THE EFFECTS OF FUTURES MARKETS ON THE SPOT PRICE VOLATILITY OF STORABLE COMMODITIES

A Thesis
Submitted to the Graduate Faculty
of the
North Dakota State University
of Agriculture and Applied Science

By

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In Partial Fulfillment of the Requirements
for the Degree of
MASTER OF SCIENCE

Major Department:
Agribusiness and Applied Economics

April 2019

Fargo, North Dakota

North Dakota State University
Graduate School

Title

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The Supervisory Committee certifies that this ***disquisition*** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

MASTER OF SCIENCE

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ABSTRACT

This thesis examines the relationship between spot prices, futures prices, and ending stocks for storable commodities. We used Granger causality and DAGs to determine causal relationships and cointegration tests to determine long-run relationships. We use VAR/VECM and consider innovation accounting techniques to see how volatility in one market affects the price behavior and volatility in the other market. Results suggest that for agricultural commodities, innovations in futures price permanently increase the level of spot prices while accounting for much of spot price variance over time. For national oil, shocks to futures price decrease the level of spot price in the long run. In regional oil markets, there are transitory impulse responses. Futures price plays a small role in the volatility of spot prices for oil over time. Overall results are mixed, with oil suggesting futures markets may have a price stabilizing effect and agriculture commodities indicating spot price destabilization.

ACKNOWLEDGMENTS

Research is a more difficult task than I had envisioned, but I am thankful that I had some great people to support me in the process of writing this thesis. First I must thank my advisor, Dr. Dragan Miljkovic; without his vision, this thesis would not have been what it eventually became. His mentorship, technical expertise, and encouragement along the way were invaluable to me, and I thank him for that. I must also thank Dr. Lei Zhang for her helpful comments and criticisms, and for serving on the thesis committee during a busy time in her life. Thirdly, thank you to Dr. Nikita Barabanov, whose help was instrumental to my understanding of much of the mathematics within this thesis. I would also like to thank my family who has supported me in the decision to pursue graduate school and throughout these last two years. Finally, thank you to Mckenna Frenzel for supporting me always, but especially during the process of writing this thesis.

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1. INTRODUCTION

Commodities, particularly energy and agricultural commodities, are known to have volatile markets that exhibit large price fluctuations. We can look back only a few years for an example of this extreme volatility; in July of 2014 the spot price of WTI crude at Cushing, OK was around $100 per barrel, by early January 2015 the price had declined by over half and was less than $50 per barrel. Other commodities tend to exhibit the same types of volatility. The nature and causes of commodity price volatility have been explored in academic literature as well as debated among politicians, market participants, and the general public.

Futures markets have received much widespread attention over the last few years. This is partly because national and international commodity markets have become more integrated and partly because of the increased financialization of the commodity markets with commodities more frequently being included as an asset class as part of a diversified investment portfolio. The general public sentiment with regards to futures trading appears to be more negative than positive with the widespread belief that speculation within futures markets leads to adverse effects on the market. However, much of the literature and empirical evidence suggests that futures trading, to include speculative trading, allows for price stabilization.

It is believed that organized futures markets developed organically to meet the needs of market participants and as a response to the price volatility seen in commodities (Telser and Higinbotham 1977). The participants within the commodity markets faced not only price risk, but quantity and quality risk, and were searching for avenues to reduce their marketing risk and make trade more efficient. It is unclear exactly when and where standardized futures trading began. However, some trace its roots back to the Dōjima Rice Market in Japan where trading standardized rice contracts began in 1730 (Kolb and Overdahl 2006; Schaede 1989). The trading of futures contracts in the U.S. is thought to have developed for the grain trade in Chicago in the 1860s. Since then, futures trading has taken off on a global scale with contracts for many different commodities and financial instruments as well as exchanges in the U.S. and around the world.

These futures markets allow for price discovery by market participants, the smoother allocation of commodities over time, as well as the transfer of risk from hedgers to speculators. Price discovery is the process of participants making bids to buy and sell the commodity which allows for the market forces to reveal information about future prices of a commodity. This process enables market participants to know the going rate for a commodity in the future, and the transparency of an organized market also allows for information to be disseminated to non-market participants. Since futures contracts for storable commodities specify the delivery of a product at some point in the future, these contracts can link someone who has the commodity to someone who needs the commodity at a set point in the future. This allows economic agents to link supply and demand needs of products while allowing for a smoother allocation of commodities. Finally, futures markets allow speculators to enter in search of profit. Since hedgers are looking to alleviate a portion of their risk, speculators provide liquidity to the market by taking the opposite side of a transaction. In doing so, speculators bear some risk of adverse price changes by providing insurance to hedgers. These speculators are generally happy to bear this risk since they believe they can profit. Thus, the distribution of products through time, price discovery and risk transfer are believed to alleviate some of the erratic price movements, or volatility, that is common in the commodity markets. However, this price stabilizing belief does not always play out in practice. For this reason, we examine the relationship between the futures price and spot prices for oil and agricultural commodities.

The objective of this study is to add to the body of literature looking at how futures markets affect spot price behavior. While not conclusive, much of the literature in this area suggests that futures markets either have no effects on spot prices or they have a price stabilizing effect. We are able to apply a methodology that, to our knowledge, has not been applied to this particular area of research.

Our study is focusing on four storable commodities: soybeans, corn, hard red spring wheat, and crude oil. We consider monthly data for futures prices, spot prices, and ending stocks. For the agricultural commodities, we are looking at spot price and stock data for North Dakota, and for oil, we look at spot price and stock data nationally as well as regionally for four of the five PAD districts. We utilize VAR/VECM as well as impulse responses and variance decompositions to examine the relationships among the variables.

Our agricultural results do not agree with much of the conclusions from previous studies in this area. For the agricultural commodities, we find that futures markets increase the level of spot prices and they account for a large portion of the variability of spot prices over time. In addition, this increased price level remains elevated up to 36 months with no sign of a decrease, leading us to conclude these effects are likely permanent. The oil results seem to conform more to the previous literature, with futures markets decreasing the level of spot prices in the long run at the national level, and futures markets accounting for a much smaller portion of that volatility overall and over time compared to the agricultural commodities. The PADD's results indicate transitory increases in spot price levels with futures prices accounting for a small portion of volatility.

The remainder of the thesis is organized as follows. Section two will discuss the relevant literature. In section three we discuss the theoretical model and its derivation. The data and data construction methods are discussed in the fourth section. The empirical methodology is discussed in the fifth section, to include unit root tests, cointegration tests, Granger causality and directed acyclic graphs, VAR/VECM, as well as impulse responses and variance decomposition. Finally, the results of our estimated models are discussed in section 6 and concluding remarks in section 7.

2. LITERATURE REVIEW

There are generally two views on commodity prices, the theory of storage and the idea that futures prices can be fragmented into an expected risk premium and an expected future spot price (Fama and French 1987). The theory of storage suggests that the basis, or the difference between the futures price and spot price, can be explained by the interest rate, the storage cost and the convenience yield (Kaldor 1939, Working 1948; Brennan 1958; Telser 1958). The convenience yield is the return of holding inventory, which arises because of inventories' role in reducing production and marketing costs as well as the reduced risk of stockout (Pindyck 2001). The idea that futures price is made up of a risk premium and the expected future spot price can be expressed in basis form, where the basis is equal to the sum of the expected risk premium and an expected change in spot price. Fama and French (1987) examined these two competing theories of commodity pricing and found that basis varies in response to interest rates and convenience yields, lending support for the theory of storage. They also found forecast power for ten commodities, and time-varying expected premiums for five commodities.

Grossman (1977) showed with the introduction of futures markets, there is a benefit to insurance, and the transfer of information from informed to uninformed traders allows for the better allocation of goods through time. Pindyck (2001) explains the short run dynamics of commodity markets and shows how spot prices, futures prices, and inventories are related. He examines these market dynamics and interrelationships for crude oil, heating oil, and gasoline.

In 1958, the U.S. banned futures trading in onions due to the belief that they increased variability in cash prices so much so that it outweighed the hedging benefits that futures provide. Working (1960) examined the onion market and found that, in years following World War II where there was substantial hedging activity, there was reduced intraseasonal and intramonth variability in spot onion prices. Gray (1963) compared seasonal ranges of onion spot prices for four different time periods, the period 1922-1941 which preceding futures trading, a period 1942-1949 with futures trading but with little trading activity, the period 1949-1958 where there was significant futures trading activity, and the period 1958-1962 for the period after futures trading was banned. Gray found that the two periods without futures trading and the period with little trading activity exhibited similar and larger seasonal price ranges than the price range found in the period where there was significant futures trading activity. Johnson (1973) also examined the onion market for the period 1930-1968. He updated both Workings' and Grays' previous studies with more recent data and found that the period of substantial hedging activity and the period following the ban of futures trading showed similar price patterns for both time periods. Johnson ultimately conducted his own analysis by using a few different methods of price performance. Ultimately, taking all results together, Johnson concluded that futures markets have no significant effect on price performance.

Powers (1970) analyzed pork bellies and live cattle weekly cash prices using the variate difference method for four years prior to futures trading and four years after the introduction of futures trading; Powers found that after the introduction of futures trading, the variance of the random element of prices was reduced for both live cattle and pork bellies. Tomek (1971) examined wheat for two different 20-year periods, 1841-1860 to represent a period before futures trading and 1891-1910 to represent a period after futures trading had been introduced. Tomek found the average cash price difference for each month in each period and discovered in 10 out of 12 months the average difference was smaller for the latter period with futures trading. Taylor and Leuthold (1974) examined cash live cattle where they analyzed Chicago and Omaha cash price for 1957-1964, a period prior to live cattle futures markets, and 1965-1972 for a period after the introduction of futures. They found that the monthly and weekly cash live cattle price variance was significantly reduced for the period 1965-1972 while the annual variation was reduced but not significantly changed. The price effects of futures trading for onions, potatoes, pork bellies, lean hogs, live cattle, and frozen concentrated orange juice were investigated by Cox (1976). Cox found that in periods of futures trading, market information was increased, and the markets were more efficient, with six of seven commodities having a reduced coefficient of variation with futures markets. Cattle and Hog prices were also examined by Tomek (1980) by the variate difference method, finding that on balance, futures trading does not have a significant effect on the variability of the random price component. Brorsen et al. (1989) studied the period 1957-1982 using daily data and found that the presence of live cattle futures increased the standard deviation of the daily cash market changes. Weaver and Banerjee (1990) found that live cattle futures did not affect the cash market volatility. Antoniou and Foster (1992) utilized a GARCH model with weekly data before and after the introduction of futures trading for Brent crude oil to find that futures had no effect of the spot market volatility while improving the efficiency of the spot market. Netz (1995) using data from 1858-1890 found that the introduction of wheat futures caused a significant decline in the coefficient of variation of spot price.

Futures trading for the housing/mortgage industry was introduced in 1975 when futures contracts based on mortgage-backed bonds guaranteed by the Government National Mortgage Association (GNMA) were introduced. These GNMA futures have been studied several times to see the effects of the futures on the spot market. Froewiss (1978) regressed weekly percent changes in spot GNMA prices against ten-year U.S. government bonds for before and after the introduction of futures trading. Froewiss concludes that the introduction of futures markets has not had a destabilizing effect of the spot market. Contrary to Froewiss, Figlewski (1981) found that GNMA futures trading led to increased volatility in the GNMA spot market. Futures trading had no effect on the GNMA spot market volatility in the study by Simpson and Ireland (1982). Corgel and Gay (1984) used intervention analysis to determine that the GNMA cash market saw a significant decline in volatility after the introduction of futures trading.

There have also been several studies done in relation to financial futures other than GNMA. Bortz (1984) found that treasury bond futures decrease the volatility of the treasury bond spot market. Simpson and Ireland (1985) found that futures markets had no effect on the cash market for treasury bills. Harris (1989) found that the introduction of S&P 500 index futures had no economically significant effect on the S&P 500 stocks' cash market volatility. Gulen and Mayhew (2000) examined stock market volatility in 25 countries before and after the introduction of equity index futures trading. For the U.S. and Japan, futures trading increased volatility while volatility was either unchanged or reduced in all other countries. Board et al. (2001) examined the effect of futures trading volume on the volatility of the equity spot market. They use a stochastic volatility (SV) model for the study. SV models, unlike a GARCH, assume that an unobserved factor is the driver of conditional volatility. They conclude that futures trading has no effect on the spot market volatility, and spot trading has no effect on spot market volatility.

Dimpfl et al. (2017) looked at the relationship between spot and futures prices for corn, wheat, soybeans, soybean oil, soybean meal, feeder cattle, live cattle, and lean hogs. They use information share methodology of Hasbrouck to estimate the contribution of spot and futures prices to price discovery. For all commodities Dimpf et al. find that the spot market is the primary determinant of the long run efficient price and futures markets contribute less than 10% to the common efficient price variance showing that futures markets play only a small role in price discovery. Thus, the authors conclude that futures speculation does not distort spot markets in the long run.

Irwin et al. (2009) claim the argument that speculation is the cause for bubbles in commodity prices is false. The authors make a few points to support their position: (1) critics of speculators have a misunderstanding of how futures markets work, (2) in times of price volatility, activity in futures markets has not been "excessive", (3) Granger causality tests show there is no causation between futures price changes and position changes. Buyuksahin and Harris (2011) used Granger causality to find that hedge funds and noncommercial traders position changes do not Granger cause crude oil price changes. Sanders and Irwin (2011) examine commodity index positions for corn, soybeans, as well as Kansas City and Chicago wheat for 2004-2009. Granger causality suggests no causal relationship between commercial index positions and commodity price changes while long-horizon regressions cannot reject the null hypothesis that commercial index positions have no impact on futures prices. The literature related to speculation and oil prices was surveyed by Fattouh et al. (2013). They find that the evidence does not support the idea that speculation drives oil spot price and instead conclude that the prices within the oil market are reflected by economic fundamentals. Killian and Murphy (2014) utilize a VAR to examine the oil markets and account for the role of inventories, which much of the previous literature has not done. They show that the business cycle is the main driver of the real oil price and the drastic increase in oil price from 2003-2008 was primarily driven by shifts in the demand for oil consumption and not speculation.

There are a few studies that examine how futures trading/speculation affects the volatility of spot prices. Crain and Lee (1996) studied how thirteen different government farm programs over the period 1950-1993 have affected wheat spot and futures price volatility. They observed that spot and futures volatility generally move together and for a majority of the period spot volatility was higher than futures volatility. Among other things, using Granger causality, they found evidence that volatility moves from the futures market to the spot market for wheat. Less prominently, volatility in the spot market also Granger causes futures market volatility. However, futures volatility has a larger and more persistent impact on spot volatility. Yang et al. (2005) studied futures trading volume and cash price volatility with Granger causality and forecast error variance decomposition. They examine seven commodities (corn, soybeans, sugar, wheat, cotton, hogs, and cattle) in two different sub-periods (1992-1995, 1997-2001) and found that unexpected futures trading volume Granger causes cash price volatility for all seven commodities in both subperiods. Looking at the Indian markets, Sehgal et al. (2012) filtered futures trading volume into expected and unexpected components using the Hodrick-Prescott filter, and then used Granger causality and GARCH to analyze the effect of futures trading volume on spot price volatility of guar seeds, turmeric, soya bean, black pepper, barley, maize, and castor seed over the period of April 2004 to March 2012. The authors found that unexpected increased futures trading volume is associated with increased spot price volatility for 5 out of the 7 commodities they studied. In the case of black pepper, the reverse was observed where an increase in spot price volatility tended to affect futures trading volume.

Bohl and Stephan (2013) examine whether futures speculation destabilizes commodity spot price. GARCH is used to examine how conditional spot price volatility is affected by speculative open interest for corn, crude oil, natural gas, soybeans, sugar, and wheat over two different ten-year periods, Oct. 1992-Sep. 2002 and Oct. 2002-Sep. 2012. They are not able to find evidence that growing futures speculation destabilize commodity spot price. Futures markets speculation and spot price destabilization was also tested by Kim (2015) for 14 different commodities. Kim used a GARCH model to measure the effect of futures trading activity on spot volatility. Kim finds that speculative trading activity either decreases or has no significant effect on spot price volatility except for live cattle. Sharma and Malhotra (2015) also examined the effect of futures trading activity on the volatility of the guar seed spot market. They also use Granger causality and find that an unexpected increase in futures volume precedes increased volatility in the guar seed spot market. Using a GARCH model, they find that there is a positive relationship between futures trading volume and spot price volatility.

Gupta and Varma (2016) looked at how futures markets affect the spot market for Indian rubber. Using Granger causality, they found two-way causality between futures volatility and spot volatility. They also found two-way Granger causality between spot volatility and futures volume. Mayer et al. (2017) looked at the effects of futures trading on spot price volatility for metals; specifically, copper, gold, palladium, platinum, and silver over the period of January 1993 – December 2013. The authors used Granger causality tests to examine the causal relationship between trading activity and spot price volatility. To observe relationships between trading positions and volatility, Mayer et al. use an EGARCH model. Their results suggested that there is less evidence that futures trading activity has a substantial effect on spot prices and volatility, but there is stronger evidence to suggest that spot prices and volatility drive changes in trading activity.

The application of DAGs has been gaining traction in the economics literature, especially in conjunction with VAR and ECM. Bessler and Akleman (1998) use a DAG and VAR to examine retail beef and pork prices. Roh and Bessler (1999) use a DAG to show that vehicle occupant death is caused by vehicle safety devices, income, and vehicle mileage. Bessler et al. (2003) look at the relationships of five international wheat markets with DAG's, VAR and VECM's, finding U.S. and Canada are the leaders in pricing wheat. Bessler and Yang (2003) employ DAG's to determine causal orderings of innovations for VAR and ECM for analysis of the world's largest stock markets.

Awokuse and Bessler (2003) use DAG's and VAR to replicate Sims' 1986 model of the U.S. Economy; interestingly, they present DAG's with a significance level of up to 30% to achieve an unambiguous causal path. Grain prices in Illinois, grain prices at the U.S. Gulf, and the barge market were examined with DAG's and ECM's by Haigh and Bessler (2004), discovering the Illinois grain market is strongly affected by the barge and commodity export markets. Awokuse (2005) used DAG's and VAR/ECM to examine how macroeconomic policy effects agricultural prices. Yang et al. (2006) use DAG's along with VAR to investigate the transmission of inflation among G-7 nations, finding U.S. inflation to have a large effect of other nations inflation. Refalo (2009) found that China had little impact on the price volatility of international oil markets using DAGs and ECM. Ji (2012) utilized DAGs along with partial least squares and VECM to look at what mechanisms are driving crude oil prices. Li et al. (2013) used DAG's to examine foreign direct investment and economic growth, finding economic growth causes foreign direct investment into developing countries, as well as foreign direct investment causing economic growth in developed countries. Ji and Fan (2015) examined five international oil markets using DAG and ECM, with results suggesting oil markets have diverged from equilibrium since 2010 and WTI beginning to reflect local supply/demand conditions. Miljkovic et al. (2016) applied DAGs to show direct causal relations among variables within the energy complex and illustrate endogeneity issues among variables. Xu (2017) employed DAG's and ECM to examine corn prices across seven Midwestern states and found that Iowa dominated corn pricing throughout the crop year. Ji et al. (2018) look at the drivers of natural gas price using a DAG and ECM and find oil price causes natural gas price.

3. ECONOMIC MODEL

We follow Kawai (1983) to derive our economic model. The use of this particular model is justified since it follows from Muth's (1961) work on rational expectations and it takes into account consumption, production, and storage while explaining the behavior of spot price volatility in the absence of futures markets as well as in the presence of futures markets. Thus, following Kawai, we derive mathematically the decision-making problems faced by risk-averse price taking consumers, producers, inventory holding dealers, and pure speculators.

3.1. Agents Optimizing Behavior

3.1.1. Consumer

The price taking consumer can maximize utility subject to a budget constraint to obtain the demand for a commodity at time t. Their demand can be expressed as,

$$C_t^i = a_0^i - a^i s_t + \varepsilon_t^i \tag{1}$$

where C_t^i is the demand, i is an individual consumer, a_0^i and a^i are fixed constants, s_t is the spot price at time t, and ε_t^i is a disturbance term that represents an individual's characteristics.

3.1.2. Commodity Producer

Producers make a production decision at time t for an output that will be produced at time $t+1$. When the decision is made at time t, the spot price in period $t+1$, s_{t+1}, is not known. However, the output in period $t+1$, Q_{t+1}, is known. We assume the producer holds no inventories. The producer will maximize their expected utility of profit:

$$E_t U^p\left(\Pi_{t+1}^p\right) \tag{2}$$

$$\text{s.t.} \;\; \Pi_{t+1}^p = s_{t+1} Q_{t+1}^p - \rho G\left(Q_{t+1}^p\right) \tag{3}$$

where p is an individual producer, E_t is the expectation operator, $U^p(\cdot)$ is a strictly concave von Neuman-Morgenstern utility function, Π_{t+1}^p is producer's profit at time $t+1$, $G(\cdot)$ is a strictly convex cost function which is known to the producer at time t, and ρ is the market rate of interest plus one. This market rate of interest plus one, ρ, is utilized since capital resources are committed at time t and profit is not realized until $t+1$.

We assume the cost function is quadratic to ensure a linear form of commodity production can be obtained. Thus, the cost can be represented as $G\left(Q_{t+1}^p\right) = \frac{1}{2}g\left(Q_{t+1}^p + \varepsilon_t^p\right)^2$, $g > 0$, where ε_t^p is a disturbance affecting the cost function. Thus, the new profit can be represented as:

$$\Pi_{t+1}^p = s_{t+1}Q_{t+1}^p - \frac{\rho g}{2}\left(Q_{t+1}^p + \varepsilon_t^p\right)^2 \tag{4}$$

We also assume constant absolute risk aversion (CARA), implying an agent will hold less risky assets as wealth increases. Hence our utility function has the form,

$$U^p(\Pi) = -e^{-r^p \Pi} \tag{5}$$

where r^p is the Arrow-Pratt coefficient of absolute risk aversion. Thus, the larger r^p is, the more risk averse the agent is. The final assumption is that s_{t+1} is normally distributed. This gives our expected utility function the form,

$$E_t U^p\left(\Pi_{t+1}^p\right) = -e^{-r^p \mathrm{E_t} \Pi_{t+1}^p + \frac{1}{2}(r^p)^2 V_t \Pi_{t+1}^p} \tag{6}$$

where V_t is the conditional variance operator such that $V_t\Pi_{t+1}^p$ is defined as $E_t\left(\Pi_{t+1}^p - E_t\Pi_{t+1}^p\right)^2$.

Mathematically, the same result as maximizing the expected utility, $E_t U^p\left(\Pi_{t+1}^p\right)$, can be achieved by maximizing:

$$E_t\Pi_{t+1}^p - \frac{1}{2}r^p V_t \Pi_{t+1}^p \tag{7}$$

Thus, we must maximize $E_t(s_{t+1})Q_{t+1}^p - \frac{\rho g}{2}\left(Q_{t+1}^p + \varepsilon_t^p\right)^2 - \frac{1}{2}r^p V_t(s_{t+1})\left(Q_{t+1}^p\right)^2$. Taking the partial derivative with respect to Q_{t+1}^p and solving yields the optimal production quantity,

$$Q_{t+1}^p = \frac{E_t s_{t+1} - \rho g \varepsilon_t^p}{\rho g + r^p V_t s_{t+1}} \tag{8}$$

which shows optimal production is positively related to the expected spot price, $E_t s_{t+1}$. It is inversely related to the discount factor ρ, the cost function g, the cost function disturbance ε_t^p, the risk aversion coefficient r^p, and the variance of spot price, $V_t s_{t+1}$.

