PROPOSED NONPARAMETRIC TESTS FOR THE SIMPLE TREE ALTERNATIVE FOR LOCATION AND SCALE TESTING

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ABSTRACT

Location-scale problems arise in many cases, such as, bioinformatics, climate dynamics, finance and medicine (Marozzi, 2013). This research focuses on developing tests to determine whether one or more treatment effects differ from the control. It will be assumed that when a treatment effect differs from the control effect, it is greater either in mean or variance (simple tree alternative). It is also assumed that a treatment effect difference results in the change of the location and / or scale parameters between two population distributions. This research will consider the area of nonparametric tests when determining whether one or more of the treatment effects is larger than the control.

Five nonparametric tests are proposed for the simple tree alternative. A simulation study will be conducted to determine how well the proposed tests maintain their significance levels. Powers will be estimated for the proposed tests under a variety of conditions for two, three and four populations. Three different types of variable parameters will be considered. The first type considered is when the location parameters are different, and the scale parameters are equal. The second type considered is when the location parameters are equal, and the scale parameters are different. The final type considered is when the location and scale parameters are both different. Results will be given as far as which test does better under certain conditions.

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DEDICATION

To my wife Meshael Alshahrani and my children Jood and Naif.

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CHAPTER 1. GENERAL INTRODUCTION

The nonparametric methods are generally used in many fields, including biostatistics, business, pharmaceutical statistics, psychology, and social sciences. Nonparametric tests are often more suitable due to the weaker assumptions they have about the underlying populations and the requirements for the measurement scales (Wang, 2011). In many cases researchers find themselves in a situation where they want to compare one or more treatments with a standard or control treatment. In these cases, the simple tree hypothesis may be the most appropriate hypothesis (Conroy, 2011). The simple tree alternative is given by:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \tag{1}$$

$$H_a: \mu_1 \leq [\mu_2, \ldots, \mu_k] \text{ (At least one strict inequality)}$$

where μ_i is the location parameter of population i.

There may be some situations in which a treatment not only may affect the location but also may affect the variance (or scale) of a distribution (Marozzi, 2013). The treatment may affect one or the other or both simultaneously.

Most of the past works regarding this area, are focused on detecting location changes or scale changes only. There is sometimes a need to test the location and scale parameters at the same time. Researches may want to know if a treatment has any effect at all. This may mean by increasing the mean of the observations or by increasing the variance of the observations.

The most common test for the two-sample location-scale problem is the Lepage test (Lepage, 1971). This test is based on a combination of the Mann-Whitney test (Mann and Whitney, 1947) and the Ansari-Bradley test (Ansari and Bradley, 1960). The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 \text{ and } \sigma_1 = \sigma_2 \quad (2)$$

$$H_a: \mu_1 \neq \mu_2 \text{ and / or } \sigma_1 \neq \sigma_2$$

where μ_i and σ_i are the location and scale parameters of population i respectively.

Hollander and Wolfe (1999) mentioned an example where the Lepage test can be applied. The example concerns the effect of maternal steroid therapy on platelet counts of newborn infants. Autoimmune Thrombocytopenic Purpura (ATP) is a disease where the patient produces antibodies to her own platelets. Children of a mother with ATP are frequently born with low platelet counts. The effect of administering corticosteroid prednisone to pregnant women with ATP was studied by (Karpatkin and Porges, 1981) in order to raise the infants' platelet counts to safe levels during their deliveries (Hollander and Wolfe, 1999). The goal of the study is to determine whether or not predelivery maternal prednisone therapy increases the platelet counts of a newborn baby. The main problem is to detect a possible change in mean platelet counts for the treated population compared to the non-treated population. However, there is also some concern that the variability in newborn baby platelet counts would be increased by the steroid therapy.

This research extends the hypotheses test given in (2) to the hypotheses test given in (3). This research is concerned with testing the hypothesis for the simple tree alternative:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k, \quad (3)$$

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k, \text{ versus}$$

$$H_a: \mu_1 \leq [\mu_2, \ldots, \mu_k] \text{ and / or}$$

$$H_a: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k] \text{ where at least one inequality is strict.}$$

where μ_i is a location parameter (the median or mean) for population i and σ_i is a scale parameter with $i = 1,2, \ldots, k$ and k are the total number of populations. Population one ($i = 1$) is usually referred to the control population, while populations 2 through k are the treatment populations.

Chapter Two will present a review of the literature regarding nonparametric statistical tests used in analyzing data. Chapter Three will describe the proposed test statistics, while Chapter Four outlines the simulation study for this research. In Chapter Five, the results obtained from the simulation study will be illustrated using tables and Chapter Six will state the conclusions concerning the proposed test statistics.

CHAPTER 2. LITERATURE REVIEW

2.1. Mann-Whitney

The Mann-Whitney test is a standard test statistic for examining the null hypothesis of equal population location parameters (Mann and Whitney, 1947). The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 \tag{4}$$

$$H_{a1}: \mu_1 \neq \mu_2,\ H_{a2}: \mu_1 < \mu_2, H_{a3}: \mu_1 > \mu_2$$

In order to compute the test statistics *MW*, it will be assumed that there is a sample of size n_1 from the population 1 and a sample size n_2 of the population 2. The measurements of combined set of $n_1 + n_2 = N$, have been arranged in order from smallest to largest. Ranks were then assigned to the ordered measurements and S_j will be the rank of jth population 2, within the set of ranks. The test statistics *MW* is the sum of the ranks of all measurements in the population 2.

$$MW = \sum S_j \tag{5}$$

The standardized version of Mann-Whitney test is given by:

$$MW^* = \frac{MW - E_0(MW)}{\sqrt{var_0(MW)}} \tag{6}$$

$$E_0(MW) = \frac{n_2(N+1)}{2}$$

$$var_0(MW) = \frac{n_1 n_2 (N+1)}{12} \tag{7}$$

When H_0 is true, the test statistics MW^* has approximately a standard normal distribution. H_0 will be rejected when $MW^* \geq Z_{\alpha/2}$ at the α level of significance where $Z_{\alpha/2}$ is the (1- $\alpha/2$) 100% of the standard normal distribution.

2.2. Wilcoxon Signed Rank

The Wilcoxon Signed Rank (WSR) test is used for testing the null hypothesis of equal population location parameters, when data are a mixture of paired observations (Wilcoxon, 1945). In contrast to the Mann-Whitney test, the WSR test does not require the independence assumption. The null hypothesis and alternative hypothesis are written below:

$$H_0: \mu_D = 0 \tag{8}$$

$$H_{a1}: \mu_D \neq 0, H_{a2}: \mu_D < 0, H_{a3}: \mu_D > 0$$

In order to obtain the WSR test, the absolute differences of the repeated measurement must first be calculated. Second, the absolute differences are ranked from smallest to largest and assign to each rank the sign of the difference. Finally, the resulting sum of the rank with positive signs T_+ and the sum of the rank with negative signs T_-. The test statistic WSR is either T_+ or T_-, depending on which is smaller.

The standardized version of Wilcoxon Signed Rank test is given by:

$$WSR^* = \frac{WSR - E_0(WSR)}{\sqrt{var_0(WSR)}} \tag{9}$$

$$E_0(WSR) = \frac{n(n+1)}{4} \tag{10}$$

$$var_0(WSR) = \frac{n(n+1)(2n+1)}{24} \tag{11}$$

When H_0 is true, the test statistics WSR^* has approximately a standard normal distribution. H_0 is rejected when $WSR^* \geq Z_{\alpha/2}$ at the α level of significance, where $Z_{\alpha/2}$ is the (1- $\alpha/2$) 100% of the standard normal distribution.

2.3. Kruskal Wallis

The Kruskal Wallis test is an extended version of the Mann-Whitney test if there are more than two populations (Kruskal and Wallis,1953). This test statistic is the nonparametric analogous

to the parametric one-way analysis of variance. The alternative hypothesis is when two or more of the treatment effects are not equal. The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k \tag{12}$$

$$H_a: \text{at least one of the two } \mu_i'\text{s are different}$$

where μ_i is a median of the i^{th} population. To compute the Kruskal-Wallis test statistic *H,* all measurements N will be catalogued into ascending order and subsequently ranked. Within each treatment the corresponding ranks will be summed, in order to attain an average rank. The Kruskal Wallis statistic is:

$$H = \frac{12}{N(N+1)} \sum_{I=1}^{k} \frac{{R_i}^2}{n_i} - 3(N+1) \tag{13}$$

R_i is the sum of the ranks assigned to observations in the i^{th} sample, n_i is the sample size of i^{th} sample and *N* is the combined sample size. When H_0 is true, H has an asymptotic chi-square distribution with *k*-1 degrees of freedom.

2.4. Jonckheere Terpstra

There are some circumstances in which one may wish to test the null hypothesis of equal group medians against an alternative hypothesis where the medians are nondecreasing.

Among the first to introduce a distribution-free test for a nondecreasing ordered alternative for location parameters were Jonckheere (1954) and Terpstra (1952). The JT test statistic is based on a comparison of pairs of treatments using their corresponding values. The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \tag{14}$$

$$H_a: \mu_1 \leq \cdots \leq \mu_k \text{ with at least one strict inequality.}$$

where $\mu_1, \ldots, \mu_k$ are the location parameters of the i^{th} sample.

Jonckheere – Terpstra test statistic, JT is:

$$JT = \sum_{i<j} U_{ij} \tag{15}$$

In order to calculate JT test, each a observation in treatment A must be combined with every b observation in treatment B until pairs of all possible combinations are formed.

If a is less than b the pair will be assigned 1 otherwise 0. U_{ij} will be the number of pairs of observations (a,b) for which A_{ia} is less than B_{ib}.

The expected value and variance of JT are given by

$$E_0(JT) = \frac{N^2 - \sum_{j=1}^{t} n_j^2}{4} \tag{16}$$

$$var_0(JT) = \frac{N^2(2N+3) - \sum_{j=1}^{t} n_j^2(2n_j+3)}{72} \tag{17}$$

The standardized JT is given by

$$Z_{JT} = \frac{JT - E_0(JT)}{\sqrt{var_0(JT)}} \tag{18}$$

Z_{JT} has an asymptotically standard normal distribution. The null hypothesis is rejected if $Z_{JT} \geq z_{\propto}$.

2.5. Mack-Wolfe

The Mack-Wolfe test statistic is designed to test the umbrella alternative that is based on simple random samples (Mack and Wolfe, 1981). The Mack-Wolfe test statistic has two versions. The first version is used when the peak is known, while the second version is used when the peak is unknown. In order to test the umbrella alternative using the hypotheses, the following will be used:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k \tag{19}$$

$H_a: [\mu_1 \leq \mu_2 \leq \ldots \leq \mu_p \geq \mu_{p+1} \ldots \geq \mu_k]$ with at least one strict inequality

where μ_i is a median of the i^{th} sample. The Mack-Wolfe test statistic when the peak p is known is given in Daniel, 1990. It is a sum of Mann-Whitney statistics by U_{uv},

$$A_p = \sum_{u=1}^{v-1}\sum_{v=2}^{p} U_{uv} + \sum_{u=p}^{v-1}\sum_{v=p+1}^{k} U_{uv} \tag{20}$$

where U_{uv} are Mann-Whitney counts for every pair of treatments with outcomes less than or equal to the hypothesized peak $1 \le u \le v \le p$ for the first part of the equation. The second part of the equation is the reverse Mann-Whitney and counts every pair of treatments with outcomes greater than or equal to the hypothesized peak $p \le u \le v \le k$. In this test the null hypothesis is used for large values of the test statistic. Further information regarding the Mack-Wolfe test can be found in Hollander and Wolfe (1999).

2.6. Dubnicka, Blair and Hettmansperger

Statistical tests using mixed designs in nonparametric statistics have been established by a variety of researchers. The researchers, Dubnicka, Blair, and Hettmansperger (2002) proposed a rank based test when data are a mixture of independent samples and paired observations. Dubnicka et al. (2002) provides a demonstration of a mixed design that is the most practical option to perform an analysis when there are two treatments present. The procedure sums the Wilcoxon signed-rank statistic and the Mann-Whitney statistic:

$$T^* = S + MW \tag{21}$$

where S represents the Wilcoxon signed-rank statistic, while MW represents the Mann-Whitney statistic.

According to Dubnicka et al. (2002), the mean and variance for the proposed test statistics under the null distribution are:

$$E_0(T^*) = \frac{n(n+1)}{4} + \frac{n_2(n_1+n_2+1)}{2} \tag{22}$$

$$Var_0(T^*) = \frac{n(n+1)(2n+1)}{24} + \frac{n_1 n_2 (n_1+n_2+1)}{12} \quad (23)$$

The values n_1 and n_2 are the sample sizes of the independent samples in the Mann-Whitney statistic, while n is the sample size for the paired data in the Wilcoxon signed-rank test. It is important to note that the expected value consist of the sum of the mean of the Wilcoxon signed-rank statistic given by $\frac{n(n+1)}{4}$ and the mean of the Mann-Whitney statistic given by $\frac{n_2(n_1+n_2+1)}{2}$. Likewise, the value $\frac{n(n+1)(2n+1)}{24}$ is the variance of the Wilcoxon signed-rank statistic, while the value $\frac{n_1 n_2 (n_1+n_2+1)}{12}$ is the variance of the Mann-Whitney statistic. The standardized version of Dubnicka et al. (2002) is given below:

$$T^+ = \frac{T - E_0(T^*)}{\sqrt{Var_0(T^*)}} \quad (24)$$

T^+ is approximately normally distributed. The null hypothesis is rejected for large values when $T^* \geq Z_\alpha$ at the α level of significance where Z_α is the (1- α) 100% of the standard normal.

2.7. Magel and Fu

Magel and Fu (2014) suggested a rank based statistic composed of a mix of paired observations and independent samples. The procedure sums the Wilcoxon signed-rank statistic and the Mann-Whitney statistic. For each of the standardized test statistics, Magel and Fu (2014) assigns equal $\frac{1}{\sqrt{2}}$. Magel and Fu's (2014) adaptation is outlined below:

$$T^* = \frac{S^* + MW^*}{\sqrt{2}} \quad (25)$$

where S^* is the standardized Wilcoxon signed-rank statistic and MW^* is the standardized Mann-Whitney statistic. Magel and Fu demonstrated that their test T^* has higher power than Dubnicka et al. (2002) test.

2.8. Fligner-Wolfe Test

Often in biological sciences it is necessary to investigate the response of treatments compared to a control. Situations in which this often occurs are clinical trials, pharmacology experiments and agricultural experiments (Olet, 2014). For instance, in clinical trials it is common practice, as well as a requirement by regulatory authorities, to evaluate the efficacy of a control / standard treatment in comparison to a new treatment. The Fligner-Wolfe test statistic is designed for use in this type of situation (Fligner and Wolfe, 1982).

The Fligner-Wolfe test statistic compares the median of the control group, to the medians of a number of other treatment groups simultaneously (Fligner and Wolfe, 1982). There are k samples with $i = 1$ denoting the control sample and the remaining $2 \leq i \leq k$ indicating treatment samples. The alternative hypothesis is fairly restrictive as it requires all treatment groups to have population location parameters that are equal to or larger than the control location parameter. The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k, \text{ versus} \tag{26}$$

$$H_a: \mu_1 \leq [\mu_2, \dots, \mu_k] \quad \text{with at least one strict inequality.}$$

In calculating the Fligner-Wolfe test statistic, it is useful to visualize two populations. One population is the control ($i = 1$) and the remaining $k - 1$ populations are the combined treatment sample. It will be assumed that there is a sample of size n_1 from the control population and a sample size n_2 of the combined treatment populations. In both the control sample and treatment sample, all of the observations will be merged together and subsequently ranked from smallest to largest. Let the rank r_{ij} with $i = 1,2$ and $j = 1,2,3, \dots, n_i$ indicate the rank of the j^{th} observation in the i^{th} sample with i equal to 1 for the control sample and i equal to 2 for the combined treatment sample.

$$T_1 = FW = \sum_{\substack{2 \le i \le k \\ 1<j<n_i}} r_{ij} \tag{27}$$

k is the number of treatments, n_i the number of observations in each treatment and r_{ij} the rank of the observation in the jth group subjected to the ith treatment. The expected value and variance of FW under the null distribution are outlined below.

$$E(T_1) = E_0(FW) = \frac{n_2(N+1)}{2} \text{ and } var(T_1) = var_0(FW) = \left\{\frac{n_1 n_2 (N+1)}{12}\right\} \tag{28}$$

where, n_1 is the size of the control population, and n_2 the number of observations in the remaining $k-1$ treatment populations $n_2 = N - n_1$. The standardized version of Fligner-Wolfe test FW^* is stated below:

$$FW^* = \frac{FW - E_0(FW)}{\sqrt{var_0(FW)}} \tag{29}$$

The null hypothesis is rejected when $FW^* \geq Z_\alpha$ at the α level of significance where Z_α is the (1- α) 100% of the standard normal distribution.

2.9. Ansari-Bradley Test

The Ansari-Bradley test is a nonparametric test designed to test for equality of variances (Ansari and Bradley, 1960). The null hypothesis and alternative hypothesis are given below:

$$H_0: \sigma_1 = \sigma_2 \tag{30}$$

$$H_{a1}: \sigma_1 \neq \sigma_2,\ H_{a2}: \sigma_1 < \sigma_2,\ H_{a3}: \sigma_1 > \sigma_2$$

In calculating the Ansari-Bradley test, all the observations from the two samples will be combined together. The combined set of $n_1 + n_2 = N$ observations will be arranged in order from smallest to largest. The ranks will be assigned to the ordered observations as follows:

- The smallest observation and the largest observation will each be given a rank of 1

- The second smallest observation and the second largest observation will each be given a rank of 2

The ordered observations will continue to be ranked in this manner until all observations have been assigned a rank. At this point R_i will be the rank of i^{th} observation in the first sample in the set of ranks. The test statistic Ansari-Bradley (*AB)* is the sum of the ranks of all observations in the first sample:

$$\text{AB} = \sum R_i \tag{31}$$

The standardized version of Ansari-Bradley test is:

$$AB^* = \frac{AB - E_0(AB)}{\sqrt{var_0(AB)}} \tag{32}$$

If *N*= n_1 + n_2 is an even number:

$$E_0(AB) = \frac{n_1(N+2)}{4} \tag{33}$$

$$var_0(AB) = \left\{\frac{n_1 n_2 (N+2)(N-2)}{48(N-1)}\right\} \tag{34}$$

If *N*= n_1 + n_2 is an odd integer:

$$E_0(AB) = \frac{n_1(N+1)^2}{4N} \tag{35}$$

$$var_0(AB) = \left\{\frac{n_1 n_2 (N+1)(3+N^2)}{48N^2}\right\} \tag{36}$$

The asymptotic null distribution of AB^* is the standard normal distribution.

2.10. Moses Test

Moses (1963) proposed a nonparametric test intended to test for equality of variances. The null hypothesis and alternative hypothesis are given below:

$$H_0: \sigma_1 = \sigma_2 \tag{37}$$

$$H_{a1}: \sigma_1 \neq \sigma_2,\ H_{a2}: \sigma_1 < \sigma_2,\ H_{a3}: \sigma_1 > \sigma_2$$

In order to calculate the test statistic for the Moses test, the first and second samples will be divided up into m_1 and m_2 subsamples of equal size q. For each of the first m_1 subsets, the sample mean will be calculated, the distance between each observation and the sample mean is found and then squared. These squared values will then be added up. The values C_1, C_2 , …, C_{m1} will be used to denote these sum of squared values for each of the m_1 subsets in the first sample. The values D_1, D_2 , …, D_{m2} denote these sum of squared values for each of the m_2 subsets in the second sample (Chen, 2011).

Next, the Mann-Whitney test (Mann and Whitney, 1947) will be applied. This means, the m_1 subsamples of C's and m_2 subsamples of D's will be combined. Following this, all observations in the combined set will be ranked from smallest to largest. The ranks of the observations from m_2 subsamples (which is, the D's) will then be added together. The Moses test statistic (M), is then the sum of the ranks assigned to the sums of squares ($\sum S_i$), computed from the subsamples of second sample, which is the sum of the ranks that is assigned to the D's (Chen, 2011).

$$M = \sum S_i \tag{38}$$

The standardized version of Moses test is given by:

$$M^* = \frac{M - E_0(M)}{\sqrt{var_0(M)}} \tag{39}$$

$$E_0(M) = m_2(m_1 + m_2 + 1)/2 \tag{40}$$

$$var_0(M) = m_1 m_2(m_1 + m_2 + 1)/12 \tag{41}$$

The asymptotic null distribution of M^* is the standard normal distribution.

2.11. Lepage's Test

A nonparametric test for the two-sample location-scale problem is the test of Lepage (Lepage, 1971). The purpose of the Lepage test is to determine whether there are differences in

either location parameters μ_1 and μ_2 or scale parameters σ_1 and σ_2. The Lepage's test is an amalgamation of the Mann-Whitney test for detecting location changes and the Ansari-Bradley test for detecting scale changes. The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 \text{ and } \sigma_1 = \sigma_2 \tag{42}$$

$$H_a: \mu_1 \neq \mu_2 \text{ and/ or } \sigma_1 \neq \sigma_2$$

The Lepage test statistics is given by:

$$\text{Lepage} = \frac{[(MW - E_0(MW)]^2}{var_0(MW)} + \frac{[(AB - E_0(AB)]^2}{var_0(AB)} = (MW^*)^2 + (AB^*)^2 \tag{43}$$

The Lepage test has a chi-square distribution with two degrees of freedom. H_0 is rejected when Lepage $\geq \chi^2_{2,\alpha}$ where $\chi^2_{2,\alpha}$ is upper a percentile point of the chi-square distribution with two degrees of freedom.

CHAPTER 3. PROPOSED TESTS

3.1. Modified Ansari-Bradley Test

A modified version of the Ansari-Bradley test will be proposed. The modified version of the Ansari-Bradley test for simple tree alternative is stated below:

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k, \tag{44}$$

$H_a: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k]$ where at least one inequality is strict.

In calculating the modified Ansari-Bradley test, it is helpful to consider a situation with two populations. One population is the control (i=1) and the remaining k−1 populations is the combined treatment sample. It will be assumed that there is a sample of size n_c from the control population and a sample size n_t of the combined treatment populations. The combined set of n_c + n_t = N observations will be arranged in order from smallest to largest. The ranks will be assigned to the ordered observations as follows:

- The smallest observation and the largest observation will each be given a rank of 1
- The second smallest observation and the second largest observation will each be given a rank of 2

The ordered observations will continue to be ranked in this manner until all observations have been assigned a rank. At this point R_i will be the rank of i^{th} control sample in the set of ranks. The test statistics *AB* is the sum of the ranks of all observations in the control sample,

$$T_2: \text{AB} = \sum R_i \tag{45}$$

The standardized version of Ansari-Bradley test is given by:

$$AB^* = \frac{AB - E_0(AB)}{\sqrt{var_0(AB)}} \tag{46}$$

If N= n_c + n_t is an even number :

$$E(T_2): E_0(AB) = \frac{n_c(N+2)}{4} \tag{47}$$

$$var(T_2): var_0(AB) = \left\{\frac{n_c n_t (N+2)(N-2)}{48(N-1)}\right\} \tag{48}$$

If $N = n_c + n_t$ is an odd integer:

$$E(T_2): E_0(AB) = \frac{n_c(N+1)^2}{4N} \tag{49}$$

$$var(T_2): var_0(AB) = \left\{\frac{n_c n_t (N+1)(3+N^2)}{48N^2}\right\} \tag{50}$$

The asymptotic null distribution of AB^* is the standard normal distribution (Ansari and Bradley, 1960).

3.2. Modified Moses Test

A modified version of the Moses test will be proposed. The modified version of the Moses test for simple tree alternative is outlined below:

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k, \tag{51}$$

$H_a: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k]$ where at least one inequality is strict.

In calculating the modified Moses test, it is useful to visualize two populations. One population is the control (i=1) and the remaining $k-1$ populations is the combined treatment samples.

In order to calculate the modified test statistic for the Moses test, first, the control and the combined treatment samples will be divided up into m_1 and m_2 subsamples of equal size q. For each of the first m_1 subsets, the sample mean will be calculated, the distance between the sample mean and each observation is found and then squared. These squared values will then added up. The values $C_1, C_2, \ldots, C_{m1}$ will be used to denote these sum of squared values for each of the m_1 subsets in the control sample. The values $D_1, D_2, \ldots, D_{m2}$ denote these sum of squared values for each of the m_2 subsets in the combined treatment samples.

Next, Mann-Whitney test (Mann and Whitney, 1947) will be applied. This means, the m_1 subsamples of C's and m_2 subsamples of D's will be combined. Following this, all observations in the combined set will be ranked from smallest to largest. The ranks of the observations from m_2 subsamples (which is, the D's) will then be added together. The Moses test statistic (*M*), is then the sum of the ranks assigned to the sums of squares $\sum S_i$, computed from the subsamples of combined treatment samples, which is the sum of the ranks that is assigned to the D's,

$$T_3: \mathrm{M} = \sum S_i \tag{52}$$

The standardized version of Moses test is given by:

$$M^* = \frac{M - E_0(M)}{\sqrt{var_0(M)}} \tag{53}$$

$$E(T_3): E_0(M) = m_2(m_1 + m_2 + 1)/2 \tag{54}$$

$$var(T_3): var_0(M) = m_1 m_2(m_1 + m_2 + 1)/12 \tag{55}$$

The asymptotic null distribution of M^* is the standard normal distribution (Moses, 1963).

3.3. Proposed Test One

The first proposed test L_1 is the sum of standardized test statistic for two tests. The first test being Fligner-Wolfe test statistic (T_1), obtained using Equation (27) and the second being the modified Ansari-Bradley test statistic (T_2) obtained using Equation (45). The mean and variance for the Fligner-Wolfe test statistic are given by $E(T_1)$ and $var(T_1)$ and obtained using Equation (28).

The standardized Fligner-Wolfe test statistics is given by:

$$Z_1 = \frac{T_1 - E(T_1)}{\sqrt{var(T_1)}} \tag{56}$$

Similarly, the mean and variance for modified Ansari-Bradley test statistics are given by $E(T_2)$ and $var(T_2)$, and obtained using Equation (47) and (48).

The standardized modified Ansari-Bradley test statistics is given by:

$$Z_2 = \frac{T_2 - E(T_2)}{\sqrt{var(T_2)}} \tag{57}$$

Both Z_1 and Z_2 have an asymptotic standard normal distribution under H_0 as given in (Fligner and Wolfe, 1982) and (Ansari and Bradley, 1960). When H_0 is true, the asymptotic distribution of $Z_1 + Z_2$ should be normal with a mean of zero (0) and a variance of two. As a result, the asymptotic distribution of the first proposed test (L_1) under H_0 is a standard normal.

$$L_1 = \frac{Z_1 + Z_2}{\sqrt{2}} \tag{58}$$

The asymptotic distribution of the test is used. H_0 is rejected for a large value which is $L_1 \geq Z_\alpha$ at the α level of significance where Z_α is the (1- α) 100% of the standard normal distribution. If the test is performed at a 5% level of significance then Z_α= 1.645.

3.4. Proposed Test Two

The second proposed test is given by:

$$L_2 = \frac{T_1 + T_2 - E(T_1 + T_2)}{\sqrt{var(T_1) + var(T_2)}} \tag{59}$$

The sum of the null distribution of the mean is given by $E(T_1 + T_2) = E(T_1) + E(T_2)$ and the null standard deviation is $\sqrt{var(T_1) + var(T_2)}$. When the null hypothesis is true, the asymptotic distribution of L_2 is also a standard normal distribution.

3.5. Proposed Test Three

The third proposed test M_1 is the sum of standardized test statistic for two tests Fligner-Wolfe test statistic (T_1) obtained using (27) Equation and the modified Moses test statistics (T_3) obtained using Equation (52). The mean and variance for Fligner-Wolfe test statistic are given by $E(T_1)$ and $var(T_1)$ and obtained using Equation (28).

The standardized Fligner-Wolfe test statistics is given by:

$$Z_1 = \frac{T_1 - E(T_1)}{\sqrt{var(T_1)}} \quad (60)$$

Similarly, the mean and variance for the modified Moses test statistics are given by $E(T_3)$ and $var(\mathrm{T}_3)$ obtained using Equation (54), (55).

The standardized modified Moses test statistics is given by:

$$Z_3 = \frac{T_3 - E(T_3)}{\sqrt{var(T_3)}} \quad (61)$$

Both Z_1 and Z_3 have an asymptotic standard normal distribution under H_0 as given in (Fligner and Wolfe, 1982) and (Moses, 1963). When H_0 is true, the asymptotic distribution of Z_1 + Z_3 should be a normal with mean zero (0) and variance (2).

$$M_1 = \frac{Z_1 + Z_3}{\sqrt{2}} \quad (62)$$

3.6. Proposed Test Four

The fourth proposed test is given by:

$$M_2 = \frac{T_1 + T_3 - E(T_1 + T_3)}{\sqrt{var(T_1) + var(T_3)}} \quad (63)$$

The sum of the null distribution of the mean is given by $E(T_1 + T_3)$= $E(T_1)$ + $E(T_3)$ and the null standard deviation is $\sqrt{var(T_1) + var(T_3)}$.

3.7. Proposed Test Five

The fifth proposed test is given by:

$$M_3 = \frac{T_1 + 3T_3 - E(T_1 + 3T_3)}{\sqrt{var(T_1 + 3T_3)}} \quad (64)$$

The asymptotic distribution of the test is used and H_0 is rejected for a large value which is $M_1 \geq Z_\alpha$, $M_2 \geq Z_\alpha$, and $M_3 \geq Z_\alpha$ at the α level of significance where Z_α is the (1- α) 100% of the standard normal distribution. If the test is performed at a 5% level of significance then Z_α= 1.645.

CHAPTER 4. DESCRIPTION OF SIMULATION STUDY

A simulation study was conducted to compare the five proposed tests. The simulation study was implemented in SAS version 9.4. The properties of the proposed test statistics were compared assuming random samples followed normal distribution, t-distribution with 3 degrees of freedom and exponential distribution. In order to generate random samples from a specific distribution, the functions RAND were used in SAS. This requires the user to state the starting point "seed". This can be done using the Call streaminit function before using the RAND function. The syntax for this function is

Call streaminit (seed)

In this research, seed = 0 is used that instructs RAND to use the system clock. This means each run of the code will produce a different set of data (Bailer, 2010). The call function for the normal distribution is

RAND ('Normal', μ, σ)

where μ is the mean and, σ is the standard deviation.

The call function for the t-distribution is

RAND ('T', 3*)*

where T is the name of the distribution and 3 is the degrees of freedom.

The call function for the exponential distribution is

RAND ('Exponential')

This function generates a random number from an exponential distribution with a mean and variance of one.

For all simulations, replications of 10,000 samples were used. The five proposed tests were compared in two parts. The first part of the simulation was to get the estimates of the alpha values

of the proposed test statistics. The stated alpha values for the proposed test statistics were all 0.05. The alpha values were estimated by counting the number of times the null hypothesis was rejected and then dividing by 10,000. This was done when the null hypothesis was true, and all distributions were the same; namely all location parameters were equal, and all scale parameters were equal.

The second part of the simulation study was to compare powers of the test statistics under various conditions. Powers were estimated by counting the number of times the proposed tests were rejected divided by 10,000.

4.1. Simulation Outline

The following outline summarizes what was done in the simulations.

1. The alpha values of each test statistics were estimated and compared to the stated alpha values for each simulation conducted. The proposed test statistics were examined in the case of k=2, k=3, and k=4 populations. In order to ensure that the proposed tests statistics maintained their type 1 error rates, scenarios were constructed in which the null hypothesis of equal group means, and variances was true. For instance, suppose that the observations in each group came from identical normal distribution with equal means and variances. To examine the type 1 error rate, the proposed tests were conducted a large number of times and counting the number of times the null hypothesis was rejected. Simulations were done with n=10,000 replications.
2. The aspect of interest in this simulation was estimating and comparing the powers of all the proposed tests. In order to do this, three varying conditions were assumed.

   Under the first condition, the location parameters were different, while the scale parameters were equal. The second condition assumed was when the location

parameters were equal, while the scale parameters were different. The final condition assumed, both the location parameters and the scale parameters were different.

3. Equal samples of sizes 9, 18, 30 were used for all populations.
4. A variety of situations where sample of sizes were unequal were considered.

4.2. Power Calculations

To compare the power of the proposed tests, different combinations of location parameters and scale parameters were considered.

For the case with two treatments:

- The two populations have different location parameters, and the same scale parameters;
- The two populations have the same location parameters, and different scale parameters;
- The two populations have different location parameters and scale parameters;

For the case with three treatments (all treatments have the same scale parameters):

- The first and second population have the same location parameters different than the third population;
- The second and third population have the same location parameters different than the first population;
- The three populations have different location parameters;

For the case with three treatments (all treatments have the same location parameters):

- The first and second population have the same scale parameters different than the third population;
- The second and third population have the same scale parameters different than the first population;
- The three populations have different scale parameters;

For the case with three treatments (treatments have different location and scale parameters):

- The first and second population have the same location parameters and scale parameters different than the third population;
- The second and third population have the same location parameters and scale parameters different than the first population;
- The three populations have different location and scale parameters;

For the case with four treatments (all treatments have the same scale parameters):

- The first three populations have the same location parameters different than the fourth population;
- The last three populations have the same location parameters different than the first population;
- The four populations have different location parameters;

For the case with four treatments (all treatments have the same location parameters):

- The first three populations have the same scale parameters different than the fourth population;
- The last three populations have the same scale parameters different than the first population;
- The four populations have different scale parameters;

For the case with four treatments (all treatments have different location and scale parameters):

- The first three populations have the same location parameters and scale parameters different than the fourth population;

- The last three populations have the same location parameters and scale parameters different than the first population;
- The four populations have different location parameters and scale parameters;

CHAPTER 5. RESULTS

In Chapter 5, the results of the simulation study which was defined in Chapter 4 will be presented. The results are separated up by distributions. The distributions were normal, t-distribution with 3 degrees of freedom and exponential. The tables show the estimated powers and significance levels for the five proposed tests.

5.1. Results for the Normal Distribution

5.1.1. Two Treatments

Tables 1-24 outline the results of simulation study for two treatments under the normal distribution. The sample size considered in the simulations are 9, 18, and 30. It appears that all the proposed tests maintained their alpha values. The estimated alpha values were around 0.05 Tables (1, 2, 7, 8, 13 and 14). When the populations have unequal location parameters and equal scale parameters, M_2 test has higher estimated power than all comparing tests Tables (2, 8, and 14). When the populations have equal location parameters and unequal scale parameters, L_1 test has higher estimated power than all comparing tests Tables (3, 9 and 15). When the populations have unequal location parameters and unequal scale parameters, L_1 test tends to have the highest estimated power Tables (5, 11 and 17). Tables 19-24 show the simulations where unequal sample sizes were considered. The sample size for the first population was 18 and the second population was 30. A similar result was found when investigating unequal sample cases.

