PROPOSED NONPARAMETRIC TESTS FOR THE UMBRELLA ALTERNATIVE IN A MIXED DESIGN

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The supervisory committee certifies that this dissertation complies with North Dakota State University's regulations and meets the accepted standards for the degree of

DOCTOR OF PHILOSOPHY

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ABSTRACT

Several nonparametric tests are proposed for a mixed design consisting of a randomized complete block design (RCBD) and a completely randomized design (CRD) under the umbrella hypothesis with a known and an unknown peak. The combination of the two statistics is based on two different methods. A simulation study was conducted to investigate the performance of the proposed mixed design tests under many different cases.

In either case of a known or an unknown peak umbrella hypothesis, the estimated power of the first method used for the proposed test statistics is better than the second method for all situations. We use a square distance as a weight in terms of assessing the power's performance of the proposed test statistics for the known peak umbrella hypothesis. The square distance modification improves in increasing the test's power; in particular, if the peak is ***indistinct*** with the first location parameter for four and five treatments, or if the location parameter on the left side of the umbrella hypothesis (*upside*) is ***greater*** than all the different location parameters on the right side of the umbrella hypothesis (*downside*) such as, (0.8 , **1.0** , 0.75 , 0.2) ; (0.75 , **0.8** , 0.6 , 0.4 , 0.2). Also, the modification improves the test's power for five treatments and peak at 3 once the underlying distribution is symmetric, as long as the peak of the umbrella hypothesis is ***distinct***.

In general, for the unknown peak umbrella hypothesis, the result of the test's power differs slightly between a modification and nonmodification cases. However, we can distinguish some cases based on the type of underlying distribution. In the case of having a symmetric distribution, the square distance modification is much better than test statistics without modification for some cases once we have four and five treatments. For the case of having three treatments; the estimated power for the proposed test statistics with a square distance modification (3.71), (3.72) is slightly different from the estimated power for the test statistic without modification (3.69), (3.70) in both cases of underlying distributions "symmetric and skewed."

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DEDICATION

This dissertation is dedicated to my family.

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1. INTRODUCTION

Many statisticians and scientists prefer the nonparametric approach since it has specific desirable properties. It requires a few assumptions about the underlying populations from which the data are obtained. In particular, most nonparametric tests assume that the underlying distributions are the same type but possibly differ in location. In many cases, the increasing or decreasing nature of the parameters is assumed to be known a priori like drug dosage levels, if the parameters are different. The effect of a drug on the experimental unit might increase to a certain level, and then its effectiveness decreases with further increasing doses. In this case, an umbrella alternative is an appropriate model. The hypothesis for an umbrella alternative is given in (1.1)

$$H_0 : \mu_1 = \mu_2 = \ldots = \mu_k$$
$$H_1 : \mu_1 \leq \mu_2 \leq \ldots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \ldots \geq \mu_k \qquad \textit{with at least one strict inequality} \qquad (1.1)$$

where μ_i is a location parameter for the i^{th} population ; $i = 1, 2, \ldots, p, \ldots, k$, and p is the peak of the umbrella alternative.

When speaking of any study's design, the first thing that a researcher should start with is thinking of the design that would be used to collect the data. It is possible that the researcher would begin an experiment with a randomized complete block design (RCBD). Then for some reason, he could not collect the data for all treatments in each block, as seen in Table 1.1.

Table 1.1. Initial Randomized Complete Block Design Missing Observations Occurred

Number of Block	**Treatment 1**	**Treatment 2**	**Treatment 3**	**Treatment 4**
1^{st}	√	√	√	√
2^{nd}	√	√	√	√
3^{rd}	√			
4^{th}	√	√	√	√
5^{th}			√	
6^{th}		√		
7^{th}				√

One of the reasons could be the experiment is so expensive, and missing observations could occur. At this point, the researcher would decide to change to a completely randomized design (CRD), and the consequence of changing the design would be losing all data collected from (RCBD), as seen in Table 1.2. However, not using all available data is a waste of resources and time.

Table 1.2. Completely Randomized Design

Treatment 1	**Treatment 2**	**Treatment 3**	**Treatment 4**
√			
		√	
	√		
			√

As a fact, the more observations that we collect, the more powerful the test will be. For this purpose, we can think of combining the two designs, as seen in Table 1.3.

Table 1.3. Mixed Design of Randomized Complete Block Design and Completely Randomized Design

Design	**Number of Block**	**Treatment 1**	**Treatment 2**	**Treatment 3**	**Treatment 4**
RCBD	1^{st}	√	√	√	√
	2^{nd}	√	√	√	√
	4^{th}	√	√	√	√
CRD		√			
				√	
			√		
					√

In this case, we need to introduce a new test statistic for this kind of design, which is called a mixed design. Magel et al. (2010) introduced a new test statistic for a mixed design consisting of (RCBD) and (CRD). We suggest adding a modification to the test statistics proposed by Magel et al. (2010) in order to improve the performance of the test's power. The modifier that we use is a square distance between compare groups, as discussed in Chapter 3 in the two cases of a known and unknown peak. In addition to this, we use two different methods of combining the test statistics for

a completely randomized design (CRD) portion and a randomized complete block design (RCBD). Test statistics vary based on the number of treatments and the specified peak, if known.

The objective of this research is to assess the performance of the test statistics for a mixed design once we use a square distance as a modification for a known and an unknown peak of the alternative umbrella hypothesis. We will compare the power of the tests proposed by Magel et al. (2010) with the proposed test statistics in this research. The rest of this research is organized as follows. In Chapter 2, we will present some of the previous studies on nonparametric statistical tests for a mixed design under a different type of alternative hypotheses. In Chapter 3, we will introduce the proposed test statistics under the umbrella hypothesis for both cases of a known and an unknown peak. In Chapter 4, the details of the simulation study will be given. In Chapter 5, we will present and discuss the results from the simulation study for all cases described in Chapter 4. Finally, Chapter 6 will contain the conclusions about the proposed test statistics and when they may be preferred.

2. PREVIOUS STUDIES

2.1. Mixed Designs

Recently, researchers have been working on mixed designs to use all the information from their data. They have developed many of the statistical tests for mixed design in nonparametric statistics. We will discuss some of these tests for location parameters in this Chapter.

2.1.1. Dubnicka, Blair and Hettmansperger

Dubnicka et al. (2002) have developed a robust nonparametric approach to testing in a mixed design. Mixtures of paired and unpaired data can be seen in a variety of experiments, especially once the researcher is comparing the effect of two different treatments on paired data; then, unfortunately, one of the treatments cannot be applied to some paired cases. In this situation, the researcher can assign those cases to one of the treatment groups.

Dubnicka et al. (2002) used a rank-based procedure to develop a nonparametric test for mixed paired and two-sample design. They combined the Wilcoxon signed-rank statistic by Wilcoxon (1945) and the Wilcoxon-Mann-Whitney statistic by Mann and Whitney (1947).

$$T^{+}(\Delta) = S^{+}(\Delta) + U^{+}(\Delta) \quad ; \quad \Delta = \theta_1 - \theta_2 \tag{2.1}$$

where $S^{+}(\Delta)$ is the Wilcoxon signed-rank statistic; $U^{+}(\Delta)$ is the Wilcoxon-Mann-Whitney statistic.

The mean and variance for $T^{+}(\Delta)$ the proposed test statistic by Dubnicka et al. (2002) for mixed design under the null distribution; $H_0 : (\Delta) = 0$ respectively are :

$$E_0 T^{+}(0) = \frac{n\,(n+1)}{4} + \frac{n_1\, n_2}{2} \tag{2.2}$$

$$Var_0 T^{+}(0) = \frac{n\,(n+1)\,(2n+1)}{24} + \frac{n_1\, n_2\,(n_1+n_2+1)}{12} \tag{2.3}$$

The mean for $T^{+}(\Delta)$ (2.2) is a combination of the mean of the Wilcoxon signed-rank statistic $\frac{n\,(n+1)}{4}$ and the mean of the Wilcoxon-Mann-Whitney statistic $\frac{n_1\, n_2}{2}$. Similarly, they combined the variance of Wilcoxon signed-rank statistic $\frac{n\,(n+1)\,(2n+1)}{24}$ and the variance of the Wilcoxon-Mann-

Whitney statistic $\frac{n_1\, n_2\, (n_1+n_2+1)}{12}$ to find the variance (2.3) for the proposed test statistic for a mixed paired and two-sample design. The values n_1 and n_2 represent the sample size for the two independent samples, whereas n represents the sample size for paired data. The standardized version of the proposed test statistic by Dubnicka et al. (2002) $T^{+}(\Delta)$, under the null distribution is:

$$T^{*}(0) = \frac{T^{+}(0) - E_0 T^{+}(0)}{\sqrt{Var_0 T^{+}(0)}} \tag{2.4}$$

which has an asymptotic standard normal distribution. Consequently, the null hypothesis will be rejected in favor of one side alternative when $T^{*}(0) > Z_{\alpha}$, where Z_{α} is the $1 - \alpha \times 100$ percentile of a standard normal distribution.

2.1.2. Magel and Fu

Magel and Fu (2014) proposed a similar test to the one developed by Dubnicka et al. (2002) for a mixed pair and two-sample design. They combined the standardized versions of the Wilcoxon signed-rank statistic by Wilcoxon (1945) and the Wilcoxon-Mann-Whitney statistic by Mann and Whitney (1947).

$$T_I^{+}(\Delta) = S^{+}(\Delta)^{*} + U^{+}(\Delta)^{*} \quad ; \quad \Delta = \theta_1 - \theta_2 \tag{2.5}$$

where $S^{+}(\Delta)^{*}$ is the standardized version of the Wilcoxon signed-rank statistic; $U^{+}(\Delta)^{*}$ is the standardized version of Wilcoxon-Mann-Whitney statistic.

The mean and variance for $T_I^{+}(\Delta)$ the proposed test statistic by Magel and Fu (2014) for mixed design under the null distribution; $H_0 : (\Delta) = 0$ respectively are :

$$E_0 T_I^{+}(0) = 0 \tag{2.6}$$

$$Var_0 T_I^{+}(0) = 2 \tag{2.7}$$

The standardized version of $T_I^+(\Delta)$; the proposed test statistic by Magel and Fu (2014) under the null distribution is:

$$T_I^*(0) = \frac{T_I^+(0) - E_0 T_I^+(0)}{\sqrt{Var_0 T_I^+(0)}} \tag{2.8}$$

which has an asymptotic standard normal distribution. Consequently, the null hypothesis will be rejected in favor of one side alternative when $T_I^*(0) > Z_\alpha$, where Z_α is the $1 - \alpha \times 100$ percentile of a standard normal distribution.

In the end, we can say the standardized version for both proposed test statistics (2.4) and (2.8) are a combination of weighted versions of the standardized Wilcoxon signed-rank statistic and the standardized Wilcoxon-Mann-Whitney statistic.

$$T^*(0) = \sqrt{(\frac{\sigma_S^2}{\sigma_S^2 + \sigma_U^2})}\, S^+(0)^* + \sqrt{(\frac{\sigma_U^2}{\sigma_S^2 + \sigma_U^2})}\, U^+(0)^* \tag{2.9}$$

$$T_I^*(0) = \frac{1}{\sqrt{2}}\, S^+(0)^* + \frac{1}{\sqrt{2}}\, U^+(0)^* \tag{2.10}$$

2.1.3. Magel, Terpstra, Canonizado and Park

Magel et al. (2010) developed various test statistics for mixed designs. They considered three different designs in the general alternative and one design in the umbrella alternative with a known peak, p.

I. **General Alternative**

- **Design 1:** Magel et al. (2010) developed a test statistic for a mixed design that is a combination of a randomized complete block design (RCBD) and a completely randomized design (CRD). The proposed test statistic T_1 is a combination of Friedman test statistic F by Friedman (1937) and Kruskal-Wallis test statistic K by Kruskal et al. (1952).

Under the null hypothesis, T_1 has an asymptotic chi-square distribution with $2k-2$ degrees of freedom.

$$
\begin{aligned}
T_1 &= F + K \\
T_1 &= \frac{12}{bk(k+1)} \sum_{j=1}^{k} \Big[R_j - \frac{b\,(k+1)}{2}\Big]^2 + \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{1}{n_i} \Big[R_i - \frac{n_i(N+1)}{2}\Big]^2
\end{aligned} \tag{2.11}
$$

- **Design 2:** Magel et al. (2010) dealt with a mixture of a randomized complete block design (RCBD), a completely randomized design (CRD), and a matched pair design. The proposed test statistic T_2 is a combination of the Friedman test statistic F by Friedman (1937), the Kruskal-Wallis test statistic K by Kruskal et al. (1952), and a square of the Wilcoxon signed rank test statistic W^2 by Wilcoxon (1945). Under the null hypothesis, T_2 has an asymptotic chi-square distribution with $2k-1$ degrees of freedom.

$$
\begin{aligned}
T_2 &= F + K + W^2 \\
T_2 &= \frac{12}{bk(k+1)} \sum_{j=1}^{k} \Big[R_j - \frac{b\,(k+1)}{2}\Big]^2 + \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{1}{n_i} \Big[R_i - \frac{n_i(N+1)}{2}\Big]^2 \\
&\quad + \Big[\frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}\Big]^2
\end{aligned} \tag{2.12}
$$

- **Design 3:** In this design Magel et al. (2010) considered a combination of a randomized complete block design (RCBD), a completely randomized design (CRD), an incomplete block design (IBD), and a matched pair design. There are two proposed test statistics T_3 and T_4.

  T_3 is a combination of the Friedman test statistic F by Friedman (1937), the Kruskal-Wallis test statistic K by Kruskal et al. (1952), a square of the Wilcoxon signed rank test statistic W^2 Wilcoxon (1945), and the Durbin test statistic D Durbin (1951) when there are four populations. Under the null hypothesis, T_3 has an asymptotic chi-square distribution with $3k-2$ degrees of freedom.

$$T_3 = F + K + W^2 + D$$

$$T_3 = \frac{12}{bk(k+1)} \sum_{j=1}^{k} \Big[R_j - \frac{b\,(k+1)}{2}\Big]^2 + \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{1}{n_i} \Big[R_i - \frac{n_i(N+1)}{2}\Big]^2 + \Big[\frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}\Big]^2 + \frac{12(k-1)}{rk(a-1)(a+1)} \sum_{j=1}^{t} R_j^2 - \frac{3r(k-1)(a+1)}{a-1} \quad (2.13)$$

T_4 is a combination of the Friedman test statistic F by Friedman (1937), the Kruskal-Wallis test statistic K by Kruskal et al. (1952), a square of the Wilcoxon signed rank test statistic W^2 Wilcoxon (1945), the Durbin test statistic $D1$ of 3 observations per block by Durbin (1951), and the Durbin test statistic $D2$ of 4 observations per block by Durbin (1951) when there are five populations. Under the null hypothesis, T_4 has an asymptotic chi-square distribution with $4k-3$ degrees of freedom.

$$T_3 = F + K + W^2 + D_1 + D_2$$

$$T_3 = \frac{12}{bk(k+1)} \sum_{j=1}^{k} \Big[R_j - \frac{b\,(k+1)}{2}\Big]^2 + \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{1}{n_i} \Big[R_i - \frac{n_i(N+1)}{2}\Big]^2 + \Big[\frac{T - \frac{n(n+1)}{4}}{\sqrt{\frac{n(n+1)(2n+1)}{24}}}\Big]^2 + \frac{12(k-1)}{r_1 k(a_1-1)(a_1+1)} \sum_{j=1}^{t} R_j^2 - \frac{3r_1(k-1)(a_1+1)}{a_1-1} + \frac{12(k-1)}{r_2 k(a_2-1)(a_2+1)} \sum_{j=1}^{t} R_j^2 - \frac{3r_2(k-1)(a_2+1)}{a_2-1} \quad (2.14)$$

II. **Umbrella Alternative**

- **Design 4:** Magel et al. (2010) considered a combination of a Kim-Kim test statistic by Kim and Kim (1992) for a randomized complete block design (RCBD) and a Mack-Wolfe test statistic by Mack and Wolfe (1981) for a completely randomized design (CRD). They considered equal sample sizes for treatments in (CRD); and one observation for i^{th} treatment and j^{th} block in (RCBD). They proposed two test statistics A_p^{**} , and A_p^{***} are given in (2.15),(2.17) respectively;

– **First proposed test statistic A_p^{**}:**

They added the standardized version of Mack-Wolfe test statistic A_p^{**}, and the standardized version of Kim-Kim test statistic A^*. Under the null hypothesis, A_p^{**} has an asymptotic normal distribution with mean zero and variance 2.

$$A_p^{**} = A_p^* + A^* \tag{2.15}$$

The standardized version of A^{**} is

$$A^{**} = \frac{A_p^{**} - 0}{\sqrt{2}} \tag{2.16}$$

which has an asymptotic standard normal distribution under the null hypothesis.

– **Second proposed test statistic A_p^{***} :**

They added Mack-Wolfe statistic A_p, and Kim-Kim statistic A.

$$A_p^{***} = A_p + A \tag{2.17}$$

The mean and variance of A_p^{***} under the null hypothesis are given in (2.18) , (2.19) respectively:

$$\begin{aligned} E_0 A_p^{***} &= E_0 A_p + E_0 A \\ &= \frac{N_1^2 + N_2^2 - \sum_{i=1}^{k} n_i^2 - n_p^2}{4} + \frac{b(p^2 + (k-p+1)^2 - k - 1)}{4} \end{aligned} \tag{2.18}$$

and

$$\begin{aligned} Var_0 A_p^{***} &= Var_0 A_p + Var_0 A \\ &= \frac{1}{72}\{2(N_1^3 + N_2^3) + 3(N_1^2 + N_2^2) - \sum_{i=1}^{k} n_i^2(2n_i + 3) - n_p^2(2n_p + 3) \\ &+ 12n_p N_1 N_2 - 12n_p^2 N\} + \frac{b}{72}\{2[p^3 + (k-p+1)^3] + 3[p_2 + (k-p+1)^2] \\ &- 5k - 5 + 12p(k-p+1) - 12k\} \end{aligned} \tag{2.19}$$

The standardized version of A_p^{***} is

$$A^{***} = \frac{A_p^{***} - E_0 A_p^{***}}{\sqrt{Var_0 A_p^{***}}} \tag{2.20}$$

which has an asymptotic standard normal distribution under the null hypothesis.

2.1.4. Magel, Terpstra, and Wen

Magel et al. (2009) developed two variations of test statistics for a mixed design of a randomized complete block (RCBD) and a completely randomized design (CRD) to test for a nondecreasing alternative which is a particular case from umbrella alternative. The proposed two test statistics Z_{comb}, and A_{combII} are given in (2.21),(2.22) respectively;

- **First proposed test statistic Z_{comb} :**

  They proposed the standardized version of a combination of a standardized version of Page statistic by Page (1963) Z_P, and the standardized version of Jonckheer-Terpstra statistic by Jonckheere (1954) Terpstra (1952) Z_{JT}. Under the null hypothesis, Z_{comb} has an asymptotic standard normal distribution.

$$Z_{comb} = \frac{Z_P + Z_{JT}}{\sqrt{2}} \tag{2.21}$$

- **Second proposed test statistic Z_{combII} :**

  They proposed the standardized version of a combination of Page statistic L by Page (1963), and Jonckheer-Terpstra statistic J by Jonckheere (1954) Terpstra (1952). Under the null hypothesis, Z_{combII} has an asymptotic standard normal distribution.

$$Z_{combII} = \frac{(L+J) - E_0(L+J)}{\sqrt{Var_0(L+J)}} \tag{2.22}$$

where

$$E_0(L+J) = \frac{bk(k+1)^2}{4} + \frac{N^2 - \sum_{i=1}^{k} n_i^2}{4} \tag{2.23}$$

$$Var_0(L+J) = \frac{b(k^3-k)^2}{144(k-1)} + \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)}{72} \tag{2.24}$$

2.2. Nondecreasing Alternative - Completely Randomized Design

Many tests have been proposed for nondecreasing alternatives, The common one is JT test statistic by Jonckheere (1954) and Terpstra (1952). They calculated the $\frac{k(k-1)}{2}$ Mann-Whitney counts, U_{ij}. where, k is the number of treatments. They sum these $\frac{k(k-1)}{2}$ of U_{ij} as

$$JT = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} U_{ij} \tag{2.25}$$

Some of the researchers proposed modified nonparametric tests for nondecreasing alternatives, such as,

- **Modified jonckheere**

  Neuhäuser et al. (1998) introduced a modified JT (MJT) test, which is weighted by the distance between groups, and the test statistic is given as

  $$MJT = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i) U_{ij} \tag{2.26}$$

  also, under the null hypothesis, the standardized version of, MJT has an asymptotic standard normal distribution.