We can now consider maximizing expected utility in the presence of futures trading. The producer can enter into a forward contract with a known price at time t to deliver or receive delivery of a quantity of the commodity at time $t+1$. The producer will now maximize $E_t U^p\left(\Pi_{t+1}^{p*}\right)$ where:

$$\Pi_{t+1}^{p*} = s_{t+1}Q_{t+1}^{p*} - \rho G\left(Q_{t+1}^{p*}\right) + R_t^p(s_{t+1} - f_t) \tag{9}$$

The quantity of futures contracts, R_t^p, can be positive, negative, or zero. The futures price f_t is for delivery in period $t+1$. The presence of the futures market is denoted by $*$ in the superscript. Assumptions are the same as before with the cost function being quadratic, constant absolute risk aversion, and normally distributed spot price. Thus, we must maximize:

$$E_t(s_{t+1})\left(Q_{t+1}^{p*} + R_t^p\right) - \frac{\rho g}{2}\left(Q_{t+1}^{p*} + \varepsilon_t^p\right)^2 - R_t^p f_t - \frac{1}{2}r^p\left(Q_{t+1}^{p*} + R_t^p\right)^2 V_t s_{t+1} \tag{10}$$

We find the optimal number of futures contracts by taking the partial derivative with respect to R_t^p to obtain,

$$R_t^p = -Q_{t+1}^{p*} + Z_t^p \tag{11a}$$

where,

$$Z_t^p = \frac{E_t s_{t+1} - f_t}{r^p V_t s_{t+1}} \tag{11b}$$

Substituting the optimal futures contracts back into the expected utility and taking the partial derivative with respect to Q_{t+1}^{p*} yields the optimal production quantity:

$$Q_{t+1}^{p*} = \frac{f_t}{\rho g} - \varepsilon_t^p \tag{11c}$$

Thus, the optimal production depends on the futures price f_t, the cost function disturbance ε_t^p, the discount factor ρ, and the cost function coefficient g. The optimal quantity of futures contracts is the sum of Z_t^p and the negative quantity of production. We can see that the optimal production does not depend on attitudes of risk or expected spot price and is independent of the futures trading decisions.

The quantity of futures contracts is divided into two parts; the hedging component which is the opposite of the production decision, $-Q_{t+1}^{p*}$, and the speculative component Z_t^p. The speculative component reflects the producer potential gain per unit of the commodity purchased in futures which is the difference of the expected future spot price $E_t s_{t+1}$ and futures price f_t. Therefore, the producer uses futures trading to hedge as well as to earn speculative profits when the opportunity arises.

3.1.3. Inventory Holding Dealer

When commodities are storable, there may exist agents who hold inventories of commodities from period to period. We can call these agents inventory holding dealers. When we take the case of a risk-averse, price taking dealer in the absence of futures trading, the dealer maximizes $E_t U^d\left(\Pi_{t+1}^d\right)$ such that,

$$\Pi_{t+1}^d = s_{t+1} I_t^d - \rho s_t I_t^d - H\left(I_t^d\right) \tag{12}$$

where d represents an individual dealer, I_t^d is the stock of commodity inventory purchased in time t, and $H(\cdot)$ is the holding cost of inventory with the usual convexity property. Like before, the discount factor ρ is multiplied to the purchase cost of the commodity in time t. Kawai assumes the holding cost of inventory is made up of the cost of deviating from a target level of inventory $\bar{I}^d + \varepsilon_t^d$ (where ε_t^d is a disturbance affecting the target stock at time t) and also has a quadratic form $H\left(I_t^d\right) = \frac{1}{2}h\left(I_t^d - \bar{I}^d - \varepsilon_t^d\right)^2, h > 0$. The holding cost is the difference between the direct cost of holding inventories of the commodity which is increasing and convex in I_t^d, and the benefit of carrying a larger inventory which reduces the probability of stockout and the loss of customers.

With the quadratic holding cost, a CARA utility function, and normally distributed spot price, we can maximize:

$$E_t(s_{t+1})I_t^d - \rho s_t I_t^d - \frac{1}{2}h\left(I_t^d - \bar{I}^d - \varepsilon_t^d\right)^2 - \frac{1}{2}r^d V_t(s_{t+1})I_t^2 \tag{13}$$

Taking the partial derivative with respect to I_t^d yields the optimal inventory,

$$I_t^d = \frac{h\left(\bar{I}^d + \varepsilon_t^d\right) + E_t s_{t+1} - \rho s_t}{h + r^d V_t s_{t+1}} \tag{14}$$

which depends positively on the target level of inventory $\bar{I}^d + \varepsilon_t^d$, and the expected capital gain of holding a unit of a commodity $E_t s_{t+1} - \rho s_t$. It depends negatively on the holding cost coefficient h, the risk aversion coefficient r^d, and the spot price variance $V_t s_{t+1}$. Inventory carrying cost allows for the "convenience yield" for a small level of inventory ($I_t^d < \bar{I}^d + \varepsilon_t^d$) so that when the expected capital gain is negative ($E_t s_{t+1} - \rho s_t < 0$), a positive quantity of stocks can be held due to the convenience of holding physical stocks. The possibility of negative inventory exists but we assume it is not probable due to high convenience yield for carrying commodities forward through time.

When we add the possibility of futures trading the inventory holding dealer can enter into one period ahead futures contracts to get a profit function of,

$$\Pi_{t+1}^{d*} = s_{t+1}I_t^{d*} - \rho s_t I_t^{d*} - H\left(I_t^{d*}\right) + R_t^d(s_{t+1} - f_t) \tag{15}$$

with R_t^d being the quantity of futures contracts purchased or sold. Thus, like the producer, the inventory holding dealer can maximize expected utility by maximizing,

$$E_t(s_{t+1})I_t^{d*} - \rho s_t I_t^{d*} - \frac{1}{2}h\left(I_t^{d*} - \bar{I}^d - \varepsilon_t^d\right)^2 + E_t(s_{t+1})R_t^d - R_t^d f_t$$

$$-\frac{1}{2}r^d\left(I_t^d + R_t^d\right)^2 V_t(s_{t+1}) \tag{16}$$

which gives us the optimal futures contracts and optimal inventory demand:

$$R_t^d = -I_t^{d*} + Z_t^d \tag{17a}$$

where,

$$Z_t^d = \frac{E_t s_{t+1} - f_t}{r^d V_t s_{t+1}} \tag{17b}$$

and,

$$I_t^{d*} = \bar{I}^d + \varepsilon_t^d + \frac{f_t - \rho s_t}{h} \tag{17c}$$

The optimal futures position is again separated into a hedging component and a speculative component like the producer. It is determined by the opposite inventory decision, the expected spot price, futures price, risk aversion coefficient, and the variance of spot price. Optimal inventory depends on the current spot price, futures price, the discount factor, and the holding cost parameters.

3.1.4. Pure Speculator

When futures trading is introduced, it is possible that agents without underlying cash positions may enter the futures market in the hopes of obtaining profits. We would categorize these agents as pure speculators; the pure designation is used to indicate they are speculating on the price movements and not making any commitments to the physical commodity market. Because we assume futures trading to be costless (no transaction costs, no capital outlays, and no margin requirements) the size of a futures position is not subject to the speculator's capital constraints. The objective of the risk-averse pure speculator "s" is to maximize their expected utility $E_t U^s(\Pi_{t+1}^s)$ where their profit function can be represented by:

$$\Pi_{t+1}^s = Z_t^s(s_{t+1} - f_t) \tag{18}$$

Under CARA (with risk aversion coefficient r^s) and normally distributed spot prices the pure speculator is maximizing:

$$E_t(s_{t+1})Z_t^s - Z_t^s f_t - \frac{1}{2} r^s (Z_t^s)^2 V_t s_{t+1} \tag{19}$$

Maximization yields the optimal volume of speculation,

$$Z_t^s = \frac{E_t s_{t+1} - f_t}{r^s V_t s_{t+1}} \tag{20}$$

which is similar to the speculative component of both the producer and dealers' optimal futures positions except for the individual risk aversion characteristics.

3.2. Determining Spot and Futures Prices

3.2.1. Commodity Markets without Futures Trading

We can now determine the equilibrium prices by taking all individual agents supply and demand behaviors together. We assume homogeneity between agents within groups; i.e., consumers are similar with respect to their demand coefficients and disturbances, producers have identical cost functions and risk aversion coefficients, dealers have identical holding costs and risk aversion coefficients, and pure speculators have the same risk attitudes. Producers, dealers, and pure speculators have rational expectations in the sense of Muth (1961), which is to say they utilize all available market information to form expectations about the next period spot price. Information is symmetric among agents.

The scenario of no futures market is considered first. We can aggregate consumer demand, production quantity, and inventory demand equations over a fixed number of agents.

$$C_t = \alpha_0 - \alpha s_t + u_t \tag{21a}$$

$$Q_t = \frac{1}{\frac{\rho}{\beta} + \frac{\theta}{\upsilon}} \left(E_{t-1} s_t + \frac{\rho}{\beta} \upsilon_{t-1} \right) \tag{21b}$$

$$I_t = \frac{1}{\frac{1}{\gamma} + \frac{\theta}{v}} \left(\frac{1}{\gamma} \bar{I} + E_t s_{t+1} - \rho s_t + \frac{1}{\gamma} w_t \right) \tag{21c}$$

$$Q_t + I_{t-1} = C_t + I_t \tag{21d}$$

where $\alpha_0, \alpha, \beta, \upsilon, \gamma, v, \bar{I}, \theta, u_t, \upsilon_t,$ and w_t are defined as,

$$\alpha_0 = n^i a_0, \qquad \alpha = n^i a, \qquad \beta = \frac{n^p}{g}, \qquad \upsilon = \frac{n^p}{r^p}, \qquad \gamma = \frac{n^d}{h}, \qquad v = \frac{n^d}{r^d}, \tag{21e}$$

$$\bar{I} = n^d \bar{I}^d, \qquad \theta = V_t s_{t+1}, \qquad u_t = n^i \varepsilon_t^i, \qquad \upsilon_t = -n^p \varepsilon_t^p, \qquad w_t = n^d \varepsilon_t^d,$$

where the fixed number of consumers, producers, and dealers are n^i, n^p, and n^d. The sources of stochastic prices are the disturbances u_t, υ_{t-1}, and w_t in equations (21a), (21b), and (21c), (u_t, υ_{t-1}, and w_t are data at time t and thereafter). We assume $u_t, \upsilon_t,$ and w_t are all pairwise uncorrelated and serially independent with means 0 and variances $\sigma_u^2, \sigma_\upsilon^2,$ and σ_w^2. The random variables assumed serial independence makes $V_t s_{t+1}$ independent of time and allows us to treat θ as a constant.

When we substitute the aggregated equations (21a), (21b), and (21c) into the spot market clearing equation (21d), some algebraic manipulation yields:

$$-\frac{1}{\frac{1}{\gamma} + \frac{\theta}{v}} (E_t s_{t+1} - \rho s_t - E_{t-1} s_t + \rho s_{t-1}) + \alpha s_t + \frac{1}{\frac{\rho}{\beta} + \frac{\theta}{\upsilon}} (E_{t-1} s_t)$$

$$= -\frac{1}{\frac{\rho}{\beta} + \frac{\theta}{\upsilon}} \left(\frac{\rho}{\beta} \upsilon_{t-1} \right) - \frac{1}{\frac{1}{\gamma} + \frac{\theta}{v}} \left[\frac{1}{\gamma} (w_{t-1} - w_t) \right] + \alpha_0 + u_t \tag{22}$$

Multiplying both sides by $-\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)$ and manipulating yields equation (23):

$$E_t s_{t+1} - \left[\rho + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)\right] s_t - \left[1 + \frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right] E_{t-1} s_t + \rho s_{t-1}$$
$$= -\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)(\alpha_0 + u_t) + \frac{\rho}{\beta}\left(\frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right) v_{t-1} - \frac{1}{\gamma}(w_t - w_{t-1}) \tag{23}$$

Since we are operating in a rational expectation's framework, market participants know that the relationships in (23) always hold. The conditional expectations operator E_{t-1} applied to both sides of (23), with the assumption $E_{t-1} u_t = E_{t-1} w_t = 0$ yields,

$$E_{t-1}[s_{t+1} - (1 + \rho + \phi)s_t + \rho s_{t-1}] = E_{t-1}\left[-\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)\alpha_0 + \frac{\rho}{\beta}\left(\frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right) v_{t-1} + \frac{1}{\gamma} w_{t-1}\right] \tag{24a}$$

where ϕ is defined as

$$\phi = \left(\alpha + \frac{1}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right)\left(\frac{1}{\gamma} + \frac{\theta}{v}\right) > 0 \tag{24b}$$

Applying the lag operator L (where $L^k s_t = s_{t-k}$) to the term in the square brackets on the left hand side of equation (9a), $[s_{t+1} - (1 + \rho + \phi)s_t + \rho s_{t-1}]$, yields,

$$L^{-1}[1 - (1 + \rho + \phi)L + \rho L^2]s_t \tag{24c}$$

We let ϕ have the form,

$$\phi = \frac{(\rho - \lambda)(1 - \lambda)}{\lambda} \tag{25}$$

where λ satisfies the restriction $0 < \lambda < 1 < \frac{\rho}{\lambda}$. Factoring (24c), we can obtain the following:

$$L^{-1}\left[1 - \left(1 + \rho + \frac{(\rho - \lambda)(1 - \lambda)}{\lambda}\right)L + \rho L^2\right] s_t = L^{-1}\left(1 - \frac{\rho}{\lambda} L\right)(1 - \lambda L)s_t$$
$$= -\frac{\rho}{\lambda}\left(1 - \frac{\lambda}{\rho} L^{-1}\right)(1 - \lambda L)s_t \tag{26}$$

Recalling the general geometric series representation $\frac{1}{1-x}=\sum_{k=0}^{\infty}x^k$, we can divide both sides of (24a) by $-\frac{\rho}{\lambda}\left(1-\frac{\lambda}{\rho}L^{-1}\right)$ to get:

$$E_{t-1}(1-\lambda L)s_t = E_{t-1}\left[\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\lambda}-1}\alpha_0-\frac{\lambda}{\beta}\left(\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\sum_{j=0}^{\infty}\left(\frac{\lambda}{\rho}\right)^j v_{t-1+j}-\frac{\lambda}{\gamma\rho}\sum_{j=0}^{\infty}\left(\frac{\lambda}{\rho}\right)^j w_{t-1+j}+\varepsilon_t\right] \quad (27)$$

The process $E_{t-1}\varepsilon_t=\frac{\rho}{\lambda}\varepsilon_{t-1}$ is denoted by ε_t, which is explosive except for the case when $\varepsilon_t=0$. For $\varepsilon_t=0$, we can eliminate speculative bubbles and ensure a unique path for $E_{t-1}s_t$. For $j>0$, $E_{t-1}v_{t-1+j}=E_{t-1}w_{t-1+j}=0$. Thus, letting $\varepsilon_t=0$, $j=0$, and solving for $E_{t-1}s_t$ yields,

$$E_{t-1}s_t=\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\lambda}-1}\alpha_0+\lambda s_{t-1}-\frac{\lambda}{\beta}\left(\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)v_{t-1}-\frac{\lambda}{\gamma\rho}w_{t-1} \quad (28)$$

We can move (28) ahead one period to obtain $E_t s_{t+1}$:

$$E_t s_{t+1}=\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\lambda}-1}\alpha_0+\lambda s_t-\frac{\lambda}{\beta}\left(\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)v_t-\frac{\lambda}{\gamma\rho}w_t \quad (29)$$

Working towards the rational expectation's equilibrium solution for spot price, we substitute (29) into (23) which yields:

$$\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\lambda}-1}\alpha_0+\lambda s_t-\frac{\lambda}{\beta}\left(\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)v_t-\frac{\lambda}{\gamma\rho}w_t-\left[\rho+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)\right]s_t-\left[1+\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right]E_{t-1}s_t+\rho s_{t-1}$$

$$=-\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)(\alpha_0+u_t)+\frac{\rho}{\beta}\left(\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)v_{t-1}-\frac{1}{\gamma}(w_t-w_{t-1}) \quad (30)$$

Manipulating (30) we can obtain,

$$\left[\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)\right]s_t=-\left[1+\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right]E_{t-1}s_t+\rho s_{t-1}+\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)(\alpha_0+u_t)-\frac{\rho}{\beta}\left(\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)v_{t-1}$$

$$+\frac{1}{\gamma}(w_t-w_{t-1})+\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\lambda}-1}\alpha_0-\frac{\lambda}{\beta}\left(\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)v_t-\frac{\lambda}{\gamma\rho}w_t \quad (31)$$

Substituting (28) into (31), dividing both sides by $\left[\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)\right]$, and simplifying allows us to obtain,

$$
\begin{aligned}
s_t = & \left[\frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right)\right]\frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\lambda} - 1}\alpha_0 \\
& + \left[\frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right)\right]\lambda s_{t-1} + \frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)u_t \\
& - \frac{\lambda}{\beta\left(\frac{\rho}{\beta} + \frac{\theta}{v}\right)}\left[\frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)v_t\right. \\
& \left. + \left\{\frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right)\right\}\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)v_{t-1}\right] \\
& + \frac{1}{\rho}\left[\frac{\frac{\rho - \lambda}{\gamma}}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}w_t\right. \\
& \left. - \left\{\frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right)\right\}\frac{\lambda}{\gamma}w_{t-1}\right] \qquad (32)
\end{aligned}
$$

Letting $\eta = \frac{\rho - \lambda}{\gamma\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}$ gives us:

$$s_t = \left[\frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right)\right]\frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\lambda} - 1}\alpha_0$$

$$+ \left[\frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right)\right]\lambda s_{t-1} + \frac{1}{\alpha + \gamma\eta}u_t$$

$$- \frac{\lambda}{\beta\left(\frac{\rho}{\beta} + \frac{\theta}{v}\right)}\left[\frac{1}{\alpha + \gamma\eta}v_t + \left\{\frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right)\right\}\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)v_{t-1}\right]$$

$$+ \frac{1}{\rho}\left[\frac{\eta}{\alpha + \gamma\eta}w_t - \left\{\frac{1}{\rho - \lambda + \alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma} + \frac{\theta}{v}}{\frac{\rho}{\beta} + \frac{\theta}{v}}\right)\right\}\frac{\lambda}{\gamma}w_{t-1}\right] \tag{33a}$$

Now setting $\bar{s} = \frac{\frac{1}{\frac{1}{\gamma}+\frac{\theta}{v}} - \frac{\lambda}{(\rho-\lambda)\left(\frac{\rho}{\beta}+\frac{\theta}{v}\right)}}{\alpha+\lambda\frac{1}{\frac{\rho}{\beta}+\frac{\theta}{v}}}\alpha_0$, and simplifying (33a) yields,

$$s_t = \bar{s} + \frac{1}{\alpha+\gamma\eta}\left(\sum_{j=0}^{\infty}\lambda^j\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^j u_{t-j} - \frac{1}{\beta\left(\frac{\rho}{\beta}+\frac{\theta}{v}\right)}\left[\lambda v_t + \left\{\alpha\left(\frac{1}{\gamma} + \frac{\theta}{v}\right) + \rho\right\}\cdot\right.\right.$$

$$\left.\sum_{j=1}^{\infty}\lambda^j\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 - \frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^j v_{t-j}\right] + \frac{1}{\rho}\left(\eta w_t - \left\{\frac{\alpha}{\gamma}\right\}\cdot\sum_{j=1}^{\infty}\lambda^j\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda} - 1 -\right.\right.\right.$$

$$\left.\left.\left.\left.\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^j w_{t-j}\right)\right) \tag{33b}$$

Thus, we have the rational expectations equilibrium spot price. Previously, we assumed normal random prices. We can see that s_t is normally distributed if $u_t, v_t,$ and w_t are also normally distributed, confirming the previous assumption. To get the equilibrium excepted spot price (34) and variance (35) T periods forward, which is conditional on information at time t, we can lead (33b) forward T periods, for $T \geq 1$:

$$E_t s_{t+T} = \bar{s} + \frac{1}{\alpha+\gamma\eta}\left\{\lambda^T\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^T\right\}\cdot\left(\sum_{j=0}^{\infty}\lambda^j\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^j\cdot u_{t-j}-\frac{\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)+\rho}{\beta\left(\frac{\rho}{\beta}+\frac{\theta}{v}\right)}\cdot\sum_{j=0}^{\infty}\lambda^j\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^j\cdot v_{t-j}-\frac{\alpha}{\gamma\rho}\cdot\sum_{j=0}^{\infty}\lambda^j\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^j\cdot w_{t-j}\right) \tag{34}$$

$$V_t s_{t+T} = \left(\frac{1}{\alpha+\gamma\eta}\right)^2\cdot\left(\frac{1-\lambda^{2T}\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2T}}{1-\lambda^2\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^2}\sigma_u^2+\frac{1}{\beta^2\left(\frac{\rho}{\beta}+\frac{\theta}{v}\right)^2}\left[\left\{\rho-\gamma\eta\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)\right\}^2+\left\{\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)+\rho\right\}^2\frac{\lambda^2\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^2-\lambda^{2T}\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2T}}{1-\lambda^2\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^2}\right]\sigma_v^2+\left[\left(\frac{\eta}{\rho}\right)^2+\left(\frac{\alpha}{\gamma\eta}\right)^2\cdot\frac{\lambda^2\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^2-\lambda^{2T}\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2T}}{1-\lambda^2\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^2}\right]\sigma_w^2\right) \tag{35}$$

3.2.2. Commodity Markets with Futures Trading

When we introduce the possibility of futures trading, commodity producers and inventory holding dealers may have their decisions altered by the ability to enter into forward contracts to maximize expected utility. Futures trading also makes it possible for pure speculators to enter the market to maximize their utility. Like before, we can aggregate consumer demand (36a), production (36b), and inventory demand (36c) over a fixed number of agents as well as aggregated futures speculation (36d). We also have the overall spot market clearing (36e) and futures market clearing condition's (36f):

$$C_t^* = \alpha_0 - \alpha s_t^* + u_t \tag{36a}$$

$$Q_t^* = \frac{\beta}{\rho} f_{t-1} + v_{t-1} \tag{36b}$$

$$I_t^* = \bar{I} + \gamma(f_t - \rho s_t^*) + w_t \tag{36c}$$

$$Z_t = \frac{\chi}{\theta^*}(E_t s_{t+1}^* - f_t) \tag{36d}$$

$$Z_{t-1} = C_t^* + I_t^* \tag{36e}$$

$$Q_{t+1}^* + I_t^* = Z_t \tag{36f}$$

where s_t^* is the spot price in the presence of a futures market and χ and $\theta^*$are defined as:

$$\chi = \upsilon + v + \omega, \qquad \omega = \frac{n^s}{r^s}, \qquad \theta^* = V_t s_{t+1}^*, \tag{36g}$$

We defined α_0, α, β, υ, γ, v, and $\bar{I}$ in (21e), stated again:

$$\alpha_0 = n^i a_0, \qquad \alpha = n^i a, \qquad \beta = \frac{n^p}{g}, \qquad \upsilon = \frac{n^p}{r^p}, \qquad \gamma = \frac{n^d}{h}, \qquad v = \frac{n^d}{r^d}, \qquad \bar{I} = n^d \bar{I}^d$$

Here, $n^s$is defined as the fixed number of pure speculators. Proceeding in a similar fashion as before, we will see that in general, the conditional variance of spot price in the presence of a futures market, θ^*, is different than the conditional variance of spot price without a futures market, θ. We will also see that θ^* is independent of time when we assume $u_t, v_t,$ and w_t have constant variance ($\sigma_u^2, \sigma_v^2,$ and σ_w^2).

