (6.8):

$$I_b = P_f + B_{PNW} - T_b - M_G \qquad (6.8)$$

where:

P_f = futures market price

B_{PNW} = terminal basis at the Pacific Northwest

T_b = rail road tariff per bushel

M_G = elevator gross margin per bushel.

The shipper's gross margin per bushel encompasses all shipper overhead costs, handling charge, and the price per bushel of the primary instrument. It is assumed that the gross margin per bushel is constant at $0.20 per bushel.

Market carry is defined as a combination of futures market spread, PNW basis spread, change in tariff rate, and the spread of daily car values (DCV). DCV is accounted for in the shortage penalty element of the model; therefore, DCV is left out of the salvage value calculation. Futures market spread is accounted for because it is assumed shippers hedge their position in the nearby futures month after each transaction and would therefore need to roll any unshipped bushels into the next futures month. Furthermore, PNW basis spread is evaluated because it is assumed shippers do not forward contract sales with PNW terminal markets; instead, they sell spot to maintain flexibility to ship or store bushels. Tariff rate per bushel is not hedged, as it is charged at the time of shipment. However, a change in tariff rate affects market carry.

Market carry is calculated as in equation (6.9):

$$C_{Market} = F_{Spread} + B_{Spread} - T_{Spread} \quad (6.9)$$

where:

C_{Market}	=	overall carry in the soybean market per bushel
F_{Spread}	=	deferred futures price minus nearby futures price per bushel
B_{Spread}	=	deferred PNW basis minus nearby PNW basis per bushel
T_{Spread}	=	deferred tariff rate minus nearby tariff rate per bushel.

Cost of carry considers interest cost on stored bushels as well as any costs associated with maintaining the condition of soybeans.

Cost of carry is calculated in equation (6.10) as:

$$C_{Cost} = [r_L * t * I_b] + [s_r * \Delta t] \quad (6.10)$$

where:

C_{Cost}	=	cost of carry
r_L	=	loan interest rate
s_r	=	weekly storage rate of soybeans
t	=	total time of storage
Δt	=	increments of time; weekly.

Salvage value for unsold soybeans is calculated by adding the market carry to investment per bushel and then subtracting cost of carry as in equation (6.11).

$$\Gamma_b = I_b + C_{Market} - C_{Cost} \quad (6.11)$$

The second element of Module 2 is the value of long call options in the call spread. The number of long calls equates marginal profit gained per one unit increase in velocity. The number of long calls, L, is calculated as:

$$L = \frac{\partial Q_D}{\partial \Psi} * (\Phi_b - \Gamma_b) \qquad (6.12)$$

where:

$\frac{\partial Q_D}{\partial \Psi}$ = increase in shipping demand due to car supply per unit of velocity

Φ_b = price received per bushel of soybeans sold.

Shipping demand due to car supply for soybeans per unit increases in velocity, $\frac{\partial Q_D}{\partial \Psi}$, depends on shuttle capacity and number of primary contracts. A shipper with a storage capacity of 5,000,000 bushels and a turnover ratio of six would plan on shipping 30,000,000 bushels annually. Rail shuttles offered by BNSF railroad have 110 cars. Each rail car holds 3,500 bushels of soybeans which means each shuttle train received holds 385,000 bushels. The number of bushels per shuttle train can be calculated in equation (6.13):

$$B_t = B_c * R_t \qquad (6.13)$$

where:

B_t = number of bushels per shuttle train

B_c = number of bushels per rail car

R_t = number of rail car per shuttle train.

As an example of the assumption made in this chapter, if expected velocity of a BNSF rail shuttle is three, a shipper can expect 36 trains in one year. Therefore, one primary contract would have an expected shipping capacity of 13,860,000 bushels per year. Therefore, a shipper which plans to ship 30,000,000 bushels of soybeans per year would purchase two primary contracts. $\frac{\partial Q_D}{\partial \Psi}$ is calculated by multiplying together number of months in purchasing strategy, number of primary rail contracts, and bushels per shuttle train as shown in equation (6.14):

$$\frac{\partial Q_D}{\partial \Psi} = M_N * C_N * B_t \tag{6.14}$$

where:

M_N = number of months in purchasing strategy

C_N = number of primary contracts owned by elevator.

Given the assumptions on the shipper, shipping demand due to car supply would increase 2,310,000 bushels of soybeans per one unit increase in velocity.

Selling price per unit is the net value for each bushel loaded by a shipper. The shipper is a "basis trader," so an elevator's profit comes from the margin. Selling price is calculated by adding margin to elevator cash price as in equation (6.15):

$$\Phi_b = I_b + M_G. \tag{6.15}$$

Price received per bushel can also be calculated by evaluating futures price, PNW basis, and tariff; however, equation (6.15) is a simplified approach. The number of long calls is now calculated using equation (6.12).

Long call premium, f_L, is found in Module 1. However, long call strike velocity, K_L, is found outside the system of equations. Long call strike velocity coincides with the velocity at which car supply would be zero. Car supply is zero when velocity is zero, so the long call strike velocity will always be zero.

Module 2's third element is the value of short calls which is found by multiplying the number of short calls by short call premium. The number of short calls equates marginal profit lost per one unit increase in velocity.

The formula for short calls as found in Chapter 3 is calculated using equation (6.16):

$$S = L + \left(\frac{\partial Q_D}{\partial \Psi} * \Lambda_b\right) \tag{6.16}$$

where:

Λ_b = shortage penalty per bushel of unmet car supply; observed as -DCV.

By default, the number of short calls equals the number of long calls if there is no shortage penalty. However, in this model, unmet car supply results in shuttle trains being sold into secondary rail markets at either a premium or discount. This value is recorded as daily car value (DCV) per bushel of soybeans. A positive DCV would result in a negative shortage penalty.

Short call strike velocity coincides with soybean purchasing strategy. As discussed in Chapter 3, this value is found in equation (6.17):

$$K_S = K_L + \left(Q_b * \frac{1}{\frac{\partial Q_D}{\partial \Psi}}\right). \tag{6.17}$$

where:

K_S = short call strike velocity

K_L = long call strike velocity is found outside the system of equations and assumed to be constant at zero.

Short call premium is now found by inputting short call strike velocity into Module 1. The delta of the short call strike may also be found using equation (3.15) to express the probability of a stockout occurring. The premium itself represents the magnitude of stockout expected to occur if the firm experiences a shortage of inventory.

Module 2's fourth element is the initial outlay. Initial outlay is the purchasing strategy multiplied by the investment per bushel of soybeans. NPV in equation 6.7 can now be calculated based on input parameters calculated in equations (6.8) through (6.17) with premiums calculated in Module 1.

6.4. Data

Data analyzed in this application comes from soybean crop marketing years of 2013/14 through 2016/17. Data is weekly and is from September 5, 2013 through August 25, 2017 for a total of 208 observations. Data obtained includes terminal basis at Pacific Northwest (PNW), secondary rail market premiums expressed as daily car value (DCV), spread in the soybean futures market, rail tariff rate per bushel shipped, and velocity. At the end of this section base case random and non-random input parameters are presented.

Data sources can be found in Table 6.4.

Table 6.4. Data Sources

Data	Source	Assembled By
Basis: PNW	Trade West Brokerage Co. 2018; Thomson Reuters Eikon 2018	Bruce Dahl Jesse Klebe
Daily Car Value (DCV)	Trade West Brokerage Co. 2018; Thomson Reuters Eikon 2018	Bruce Dahl Jesse Klebe
Soybean Futures	CBOT 2018a	Jesse Klebe
Tariff Rate	USDA-AMS 2018	Jesse Klebe
Velocity	Trade West Brokerage Co. 2018	Bruce Dahl

6.4.1. Basis at PNW

Terminal basis at PNW is the selling point of soybeans relative to the soybean futures. Of the 208 data points, there were 14 missing observations. An instrument variable from Thompson Reuters Eikon is implemented to fit the missing observations using symbol "SYB-TERM-PORT" and subtracting the active soybean futures contract to obtain PNW basis. Appendix D shows the regression equation used to generate missing values. Figure 6.3 shows PNW Basis behavior through time and across different years.

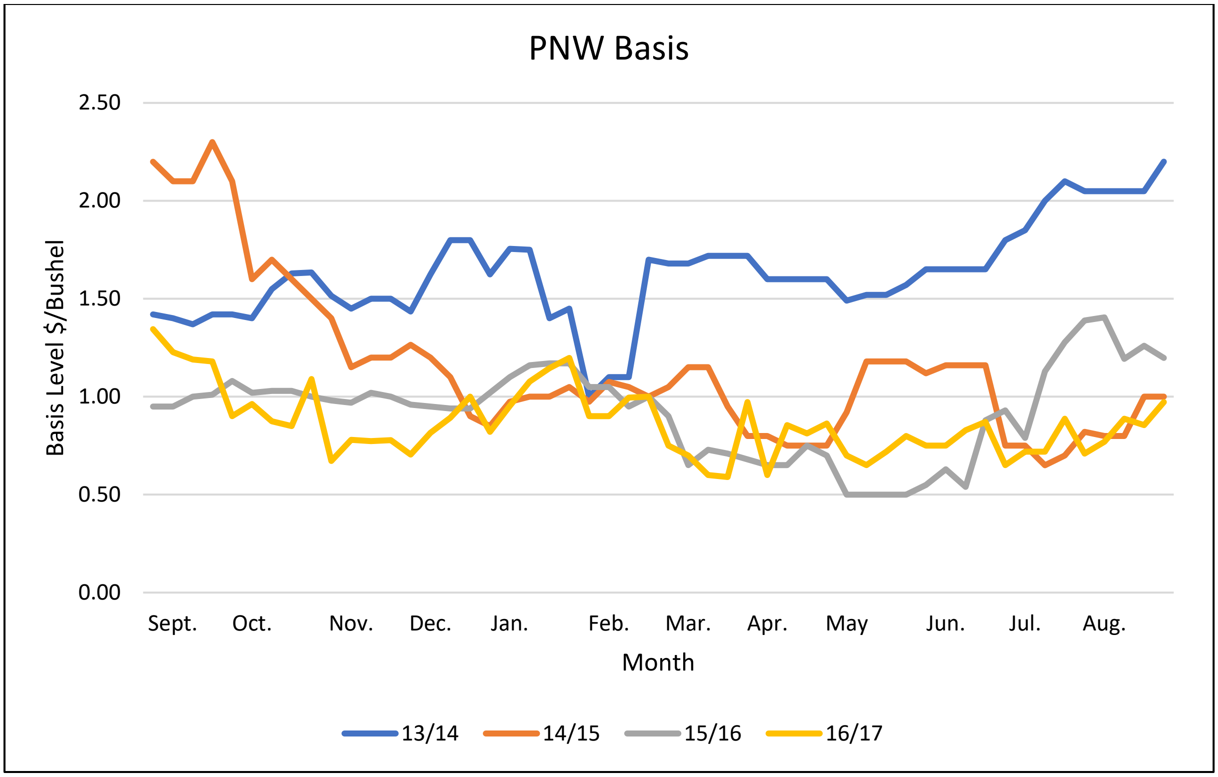

Figure 6.3. PNW Basis (TradeWest Brokerage Co. 2018; compiled by Klebe 2018)

6.4.2. Daily Car Value

Daily car value (DCV) is expressed in dollars per bushel by dividing rail car values by 3,500. Of the 208 data points, there were 18 missing observations. An instrument variable form Thompson Reuters Eikon is implemented to fit missing observations using symbol "BNSF-RCSHT-C1". The regression equation used to fit missing values is in Appendix E. Figure 6.4 shows behavior of DCV through time. There were excessively high DCV rates in the soybean marketing year of 2013/14 and at the beginning of 2014/15, as well as high rates during winter months of 2016/17. These excessively high rates come from fundamental factors in the market and cannot be predicted with certainty.

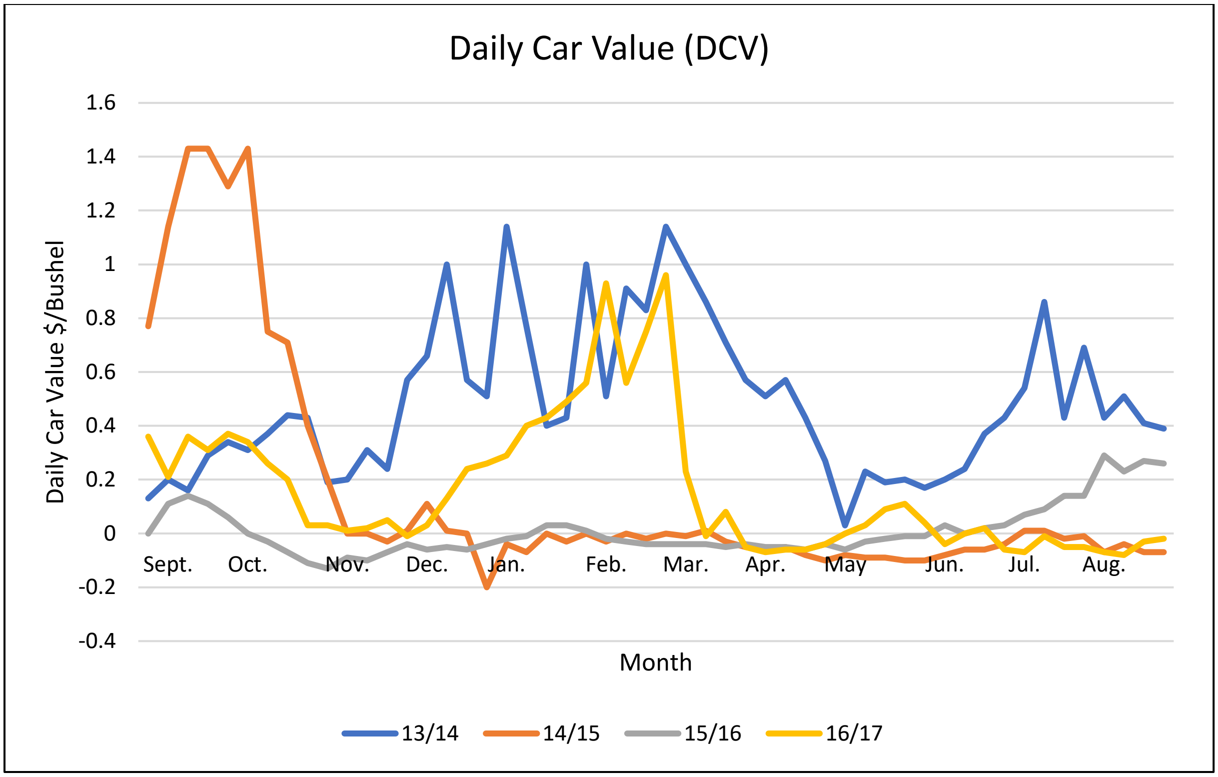

Figure 6.4. Daily Car Value (TradeWest Brokerage Co. 2018; compiled by Klebe 2018)

6.4.3. Futures Spread

Futures spread in the soybean market compares deferred soybean price to nearby futures price on a weekly basis. Data was extracted from Data Transmission Network (DTN) ProphetX. Futures spread between nearby and deferred months is recorded until the second Monday of delivery month. After the second Monday, spread is reported as the difference between the next two future month contracts. Spread represents the forward curve in the market. Futures markets holds a carry when deferred futures price is higher than nearby futures price. Figure 6.5 shows how soybean future spread behaves through time.

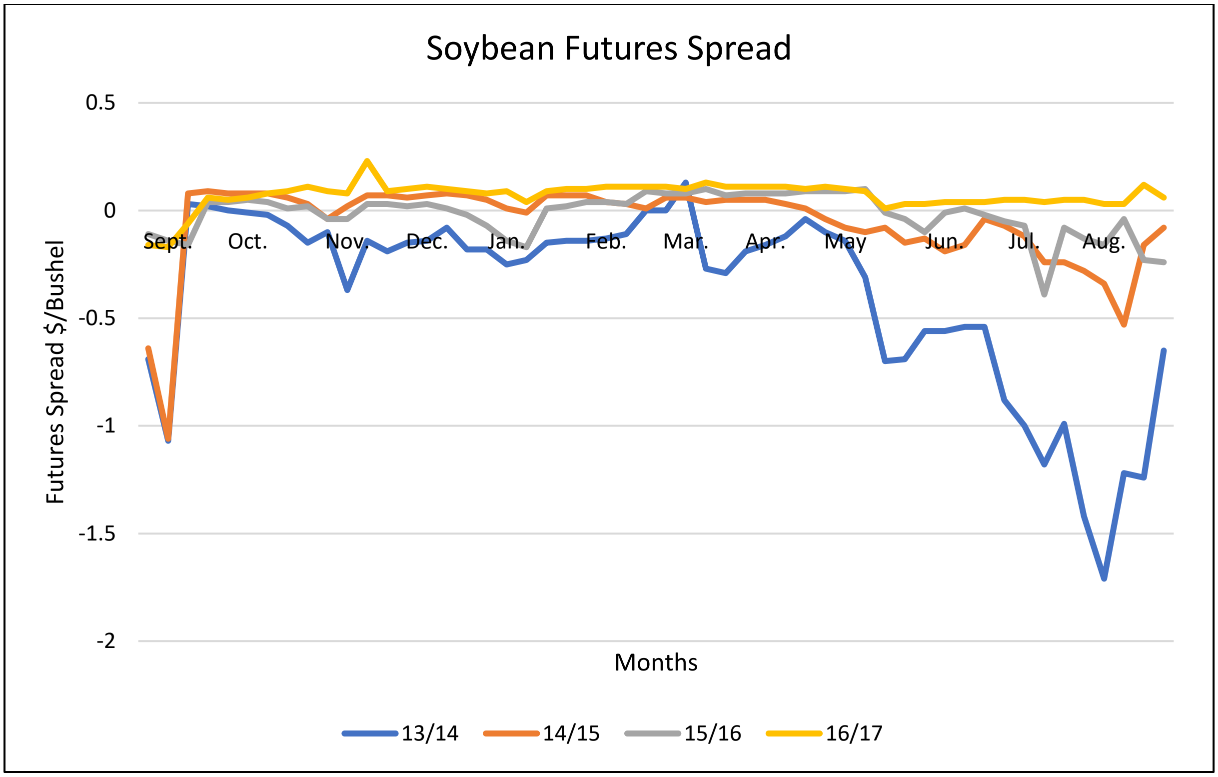

Figure 6.5. Soybean Futures Spread (CBOT 2018a; compiled by Klebe 2018)

A major inverse at the end of 2013/14 crop year and beginning of 2014/15 crop year resulted from several factors including a large Brazil soybean crop in those years; and concurrent, a substantially improved logistical performance. Improved logistics put downward pressure on port basis values in the PNW which competes directly with Brazil.

6.4.4. Tariff Rate

Tariff rate is how much BNSF railroad charges per car to ship grain from Fargo, ND to Tacoma, WA (GTR-AMS 2018). Tariff values per car is divided by 3,500 to report data in terms of dollars per bushel of soybeans. Tariff rate generally only changes once a year. Still, a change in tariff affects margin to ship grain and is accounted for in the model. Figure 6.6 illustrates how tariff rate has changed through time.