- **Rank modification**

  Shan et al. (2014) proposed a nonparametric for an ordered alternative hypothesis based on the rank difference between two observations from different independent groups as given in (2.27)

  $$S = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} D_{ij} \tag{2.27}$$

  where $D_{ij} = \sum_{l=1}^{n_i} \sum_{m=1}^{n_j} Z_{ijlm} = (R_{jm} - R_{il}) I(X_{jm} > X_{il})$, R_{jm} and R_{il} denote to the rank of the observation X_{il} and X_{jm} in a combined data set, respectively.

2.3. Umbrella Alternatives

There are several nonparametric tests that have been developed for a known and an unknown peak for umbrella alternatives. The most familiar one is Mack and Wolfe (1981), and it is based on pairwise Mann and Whitney (1947) statistics for a known peak, while Hettmansperger and Norton

(1987) suggested a class of rank tests using a different weighting scheme. Shi (1988) proposed a rank test similar to the Hettmansperger and Norton (1987) test. For a case of an unknown peak, Mack and Wolfe (1981) estimated the peak to be at the treatment that maximizes a collection of combined Mann-Whitney statistics, but on the other hand Hettmansperger and Norton (1987), Shi (1988), Chen and Wolfe (1990), and Chen (1991) estimated the peak to be at the treatment that maximizes their test statistics. Also, Gökpinar and Gökpinar (2016) developed a test for both a known and an unknown peak, and it is based on linear ranks. Here some details about it.

- **Esra Gokpinar and Fikri Gokpinar**

  Gökpinar and Gökpinar (2016) proposed a modified Mack-Wolfe (MMW_p) test statistic for a known and an unknown umbrella peak.

  In the case of a known peak *(p)*, they combined the two modified JT statistics (MJT_{up}) and (MJT_{down}), the test statistic is given as (2.28)

$$\begin{aligned} MMW_p &= MJT_{up} + MJT_{down} \\ MMW_p &= \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (j-i)U_{ij} + \sum_{i=p}^{k-1} \sum_{j=i+1}^{k} (j-i)U_{ji} \end{aligned} \tag{2.28}$$

  The null mean and variance of MMW_p are given in (2.29), (2.30) respectively; when the sample sizes are equal for treatments:

$$E_0 MMW_p = \frac{n^2}{2}\Big[\binom{p+1}{3} + \binom{k-p+2}{3}\Big] \tag{2.29}$$

$$\begin{aligned} Var_0 MMW_p = &\frac{n^2p^2(p^2-1)(np+1) + n^2(k-p+1)^2[((k-p+1)^2)-1][n(k-p+1)+1]}{144} \\ &+ \frac{n^3p(p-1)(k-p)(k-p+1)}{24} \end{aligned} \tag{2.30}$$

  as a result, the modified Mack-Wolfe (MMW_p) test statistic is asymptotically a standard normal distribution under the null hypothesis when it is standardized.

In the case of an unknown peak, they applied the idea of Mack and Wolfe (1981) for an unknown umbrella peak. They first estimated the peak (p) from the sample data, and this could be accomplished by calculating k combined-samples Mann-Whitney statistics

$$U_{.q} = \sum_{i \neq q} U_{iq}, \qquad for\ q = 1, 2, \ldots, k \tag{2.31}$$

where U_{iq} is the number of i^{th} sample observations that precede q^{th} sample observations which is the Mann-Whitney statistic for i^{th} and q^{th} samples. Therefore, $U_{.q}$ is a single Mann-Whitney statistic computed between the q^{th} sample and the remaining (k-1) samples combined. Then after that, they standardize each of the $U_{.q}$'s

$$U^*_{.q} = \frac{U_{.q} - E_0(U_{.q})}{\sqrt{Var_0(U_{.q})}}, \qquad for\ q = 1, 2, \ldots, k \tag{2.32}$$

where, $E_0(U_{.q}) = \frac{n_q(N-n_q)}{2}$ and $Var_0(U_{.q}) = \frac{n_q(N-n_q)(N+1)}{12}$.

The test statistic can be written as

$$MMW^*_{\hat{p}} = \frac{MMW_{\hat{p}} - E_0(MMW_{\hat{p}})}{\sqrt{Var_0(MMW_{\hat{p}})}} \tag{2.33}$$

The null hypothesis can be rejected for large values of $MMW^*_{\hat{p}}$. $\hat{p}$ is the estimate of the unknown umbrella peak p corresponding to the maximum $U^*_{.q}$. However, there is a chance of getting r treatments tied for the maximum of $U^*_{.q}$. In this case, the test statistic $MMW^*_{\hat{p}}$ is equal to the average of those standardized peak known statistics corresponding to peaks at each of the r samples tied for the maximum $U^*_{.q}$.

3. METHODOLOGY

In this research, we are modifying a nonparametric test statistic for a mixed design of a randomized complete block design (RCBD) and a completely randomized design (CRD) based on the umbrella hypothesis given in (1.1) in the case of a known and unknown peak. As in Dubnicka et al. (2002) and Magel et al. (2010), we are considering test statistics in terms of combining a modified version of test statistics weighted by the squared distance between groups for the (RCBD) and (CRD).

3.1. Modified Mack-Wolfe Test Statistic II

We will introduce another modification of the Mack-Wolfe test statistic for the (CRD) portion, which differs from the one investigated by Gökpinar and Gökpinar (2016). The modified version of the Mack-Wolfe test statistic MMW_{pII} in (3.1) gives weight to the Mann-Whitney statistics, which is a squared distance between groups.

$$MMW_{pII} = \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (j-i)^2 U_{ij} + \sum_{i=p}^{k-1} \sum_{j=i+1}^{k} (j-i)^2 U_{ji} \tag{3.1}$$

where U_{ij} is a Mann-Whitney statistic applied to the observations in i^{th} and j^{th} groups.

3.1.1. The Mean and Variance

Mack and Wolfe (1981) proposed a test statistic that has a null asymptotic normality, so the modified Mack-Wolfe test statistic MMW_{pII} in (3.1), when it is standardized, is asymptotically a standard normal distribution under H_0. For this purpose, we need to know the expected value and variance of MMW_{pII} when H_0 is true, and for simplicity, we will use the same sample size for all treatments.

Theorem 3.1.1. *The null expected value of a modified version of Mack-Wolfe test statistic MMW_{pII}, when the sample sizes are equal is given by*

$$E_0(MMW_{pII}) = \frac{n^2}{24}\Big\{p^2(p^2-1) + (k-p+1)^2\big[(k-p+1)^2-1\big]\Big\} \tag{3.2}$$

□

Proof : In terms of finding the null expected value of MMW_{pII}, we need first to define the null expected values for the Mann-Whitney statistic U_{ij} from Tryon and Hettmansperger (1973). The null expected value of U_{ij} is

$$E_0(U_{ij}) = \frac{1}{2} n_i n_j \; ; \; \forall \; i \neq j \tag{3.3}$$

where n_i and n_j are the sample sizes for the i^{th} and j^{th} group respectively.

Once the sample sizes are all equal to n, then the null expected value of U_{ij} in (3.3) reduces to

$$E_0(U_{ij}) = \frac{n^2}{2} \; ; \; \forall \; i \neq j \tag{3.4}$$

The null expected value of MMW_{pII} is

$$E_0(MMW_{pII}) = \left[\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (j-i)^2 E_0(U_{ij}) + \sum_{i=p}^{k-1} \sum_{j=i+1}^{k} (j-i)^2 E_0(U_{ji}) \right] \tag{3.5}$$

Since we consider the equal sample size, we substitute the null expected value of U_{ij} as given in (3.4) to equation (3.5).

$$E_0(MMW_{pII}) = \frac{n^2}{2} \left[\underbrace{\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (j-i)^2}_{1} + \underbrace{\sum_{i=p}^{k-1} \sum_{j=i+1}^{k} (j-i)^2}_{2} \right] \tag{3.6}$$

we can simplify part 1 and part 2 in (3.6) as follows:

$$\begin{aligned} \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (j-i)^2 &= \sum_{i=1}^{p-1} \left[1^2 + 2^2 + \ldots + (p-i)^2 \right] \\ &= \sum_{i=1}^{p-1} \frac{(p-i)(p-i+1)(2p-2i+1)}{6} \\ &= \frac{p^2(p^2-1)}{12} \end{aligned} \tag{3.7}$$

$$\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^2 = \sum_{i=p}^{k-1}\left[1^2+2^2+\ldots+(k-i)^2\right]$$
$$= \sum_{i=p}^{k-1}\frac{(k-i)(k-i+1)(2k-2i+1)}{6}$$
$$= \frac{(k-p+1)^2[(k-p+1)^2-1]}{12} \tag{3.8}$$

Now, after we simplified part 1 and part 2, we will substitute them in (3.6) to get the final form for the null expected value of MMW_{pII} as given in (3.9)

$$E_0(MMW_p) = \frac{n^2}{24}\Big\{p^2(p^2-1)+(k-p+1)^2\big[(k-p+1)^2-1\big]\Big\} \tag{3.9}$$

□

Theorem 3.1.2. *The null variance of a modified version of Mack-Wolfe test statistic MMW_{pII}, when the sample sizes are equal is given by*

$$Var_0(MMW_{pII}) = \frac{n^2(2n+1)}{12}\Big\{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^4+\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^4\Big\}$$
$$+\frac{n^3}{12}\Big\{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}sum_{ij}+\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}sum_{ji}\Big\}$$
$$+\frac{n^3}{6}\Big\{\frac{p(p-1)(k-p)(k-p+1)[2(k-p)+1][2(p-1)+1]}{36}\Big\} \tag{3.10}$$

where,

$$sum_{ij} = (j-i)^2\Big[-(j-i)^2-\sum_{t=1}^{i-1}t^2+\sum_{t=1}^{i-1}(j-t)^2+\sum_{t=1}^{p-i}t^2+\sum_{t=0}^{j-i-1}t^2-\sum_{t=1}^{p-j}t^2\Big]$$

$$sum_{ji} = (j-i)^2\Big[-(j-i)^2-\sum_{t=1}^{i-p}t^2+\sum_{t=p}^{i-1}(j-t)^2+\sum_{t=1}^{k-i}t^2+\sum_{t=0}^{j-i-1}t^2-\sum_{t=1}^{k-j}t^2\Big]$$

□

Proof : In terms of finding the null variance of MMW_{pII}, we need first to define the null variance and covariances of the Mann-Whitney statistic U_{ij} from Tryon and Hettmansperger (1973).

- The null variance of U_{ij} is

$$Var_0(U_{ij}) = \frac{1}{12} n_i n_j (n_i + n_j + 1) \; ; \; \forall \; i \neq j \tag{3.11}$$

  where n_i and n_j are the sample sizes for the i^{th} and j^{th} group respectively.

- The null covariances of U_{ij}'s are

$$Cov_0(U_{ij}, U_{il}) = Cov_0(U_{ji}, U_{li}) = \frac{1}{12} n_i n_j n_l \qquad \textit{if all i, j, l are different} \tag{3.12}$$

$$Cov_0(U_{ij}, U_{li}) = Cov_0(U_{ji}, U_{il}) = -\frac{1}{12} n_i n_j n_l \qquad \textit{if all i, j, l are different} \tag{3.13}$$

$$Cov_0(U_{ij}, U_{lm}) = 0 \qquad \textit{if all i, j, l, m are different} \tag{3.14}$$

  where n_i, n_j, and n_l are the sample sizes for the i^{th}, j^{th}, and l^{th} group respectively.

Once the sample sizes are all equal to n, then the null variance of U_{ij} in (3.11) reduces to

$$Var_0(U_{ij}) = \frac{1}{12} n^2 (2n + 1) \; ; \; \forall \; i \neq j \tag{3.15}$$

and the null covariances of U_{ij}'s in (3.12), (3.13), (3.14) reduce to

$$Cov_0(U_{ij}, U_{il}) = Cov_0(U_{ji}, U_{li}) = \frac{1}{12} n^3 \qquad \textit{if all i, j, l are different} \tag{3.16}$$

$$Cov_0(U_{ij}, U_{li}) = Cov_0(U_{ji}, U_{il}) = -\frac{1}{12} n^3 \qquad \textit{if all i, j, l are different} \tag{3.17}$$

$$Cov_0(U_{ij}, U_{lm}) = 0 \qquad \textit{if all i, j, l, m are different} \tag{3.18}$$

Now, we will use the fact in (3.19) to find the null variance of MMW_{pII}.

$$\begin{aligned} Var_0(MMW_{pII}) &= Var_0(MJT_{upII}) + Var_0(MJT_{downII}) \\ &\quad + 2\,Cov_0(MJT_{upII}\,,\, MJT_{downII}) \end{aligned} \tag{3.19}$$

1. **Finding the** $Var_0(MJT_{upII})$:

   The $Var_0(MJT_{upII})$ is given by

$$\begin{aligned} Var_0(MJT_{upII}) &= \sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^4 Var_0(U_{ij}) \\ &\quad + 2\sum_{A}(j-i)^2(m-l)^2 Cov_0(U_{ij}, U_{lm}) \end{aligned} \tag{3.20}$$

   Since we consider the equal sample size, we substitute the null variance of U_{ij} and the null covariances of U_{ij}'s as given in (3.15) (3.16), (3.17), (3.18) respectively to equation (3.20).

$$Var_0(MJT_{upII}) = \frac{n^2(2n+1)}{12}\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^4 + \frac{n^3}{12}\underbrace{\sum_{A} 2\,(j-i)^2(m-l)^2}_{1} \tag{3.21}$$

   where $A = \{(i,j)(l,m)|\{(1,2),(1,3)\},\{(1,2),(1,4)\},\ldots,\{(p-2,p),(p-1,p)\}\}$

   For part 1 in (3.21), the null covariances of the Mann-Whitney statistics are the same with different coefficients for each term. The appropriate coefficients are given in Table 3.1. The sums of the rows of Table 3.1 are obtained as follows:

   - For row $1j; (j = 2, 3, \ldots, p)$

$$\begin{aligned} sum_{1j} &= (j-1)^2\Big[\sum_{t=1}^{p-1}t^2 - (j-1)^2 + \sum_{t=0}^{j-2}t^2 - \sum_{t=1}^{p-j}t^2\Big] \\ &= (j-1)^2\Big[-(j-1)^2 + \sum_{t=1}^{p-1}t^2 + \sum_{t=0}^{j-2}t^2 - \sum_{t=1}^{p-j}t^2\Big],\ j = 2,3,\ldots,p \end{aligned} \tag{3.22}$$

- For row $2j; (j = 3, 4, \ldots, p)$

$$sum_{2j} = (j-2)^2\Big[-1 + (j-1)^2 + \sum_{t=1}^{p-2} t^2 - (j-2)^2 + \sum_{t=0}^{j-3} t^2 - \sum_{t=1}^{p-j} t^2\Big]$$
$$= (j-2)^2\Big[-1 + (j-1)^2 - (j-2)^2 + \sum_{t=1}^{p-2} t^2 + \sum_{t=0}^{j-3} t^2 - \sum_{t=1}^{p-j} t^2\Big]$$
$$, \; j = 3, 4, \ldots, p \tag{3.23}$$

- For row $3j; (j = 4, 5, \ldots, p)$

$$sum_{3j} = (j-3)^2\Big[-(2)^2 - 1 + (j-1)^2 + (j-2)^2 - (j-3)^2 + \sum_{t=1}^{p-3} t^2 + \sum_{t=0}^{j-4} t^2 - \sum_{t=1}^{p-j} t^2\Big]$$
$$= (j-3)^2\Big[-\sum_{t=1}^{2} t^2 + \sum_{t=1}^{2}(j-t)^2 - (j-3)^2 + \sum_{t=1}^{p-3} t^2 + \sum_{t=0}^{j-4} t^2 - \sum_{t=1}^{p-j} t^2\Big]$$
$$, \; j = 4, 5, \ldots, p \tag{3.24}$$

- For general row ij

$$sum_{ij} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-1} t^2 + \sum_{t=1}^{i-1}(j-t)^2 + \sum_{t=1}^{p-i} t^2 + \sum_{t=0}^{j-i-1} t^2 - \sum_{t=1}^{p-j} t^2\Big]$$
$$, \; i = 1, 2, \ldots, p-1 \; ; \; j = i+1, i+2, \ldots, p \tag{3.25}$$

and by summing the row sums in Table 3.1, part 1 in (3.21) can be written as

$$\sum_{A} \; 2\,(j-i)^2(m-l)^2 = \Big\{\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} sum_{ij}\Big\} \tag{3.26}$$

Then by substituting (3.26) in (3.21) the $Var_0(MJT_{upII})$ is

$$Var_0(MJT_{up}) = \frac{1}{12}\Big\{n^2(2n+1) \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (j-i)^4 + n^3 \sum_{i=1}^{p-1} \sum_{j=i+1}^{p} sum_{ij}\Big\} \tag{3.27}$$

where,

$$sum_{ij} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-1} t^2 + \sum_{t=1}^{i-1}(j-t)^2 + \sum_{t=1}^{p-i} t^2 + \sum_{t=0}^{j-i-1} t^2 - \sum_{t=1}^{p-j} t^2\Big]$$

Table 3.1. Coefficients of Covariance Terms for Up Side of The Umbrella Hypothesis

j-i	1	2	...	j-1	...	p-2	p-1	1	2	...	j-2	...	p-3	p-2	...	1
j-i	U_{ij}	U_{12}	U_{13}	...	U_{1j}	...	$U_{1(p-1)}$	U_{1p}	U_{23}	U_{24}	...	U_{2j}	...	$U_{2(p-1)}$	U_{2p}	...	$U_{(p-1)p}$
1	U_{12}	-	$1^2 2^2$	...	$1^2(j-1)^2$	...	$1^2(p-2)^2$	$1^2(p-1)^2$	$-(1)^2 1^2$	$-(1)^2(2)^2$	...	$-(1)^2(j-2)^2$	...	$-(1)^2(p-3)^2$	$-(1)^2(p-2)^2$	...	0
2	U_{13}	$2^2 1^2$	-	...	$2^2(j-1)^2$	...	$2^2(p-2)^2$	$2^2(p-1)^2$	$2^2 1^2$	0	...	0	...	0	0	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
j-1	U_{1j}	$(j-1)^2 1^2$	$(j-1)^2 2^2$	...	-	...	$(j-1)^2(p-2)^2$	$(j-1)^2(p-1)^2$	0	0	...	$(j-1)^2(j-2)^2$	...	0	0	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
p-2	$U_{1(p-1)}$	$(p-2)^2 1^2$	$(p-2)^2 2^2$	...	$(p-2)^2(j-1)^2$	...	-	$(p-2)^2(p-1)^2$	0	0	...	0	...	$(p-2)^2(p-3)^2$	0	...	$-(p-2)^2 1^2$
p-1	U_{1p}	$(p-1)^2 1^2$	$(p-1)^2 2^2$	...	$(p-1)^2(j-1)^2$	...	$(p-1)^2(p-2)^2$	-	0	0	...	0	...	0	$(p-1)^2(p-2)^2$	...	$(p-1)^2 1^2$
1	U_{23}	$-(1)^2 1^2$	$(1)^2 2^2$	...	0	...	0	0	-	$(1)^2 2^2$	...	$(1)^2(j-2)^2$	...	$(1)^2(p-3)^2$	$(1)^2(p-2)^2$	...	0
2	U_{24}	$-(2)^2 1^2$	0	...	0	...	0	0	$(2)^2(1)^2$	-	...	$(2)^2(j-2)^2$	...	$(2)^2(p-3)^2$	$(2)^2(p-2)^2$	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
j-2	U_{2j}	$-(j-2)^2 1^2$	0	...	$(j-2)^2(j-1)^2$	...	0	0	$(j-2)^2(1)^2$	$(j-2)^2(2)^2$	...	-	...	$(j-2)^2(p-3)^2$	$(j-2)^2(p-2)^2$	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
p-3	$U_{2(p-1)}$	$-(p-3)^2(1)^2$	0	...	0	...	$(p-3)^2(p-2)^2$	0	$(p-3)^2(1)^2$	$(p-3)^2(2)^2$	...	$(p-3)^2(j-2)^2$	...	-	$(p-3)^2(p-2)^2$	...	$-(p-3)^2(1)^2$
p-2	U_{2p}	$-(p-2)^2(1)^2$	0	...	0	...	0	$(p-2)^2(p-1)^2$	$(p-2)^2(1)^2$	$(p-2)^2(2)^2$	...	$(p-2)^2(j-2)^2$	...	$(p-2)^2(p-3)^2$	-	...	$(p-2)^2(1)^2$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	$U_{(p-1)p}$	0	0	...	0	...	$-(1)^2(p-2)^2$	$(1)^2(p-1)^2$	0	0	...	0	...	$-(1)^2(p-3)^2$	$(1)^2(p-2)^2$	...	-

2. **Finding the** $Var_0(MJT_{downII})$.