Consumer demand (36a) is identical to before. Summing (11c) and (17c) over identical agents gives us the production (36b) and inventory demand (36c) equations. We can see that unlike before [eq. (21b) and (21c)], production and inventory demand do not depend on price uncertainty. We showed previously there are speculative components in the futures contract decisions for producers and inventory holding dealers. Thus, to obtain the market demand for futures speculation (36d), we must aggregate the speculative demand of producers, inventory holding dealers, as well as that of pure speculators: Z_t^p, Z_t^d, and Z_t^s.

The amount purchased by speculators (to include pure speculators as well as the speculative activities of producers and dealers) in one period, say $t-1$, does not appear until the next period, t. Since producers and dealers are committed as hedgers in period $t-1$ for delivery in period t, only speculators have a supply of spot commodities for sale in time t. We get the spot market clearing condition (36e) since speculators are the only agents with a supply of spot commodities for sale and consumers and dealers are the only agents who demand the spot commodities.

For the futures market clearing condition, we see that the supply of futures contracts is made up of the producers and dealers hedging activities, $Q_{t+1}^* + I_t^*$. That is equal to the futures contracts demanded by speculators, Z_t. Thus, we have:

$$Q_{t+1}^* + I_t^* = Z_t = \frac{\chi}{\theta^*}(E_t s_{t+1}^* - f_t), \qquad \frac{\chi}{\theta^*} \geq 0 \tag{37}$$

Therefore, we can see that $E_t s_{t+1}^* - f_t$ and $Q_{t+1}^* + I_t^*$ must have the same sign. Since hedgers are likely to take a net short position ($Q_{t+1}^* + I_t^* > 0$ as long as $I_t^* > 0$), the expected gain on the price movements ($E_t s_{t+1}^* - f_t > 0$) will compel speculators to take a net long position ($Z_t > 0$). Speculators take net long positions when the expected future spot price is greater than the current futures price. We can see that this corresponds to the case of a market in normal backwardation. Normal backwardation is the belief that futures prices tend to rise over the life of a contract due to the desire of hedgers to collectively be net short (Kolb and Overdahl 2006). Hence, we can only have contango when there is a short stock of inventory. Contango is the belief that prices tend to fall over the life of a contract, occurring when hedgers are net long with futures prices above the expected future spot price (Kolb and Overdahl 2006). Thus, the futures market allows hedgers to be insured by speculators where hedgers pay an insurance price, or risk premium, of $E_t s_{t+1}^* - f_t$ to the speculators to accept the risk of open positions.

To obtain the physical resource constraint, we can lag the futures market clearing condition (36f) by one period, substitute the spot market clearing conditions (36e) and manipulate the expression to obtain:

$$Q_t^* = C_t^* + (I_t^* - I_{t-1}^*) \tag{38}$$

This tells us that the quantity produced is expended by consumer demand and the change in inventory.

Toward finding the rational expectations equilibrium spot and futures price, we can begin by substituting Q_{t+1}^* (36b moved ahead one period), I_t^*, and Z_t into (36f):

$$f_t = \frac{1}{\frac{\beta}{\rho} + \gamma + \frac{\chi}{\theta^*}} \left(-\bar{I} + \frac{\chi}{\theta^*} E_t s_{t+1}^* + \gamma \rho s_t^* - v_t - w_t \right) \tag{39}$$

Lagging (39) by one period and then substituting into (36d) also lagged by one period yields:

$$Z_{t-1} = \frac{\chi}{\theta^*} \left[E_{t-1} s_t^* - \frac{1}{\frac{\beta}{\rho} + \gamma + \frac{\chi}{\theta^*}} \left(-\bar{I} + \frac{\chi}{\theta^*} E_{t-1} s_t^* + \gamma \rho s_{t-1}^* - v_{t-1} - w_{t-1} \right) \right] \tag{40}$$

Substituting (36a) and (36c) into (36e) yields,

$$Z_{t-1} = \alpha_0 + \alpha s_t^* + u_t + \bar{I} + \gamma (f_t - \rho s_t^*) + w_t \tag{41}$$

and substituting (39) into (41) gives us:

$$Z_{t-1} = \alpha_0 + \alpha s_t^* + u_t + \bar{I} + \gamma \left[\frac{1}{\frac{\beta}{\rho} + \gamma + \frac{\chi}{\theta^*}} \left(-\bar{I} + \frac{\chi}{\theta^*} E_t s_{t+1}^* + \gamma \rho s_t^* - v_t - w_t \right) - \rho s_t^* \right] + w_t \tag{42}$$

Setting (42) equal to (40) and manipulating yields:

$$\frac{\chi}{\theta^*}\left[E_{t-1}s_t^* - \frac{1}{\frac{\beta}{\rho}+\gamma+\frac{\chi}{\theta^*}}\left(-\bar{I}+\frac{\chi}{\theta^*}E_{t-1}s_t^*+\gamma\rho s_{t-1}^* - v_{t-1} - w_{t-1}\right)\right]$$

$$= \alpha_0 + \alpha s_t^* + u_t + \bar{I}$$

$$+\gamma\left[\frac{1}{\frac{\beta}{\rho}+\gamma+\frac{\chi}{\theta^*}}\left(-\bar{I}+\frac{\chi}{\theta^*}E_t s_{t+1}^* + \gamma\rho s_t^* - v_t - w_t\right) - \rho s_t^*\right] + w_t \tag{43}$$

Continued manipulating gives us:

$$E_t s_{t+1}^* - \left[\rho + \alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}\right]s_t^* - \left(1+\frac{\beta}{\gamma\rho}\right)E_{t-1}s_t^* + \rho s_{t-1}^*$$

$$= -\frac{\beta}{\gamma\rho}\cdot\frac{\theta^*}{\chi}\bar{I} - \left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}(\alpha_0+u_t)$$

$$+\frac{\theta^*}{\chi}v_t + \frac{1}{\gamma}v_{t-1} - \frac{1}{\gamma}\left\{\left(1+\frac{\beta}{\rho}\cdot\frac{\theta^*}{\chi}\right)w_t - w_{t-1}\right\} \tag{44}$$

We can see that (44) has different coefficients but takes a similar form to the stochastic difference equation in the absence of futures trading, i.e., equation (23). Thus, the magnitude of the coefficients is changed when futures trading is introduced.

We employ the same procedure to obtain the rational expectations spot price in the presence of futures markets:

$$s_t^* = \bar{s}^* + \frac{1}{\alpha+\gamma\eta^*}\Bigg(\sum_{j=0}^{\infty}\lambda^{*j}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^j\cdot u_{t-j} - \left(\frac{\rho-\eta^*}{\rho}\right)v_t - \Big(1+$$

$$\frac{\alpha}{\gamma\rho}\Big)\sum_{j=1}^{\infty}\lambda^{*j}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^j v_{t-j} + \frac{1}{\rho}\Bigg(\eta^* w_t -$$

$$\frac{\alpha}{\gamma}\sum_{j=1}^{\infty}\lambda^{*j}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^j w_{t-j}\Bigg)\Bigg) \tag{45a}$$

We define $\bar{s}^*$, λ^*, and η^* in (45b).

$$\left.\begin{aligned}
&\bar{s}^* = \left[\frac{\alpha_0\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\frac{\beta}{\gamma\rho}\frac{\theta^*}{\chi}\bar{I}}{\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\left(\frac{\lambda^*}{\gamma\rho}+\frac{\theta^*}{\chi}\right)}\right]\left(1-\frac{\beta\lambda^*}{\gamma\rho^2}\right),\\
&\frac{(\rho-\lambda^*)(1-\lambda^*)}{\lambda^*} = \alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\left(\frac{1}{\gamma\rho}+\frac{\theta^*}{\chi}\right),\;\; 0<\lambda^*<1<\frac{\rho}{\lambda^*}\,,\\
&\eta^* = \frac{\rho-\lambda^*+\beta\frac{\theta^*}{\chi}}{\gamma\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}}\,.
\end{aligned}\right\} \tag{45b}$$

The expected spot price and variance in period $t+T$ conditional on information at time t are:

$$E_t s^*_{t+T} = \bar{s}^* + \frac{1}{\alpha+\gamma\eta^*}\left\{\lambda^{*T}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^T\right\}\left(\sum_{j=0}^{\infty}\lambda^{*j}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^j\cdot u_{t-j}-\left(1+\frac{\alpha}{\gamma\rho}\right)\cdot\right.$$

$$\sum_{j=0}^{\infty}\lambda^{*j}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^j\cdot v_{t-j}-\frac{\alpha}{\gamma\rho}\cdot$$

$$\left.\sum_{j=0}^{\infty}\lambda^{*j}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^j w_{t-j}\right) \tag{46}$$

$$V_t s^*_{t+T} = \left(\frac{1}{\alpha+\gamma\eta^*}\right)^2\left[\frac{1-\lambda^{*2T}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2T}}{1-\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^2}\sigma_u^2+\left\{\left(\frac{\rho-\eta^*}{\rho}\right)^2+\left(1+\frac{\alpha}{\gamma\rho}\right)^2\cdot\right.\right.$$

$$\left.\frac{\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^2-\lambda^{*2T}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2T}}{1-\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^2}\right\}\cdot\sigma_v^2+$$

$$\left\{\left(\frac{\eta^*}{\rho}\right)^2 + \left(\frac{\alpha}{\gamma\rho}\right)^2 \cdot\right.$$

$$\left.\frac{\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^2-\lambda^{*2T}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2T}}{1-\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^2}\right\} \cdot \sigma_w^2 \Bigg] \tag{47}$$

We can get $E_t s_{t+1}^*$ from (46), and substitute it along with s_t^* into (39) to obtain the rational expectations futures price:

$$f_t = \bar{f} + \frac{1}{\alpha+\gamma\eta^*}\left\{\frac{\frac{\chi}{\theta^*}\lambda^*\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]+\gamma\rho}{\frac{\beta}{\rho}+\gamma+\frac{\chi}{\theta^*}} \cdot \left[\sum_{j=0}^{\infty}\lambda^{*j}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-\right.\right.\right.\right.$$

$$\left.\left.1-\frac{\beta}{\gamma\rho}\right)\right]^j \cdot u_{t-j} - \left(1+\frac{\alpha}{\gamma\rho}\right)\cdot\sum_{j=0}^{\infty}\lambda^{*j}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^j \cdot v_{t-j} -$$

$$\left.\left.\frac{\alpha}{\gamma\rho}\cdot\sum_{j=0}^{\infty}\lambda^{*j}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^j w_{t-j}\right]\right\} \tag{48a}$$

The long term expected value of the futures price, $\bar{f}$, can be expressed as:

$$\bar{f} = \lim_{T\to\infty} E_t f_{t+T} = \frac{1}{\frac{\beta}{\rho}+\gamma+\frac{\chi}{\theta^*}}\left(-\bar{I} + \left(\frac{\chi}{\theta^*}+\gamma\rho\right)\left[\frac{\alpha_0\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\frac{\beta}{\gamma\rho}\frac{\theta^*}{\chi}\bar{I}}{\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\left(\frac{\lambda^*}{\gamma\rho}+\frac{\theta^*}{\chi}\right)}\right]\left(1-\frac{\beta\lambda^*}{\gamma\rho^2}\right)\right) \tag{48b}$$

The conditional variance of futures price is found in the same fashion as for spot price, and is expressed as:

$$V_t f_{t+T} = \left(\frac{1}{\alpha+\gamma\eta^*}\right)^2 \left\{ \frac{1}{\left(\frac{\beta}{\rho}+\gamma+\frac{\chi}{\theta^*}\right)^2} \left(\frac{\chi}{\theta^*}\lambda^* \left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}} \left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right) \right] + \gamma\rho \right)^2 \cdot \left[\sigma_u^2 + \left(1+\frac{\alpha}{\gamma\rho}\right)^2 \sigma_v^2 + \left(\frac{\alpha}{\gamma\rho}\right)^2 \sigma_w^2 \right] \cdot \frac{1-\lambda^{*2T}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2T}}{1-\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2}} \right\} \tag{49}$$

3.3. Spot Price Volatility with and Without Futures Trading

3.3.1. General Comparison of Price Volatility

Continuing from Kawai, we examine if the introduction of a futures market changes the volatility of spot prices. To examine this volatility, we take into consideration the conditional variance of spot price ($V_t s_{t+T}$ and $V_t s_{t+T}^*$). We deem the short-term variance as that which pertains to $T = 1$ and the long-term variance when $T > 1$.

We can see that a general comparison between the conditional variance of spot price with a futures market and without a futures market is quite difficult. As Kawai points out, "the difficulty arises from the fact that an indicator of price uncertainty θ (or θ^*) is one of the structural coefficients, which in turn determines the equilibrium spot price and its conditional variances including $\theta = V_t s_{t+1}$ (or $\theta^* = V_t s_{t+1}^*$) itself, so that complicated nonlinear relationships exist among the structural parameters" (Kawai 1983, pg. 449). This nonlinear relationship may lead to problems of nonexistence and nonuniqueness of a rational expectations solution (McCafferty and Driskill 1980; Kawai 1983).

Thus, we can express again our conditional variances of spot price that we found earlier:

$$
\begin{aligned}
V_t s_{t+T} = {} & \left(\frac{1}{\alpha+\gamma\eta}\right)^2 \cdot \left[\frac{1-\lambda^{2T}\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2T}}{1-\lambda^{2}\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2}}\sigma_u^2 + \left(\left\{\frac{\rho-\gamma\eta\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}{\beta\left(\frac{\rho}{\beta}+\frac{\theta}{v}\right)}\right\}^2 + \right.\right. \\
& \left. + \left\{\frac{\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)+\rho}{\beta\left(\frac{\rho}{\beta}+\frac{\theta}{v}\right)}\right\}^2 \frac{\lambda^2\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2}-\lambda^{2T}\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2T}}{1-\lambda^{2}\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2}}\right)\sigma_v^2 + \left\{\left(\frac{\eta}{\rho}\right)^2 + \right. \\
& \left.\left. + \left(\frac{\alpha}{\gamma\rho}\right)^2 \frac{\lambda^2\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2}-\lambda^{2T}\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2T}}{1-\lambda^{2}\left[\frac{1}{\rho-\lambda+\alpha\left(\frac{1}{\gamma}+\frac{\theta}{v}\right)}\left(\frac{\rho}{\lambda}-1-\frac{\frac{1}{\gamma}+\frac{\theta}{v}}{\frac{\rho}{\beta}+\frac{\theta}{v}}\right)\right]^{2}}\right\}\sigma_w^2\right]
\end{aligned}
\tag{35}
$$

$$
\begin{aligned}
V_t s^*_{t+T} = {} & \left(\frac{1}{\alpha+\gamma\eta^*}\right)^2 \left[\frac{1-\lambda^{*2T}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2T}}{1-\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2}}\sigma_u^2 + \left\{\left(\frac{\rho-\eta^*}{\rho}\right)^2 + \left(1+\frac{\alpha}{\gamma\rho}\right)^2 \cdot \right.\right. \\
& \left. \frac{\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2}-\lambda^{*2T}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2T}}{1-\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2}}\right\}\sigma_v^2 + \left\{\left(\frac{\eta^*}{\rho}\right)^2 + \right. \\
& \left(\frac{\alpha}{\gamma\rho}\right)^2 \cdot \\
& \left.\left. \frac{\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2}-\lambda^{*2T}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2T}}{1-\lambda^{*2}\left[\frac{1}{\rho-\lambda^*+\alpha\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}+\beta\frac{\theta^*}{\chi}}\left(\frac{\rho}{\lambda^*}-1-\frac{\beta}{\gamma\rho}\right)\right]^{2}}\right\}\sigma_w^2\right]
\end{aligned}
\tag{47}
$$

where parameters $\{\theta, \lambda, \eta\}$ and $\{\theta^*, \lambda^*, \eta^*\}$ are related through the relationships:

$$\theta = \left(\frac{1}{\alpha+\gamma\eta}\right)^2 \cdot \left[\sigma_u^2 + \left\{\frac{\rho - \gamma\eta\left(\frac{1}{\gamma}+\frac{\theta}{\upsilon}\right)}{\beta\left(\frac{\rho}{\beta}+\frac{\theta}{\upsilon}\right)}\right\}^2 \cdot \sigma_v^2 + \left(\frac{\eta}{\rho}\right)^2 \cdot \sigma_w^2\right] \tag{50}$$

$$\frac{(\rho-\lambda)(1-\lambda)}{\lambda} = \left(\alpha + \frac{1}{\frac{\rho}{\beta}+\frac{\theta}{\upsilon}}\right)\left(\frac{1}{\gamma}+\frac{\theta}{\upsilon}\right), \qquad 0<\lambda<1 \tag{51}$$

$$\eta = \frac{\rho-\lambda}{\gamma\left(\frac{1}{\gamma}+\frac{\theta}{\upsilon}\right)} \tag{52}$$

$$\theta^* = \left(\frac{1}{\alpha+\gamma\eta^*}\right)^2 \cdot \left[\sigma_u^2 + \left(\frac{\rho-\eta^*}{\rho}\right)^2 \cdot \sigma_v^2 + \left(\frac{\eta^*}{\rho}\right)^2 \cdot \sigma_w^2\right] \tag{53}$$

$$\frac{(\rho-\lambda^*)(1-\lambda^*)}{\lambda^*} = \alpha\left\{\frac{1}{\gamma} + \left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\} + \beta\left(\frac{1}{\gamma\rho}+\frac{\theta^*}{\chi}\right), \quad 0<\lambda^*<1 \tag{54}$$

$$\eta^* = \frac{\rho-\lambda^*+\beta\frac{\theta^*}{\chi}}{\gamma\left\{\frac{1}{\gamma}+\left(1+\frac{\beta}{\gamma\rho}\right)\frac{\theta^*}{\chi}\right\}} \tag{55}$$

Due to the problematic nature of simultaneously solving for the triplets $\{\theta, \lambda, \eta\}$ and $\{\theta^*, \lambda^*, \eta^*\}$, a general comparison of $V_t s_{t+T}$ and $V_t s_{t+T}^*$ is "virtually impossible" (Kawai 1983). Also, due to nonuniquenss and nonexistence mentioned earlier, it could also be the case that these triplets may not exist, or multiple solutions could be obtained. Kawai notes that the origins of the disturbances are what is important for making meaningful comparisons between the different spot price variances. To do this, we resort to numerical illustrations.

3.3.2. Numerical Illustrations

We seek to quantitatively compare our derivation results to Kawai's, as well as examine what happens to the spot price variance when different disturbances are dominant within the market. We utilize identical parameter values as Kawai. Thus, we try different combinations of parameter values with $\gamma = 1.2, 1.4,$ or 1.8, α and β taking values between 0.4 and 3.0, $\upsilon = v = 1$, $\rho = 1.1$, and $\sigma_u^2, \sigma_v^2, \sigma_w^2 = 1$ or 100. Different speculator risk attitudes are also accounted for, with $\omega = 0$ representing infinitely risk adverse speculators and $\omega = 100$ representing approximately risk neutral pure speculators. We utilize the python language to construct a program to solve for the triplets $\{\theta, \lambda, \eta\}$ and $\{\theta^*, \lambda^*, \eta^*\}$ as well as compute $V_t s_{t+T}$, $V_t s_{t+T}^*$, and $V_t f_{t+T}$. For all cases, we are able to obtain the same results as Kawai indicating that for the chosen parameter values, our derivation is essentially equivalent to Kawai. The results of our numerical illustration can be seen in table 1.

We can see that when consumption, production, and inventory disturbances are all equal ($\sigma_u^2, \sigma_v^2, \sigma_w^2 = 1$), we cannot say whether spot price is stabilized in the presence of a futures market since we have some ambiguity depending on the parameter choice. If the consumption disturbance is dominant ($\sigma_u^2 = 100$ and $\sigma_v^2, \sigma_w^2 = 1$) the presence of a futures market stabilizes the volatility of spot price in the short and long term no matter the risk attitudes of speculators ($V_t s_{t+T}^* < V_t s_{t+T}$ for $T = 1$ or $T \to \infty$). When production is the dominant disturbance ($\sigma_v^2 = 100$ and $\sigma_u^2, \sigma_w^2 = 1$) we see that spot price is destabilized in the short run, but may or may not be stabilized in the long run ($V_t s_{t+T}^* > V_t s_{t+T}$ for $T = 1$ and $V_t s_{t+T}^* \gtreqless V_t s_{t+T}$ for $T \to \infty$). Finally, we can see that when the inventory demand disturbance is the main stochastic factor ($\sigma_w^2 = 100$ and $\sigma_u^2, \sigma_v^2 = 1$) futures markets tend to destabilize spot price at both time horizons and speculative risk attitudes ($V_t s_{t+T}^* > V_t s_{t+T}$ for $T = 1$ and $T \to \infty$). Thus, we cannot make any blanket statements in regards to the price stabilizing/destabilizing nature of futures markets.