Table 1. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
9	9	0	1	0	1	0.0554	0.0507	0.0413
9	9	0	1	0.5	1	0.1771	0.2185	0.1039
9	9	0	1	0.75	1	0.2618	0.3585	0.1933
9	9	0	1	1	1	0.3837	0.5342	0.3287
9	9	0	1	1.25	1	0.5149	0.6923	0.4942
9	9	0	1	1.50	1	0.6432	0.8246	0.6668
9	9	0	1	1.75	1	0.7585	0.9146	0.8155

Table 2. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
9	9	0	1	0	1	0.0540	0.0472	0.0525
9	9	0	1	0.5	1	0.1789	**0.2365**	0.2145
9	9	0	1	0.75	1	0.2719	**0.4118**	0.3502
9	9	0	1	1	1	0.3661	**0.5834**	0.5007
9	9	0	1	1.25	1	0.4738	**0.7610**	0.6623
9	9	0	1	1.50	1	0.5792	**0.8808**	0.7692
9	9	0	1	1.75	1	0.6550	**0.9481**	0.8642

Table 3. Percentage of Rejection for k=2 Populations; Normal Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
9	9	0	1	0	1.5	**0.1558**	0.1004	0.0850
9	9	0	1	0	1.75	**0.2039**	0.1181	0.1386
9	9	0	1	0	2	**0.2644**	0.1471	0.1864
9	9	0	1	0	2.25	**0.3120**	0.1731	0.2543
9	9	0	1	0	2.50	**0.3584**	0.1898	0.3077
9	9	0	1	0	2.75	**0.4016**	0.2131	0.3608
9	9	0	1	0	3	**0.4364**	0.2352	0.4157

Table 4. Percentage of Rejection for k=2 Populations; Normal Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
9	9	0	1	0	1.5	0.1275	0.0636	0.1065
9	9	0	1	0	1.75	0.1626	0.0729	0.1271
9	9	0	1	0	2	0.1952	0.0784	0.1507
9	9	0	1	0	2.25	0.2182	0.0806	0.1622
9	9	0	1	0	2.50	0.2418	0.0880	0.1862
9	9	0	1	0	2.75	0.2624	0.0944	0.1977
9	9	0	1	0	3	0.2790	0.0992	0.2002

Table 5. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
9	9	0	1	0.5	1.5	**0.3104**	0.2758	0.1317
9	9	0	1	0.5	1.75	**0.3763**	0.3024	0.1730
9	9	0	1	0.5	2	**0.4297**	0.3205	0.2184
9	9	0	1	0.5	2.25	**0.4706**	0.3331	0.2726
9	9	0	1	0.5	2.50	**0.5083**	0.3490	0.3331
9	9	0	1	0.5	2.75	**0.5382**	0.3585	0.3830
9	9	0	1	0.5	3	**0.5686**	0.3715	0.4317

Table 6. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
9	9	0	1	0.5	1.5	0.2867	0.2204	0.2848
9	9	0	1	0.5	1.75	0.3374	0.2125	0.3041
9	9	0	1	0.5	2	0.3549	0.2042	0.3257
9	9	0	1	0.5	2.25	0.3880	0.1982	0.3394
9	9	0	1	0.5	2.50	0.4091	0.2002	0.3423
9	9	0	1	0.5	2.75	0.4181	0.1975	0.3312
9	9	0	1	0.5	3	0.4216	0.1862	0.3347

Table 7. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	18	0	1	0	1	0.0512	0.0475	0.0471
18	18	0	1	0.5	1	0.2612	0.3544	0.2070
18	18	0	1	0.75	1	0.4354	0.5984	0.4415
18	18	0	1	1	1	0.6375	0.8182	0.6973
18	18	0	1	1.25	1	0.7984	0.9326	0.8835
18	18	0	1	1.50	1	0.9055	0.9834	0.9689
18	18	0	1	1.75	1	0.9643	0.9974	0.9950

Table 8. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	18	0	1	0	1	0.0488	0.0522	0.0480
18	18	0	1	0.5	1	0.2628	**0.4134**	0.3365
18	18	0	1	0.75	1	0.4365	**0.6790**	0.5780
18	18	0	1	1	1	0.6083	**0.8772**	0.7718
18	18	0	1	1.25	1	0.7593	**0.9685**	0.9038
18	18	0	1	1.50	1	0.8678	**0.9940**	0.9627
18	18	0	1	1.75	1	0.9276	**0.9997**	0.9914

Table 9. Percentage of Rejection for k=2 Populations; Normal Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	18	0	1	0	1.5	**0.2201**	0.1337	0.1742
18	18	0	1	0	1.75	**0.3258**	0.1873	0.3035
18	18	0	1	0	2	**0.4350**	0.2337	0.4418
18	18	0	1	0	2.25	**0.5173**	0.2716	0.5641
18	18	0	1	0	2.50	**0.5752**	0.3145	0.6687
18	18	0	1	0	2.75	**0.6404**	0.3570	0.7510
18	18	0	1	0	3	**0.6878**	0.3870	0.8193

Table 10. Percentage of Rejection for k=2 Populations; Normal Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	18	0	1	0	1.5	0.1927	0.0814	0.1342
18	18	0	1	0	1.75	0.2687	0.0952	0.1854
18	18	0	1	0	2	0.3358	0.1045	0.2136
18	18	0	1	0	2.25	0.3971	0.1134	0.2495
18	18	0	1	0	2.50	0.4423	0.1272	0.2846
18	18	0	1	0	2.75	0.4750	0.1293	0.3022
18	18	0	1	0	3	0.5128	0.1388	0.3277

Table 11. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	18	0	1	0.5	1.5	**0.5175**	0.4680	0.2917
18	18	0	1	0.5	1.75	**0.6119**	0.5052	0.3925
18	18	0	1	0.5	2	**0.6782**	0.5309	0.5049
18	18	0	1	0.5	2.25	**0.7466**	0.5646	0.6164
18	18	0	1	0.5	2.50	**0.7758**	0.5814	0.7030
18	18	0	1	0.5	2.75	**0.8183**	0.6035	0.7775
18	18	0	1	0.5	3	**0.8417**	0.6116	0.8374

Table 12. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	18	0	1	0.5	1.5	0.4917	0.3817	0.4847
18	18	0	1	0.5	1.75	0.5653	0.3665	0.5016
18	18	0	1	0.5	2	0.6188	0.3532	0.5261
18	18	0	1	0.5	2.25	0.6519	0.3323	0.5361
18	18	0	1	0.5	2.50	0.6838	0.3251	0.5479
18	18	0	1	0.5	2.75	0.7016	0.3217	0.5492
18	18	0	1	0.5	3	0.7080	0.3032	0.5512

Table 13. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
30	30	0	1	0	1	0.0521	0.0501	0.0506
30	30	0	1	0.5	1	0.3624	0.5048	0.3506
30	30	0	1	0.75	1	0.6241	0.8072	0.6965
30	30	0	1	1	1	0.8366	0.9590	0.9216
30	30	0	1	1.25	1	0.9449	0.9927	0.9871
30	30	0	1	1.50	1	0.9882	0.9991	0.9988
30	30	0	1	1.75	1	0.9975	1	1

Table 14. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
30	30	0	1	0	1	0.0517	0.0512	0.0507
30	30	0	1	0.5	1	0.3718	**0.5792**	0.4882
30	30	0	1	0.75	1	0.6161	**0.8591**	0.7694
30	30	0	1	1	1	0.8070	**0.9784**	0.9324
30	30	0	1	1.25	1	0.9240	**0.9983**	0.9884
30	30	0	1	1.50	1	0.9762	**1**	0.9983
30	30	0	1	1.75	1	0.9946	**1**	0.9999

Table 15. Percentage of Rejection for k=2 Populations; Normal Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
30	30	0	1	0	1.5	**0.3119**	0.1772	0.2886
30	30	0	1	0	1.75	**0.4754**	0.2606	0.5060
30	30	0	1	0	2	**0.6132**	0.3412	0.6988
30	30	0	1	0	2.25	**0.7056**	0.4026	0.8316
30	30	0	1	0	2.50	**0.7725**	0.4541	0.8991
30	30	0	1	0	2.75	**0.8285**	0.5027	0.9523
30	30	0	1	0	3	**0.8644**	0.5520	0.9687

Table 16. Percentage of Rejection for k=2 Populations; Normal Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
30	30	0	1	0	1.5	0.2655	0.0834	0.1715
30	30	0	1	0	1.75	0.3854	0.0989	0.2425
30	30	0	1	0	2	0.4956	0.1154	0.3066
30	30	0	1	0	2.25	0.5776	0.1369	0.3556
30	30	0	1	0	2.50	0.6392	0.1449	0.3946
30	30	0	1	0	2.75	0.6843	0.1499	0.4282
30	30	0	1	0	3	0.7180	0.1620	0.4567

Table 17. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
30	30	0	1	0.5	1.5	**0.7133**	0.6600	0.4804
30	30	0	1	0.5	1.75	**0.8097**	0.7012	0.6332
30	30	0	1	0.5	2	**0.8696**	0.7301	0.7675
30	30	0	1	0.5	2.25	**0.9113**	0.7611	0.8659
30	30	0	1	0.5	2.50	**0.9325**	0.7802	0.9201
30	30	0	1	0.5	2.75	**0.9490**	0.7868	0.9564
30	30	0	1	0.5	3	**0.9583**	0.8051	0.9788

Table 18. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
30	30	0	1	0.5	1.5	0.6871	0.5399	0.6597
30	30	0	1	0.5	1.75	0.7635	0.5107	0.7084
30	30	0	1	0.5	2	0.8196	0.4840	0.7106
30	30	0	1	0.5	2.25	0.8535	0.4742	0.7284
30	30	0	1	0.5	2.50	0.8774	0.4431	0.7338
30	30	0	1	0.5	2.75	0.8903	0.4385	0.7352
30	30	0	1	0.5	3	0.8987	0.4099	0.7397

Table 19. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	30	0	1	0	1	0.0475	0.0476	0.0479
18	30	0	1	0.5	1	0.2635	0.3961	0.2639
18	30	0	1	0.75	1	0.4337	0.6713	0.5514
18	30	0	1	1	1	0.6132	0.8630	0.8158
18	30	0	1	1.25	1	0.7633	0.9657	0.9538
18	30	0	1	1.50	1	0.8747	0.9945	0.9927
18	30	0	1	1.75	1	0.9386	0.9988	0.9993

Table 20. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	30	0	1	0	1	0.0517	**0.0519**	0.0538
18	30	0	1	0.5	1	0.3049	**0.4705**	0.3918
18	30	0	1	0.75	1	0.5038	**0.7682**	0.6578
18	30	0	1	1	1	0.6884	**0.9301**	0.8518
18	30	0	1	1.25	1	0.8332	**0.9885**	0.9538
18	30	0	1	1.50	1	0.9216	**0.9988**	0.9896
18	30	0	1	1.75	1	0.9644	**0.9999**	0.9976

Table 21. Percentage of Rejection for k=2 Populations; Normal Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	30	0	1	0	1.5	**0.2493**	0.1393	0.1956
18	30	0	1	0	1.75	**0.3912**	0.2095	0.3739
18	30	0	1	0	2	**0.5124**	0.2670	0.5315
18	30	0	1	0	2.25	**0.6063**	0.3121	0.6725
18	30	0	1	0	2.50	**0.6868**	0.3733	0.7908
18	30	0	1	0	2.75	**0.7464**	0.4064	0.8655
18	30	0	1	0	3	**0.7899**	0.4509	0.9141

Table 22. Percentage of Rejection for k=2 Populations; Normal Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	30	0	1	0	1.5	0.2051	0.0690	0.1374
18	30	0	1	0	1.75	0.3069	0.0839	0.1986
18	30	0	1	0	2	0.3986	0.0954	0.2479
18	30	0	1	0	2.25	0.4668	0.0984	0.2791
18	30	0	1	0	2.50	0.5207	0.1075	0.3159
18	30	0	1	0	2.75	0.5774	0.1086	0.3363
18	30	0	1	0	3	0.6149	0.1109	0.3584

Table 23. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	30	0	1	0.5	1.5	**0.5851**	0.5386	0.3333
18	30	0	1	0.5	1.75	**0.6986**	0.5938	0.4627
18	30	0	1	0.5	2	**0.7865**	0.6339	0.6050
18	30	0	1	0.5	2.25	**0.8460**	0.6630	0.7279
18	30	0	1	0.5	2.50	**0.8792**	0.6783	0.8102
18	30	0	1	0.5	2.75	**0.9036**	0.6995	0.8751
18	30	0	1	0.5	3	**0.9204**	0.7112	0.9243

Table 24. Percentage of Rejection for k=2 Populations; Normal Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	30	0	1	0.5	1.5	0.5623	0.4345	0.5374
18	30	0	1	0.5	1.75	0.6533	0.4163	0.5794
18	30	0	1	0.5	2	0.7175	0.3949	0.6114
18	30	0	1	0.5	2.25	0.7624	0.3724	0.6221
18	30	0	1	0.5	2.50	0.7912	0.3487	0.6282
18	30	0	1	0.5	2.75	0.8101	0.3457	0.6396
18	30	0	1	0.5	3	0.8254	0.3301	0.6413

5.1.2. Three Treatments

Tables 25-54 present the results of simulation study for three treatments under the normal distribution. The sample size considered in the simulations are 9, 18 and 30. It can be seen that all the proposed tests maintained their alpha values. The estimated alpha values were around 0.05 Tables (25, 26, 31, 32, 37 and 38). When the three populations have unequal location parameters and equal scale parameters, the M_2 test has higher estimated power than all comparing tests Tables (26, 32 and 38). The only exception to this is when the first two location parameters are equal and the last one is different. In this situation, the L_2 has a higher estimated power (see Appendix A, Tables 1, 13 and 25). When the three populations have equal location parameters and unequal scale parameters, the L_1 test has higher estimated power than all comparing tests Tables (27, 33 and 39). When the three populations have unequal location parameters and unequal scale parameters, the L_1 test tends to have the highest estimated power Tables (29, 35 and 41). Tables 19-24 present the simulations where unequal sample sizes were considered. The same results were found in the case with unequal sample sizes.

Table 25. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	0	1	0	1	0	1	0.0483	0.0468	0.0449
9	9	9	0	1	0.25	1	0.5	1	0.1294	0.1791	0.0977
9	9	9	0	1	0.5	1	0.75	1	0.2043	0.3262	0.2015
9	9	9	0	1	0.75	1	1	1	0.2859	0.5132	0.3763
9	9	9	0	1	1	1	1.25	1	0.3747	0.6831	0.5840
9	9	9	0	1	1.25	1	1.5	1	0.4520	0.8288	0.7754
9	9	9	0	1	1.5	1	1.75	1	0.5146	0.9209	0.9007

Table 26. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	0	1	0	1	0	1	0.0514	0.0534	0.0520
9	9	9	0	1	0.25	1	0.5	1	0.1564	**0.2162**	0.1851
9	9	9	0	1	0.5	1	0.75	1	0.2583	**0.4028**	0.3269
9	9	9	0	1	0.75	1	1	1	0.3898	**0.6234**	0.5125
9	9	9	0	1	1	1	1.25	1	0.5131	**0.8016**	0.6710
9	9	9	0	1	1.25	1	1.5	1	0.6368	**0.9229**	0.8124
9	9	9	0	1	1.5	1	1.75	1	0.7296	**0.9748**	0.9034

Table 27. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	0	1	0	1.5	0	2	**0.2250**	0.1151	0.1378
9	9	9	0	1	0	1.75	0	2.25	**0.2979**	0.1492	0.2231
9	9	9	0	1	0	2	0	2.5	**0.3797**	0.1956	0.3013
9	9	9	0	1	0	2.25	0	2.75	**0.4369**	0.2117	0.3898
9	9	9	0	1	0	2.5	0	3	**0.5006**	0.2414	0.4754
9	9	9	0	1	0	2.75	0	3.25	**0.5354**	0.2658	0.5446

Table 28. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	0	1	0	1.5	0	2	0.1654	0.0639	0.1117
9	9	9	0	1	0	1.75	0	2.25	0.2032	0.0648	0.1394
9	9	9	0	1	0	2	0	2.5	0.2477	0.0684	0.1653
9	9	9	0	1	0	2.25	0	2.75	0.2761	0.0764	0.1681
9	9	9	0	1	0	2.5	0	3	0.3044	0.0783	0.1816
9	9	9	0	1	0	2.75	0	3.25	0.3312	0.0747	0.1985

Table 29. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	0	1	0.25	1.5	0.5	2	**0.3757**	0.2773	0.1634
9	9	9	0	1	0.25	1.75	0.5	2.25	**0.4550**	0.3080	0.2336
9	9	9	0	1	0.25	2	0.5	2.5	**0.5265**	0.3396	0.3079
9	9	9	0	1	0.25	2.25	0.5	2.75	**0.5749**	0.3561	0.3911
9	9	9	0	1	0.25	2.5	0.5	3	**0.6218**	0.3778	0.4631
9	9	9	0	1	0.25	2.75	0.5	3.25	**0.6624**	0.3958	0.5522

Table 30. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	0	1	0.25	1.5	0.5	2	0.3283	0.1864	0.2690
9	9	9	0	1	0.25	1.75	0.5	2.25	0.3611	0.1786	0.2820
9	9	9	0	1	0.25	2	0.5	2.5	0.3999	0.1778	0.3125
9	9	9	0	1	0.25	2.25	0.5	2.75	0.4321	0.1701	0.3128
9	9	9	0	1	0.25	2.50	0.5	3	0.4596	0.1598	0.3233
9	9	9	0	1	0.25	2.75	0.5	3.25	0.4784	0.1587	0.3269

Table 31. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	0	1	0	1	0	1	0.0488	0.0459	0.0482
18	18	18	0	1	0.25	1	0.5	1	0.1880	0.2767	0.1655
18	18	18	0	1	0.5	1	0.75	1	0.3517	0.5496	0.4233
18	18	18	0	1	0.75	1	1	1	0.5235	0.7993	0.7212
18	18	18	0	1	1	1	1.25	1	0.6805	0.9339	0.9182
18	18	18	0	1	1.25	1	1.5	1	0.8089	0.9874	0.9872
18	18	18	0	1	1.5	1	1.75	1	0.8889	0.9983	0.9990

Table 32. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	0	1	0	1	0	1	0.0497	0.0508	0.0513
18	18	18	0	1	0.25	1	0.5	1	0.2204	**0.3288**	0.2826
18	18	18	0	1	0.5	1	0.75	1	0.4257	**0.6510**	0.5553
18	18	18	0	1	0.75	1	1	1	0.6237	**0.8867**	0.7873
18	18	18	0	1	1	1	1.25	1	0.7930	**0.9739**	0.9241
18	18	18	0	1	1.25	1	1.5	1	0.8990	**0.9979**	0.9824
18	18	18	0	1	1.5	1	1.75	1	0.9547	**0.9999**	0.9967

Table 33. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	0	1	0	1.5	0	2	**0.3920**	0.2054	0.3601
18	18	18	0	1	0	1.75	0	2.25	**0.5158**	0.2538	0.5438
18	18	18	0	1	0	2	0	2.50	**0.6324**	0.3261	0.7011
18	18	18	0	1	0	2.25	0	2.75	**0.7144**	0.3749	0.8083
18	18	18	0	1	0	2.5	0	3	**0.7785**	0.4276	0.8926
18	18	18	0	1	0	2.75	0	3.25	**0.8227**	0.4650	0.9358

Table 34. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	0	1	0	1.5	0	2	0.3005	0.0846	0.1913
18	18	18	0	1	0	1.75	0	2.25	0.3974	0.0933	0.2423
18	18	18	0	1	0	2	0	2.50	0.4740	0.1041	0.3067
18	18	18	0	1	0	2.25	0	2.75	0.5499	0.1077	0.3327
18	18	18	0	1	0	2.5	0	3	0.6022	0.1126	0.3543
18	18	18	0	1	0	2.75	0	3.25	0.6528	0.1257	0.3983

Table 35. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	0	1	0.25	1.5	0.5	2	**0.6448**	0.4907	0.4137
18	18	18	0	1	0.25	1.75	0.5	2.25	**0.7525**	0.5496	0.5952
18	18	18	0	1	0.25	2	0.5	2.5	**0.8148**	0.5890	0.7330
18	18	18	0	1	0.25	2.25	0.5	2.75	**0.8718**	0.6337	0.8315
18	18	18	0	1	0.25	2.5	0.5	3	**0.8955**	0.6514	0.8962
18	18	18	0	1	0.25	2.75	0.5	3.25	**0.9201**	0.6738	0.9382

Table 36. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	0	1	0.25	1.5	0.5	2	0.5745	0.2982	0.4962
18	18	18	0	1	0.25	1.75	0.5	2.25	0.6506	0.2858	0.5375
18	18	18	0	1	0.25	2	0.5	2.50	0.7135	0.2808	0.5681
18	18	18	0	1	0.25	2.25	0.5	2.75	0.7659	0.2749	0.5865
18	18	18	0	1	0.25	2.50	0.5	3	0.7885	0.2606	0.6097
18	18	18	0	1	0.25	2.75	0.5	3.25	0.8139	0.2597	0.6229

Table 37. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	0	1	0	1	0	1	0.0502	0.0492	0.0501
30	30	30	0	1	0.25	1	0.5	1	0.2787	0.4114	0.2748
30	30	30	0	1	0.5	1	0.75	1	0.5232	0.7540	0.6569
30	30	30	0	1	0.75	1	1	1	0.7487	0.9494	0.9238
30	30	30	0	1	1	1	1.25	1	0.8939	0.9941	0.9945
30	30	30	0	1	1.25	1	1.5	1	0.9659	0.9996	0.9996
30	30	30	0	1	1.5	1	1.75	1	0.9924	1	1

Table 38. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	0	1	0	1	0	1	0.0504	0.0513	0.0505
30	30	30	0	1	0.25	1	0.5	1	0.3102	**0.4790**	0.4010
30	30	30	0	1	0.5	1	0.75	1	0.5767	**0.8406**	0.7360
30	30	30	0	1	0.75	1	1	1	0.8151	**0.9802**	0.9387
30	30	30	0	1	1	1	1.25	1	0.9415	**0.9985**	0.9918
30	30	30	0	1	1.25	1	1.5	1	0.9860	**1**	0.9991
30	30	30	0	1	1.5	1	1.75	1	0.9972	**1**	1

Table 39. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	0	1	0	1.5	0	2	**0.5704**	0.2948	0.6196
30	30	30	0	1	0	1.75	0	2.25	**0.7362**	0.4009	0.8265
30	30	30	0	1	0	2	0	2.5	**0.8293**	0.4814	0.9284
30	30	30	0	1	0	2.25	0	2.75	**0.8934**	0.5578	0.9760
30	30	30	0	1	0	2.5	0	3	**0.9348**	0.6250	0.9930
30	30	30	0	1	0	2.75	0	3.25	**0.9521**	0.6618	0.9981

Table 40. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	0	1	0	1.5	0	2	0.4554	0.0896	0.2753
30	30	30	0	1	0	1.75	0	2.25	0.5905	0.1114	0.3519
30	30	30	0	1	0	2	0	2.5	0.7039	0.1166	0.4104
30	30	30	0	1	0	2.25	0	2.75	0.7780	0.1280	0.4809
30	30	30	0	1	0	2.5	0	3	0.8333	0.1307	0.5373
30	30	30	0	1	0	2.75	0	3.25	0.8701	0.1350	0.5753

Table 41. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	0	1	0.25	1.5	0.5	2	**0.8472**	0.6995	0.6897
30	30	30	0	1	0.25	1.75	0.5	2.25	**0.9207**	0.7637	0.8569
30	30	30	0	1	0.25	2	0.5	2.5	**0.9601**	0.8131	0.9461
30	30	30	0	1	0.25	2.25	0.5	2.75	**0.9766**	0.8402	0.9827
30	30	30	0	1	0.25	2.5	0.5	3	**0.9843**	0.8599	0.9941
30	30	30	0	1	0.25	2.75	0.5	3.25	**0.9898**	0.8787	0.9974

Table 42. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	0	1	0.25	1.5	0.5	2	0.7896	0.4392	0.7201
30	30	30	0	1	0.25	1.75	0.5	2.25	0.8618	0.4274	0.7665
30	30	30	0	1	0.25	2	0.5	2.5	0.9061	0.4281	0.7735
30	30	30	0	1	0.25	2.25	0.5	2.75	0.9368	0.4027	0.8004
30	30	30	0	1	0.25	2.5	0.5	3	0.9528	0.3948	0.8169
30	30	30	0	1	0.25	2.75	0.5	3.25	0.9605	0.3868	0.8264

Table 43. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	9	9	0	1	0	1	0	1	0.0514	0.0501	0.0485
18	9	9	0	1	0.25	1	0.5	1	0.1862	0.2419	0.1278
18	9	9	0	1	0.5	1	0.75	1	0.3533	0.4789	0.3152
18	9	9	0	1	0.75	1	1	1	0.5427	0.7171	0.5682
18	9	9	0	1	1	1	1.25	1	0.7254	0.8862	0.8008
18	9	9	0	1	1.25	1	1.5	1	0.8626	0.9660	0.9360
18	9	9	0	1	1.5	1	1.75	1	0.9448	0.9924	0.9881

Table 44. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	9	9	0	1	0	1	0	1	0.0496	0.0528	0.0482
18	9	9	0	1	0.25	1	0.5	1	0.1824	**0.2759**	0.2226
18	9	9	0	1	0.5	1	0.75	1	0.3443	**0.5456**	0.4435
18	9	9	0	1	0.75	1	1	1	0.5213	**0.7879**	0.6746
18	9	9	0	1	1	1	1.25	1	0.6833	**0.9333**	0.8491
18	9	9	0	1	1.25	1	1.5	1	0.8144	**0.9872**	0.9420
18	9	9	0	1	1.5	1	1.75	1	0.8975	**0.9971**	0.9822

Table 45. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	9	9	0	1	0	1.5	0	2	**0.3313**	0.1878	0.2853
18	9	9	0	1	0	1.75	0	2.25	**0.4246**	0.2316	0.4196
18	9	9	0	1	0	2	0	2.5	**0.5107**	0.2731	0.5465
18	9	9	0	1	0	2.25	0	2.75	**0.5802**	0.3151	0.6580
18	9	9	0	1	0	2.5	0	3	**0.6390**	0.3521	0.7503
18	9	9	0	1	0	2.75	0	3.25	**0.6793**	0.3767	0.8102

Table 46. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	9	9	0	1	0	1.5	0	2	0.2524	0.0902	0.1736
18	9	9	0	1	0	1.75	0	2.25	0.3402	0.1022	0.2199
18	9	9	0	1	0	2	0	2.5	0.3901	0.1094	0.2562
18	9	9	0	1	0	2.25	0	2.75	0.4454	0.1169	0.2795
18	9	9	0	1	0	2.5	0	3	0.4838	0.1307	0.2983
18	9	9	0	1	0	2.75	0	3.25	0.5109	0.1315	0.3296

Table 47. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	9	9	0	1	0.25	1.5	0.5	2	**0.5214**	0.3974	0.3289
18	9	9	0	1	0.25	1.75	0.5	2.25	**0.6173**	0.4504	0.4581
18	9	9	0	1	0.25	2	0.5	2.5	**0.6848**	0.4787	0.5864
18	9	9	0	1	0.25	2.25	0.5	2.75	**0.7220**	0.5011	0.6802
18	9	9	0	1	0.25	2.5	0.5	3	**0.7714**	0.5296	0.7653
18	9	9	0	1	0.25	2.75	0.5	3.25	**0.7983**	0.5463	0.8216

Table 48. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	9	9	0	1	0.25	1.5	0.5	2	0.4731	0.2746	0.3873
18	9	9	0	1	0.25	1.75	0.5	2.25	0.5289	0.2631	0.4270
18	9	9	0	1	0.25	2	0.5	2.5	0.5738	0.2573	0.4573
18	9	9	0	1	0.25	2.25	0.5	2.75	0.6124	0.2619	0.4652
18	9	9	0	1	0.25	2.5	0.5	3	0.6421	0.2548	0.4762
18	9	9	0	1	0.25	2.75	0.5	3.25	0.6603	0.2445	0.4815

Table 49. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	30	30	0	1	0	1	0	1	0.0427	0.0453	0.0486
18	30	30	0	1	0.25	1	0.5	1	0.1926	0.3016	0.1927
18	30	30	0	1	0.5	1	0.75	1	0.3221	0.5781	0.4809
18	30	30	0	1	0.75	1	1	1	0.4773	0.8320	0.7866
18	30	30	0	1	1	1	1.25	1	0.6011	0.9534	0.9538
18	30	30	0	1	1.25	1	1.5	1	0.6875	0.9910	0.9937
18	30	30	0	1	1.5	1	1.75	1	0.7293	0.9985	0.9998

Table 50. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	30	30	0	1	0	1	0	1	0.0544	0.0537	0.0552
18	30	30	0	1	0.25	1	0.5	1	0.2505	**0.3764**	0.3205
18	30	30	0	1	0.5	1	0.75	1	0.4521	**0.6970**	0.5932
18	30	30	0	1	0.75	1	1	1	0.6727	**0.9155**	0.8303
18	30	30	0	1	1	1	1.25	1	0.8447	**0.9893**	0.9519
18	30	30	0	1	1.25	1	1.5	1	0.9278	**0.9990**	0.9890
18	30	30	0	1	1.5	1	1.75	1	0.9727	**1**	0.9985

Table 51. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	30	30	0	1	0	1.5	0	2	**0.4362**	0.2091	0.4187
18	30	30	0	1	0	1.75	0	2.25	**0.5936**	0.2866	0.6222
18	30	30	0	1	0	2	0	2.50	**0.7060**	0.3490	0.7905
18	30	30	0	1	0	2.25	0	2.75	**0.7921**	0.4098	0.8917
18	30	30	0	1	0	2.5	0	3	**0.8465**	0.4738	0.9457
18	30	30	0	1	0	2.75	0	3.25	**0.8872**	0.5290	0.9744

Table 52. Percentage of Rejection for k=3 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	30	30	0	1	0	1.5	0	2	0.3257	0.0632	0.1939
18	30	30	0	1	0	1.75	0	2.25	0.4462	0.0705	0.2419
18	30	30	0	1	0	2	0	2.50	0.5496	0.0773	0.3091
18	30	30	0	1	0	2.25	0	2.75	0.6274	0.0762	0.3372
18	30	30	0	1	0	2.50	0	3	0.6855	0.0788	0.3798
18	30	30	0	1	0	2.75	0	3.25	0.7421	0.0797	0.4032

Table 53. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	30	30	0	1	0.25	1.5	0.5	2	**0.7137**	0.5477	0.4683
18	30	30	0	1	0.25	1.75	0.5	2.25	**0.8298**	0.6239	0.6634
18	30	30	0	1	0.25	2	0.5	2.5	**0.8878**	0.6740	0.8038
18	30	30	0	1	0.25	2.25	0.5	2.75	**0.9270**	0.7188	0.8963
18	30	30	0	1	0.25	2.5	0.5	3	**0.9530**	0.7509	0.9465
18	30	30	0	1	0.25	2.75	0.5	3.25	**0.9679**	0.7744	0.9790

Table 54. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	30	30	0	1	0.25	1.5	0.5	2	0.6372	0.3190	0.5415
18	30	30	0	1	0.25	1.75	0.5	2.25	0.7323	0.3051	0.5728
18	30	30	0	1	0.25	2	0.5	2.50	0.8019	0.2973	0.6273
18	30	30	0	1	0.25	2.25	0.5	2.75	0.8442	0.2889	0.6501
18	30	30	0	1	0.25	2.5	0.5	3	0.8789	0.2685	0.6721
18	30	30	0	1	0.25	2.75	0.5	3.25	0.8973	0.2610	0.6811

5.1.3. Four Treatments

Tables 55-78 give the results of simulation study four treatments under the normal distribution. The sample size considered in the simulations are 9, 18, 30. It can be seen that all the proposed tests maintained their alpha values. The estimated alpha values were around 0.05 Tables (55, 56, 61, 62, 67 and 68). When the four populations have unequal location parameters and equal scale parameters, M_2 test has higher estimated power than all comparing tests Tables (56, 62 and 68). The only exception to this is when the first three location parameters equal and the last one is different. In this situation, the L_2 has a higher estimated power (see Appendix D, Tables 1, 13 and 25). When the four populations have equal location parameters and unequal scale parameters, L_1 test has higher estimated power than all comparing tests Tables (57, 63 and 69). When the three populations have unequal location parameters and unequal scale parameters, L_1 test tends to have the highest estimated power Tables (59, 65 and 71). Tables 73-78 present the simulations where unequal sample sizes were considered. The case with unequal sample sizes gave the same results.

Table 55. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	0	1	0	1	0	1	0.0493	0.0503	0.0464
9	9	9	9	0	1	0.1	1	0.2	1	0.3	1	0.0768	0.0979	0.0619
9	9	9	9	0	1	0.3	1	0.4	1	0.5	1	0.1141	0.1852	0.1145
9	9	9	9	0	1	0.5	1	0.6	1	0.7	1	0.1687	0.3199	0.2217
9	9	9	9	0	1	0.7	1	0.8	1	0.9	1	0.2064	0.4636	0.3730
9	9	9	9	0	1	0.9	1	1	1	1.1	1	0.2546	0.6174	0.5452
9	9	9	9	0	1	1.1	1	1.2	1	1.3	1	0.2661	0.7580	0.7283

Table 56. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	0	1	0	1	0	1	0.0527	0.0534	0.0544
9	9	9	9	0	1	0.1	1	0.2	1	0.3	1	0.1067	**0.1281**	0.1174
9	9	9	9	0	1	0.3	1	0.4	1	0.5	1	0.1844	**0.2584**	0.2158
9	9	9	9	0	1	0.5	1	0.6	1	0.7	1	0.2728	**0.4214**	0.3438
9	9	9	9	0	1	0.7	1	0.8	1	0.9	1	0.3862	**0.6133**	0.5007
9	9	9	9	0	1	0.9	1	1	1	1.1	1	0.5001	**0.7759**	0.6587
9	9	9	9	0	1	1.1	1	1.2	1	1.3	1	0.6118	**0.8878**	0.7788

Table 57. Percentage of Rejection for k=4 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	0	1.25	0	1.50	0	1.75	**0.1521**	0.0817	0.0887
9	9	9	9	0	1	0	1.50	0	1.75	0	2	**0.2348**	0.1122	0.1607
9	9	9	9	0	1	0	1.75	0	2	0	2.25	**0.3214**	0.1484	0.2462
9	9	9	9	0	1	0	2	0	2.25	0	2.50	**0.4013**	0.1771	0.3501
9	9	9	9	0	1	0	2.25	0	2.50	0	2.75	**0.4777**	0.2124	0.4421
9	9	9	9	0	1	0	2.50	0	2.75	0	3	**0.5442**	0.2417	0.5524

Table 58. Percentage of Rejection for k=4 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	0	1.25	0	1.50	0	1.75	0.1298	0.0568	0.1157
9	9	9	9	0	1	0	1.50	0	1.75	0	2	0.1799	0.0603	0.1272
9	9	9	9	0	1	0	1.75	0	2	0	2.25	0.2250	0.0571	0.1451
9	9	9	9	0	1	0	2	0	2.25	0	2.50	0.2685	0.0517	0.1659
9	9	9	9	0	1	0	2.25	0	2.50	0	2.75	0.3078	0.0586	0.1744
9	9	9	9	0	1	0	2.50	0	2.75	0	3	0.3436	0.0566	0.1877

Table 59. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	0.3	1.25	0.4	1.50	0.5	1.75	**0.3070**	0.2703	0.1136
9	9	9	9	0	1	0.3	1.50	0.4	1.75	0.5	2	**0.4092**	0.3055	0.1661
9	9	9	9	0	1	0.3	1.75	0.4	2	0.5	2.25	**0.5124**	0.3401	0.2584
9	9	9	9	0	1	0.3	2	0.4	2.25	0.5	2.50	**0.5831**	0.3721	0.3477
9	9	9	9	0	1	0.3	2.25	0.4	2.50	0.5	2.75	**0.6469**	0.3968	0.4473
9	9	9	9	0	1	0.3	2.50	0.4	2.75	0.5	3	**0.6993**	0.4288	0.5420

Table 60. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	0.3	1.25	0.4	1.50	0.5	1.75	0.3057	0.2138	0.2891
9	9	9	9	0	1	0.3	1.50	0.4	1.75	0.5	2	0.3788	0.2074	0.3175
9	9	9	9	0	1	0.3	1.75	0.4	2	0.5	2.25	0.4338	0.1957	0.3389
9	9	9	9	0	1	0.3	2	0.4	2.25	0.5	2.50	0.4761	0.1781	0.3408
9	9	9	9	0	1	0.3	2.25	0.4	2.50	0.5	2.75	0.5081	0.1751	0.3630
9	9	9	9	0	1	0.3	2.50	0.4	2.75	0.5	3	0.5342	0.1589	0.3702

Table 61. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	0	1	0	1	0	1	0.0474	0.0464	0.0477
18	18	18	18	0	1	0.1	1	0.2	1	0.3	1	0.1105	0.1398	0.0832
18	18	18	18	0	1	0.3	1	0.4	1	0.5	1	0.1947	0.3141	0.2064
18	18	18	18	0	1	0.5	1	0.6	1	0.7	1	0.3244	0.5617	0.4490
18	18	18	18	0	1	0.7	1	0.8	1	0.9	1	0.4534	0.7588	0.7038
18	18	18	18	0	1	0.9	1	1	1	1.1	1	0.5586	0.9032	0.8836
18	18	18	18	0	1	1.1	1	1.2	1	1.3	1	0.6516	0.9671	0.9716

Table 62. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	0	1	0	1	0	1	0.0530	0.0536	0.0529
18	18	18	18	0	1	0.1	1	0.2	1	0.3	1	0.1315	**0.1728**	0.1573
18	18	18	18	0	1	0.3	1	0.4	1	0.5	1	0.2644	**0.4005**	0.3360
18	18	18	18	0	1	0.5	1	0.6	1	0.7	1	0.4314	**0.6671**	0.5595
18	18	18	18	0	1	0.7	1	0.8	1	0.9	1	0.6077	**0.8718**	0.7624
18	18	18	18	0	1	0.9	1	1	1	1.1	1	0.7562	**0.9647**	0.8993
18	18	18	18	0	1	1.1	1	1.2	1	1.3	1	0.8605	**0.9928**	0.9625

Table 63. Percentage of Rejection for k=4 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	0	1.25	0	1.50	0	1.75	**0.2724**	0.1407	0.2101
18	18	18	18	0	1	0	1.50	0	1.75	0	2	**0.4312**	0.2072	0.4137
18	18	18	18	0	1	0	1.75	0	2	0	2.25	**0.5772**	0.2767	0.6151
18	18	18	18	0	1	0	2	0	2.25	0	2.50	**0.7020**	0.3547	0.7776
18	18	18	18	0	1	0	2.25	0	2.50	0	2.75	**0.7854**	0.4151	0.8816
18	18	18	18	0	1	0	2.50	0	2.75	0	3	**0.8436**	0.4731	0.9412

Table 64. Percentage of Rejection for k=4 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	0	1.25	0	1.50	0	1.75	0.2191	0.0648	0.1313
18	18	18	18	0	1	0	1.50	0	1.75	0	2	0.3264	0.0674	0.1992
18	18	18	18	0	1	0	1.75	0	2	0	2.25	0.4425	0.0782	0.2510
18	18	18	18	0	1	0	2	0	2.25	0	2.50	0.5358	0.0778	0.2930
18	18	18	18	0	1	0	2.25	0	2.50	0	2.75	0.6166	0.0827	0.3304
18	18	18	18	0	1	0	2.50	0	2.75	0	3	0.6210	0.0891	0.3721

Table 65. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	0.3	1.25	0.4	1.50	0.5	1.75	**0.5812**	0.5049	0.3001
18	18	18	18	0	1	0.3	1.50	0.4	1.75	0.5	2	**0.7181**	0.5685	0.4690
18	18	18	18	0	1	0.3	1.75	0.4	2	0.5	2.25	**0.8253**	0.6275	0.6521
18	18	18	18	0	1	0.3	2	0.4	2.25	0.5	2.50	**0.8910**	0.6826	0.7949
18	18	18	18	0	1	0.3	2.25	0.4	2.50	0.5	2.75	**0.9264**	0.7169	0.8924
18	18	18	18	0	1	0.3	2.50	0.4	2.75	0.5	3	**0.9496**	0.7532	0.9443

Table 66. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	0.3	1.25	0.4	1.50	0.5	1.75	0.5462	0.3730	0.4980
18	18	18	18	0	1	0.3	1.50	0.4	1.75	0.5	2	0.6540	0.3471	0.5612
18	18	18	18	0	1	0.3	1.75	0.4	2	0.5	2.25	0.7459	0.3366	0.5921
18	18	18	18	0	1	0.3	2	0.4	2.25	0.5	2.50	0.8070	0.3301	0.6357
18	18	18	18	0	1	0.3	2.25	0.4	2.50	0.5	2.75	0.8433	0.3080	0.6641
18	18	18	18	0	1	0.3	2.50	0.4	2.75	0.5	3	0.8780	0.2946	0.6734

Table 67. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	0	1	0	1	0	1	0.0524	0.0492	0.0493
30	30	30	30	0	1	0.1	1	0.2	1	0.3	1	0.1402	0.1942	0.1226
30	30	30	30	0	1	0.3	1	0.4	1	0.5	1	0.2950	0.4644	0.3463
30	30	30	30	0	1	0.5	1	0.6	1	0.7	1	0.4880	0.7568	0.6853
30	30	30	30	0	1	0.7	1	0.8	1	0.9	1	0.6782	0.9315	0.9115
30	30	30	30	0	1	0.9	1	1	1	1.1	1	0.8068	0.9881	0.9878
30	30	30	30	0	1	1.1	1	1.2	1	1.3	1	0.8935	0.9991	0.9990