The $Var_0(MJT_{downII})$ is given as

$$Var_0(MJT_{downII}) = \sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^4 Var_0(U_{ji}) + 2\sum_{A}(j-i)^2(m-l)^2 Cov_0(U_{ji},U_{ml}) \tag{3.28}$$

Since we consider the equal sample size, we substitute the null variance of U_{ij} and the null covariances of U_{ij}'s as given in (3.15) (3.16), (3.17), (3.18) respectively to equation (3.28).

$$Var_0(MJT_{downII}) = \frac{n^2(2n+1)}{12}\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^4 + \frac{n^3}{12}\underbrace{\sum_{A} 2\,(j-i)^2(m-l)^2}_{1} \tag{3.29}$$

where $A = \{(i,j)(l,m)|\{(p,p+1),(p,p+2)\},\{(p,p+1),(p,p+3)\},\ldots,\{(k-2,k),(k-1,k)\}\}$ For part 1 in (3.29), the null covariances of the Mann-Whitney statistics are the same with different coefficients for each term. The appropriate coefficients are given in Table 3.2. The sums of the rows of Table 3.2 are obtained as follows:

- For row $j1; (j = 2,3,\ldots,k)$

$$\begin{aligned} sum_{j1} &= (j-1)^2\Big[-\sum_{t=1}^{1-p}t^2 - (j-1)^2 + \sum_{t=1}^{k-1}t^2 + \sum_{t=0}^{j-2}t^2 - \sum_{t=1}^{k-j}t^2\Big] \\ &= (j-1)^2\Big[-(j-1)^2 - \sum_{t=1}^{1-p}t^2 + \sum_{t=1}^{k-1}t^2 + \sum_{t=0}^{j-2}t^2 - \sum_{t=1}^{k-j}t^2\Big] \\ &\qquad\qquad , \; j = 2,3,\ldots,k \end{aligned} \tag{3.30}$$

- For row $j2; (j = 3, 4, \ldots, k)$

$$sum_{j2} = (j-2)^2\Big[-\sum_{t=1}^{2-p} t^2 + \sum_{t=p}^{1}(j-t)^2 - (j-2)^2 + \sum_{t=1}^{k-2} t^2 + \sum_{t=0}^{j-3} t^2 - \sum_{t=1}^{k-j} t^2\Big]$$
$$= (j-2)^2\Big[-(j-2)^2 - \sum_{t=1}^{2-p} t^2 + \sum_{t=p}^{1}(j-t)^2 + \sum_{t=1}^{k-2} t^2 + \sum_{t=0}^{j-3} t^2 - \sum_{t=1}^{k-j} t^2\Big]$$
$$, \; j = 3, 4, \ldots, k \qquad (3.31)$$

- For row $j3; (j = 4, 5, \ldots, k)$

$$sum_{j3} = (j-3)^2\Big[-\sum_{t=1}^{3-p} t^2 + \sum_{t=p}^{2}(j-t)^2 - (j-3)^2 + \sum_{t=1}^{k-3} t^2 + \sum_{t=0}^{j-4} t^2 - \sum_{t=1}^{k-j} t^2\Big]$$
$$= (j-3)^2\Big[-(j-3)^2 - \sum_{t=1}^{3-p} t^2 + \sum_{t=p}^{2}(j-t)^2 + \sum_{t=1}^{k-3} t^2 + \sum_{t=0}^{j-4} t^2 - \sum_{t=1}^{k-j} t^2\Big]$$
$$, \; j = 4, 5, \ldots, k \qquad (3.32)$$

- For general row ji

$$sum_{ji} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-p} t^2 + \sum_{t=p}^{i-1}(j-t)^2 + \sum_{t=1}^{k-i} t^2 + \sum_{t=0}^{j-i-1} t^2 - \sum_{t=1}^{k-j} t^2\Big]$$
$$, i = p, p+1, \ldots, k-1 \; ; \; j = i+1, i+2, \ldots, k \qquad (3.33)$$

and by summing the row sums in Table 3.2, the part 1 in (3.29) can be written as

$$\sum_{A} 2\,(j-i)^2(m-l)^2 = \frac{n^3}{12}\Big\{\sum_{i=p}^{k-1}\sum_{j=i+1}^{k} sum_{ji}\Big\} \qquad (3.34)$$

Then by substituting (3.34) in (3.29) the $Var_0(MJT_{downII})$ is

$$Var_0(MJT_{downII}) = \frac{1}{12}\Big\{n^2(2n+1)\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^4 + n^3\sum_{i=p}^{k-1}\sum_{j=i+1}^{k} sum_{ji}\Big\} \qquad (3.35)$$

where,

$$sum_{ji} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-p} t^2 + \sum_{t=p}^{i-1}(j-t)^2 + \sum_{t=1}^{k-i} t^2 + \sum_{t=0}^{j-i-1} t^2 - \sum_{t=1}^{k-j} t^2\Big]$$

Table 3.2. Coefficients of Covariance Terms for Down Side of The Umbrella Hypothesis

j-i		1	2	...	j-p	...	k-1-p	k-p	1	2	...	j-p-1	...	k-2-p	k-1-p	...	1
j-i	U_{ji}	$U_{(p+1)p}$	$U_{(p+2)p}$	...	U_{jp}	...	$U_{(k-1)p}$	U_{kp}	$U_{(p+2)(p+1)}$	$U_{(p+3)(p+1)}$	...	$U_{j(p+1)}$	...	$U_{(k-1)(p+1)}$	$U_{k(p+1)}$	...	$U_{k(k-1)}$
1	$U_{(p+1)p}$	-	$1^2 2^2$	...	$1^2(j-p)^2$	...	$1^2(k-1-p)^2$	$1^2(k-p)^2$	$-(1)^2 1^2$	$-(1)^2 2^2$	...	$-(1)^2(j-p-1)^2$	...	$-(1)^2(k-2-p)^2$	$-(1)^2(k-1-p)^2$	...	0
2	$U_{(p+2)p}$	$2^2 1^2$	-	...	$2^2(j-p)^2$	...	$2^2(k-1-p)^2$	$2^2(k-p)^2$	$2^2 1^2$	0	...	0	...	0	0	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
j-p	U_{jp}	$(j-p)^2 1^2$	$(j-p)^2 2^2$	...	-	...	$(j-p)^2(k-1-p)^2$	$(j-p)^2(k-p)^2$	0	0	...	$(j-p)^2(j-p-1)^2$	...	0	0	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
k-1-p	$U_{(k-1)p}$	$(k-1-p)^2 1^2$	$(k-1-p)^2 2^2$	...	$(k-1-p)^2(j-p)^2$	...	-	$(k-1-p)^2(k-p)^2$	0	0	...	0	...	$(k-1-p)^2(k-2-p)^2$	0	...	$-(k-1-p)^2 1^2$
k-p	U_{kp}	$(k-p)^2 1^2$	$(k-p)^2 2^2$	...	$(k-p)^2(j-p)^2$	...	$(k-p)^2(k-1-p)^2$	-	0	0	...	0	...	0	$(k-p)^2(k-1-p)^2$	...	$(k-p)^2 1^2$
1	$U_{(p+2)(p+1)}$	$-(1)^2 1^2$	$1^2 2^2$	...	0	...	0	0	-	$1^2 2^2$	...	$1^2(j-p-1)^2$	...	$1^2(k-2-p)^2$	$1^2(k-1-p)^2$	...	0
2	$U_{(p+3)(p+1)}$	$-(2)^2 1^2$	0	...	0	...	0	0	$2^2 1^2$	-	...	$2^2(j-p-1)^2$	...	$2^2(k-2-p)^2$	$2^2(k-1-p)^2$	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
j-p-1	$U_{j(p+1)}$	$-(j-p-1)^2 1^2$	0	...	$(j-p-1)^2(j-p)^2$	...	0	0	$(j-p-1)^2 1^2$	$(j-p-1)^2 2^2$	...	-	...	$(j-p-1)^2(k-2-p)^2$	$(j-p-1)^2(k-1-p)^2$	...	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
k-2-p	$U_{(k-1)(p+1)}$	$-(k-2-p)^2 1^2$	0	...	0	...	$(k-2-p)^2(k-1-p)^2$	0	$(k-2-p)^2 1^2$	$(k-2-p)^2 2^2$	...	$(k-2-p)^2(j-p-1)^2$	...	-	$(k-2-p)^2(k-1-p)^2$	...	$-(k-2-p)^2 1^2$
k-1-p	$U_{k(p+1)}$	$-(k-1-p)^2 1^2$	0	...	0	...	0	$(k-1-p)^2(k-p)^2$	$(k-1-p)^2 1^2$	$(k-1-p)^2 2^2$	...	$(k-1-p)^2(j-p-1)^2$	...	$(k-1-p)^2(k-2-p)^2$	-	...	$(k-1-p)^2 1^2$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	$U_{k(k-1)}$	0	0	...	0	...	$-(1)^2(k-1-p)^2$	$1^2(k-p)^2$	0	0	...	0	...	$-(1)^2(k-2-p)^2$	$1^2(k-1-p)^2$	...	-

3. **Finding the** $Cov_0(MJT_{upII}\,,\,MJT_{downII})$.

The $Cov_0(MJT_{upII}\,,\,MJT_{downII})$ can be found as follows

$$Cov_0(MJT_{upII}\,,\,MJT_{downII}) = \sum_{i=1}^{p-1}\sum_{j=p+1}^{k}(j-p)^2(p-i)^2 Cov_0(U_{ip},U_{jp}) \tag{3.36}$$

Since we consider the equal sample size, we substitute the null covariances of U_{ij}'s as given in (3.16) to equation (3.36).

$$\begin{aligned}
Cov_0(MJT_{upII}\,,\,MJT_{downII}) &= \frac{n^3}{12}\sum_{i=1}^{p-1}\sum_{j=p+1}^{k}(j-p)^2(p-i)^2 \\
&= \frac{n^3}{12}\sum_{i=1}^{p-1}(p-i)^2\sum_{j=p+1}^{k}(j-p)^2 \\
&= \frac{n^3}{12}\Big[(p-1)^2+(p-2)^2+\ldots+1^2\Big]\Big[1^2+2^2+\ldots+(k-p)^2\Big] \\
&= \frac{n^3}{12}\Big\{\frac{p(p-1)[2(p-1)+1]}{6}\Big\}\Big\{\frac{(k-p)(k-p+1)[2(k-p)+1]}{6}\Big\} \\
&= \frac{n^3}{12}\,\frac{p(p-1)(k-p)(k-p+1)[2(k-p)+1][2(p-1)+1]}{36}
\end{aligned} \tag{3.37}$$

Then,

$$2\,Cov_0(MJT_{upII}\,,\,MJT_{downII}) = \frac{n^3}{6}\Big\{\frac{p(p-1)(k-p)(k-p+1)[2(k-p)+1][2(p-1)+1]}{36}\Big\} \tag{3.38}$$

Now, we will substitute the (3.27), (3.35), (3.38), to equation (3.19), then the null variance of MMW_{pII} can be written as follows

$$\begin{aligned}
Var_0(MMW_{pII}) &= \frac{n^2(2n+1)}{12}\Big\{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^4+\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^4\Big\} \\
&\quad+\frac{n^3}{12}\Big\{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}sum_{ij}+\sum_{i=p}^{k-1}\sum_{j=i+1}^{k}sum_{ji}\Big\} \\
&\quad+\frac{n^3}{6}\Big\{\frac{p(p-1)(k-p)(k-p+1)[2(k-p)+1][2(p-1)+1]}{36}\Big\}
\end{aligned} \tag{3.39}$$

where,

$$sum_{ij} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-1} t^2 + \sum_{t=1}^{i-1}(j-t)^2 + \sum_{t=1}^{p-i} t^2 + \sum_{t=0}^{j-i-1} t^2 - \sum_{t=1}^{p-j} t^2\Big]$$

$$sum_{ji} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-p} t^2 + \sum_{t=p}^{i-1}(j-t)^2 + \sum_{t=1}^{k-i} t^2 + \sum_{t=0}^{j-i-1} t^2 - \sum_{t=1}^{k-j} t^2\Big]$$

□

3.1.2. Special Cases for Mean and Variance

We will consider in this research some of the practical cases for the mean and the variance under the null distribution :

- **Four treatments at a known peak ($p = 2$):** Under H_0, the mean and the variance of MMW_{pII} are given in (3.40) ,(3.41) when we are considering equal sample sizes ($n_1 = n_2 = n_3 = n_4 = n$) respectively.

$$E_0(MMW_{pII}) = \frac{7}{2}\,n^2 \tag{3.40}$$

$$Var_0(MMW_{pII}) = \frac{1}{12}[19\,n^2(2n+1) + 24\,n^3] \tag{3.41}$$

- **Five treatments at a known peak ($p = 2$):** Under H_0, the mean and the variance of MMW_{pII} are given in (3.42) ,(3.43) when we are considering equal sample sizes ($n_1 = n_2 = n_3 = n_4 = n_5 = n$) respectively.

$$E_0(MMW_{pII}) = \frac{21}{2}\,n^2 \tag{3.42}$$

$$Var_0(MMW_{pII}) = \frac{1}{12}[117\,n^2(2n+1) + 220\,n^3] \tag{3.43}$$

- **Five treatments at a known peak ($p = 3$):**

  Under H_0, the mean and the variance of MMW_{pII} are given in (3.44) ,(3.45) when we are considering equal sample sizes ($n_1 = n_2 = n_3 = n_4 = n_5 = n$) respectively.

$$E_0(MMW_{pII}) = \frac{12}{2}\, n^2 \tag{3.44}$$

$$Var_0(MMW_{pII}) = \frac{1}{12}[36\, n^2(2n+1) + 78\, n^3] \tag{3.45}$$

The standardized version of MMW_{pII} is given in (3.46)

$$MMW^*_{pII} = \frac{MMW_{pII} - E_0(MMW_{pII})}{\sqrt{Var_0(MMW_{pII})}} \tag{3.46}$$

When H_0 is true, MMW^*_{pII} has an asymptotic standard normal distribution. We reject H_0 if $MMW^*_{pII} \geq Z_\alpha$ at α significant level, where Z_α is the upper α quantile of the standard normal distribution.

3.2. Modified Kim-Kim

The modified Kim-Kim test statistic for the (RCBD) portion is a sum of a modified Mack-Wolfe MMW_{pII} over b blocks in case of known umbrella peak.

$$\begin{aligned} MKK &= \sum_{s=1}^{b} MMW_{spII} \\ &= \sum_{s=1}^{b} \Big[\sum_{i=1}^{p-1} \sum_{j=i+1}^{p} (j-i)^2\, U_{sij} + \sum_{i=p}^{k-1} \sum_{j=i+1}^{k} (j-i)^2\, U_{sji} \Big] \end{aligned} \tag{3.47}$$

Where U_{sij} is a Mann-Whitney statistic applied to the observations in cells (s, i) and (s, j).

3.2.1. The Mean and Variance

Kim and Kim (1992) proposed test statistic has an asymptotic normal distribution when H_0 is true, so the modified Kim-Kim test statistic MKK in (3.47), when it is standardized, is asymptotically a standard normal distribution under the null hypothesis. For this purpose, we need to know the expected value and variance of MKK when H_0 is true; for simplicity, we will use the same sample size for all cells. Kim and Kim (1992) derived the mean and variance for a

randomized complete block design by summing the mean and variance of Mack-Wolfe over blocks. As a result, the expected value and variance of MKK are, respectively,

$$\begin{aligned} E_0(MKK) &= \sum_{s=1}^{b} E_0(MMW_{spII}) \\ &= \frac{b\,n^2}{24}\Big\{p^2(p^2-1) + (k-p+1)^2\big[(k-p+1)^2-1\big]\Big\} \end{aligned} \tag{3.48}$$

$$\begin{aligned} Var_0(MKK) &= \sum_{s=1}^{b} Var_0(MMW_{spII}) \\ &= \frac{b\,n^2(2n+1)}{12}\Big\{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^4 + \sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^4\Big\} \\ &+ \frac{b\,n^3}{12}\Big\{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} sum_{ij} + \sum_{i=p}^{k-1}\sum_{j=i+1}^{k} sum_{ji}\Big\} \\ &+ \frac{b\,n^3}{6}\Big\{\frac{p(p-1)(k-p)(k-p+1)[2(k-p)+1][2(p-1)+1]}{36}\Big\} \end{aligned} \tag{3.49}$$

where,

$$sum_{ij} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-1} t^2 + \sum_{t=1}^{i-1}(j-t)^2 + \sum_{t=1}^{p-i} t^2 + \sum_{t=0}^{j-i-1} t^2 - \sum_{t=1}^{p-j} t^2\Big]$$

$$sum_{ji} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-p} t^2 + \sum_{t=p}^{i-1}(j-t)^2 + \sum_{t=1}^{k-i} t^2 + \sum_{t=0}^{j-i-1} t^2 - \sum_{t=1}^{k-j} t^2\Big]$$

3.2.2. Special Cases for Mean and Variance

We will consider in this research some of the practical cases for the mean and the variance under the null distribution :

- **Four treatments at a known peak ($p = 2$):** Under H_0, the mean and the variance of MKK are given in (3.50), (3.51) when we are considering one observation for each cell respectively.

$$E_0(MKK) = \frac{7}{2}\,b \tag{3.50}$$

$$Var_0(MKK) = \frac{81}{12}\, b \tag{3.51}$$

- **Five treatments at a known peak ($p = 2$):** Under H_0, the mean and the variance of MKK are given in (3.52), (3.53) when we are considering one observation for each cell respectively.

$$E_0(MKK) = \frac{21}{2}\, b \tag{3.52}$$

$$Var_0(MKK) = \frac{571}{12}\, b \tag{3.53}$$

- **Five treatments at known peak ($p = 3$):** Under H_0, the mean and the variance of MKK are given in (3.54) , (3.55) when we are considering one observation for each cell respectively.

$$E_0(MKK) = \frac{12}{2}\, b \tag{3.54}$$

$$Var_0(MKK) = \frac{186}{12}\, b \tag{3.55}$$

The standardized version of MKK is given in (3.56)

$$MKK^* = \frac{MKK - E_0(MKK)}{\sqrt{Var_0(MKK)}} \tag{3.56}$$

When H_0 is true, MKK^* has an asymptotic standard normal distribution. We reject H_0 if $MKK^* \geq Z_\alpha$ at α significant level, where Z_α is the upper α quantile of the standard normal distribution.