Table 1. Numerical Illustrations

Numerical Illustrations: υ = ν = 1.0, ρ = 1.1													
						Commodity Market without Futures		Commodity Market with Futures (ω = 0)			Commodity Market with Futures (ω = 100)		
α	β	γ	σ_u^2	σ_v^2	σ_w^2	$V_t s_{t+1}$	$\lim_{T\to\infty} V_t s_{t+T}$	$V_t s_{t+1}^*$	$\lim_{T\to\infty} V_t s_{t+T}^*$	$\lim_{T\to\infty} V_t f_{t+T}$	$V_t s_{t+1}^*$	$\lim_{T\to\infty} V_t s_{t+T}^*$	$\lim_{T\to\infty} V_t f_{t+T}$
0.4	0.4	1.2	1.0	1.0	1.0	2.5844	3.1246	1.6091	2.0295	1.3308	1.1823	1.8621	0.6978
1.0	3.0	1.4	1.0	1.0	1.0	0.4051	0.4549	0.3487	0.3842	0.0674	0.3424	0.3959	0.0542
1.6	1.2	1.4	1.0	1.0	1.0	0.2211	0.2902	0.2214	0.2858	0.1079	0.2163	0.2960	0.0806
2.6	2.2	1.8	1.0	1.0	1.0	0.0961	0.1240	0.0974	0.1242	0.0385	0.0967	0.1280	0.0316
3.0	2.6	1.4	1.0	1.0	1.0	0.0914	0.1161	0.0938	0.1177	0.0343	0.0938	0.1219	0.0283
0.4	0.4	1.2	100.0	1.0	1.0	619.5172	619.5292	197.7421	197.7696	140.7523	114.8432	127.9596	55.3020
1.0	3.0	1.4	100.0	1.0	1.0	97.8255	97.8371	24.4645	24.4696	3.3650	21.6595	22.4770	1.7371
1.6	1.2	1.4	100.0	1.0	1.0	37.7224	37.7336	18.8623	18.8758	6.9752	14.2910	15.3200	1.9360
2.6	2.2	1.8	100.0	1.0	1.0	13.9879	13.9984	7.3383	7.3471	1.8671	6.0541	6.3980	0.5304
3.0	2.6	1.4	100.0	1.0	1.0	10.4333	10.4433	6.2563	6.2618	1.0079	5.5988	5.7928	0.2947
0.4	0.4	1.2	1.0	100.0	1.0	5.0200	26.0402	113.8442	113.9761	231.2386	22.5557	58.5602	55.0074
1.0	3.0	1.4	1.0	100.0	1.0	0.6695	2.1900	2.3617	2.9239	6.7348	1.3700	4.8054	3.6410
1.6	1.2	1.4	1.0	100.0	1.0	0.5564	3.9690	4.3729	4.9184	21.1459	1.2159	6.4457	5.5744
2.6	2.2	1.8	1.0	100.0	1.0	0.2308	1.6472	0.8664	1.5748	5.1263	0.4016	2.4687	2.1349
3.0	2.6	1.4	1.0	100.0	1.0	0.1731	1.4117	0.3969	1.3819	3.2994	0.2606	2.0697	1.8488
0.4	0.4	1.2	1.0	1.0	100.0	6.7564	8.3592	15.8858	16.1325	12.5615	23.4801	25.9686	3.8585
1.0	3.0	1.4	1.0	1.0	100.0	2.6141	3.4296	11.2006	11.2098	1.4244	12.4914	12.9327	0.7071
1.6	1.2	1.4	1.0	1.0	100.0	1.7842	2.5366	4.7698	4.9011	5.8648	7.0493	8.3114	1.7744
2.6	2.2	1.8	1.0	1.0	100.0	1.0055	1.4341	2.5629	2.6361	2.4972	3.4719	4.0937	0.8074
3.0	2.6	1.4	1.0	1.0	100.0	1.0719	1.5888	2.8818	2.9458	2.9808	3.7066	4.4210	0.9522

4. DATA

4.1. Oil

There are five Petroleum Administration for Defense Districts (PADDs) in the US, each corresponding to a different geographic region. They are conveniently named PADD I (eastern US), II (Midwest and central plains region), III (southern US from AL to NM), IV (Rocky Mountain region), and V (western US). The Energy Information Administration (EIA) keeps data on monthly ending stocks by PADD for Crude Oil in the United States. We obtain a monthly time series of ending stocks by PADD for crude oil from January 1990-December 2017. The sum of all PADD's ending stocks is the national ending stocks. All the stock data was obtained from EIA.

The EIA keeps track of data for U.S. Crude Oil Domestic Acquisition Cost by Refiners. Refiner acquisition cost (RAC) of crude oil refers to the cost of crude oil, including transportation and other fees paid by the refiner. The refiner acquisition cost does not include the cost of crude oil purchased for the Strategic Petroleum Reserve (SPR), and the domestic portion refers to Crude oil produced in the U.S. or from its "outer continental shelf" (EIA; Petroleum & Other Liquid). This cost was chosen because the EIA keeps track of it for every PADD as well as nationally which allows us to examine regional and national oil markets. RAC will be used as a to represent the spot price of crude oil. We have obtained a monthly series for the national price from January 1990 to December 2017 and by PADD from January 2004 to December 2017. For West Texas Intermediate (WTI) futures prices, we obtained a monthly series of closing prices for the NYMEX (New York Mercantile Exchange) WTI front contract (the contract nearest expiration) from Bloomberg for January 1990-December 2017.

4.2. Corn

There is not monthly corn stock data available for North Dakota. We constructed the monthly ending stock data manually which runs from January 2002-December 2016. In order to construct our monthly corn stocks data, we begin with the quarterly stocks of North Dakota corn measured in bushels. These quarterly stocks are available from the United States Department of Agriculture (USDA) National Agricultural Statistics Service (NASS) quick stats for the first of March, the first of June, the first of September, and the first of December of each year.

Next, we find the monthly production of corn; which is the number of bushels of corn harvested in a given month. To accomplish this, we get the yearly total production of corn in bushels for North Dakota from NASS. We obtain the weekly percentage harvested for North Dakota from the USDA crop progress reports (NASS Quick Stats). Since these reports are weekly, we choose the report for the week which has an ending date that is the closest to the last day of the month, which sometimes results in the choice of a report for a week which ends in the next month. Since the report's percentages are cumulative, we subtract the percent harvested in earlier months from a given report to obtain the percent harvested in that particular month. However, since the last crop progress report that corn appears on usually does not show that 100% of corn has been harvested, we add the remaining percentage to the last month of harvest (usually November) to make the total percentage harvested equal to 100%. Once we have a monthly percentage harvested, we take that percentage multiplied with the yearly bushels of corn produced to get a monthly number of bushels produced.

To find out the amount of corn which is exported from North Dakota each month we use the North Dakota Grain and Oilseed Transportation Statistics report which is produced each year by the Upper Great Plains Transportation Institute (UGPTI) (Vachal and Benson 2017). In these reports, we find the grain movements by month which show the total number of bushels for each particular grain that are transported each month and also the total number of bushels transported for a given year. Next, we find the North Dakota corn shipments by destination which shows the total number of bushels shipped yearly to specific destinations. Within these destinations is a section for the number of bushels shipped to a destination in North Dakota. Since we want to know the number of bushels that are exported from ND, we take the number of bushels with a destination of ND and divide it by 12 to get an average number of bushels with a destination of ND per month. We subtract this average from the number of bushels moved in each month. Now we have an estimate for the number of bushels exported from ND each month.

Next, we must get the number of bushels of corn that are used for ethanol production in ND. There are currently five ethanol plants in ND with each coming online since 2007. Previously there were two small ethanol production facilities in the state with one ceasing operation around July 2012 and the other around October 2007. To get an idea of the number of bushels these facilities use, we get each plant's yearly production capacity. The Nebraska Energy Office keeps track of each plant's yearly ethanol production capacity in millions of gallons of ethanol for each month (Ethanol Production Capacity by Plant). However, the Nebraska Energy Office only has this data from January 2005 onwards, so we must assume that the production capacity was the same going back to December 2001 which is reasonable since the capacity is small to begin with and also since we are unable to obtain any information on capacity changes at the plants operating over that time period. Therefore, we assume that each plant is operating at full capacity for each month it is in operation. To convert this ethanol production capacity into an amount of bushels used we take the annualized capacity every month, multiply it by 1 million to show the total capacity (not represented in millions of gallons), divide this by 2.8 (to represent 1 bushel of corn producing 2.8 gallons of ethanol; North Dakota Ethanol Industry), and then divide that number by 12 in order to get a monthly value of the bushels of corn used for ethanol production. It is important to note here that we are assuming that each plant is operating at full capacity for each month since we do not know at what percent capacity they are operating at in each month. The North Dakota Ethanol Council (NDEC) says on their website that over 80% of the corn used for ethanol production in ND is purchased from ND farmers (North Dakota Ethanol Industry). Therefore, we take our previous number of bushels used and multiply it by 85%; the extra 5% is included since the NDEC only says over 80%.

Finally, to construct our monthly series of ending stocks, we start with the NASS beginning stocks for December 2001, we add our calculated value for production in that month, subtract the calculated exports for that month, and subtract the estimated bushels used for ethanol for that month. The result is our estimated ending stocks for December 2001. Intuitively it makes sense that the ending stocks of one month are also the beginning stocks for the next month; hence, our estimated ending stocks for December 2001 are also our beginning stocks for January 2002. We find January 2002 ending stocks in the same fashion that we found the December 2001 ending stocks. We incorporate the NASS quarterly stock data by utilizing their stock number for the beginning stocks for each month that they have data and using it as the ending stocks of the preceding month. We calculate our stocks in this way and utilize the NASS data for the months that it is available and then rely on our calculated estimates for all other months.

For spot price, we utilize the monthly price receive in dollars per bushel for North Dakota which was obtained from NASS. For futures prices, we use a series of the monthly closing price for the front contract of the Chicago Board of Trade (CBOT) corn futures contract which was obtained from Bloomberg.

4.3. Soybeans

Like corn, we were unable to obtain a previously compiled series of monthly stocks for soybeans. We construct the monthly ending stocks in much the same way that we previously did for corn using the same type of NASS and UGPTI data. The major differences are that there may be no significant users of soybeans within the state that we are aware of, so we have no inclusion of "use" data in our monthly soybean's stocks. However, even in the absence of "use" data, we are still able to obtain an estimate of monthly inventories. Our constructed series for soybeans also runs from January 2002-December 2016.

Again, for spot price, we utilize the monthly price received in dollars per bushel for North Dakota obtained from NASS. For futures prices, we use a series of the monthly closing price for the front contract of the Chicago Board of Trade (CBOT) soybeans futures contract which was obtained from Bloomberg.

4.4. Hard Red Spring Wheat (HRS)

Like the other agricultural commodities, there was no monthly stock data for HRS for North Dakota. Therefore, to construct our stock data for HRS, we once again use the same NASS and UGPTI data for wheat. Similar to soybeans, we cannot find sources or proxies for the number of bushels of hard red spring wheat used in ND, so we have not included "use" figures in our monthly ending stocks calculation. Another issue we faced for HRS is that the NASS quarterly stocks has data for durum wheat and total wheat. According to the NASS's small grains annual summary (usda.library.cornell.edu), they estimate that the production distribution of other spring wheat (excluding durum) by class is 100% HRS for the state of ND for every year of our study. Also, based on the small grain's annual summary, the amount of winter wheat produced yearly is minuscule compared to HRS and Durum. Therefore, since HRSW is the dominant spring wheat in

ND and winter wheat is minuscule we can obtain the quarterly stock estimate for HRS by taking the NASS total wheat stock minus the NASS Durum stock for each observation to obtain an HRS estimated quarterly stock. The calculation of the monthly ending stocks continues in the same fashion as we did for the other two commodities. We obtain monthly stocks of HRS for the period as the other two commodities, January 2002-December 2016.

For spot price, we obtain the monthly price received (in dollars per bushel) for spring wheat (excluding Durum) for North Dakota from NASS. For futures prices, we use a series of the monthly closing price for the front contract of the Minneapolis Grain Exchange (MGE) HRS futures contract which was obtained from Bloomberg.

5. METHODOLOGY

5.1. Unit Root Tests

Unit root testing provides a way to understand if a series of data has changing mean and/or variance over time. If it is the case that mean or variance of a series are changing over time, we can say that the series has a unit root. A nonstationary time series contains one or more unit roots while a stationary series does not have a unit root. Nonstationary time series do not have a constant probability distribution in time, and hence, they exhibit a trend. Stationary time series have a constant probability distribution in time which is favorable for analysis. Therefore, when dealing with time series data, it is essential to know if the series is stationary. To do so, we utilize unit root tests.

The two tests we employ are the augmented Dickey-Fuller (ADF) test and the Kwiatkowski, Phillips, Schmidt, Shin (KPSS) test (Dickey and Fuller 1979; Kwiatkowski et al. 1992). The ADF tests a null hypothesis that the given series has a unit root while the KPSS test has a null hypothesis that the given series is stationary, essentially testing for no unit root. For our purposes, the ADF test is employed in the next section when examining cointegration, while we use the KPSS test for our series of data. The KPSS test is generally believed to be a higher-powered unit root test, and we use it here to identify which series are stationary and their order of integration.

When dealing with time series data, the unit root tests are first performed on the level data. If the level data is determined to be nonstationary, the series can be differenced and retested which will usually solve the nonstationary issue and result in a stationary series. A series that has not been differenced is said to be integrated of order zero, or $I(0)$. When a series has been differenced once, it is said to be integrated of order one, or $I(1)$. The order of integration is important because variables should be integrated of the same order when estimating a model. Table 2 has results of the KPSS test for our oil data while Table 3 carries results for the agricultural data. We ran the KPSS test for each variable with a constant as the only exogenous variable. If the level data appeared to be nonstationary, we also performed the test on the first differences.

We see that for the national oil data we can soundly reject the null hypothesis of stationarity on the levels for all three variables. When differenced once, we fail to reject the null hypothesis for the national oil variables which lends support that the first difference of the variables is now stationary. When investigating the PADD oil data, we do not see the same type of results. It appears that for the PADD data, the futures price is now stationary in the level (we are testing over a different period), as well as the RAC for each PADD. However, it appears that the ending stocks for each PADD are not stationary in the level but are stationary in the first difference.

The results of the KPSS tests for the agricultural data suggests that most of the data contain a unit root. For corn, we can reject the null hypothesis for all three variables at the 1% significance level for the level variables, while the first differences appear stationary. Soybean results also suggest nonstationary in the levels, with futures price and price received able to reject the null at the 1% level, while ending stocks can do so at the 5% level. Once again, all three soybean variables are stationary in the first difference. HRS Wheat futures price and price received are also nonstationary in the levels with 5% significance while ending stocks appears to be stationary. Both futures price and price received are stationary in the first difference.

Table 2. Kwiatkowski-Phillips-Schmidt-Shin test - Oil

Null Hypothesis: The series is stationary

Variables	Time Period	Exogenous Variables	LM-stat (level)	LM-stat (first diff.)
Oil Futures Price	1990-2017	Constant	1.543756***	0.051773
National RAC	1990-2017	Constant	1.540272***	0.069808
National Oil Ending Stocks	1990-2017	Constant	0.761872***	0.131924
Oil Futures Price	2004-2017	Constant	0.284237	
PADD 2 RAC	2004-2017	Constant	0.283311	
PADD 2 Ending Stocks	2004-2017	Constant	1.427856***	0.04278
PADD 3 RAC	2004-2017	Constant	0.302991	
PADD 3 Ending Stocks	2004-2017	Constant	0.99518***	0.072617
PADD 4 RAC	2004-2017	Constant	0.270123	
PADD 4 Ending Stocks	2004-2017	Constant	1.345832***	0.197414
PADD 5 RAC	2004-2017	Constant	0.321593	
PADD 5 Ending Stocks	2004-2017	Constant	0.060231	

* 10% Significance

** 5 % Significance

*** 1% Significance

Table 3. Kwiatkowski-Phillips-Schmidt-Shin test - Agriculture

Null Hypothesis: The series is stationary

Variables	Time Period	Exogenous Variables	LM-stat (level)	LM-stat (first diff.)
Corn Futures Price	2002-2016	Constant	0.796674***	0.106941
Corn Price Received	2002-2016	Constant	0.821863***	0.16515
Corn Ending Stocks	2002-2016	Constant	1.494899***	0.110254
Soybeans Futures Price	2002-2016	Constant	1.055223***	0.095598
Soybeans Price Received	2002-2016	Constant	1.084456***	0.152715
Soybeans Ending Stocks	2002-2016	Constant	0.481147**	0.188342
HRS Wheat Futures Price	2002-2016	Constant	0.574659**	0.078475
HRS Wheat Price Received	2002-2016	Constant	0.605427**	0.112178
HRS Wheat Ending Stocks	2002-2016	Constant	0.307603	

* 10% Significance

** 5 % Significance

*** 1% Significance

5.2. Engle-Granger Cointegration Test

For variables that are integrated of the same order, if a linear combination of these variables is stationary, the variables are said to be cointegrated. More formally, the components of some vector $v_t = (v_{1t}, v_{2t}, \ldots, v_{nt})'$ are cointegrated of order d, b, denoted $v_t \sim CI(d, b)$ if: (1) all elements of vector v_t are integrated of order d, $I(d)$, and (2) there exists some cointegrating vector $\delta = (\delta_1, \delta_2, \ldots, \delta_n)$ such that a linear combination $\delta v_t = \delta_1 v_{1t} + \delta_2 v_{2t} + \cdots + \delta_n v_{nt}$ is integrated of order $(d - b)$ where $b > 0$ (Engle and Granger 1987; Enders 2010). Hence, variables with a different order of integration cannot be cointegrated.

We test for cointegration using the Engle-Granger (1987) method. The test is a unit root test on residuals obtained from a regression. The Engle-Granger method employs the ADF unit root test mentioned in the previous section. The first step in the procedure is to employ unit root tests to ensure all variables are integrated of the same order, which was done in the previous section (see Table 2 and Table 3). Next, we estimate the long-run equilibrium relationship for the different time series. For two arbitrary series integrated of order one, x_t and y_t, this can be done by estimating the regression $y_t = \beta_0 + \beta_1 x_t + \varepsilon_t$. The series of regression residuals is then tested for a unit root. The ADF test is used in determining cointegration, where we obtain two test statistics, the tau-statistic (t-statistic) and the normalized autocorrelation coefficient which we refer to as the z-statistic. Critical values for the statistics can be found in MacKinnon (1996). If the residuals do not contain a unit root (are stationary), we can say that y_t and x_t are cointegrated of order (1,1) (Enders 2010). Overall, the Engle-Granger test has a null hypothesis that the series are not cointegrated since the null hypothesis of the unit root test is that the series contains a unit root. The results of our cointegration tests are contained in Table 4.

We can see that for oil futures price and national RAC, both the tau-statistic and the z-statistic suggest we can soundly reject the null hypothesis, letting us conclude that they are cointegrated. For national oil ending stocks and oil futures, when ending stocks is the dependent variable, we can reject the null hypothesis; however, on balance, the results suggest they are not cointegrated. Similarly, for national oil ending stocks and national RAC, only when ending stocks is the dependent variable, we can reject the null. Therefore, the series do not have a cointegrating relationship. For corn futures price and corn price received, we can soundly conclude they are cointegrated. The null hypothesis is also rejected for soybeans futures, and soybeans price received, indicating a cointegrating relationship. Lastly, there is strong evidence that HRS wheat futures and price received are also cointegrated.

Table 4. Engle-Granger Cointegration Test

Null Hypothesis: Series are not cointegrated

Dependent Variable	tau-statistic	Prob.^	z-statistic	Prob.^
Oil Futures Price	-6.734058	0.0000***	-92.43425	0.0000***
National RAC	-6.655748	0.0000***	-90.40870	0.0000***
Oil Futures Price	-2.043983	0.5058	-8.117879	0.4841
National Oil Ending Stocks	-2.439838	0.3083	-24.74891	0.0188**
National RAC	-2.366635	0.3423	-11.05719	0.2998
National Oil Ending Stocks	-2.462531	0.2981	-25.28565	0.0167**
Corn Futures Price	-4.489999	0.0018***	-40.76904	0.0003***
Corn Price Received	-4.322672	0.0031***	-37.78037	0.0007***
Corn Futures Price	-2.031089	0.5133	-8.003023	0.4885
Corn Ending Stocks	-0.301972	0.9755	-1.020473	0.9669
Corn Price Received	-1.699799	0.6787	-5.434000	0.6939
Corn Ending Stocks	-0.364870	0.9719	-1.222639	0.9609
Soybeans Futures Price	-5.658662	0.0000***	-64.87981	0.0000***
Soybeans Price Received	-5.401981	0.0001***	-58.96047	0.0000***
Soybeans Futures Price	-2.246499	0.4026	-8.190861	0.4747
Soybeans Ending Stocks	-0.612376	0.9531	-2.258879	0.9191
Soybeans Price Received	-2.002835	0.5280	-5.934239	0.6525
Soybeans Ending Stocks	-0.616608	0.9527	-2.216826	0.9212

Table 4. Engle-Granger Cointegration Test (continued)

Dependent Variable	tau-statistic	Prob.^	z-statistic	Prob.^
HRS Wheat Futures Price	-7.124047	0.0000***	-79.48459	0.0000***
HRS Wheat Price Received	-6.709667	0.0000***	-72.32971	0.0000***
HRS Wheat Futures Price	-2.787629	0.1748	-14.09031	0.1671
HRS Wheat Ending Stocks	-1.361484	0.8124	-5.649003	0.6758
HRS Wheat Price Received	-2.392476	0.3317	-10.37244	0.3320
HRS Wheat Ending Stocks	-1.329392	0.8225	-4.986089	0.7304

^MacKinnon (1996) p-values.
* 10% Significance
** 5% Significance
*** 1% Significance

5.3. Granger Causality

Granger (1969) causality tests are commonly utilized to discover causal relationships among different time series. These tests can also be useful in informing about the endogeneity of certain variables in a system. To do this, the test examines if the lags of one variable are useful in explaining another variable. Thus, in performing a Granger causality test, two regressions are estimated. For two series Y and X, we regress Y against lagged values of Y and lagged values of X which gives us our unrestricted regression. For the restricted regression, we regress Y against only lagged values of itself. We then perform an F-test to see if the group of coefficients related to the lagged values of X are significantly different from zero. If they are significant, we can reject the hypothesis that X does not Granger cause Y, since past values of X help explain the current level of Y. When the lags of one variable are useful in explaining another variable, we can then say that X

Granger causes Y. If the results indicate that X Granger causes Y, and Y Granger causes X, then we can say that it is likely one or more variables cause X and Y, indicating they are endogenous variables in the system. Table 5 contains our results of the Granger causality test for the oil related variables while Table 6 contains the results for the agricultural variables.

We can reject the null hypothesis for national oil RAC Granger causing oil futures price, and in turn, we can also reject the null hypothesis that oil futures price Granger causes national oil RAC. Hence, Granger causality suggests that national oil RAC and oil futures price are endogenous variables. There is also some weaker evidence (10% significance) that national ending stocks and futures price may be endogenous. At 1% significance, we can see two-way Granger causality between PADD 2 RAC and futures price. For PADD 3 RAC and futures price, we have two-way Granger causality with 10% significance. Similarly, we can see that PADD 4 RAC and futures price are also endogenous variables. Lastly, we can see that futures price and PADD 5 RAC Granger cause each other.

For our agricultural variables, we can see that they follow a similar pattern with regards to Granger causality as did the oil variables. For corn, we can see that futures price Granger causes price received but somewhat unexpectedly we fail to reject the null hypothesis for price received Granger causing futures price. However, we do find two-way causality between soybeans futures price and soybeans price received. Lastly, we also see strong evidence that HRS wheat futures and price received are endogenous variables.