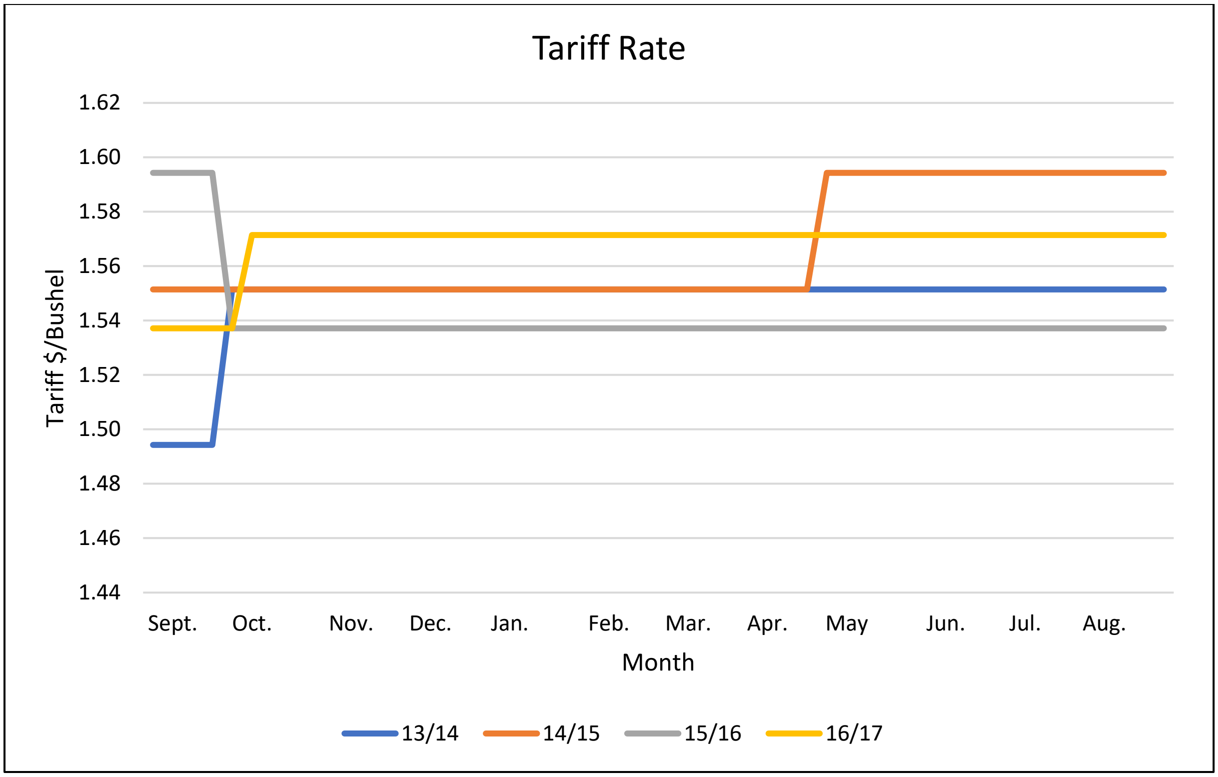

Figure 6.6. Tariff Rate (USDA-AMS 2018)

6.4.5. Velocity

Velocity at which shuttle trains arrive is taken from TradeWest Brokerage Co. The data set only has one missing value. The average of the week before and after is used to fill in the missing value. As shown in Figure 6.7, velocity of shuttle trains generally is in the range of 1.9 to 3.3. Typically, low levels of velocity are matched with high secondary rail market prices. This relationship exists because velocity also functions as a supply of transportation.

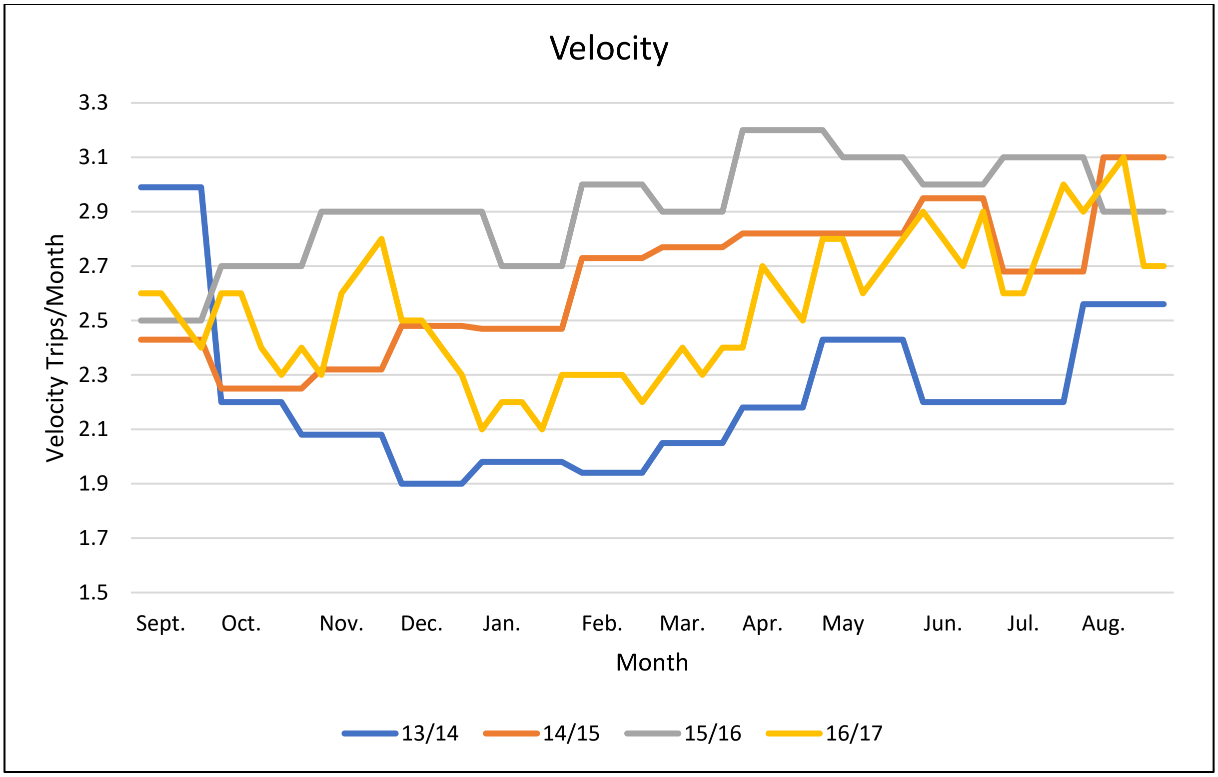

Figure 6.7. Velocity (TradeWest Brokerage Co. 2018; compiled by Dahl 2018)

6.4.6. Stochastic Distributions

Data from the 2015/16 soybean crop year is used for base case results. Fundamentally, 2015/16 did not have any abnormalities regarding transportation or supply and demand fluctuations. Stochastic variables for PNW basis, DCV, soybean futures spread, and tariff rate are inputs of pricing variables in the model. For price variables, the model uses ex post probability density functions based on past year's price behavior because it contains the most relevant information regarding current fundamental factors. Velocity is the car supply variable in the model; therefore, a time series forecasting method is used to generate estimates. @Risk™ uses Bestfit™ to fit distributions automatically. @Risk™ compares and chooses the best fit distribution based on Akaike Information Cristea (AIC). Appendix F shows the distributions used by Bestfit™ with descriptions. @Risk™ uses Spearman Rank-Order Correlations to fit a correlation matrix to the distributions.

Time series distributions require different fitting practices when developing a forecast. Bestfit™ compares variations of autoregressive, moving average, Brownian motion, auto regressive conditional heteroscedasticity (ARCH), and generalized auto regressive conditional heteroscedasticity (GARCH) models when fitting time series data. Appendix G shows a complete description of the times series distributions compared by Bestfit™. Bestfit™ detects seasonality, trend, and stationarity to make proper transformations before fitting data. After a proper time series models has been fit, @Risk™ formulates a forecast based on specifications of the user. Tables 6.5, 6.6, and 6.7 report distributions and correlations of stochastic variables.

Table 6.5. Base Case Distribution Fits (@Risk™)

Variable:	**Basis PNW**	**DCV**	**Futures Spread**	**Tariff**
Distribution:	Triangular	Log-Logistic	Gumbel	Inverse Gaussian
Function:	RiskTriang (0.36211,1,1 .4551)	RiskLogLogistic (-0.15605, 0.14238,3.3535)	RiskExtValueMin (0.025535, 0.070599)	RiskInvGauss (0.0043971, 1.65019e-006, RiskShift(1.5371413))
AIC Score	-4.02	-112.14	-102.62	-1116.80
Mean	0.94	0.01	-0.02	1.54
Standard Deviation	0.22	0.11	0.09	0.23

Table 6.6. Base Case Correlation Matrix (@Risk™)

	Basis PNW	**DCV**	**Futures Spread**	**Tariff**
Basis PNW	1.000			
DCV	0.426	1.000		
Futures Spread	-0.461	-0.538	1.000	
Tariff	0.022	0.304	-0.222	1.000

Table 6.7. Base Case Time Series Function (@Risk™)

Variable	Distribution	Function	AIC	Transformation
Velocity	Moving Average	RiskMA1(0.0078431,0.085296,-0.0057506,-0.0078885)	-105.567	First Difference

The time series distribution in Figure 6.8 provides a sample path of forecast velocity. BestFit™ ranks the MA1 process on the first difference transformation of velocity as the best forecasting function. MA1 is a moving average of forecast errors with one lag and four @Risk™ parameters. The first parameter, 0.0078431, is the mean first difference of the stochastic process. Second parameter, 0.085296, is the standard deviation of first difference errors. Third parameter, -0.0057506, is the β_1 coefficient which is multiplied by the lagged one period error term. The final parameter, -0.0078885 is the initial error of the MA1 process (Palisade, 2016). Negative values of the X-axis represent historical data. Observations greater than zero on the X-axis are forecast velocity values. The dark line represents mean of forecast velocity, the gray areas above and below the mean represent confidence intervals, and the red line is a sample path.

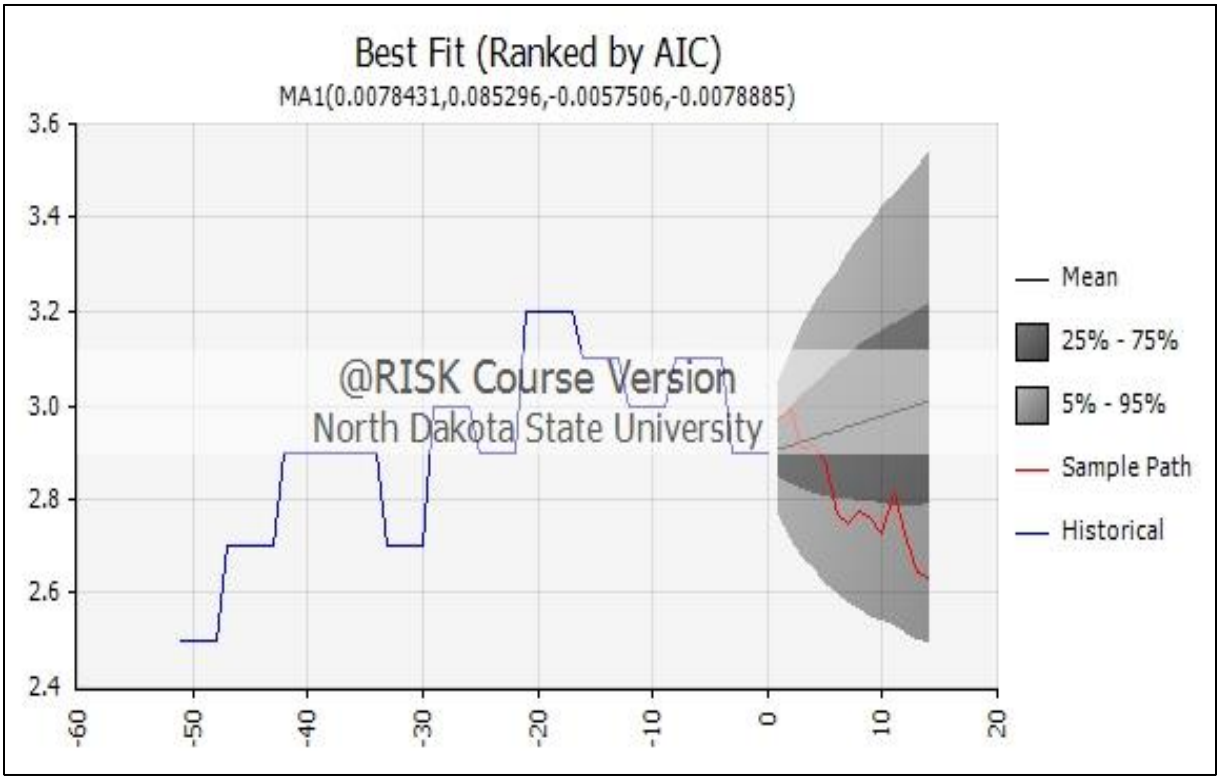

Figure 6.8. Velocity Time Series Distribution (@Risk™)

Distributions for PNW basis, DCV, futures spread, and tariff are in Figures 6.9 through 6.13. Histograms show clumping of historical data. The red line shows best fit distribution to the data.

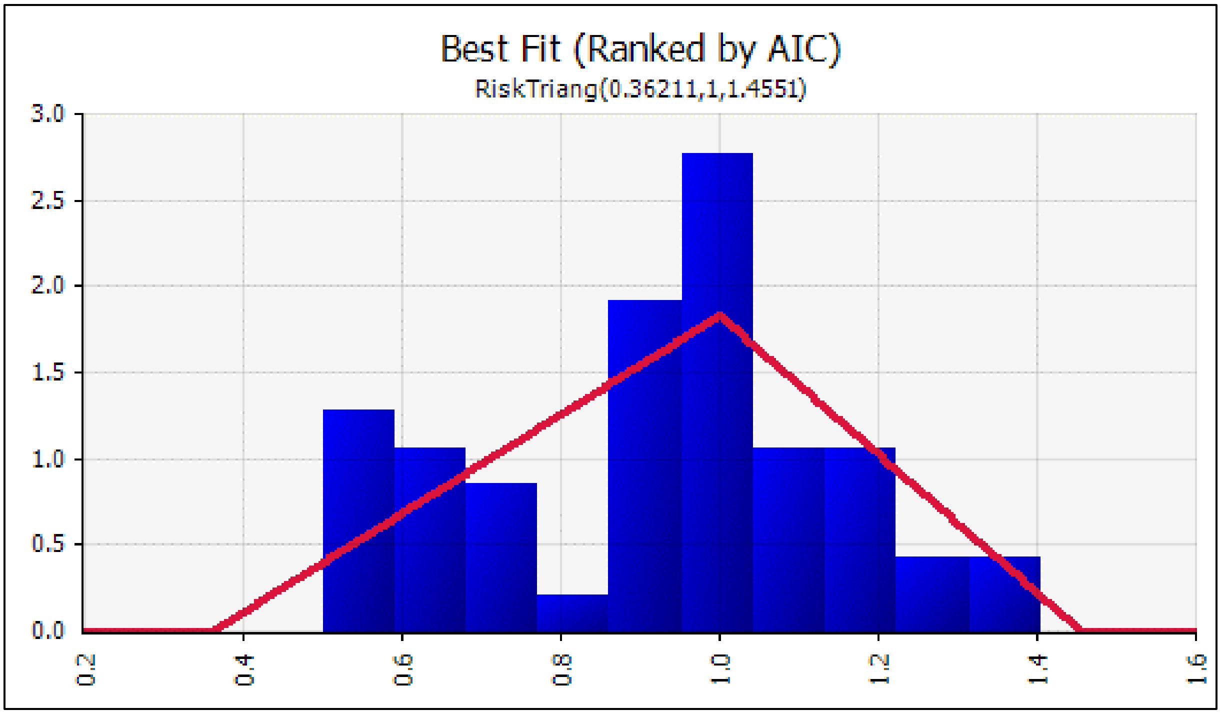

Figure 6.9. PNW Basis Distribution Fit (@Risk™)

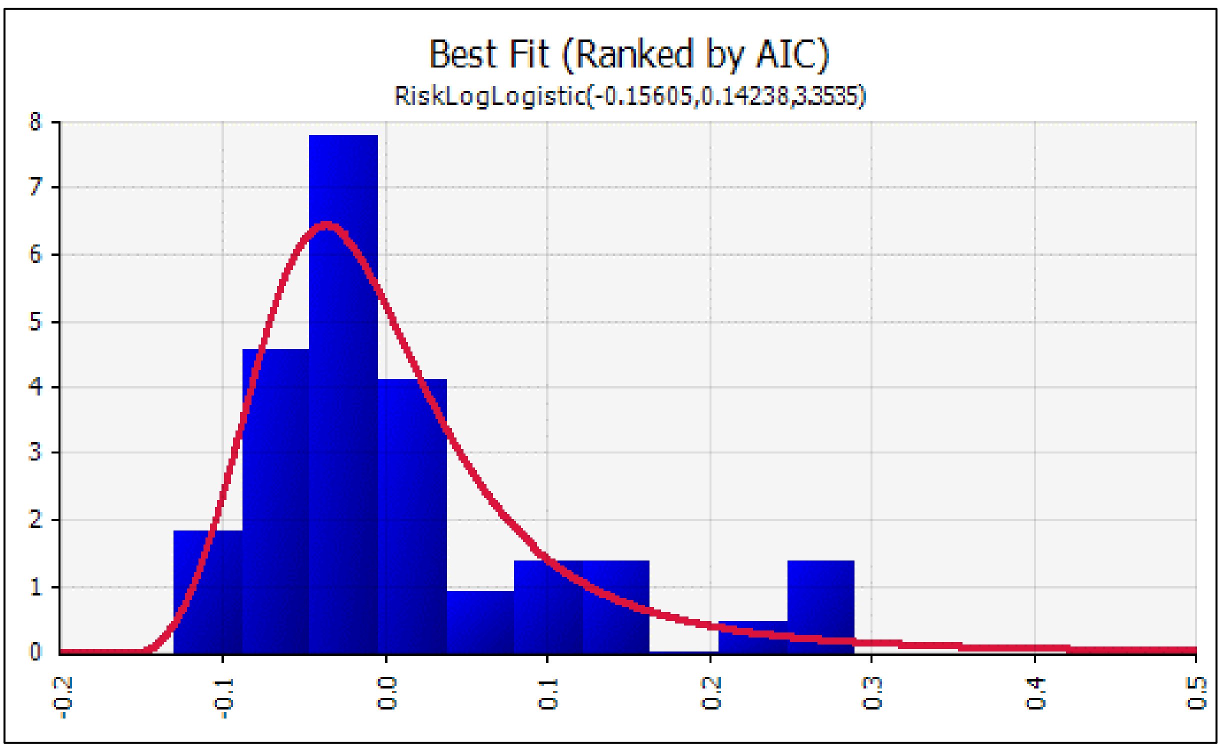

Figure 6.10. Daily Car Value Distribution Fit (@Risk™)

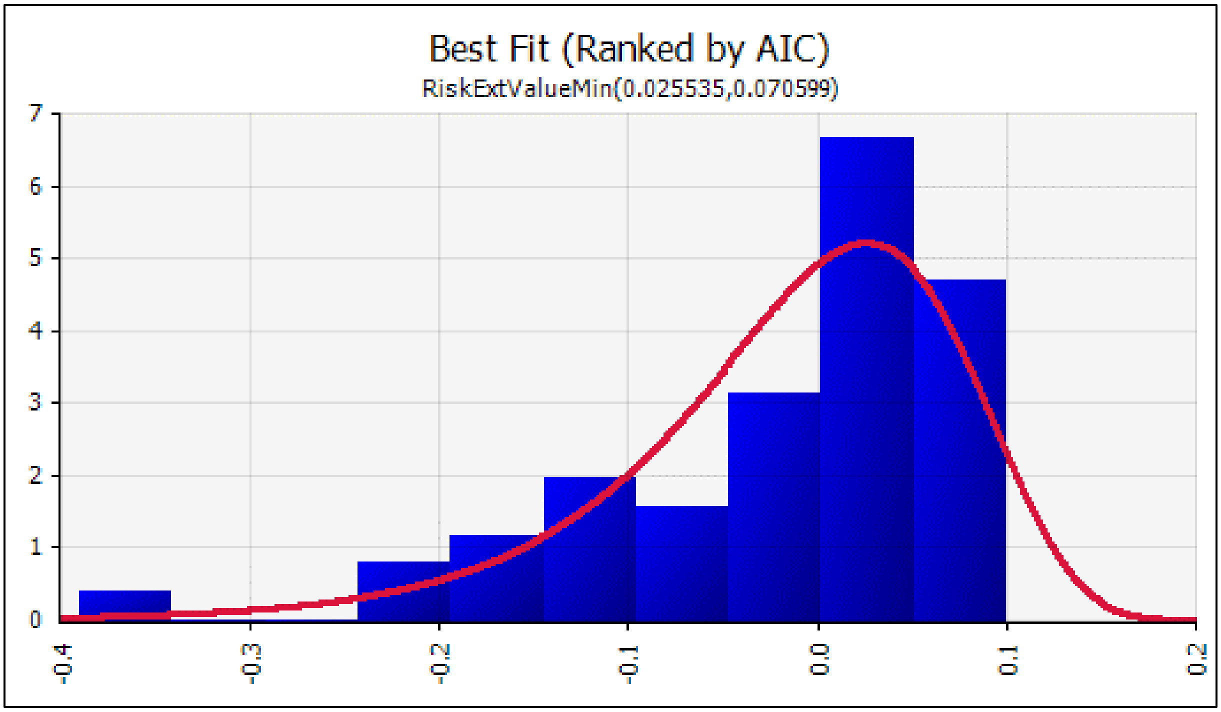

Figure 6.11. Soybean Futures Spread Distribution Fit (@Risk™)