Table 68. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	0	1	0	1	0	1	0.0512	0.0509	0.0510
30	30	30	30	0	1	0.1	1	0.2	1	0.3	1	0.1628	**0.2259**	0.1926
30	30	30	30	0	1	0.3	1	0.4	1	0.5	1	0.3557	**0.5581**	0.4751
30	30	30	30	0	1	0.5	1	0.6	1	0.7	1	0.5956	**0.8499**	0.7278
30	30	30	30	0	1	0.7	1	0.8	1	0.9	1	0.7976	**0.9741**	0.9274
30	30	30	30	0	1	0.9	1	1	1	1.1	1	0.9208	**0.9975**	0.9864
30	30	30	30	0	1	1.1	1	1.2	1	1.3	1	0.9707	**0.9999**	0.9977

Table 69. Percentage of Rejection for k=4 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	0	1.25	0	1.50	0	1.75	**0.4076**	0.2067	0.3845
30	30	30	30	0	1	0	1.50	0	1.75	0	2	**0.6308**	0.3142	0.6850
30	30	30	30	0	1	0	1.75	0	2	0	2.25	**0.7906**	0.4299	0.8820
30	30	30	30	0	1	0	2	0	2.25	0	2.50	**0.8873**	0.5342	0.9627
30	30	30	30	0	1	0	2.25	0	2.50	0	2.75	**0.9388**	0.6136	0.9911
30	30	30	30	0	1	0	2.50	0	2.75	0	3	**0.9629**	0.6907	0.9983

Table 70. Percentage of Rejection for k=4 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	0	1.25	0	1.50	0	1.75	0.3169	0.0727	0.1958
30	30	30	30	0	1	0	1.50	0	1.75	0	2	0.5165	0.0821	0.2914
30	30	30	30	0	1	0	1.75	0	2	0	2.25	0.6610	0.0955	0.3775
30	30	30	30	0	1	0	2	0	2.25	0	2.50	0.7763	0.1002	0.4647
30	30	30	30	0	1	0	2.25	0	2.50	0	2.75	0.8569	0.1099	0.5230
30	30	30	30	0	1	0	2.50	0	2.75	0	3	0.8987	0.1190	0.5778

Table 71. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	0.3	1.25	0.4	1.50	0.5	1.75	**0.7968**	0.7145	0.5357
30	30	30	30	0	1	0.3	1.50	0.4	1.75	0.5	2	**0.9125**	0.7965	0.7781
30	30	30	30	0	1	0.3	1.75	0.4	2	0.5	2.25	**0.9650**	0.8530	0.9115
30	30	30	30	0	1	0.3	2	0.4	2.25	0.5	2.50	**0.9848**	0.8801	0.9757
30	30	30	30	0	1	0.3	2.25	0.4	2.50	0.5	2.75	**0.9935**	0.9157	0.9941
30	30	30	30	0	1	0.3	2.50	0.4	2.75	0.5	3	**0.9961**	0.9267	0.9974

Table 72. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	0.3	1.25	0.4	1.50	0.5	1.75	0.7589	0.5491	0.7092
30	30	30	30	0	1	0.3	1.50	0.4	1.75	0.5	2	0.8635	0.5201	0.7675
30	30	30	30	0	1	0.3	1.75	0.4	2	0.5	2.25	0.9290	0.5041	0.8206
30	30	30	30	0	1	0.3	2	0.4	2.25	0.5	2.50	0.9634	0.4945	0.8594
30	30	30	30	0	1	0.3	2.25	0.4	2.50	0.5	2.75	0.9782	0.4639	0.8781
30	30	30	30	0	1	0.3	2.50	0.4	2.75	0.5	3	0.9839	0.4539	0.8901

Table 73. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	9	9	9	0	1	0	1	0	1	0	1	0.0444	0.0425	0.0486
18	9	9	9	0	1	0.1	1	0.2	1	0.3	1	0.1131	0.1370	0.0739
18	9	9	9	0	1	0.3	1	0.4	1	0.5	1	0.2033	0.2863	0.1747
18	9	9	9	0	1	0.5	1	0.6	1	0.7	1	0.3319	0.4963	0.3524
18	9	9	9	0	1	0.7	1	0.8	1	0.9	1	0.4842	0.7084	0.5825
18	9	9	9	0	1	0.9	1	1	1	1.1	1	0.6228	0.8581	0.7831
18	9	9	9	0	1	1.1	1	1.2	1	1.3	1	0.7474	0.9447	0.9189

Table 74. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	9	9	9	0	1	0	1	0	1	0	1	0.0512	0.0516	0.0515
18	9	9	9	0	1	0.1	1	0.2	1	0.3	1	0.1119	**0.1542**	0.1220
18	9	9	9	0	1	0.3	1	0.4	1	0.5	1	0.2265	**0.3377**	0.2561
18	9	9	9	0	1	0.5	1	0.6	1	0.7	1	0.3703	**0.5753**	0.4259
18	9	9	9	0	1	0.7	1	0.8	1	0.9	1	0.5312	**0.7965**	0.6126
18	9	9	9	0	1	0.9	1	1	1	1.1	1	0.6757	**0.9189**	0.7622
18	9	9	9	0	1	1.1	1	1.2	1	1.3	1	0.7929	**0.9751**	0.8737

Table 75. Percentage of Rejection for k=4 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	9	9	9	0	1	0	1.25	0	1.50	0	1.75	**0.2382**	0.1401	0.1809
18	9	9	9	0	1	0	1.50	0	1.75	0	2	**0.3681**	0.1922	0.3410
18	9	9	9	0	1	0	1.75	0	2	0	2.25	**0.4754**	0.2547	0.4956
18	9	9	9	0	1	0	2	0	2.25	0	2.50	**0.5787**	0.3039	0.6466
18	9	9	9	0	1	0	2.25	0	2.50	0	2.75	**0.6595**	0.3511	0.7610
18	9	9	9	0	1	0	2.50	0	2.75	0	3	**0.7236**	0.3959	0.8443

Table 76. Percentage of Rejection for k=4 Populations; Normal Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	9	9	9	0	1	0	1.25	0	1.50	0	1.75	0.1881	0.0660	0.1599
18	9	9	9	0	1	0	1.50	0	1.75	0	2	0.2912	0.0834	0.2415
18	9	9	9	0	1	0	1.75	0	2	0	2.25	0.3746	0.0837	0.3008
18	9	9	9	0	1	0	2	0	2.25	0	2.50	0.4348	0.0954	0.3535
18	9	9	9	0	1	0	2.25	0	2.50	0	2.75	0.4981	0.1037	0.4042
18	9	9	9	0	1	0	2.50	0	2.75	0	3	0.5605	0.1089	0.4429

Table 77. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	9	9	9	0	1	0.3	1.25	0.4	1.50	0.5	1.75	**0.4880**	0.4227	0.2587
18	9	9	9	0	1	0.3	1.50	0.4	1.75	0.5	2	**0.6063**	0.4791	0.3942
18	9	9	9	0	1	0.3	1.75	0.4	2	0.5	2.25	**0.7108**	0.5265	0.5443
18	9	9	9	0	1	0.3	2	0.4	2.25	0.5	2.50	**0.7866**	0.5732	0.6757
18	9	9	9	0	1	0.3	2.25	0.4	2.50	0.5	2.75	**0.8219**	0.5931	0.7792
18	9	9	9	0	1	0.3	2.50	0.4	2.75	0.5	3	**0.8665**	0.6334	0.8565

Table 78. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	9	9	9	0	1	0.3	1.25	0.4	1.50	0.5	1.75	0.4622	0.3188	0.4450
18	9	9	9	0	1	0.3	1.50	0.4	1.75	0.5	2	0.5654	0.3111	0.5260
18	9	9	9	0	1	0.3	1.75	0.4	2	0.5	2.25	0.6301	0.2963	0.5782
18	9	9	9	0	1	0.3	2	0.4	2.25	0.5	2.50	0.6805	0.2926	0.6149
18	9	9	9	0	1	0.3	2.25	0.4	2.50	0.5	2.75	0.7236	0.2914	0.6486
18	9	9	9	0	1	0.3	2.50	0.4	2.75	0.5	3	0.7492	0.2808	0.6625

5.2. Results for the T-Distribution with Three Degrees of Freedom

5.2.1. Two Treatments

Tables 79-102 outline the results of simulation study for two treatments under the t-distribution. The sample size considered in the simulations are 9, 18, and 30. In the tables, σ represents the standard deviation of a t random variable. The standard deviation formula is $\sqrt{\frac{v}{v-2}}$ where v is the degrees of freedom (Taboga, 2017). It appears that all the proposed tests maintained their alpha values. The estimated alpha values were around 0.05 Tables (79, 80, 85, 86, 91 and 92). When the populations have unequal location parameters and equal scale parameters, M_2 test has higher estimated power than all comparing tests Tables (80, 86, and 92). When the populations have equal location parameters and unequal scale parameters, L_1 test has higher estimated power than all comparing tests Tables (81, 87 and 93). When the populations have unequal location parameters and unequal scale parameters, L_1 test tends to have the highest estimated power Tables (83, 89 and 95). Tables 97-102 show the simulations where unequal sample sizes were considered. The sample size for the first population was 18 and the second population was 30. A similar result was found when investigating unequal sample cases.

Table 79. Percentage of Rejection for k=2 Populations; T Distribution with different means and equal variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
9	9	0	0	σ	σ	0.0521	0.0480	0.0438
9	9	0	0.5	σ	σ	0.1504	0.1770	0.0905
9	9	0	0.75	σ	σ	0.2151	0.2801	0.1481
9	9	0	1	σ	σ	0.2991	0.4007	0.2318
9	9	0	1.25	σ	σ	0.3840	0.5187	0.3455
9	9	0	1.5	σ	σ	0.4816	0.6392	0.4809
9	9	0	1.75	σ	σ	0.5652	0.7363	0.5965

Table 80. Percentage of Rejection for k=2 Populations; T Distribution with different means and equal variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
9	9	0	0	σ	σ	0.0518	0.0461	0.0504
9	9	0	0.5	σ	σ	0.1374	**0.1809**	0.1717
9	9	0	0.75	σ	σ	0.2063	**0.2964**	0.2782
9	9	0	1	σ	σ	0.2850	**0.4454**	0.3843
9	9	0	1.25	σ	σ	0.3623	**0.5771**	0.4977
9	9	0	1.5	σ	σ	0.4458	**0.6980**	0.6038
9	9	0	1.75	σ	σ	0.5111	**0.8032**	0.6991

Table 81. Percentage of Rejection for k=2 Populations; T Distribution with same means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
9	9	0	0	σ	2σ	**0.2201**	0.1269	0.1507
9	9	0	0	σ	3σ	**0.3696**	0.2029	0.3254
9	9	0	0	σ	4σ	**0.4781**	0.2501	0.4745
9	9	0	0	σ	5σ	**0.5395**	0.2823	0.6025
9	9	0	0	σ	6σ	**0.5843**	0.3121	0.6897
9	9	0	0	σ	7σ	**0.6202**	0.3460	0.7497

Table 82. Percentage of Rejection for k=2 Populations; T Distribution with same means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
9	9	0	0	σ	2σ	0.1629	0.0734	0.1257
9	9	0	0	σ	3σ	0.2436	0.0899	0.1859
9	9	0	0	σ	4σ	0.2893	0.0986	0.2064
9	9	0	0	σ	5σ	0.3195	0.1045	0.2220
9	9	0	0	σ	6σ	0.3408	0.1081	0.2407
9	9	0	0	σ	7σ	0.3545	0.1111	0.2424

Table 83. Percentage of Rejection for k=2 Populations; T Distribution with different means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
9	9	0	0.25	σ	2σ	**0.2710**	0.1808	0.1542
9	9	0	0.25	σ	3σ	**0.4166**	0.2455	0.3231
9	9	0	0.25	σ	4σ	**0.5220**	0.2921	0.4756
9	9	0	0.25	σ	5σ	**0.5831**	0.3286	0.6004
9	9	0	0.25	σ	6σ	**0.6201**	0.3502	0.6884
9	9	0	0.25	σ	7σ	**0.6449**	0.3645	0.7531

Table 84. Percentage of Rejection for k=2 Populations; T Distribution with different means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
9	9	0	0.25	σ	2σ	0.2215	0.1174	0.1855
9	9	0	0.25	σ	3σ	0.2941	0.1219	0.2307
9	9	0	0.25	σ	4σ	0.3355	0.1243	0.2521
9	9	0	0.25	σ	5σ	0.3579	0.1278	0.2701
9	9	0	0.25	σ	6σ	0.3769	0.1239	0.2702
9	9	0	0.25	σ	7σ	0.3885	0.1293	0.2664

Table 85. Percentage of Rejection for k=2 Populations; T Distribution with different means and equal variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
18	18	0	0	σ	σ	0.0481	0.0474	0.0480
18	18	0	0.5	σ	σ	0.2129	0.2801	0.1570
18	18	0	0.75	σ	σ	0.3201	0.4460	0.3005
18	18	0	1	σ	σ	0.4828	0.6402	0.5011
18	18	0	1.25	σ	σ	0.6131	0.7960	0.7041
18	18	0	1.5	σ	σ	0.7268	0.8857	0.8303
18	18	0	1.75	σ	σ	0.8108	0.9486	0.9235

Table 86. Percentage of Rejection for k=2 Populations; T Distribution with different means and equal variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
18	18	0	0	σ	σ	0.0501	0.0498	0.0493
18	18	0	0.5	σ	σ	0.1970	**0.3072**	0.2513
18	18	0	0.75	σ	σ	0.3156	**0.5144**	0.4271
18	18	0	1	σ	σ	0.4603	**0.7157**	0.5908
18	18	0	1.25	σ	σ	0.6023	**0.8632**	0.7579
18	18	0	1.5	σ	σ	0.7074	**0.9425**	0.8612
18	18	0	1.75	σ	σ	0.7972	**0.9817**	0.9265

Table 87. Percentage of Rejection for k=2 Populations; T Distribution with same means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
18	18	0	0	σ	2σ	**0.3536**	0.2032	0.3356
18	18	0	0	σ	3σ	**0.6084**	0.3301	0.7025
18	18	0	0	σ	4σ	**0.7299**	0.4180	0.8746
18	18	0	0	σ	5σ	**0.8040**	0.4753	0.9453
18	18	0	0	σ	6σ	**0.8383**	0.5192	0.9793
18	18	0	0	σ	7σ	**0.8623**	0.5542	0.9896

Table 88. Percentage of Rejection for k=2 Populations; T Distribution with same means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
18	18	0	0	σ	2σ	0.2615	0.0925	0.1894
18	18	0	0	σ	3σ	0.4310	0.1231	0.2776
18	18	0	0	σ	4σ	0.5229	0.1426	0.3260
18	18	0	0	σ	5σ	0.5746	0.1447	0.3685
18	18	0	0	σ	6σ	0.6106	0.1537	0.3731
18	18	0	0	σ	7σ	0.6218	0.1609	0.3959

Table 89. Percentage of Rejection for k=2 Populations; T Distribution with different means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
18	18	0	0.25	σ	2σ	**0.4427**	0.2977	0.3341
18	18	0	0.25	σ	3σ	**0.6722**	0.4157	0.6975
18	18	0	0.25	σ	4σ	**0.7835**	0.4942	0.8807
18	18	0	0.25	σ	5σ	**0.8332**	0.5381	0.9458
18	18	0	0.25	σ	6σ	**0.8632**	0.5724	0.9782
18	18	0	0.25	σ	7σ	**0.8853**	0.5961	0.9889

Table 90. Percentage of Rejection for k=2 Populations; T Distribution with different means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
18	18	0	0.25	σ	2σ	0.3687	0.1606	0.2842
18	18	0	0.25	σ	3σ	0.5199	0.1847	0.3616
18	18	0	0.25	σ	4σ	0.6013	0.1835	0.4121
18	18	0	0.25	σ	5σ	0.6423	0.1889	0.4292
18	18	0	0.25	σ	6σ	0.6490	0.1896	0.4194
18	18	0	0.25	σ	7σ	0.6645	0.1898	0.4344

Table 91. Percentage of Rejection for k=2 Populations; T Distribution with different means and equal variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
30	30	0	0	σ	σ	0.0514	0.0515	0.0467
30	30	0	0.5	σ	σ	0.2893	0.3893	0.2494
30	30	0	0.75	σ	σ	0.4716	0.6345	0.4961
30	30	0	1	σ	σ	0.6729	0.8453	0.7548
30	30	0	1.25	σ	σ	0.8031	0.9390	0.9077
30	30	0	1.5	σ	σ	0.8957	0.9820	0.9730
30	30	0	1.75	σ	σ	0.9516	0.9965	0.9952

Table 92. Percentage of Rejection for k=2 Populations; T Distribution with different means and equal variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
30	30	0	0	σ	σ	0.0502	0.0473	0.0490
30	30	0	0.5	σ	σ	0.2817	**0.4359**	0.3518
30	30	0	0.75	σ	σ	0.4712	**0.7126**	0.6064
30	30	0	1	σ	σ	0.6492	**0.8934**	0.8102
30	30	0	1.25	σ	σ	0.7971	**0.9723**	0.9221
30	30	0	1.5	σ	σ	0.8915	**0.9942**	0.9763
30	30	0	1.75	σ	σ	0.9462	**0.9992**	0.9919

Table 93. Percentage of Rejection for k=2 Populations; T Distribution with same means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
30	30	0	0	σ	2σ	**0.5051**	0.2756	0.5549
30	30	0	0	σ	3σ	**0.7929**	0.4682	0.9202
30	30	0	0	σ	4σ	**0.9031**	0.5935	0.9883
30	30	0	0	σ	5σ	**0.9414**	0.6626	0.9984
30	30	0	0	σ	6σ	**0.9569**	0.7030	0.9995
30	30	0	0	σ	7σ	**0.9674**	0.7291	1

Table 94. Percentage of Rejection for k=2 Populations; T Distribution with same means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
30	30	0	0	σ	2σ	0.3834	0.1014	0.2425
30	30	0	0	σ	3σ	0.6392	0.1392	0.3962
30	30	0	0	σ	4σ	0.7403	0.1658	0.4718
30	30	0	0	σ	5σ	0.7914	0.1749	0.5189
30	30	0	0	σ	6σ	0.8285	0.1916	0.5420
30	30	0	0	σ	7σ	0.8444	0.1941	0.5562

Table 95. Percentage of Rejection for k=2 Populations; T Distribution with different means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
30	30	0	0.25	σ	2σ	**0.6528**	0.4371	0.5826
30	30	0	0.25	σ	3σ	**0.8616**	0.5931	0.9203
30	30	0	0.25	σ	4σ	**0.9281**	0.6757	0.9897
30	30	0	0.25	σ	5σ	**0.9592**	0.7325	0.9986
30	30	0	0.25	σ	6σ	**0.9704**	0.7620	0.9996
30	30	0	0.25	σ	7σ	**0.9751**	0.7691	0.9997

Table 96. Percentage of Rejection for k=2 Populations; T Distribution with different means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
30	30	0	0.25	σ	2σ	0.5465	0.2227	0.4042
30	30	0	0.25	σ	3σ	0.7266	0.2339	0.5272
30	30	0	0.25	σ	4σ	0.8168	0.2393	0.5711
30	30	0	0.25	σ	5σ	0.8498	0.2376	0.5922
30	30	0	0.25	σ	6σ	0.8661	0.2347	0.6137
30	30	0	0.25	σ	7σ	0.8769	0.2301	0.6179

Table 97. Percentage of Rejection for k=2 Populations; T Distribution with different means and equal variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
18	30	0	0	σ	σ	0.0442	0.0442	0.0455
18	30	0	0.5	σ	σ	0.2050	0.2980	0.1863
18	30	0	0.75	σ	σ	0.3264	0.4975	0.3838
18	30	0	1	σ	σ	0.4478	0.6974	0.6174
18	30	0	1.25	σ	σ	0.5706	0.8297	0.8033
18	30	0	1.5	σ	σ	0.6561	0.9099	0.9190
18	30	0	1.75	σ	σ	0.7208	0.9569	0.9738

Table 98. Percentage of Rejection for k=2 Populations; T Distribution with different means and equal variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
18	30	0	0	σ	σ	0.0443	0.0467	0.0453
18	30	0	0.5	σ	σ	0.2367	**0.3604**	0.3025
18	30	0	0.75	σ	σ	0.3874	**0.6064**	0.5077
18	30	0	1	σ	σ	0.5537	**0.8065**	0.7082
18	30	0	1.25	σ	σ	0.6864	**0.9212**	0.8378
18	30	0	1.5	σ	σ	0.7852	**0.9715**	0.9190
18	30	0	1.75	σ	σ	0.8545	**0.9935**	0.9623

Table 99. Percentage of Rejection for k=2 Populations; T Distribution with same means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
18	30	0	0	σ	2σ	**0.4066**	0.2134	0.3967
18	30	0	0	σ	3σ	**0.6983**	0.3806	0.8049
18	30	0	0	σ	4σ	**0.8229**	0.4826	0.9520
18	30	0	0	σ	5σ	**0.8896**	0.5685	0.9881
18	30	0	0	σ	6σ	**0.9173**	0.6086	0.9955
18	30	0	0	σ	7σ	**0.9341**	0.6391	0.9985

Table 100. Percentage of Rejection for k=2 Populations; T Distribution with same means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
18	30	0	0	σ	2σ	0.3051	0.0829	0.1964
18	30	0	0	σ	3σ	0.5139	0.1047	0.3103
18	30	0	0	σ	4σ	0.6266	0.1184	0.3769
18	30	0	0	σ	5σ	0.7020	0.1276	0.4176
18	30	0	0	σ	6σ	0.7295	0.1293	0.4371
18	30	0	0	σ	7σ	0.7531	0.1366	0.4539

Table 101. Percentage of Rejection for k=2 Populations; T Distribution with different means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	L_1	L_2	Lepage
18	30	0	0.25	σ	2σ	**0.5367**	0.3554	0.4068
18	30	0	0.25	σ	3σ	**0.7841**	0.4952	0.8114
18	30	0	0.25	σ	4σ	**0.8812**	0.5849	0.9556
18	30	0	0.25	σ	5σ	**0.9124**	0.6308	0.9860
18	30	0	0.25	σ	6σ	**0.9359**	0.6643	0.9964
18	30	0	0.25	σ	7σ	**0.9518**	0.6967	0.9994

Table 102. Percentage of Rejection for k=2 Populations; T Distribution with different means and different variances

n_1	n_2	μ_1	μ_2	σ_1	σ_2	M_1	M_2	M_3
18	30	0	0.25	σ	2σ	0.4357	0.1788	0.3303
18	30	0	0.25	σ	3σ	0.6248	0.1793	0.4215
18	30	0	0.25	σ	4σ	0.7164	0.1791	0.4792
18	30	0	0.25	σ	5σ	0.7505	0.1696	0.4868
18	30	0	0.25	σ	6σ	0.7767	0.1746	0.5049
18	30	0	0.25	σ	7σ	0.7940	0.1718	0.5097

5.2.2. Three Treatments

Tables 103-132 show the results of simulation study for three treatments under the t-distribution. The sample size considered in the simulations are 9, 18, 30. It can be seen that all the proposed tests maintained their alpha values. The estimated alpha values were around 0.05 Tables (103, 104, 109, 110, 115 and 116). When the three populations have unequal location parameters and equal scale parameters, M_2 test has higher estimated power than all comparing tests Tables (104, 110 and 116). The only exception to this is when the first two location parameters equal and the last one is different. In this situation, the L_2 has a higher estimated power (see Appendix B, Tables 1, 13 and 25). When the three populations have equal location parameters and unequal scale parameters, L_1 test has higher estimated power than all comparing tests Tables (105, 111 and 117). When the three populations have unequal location parameters and unequal scale parameters, L_1 test tends to have the highest estimated power Tables (107, 113 and 119). Tables 121-131 present the simulations where unequal sample sizes were considered.

Table 103. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
9	9	9	0	0	0	σ	σ	σ	0.0466	0.0471	0.0416
9	9	9	0	0.25	0.5	σ	σ	σ	0.1158	0.1474	0.0739
9	9	9	0	0.5	0.75	σ	σ	σ	0.1693	0.2557	0.1547
9	9	9	0	0.75	1	σ	σ	σ	0.2286	0.3738	0.2635
9	9	9	0	1	1.25	σ	σ	σ	0.3014	0.5080	0.4102
9	9	9	0	1.25	1.5	σ	σ	σ	0.3560	0.6189	0.5833
9	9	9	0	1.5	1.75	σ	σ	σ	0.4224	0.7229	0.7120

Table 104. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
9	9	9	0	0	0	σ	σ	σ	0.0501	0.0520	0.0520
9	9	9	0	0.25	0.5	σ	σ	σ	0.1292	**0.1737**	0.1457
9	9	9	0	0.5	0.75	σ	σ	σ	0.2071	**0.3123**	0.2598
9	9	9	0	0.75	1	σ	σ	σ	0.3045	**0.4763**	0.3992
9	9	9	0	1	1.25	σ	σ	σ	0.3981	**0.6323**	0.5189
9	9	9	0	1.25	1.5	σ	σ	σ	0.5053	**0.7769**	0.6510
9	9	9	0	1.5	1.75	σ	σ	σ	0.5783	**0.8638**	0.7467

Table 105. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
9	9	9	0	0	0	σ	1.5σ	2σ	**0.1941**	0.1110	0.1130
9	9	9	0	0	0	σ	2σ	2.5σ	**0.3034**	0.1528	0.2286
9	9	9	0	0	0	σ	2.5σ	3σ	**0.4049**	0.2039	0.3532
9	9	9	0	0	0	σ	3σ	3.5σ	**0.4878**	0.2374	0.4644
9	9	9	0	0	0	σ	3.5σ	4σ	**0.5658**	0.2846	0.5817
9	9	9	0	0	0	σ	4σ	4.5σ	**0.5996**	0.3003	0.6628

Table 106. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
9	9	9	0	0	0	σ	1.5σ	2σ	0.1333	0.0588	0.1041
9	9	9	0	0	0	σ	2σ	2.5σ	0.2032	0.0640	0.1333
9	9	9	0	0	0	σ	2.5σ	3σ	0.2562	0.0749	0.1626
9	9	9	0	0	0	σ	3σ	3.5σ	0.2936	0.0771	0.1820
9	9	9	0	0	0	σ	3.5σ	4σ	0.3277	0.0815	0.2071
9	9	9	0	0	0	σ	4σ	4.5σ	0.3675	0.0854	0.2211

Table 107. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
9	9	9	0	0.25	0.5	σ	1.5σ	2σ	**0.3058**	0.2293	0.1312
9	9	9	0	0.25	0.5	σ	2σ	2.5σ	**0.4150**	0.2649	0.2423
9	9	9	0	0.25	0.5	σ	2.5σ	3σ	**0.5143**	0.3090	0.3627
9	9	9	0	0.25	0.5	σ	3σ	3.5σ	**0.5922**	0.3497	0.4842
9	9	9	0	0.25	0.5	σ	3.5σ	4σ	**0.6459**	0.3813	0.5922
9	9	9	0	0.25	0.5	σ	4σ	4.5σ	**0.6878**	0.3977	0.6665

Table 108. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
9	9	9	0	0.25	0.5	σ	1.5σ	2σ	0.2535	0.1517	0.2192
9	9	9	0	0.25	0.5	σ	2σ	2.5σ	0.3273	0.1401	0.2476
9	9	9	0	0.25	0.5	σ	2.5σ	3σ	0.3741	0.1452	0.2710
9	9	9	0	0.25	0.5	σ	3σ	3.5σ	0.4027	0.1387	0.2796
9	9	9	0	0.25	0.5	σ	3.5σ	4σ	0.4344	0.1328	0.2926
9	9	9	0	0.25	0.5	σ	4σ	4.5σ	0.4591	0.1308	0.2977

Table 109. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	18	18	0	0	0	σ	σ	σ	0.0494	0.0493	0.0440
18	18	18	0	0.25	0.5	σ	σ	σ	0.1542	0.2189	0.1306
18	18	18	0	0.5	0.75	σ	σ	σ	0.2717	0.4097	0.2927
18	18	18	0	0.75	1	σ	σ	σ	0.3890	0.6180	0.5221
18	18	18	0	1	1.25	σ	σ	σ	0.4902	0.7721	0.7496
18	18	18	0	1.25	1.5	σ	σ	σ	0.5848	0.8755	0.8962
18	18	18	0	1.5	1.75	σ	σ	σ	0.6561	0.9393	0.9657

Table 110. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	18	18	0	0	0	σ	σ	σ	0.0512	0.0504	0.0516
18	18	18	0	0.25	0.5	σ	σ	σ	0.1851	**0.2615**	0.2266
18	18	18	0	0.5	0.75	σ	σ	σ	0.3252	**0.4956**	0.4209
18	18	18	0	0.75	1	σ	σ	σ	0.4876	**0.7292**	0.6329
18	18	18	0	1	1.25	σ	σ	σ	0.6384	**0.8869**	0.7891
18	18	18	0	1.25	1.5	σ	σ	σ	0.7522	**0.9592**	0.8998
18	18	18	0	1.5	1.75	σ	σ	σ	0.8491	**0.9882**	0.9548

Table 111. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	18	18	0	0	0	σ	1.5σ	2σ	**0.3214**	0.1690	0.2774
18	18	18	0	0	0	σ	2σ	2.5σ	**0.5224**	0.2623	0.5550
18	18	18	0	0	0	σ	2.5σ	3σ	**0.6807**	0.3504	0.7672
18	18	18	0	0	0	σ	3σ	3.5σ	**0.7747**	0.4251	0.8877
18	18	18	0	0	0	σ	3.5σ	4σ	**0.8415**	0.4901	0.9494
18	18	18	0	0	0	σ	4σ	4.5σ	**0.8761**	0.5326	0.9746

Table 112. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	18	18	0	0	0	σ	1.5σ	2σ	0.2344	0.0678	0.1507
18	18	18	0	0	0	σ	2σ	2.5σ	0.3779	0.0853	0.2327
18	18	18	0	0	0	σ	2.5σ	3σ	0.4911	0.0908	0.2942
18	18	18	0	0	0	σ	3σ	3.5σ	0.5874	0.1039	0.3440
18	18	18	0	0	0	σ	3.5σ	4σ	0.6474	0.1050	0.3705
18	18	18	0	0	0	σ	4σ	4.5σ	0.6834	0.1094	0.4060

Table 113. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	18	18	0	0.25	0.5	σ	1.5σ	2σ	**0.5082**	0.3840	0.2988
18	18	18	0	0.25	0.5	σ	2σ	2.5σ	**0.6975**	0.4715	0.5755
18	18	18	0	0.25	0.5	σ	2.5σ	3σ	**0.8109**	0.5497	0.7759
18	18	18	0	0.25	0.5	σ	3σ	3.5σ	**0.8739**	0.6080	0.8894
18	18	18	0	0.25	0.5	σ	3.5σ	4σ	**0.9074**	0.6374	0.9474
18	18	18	0	0.25	0.5	σ	4σ	4.5σ	**0.9293**	0.6751	0.9746

Table 114. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	18	18	0	0.25	0.5	σ	1.5σ	2σ	0.4384	0.2342	0.3695
18	18	18	0	0.25	0.5	σ	2σ	2.5σ	0.5819	0.2306	0.4403
18	18	18	0	0.25	0.5	σ	2.5σ	3σ	0.6671	0.2157	0.4940
18	18	18	0	0.25	0.5	σ	3σ	3.5σ	0.7236	0.2165	0.5219
18	18	18	0	0.25	0.5	σ	3.5σ	4σ	0.7727	0.2028	0.5364
18	18	18	0	0.25	0.5	σ	4σ	4.5σ	0.7933	0.2041	0.5430

Table 115. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
30	30	30	0	0	0	σ	σ	σ	0.0486	0.0491	0.0506
30	30	30	0	0.25	0.5	σ	σ	σ	0.2207	0.3098	0.1895
30	30	30	0	0.5	0.75	σ	σ	σ	0.3830	0.5808	0.4697
30	30	30	0	0.75	1	σ	σ	σ	0.5472	0.8060	0.7687
30	30	30	0	1	1.25	σ	σ	σ	0.6961	0.9321	0.9419
30	30	30	0	1.25	1.5	σ	σ	σ	0.7932	0.9815	0.9904
30	30	30	0	1.5	1.75	σ	σ	σ	0.8639	0.9955	0.9993

Table 116. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
30	30	30	0	0	0	σ	σ	σ	0.0517	0.0513	0.0520
30	30	30	0	0.25	0.5	σ	σ	σ	0.2383	**0.3574**	0.3062
30	30	30	0	0.5	0.75	σ	σ	σ	0.4550	**0.6874**	0.5886
30	30	30	0	0.75	1	σ	σ	σ	0.6620	**0.8997**	0.8150
30	30	30	0	1	1.25	σ	σ	σ	0.8213	**0.9801**	0.9410
30	30	30	0	1.25	1.5	σ	σ	σ	0.9246	**0.9976**	0.9830
30	30	30	0	1.5	1.75	σ	σ	σ	0.9628	**0.9997**	0.9955

Table 117. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
30	30	30	0	0	0	σ	1.5σ	2σ	**0.4664**	0.2481	0.4714
30	30	30	0	0	0	σ	2σ	2.5σ	**0.7255**	0.3901	0.8233
30	30	30	0	0	0	σ	2.5σ	3σ	**0.8746**	0.5378	0.9606
30	30	30	0	0	0	σ	3σ	3.5σ	**0.9334**	0.6283	0.9919
30	30	30	0	0	0	σ	3.5σ	4σ	**0.9595**	0.7002	0.9986
30	30	30	0	0	0	σ	4σ	4.5σ	**0.9791**	0.7458	0.9999

Table 118. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
30	30	30	0	0	0	σ	1.5σ	2σ	0.3457	0.0766	0.2172
30	30	30	0	0	0	σ	2σ	2.5σ	0.5723	0.1024	0.3421
30	30	30	0	0	0	σ	2.5σ	3σ	0.7277	0.1216	0.4403
30	30	30	0	0	0	σ	3σ	3.5σ	0.8132	0.1323	0.5131
30	30	30	0	0	0	σ	3.5σ	4σ	0.8635	0.1305	0.5614
30	30	30	0	0	0	σ	4σ	4.5σ	0.9065	0.1543	0.6031

Table 119. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
30	30	30	0	0.25	0.5	σ	1.5σ	2σ	**0.7257**	0.5679	0.5295
30	30	30	0	0.25	0.5	σ	2σ	2.5σ	**0.8917**	0.6865	0.8464
30	30	30	0	0.25	0.5	σ	2.5σ	3σ	**0.9568**	0.7669	0.9604
30	30	30	0	0.25	0.5	σ	3σ	3.5σ	**0.9801**	0.8170	0.9912
30	30	30	0	0.25	0.5	σ	3.5σ	4σ	**0.9882**	0.8470	0.9979
30	30	30	0	0.25	0.5	σ	4σ	4.5σ	**0.9923**	0.8682	0.9995

Table 120. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
30	30	30	0	0.25	0.5	σ	1.5σ	2σ	0.6346	0.3329	0.5292
30	30	30	0	0.25	0.5	σ	2σ	2.5σ	0.7944	0.3354	0.6356
30	30	30	0	0.25	0.5	σ	2.5σ	3σ	0.8728	0.3309	0.6857
30	30	30	0	0.25	0.5	σ	3σ	3.5σ	0.9214	0.3146	0.7293
30	30	30	0	0.25	0.5	σ	3.5σ	4σ	0.9406	0.3022	0.7505
30	30	30	0	0.25	0.5	σ	4σ	4.5σ	0.9534	0.2949	0.7689

Table 121. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	9	9	0	0	0	σ	σ	σ	0.0502	0.0485	0.0450
18	9	9	0	0.25	0.5	σ	σ	σ	0.1569	0.1935	0.1093
18	9	9	0	0.5	0.75	σ	σ	σ	0.2739	0.3696	0.2189
18	9	9	0	0.75	1	σ	σ	σ	0.4036	0.5443	0.3921
18	9	9	0	1	1.25	σ	σ	σ	0.5496	0.7189	0.5925
18	9	9	0	1.25	1.5	σ	σ	σ	0.6743	0.8445	0.7672
18	9	9	0	1.5	1.75	σ	σ	σ	0.7856	0.9286	0.8820

Table 122. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	9	9	0	0	0	σ	σ	σ	0.0519	0.0510	0.0514
18	9	9	0	0.25	0.5	σ	σ	σ	0.1486	**0.2164**	0.1855
18	9	9	0	0.5	0.75	σ	σ	σ	0.2516	**0.4002**	0.3346
18	9	9	0	0.75	1	σ	σ	σ	0.4024	**0.6199**	0.5148
18	9	9	0	1	1.25	σ	σ	σ	0.5343	**0.8100**	0.6839
18	9	9	0	1.25	1.5	σ	σ	σ	0.6504	**0.9099**	0.8155
18	9	9	0	1.5	1.75	σ	σ	σ	0.7651	**0.9667**	0.9031

Table 123. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	9	9	0	0	0	σ	1.5σ	2σ	**0.2575**	0.1534	0.2177
18	9	9	0	0	0	σ	2σ	2.5σ	**0.4255**	0.2335	0.4279
18	9	9	0	0	0	σ	2.5σ	3σ	**0.5498**	0.2969	0.6120
18	9	9	0	0	0	σ	3σ	3.5σ	**0.6409**	0.3551	0.7529
18	9	9	0	0	0	σ	3.5σ	4σ	**0.7014**	0.3979	0.8421
18	9	9	0	0	0	σ	4σ	4.5σ	**0.7589**	0.4404	0.9008

Table 124. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	9	9	0	0	0	σ	1.5σ	2σ	0.2071	0.0806	0.1421
18	9	9	0	0	0	σ	2σ	2.5σ	0.3082	0.1002	0.2144
18	9	9	0	0	0	σ	2.5σ	3σ	0.4039	0.1147	0.2528
18	9	9	0	0	0	σ	3σ	3.5σ	0.4731	0.1286	0.2993
18	9	9	0	0	0	σ	3.5σ	4σ	0.5135	0.1449	0.3088
18	9	9	0	0	0	σ	4σ	4.5σ	0.5407	0.1413	0.3409

Table 125. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	9	9	0	0.25	0.5	σ	1.5σ	2σ	**0.4205**	0.3161	0.2467
18	9	9	0	0.25	0.5	σ	2σ	2.5σ	**0.5692**	0.3970	0.4545
18	9	9	0	0.25	0.5	σ	2.5σ	3σ	**0.6732**	0.4458	0.6310
18	9	9	0	0.25	0.5	σ	3σ	3.5σ	**0.7472**	0.4871	0.7668
18	9	9	0	0.25	0.5	σ	3.5σ	4σ	**0.7918**	0.5166	0.8486
18	9	9	0	0.25	0.5	σ	4σ	4.5σ	**0.8219**	0.5368	0.9079

Table 126. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	9	9	0	0.25	0.5	σ	1.5σ	2σ	0.3687	0.2209	0.3082
18	9	9	0	0.25	0.5	σ	2σ	2.5σ	0.4780	0.2193	0.3504
18	9	9	0	0.25	0.5	σ	2.5σ	3σ	0.5422	0.2205	0.3910
18	9	9	0	0.25	0.5	σ	3σ	3.5σ	0.5923	0.2153	0.4169
18	9	9	0	0.25	0.5	σ	3.5σ	4σ	0.6074	0.2084	0.4311
18	9	9	0	0.25	0.5	σ	4σ	4.5σ	0.6472	0.2169	0.4341

Table 127. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	30	30	0	0	0	σ	σ	σ	0.0457	0.0450	0.0482
18	30	30	0	0.25	0.5	σ	σ	σ	0.1633	0.2362	0.1514
18	30	30	0	0.5	0.75	σ	σ	σ	0.2601	0.4304	0.3381
18	30	30	0	0.75	1	σ	σ	σ	0.3471	0.6370	0.5984
18	30	30	0	1	1.25	σ	σ	σ	0.4186	0.7909	0.8211
18	30	30	0	1.25	1.5	σ	σ	σ	0.4793	0.8878	0.9426
18	30	30	0	1.5	1.75	σ	σ	σ	0.5093	0.9443	0.9862

Table 128. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	30	30	0	0	0	σ	σ	σ	0.0481	0.0473	0.0483
18	30	30	0	0.25	0.5	σ	σ	σ	0.1886	**0.2709**	0.2443
18	30	30	0	0.5	0.75	σ	σ	σ	0.3485	**0.5427**	0.4654
18	30	30	0	0.75	1	σ	σ	σ	0.5373	**0.7875**	0.6859
18	30	30	0	1	1.25	σ	σ	σ	0.6965	**0.9203**	0.8482
18	30	30	0	1.25	1.5	σ	σ	σ	0.7957	**0.9784**	0.9346
18	30	30	0	1.5	1.75	σ	σ	σ	0.8823	**0.9942**	0.9755

Table 129. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	30	30	0	0	0	σ	1.5σ	2σ	**0.3538**	0.1740	0.3048
18	30	30	0	0	0	σ	2σ	2.5σ	**0.5912**	0.2844	0.6305
18	30	30	0	0	0	σ	2.5σ	3σ	**0.7513**	0.3883	0.8401
18	30	30	0	0	0	σ	3σ	3.5σ	**0.8504**	0.4780	0.9367
18	30	30	0	0	0	σ	3.5σ	4σ	**0.9023**	0.5543	0.9804
18	30	30	0	0	0	σ	4σ	4.5σ	**0.9307**	0.6034	0.9911

Table 130. Percentage of Rejection for k=3 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	30	30	0	0	0	σ	1.5σ	2σ	0.2616	0.0638	0.1550
18	30	30	0	0	0	σ	2σ	2.5σ	0.4285	0.0727	0.2409
18	30	30	0	0	0	σ	2.5σ	3σ	0.5698	0.0765	0.3146
18	30	30	0	0	0	σ	3σ	3.5σ	0.6695	0.0827	0.3675
18	30	30	0	0	0	σ	3.5σ	4σ	0.7432	0.0842	0.4093
18	30	30	0	0	0	σ	4σ	4.5σ	0.7917	0.0884	0.4413

Table 131. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	30	30	0	0.25	0.5	σ	1.5σ	2σ	**0.5633**	0.4214	0.3409
18	30	30	0	0.25	0.5	σ	2σ	2.5σ	**0.7717**	0.5408	0.6501
18	30	30	0	0.25	0.5	σ	2.5σ	3σ	**0.8695**	0.6178	0.8457
18	30	30	0	0.25	0.5	σ	3σ	3.5σ	**0.9308**	0.6780	0.9370
18	30	30	0	0.25	0.5	σ	3.5σ	4σ	**0.9607**	0.7376	0.9789
18	30	30	0	0.25	0.5	σ	4σ	4.5σ	**0.9745**	0.7546	0.9927

Table 132. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	30	30	0	0.25	0.5	σ	1.5σ	2σ	0.4934	0.2459	0.4104
18	30	30	0	0.25	0.5	σ	2σ	2.5σ	0.6511	0.2330	0.4819
18	30	30	0	0.25	0.5	σ	2.5σ	3σ	0.7525	0.2190	0.5485
18	30	30	0	0.25	0.5	σ	3σ	3.5σ	0.8166	0.2091	0.5744
18	30	30	0	0.25	0.5	σ	3.5σ	4σ	0.8563	0.2025	0.6133
18	30	30	0	0.25	0.5	σ	4σ	4.5σ	0.8855	0.1905	0.6359

5.2.3. Four Treatments

Tables 133-156 outline the results of simulation study for four treatments under the t-distribution. The sample size considered in the simulations are 9, 18, 30. It can be seen that all the proposed tests maintained their alpha values. The estimated alpha values were around 0.05 Tables (133, 134, 139, 140, 145 and 146). When the four populations have unequal location parameters and equal scale parameters, M_2 test has higher estimated power than all comparing tests Tables (134, 140 and 146). The only exception to this is when the first three location parameters equal and the last one is different. In this situation, the L_2 has a higher estimated power (see Appendix E, Tables 1, 13 and 25). When the four populations have equal location parameters and unequal scale parameters, L_1 test has higher estimated power than all comparing tests Tables (135, 141 and 147). When the three populations have unequal location parameters and unequal scale parameters, L_1 test tends to have the highest estimated power Tables (137, 143 and 149).