In Section 3.3, we will illustrate the proposed test statistic for a mixed design in the case of a known and unknown peak. It is based on combining a modified Mack-Wolfe test statistic in (3.1) for a completely randomized design portion and a modified Kim-Kim test statistic in (3.47) for a randomized complete block design portion.

3.3. Proposed Mixed Design Test

3.3.1. Peak Known

We will propose a test statistic for a mixed design for a case for a known peak with two different methods of combining the modified test statistics for the (RCBD) and (CRD) portions.

- ***First Method*** : We suggest combining the standardized versions of a modified Mack-Wolfe test statistic and a modified Kim-Kim test statistic as given in (3.57)

$$MD_I = MMW^*_{pII} + MKK^* \tag{3.57}$$

Under H_0, the asymptotic distribution of MD_I is a normal distribution with a mean of zero and a variance of 2. The standardized version of MD_I is given in (3.58)

$$MD^*_I = \frac{MD_I - 0}{\sqrt{2}} \tag{3.58}$$

Under H_0, MD^*_I has an asymptotic standard normal distribution, and we reject H_0 when $MD^*_I \geq Z_\alpha$ at α significant level, where Z_α is the upper α quantile of the standard normal distribution.

- ***Second Method*** : We suggest combining the two modified versions of the Mack-Wolfe test statistic and the Kim-Kim test statistic as given in (3.59)

$$MD_{II} = MMW_{pII} + MKK \tag{3.59}$$

It is noted that the sample sizes will be equal for all treatments for the (CRD) portion and one observation for i^{th} treatment and the s^{th} block in (RCBD) portion. Therefore, when H_0 is true, the mean and the variance of MD_{II} once we have the same sample size for all treatments in (CRD) portion and one observation for each cell in (RCBD) portion are, respectively,

$$\begin{aligned} E_0(MD_{II}) &= E_0(MMW_{pII}) + E_0(MKK) \\ &= (n^2 + b)\left\{\frac{p^2(p^2-1) + (k-p+1)^2\left[(k-p+1)^2 - 1\right]}{24}\right\} \end{aligned} \tag{3.60}$$

$$
\begin{aligned}
Var_0(MD_{II}) &= Var_0(MMW_{pII}) + Var_0(MKK) \\
&= \frac{n^2(2n+1)+3b}{12}\Big\{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p}(j-i)^4 + \sum_{i=p}^{k-1}\sum_{j=i+1}^{k}(j-i)^4\Big\} \\
&+ \frac{n^3+b}{12}\Big\{\sum_{i=1}^{p-1}\sum_{j=i+1}^{p} sum_{ij} + \sum_{i=p}^{k-1}\sum_{j=i+1}^{k} sum_{ji}\Big\} \\
&+ \frac{n^3+b}{6}\Big\{\frac{p(p-1)(k-p)(k-p+1)[2(k-p)+1][2(p-1)+1]}{36}\Big\} \qquad (3.61)
\end{aligned}
$$

where,

$$
sum_{ij} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-1}t^2 + \sum_{t=1}^{i-1}(j-t)^2 + \sum_{t=1}^{p-i}t^2 + \sum_{t=0}^{j-i-1}t^2 - \sum_{t=1}^{p-j}t^2\Big]
$$

$$
sum_{ji} = (j-i)^2\Big[-(j-i)^2 - \sum_{t=1}^{i-p}t^2 + \sum_{t=p}^{i-1}(j-t)^2 + \sum_{t=1}^{k-i}t^2 + \sum_{t=0}^{j-i-1}t^2 - \sum_{t=1}^{k-j}t^2\Big]
$$

3.3.1.1. Special cases for mean and variance

- **Four treatments at a known peak ($p = 2$):** Under H_0, the mean and the variance of MD_{II} once we have the same sample size for all treatments in (CRD) portion and one observation for each cell in (RCBD) portion are given in (3.62), (3.63).

$$
E_0(MD_{II}) = \frac{7}{2}(n^2 + b) \qquad (3.62)
$$

$$
Var_0(MD_{II}) = \frac{1}{12}[19\,n^2(2n+1) + 24n^3] + \frac{81}{12}\,b \qquad (3.63)
$$

- **Five treatments at a known peak ($p = 2$):** Under H_0, the mean and the variance of MD_{II} once we have the same sample size for all treatments in (CRD) portion and one observation for each cell in (RCBD) portion are given in (3.64), (3.65).

$$
E_0(MD_{II}) = \frac{21}{2}(n^2 + b) \qquad (3.64)
$$

$$Var_0(MD_{II}) = \frac{1}{12}[117\, n^2(2n+1) + 220n^3] + \frac{571}{12}\, b \tag{3.65}$$

- **Five treatments at a known peak ($p = 3$):** Under H_0, the mean and the variance of MD_{II} once we have the same sample size for all treatments in (CRD) portion and one observation for each cell in (RCBD) portion are given in (3.66), (3.67).

$$E_0(MD_{II}) = \frac{12}{2}\,(n^2 + \, b) \tag{3.66}$$

$$Var_0(MD_{II}) = \frac{1}{12}[36\, n^2(2n+1) + 78n^3] + \frac{186}{12}\, b \tag{3.67}$$

The standardized version of MD_{II} is given in (3.68).

$$MD^*_{II} = \frac{MD_{II} - E_0(MD_{II})}{\sqrt{Var_0(MD_{II})}} \tag{3.68}$$

Under H_0, MD^*_{II} has an asymptotic standard normal distribution, and we reject H_0 when $MD^*_{II} \geq Z_\alpha$ at α significant level, where Z_α is the upper α quantile of the standard normal distribution.

3.3.2. Peak Unknown

In this section, we will introduce two different cases of the proposed test statistic for a mixed design of an unknown peak. The first case is one in which we use the nonmodified version of the Mack-Wolfe and Kim-Kim test statistics. The second case is one in which we use a square distance modification for both Mack-Wolfe and Kim-Kim statistics.

In order to work with an unknown peak for the umbrella hypothesis, we need to estimate the peak. For this purpose, we apply the idea introduced by Chen and Wolfe (1990) and Chen (1991), which is the test statistic based on the maximum of the standardized test statistics calculated for all assumed known peaks. Then, the peak would be at the treatment that has the maximum of the standardized test statistics. The two cases of the proposed test statistic for a mixed design are:

- ***Non-Modification case :*** This is a case of combining nonmodified test statistics for a randomized complete block design (RCBD) portion and completely randomized design (CRD) portion as illustrated in Magel et al. (2010) in two different methods.

– ***First Method:*** We combine the standardized versions of a nonmodified Mack-Wolfe test statistic and nonmodified Kim-Kim test statistic as given in (2.15). Then the proposed test statistic for a mixed design is given as in (3.69)

$$A^*_{maxI} = max(A^{**}{}_1, A^{**}{}_2, \ldots, A^{**}{}_k) \tag{3.69}$$

where $A^{**}{}_i$ is the standardized version of combining the standardized versions of nonmodified Mack-Wolfe and Kim-Kim test statistics, introduced by Magel et al. (2010), at i^{th} peak; $i = 1, 2, \ldots, k$.

The peak can be estimated at the treatment that maximizes the test statistic as in Chen (1991) for a mixed design. The null hypothesis is rejected if $A^*_{maxI} \geq C_{I(k,n,b,\alpha)}$ at α significant level. $C_{I(k,n,b,\alpha)}$ is an asymptotical critical value for the statistic A^*_{maxI} in Table 3.3, explained in Section 4.4 Chapter 4 and discussed in Section 5.7.1 Chapter 5.

– ***Second Method:*** We combine the nonmodified Mack-Wolfe test statistic and nonmodified Kim-Kim test statistic as given in (2.17). Then the proposed test statistic for a mixed design is given as in (3.70)

$$A^*_{maxII} = max(A^{***}{}_1, A^{***}{}_2, \ldots, A^{***}{}_k) \tag{3.70}$$

where $A^{***}{}_i$ is the standardized version of combining the nonmodified Mack-Wolfe test statistic and nonmodified Kim-Kim test statistic, introduced by Magel et al. (2010), at i^{th} peak; $i = 1, 2, \ldots, k$.

The peak can be estimated at the treatment that maximizes the test statistic for a mixed design. The null hypothesis is rejected if $A^*_{maxII} \geq C_{II(k,n,b,\alpha)}$ at α significant level. $C_{II(k,n,b,\alpha)}$ is an asymptotical critical value for the statistic A^*_{maxII} in Table 3.3 , explained in Section 4.4 Chapter 4 and discussed in Section 5.7.1 Chapter 5.

Table 3.3. Asymptotical Critical Values for The Proposed Test Statistic in **a Case of Non-Modification** for a Mixed Design. k= The Number of Treatments ; n=The Sample Size for The CRD Design ; b= The Number of Blocks for The RCBD Design.

k	n	b	$\boldsymbol{\alpha}$	$C_{I(k,n,b,\alpha)}$ *	$C_{II(k,n,b,\alpha)}$ **
3	10	5	0.10	1.82053	1.83806
			0.05	2.09963	2.10494
			0.01	2.65511	2.67182
		10	0.10	1.83593	1.83595
			0.05	2.11244	2.11534
			0.01	2.68530	2.66649
		20	0.10	1.82435	1.80141
			0.05	2.12776	2.08499
			0.01	2.70906	2.66458
4	10	5	0.10	1.91053	1.90072
			0.05	2.20239	2.21350
			0.01	2.72935	2.73388
		10	0.10	1.91535	1.91877
			0.05	2.22474	2.18260
			0.01	2.78202	2.75824
		20	0.10	1.89012	1.91039
			0.05	2.17594	2.16913
			0.01	2.74525	2.73728
5	10	5	0.10	1.98269	2.01261
			0.05	2.23835	2.30257
			0.01	2.80491	2.84049
		10	0.10	1.97648	1.97576
			0.05	2.27743	2.27862
			0.01	2.83507	2.82509
		20	0.10	2.00883	1.99456
			0.05	2.29650	2.28243
			0.01	2.90447	2.80822

- ***Modification case :*** This is a case of combining modified test statistics for a randomized complete block design (RCBD) portion and completely randomized design (CRD) portion as illustrated in section 3.3.1 in two different methods.

*The critical value for proposed test statistic A^*_{maxI} for a mixed design with first method.

**The critical value for proposed test statistic A^*_{maxII} for a mixed design with second method.

– ***First Method:*** We combine the standardized versions of a modified Mack-Wolfe and Kim-Kim test statistics as given in (3.57). Then the proposed test statistic for a mixed design is given as in (3.71)

$$MA^*_{maxI} = max(MD^*_{I_1}, MD^*_{I_2}, \ldots, MD^*_{I_k}) \tag{3.71}$$

where, $MD^*_{I_i}$ is the standardized version of combining the standardized versions of the modified Mack-Wolfe and modified Kim-Kim test statistics, the test statistic in (3.58), at i^{th} peak; $i = 1, 2, \ldots, k$. As in Chen (1991), the peak is estimated to be at the treatment that maximizes the test statistic for a mixed design. The null hypothesis is rejected if $MA^*_{maxI} \geq CM_{I(k,n,b,\alpha)}$ at α significant level. $CM_{I(k,n,b,\alpha)}$ is an asymptotical critical value for the statistic MA^*_{maxI} in Table 3.4, explained in Section 4.4 Chapter 4 and discussed in Section 5.7.2 Chapter 5.

– ***Second Method:*** We combine the modified Mack-Wolfe and modified Kim-Kim test statistics as given in (3.59). Then the proposed test statistic for a mixed design is given as in (3.72)

$$MA^*_{maxII} = max(MD^*_{II_1}, MD^*_{II_2}, \ldots, MD^*_{II_k}) \tag{3.72}$$

where, $MD^*_{II_i}$ is the standardized version of combining the modified versions of Mack-Wolfe and the Kim-Kim test statistics, the test statistic in (3.68), at i^{th} peak; $i = 1, 2, \ldots, k$. As in Chen (1991) the peak would be estimated at the treatment that maximizes the test statistic for a mixed design. The null hypothesis is rejected if $MA^*_{maxI} \geq CM_{II(k,n,b,\alpha)}$ at α significant level. $CM_{II(k,n,b,\alpha)}$ is an asymptotical critical value for the statistic MA^*_{maxII} in Table 3.4, explained in Section 4.4 Chapter 4 and discussed in Section 5.7.2 Chapter 5.

Table 3.4. Asymptotical Critical Values for The Proposed Test Statistic in **a Case of Modification** for a Mixed Design. k= The Number of Treatments ; n=The Sample Size for The CRD Design ; b= The Number of Blocks for The RCBD Design.

k	n	b	$\boldsymbol{\alpha}$	$CM_{I(k,n,b,\alpha)}$ ‡	$CM_{II(k,n,b,\alpha)}$ ‡‡
3	10	5	0.10	1.84054	1.80531
			0.05	2.11007	2.09355
			0.01	2.67157	2.65487
		10	0.10	1.81113	1.81458
			0.05	2.10612	2.10188
			0.01	2.66426	2.60089
		20	0.10	1.83394	1.81796
			0.05	2.11791	2.11845
			0.01	2.70585	2.64967
4	10	5	0.10	1.89759	1.87845
			0.05	2.18999	2.14269
			0.01	2.68560	2.65698
		10	0.10	1.90159	1.87925
			0.05	2.20213	2.15698
			0.01	2.76447	2.66908
		20	0.10	1.90858	1.90809
			0.05	2.19475	2.17886
			0.01	2.72999	2.69313
5	10	5	0.10	1.97542	1.95218
			0.05	2.25993	2.26925
			0.01	2.76845	2.78085
		10	0.10	1.96769	1.95018
			0.05	2.27330	2.23159
			0.01	2.83780	2.77227
		20	0.10	1.97724	1.97093
			0.05	2.25648	2.25812
			0.01	2.82158	2.83995

‡The critical value for proposed test statistic MA^*_{maxI} for a mixed design with first method.

‡‡The critical value for proposed test statistic MA^*_{maxII} for a mixed design with second method.

4. SIMULATION STUDY

In this chapter, we will characterize the procedures and criteria used to investigate the performance of the proposed mixed design tests; using a simulation study performed in SAS version 9.4. We will assess the performance of the proposed test statistics for a mixed design in the case of a known and unknown peak.

The observations will be assumed to come from two different types of underlying distributions. The first type is a symmetric distribution, like Normal, and student's t with 3 degrees of freedom. The second type is a skewed distribution, like the exponential. For clarification, the observations from all populations were generated from a standard normal, standard exponential, or student's t with 3 degrees of freedom. The appropriate location shift was added to each observation in a particular sample based on the population we were sampling from. Namely, added the first location shift to all observations from the first sample, the second location shift to all observations from the second sample, and finally the k^{th} location shift to all observations from the k^{th} sample. We assume that the variance of the error terms in (RCBD) and (CRD) are equal for a mixed design.

The performance of the test statistics was assessed based on replications of 5,000 samples. The first part of the simulation study is to estimate the significant level α for the proposed mixed design tests in this research and the proposed test by Magel et al. (2010) under the umbrella hypothesis. The significant level α is estimated by counting the number of times that the null hypothesis rejected under H_0 is true and divided by the number of replications (5,000). The second part of the simulation study is to estimate the power of the test statistics in two cases:

- ***Known Peak:*** in this case, we make a comparison between the proposed test statistics as given in (3.58), (3.68), and the test statistics introduced by Magel et al. (2010) under the umbrella hypothesis as given in (2.16), (2.20).

- ***Unknown Peak:*** in this case, we make a comparison between the proposed test statistics without modification as given in (3.69), (3.70), and the proposed test statistics with a square distance modification as given in (3.71), (3.72).

The power is estimated by counting the number of times the null hypothesis is rejected when the H_1 is true and divided by the number of replications (5,000). We consider a variety of numbers of treatments, sample sizes for the (CRD) portion, and numbers of blocks for the (RCBD) portion. Here they are in detail.

4.1. Number of Treatments

1. **Peak Known**

   (a) Number of treatments $k = 4$ at $p = 2$.

   (b) Number of treatments $k = 5$ at $p = 2, 3$.

2. **Peak Unknown**

   (a) Number of treatments $k = 3$.

   (b) Number of treatments $k = 4$.

   (c) Number of treatments $k = 5$.

4.2. Sample Sizes and Number of Blocks

1. **Peak Known**

   (a) Once the sample sizes of all treatments are equal for (CRD) portion, and the sample size for each treatment is ***twice*** the number of the blocks for (RCBD) portion.

   - $n = 6, 10, 16, 20$ and $b = 3, 5, 8, 10$

   (b) Once the sample sizes of all treatments are equal for (CRD) portion, and the sample size for each treatment is ***equal*** to the number of the blocks for (RCBD) portion.

   - $n = 6, 10, 16, 20$ and $b = 6, 10, 16, 20$

   (c) Once the sample sizes of all treatments are equal for (CRD) portion, and the sample size for each treatment is ***half*** the number of the blocks for (RCBD) portion.

   - $n = 6, 10, 16, 20$ and $b = 12, 20, 32, 40$

2. **Peak Unknown**

   (a) Once the sample sizes of all treatments are equal for (CRD) portion, and ***increase*** the number of the blocks for (RCBD) portion.

   - $n = 10$ and $b = 5, 10, 20$

4.3. Location Parameters

We consider a variety of configurations for the location parameters in a case of a known and unknown peak to estimate the power of the test statistics as given in (2.16), (2.20), (3.58), (3.68), (3.69), (3.70), (3.71), and (3.72) for a mixed design.

1. **Peak Known**

   (a) Four treatments at a known peak ($p = 2$).

   - Distinct peak
     - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***less*** than all the different location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , **1.0** , 0.75 , 0.2).
     - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***greater*** than all the different location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.8 , **1.0** , 0.75 , 0.2).
     - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***less*** than the equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0 , **1.0** , 0.5 , 0.5).
     - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.5 , **1.0** , 0.5 , 0.5).
     - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***greater*** than the equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.5 , **1.0** , 0.2 , 0.2).

- The location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the first one and ***greater*** than the last one of the location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.75 , **1**, 0.75 , 0.2).

- Indistinct peak
  - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the peak with a different configuration of the right side of the umbrella hypothesis (*downside*), such as,
    - (0.5 , **0.5**, 0.2 , 0.0)
    - (0.5 , **0.5** , 0.0 , 0.0)
    - (0.5 , **0.5** , 0.5 , 0.0)
  - Once the peak is ***equal*** to all the equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , **0.5** , 0.5 , 0.5).

(b) Five treatments at a known peak ($p = 2$).

- Distinct peak
  - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***less*** than all the different location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , **0.8** , 0.6 , 0.4 , 0.2).
  - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***greater*** than all the different location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.75 , **0.8** , 0.5 , 0.2 , 0).
  - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***less*** than the first and the last two equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , **0.7** , 0.4 , 0.2 , 0.2).
  - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***greater*** than the first and the last two equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.5 , **1.0** , 0.2 , 0.0 , 0.0).