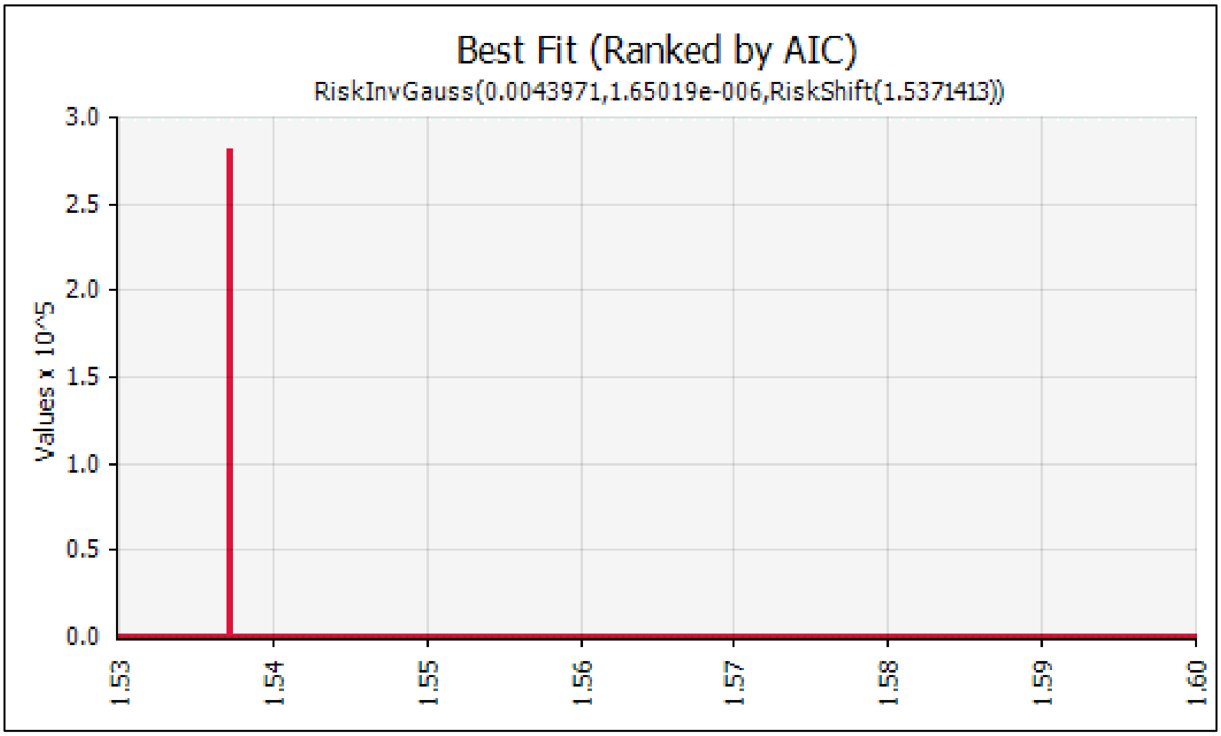

Figure 6.12. Tariff Distribution Fit (@Risk™)

6.4.7. Input Parameters

Input parameters are split into two groups: random and non-random inputs. Random input parameters are either linked, or have calculations linked, to distributions in @Risk™. Non-random inputs are static and do not change during sensitivity analysis. Non-random inputs are summarized in Table 6.8 and random input parameters are summarized in Table 6.9. In Table 6.8 the returns to storage equals soybean futures spread; this is a coincidence. In the base case, market carry equals $0.24 per bushel as wells as storage and interest equals $0.26 per bushel. Subtracting market carry from storage and interest equals -$0.02 per bushel which is the same as soybean futures spread.

Table 6.8. Random Model Inputs

Random Inputs	Input Mean	Units	Source
Forecast State Velocity:	2.96	Shuttle Trains Per Month	TradeWest Brokerage Co. 2018
Velocity Volatility:	21%	Annual Percentage Change	Calculation
Deferred PNW Basis:	$0.94	Dollars Per Bushel	TradeWest Brokerage Co. 2018
DCV:	$0.01	Dollars Per Bushel	TradeWest Brokerage Co. 2018; Calculation
Soybean Futures Spread:	-$0.02	Dollars Per Bushel	CBOT 208
Deferred Tariff Rate:	$1.54	Dollars Per Bushel	USDA-AMS
PNW Basis Spread:	$0.26	Dollars Per Bushel	Calculation
Tariff Spread:	$0.00	Dollars Per Bushel	Calculation
Market Carry:	$0.24	Dollars Per Bushel	Calculation
Returns to Storage:	-$0.02	Dollars Per Bushel	Calculation
Shortage Penalty:	-$0.01	Dollars Per Bushel	Calculation
Salvage Value:	$8.60	Dollars Per Bushel	Calculation

Table 6.9. Non-Random Model Inputs

Non-Random Inputs	Value	Units	Source
Current State Velocity:	2.9	Trains Per Months	TradeWest Brokerage Co. 2018
SB Bushels/Car:	3500	Soybean Bushels	BNSF 2016
Cars Per Shuttle:	110	Rail Cars	BNSF 2016
Bushels/Shuttle:	385000	Soybean Bushels	Calculation
Number of Contracts:	2	Primary Rail Contract	Assumption
Number of Months in Purchasing Strategy:	3	Months	Assumption
Min # Trains:	0	Shuttle Trains	Assumption
Max # Trains:	24	Shuttle Trains	Assumption
Increase Shipping Demand due to Car Supply per Velocity Increase:	2,310,000	Soybean Bushels	Calculation
Risk Free Interest Rate:	2.7%	Interest Rate	USDT 2018
Loan Interest Rate:	5.0%	Interest Rate	Assumption
Purchasing Strategy Maturity:	0.27	Years	Calculation
Nearby Futures:	$9.67	Dollars Per Bushel	CBOT 2018
Nearby PNW Basis:	$0.68	Dollars Per Bushel	TradeWest 2018
Nearby RR Tariff:	$1.54	Dollars Per Bushel	USDA-AMS 2018
Elevator Margin:	$0.20	Dollars Per Bushel	Assumption
Investment/Bushel:	$8.62	Dollars Per Bushel	Calculation
Net Price Per Bushel Sold:	$8.82	Dollars Per Bushel	Calculation
Weekly Storage Rate:	$0.01	Dollars Per Bushel	Assumption
Storage and Interest of Unsold Bushels:	$0.26	Dollars Per Bushel	Calculation

6.5. Base Case Results

Monte Carlo simulation is implemented using @Risk™ to run 1,000 iterations of the model based on structural and stochastic variables. Specific @Risk™ settings are shown in Table 6.10.

Table 6.10. @Risk™ Settings

@Risk™ Specification	@Risk™ Setting
Sampling Type	Latin Hypercube
Generator	Mersenne Twister
Initial Seed Value	150,000
Macros	VBA

Results of the base case, and subsequent sensitives, reflect mean values of stochastic simulation for a specific purchasing strategy. RiskOptimizer™ maximizes mean NPV by changing the quantity of bushels to purchase. Purchasing strategy is changed in discrete step sizes of 10,000 bushels. Step sizes equivalent to a full shuttle train were not used to allow for flexibility in farmer deliveries on forward contracts. Using a smaller step size also increases sensitivity to random variables and gives a result that possesses a higher level of accuracy. Constraints are set on RiskOptimizer™ to reflect a minimum purchasing strategy of zero bushels and a maximum strategy of 9,240,000. 9,240,000 bushels would be enough to meet a car supply of 24 trains over the course of three months. 24 trains are chosen because it is assumed the maximum a shipper can load is four trips per months per primary contract. The base case results are formulated using distributions from data collected in the soybean crop marketing year of 2015/16.

Base case results are in Table 6.11.

Table 6.11. Base Case Results

Observation	Value
Purchasing Strategy	7,150,000
Trains Prepared for Based on Purchasing Strategy	19
Percent of Forecast	105%
NPV	$874,873
Standard Deviation	$139,087
Short Call Strike Velocity	3.10
Number Short Call	463,854
Short Call Premium	0.091
Number Long Calls	487,218
Long Call Premium	2.96

The optimal purchasing strategy is 7,150,000 bushels of soybeans for a shipper possessing two primary contracts over the course of three months (assuming initial inventory is zero). This value is 105% of forecast car supply. Forecast car supply is 2.96 trains per month per contract. A purchasing strategy of 7,150,000 bushels is enough to meet a rail velocity of 3.1, which exceeds the forecast car supply.

Figure 6.13 shows a payoff function for the optimal purchasing strategy which reflects a call spread on the option to ship. The shuttle elevator is long 487,218 call options at a strike velocity of zero. Being long 487,218 contracts at a strike velocity of zero means the elevator possesses the right to ship grain whenever velocity is greater than zero. Elevator's profit would increase $487,218 per one unit increase in velocity. In Figure 6.13 the shipper has a minimum profit level of -$73,500 when velocity is zero. This value reflects a moderate salvage value for unshipped bushels.

The shipper is short 463,854 call options at a strike velocity of 3.1. This means if velocity is at or above 3.1, the shipper would lose $463,854 in profit per one unit increase in velocity. However, the shipper still possesses 487,218 long call options at a strike velocity of zero. Therefore, the shipper's net profit per unit increase in velocity is $23,364 when velocity is above 3.1. Figure 6.13 reflects the decrease in marginal profit by flattening the payoff function after a velocity of 3.1.

The payoff function in Figure 6.13 shows how a shipper's profit would change (Y-axis) relative to changes in Velocity (X-axis). The X-axis shows a possibility for velocity to be below zero, however this is impossible. Velocity values below zero are depicted graphically to show relevance in possessing long calls with a strike velocity of zero.

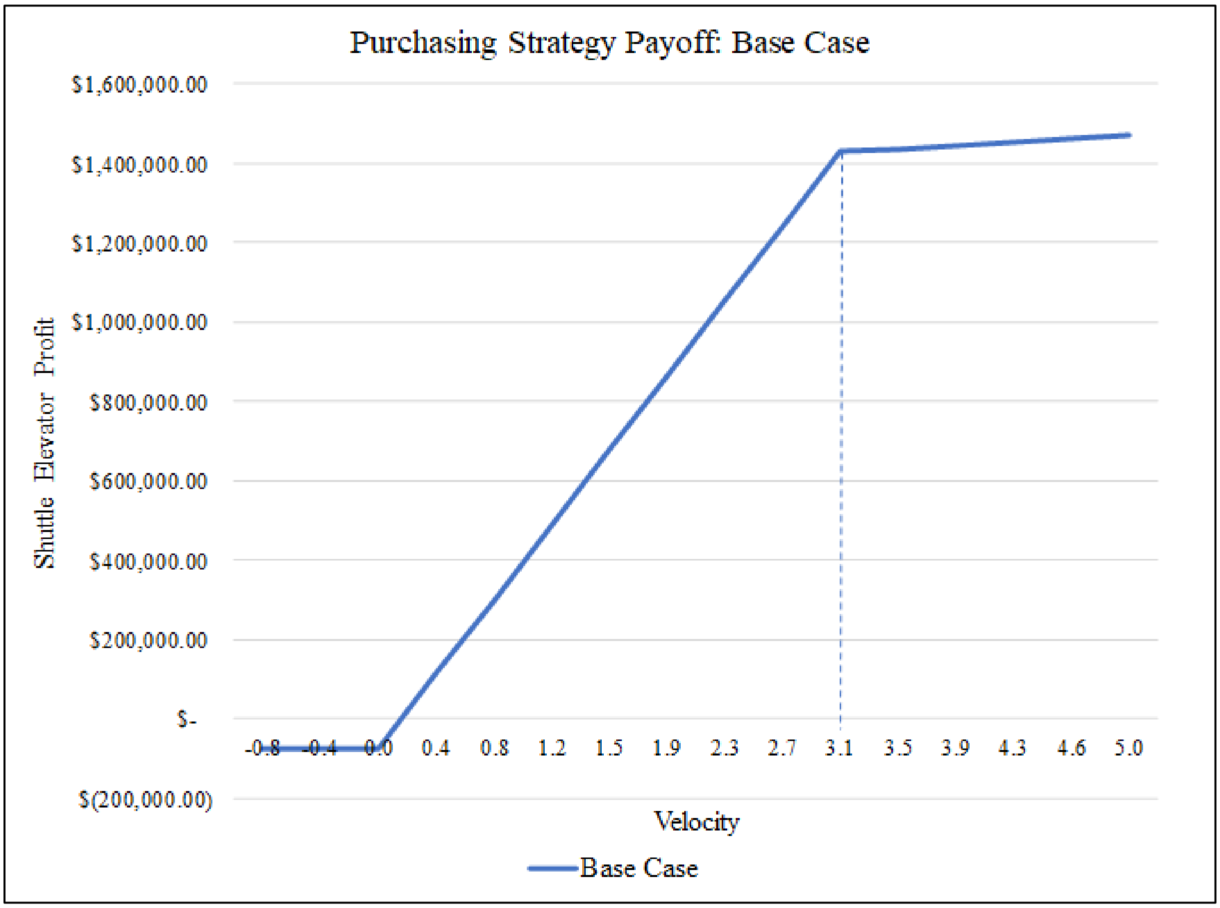

Figure 6.13. Base Case Payoff Function.

Figure 6.14 shows the probability distribution of NPV for 1,000 iterations. Base case results are highly clustered between $800,000 and $1,000,000. Standard deviation of NPV is

$139,086 and is slightly skewed to the left. The 90% confidence interval is between $601,000 and $1,036,000 which further demonstrates the NPV distribution skewness.

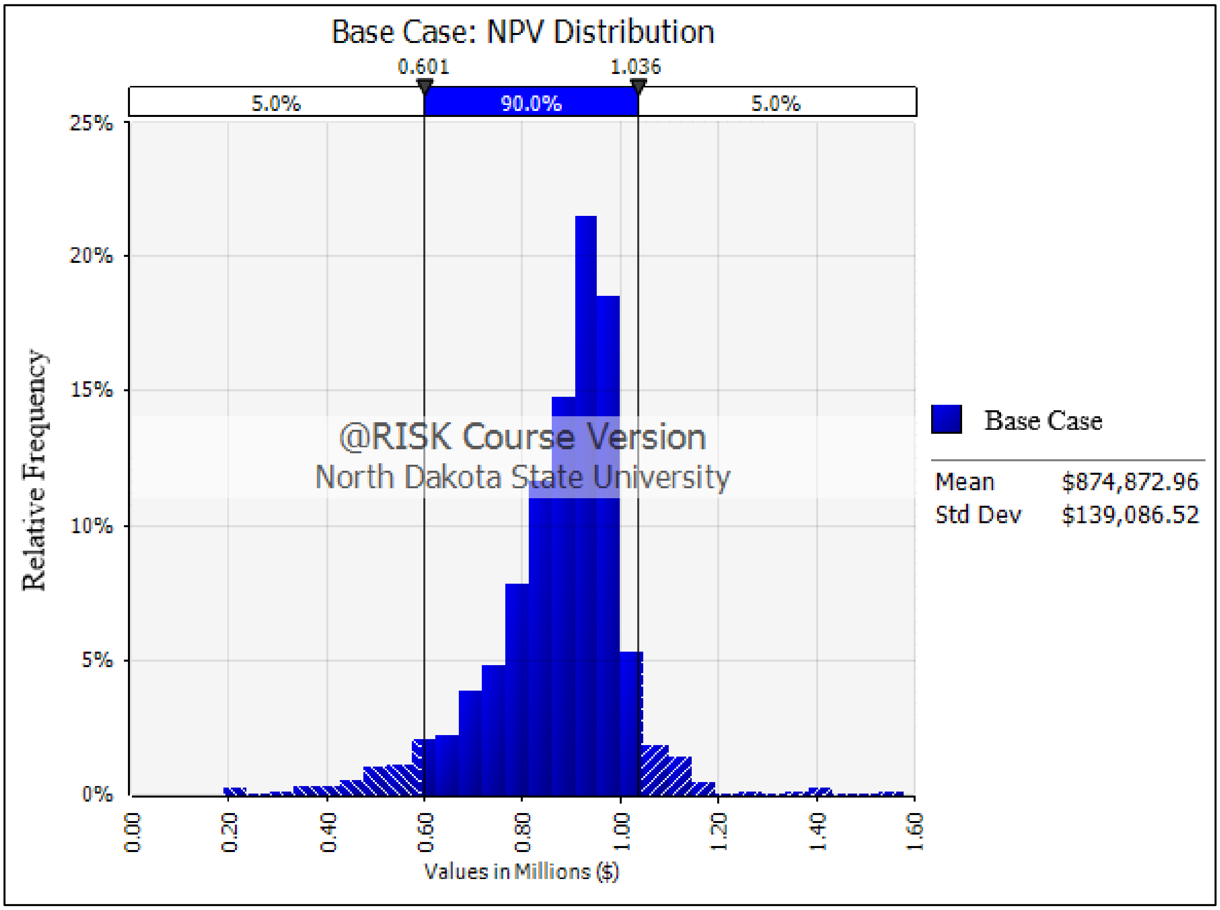

Figure 6.14. Base Case: NPV Distribution (@Risk™)

The tornado graph in Figure 6.15 ranks input variables by their effect on the mean NPV. PNW basis has the greatest effect followed by DCV and velocity volatility. These are inputs which sensitivity analysis are conducted on later in the chapter.

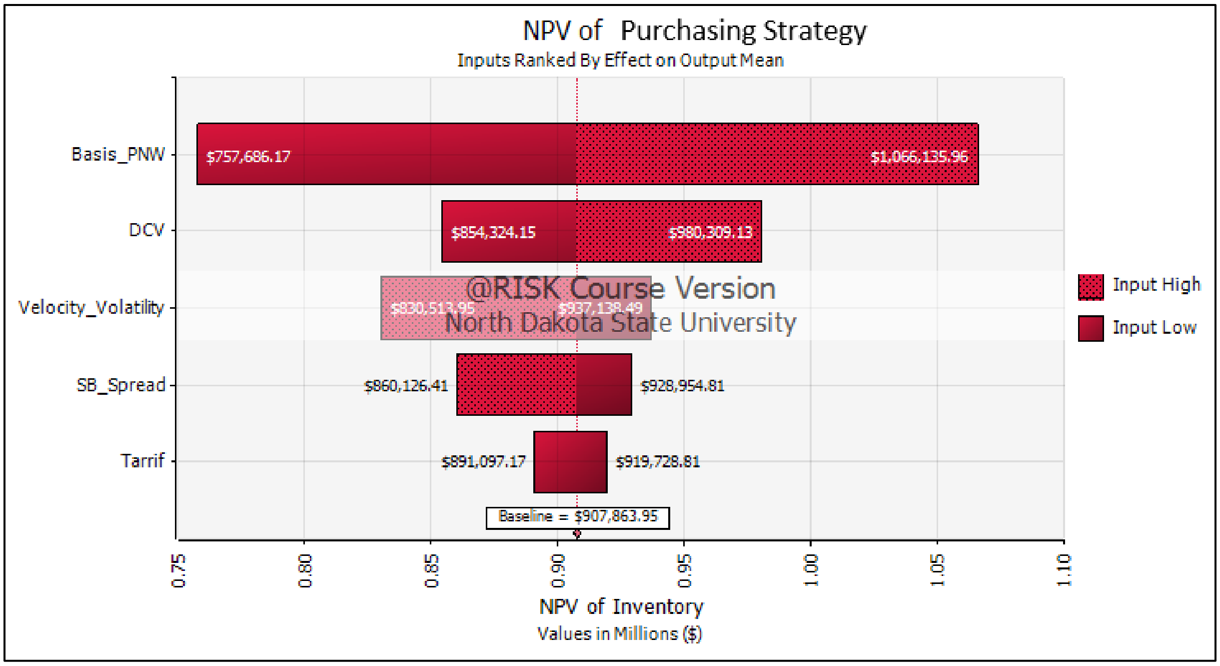

Figure 6.15. Tornado Graph of NPV Input Effects (@Risk™)

Figure 6.16 is an E-V frontier of different purchasing strategies. Strategies are reported as a percentage of the forecast car supply. The X-axis represents the standard deviation of purchasing strategies in 100,000's. The Y-axis represents expected NPV of the purchasing strategy. The base case purchasing strategy of 105% has a maximized mean NPV of $874,873 and a standard deviation of $139,087. However, a purchasing strategy of 93% has the lowest risk with a standard deviation of $87,251 but a mean NPV of $851,234. A 93% purchasing strategy has an expected profit $23,639 less than the optimal strategy of 105%; however, risk in expected profit is reduced by more than $50,000.