Table 5. Granger Causality test – Oil

Null Hypothesis: Lags: 3	F-Statistic	Prob.
National RAC does not Granger Cause Futures Price	278.342	0.0000***
Futures Price does not Granger Cause National RAC	10.3409	0.0000***
National Ending Stocks does not Granger Cause Futures Price	2.43778	0.0645*
Futures Price does not Granger Cause National Ending Stocks	4.91727	0.0024***
Ending Stocks does not Granger Cause National RAC	0.70106	0.5520
National RAC does not Granger Cause National Ending Stocks	3.90972	0.0091***
PADD2 RAC does not Granger Cause Futures Price	149.507	0.0000***
Futures Price does not Granger Cause PADD2 RAC	4.73155	0.0034***
PADD2 Ending Stocks does not Granger Cause Futures Price	0.88012	0.4528
Futures Price does not Granger Cause PADD2 Ending Stocks	2.9547	0.0343**
PADD2 Ending Stocks does not Granger Cause PADD2 RAC	0.1224	0.9468
PADD2 RAC does not Granger Cause PADD2 Ending Stocks	5.4585	0.0014***
PADD3 RAC does not Granger Cause Futures Price	97.9619	0.0000***
Futures Price does not Granger Cause PADD3 RAC	2.43538	0.0668*
PADD3 Ending Stocks does not Granger Cause Futures Price	1.91392	0.1295
Futures Price does not Granger Cause PADD3 Ending Stocks	2.26213	0.0834*
PADD3 Ending Stocks does not Granger Cause PADD3 RAC	1.79852	0.1497
PADD3 RAC does not Granger Cause PADD3 Ending Stocks	1.07634	0.3609
PADD4 RAC does not Granger Cause Futures Price	105.253	0.0000***

Table 5. Granger Causality test – Oil (continued)

Null Hypothesis: Lags: 3	F-Statistic	Prob.
Futures Price does not Granger Cause PADD4 RAC	3.56641	0.0156**
PADD4 Ending Stocks does not Granger Cause Futures Price	3.18091	0.0256**
Futures Price does not Granger Cause PADD4 Ending Stocks	1.29596	0.2778
PADD4 Ending Stocks does not Granger Cause PADD4 RAC	3.23118	0.024***
PADD4 RAC does not Granger Cause PADD4 Ending Stocks	1.12416	0.3411
PADD5 RAC does not Granger Cause Futures Price	65.6667	0.0000***
Futures Price does not Granger Cause PADD5 RAC	4.73045	0.0035***
PADD5 Ending Stocks does not Granger Cause Futures Price	0.03242	0.9921
Futures Price does not Granger Cause PADD5 Ending Stocks	0.25966	0.8544
PADD5 Ending Stocks does not Granger Cause PADD5 RAC	0.04459	0.9874
PADD5 RAC does not Granger Cause PADD5 Ending Stocks	0.12175	0.9472

* 10% Significance
** 5% Significance
*** 1% Significance

Table 6. Granger Causality test – Agriculture

Null Hypothesis: Lags: 3	F-Statistic	Prob.
Corn Price Received does not Granger Cause Corn Futures Price	0.43585	0.7276
Corn Futures Price does not Granger Cause Corn Price Received	47.5034	0.0000***
Corn Ending Stocks does not Granger Cause Corn Futures Price	0.4745	0.7004
Corn Futures Price does not Granger Cause Corn Ending Stocks	0.90792	0.4385
Corn Ending Stocks does not Granger Cause Corn Price Received	1.67607	0.1740
Corn Price Received does not Granger Cause Corn Ending Stocks	1.39044	0.2475
Soybeans Price Received does not Granger Cause Soybeans Futures Price	4.05667	0.0081***
Soybeans Futures Price does not Granger Cause Soybeans Price Received	27.3454	0.0000***
Soybeans Ending Stocks does not Granger Cause Soybeans Futures Prices	1.66839	0.1757
Soybeans Futures Price does not Granger Cause Soybeans Ending Stocks	2.33926	0.0753*
Soybeans Ending Stocks does not Granger Cause Soybeans Price Received	2.67468	0.0489**
Soybeans Price Received does not Granger Cause Soybeans Ending Stocks	2.02005	0.1130
HRS Wheat Price Received does not Granger Cause HRS Wheat Futures Price	8.92827	0.0000***
HRS Wheat Futures Price does not Granger Cause HRS Wheat Price Received	66.6385	0.0000***
HRS Wheat Ending Stocks does not Granger Cause HRS Wheat Futures Price	0.01928	0.9963
HRS Wheat Futures Price does not Granger Cause HRS Wheat Ending Stocks	0.89517	0.4449
HRS Wheat Ending Stocks does not Granger Cause HRS Wheat Price Received	0.58933	0.6228
HRS Wheat Price Received does not Granger Cause HRS Wheat Ending Stocks	1.79332	0.1503

* 10% Significance

** 5% Significance

*** 1% Significance

5.4. Directed Acyclic Graphs

A directed acyclic graph (DAG) is an alternative technique to determine causal relations among variables. DAGs present an alternative to the Granger causality tests insofar as DAGs explore non-time sequence asymmetry in causal relations as opposed to the Granger test which exploits the time sequence asymmetry (Yang et al. 2006). These causal relations are determined by computer algorithms which produce graphs with nodes (vertices, variables) and edges between nodes. Visually, a DAG is a graph, which is an ordered triple $\langle V, M, E \rangle$. Here, V is the vertex set, which is a non-empty set that contains nodes, M is a non-empty set of marks which shows the directedness of an edge, and E is the edge set, containing ordered pairs representing edges between nodes (Bessler and Yang 2003). These edges indicate a causal relationship between nodes and can be either directed or undirected edges (indicated by the marks). For two arbitrary nodes A and B, with a directed edge (indicated by a line with an arrow) from node A to node B, we can say that node A is a cause of node B. For an undirected edge (indicated by a line between nodes) between node A and node B, we can say one of the following: a.) node A is a cause of node B, b.) node B is a cause of node A, c.) there is some unmeasured confounder of A and B, d.) both a. and b., or e.) both b. and c. In determining endogeneity from these graphs, we can say that variables which have no causal input are exogenous while variables that are not exogenous are endogenous (Spirtes et al., 2000).

Mathematically, following Miljkovic et al. (2016), a DAG is represented as the conditional independence by the recursive product decomposition:

$$Pr(v_1, v_2, \dots, v_n) = \prod_{i=1}^{n} Pr(v_i | p\pi_i) \tag{56}$$

in which Pr represents the probability of variables $(v_1, v_2, \dots, v_n)$. The product operator is represented by Π, and $p\pi_i$ represents the realization of some subset of variables that causes V_i in order $(i = 1, 2, \ldots, n)$. Pearls' (1995) work on d-separation allows independencies and causes to be translated graphically. In explaining d-separation, consider the three variable set X, Y, and Z. If the flow of information between these nodes is blocked, we can say these variables are d-separated. This so-called d-separation can occur in two ways: (1) if one variable is the cause of the other two variables, i.e. Y in $X \leftarrow Y \longrightarrow Z$, or if there is a passthrough variable, i.e. Y in $X \rightarrow Y \rightarrow Z$; (2) when a variable is caused (effected) by two variables, i.e. Y in $X \rightarrow Y \leftarrow Z$. This notion of d-separation was incorporated into the PC algorithm by Spirtes et al. (1993).

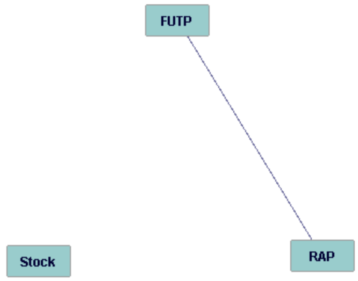
For our DAG's, we utilized the PC algorithm and FGES algorithm with the TETRAD software version 6.5.4. We chose these two algorithms because they complement each other nicely while providing alternatives for discovering relationships within the data. The PC algorithm begins with a connected graph containing undirected edges between all nodes. The algorithm proceeds by performing independence and conditional independence tests on edges between nodes in order to remove edges. For zero order conditioning, the algorithm tests if the conditional correlation between nodes is significantly different from zero using Fisher's z (Awokuse and Bessler 2003). If the algorithm fails to reject the null hypothesis that the correlation between nodes is not significantly different from zero, then the edge between the nodes is removed (Li et al. 2013). The surviving edges are tested with first-order partial correlation; the edges between two nodes where first-order partial correlation is not statistically different from zero are removed (Ji et al., 2018). For N variables, this process continues for higher order partial correlation until an $\mathrm{N} - 2$ order partial correlation test is finished or if no edges are remaining (Ji et al., 2018). Any remaining edges are directed via the theory of sepset. For a more in-depth explanation of sepset's see Yang and Bessler (2008).

The FGES algorithm is an optimized version of the GES algorithm that searches over equivalence classes of DAG's and returns a model with the best Bayesian score (Chickering 2003; Ramsey et al. 2017). The algorithm begins its search with an unconnected graph. It then performs a forward search, adding the edge with the largest improvement in the Bayesian Information Criterion (BIC) in each step. Once the algorithm reaches a point where no edges will improve the BIC, the algorithm moves to the backward stepping search. For the backward stepping search, it begins with the previously discovered graph and then iteratively removes an edge in each step that gives the largest improvement in the Bayesian score until no more deletions will improve the score (Ramsey et al. 2017). When the BIC cannot be improved by removing more edges, the algorithm returns the resulting graph.

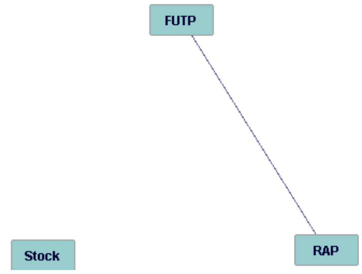
Our graphs produced by each algorithm can be seen in table 7. For the PC algorithm, we have a significance level of 10%. For total oil, we see both FGES and PC algorithms produce similar graphs containing an undirected edge between the futures price and RAC, suggesting that they are endogenous variables while ending stocks appears to be an exogenous variable. The PADD 2 FGES algorithm also produces a graph with an undirected edge between futures price and RAC, while the PC produces a cyclic undirected graph with all variables connected. While the DAG results for PADD 2 appear to be ambiguous, if we also take the findings from PADD 2's Granger causality test into account, we see that both the Granger test as well as the FGES algorithm lend credence to futures price and RAC being endogenous variables. PADD 3 has both algorithms producing a connected graph, with undirected edges between both futures price and price received, as well as futures price and ending stocks, suggesting three endogenous variables. The graphs for PADD 4 and PADD 5 are similar to each other for both algorithms, with an undirected edge between futures price and RAC, suggesting endogeneity of futures and RAC.

The agricultural graphs are similar to the oil graphs with many suggesting endogeneities between futures price and price received. The corn graph suggests endogeneity for futures price and price received with the FGES algorithm, while the PC algorithm produces a graph with undirected edges between futures price and price received, as well as between ending stocks and futures price. The algorithms for soybeans both suggest futures price and price received are endogenous. The algorithms achieve a similar result for HRS wheat as they did for corn, with FGES suggesting futures price and price received are endogenous, and the PC showing undirected edges between price received and futures as well as ending stocks and futures.

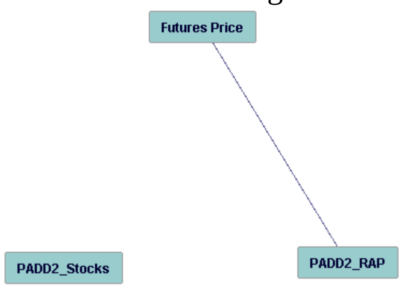
Table 7. Directed Acyclic Graph Results



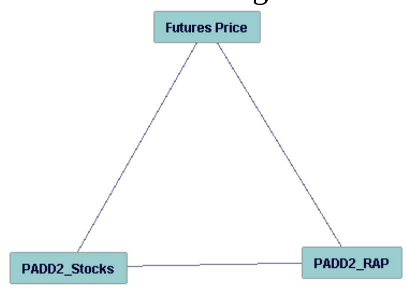
Total Oil FGES algorithm



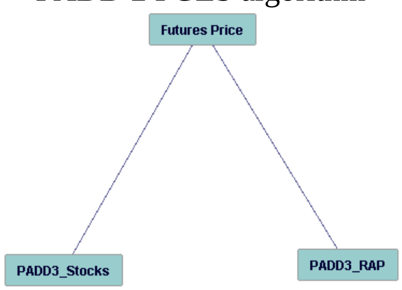
Total Oil PC algorithm



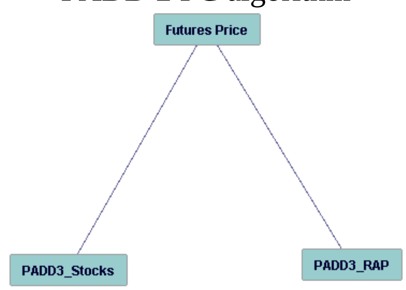
PADD 2 FGES algorithm



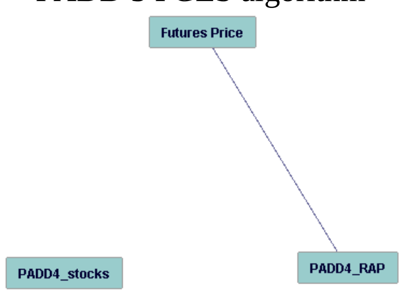
PADD 2 PC algorithm



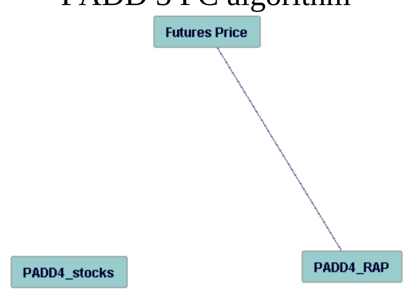
PADD 3 FGES algorithm



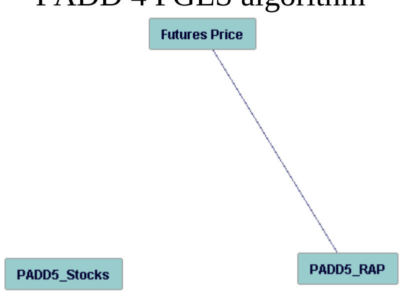
PADD 3 PC algorithm



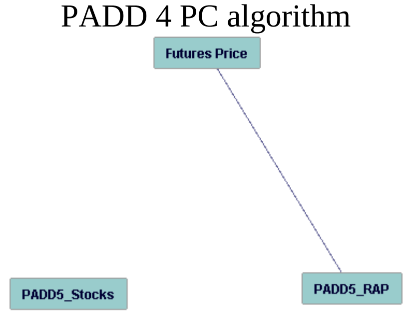
PADD 4 FGES algorithm



PADD 4 PC algorithm

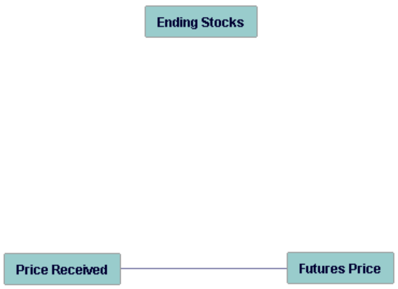


PADD 5 FGES algorithm

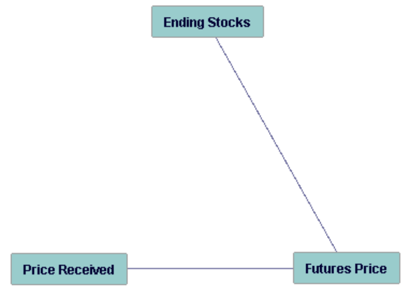


PADD 5 PC algorithm

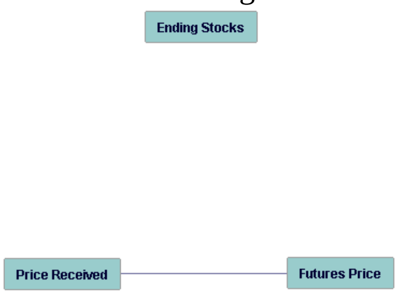
Table 7. Directed Acyclic Graph Results (continued)



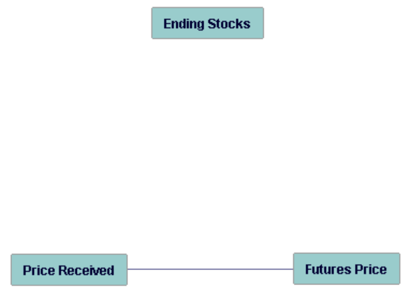
Corn FGES algorithm



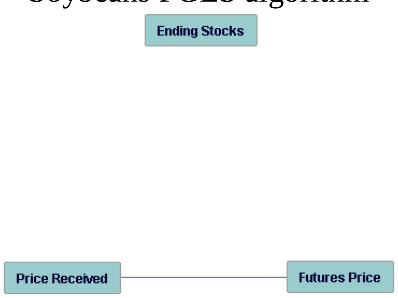
Corn PC algorithm



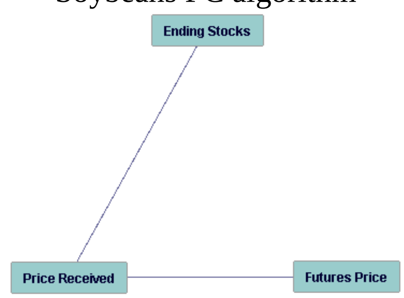
Soybeans FGES algorithm



Soybeans PC algorithm



HRS Wheat FGES algorithm



HRS Wheat PC algorithm

5.5. Vector Autoregression

One of the goals of this thesis is to examine the dynamic relationships and interrelationships between ending stocks, and spot and futures price. Given that aim and the fact that much of the previous results suggest that spot price and futures price are endogenous, we can utilize a vector autoregression (VAR) or vector error correction model (VECM). Sims (1980) introduced VAR as a new approach to estimating multiple equation models.

Interrelated time series are commonly estimated using VAR (Wilson and Miljkovic 2013). A VAR is a system of equations where all endogenous variables are a function of lagged values of itself, lagged values of the other endogenous variables, and any other exogenous explanatory variables that are deemed appropriate for the model. Thus, mathematically we can represent a VAR as:

$$Y_t = B_0 + B_1 Y_{t-1} + \cdots + B_p Y_{t-p} + A_0 X_t + A_1 X_{t-1} + \cdots + A_p X_{t-p} + \varepsilon_t \qquad (57)$$

where Y_t is an $n \times 1$ vector of endogenous variables, B_0, is an $n \times 1$ vector of intercept terms, $B_1,\ldots, B_p$ are $n \times n$ matrices of coefficients to be estimated for lagged endogenous variables, $A_0,\ldots,A_p$ are $n \times n$ matrices related to coefficients of current and lagged exogenous variables, and ε_t is an $n \times 1$ vector of innovations. The innovations are not correlated with their own lagged values and are uncorrelated with all explanatory variables but may be contemporaneously correlated (Wilson and Miljkovic 2013). Since all right-hand side explanatory variables are the same, ordinary least squares (OLS) yields efficient estimates (Enders 2010).

5.6. Vector Error Correction

When cointegration is present, we must change our estimation approach. Since VAR models cannot deal with cointegration, we can restrict the VAR to achieve an error correcting approach. VECM allow us to examine the short-term adjustments of cointegrated variables to their long-run equilibrium. Similar to a VAR, in a VECM the differenced endogenous variables are a function of lagged differenced values of itself, lagged differenced values of other endogenous variables, differenced exogenous variables, and one or more cointegrating vectors which are the difference between the two cointegrated variables. Thus, we can represent a VECM mathematically as:

$$\Delta Y_t = B_0 + \pi z_{t-1} + B_1 \Delta Y_{t-1} + \cdots + B_p \Delta Y_{t-p} + A_0 \Delta X_t + \varepsilon_t \qquad (58)$$

where Δ is the difference operator, B_0, is an $n \times 1$ vector of intercept terms, $B_1, \ldots, B_p$ are $n \times n$ matrices of coefficients to be estimated for lagged endogenous variables, Y_t is an $n \times 1$ vector of endogenous variables, z_{t-1} is a $1 \times n$ vector containing the difference of cointegrating variables forming our cointegrating vector, π is an $n \times 1$ vector of adjustment coefficients related to our cointegrating vector, A_0 is $n \times n$ matrices related to coefficients of our exogenous variables (we may include lagged exogenous variables in a VECM as well), and ε_t is an $n \times 1$ vector of innovations. Note that if all elements of π are zero, we simply have a VAR in first differences (Enders 2010).

5.7. Impulse Response

Impulse response functions allow us to observe over time how endogenous variables respond to an exogenous shock to itself as well as a shock to other endogenous variables. Based on the dynamic structure of a VAR or VECM, a shock to one endogenous variable will affect that variable but can also affect other endogenous variables. Thus, with impulse response functions, we can observe how a shock to one variable filters through the model to affect the other variables within the model (Pindyck and Rubinfeld 1998). This technique ultimately allows us to examine what effect a shock in the futures market has on the time path of the spot market and vice versa.

Following Enders (2010), we can express a VAR as a vector moving average (VMA) in matrix form for two arbitrary variables y_t and z_t:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} e_{1t-i} \\ e_{2t-i} \end{bmatrix} \tag{59}$$

We can rewrite the vector of errors as:

$$\begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix} = \frac{1}{1 - b_{12}b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{zt} \end{bmatrix} \tag{60}$$

Combining (59) and (60) to express y_t and z_t in terms of ε_{yt} and ε_{zt} gives us:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \frac{1}{1 - b_{12}b_{21}} \sum_{i=0}^{\infty} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}^i \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{bmatrix} \tag{61}$$

To simplify the notation, we can define the matrix ϕ_i, whose elements are $\phi_{mn}(i)$, as follows:

$$\phi_i = \frac{A_1^i}{1 - b_{12}b_{21}} \begin{bmatrix} 1 & -b_{12} \\ -b_{21} & 1 \end{bmatrix} \tag{62}$$

Thus, the vector moving average of a VAR in matrix form can be expressed as:

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = \begin{bmatrix} \bar{y} \\ \bar{z} \end{bmatrix} + \sum_{i=0}^{\infty} \begin{bmatrix} \phi_{11}(i) & \phi_{12}(i) \\ \phi_{21}(i) & \phi_{22}(i) \end{bmatrix} \begin{bmatrix} \varepsilon_{yt-i} \\ \varepsilon_{zt-i} \end{bmatrix} \tag{63}$$

Here the coefficients $\phi_{11}(i), \ldots, \phi_{22}(i)$ are the impulse response functions. We can see that $\phi_{11}(0)$ is the instantaneous impact of a one unit change in ε_{yt} on y_t while $\phi_{11}(i)$ represents the i-th period impact of a one unit change in ε_{yt-i} on y_t (Enders 2010). We can plot the impulse response functions to see the time path of the responses to shocks.

Since an estimated VAR is under-identified, the impulse responses require additional restrictions to be identified (Enders 2010). We utilize Choleski decomposition to orthogonalize the innovations to obtain our impulse responses (Wilson and Miljkovic 2013). Thus, the restriction alters the system so that y_t will not contemporaneously effect z_t. Again, following Enders (2010), we decompose the error terms in (60) such that:

$$e_{1t} = \varepsilon_{yt} - b_{12}\varepsilon_{zt} \tag{64}$$

$$e_{2t} = \varepsilon_{zt} \tag{65}$$

Thus, ε_{zt} has a contemporaneous direct effect on both z_t and y_t, while ε_{yt} has a direct effect on y_t and an indirect effect on z_t through lagged values of y_t. Hence, the impulse response functions allow us to observe how endogenous variables respond to shocks within the system. The results of our impulse responses can be found in section 6, along with the results of our estimated models.

5.8. Variance Decomposition

The dynamic structure of our models can also be examined using variance decomposition, which breaks down the variance of the forecast errors for every endogenous variable into the percentage of the variance that can be credited to the other endogenous variables (Pindyck and Rubinfeld 1998). This can be useful in identifying how large a role one variable has in effecting the variation of another variable over time.