Table 133. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
9	9	9	9	0	0	0	0	σ	σ	σ	σ	0.0463	0.0460	0.0489
9	9	9	9	0	0.25	0.5	0.75	σ	σ	σ	σ	0.1335	0.2064	0.1223
9	9	9	9	0	0.5	0.75	1	σ	σ	σ	σ	0.1830	0.3222	0.2363
9	9	9	9	0	0.75	1	1.25	σ	σ	σ	σ	0.2233	0.4496	0.3858
9	9	9	9	0	1	1.25	1.5	σ	σ	σ	σ	0.2530	0.5608	0.5525
9	9	9	9	0	1.25	1.5	1.75	σ	σ	σ	σ	0.2586	0.6524	0.7132
9	9	9	9	0	1.5	1.75	2	σ	σ	σ	σ	0.2502	0.7406	0.8389

Table 134. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
9	9	9	9	0	0	0	0	σ	σ	σ	σ	0.0504	0.0518	0.0518
9	9	9	9	0	0.25	0.5	0.75	σ	σ	σ	σ	0.1851	**0.2630**	0.2281
9	9	9	9	0	0.5	0.75	1	σ	σ	σ	σ	0.2807	**0.4352**	0.3545
9	9	9	9	0	0.75	1	1.25	σ	σ	σ	σ	0.3898	**0.6061**	0.5015
9	9	9	9	0	1	1.25	1.5	σ	σ	σ	σ	0.4899	**0.7476**	0.6353
9	9	9	9	0	1.25	1.5	1.75	σ	σ	σ	σ	0.6003	**0.8654**	0.7422
9	9	9	9	0	1.5	1.75	2	σ	σ	σ	σ	0.6704	**0.9255**	0.8236

Table 135. Percentage of Rejection for k=4 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
9	9	9	9	0	0	0	0	σ	1.5σ	2σ	2.5σ	**0.2541**	0.1274	0.1763
9	9	9	9	0	0	0	0	σ	2σ	2.5σ	3σ	**0.3806**	0.1755	0.3342
9	9	9	9	0	0	0	0	σ	2.5σ	3σ	3.5σ	**0.4879**	0.2175	0.4722
9	9	9	9	0	0	0	0	σ	3σ	3.5σ	4σ	**0.5815**	0.2677	0.6081
9	9	9	9	0	0	0	0	σ	3.5σ	4σ	4.5σ	**0.6385**	0.3014	0.7101
9	9	9	9	0	0	0	0	σ	4σ	4.5σ	5σ	**0.6927**	0.3424	0.7916

Table 136. Percentage of Rejection for k=4 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
9	9	9	9	0	0	0	0	σ	1.5σ	2σ	2.5σ	0.1779	0.0598	0.1153
9	9	9	9	0	0	0	0	σ	2σ	2.5σ	3σ	0.2453	0.0553	0.1406
9	9	9	9	0	0	0	0	σ	2.5σ	3σ	3.5σ	0.3073	0.0594	0.1705
9	9	9	9	0	0	0	0	σ	3σ	3.5σ	4σ	0.3611	0.0653	0.1989
9	9	9	9	0	0	0	0	σ	3.5σ	4σ	4.5σ	0.3954	0.0656	0.2157
9	9	9	9	0	0	0	0	σ	4σ	4.5σ	5σ	0.4235	0.0684	0.2256

Table 137. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
9	9	9	9	0	0.2	0.4	0.6	σ	1.5σ	2σ	2.5σ	**0.3789**	0.2539	0.1856
9	9	9	9	0	0.2	0.4	0.6	σ	2σ	2.5σ	3σ	**0.5130**	0.3058	0.3296
9	9	9	9	0	0.2	0.4	0.6	σ	2.5σ	3σ	3.5σ	**0.6174**	0.3628	0.4772
9	9	9	9	0	0.2	0.4	0.6	σ	3σ	3.5σ	4σ	**0.6835**	0.3873	0.6043
9	9	9	9	0	0.2	0.4	0.6	σ	3.5σ	4σ	4.5σ	**0.7291**	0.4202	0.6953
9	9	9	9	0	0.2	0.4	0.6	σ	4σ	4.5σ	5σ	**0.7767**	0.4468	0.7779

Table 138. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
9	9	9	9	0	0.2	0.4	0.6	σ	1.5σ	2σ	2.5σ	0.3278	0.1581	0.2457
9	9	9	9	0	0.2	0.4	0.6	σ	2σ	2.5σ	3σ	0.4018	0.1452	0.2829
9	9	9	9	0	0.2	0.4	0.6	σ	2.5σ	3σ	3.5σ	0.4540	0.1387	0.3078
9	9	9	9	0	0.2	0.4	0.6	σ	3σ	3.5σ	4σ	0.4907	0.1211	0.3184
9	9	9	9	0	0.2	0.4	0.6	σ	3.5σ	4σ	4.5σ	0.5089	0.1216	0.3144
9	9	9	9	0	0.2	0.4	0.6	σ	4σ	4.5σ	5σ	0.5426	0.1230	0.3313

Table 139. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	18	18	18	0	0	0	0	σ	σ	σ	σ	0.0482	0.0488	0.0454
18	18	18	18	0	0.25	0.5	0.75	σ	σ	σ	σ	0.2163	0.3265	0.2166
18	18	18	18	0	0.5	0.75	1	σ	σ	σ	σ	0.3141	0.5365	0.4611
18	18	18	18	0	0.75	1	1.25	σ	σ	σ	σ	0.4096	0.7182	0.7032
18	18	18	18	0	1	1.25	1.5	σ	σ	σ	σ	0.4831	0.8453	0.8804
18	18	18	18	0	1.25	1.5	1.75	σ	σ	σ	σ	0.5323	0.9239	0.9675
18	18	18	18	0	1.5	1.75	2	σ	σ	σ	σ	0.5606	0.9594	0.9920

Table 140. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	18	18	18	0	0	0	0	σ	σ	σ	σ	0.0503	0.0493	0.0510
18	18	18	18	0	0.25	0.5	0.75	σ	σ	σ	σ	0.2644	**0.4004**	0.3301
18	18	18	18	0	0.5	0.75	1	σ	σ	σ	σ	0.4314	**0.6628**	0.5612
18	18	18	18	0	0.75	1	1.25	σ	σ	σ	σ	0.5994	**0.8540**	0.7530
18	18	18	18	0	1	1.25	1.5	σ	σ	σ	σ	0.7480	**0.9559**	0.8801
18	18	18	18	0	1.25	1.5	1.75	σ	σ	σ	σ	0.8416	**0.9856**	0.9517
18	18	18	18	0	1.5	1.75	2	σ	σ	σ	σ	0.9052	**0.9961**	0.9793

Table 141. Percentage of Rejection for k=4 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	18	18	18	0	0	0	0	σ	1.5σ	2σ	2.5σ	**0.4577**	0.2202	0.4370
18	18	18	18	0	0	0	0	σ	2σ	2.5σ	3σ	**0.6606**	0.3264	0.7272
18	18	18	18	0	0	0	0	σ	2.5σ	3σ	3.5σ	**0.7884**	0.4227	0.8832
18	18	18	18	0	0	0	0	σ	3σ	3.5σ	4σ	**0.8616**	0.5005	0.9548
18	18	18	18	0	0	0	0	σ	3.5σ	4σ	4.5σ	**0.9124**	0.5798	0.9846
18	18	18	18	0	0	0	0	σ	4σ	4.5σ	5σ	**0.9312**	0.6110	0.9943

Table 142. Percentage of Rejection for k=4 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	18	18	18	0	0	0	0	σ	1.5σ	2σ	2.5σ	0.3234	0.0653	0.1855
18	18	18	18	0	0	0	0	σ	2σ	2.5σ	3σ	0.4753	0.0734	0.2601
18	18	18	18	0	0	0	0	σ	2.5σ	3σ	3.5σ	0.6037	0.0879	0.3246
18	18	18	18	0	0	0	0	σ	3σ	3.5σ	4σ	0.6825	0.0925	0.3824
18	18	18	18	0	0	0	0	σ	3.5σ	4σ	4.5σ	0.7445	0.0938	0.4177
18	18	18	18	0	0	0	0	σ	4σ	4.5σ	5σ	0.7887	0.0969	0.4534

Table 143. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	18	18	18	0	0.2	0.4	0.6	σ	1.5σ	2σ	2.5σ	**0.6725**	0.4816	0.4721
18	18	18	18	0	0.2	0.4	0.6	σ	2σ	2.5σ	3σ	**0.8275**	0.5808	0.7450
18	18	18	18	0	0.2	0.4	0.6	σ	2.5σ	3σ	3.5σ	**0.9034**	0.6544	0.8881
18	18	18	18	0	0.2	0.4	0.6	σ	3σ	3.5σ	4σ	**0.9446**	0.6999	0.9579
18	18	18	18	0	0.2	0.4	0.6	σ	3.5σ	4σ	4.5σ	**0.9622**	0.7369	0.9850
18	18	18	18	0	0.2	0.4	0.6	σ	4σ	4.5σ	5σ	**0.9706**	0.7622	0.9958

Table 144. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	18	18	18	0	0.2	0.4	0.6	σ	1.5σ	2σ	2.5σ	0.5680	0.2571	0.4474
18	18	18	18	0	0.2	0.4	0.6	σ	2σ	2.5σ	3σ	0.6905	0.2392	0.5157
18	18	18	18	0	0.2	0.4	0.6	σ	2.5σ	3σ	3.5σ	0.7735	0.2246	0.5556
18	18	18	18	0	0.2	0.4	0.6	σ	3σ	3.5σ	4σ	0.8224	0.2119	0.5813
18	18	18	18	0	0.2	0.4	0.6	σ	3.5σ	4σ	4.5σ	0.8611	0.2028	0.6152
18	18	18	18	0	0.2	0.4	0.6	σ	4σ	4.5σ	5σ	0.8777	0.2012	0.6147

Table 145. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
30	30	30	30	0	0	0	0	σ	σ	σ	σ	0.0459	0.0466	0.0513
30	30	30	30	0	0.25	0.5	0.75	σ	σ	σ	σ	0.3012	0.4688	0.3504
30	30	30	30	0	0.5	0.75	1	σ	σ	σ	σ	0.4690	0.7395	0.6913
30	30	30	30	0	0.75	1	1.25	σ	σ	σ	σ	0.6081	0.9007	0.9118
30	30	30	30	0	1	1.25	1.5	σ	σ	σ	σ	0.6967	0.9656	0.9844
30	30	30	30	0	1.25	1.5	1.75	σ	σ	σ	σ	0.7649	0.9909	0.9988
30	30	30	30	0	1.5	1.75	2	σ	σ	σ	σ	0.8024	0.9976	1

Table 146. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
30	30	30	30	0	0	0	0	σ	σ	σ	σ	0.0502	0.0508	0.0507
30	30	30	30	0	0.25	0.5	0.75	σ	σ	σ	σ	0.3770	**0.5669**	0.4711
30	30	30	30	0	0.5	0.75	1	σ	σ	σ	σ	0.5955	**0.8461**	0.7568
30	30	30	30	0	0.75	1	1.25	σ	σ	σ	σ	0.7844	**0.9677**	0.9116
30	30	30	30	0	1	1.25	1.5	σ	σ	σ	σ	0.9038	**0.9949**	0.9759
30	30	30	30	0	1.25	1.5	1.75	σ	σ	σ	σ	0.9611	**0.9995**	0.9918
30	30	30	30	0	1.5	1.75	2	σ	σ	σ	σ	0.9852	**1**	0.9969

Table 147. Percentage of Rejection for k=4 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
30	30	30	30	0	0	0	0	σ	1.5σ	2σ	2.5σ	**0.6466**	0.3234	0.7136
30	30	30	30	0	0	0	0	σ	2σ	2.5σ	3σ	**0.8525**	0.5019	0.9430
30	30	30	30	0	0	0	0	σ	2.5σ	3σ	3.5σ	**0.9438**	0.6334	0.9919
30	30	30	30	0	0	0	0	σ	3σ	3.5σ	4σ	**0.9741**	0.7308	0.9986
30	30	30	30	0	0	0	0	σ	3.5σ	4σ	4.5σ	**0.9872**	0.7808	0.9997
30	30	30	30	0	0	0	0	σ	4σ	4.5σ	5σ	**0.9916**	0.8224	1

Table 148. Percentage of Rejection for k=4 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
30	30	30	30	0	0	0	0	σ	1.5σ	2σ	2.5σ	0.4934	0.0866	0.2872
30	30	30	30	0	0	0	0	σ	2σ	2.5σ	3σ	0.7179	0.1017	0.4183
30	30	30	30	0	0	0	0	σ	2.5σ	3σ	3.5σ	0.8273	0.1151	0.5149
30	30	30	30	0	0	0	0	σ	3σ	3.5σ	4σ	0.9039	0.1184	0.5918
30	30	30	30	0	0	0	0	σ	3.5σ	4σ	4.5σ	0.9350	0.1355	0.6390
30	30	30	30	0	0	0	0	σ	4σ	4.5σ	5σ	0.9591	0.1367	0.6818

Table 149. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
30	30	30	30	0	0.2	0.4	0.6	σ	1.5σ	2σ	2.5σ	**0.8717**	0.6958	0.7509
30	30	30	30	0	0.2	0.4	0.6	σ	2σ	2.5σ	3σ	**0.9616**	0.7934	0.9490
30	30	30	30	0	0.2	0.4	0.6	σ	2.5σ	3σ	3.5σ	**0.9858**	0.8608	0.9920
30	30	30	30	0	0.2	0.4	0.6	σ	3σ	3.5σ	4σ	**0.9933**	0.8990	0.9987
30	30	30	30	0	0.2	0.4	0.6	σ	3.5σ	4σ	4.5σ	**0.9974**	0.9178	1
30	30	30	30	0	0.2	0.4	0.6	σ	4σ	4.5σ	5σ	**0.9984**	0.9315	1

Table 150. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
30	30	30	30	0	0.2	0.4	0.6	σ	1.5σ	2σ	2.5σ	0.7715	0.3634	0.6425
30	30	30	30	0	0.2	0.4	0.6	σ	2σ	2.5σ	3σ	0.8927	0.3673	0.7315
30	30	30	30	0	0.2	0.4	0.6	σ	2.5σ	3σ	3.5σ	0.9441	0.3430	0.7875
30	30	30	30	0	0.2	0.4	0.6	σ	3σ	3.5σ	4σ	0.9672	0.3282	0.8162
30	30	30	30	0	0.2	0.4	0.6	σ	3.5σ	4σ	4.5σ	0.9810	0.3185	0.8373
30	30	30	30	0	0.2	0.4	0.6	σ	4σ	4.5σ	5σ	0.9873	0.3039	0.8564

Table 151. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	9	9	9	0	0	0	0	σ	σ	σ	σ	0.0478	0.0460	0.0433
18	9	9	9	0	0.25	0.5	0.75	σ	σ	σ	σ	0.2155	0.3025	0.1773
18	9	9	9	0	0.5	0.75	1	σ	σ	σ	σ	0.3387	0.4961	0.3580
18	9	9	9	0	0.75	1	1.25	σ	σ	σ	σ	0.4631	0.6766	0.5682
18	9	9	9	0	1	1.25	1.5	σ	σ	σ	σ	0.5806	0.8180	0.7711
18	9	9	9	0	1.25	1.5	1.75	σ	σ	σ	σ	0.6761	0.9112	0.9044
18	9	9	9	0	1.5	1.75	2	σ	σ	σ	σ	0.7565	0.9527	0.9610

Table 152. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	9	9	9	0	0	0	0	σ	σ	σ	σ	0.0488	0.0503	0.0496
18	9	9	9	0	0.25	0.5	0.75	σ	σ	σ	σ	0.2304	**0.3441**	0.2903
18	9	9	9	0	0.5	0.75	1	σ	σ	σ	σ	0.3719	**0.5756**	0.4846
18	9	9	9	0	0.75	1	1.25	σ	σ	σ	σ	0.5280	**0.7803**	0.6762
18	9	9	9	0	1	1.25	1.5	σ	σ	σ	σ	0.6680	**0.9126**	0.8239
18	9	9	9	0	1.25	1.5	1.75	σ	σ	σ	σ	0.7712	**0.9675**	0.9067
18	9	9	9	0	1.5	1.75	2	σ	σ	σ	σ	0.8511	**0.9921**	0.9624

Table 153. Percentage of Rejection for k=4 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	9	9	9	0	0	0	0	σ	1.5σ	2σ	2.5σ	**0.3790**	0.1980	0.3579
18	9	9	9	0	0	0	0	σ	2σ	2.5σ	3σ	**0.5603**	0.2936	0.6066
18	9	9	9	0	0	0	0	σ	2.5σ	3σ	3.5σ	**0.6708**	0.3638	0.7753
18	9	9	9	0	0	0	0	σ	3σ	3.5σ	4σ	**0.7583**	0.4223	0.8759
18	9	9	9	0	0	0	0	σ	3.5σ	4σ	4.5σ	**0.8128**	0.4754	0.9388
18	9	9	9	0	0	0	0	σ	4σ	4.5σ	5σ	**0.8488**	0.5138	0.9705

Table 154. Percentage of Rejection for k=4 Populations; T Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	9	9	9	0	0	0	0	σ	1.5σ	2σ	2.5σ	0.2784	0.0799	0.1749
18	9	9	9	0	0	0	0	σ	2σ	2.5σ	3σ	0.4044	0.0976	0.2487
18	9	9	9	0	0	0	0	σ	2.5σ	3σ	3.5σ	0.4891	0.1062	0.2937
18	9	9	9	0	0	0	0	σ	3σ	3.5σ	4σ	0.5610	0.1132	0.3360
18	9	9	9	0	0	0	0	σ	3.5σ	4σ	4.5σ	0.5999	0.1232	0.3614
18	9	9	9	0	0	0	0	σ	4σ	4.5σ	5σ	0.6380	0.1249	0.3833

Table 155. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	9	9	9	0	0.2	0.4	0.6	σ	1.5σ	2σ	2.5σ	**0.5724**	0.4146	0.3927
18	9	9	9	0	0.2	0.4	0.6	σ	2σ	2.5σ	3σ	**0.7075**	0.4848	0.6201
18	9	9	9	0	0.2	0.4	0.6	σ	2.5σ	3σ	3.5σ	**0.7969**	0.5396	0.7838
18	9	9	9	0	0.2	0.4	0.6	σ	3σ	3.5σ	4σ	**0.8516**	0.5905	0.8780
18	9	9	9	0	0.2	0.4	0.6	σ	3.5σ	4σ	4.5σ	**0.8796**	0.6178	0.9368
18	9	9	9	0	0.2	0.4	0.6	σ	4σ	4.5σ	5σ	**0.9112**	0.6394	0.9682

Table 156. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	9	9	9	0	0.2	0.4	0.6	σ	1.5σ	2σ	2.5σ	0.4637	0.2250	0.3763
18	9	9	9	0	0.2	0.4	0.6	σ	2σ	2.5σ	3σ	0.5799	0.2251	0.4322
18	9	9	9	0	0.2	0.4	0.6	σ	2.5σ	3σ	3.5σ	0.6443	0.2225	0.4702
18	9	9	9	0	0.2	0.4	0.6	σ	3σ	3.5σ	4σ	0.6901	0.2160	0.4891
18	9	9	9	0	0.2	0.4	0.6	σ	3.5σ	4σ	4.5σ	0.7220	0.2055	0.4974
18	9	9	9	0	0.2	0.4	0.6	σ	4σ	4.5σ	5σ	0.7511	0.2117	0.5152

5.3. Results for the Exponential Distribution

5.3.1. Two Treatments

Tables 157-174 outline the results of simulation study for two treatments under the exponential distribution. The sample size considered in the simulations are 9, 18, 30. It appeared that both of L_1 and L_2 maintained their alpha values. The estimated alpha values were around 0.05 Tables (157, 163 and 169). On the other hand, it is noted that both of M_1, M_2 and M_3 didn't maintain their alpha values Tables (158, 164 and 170). As a result, the results for M_1, M_2 and M_3 listed for completion purposes not to be compared with L_1 and L_2.

When the populations have unequal location parameters and equal scale parameters, L_2 test has higher estimated power than L_1 Tables (157, 163, and 169). When the populations have equal location parameters and unequal scale parameters, L_1 test has higher estimated power than L_2 Tables (159, 165 and 171). When the populations have unequal location parameters and unequal scale parameters, L_2 test tends to have the highest estimated power Tables (161, 167 and 173). Tables 175-180 show the simulations where unequal sample sizes were considered. The sample size for the first population was 18 and the second population was 30. A similar result was found when investigating unequal sample cases.

Table 157. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
9	9	1	1	1	1	0.0497	0.0466	0.0444
9	9	1	1	1.25	1	0.0650	**0.1118**	0.1101
9	9	1	1	1.5	1	0.0920	**0.2080**	0.3057
9	9	1	1	1.75	1	0.1467	**0.3363**	0.5361
9	9	1	1	2	1	0.2150	**0.4623**	0.7284
9	9	1	1	2.25	1	0.3162	**0.6146**	0.8689
9	9	1	1	2.5	1	0.4147	**0.7250**	0.9368

Table 158. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
9	9	1	1	1	1	0.0946	0.0634	0.0928
9	9	1	1	1.25	1	0.1989	0.2061	0.2101
9	9	1	1	1.5	1	0.3068	0.4066	0.3366
9	9	1	1	1.75	1	0.4120	0.5961	0.4572
9	9	1	1	2	1	0.4960	0.7393	0.5579
9	9	1	1	2.25	1	0.5677	0.8401	0.6356
9	9	1	1	2.5	1	0.6222	0.9076	0.6987

Table 159. Percentage of Rejection for k=2 Populations; Exponential Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
9	9	1	1	1	1.25^2	**0.0923**	0.0544	0.0858
9	9	1	1	1	1.5^2	**0.1309**	0.0552	0.1746
9	9	1	1	1	1.75^2	**0.1584**	0.0584	0.2722
9	9	1	1	1	2^2	**0.1902**	0.0711	0.3625
9	9	1	1	1	2.25^2	**0.2155**	0.0710	0.4490
9	9	1	1	1	2.5^2	**0.2356**	0.0802	0.5232

Table 160. Percentage of Rejection for k=2 Populations; Exponential Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
9	9	1	1	1	1.25^2	0.0873	0.0417	0.0805
9	9	1	1	1	1.5^2	0.0867	0.0277	0.0742
9	9	1	1	1	1.75^2	0.0905	0.0249	0.0750
9	9	1	1	1	2^2	0.0981	0.0244	0.0825
9	9	1	1	1	2.25^2	0.1038	0.0223	0.0826
9	9	1	1	1	2.5^2	0.0991	0.0184	0.0785

Table 161. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
9	9	1	1	1.25	1.25^2	0.1114	**0.1113**	0.0561
9	9	1	1	1.5	1.5^2	0.1735	**0.1864**	0.0781
9	9	1	1	1.75	1.75^2	0.2392	**0.2645**	0.1101
9	9	1	1	2	2^2	0.3090	**0.3492**	0.1486
9	9	1	1	2.25	2.25^2	0.3696	**0.4209**	0.1945
9	9	1	1	2.5	2.5^2	0.4493	**0.5058**	0.2427

Table 162. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
9	9	1	1	1.25	1.25^2	0.1775	0.1209	0.1744
9	9	1	1	1.5	1.5^2	0.2824	0.2084	0.2816
9	9	1	1	1.75	1.75^2	0.3893	0.2994	0.3885
9	9	1	1	2	2^2	0.4778	0.3878	0.4799
9	9	1	1	2.25	2.25^2	0.5730	0.4797	0.5758
9	9	1	1	2.5	2.5^2	0.6381	0.5508	0.6426

Table 163. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	18	1	1	1	1	0.0488	0.0487	0.0445
18	18	1	1	1.25	1	0.0677	**0.1460**	0.2346
18	18	1	1	1.5	1	0.1011	**0.3185**	0.6559
18	18	1	1	1.75	1	0.1848	**0.5261**	0.9019
18	18	1	1	2	1	0.3155	**0.7107**	0.9791
18	18	1	1	2.25	1	0.4788	**0.8506**	0.9960
18	18	1	1	2.5	1	0.6144	**0.9279**	0.9993

Table 164. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	18	1	1	1	1	0.0870	0.0616	0.0859
18	18	1	1	1.25	1	0.2526	0.3279	0.2728
18	18	1	1	1.5	1	0.4399	0.6422	0.4886
18	18	1	1	1.75	1	0.5969	0.8472	0.6629
18	18	1	1	2	1	0.7196	0.9524	0.7885
18	18	1	1	2.25	1	0.8034	0.9851	0.8662
18	18	1	1	2.5	1	0.8696	0.9944	0.9275

Table 165. Percentage of Rejection for k=2 Populations; Exponential Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	18	1	1	1	1.25^2	**0.1160**	0.0564	0.1672
18	18	1	1	1	1.5^2	**0.1877**	0.0680	0.4005
18	18	1	1	1	1.75^2	**0.2426**	0.0760	0.5987
18	18	1	1	1	2^2	**0.3003**	0.0887	0.7409
18	18	1	1	1	2.25^2	**0.3436**	0.0926	0.8212
18	18	1	1	1	2.5^2	**0.3805**	0.1066	0.8871

Table 166. Percentage of Rejection for k=2 Populations; Exponential Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	18	1	1	1	1.25^2	0.0808	0.0329	0.0723
18	18	1	1	1	1.5^2	0.0948	0.0230	0.0751
18	18	1	1	1	1.75^2	0.1034	0.0197	0.0772
18	18	1	1	1	2^2	0.1179	0.0199	0.0883
18	18	1	1	1	2.25^2	0.1255	0.0166	0.0888
18	18	1	1	1	2.5^2	0.1422	0.0174	0.0988

Table 167. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	18	1	1	1.25	1.25^2	0.1396	**0.1503**	0.0752
18	18	1	1	1.5	1.5^2	0.2602	**0.2886**	0.1364
18	18	1	1	1.75	1.75^2	0.3953	**0.4426**	0.2338
18	18	1	1	2	2^2	0.5075	**0.5709**	0.3331
18	18	1	1	2.25	2.25^2	0.6141	**0.6924**	0.4387
18	18	1	1	2.5	2.5^2	0.7116	**0.7802**	0.5470

Table 168. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	18	1	1	1.25	1.25^2	0.2249	0.1825	0.2242
18	18	1	1	1.5	1.5^2	0.3995	0.3438	0.4018
18	18	1	1	1.75	1.75^2	0.5697	0.5011	0.5703
18	18	1	1	2	2^2	0.6998	0.6408	0.7029
18	18	1	1	2.25	2.25^2	0.7867	0.7304	0.7915
18	18	1	1	2.5	2.5^2	0.8609	0.8230	0.8640

Table 169. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
30	30	1	1	1	1	0.0473	0.0460	0.0442
30	30	1	1	1.25	1	0.0665	**0.1843**	0.3991
30	30	1	1	1.5	1	0.1236	**0.4404**	0.8845
30	30	1	1	1.75	1	0.2404	**0.7134**	0.9908
30	30	1	1	2	1	0.4225	**0.8842**	0.9996
30	30	1	1	2.25	1	0.6186	**0.9652**	1
30	30	1	1	2.5	1	0.7891	**0.9891**	1

Table 170. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
30	30	1	1	1	1	0.0927	0.0621	0.0910
30	30	1	1	1.25	1	0.3164	0.4255	0.3471
30	30	1	1	1.5	1	0.5759	0.8250	0.6469
30	30	1	1	1.75	1	0.7625	0.9639	0.8324
30	30	1	1	2	1	0.8736	0.9938	0.9305
30	30	1	1	2.25	1	0.9371	0.9995	0.9701
30	30	1	1	2.5	1	0.9734	1	0.9915

Table 171. Percentage of Rejection for k=2 Populations; Exponential Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
30	30	1	1	1	1.25^2	**0.1533**	0.0632	0.2736
30	30	1	1	1	1.5^2	**0.2582**	0.0785	0.6267
30	30	1	1	1	1.75^2	**0.3438**	0.0848	0.8447
30	30	1	1	1	2^2	**0.4231**	0.1034	0.9312
30	30	1	1	1	2.25^2	**0.4923**	0.1113	0.9682
30	30	1	1	1	2.5^2	**0.5476**	0.1276	0.9895

Table 172. Percentage of Rejection for k=2 Populations; Exponential Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
30	30	1	1	1	1.25^2	0.0932	0.0285	0.0788
30	30	1	1	1	1.5^2	0.0928	0.0148	0.0707
30	30	1	1	1	1.75^2	0.1143	0.0112	0.0816
30	30	1	1	1	2^2	0.1367	0.0110	0.0937
30	30	1	1	1	2.25^2	0.1502	0.0097	0.0988
30	30	1	1	1	2.5^2	0.1681	0.0110	0.1093

Table 173. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
30	30	1	1	1.25	1.25^2	0.1715	**0.1892**	0.0919
30	30	1	1	1.5	1.5^2	0.3607	**0.4103**	0.2153
30	30	1	1	1.75	1.75^2	0.5558	**0.6253**	0.3796
30	30	1	1	2	2^2	0.7167	**0.7859**	0.5607
30	30	1	1	2.25	2.25^2	0.8190	**0.8788**	0.6951
30	30	1	1	2.5	2.5^2	0.8951	**0.9383**	0.8071

Table 174. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
30	30	1	1	1.25	1.25^2	0.2943	0.2399	0.2945
30	30	1	1	1.5	1.5^2	0.5220	0.4536	0.5240
30	30	1	1	1.75	1.75^2	0.7230	0.6496	0.7250
30	30	1	1	2	2^2	0.8565	0.8093	0.8589
30	30	1	1	2.25	2.25^2	0.9291	0.8932	0.9295
30	30	1	1	2.5	2.5^2	0.9642	0.9454	0.9660

Table 175. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	30	1	1	1	1	0.0434	0.0462	0.0448
18	30	1	1	1.25	1	0.0508	**0.1484**	0.3164
18	30	1	1	1.5	1	0.0691	**0.3334**	0.7542
18	30	1	1	1.75	1	0.1123	**0.5496**	0.9495
18	30	1	1	2	1	0.1729	**0.7279**	0.9916
18	30	1	1	2.25	1	0.2697	**0.8573**	0.9993
18	30	1	1	2.5	1	0.3719	**0.9343**	0.9998

Table 176. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	30	1	1	1	1	0.0907	0.0677	0.0877
18	30	1	1	1.25	1	0.2751	0.3679	0.3301
18	30	1	1	1.5	1	0.4871	0.7060	0.6016
18	30	1	1	1.75	1	0.6625	0.8998	0.7959
18	30	1	1	2	1	0.7774	0.9715	0.9044
18	30	1	1	2.25	1	0.8655	0.9926	0.9570
18	30	1	1	2.5	1	0.9141	0.9971	0.9815

Table 177. Percentage of Rejection for k=2 Populations; Exponential Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	30	1	1	1	1.25^2	**0.1252**	0.0524	0.1789
18	30	1	1	1	1.5^2	**0.2087**	0.0608	0.4885
18	30	1	1	1	1.75^2	**0.2680**	0.0661	0.7165
18	30	1	1	1	2^2	**0.3411**	0.0813	0.8551
18	30	1	1	1	2.25^2	**0.3954**	0.0851	0.9263
18	30	1	1	1	2.5^2	**0.4241**	0.0988	0.9616

Table 178. Percentage of Rejection for k=2 Populations; Exponential Distribution with same means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	30	1	1	1	1.25^2	0.0811	0.0245	0.0543
18	30	1	1	1	1.5^2	0.0810	0.0131	0.0435
18	30	1	1	1	1.75^2	0.0894	0.0104	0.0418
18	30	1	1	1	2^2	0.1040	0.0082	0.0438
18	30	1	1	1	2.25^2	0.1107	0.0074	0.0424
18	30	1	1	1	2.5^2	0.1310	0.0071	0.0465

Table 179. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	L_1	L_2	Lepage
18	30	1	1	1.25	1.25^2	0.1410	**0.1559**	0.0782
18	30	1	1	1.5	1.5^2	0.2679	**0.3189**	0.1596
18	30	1	1	1.75	1.75^2	0.4178	**0.4964**	0.2643
18	30	1	1	2	2^2	0.5509	**0.6593**	0.4048
18	30	1	1	2.25	2.25^2	0.6666	**0.7775**	0.5383
18	30	1	1	2.5	2.5^2	0.7670	**0.8629**	0.6633

Table 180. Percentage of Rejection for k=2 Populations; Exponential Distribution with different means and different variances

n_1	n_2	μ_1	σ_1	μ_2	σ_2	M_1	M_2	M_3
18	30	1	1	1.25	1.25^2	0.2503	0.1954	0.2416
18	30	1	1	1.5	1.5^2	0.4524	0.3809	0.4474
18	30	1	1	1.75	1.75^2	0.6327	0.5606	0.6260
18	30	1	1	2	2^2	0.7720	0.7177	0.7737
18	30	1	1	2.25	2.25^2	0.8641	0.8160	0.8638
18	30	1	1	2.5	2.5^2	0.9223	0.8902	0.9233

5.3.2. Three Treatments

Tables 181-198 outline the results of simulation study for three treatments under the exponential distribution. The sample size considered in the simulations are 9, 18, 30. It appeared that both of L_1 and L_2 maintained their alpha values. The estimated alpha values were around 0.05 Tables (181, 187 and 193). On the other hand, it is noted that both of M_1, M_2 and M_3 didn't maintained their alpha values Tables (182, 188 and 194). As a result, the results for M_1, M_2 and M_3 listed for completion purposes not to be compared with L_1 and L_2.