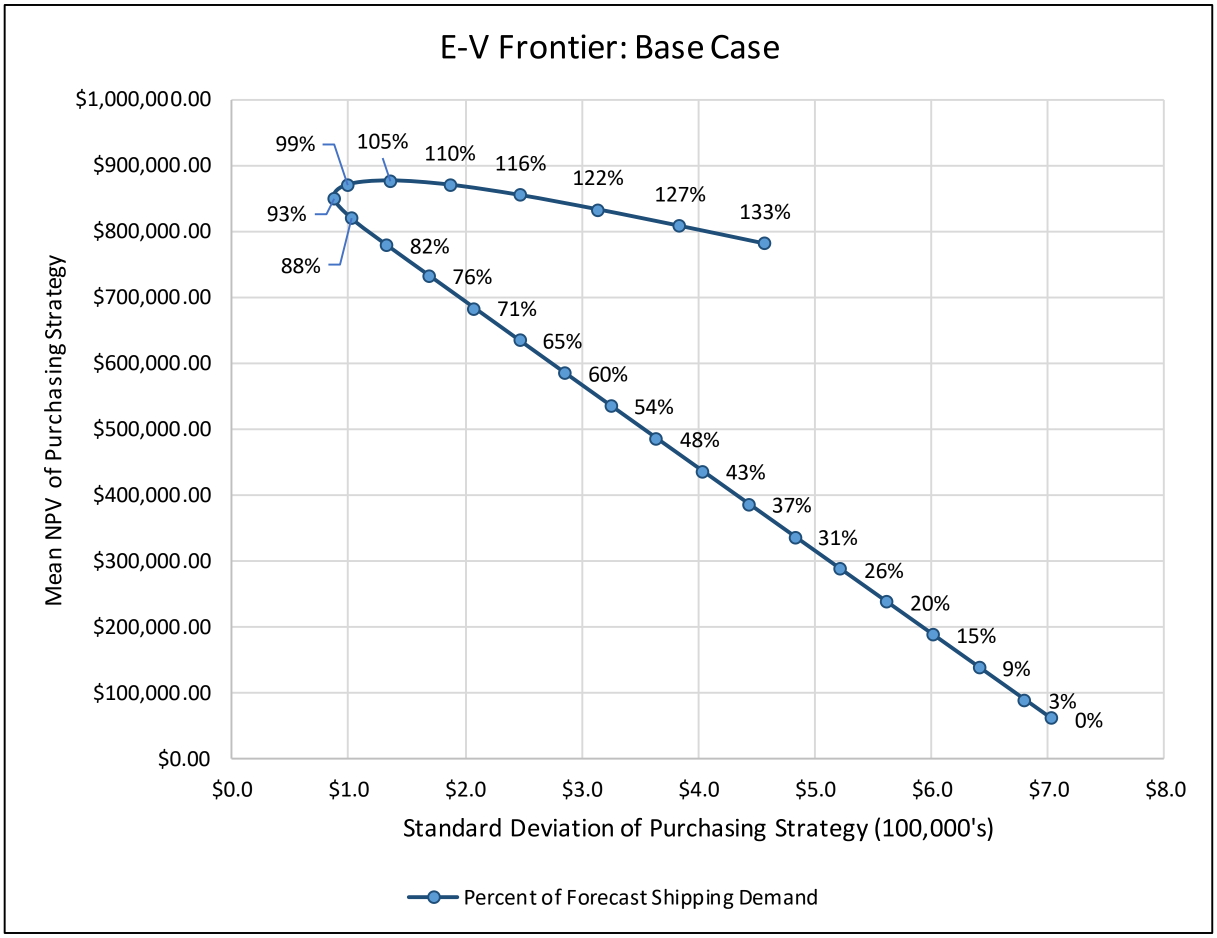

Figure 6.16. E-V Frontier: Base Case

6.6. Sensitivities

Variables which have the greatest effect on mean NPV of the purchasing strategy are PNW basis, DCV, and velocity volatility. Sensitivity analysis is conducted by shifting the input means of stochastic variables. PNW basis is an input to market carry which directly influences salvage value. Market carry is equally affected through changing any one distribution for PNW basis, futures spread, or change in tariff. Therefore, sensitivity analysis on market carry is equally explained through any one of these three variables.

Sensitivity analysis is also conducted on DCV, which has a direct effect on stockout penalty; as well as velocity volatility which influences option premium. A final sensitivity, which alters transferability of the primary instrument, changes the logic of salvage value calculation as well as stockout penalty. Table 6.12 summarizes the sensitivity analyses conducted on optimal purchasing strategy.

Table 6.12. Rail Sensitivity Analysis Summary

Sensitivity	Variable Mean	Sensitivity Analysis
Market Carry	$0.25	+/- $0.10
Daily Car Value	$0.01	+/- $0.15
Velocity Volatility	21%	0% and 50%
Transfer Option	Non-Transferable	Fully-Transferable

6.6.1. Market Carry

Market carry affects the salvage value for grain which is carried into the next purchasing period.   Several variables affect market carry in this application which is comprised of PNW basis, soybean futures spread, and change in railroad tariff. The combined effect influences market carry and therefore the incentive to store. Market carry is compared to the cost of storage and interest to generate the returns to storage.

In the base case, market carry equals $0.24 per bushel and cost of storage and interest equals $0.26 per bushel. Return to storage is therefore -$0.02 per bushel. In Figure 6.13, salvage value causes the profit function to be near zero if velocity were to be 0. This is because market carry, $0.24 per bushel, is very close to cost of storage and interest for unshipped bushels. Table 6.13 shows how shifting the distribution of carry affects the optimal purchasing strategy.

Table 6.13. Sensitivity to Change in Carry

Observation	Decrease Carry $0.10	Base Carry	Increase Carry $0.10
Gross Market Carry	$0.14	$0.24	$0.34
Storage and Interest	$0.26	$0.26	$0.26
Returns to Storage	-$0.12	-$0.02	$0.08
Purchasing Strategy	6,670,000	7,150,000	9,240,000
Trains Prepared for Based on Purchasing Strategy	17	19	24
Percent of Forecast	98%	105%	135%
NPV	$844,808	$874,873	$994,757
Standard Deviation	$111,397	$139,087	$494,313
Short Strike Velocity	2.89	3.10	4.00
Number Short Call	694,854	463,854	232,854
Short Call Premium	0.184	0.091	0.001
Number Long Calls	718,218	487,218	256,218

Increasing market carry by $0.10 per bushel causes optimal purchasing strategy to be 135% of the forecast velocity. The assumption that the shipper does not forward contract soybeans to be delivered to PNW allows the option to ship or store to be retained by the shipper. When optimal purchasing strategy is 135% of forecast car supply, the shipper has the option to either ship excess soybeans as trains arrive or store soybeans until the next shipping period. A shipper maintains this flexibility to ship or store bushels but assumes added risk of being at the mercy of PNW basis. The drastic increase in purchasing strategy is due to salvage value being raised to levels where a shipper would benefit from both shipping or storing the grain. Simply, when market carry is large and positive, a shipper would over-purchase grain relative to expected car supply. If they receive more cars than expected, they simply ship. If they do not, they store the extra grain and accrue earnings to storage. Hence, the incentive to buy more grain than forecast car supply.

Similarly, if market carry decreases, there is less incentive to store and a shipper would purchase less soybeans. A lower purchasing strategy creates a greater probability of not meeting car supply and thus not storing bushels for a loss in value. Figure 6.17 shows how shifts in market carry affect minimum level of profit.

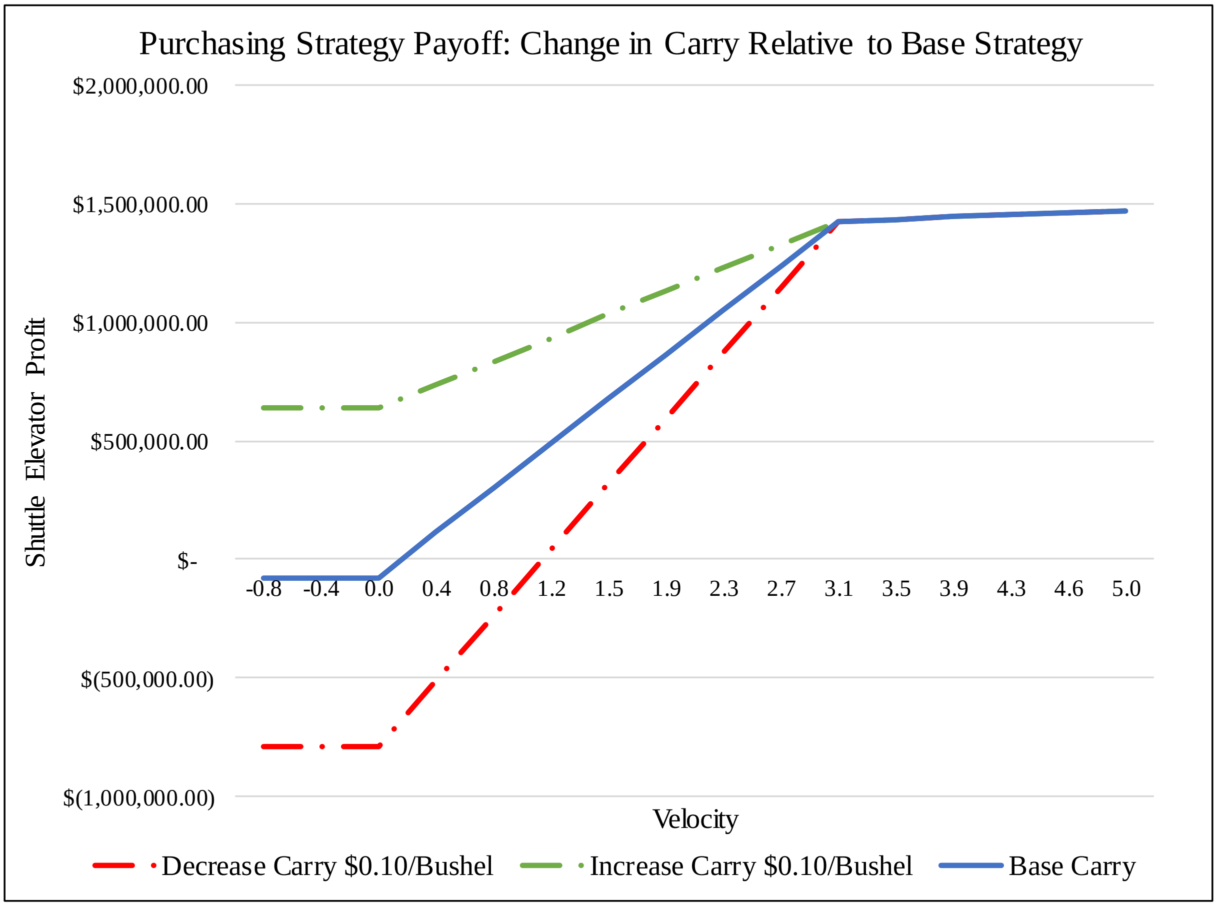

Figure 6.17. Purchasing Strategy Payoff: Change in Carry Relative to Base Strategy

The green dashed line in Figure 6.17 shows that a shuttle elevator would still gain over $500,000 in profit if no bushels were shipped. Decreasing market carry lowers the vertical discount, which exposes the shuttle elevator to more risk if they overestimate car supply and are forced to carry bushels into the next shipping period. A decrease in salvage value thus discourages excess grain to be stored.

Figure 6.18 shows how NPV distribution of the base case purchasing strategy changes with differences in market carry. An increase in market carry raises mean NPV and lowers the risk of expected profit. Contrary, a decrease in market carry decreases mean NPV while increasing standard deviation.

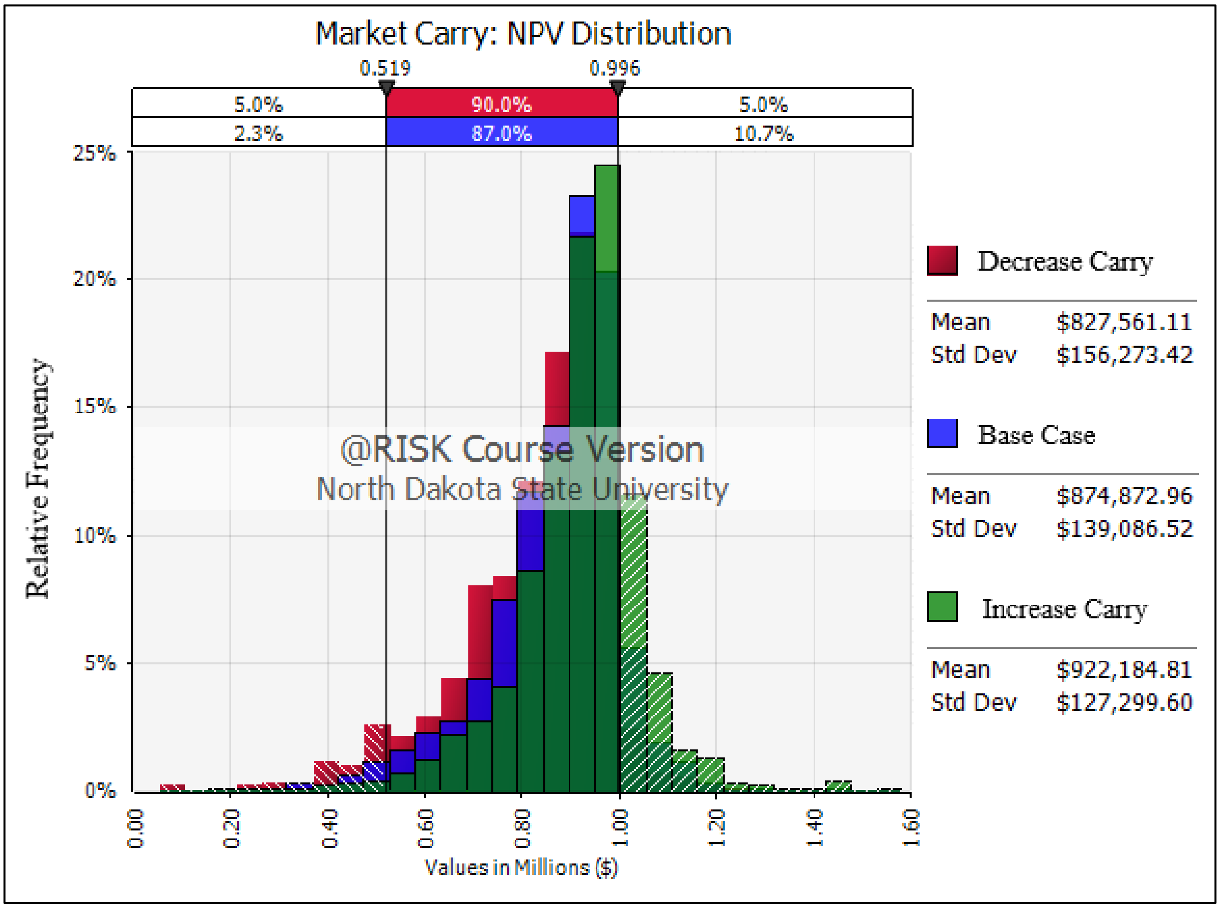

Figure 6.18. Carry Change NPV Distribution (@Risk™)

Figure 6.19 displays the E-V frontier of different purchasing strategies under each market carry sensitivity. When market carry decreases, an optimal purchasing strategy of 98% is relatively close to the least risky strategy of 92% with very little difference in expected profit or risk. When market carry increases, optimal purchasing strategy gains $400,000 in risk to gain $133,000 in expected profit. Added benefit to risk parameter in this situation is very low and should be considered before an optimal purchasing strategy is made.

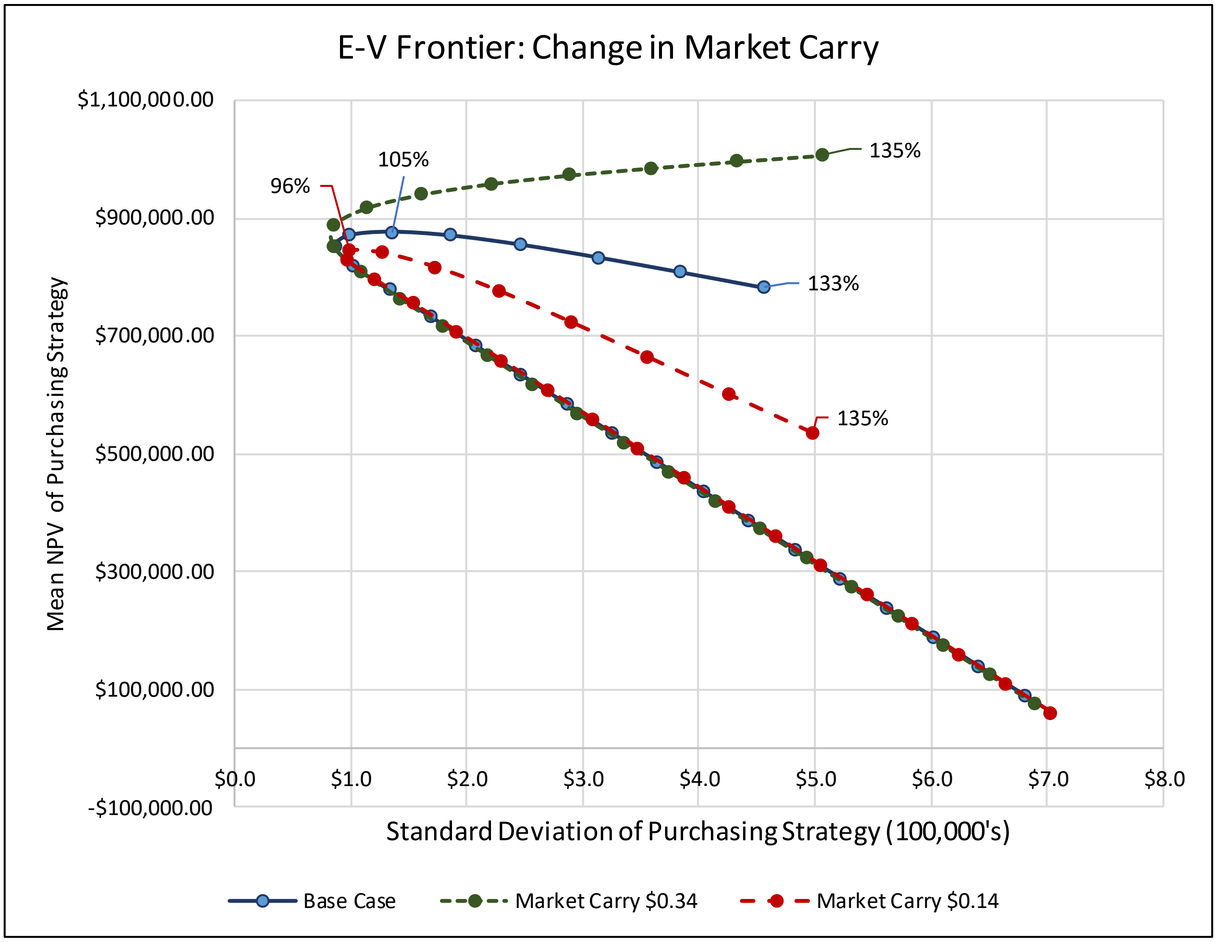

Figure 6.19. E-V Frontier: Market Carry

6.6.2. Sensitivity: Daily Car Value (DCV)

Daily car value (DCV) reflects the market value of excess shuttle trains on the secondary rail market. DCV represents stockout penalty in this application. If a shipper underestimates rail velocity, car supply would be greater than inventory. When this happens, a shipper would sell excess rail cars into the secondary market at either a premium or discount. The base case example has DCV at $31/car or about $0.01/bushel. A premium or discount affects the number of short calls. When there is no stockout penalty, the number of short calls is equal to the number of long calls. When the two values are equal, slope in profit is flat. The base case of

$0.01/bushel acts as a negative stockout penalty. A negative stockout penalty reduces the number of short calls and causes net profit of unmet car supply to be slightly increasing.

Sensitivity on DCV shifts the mean value to a negative $0.14 per bushel and a positive $0.16 per bushel as shown in Table 6.14.