Once again, following Enders (2010), we can express the VMA in terms of its forecast errors. First, (63) can be expressed more compactly as:

$$x_t = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t-i} \tag{66}$$

The n-period ahead forecast of (66) can be expressed as:

$$x_{t+n} = \mu + \sum_{i=0}^{\infty} \phi_i \varepsilon_{t+n-i} \tag{67}$$

Hence, we can represent the n-period ahead forecast error as:

$$x_{t+n} - E_t x_{t+n} = \sum_{i=0}^{n-1} \phi_i \varepsilon_{t+n-i} \tag{68}$$

Thus, for the variable y_t, we see the n-step ahead forecast error:

$$\begin{aligned} y_{t+n} - E_t y_{t+n} &= \phi_{11}(0)\varepsilon_{yt+n} + \phi_{11}(1)\varepsilon_{yt+n-1} + \cdots + \phi_{11}(n-1)\varepsilon_{yt+1} \\ &\quad + \phi_{12}(0)\varepsilon_{zt+n} + \phi_{12}(1)\varepsilon_{zt+n-1} + \cdots + \phi_{12}(n-1)\varepsilon_{zt+1} \end{aligned} \tag{69}$$

Where the n-period forecast error variance of y_t is $\sigma_y(n)^2$:

$$\begin{aligned} \sigma_y(n)^2 &= \sigma_y^2[\phi_{11}(0)^2 + \phi_{11}(1)^2 + \cdots + \phi_{11}(n-1)^2] \\ &\quad + \sigma_z^2[\phi_{12}(0)^2 + \phi_{12}(1)^2 + \cdots + \phi_{12}(n-1)^2] \end{aligned} \tag{70}$$

Note that as the forecast horizon increases, so too will the forecast error variance due to the nonnegativity of all $\phi_{mn}(i)^2$ terms. We can represent the proportion of y_t's n-period forecast error variance due to shocks to ε_{yt} as follows,

$$\frac{\sigma_y^2[\phi_{11}(0)^2 + \phi_{11}(1)^2 + \cdots + \phi_{11}(n-1)^2]}{\sigma_y(n)^2} \tag{71}$$

also, the proportion due to shocks to ε_{zt}:

$$\frac{\sigma_z^2[\phi_{12}(0)^2 + \phi_{12}(1)^2 + \cdots + \phi_{12}(n-1)^2]}{\sigma_y(n)^2} \tag{72}$$

The proportion of z_t's n-period ahead forecast error variance can be decomposed in a similar fashion. We also employ Choleski decomposition for our variance decomposition in the same manner as (64) and (65). Thus, for the one period ahead forecast error variance, all the variation in z_t is due to ε_{zt} (Enders 2010). We use this variance decomposition to see what percentage of the variance in futures price can be attributed to spot price over time or what percentage of the variation in spot price can be attributed to futures price. The results of our variance decompositions are located in section 6 along with the results of our estimated models.

6. RESULTS

The first step in our estimation process uses unit root testing to determine if each variable is stationary in levels or first differences. We use the Engle-Granger cointegration test to determine if we estimate a VAR or a VECM. If cointegration is present, we estimate a VECM; otherwise, we estimate a VAR. For the VECM, we first estimate a restricted model where we impose a restriction on the speed of adjustment coefficients by setting them equal to test if futures price and spot price adjust at the same rate. If we can reject that restriction, we then estimate a VECM with no restrictions on the speed of adjustment coefficients. The endogenous variables are determined based on the results of the Granger causality and DAGs, keeping in mind that DAGs are a more powerful method since there are no a priori assumptions of the causal nature of the data (Li et al. 2013). The number of endogenous lags to include is determined by AIC, except for our national oil VECM, where we used our intuition to select the second-best model based on AIC.

Once we estimate a model, we examine the impulse responses and variance decomposition. The impulse responses allow us to examine the size and duration of the impact a shock to one endogenous variable has in another endogenous variable. The variance decomposition allows us to account for the percentage of variation in an endogenous variable which is made up by itself as well as by other endogenous variables over time. For the innovation accounting, we are interested in how the futures market affects the spot market over a period of 36 months.

6.1. National Oil

For national oil, we found that futures price, RAC, and ending stocks are stationary in the first differences. The Engle-Granger cointegration test indicated that futures price and RAC are cointegrated. The Granger tests had two-way causality between futures price and RAC significant at the 1% level as well as two-way causality between ending stocks and futures price at the 10%

level. The DAG results suggest endogeneity between futures price and RAC. Thus, on balance, futures price and RAC are endogenous and ending stocks can be treated as exogenous. These results indicate that a VECM is appropriate; we estimate a model with three lags. The results of the estimation are in table 8. We see that we fail to reject the restriction on the speed of adjustment coefficients, indicating that futures and spot prices adjust at the same rate.

We see that for the explanation of both the change in futures price and the change in RAC, the cointegration equation is significant indicating both RAC and FUTP adjust towards their long-run equilibrium. In the explanation of the first difference in futures price, we see that all three lags of ΔFUTP are significant with negative coefficients, while all three lags of RAC are significant with positive signs. In the explanation of ΔRAC, the first two lags of futures price are significant with negative signs while the first two lags of RAC are significant with positive coefficients. We see that the change in ending stocks is also significant at the 10% level with a negative coefficient.

We can see the impulse responses and variance decompositions in Figure 1 and 2 below. The response of futures price to a shock in RAC leads to an increase in FUTP which appears to be permanent. RAC response to a shock in futures price indicates an initial increase but ultimately a decreased price level around 15 months after the shock which remains permanently lower thereafter. For the variance decompositions, we see that RAC accounts for nearly 90% of the long-term variance of futures price. Futures price accounts for about 25% of RAC variance initially but accounts for only about 3% of the variance in the long run.

Table 8. National Oil Restricted VECM

Cointegration Restrictions:		
$\pi_{11} = \pi_{21}$		
Convergence achieved after 3 iterations.		
Not all cointegrating vectors are identified		
LR test for binding restrictions (rank = 1):		
Chi-square(1)	2.4606	
Probability	0.1167	
Cointegrating Eq:	CointEq1	
FUTP(-1)	-0.3761	
RAC(-1)	0.3440	
C	1.8085	
Error Correction:	ΔFUTP	ΔRAC
CointEq1	0.3682***	0.3682***
	[2.91725]	[2.91725]
ΔFUTP(-1)	-0.8797***	-0.3020***
	[-11.7452]	[-3.26785]
ΔFUTP(-2)	-0.5223***	-0.2095**
	[-6.35323]	[-2.06536]
ΔFUTP(-3)	-0.1170**	-0.0197
	[-2.01945]	[-0.27571]
ΔRAC(-1)	1.3365***	0.6976***
	[19.3245]	[8.17531]
ΔRAC(-2)	0.7506***	0.3468***
	[7.44699]	[2.78870]
ΔRAC(-3)	0.1874*	-0.0761
	[1.94051]	[-0.63854]
C	0.0006	0.0751
	[0.00427]	[0.43173]
ΔSTOCK	0.0000	-0.00003*
	[0.95533]	[-1.77404]
R-squared	0.7432	0.4041

t - statistics in [ ]

* 10% Significance

** 5% Significance

*** 1% Significance

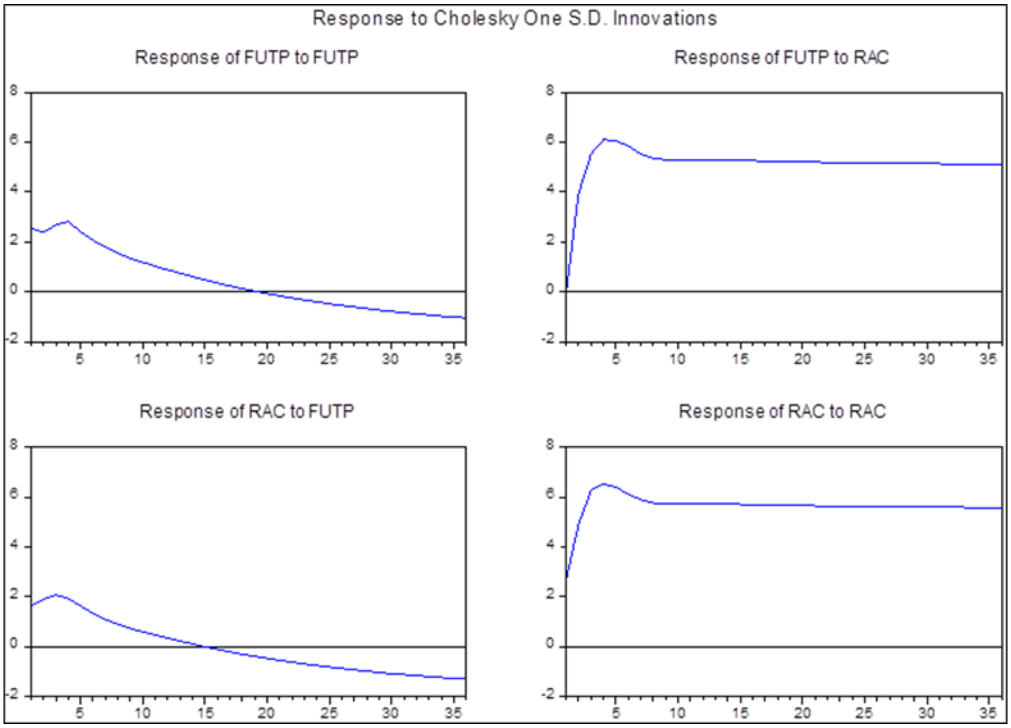


Figure 1. Restricted National Oil Impulse Responses

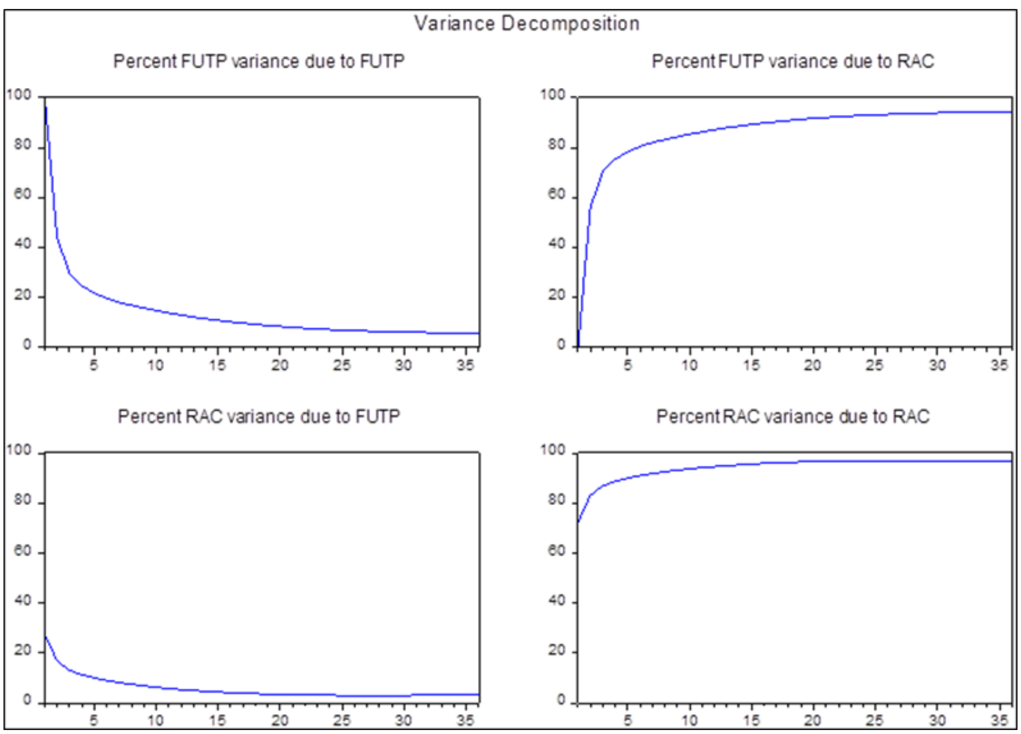


Figure 2. Restricted National Oil Variance Decomposition

6.2. Regional Oil

6.2.1. PADD 2

For PADD 2, we found that futures price and PADD 2 RAC are stationary in the levels while PADD 2 ending stocks is stationary in the first difference. Thus, since 2 of the three variables are stationary, we cannot have a cointegrating vector between any of the three variables. Granger causality indicates two-way causality between futures price and RAC. The FGES algorithm suggests endogeneity between futures price and RAC while the PC algorithm resulted in a cyclical graph. Thus, 2 of the three causality tests suggest we can treat futures price and RAC as endogenous. Since cointegration is not possible, we estimate a VAR model in the levels for PADD 2 with three endogenous lags suggested by AIC. The results are contained in Table 9.

In the explanation of FUTP, the second and third lags of FUTP are significant with positive coefficients, all three lags of RAC are significant, where the first lag has a positive coefficient and the second and third lags have negative coefficients. In explaining RAC, the first lag of FUTP is significant with a negative coefficient, and the third lag of FUTP is significant with a positive coefficient. The first lag of RAC is significant with a positive coefficient while the third lag is significant at the 10% level with a negative coefficient. Finally, the change in ending stocks is significant with a negative coefficient.

Impulse responses in Figure 3 indicate that innovations in either endogenous variable have transitory effects. The variance decomposition in Figure 4 indicates that RAC plays an increasing role in the variation in FUTP, accounting for nearly 90% by the 36th period. FUTP also plays a decreasing role in the variation of RAC while accounting for 27% in the first period and only 7$^{\%}$ by the 36th period.

Table 9. PADD 2 VAR

	FUTP	PADD2 RAC
FUTP(-1)	-0.1047	-0.3794***
	[-1.15603]	[-3.20409]
FUTP(-2)	0.2265**	-0.0355
	[2.59759]	[-0.31127]
FUTP(-3)	0.3284***	0.2171**
	[4.28703]	[2.16743]
PADD2 RAC(-1)	1.4506***	1.6677***
	[20.4559]	[17.9834]
PADD2 RAC(-2)	-0.4053***	-0.1996
	[-2.97043]	[-1.11864]
PADD2 RAC(-3)	-0.5258***	-0.3197*
	[-4.12921]	[-1.91980]
C	2.5901	3.8435
	[2.76032]	[3.13229]
ΔPADD2 STOCKS	0.0000	-0.0002**
	[-0.52013]	[-2.44462]
R-squared	0.9808	0.9683

t - statistics in [ ]

* 10% Significance

** 5% Significance

*** 1% Significance

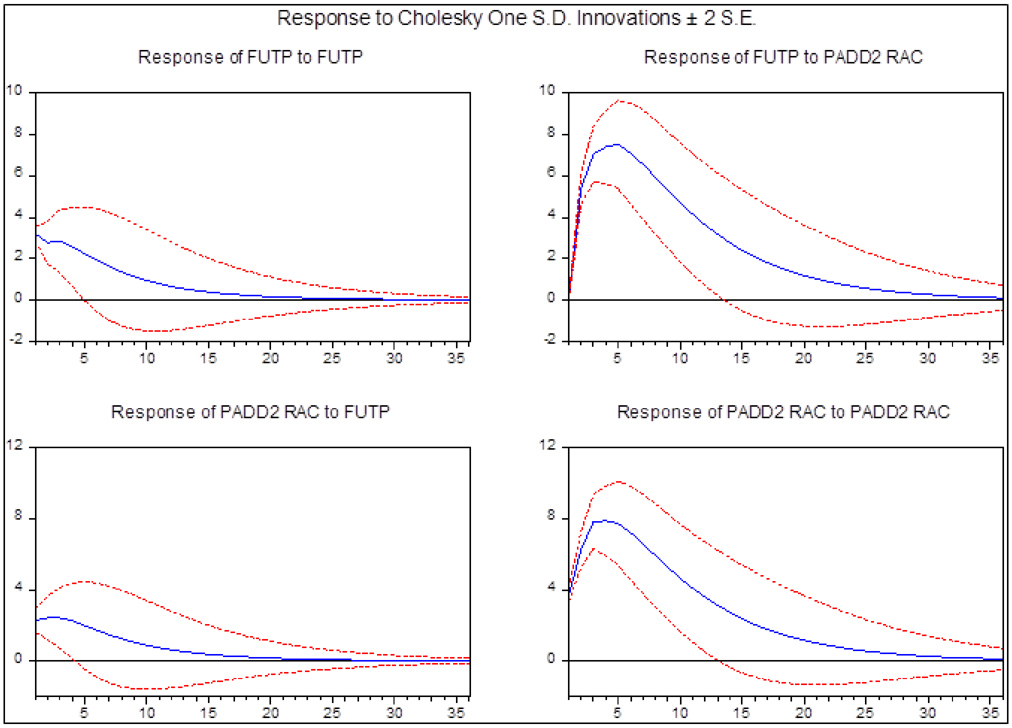


Figure 3. PADD 2 Impulse Responses

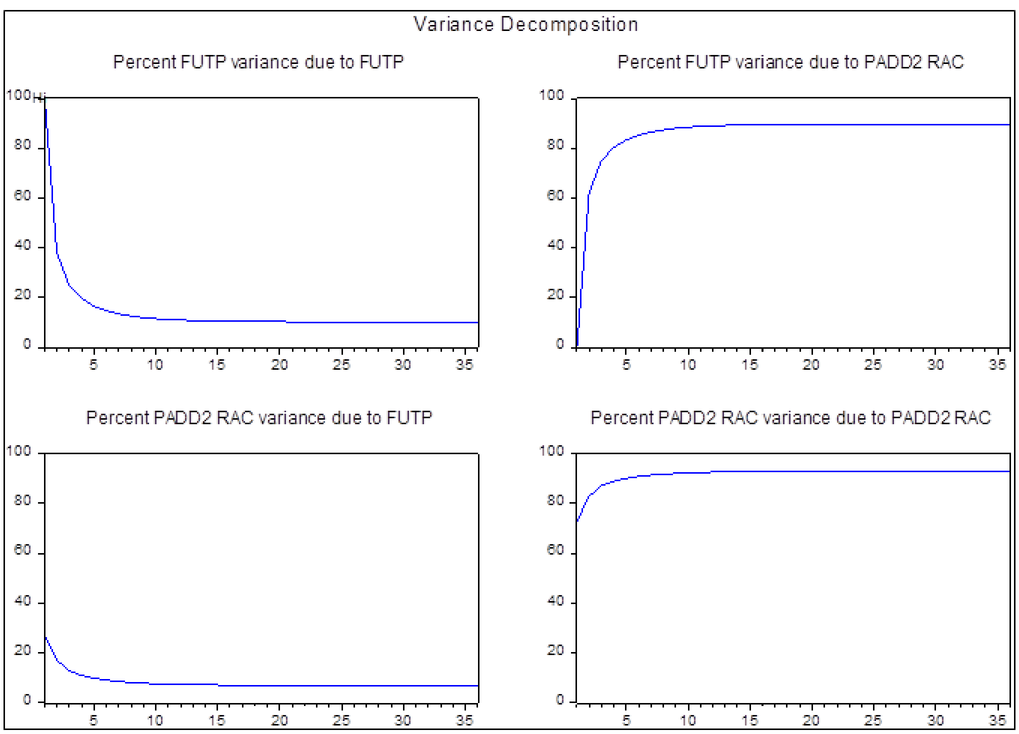


Figure 4. PADD 2 Variance Decomposition

6.2.2. PADD 3

Futures price and PADD 3 RAC are stationary in the levels while PADD 3 ending stocks is stationary in the first difference. Once again, two of the three variables are stationary, so we cannot have a cointegrating vector between any of the three variables. RAC Granger causes futures price at the 1% level while futures price Granger causes RAC at the 10% level. For DAGs, both algorithms produce graphs with an undirected edge between futures price and RAC, and between futures price and ending stocks. Thus, DAGs suggest three endogenous variables, so we estimate a VAR with all three variables as endogenous where AIC suggests three lags.

The results of PADD 3 VAR can be seen in Table 10. For the explanation of the change in ending stocks, we can see that the differenced third lag of PADD3 STOCKS is significant with a negative sign, while the first lag of FUTP is significant with a positive sign and the second lag of FUTP is significant with a negative sign. For FUTP we see the differenced first lag of ending stocks is significant, all three lags of FUTP are significant with positive signs with the first significant at the 10% level, while all three lags of RAC are significantly different from zero with the first having a positive sign and the latter two having negative signs. In the explanation of RAC, we see that only the first and second lag of RAC are significant with the first having a positive sign and the second having a negative sign.

The impulse responses and variance decompositions of our second PADD 3 model can be seen in Figure's 5 and 6. We can see that a shock to FUTP or RAC does not prompt much of a response in PADD 3 stocks and the minor effects are over within a few months. The response of FUTP to an innovation in ending stocks causes a decrease in price which appears to be slowly decaying back towards zero in the long run while a shock to RAC causes an increased price which also appears to be slowly reverting towards zero in the long run. A shock to ending stocks prompts a similar response in RAC as it did in FUTP, while a shock to futures price causes an initial increase in RAC's before returning to near zero. For the variance decompositions, we can see that ending stock variance is mostly accounted for due to its own innovations, while the variance of futures price is about 70% due to RAC in the long term. RAC's variance is also mostly accounted for by its own innovations.