- The location parameter on the left side of the umbrella hypothesis (*upside*) is ***less*** than the first two equal and the last of the location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , **0.6** , 0.4 , 0.4 , 0.2).
- The location parameter on the left side of the umbrella hypothesis (*upside*) is ***greater*** than the first two equal and the last of the location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.5 , **1.0** , 0.2 , 0.2 , 0).
- The location parameter on the left side of the umbrella hypothesis (*upside*) is ***less*** than all the equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , **0.6** , 0.2 , 0.2 , 0.2).
- The location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to all the equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , **0.5** , 0.0 , 0.0 , 0.0).
- The location parameter on the left side of the umbrella hypothesis (*upside*) is ***greater*** than all the equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.2 , **0.5** , 0.0 , 0.0 , 0.0).
- The location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the first and ***greater*** than the last two of the location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.4 , **0.8** , 0.4 , 0.2 , 0.0).
- The location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the first one and ***greater*** than the last two equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.2 , **0.7** , 0.2 , 0.0 , 0.0).
- The location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the first two equal location parameters and ***greater*** than the last of the location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.2 , **0.5** , 0.2 , 0.2 , 0.0).

- Indistinct peak
  - The location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the peak with a different configuration of the right side of the umbrella hypothesis (*downside*), such as,
    - (0.8 , **0.8** , 0.5 , 0.2 , 0.0)
    - (0.5 , **0.5** , 0.2 , 0.0 , 0.0)
    - (0.5 , **0.5** , 0.2 , 0.2 , 0.0)
    - (0.5 , **0.5** , 0.0 , 0.0 , 0.0)
    - (0.5 , **0.5** , 0.5 , 0.2 , 0.0)
    - (0.5 , **0.5** , 0.5 , 0.0 , 0.0)
    - (0.5 , **0.5** , 0.5 , 0.5 , 0.0)
  - Once the peak is ***equal*** to all the equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , **0.5** , 0.5 , 0.5 , 0.5).

(c) Five treatments at a known peak $p = 3$.

- Distinct peak
  - The two different location parameters on the left side of the umbrella hypothesis (*upside*) are ***less*** than the two different location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , 0.2 , **0.8** , 0.5 , 0.3).
  - The two equal location parameters on the left side of the umbrella hypothesis (*upside*) are ***same*** as the two equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , 0.0 , **0.5** , 0.0 , 0.0).
  - The two equal location parameters on the left side of the umbrella hypothesis (*upside*) are ***less*** than the two equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , 0.0 , **0.5** , 0.2 , 0.2).
  - The two equal location parameters on the left side of the umbrella hypothesis (*upside*) are ***greater*** than the two equal location parameters on the right side of the umbrella hypothesis (*downside*), such as, (0.2 , 0.2 , **0.5** , 0.0 , 0.0).

- The first location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the last location parameter on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , 0.2 , **0.5** , 0.2 , 0.0) , (0.0 , 0.4 , **0.7** , 0.2 , 0.0).
- The second location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the fourth location parameter on the right side of the umbrella hypothesis (*downside*), such as, (0.0 , 0.4 , **0.7** , 0.4 , 0.2) , (0.2 , 0.4 , **0.7** , 0.4 , 0.0).

- Indistinct peak
  - The second location parameter on the left side of the umbrella hypothesis (*upside*) is ***equal*** to the peak in several configurations of the umbrella hypothesis, such as,
    - (0.0 , 0.5 , **0.5** , 0.0 , 0.0)
    - (0.0 , 0.5 , **0.5** , 0.2 , 0.0)
    - (0.2 , 0.5 , **0.5** , 0.2 , 0.0)
    - (0.0 , 0.8 , **0.8** , 0.5 , 0.2)
  - The first and second location parameters on the left side of the umbrella hypothesis (*upside*) are ***equal*** to the peak, such as,
    - (0.5 , 0.5 , **0.5** , 0.0 , 0.0)
    - (0.5 , 0.5 , **0.5** , 0.2 , 0.0)
  - The second and fourth location parameters in the umbrella hypothesis are ***equal*** to the peak in several configurations of the umbrella hypothesis, such as,
    - (0.0 , 0.5 , **0.5** , 0.5 , 0.0)
    - (0.0 , 0.5 , **0.5** , 0.5 , 0.2)
  - The fourth and fifth location parameters of the umbrella hypothesis are ***equal*** to the peak in several configurations of the umbrella hypothesis, such as,
    - (0.0 , 0.0 , **0.5** , 0.5 , 0.5)
    - (0.0 , 0.2 , **0.5** , 0.5 , 0.5)
  - Once the peak is ***equal*** to all the rest of the location parameters in the umbrella hypothesis except the first location parameter, such as, (0.0 , 0.5 , **0.5** , 0.5 , 0.5).

- Once the peak is ***equal*** to all the rest of the location parameters in the umbrella hypothesis except the last location parameter, such as, (0.5 , 0.5 , **0.5** , 0.5 , 0.0).

2. **Peak Unknown**

(a) Three treatments.

- Once the peak is at ***first*** or ***last*** treatment of the hypothesis, such as,
  - (**0.7** , 0.5 , 0.0)
  - (0.0 , 0.5 , **1.0**)
- Once the peak ***equals*** the treatment next to it, such as, (0.0 , **0.5** , **0.5**)
- Once the peak is ***distinct***, such as,
  - (0.0 , **0.7** , 0.5)
  - (0.0 , **0.7** , 0.0)

(b) Four treatments.

- Once the peak is at the ***first*** or ***last*** treatment of the hypothesis, such as,
  - (**1.0** , 0.0 , 0.0 , 0.0)
  - (**1.0** , 0.75 , 0.5 , 0.25)
  - (0.0 , 0.0 , 0.25 , **1.0**)
- Once the peak ***equals*** the treatment next to it, such as,
  - (**0.75** , **0.75** , 0.5 , 0.0)
  - (**0.5** , **0.5** , **0.5** , 0.0)
  - (0.0 , 0.0 , **0.75** , **0.75**)
- Once the peak is ***distinct***, such as,
  - (0.5 , **1.0** , 0.2 , 0.2)
  - (0.0 , 0.25 , **0.5** , 0.25)
  - (0.2 , 0.75 , **1.0** , 0.75)
  - (0.8 , **1.0** , 0.75 , 0.2)

(c) Five treatments.

- Once the peak is at the ***first*** or ***last*** treatment of the hypothesis, such as,
  - (**1.0** , 0.0 , 0.0 , 0.0 , 0.0)
  - (**1.0** , 0.2 , 0.2 , 0.2 , 0.0)
  - (0.0 , 0.0 , 0.0 , 0.5 , **1.0**)
- Once the peak ***equals*** the treatment that next to it, such as,
  - (**0.8** , **0.8** , 0.5 , 0.2 , 0.0)
  - (**0.5** , **0.5** , 0.2 , 0.2 , 0.0)
  - (**0.5** , **0.5** , 0.2 , 0.0 , 0.0)
  - (**0.5** , **0.5** , **0.5** , 0.0 , 0.0)
  - (0.0 , 0.0 , 0.0 , **0.5** , **0.5**)
  - (0.0 , 0.0 , **0.5** , **0.5** , **0.5**)
- Once the peak is ***distinct***, such as, (0.2 , 0.2 , **0.5** , 0.0 , 0.0)

4.4. Critical Values for Unknown Peak Case

In this section, we will explain the procedures of estimating the critical values for the proposed test statistics without modification, as given in (3.69), (3.70), and with a square distance modification, as given in (3.71), (3.72) for a mixed design once the peak is unknown; using a simulation study performed in SAS version 9.4.

We generate an empirical cumulative distribution for each of the test statistics (3.69), (3.70), (3.71), and (3.72) based on a sample size of 10,000 from the corresponding true distribution. The estimated critical values for each test statistic are the percentiles of the empirical distribution. Moreover, we consider three different levels of percentiles (0.99 , 0.95 , 0.90). The cases that are considered in the simulation study for estimating the critical values are :

(a) Number of treatments $k = 3, 4, 5$.

(b) Same sample sizes for each treatment $n = 10$ for a completely randomized design (CRD) portion, and one observation for each cell for a randomized complete block design (RCBD).

(c) Number of blocks $b = 5, 10, 20$.

5. RESULTS AND DISCUSSION

In this Chapter, we present the results of the proposed test statistics described in Chapter 3 for a known and unknown peak of the umbrella hypothesis; as well as the results of the test statistics introduced by Magel et al. (2010) under the umbrella hypothesis once the peak is known for the mixed design of combining a completely randomized design portion (CRD) and a randomized complete block design portion (RCBD).

As an illustration, the results are based on the cases described in detail in Chapter 4. We will present the estimated significant level α, and the estimated power for proposed test statistics at each configuration for the location parameters of the umbrella hypothesis; once we have taken into account the relationship between the sample size for the (CRD) portion and the numbers of blocks for the (RCBD) portion. We assume three different underlying distributions, including normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution. Additionally, the variance for a completely randomized design portion (CRD) is equal to the variance for a randomized complete block design portion (RCBD).

5.1. Four Treatments at Known Peak ($p = 2$)

In this section, we present the results from the simulation study estimating the significant level and the power for proposed test statistics; once there are four treatments and the peak at second treatment.

5.1.1. Estimated Significant Level α

We estimate the significant level α by finding the percentage of rejection of the null hypothesis when it is true for each test statistic, as given in (3.58), (3.68), (2.16), (2.20). All these statistics have an asymptotic standard normal distribution under the null hypothesis. The significant level for each test is stated to be 0.05.

Tables from 5.1 through 5.9 show the results of the estimated significant level based on three different relationships between the sample size for the (CRD) portion and the number of blocks for (RCBD) portion under a variety of underlying distributions:

1. **The sample size of all treatments for the (CRD) portion is *twice* the number of blocks for the (RCBD) portion:** Tables from 5.1 through 5.3 present the estimated

significant level when the sample size for the (CRD) portion is $n = 6, 10, 16, 20$ and the number of blocks for the (RCBD) portion is $b = 3, 5, 8, 10$. The results are based on assuming three different underlying distributions, including normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution for a simulation study. As a result of the simulation study, the estimated significant level α is around 0.05 for the proposed test statistics (3.58), (3.68), and the test statistics (2.16), (2.20) introduced by Magel et al. (2010).

Table 5.1. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is ***twice*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	3	**First**	0.0510	0.0500
		Second	0.0570	0.0472
10	5	**First**	0.0570	0.0448
		Second	0.0512	0.0516
16	8	**First**	0.0520	0.0570
		Second	0.0516	0.0516
20	10	**First**	0.0512	0.0522
		Second	0.0520	0.0544

Table 5.2. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is ***twice*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	3	**First**	0.0464	0.0516
		Second	0.0556	0.0486
10	5	**First**	0.0514	0.0492
		Second	0.0442	0.0518
16	8	**First**	0.0504	0.0470
		Second	0.0484	0.0500
20	10	**First**	0.0486	0.0478
		Second	0.0512	0.0516

Table 5.3. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is ***twice*** The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
6	3	**First**	0.0460	0.0526
		Second	0.0504	0.0542
10	5	**First**	0.0550	0.0546
		Second	0.0538	0.0502
16	8	**First**	0.0508	0.0482
		Second	0.0558	0.0484
20	10	**First**	0.0462	0.0426
		Second	0.0490	0.0470

2. **The sample size of all treatments for the (CRD) portion is *equal* to the number of blocks for the (RCBD) portion:** Tables 5.4 through 5.6 show the estimated significant level when the sample size for the (CRD) portion is $n = 6, 10, 16, 20$ and the number of blocks for the (RCBD) portion is $b = 6, 10, 16, 20$. The assumed underlying distributions for a simulation study are normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution. As a result of the simulation study, the estimated significant level α is around 0.05 for all test statistics (3.58), (3.68), (2.16), (2.20).

Table 5.4. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is ***equal*** to The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
6	6	**First**	0.0494	0.0522
		Second	0.0476	0.0498
10	10	**First**	0.0494	0.0556
		Second	0.0484	0.0534
16	16	**First**	0.0504	0.0506
		Second	0.0490	0.0506
20	20	**First**	0.0520	0.0534
		Second	0.0518	0.0488

Table 5.5. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is ***equal*** to The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	6	**First**	0.0486	0.0522
		Second	0.0442	0.0532
10	10	**First**	0.0534	0.0532
		Second	0.0504	0.0518
16	16	**First**	0.0516	0.0528
		Second	0.0502	0.0506
20	20	**First**	0.0500	0.0498
		Second	0.0450	0.0506

Table 5.6. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is ***equal*** to The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	6	**First**	0.0506	0.0514
		Second	0.0476	0.0498
10	10	**First**	0.0466	0.0580
		Second	0.0408	0.0508
16	16	**First**	0.0430	0.0510
		Second	0.0520	0.0532
20	20	**First**	0.0544	0.0402
		Second	0.0496	0.0442

3. **The sample size of all treatments for the (CRD) portion is *half* the number of blocks for the (RCBD) portion:** Tables 5.7 through 5.9 present the estimated significant level when the sample size for the (CRD) portion is $n = 6, 10, 16, 20$ and the number of blocks for the (RCBD) portion is $b = 12, 20, 32, 40$. We assume three different underlying distributions, including normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution for a simulation study. As a result of the simulation study, the estimated significant level α is around 0.05 for all test statistics (3.58),(3.68),(2.16),(2.20).

Table 5.7. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is ***half*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	12	**First**	0.0526	0.0506
		Second	0.0506	0.0494
10	20	**First**	0.0498	0.0538
		Second	0.0538	0.0512
16	32	**First**	0.0502	0.0526
		Second	0.0474	0.0498
20	40	**First**	0.0456	0.0470
		Second	0.0504	0.0524

Table 5.8. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is ***half*** The Number of Blocks for The RCBD portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	12	**First**	0.0528	0.0514
		Second	0.0530	0.0522
10	20	**First**	0.0492	0.0488
		Second	0.0490	0.0532
16	32	**First**	0.0480	0.0514
		Second	0.0490	0.0546
20	40	**First**	0.0562	0.0528
		Second	0.0460	0.0498

Table 5.9. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is ***half*** The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
6	12	**First**	0.0534	0.0442
		Second	0.0482	0.0494
10	20	**First**	0.0522	0.0524
		Second	0.0562	0.0480
16	32	**First**	0.0508	0.0514
		Second	0.0494	0.0558
20	40	**First**	0.0544	0.0402
		Second	0.0496	0.0442

5.1.2. Estimated Power

We estimate the power of the test statistics as given in (3.58), (3.68), (2.16), (2.20) by finding the percentage of rejection of the null hypothesis when the alternative hypothesis is true. As has been mentioned in Chapter 4, we consider three different situations for the relationship between the sample size of treatments for a completely randomized design (CRD) portion and the number of blocks for a randomized complete block design (RCBD) portion, as presented in Tables 5.10 through 5.18 under a variety of underlying distributions.

We will present in this section the results of the estimated power once the sample size for each treatment is $n = 10$ and we change the number of blocks to be $b = 5, 10, 20$.

1. **The sample size of all treatments for the (CRD) portion is *twice* the number of blocks for the (RCBD) portion:** Tables 5.10 through 5.12 show the estimated power when the sample size for the (CRD) portion is $n = 10$, and the number of blocks for the (RCBD) portion is $b = 5$. There are a variety of underlying distributions, including normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution. The other results for different sample sizes and number of blocks are presented in Appendix A. As a result of the simulation study, the estimated power for the proposed test statistics in Chapter 3 (3.58), (3.68) with a squared distance modification is high for all cases of the ***indistinct peak with the first location parameter*** of the umbrella hypothesis. Besides

that, there are a few cases of ***distinct peak*** that have high estimated power for the proposed test statistics (3.58), (3.68) like (0.8 , **1.0** , 0.75 , 0.2), and (0.75 , **1.0** , 0.75 , 0.2). In either case of modification or nonmodification, the estimated power by the first method used to propose the test statistics, as given in (3.58) and (2.16), is better than the second method, as given in (3.68) and (2.20), for all different distributions.

Table 5.10. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	**0.7730**	0.7042
	Second	**0.6976**	0.6284
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.5040	**0.5870**
	Second	0.4394	**0.5252**
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2434	**0.2924**
	Second	0.2122	**0.2554**
(0 , **1.0** , 0.5 , 0.5)	**First**	**0.5672**	0.4520
	Second	**0.5022**	0.4040
(0.5 , **1.0** , 0.5 , 0.5)	**First**	**0.3974**	0.3764
	Second	0.3488	0.3320
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.6020	**0.6180**
	Second	0.5346	**0.5566**
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2296	**0.2876**
	Second	0.1998	**0.2556**
(0.0 , **0.5** , 0.5 , 0.5)	**First**	**0.1130**	0.0734
	Second	**0.1044**	0.0738
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2236	**0.2938**
	Second	0.2008	**0.2634**
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.5210	**0.5962**
	Second	0.4610	**0.5330**

Table 5.11. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	**0.6046**	0.5442
	Second	**0.5296**	0.4876
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.3848	**0.4392**
	Second	0.3380	**0.3806**
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1922	**0.2234**
	Second	0.1672	**0.1992**
(0 , **1.0** , 0.5 , 0.5)	**First**	**0.4272**	0.3458
	Second	**0.3704**	0.3144
(0.5 , **1.0** , 0.5 , 0.5)	**First**	**0.3052**	0.2908
	Second	**0.2628**	0.2598
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.4796	0.4770
	Second	0.4038	0.4102
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1840	**0.2204**
	Second	0.1584	**0.1988**
(0.0 , **0.5** , 0.5 , 0.5)	**First**	**0.0954**	0.0662
	Second	**0.0948**	0.0662
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1824	**0.2328**
	Second	0.1616	**0.2110**
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.4008	**0.4472**
	Second	0.3448	**0.3992**

Table 5.12. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **1.0** , 0.75 , 0.2)	**First**	**0.9556**	0.9070
	Second	**0.9130**	0.8466
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.7670	**0.8038**
	Second	0.6918	**0.7516**
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3962	**0.4924**
	Second	0.3442	**0.4384**
(0 , **1.0** , 0.5 , 0.5)	**First**	**0.8152**	0.6824
	Second	**0.7414**	0.6164
(0.5 , **1.0** , 0.5 , 0.5)	**First**	**0.6516**	0.6052
	Second	**0.5730**	0.5494
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.8694	0.8572
	Second	0.8034	0.7874
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3584	**0.4636**
	Second	0.3038	**0.4124**
(0.0 , **0.5** , 0.5 , 0.5)	**First**	**0.1482**	0.0854
	Second	**0.1344**	0.0822
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3810	**0.4648**
	Second	0.3260	**0.4112**
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.7926	**0.8302**
	Second	0.7070	**0.7540**

2. **The sample size of all treatments for the (CRD) portion is *equal* to the number of blocks for the (RCBD) portion:** Tables 5.13 through 5.15 show the estimated power when the sample size for the (CRD) portion is $n = 10$, and the number of blocks for the (RCBD) portion is $b = 10$. We assume three different underlying distributions, including normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution. The other results for different sample sizes and number of blocks are presented in Appendix A. As a result of the simulation study, there is a similar pattern with the previous case, in which the estimated power for the proposed test statistics (3.58), (3.68) with a squared distance modification is high for all cases of the ***indistinct peak with the first location***

parameter of the umbrella hypothesis. Besides that, there are a few cases of ***distinct peak*** that have high estimated power for the proposed test statistics (3.58), (3.68), like (0.8 , **1.0** , 0.75 , 0.2), and (0.75 , **1.0** , 0.75 , 0.2). It is good to mention that, the estimated power by using the first method of combining the two test statistics for (CRD) and (RCBD), as given in (3.58) and (2.16), is better than the second method, as given in (3.68) and (2.20) for all different distributions.