Table 6.14. Change in DCV

Observation	Decrease DCV $0.15/Bu	Base DCV	Increase DCV $0.15/Bu
DCV $/Bu	-$0.14	$0.01	$0.16
DCV $/Car	-$494	$31	$556
Purchasing Strategy	7,540,000	7,150,000	0
Trains Prepared for Based on Purchasing Strategy	20	19	0
Percent of Forecast	110%	105%	0%
NPV	$848,940	$874,873	$1,094,446
Standard Deviation	$188,820	$139,087	$813,682
Short Strike Velocity	$3	$3	$0
Number Short Call	810,354	463,854	117,354
Short Call Premium	0.046	0.091	2.958
Number Long Calls	487,218	487,218	487,218

A decrease in DCV from $31/car to -$494/car results in a large stockout penalty. A large stockout penalty increases the number of short calls which decreases overall profit level when car supply is not met as shown by the red dashed line in Figure 6.20. When DCV decreases, optimal purchasing strategy increases to 110%; however, expected profit decreases by $25,900. This occurs due to a high stockout penalty, i.e., selling cars at a discount, which increases number of short calls by 346,500 while number of long calls stays the same. An increase in short call options causes net profit gained after a strike velocity of 3.1 to decreases to a negative $323,136. The red dashed line in Figure 6.20 shows this change in slope. Simply put, when

DCV becomes negative; the shipper will purchase more bushels to avoid incurring a stockout penalty.

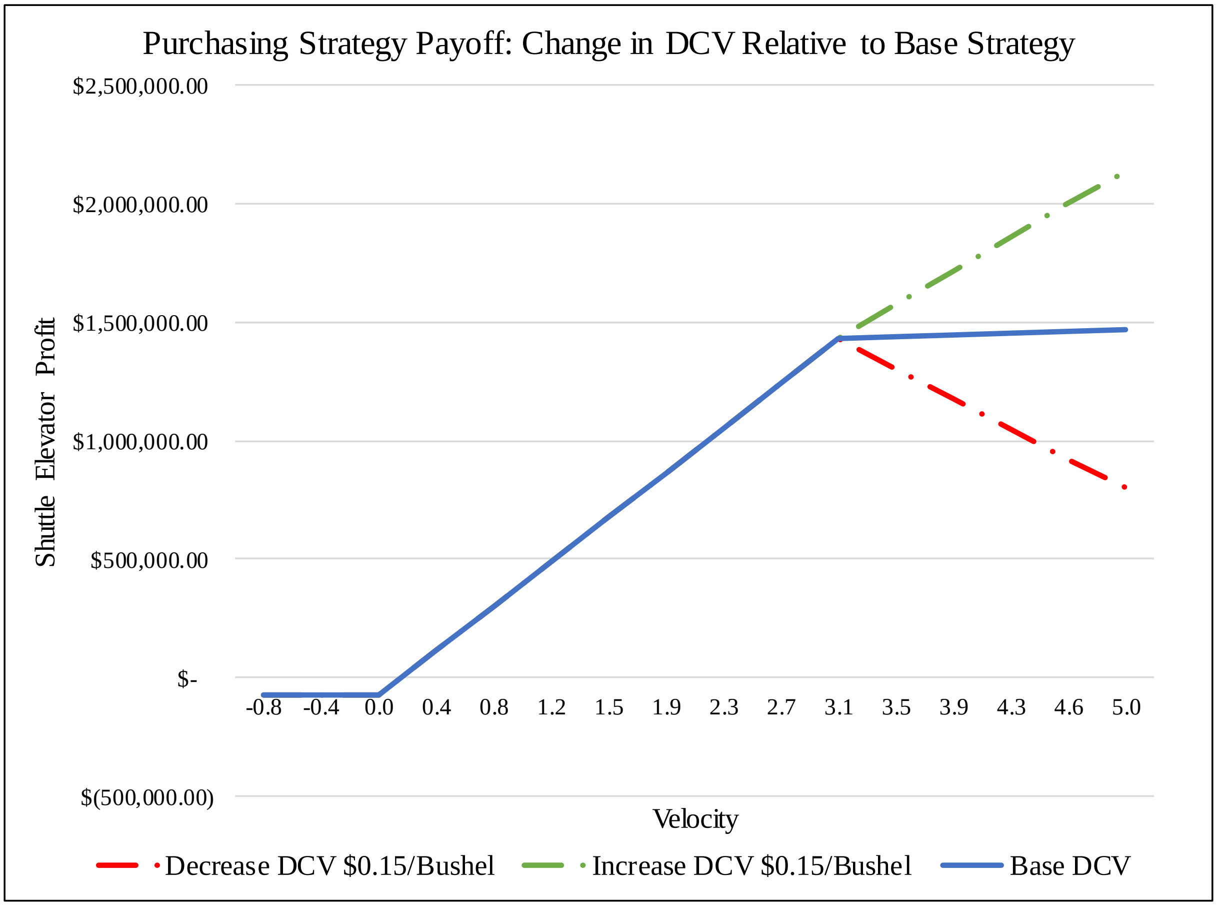

Figure 6.20. Purchasing Strategy Payoff: Change in DCV Relative to Base Case

Alternatively, an increase in DCV reduces the number of short calls and results in a profit increase when car supply is not met. When supply is not met the shipper makes money by selling their primary instrument into the secondary market, despite not having inventory to sell. The shipper thus sells excess rail cars for a profit. In Table 6.14, optimal purchasing strategy is reduced to zero when DCV increases by $0.15/bushel. The shipper intends to sell all shuttle trains which arrive into the secondary market for spot DCV and make more money on selling freight than shipping grain.

Figure 6.21 shows how the NPV distribution of the base case purchasing strategy changes with differences in DCV. An increase in DCV increases mean NPV and increases standard deviation. The increase in standard deviation is caused from instances of stockout resulting in higher profit and thus widening the distribution to the right. A decrease in DCV lowers mean NPV and lowers standard deviation. When DCV is lower, instances of stockout have a great negative effect on profit and narrow NPV distribution to the left.

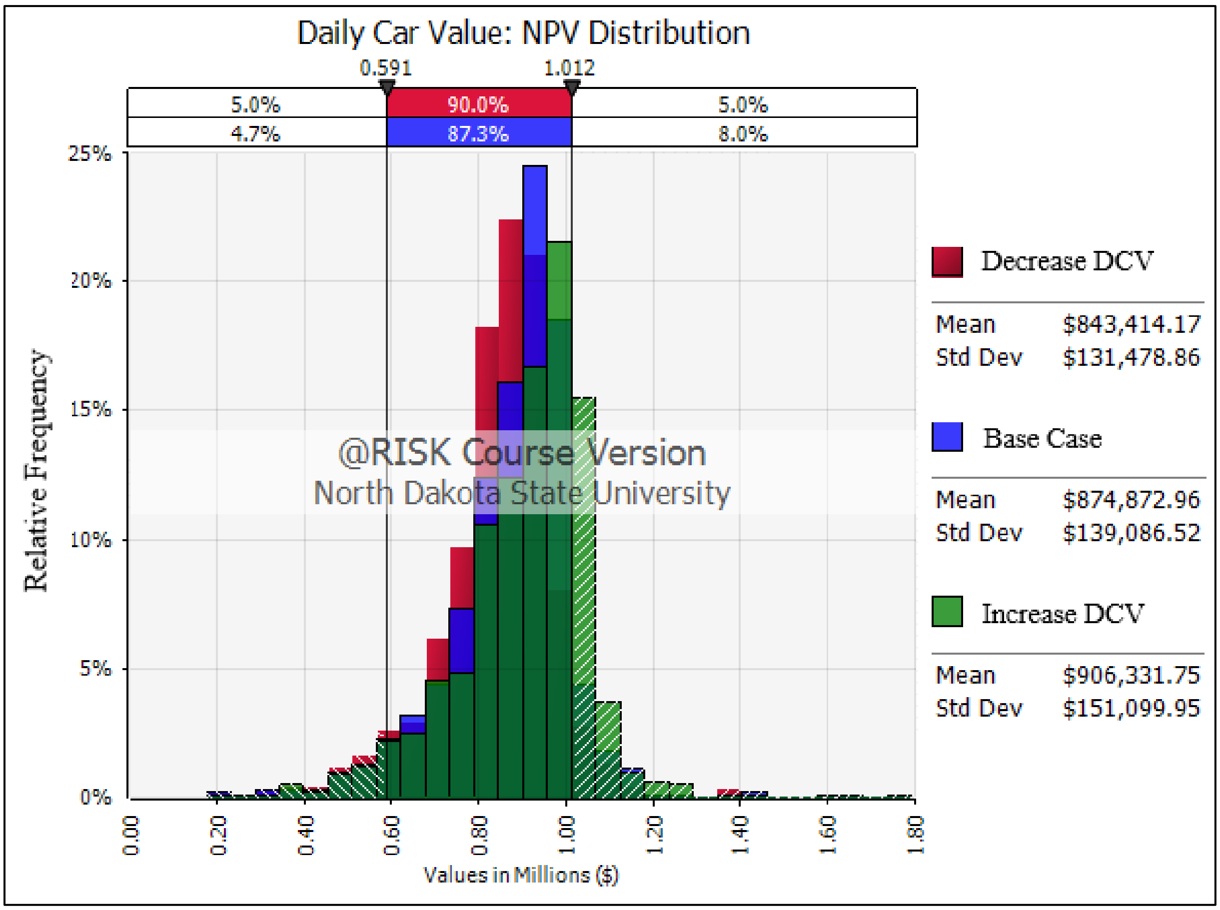

Figure 6.21. DCV Change: NPV Distribution (@Risk™)

The E-V frontier in Figure 6.22 shows different purchasing strategies under each DCV sensitivity. A decrease in DCV has an optimal purchasing strategy which is maximized near the upper left of the frontier. If DCV decreases and the shipper were to decreases their purchasing strategy to 0%, they would have a mean NPV of negative $964,513. A negative expected profit occurs because all shuttle trains are sold into secondary market and the shipper must pay other entities to take all shuttle trains.

When DCV increases, mean NPV is maximized to the upper right of the frontier with a purchasing strategy of 0% and a high level of risk. Not only is added benefit to risk ratio .2, but the frontier is flipped relative to the base case. A flipped frontier means a decrease in purchasing strategy along the frontier causes an increase in expected profit.

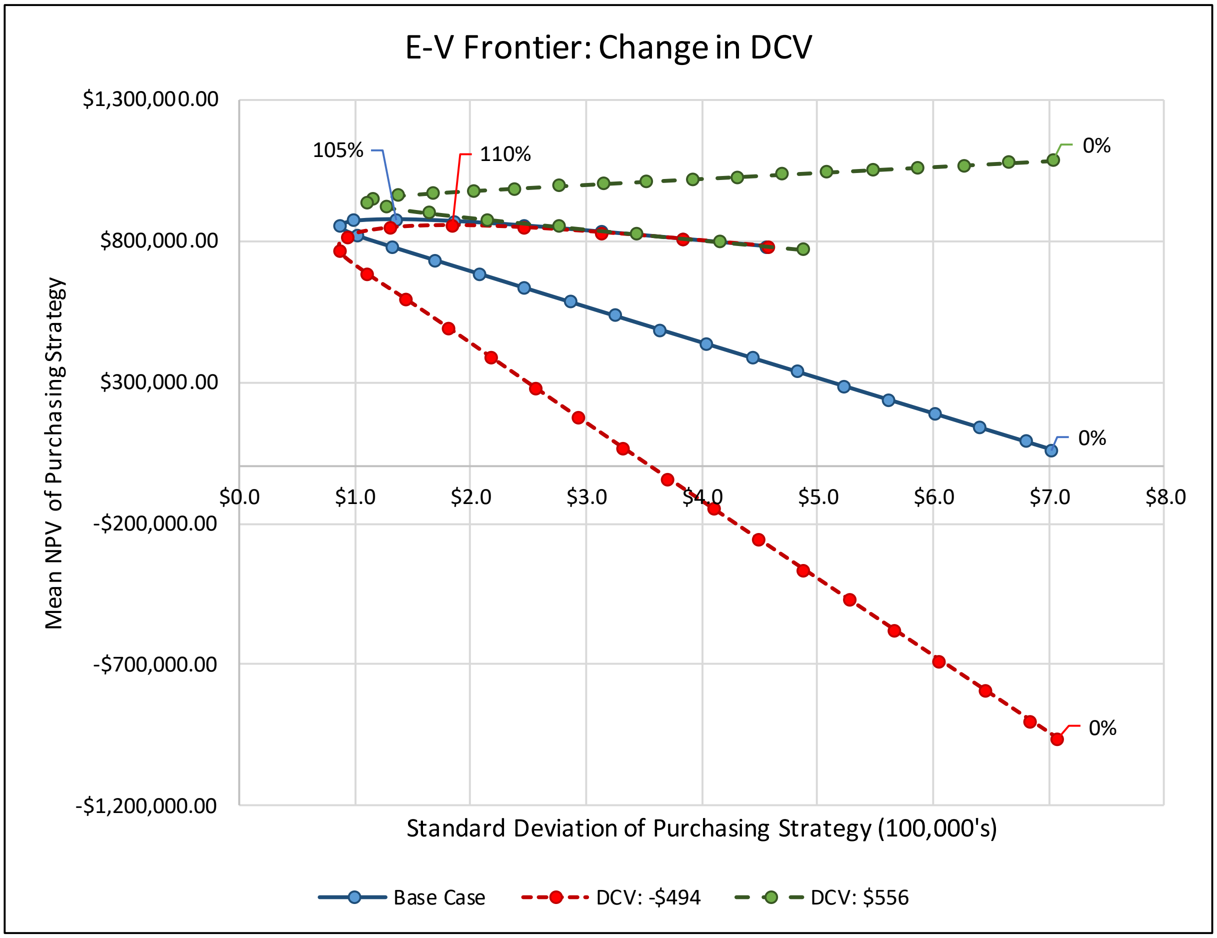

Figure 6.22. E-V Frontier: Daily Car Value

6.6.3. Sensitivity: Velocity Volatility

Velocity volatility affects the riskiness of how many shuttle trains will arrive per month. Increasing velocity volatility adds uncertainty to car supply. This uncertainty greatly impacts the expected profit of a shipper's purchasing strategy. Unlike the previous two sensitivities, changing velocity volatilely does not affect the shape of the payoff function. However, it does have a great impact on standard deviation of NPV as well as short call premium.

Table 6.15 shows how changing velocity volatility affects NPV of the purchasing strategy.

Table 6.15. Sensitivity to Velocity Volatility

Observation	Decrease Volatility	Base Volatility	Increase Volatility
Velocity Volatility	0.00	0.21	0.50
Purchasing Strategy	7,010,000	7,150,000	7,340,000
Trains Prepared for Based on Purchasing Strategy	18	19	19
Percent of Forecast	103%	105%	107%
NPV	$904,257	$874,873	$795,878
Standard Deviation	$108,419	$139,087	$234,899
Short Strike Velocity	3.04	3.10	3.18
Number Short Call	463,854	463,854	463,854
Short Call Premium	0.043	0.091	0.228
Number Long Calls	487,218	487,218	487,218

Table 6.15 shows if velocity volatility decreases to zero, the optimal purchasing strategy decreases, expected profit increases, and the standard deviation decreases. Simply, if there is no risk in velocity, a shipper has a high degree of certainty in the number of shipments. As a result, the shuttle elevator would buy fewer bushels. A lower purchasing strategy of 103% would have an increase in expected profit and a decrease in standard deviation.

When velocity volatility increases, the purchasing strategy increases to compensate for the added risk of stockout. The NPV decreases even though the purchasing strategy increases because an increase in velocity volatility increases short call premium which reflects the likelihood of incurring a stockout. A shipper would increase its purchasing strategy to increases strike velocity of short calls which has a lower option premium. Even so, a velocity volatility of .5 causes option premium to be more than double that of base case. This increase in call premium increases the effect of short calls on NPV and thus lowers NPV even though the purchasing strategy is increased.

Figure 6.23 shows how the NPV distribution of base case changes with differences in velocity volatility. An increase in velocity volatility decreases mean NPV and increases standard deviation. Alternatively, when velocity volatility decreases, mean NPV increases and standard deviation decreases. The NPV distribution is negatively skewed, so a decrease in standard deviation narrows the distribution to the right and increases mean expected profit.

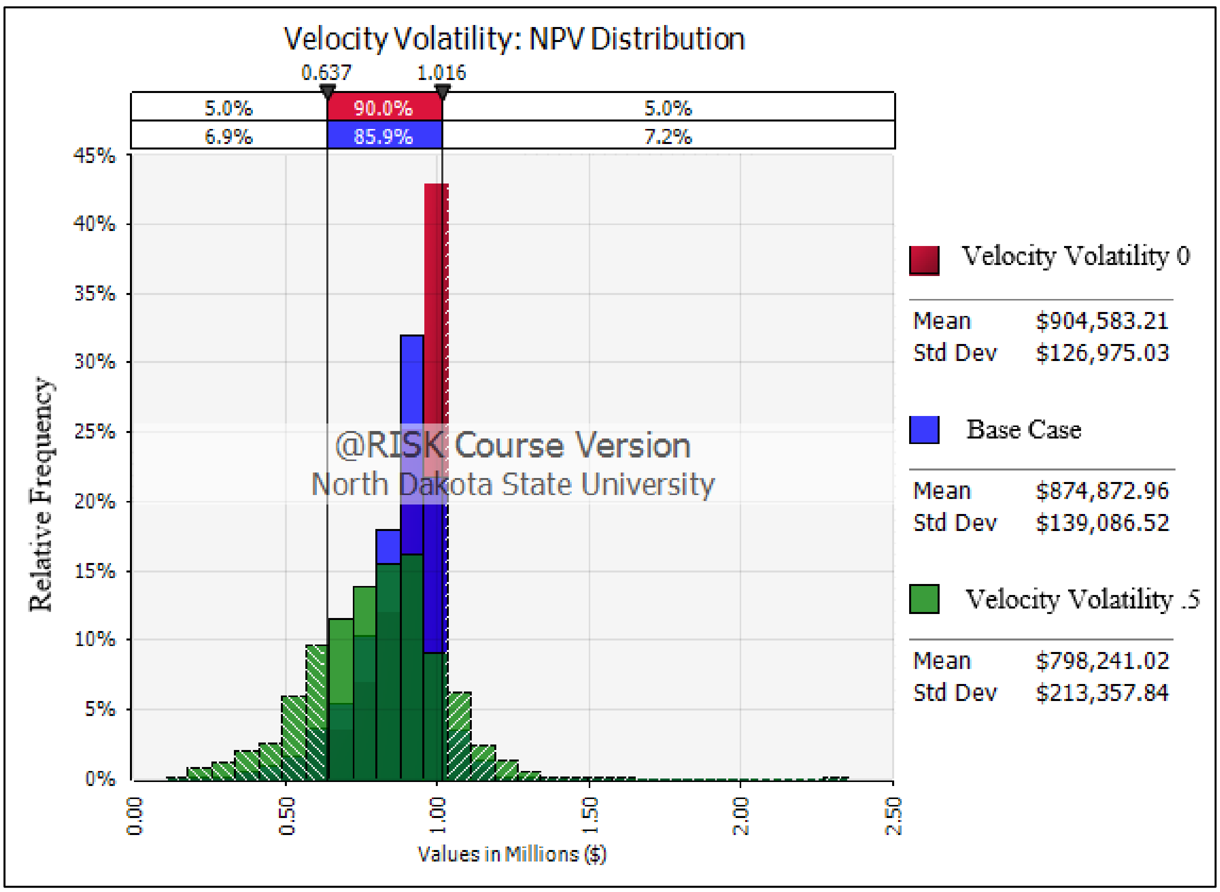

Figure 6.23. Velocity Volatility Change: NPV Distribution (@Risk™)

The E-V frontier in Figure 6.24 show different purchasing strategies under each velocity volatility. The E-V frontier for a decrease in velocity volatility causes the frontier to shift up and to the left. Expected profit is both higher and has a lower standard deviation. Conversely, a higher velocity volatility shifts down and to the right. Expected profit thus decreases with an increase in standard deviation. The E-V frontier further supports why an increase in purchasing strategy causes a decrease in NPV when velocity volatility increases.