When the populations have unequal location parameters and equal scale parameters, L_2 test has higher estimated power than L_1 Tables (181, 187, and 193). When the populations have equal location parameters and unequal scale parameters, L_1 test has higher estimated power than L_2 Tables (183, 189 and 195). When the populations have unequal location parameters and unequal scale parameters, L_2 test tends to have the highest estimated power Tables (185, 191 and 197). Tables 199-210 show the simulations where unequal sample sizes were considered. A similar result was found when investigating unequal sample cases.

Table 181. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	1	1	1	1	1	1	0.0546	0.0507	0.0451
9	9	9	1	1	1.25	1	1.5	1	0.0535	**0.1681**	0.3094
9	9	9	1	1	1.5	1	1.75	1	0.0614	**0.2887**	0.5972
9	9	9	1	1	1.75	1	2	1	0.0738	**0.4252**	0.8012
9	9	9	1	1	2	1	2.25	1	0.0740	**0.5517**	0.9129
9	9	9	1	1	2.25	1	2.5	1	0.0759	**0.6699**	0.9682
9	9	9	1	1	2.5	1	2.75	1	0.0776	**0.7546**	0.9858

Table 182. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	1	1	1	1	1	1	0.0909	0.0680	0.0935
9	9	9	1	1	1.25	1	1.5	1	0.2858	0.3801	0.3183
9	9	9	1	1	1.5	1	1.75	1	0.4198	0.6056	0.4712
9	9	9	1	1	1.75	1	2	1	0.5237	0.7661	0.5915
9	9	9	1	1	2	1	2.25	1	0.6116	0.8701	0.6880
9	9	9	1	1	2.25	1	2.5	1	0.6932	0.9321	0.7692
9	9	9	1	1	2.5	1	2.75	1	0.7432	0.9647	0.8208

Table 183. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	1	1	1	2^2	1	3^2	**0.2577**	0.0631	0.6304
9	9	9	1	1	1	2.25^2	1	3.5^2	**0.2930**	0.0711	0.7434
9	9	9	1	1	1	2.5^2	1	3.75^2	**0.2982**	0.0731	0.7880
9	9	9	1	1	1	2.75^2	1	4^2	**0.3188**	0.0761	0.8377
9	9	9	1	1	1	3^2	1	4.25^2	**0.3372**	0.0820	0.8647
9	9	9	1	1	1	3.25^2	1	4.5^2	**0.3539**	0.0876	0.8914

Table 184. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	1	1	1	2^2	1	3^2	0.0820	0.0105	0.0612
9	9	9	1	1	1	2.25^2	1	3.5^2	0.0880	0.0076	0.0691
9	9	9	1	1	1	2.5^2	1	3.75^2	0.0795	0.0081	0.0598
9	9	9	1	1	1	2.75^2	1	4^2	0.0878	0.0080	0.0652
9	9	9	1	1	1	3^2	1	4.25^2	0.0857	0.0071	0.0623
9	9	9	1	1	1	3.25^2	1	4.5^2	0.0903	0.0075	0.0653

Table 185. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	1	1	1.5	1.5^2	1.75	1.75^2	0.1999	**0.2418**	0.1022
9	9	9	1	1	1.75	1.75^2	2	2^2	0.2748	**0.3474**	0.1497
9	9	9	1	1	2	2^2	2.25	2.25^2	0.3508	**0.4566**	0.2158
9	9	9	1	1	2.25	2.25^2	2.5	2.5^2	0.4203	**0.5516**	0.2782
9	9	9	1	1	2.5	2.5^2	2.75	2.75^2	0.4842	**0.6391**	0.3420
9	9	9	1	1	2.75	2.75^2	3	3^2	0.5376	**0.7096**	0.4145

Table 186. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	1	1	1.5	1.5^2	1.75	1.75^2	0.3744	0.3151	0.3850
9	9	9	1	1	1.75	1.75^2	2	2^2	0.4988	0.4293	0.5082
9	9	9	1	1	2	2^2	2.25	2.25^2	0.6093	0.5413	0.6213
9	9	9	1	1	2.25	2.25^2	2.5	2.5^2	0.6973	0.6384	0.7111
9	9	9	1	1	2.5	2.5^2	2.75	2.75^2	0.7689	0.7139	0.7815
9	9	9	1	1	2.75	2.75^2	3	3^2	0.8231	0.7701	0.8329

Table 187. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	1	1	1	1	1	1	0.0474	0.0469	0.0469
18	18	18	1	1	1.25	1	1.5	1	0.0572	**0.2452**	0.5603
18	18	18	1	1	1.5	1	1.75	1	0.0822	**0.4465**	0.8862
18	18	18	1	1	1.75	1	2	1	0.1258	**0.6531**	0.9830
18	18	18	1	1	2	1	2.25	1	0.1726	**0.8050**	0.9977
18	18	18	1	1	2.25	1	2.5	1	0.2437	**0.8979**	0.9997
18	18	18	1	1	2.5	1	2.75	1	0.3256	**0.9554**	0.9999

Table 188. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	1	1	1	1	1	1	0.0859	0.0631	0.0851
18	18	18	1	1	1.25	1	1.5	1	0.3894	0.5574	0.4377
18	18	18	1	1	1.5	1	1.75	1	0.5912	0.8279	0.6568
18	18	18	1	1	1.75	1	2	1	0.7405	0.9479	0.8108
18	18	18	1	1	2	1	2.25	1	0.8413	0.9877	0.9001
18	18	18	1	1	2.25	1	2.5	1	0.9033	0.9979	0.9497
18	18	18	1	1	2.5	1	2.75	1	0.9388	0.9990	0.9725

Table 189. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	1	1	1	2^2	1	3^2	**0.4333**	0.0908	0.9674
18	18	18	1	1	1	2.25^2	1	3.5^2	**0.4840**	0.1052	0.9885
18	18	18	1	1	1	2.5^2	1	3.75^2	**0.5068**	0.1038	0.9946
18	18	18	1	1	1	2.75^2	1	4^2	**0.5285**	0.1196	0.9964
18	18	18	1	1	1	3^2	1	4.25^2	**0.5426**	0.1194	0.9986
18	18	18	1	1	1	3.25^2	1	4.5^2	**0.5727**	0.1317	0.9995

Table 190. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	1	1	1	2^2	1	3^2	0.1179	0.0051	0.0724
18	18	18	1	1	1	2.25^2	1	3.5^2	0.1304	0.0033	0.0756
18	18	18	1	1	1	2.5^2	1	3.75^2	0.1410	0.0043	0.0813
18	18	18	1	1	1	2.75^2	1	4^2	0.1460	0.0035	0.0828
18	18	18	1	1	1	3^2	1	4.25^2	0.1641	0.0037	0.0901
18	18	18	1	1	1	3.25^2	1	4.5^2	0.1662	0.0035	0.0934

Table 191. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	1	1	1.5	1.5^2	1.75	1.75^2	0.3460	**0.4160**	0.2173
18	18	18	1	1	1.75	1.75^2	2	2^2	0.5127	**0.6063**	0.3565
18	18	18	1	1	2	2^2	2.25	2.25^2	0.6349	**0.7511**	0.5011
18	18	18	1	1	2.25	2.25^2	2.5	2.5^2	0.7358	**0.8436**	0.6208
18	18	18	1	1	2.5	2.5^2	2.75	2.75^2	0.8106	**0.9118**	0.7361
18	18	18	1	1	2.75	2.75^2	3	3^2	0.8684	**0.9478**	0.8145

Table 192. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	1	1	1.5	1.5^2	1.75	1.75^2	0.5579	0.4852	0.5639
18	18	18	1	1	1.75	1.75^2	2	2^2	0.7269	0.6616	0.7332
18	18	18	1	1	2	2^2	2.25	2.25^2	0.8440	0.7940	0.8495
18	18	18	1	1	2.25	2.25^2	2.5	2.5^2	0.9084	0.8753	0.9127
18	18	18	1	1	2.5	2.5^2	2.75	2.75^2	0.9547	0.9316	0.9568
18	18	18	1	1	2.75	2.75^2	3	3^2	0.9751	0.9633	0.9766

Table 193. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	1	1	1	1	1	1	0.0459	0.0464	0.0489
30	30	30	1	1	1.25	1	1.5	1	0.0681	**0.3483**	0.7919
30	30	30	1	1	1.5	1	1.75	1	0.1048	**0.6304**	0.9848
30	30	30	1	1	1.75	1	2	1	0.1749	**0.8401**	0.9995
30	30	30	1	1	2	1	2.25	1	0.2698	**0.9475**	1
30	30	30	1	1	2.25	1	2.5	1	0.4019	**0.9848**	1
30	30	30	1	1	2.5	1	2.75	1	0.5228	**0.9970**	1

Table 194. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	1	1	1	1	1	1	0.0905	0.0662	0.0892
30	30	30	1	1	1.25	1	1.5	1	0.5085	0.7365	0.5730
30	30	30	1	1	1.5	1	1.75	1	0.7505	0.9492	0.8204
30	30	30	1	1	1.75	1	2	1	0.8922	0.9943	0.9387
30	30	30	1	1	2	1	2.25	1	0.9525	0.9996	0.9790
30	30	30	1	1	2.25	1	2.5	1	0.9825	1	0.9954
30	30	30	1	1	2.5	1	2.75	1	0.9911	1	0.9977

Table 195. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	1	1	1	2^2	1	3^2	**0.6123**	0.1189	0.9992
30	30	30	1	1	1	2.25^2	1	3.5^2	**0.6699**	0.1395	1
30	30	30	1	1	1	2.5^2	1	3.75^2	**0.6964**	0.1505	1
30	30	30	1	1	1	2.75^2	1	4^2	**0.7242**	0.1555	1
30	30	30	1	1	1	3^2	1	4.25^2	**0.7379**	0.1605	1
30	30	30	1	1	1	3.25^2	1	4.5^2	**0.7485**	0.1728	1

Table 196. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	1	1	1	2^2	1	3^2	0.1557	0.0018	0.0854
30	30	30	1	1	1	2.25^2	1	3.5^2	0.1779	0.0021	0.0927
30	30	30	1	1	1	2.5^2	1	3.75^2	0.2079	0.0026	0.1034
30	30	30	1	1	1	2.75^2	1	4^2	0.2195	0.0013	0.1062
30	30	30	1	1	1	3^2	1	4.25^2	0.2373	0.0013	0.1192
30	30	30	1	1	1	3.25^2	1	4.5^2	0.2492	0.0016	0.1209

Table 197. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	1	1	1.5	1.5^2	1.75	1.75^2	0.5242	**0.6137**	0.3720
30	30	30	1	1	1.75	1.75^2	2	2^2	0.7225	**0.8147**	0.5937
30	30	30	1	1	2	2^2	2.25	2.25^2	0.8462	**0.9250**	0.7691
30	30	30	1	1	2.25	2.25^2	2.5	2.5^2	0.9189	**0.9714**	0.8851
30	30	30	1	1	2.5	2.5^2	2.75	2.75^2	0.9570	**0.9907**	0.9444
30	30	30	1	1	2.75	2.75^2	3	3^2	0.9821	**0.9965**	0.9722

Table 198. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	1	1	1.5	1.5^2	1.75	1.75^2	0.7211	0.6619	0.7264
30	30	30	1	1	1.75	1.75^2	2	2^2	0.8785	0.8389	0.8821
30	30	30	1	1	2	2^2	2.25	2.25^2	0.9572	0.9354	0.9602
30	30	30	1	1	2.25	2.25^2	2.5	2.5^2	0.9838	0.9746	0.9854
30	30	30	1	1	2.5	2.5^2	2.75	2.75^2	0.9936	0.9910	0.9940
30	30	30	1	1	2.75	2.75^2	3	3^2	0.9983	0.9972	0.9984

Table 199. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	9	9	1	1	1	1	1	1	0.0518	0.0480	0.0469
18	9	9	1	1	1.25	1	1.5	1	0.0895	**0.2270**	0.4023
18	9	9	1	1	1.5	1	1.75	1	0.1499	**0.4247**	0.7812
18	9	9	1	1	1.75	1	2	1	0.2682	**0.6339**	0.9515
18	9	9	1	1	2	1	2.25	1	0.4042	**0.7914**	0.9913
18	9	9	1	1	2.25	1	2.5	1	0.5456	**0.8959**	0.9989
18	9	9	1	1	2.5	1	2.75	1	0.6889	**0.9494**	1

Table 200. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	9	9	1	1	1	1	1	1	0.0955	0.0688	0.0907
18	9	9	1	1	1.25	1	1.5	1	0.3396	0.4828	0.3754
18	9	9	1	1	1.5	1	1.75	1	0.5120	0.7603	0.5736
18	9	9	1	1	1.75	1	2	1	0.6651	0.9122	0.7342
18	9	9	1	1	2	1	2.25	1	0.7627	0.9741	0.8331
18	9	9	1	1	2.25	1	2.5	1	0.8390	0.9924	0.9020
18	9	9	1	1	2.5	1	2.75	1	0.8905	0.9970	0.9423

Table 201. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	9	9	1	1	1	2^2	1	3^2	**0.3687**	0.1070	0.8659
18	9	9	1	1	1	2.25^2	1	3.5^2	**0.4114**	0.1127	0.9217
18	9	9	1	1	1	2.5^2	1	3.75^2	**0.4435**	0.1254	0.9448
18	9	9	1	1	1	2.75^2	1	4^2	**0.4583**	0.1261	0.9644
18	9	9	1	1	1	3^2	1	4.25^2	**0.4848**	0.1292	0.9759
18	9	9	1	1	1	3.25^2	1	4.5^2	**0.4911**	0.1436	0.9825

Table 202. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	9	9	1	1	1	2^2	1	3^2	0.1317	0.0162	0.0941
18	9	9	1	1	1	2.25^2	1	3.5^2	0.1390	0.0172	0.0979
18	9	9	1	1	1	2.5^2	1	3.75^2	0.1511	0.0170	0.1011
18	9	9	1	1	1	2.75^2	1	4^2	0.1618	0.0164	0.1098
18	9	9	1	1	1	3^2	1	4.25^2	0.1654	0.0140	0.1097
18	9	9	1	1	1	3.25^2	1	4.5^2	0.1741	0.0160	0.1202

Table 203. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	9	9	1	1	1.5	1.5^2	1.75	1.75^2	0.3162	**0.3564**	0.1767
18	9	9	1	1	1.75	1.75^2	2	2^2	0.4502	**0.5099**	0.2780
18	9	9	1	1	2	2^2	2.25	2.25^2	0.5685	**0.6432**	0.3894
18	9	9	1	1	2.25	2.25^2	2.5	2.5^2	0.6672	**0.7383**	0.4957
18	9	9	1	1	2.5	2.5^2	2.75	2.75^2	0.7510	**0.8169**	0.5936
18	9	9	1	1	2.75	2.75^2	3	3^2	0.8103	**0.8745**	0.6747

Table 204. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	9	9	1	1	1.5	1.5^2	1.75	1.75^2	0.5579	0.4852	0.5639
18	9	9	1	1	1.75	1.75^2	2	2^2	0.7269	0.6616	0.7332
18	9	9	1	1	2	2^2	2.25	2.25^2	0.8440	0.7940	0.8495
18	9	9	1	1	2.25	2.25^2	2.5	2.5^2	0.9084	0.8753	0.9127
18	9	9	1	1	2.5	2.5^2	2.75	2.75^2	0.9547	0.9316	0.9568
18	9	9	1	1	2.75	2.75^2	3	3^2	0.9751	0.9633	0.9766

Table 205. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	30	30	1	1	1	1	1	1	0.0442	0.0448	0.0426
18	30	30	1	1	1.25	1	1.5	1	0.0457	**0.2488**	0.6218
18	30	30	1	1	1.5	1	1.75	1	0.0439	**0.4584**	0.9183
18	30	30	1	1	1.75	1	2	1	0.0445	**0.663**	0.9881
18	30	30	1	1	2	1	2.25	1	0.0529	**0.8061**	0.9992
18	30	30	1	1	2.25	1	2.5	1	0.0513	**0.8985**	1
18	30	30	1	1	2.5	1	2.75	1	0.0463	**0.9535**	1

Table 206. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	30	30	1	1	1	1	1	1	0.0914	0.0632	0.0841
18	30	30	1	1	1.25	1	1.5	1	0.4312	0.6054	0.5243
18	30	30	1	1	1.5	1	1.75	1	0.6347	0.8656	0.7656
18	30	30	1	1	1.75	1	2	1	0.7740	0.9582	0.8920
18	30	30	1	1	2	1	2.25	1	0.8618	0.9905	0.9539
18	30	30	1	1	2.25	1	2.5	1	0.9228	0.9981	0.9846
18	30	30	1	1	2.5	1	2.75	1	0.9587	0.9995	0.9943

Table 207. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	30	30	1	1	1	2^2	1	3^2	**0.4637**	0.0789	0.9906
18	30	30	1	1	1	2.25^2	1	3.5^2	**0.5092**	0.0856	0.9985
18	30	30	1	1	1	2.5^2	1	3.75^2	**0.5337**	0.0934	0.9997
18	30	30	1	1	1	2.75^2	1	4^2	**0.5555**	0.0959	0.9999
18	30	30	1	1	1	3^2	1	4.25^2	**0.5775**	0.1026	0.9997
18	30	30	1	1	1	3.25^2	1	4.5^2	**0.5784**	0.1064	1

Table 208. Percentage of Rejection for k=3 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	30	30	1	1	1	2^2	1	3^2	0.1074	0.0014	0.0252
18	30	30	1	1	1	2.25^2	1	3.5^2	0.1189	0.0010	0.0229
18	30	30	1	1	1	2.5^2	1	3.75^2	0.1285	0.0008	0.0221
18	30	30	1	1	1	2.75^2	1	4^2	0.1424	0.0008	0.0224
18	30	30	1	1	1	3^2	1	4.25^2	0.1469	0.0007	0.0272
18	30	30	1	1	1	3.25^2	1	4.5^2	0.1529	0.0007	0.0233

Table 209. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	30	30	1	1	1.5	1.5^2	1.75	1.75^2	0.3699	**0.4663**	0.2520
18	30	30	1	1	1.75	1.75^2	2	2^2	0.5191	**0.6554**	0.4033
18	30	30	1	1	2	2^2	2.25	2.25^2	0.6490	**0.7945**	0.5559
18	30	30	1	1	2.25	2.25^2	2.5	2.5^2	0.7423	**0.8856**	0.6929
18	30	30	1	1	2.5	2.5^2	2.75	2.75^2	0.8247	**0.9459**	0.8032
18	30	30	1	1	2.75	2.75^2	3	3^2	0.8698	**0.9726**	0.8701

Table 210. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	30	30	1	1	1.5	1.5^2	1.75	1.75^2	0.6136	0.5319	0.6052
18	30	30	1	1	1.75	1.75^2	2	2^2	0.7775	0.7222	0.7790
18	30	30	1	1	2	2^2	2.25	2.25^2	0.8889	0.8469	0.8897
18	30	30	1	1	2.25	2.25^2	2.5	2.5^2	0.9450	0.9212	0.9466
18	30	30	1	1	2.5	2.5^2	2.75	2.75^2	0.9760	0.9625	0.9772
18	30	30	1	1	2.75	2.75^2	3	3^2	0.9876	0.9811	0.9892

5.3.3. Four Treatments

Tables 211-228 outline the results of simulation study for three treatments under the exponential distribution. The sample size considered in the simulations are 9, 18, 30. It appeared that both of L_1 and L_2 maintained their alpha values. The estimated alpha values were around 0.05 Tables (211, 217 and 223). On the other hand, it is noted that both of M_1, M_2 and M_3 didn't maintained their alpha values Tables (212, 218 and 224). As a result, the results for M_1, M_2 and M_3 listed for completion purposes not to be compared with L_1 and L_2.

When the populations have unequal location parameters and equal scale parameters, L_2 test has higher estimated power than L_1 Tables (211, 217, and 223). When the populations have equal location parameters and unequal scale parameters, L_1 test has higher estimated power than L_2 Tables (213, 219 and 225). When the populations have unequal location parameters and unequal scale parameters, L_2 test tends to have the highest estimated power Tables (215, 221 and 227). Tables 229-234 show the simulations where unequal sample sizes were considered. A similar result was found when investigating unequal sample cases.

Table 211. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	1	1	1	1	1	1	1	1	0.0441	0.0483	0.0511
9	9	9	9	1	1	1.25	1	1.5	1	1.75	1	0.0390	**0.2215**	0.4625
9	9	9	9	1	1	1.5	1	1.75	1	2	1	0.0232	**0.3432**	0.7429
9	9	9	9	1	1	1.75	1	2	1	2.25	1	0.0153	**0.4738**	0.8911
9	9	9	9	1	1	2	1	2.25	1	2.5	1	0.0083	**0.5990**	0.9612
9	9	9	9	1	1	2.25	1	2.5	1	2.75	1	0.0050	**0.6895**	0.9840
9	9	9	9	1	1	2.5	1	2.75	1	3	1	0.0020	**0.7669**	0.9951

Table 212. Percentage of Rejection for k=4 Populations; Exponential Distribution with different mean and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	1	1	1	1	1	1	1	1	0.0928	0.0704	0.0877
9	9	9	9	1	1	1.25	1	1.5	1	1.75	1	0.3719	0.5222	0.4491
9	9	9	9	1	1	1.5	1	1.75	1	2	1	0.4968	0.7153	0.6081
9	9	9	9	1	1	1.75	1	2	1	2.25	1	0.6026	0.8512	0.7330
9	9	9	9	1	1	2	1	2.25	1	2.5	1	0.6890	0.9243	0.8234
9	9	9	9	1	1	2.25	1	2.5	1	2.75	1	0.7577	0.9674	0.8826
9	9	9	9	1	1	2.5	1	2.75	1	3	1	0.8013	0.9822	0.9217

Table 213. Percentage of Rejection for k=4 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	1	1	1	1.5^2	1	2^2	1	2.5^2	**0.2173**	0.0462	0.4765
9	9	9	9	1	1	1	2^2	1	2.5^2	1	3^2	**0.2695**	0.0549	0.7192
9	9	9	9	1	1	1	2.5^2	1	3^2	1	3.5^2	**0.3141**	0.0548	0.8638
9	9	9	9	1	1	1	3^2	1	3.5^2	1	4^2	**0.3397**	0.0676	0.9262
9	9	9	9	1	1	1	3.5^2	1	4^2	1	4.5^2	**0.3660**	0.0697	0.9551
9	9	9	9	1	1	1	4^2	1	4.5^2	1	5^2	**0.3952**	0.0787	0.9764

Table 214. Percentage of Rejection for k=4 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	1	1	1	1.5^2	1	2^2	1	2.5^2	0.0746	0.0073	0.0542
9	9	9	9	1	1	1	2^2	1	2.5^2	1	3^2	0.0738	0.0037	0.0487
9	9	9	9	1	1	1	2.5^2	1	3^2	1	3.5^2	0.0788	0.0033	0.0473
9	9	9	9	1	1	1	3^2	1	3.5^2	1	4^2	0.0803	0.0025	0.0462
9	9	9	9	1	1	1	3.5^2	1	4^2	1	4.5^2	0.0869	0.0027	0.0524
9	9	9	9	1	1	1	4^2	1	4.5^2	1	5^2	0.0848	0.0026	0.0497

Table 215. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	1	1	1.25	1.25^2	1.5	1.5^2	1.75	1.75^2	0.1611	**0.2014**	0.0880
9	9	9	9	1	1	1.5	1.5^2	1.75	1.75^2	2	2^2	0.2289	**0.3070**	0.1348
9	9	9	9	1	1	1.75	1.75^2	2	2^2	2.25	2.25^2	0.2978	**0.4237**	0.2064
9	9	9	9	1	1	2	2^2	2.25	2.25^2	2.5	2.5^2	0.3590	**0.5240**	0.2617
9	9	9	9	1	1	2.25	2.25^2	2.5	2.5^2	2.75	2.75^2	0.4187	**0.6207**	0.3383
9	9	9	9	1	1	2.5	2.5^2	2.75	2.75^2	3	3^2	0.4707	**0.7057**	0.4137

Table 216. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	1	1	1.25	1.25^2	1.5	1.5^2	1.75	1.75^2	0.3323	0.2744	0.3388
9	9	9	9	1	1	1.5	1.5^2	1.75	1.75^2	2	2^2	0.4593	0.3904	0.4643
9	9	9	9	1	1	1.75	1.75^2	2	2^2	2.25	2.25^2	0.5955	0.5276	0.6049
9	9	9	9	1	1	2	2^2	2.25	2.25^2	2.5	2.5^2	0.6913	0.6305	0.7025
9	9	9	9	1	1	2.25	2.25^2	2.5	2.5^2	2.75	2.75^2	0.7796	0.7253	0.7881
9	9	9	9	1	1	2.5	2.5^2	2.75	2.75^2	3	3^2	0.8405	0.7947	0.8464

Table 217. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	1	1	1	1	1	1	1	1	0.0461	0.0478	0.0489
18	18	18	18	1	1	1.25	1	1.5	1	1.75	1	0.0609	**0.3698**	0.7801
18	18	18	18	1	1	1.5	1	1.75	1	2	1	0.0657	**0.5833**	0.9598
18	18	18	18	1	1	1.75	1	2	1	2.25	1	0.0840	**0.7540**	0.9945
18	18	18	18	1	1	2	1	2.25	1	2.5	1	0.0967	**0.8640**	0.9988
18	18	18	18	1	1	2.25	1	2.5	1	2.75	1	0.1122	**0.9337**	0.9998
18	18	18	18	1	1	2.5	1	2.75	1	3	1	0.1192	**0.9681**	1

Table 218. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	1	1	1	1	1	1	1	1	0.0908	0.0628	0.0840
18	18	18	18	1	1	1.25	1	1.5	1	1.75	1	0.5196	0.7407	0.6320
18	18	18	18	1	1	1.5	1	1.75	1	2	1	0.7011	0.9210	0.8313
18	18	18	18	1	1	1.75	1	2	1	2.25	1	0.8162	0.9802	0.9248
18	18	18	18	1	1	2	1	2.25	1	2.5	1	0.8895	0.9956	0.9679
18	18	18	18	1	1	2.25	1	2.5	1	2.75	1	0.9356	0.9991	0.9885
18	18	18	18	1	1	2.5	1	2.75	1	3	1	0.9640	0.9997	0.9961

Table 219. Percentage of Rejection for k=4 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	1	1	1	1.5^2	1	2^2	1	2.5^2	**0.3698**	0.0726	0.9155
18	18	18	18	1	1	1	2^2	1	2.5^2	1	3^2	**0.4671**	0.0850	0.9917
18	18	18	18	1	1	1	2.5^2	1	3^2	1	3.5^2	**0.5325**	0.0952	0.9983
18	18	18	18	1	1	1	3^2	1	3.5^2	1	4^2	**0.5663**	0.1067	0.9999
18	18	18	18	1	1	1	3.5^2	1	4^2	1	4.5^2	**0.5818**	0.1143	0.9999
18	18	18	18	1	1	1	4^2	1	4.5^2	1	5^2	**0.6082**	0.1298	1

Table 220. Percentage of Rejection for k=4 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	1	1	1	1.5^2	1	2^2	1	2.5^2	0.0893	0.0032	0.0545
18	18	18	18	1	1	1	2^2	1	2.5^2	1	3^2	0.1131	0.0017	0.0604
18	18	18	18	1	1	1	2.5^2	1	3^2	1	3.5^2	0.1349	0.0013	0.0668
18	18	18	18	1	1	1	3^2	1	3.5^2	1	4^2	0.1459	0.0009	0.0693
18	18	18	18	1	1	1	3.5^2	1	4^2	1	4.5^2	0.1625	0.0003	0.0754
18	18	18	18	1	1	1	4^2	1	4.5^2	1	5^2	0.1777	0.0003	0.0766

Table 221. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	1	1	1.25	1.25^2	1.5	1.5^2	1.75	1.75^2	0.2858	**0.3431**	0.1672
18	18	18	18	1	1	1.5	1.5^2	1.75	1.75^2	2	2^2	0.4365	**0.5461**	0.2965
18	18	18	18	1	1	1.75	1.75^2	2	2^2	2.25	2.25^2	0.5722	**0.7088**	0.4496
18	18	18	18	1	1	2	2^2	2.25	2.25^2	2.5	2.5^2	0.6948	**0.8365**	0.6085
18	18	18	18	1	1	2.25	2.25^2	2.5	2.5^2	2.75	2.75^2	0.7820	**0.9109**	0.7321
18	18	18	18	1	1	2.5	2.5^2	2.75	2.75^2	3	3^2	0.8466	**0.9571**	0.8304

Table 222. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	1	1	1.25	1.25^2	1.5	1.5^2	1.75	1.75^2	0.4812	0.4072	0.4825
18	18	18	18	1	1	1.5	1.5^2	1.75	1.75^2	2	2^2	0.6812	0.6147	0.6824
18	18	18	18	1	1	1.75	1.75^2	2	2^2	2.25	2.25^2	0.8294	0.7734	0.8349
18	18	18	18	1	1	2	2^2	2.25	2.25^2	2.5	2.5^2	0.9141	0.8838	0.9175
18	18	18	18	1	1	2.25	2.25^2	2.5	2.5^2	2.75	2.75^2	0.9595	0.9413	0.9631
18	18	18	18	1	1	2.5	2.5^2	2.75	2.75^2	3	3^2	0.9804	0.9702	0.9812

Table 223. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	1	1	1	1	1	1	1	1	0.0482	0.0445	0.0491
30	30	30	30	1	1	1.25	1	1.5	1	1.75	1	0.0785	**0.5233**	0.9440
30	30	30	30	1	1	1.5	1	1.75	1	2	1	0.0976	**0.7752**	0.9978
30	30	30	30	1	1	1.75	1	2	1	2.25	1	0.1281	**0.9112**	1
30	30	30	30	1	1	2	1	2.25	1	2.5	1	0.1697	**0.9734**	1
30	30	30	30	1	1	2.25	1	2.5	1	2.75	1	0.2335	**0.9927**	1
30	30	30	30	1	1	2.5	1	2.75	1	3	1	0.3065	**0.9985**	1

Table 224. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	1	1	1	1	1	1	1	1	0.0929	0.0667	0.0857
30	30	30	30	1	1	1.25	1	1.5	1	1.75	1	0.6651	0.8910	0.7938
30	30	30	30	1	1	1.5	1	1.75	1	2	1	0.8486	0.9856	0.9446
30	30	30	30	1	1	1.75	1	2	1	2.25	1	0.9393	0.9987	0.9869
30	30	30	30	1	1	2	1	2.25	1	2.5	1	0.9781	1	0.9980
30	30	30	30	1	1	2.25	1	2.5	1	2.75	1	0.9920	1	0.9998
30	30	30	30	1	1	2.5	1	2.75	1	3	1	0.9967	1	1

Table 225. Percentage of Rejection for k=4 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	1	1	1	1.5^2	1	2^2	1	2.5^2	**0.5260**	0.0873	0.9956
30	30	30	30	1	1	1	2^2	1	2.5^2	1	3^2	**0.6508**	0.1043	1
30	30	30	30	1	1	1	2.5^2	1	3^2	1	3.5^2	**0.7166**	0.1285	1
30	30	30	30	1	1	1	3^2	1	3.5^2	1	4^2	**0.7453**	0.1464	1
30	30	30	30	1	1	1	3.5^2	1	4^2	1	4.5^2	**0.7717**	0.1576	1
30	30	30	30	1	1	1	4^2	1	4.5^2	1	5^2	**0.7774**	0.1684	1

Table 226. Percentage of Rejection for k=4 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	1	1	1	1.5^2	1	2^2	1	2.5^2	0.1112	0.0007	0.0601
30	30	30	30	1	1	1	2^2	1	2.5^2	1	3^2	0.1497	0.0005	0.0693
30	30	30	30	1	1	1	2.5^2	1	3^2	1	3.5^2	0.1910	0.0003	0.0823
30	30	30	30	1	1	1	3^2	1	3.5^2	1	4^2	0.2261	0.0005	0.0923
30	30	30	30	1	1	1	3.5^2	1	4^2	1	4.5^2	0.2649	0.0001	0.1085
30	30	30	30	1	1	1	4^2	1	4.5^2	1	5^2	0.2947	0.0003	0.1149

Table 227. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	1	1	1.25	1.25^2	1.5	1.5^2	1.75	1.75^2	0.4234	**0.5078**	0.2783
30	30	30	30	1	1	1.5	1.5^2	1.75	1.75^2	2	2^2	0.6560	**0.7644**	0.5222
30	30	30	30	1	1	1.75	1.75^2	2	2^2	2.25	2.25^2	0.8082	**0.9102**	0.7368
30	30	30	30	1	1	2	2^2	2.25	2.25^2	2.5	2.5^2	0.9090	**0.9704**	0.8796
30	30	30	30	1	1	2.25	2.25^2	2.5	2.5^2	2.75	2.75^2	0.9549	**0.9904**	0.9448
30	30	30	30	1	1	2.5	2.5^2	2.75	2.75^2	3	3^2	0.9817	**0.9970**	0.9807

Table 228. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	1	1	1.25	1.25^2	1.5	1.5^2	1.75	1.75^2	0.6347	0.5655	0.6412
30	30	30	30	1	1	1.5	1.5^2	1.75	1.75^2	2	2^2	0.8483	0.8012	0.8527
30	30	30	30	1	1	1.75	1.75^2	2	2^2	2.25	2.25^2	0.9559	0.9305	0.9564
30	30	30	30	1	1	2	2^2	2.25	2.25^2	2.5	2.5^2	0.9853	0.9794	0.9869
30	30	30	30	1	1	2.25	2.25^2	2.5	2.5^2	2.75	2.75^2	0.9967	0.9944	0.9975
30	30	30	30	1	1	2.5	2.5^2	2.75	2.75^2	3	3^2	0.9991	0.9980	0.9991

Table 229. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	9	9	9	1	1	1	1	1	1	1	1	0.0469	0.0468	0.0447
18	9	9	9	1	1	1.25	1	1.5	1	1.75	1	0.1001	**0.3491**	0.6638
18	9	9	9	1	1	1.5	1	1.75	1	2	1	0.1538	**0.5566**	0.9265
18	9	9	9	1	1	1.75	1	2	1	2.25	1	0.2392	**0.7432**	0.9885
18	9	9	9	1	1	2	1	2.25	1	2.5	1	0.3486	**0.8618**	0.9980
18	9	9	9	1	1	2.25	1	2.5	1	2.75	1	0.4762	**0.9342**	0.9997
18	9	9	9	1	1	2.5	1	2.75	1	3	1	0.5787	**0.9671**	1

Table 230. Percentage of Rejection for k=3 Populations; Exponential Distribution with different mean and equal variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	9	9	9	1	1	1	1	1	1	1	1	0.0961	0.0662	0.0871
18	9	9	9	1	1	1.25	1	1.5	1	1.75	1	0.4666	0.6760	0.5782
18	9	9	9	1	1	1.5	1	1.75	1	2	1	0.6406	0.8818	0.7764
18	9	9	9	1	1	1.75	1	2	1	2.25	1	0.7727	0.9669	0.8914
18	9	9	9	1	1	2	1	2.25	1	2.5	1	0.8457	0.9896	0.9476
18	9	9	9	1	1	2.25	1	2.5	1	2.75	1	0.8993	0.9979	0.9764
18	9	9	9	1	1	2.5	1	2.75	1	3	1	0.9363	0.9997	0.9905

Table 231. Percentage of Rejection for k=4 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	9	9	9	1	1	1	1.5^2	1	2^2	1	2.5^2	**0.3177**	0.0835	0.7908
18	9	9	9	1	1	1	2^2	1	2.5^2	1	3^2	**0.4202**	0.0994	0.9391
18	9	9	9	1	1	1	2.5^2	1	3^2	1	3.5^2	**0.4884**	0.1179	0.9812
18	9	9	9	1	1	1	3^2	1	3.5^2	1	4^2	**0.5327**	0.1278	0.9936
18	9	9	9	1	1	1	3.5^2	1	4^2	1	4.5^2	**0.5610**	0.1408	0.9971
18	9	9	9	1	1	1	4^2	1	4.5^2	1	5^2	**0.5835**	0.1506	0.9991

Table 232. Percentage of Rejection for k=2 Populations; Exponential Distribution with same means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	9	9	9	1	1	1	1.5^2	1	2^2	1	2.5^2	0.1056	0.0101	0.0724
18	9	9	9	1	1	1	2^2	1	2.5^2	1	3^2	0.1272	0.0079	0.0847
18	9	9	9	1	1	1	2.5^2	1	3^2	1	3.5^2	0.1415	0.0076	0.0879
18	9	9	9	1	1	1	3^2	1	3.5^2	1	4^2	0.1563	0.0072	0.0988
18	9	9	9	1	1	1	3.5^2	1	4^2	1	4.5^2	0.1771	0.0057	0.1059
18	9	9	9	1	1	1	4^2	1	4.5^2	1	5^2	0.1889	0.0072	0.1129

Table 233. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	9	9	9	1	1	1.25	1.25^2	1.5	1.5^2	1.75	1.75^2	0.2690	**0.3131**	0.1461
18	9	9	9	1	1	1.5	1.5^2	1.75	1.75^2	2	2^2	0.4073	**0.4835**	0.2588
18	9	9	9	1	1	1.75	1.75^2	2	2^2	2.25	2.25^2	0.5409	**0.6322**	0.3825
18	9	9	9	1	1	2	2^2	2.25	2.25^2	2.5	2.5^2	0.6618	**0.7678**	0.5120
18	9	9	9	1	1	2.25	2.25^2	2.5	2.5^2	2.75	2.75^2	0.7518	**0.8504**	0.6254
18	9	9	9	1	1	2.5	2.5^2	2.75	2.75^2	3	3^2	0.8218	**0.9027**	0.7215

Table 234. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	9	9	9	1	1	1.25	1.25^2	1.5	1.5^2	1.75	1.75^2	0.4189	0.3550	0.4185
18	9	9	9	1	1	1.5	1.5^2	1.75	1.75^2	2	2^2	0.6056	0.5333	0.6075
18	9	9	9	1	1	1.75	1.75^2	2	2^2	2.25	2.25^2	0.7551	0.6844	0.7562
18	9	9	9	1	1	2	2^2	2.25	2.25^2	2.5	2.5^2	0.8486	0.7953	0.8529
18	9	9	9	1	1	2.25	2.25^2	2.5	2.5^2	2.75	2.75^2	0.9091	0.8729	0.9125
18	9	9	9	1	1	2.5	2.5^2	2.75	2.75^2	3	3^2	0.9440	0.9173	0.9455

CHAPTER 6. CONCLUSION

To sum up, this research proposed five nonparametric tests L_1, L_2, M_1, M_2, and M_3 for the simple tree alternative. The sample sizes considered in this research are 9, 18, and 30. Unequal sample sizes were also considered. The subsample size used in M_1, M_2, and M_3 tests is 3. This research is intended to compare the powers of the proposed tests assuming random samples following normal distributions, t-distributions with 3 degrees of freedom and exponential distributions.