Table 5.13. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	**0.8790**	0.8194
	Second	**0.7426**	0.6848
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.6060	**0.6966**
	Second	0.4566	**0.5420**
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2868	**0.3628**
	Second	0.2212	**0.2722**
(0.0 , **1.0** , 0.5 , 0.5)	**First**	**0.6704**	0.5572
	Second	**0.5190**	0.4274
(0.5 , **1.0** , 0.5 , 0.5)	**First**	**0.4866**	0.4670
	Second	**0.3766**	0.3410
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.7314	0.7336
	Second	0.5620	**0.5994**
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2738	**0.3490**
	Second	0.2130	**0.2692**
(0.0 , **0.5** , 0.5 , 0.5)	**First**	**0.1328**	0.0816
	Second	**0.1084**	0.0722
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2820	**0.3472**
	Second	0.2076	**0.2720**
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.6358	**0.6978**
	Second	0.4822	**0.5476**

Table 5.14. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	**0.7326**	0.6436
	Second	**0.5648**	0.5034
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.4666	**0.5348**
	Second	0.3494	**0.4070**
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2314	**0.2804**
	Second	0.1762	**0.2196**
(0.0 , **1.0** , 0.5 , 0.5)	**First**	**0.5324**	0.4096
	Second	**0.4008**	0.3170
(0.5 , **1.0** , 0.5 , 0.5)	**First**	**0.3726**	0.3436
	Second	**0.2804**	0.2730
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.5774	0.5766
	Second	0.4308	0.4438
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2198	**0.2760**
	Second	0.1714	**0.2144**
(0.0 , **0.5** , 0.5 , 0.5)	**First**	**0.1100**	0.0692
	Second	**0.0966**	0.0702
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2250	**0.2744**
	Second	0.1764	**0.2222**
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.4926	**0.5420**
	Second	0.3708	**0.4246**

Table 5.15. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **1.0** , 0.75 , 0.2)	**First**	**0.9880**	0.9646
	Second	**0.9346**	0.8834
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.8742	**0.9052**
	Second	0.7320	**0.7630**
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4924	**0.5918**
	Second	0.3648	**0.4538**
(0.0 , **1.0** , 0.5 , 0.5)	**First**	**0.9142**	0.7968
	Second	**0.7880**	0.6452
(0.5 , **1.0** , 0.5 , 0.5)	**First**	**0.7758**	0.7270
	Second	**0.6162**	0.5772
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9430	0.9358
	Second	0.8258	0.8226
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4518	**0.5716**
	Second	0.3332	**0.4412**
(0.0 , **0.5** , 0.5 , 0.5)	**First**	**0.1760**	0.0924
	Second	**0.1434**	0.0826
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4538	**0.5848**
	Second	0.3488	**0.4522**
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.8886	**0.9150**
	Second	0.7538	**0.7982**

3. **The sample size of all treatments for the (CRD) portion is *half* the number of blocks for the (RCBD) portion:** Tables 5.16 through 5.18 present the estimated power when the sample size for the (CRD) portion is $n = 10$, and the number of blocks for the (RCBD) portion is $b = 20$; assuming a variety of underlying distribution, such as normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution. The other results for different sample sizes and number of blocks are presented in Appendix A. The results still have a similar pattern with the two previous cases. The estimated power for proposed test statistics (3.58), (3.68) with a squared distance modification is high for all cases of the ***indistinct peak with the first location parameter*** of the umbrella hypothesis.

Besides that, there are a few cases of ***distinct peak*** that have high estimated power for proposed test statistics (3.58), (3.68), like (0.8 , **1.0** , 0.75 , 0.2), and (0.75 , **1.0** , 0.75 , 0.2). In either case, the estimated power by the first method used for the proposed test statistics, as given in (3.58) and (2.16), is better than the second method, as given in (3.68) and (2.20) for all different distributions.

Table 5.16. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	**0.9576**	0.9208
	Second	**0.7982**	0.7324
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.7516	**0.8170**
	Second	0.5288	**0.5950**
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3732	**0.4590**
	Second	0.2548	**0.3110**
(0.0 , **1.0** , 0.5 , 0.5)	**First**	**0.8136**	0.6752
	Second	**0.6010**	0.4626
(0.5 , **1.0** , 0.5 , 0.5)	**First**	**0.6146**	0.5592
	Second	**0.4116**	0.3782
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.8482	**0.8662**
	Second	0.6326	**0.6632**
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3550	**0.4536**
	Second	0.2338	**0.3010**
(0.0 , **0.5** , 0.5 , 0.5)	**First**	**0.1568**	0.0824
	Second	**0.1170**	0.0746
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3600	**0.4518**
	Second	0.2506	**0.3060**
(0.75, **1.0** , 0.75 , 0.2)	**First**	0.7744	**0.8154**
	Second	0.5586	**0.6034**

Table 5.17. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	**0.8458**	0.7992
	Second	**0.6366**	0.5804
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.5886	**0.6838**
	Second	0.3932	**0.4704**
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2732	**0.3488**
	Second	0.1944	**0.2342**
(0.0 , **1.0** , 0.5 , 0.5)	**First**	**0.6612**	0.5290
	Second	**0.4506**	0.3504
(0.5 , **1.0** , 0.5 , 0.5)	**First**	**0.4700**	0.4300
	Second	**0.3190**	0.2958
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.6956	0.7060
	Second	0.4856	0.4986
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2838	**0.3310**
	Second	0.1978	**0.2336**
(0.0 , **0.5** , 0.5 , 0.5)	**First**	**0.1236**	0.0866
	Second	**0.1006**	0.0726
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2820	**0.3384**
	Second	0.1956	**0.2334**
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.6030	**0.6666**
	Second	0.4056	**0.4628**

Table 5.18. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9986	0.9928
	Second	0.9584	0.9180
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.9622	**0.9728**
	Second	0.8014	**0.8336**
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6208	**0.7426**
	Second	0.4212	**0.5258**
(0.0 , **1.0** , 0.5 , 0.5)	**First**	**0.9786**	0.9174
	Second	**0.8444**	0.7236
(0.5 , **1.0** , 0.5 , 0.5)	**First**	**0.9012**	0.8544
	Second	**0.6888**	0.6306
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9856	0.9880
	Second	0.8836	0.8758
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.5904	**0.7236**
	Second	0.3770	**0.4998**
(0.0 , **0.5** , 0.5 , 0.5)	**First**	**0.2160**	0.1090
	Second	**0.1494**	0.0838
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5868	**0.6986**
	Second	0.3930	**0.4986**
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.9658	**0.9764**
	Second	0.8150	**0.8518**

5.2. Five Treatments at Known Peak ($p = 2$)

In this section, we present the results from the simulation study of estimating the significant level and the power for proposed test statistics; there are five treatments and the peak is at the second treatment.

5.2.1. Estimated Significant Level α

Tables 5.19 through 5.27 show the results of the estimated significant level when there are five treatments and the peak is at the second treatment. The results are based on the consideration of the relationship between the sample size for the (CRD) portion and the number of blocks for the

(RCBD) portion under a variety of underlying distributions, such as normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution:

1. **The sample size of all treatments for the (CRD) portion is *twice* the number of blocks for the (RCBD) portion:** Tables 5.19 through 5.21 present the estimated significant level when the sample size for the (CRD) portion is $n = 6, 10, 16, 20$ and the number of blocks for the (RCBD) portion is $b = 3, 5, 8, 10$. The results are based on assuming three different underlying distributions. As a result of the simulation study, the estimated significant level α is around 0.05 for proposed test statistics with a squared distance modification (3.58), (3.68), and the test statistics (2.16), (2.20) introduced by Magel et al. (2010).

Table 5.19. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is ***twice*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	3	**First**	0.0518	0.0470
		Second	0.0506	0.0498
10	5	**First**	0.0512	0.0566
		Second	0.0500	0.0530
16	8	**First**	0.0498	0.0546
		Second	0.0506	0.0488
20	10	**First**	0.0512	0.0496
		Second	0.0530	0.0460

Table 5.20. Estimated Significant Level α for a Test Statistic of a Mixed Design Under the Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is ***twice*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	3	**First**	0.0466	0.0462
		Second	0.0472	0.0530
10	5	**First**	0.0520	0.0502
		Second	0.0486	0.0542
16	8	**First**	0.0522	0.0474
		Second	0.0526	0.0512
20	10	**First**	0.0526	0.0524
		Second	0.0486	0.0488

Table 5.21. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is ***twice*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	3	**First**	0.0472	0.0488
		Second	0.0506	0.0474
10	5	**First**	0.0558	0.0576
		Second	0.0490	0.0538
16	8	**First**	0.0468	0.0466
		Second	0.0520	0.0498
20	10	**First**	0.0554	0.0478
		Second	0.0542	0.0498

2. **The sample size of all treatments for the (CRD) portion is *equal* to the number of blocks for the (RCBD) portion:** Tables 5.22 through 5.24 show the estimated significant level when the sample size for the (CRD) portion is $n = 6, 10, 16, 20$, and the number of blocks for the (RCBD) portion is $b = 6, 10, 16, 20$. We assume a variety of underlying distributions. As a result of the simulation study, the estimated significant level α is around 0.05 for all test statistics (3.58), (3.68), (2.16), (2.20).

Table 5.22. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is ***equal*** to The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	6	**First**	0.0464	0.0480
		Second	0.0476	0.0478
10	10	**First**	0.0534	0.0508
		Second	0.0524	0.0540
16	16	**First**	0.0492	0.0468
		Second	0.0496	0.0530
20	20	**First**	0.0482	0.0510
		Second	0.0478	0.0470

Table 5.23. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is ***equal*** to The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	6	**First**	0.0476	0.0518
		Second	0.0502	0.0520
10	10	**First**	0.0528	0.0540
		Second	0.0520	0.0562
16	16	**First**	0.0526	0.0494
		Second	0.0502	0.0500
20	20	**First**	0.0422	0.0520
		Second	0.0510	0.0484

Table 5.24. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at peak 2; The Sample Size for The CRD Portion is ***equal*** to The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	6	**First**	0.0508	0.0544
		Second	0.0470	0.0540
10	10	**First**	0.0570	0.0532
		Second	0.0534	0.0482
16	16	**First**	0.0480	0.0480
		Second	0.0490	0.0494
20	20	**First**	0.0486	0.0502
		Second	0.0534	0.0482

3. **The sample size of all treatments for the (CRD) portion is *half* the number of blocks for the (RCBD) portion:** Tables 5.25 through 5.27 present the estimated significant level when the sample size for the (CRD) portion is $n = 6, 10, 16, 20$ and the number of blocks for the (RCBD) portion is $b = 12, 20, 32, 40$. The results are based on assuming three different underlying distributions, such as normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution for a simulation study. As a result of the simulation study, the estimated significant level α is around 0.05 for all test statistics (3.58), (3.68), (2.16), (2.20).

Table 5.25. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for CRD Portion is ***half*** The Number of Blocks for RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	12	**First**	0.0478	0.0496
		Second	0.0486	0.0524
10	20	**First**	0.0522	0.0550
		Second	0.0524	0.0530
16	32	**First**	0.0484	0.0446
		Second	0.0516	0.0522
20	40	**First**	0.0470	0.0490
		Second	0.0496	0.0506

Table 5.26. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degree of Freedom for 5 Treatments at Peak 2; The Sample Size for CRD Portion is ***half*** The Number of Blocks for RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	12	**First**	0.0534	0.0496
		Second	0.0522	0.0510
10	20	**First**	0.0582	0.0466
		Second	0.0522	0.0430
16	32	**First**	0.0472	0.0542
		Second	0.0494	0.0568
20	40	**First**	0.0518	0.0516
		Second	0.0500	0.0534

Table 5.27. Estimated Significant Level α for a Test Statistic of a Mixed Design Under the Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for CRD Portion is ***half*** The number of Blocks for RCBD Portion.

n	b	Method	Non Modification	Modification
6	12	**First**	0.0456	0.0516
		Second	0.0520	0.0510
10	20	**First**	0.0462	0.0486
		Second	0.0536	0.0468
16	32	**First**	0.0484	0.0496
		Second	0.0502	0.0452
20	40	**First**	0.0432	0.0538
		Second	0.0430	0.0476

5.2.2. Estimated Power

In this section, we estimate the power for each test statistic as given in (3.58), (3.68), (2.16), (2.20). There are three different situations for the relationship between the sample size of treatments for the completely randomized design (CRD) portion and the number of blocks for the randomized complete block design (RCBD) portion, as presented in Tables 5.28 through 5.36 under a variety of underlying distributions.

We present the results of the estimated power once the sample size for each treatment is $n = 10$ and change the number of blocks to be $b = 5, 10, 20$.

1. **The sample size of all treatments for the (CRD) portion is *twice* the number of blocks for the (RCBD) portion:** Tables 5.28 through 5.30 show the estimated power when the sample size for the (CRD) portion is $n = 10$, and the number of blocks for the (RCBD) portion is $b = 5$. We use a variety of underlying distributions, such as normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution. The other results for different sample sizes and number of blocks are presented in Appendix B. As a result of the simulation study, the estimated power for the proposed test statistics in Chapter 3 (3.58), (3.68) with a squared distance modification is high for all cases of the ***indistinct peak with the first location parameter*** of the umbrella hypothesis. On the other hand, for the case of the ***distinct peak***, the estimated power for test statistics (2.16), (2.20) introduced

by Magel et al. (2010) is high unless the umbrella hypothesis in this type is (0.75 , **0.8** , 0.6 , 0.4 , 0.2). In either case of modification or nonmodification, the estimated power by using the first method of combining the two test statistics, as given in (3.58) and (2.16), is better than the second method, as given in (3.68) and (2.20), for all different distributions.

Table 5.28. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	**0.5800**	0.4970
	Second	**0.5134**	0.4478
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.5754	**0.6512**
	Second	0.4956	**0.5814**
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.5488	**0.6518**
	Second	0.4776	**0.5786**
(0.0 , **0.7** , 0.4 , 0.2 , 0.2)	**First**	**0.4870**	0.3938
	Second	**0.4180**	0.3458
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.7876	0.7934
	Second	0.7090	0.7204
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.3026	**0.3486**
	Second	0.2528	**0.3150**
(0.0 , **0.6** , 0.4 , 0.4 , 0.2)	**First**	**0.3250**	0.2618
	Second	**0.2716**	0.2326
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.7566	**0.7732**
	Second	0.6666	**0.6984**
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2630	**0.3086**
	Second	0.2276	**0.2670**
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	**0.3236**	0.2622
	Second	**0.2784**	0.2188
(0.0 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.3714**	0.3418
	Second	**0.3278**	0.2974
(0.2 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3160	0.3244
	Second	0.2772	0.2734
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2466	**0.3156**
	Second	0.2216	**0.2882**

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Table 5.28. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$. (Continued)

Location Parameter	Method	Non Modification	Modification
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.6298	**0.6426**
	Second	0.5544	**0.5770**
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.3326	**0.3800**
	Second	0.2900	**0.3220**
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	**0.5834**	0.5576
	Second	0.4994	0.4896
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.3620	**0.4376**
	Second	0.3092	**0.3794**
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3300	0.3378
	Second	0.2770	0.2908
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.2484	**0.2980**
	Second	0.2138	**0.2706**
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	**0.0890**	0.0584
	Second	**0.0804**	0.0580

Table 5.29. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	**0.4396**	0.3748
	Second	**0.3824**	0.3224
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.4236	**0.4988**
	Second	0.3690	**0.4302**
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.4134	**0.5068**
	Second	0.3680	**0.4488**
(0.0 , **0.7** , 0.4 , 0.2 , 0.2)	**First**	**0.3652**	0.3002
	Second	**0.3144**	0.2756
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.6228	**0.6440**
	Second	0.5514	**0.5664**
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.2276	**0.2690**
	Second	0.1988	**0.2436**
(0.0 , **0.6** , 0.4 , 0.4 , 0.2)	**First**	**0.2516**	0.2008
	Second	**0.2212**	0.1810
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.5708	**0.6034**
	Second	0.5034	**0.5348**
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2014	**0.2348**
	Second	0.1782	**0.2098**
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	**0.2598**	0.2012
	Second	**0.2202**	0.1896
(0.0 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.2786**	0.2576
	Second	0.2378	0.2264
(0.2 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.2568**	0.2336
	Second	0.2246	0.2228
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.1996	**0.2336**
	Second	0.1770	**0.2100**
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.5018	0.5100
	Second	0.4330	0.4400
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.2412	**0.2890**
	Second	0.1996	**0.2492**

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Table 5.29. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.4446	0.4310
	Second	0.3764	0.3788
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.2894	**0.3464**
	Second	0.2528	**0.2962**
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2556	0.2494
	Second	0.2252	0.2240
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.2018	**0.2402**
	Second	0.1800	**0.2122**
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	**0.0770**	0.0550
	Second	**0.0702**	0.0574

Table 5.30. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	**0.8480**	0.7532
	Second	**0.7936**	0.6872
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8360	**0.8884**
	Second	0.7644	**0.8230**
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8260	**0.8798**
	Second	0.7556	**0.8192**
(0.0 , **0.7** , 0.4 , 0.2 , 0.2)	**First**	**0.7586**	0.6654
	Second	**0.6786**	0.5744
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9596	0.9504
	Second	0.9240	0.9068
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.5150	**0.6022**
	Second	0.4526	**0.5382**
(0.0 , **0.6** , 0.4 , 0.4 , 0.2)	**First**	**0.5658**	0.4592
	Second	**0.4908**	0.4044
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	**0.9444**	0.9274
	Second	**0.8942**	0.8738
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4384	**0.5236**
	Second	0.3828	**0.4590**
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	**0.5306**	0.4446
	Second	**0.4566**	0.3814
(0.0 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.6044**	0.5576
	Second	**0.5160**	0.4862
(0.2 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.5304	0.5356
	Second	0.4408	0.4660
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3904	**0.4962**
	Second	0.3328	**0.4268**
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	**0.9060**	0.8808
	Second	**0.8528**	0.8178
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.5524	**0.6258**
	Second	0.4828	**0.5568**

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Table 5.30. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$. (Continued)

Location Parameter	Method	Non Modification	Modification
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	**0.8634**	0.8254
	Second	**0.7940**	0.7546
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.6054	**0.7064**
	Second	0.5270	**0.6304**
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	**0.5792**	0.5612
	Second	0.4980	0.5004
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.3946	**0.4948**
	Second	0.3474	**0.4360**
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	**0.1092**	0.0580
	Second	**0.0984**	0.0576

2. **The sample size of all treatments for the (CRD) portion is *equal* to the number of blocks for the (RCBD) portion:** Tables 5.31 through 5.33 show the estimated power when the sample size for the (CRD) portion is $n = 10$, and the number of blocks for the (RCBD) portion is $b = 10$. Also, there are three different underlying distributions. The other results for different sample sizes and number of blocks are presented in Appendix B. As a result of the simulation study, there is a similar pattern with the previous case, in which the estimated power for proposed test statistics in Chapter 3 (3.58), (3.68) with a squared distance modification is high for all cases of the ***indistinct peak with the first location parameter*** of the umbrella hypothesis. On the other hand, for the case of the ***distinct peak***, the estimated power for test statistics (2.16), (2.20) introduced by Magel et al. (2010) is high unless the umbrella hypothesis in this type is (0.75 , **0.8** , 0.6 , 0.4 , 0.2). In either case, the estimated power by the first method used for proposing the test statistics, as given in (3.58) and (2.16), is better than the second method, as given in (3.68) and (2.20), for all different distributions.