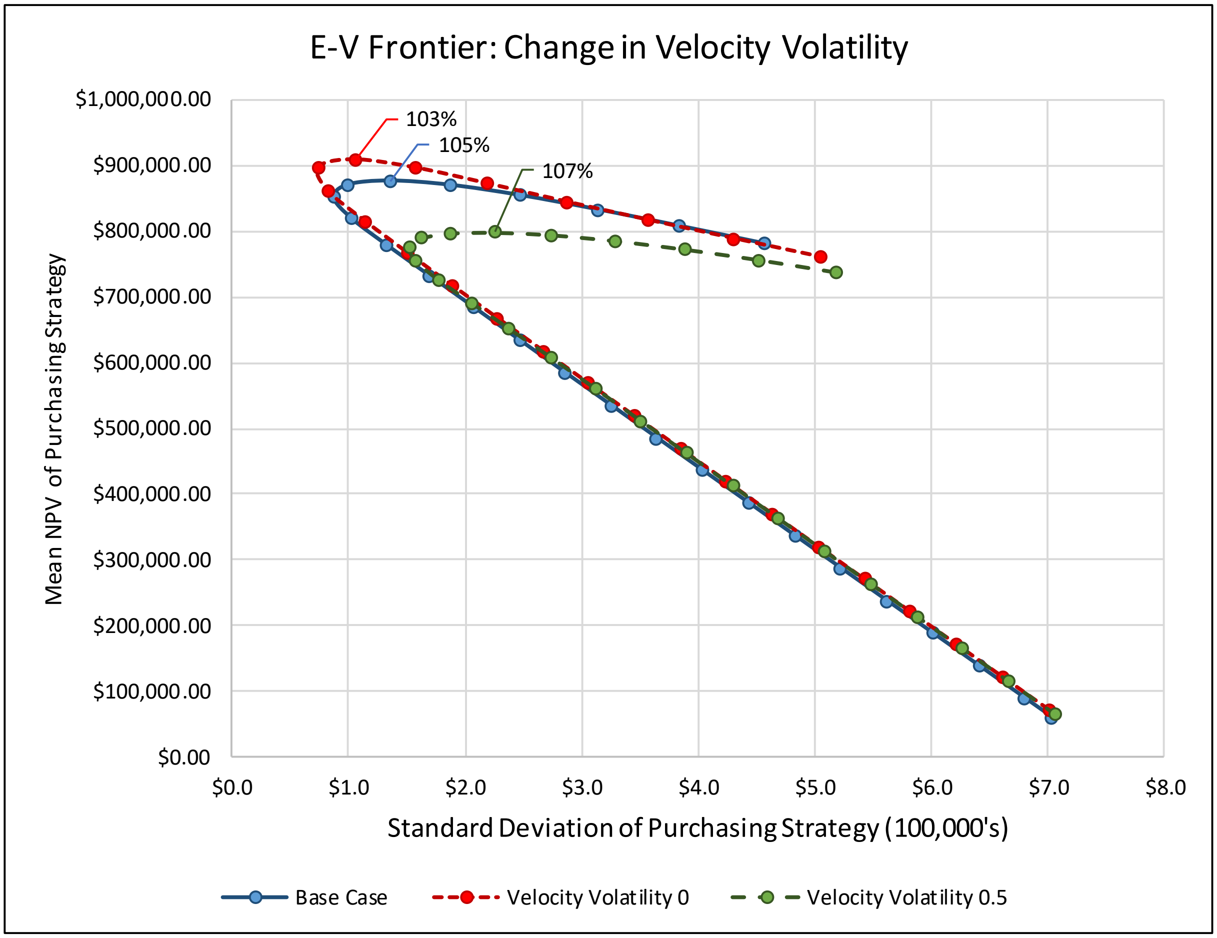

Figure 6.24. E-V Frontier: Velocity Volatility

6.6.4. Sensitivity: Transfer Option

Transferability of the primary rail instruments play a key role in both salvage value and stockout penalty for a shipper. The base case model assumes a shipper can sell unused cars into the secondary market but does not possess the option to purchase additional secondary cars if they want to continue to ship. There are two sensitivities in this section: the first sensitivity which does not allow any transferability of the primary instrument, and a second which grants a shipper the right to both buy and sell secondary rail shuttles.

The first sensitivity shows how the payoff function changes when the primary rail instrument is non-transferable. Some railroads do not give primary contract holders the ability to buy or sell secondary shuttles. Shippers in this situation have limited options to: cancel primary contract in its entirety, buy spot bushels at an inflated value, or pay demurrage fees in dollars per car per day. Cancelling a primary contract will cancel all subsequent shuttle shipments; shippers normally do not choose this option and therefore it is not considered a viable option (Landman 2017). The other two options are to buy spot bushels or pay demurrage fees. When elevators run out of grain they may buy bushels at an inflated price less than or equal to potential demurrage fees. Demurrage fees for BNSF railroad equal $75 per car per day which is just over $0.02 per bushel per day (BNSF 2018b). The number of days a shuttle train may sit in demurrage follows a Pearson VI distribution with a mean of 7.6 days and standard deviation of 1.9 (Wilson et al. 2004). The amount an elevator would pay in addition to spot price varies with expected amount of demurrage. Therefore, the sensitivity when the primary instrument is non-transferable only evaluates stockout penalty due to demurrage fees when a shipper runs out of bushels and does not have the option to transfer rail ownership.

The distribution specification for number of days in demurrage is shown in Table 6.16 (Wilson et al. 2004).

Table 6.16. Days of Demurrage Distribution (Wilson et al., 2004)

Variable	Distribution	Function	Mean	Standard Deviation
Days of Demurrage	Pearson VI	RiskPearson6 (280,17.61,0.45)	7.6	1.9

The stockout penalty under the first sensitivity is calculated as shown in equation (6.18):

$$\Lambda_b = \delta_d * f_{\delta,b} \tag{6.18}$$

where:

Λ_b = shortage penalty per bushel of unmet car supply

δ_d = days of demurrage

$f_{\delta,b}$ = demurrage fee per bushel per day.

The second sensitivity considers full transferability of the primary instrument which gives a shipper the option to purchase secondary cars at spot DCV in addition to selling surplus cars in secondary markets. This option affects salvage value of the CCI model by adding a logic function into salvage value calculation as shown in equation (6.19):

$$\Gamma_b = I_b + \max(C_{Market} - C_{Cost}, -DCV_b) \tag{6.19}$$

where:

Γ_b = salvage value per bushel

I_b = investment per bushel

C_{Market} = market carry in dollars per bushel

C_{Cost} = cost of carry in dollars per bushel

DCV_b = daily car value in dollars per bushel.

DCV per bushel is negative because a negative DCV indicates that the shipper would receive a premium when purchasing rail cars from secondary markets if they are currently selling at a discount.

Table 6.17 shows optimal purchasing strategy results from the two transfer option sensitivities relative to the base case.

Table 6.17. Sensitivity: Transfer Option

Observation	Primary Instrument Non-Transferable	Base Case	Primary Instrument Fully-Transferable
Option to Buy Spot Secondary Shuttles	No	No	Yes
Option to Sell Spot Secondary Shuttles	No	Yes	Yes
Purchasing Strategy	7,550,000	7,150,000	9,240,000
Trains Prepared for Based on Purchasing Strategy	20	19	24
Percent of Forecast	110%	105%	135%
NPV	$845,099	$874,873	$993,054
Standard Deviation	$196,499	$139,087	$273,644
Short Call Strike Velocity	3.27	3.1	4
Number Short Call	862,764	463,854	237,637
Short Call Premium	0.046	0.091	0.001
Number Long Calls	487,218	487,218	261,001

Table 6.17 shows optimal purchasing strategy increases to 110% when the primary instrument is non-transferable and increases to 135% when the option is fully-transferable. Optimal purchasing strategy increases in both instances; however, the reasons for an increase are quite different.

The purchasing strategy increases when the primary instrument is non-transferable due to a high stockout penalty which increases the number of short calls to 862,764. The high stockout penalty is because the shipper will pay demurrage fees with certainty if they run out of inventory. Simply, if the shipper is certain of incurring a stockout penalty, the shipper will purchase more bushels to ensure a stockout doesn't happen. An increase in number of short calls increases the negative effect of short call element in the CCI model. A shipper thus increases purchasing strategy to lower the effect of short call element and increase mean NPV to $845,099 which is still almost $30,000 lower than base case.

When primary instrument is fully-transferable, the purchasing strategy increases to 135% due to an increase in mean salvage value relative to base case. Option to ship or store excess bushels causes salvage value to increase relative to bases case. Increase occurs because a shipper can now maximize salvage value by either shipping or storing the excess bushels until the next shipping period; whichever option returns a greater return.

The orange dashed line in Figure 6.25 shows how the increase in salvage value increases vertical shift of long call option payoff when primary instrument is fully-transferable. Purchasing additional bushels would further increase vertical shift which is why a shipper would choose to max out inventory and purchases 135% of forecast car supply. The green dashed line shows how demurrage fees cause a negative slope of profit when shipping demand due to car supply is greater than purchased bushels. The green dashed line further shows why a shipper would want to purchase more bushels to increase their short call strike and lower their probability of experiencing a stockout.

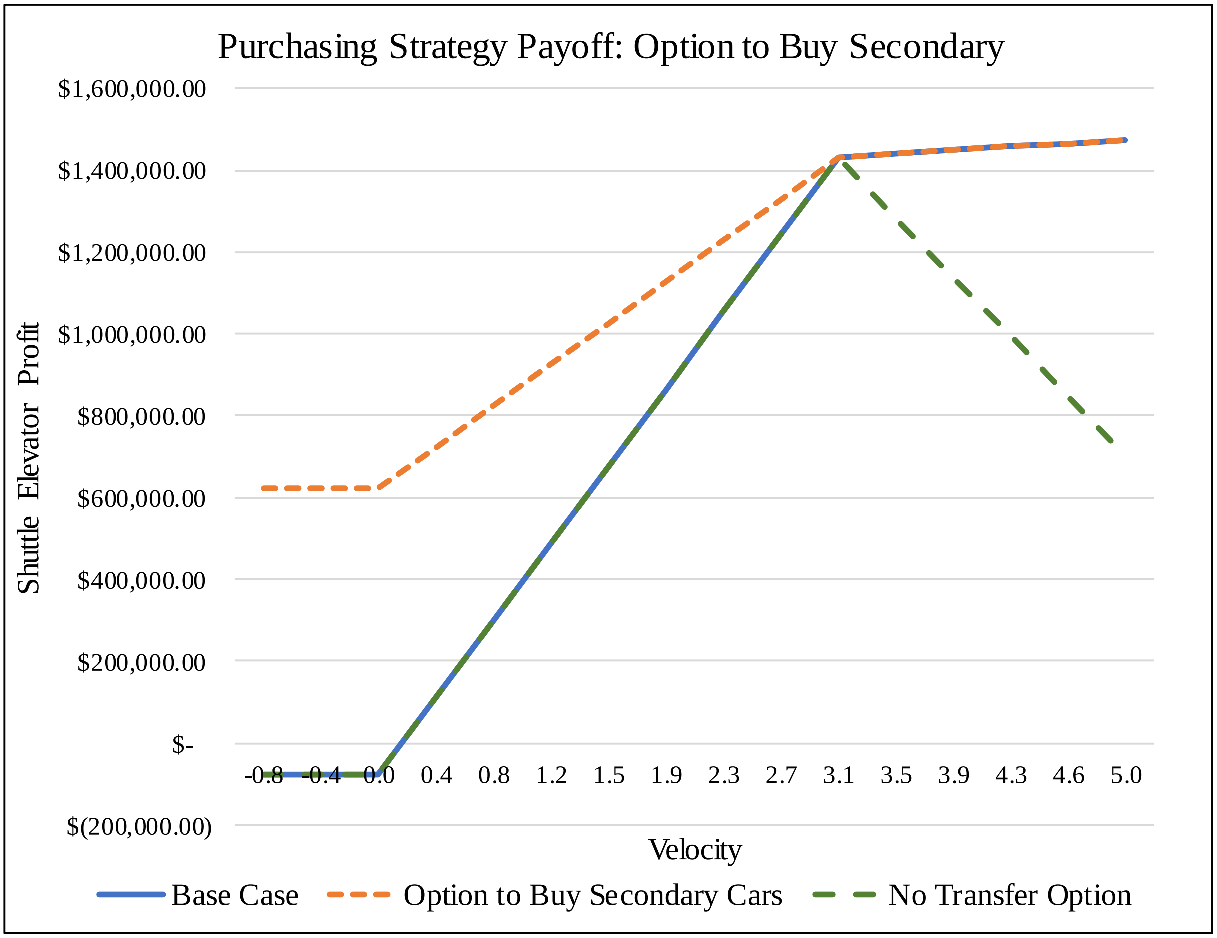

Figure 6.25. Purchasing Strategy Payoff: Option to Buy Secondary Cars

The pie chart in Figure 6.26 compares which option returns the highest salvage value when primary instrument is fully-transferable. Buying spot secondary cars is the best strategy 48% of the time under base case price distributions.

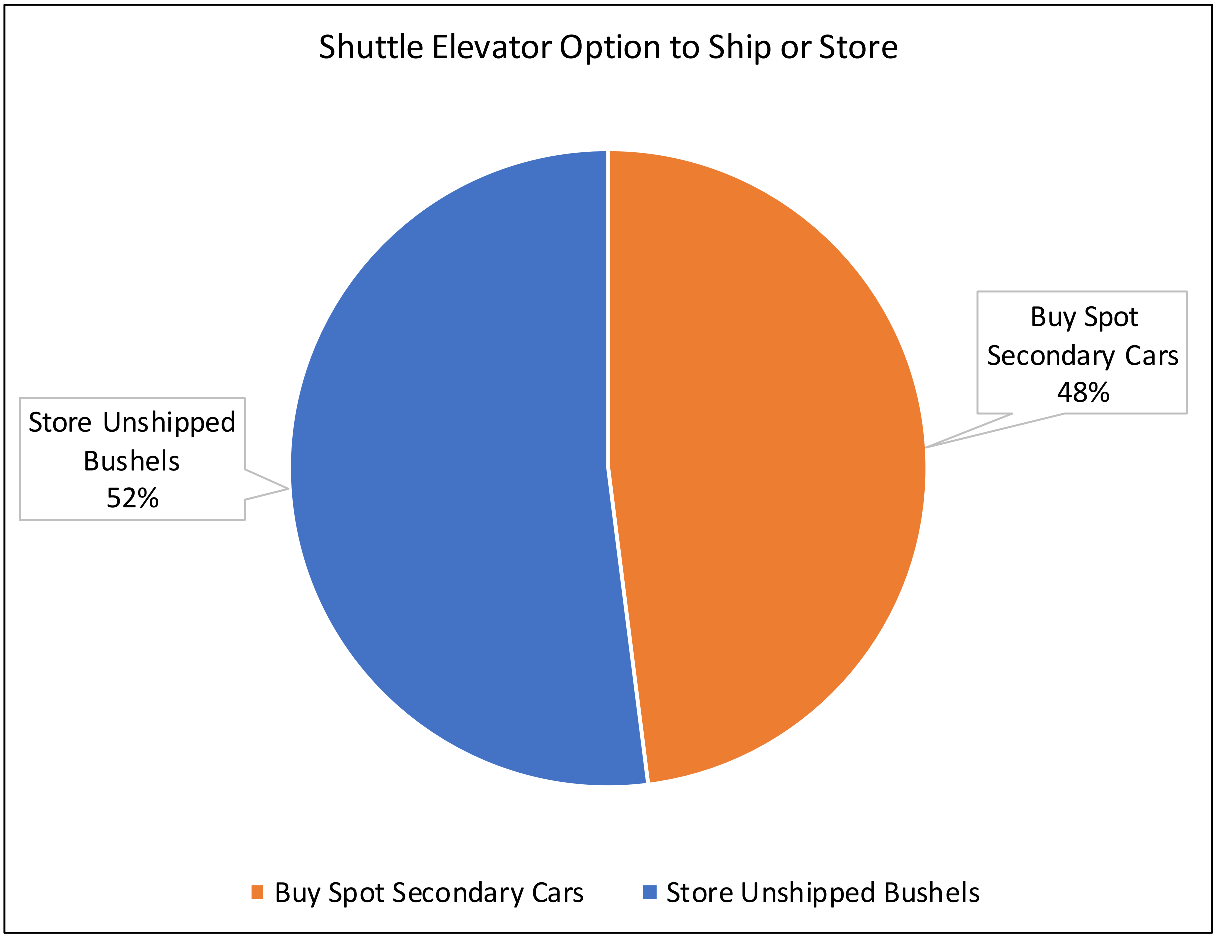

Figure 6.26. Shuttle Elevator's Best Option: Purchase Secondary Cars or Store

In Figure 6.27 the NPV distributions of both sensitives are compared to the base case strategy. When primary instrument is fully-transferable, expected profit increases by $47,500 and risk is reduced by over $63,000. When the primary instrument is non-transferable, the expected profit decreases close to $40,000 but the standard deviation also decreases by $16,000. Standard deviation decreases because a shipper now has certainty in experiencing a positive stockout penalty when car supply is not met. This certainty also causes the NPV distribution to move closer to normal which further explains why a decreased standard deviation would result in a lower expected profit level relative to the negatively skewed base case.

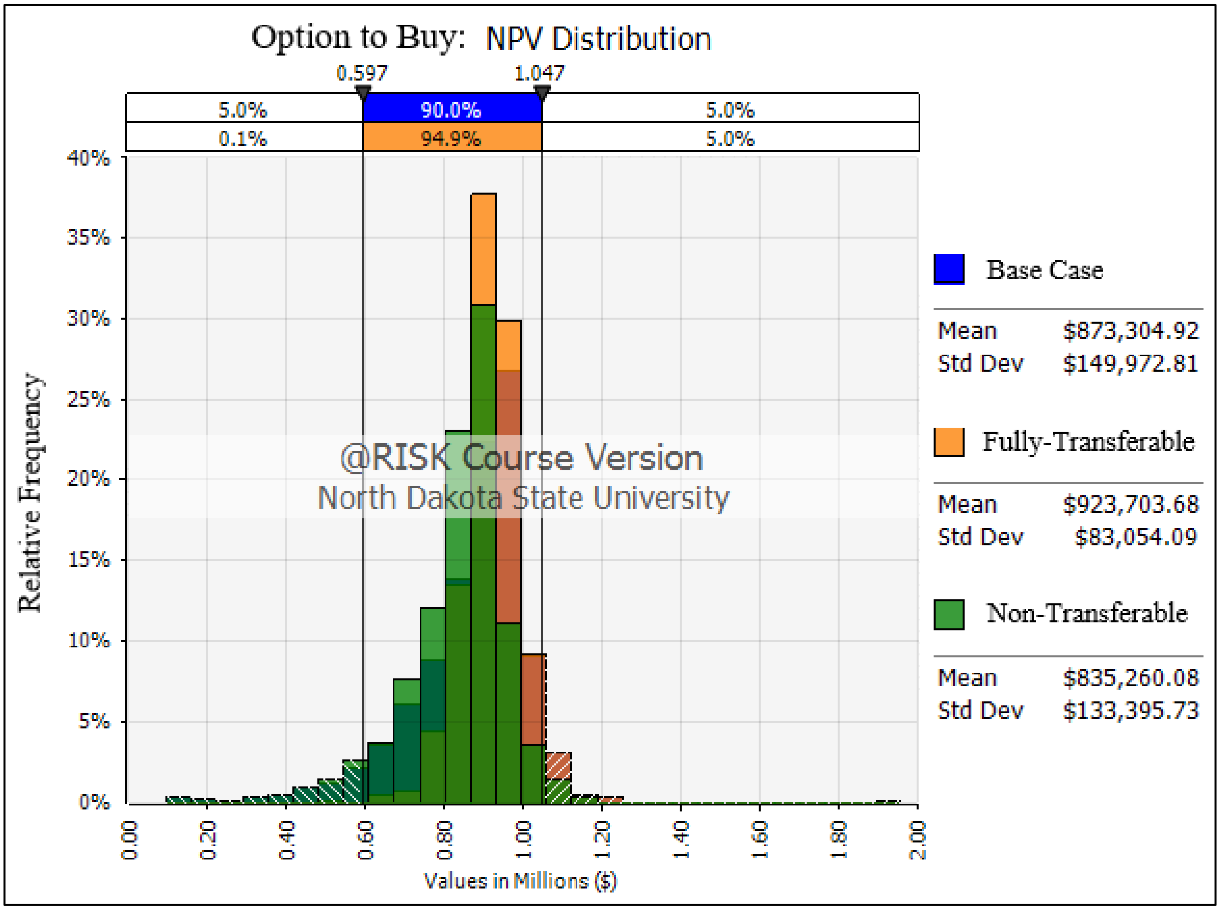

Figure 6.27. Option to Purchase Secondary Cars: NPV Distribution (@Risk™)

The E-V frontier in Figure 6.28 shows expected profit and risk of different purchasing strategies under the transfer option sensitivity. When the primary contracts are fully-transferable, every point on the E-V frontier has a higher mean NPV than the bases case. Furthermore, a fully-transferable contract is the only scenario where least risky purchasing strategy occurs above 100% of forecast car supply. When the primary instrument is non-transferable, the E-V frontier shifts down and to the left. Expected profit also decreases rapidly when the purchasing strategy is decreased. This occurs because there is a higher certainty of experiencing a stockout and incurring demurrage fees.