For the symmetric distributions considered (Normal and T), all the proposed tests maintained their alpha values. When the difference was only in treatment means, this research showed that M_2 test was more powerful than the other tests. When the difference was only in treatment variances, this research showed that L_1 test was more powerful than the other tests. When the difference was in both treatment means and variances, this research presented that L_1 was more powerful than the other tests.

For the asymmetric distribution considered (Exponential), the proposed tests L_1, L_2 maintained their alpha values well. On the other hand, M_1, M_2, and M_3 tests did not maintain their alpha values. This is because the Mann-Whitney test and Moses test are not independent when distributions are not symmetric. In this situation, the comparison has been only made between L_1 and L_2 tests. When the difference was only in treatment means, L_2 test had powers much higher than L_1 test. For instance, when the location parameters were 1, 1.75, 2 in Table 187, the power of the L_2 test was 0.6531 while the power of the L_1 test was 0.1258. When the difference is only in treatment variances, the L_1 test had powers far higher than the L_2 test. For example, when the scale parameters were 1, 3.25^2, 4.5^2 in Table 195, the power of the L_1 test was 0.7485 while the power

of the L_2 test was 0.1728. When the difference was in both treatment means and variances, L_2 test was more powerful than L_1 test.

If the distribution that one is sampling from is assumed to be approximately symmetric, L_1 is recommended to test for both an increasing change in the location and/or scale when treatments are applied. L_1 did have lower powers if only the locations (means) were different, but did have higher powers in the other two cases. If one expects the underlying distribution to be relatively skewed, then L_2 is the recommended test statistic to test for both increasing changes in the location and scale when treatments are applied.

Future work could extend the comparison between the proposed tests using another asymmetric distribution with less skewness than the exponential distribution. Also, this research could be extended to another alternative such as the umbrella alternative and non-decreasing alternative.

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APPENDIX A. PERCENTAGE OF REJECTION FOR K=3 POPULATIONS (NORMAL DISTRIBUTION)

Table A1. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	0	1	0	1	0.75	1	0.1430	**0.1801**	0.0804
9	9	9	0	1	0	1	1	1	0.2043	**0.2552**	0.1091
9	9	9	0	1	0	1	1.25	1	0.2864	**0.3603**	0.1544
9	9	9	0	1	0	1	1.5	1	0.3706	**0.4589**	0.1822
9	9	9	0	1	0	1	1.75	1	0.4808	**0.5822**	0.2411
9	9	9	0	1	0	1	2	1	0.5737	**0.6775**	0.2902

Table A2. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	0	1	0	1	0.75	1	0.1356	0.1966	0.1633
9	9	9	0	1	0	1	1	1	0.1865	0.2708	0.2268
9	9	9	0	1	0	1	1.25	1	0.2305	0.3480	0.2883
9	9	9	0	1	0	1	1.5	1	0.2633	0.4153	0.3385
9	9	9	0	1	0	1	1.75	1	0.3092	0.4951	0.3957
9	9	9	0	1	0	1	2	1	0.3215	0.5504	0.4238

Table A3. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	0	1	0	1	0	2	**0.1257**	0.0808	0.0641
9	9	9	0	1	0	1	0	2.5	**0.1619**	0.0939	0.0849
9	9	9	0	1	0	1	0	3	**0.196**0	0.1118	0.1009
9	9	9	0	1	0	1	0	3.5	**0.2124**	0.1147	0.1192
9	9	9	0	1	0	1	0	4	**0.2249**	0.1175	0.1265
9	9	9	0	1	0	1	0	4.5	**0.2492**	0.1286	0.1434

Table A4. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	0	1	0	1	0	2	0.1049	0.0550	0.0814
9	9	9	0	1	0	1	0	2.5	0.1068	0.0538	0.0850
9	9	9	0	1	0	1	0	3	0.1080	0.0541	0.0833
9	9	9	0	1	0	1	0	3.5	0.1182	0.0564	0.0902
9	9	9	0	1	0	1	0	4	0.1285	0.0586	0.0978
9	9	9	0	1	0	1	0	4.5	0.1259	0.0607	0.0957

Table A5. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	0	1	0	1	0.25	2	**0.1587**	0.1103	0.0686
9	9	9	0	1	0	1	0.5	2.5	**0.2184**	0.1530	0.0839
9	9	9	0	1	0	1	0.75	3	**0.2746**	0.1825	0.1099
9	9	9	0	1	0	1	1	3.5	**0.3198**	0.2165	0.1265
9	9	9	0	1	0	1	1.25	4	**0.3523**	0.2448	0.1439
9	9	9	0	1	0	1	1.5	4.5	**0.3893**	0.2614	0.1635

Table A6. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	0	1	0	1	0.25	2	0.1217	0.0791	0.1030
9	9	9	0	1	0	1	0.5	2.5	0.1633	0.1042	0.1401
9	9	9	0	1	0	1	0.75	3	0.1957	0.1196	0.1677
9	9	9	0	1	0	1	1	3.5	0.2158	0.1317	0.1887
9	9	9	0	1	0	1	1.25	4	0.2299	0.1440	0.2017
9	9	9	0	1	0	1	1.5	4.5	0.2526	0.1483	0.2166

Table A7. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	0	1	0.5	1	0.5	1	0.1643	0.2501	0.1434
9	9	9	0	1	0.75	1	0.75	1	0.2405	0.4168	0.2839
9	9	9	0	1	1	1	1	1	0.3236	0.6038	0.4911
9	9	9	0	1	1.25	1	1.25	1	0.4176	0.7708	0.6964
9	9	9	0	1	1.5	1	1.5	1	0.4895	0.8865	0.8558
9	9	9	0	1	1.75	1	1.75	1	0.5215	0.9506	0.9445

Table A8. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	0	1	0.5	1	0.5	1	0.2123	**0.3109**	0.2642
9	9	9	0	1	0.75	1	0.75	1	0.3244	**0.5147**	0.4197
9	9	9	0	1	1	1	1	1	0.4597	**0.7279**	0.6013
9	9	9	0	1	1.25	1	1.25	1	0.5879	**0.8804**	0.7604
9	9	9	0	1	1.5	1	1.5	1	0.6954	**0.9532**	0.8643
9	9	9	0	1	1.75	1	1.75	1	0.7772	**0.987**0	0.9310

Table A9. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	0	1	0	2	0	2	**0.316**0	0.1616	0.2244
9	9	9	0	1	0	2.5	0	2.5	**0.4427**	0.2116	0.3929
9	9	9	0	1	0	3	0	3	**0.5378**	0.2635	0.5421
9	9	9	0	1	0	3.5	0	3.5	**0.6118**	0.3012	0.6666
9	9	9	0	1	0	4	0	4	**0.6687**	0.3410	0.7592
9	9	9	0	1	0	4.5	0	4.5	**0.7004**	0.3627	0.8248

Table A10. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	0	1	0	2	0	2	0.2120	0.0669	0.1448
9	9	9	0	1	0	2.5	0	2.5	0.2831	0.0699	0.1724
9	9	9	0	1	0	3	0	3	0.3404	0.0797	0.2063
9	9	9	0	1	0	3.5	0	3.5	0.3595	0.0829	0.2139
9	9	9	0	1	0	4	0	4	0.3944	0.0854	0.2337
9	9	9	0	1	0	4.5	0	4.5	0.4177	0.0911	0.2479

Table A11. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	0	1	0.5	1.5	0.5	1.5	**0.3427**	0.3192	0.1295
9	9	9	0	1	0.75	1.75	0.75	1.75	**0.5513**	0.5141	0.2368
9	9	9	0	1	1	2	1	2	**0.6992**	0.6573	0.3348
9	9	9	0	1	1.25	2.25	1.25	2.25	**0.8144**	0.7822	0.4775
9	9	9	0	1	1.5	2.5	1.5	2.5	**0.8858**	0.8571	0.5689
9	9	9	0	1	1.75	2.75	1.75	2.75	**0.9258**	0.8999	0.6641

Table A12. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
9	9	9	0	1	0.5	1.5	0.5	1.5	0.3491	0.2759	0.3336
9	9	9	0	1	0.75	1.75	0.75	1.75	0.5416	0.4168	0.5143
9	9	9	0	1	1	2	1	2	0.6917	0.5366	0.6520
9	9	9	0	1	1.25	2.25	1.25	2.25	0.8045	0.6362	0.7690
9	9	9	0	1	1.5	2.5	1.5	2.5	0.8720	0.7049	0.8379
9	9	9	0	1	1.75	2.75	1.75	2.75	0.9116	0.7598	0.8803

Table A13. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	0	1	0	1	0.75	1	0.2369	**0.3085**	0.1572
18	18	18	0	1	0	1	1	1	0.3533	**0.4465**	0.2334
18	18	18	0	1	0	1	1.25	1	0.5103	**0.6157**	0.3626
18	18	18	0	1	0	1	1.5	1	0.6586	**0.768**0	0.4863
18	18	18	0	1	0	1	1.75	1	0.7997	**0.8841**	0.6285
18	18	18	0	1	0	1	2	1	0.8944	**0.9505**	0.7529

Table A14. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	0	1	0	1	0.75	1	0.2055	0.3075	0.2630
18	18	18	0	1	0	1	1	1	0.2889	0.4498	0.3734
18	18	18	0	1	0	1	1.25	1	0.3701	0.5887	0.4938
18	18	18	0	1	0	1	1.5	1	0.4469	0.7123	0.6002
18	18	18	0	1	0	1	1.75	1	0.5078	0.7913	0.6701
18	18	18	0	1	0	1	2	1	0.5530	0.8622	0.7319

Table A15. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	0	1	0	1	0	2	**0.1932**	0.1099	0.1329
18	18	18	0	1	0	1	0	2.5	**0.2631**	0.1372	0.2033
18	18	18	0	1	0	1	0	3	**0.3287**	0.1684	0.2774
18	18	18	0	1	0	1	0	3.5	**0.3745**	0.1893	0.3299
18	18	18	0	1	0	1	0	4	**0.4188**	0.2078	0.3878
18	18	18	0	1	0	1	0	4.5	**0.4483**	0.2174	0.4285

Table A16. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	0	1	0	1	0	2	0.1498	0.0594	0.1094
18	18	18	0	1	0	1	0	2.5	0.1808	0.0651	0.1297
18	18	18	0	1	0	1	0	3	0.2079	0.0647	0.1383
18	18	18	0	1	0	1	0	3.5	0.2227	0.0654	0.1415
18	18	18	0	1	0	1	0	4	0.2337	0.0663	0.1564
18	18	18	0	1	0	1	0	4.5	0.2512	0.0726	0.1622

Table A17. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	0	1	0	1	0.25	2	**0.2485**	0.1647	0.1364
18	18	18	0	1	0	1	0.5	2.5	**0.3862**	0.2611	0.2194
18	18	18	0	1	0	1	0.75	3	**0.4988**	0.3437	0.3040
18	18	18	0	1	0	1	1	3.5	**0.586**0	0.3982	0.3693
18	18	18	0	1	0	1	1.25	4	**0.6512**	0.4548	0.4236
18	18	18	0	1	0	1	1.5	4.5	**0.7014**	0.4981	0.4830

Table A18. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	0	1	0	1	0.25	2	0.2029	0.1027	0.1660
18	18	18	0	1	0	1	0.5	2.5	0.2932	0.1441	0.2341
18	18	18	0	1	0	1	0.75	3	0.3678	0.1829	0.2987
18	18	18	0	1	0	1	1	3.5	0.4228	0.2102	0.3452
18	18	18	0	1	0	1	1.25	4	0.4523	0.2269	0.3739
18	18	18	0	1	0	1	1.5	4.5	0.4898	0.2534	0.4070

Table A19. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	0	1	0.5	1	0.5	1	0.2586	0.4101	0.2903
18	18	18	0	1	0.75	1	0.75	1	0.4320	0.6873	0.5784
18	18	18	0	1	1	1	1	1	0.6125	0.8759	0.8373
18	18	18	0	1	1.25	1	1.25	1	0.7427	0.9705	0.9664
18	18	18	0	1	1.5	1	1.5	1	0.8444	0.9942	0.9953
18	18	18	0	1	1.75	1	1.75	1	0.9154	0.9995	0.9995

Table A20. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	0	1	0.5	1	0.5	1	0.3225	**0.4948**	0.4223
18	18	18	0	1	0.75	1	0.75	1	0.5227	**0.7834**	0.6785
18	18	18	0	1	1	1	1	1	0.7164	**0.9457**	0.8722
18	18	18	0	1	1.25	1	1.25	1	0.8515	**0.9905**	0.9599
18	18	18	0	1	1.5	1	1.5	1	0.9308	**0.9982**	0.9901
18	18	18	0	1	1.75	1	1.75	1	0.9731	**1**	0.9987

Table A21. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	0	1	0	2	0	2	**0.5369**	0.2682	0.5646
18	18	18	0	1	0	2.5	0	2.5	**0.7118**	0.3798	0.8148
18	18	18	0	1	0	3	0	3	**0.8264**	0.4665	0.9367
18	18	18	0	1	0	3.5	0	3.5	**0.8766**	0.5325	0.9801
18	18	18	0	1	0	4	0	4	**0.9101**	0.5745	0.9931
18	18	18	0	1	0	4.5	0	4.5	**0.93**02	0.6208	0.9978

Table A22. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	0	1	0	2	0	2	0.4179	0.0869	0.2525
18	18	18	0	1	0	2.5	0	2.5	0.5502	0.0958	0.3191
18	18	18	0	1	0	3	0	3	0.6433	0.1054	0.3772
18	18	18	0	1	0	3.5	0	3.5	0.7047	0.1110	0.4125
18	18	18	0	1	0	4	0	4	0.7484	0.1156	0.4388
18	18	18	0	1	0	4.5	0	4.5	0.7656	0.1225	0.4545

Table A23. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	0	1	0.5	1.5	0.5	1.5	**0.6169**	0.5644	0.3447
18	18	18	0	1	0.75	1.75	0.75	1.75	**0.8556**	0.8135	0.6152
18	18	18	0	1	1	2	1	2	**0.9598**	0.9339	0.8109
18	18	18	0	1	1.25	2.25	1.25	2.25	**0.9904**	0.9779	0.9204
18	18	18	0	1	1.5	2.5	1.5	2.5	**0.997**0	0.9914	0.9659
18	18	18	0	1	1.75	2.75	1.75	2.75	**0.9992**	0.9964	0.9883

Table A24. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
18	18	18	0	1	0.5	1.5	0.5	1.5	0.5976	0.4532	0.5721
18	18	18	0	1	0.75	1.75	0.75	1.75	0.8303	0.6782	0.8063
18	18	18	0	1	1	2	1	2	0.9419	0.8244	0.9237
18	18	18	0	1	1.25	2.25	1.25	2.25	0.9801	0.8984	0.9684
18	18	18	0	1	1.5	2.5	1.5	2.5	0.9946	0.9468	0.9897
18	18	18	0	1	1.75	2.75	1.75	2.75	0.9971	0.9605	0.9945

Table A25. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	0	1	0	1	0.75	1	0.3444	**0.4522**	0.2609
30	30	30	0	1	0	1	1	1	0.5333	**0.6622**	0.4230
30	30	30	0	1	0	1	1.25	1	0.7232	**0.8312**	0.6152
30	30	30	0	1	0	1	1.5	1	0.8779	**0.9445**	0.7930
30	30	30	0	1	0	1	1.75	1	0.9597	**0.9863**	0.9098
30	30	30	0	1	0	1	2	1	0.9915	**0.9979**	0.9701

Table A26. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	0	1	0	1	0.75	1	0.2889	0.4599	0.3802
30	30	30	0	1	0	1	1	1	0.4040	0.6346	0.5301
30	30	30	0	1	0	1	1.25	1	0.5243	0.7943	0.6817
30	30	30	0	1	0	1	1.5	1	0.6267	0.9011	0.7949
30	30	30	0	1	0	1	1.75	1	0.7048	0.9536	0.8716
30	30	30	0	1	0	1	2	1	0.7691	0.9781	0.9174

Table A27. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	0	1	0	1	0	2	**0.2844**	0.1567	0.2369
30	30	30	0	1	0	1	0	2.5	**0.4001**	0.2104	0.3818
30	30	30	0	1	0	1	0	3	**0.4911**	0.2490	0.5080
30	30	30	0	1	0	1	0	3.5	**0.5627**	0.2830	0.6013
30	30	30	0	1	0	1	0	4	**0.6081**	0.3085	0.6712
30	30	30	0	1	0	1	0	4.5	**0.6522**	0.3337	0.7414

Table A28. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	0	1	0	1	0	2	0.2173	0.0699	0.1464
30	30	30	0	1	0	1	0	2.5	0.2811	0.0739	0.1764
30	30	30	0	1	0	1	0	3	0.3169	0.0795	0.1902
30	30	30	0	1	0	1	0	3.5	0.3493	0.0802	0.2074
30	30	30	0	1	0	1	0	4	0.3681	0.0801	0.2204
30	30	30	0	1	0	1	0	4.5	0.3841	0.0787	0.2301

Table A29. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	0	1	0	1	0.25	2	**0.3783**	0.2494	0.2537
30	30	30	0	1	0	1	0.5	2.5	**0.58**01	0.3978	0.4079
30	30	30	0	1	0	1	0.75	3	**0.7183**	0.5178	0.5579
30	30	30	0	1	0	1	1	3.5	**0.8086**	0.6089	0.6549
30	30	30	0	1	0	1	1.25	4	**0.8623**	0.6788	0.7438
30	30	30	0	1	0	1	1.5	4.5	**0.9043**	0.7266	0.7967

Table A30. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	0	1	0	1	0.25	2	0.3015	0.1287	0.2317
30	30	30	0	1	0	1	0.5	2.5	0.4471	0.1994	0.3401
30	30	30	0	1	0	1	0.75	3	0.5565	0.2557	0.4402
30	30	30	0	1	0	1	1	3.5	0.6278	0.3035	0.5069
30	30	30	0	1	0	1	1.25	4	0.6866	0.3379	0.5683
30	30	30	0	1	0	1	1.5	4.5	0.7207	0.3734	0.6017

Table A31. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	0	1	0.5	1	0.5	1	0.3870	0.5847	0.4647
30	30	30	0	1	0.75	1	0.75	1	0.6389	0.8765	0.8230
30	30	30	0	1	1	1	1	1	0.8241	0.9816	0.9764
30	30	30	0	1	1.25	1	1.25	1	0.9354	0.9984	0.9991
30	30	30	0	1	1.5	1	1.5	1	0.9788	1	1
30	30	30	0	1	1.75	1	1.75	1	0.9945	1	1

Table A32. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	0	1	0.5	1	0.5	1	0.4460	**0.6803**	0.5752
30	30	30	0	1	0.75	1	0.75	1	0.7185	**0.9358**	0.8621
30	30	30	0	1	1	1	1	1	0.8883	**0.9948**	0.9737
30	30	30	0	1	1.25	1	1.25	1	0.9699	**0.9998**	0.9974
30	30	30	0	1	1.5	1	1.5	1	0.9921	**1**	0.9996
30	30	30	0	1	1.75	1	1.75	1	0.9993	**1**	1

Table A33. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	0	1	0	2	0	2	**0.7327**	0.3931	0.8332
30	30	30	0	1	0	2.5	0	2.5	**0.8947**	0.5617	0.9799
30	30	30	0	1	0	3	0	3	**0.9528**	0.6688	0.9976
30	30	30	0	1	0	3.5	0	3.5	**0.9784**	0.7394	0.9998
30	30	30	0	1	0	4	0	4	**0.9866**	0.7862	1
30	30	30	0	1	0	4.5	0	4.5	**0.9907**	0.8201	1

Table A34. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	0	1	0	2	0	2	0.6175	0.1040	0.3703
30	30	30	0	1	0	2.5	0	2.5	0.7850	0.1265	0.4798
30	30	30	0	1	0	3	0	3	0.8756	0.1413	0.5637
30	30	30	0	1	0	3.5	0	3.5	0.9151	0.1512	0.6169
30	30	30	0	1	0	4	0	4	0.9391	0.1533	0.6467
30	30	30	0	1	0	4.5	0	4.5	0.9503	0.1599	0.6650

Table A35. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	0	1	0.5	1.5	0.5	1.5	**0.8272**	0.7813	0.5882
30	30	30	0	1	0.75	1.75	0.75	1.75	**0.9723**	0.9528	0.8796
30	30	30	0	1	1	2	1	2	**0.9975**	0.9938	0.9770
30	30	30	0	1	1.25	2.25	1.25	2.25	**0.9998**	0.9991	0.9980
30	30	30	0	1	1.5	2.5	1.5	2.5	**1**	0.9998	1
30	30	30	0	1	1.75	2.75	1.75	2.75	**1**	1	1

Table A36. Percentage of Rejection for k=3 Populations; Normal Distribution. Populations 2 and 3 have the same mean and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	M_1	M_2	M_3
30	30	30	0	1	0.5	1.5	0.5	1.5	0.8012	0.6460	0.7712
30	30	30	0	1	0.75	1.75	0.75	1.75	0.9652	0.8615	0.9509
30	30	30	0	1	1	2	1	2	0.9952	0.9610	0.9930
30	30	30	0	1	1.25	2.25	1.25	2.25	0.9993	0.9842	0.9981
30	30	30	0	1	1.5	2.5	1.5	2.5	0.9999	0.9939	0.9996
30	30	30	0	1	1.75	2.75	1.75	2.75	1	0.9988	1

APPENDIX B. PERCENTAGE OF REJECTION FOR K=3 POPULATIONS

(T- DISTRIBUTION)

Table B1. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
9	9	9	0	0	1	σ	σ	σ	0.1708	**0.2072**	0.0920
9	9	9	0	0	1.25	σ	σ	σ	0.2261	**0.2676**	0.1220
9	9	9	0	0	1.5	σ	σ	σ	0.2856	**0.3365**	0.1487
9	9	9	0	0	1.75	σ	σ	σ	0.3516	**0.4124**	0.1809
9	9	9	0	0	2	σ	σ	σ	0.4256	**0.4853**	0.2134
9	9	9	0	0	2.25	σ	σ	σ	0.4747	**0.5489**	0.2497

Table B2. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
9	9	9	0	0	1	σ	σ	σ	0.1570	0.2191	0.1872
9	9	9	0	0	1.25	σ	σ	σ	0.1825	0.2631	0.2209
9	9	9	0	0	1.5	σ	σ	σ	0.2143	0.3229	0.2701
9	9	9	0	0	1.75	σ	σ	σ	0.2445	0.3751	0.3081
9	9	9	0	0	2	σ	σ	σ	0.2664	0.4243	0.3456
9	9	9	0	0	2.25	σ	σ	σ	0.2937	0.4713	0.3815

Table B3. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
9	9	9	0	0	0	σ	σ	4σ	**0.2118**	0.1151	0.1218
9	9	9	0	0	0	σ	σ	4.5σ	**0.2177**	0.1141	0.1255
9	9	9	0	0	0	σ	σ	5σ	**0.2353**	0.1294	0.1426
9	9	9	0	0	0	σ	σ	5.5σ	**0.2484**	0.1336	0.1579
9	9	9	0	0	0	σ	σ	6σ	**0.2618**	0.1388	0.1570
9	9	9	0	0	0	σ	σ	6.5σ	**0.269**0	0.1369	0.1723

Table B4. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
9	9	9	0	0	0	σ	σ	4σ	0.1152	0.0528	0.0886
9	9	9	0	0	0	σ	σ	4.5σ	0.1245	0.0574	0.0945
9	9	9	0	0	0	σ	σ	5σ	0.1287	0.0584	0.0985
9	9	9	0	0	0	σ	σ	5.5σ	0.1242	0.0539	0.0955
9	9	9	0	0	0	σ	σ	6σ	0.1290	0.0590	0.1007
9	9	9	0	0	0	σ	σ	6.5σ	0.1379	0.0636	0.1046

Table B5. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
9	9	9	0	0	0.5	σ	σ	4σ	**0.2446**	0.1501	0.1202
9	9	9	0	0	0.75	σ	σ	4.5σ	**0.2809**	0.1732	0.1356
9	9	9	0	0	1	σ	σ	5σ	**0.3064**	0.1918	0.1388
9	9	9	0	0	1.25	σ	σ	5.5σ	**0.3385**	0.2102	0.1569
9	9	9	0	0	1.5	σ	σ	6σ	**0.3554**	0.2227	0.1684
9	9	9	0	0	1.75	σ	σ	6.5σ	**0.3761**	0.2343	0.1767

Table B6. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
9	9	9	0	0	0.5	σ	σ	4σ	0.1511	0.0788	0.1218
9	9	9	0	0	0.75	σ	σ	4.5σ	0.1691	0.0925	0.1398
9	9	9	0	0	1	σ	σ	5σ	0.1895	0.1015	0.1566
9	9	9	0	0	1.25	σ	σ	5.5σ	0.2019	0.1104	0.1704
9	9	9	0	0	1.5	σ	σ	6σ	0.2072	0.1184	0.1777
9	9	9	0	0	1.75	σ	σ	6.5σ	0.2231	0.1269	0.1862

Table B7. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
9	9	9	0	0.5	0.5	σ	σ	σ	0.1358	0.1911	0.1140
9	9	9	0	0.75	0.75	σ	σ	σ	0.2036	0.3090	0.1988
9	9	9	0	1	1	σ	σ	σ	0.2689	0.4390	0.3405
9	9	9	0	1.25	1.25	σ	σ	σ	0.3343	0.5697	0.4990
9	9	9	0	1.5	1.5	σ	σ	σ	0.3810	0.6716	0.6551
9	9	9	0	1.75	1.75	σ	σ	σ	0.4314	0.7599	0.7812

Table B8. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
9	9	9	0	0.5	0.5	σ	σ	σ	0.1681	**0.2349**	0.2024
9	9	9	0	0.75	0.75	σ	σ	σ	0.2534	**0.3983**	0.3266
9	9	9	0	1	1	σ	σ	σ	0.3520	**0.561**0	0.4589
9	9	9	0	1.25	1.25	σ	σ	σ	0.4621	**0.7136**	0.5962
9	9	9	0	1.5	1.5	σ	σ	σ	0.5445	**0.8246**	0.7029
9	9	9	0	1.75	1.75	σ	σ	σ	0.6267	**0.8991**	0.7918

Table B9. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
9	9	9	0	0	0	σ	2σ	2σ	**0.254**0	0.1375	0.1742
9	9	9	0	0	0	σ	2.5σ	2.5σ	**0.3677**	0.1852	0.3016
9	9	9	0	0	0	σ	3σ	3σ	**0.4545**	0.2224	0.4239
9	9	9	0	0	0	σ	3.5σ	3.5σ	**0.5289**	0.2566	0.5304
9	9	9	0	0	0	σ	4σ	4σ	**0.5829**	0.2862	0.6266
9	9	9	0	0	0	σ	4.5σ	4.5σ	**0.6289**	0.3199	0.7066

Table B10. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
9	9	9	0	0	0	σ	2σ	2σ	0.1742	0.0608	0.1197
9	9	9	0	0	0	σ	2.5σ	2.5σ	0.2340	0.0687	0.1502
9	9	9	0	0	0	σ	3σ	3σ	0.2822	0.0732	0.1753
9	9	9	0	0	0	σ	3.5σ	3.5σ	0.3141	0.0739	0.1926
9	9	9	0	0	0	σ	4σ	4σ	0.3463	0.0786	0.2049
9	9	9	0	0	0	σ	4.5σ	4.5σ	0.3639	0.0776	0.2145

Table B11. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
9	9	9	0	0.25	0.25	σ	1.5σ	1.5σ	**0.2058**	0.1634	0.0832
9	9	9	0	0.5	0.5	σ	1.75σ	1.75σ	**0.3465**	0.2781	0.1455
9	9	9	0	0.75	0.75	σ	2σ	2σ	**0.4792**	0.4034	0.2166
9	9	9	0	1	1	σ	2.25σ	2.25σ	**0.6033**	0.5175	0.3092
9	9	9	0	1.25	1.25	σ	2.5σ	2.5σ	**0.7031**	0.6201	0.3946
9	9	9	0	1.5	1.5	σ	2.75σ	2.75σ	**0.7821**	0.7114	0.4792

Table B12. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
9	9	9	0	0.25	0.25	σ	1.5σ	1.5σ	0.1840	0.1193	0.1608
9	9	9	0	0.5	0.5	σ	1.75σ	1.75σ	0.3085	0.2016	0.2754
9	9	9	0	0.75	0.75	σ	2σ	2σ	0.4511	0.3011	0.4054
9	9	9	0	1	1	σ	2.25σ	2.25σ	0.5614	0.3847	0.5126
9	9	9	0	1.25	1.25	σ	2.5σ	2.5σ	0.6637	0.4609	0.6044
9	9	9	0	1.5	1.5	σ	2.75σ	2.75σ	0.7338	0.5243	0.6802

Table B13. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	18	18	0	0	1	σ	σ	σ	0.2785	**0.3425**	0.1839
18	18	18	0	0	1.25	σ	σ	σ	0.3779	**0.4609**	0.2563
18	18	18	0	0	1.5	σ	σ	σ	0.4791	**0.5726**	0.3297
18	18	18	0	0	1.75	σ	σ	σ	0.5952	**0.6924**	0.4246
18	18	18	0	0	2	σ	σ	σ	0.7083	**0.7957**	0.5374
18	18	18	0	0	2.25	σ	σ	σ	0.7759	**0.8533**	0.6089

Table B14. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	18	18	0	0	1	σ	σ	σ	0.2232	0.3383	0.2886
18	18	18	0	0	1.25	σ	σ	σ	0.2839	0.4451	0.3718
18	18	18	0	0	1.5	σ	σ	σ	0.3401	0.5361	0.4457
18	18	18	0	0	1.75	σ	σ	σ	0.3928	0.6265	0.5216
18	18	18	0	0	2	σ	σ	σ	0.4479	0.7075	0.5966
18	18	18	0	0	2.25	σ	σ	σ	0.4824	0.7628	0.6381

Table B15. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	18	18	0	0	0	σ	σ	4σ	**0.3626**	0.1867	0.3179
18	18	18	0	0	0	σ	σ	4.5σ	**0.3942**	0.1942	0.3612
18	18	18	0	0	0	σ	σ	5σ	**0.4142**	0.2036	0.3893
18	18	18	0	0	0	σ	σ	5.5σ	**0.4409**	0.2125	0.4235
18	18	18	0	0	0	σ	σ	6σ	**0.4643**	0.2230	0.4605
18	18	18	0	0	0	σ	σ	6.5σ	**0.4889**	0.2368	0.4815

Table B16. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	18	18	0	0	0	σ	σ	4σ	0.2143	0.0679	0.1441
18	18	18	0	0	0	σ	σ	4.5σ	0.2215	0.0665	0.1486
18	18	18	0	0	0	σ	σ	5σ	0.2237	0.0709	0.1504
18	18	18	0	0	0	σ	σ	5.5σ	0.2335	0.0651	0.1533
18	18	18	0	0	0	σ	σ	6σ	0.2525	0.0723	0.1648
18	18	18	0	0	0	σ	σ	6.5σ	0.2464	0.0699	0.1625

Table B17. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	18	18	0	0	0.5	σ	σ	4σ	**0.4425**	0.2577	0.3269
18	18	18	0	0	0.75	σ	σ	4.5σ	**0.5008**	0.2981	0.3732
18	18	18	0	0	1	σ	σ	5σ	**0.5575**	0.3453	0.4130
18	18	18	0	0	1.25	σ	σ	5.5σ	**0.5993**	0.3764	0.4424
18	18	18	0	0	1.5	σ	σ	6σ	**0.6414**	0.4132	0.4857
18	18	18	0	0	1.75	σ	σ	6.5σ	**0.6785**	0.4425	0.5229

Table B18. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	18	18	0	0	0.5	σ	σ	4σ	0.2863	0.1092	0.2086
18	18	18	0	0	0.75	σ	σ	4.5σ	0.3207	0.1283	0.2405
18	18	18	0	0	1	σ	σ	5σ	0.3538	0.1469	0.2694
18	18	18	0	0	1.25	σ	σ	5.5σ	0.3857	0.1593	0.2923
18	18	18	0	0	1.5	σ	σ	6σ	0.3970	0.1703	0.3090
18	18	18	0	0	1.75	σ	σ	6.5σ	0.4291	0.1856	0.3378

Table B19. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	18	18	0	0.5	0.5	σ	σ	σ	0.2089	0.3047	0.1962
18	18	18	0	0.75	0.75	σ	σ	σ	0.3189	0.5098	0.4063
18	18	18	0	1	1	σ	σ	σ	0.4431	0.7067	0.6512
18	18	18	0	1.25	1.25	σ	σ	σ	0.5433	0.8417	0.8350
18	18	18	0	1.5	1.5	σ	σ	σ	0.6170	0.9161	0.9408
18	18	18	0	1.75	1.75	σ	σ	σ	0.6948	0.9615	0.9814

Table B20. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	18	18	0	0.5	0.5	σ	σ	σ	0.2503	**0.3719**	0.3229
18	18	18	0	0.75	0.75	σ	σ	σ	0.4040	**0.6173**	0.5241
18	18	18	0	1	1	σ	σ	σ	0.5692	**0.819**0	0.7179
18	18	18	0	1.25	1.25	σ	σ	σ	0.7104	**0.9306**	0.8567
18	18	18	0	1.5	1.5	σ	σ	σ	0.8098	**0.9786**	0.9299
18	18	18	0	1.75	1.75	σ	σ	σ	0.8819	**0.9938**	0.9703

Table B21. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	18	18	0	0	0	σ	2σ	2σ	**0.4292**	0.2165	0.4215
18	18	18	0	0	0	σ	2.5σ	2.5σ	**0.6183**	0.3110	0.6737
18	18	18	0	0	0	σ	3σ	3σ	**0.7412**	0.3927	0.8432
18	18	18	0	0	0	σ	3.5σ	3.5σ	**0.8137**	0.4610	0.9275
18	18	18	0	0	0	σ	4σ	4σ	**0.8541**	0.5076	0.9640
18	18	18	0	0	0	σ	4.5σ	4.5σ	**0.888**0	0.5619	0.9834

Table B22. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	18	18	0	0	0	σ	2σ	2σ	0.3201	0.0820	0.2043
18	18	18	0	0	0	σ	2.5σ	2.5σ	0.4538	0.0906	0.2717
18	18	18	0	0	0	σ	3σ	3σ	0.5458	0.0953	0.3165
18	18	18	0	0	0	σ	3.5σ	3.5σ	0.6204	0.1054	0.3658
18	18	18	0	0	0	σ	4σ	4σ	0.6569	0.1087	0.3891
18	18	18	0	0	0	σ	4.5σ	4.5σ	0.7091	0.1153	0.4207

Table B23. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
18	18	18	0	0.25	0.25	σ	1.5σ	1.5σ	**0.3475**	0.2656	0.1849
18	18	18	0	0.5	0.5	σ	1.75σ	1.75σ	**0.5963**	0.4885	0.3606
18	18	18	0	0.75	0.75	σ	2σ	2σ	**0.7907**	0.6887	0.5494
18	18	18	0	1	1	σ	2.25σ	2.25σ	**0.8922**	0.8149	0.7099
18	18	18	0	1.25	1.25	σ	2.5σ	2.5σ	**0.9466**	0.9009	0.8135
18	18	18	0	1.5	1.5	σ	2.75σ	2.75σ	**0.9768**	0.9481	0.8948

Table B24. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
18	18	18	0	0.25	0.25	σ	1.5σ	1.5σ	0.2961	0.1638	0.2486
18	18	18	0	0.5	0.5	σ	1.75σ	1.75σ	0.5229	0.3294	0.4671
18	18	18	0	0.75	0.75	σ	2σ	2σ	0.7262	0.4949	0.6682
18	18	18	0	1	1	σ	2.25σ	2.25σ	0.8509	0.6290	0.8078
18	18	18	0	1.25	1.25	σ	2.5σ	2.5σ	0.9225	0.7399	0.8871
18	18	18	0	1.5	1.5	σ	2.75σ	2.75σ	0.9601	0.8139	0.9337

Table B25. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
30	30	30	0	0	1	σ	σ	σ	0.4032	**0.5006**	0.2938
30	30	30	0	0	1.25	σ	σ	σ	0.5466	**0.6577**	0.4386
30	30	30	0	0	1.5	σ	σ	σ	0.6841	**0.7946**	0.5757
30	30	30	0	0	1.75	σ	σ	σ	0.8041	**0.8865**	0.7031
30	30	30	0	0	2	σ	σ	σ	0.8888	**0.9443**	0.8053
30	30	30	0	0	2.25	σ	σ	σ	0.9393	**0.9751**	0.8871

Table B26. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
30	30	30	0	0	1	σ	σ	σ	0.3116	0.4837	0.4085
30	30	30	0	0	1.25	σ	σ	σ	0.4093	0.6296	0.5373
30	30	30	0	0	1.5	σ	σ	σ	0.4830	0.7418	0.6333
30	30	30	0	0	1.75	σ	σ	σ	0.5668	0.8407	0.7291
30	30	30	0	0	2	σ	σ	σ	0.6220	0.8956	0.7903
30	30	30	0	0	2.25	σ	σ	σ	0.6742	0.9357	0.8429

Table B27. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
30	30	30	0	0	0	σ	σ	4σ	**0.5356**	0.2725	0.5810
30	30	30	0	0	0	σ	σ	4.5σ	**0.5839**	0.2933	0.6415
30	30	30	0	0	0	σ	σ	5σ	**0.6199**	0.3203	0.6917
30	30	30	0	0	0	σ	σ	5.5σ	**0.6442**	0.3333	0.7284
30	30	30	0	0	0	σ	σ	6σ	**0.6695**	0.3428	0.7621
30	30	30	0	0	0	σ	σ	6.5σ	**0.6901**	0.3579	0.7924