Table 5.31. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	**0.6946**	0.6016
	Second	**0.5388**	0.4650
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6658	**0.7668**
	Second	0.5254	**0.6188**
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6634	**0.7772**
	Second	0.5174	**0.6256**
(0.0 , **0.7** , 0.4 , 0.2 , 0.2)	**First**	**0.5752**	0.4862
	Second	**0.4390**	0.3680
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.8816	0.8918
	Second	0.7432	0.7600
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.3700	**0.4422**
	Second	0.2746	**0.3384**
(0.0 , **0.6** , 0.4 , 0.4 , 0.2)	**First**	**0.3942**	0.3112
	Second	**0.2962**	0.2484
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.8442	**0.8638**
	Second	0.7038	0.7174
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3196	**0.3732**
	Second	0.2314	**0.2846**
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	**0.4038**	0.3272
	Second	**0.3050**	0.2508
(0.0 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.4642**	0.4160
	Second	**0.3424**	0.3106
(0.2 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3932	0.4046
	Second	0.3040	0.3010
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2944	**0.3764**
	Second	0.2300	**0.2830**

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Table 5.31. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$. (Continued)

Location Parameter	Method	Non Modification	Modification
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.7448	**0.7726**
	Second	0.5980	**0.6126**
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.3932	**0.4660**
	Second	0.3020	**0.3558**
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.6946	0.6816
	Second	0.5382	0.5310
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.4650	**0.5354**
	Second	0.3540	**0.4049**
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3898	0.3970
	Second	0.3076	0.2964
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.3084	**0.3762**
	Second	0.2376	**0.2830**
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	**0.1022**	0.0578
	Second	**0.0930**	0.0632

Table 5.32. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	**0.5396**	0.4518
	Second	**0.4102**	0.3472
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.5220	**0.6096**
	Second	0.3966	**0.4652**
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.5168	**0.5924**
	Second	0.3994	**0.4588**
(0.0 , **0.7** , 0.4 , 0.2 , 0.2)	**First**	**0.4492**	0.3652
	Second	**0.3406**	0.2828
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.7272	**0.7410**
	Second	0.5816	0.5922
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.2696	**0.3298**
	Second	0.2074	**0.2506**
(0.0 , **0.6** , 0.4 , 0.4 , 0.2)	**First**	**0.3136**	0.2436
	Second	**0.2450**	0.1868
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.6782	**0.7178**
	Second	0.5390	**0.5634**
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2368	**0.2884**
	Second	0.1930	**0.2100**
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	**0.2900**	0.2492
	Second	**0.2338**	0.1920
(0.0 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.3506**	0.3290
	Second	**0.2626**	0.2404
(0.2 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2896	0.2890
	Second	0.2236	0.2258
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2358	**0.2842**
	Second	0.1718	**0.2128**
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.5992	0.6030
	Second	0.4584	0.4608
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.3014	**0.3544**
	Second	0.2404	**0.2736**

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Table 5.32. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.5360	0.5350
	Second	0.4062	0.4020
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.3476	**0.4006**
	Second	0.2652	**0.3124**
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3026	0.3086
	Second	0.2298	0.2342
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.2488	**0.2792**
	Second	0.1956	**0.2110**
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	**0.0924**	0.0578
	Second	**0.0820**	0.0616

Table 5.33. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	**0.9414**	0.8524
	Second	**0.8338**	0.7202
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9272	**0.9532**
	Second	0.8056	**0.8556**
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9170	**0.9458**
	Second	0.7892	**0.8404**
(0.0 , **0.7** , 0.4 , 0.2 , 0.2)	**First**	**0.8706**	0.7724
	Second	**0.7230**	0.6134
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9924	0.9864
	Second	0.9430	0.9236
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.6330	**0.7136**
	Second	0.4784	**0.5488**
(0.0 , **0.6** , 0.4 , 0.4 , 0.2)	**First**	**0.6740**	0.5714
	Second	**0.5246**	0.4338
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9826	0.9766
	Second	0.9188	0.9014
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.5510	**0.6338**
	Second	0.4172	**0.4934**
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	**0.6514**	0.5450
	Second	**0.4942**	0.4006
(0.0 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.7318**	0.6778
	Second	**0.5742**	0.5100
(0.2 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.6706**	0.6556
	Second	0.5046	0.4988
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.4950	**0.6182**
	Second	0.3744	**0.4696**
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9622	0.9512
	Second	0.8692	0.8558
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.6572	**0.7562**
	Second	0.5190	**0.5952**

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Table 5.33. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$. (Continued)

Location Parameter	Method	Non Modification	Modification
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	**0.9430**	0.9120
	Second	**0.8250**	0.7802
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.7238	**0.7966**
	Second	0.5660	**0.6404**
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	**0.6982**	0.6756
	Second	0.5446	0.5314
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.4906	**0.6166**
	Second	0.3924	**0.4748**
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	**0.1236**	0.0608
	Second	**0.1062**	0.0546

3. **The sample size of all treatments for the (CRD) portion is *half* the number of blocks for the (RCBD) portion:** Tables 5.34 through 5.36 show the estimated power when the sample size for the (CRD) portion is $n = 10$, and the number of blocks for the (RCBD) portion is $b = 20$; under a variety of underlying distributions. The other results for different sample sizes and number of blocks are presented in Appendix B. The results still have a similar pattern with the two previous cases. The estimated power for the proposed test statistics in Chapter 3 (3.58), (3.68) with a squared distance modification is high for all cases of the ***indistinct peak with the first location parameter*** of the umbrella hypothesis. On the other hand, for the case of the ***distinct peak***, the estimated power for test statistics (2.16), (2.20) introduced by Magel et al. (2010) is high unless the umbrella hypothesis in this type is (0.75 , **0.8** , 0.6 , 0.4 , 0.2). In either case, the estimated power by the first method used to propose test statistics, as given in (3.58) and (2.16), is better than the second method, as given in (3.68) and (2.20), for all different distributions.

Table 5.34. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	**0.8276**	0.7382
	Second	**0.5974**	0.5164
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8102	**0.8906**
	Second	0.5862	**0.6732**
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.7984	**0.8776**
	Second	0.5762	**0.6728**
(0.0 , **0.7** , 0.4 , 0.2 , 0.2)	**First**	**0.7214**	0.6200
	Second	**0.4802**	0.4208
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9592	0.9694
	Second	0.7978	0.8256
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.4690	**0.5566**
	Second	0.3048	**0.3802**
(0.0 , **0.6** , 0.4 , 0.4 , 0.2)	**First**	**0.5180**	0.4168
	Second	**0.3382**	0.2734
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9442	0.9482
	Second	0.7648	0.7802
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3974	**0.4746**
	Second	0.2658	**0.3100**
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	**0.5114**	0.4004
	Second	**0.3234**	0.2676
(0.0 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.5720**	0.5214
	Second	**0.3836**	0.3478
(0.2 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.5132	0.5110
	Second	0.3386	0.3462
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3826	**0.4848**
	Second	0.2536	**0.3124**
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.8704	**0.8840**
	Second	0.6448	**0.6692**
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.5088	**0.5902**
	Second	0.3348	**0.3912**

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Table 5.34. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	**0.8262**	0.8042
	Second	0.5984	0.5902
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.5726	**0.6652**
	Second	0.3862	**0.4570**
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.5170	0.5070
	Second	0.3396	0.3340
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.3978	**0.4778**
	Second	0.2498	**0.3106**
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	**0.1006**	0.0598
	Second	**0.0918**	0.0584

Table 5.35. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	**0.6740**	0.5804
	Second	**0.4622**	0.3896
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6702	**0.7428**
	Second	0.4466	**0.5168**
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6484	**0.7262**
	Second	0.4382	**0.5074**
(0.0 , **0.7** , 0.4 , 0.2 , 0.2)	**First**	**0.5650**	0.4754
	Second	**0.3770**	0.3190
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.8582	**0.8738**
	Second	0.6290	**0.6664**
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.3608	**0.4276**
	Second	0.2366	**0.2846**
(0.0 , **0.6** , 0.4 , 0.4 , 0.2)	**First**	**0.3818**	0.3042
	Second	**0.2490**	0.2116
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.8296	**0.8358**
	Second	0.6050	0.6118
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2936	**0.3574**
	Second	0.2046	**0.2460**
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	**0.3828**	0.3262
	Second	**0.2550**	0.2096
(0.0 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.4470**	0.3966
	Second	**0.3042**	0.2684
(0.2 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.3982**	0.3824
	Second	0.2610	0.2590
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2996	**0.3568**
	Second	0.2040	**0.2346**
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.7322	0.7432
	Second	0.5004	0.5168
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.4008	**0.4552**
	Second	0.2596	**0.3106**

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Table 5.35. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	**0.6744**	0.6488
	Second	**0.4586**	0.4354
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.4284	**0.5088**
	Second	0.2890	**0.3370**
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3732	0.3832
	Second	0.2532	0.2604
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.2988	**0.3570**
	Second	0.2046	**0.2398**
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	**0.0978**	0.0632
	Second	**0.0802**	0.0548

Table 5.36. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	**0.9884**	0.9558
	Second	**0.8756**	0.7798
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9832	**0.9878**
	Second	0.8544	**0.8950**
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9774	**0.9886**
	Second	0.8416	**0.8892**
(0.0 , **0.7** , 0.4 , 0.2 , 0.2)	**First**	**0.9602**	0.8918
	Second	**0.7850**	0.6724
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9992	0.9988
	Second	0.9724	0.9554
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.7666	**0.8470**
	Second	0.5316	**0.6108**
(0.0 , **0.6** , 0.4 , 0.4 , 0.2)	**First**	**0.8188**	0.7152
	Second	**0.5884**	0.4950
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9984	0.9978
	Second	0.9534	0.9344
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.6990	**0.7648**
	Second	0.4732	**0.5392**
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	**0.8074**	0.6714
	Second	**0.5570**	0.4436
(0.0 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	**0.8686**	0.8072
	Second	**0.6400**	0.5728
(0.2 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.7966	0.8058
	Second	0.5510	0.5672
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.6394	**0.7464**
	Second	0.4010	**0.5120**
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9954	0.9922
	Second	0.9130	0.8926
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.7944	**0.8644**
	Second	0.5756	**0.6574**

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Table 5.36. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$. (Continued)

Location Parameter	Method	Non Modification	Modification
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	**0.9906**	0.9752
	Second	**0.8798**	0.8390
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.8472	**0.9142**
	Second	0.6164	**0.6912**
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	**0.8316**	0.8098
	Second	**0.6068**	0.5804
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.6168	**0.7394**
	Second	0.4278	**0.5202**
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	**0.1434**	0.0722
	Second	**0.1082**	0.0608

5.3. Five Treatments at Known Peak ($p = 3$)

In this section, we present the results from the simulation study of estimating the significant level and the power for the proposed test statistics, once there are five treatments and the peak is at the third treatment.

5.3.1. Estimated Significant Level α

Tables 5.37 through 5.45 show the results of the estimated significant level. Besides that, we consider three different relationships between the sample size for the (CRD) portion and the number of blocks for the (RCBD) portion under a variety of underlying distributions, such as normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution:

1. **The sample size of all treatments for the (CRD) portion is *twice* the number of blocks for the (RCBD) portion:** Tables 5.37 through 5.39 present the estimated significant level when the sample size for the (CRD) portion is $n = 6, 10, 16, 20$ and the number of blocks for the (RCBD) portion is $b = 3, 5, 8, 10$. The results are based on assuming three different underlying distributions. As a result of the simulation study, the estimated significant level α is around 0.05 for the proposed test statistics (3.58), (3.68), and the test statistics (2.16), (2.20) introduced by Magel et al. (2010).

Table 5.37. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is ***twice*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	3	**First**	0.0468	0.0464
		Second	0.0504	0.0498
10	5	**First**	0.0512	0.0484
		Second	0.0452	0.0458
16	8	**First**	0.0478	0.0522
		Second	0.0478	0.0524
20	10	**First**	0.0496	0.0534
		Second	0.0446	0.0556

Table 5.38. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is ***twice*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	3	**First**	0.0582	0.0520
		Second	0.0548	0.0542
10	5	**First**	0.0468	0.0516
		Second	0.0518	0.0522
16	8	**First**	0.0488	0.0502
		Second	0.0496	0.0480
20	10	**First**	0.0480	0.0478
		Second	0.0534	0.0466

Table 5.39. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is ***twice*** The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
6	3	**First**	0.0494	0.0510
		Second	0.0482	0.0544
10	5	**First**	0.0462	0.0512
		Second	0.0460	0.0566
16	8	**First**	0.0552	0.0540
		Second	0.0478	0.0536
20	10	**First**	0.0460	0.0512
		Second	0.0546	0.0518

2. **The sample size of all treatments for the (CRD) portion is *equal* to the number of blocks for the (RCBD) portion:** Tables 5.40 through 5.42 show the estimated significant level when the sample size for the (CRD) portion is $n = 6, 10, 16, 20$ and the number of blocks for the (RCBD) portion is $b = 6, 10, 16, 20$. As a result, the estimated significant level α is around 0.05 for all test statistics (3.58), (3.68), (2.16), (2.20) with a variety of underlying distributions.

Table 5.40. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is ***equal*** to The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
6	6	**First**	0.0456	0.0478
		Second	0.0504	0.0468
10	10	**First**	0.0468	0.0502
		Second	0.0430	0.0490
16	16	**First**	0.0510	0.0510
		Second	0.0452	0.0516
20	20	**First**	0.0534	0.0512
		Second	0.0594	0.0452

Table 5.41. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is ***equal*** to The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	6	**First**	0.0510	0.0492
		Second	0.0508	0.0478
10	10	**First**	0.0514	0.0500
		Second	0.0534	0.0478
16	16	**First**	0.0478	0.0506
		Second	0.0444	0.0508
20	20	**First**	0.0432	0.0492
		Second	0.0476	0.0482

Table 5.42. Estimated Significant Level α for a Test Statistic of a Mixed Design under the Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is ***equal*** to The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	6	**First**	0.0486	0.0524
		Second	0.0486	0.0468
10	10	**First**	0.0552	0.0508
		Second	0.0552	0.0520
16	16	**First**	0.0496	0.0488
		Second	0.0508	0.0478
20	20	**First**	0.0472	0.0510
		Second	0.0514	0.0530

3. **The sample size of all treatments for the (CRD) portion is *half* the number of blocks for the (RCBD) portion:** Tables 5.43 through 5.45 present the estimated significant level when the sample size for the (CRD) portion is $n = 6, 10, 16, 20$ and the number of blocks for the (RCBD) portion is $b = 12, 20, 32, 40$. The results are based on assuming three different underlying distributions, such as normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution for a simulation study.

The simulation study shows that the estimated significant level α is around 0.05 for all test statistics (3.58),(3.68),(2.16),(2.20).

Table 5.43. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is ***half*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	12	**First**	0.0460	0.0542
		Second	0.0450	0.0500
10	20	**First**	0.0526	0.0446
		Second	0.0520	0.0470
16	32	**First**	0.0496	0.0466
		Second	0.0472	0.0496
20	40	**First**	0.0436	0.0506
		Second	0.0458	0.0552

Table 5.44. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is ***half*** The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
6	12	**First**	0.0494	0.0482
		Second	0.0432	0.0468
10	20	**First**	0.0472	0.0526
		Second	0.0474	0.0544
16	32	**First**	0.0438	0.0478
		Second	0.0412	0.0520
20	40	**First**	0.0504	0.0550
		Second	0.0538	0.0532

Table 5.45. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is ***half*** The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
6	12	**First**	0.0490	0.0466
		Second	0.0518	0.0500
10	20	**First**	0.0484	0.0522
		Second	0.0546	0.0532
16	32	**First**	0.0426	0.0504
		Second	0.0468	0.0526
20	40	**First**	0.0510	0.0510
		Second	0.0460	0.0552

5.3.2. Estimated Power

We estimate the power of test statistics in (3.58), (3.68), (2.16), (2.20). As before, we will consider three different situations for the relationship between the sample size of treatments for a completely randomized design (CRD) portion and the number of blocks for a randomized complete block design (RCBD) portion as presented in Tables 5.46 through 5.63 under a variety of underlying distributions. We present the results of estimated power for two different sample sizes $n = 6, 10$ for (CRD) portion with a different number of blocks for (RCBD) portion:

1. **The sample size of all treatments for the (CRD) portion is *twice* the number of blocks for the (RCBD) portion:** Tables 5.46 through 5.51 show the estimated power when the sample sizes for the (CRD) portion are $n = 6, 10$, and the number of blocks for the (RCBD) portion are $b = 3, 5$. A variety of underlying distributions are used, such as normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution. The other results for different sample sizes and number of blocks are presented in Appendix C.

   As a result of the simulation study, once the underlying distribution is symmetric (normal, student's t). The estimated power for proposed test statistics (3.58), (3.68) with a squared distance modification is slightly different from the estimated power for test statistics (2.16), (2.20) introduced by Magel et al. (2010) in most of the location parameters configurations of

the umbrella hypothesis as presented in Tables 5.46 through 5.47, and Tables 5.49 through 5.50. However, once the sample size of the (CRD) portion is $n = 6$, and the peak of the umbrella hypothesis is ***distinct*** the estimated power for the proposed test statistics with modification, as given in (3.58), (3.68), is higher than the estimated power for the test statistics without modification, as given in (2.16), (2.20), as presented in Tables 5.46 and 5.47.

On the other hand, the results for the skewed underlying distribution (exponential) vary from symmetric underlying distribution for $n = 6, 10$. Table 5.48 shows there is an unclear pattern for the result of the estimated power for all test statistics (3.58), (3.68), (2.16), (2.20) once the sample size of the (CRD) portion is $n = 6$ for the case of a ***distinct peak*** of the umbrella hypothesis. While the estimated power for the test statistics without modification is better than the estimated power for the test statistics with a squared distance modification, once the sample size for the (CRD) portion is $n = 10$ for a ***distinct peak*** case as presented in Table 5.51. In either case of combining the Mack-Wolfe and Kim-Kim test statistics, the estimated power by using the first method for proposed test statistics, as given in (3.58) and (2.16), is better than the second method, as given in (3.68) and (2.20), for all different distributions.