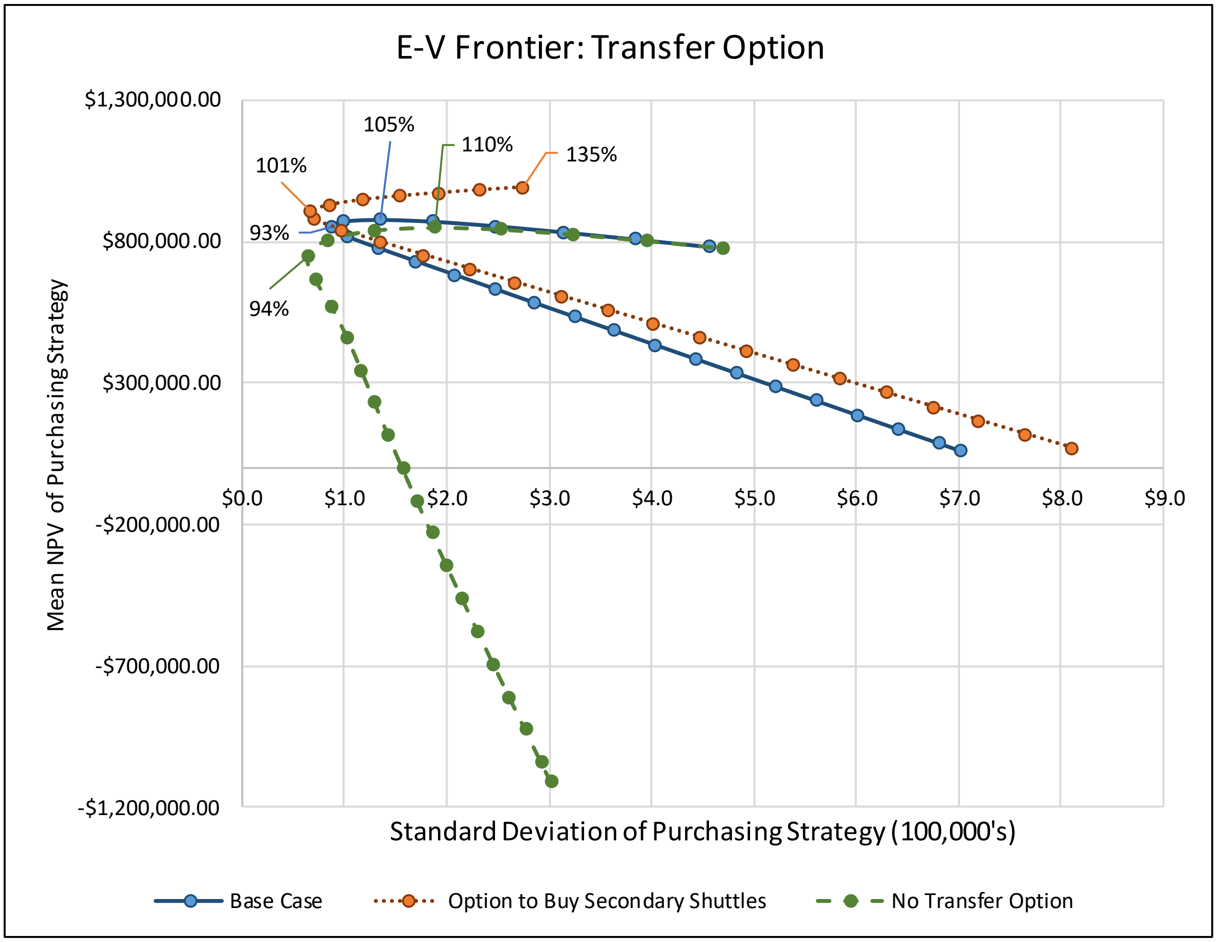

Figure 6.28. E-V Frontier: Option to Purchase Secondary Rail Cars

6.7. Conclusion

Shuttle elevators with primary rail contracts have several risks to consider when developing a grain purchasing strategy. Sources of uncertainty arise from the market spread of soybeans, changes in secondary rail car values, and fluctuation in velocity. Shuttle elevators are left with the task of developing an optimal purchasing strategy which would maximize their expected profit.

This chapter applies the contingent claim inventory (CCI) model as developed by Stowe and Su (1997) to aid shuttle elevators in developing an optimal purchasing strategy. Real option methodology is used to value uncertainty in velocity, which is the demand to ship grain due to car supply. A shipper gains the right to ship grain when bushels are purchased. A shipper also loses the right to ship grain if he/she runs out of inventory when velocity increases beyond short exercise velocity. This relationship results in a call spread.

Base case results use data from soybean crop marketing year of 2015/16 when relatively stable market conditions existed. The optimal purchasing strategy from base case shows that an elevator should purchase 5% more bushels than forecast velocity to account for volatility in car supply.

Sensitivities on the input parameters of market carry, daily car value, and velocity volatility change the optimal purchasing strategy in predictable ways. An increase in market carry by $0.10 causes the shipper to max out their storage capacity and purchase 135% of forecast car supply. In this situation, a shipper possesses the right to either ship bushels for their marginal value or store bushels and collect carry. Either way, a shipper would make money, and thus encourages an excessive purchasing strategy. When DCV is increased by $0.15, the optimal purchasing strategy was to not buy any bushels and sell all available shuttle trains into the secondary market for a profit. This strategy is profit maximizing but is also very risky. Changing velocity volatility from 21% to 50% causes the elevator to purchase more bushels to avoid possibility of stockout but will also decrease the expected profit.

A fourth sensitivity focused on transferability of the primary instrument. When the primary instrument is non-transferable, the optimal purchasing strategy increases to 110% and expected profit decreases. Conversely, when primary instrument is fully-transferable, expected profit increases and the risk is reduced substantially.

The overall result from this analysis is that due to uncertainties, from numerous sources, shippers would buy more grain than forecast need. This is not an obscure idea in grain trading and marketing. Indeed, processors would routinely buy or store more grain than needed; growers would normally under-hedge their production in anticipation of random yields (Blank et al. 1991; McKinnon 1967); traders would under hedge their position, or offset it with an option strategy, if they anticipate counterparty risk; among other examples. In all these cases there is some type of uncertainty and it affects a risk mitigation decision. In this case, there are several uncertainties and the shipper would appropriately respond in most cases by either overbuying or assuring he/she has more grain available than expected car supply. Hence, in this application, excess inventory of grain can be viewed as a real option.

Implications of the model developed in this application provide shippers a tool which aids in formulating an optimal grain purchasing strategy. Stochastic simulation and optimization enable managers to see possible outcomes based on distribution of PNW basis, futures spread, changes in rail tariff, daily car values, and velocity. The purchasing strategy is adjusted to maximize expected profit of the shipper; however, a shipper should also evaluate the E-V frontier of different strategies. Sensitivities are run on the model to account for a shipper's bias to market values to see how they should adjust their grain purchasing strategy.

CHAPTER 7. CONCLUSION

7.1. Review of Problem

Issues in inventory management have been around since before 600 B.C. (Kokukula and Papudesu 2006). Holding too much inventory ties up capital and accrues storage and interest while not having enough may lead to company shutdown and foregone profits. In recent times, Just-in-Time manufacturing (JIT) concepts and lean production have moved industries towards inventory strategies which hold zero buffer stocks (Ballou [1973] 1992; Jacobs and Chase [2008] 2017). In commodity marketing industries, there are important differences including those related to uncertainties in supply and demand, logistical performance and costs, as well as margins; all of which play a major role in inventory strategy. Whether it is wheat in flour milling, urea in fertilizer merchandising, or shipping soybeans via rail; effective inventory management and purchasing strategies play a key role in profitability. The methodology developed by Stowe and Su (1997) evaluates the real option to operate embedded in holding excess inventories. In this thesis, this methodology along with stochastic binomial real options, has been applied to three industry applications to develop optimal strategies which maximize expected profit.

7.2. Review of Procedures

In Chapter 2 inventory management and real options were explained to develop an adequate background for addressing the problem. Relevant literature in both areas was presented to identity the significance of inventory strategy and the role real options play in decision making. Chapter three provided the theoretical framework of the procedures used in this thesis paper.

The methodology developed by Stowe and Su (1997) maps the payoff function of inventories as a call spread combined with discounted net salvage value and initial inventory value. The value of both long and short call options was calculated using stochastic binomial real option valuation techniques similar to Churchill (2016) and Landman (2017). The model was simulated using Monte Carlo simulation and optimized using RiskOptimizer™. RiskOptimizer™ would change the strategy until expected profit was maximized.

7.3. Review of Results

Chapters 4, 5, and 6 were independent applications of contingent claims inventory (CCI) analysis in agricultural related industries. Chapter 4 addressed optimal purchasing strategies for a processor which milled wheat into flour and byproducts. Chapter 5 applied this methodology to the fertilizer industry and used competitive arbitrage pricing to address uncertainties of margin and demand. Chapter 6 applied the methodology to develop a three-month purchasing strategy for a bulk soybean shipper using primary rail contracts.

7.3.1. Wheat Flour Mill Results and Sensitivities

In chapter 4 a representative flour mill would utilize a purchasing strategy of 120% expected demand which would be replenished at the end of each processing month. This strategy is a result of large margins and a relatively low storage and interest cost. When the futures market is in backwardation, the optimal purchasing strategy was still 114% of expected milling demand. Though the optimal inventory strategy was reduced due to the inverse, it is still large and is s attributed to the high convenience yield in maintaining stocks when margins are high (Working 1949).

The current extraction rate behavior has little effect on optimal purchasing strategy; however, if it were to increase in standard deviation even a marginal amount, there would be major impacts on the flour milling industry. The current extraction rate of flour from hard red winter wheat has a standard deviation of only 0.8% (USDA-ERS 2019). When the standard deviation is reduced to 0%, the optimal purchasing strategy is reduced only .5%; however, when the standard deviation is increased to 3%, the optimal purchasing strategy increases to 125.8% of expected milling requirements.

7.3.2. Urea Merchandizing Results and Sensitivities

Fertilizer is the unique application of the thesis in which demand is very lumpy. If fertilizer demand is overestimated, the storage and interest costs associated with caring inventory until the next purchasing period are relatively high. This causes the base case purchasing strategy to be only 102.9% of forecast demand.

Competitive arbitrage pricing was implemented in the application. The base case assumes the representative fertilizer location would maintain a minimum of 30% market share from any competitive region. When minimum market share was decreased from 30% to 20%, the purchasing strategy increased to 107.6% of expected demand; however, the total purchasing quantity decreased because expected demand is lower when a county centroid is competing for less market share. The merchant is also able to charge a higher price for their fertilizer which increases expected margin and the probability of a positive profit.

Currently, the only futures hedging instrument in urea is a urea Swap Futures contract which is based off the spot price at the US Gulf. This is far from a perfect hedging instrument but may be used for a hedge over a longer duration of time. If the county centroid uses a hedging mechanism, they would take an equal and opposite position by selling Swap Future contracts and applying any gains or losses from the Swap Futures contract to their net revenue from selling cash urea. They would also need to consider the spread of urea Swap Futures contracts for any fertilizer which was not sold. The spread of urea Swap Futures contracts in the spring of the year is generally highly inverted. The inversion decreases the net salvage value which would decrease the optimal inventory strategy ceteris paribus (Stowe and Su 1997). Implementing the hedging instrument reduced the risk by 90% and increased positive expected profit to 95% of the time. However, due to the seasonal trend of Swap Futures spread, and its effect on net salvage value, the optimal purchasing strategy was reduced to 98.6% of expected demand, i.e., the country centroid would purchase less fertilizer than expected demand to avoid an inverted Swap Future spread as well as high storage and interest cost effects on net salvage value for any unsold urea.

7.3.3. Bulk Soybean Shipper Results and Sensitivities

The representative shipper in Chapter 6 was that of a shuttle elevator located in interior markets who ships soybeans via rail to terminal markets located on the coast. The shipper must develop a three-month purchasing strategy of soybeans under uncertainties in both car supply and margin. The shipping demand due to rail car supply is directly dependent on the performance of the railroad measured in velocity of total trips per month. The shipper is also susceptible to uncertainties in margin which result from terminal market basis spread, futures market spread, secondary market daily car value (DCV), and changes in tariff rate. Under base case assumptions and distributions, the optimal purchasing strategy occurred at 105% of expected shipping demand due to car supply.

Changes in futures market spreads, the carry in the market, have a great impact on the salvage value of unshipped soybeans. When the soybean futures market is inverted, the optimal purchasing strategy dropped to 95%, i.e., the shipper would buy less soybeans than forecast car supply because storing any unshipped bushels would lose more value than the foregone profits from stocking out. Contrary, when the spread in the futures market increases, i.e., there were great returns to storage, the shipper would max out their storage capacity and purchase 135% of forecast car supply. In this situation the shipper gains money from shipping soybeans but also gains money from storing soybeans until the next shipping period.

The secondary market daily-car-value (DCV) represented the price which the elevator could transfer any unused shuttle trains to another market participant. DCV acted as a stockout penalty to transfer any trains which arrive when the shipper ran out of soybean inventory; however, a positive DCV would act as a negative salvage value which meant profit would continue to increase if a stockout occurred. When DCV was decreased by $0.15 per bushel the optimal purchasing strategy increased to 110% of forecast car supply. When DCV was increased, the optimal purchasing strategy was reduced to 0% of forecast car supply. A 0% purchasing strategy occurs because it would be more profitable to sell all primary rail trains into the secondary market than assume the risk of purchasing soybeans to ship.

The performance of the railroad, measure in velocity, dictates how many shuttle trains arrived each month. A change in velocity volatility would affect the uncertainty in car supply and thus the riskiness of expected profit. When velocity volatility increased, the optimal purchasing strategy increased to 107% of forecast car supply. An increase in purchasing strategy would be used as a real option to hedge against the increased volatility of car supply.

The final sensitivity on a bulk shipper dealt with the transferability of the primary instrument. In the base case, the shipper was only allowed to sell any unused shuttle trains into the secondary market. The shipper was not given the option to purchase additional secondary rail cars if buying secondary rail cars to ship grain returned a greater profit than storing excess bushels. Granting the shipper this option increased the purchasing strategy to 135% of forecast car supply and increased expected profit by 12.7%.

7.3.4. Generalization of Results

In several cases of agricultural marketing there are an array of uncertainties in demand and margins. The factors impacting demand and margin will remain volatile, but inventory choice and targets are a management decision. If managers become short inventory they would forgo margin and possibly accrue stockout penalties. If managers have excess inventory they would accrue storage and interest costs. This thesis provides the framework of contingent claims inventory (CCI) analysis which may aide in this crucial management decision. CCI analysis finds, in most cases, that it is optimal to carry excess inventory because inventory holds a real option on the ability to operate. Sensitivity analysis finds the optimal level of inventory varies with demand volatility, net salvage values, margins, and possible stockout penalties. There are a number of commonalities among these applications, including:

- Increasing volatility in supply or demand increases the optimal inventory strategy.
- As net salvage values increase the optimal inventory strategy increases.
- Increased margins increase the optimal inventory strategy and make the optimal strategy less sensitive to changes in other aspects of inventory management.
- An addition of stockout penalties increase optimal inventory strategy.

Inventory management becomes very complex when factors relating to supply and demand as well as margins are considered. A real option on inventory using CCI analysis helps value strategies under ever changing market conditions. Stochastic simulation and optimization aide in developing a strategy which maximizes expected profit while also accounting for risky distributions of random inputs. CCI analysis using stochastic binormal real option models serves as an effective tool which may be utilized in strategic supply chain management decisions.

7.4. Contributions to Literature

The idea of applying financial theory to inventory management decisions started at the end of the 1980s when Kim and Chung (1989) applied a capital asset pricing model (CAPM) as an alternative to the profit maximization approach. Stowe and Su (1997) use the Black-Scholes (1973) model in a contingent claims approach. Goel and Gutierrez (2006) were one of the first to apply Monte Carlo Simulation and convenience yield. In more recent times, studies have become continuously more interested in the contingent claims approach and the different valuation methods in which it is accomplished (Shi et al. 2011; Chang et al. 2015; Li and Arreola-Risa 2017).

This thesis is an application of Stowe and Su (1997) paper which models inventory as a real option on the ability to operate. This thesis applies stochastic binomial real options to capture uncertainties in margin as well as supply and demand. The real option valuation techniques are extensions of Landman (2017) and Churchill (2016) in capturing uncertainties in the forecast underlying state variable and changing volatility.

This thesis applies real option premiums in a different way than previous literature. Real option techniques used in this paper use demand as the underlying state variable rather than an asset with monetary value. The term keyed "option demand premium" is used as a proxy for demand uncertainty with relation to time-value and volatility. Bhattacharya and Wright (2005) extend real options in a similar way through valuing human capital which does not explicitly hold monetary value. Option demand premium, when combined with Stowe and Su's contingent claim inventory model, results in a net present value (NPV) which is used to evaluate strategy.

This is possible because the number of options in Stowe and Su's paper represent the slope of increased revenue per increase in the underlying state variable. Multiplying the number of options, which have monetary meaning, by an option demand premium, which is discounted for time value, outputs a value which represents expected value of the real option embedded in holding inventory.

7.5. Limitations

The results found in this thesis follow the theoretical framework of Stowe and Su (1997). However, there are assumptions made in this thesis which may be disputable by industry firms and should be accounted for before key management decisions are made.

In wheat, it is assumed all flour produced forward contracted at the beginning of the purchasing month. In reality, flour is contracted through out time with varying contract terms in regard to time, quantity, delivery, among other specifications.

In fertilizer, aggregate demand for urea is assumed to follow an aggregate trendline for each individual county. However, aggerate demand is susceptible to shifts in crops planted due to broken crop rotations and changes in application rate. The assumption that commercial trucking costs are $0.60 per mile in the current environment is arguably unrealistic (Rolf 2019); however, the high assumption was needed to generate results that adequately satisfied competitive arbitrage pricing.

In rail shipments, there are a few key assumptions which need to be taken lightly. First, most country shuttle elevators handle more commodities than only soybeans. Secondly, this application assumes all soybean deliveries are forward contracted. In reality, the randomness in farmer spot deliveries should also be considered. Finally, interior market shuttle elevators generally have other options in terminal markets than just the Pacific Northwest.

7.6. Further Research

This thesis provides the framework for many areas of further research in inventory management using contingent claims inventory (CCI) analysis paired with stochastic binomial real option valuation. Extensions and further research may also be conducted on the applications in this thesis through the following ways:

- In wheat, the application assumes a stockout penalty could occur that may reflect the need to reroute flour. Many companies own multiple facilities throughout the United States. Therefore, an application may be developed which encompasses the option to switch contract fulfilment to an alternative location which would thus lower the need for buffer stocks.
- For fertilizer, competitive arbitrage pricing may be extended to encompass not only county centroids, but also Canada-USA boarder points and USA domestic fertilizer plants (Wilson et al. 2014). Further research may be required to acquire a better hedging instrument for urea. Research into a better hedging instrument which follows interior market prices would help further lower the risk in fertilizer merchandizing. The current relationship between interior country centroids and urea Swap Futures is lagged which cause poor correlation when first difference. The poor relationship would make short term hedging ineffective (Bland et al. 1991).
- As stated in Chapter 6 for rail shipments, to properly assess a one-year primary rail contract, an MRP model should be built to encompass 12 months of shipping. However, this type of extension, given the complexity of the existing application, becomes strenuous on Monte Carlo simulation and optimization using current software packages. The application could also be extended account for multiple

terminal locations which would offer competitive bidding. Select sensitives in rail shipping suggest a nil purchasing. This strategy may have effects which are not accounted I the model which pertain to customer relationships and ethics.

From a risk perspective, there are several areas which could be either improved or extended:

- Value at Risk (VaR) could be implemented to address a minimum expected profit with a certain degree of confidence.
- This thesis correlates stochastic variables using Spearman Rank Order Correlations. Research has found that using a Copula may accomplish results which capture risk in a way which may outperform conventional correlation matrixes (Durrleman et al. 2000). At best, Copula's may also be used as tool which proves how robust results are based on correlation matrices.
- Inventory strategies may be compared using stochastic dominance rather than simply maximizing mean expected profit.
- Stochastic efficiency with respect to a function (SERF) may be used to determine the worth of switching from one strategy to another under the same market conditions.