Table B28. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
30	30	30	0	0	0	σ	σ	4σ	0.3255	0.0751	0.1997
30	30	30	0	0	0	σ	σ	4.5σ	0.3562	0.0803	0.2072
30	30	30	0	0	0	σ	σ	5σ	0.3678	0.0844	0.2205
30	30	30	0	0	0	σ	σ	5.5σ	0.3790	0.0870	0.2280
30	30	30	0	0	0	σ	σ	6σ	0.3851	0.0812	0.2284
30	30	30	0	0	0	σ	σ	6.5σ	0.4072	0.0851	0.2403

Table B29. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
30	30	30	0	0	0.5	σ	σ	4σ	**0.6413**	0.3985	0.5898
30	30	30	0	0	0.75	σ	σ	4.5σ	**0.7239**	0.4734	0.6575
30	30	30	0	0	1	σ	σ	5σ	**0.7863**	0.5255	0.7163
30	30	30	0	0	1.25	σ	σ	5.5σ	**0.8225**	0.5792	0.7623
30	30	30	0	0	1.5	σ	σ	6σ	**0.8543**	0.6109	0.8011
30	30	30	0	0	1.75	σ	σ	6.5σ	**0.8875**	0.6635	0.8339

Table B30. Percentage of Rejection for k=3 Populations; T Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
30	30	30	0	0	0.5	σ	σ	4σ	0.4363	0.1452	0.3046
30	30	30	0	0	0.75	σ	σ	4.5σ	0.5009	0.1769	0.3599
30	30	30	0	0	1	σ	σ	5σ	0.5548	0.2048	0.4085
30	30	30	0	0	1.25	σ	σ	5.5σ	0.5838	0.2209	0.4328
30	30	30	0	0	1.5	σ	σ	6σ	0.6240	0.2490	0.4794
30	30	30	0	0	1.75	σ	σ	6.5σ	0.6518	0.2764	0.5065

Table B31. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
30	30	30	0	0.5	0.5	σ	σ	σ	0.2924	0.4431	0.3230
30	30	30	0	0.75	0.75	σ	σ	σ	0.4674	0.7049	0.6315
30	30	30	0	1	1	σ	σ	σ	0.6317	0.8839	0.8765
30	30	30	0	1.25	1.25	σ	σ	σ	0.7430	0.9626	0.9735
30	30	30	0	1.5	1.5	σ	σ	σ	0.8319	0.9904	0.9964
30	30	30	0	1.75	1.75	σ	σ	σ	0.8828	0.9972	0.9997

Table B32. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
30	30	30	0	0.5	0.5	σ	σ	σ	0.3441	**0.5348**	0.4431
30	30	30	0	0.75	0.75	σ	σ	σ	0.5560	**0.8146**	0.7122
30	30	30	0	1	1	σ	σ	σ	0.7598	**0.9552**	0.8932
30	30	30	0	1.25	1.25	σ	σ	σ	0.8828	**0.9918**	0.9659
30	30	30	0	1.5	1.5	σ	σ	σ	0.9465	**0.9986**	0.9919
30	30	30	0	1.75	1.75	σ	σ	σ	0.9803	**0.9999**	0.9982

Table B33. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
30	30	30	0	0	0	σ	2σ	2σ	**0.6253**	0.3223	0.6950
30	30	30	0	0	0	σ	2.5σ	2.5σ	**0.8208**	0.4753	0.9166
30	30	30	0	0	0	σ	3σ	3σ	**0.9104**	0.5876	0.9820
30	30	30	0	0	0	σ	3.5σ	3.5σ	**0.9464**	0.6540	0.9972
30	30	30	0	0	0	σ	4σ	4σ	**0.9691**	0.7194	0.9990
30	30	30	0	0	0	σ	4.5σ	4.5σ	**0.9809**	0.7632	1

Table B34. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
30	30	30	0	0	0	σ	2σ	2σ	0.4898	0.0973	0.2895
30	30	30	0	0	0	σ	2.5σ	2.5σ	0.6617	0.1107	0.3987
30	30	30	0	0	0	σ	3σ	3σ	0.7735	0.1220	0.4772
30	30	30	0	0	0	σ	3.5σ	3.5σ	0.8493	0.1384	0.5397
30	30	30	0	0	0	σ	4σ	4σ	0.8831	0.1458	0.5750
30	30	30	0	0	0	σ	4.5σ	4.5σ	0.9172	0.1601	0.6203

Table B35. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	L_1	L_2	Lepage
30	30	30	0	0.25	0.25	σ	1.5σ	1.5σ	**0.4983**	0.3753	0.3064
30	30	30	0	0.5	0.5	σ	1.75σ	1.75σ	**0.8103**	0.6963	0.5951
30	30	30	0	0.75	0.75	σ	2σ	2σ	**0.9419**	0.8854	0.8233
30	30	30	0	1	1	σ	2.25σ	2.25σ	**0.9843**	0.9617	0.9308
30	30	30	0	1.25	1.25	σ	2.5σ	2.5σ	**0.9974**	0.9891	0.9765
30	30	30	0	1.5	1.5	σ	2.75σ	2.75σ	**0.9997**	0.9964	0.9926

Table B36. Percentage of Rejection for k=3 Populations; T Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	μ_2	μ_3	σ_1	σ_2	σ_3	M_1	M_2	M_3
30	30	30	0	0.25	0.25	σ	1.5σ	1.5σ	0.4315	0.2281	0.3601
30	30	30	0	0.5	0.5	σ	1.75σ	1.75σ	0.7388	0.4879	0.6705
30	30	30	0	0.75	0.75	σ	2σ	2σ	0.9081	0.6946	0.8631
30	30	30	0	1	1	σ	2.25σ	2.25σ	0.9698	0.8417	0.9512
30	30	30	0	1.25	1.25	σ	2.5σ	2.5σ	0.9923	0.9150	0.9825
30	30	30	0	1.5	1.5	σ	2.75σ	2.75σ	0.9979	0.9543	0.9938

APPENDIX C. PERCENTAGE OF REJECTION FOR K=3 POPULATIONS (EXPONENTIAL DISTRIBUTION)

Table C1. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	1	1	1	1	1.5	1	0.0727	**0.125**0	0.1113
9	9	9	1	1	1	1	1.75	1	0.1122	**0.1929**	0.1479
9	9	9	1	1	1	1	2	1	0.1773	**0.2776**	0.1828
9	9	9	1	1	1	1	2.25	1	0.2634	**0.3747**	0.2202
9	9	9	1	1	1	1	2.5	1	0.3529	**0.4683**	0.2670
9	9	9	1	1	1	1	2.75	1	0.4515	**0.5568**	0.3044

Table C2. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	1	1	1	1	1	2^2	**0.1114**	0.0518	0.0926
9	9	9	1	1	1	1	1	3^2	**0.1381**	0.0539	0.1363
9	9	9	1	1	1	1	1	4^2	**0.1601**	0.0559	0.1706
9	9	9	1	1	1	1	1	5^2	**0.1734**	0.0640	0.1985
9	9	9	1	1	1	1	1	6^2	**0.1894**	0.0661	0.2174
9	9	9	1	1	1	1	1	7^2	**0.1844**	0.0630	0.2166

Table C3. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	1	1	1	1	2	2^2	0.1534	**0.1658**	0.0699
9	9	9	1	1	1	1	2.5	2.5^2	0.2024	**0.2298**	0.0896
9	9	9	1	1	1	1	3	3^2	0.2622	**0.2895**	0.1061
9	9	9	1	1	1	1	3.5	3.5^2	0.3132	**0.3528**	0.1325
9	9	9	1	1	1	1	4	4^2	0.3575	**0.4031**	0.1512
9	9	9	1	1	1	1	4.5	4.5^2	0.3952	**0.4432**	0.1601

Table C4. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	1	1	1.25	1	1.25	1	0.0475	**0.1101**	0.1920
9	9	9	1	1	1.5	1	1.5	1	0.0559	**0.2263**	0.4876
9	9	9	1	1	1.75	1	1.75	1	0.0596	**0.354**0	0.7276
9	9	9	1	1	2	1	2	1	0.0608	**0.4831**	0.8733
9	9	9	1	1	2.25	1	2.25	1	0.0716	**0.603**0	0.9443
9	9	9	1	1	2.5	1	2.5	1	0.0662	**0.7064**	0.9767

Table C5. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	1	1	1	2^2	1	2^2	**0.2167**	0.0624	0.4675
9	9	9	1	1	1	2.5^2	1	2.5^2	**0.27**01	0.0697	0.6728
9	9	9	1	1	1	3^2	1	3^2	**0.3039**	0.0758	0.7909
9	9	9	1	1	1	3.5^2	1	3.5^2	**0.3411**	0.0844	0.8642
9	9	9	1	1	1	4^2	1	4^2	**0.3618**	0.0924	0.9140
9	9	9	1	1	1	4.5^2	1	4.5^2	**0.3862**	0.1028	0.9386

Table C6. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
9	9	9	1	1	1.5	1.5^2	1.5	1.5^2	0.1611	**0.1954**	0.0784
9	9	9	1	1	2	2^2	2	2^2	0.3135	**0.4004**	0.1803
9	9	9	1	1	2.5	2.5^2	2.5	2.5^2	0.4587	**0.6019**	0.3102
9	9	9	1	1	3	3^2	3	3^2	0.5518	**0.7309**	0.4308
9	9	9	1	1	3.5	3.5^2	3.5	3.5^2	0.6570	**0.837**0	0.5513
9	9	9	1	1	4	4^2	4	4^2	0.7075	**0.8945**	0.6401

Table C7. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	1	1	1	1	1.5	1	0.0837	**0.1744**	0.1997
18	18	18	1	1	1	1	1.75	1	0.1470	**0.2953**	0.3067
18	18	18	1	1	1	1	2	1	0.2625	**0.4457**	0.3815
18	18	18	1	1	1	1	2.25	1	0.4136	**0.5907**	0.4607
18	18	18	1	1	1	1	2.5	1	0.5760	**0.738**0	0.5621
18	18	18	1	1	1	1	2.75	1	0.7103	**0.8386**	0.6506

Table C8. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	1	1	1	1	1	2^2	**0.1594**	0.0632	0.2342
18	18	18	1	1	1	1	1	3^2	**0.2192**	0.0693	0.4076
18	18	18	1	1	1	1	1	4^2	**0.2647**	0.0832	0.5332
18	18	18	1	1	1	1	1	5^2	**0.2933**	0.0819	0.6024
18	18	18	1	1	1	1	1	6^2	**0.3096**	0.0881	0.6460
18	18	18	1	1	1	1	1	7^2	**0.314**0	0.0886	0.6678

Table C9. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	1	1	1	1	2	2^2	0.2517	**0.2735**	0.1270
18	18	18	1	1	1	1	2.5	2.5^2	0.3635	**0.4011**	0.1925
18	18	18	1	1	1	1	3	3^2	0.4812	**0.523**0	0.2640
18	18	18	1	1	1	1	3.5	3.5^2	0.5738	**0.6218**	0.3296
18	18	18	1	1	1	1	4	4^2	0.6575	**0.7014**	0.4022
18	18	18	1	1	1	1	4.5	4.5^2	0.7282	**0.7713**	0.4679

Table C10. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	1	1	1.25	1	1.25	1	0.0510	**0.157**0	0.3531
18	18	18	1	1	1.5	1	1.5	1	0.0618	**0.3332**	0.7758
18	18	18	1	1	1.75	1	1.75	1	0.0862	**0.5439**	0.9577
18	18	18	1	1	2	1	2	1	0.1278	**0.7389**	0.9939
18	18	18	1	1	2.25	1	2.25	1	0.1912	**0.8546**	0.9996
18	18	18	1	1	2.5	1	2.5	1	0.2761	**0.9314**	0.9998

Table C11. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	1	1	1	2^2	1	2^2	**0.3469**	0.0701	0.8894
18	18	18	1	1	1	2.5^2	1	2.5^2	**0.3734**	0.0602	0.9797
18	18	18	1	1	1	3^2	1	3^2	**0.5105**	0.1079	0.9943
18	18	18	1	1	1	3.5^2	1	3.5^2	**0.5508**	0.1208	0.9988
18	18	18	1	1	1	4^2	1	4^2	**0.5795**	0.1241	0.9997
18	18	18	1	1	1	4.5^2	1	4.5^2	**0.604**0	0.1423	0.9997

Table C12. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
18	18	18	1	1	1.5	1.5^2	1.5	1.5^2	0.2873	**0.3399**	0.1669
18	18	18	1	1	2	2^2	2	2^2	0.5796	**0.688**0	0.4339
18	18	18	1	1	2.5	2.5^2	2.5	2.5^2	0.7735	**0.881**0	0.6777
18	18	18	1	1	3	3^2	3	3^2	0.8902	**0.9629**	0.8432
18	18	18	1	1	3.5	3.5^2	3.5	3.5^2	0.9515	**0.9878**	0.9355
18	18	18	1	1	4	4^2	4	4^2	0.9749	**0.9956**	0.9704

Table C13. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 1 and 2 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	1	1	1	1	1.5	1	0.0975	**0.2383**	0.3422
30	30	30	1	1	1	1	1.75	1	0.1908	**0.4129**	0.4889
30	30	30	1	1	1	1	2	1	0.3625	**0.6251**	0.6117
30	30	30	1	1	1	1	2.25	1	0.5694	**0.7935**	0.7287
30	30	30	1	1	1	1	2.5	1	0.7697	**0.9102**	0.8263
30	30	30	1	1	1	1	2.75	1	0.8877	**0.9648**	0.9006

Table C14. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 1 and 2 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	1	1	1	1	1	2^2	**0.2268**	0.0742	0.4485
30	30	30	1	1	1	1	1	3^2	**0.3074**	0.0806	0.7102
30	30	30	1	1	1	1	1	4^2	**0.3710**	0.0968	0.8354
30	30	30	1	1	1	1	1	5^2	**0.4150**	0.1115	0.8898
30	30	30	1	1	1	1	1	6^2	**0.4455**	0.1157	0.9247
30	30	30	1	1	1	1	1	7^2	**0.4787**	0.1267	0.9429

Table C15. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 1 and 2 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	1	1	1	1	2	2^2	0.3707	**0.4084**	0.2006
30	30	30	1	1	1	1	2.5	2.5^2	0.5495	**0.5934**	0.3390
30	30	30	1	1	1	1	3	3^2	0.7011	**0.7496**	0.4758
30	30	30	1	1	1	1	3.5	3.5^2	0.8056	**0.8386**	0.5828
30	30	30	1	1	1	1	4	4^2	0.8698	**0.8991**	0.6880
30	30	30	1	1	1	1	4.5	4.5^2	0.9176	**0.9392**	0.7631

Table C16. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 2 and 3 have the same means

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	1	1	1.25	1	1.25	1	0.0502	**0.2078**	0.5307
30	30	30	1	1	1.5	1	1.5	1	0.0704	**0.4903**	0.9406
30	30	30	1	1	1.75	1	1.75	1	0.1222	**0.7417**	0.9970
30	30	30	1	1	2	1	2	1	0.1978	**0.9007**	0.9999
30	30	30	1	1	2.25	1	2.25	1	0.3147	**0.969**0	1
30	30	30	1	1	2.5	1	2.5	1	0.4449	**0.9905**	1

Table C17. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 2 and 3 have the same variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	1	1	1	2^2	1	2^2	**0.5138**	0.0950	0.9903
30	30	30	1	1	1	2.5^2	1	2.5^2	**0.634**0	0.1194	0.9991
30	30	30	1	1	1	3^2	1	3^2	**0.6978**	0.1401	1
30	30	30	1	1	1	3.5^2	1	3.5^2	**0.7356**	0.1652	1
30	30	30	1	1	1	4^2	1	4^2	**0.7598**	0.1824	1
30	30	30	1	1	1	4.5^2	1	4.5^2	**0.7738**	0.1944	1

Table C18. Percentage of Rejection for k=3 Populations; Exponential Distribution. Populations 2 and 3 have the same means and variances

n_1	n_2	n_3	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	L_1	L_2	Lepage
30	30	30	1	1	1.5	1.5^2	1.5	1.5^2	0.4147	**0.4885**	0.2706
30	30	30	1	1	2	2^2	2	2^2	0.7862	**0.8776**	0.6852
30	30	30	1	1	2.5	2.5^2	2.5	2.5^2	0.9467	**0.9834**	0.9215
30	30	30	1	1	3	3^2	3	3^2	0.9867	**0.998**0	0.9828
30	30	30	1	1	3.5	3.5^2	3.5	3.5^2	0.9965	**0.9999**	0.9971
30	30	30	1	1	4	4^2	4	4^2	0.9996	**1**	0.9994

APPENDIX D. PERCENTAGE OF REJECTION FOR K=4 POPULATIONS (NORMAL DISTRIBUTION)

Table D1. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	0	1	0	1	1.5	1	0.2228	**0.2661**	0.1051
9	9	9	9	0	1	0	1	0	1	1.75	1	0.2801	**0.3221**	0.1211
9	9	9	9	0	1	0	1	0	1	2	1	0.3298	**0.3789**	0.1332
9	9	9	9	0	1	0	1	0	1	2.25	1	0.3775	**0.4227**	0.1459
9	9	9	9	0	1	0	1	0	1	2.5	1	0.4324	**0.4777**	0.1612
9	9	9	9	0	1	0	1	0	1	2.75	1	0.4754	**0.5143**	0.1729

Table D2. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	0	1	0	1	1.5	1	0.1802	0.2561	0.2182
9	9	9	9	0	1	0	1	0	1	1.75	1	0.1977	0.2874	0.2403
9	9	9	9	0	1	0	1	0	1	2	1	0.2083	0.3202	0.2566
9	9	9	9	0	1	0	1	0	1	2.25	1	0.2173	0.3384	0.2706
9	9	9	9	0	1	0	1	0	1	2.5	1	0.2287	0.3515	0.2850
9	9	9	9	0	1	0	1	0	1	2.75	1	0.2391	0.3739	0.2992

Table D3. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	0	1	0	1	0	2	**0.0905**	0.0652	0.0428
9	9	9	9	0	1	0	1	0	1	0	3	**0.1214**	0.0765	0.0604
9	9	9	9	0	1	0	1	0	1	0	4	**0.1368**	0.0796	0.0665
9	9	9	9	0	1	0	1	0	1	0	5	**0.1555**	0.0825	0.0791
9	9	9	9	0	1	0	1	0	1	0	6	**0.1551**	0.0852	0.0804
9	9	9	9	0	1	0	1	0	1	0	7	**0.1701**	0.0934	0.0869

Table D4. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	0	1	0	1	0	2	0.0773	0.0519	0.0653
9	9	9	9	0	1	0	1	0	1	0	3	0.0880	0.0545	0.0719
9	9	9	9	0	1	0	1	0	1	0	4	0.0905	0.0486	0.0734
9	9	9	9	0	1	0	1	0	1	0	5	0.0887	0.0446	0.0687
9	9	9	9	0	1	0	1	0	1	0	6	0.0871	0.0469	0.0708
9	9	9	9	0	1	0	1	0	1	0	7	0.0963	0.0481	0.0760

Table D5. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	0	1	0	1	0.75	2	**0.1293**	0.1093	0.0529
9	9	9	9	0	1	0	1	0	1	1	2.5	**0.1598**	0.1314	0.0627
9	9	9	9	0	1	0	1	0	1	1.25	3	**0.1876**	0.1480	0.0750
9	9	9	9	0	1	0	1	0	1	1.5	3.5	**0.2001**	0.1563	0.0732
9	9	9	9	0	1	0	1	0	1	1.75	4	**0.2167**	0.1660	0.0725
9	9	9	9	0	1	0	1	0	1	2	4.5	**0.2279**	0.1704	0.0801

Table D6. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	0	1	0	1	0.75	2	0.1344	0.1101	0.1245
9	9	9	9	0	1	0	1	0	1	1	2.5	0.1560	0.1151	0.1446
9	9	9	9	0	1	0	1	0	1	1.25	3	0.1591	0.1197	0.1508
9	9	9	9	0	1	0	1	0	1	1.5	3.5	0.1710	0.1259	0.1589
9	9	9	9	0	1	0	1	0	1	1.75	4	0.1748	0.1282	0.1599
9	9	9	9	0	1	0	1	0	1	2	4.5	0.1845	0.1322	0.1691

Table D7. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	0.5	1	0.5	1	0.5	1	0.1436	0.2529	0.1685
9	9	9	9	0	1	0.75	1	0.75	1	0.75	1	0.2025	0.4253	0.3302
9	9	9	9	0	1	1	1	1	1	1	1	0.2386	0.6224	0.5514
9	9	9	9	0	1	1.25	1	1.25	1	1.25	1	0.2541	0.7897	0.7723
9	9	9	9	0	1	1.5	1	1.5	1	1.5	1	0.2895	0.8876	0.8987
9	9	9	9	0	1	1.75	1	1.75	1	1.75	1	0.3369	0.9526	0.9674

Table D8. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	0.5	1	0.5	1	0.5	1	0.2333	**0.3439**	0.2889
9	9	9	9	0	1	0.75	1	0.75	1	0.75	1	0.3577	**0.5667**	0.4632
9	9	9	9	0	1	1	1	1	1	1	1	0.4999	**0.7746**	0.6510
9	9	9	9	0	1	1.25	1	1.25	1	1.25	1	0.6384	**0.9088**	0.7952
9	9	9	9	0	1	1.5	1	1.5	1	1.5	1	0.7396	**0.9752**	0.9004
9	9	9	9	0	1	1.75	1	1.75	1	1.75	1	0.8282	**0.9928**	0.9549

Table D9. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	0	1.5	0	1.5	0	1.5	**0.1619**	0.0892	0.0934
9	9	9	9	0	1	0	2	0	2	0	2	**0.3284**	0.1465	0.2566
9	9	9	9	0	1	0	2.5	0	2.5	0	2.5	**0.4835**	0.2106	0.4555
9	9	9	9	0	1	0	3	0	3	0	3	**0.6096**	0.2745	0.6282
9	9	9	9	0	1	0	3.5	0	3.5	0	3.5	**0.6706**	0.3101	0.7560
9	9	9	9	0	1	0	4	0	4	0	4	**0.7328**	0.3532	0.8450

Table D10. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	0	1.5	0	1.5	0	1.5	0.1359	0.0574	0.0969
9	9	9	9	0	1	0	2	0	2	0	2	0.2306	0.0585	0.1428
9	9	9	9	0	1	0	2.5	0	2.5	0	2.5	0.3127	0.0539	0.1710
9	9	9	9	0	1	0	3	0	3	0	3	0.3760	0.0602	0.1999
9	9	9	9	0	1	0	3.5	0	3.5	0	3.5	0.4205	0.0627	0.2171
9	9	9	9	0	1	0	4	0	4	0	4	0.4493	0.0644	0.2320

Table D11. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	0.25	1.5	0.25	1.5	0.25	1.5	**0.2555**	0.1897	0.1017
9	9	9	9	0	1	0.5	1.75	0.5	1.75	0.5	1.75	**0.4635**	0.3803	0.1910
9	9	9	9	0	1	0.75	2	0.75	2	0.75	2	**0.6581**	0.5674	0.2936
9	9	9	9	0	1	1	2.25	1	2.25	1	2.25	**0.7925**	0.7211	0.4159
9	9	9	9	0	1	1.25	2.5	1.25	2.5	1.25	2.5	**0.8854**	0.8309	0.5371
9	9	9	9	0	1	1.5	2.75	1.5	2.75	1.5	2.75	**0.932**0	0.8999	0.6297

Table D12. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	0.25	1.5	0.25	1.5	0.25	1.5	0.2466	0.1386	0.2062
9	9	9	9	0	1	0.5	1.75	0.5	1.75	0.5	1.75	0.4496	0.2648	0.3907
9	9	9	9	0	1	0.75	2	0.75	2	0.75	2	0.6323	0.4082	0.5622
9	9	9	9	0	1	1	2.25	1	2.25	1	2.25	0.7802	0.5354	0.7140
9	9	9	9	0	1	1.25	2.5	1.25	2.5	1.25	2.5	0.8730	0.6541	0.8179
9	9	9	9	0	1	1.5	2.75	1.5	2.75	1.5	2.75	0.9316	0.7381	0.8897

Table D13. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	0	1	0	1	1.5	1	0.4317	**0.5009**	0.2429
18	18	18	18	0	1	0	1	0	1	1.75	1	0.5576	**0.6146**	0.3126
18	18	18	18	0	1	0	1	0	1	2	1	0.6702	**0.7161**	0.3902
18	18	18	18	0	1	0	1	0	1	2.25	1	0.7479	**0.7997**	0.4675
18	18	18	18	0	1	0	1	0	1	2.5	1	0.8330	**0.8603**	0.5323
18	18	18	18	0	1	0	1	0	1	2.75	1	0.8878	**0.9047**	0.5887

Table D14. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	0	1	0	1	1.5	1	0.2733	0.4283	0.3537
18	18	18	18	0	1	0	1	0	1	1.75	1	0.3089	0.4846	0.3993
18	18	18	18	0	1	0	1	0	1	2	1	0.3376	0.5529	0.4472
18	18	18	18	0	1	0	1	0	1	2.25	1	0.3637	0.6016	0.4776
18	18	18	18	0	1	0	1	0	1	2.5	1	0.3788	0.6286	0.5085
18	18	18	18	0	1	0	1	0	1	2.75	1	0.3902	0.6594	0.5312

Table D15. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	0	1	0	1	0	2	**0.1352**	0.0893	0.0801
18	18	18	18	0	1	0	1	0	1	0	3	**0.2004**	0.1148	0.1284
18	18	18	18	0	1	0	1	0	1	0	4	**0.2405**	0.1299	0.1673
18	18	18	18	0	1	0	1	0	1	0	5	**0.2779**	0.1425	0.1988
18	18	18	18	0	1	0	1	0	1	0	6	**0.2978**	0.1472	0.2336
18	18	18	18	0	1	0	1	0	1	0	7	**0.3113**	0.1551	0.2406

Table D16. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	0	1	0	1	0	2	0.1055	0.0545	0.0824
18	18	18	18	0	1	0	1	0	1	0	3	0.1305	0.0537	0.0957
18	18	18	18	0	1	0	1	0	1	0	4	0.1439	0.0571	0.1029
18	18	18	18	0	1	0	1	0	1	0	5	0.1546	0.0536	0.1103
18	18	18	18	0	1	0	1	0	1	0	6	0.1546	0.0508	0.1051
18	18	18	18	0	1	0	1	0	1	0	7	0.1525	0.0521	0.1054

Table D17. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	0	1	0	1	0.75	2	**0.2442**	0.1947	0.1019
18	18	18	18	0	1	0	1	0	1	1	2.5	**0.3048**	0.2405	0.1322
18	18	18	18	0	1	0	1	0	1	1.25	3	**0.3541**	0.2710	0.1627
18	18	18	18	0	1	0	1	0	1	1.5	3.5	**0.4047**	0.3035	0.1876
18	18	18	18	0	1	0	1	0	1	1.75	4	**0.4355**	0.3286	0.2048
18	18	18	18	0	1	0	1	0	1	2	4.5	**0.4704**	0.3402	0.2230

Table D18. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	0	1	0	1	0.75	2	0.2127	0.1521	0.1971
18	18	18	18	0	1	0	1	0	1	1	2.5	0.2448	0.1704	0.2229
18	18	18	18	0	1	0	1	0	1	1.25	3	0.2805	0.1792	0.2486
18	18	18	18	0	1	0	1	0	1	1.5	3.5	0.2943	0.1799	0.2585
18	18	18	18	0	1	0	1	0	1	1.75	4	0.3181	0.2006	0.2792
18	18	18	18	0	1	0	1	0	1	2	4.5	0.3184	0.1927	0.2750

Table D19. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	0.5	1	0.5	1	0.5	1	0.2540	0.4289	0.3190
18	18	18	18	0	1	0.75	1	0.75	1	0.75	1	0.4041	0.7116	0.6433
18	18	18	18	0	1	1	1	1	1	1	1	0.5563	0.8990	0.8840
18	18	18	18	0	1	1.25	1	1.25	1	1.25	1	0.6705	0.9772	0.9801
18	18	18	18	0	1	1.5	1	1.5	1	1.5	1	0.7506	0.9959	0.9984
18	18	18	18	0	1	1.75	1	1.75	1	1.75	1	0.7968	0.9996	0.9999

Table D20. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	0.5	1	0.5	1	0.5	1	0.3499	**0.5403**	0.4530
18	18	18	18	0	1	0.75	1	0.75	1	0.75	1	0.5637	**0.8254**	0.7205
18	18	18	18	0	1	1	1	1	1	1	1	0.7538	**0.962**0	0.8975
18	18	18	18	0	1	1.25	1	1.25	1	1.25	1	0.8897	**0.9968**	0.9762
18	18	18	18	0	1	1.5	1	1.5	1	1.5	1	0.9531	**0.9998**	0.9948
18	18	18	18	0	1	1.75	1	1.75	1	1.75	1	0.9830	**1**	0.9994

Table D21. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	0	1.5	0	1.5	0	1.5	**0.28**01	0.1411	0.2241
18	18	18	18	0	1	0	2	0	2	0	2	**0.5956**	0.2860	0.6347
18	18	18	18	0	1	0	2.5	0	2.5	0	2.5	**0.7817**	0.4158	0.8850
18	18	18	18	0	1	0	3	0	3	0	3	**0.8802**	0.5208	0.9722
18	18	18	18	0	1	0	3.5	0	3.5	0	3.5	**0.9317**	0.5978	0.9934
18	18	18	18	0	1	0	4	0	4	0	4	**0.9523**	0.6401	0.9986

Table D22. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	0	1.5	0	1.5	0	1.5	0.2202	0.0611	0.1431
18	18	18	18	0	1	0	2	0	2	0	2	0.4409	0.0736	0.2473
18	18	18	18	0	1	0	2.5	0	2.5	0	2.5	0.6131	0.0805	0.3328
18	18	18	18	0	1	0	3	0	3	0	3	0.7279	0.0892	0.4136
18	18	18	18	0	1	0	3.5	0	3.5	0	3.5	0.7985	0.0985	0.4487
18	18	18	18	0	1	0	4	0	4	0	4	0.8349	0.1032	0.4839

Table D23. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	0.25	1.5	0.25	1.5	0.25	1.5	**0.4724**	0.3499	0.2511
18	18	18	18	0	1	0.5	1.75	0.5	1.75	0.5	1.75	**0.7969**	0.6737	0.5292
18	18	18	18	0	1	0.75	2	0.75	2	0.75	2	**0.9422**	0.8824	0.7655
18	18	18	18	0	1	1	2.25	1	2.25	1	2.25	**0.9874**	0.9652	0.9083
18	18	18	18	0	1	1.25	2.5	1.25	2.5	1.25	2.5	**0.9977**	0.9893	0.9676
18	18	18	18	0	1	1.5	2.75	1.5	2.75	1.5	2.75	**0.9994**	0.9977	0.9919

Table D24. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	0.25	1.5	0.25	1.5	0.25	1.5	0.4288	0.2205	0.3527
18	18	18	18	0	1	0.5	1.75	0.5	1.75	0.5	1.75	0.7476	0.4659	0.6681
18	18	18	18	0	1	0.75	2	0.75	2	0.75	2	0.9214	0.6952	0.8718
18	18	18	18	0	1	1	2.25	1	2.25	1	2.25	0.9773	0.8384	0.9540
18	18	18	18	0	1	1.25	2.5	1.25	2.5	1.25	2.5	0.9933	0.9192	0.9843
18	18	18	18	0	1	1.5	2.75	1.5	2.75	1.5	2.75	0.9991	0.9576	0.9941

Table D25. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	0	1	0	1	1.5	1	0.6586	**0.7268**	0.4477
30	30	30	30	0	1	0	1	0	1	1.75	1	0.7902	**0.8397**	0.5740
30	30	30	30	0	1	0	1	0	1	2	1	0.8872	**0.9151**	0.6965
30	30	30	30	0	1	0	1	0	1	2.25	1	0.9513	**0.9635**	0.8007
30	30	30	30	0	1	0	1	0	1	2.5	1	0.9802	**0.9844**	0.8716
30	30	30	30	0	1	0	1	0	1	2.75	1	0.9920	**0.9938**	0.9266

Table D26. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	0	1	0	1	1.5	1	0.3888	0.6194	0.5115
30	30	30	30	0	1	0	1	0	1	1.75	1	0.4405	0.7102	0.5890
30	30	30	30	0	1	0	1	0	1	2	1	0.4878	0.7714	0.6516
30	30	30	30	0	1	0	1	0	1	2.25	1	0.5312	0.8283	0.7055
30	30	30	30	0	1	0	1	0	1	2.5	1	0.5531	0.8582	0.7289
30	30	30	30	0	1	0	1	0	1	2.75	1	0.5784	0.884	0.7645

Table D27. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	0	1	0	1	0	2	**0.1808**	0.1062	0.1190
30	30	30	30	0	1	0	1	0	1	0	3	**0.2872**	0.1546	0.2401
30	30	30	30	0	1	0	1	0	1	0	4	**0.3612**	0.1808	0.3201
30	30	30	30	0	1	0	1	0	1	0	5	**0.4248**	0.2122	0.3992
30	30	30	30	0	1	0	1	0	1	0	6	**0.4523**	0.2185	0.4426
30	30	30	30	0	1	0	1	0	1	0	7	**0.4698**	0.2268	0.4692

Table D28. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	0	1	0	1	0	2	0.1439	0.0614	0.1043
30	30	30	30	0	1	0	1	0	1	0	3	0.1884	0.0622	0.1270
30	30	30	30	0	1	0	1	0	1	0	4	0.2171	0.0626	0.1382
30	30	30	30	0	1	0	1	0	1	0	5	0.2240	0.0616	0.1458
30	30	30	30	0	1	0	1	0	1	0	6	0.2217	0.0559	0.1396
30	30	30	30	0	1	0	1	0	1	0	7	0.2380	0.0647	0.1491

Table D29. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	0	1	0	1	0.75	2	**0.3582**	0.2936	0.1771
30	30	30	30	0	1	0	1	0	1	1	2.5	**0.4666**	0.3662	0.2415
30	30	30	30	0	1	0	1	0	1	1.25	3	**0.5519**	0.4236	0.3079
30	30	30	30	0	1	0	1	0	1	1.5	3.5	**0.616**0	0.4724	0.3677
30	30	30	30	0	1	0	1	0	1	1.75	4	**0.6656**	0.4982	0.4081
30	30	30	30	0	1	0	1	0	1	2	4.5	**0.6893**	0.5301	0.4424

Table D30. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	0	1	0	1	0.75	2	0.3029	0.2087	0.2760
30	30	30	30	0	1	0	1	0	1	1	2.5	0.3680	0.2359	0.3262
30	30	30	30	0	1	0	1	0	1	1.25	3	0.4151	0.2516	0.3624
30	30	30	30	0	1	0	1	0	1	1.5	3.5	0.4592	0.2805	0.3974
30	30	30	30	0	1	0	1	0	1	1.75	4	0.4839	0.2873	0.4229
30	30	30	30	0	1	0	1	0	1	2	4.5	0.5071	0.3004	0.4368

Table D31. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	0.5	1	0.5	1	0.5	1	0.3884	0.6177	0.5109
30	30	30	30	0	1	0.75	1	0.75	1	0.75	1	0.6153	0.9001	0.8677
30	30	30	30	0	1	1	1	1	1	1	1	0.7992	0.9878	0.9890
30	30	30	30	0	1	1.25	1	1.25	1	1.25	1	0.8990	0.9992	0.9997
30	30	30	30	0	1	1.5	1	1.5	1	1.5	1	0.9526	1	1
30	30	30	30	0	1	1.75	1	1.75	1	1.75	1	0.9753	1	1

Table D32. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	0.5	1	0.5	1	0.5	1	0.4763	**0.7191**	0.6162
30	30	30	30	0	1	0.75	1	0.75	1	0.75	1	0.7639	**0.9622**	0.8989
30	30	30	30	0	1	1	1	1	1	1	1	0.9161	**0.9977**	0.9839
30	30	30	30	0	1	1.25	1	1.25	1	1.25	1	0.9811	**0.9998**	0.9987
30	30	30	30	0	1	1.5	1	1.5	1	1.5	1	0.9961	**1**	0.9998
30	30	30	30	0	1	1.75	1	1.75	1	1.75	1	0.9994	**1**	1

Table D33. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	0	1.5	0	1.5	0	1.5	**0.4217**	0.2128	0.4023
30	30	30	30	0	1	0	2	0	2	0	2	**0.7988**	0.4337	0.8889
30	30	30	30	0	1	0	2.5	0	2.5	0	2.5	**0.9403**	0.6221	0.9910
30	30	30	30	0	1	0	3	0	3	0	3	**0.979**0	0.7390	0.9996
30	30	30	30	0	1	0	3.5	0	3.5	0	3.5	**0.9909**	0.8120	1
30	30	30	30	0	1	0	4	0	4	0	4	**0.9952**	0.8525	1

Table D34. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	0	1.5	0	1.5	0	1.5	0.3401	0.0778	0.2035
30	30	30	30	0	1	0	2	0	2	0	2	0.6749	0.0949	0.3896
30	30	30	30	0	1	0	2.5	0	2.5	0	2.5	0.8553	0.1137	0.5356
30	30	30	30	0	1	0	3	0	3	0	3	0.9318	0.1279	0.6217
30	30	30	30	0	1	0	3.5	0	3.5	0	3.5	0.9611	0.1438	0.6932
30	30	30	30	0	1	0	4	0	4	0	4	0.9781	0.1385	0.7233

Table D35. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	0.25	1.5	0.25	1.5	0.25	1.5	**0.691**0	0.5401	0.4766
30	30	30	30	0	1	0.5	1.75	0.5	1.75	0.5	1.75	**0.9475**	0.8793	0.8235
30	30	30	30	0	1	0.75	2	0.75	2	0.75	2	**0.996**0	0.9846	0.9660
30	30	30	30	0	1	1	2.25	1	2.25	1	2.25	**1**	0.9990	0.9963
30	30	30	30	0	1	1.25	2.5	1.25	2.5	1.25	2.5	**1**	0.9999	0.9997
30	30	30	30	0	1	1.5	2.75	1.5	2.75	1.5	2.75	**1**	1	0.9999

Table D36. Percentage of Rejection for k=4 Populations; Normal Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	0.25	1.5	0.25	1.5	0.25	1.5	0.6250	0.3217	0.5181
30	30	30	30	0	1	0.5	1.75	0.5	1.75	0.5	1.75	0.9262	0.6857	0.8732
30	30	30	30	0	1	0.75	2	0.75	2	0.75	2	0.9922	0.8925	0.9781
30	30	30	30	0	1	1	2.25	1	2.25	1	2.25	0.9996	0.9675	0.9977
30	30	30	30	0	1	1.25	2.5	1.25	2.5	1.25	2.5	0.9999	0.9916	0.9994
30	30	30	30	0	1	1.5	2.75	1.5	2.75	1.5	2.75	1	0.9976	1