Table 10. PADD 3 VAR

	ΔPADD3 STOCKS	FUTP	PADD3 RAC
ΔPADD3 STOCKS(-1)	0.1267	-0.00008**	0.0000
	[1.61619]	[-2.13913]	[-1.52744]
ΔPADD3 STOCKS(-2)	0.0108	0.0000	0.0000
	[0.13580]	[0.42115]	[0.47446]
ΔPADD3 STOCKS(-3)	-0.2168***	0.0000	0.0000
	[-2.72393]	[-1.04923]	[-0.50655]
FUTP(-1)	408.8833**	0.1783*	-0.1766
	[2.03406]	[1.93884]	[-1.61193]
FUTP(-2)	-423.9653**	0.2674***	-0.0927
	[-2.33122]	[3.21488]	[-0.93533]
FUTP(-3)	148.7819	0.2949***	0.1051
	[0.85927]	[3.72384]	[1.11399]
PADD3 RAC(-1)	-230.2248	1.359***	1.6061***
	[-1.30829]	[16.8821]	[16.7446]
PADD3 RAC(-2)	-152.2992	-0.7224***	-0.3676**
	[-0.49394]	[-5.12212]	[-2.1872]
PADD3 RAC(-3)	267.8779	-0.4278***	-0.1325
	[0.99388]	[-3.46948]	[-0.90207]
C	-915.0586	3.5975	4.267
	[-0.37020]	[3.18174]	[3.16705]
R-squared	0.1178	0.9747	0.9704

t - statistics in [ ]

* 10% Significance

** 5% Significance

*** 1% Significance

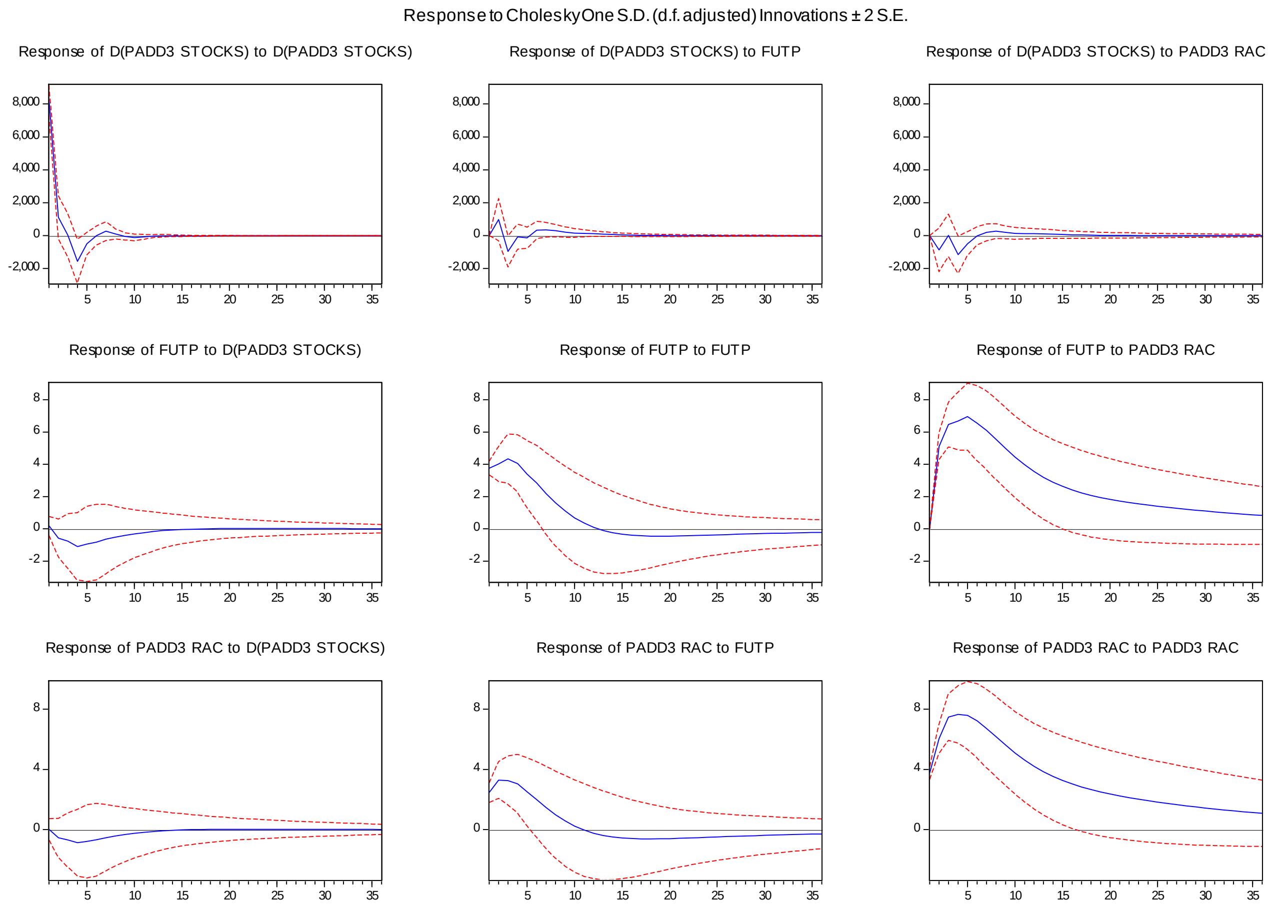


Figure 5. PADD 3 Impulse Responses

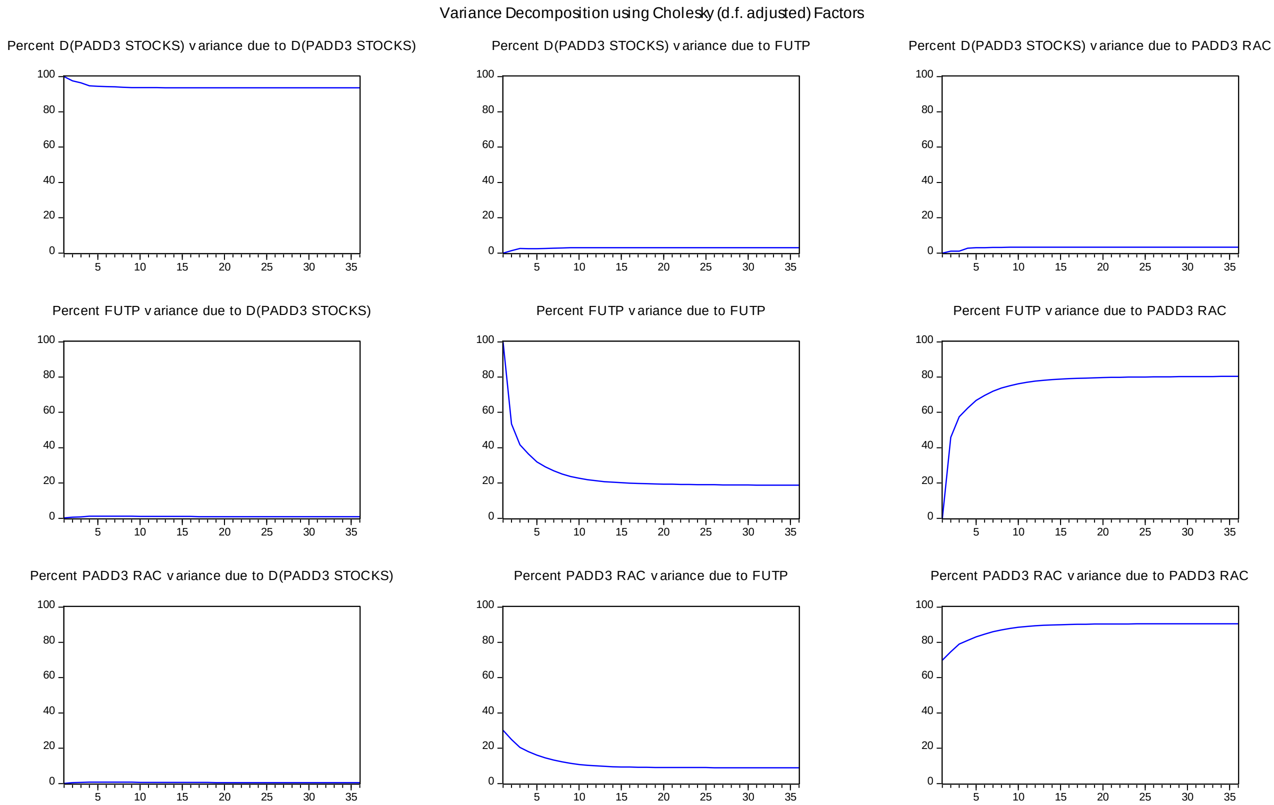


Figure 6. PADD 3 Variance Decomposition

6.2.3. PADD 4

For PADD 4, again we have futures price and PADD 4 RAC as stationary in the levels while PADD 4 ending stocks is stationary in the first difference. Granger causality indicates two-way causality between futures price and RAC. The DAGs suggest endogeneity between futures price and RAC. Thus, we estimate a VAR in the levels with three lags based on AIC, with the results in Table 11.

In explaining FUTP, the second and third lag of FUTP are significant with positive coefficients, the first and third lag of RAC are significant with the first having a positive coefficient and the third having a negative coefficient. In the explanation of RAC, the first and third lag of FUTP is significant with the first having a negative coefficient and the third lag having a positive coefficient. The first and third lag of RAC are also significant with the first having a positive coefficient and the third having a negative coefficient.

Once again, in Figure 7, we see that a shock to an endogenous variable has increasing effects in the short term, but they are ultimately transitory as the effects are practically nonexistent by the 36^{th} month. For the variance decomposition results in Figure 8, we find that once again RAC plays an increasing role in the variation of FUTP as time increases. FUTP accounts for about 33% of RAC variance in the first period, dips to about 24% in the third period, and rises to about 30% again by the 36^{th} period.

Table 11. PADD 4 VAR

	FUTP	PADD4 RAC
FUTP(-1)	-0.0093	-0.3219**
	[-0.09629]	[-2.36133]
FUTP(-2)	0.2339**	0.1606
	[2.41308]	[1.17159]
FUTP(-3)	0.1920***	0.2043**
	[2.98048]	[2.24168]
PADD4 RAC(-1)	1.1771***	1.4907***
	[16.9714]	[15.1951]
PADD4 RAC(-2)	-0.1619	-0.1336
	[-1.3628]	[-0.79502]
PADD4 RAC(-3)	-0.3866***	-0.4596***
	[-3.50636]	[-2.94706]
C	0.4341	3.7973
	[0.43726]	[2.70422]
ΔPADD4 STOCKS	0.0004	-0.0002
	[0.92816]	[-0.26875]
R-squared	0.9755	0.9411

t - statistics in [ ]

* 10% Significance

** 5% Significance

*** 1% Significance

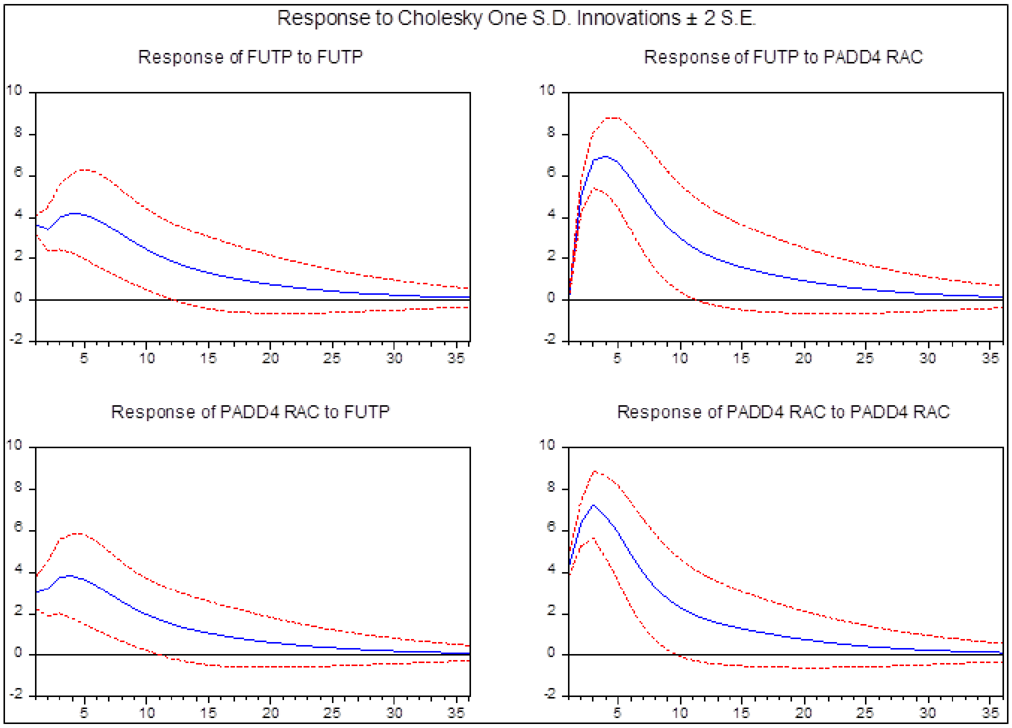


Figure 7. PADD 4 Impulse Responses

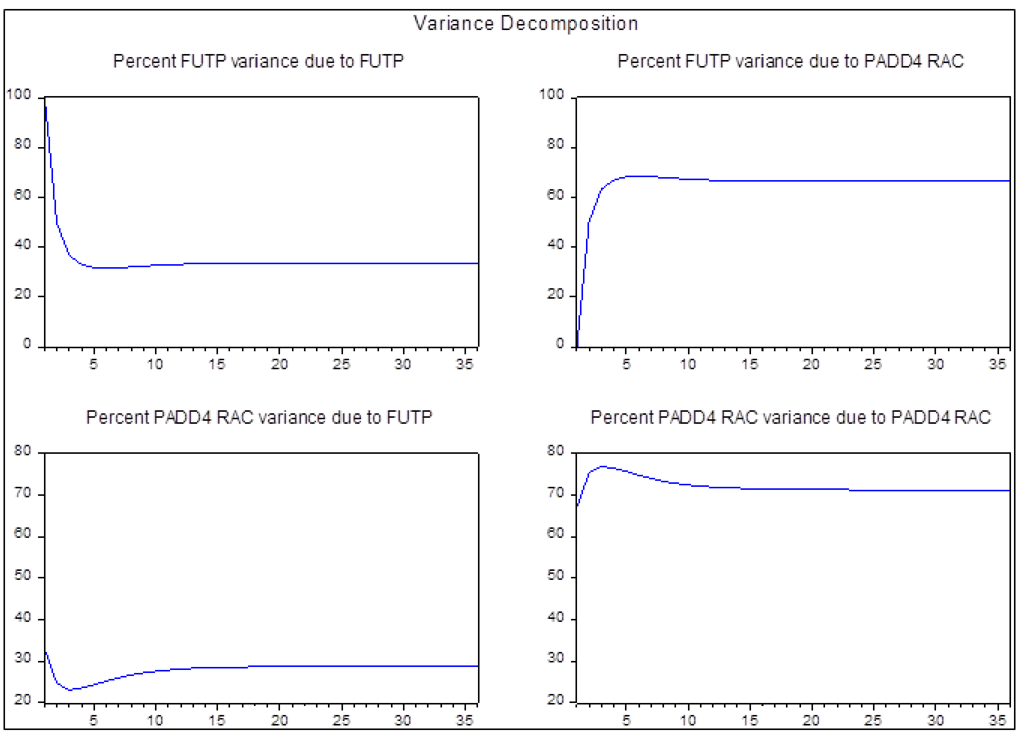


Figure 8. PADD 4 Variance Decomposition

6.2.4. PADD 5

In PADD 5, the final PADD, we see that futures price, RAC and ending stocks are all stationary in the levels. Granger causality indicates two-way causality between futures price and RAC while the DAGs indicate endogeneity between futures price and RAC. Therefore, we estimate a VAR in the levels with three lags based on AIC, with the results in Table 12.

The results for PADD 5 suggests that all three lags of FUTP and RAC are all significant in explaining FUTP. We see that the signs of the three FUTP lags and the first RAC lag are all positive while the second and third lags of RAC have negative signs. In the explanation of RAC, the first lag of FUTP and third lag of RAC are significant with both having a negative coefficient, while the first lag of RAC is also significant with a positive sign.

PADD 5 impulse responses in Figure 9 show similar patterns as other PADDs. A shock to RAC has an increasing effect in the first few months but begins decreasing thereafter. While it does not entirely return to zero, FUTP response appears to be trending toward zero by the 36^{th} period. A shock to FUTP also appears to have transitory effects on RAC after an early increase in standard deviation dissipates. Again, variance decomposition for PADD 5, indicated in Figure 10, suggests RAC plays an increasing role in FUTP variance as time passes. Conversely, FUTP plays a decreasing role in RAC variance as time passes.

Table 12. PADD 5 VAR

	FUTP	PADD5 RAC
FUTP(-1)	0.2342**	-0.3978***
	[2.48305]	[-3.59301]
FUTP(-2)	0.3929***	0.1337
	[3.95812]	[1.14756]
FUTP(-3)	0.1694**	0.1436
	[2.03740]	[1.47141]
PADD5 RAC(-1)	1.1047***	1.6176***
	[13.2874]	[16.5750]
PADD5 RAC(-2)	-0.4910***	-0.2588
	[-3.54446]	[-1.59164]
PADD5 RAC(-3)	-0.4511***	-0.2935**
	[-3.77787]	[-2.09373]
C	0.2519	1.9961
	[0.03893]	[0.26281]
PADD5 STOCKS	0.0001	0.0000
	[0.44847]	[0.29025]
R-squared	0.9671	0.9630

t - statistics in [ ]

* 10% Significance

** 5% Significance

*** 1% Significance

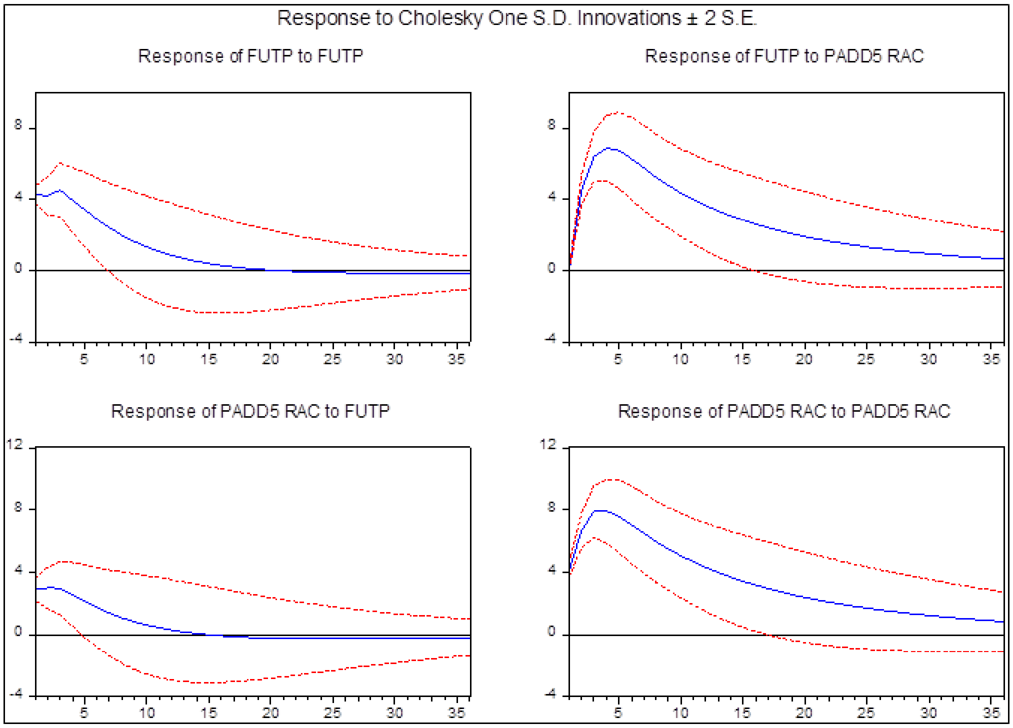


Figure 9. PADD 5 Impulse Responses

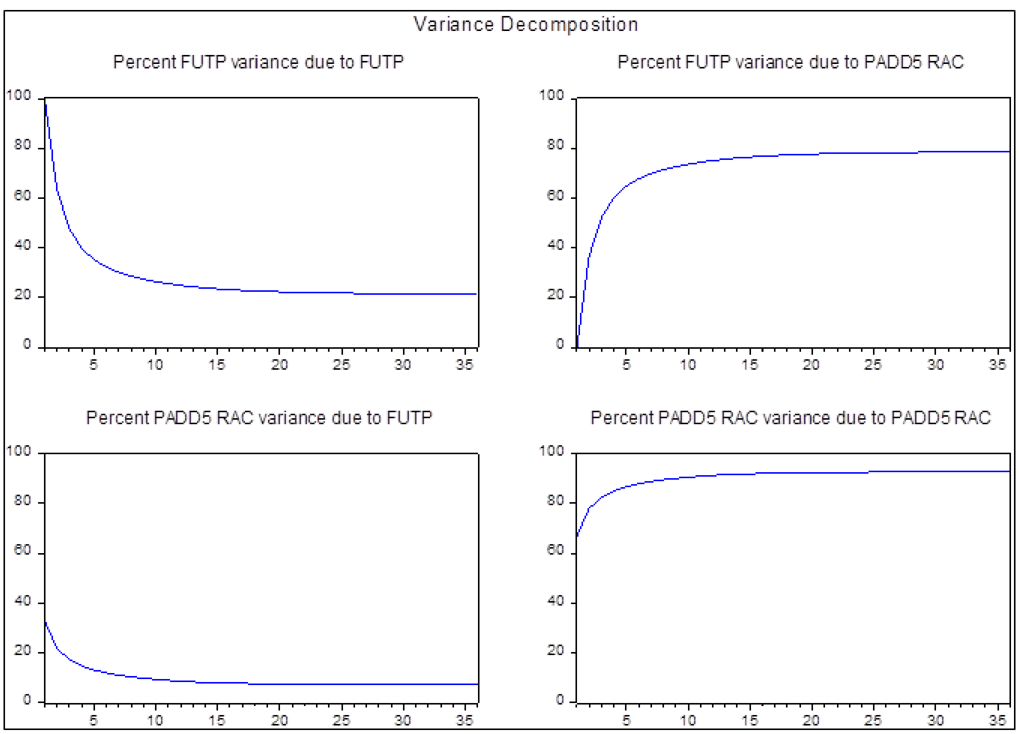


Figure 10. PADD 5 Variance Decomposition

6.3. Corn

Unit root testing for corn indicated that futures price, price received (SPOT), and ending stocks are all stationary in their first differences. Cointegration testing suggests the presence of a cointegrating vector between futures price and SPOT. Granger causality does not indicate any two-way causality, only futures price Granger causing spot price. The FGES algorithm suggests endogeneity of futures price and SPOT, while the PC algorithm produced a graph with undirected edges between futures price and SPOT as well as between futures price and ending stocks. Although the endogeneity is not entirely clear, given the data-driven approach and higher power of DAGs, we are comfortable moving forward with endogenous variables of FUTP and SPOT. Thus, we estimate a VECM where AIC indicated one lag is the best specification. In Table 13, we see that we can reject the speed of adjustment coefficient, indicating futures and spot price adjust at different rates. The results of our unrestricted VECM are contained in Table 14.

For the cointegrating vector, we see that the lag of SPOT is significant with a negative coefficient. In explaining the first difference of futures price, we find that only the first lag of FUTP price is significant with a negative sign. In explaining the first difference of spot price, we see that the cointegrating vector is significant with a positive coefficient suggesting spot price adjusts towards the long run relationship with futures price. Also, the first lag of FUTP is significant with a positive coefficient, the first lag of SPOT is significant with a negative sign, and the first difference of ending stocks is significant while the coefficient rounded to 4 decimal places is 0; however, we can see that it is negative, indicating an increase in ending stocks has a very small negative impact on spot price.

The impulse responses for corn are located in Figure 11. We see that an innovation in SPOT has a permanent decreasing effect to FUTP level by nearly $0.10 in the long run. Conversely, a shock to FUTP appears to have a permanent increasing effect to the level of SPOT by over $0.40 in the long run. Investigating the variance decompositions in Figure 12, we see that SPOT accounts for a tiny percentage of the variability of FUTP at only about 2% by the 36^{th} period. On the other hand, FUTP has a significant increasing role in the volatility of SPOT through time, accounting for 97% of the variance in the 36^{th} period. Thus, in the long run, futures markets increase the level of spot price and are a key driver of volatility.