From a policy prospective, this model could be used to capture the effects of foreign trade policy on U.S. ending stocks. The nature of this model, which views volatility as a forward calculation on random forecasts, captures several different scenarios on what could happen given a certain trade policy. Sensitives on the model can be run to see how adjustments to the trade policy, eg., how many metric tons of soybeans China pledges to import from the United States, would change the level of either supply and demand as well as its distribution.

REFERENCES

Amram, Martha and Kulatilaka, Nalin. 1999. *Real Options: Managing Strategic Investment in an Uncertain World.* Boston: Harvard Business School Press.

Association of American Plant Food Control Officials (AAPFCO). 2011. Commercial Fertilizers 2011 Association of American Plant Food Control Officials and The Fertilizer Institute. Washington D.C.

Association of American Railroads (AAR). 2018. Rail Cost Adjustment Factor-2017Q4 Base. https://www.aar.org/rail-cost-indexes/. Accessed December 19, 2018.

Ballou, Ronald H. (1973) 1992. *Business Logistics Management.* Englewood Cliffs: Prentice Hall.

Bhattacharya, Mousumi and Wright, Patrick M. 2005. "Managing Human Assets in an Uncertain World: Applying Real Options Theory to HRM." *The International Journal of Human Resource Management* 16(6): 929-948. http://www.tandfonline.com/doi/abs/10.1080/09585190500120574

Black, Fischer, and Myron Scholes. 1973. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81(3): 637-54. http://www.jstor.org/stable/1831029.

Blank, Steven C., Carter, Colin A., and Schmiesing Brian H. 1991. "Practical and Theoretical Aspects of Hedging Commodities." In *Futures and Options Markets: Trading in Commodities and Financials,* 226-231. Englewood Cliffs: Prentice Hall.

Boyle, Phelim P. 1977. "Options: A Monte Carlo Approach." *Journal of Financial Economics* 4 (3): 323–38. https://doi.org/10.1016/0304-405X(77)90005-8.

Burlington Northern Santa Fe (BNSF). 2019. BNSF 6003 Rail Miles Inquiry. http://www.bnsf.com/bnsf.was6/RailMiles/

Burlington Northern Santa Fe (BNSF). 2018a. BNSF 90004 Item 1000 Grain Product Rate Item Price List: Urea, Other Than Liquor or Liquid. file:///C:/Users/jesse/AppData/Local/Microsoft/Windows/INetCache/Content.Outlook/2RXQFHWB/Fertilizer%20BNSF%20Freight.pdf. Accessed February 15, 2019.

Burlington Northern Santa Fe (BNSF). 2018b. Demurrage and Storage. https://www.bnsf.com/ship-with-bnsf/support-services/demurrage-storage-and-extended-services.html. Accessed November 18, 2018.

Burlington Northern Santa Fe (BNSF). 2018c. Jumbo Hopper Car Dimensions. https://www.bnsf.com/ship-with-bnsf/ways-of-shipping/equipment/pdf/Jumbo.pdf. Accessed November 18, 2018.

Casassus, Jaime, and Collin-Dufresne, Pierre. 2005. "Stochastic Convenience Yield Implied from Commodity Futures and Interest Rates." The Journal of Finance 60 (5): 2283–2331.

Chang, Jack SK, Chang, Carolyn and Shi, Min. 2015. "A Market-Based Martingale Valuation Approach to Optimum Inventory Control in a Doubly Stochastic Jump-Diffusion Economy." *Journal of the Operational Research Society* 66 (3): 405–20. https://doi.org/10.1057/jors.2014.4.

Chicago Board of Trade (CBOT). 2018a. Soybean Futures Prices. Retrieved from Data Transmission Network (DTN) Prophet X.

Chicago Board of Trade (CBOT). 2018b. Urea FOB US Gulf Coast Swap Futures. Retrieved from Bloomberg.

Chicago Mercantile Exchange (CME). 2019. Kansas City Hard Red Winter Wheat Futures. Retrieved from Data Transmission Network (DTN) ProphetX.

Churchill, Jason C. 2016. "Valuation of Licensing Agreements in Agriculture Biotechnology." Order No. 10150582, North Dakota State University. https://ezproxy.lib.ndsu.nodak.edu/login?url=https://search-proquest-com.ezproxy.lib.ndsu.nodak.edu/docview/1834113142?accountid=6766.

Cox, John C., Stephen A. Ross, and Mark Rubinstein. 1979. "Option Pricing: A Simplified Approach." *Journal of Financial Economics* 7 (3): 229–63. https://doi.org/10.1016/0304-405X(79)90015-1.

Coyle, John J. and Bardi, Edward J. (1976) 1984. *The Management of Business Logistics.* St. Paul: West Publishing.

Data Transmission Network (DTN). 2018a. Cargill Milling Inc Urea prices. Retrieved from Bloomberg.

Data Transmission Network (DTN). 2018b. Northern Plains Cooperative Urea prices. Retrieved from Bloomberg.

Data Transmission Network (DTN). 2018c. Woodworth Farmers Grain Urea prices. Retrieved from Bloomberg.

Dixit, Aninash K. and Pindyck, Robert S. 1994. *Investment Under Uncertainty.* Princeton: Princeton University Press.

Durrleman, Valdo, Ashkan Nikeghbali, and Thierry Roncalli. 2000. "Which Copula Is the Right One?" *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.1032545.

Gibson, Rajna, and Schwartz , Eduardo S. 1990. "Stochastic Convenience Yield and the Pricing of Oil Contingent Claims." *The Journal of Finance* 45 (3): 959–76. https://doi.org/10.2307/2328801.

Goel, Ankur and Gutierrez, Gutierrez J. 2006. "Integrating commodity markets in the optimal procurement policies of a stochastic inventory system." Working paper, University of Texas at Austin.

Grain & Milling Annual. 2015. Kansas City, MO: Sosland Publishing Co.

Green Markets. 2018a. NOLA Spot Urea Prices. Retrieved from Bloomberg.

Green Markets. 2018b. South East Spot Urea Prices. Retrieved from Bloomberg.

Green Markets. 2018c. Western Corn Belt Spot Urea Prices. Retrieved from Bloomberg.

Guthrie, Graeme. 2009. *Real Options in Theory and Practice*. New York: Oxford University Press.

Harris, Ford W. 1913. "How Many Parts to Make at Once." *Operations Research* 38 (6): 947–950.

Hull, John. (1995) 2008. *Fundamentals of Futures and Options Markets*. Upper Saddle River: Pearson Prentice Hall.

Jacobs, Robert F. and Chase, Richard B. (2008) 2017. *Operations and Supply Chain Management: The Core.* New York: McGraw-Hill Education.

Kaldor, Nicholas. 1939. "Speculation and Economic Stability." *The Review of Economic Studies* 7 (1): 1–27. https://doi.org/10.2307/2967593.

Kim, Yong H., and Chung, Kee H. 1989. "Inventory Management under Uncertainty: A Financial Theory for the Transactions Motive." *Managerial and Decision Economics* 10 (4): 291–98.

Kodukula, Prasad and Chandra, Papudesu. 2006. *Project Valuation Using Real Options: A Practitioner's Guide*. Fort Lauderdale: J Ross Publishing

Kolb, Robert W. and Overdahl, James A. (1985) 2006. *Understanding Futures Markets.* Malden: Blackwell Publishing.

Landman, Daniel J. 2017. "Real Option Analysis of Primary Rail Contracts in Grain Shipping." Order No. 10269230, North Dakota State University. https://ezproxy.lib.ndsu.nodak.edu/login?url=https://search-proquest-com.ezproxy.lib.ndsu.nodak.edu/docview/1907030235?accountid=6766.

Leiblein, M. 2003. "The Choice of Organizational Governance Form and Performance: Predictions from Transaction Cost, Resource-Based, and Real Options Theories." *Journal of Management* 29 (6): 937–61. https://doi.org/10.1016/S0149-2063(03)00085-0.

Li, Bo, and Arreola-Risa, Antonio. 2017. "Financial Risk, Inventory Decision and Process Improvement for a Firm with Random Capacity." *European Journal of Operational Research* 260 (1): 183–94. https://doi.org/10.1016/j.ejor.2016.12.007.

Luehrman, Timothy A. 1998. "Strategy as a Portfolio of Real Options." *Harvard Business Review* 76 (5): 89–99.

Ma, Shihua, Zhe Yin, and Xu Guan. 2013. "The Role of Spot Market in a Decentralised Supply Chain under Random Yield." *International Journal of Production Research* 51 (21): 6410–34. doi:10.1080/00207543.2013.813987.

MapChart. 2019. https://mapchart.net/usa.html. Accessed February 13, 2019.

McKinnon, Ronald I. 1967. "Futures Markets, Buffer Stocks, and Income Stability for Primary Producers." *Journal of Political Economy* 75 (6): 844-861.

Palisade Corp. 2016. *@Risk: Risk Analysis and Simulation Add-In for Microsoft® Excel.* Ithaca: Palisade Corporation.

Ptak, Carol, and Smith, Chad. (1975) 2011. *Orlicky's Material Requirements Planning.* New York: McGraw Hill.

Osowski, Nicholas R. 2004. "Inventory Valuation Decisions and Strategy Analysis." North Dakota State University.

Rolf, Jeremy. 2019. Interviewed by Jesse Klebe. Personal Interview. Fargo, February 15.

Rosing, Anthony. 2018. Interviewed by Jesse Klebe. Personal Interview. Moorhead, April 23.

Shi, Y, Wu, F, Chu, L K, Sculli, D and Xu, Y H. 2011. "A Portfolio Approach to Managing Procurement Risk Using Multi-Stage Stochastic Programming." *Journal of the Operational Research Society* 62 (11): 1958–70. https://doi.org/10.1057/jors.2010.149.

Shreve, Steven. 2004. *Stochastic Calculus for Finance I: The Binomial Asset Pricing Model.* Verlag, New York.: Springer Science + Business Media.

Stowe, John D., and Su, Tie. 1997. "A Contingent Claims Approach to the Inventory-Stocking Decision." *Financial Management* 26 (4): 42-55.

Thomson Reuters. 2018a. PNW Terminal Market Soybean Prices. Retrieved from Thomson Reuters Eikon.

Thomson Reuters. 2018b. Secondary Rail Market Daily Car Values (DCV). Retrieved from Thomson Reuters Eikon.

Tomek, William G., and Kaiser, Harry M. (1972) 2014. "Spatial Price Relationships." In *Agricultural Product Prices*, 145-167. 5th ed. Ithaca: Cornell University Press.

TradeWest Brokerage Co. 2018. *Evening Market Recap*. Hillsboro, OR: TradeWest Brokerage Co.

Trigeorgis, Lenos. (1996) 1999. *Real Options: Managerial Flexibility and Strategy in Resource Allocation.* Cambridge: The MIT Press.

U.S. Department of Agriculture (USDA)-Agriculture Marketing Service (AMS). 2018. Grain Transportation Report (GTR) Data sets. Raw data. Washington DC. November.

U.S. Department of Agriculture (USDA)-Economic Research Service (ERS). 2013. Fertilizer use and Price. http://www.ers.usda.gov/data- products/fertilizer-use-and-price.aspx#.UsxvDVAXZmp. Accessed May 17, 2013.

U.S. Department of Agriculture (USDA)- Economic Research Service (ERS). 2018. U.S. Consumption of Selected Nitrogen Materials. https://www.ers.usda.gov/data-products/fertilizer-use-and-price.aspx. Accessed November 15, 2018.

U.S. Department of Agriculture (USDA)-Economic Research Service (ERS). 2019a. Wheat and Flour Price Relationships, Kansas City. https://www.ers.usda.gov/data-products/wheat-data/. Accessed February 11, 2019.

U.S. Department of Agriculture (USDA)-Economic Research Service (ERS). 2019b. Wheat Flour Production. https://www.ers.usda.gov/data-products/wheat-data/. Accessed February 11, 2019.

U.S. Department of Agriculture (USDA)-National Agriculture Statistics Service (NASS). 2013a. Planted acres by crop by county. http://quickstats.nass.usda.gov/. Accessed May 16, 2013.

U.S. Department of Agriculture (USDA)-National Agriculture Statistics Service (NASS). 2013b. Agricultural Chemical Use Program Survey Data on Fertilizer use by crop. http://www.nass.usda.gov/Surveys/Guide_to_NASS_Surveys/Chemical_Use/ind ex.asp. Accessed May 17, 2013.

U.S. Department of the Treasury (USDT). 2018. 52-Week Treasury Yield Rates. Web.

Wilson, William W. 2019. Interviewed by Jesse Klebe. Personal Interview. Fargo, February 11.

Wilson, William W., Carlson, Donald C.E., and Dahl, Bruce L. 2004. "Logistics and Supply Chain Strategies in Grain Exporting." *Agribusiness* 20 (4): 449-464.

Wilson, William W., and Bruce Dahl. 2011. "Grain Pricing and Transportation: Dynamics and Changes in Markets." *Agribusiness* 27 (4): 420–34. https://doi.org/10.1002/agr.20277.

Wilson, William W., Shakya, Sumadhur, and Dahl, Bruce. 2014. "Dynamic Changes in Spatial Competition for Fertilizer." *Agribusiness & Applied Economics Report* No. 726. North Dakota State University.

Winston, Wayne. 2008. *Financial Models using Simulation and Optimization II*. New York: Palisade Corporation.

Working, Holbrook. 1949. "The Theory of Price of Storage." *The American Economic Review* 39 (6): 1254–62.

APPENDIX A. DETRENDED EXTRACTION RATE (OLS)

Regression Statistics	
Multiple R	0.78367
R Square	0.614139
Adjusted R Square	0.600834
Standard Error	0.007739
Observations	31

ANOVA

	df	**SS**	**MS**	**F**	**Significance F**
Regression	1	0.002764	0.002764	46.15668	1.85E-07
Residual	29	0.001737	5.99E-05		
Total	30	0.004501			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.738633	0.002714	272.167	5.74E-51
Time T	0.001056	0.000155	6.793871	1.85E-07

APPENDIX B. ST. LOUIS INSTRUMENT VARIABLE (OLS)

Regression Statistics	
Multiple R	0.973514
R Square	0.947729
Adjusted R Square	0.946961
Standard Error	17.03375
Observations	70

ANOVA

	df	SS	MS	F	Significance F
Regression	1	357729.7	357729.7	1232.919	2.61E-45
Residual	68	19730.11	290.1486		
Total	69	377459.8			

	Coefficients	Standard Error	t Stat	P-value
Intercept	-22.6792	10.27033	-2.20822	0.030604
South East Spot (Florida)	1.028668	0.029296	35.11294	2.61E-45

APPENDIX C. NORTHERN PLAINS INSTRUMENT VARIABLE (OLS)

Regression Statistics	
Multiple R	0.936942
R Square	0.877859
Adjusted R Square	0.875951
Standard Error	33.65248
Observations	66

ANOVA

	df	**SS**	**MS**	**F**	**Significance F**
Regression	1	520930	520930	459.9865	6.37E-31
Residual	64	72479.34	1132.49		
Total	65	593409.4			

	Coefficients	Standard Error	t Stat	P-value
Intercept	116.66	16.55255	7.047859	1.53E-09
NOLA Spot	1.188751	0.055427	21.4473	6.37E-31

APPENDIX D. PNW INSTRUMENT VARIABLE (OLS)

Regression Statistics					
Multiple R	0.75				
R Square	0.56				
Adjusted R Square	0.56				
Standard Error	0.28				
Observations	193.00				
ANOVA					
	df	**SS**	**MS**	**F**	**Significance F**
Regression	1.00	18.63	18.63	244.71	0.00
Residual	191.00	14.54	0.08		
Total	192.00	33.17			
	Coefficients	Standard Error	t Stat	P-value	
Intercept	0.26	0.06	4.33	0.00	
TR_Basis_PNW	0.85	0.05	15.64	0.00	

APPENDIX E. DAILY CAR VALUE INSTRUMENT VARIABLE (OLS)

Regression Statistics					
Multiple R	0.93				
R Square	0.86				
Adjusted R Square	0.86				
Standard Error	449.55				
Observations	184.00				
ANOVA					
	df	**SS**	**MS**	**F**	**Significance F**
Regression	1.00	227340375.58	227340375.58	1124.91	0.00
Residual	182.00	36781461.08	202095.94		
Total	183.00	264121836.65			
	Coefficients	Standard Error	t Stat	P-value	
Intercept	35.25	38.79	0.91	0.36	
TR_DCV	0.92	0.03	33.54	0.00	

APPENDIX F. DISTRIBUTION FITS (@RISK™)

@Risk™ Distribution	Distribution Description
RiskBetaGeneral(alpha1,alpha2, minimum,maximum)	beta distribution with defined minimum and maximum, and shape parameters alpha1 and alpha2
RiskExpon(beta)	exponential distribution with mean beta
RiskExtvalue(alpha,beta)	extreme value (or Gumbel) distribution with location parameter alpha and scale parameter beta
RiskExtValueMin(alpha, beta)	extreme value min distribution with location parameter alpha and shape parameter beta
RiskGamma(alpha,beta)	gamma distribution with shape parameter alpha and scale parameter beta
RiskInvGauss(mu,lambda)	inverse gaussian (or Wald) distribution with mean mu and shape parameter lambda
RiskKumaraswamy(alpha1,alpha2, minimum,maximum)	Kumaraswamy distribution with shape parameters alpha1 and alpha2 and minimum and maximum.
RiskLaplace(μ,σ)	Laplace distribution with location parameter μ and scale parameter σ
RiskLevy(a,c)	Levy distribution with location a and continuous scale parameter c
RiskLogistic(alpha,beta)	logistic distribution with location parameter alpha and scale parameter beta
RiskLoglogistic(gamma,beta, alpha)	log-logistic distribution with location parameter gamma, scale parameter beta, and shape parameter alpha
RiskLognorm(mean,standard deviation)	lognormal distribution with specified mean and standard deviation
RiskNormal(mean,standard deviation)	normal distribution with given mean and standard deviation
RiskPareto(theta,alpha)	Pareto distribution with parameters theta and alpha
RiskPearson5(alpha,beta)	Pearson type V (or inverse gamma) distribution with shape parameter alpha and scale parameter beta
RiskPearson6(beta,alpha1, alpha2)	Pearson type VI distribution with scale parameter beta and shape parameters alpha1 and alpha2
RiskTriang(minimum,most likely, maximum)	triangular distribution with given minimum, most likely, and maximum values
RiskUniform(minimum, maximum)	uniform distribution between minimum and maximum
RiskWeibull(alpha,beta)	Weibull distribution with shape parameter alpha and scale parameter beta

APPENDIX G. TIME SERIES FUNCTIONS (@RISK™)

@Risk™ Time Series Function	Description
RiskAR1(mu,Sigma,A,R0, StartValue,WhatToReturn)	Calculates an auto-regressive AR(1) series with these parameters
RiskAR2(mu,Sigma,A1,A2,R0, RNeg1, StartValue,WhatToReturn)	Calculates an auto-regressive AR(2) series with these parameters
RiskARCH(mu,Omega,A,R0, StartValue,WhatToReturn)	Calculates an auto-regressive conditional heteroskedastic series with these parameters
RiskARMA(mu,Sigma,A1,B1,R0, StartValue,WhatToReturn)	Calculates an auto-regressive moving average series with these parameters
RiskGARCH(mu, Omega, A,B,R0, Sigma0, StartValue,WhatToReturn)	Calculates a generalized auto-regressive conditional heteroskedastic series with these parameters
RiskGBM(mu,Sigma, StartValue,WhatToReturn)	Calculates a geometric brownian motion series with these parameters
RiskGBMJD(mu,Sigma,Lambda, JumpMu,JumpSigma, , StartValue,WhatToReturn)	Calculates a geometric brownian motion with jump diffusion series with these parameters
RiskBMMR(mu,Sigma,Alpha,R0, StartValue,WhatToReturn)	Calculates a geometric brownian motion with mean reversion series with these parameters
RiskBMMRJD(mu,Sigma,Alpha,R0, Lambda,JumpMu, JumpSigma, , StartValue,WhatToReturn)	Calculates a geometric brownian motion with mean reversion and jump diffusion series with these parameters
RiskMA1(mu,Sigma, B1, StartValue,WhatToReturn)	Calculates a moving average MA(1) series with these parameters
RiskMA2(mu,Sigma, B1, B2, StartValue,WhatToReturn)	Calculates a moving average MA(2) series with these parameters