APPENDIX E. PERCENTAGE OF REJECTION FOR K=4 POPULATIONS

(T- DISTRIBUTION)

Table E1. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
9	9	9	9	0	0	0	1	σ	σ	σ	σ	0.1202	**0.1406**	0.0626
9	9	9	9	0	0	0	1.5	σ	σ	σ	σ	0.1810	**0.2081**	0.0875
9	9	9	9	0	0	0	2	σ	σ	σ	σ	0.2589	**0.2894**	0.1167
9	9	9	9	0	0	0	2.5	σ	σ	σ	σ	0.3313	**0.3609**	0.1355
9	9	9	9	0	0	0	3	σ	σ	σ	σ	0.3872	**0.4214**	0.1473
9	9	9	9	0	0	0	3.5	σ	σ	σ	σ	0.4342	**0.4652**	0.1645

Table E2. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
9	9	9	9	0	0	0	1	σ	σ	σ	σ	0.1159	0.1606	0.1288
9	9	9	9	0	0	0	1.5	σ	σ	σ	σ	0.1461	0.2009	0.1649
9	9	9	9	0	0	0	2	σ	σ	σ	σ	0.1815	0.2594	0.2049
9	9	9	9	0	0	0	2.5	σ	σ	σ	σ	0.2049	0.2992	0.2310
9	9	9	9	0	0	0	3	σ	σ	σ	σ	0.2159	0.3251	0.2458
9	9	9	9	0	0	0	3.5	σ	σ	σ	σ	0.2188	0.3452	0.2543

Table E3. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
9	9	9	9	0	0	0	0	σ	σ	σ	4σ	**0.1265**	0.0818	0.0642
9	9	9	9	0	0	0	0	σ	σ	σ	5σ	**0.1404**	0.0785	0.0723
9	9	9	9	0	0	0	0	σ	σ	σ	6σ	**0.1457**	0.0795	0.0763
9	9	9	9	0	0	0	0	σ	σ	σ	7σ	**0.1549**	0.0881	0.0824
9	9	9	9	0	0	0	0	σ	σ	σ	8σ	**0.1578**	0.0860	0.0838
9	9	9	9	0	0	0	0	σ	σ	σ	9σ	**0.1668**	0.0885	0.0916

Table E4. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
9	9	9	9	0	0	0	0	σ	σ	σ	4σ	0.0866	0.0533	0.0840
9	9	9	9	0	0	0	0	σ	σ	σ	5σ	0.0871	0.0455	0.0804
9	9	9	9	0	0	0	0	σ	σ	σ	6σ	0.0902	0.0471	0.0852
9	9	9	9	0	0	0	0	σ	σ	σ	7σ	0.0898	0.0497	0.0853
9	9	9	9	0	0	0	0	σ	σ	σ	8σ	0.0949	0.0463	0.0879
9	9	9	9	0	0	0	0	σ	σ	σ	9σ	0.0847	0.0420	0.0789

Table E5. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
9	9	9	9	0	0	0	1	σ	σ	σ	4σ	**0.158**0	0.1149	0.0665
9	9	9	9	0	0	0	2	σ	σ	σ	5σ	**0.2087**	0.1522	0.0784
9	9	9	9	0	0	0	3	σ	σ	σ	6σ	**0.2399**	0.1722	0.0863
9	9	9	9	0	0	0	4	σ	σ	σ	7σ	**0.2661**	0.2009	0.0917
9	9	9	9	0	0	0	5	σ	σ	σ	8σ	**0.2941**	0.2241	0.1034
9	9	9	9	0	0	0	6	σ	σ	σ	9σ	**0.2992**	0.2279	0.1064

Table E6. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
9	9	9	9	0	0	0	1	σ	σ	σ	4σ	0.1260	0.0846	0.1269
9	9	9	9	0	0	0	2	σ	σ	σ	5σ	0.1571	0.1102	0.1543
9	9	9	9	0	0	0	3	σ	σ	σ	6σ	0.1863	0.1344	0.1852
9	9	9	9	0	0	0	4	σ	σ	σ	7σ	0.1904	0.1437	0.1911
9	9	9	9	0	0	0	5	σ	σ	σ	8σ	0.2091	0.1525	0.2098
9	9	9	9	0	0	0	6	σ	σ	σ	9σ	0.2270	0.1662	0.2276

Table E7. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
9	9	9	9	0	1	1	1	σ	σ	σ	σ	0.2226	0.4481	0.3899
9	9	9	9	0	1.25	1.25	1.25	σ	σ	σ	σ	0.2450	0.5686	0.5701
9	9	9	9	0	1.5	1.5	1.5	σ	σ	σ	σ	0.2485	0.6513	0.7247
9	9	9	9	0	1.75	1.75	1.75	σ	σ	σ	σ	0.2412	0.7375	0.8493
9	9	9	9	0	2	2	2	σ	σ	σ	σ	0.2049	0.7861	0.9198
9	9	9	9	0	2.25	2.25	2.25	σ	σ	σ	σ	0.1596	0.8338	0.9682

Table E8. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
9	9	9	9	0	1	1	1	σ	σ	σ	σ	0.3914	**0.6055**	0.4479
9	9	9	9	0	1.25	1.25	1.25	σ	σ	σ	σ	0.5065	**0.7602**	0.5796
9	9	9	9	0	1.5	1.5	1.5	σ	σ	σ	σ	0.5952	**0.866**0	0.6817
9	9	9	9	0	1.75	1.75	1.75	σ	σ	σ	σ	0.6811	**0.9314**	0.7701
9	9	9	9	0	2	2	2	σ	σ	σ	σ	0.7413	**0.9596**	0.8254
9	9	9	9	0	2.25	2.25	2.25	σ	σ	σ	σ	0.7911	**0.9788**	0.8689

Table E9. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
9	9	9	9	0	0	0	0	σ	2σ	2σ	2σ	**0.267**0	0.1282	0.1952
9	9	9	9	0	0	0	0	σ	2.5σ	2.5σ	2.5σ	**0.3911**	0.1801	0.3327
9	9	9	9	0	0	0	0	σ	3σ	3σ	3σ	**0.5042**	0.2230	0.4930
9	9	9	9	0	0	0	0	σ	3.5σ	3.5σ	3.5σ	**0.5767**	0.2653	0.6149
9	9	9	9	0	0	0	0	σ	4σ	4σ	4σ	**0.6378**	0.3042	0.7157
9	9	9	9	0	0	0	0	σ	4.5σ	4.5σ	4.5σ	**0.6897**	0.3378	0.7804

Table E10. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
9	9	9	9	0	0	0	0	σ	2σ	2σ	2σ	0.1993	0.0604	0.1730
9	9	9	9	0	0	0	0	σ	2.5σ	2.5σ	2.5σ	0.2515	0.0610	0.2044
9	9	9	9	0	0	0	0	σ	3σ	3σ	3σ	0.3235	0.0606	0.2560
9	9	9	9	0	0	0	0	σ	3.5σ	3.5σ	3.5σ	0.3676	0.0663	0.2916
9	9	9	9	0	0	0	0	σ	4σ	4σ	4σ	0.4069	0.0646	0.3163
9	9	9	9	0	0	0	0	σ	4.5σ	4.5σ	4.5σ	0.4340	0.0688	0.3348

Table E11. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
9	9	9	9	0	0.25	0.25	0.25	σ	2σ	2σ	2σ	**0.3502**	0.2079	0.1983
9	9	9	9	0	0.5	0.5	0.5	σ	2.5σ	2.5σ	2.5σ	**0.5576**	0.3593	0.3489
9	9	9	9	0	0.75	0.75	0.75	σ	3σ	3σ	3σ	**0.7121**	0.5007	0.4947
9	9	9	9	0	1	1	1	σ	3.5σ	3.5σ	3.5σ	**0.8168**	0.6080	0.6119
9	9	9	9	0	1.25	1.25	1.25	σ	4σ	4σ	4σ	**0.8791**	0.6891	0.7010
9	9	9	9	0	1.5	1.5	1.5	σ	4.5σ	4.5σ	4.5σ	**0.9256**	0.7636	0.7690

Table E12. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
9	9	9	9	0	0.25	0.25	0.25	σ	2σ	2σ	2σ	0.2928	0.1218	0.2648
9	9	9	9	0	0.5	0.5	0.5	σ	2.5σ	2.5σ	2.5σ	0.4572	0.1752	0.4072
9	9	9	9	0	0.75	0.75	0.75	σ	3σ	3σ	3σ	0.5962	0.2482	0.5413
9	9	9	9	0	1	1	1	σ	3.5σ	3.5σ	3.5σ	0.6960	0.3004	0.6384
9	9	9	9	0	1.25	1.25	1.25	σ	4σ	4σ	4σ	0.7732	0.3536	0.7163
9	9	9	9	0	1.5	1.5	1.5	σ	4.5σ	4.5σ	4.5σ	0.8218	0.3962	0.7637

Table E13. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	18	18	18	0	0	0	1	σ	σ	σ	σ	0.1887	**0.2189**	0.1118
18	18	18	18	0	0	0	1.5	σ	σ	σ	σ	0.3233	**0.3668**	0.1782
18	18	18	18	0	0	0	2	σ	σ	σ	σ	0.4890	**0.531**0	0.2686
18	18	18	18	0	0	0	2.5	σ	σ	σ	σ	0.6241	**0.6634**	0.3664
18	18	18	18	0	0	0	3	σ	σ	σ	σ	0.7245	**0.7538**	0.4461
18	18	18	18	0	0	0	3.5	σ	σ	σ	σ	0.8113	**0.8319**	0.5272

Table E14. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	18	18	18	0	0	0	1	σ	σ	σ	σ	0.1593	0.2196	0.1902
18	18	18	18	0	0	0	1.5	σ	σ	σ	σ	0.2153	0.3292	0.2743
18	18	18	18	0	0	0	2	σ	σ	σ	σ	0.2762	0.4243	0.3527
18	18	18	18	0	0	0	2.5	σ	σ	σ	σ	0.3178	0.5141	0.4243
18	18	18	18	0	0	0	3	σ	σ	σ	σ	0.3437	0.5671	0.4591
18	18	18	18	0	0	0	3.5	σ	σ	σ	σ	0.3715	0.6170	0.4968

Table E15. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	18	18	18	0	0	0	0	σ	σ	σ	4σ	**0.2222**	0.1209	0.1485
18	18	18	18	0	0	0	0	σ	σ	σ	5σ	**0.2517**	0.1327	0.1733
18	18	18	18	0	0	0	0	σ	σ	σ	6σ	**0.2714**	0.1379	0.1973
18	18	18	18	0	0	0	0	σ	σ	σ	7σ	**0.2902**	0.1458	0.2239
18	18	18	18	0	0	0	0	σ	σ	σ	8σ	**0.3051**	0.1523	0.2367
18	18	18	18	0	0	0	0	σ	σ	σ	9σ	**0.3203**	0.1614	0.2499

Table E16. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	18	18	18	0	0	0	0	σ	σ	σ	4σ	0.1386	0.0573	0.1003
18	18	18	18	0	0	0	0	σ	σ	σ	5σ	0.1424	0.0518	0.0987
18	18	18	18	0	0	0	0	σ	σ	σ	6σ	0.1402	0.0505	0.0990
18	18	18	18	0	0	0	0	σ	σ	σ	7σ	0.1503	0.0499	0.1010
18	18	18	18	0	0	0	0	σ	σ	σ	8σ	0.1602	0.0547	0.1101
18	18	18	18	0	0	0	0	σ	σ	σ	9σ	0.1470	0.0534	0.1014

Table E17. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	18	18	18	0	0	0	1	σ	σ	σ	4σ	**0.3083**	0.2025	0.1607
18	18	18	18	0	0	0	2	σ	σ	σ	5σ	**0.4162**	0.2890	0.2138
18	18	18	18	0	0	0	3	σ	σ	σ	6σ	**0.4893**	0.3538	0.2432
18	18	18	18	0	0	0	4	σ	σ	σ	7σ	**0.5406**	0.4057	0.2761
18	18	18	18	0	0	0	5	σ	σ	σ	8σ	**0.5986**	0.4507	0.3065
18	18	18	18	0	0	0	6	σ	σ	σ	9σ	**0.6286**	0.4802	0.3206

Table E18. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	18	18	18	0	0	0	1	σ	σ	σ	4σ	0.2069	0.1106	0.1690
18	18	18	18	0	0	0	2	σ	σ	σ	5σ	0.2791	0.1566	0.2362
18	18	18	18	0	0	0	3	σ	σ	σ	6σ	0.3205	0.1975	0.2775
18	18	18	18	0	0	0	4	σ	σ	σ	7σ	0.3622	0.2351	0.3261
18	18	18	18	0	0	0	5	σ	σ	σ	8σ	0.3882	0.2541	0.3469
18	18	18	18	0	0	0	6	σ	σ	σ	9σ	0.4112	0.2796	0.3771

Table E19. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	18	18	18	0	1	1	1	σ	σ	σ	σ	0.3975	0.7174	0.7089
18	18	18	18	0	1.25	1.25	1.25	σ	σ	σ	σ	0.4622	0.8405	0.8836
18	18	18	18	0	1.5	1.5	1.5	σ	σ	σ	σ	0.5188	0.9169	0.9714
18	18	18	18	0	1.75	1.75	1.75	σ	σ	σ	σ	0.5488	0.9603	0.9929
18	18	18	18	0	2	2	2	σ	σ	σ	σ	0.5741	0.9790	0.9995
18	18	18	18	0	2.25	2.25	2.25	σ	σ	σ	σ	0.5888	0.9891	0.9997

Table E20. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	18	18	18	0	1	1	1	σ	σ	σ	σ	0.6130	**0.8566**	0.7595
18	18	18	18	0	1.25	1.25	1.25	σ	σ	σ	σ	0.7478	**0.9556**	0.8870
18	18	18	18	0	1.5	1.5	1.5	σ	σ	σ	σ	0.8433	**0.9861**	0.9496
18	18	18	18	0	1.75	1.75	1.75	σ	σ	σ	σ	0.9115	**0.9955**	0.9773
18	18	18	18	0	2	2	2	σ	σ	σ	σ	0.9460	**0.9991**	0.9928
18	18	18	18	0	2.25	2.25	2.25	σ	σ	σ	σ	0.9651	**1**	0.9971

Table E21. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	18	18	18	0	0	0	0	σ	2σ	2σ	2σ	**0.4866**	0.2361	0.4797
18	18	18	18	0	0	0	0	σ	2.5σ	2.5σ	2.5σ	**0.6797**	0.3387	0.7466
18	18	18	18	0	0	0	0	σ	3σ	3σ	3σ	**0.7971**	0.4326	0.8903
18	18	18	18	0	0	0	0	σ	3.5σ	3.5σ	3.5σ	**0.8676**	0.5110	0.9595
18	18	18	18	0	0	0	0	σ	4σ	4σ	4σ	**0.9129**	0.5687	0.9870
18	18	18	18	0	0	0	0	σ	4.5σ	4.5σ	4.5σ	**0.936**0	0.6237	0.9940

Table E22. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	18	18	18	0	0	0	0	σ	2σ	2σ	2σ	0.3518	0.0733	0.2064
18	18	18	18	0	0	0	0	σ	2.5σ	2.5σ	2.5σ	0.4977	0.0769	0.2778
18	18	18	18	0	0	0	0	σ	3σ	3σ	3σ	0.6104	0.0857	0.3405
18	18	18	18	0	0	0	0	σ	3.5σ	3.5σ	3.5σ	0.6905	0.0887	0.3889
18	18	18	18	0	0	0	0	σ	4σ	4σ	4σ	0.7514	0.0990	0.4226
18	18	18	18	0	0	0	0	σ	4.5σ	4.5σ	4.5σ	0.7886	0.0975	0.4527

Table E23. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
18	18	18	18	0	0.25	0.25	0.25	σ	2σ	2σ	2σ	**0.6262**	0.4027	0.4844
18	18	18	18	0	0.5	0.5	0.5	σ	2.5σ	2.5σ	2.5σ	**0.8665**	0.6595	0.7629
18	18	18	18	0	0.75	0.75	0.75	σ	3σ	3σ	3σ	**0.9604**	0.8259	0.9067
18	18	18	18	0	1	1	1	σ	3.5σ	3.5σ	3.5σ	**0.9862**	0.9175	0.9656
18	18	18	18	0	1.25	1.25	1.25	σ	4σ	4σ	4σ	**0.9956**	0.9513	0.9862
18	18	18	18	0	1.5	1.5	1.5	σ	4.5σ	4.5σ	4.5σ	**0.9986**	0.9738	0.9949

Table E24. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
18	18	18	18	0	0.25	0.25	0.25	σ	2σ	2σ	2σ	0.4988	0.1694	0.3630
18	18	18	18	0	0.5	0.5	0.5	σ	2.5σ	2.5σ	2.5σ	0.7621	0.3217	0.6109
18	18	18	18	0	0.75	0.75	0.75	σ	3σ	3σ	3σ	0.8925	0.4374	0.7609
18	18	18	18	0	1	1	1	σ	3.5σ	3.5σ	3.5σ	0.9548	0.5436	0.8646
18	18	18	18	0	1.25	1.25	1.25	σ	4σ	4σ	4σ	0.9742	0.6003	0.9058
18	18	18	18	0	1.5	1.5	1.5	σ	4.5σ	4.5σ	4.5σ	0.9886	0.6850	0.9435

Table E25. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
30	30	30	30	0	0	0	1	σ	σ	σ	σ	0.2623	**0.312**0	0.1645
30	30	30	30	0	0	0	1.5	σ	σ	σ	σ	0.4853	**0.5517**	0.3056
30	30	30	30	0	0	0	2	σ	σ	σ	σ	0.6926	**0.7493**	0.4828
30	30	30	30	0	0	0	2.5	σ	σ	σ	σ	0.8462	**0.8793**	0.6423
30	30	30	30	0	0	0	3	σ	σ	σ	σ	0.9256	**0.9428**	0.7715
30	30	30	30	0	0	0	3.5	σ	σ	σ	σ	0.9664	**0.9745**	0.8544

Table E26. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
30	30	30	30	0	0	0	1	σ	σ	σ	σ	0.2135	0.3084	0.2626
30	30	30	30	0	0	0	1.5	σ	σ	σ	σ	0.2974	0.4732	0.3955
30	30	30	30	0	0	0	2	σ	σ	σ	σ	0.3901	0.6198	0.5136
30	30	30	30	0	0	0	2.5	σ	σ	σ	σ	0.4604	0.7242	0.6015
30	30	30	30	0	0	0	3	σ	σ	σ	σ	0.5140	0.7931	0.6764
30	30	30	30	0	0	0	3.5	σ	σ	σ	σ	0.5438	0.8338	0.7097

Table E27. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
30	30	30	30	0	0	0	0	σ	σ	σ	4σ	**0.3228**	0.1677	0.2736
30	30	30	30	0	0	0	0	σ	σ	σ	5σ	**0.3673**	0.1788	0.3289
30	30	30	30	0	0	0	0	σ	σ	σ	6σ	**0.409**0	0.2025	0.3849
30	30	30	30	0	0	0	0	σ	σ	σ	7σ	**0.4383**	0.2151	0.4222
30	30	30	30	0	0	0	0	σ	σ	σ	8σ	**0.4623**	0.2277	0.4507
30	30	30	30	0	0	0	0	σ	σ	σ	9σ	**0.4833**	0.2308	0.4852

Table E28. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
30	30	30	30	0	0	0	0	σ	σ	σ	4σ	0.2001	0.0632	0.1324
30	30	30	30	0	0	0	0	σ	σ	σ	5σ	0.2075	0.0608	0.1337
30	30	30	30	0	0	0	0	σ	σ	σ	6σ	0.2206	0.0611	0.1399
30	30	30	30	0	0	0	0	σ	σ	σ	7σ	0.2257	0.0608	0.1431
30	30	30	30	0	0	0	0	σ	σ	σ	8σ	0.2357	0.0601	0.1459
30	30	30	30	0	0	0	0	σ	σ	σ	9σ	0.2363	0.0621	0.1420

Table E29. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
30	30	30	30	0	0	0	1	σ	σ	σ	4σ	**0.4674**	0.3098	0.2955
30	30	30	30	0	0	0	2	σ	σ	σ	5σ	**0.6113**	0.4365	0.3869
30	30	30	30	0	0	0	3	σ	σ	σ	6σ	**0.7214**	0.5486	0.4805
30	30	30	30	0	0	0	4	σ	σ	σ	7σ	**0.7919**	0.6311	0.5414
30	30	30	30	0	0	0	5	σ	σ	σ	8σ	**0.8374**	0.688	0.6024
30	30	30	30	0	0	0	6	σ	σ	σ	9σ	**0.8656**	0.7189	0.6343

Table E30. Percentage of Rejection for k=4 Populations; T Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
30	30	30	30	0	0	0	1	σ	σ	σ	4σ	0.3236	0.1440	0.2527
30	30	30	30	0	0	0	2	σ	σ	σ	5σ	0.4308	0.2338	0.3552
30	30	30	30	0	0	0	3	σ	σ	σ	6σ	0.4986	0.2916	0.4311
30	30	30	30	0	0	0	4	σ	σ	σ	7σ	0.5548	0.3448	0.4862
30	30	30	30	0	0	0	5	σ	σ	σ	8σ	0.6054	0.3978	0.5448
30	30	30	30	0	0	0	6	σ	σ	σ	9σ	0.6303	0.4232	0.5765

Table E31. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
30	30	30	30	0	1	1	1	σ	σ	σ	σ	0.5914	0.8960	0.9171
30	30	30	30	0	1.25	1.25	1.25	σ	σ	σ	σ	0.6866	0.9668	0.9889
30	30	30	30	0	1.5	1.5	1.5	σ	σ	σ	σ	0.7587	0.9905	0.9983
30	30	30	30	0	1.75	1.75	1.75	σ	σ	σ	σ	0.7932	0.9970	1
30	30	30	30	0	2	2	2	σ	σ	σ	σ	0.8235	0.9989	1
30	30	30	30	0	2.25	2.25	2.25	σ	σ	σ	σ	0.8334	0.9996	1

Table E32. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
30	30	30	30	0	1	1	1	σ	σ	σ	σ	0.7962	**0.9685**	0.9179
30	30	30	30	0	1.25	1.25	1.25	σ	σ	σ	σ	0.9067	**0.9955**	0.9792
30	30	30	30	0	1.5	1.5	1.5	σ	σ	σ	σ	0.9628	**0.9992**	0.9956
30	30	30	30	0	1.75	1.75	1.75	σ	σ	σ	σ	0.9835	**1**	0.9990
30	30	30	30	0	2	2	2	σ	σ	σ	σ	0.9957	**1**	0.9999
30	30	30	30	0	2.25	2.25	2.25	σ	σ	σ	σ	0.9978	**1**	1

Table E33. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
30	30	30	30	0	0	0	0	σ	2σ	2σ	2σ	**0.6755**	0.3449	0.7535
30	30	30	30	0	0	0	0	σ	2.5σ	2.5σ	2.5σ	**0.8715**	0.5220	0.9498
30	30	30	30	0	0	0	0	σ	3σ	3σ	3σ	**0.9471**	0.6445	0.9924
30	30	30	30	0	0	0	0	σ	3.5σ	3.5σ	3.5σ	**0.9757**	0.7342	0.9987
30	30	30	30	0	0	0	0	σ	4σ	4σ	4σ	**0.9871**	0.7829	1
30	30	30	30	0	0	0	0	σ	4.5σ	4.5σ	4.5σ	**0.9922**	0.8268	1

Table E34. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
30	30	30	30	0	0	0	0	σ	2σ	2σ	2σ	0.5264	0.0915	0.3063
30	30	30	30	0	0	0	0	σ	2.5σ	2.5σ	2.5σ	0.7226	0.0974	0.4228
30	30	30	30	0	0	0	0	σ	3σ	3σ	3σ	0.8442	0.1134	0.5352
30	30	30	30	0	0	0	0	σ	3.5σ	3.5σ	3.5σ	0.9074	0.1244	0.5997
30	30	30	30	0	0	0	0	σ	4σ	4σ	4σ	0.9402	0.1315	0.6549
30	30	30	30	0	0	0	0	σ	4.5σ	4.5σ	4.5σ	0.9616	0.1430	0.6925

Table E35. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	L_1	L_2	Lepage
30	30	30	30	0	0.25	0.25	0.25	σ	2σ	2σ	2σ	**0.8298**	0.5905	0.7636
30	30	30	30	0	0.5	0.5	0.5	σ	2.5σ	2.5σ	2.5σ	**0.9773**	0.8724	0.9561
30	30	30	30	0	0.75	0.75	0.75	σ	3σ	3σ	3σ	**0.9976**	0.9624	0.9941
30	30	30	30	0	1	1	1	σ	3.5σ	3.5σ	3.5σ	**0.9997**	0.9906	0.9997
30	30	30	30	0	1.25	1.25	1.25	σ	4σ	4σ	4σ	**1**	0.9971	1
30	30	30	30	0	1.5	1.5	1.5	σ	4.5σ	4.5σ	4.5σ	**1**	0.9993	1

Table E36. Percentage of Rejection for k=4 Populations; T Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	μ_2	μ_3	μ_4	σ_1	σ_2	σ_3	σ_4	M_1	M_2	M_3
30	30	30	30	0	0.25	0.25	0.25	σ	2σ	2σ	2σ	0.7227	0.2466	0.5381
30	30	30	30	0	0.5	0.5	0.5	σ	2.5σ	2.5σ	2.5σ	0.9334	0.4738	0.8217
30	30	30	30	0	0.75	0.75	0.75	σ	3σ	3σ	3σ	0.9861	0.6534	0.9348
30	30	30	30	0	1	1	1	σ	3.5σ	3.5σ	3.5σ	0.9970	0.7580	0.9756
30	30	30	30	0	1.25	1.25	1.25	σ	4σ	4σ	4σ	0.9994	0.8344	0.9903
30	30	30	30	0	1.5	1.5	1.5	σ	4.5σ	4.5σ	4.5σ	0.9999	0.8794	0.9951

APPENDIX F. PERCENTAGE OF REJECTION FOR K=4 POPULATIONS (EXPONENTIAL DISTRIBUTION)

Table F1. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	1	1	1	1	1	1	1.75	1	0.0899	**0.141**0	0.0886
9	9	9	9	1	1	1	1	1	1	2	1	0.1354	**0.1891**	0.0999
9	9	9	9	1	1	1	1	1	1	2.25	1	0.1899	**0.242**0	0.1159
9	9	9	9	1	1	1	1	1	1	2.5	1	0.2530	**0.3029**	0.1398
9	9	9	9	1	1	1	1	1	1	2.75	1	0.3061	**0.3443**	0.1476
9	9	9	9	1	1	1	1	1	1	3	1	0.3568	**0.4016**	0.1610

Table F2. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	1	1	1	1	1	1	1	3^2	**0.0981**	0.0455	0.0701
9	9	9	9	1	1	1	1	1	1	1	4^2	**0.1074**	0.0443	0.0838
9	9	9	9	1	1	1	1	1	1	1	5^2	**0.1099**	0.0490	0.0887
9	9	9	9	1	1	1	1	1	1	1	6^2	**0.1201**	0.0509	0.0952
9	9	9	9	1	1	1	1	1	1	1	7^2	**0.1188**	0.0473	0.0946
9	9	9	9	1	1	1	1	1	1	1	8^2	**0.1255**	0.0512	0.1006

Table F3. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	1	1	1	1	1	1	2	2^2	0.1019	**0.1108**	0.0526
9	9	9	9	1	1	1	1	1	1	3	3^2	0.1521	**0.1679**	0.0672
9	9	9	9	1	1	1	1	1	1	4	4^2	0.1926	**0.2141**	0.0852
9	9	9	9	1	1	1	1	1	1	5	5^2	0.2307	**0.2533**	0.0929
9	9	9	9	1	1	1	1	1	1	6	6^2	0.2661	**0.2934**	0.1014
9	9	9	9	1	1	1	1	1	1	7	7^2	0.2940	**0.3195**	0.1138

Table F4. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	1	1	1.25	1	1.25	1	1.25	1	0.0330	**0.1082**	0.2227
9	9	9	9	1	1	1.5	1	1.5	1	1.5	1	0.0266	**0.2102**	0.5356
9	9	9	9	1	1	1.75	1	1.75	1	1.75	1	0.0205	**0.3373**	0.7739
9	9	9	9	1	1	2	1	2	1	2	1	0.0106	**0.4671**	0.9075
9	9	9	9	1	1	2.25	1	2.25	1	2.25	1	0.0059	**0.5812**	0.9651
9	9	9	9	1	1	2.5	1	2.5	1	2.5	1	0.0036	**0.6787**	0.9886

Table F5. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	1	1	1	2^2	1	2^2	1	2^2	**0.2207**	0.0459	0.5203
9	9	9	9	1	1	1	2.5^2	1	2.5^2	1	2.5^2	**0.2813**	0.0573	0.7488
9	9	9	9	1	1	1	3^2	1	3^2	1	3^2	**0.3105**	0.0593	0.8661
9	9	9	9	1	1	1	3.5^2	1	3.5^2	1	3.5^2	**0.3438**	0.0627	0.9215
9	9	9	9	1	1	1	4^2	1	4^2	1	4^2	**0.3726**	0.0747	0.9582
9	9	9	9	1	1	1	4.5^2	1	4.5^2	1	4.5^2	**0.3870**	0.0781	0.9738

Table F6. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
9	9	9	9	1	1	1.5	1.5^2	1.5	1.5^2	1.5	1.5^2	0.1634	**0.2056**	0.0935
9	9	9	9	1	1	2	2^2	2	2^2	2	2^2	0.3033	**0.4301**	0.1994
9	9	9	9	1	1	2.5	2.5^2	2.5	2.5^2	2.5	2.5^2	0.4157	**0.6322**	0.3392
9	9	9	9	1	1	3	3^2	3	3^2	3	3^2	0.5078	**0.7716**	0.4826
9	9	9	9	1	1	3.5	3.5^2	3.5	3.5^2	3.5	3.5^2	0.5771	**0.8709**	0.6116
9	9	9	9	1	1	4	4^2	4	4^2	4	4^2	0.5985	**0.9286**	0.7172

Table F7. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	1	1	1	1	1	1	1.75	1	0.1192	**0.2061**	0.1562
18	18	18	18	1	1	1	1	1	1	2	1	0.2040	**0.3049**	0.1955
18	18	18	18	1	1	1	1	1	1	2.25	1	0.3311	**0.4312**	0.2436
18	18	18	18	1	1	1	1	1	1	2.5	1	0.4532	**0.5322**	0.2897
18	18	18	18	1	1	1	1	1	1	2.75	1	0.5519	**0.6231**	0.3591
18	18	18	18	1	1	1	1	1	1	3	1	0.6443	**0.7018**	0.4256

Table F8. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	1	1	1	1	1	1	1	3^2	**0.1525**	0.0582	0.1811
18	18	18	18	1	1	1	1	1	1	1	4^2	**0.1687**	0.0622	0.2218
18	18	18	18	1	1	1	1	1	1	1	5^2	**0.1886**	0.0653	0.2557
18	18	18	18	1	1	1	1	1	1	1	6^2	**0.1999**	0.0647	0.2764
18	18	18	18	1	1	1	1	1	1	1	7^2	**0.1975**	0.0669	0.2846
18	18	18	18	1	1	1	1	1	1	1	8^2	**0.2035**	0.0685	0.3026

Table F9. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	1	1	1	1	1	1	2	2^2	0.1601	**0.1717**	0.0818
18	18	18	18	1	1	1	1	1	1	3	3^2	0.2960	**0.3149**	0.1374
18	18	18	18	1	1	1	1	1	1	4	4^2	0.4057	**0.4249**	0.1887
18	18	18	18	1	1	1	1	1	1	5	5^2	0.4942	**0.5177**	0.2370
18	18	18	18	1	1	1	1	1	1	6	6^2	0.5602	**0.5811**	0.2832
18	18	18	18	1	1	1	1	1	1	7	7^2	0.6167	**0.6425**	0.3188

Table F10. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	1	1	1.25	1	1.25	1	1.25	1	0.0417	**0.1578**	0.3888
18	18	18	18	1	1	1.5	1	1.5	1	1.5	1	0.0423	**0.3484**	0.8210
18	18	18	18	1	1	1.75	1	1.75	1	1.75	1	0.0514	**0.5594**	0.9666
18	18	18	18	1	1	2	1	2	1	2	1	0.0618	**0.7384**	0.9953
18	18	18	18	1	1	2.25	1	2.25	1	2.25	1	0.0750	**0.858**0	0.9999
18	18	18	18	1	1	2.5	1	2.5	1	2.5	1	0.0890	**0.9306**	1

Table F11. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	1	1	1	2^2	1	2^2	1	2^2	**0.382**0	0.0707	0.9389
18	18	18	18	1	1	1	2.5^2	1	2.5^2	1	2.5^2	**0.479**0	0.0807	0.9927
18	18	18	18	1	1	1	3^2	1	3^2	1	3^2	**0.53**01	0.0925	0.9987
18	18	18	18	1	1	1	3.5^2	1	3.5^2	1	3.5^2	**0.573**0	0.1082	0.9999
18	18	18	18	1	1	1	4^2	1	4^2	1	4^2	**0.5897**	0.1160	1
18	18	18	18	1	1	1	4.5^2	1	4.5^2	1	4.5^2	**0.6072**	0.1282	1

Table F12. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
18	18	18	18	1	1	1.5	1.5^2	1.5	1.5^2	1.5	1.5^2	0.2941	**0.3554**	0.1707
18	18	18	18	1	1	2	2^2	2	2^2	2	2^2	0.5857	**0.7244**	0.4667
18	18	18	18	1	1	2.5	2.5^2	2.5	2.5^2	2.5	2.5^2	0.7933	**0.9134**	0.7350
18	18	18	18	1	1	3	3^2	3	3^2	3	3^2	0.8991	**0.978**0	0.8930
18	18	18	18	1	1	3.5	3.5^2	3.5	3.5^2	3.5	3.5^2	0.9449	**0.9953**	0.9634
18	18	18	18	1	1	4	4^2	4	4^2	4	4^2	0.9718	**0.9987**	0.9857

Table F13. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 1, 2 and 3 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	1	1	1	1	1	1	1.75	1	0.1625	**0.2874**	0.2431
30	30	30	30	1	1	1	1	1	1	2	1	0.2934	**0.4418**	0.3137
30	30	30	30	1	1	1	1	1	1	2.25	1	0.4701	**0.6052**	0.4016
30	30	30	30	1	1	1	1	1	1	2.5	1	0.6457	**0.7488**	0.5131
30	30	30	30	1	1	1	1	1	1	2.75	1	0.7648	**0.8411**	0.6139
30	30	30	30	1	1	1	1	1	1	3	1	0.8549	**0.9021**	0.7014

Table F14. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 1, 2 and 3 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	1	1	1	1	1	1	1	3^2	**0.207**0	0.0750	0.3558
30	30	30	30	1	1	1	1	1	1	1	4^2	**0.2420**	0.0746	0.4384
30	30	30	30	1	1	1	1	1	1	1	5^2	**0.2621**	0.0799	0.4990
30	30	30	30	1	1	1	1	1	1	1	6^2	**0.2782**	0.0815	0.5349
30	30	30	30	1	1	1	1	1	1	1	7^2	**0.2945**	0.0860	0.5701
30	30	30	30	1	1	1	1	1	1	1	8^2	**0.2973**	0.0904	0.5922

Table F15. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 1, 2 and 3 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	1	1	1	1	1	1	2	2^2	0.2283	**0.2460**	0.1151
30	30	30	30	1	1	1	1	1	1	3	3^2	0.4470	**0.4755**	0.2464
30	30	30	30	1	1	1	1	1	1	4	4^2	0.6183	**0.6461**	0.3636
30	30	30	30	1	1	1	1	1	1	5	5^2	0.7243	**0.7423**	0.4590
30	30	30	30	1	1	1	1	1	1	6	6^2	0.8117	**0.8264**	0.5516
30	30	30	30	1	1	1	1	1	1	7	7^2	0.8619	**0.87**01	0.6242

Table F16. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 2, 3 and 4 have the same means

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	1	1	1.25	1	1.25	1	1.25	1	0.0454	**0.2159**	0.5767
30	30	30	30	1	1	1.5	1	1.5	1	1.5	1	0.0510	**0.4961**	0.9611
30	30	30	30	1	1	1.75	1	1.75	1	1.75	1	0.0686	**0.7556**	0.9982
30	30	30	30	1	1	2	1	2	1	2	1	0.0998	**0.9064**	0.9999
30	30	30	30	1	1	2.25	1	2.25	1	2.25	1	0.1419	**0.9718**	1
30	30	30	30	1	1	2.5	1	2.5	1	2.5	1	0.2016	**0.9908**	1

Table F17. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 2, 3 and 4 have the same variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	1	1	1	2^2	1	2^2	1	2^2	**0.5529**	0.0875	0.9976
30	30	30	30	1	1	1	2.5^2	1	2.5^2	1	2.5^2	**0.6702**	0.1112	1
30	30	30	30	1	1	1	3^2	1	3^2	1	3^2	**0.716**0	0.1298	1
30	30	30	30	1	1	1	3.5^2	1	3.5^2	1	3.5^2	**0.7544**	0.1423	1
30	30	30	30	1	1	1	4^2	1	4^2	1	4^2	**0.7683**	0.1589	1
30	30	30	30	1	1	1	4.5^2	1	4.5^2	1	4.5^2	**0.7836**	0.1665	1

Table F18. Percentage of Rejection for k=4 Populations; Exponential Distribution. Populations 2, 3 and 4 have the same means and variances

n_1	n_2	n_3	n_4	μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	L_1	L_2	Lepage
30	30	30	30	1	1	1.5	1.5^2	1.5	1.5^2	1.5	1.5^2	0.4262	**0.5222**	0.2996
30	30	30	30	1	1	2	2^2	2	2^2	2	2^2	0.8202	**0.9117**	0.7555
30	30	30	30	1	1	2.5	2.5^2	2.5	2.5^2	2.5	2.5^2	0.9530	**0.9916**	0.9529
30	30	30	30	1	1	3	3^2	3	3^2	3	3^2	0.9924	**0.9994**	0.9928
30	30	30	30	1	1	3.5	3.5^2	3.5	3.5^2	3.5	3.5^2	0.9979	**1**	0.9992
30	30	30	30	1	1	4	4^2	4	4^2	4	4^2	0.9997	**1**	0.9999