Table 13. Corn Restricted VECM

Cointegration Restrictions:		
$\pi_{11} = \pi_{21}$		
Convergence achieved after 4 iterations.		
Not all cointegrating vectors are identified		
LR test for binding restrictions (rank = 1):		
Chi-square(1)	5.3135	
Probability	0.02116**	
Cointegrating Eq:	CointEq1	
FUTP(-1)	-2.5251	
SPOT(-1)	2.835474	
C	0.0780	
Error Correction:	ΔFUTP	ΔSPOT
CointEq1	-0.1125*** [-7.63874]	-0.1125*** [-7.63874]
ΔFUTP(-1)	-0.1966* [-1.94025]	0.1050** [2.29121]
ΔSPOT(-1)	0.0034 [0.02733]	-0.1715*** [-3.02790]
C	0.0091 [0.27721]	0.0093 [0.63005]
ΔSTOCK	0.0000 [0.76798]	0.0000*** [-3.66922]
R-squared	0.0240	0.5033

t - statistics in [ ]
* 10% Significance
** 5% Significance
*** 1% Significance

Table 14. Corn VECM

Cointegrating Eq:	CointEq1	
FUTP(-1)	1.0000	
SPOT(-1)	-1.0994***	
	[-32.4578]	
C	-0.1143	
Error Correction:	ΔFUTP	ΔSPOT
CointEq1	0.1093	0.2912***
	[1.31364]	[7.74185]
ΔFUTP(-1)	-0.1847**	0.1064***
	[-1.82344]	[2.32353]
ΔSPOT(-1)	0.0052	-0.1759***
	[0.04149]	[-3.09864]
C	0.0090	0.0093
	[0.27453]	[0.63093]
ΔSTOCK	0.0000	0.0000***
	[0.74490]	[-3.66512]
R-squared	0.0212	0.5026

t - statistics in [ ]

* 10% Significance

** 5% Significance

*** 1% Significance

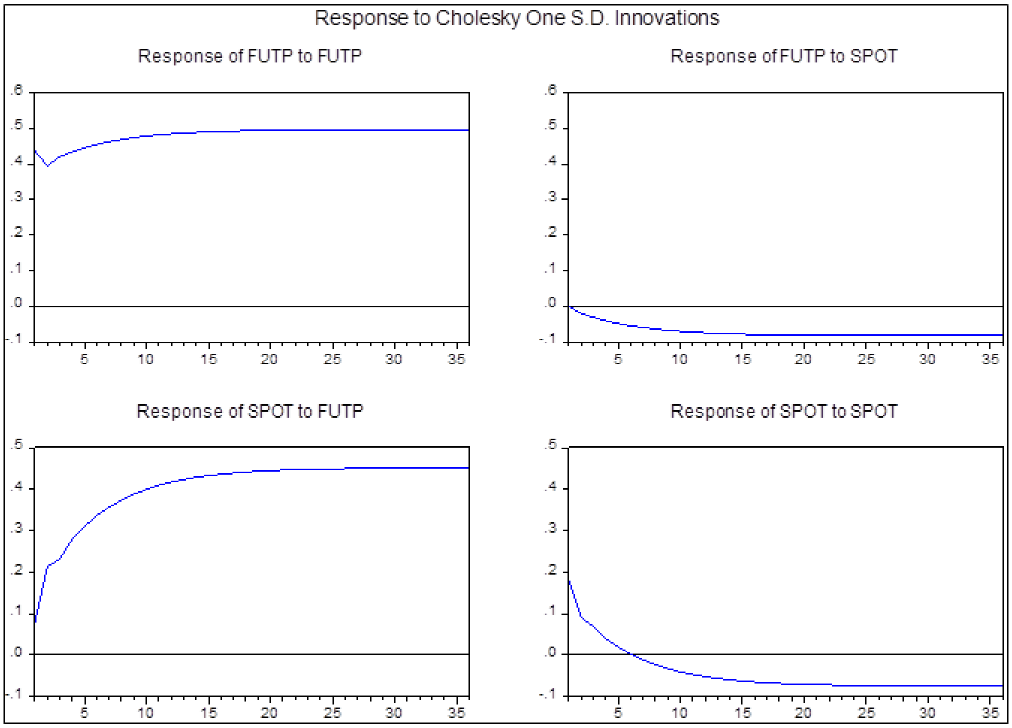


Figure 11. Corn Impulse Responses

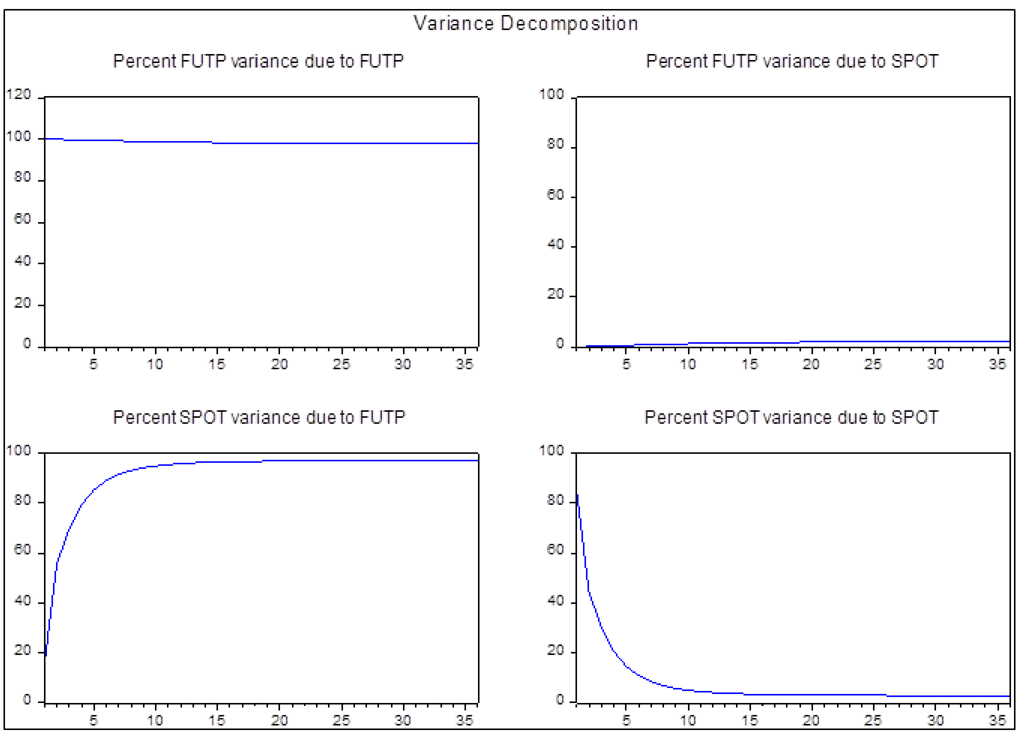


Figure 12. Corn Variance Decomposition

6.4. Soybeans

The KPSS test for soybeans suggests that all three variables are stationary in the first differences. Engle-Granger indicates a cointegrating relationship between futures price and price received (SPOT). Two way Granger causality exists between futures price and price received. Both DAG algorithms suggest endogeneity between FUTP and SPOT. Thus, we estimate a VECM with three lags. Table 15 indicates we can reject the speed of adjustment restriction; results for the unrestricted VECM are located in Table 16.

In the cointegrating equation, lagged SPOT is significant with a negative coefficient. In explaining the first difference of FUTP, the third lag of FUTP is significant with a negative sign while the first two lags of SPOT are significant with positive signs. In explaining the first difference of SPOT, the cointegrating vector is significant with a positive adjustment coefficient, while the first lag of FUTP and the first difference of STOCK are significant at the 10% level.

The soybean impulse responses are located in Figure 13 while the variance decomposition is located in Figure 14. A shock to SPOT prompts a permanent increase in FUTP by nearly \$0.20 in the long run. Similarly, an innovation in FUTP elicits a permanently increased response in the standard deviation of SPOT by almost \$0.80 in the long run. SPOT also comprises only a small portion of FUTP variance, accounting for only 4% in the 36th period. A large portion of SPOT variance is made up of FUTP, which accounts for 95% of the variance by the 36th month.

Table 15. Soybeans Restricted VECM

Cointegration Restrictions:		
$\pi_{11} = \pi_{21}$		
Convergence achieved after 4 iterations.		
Not all cointegrating vectors are identified		
LR test for binding restrictions (rank = 1):		
Chi-square(1)	12.1863	
Probability	0.0005***	
Cointegrating Eq:	CointEq1	
FUTP(-1)	-2.4268	
SPOT(-1)	2.7277	
C	-0.6460	
Error Correction:	ΔFUTP	ΔSPOT
CointEq1	-0.1414***	-0.1414***
	[-5.26514]	[-5.26514]
ΔFUTP(-1)	-0.0993	0.0972
	[-0.69228]	[1.55799]
ΔFUTP(-2)	-0.1580	0.0122
	[-1.20074]	[0.21322]
ΔFUTP(-3)	-0.3841***	-0.0829*
	[-3.52441]	[-1.74966]
ΔSPOT(-1)	0.5591***	0.0386
	[2.64322]	[0.42009]
ΔSPOT(-2)	0.6224***	0.0071
	[3.16849]	[0.08350]
ΔSPOT(-3)	-0.1346	-0.0596
	[-0.84841]	[-0.86412]
C	0.0199	0.0279
	[0.30284]	[0.97655]
ΔSTOCK	0.0000	0.0000*
	[0.22118]	[-1.79880]
R-squared	0.1519	0.4335

t - statistics in [ ]

*   10% Significance

**   5% Significance

*** 1% Significance

Table 16. Soybeans VECM

Cointegrating Eq:	CointEq1	
FUTP(-1)	1.0000	
SPOT(-1)	-1.0870*** [-48.1708]	
C	-0.0677	
Error Correction:	ΔFUTP	ΔSPOT
CointEq1	-0.1888 [-1.18789]	0.2791*** [3.99476]
ΔFUTP(-1)	-0.0328 [-0.22842]	0.1105* [1.74986]
ΔFUTP(-2)	-0.1175 [-0.89542]	0.0214 [0.37160]
ΔFUTP(-3)	-0.3637*** [-3.34896]	-0.0780 [-1.63313]
ΔSPOT(-1)	0.5212** [2.47882]	0.0263 [0.28444]
ΔSPOT(-2)	0.6140*** [3.14131]	-0.0018 [-0.02114]
ΔSPOT(-3)	-0.1022 [-0.63985]	-0.0631 [-0.89862]
C	0.0163 [0.24854]	0.0277 [0.96286]
ΔSTOCK	0.0000 [0.21704]	0.0000* [-1.76599]
R-squared	0.1572	0.4249

t - statistics in [ ]

* 10% Significance

** 5% Significance

*** 1% Significance

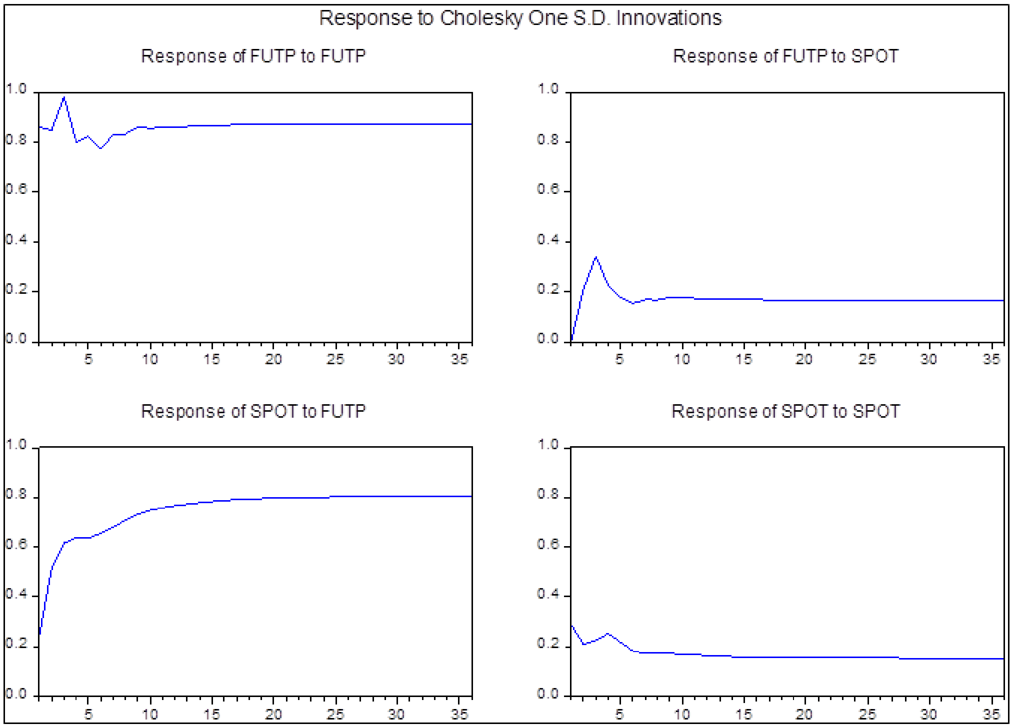


Figure 13. Soybeans Impulse Responses

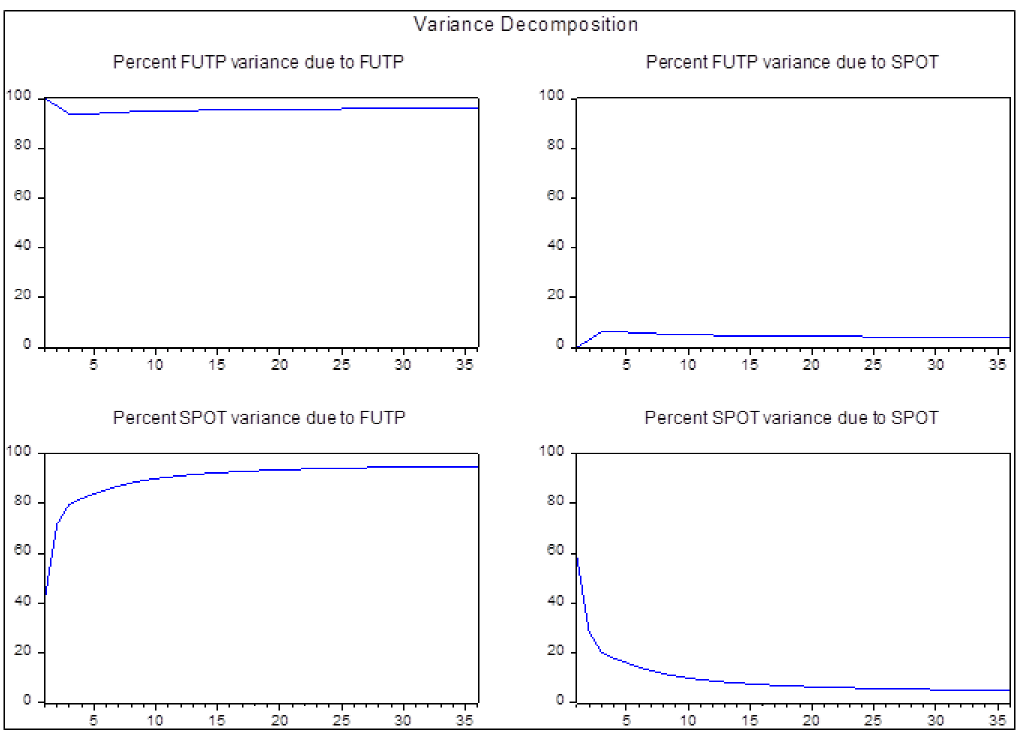


Figure 14. Soybeans Variance Decomposition

6.5. Hard Red Spring Wheat

For HRS wheat, futures price and price received are stationary in the first differences, while ending stocks are stationary in the levels. The cointegration tests provide evidence of a cointegrating relationship between futures price and price received. Granger causality tests indicate two-way causality between futures price and price received. The FGES algorithm suggests endogeneity between futures price and price received while the PC algorithm provides a graph with an undirected edge between futures price and price received and an undirected edge between futures price and ending stocks. On balance, the evidence suggests futures price and price received are endogenous variables. We estimate a VECM with four lags, with the results in Table 17. We can see that we fail to reject the restriction the speed of adjustment coefficient, indicating futures and spot price adjust at the same rate.

We see that the cointegration equation is significant with a negative coefficient in explaining the first difference of futures price and spot price. For futures price, the fourth lag of FUTP is significant with a positive coefficient, and the first and third lags of SPOT are significant with negative coefficients. In explaining ΔSPOT, the first lag of futures price is significant with a positive coefficient, the first and second lags of SPOT are significant with the first having a positive coefficient and the second having a negative coefficient. The change in ending stocks is also significant with a negative coefficient rounded to zero at decimal four places suggesting an increase in ending stocks has a small inverse effect on spot price.

Impulse responses and variance decompositions are contained in Figure 15 and 16. We can see that a shock to SPOT decreases FUTP by about $0.50 in the long term. An innovation in FUTP prompts a permanent long-term increase in the level of spot price by about $1.20. For the decomposition of FUTP variance, SPOT accounts for about 13% of the long run variance. Futures price accounts for about 87% of the variance of spot price in the long run.

Table 17. HRS Wheat Restricted VECM

Cointegration Restrictions:		
$\pi_{11} = \pi_{21}$		
Convergence achieved after 3 iterations.		
Not all cointegrating vectors are identified		
LR test for binding restrictions (rank = 1):		
Chi-square(1)	1.8634	
Probability	0.1722	
Cointegrating Eq:	CointEq1	
FUTP(-1)	-3.1454	
SPOT(-1)	3.4326	
C	-0.0021	
Error Correction:	ΔFUTP	ΔSPOT
CointEq1	-0.1227*** [-5.98533]	-0.1227*** [-5.98533]
ΔFUTP(-1)	0.1291 [0.64163]	0.1930** [2.52463]
ΔFUTP(-2)	0.3201 [1.63952]	0.0427 [0.57512]
ΔFUTP(-3)	0.2536 [1.42089]	-0.0136 [-0.20008]
ΔFUTP(-4)	0.3823*** [3.25612]	0.0314 [0.70520]
ΔSPOT(-1)	-1.2193*** [-4.40519]	-0.3538*** [-3.36563]
ΔSPOT(-2)	-0.0886 [-0.33096]	0.2362** [2.32307]
ΔSPOT(-3)	-0.4803** [-2.01267]	-0.0834 [-0.92018]
ΔSPOT(-4)	0.2504 [1.44574]	-0.0450 [-0.68481]
C	0.0154 [0.25960]	0.0104 [0.46373]
ΔSTOCK	0.0000 [-1.42095]	0.0000*** [-3.27995]
R-squared	0.2239	0.6253

t - statistics in [ ]

* 10% Significance

** 5% Significance

*** 1% Significance

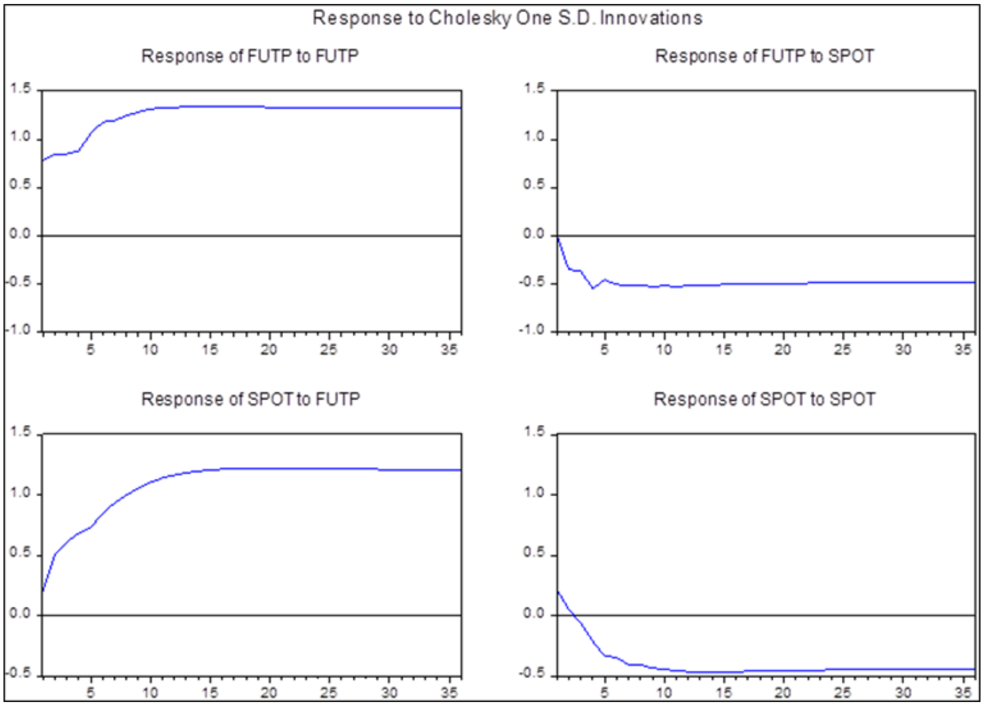


Figure 15. Restricted HRS Wheat Impulse Responses

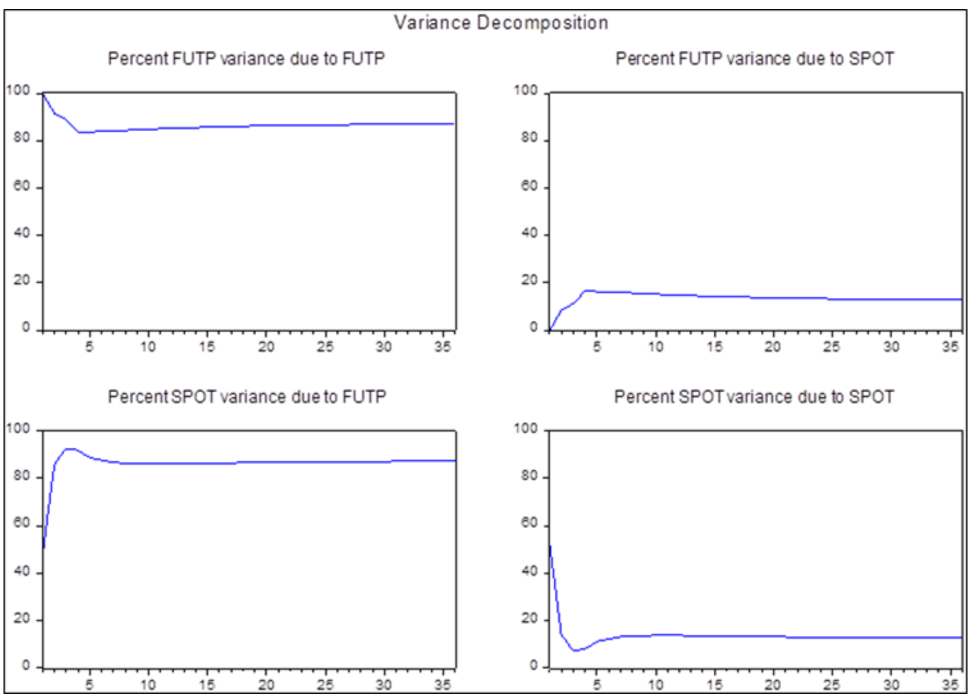


Figure 16. Restricted HRS Wheat Variance Decomposition

7. CONCLUSIONS

To examine the relationships between futures prices, spot prices, and inventory levels we utilized VAR/VEC models. We used unit root testing which, for the most part, suggested prices and inventories were stationary in the first differences. Cointegration testing revealed that futures prices and spot prices have a long-run equilibrium relationship for all commodities except for the regional oil PADD's. We determined endogeneity of variables using both Granger causality and DAG techniques which mostly suggested futures price and spot price were endogenous while ending stocks are exogenous. Impulse responses revealed that shocks in the futures market have a permanent increasing effect on the price level of the agricultural spot markets while national oil spot prices decrease in the long run and the regional oil PADD's impulses have transitory increasing effects. From our variance decompositions, we see that futures markets account for over 90% of the long-term variance in spot prices for the corn, soybeans, and nearly 90% for wheat. For oil, we see that futures markets play a much small role in the spot price variance over time while most of the long-term futures price variance can be attributed to the spot market.

It has generally been believed among scholars and experts that futures markets have a stabilizing effect on spot prices. The underpinnings of this price stabilizing relationship seem to make sense on an intuitive level. However, our empirical results for the agricultural commodities run counter to what one would expect to find. Our findings of price destabilization coupled with the theory would indicate that the dominant stochastic factor in agricultural markets is either inventory demand disturbances or production disturbances. We see oil price is destabilized initially, while the national oil is stabilized in the long run and regional oil returns to nearly no effects long term. The theory would seem to indicate that production disturbances are the dominant stochastic factor in the oil markets. To us, the contradictory nature of some of the results indicates that there may still be phenomena at play in the related markets for storable commodities which are not fully understood. We do not believe our results should serve as the end of the discussion on the stabilizing/destabilizing nature of futures markets; they should serve as a prompt that more work should be done to better understand the effects of futures markets on the volatility and behavior of spot prices of storable commodities.

Given that there is no clear consensus on the effects of futures markets on spot markets in this thesis or the field in general, we feel it to be entirely premature to draw any type of policy conclusions from our results. As mentioned, more research must be done in this area so we may at some point come to a uniform understanding of the relationship between futures market and spot market volatility. It is also possible that, given our differing results between oil and agricultural commodities, this area may be too nuanced for a uniform policy response (if one is necessary) across the board. We hope that this thesis and future works by scholars in this area will reveal more clearly any policy implications that arise due to the relationship between futures and spot markets.

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