Table 5.46. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.2916	0.2970
	Second	0.2636	0.2792
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.2182	**0.2338**
	Second	0.1978	**0.2160**
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.2306	0.2302
	Second	0.2046	0.2034
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.2974	0.3026
	Second	0.2718	0.2754
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.4560	**0.4750**
	Second	0.4036	**0.4240**
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.3592	**0.3734**
	Second	0.3136	**0.3374**
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.3730	0.3788
	Second	0.3308	0.3416
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.4154	0.4110
	Second	0.3712	0.3772
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2812	**0.2988**
	Second	0.2532	**0.2666**
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2806	**0.2982**
	Second	0.2500	**0.2702**
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	**0.2252**	0.2116
	Second	0.1992	0.2008
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.4454	0.4574
	Second	0.3924	0.4142
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2870	0.2862
	Second	0.2568	0.2644
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.2264	0.2236
	Second	0.1954	0.2030
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.1296	0.1336
	Second	0.1164	0.1264
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1320	0.1394
	Second	0.1222	0.1264
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1344	0.1364
	Second	0.1256	0.1248

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Table 5.46. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1278	0.1310
	Second	0.1112	0.1260
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1268	0.1266
	Second	0.1214	0.1210
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1338	0.1414
	Second	0.1176	0.1222

Table 5.47. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degree of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.2200	0.2232
	Second	0.1968	0.2076
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.1750	0.1804
	Second	0.1532	0.1746
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.1884	0.1794
	Second	0.1656	0.1586
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.2214	0.2280
	Second	0.2008	0.2190
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.3482	0.3330
	Second	0.3044	0.3090
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.2848	0.2832
	Second	0.2552	0.2630
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.2794	0.2856
	Second	0.2464	0.2594
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.3054	0.3168
	Second	0.2716	0.2954
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	**0.2354**	0.2170
	Second	0.2108	0.2066
(0.0 , 0.5 , 0.5 , 0.2 ,0.0)	**First**	0.2248	0.2356
	Second	0.2012	0.2158
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1782	0.1846
	Second	0.1546	0.1754
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.3446	0.3444
	Second	0.3036	0.3084
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2276	0.2298
	Second	0.2034	0.2080
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.1734	0.1698
	Second	0.1584	0.1578
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.1108	0.1132
	Second	0.1082	0.1082
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1142	0.1130
	Second	0.1096	0.1100
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1134	0.1122
	Second	0.1014	0.1060

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Table 5.47. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degree of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1150	0.1136
	Second	0.1022	0.1070
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1120	0.1170
	Second	0.1020	0.1072
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1150	0.1110
	Second	0.1066	0.1080

Table 5.48. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.4670	**0.4828**
	Second	0.4040	**0.4346**
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.3408	0.3530
	Second	0.2818	0.3092
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.3380	**0.3526**
	Second	0.2916	**0.3144**
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.5078	0.4994
	Second	0.4502	0.4478
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	**0.7266**	0.6976
	Second	**0.6634**	0.6428
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	**0.6240**	0.5974
	Second	**0.5536**	0.5370
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	**0.6238**	0.6082
	Second	0.5514	0.5502
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	**0.6676**	0.6534
	Second	0.5970	0.5868
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4704	0.4752
	Second	0.4140	0.4216
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	**0.5026**	0.4874
	Second	0.4424	0.4370
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	**0.3680**	0.3516
	Second	0.3236	0.3110
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.6930	0.6840
	Second	0.6282	0.6170
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4784	0.4816
	Second	0.4218	0.4234
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.3662	0.3616
	Second	0.3174	0.3208
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.1874	0.1858
	Second	0.1752	0.1732
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1938	0.1850
	Second	0.1730	0.1694
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1740	0.1638
	Second	0.1464	0.1536

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Table 5.48. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1692	0.1706
	Second	0.1470	0.1496
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1938	0.2004
	Second	0.1660	0.1756
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1916	0.1836
	Second	0.1696	0.1716

Table 5.49. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.4084	**0.4254**
	Second	0.3558	0.3690
(0.0 , 0.0 , **0.5**, 0.2 , 0.2)	**First**	0.3112	0.3076
	Second	0.2726	0.2688
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.3040	0.3126
	Second	0.2630	0.2686
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.4208	0.4158
	Second	0.3592	0.3606
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.6434	0.6414
	Second	0.5692	0.5606
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.5284	0.5326
	Second	0.4794	0.4552
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	**0.5418**	0.5274
	Second	0.4740	0.4710
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.5822	0.5826
	Second	0.5074	0.5100
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4088	0.4146
	Second	0.3636	0.3632
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4124	0.4128
	Second	0.3638	0.3596
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3096	0.3110
	Second	0.2730	0.2728
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.6414	0.6376
	Second	0.5696	0.5676
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4182	0.4146
	Second	0.3572	0.3624
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.3138	0.3028
	Second	0.2786	0.2636
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.1600	**0.1772**
	Second	0.1480	**0.1608**
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1722	0.1656
	Second	0.1558	0.1496
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1738	0.1744
	Second	0.1604	0.1480

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Table 5.49. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1718	0.1714
	Second	0.1512	0.1514
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1774	0.1722
	Second	0.1632	0.1490
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1752	0.1740
	Second	0.1596	0.1570

Table 5.50. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degree of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.3146	0.3208
	Second	0.2748	0.2754
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.2386	0.2344
	Second	0.2186	0.2064
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.2372	0.2424
	Second	0.2108	0.2162
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.3084	**0.3232**
	Second	0.2776	0.2780
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.4948	0.4886
	Second	0.4306	0.4228
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.3958	0.4018
	Second	0.3492	0.3552
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.3974	0.4078
	Second	0.3466	0.3512
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.4324	0.4348
	Second	0.3792	0.3756
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3072	0.3002
	Second	0.2736	0.2712
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3174	0.3258
	Second	0.2900	0.2838
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2448	0.2496
	Second	0.2122	0.2198
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	**0.4912**	0.4710
	Second	**0.4310**	0.4138
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3110	0.3212
	Second	0.2732	0.2842
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.2408	0.2408
	Second	0.2102	0.2164
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.1432	0.1302
	Second	0.1260	0.1232
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1372	0.1430
	Second	0.1264	0.1324
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1368	**0.1514**
	Second	0.1312	0.1280

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Table 5.50. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degree of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1304	0.1442
	Second	0.1268	0.1236
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1458	0.1380
	Second	0.1300	0.1318
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1476	0.1406
	Second	0.1260	0.1284

Table 5.51. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.6688	0.6790
	Second	0.5950	0.5922
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.5108	0.5202
	Second	0.4344	0.4390
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.5212	0.5154
	Second	0.4436	0.4490
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	**0.7102**	0.6994
	Second	**0.6348**	0.6134
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	**0.9118**	0.8958
	Second	**0.8506**	0.8390
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	**0.8372**	0.8018
	Second	**0.7680**	0.7392
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.8240	0.8130
	Second	0.7500	0.7386
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	**0.8696**	0.8526
	Second	0.7996	0.7872
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.6658	0.6742
	Second	0.5934	0.5994
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6778	0.6840
	Second	0.6040	0.5938
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5398	0.5378
	Second	0.4808	0.4602
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.8720	0.8798
	Second	0.8154	0.8210
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.6600	0.6668
	Second	0.5914	0.5950
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.5236	0.5226
	Second	0.4576	0.4546
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2532	0.2624
	Second	0.2274	0.2210
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2536	0.2436
	Second	0.2278	0.2168
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2534	0.2400
	Second	0.2112	0.2096

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Table 5.51. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$. (Continued)

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2544	0.2452
	Second	0.2104	0.2046
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	**0.2742**	0.2588
	Second	0.2380	0.2266
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2712	0.2620
	Second	0.2384	0.2298

2. **The sample size of all treatments for the (CRD) portion is *equal* to the number of blocks for the (RCBD) portion:** Tables 5.52 through 5.57 show the estimated power when the sample size for the (CRD) portion is $n = 6, 10$, and the number of blocks for the (RCBD) portion is $b = 6, 10$. We assume a variety of underlying distributions. The other results for different sample sizes and number of blocks are presented in Appendix C.

   The simulation study shows that the estimated power for proposed test statistics with a squared distance modification, as given in (3.58), (3.68), differs slightly from the estimated power for test statistics (2.16), (2.20) introduced by Magel et al. (2010), as presented in Tables 5.52 through 5.53 and Tables 5.55 through 5.56, once the underlying distribution is symmetric.

   On the other hand, the results for skewed underlying distribution show the estimated power for the test statistics without modification is better than the estimated power with a squared distance modification for the case of the ***distinct peak***, once $n = 6, 10$ for the (CRD) portion as shown in Tables 5.54 and 5.57.

   In either case, the estimated power by using the first method of proposing the test statistics, as given in (3.58) and (2.16), is better than the second method, as given in (3.68) and (2.20), for all different distributions.

Table 5.52. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.3480	0.3560
	Second	0.2752	0.2854
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.2570	0.2654
	Second	0.2076	0.2178
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.2646	0.2642
	Second	0.2170	0.2136
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.3602	0.3618
	Second	0.3036	0.2990
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.5586	**0.5718**
	Second	0.4580	0.4678
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.4666	0.4602
	Second	0.3710	0.3698
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.4554	0.4580
	Second	0.3662	0.3658
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.5194	0.5098
	Second	0.4132	0.4126
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3538	**0.3714**
	Second	0.2806	**0.2954**
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3664	0.3560
	Second	0.2954	0.2850
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2636	0.2700
	Second	0.2096	0.2182
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.5430	**0.5578**
	Second	0.4536	0.4502
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3564	0.3554
	Second	0.2928	0.2976
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.2716	0.2722
	Second	0.2278	0.2190
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	**0.1604**	0.1448
	Second	0.1312	0.1274
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1410	**0.1550**
	Second	0.1254	0.1330
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	**0.1622**	0.1426
	Second	0.1314	0.1302

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Table 5.52. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1510	0.1544
	Second	0.1324	0.1346
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	**0.1626**	0.1474
	Second	0.1378	0.1262
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1544	0.1578
	Second	0.1290	0.1324

Table 5.53. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.2724	0.2622
	Second	0.2204	0.2276
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.2084	0.2028
	Second	0.1782	0.1688
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.2022	0.2134
	Second	0.1634	0.1798
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.2736	0.2796
	Second	0.2204	0.2286
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.4090	**0.4278**
	Second	0.3378	0.3432
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.3336	0.3426
	Second	0.2714	0.2814
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.3462	0.3452
	Second	0.2776	0.2820
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.3834	0.3840
	Second	0.3030	0.3064
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2734	0.2742
	Second	0.2226	0.2320
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2704	0.2722
	Second	0.2172	0.2270
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2104	0.2068
	Second	0.1730	0.1684
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.4210	0.4136
	Second	0.3306	0.3268
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2596	**0.2790**
	Second	0.2130	**0.2304**
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.2122	0.2100
	Second	0.1742	0.1746
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.1348	0.1234
	Second	0.1160	0.1120
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1342	0.1298
	Second	0.1174	0.1158
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1224	0.1274
	Second	0.1074	0.1112

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Table 5.53. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1264	0.1288
	Second	0.1068	0.1122
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1222	0.1196
	Second	0.1028	0.1122
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	**0.1372**	0.1184
	Second	0.1178	0.1084

Table 5.54. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.6024	0.5900
	Second	0.4676	0.4662
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.4448	0.4424
	Second	0.3418	0.3486
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.4358	0.4492
	Second	0.3308	0.3574
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	**0.6368**	0.6102
	Second	**0.5072**	0.4822
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	**0.8458**	0.8332
	Second	**0.7274**	0.7108
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	**0.7522**	0.7310
	Second	**0.6242**	0.5958
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	**0.7494**	0.7276
	Second	**0.6192**	0.5972
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	**0.8064**	0.7808
	Second	**0.6708**	0.6472
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.5772	0.5894
	Second	0.4532	0.4662
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6070	0.5978
	Second	0.4880	0.4792
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4670	0.4570
	Second	0.3678	0.3606
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.7944	**0.8086**
	Second	0.6812	0.6882
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5824	0.5900
	Second	0.4684	0.4770
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.4474	0.4486
	Second	0.3556	0.3556
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2290	0.2110
	Second	0.1902	0.1702
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2230	0.2170
	Second	0.1922	0.1764
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2012	0.2148
	Second	0.1624	0.1676

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Table 5.54. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2158	0.2132
	Second	0.1642	0.1638
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2336	0.2326
	Second	0.1796	0.1832
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2232	0.2308
	Second	0.1856	0.1794

Table 5.55. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.4994	0.5042
	Second	0.3826	0.3820
(0.0 , 0.0 , **0.5**, 0.2, 0.2)	**First**	0.3768	0.3714
	Second	0.2842	0.2864
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	**0.3922**	0.3798
	Second	0.2896	0.2866
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.5072	0.5006
	Second	0.3922	0.3938
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.7578	0.7570
	Second	0.6090	0.5970
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.6342	0.6276
	Second	0.4988	0.4786
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.6288	0.6384
	Second	0.4860	0.4842
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.7084	0.7010
	Second	0.5566	0.5502
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4988	0.5094
	Second	0.3894	0.3818
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5120	0.5098
	Second	0.3834	0.3864
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3812	0.3700
	Second	0.2836	0.2850
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.7468	0.7414
	Second	0.5958	0.5862
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5032	0.5110
	Second	0.3810	0.3870
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.3734	0.3828
	Second	0.2914	0.2898
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2020	0.1966
	Second	0.1626	0.1602
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	**0.2126**	0.1992
	Second	0.1660	0.1594
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1972	0.1988
	Second	0.1670	0.1612

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Table 5.55. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2012	0.2054
	Second	0.1570	0.1620
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2008	0.2028
	Second	0.1638	0.1680
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2070	0.2046
	Second	0.1662	0.1558

Table 5.56. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.3798	**0.3960**
	Second	0.2924	0.3016
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.2766	**0.2906**
	Second	0.2158	0.2242
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.2962	0.2822
	Second	0.2306	0.2262
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.3744	0.3812
	Second	0.2964	0.2890
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.5870	0.5970
	Second	0.4560	0.4514
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.5062	0.4932
	Second	0.3910	0.3700
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.4868	0.4946
	Second	0.3716	0.3778
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.5530	0.5550
	Second	0.4202	0.4108
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3760	0.3844
	Second	0.2996	0.3034
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3864	0.3860
	Second	0.2982	0.2818
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2958	0.2930
	Second	0.2324	0.2188
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.5858	0.5908
	Second	0.4486	0.4506
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3810	0.3784
	Second	0.2944	0.2878
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.2886	0.2904
	Second	0.2234	0.2220
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.1566	0.1564
	Second	0.1344	0.1286
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1628	0.1698
	Second	0.1338	0.1388
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1656	0.1726
	Second	0.1318	0.1378

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Table 5.56. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1668	0.1608
	Second	0.1276	0.1300
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1630	0.1640
	Second	0.1320	0.1298
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1656	0.1658
	Second	0.1464	0.1452

Table 5.57. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.8020	0.8092
	Second	0.6402	0.6444
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.6458	0.6370
	Second	0.4690	0.4864
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.6304	0.6284
	Second	0.4762	0.4748
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	**0.8458**	0.8050
	Second	**0.6760**	0.6440
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	**0.9742**	0.9594
	Second	**0.8826**	0.8650
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9150	0.9092
	Second	0.7908	0.7594
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	**0.9234**	0.9056
	Second	**0.7872**	0.7600
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	**0.9524**	0.9398
	Second	**0.8366**	0.8174
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.7936	0.8008
	Second	0.6226	0.6446
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.7926	0.7988
	Second	0.6374	0.6414
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6544	0.6470
	Second	0.5010	0.4864
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9486	0.9540
	Second	0.8342	0.8324
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.7698	**0.7912**
	Second	0.6202	0.6234
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.6322	**0.6532**
	Second	0.4828	0.4808
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.3170	0.3130
	Second	0.2462	0.2368
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	**0.3278**	0.3034
	Second	**0.2484**	0.2260
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3064	0.3080
	Second	0.2220	0.2246

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Table 5.57. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$. (Continued)

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.3038	0.3108
	Second	0.2252	0.2228
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	**0.3318**	0.3084
	Second	**0.2504**	0.2320
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	**0.3300**	0.3174
	Second	**0.2452**	0.2310

3. **The sample size of all treatments for the (CRD) portion is *half* the number of blocks for the (RCBD) portion:** Tables 5.58 through 5.63 show the estimated power when the sample size for the (CRD) portion is $n = 6, 10$, and the number of blocks for the (RCBD) portion is $b = 12, 20$. We assume a variety of underlying distributions, such as normal distribution, student's t distribution with 3 degrees of freedom, and exponential distribution. The other results for different sample sizes and number of blocks are presented in Appendix C.

   In general, once we have underlying symmetric distribution, the estimated power for proposed test statistics with modification, as given in (3.58), (3.68), is slightly different from the estimated power for test statistics in (2.16), (2.20) introduced by Magel et al. (2010) in most of the location parameters configurations of the umbrella hypothesis. However, once the peak of the umbrella hypothesis is ***distinct*** the estimated power for the proposed test statistics with modification, as given in (3.58), (3.68), is higher than the estimated power for the test statistics without modification, as given in (2.16), (2.20), for some cases as presented in Tables 5.58, 5.59, 5.61 and 5.62.

   On the other hand, the results for skewed underlying distribution show that the estimated power for the test statistics without modification varies slightly from the estimated power with a squared distance modification for the case of the ***distinct peak*** once the sample size for the (CRD) portion is $n = 10, 16, 20$. While the results have no clear pattern once sample size for the (CRD) portion is $n = 6$, as shown in Tables 5.60 and 5.63.

In either case of combining the two test statistics for (CRD) and (RCBD), the estimated power by using the first method to propose the test statistics, as given in (3.58) and (2.16), is better than the second method, as given in (3.68) and (2.20), for all different distributions.

Table 5.58. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.4568	0.4608
	Second	0.3396	0.3280
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.3260	**0.3436**
	Second	0.2426	0.2552
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.3336	0.3328
	Second	0.2462	0.2498
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.4428	**0.4682**
	Second	0.3226	0.3350
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.6874	**0.7030**
	Second	0.5144	**0.5378**
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.5762	0.5796
	Second	0.4306	0.4230
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.5824	0.5760
	Second	0.4334	0.4188
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.6404	0.6314
	Second	0.4652	0.4664
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4482	0.4584
	Second	0.3174	0.3332
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4392	**0.4640**
	Second	0.3266	0.3354
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3422	0.3482
	Second	0.2436	0.2554
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.6846	0.6880
	Second	0.5132	0.5290
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4496	**0.4626**
	Second	0.3162	**0.3370**
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.3376	0.3270
	Second	0.2420	0.2364
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	**0.1906**	0.1776
	Second	0.1460	0.1366
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1998	0.1836
	Second	0.1496	0.1424
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1794	0.1866
	Second	0.1402	0.1400

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Table 5.58. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1888	0.1840
	Second	0.1382	0.1438
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1878	0.1894
	Second	0.1438	0.1370
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1880	**0.2028**
	Second	0.1458	0.1456

Table 5.59. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.3438	0.3510
	Second	0.2518	0.2530
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.2590	0.2534
	Second	0.1924	0.1890
(0.2 , 0.2 , **0.5** , 0.0, 0.0)	**First**	0.2618	0.2526
	Second	0.1958	0.1972
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.3558	0.3422
	Second	0.2528	0.2448
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.5396	0.5382
	Second	0.3900	0.4018
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.4390	0.4502
	Second	0.3242	0.3304
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.4472	0.4460
	Second	0.3174	0.3342
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.4976	0.4984
	Second	0.3490	0.3664
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3350	**0.3504**
	Second	0.2470	0.2574
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3480	0.3446
	Second	0.2482	0.2606
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2502	0.2594
	Second	0.1920	0.2018
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.5348	0.5366
	Second	0.3754	0.3970
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3498	0.3504
	Second	0.2494	0.2626
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.2614	0.2634
	Second	0.1992	0.2024
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.1564	0.1538
	Second	0.1216	0.1252
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1436	0.1476
	Second	0.1182	0.1130
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1476	0.1528
	Second	0.1178	0.1270

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Table 5.59. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1586	0.1532
	Second	0.1260	0.1254
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1522	0.1478
	Second	0.1184	0.1236
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1530	0.1536
	Second	0.1164	0.1202

Table 5.60. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.7342	**0.7524**
	Second	0.5392	**0.5540**
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.5770	0.5786
	Second	0.3964	0.4108
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.5790	0.5820
	Second	0.3952	0.4178
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	**0.7722**	0.7468
	Second	0.5744	0.5604
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9426	0.9342
	Second	0.8130	0.7992
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	**0.8810**	0.8566
	Second	**0.7252**	0.6850
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	**0.8744**	0.8570
	Second	**0.7028**	0.6834
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	**0.9166**	0.8968
	Second	0.7580	0.7502
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.7326	0.7304
	Second	0.5390	0.5462
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.7486	0.7502
	Second	0.5612	0.5650
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	**0.6046**	0.5800
	Second	**0.4278**	0.4032
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9160	0.9216
	Second	0.7550	0.7750
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.7122	**0.7414**
	Second	0.5344	**0.5672**
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.5658	0.5758
	Second	0.4024	0.4202
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2808	0.2758
	Second	0.2072	0.2062
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2898	0.2844
	Second	0.2104	0.2124
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2654	0.2528
	Second	0.1808	0.1786

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Table 5.60. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2714	0.2750
	Second	0.1808	0.1992
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	**0.3128**	0.2846
	Second	0.2164	0.2076
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	**0.2954**	0.2774
	Second	0.2102	0.2070

Table 5.61. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0, 0.0)	**First**	0.6316	0.6366
	Second	0.4246	0.4302
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.4846	0.4756
	Second	0.3108	0.3166
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.4778	0.4798
	Second	0.3100	0.3104
(0.0 , 0.2 , **0.5** , 0.2, 0.0)	**First**	0.6350	0.6472
	Second	0.4272	0.4308
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.8846	0.8742
	Second	0.6614	0.6474
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.7654	0.7750
	Second	0.5292	0.5496
(0.2 , 0.4 , **0.7** , 0.4, 0.0)	**First**	0.7732	0.7684
	Second	0.5416	0.5468
(0.0 , 0.2 , **0.8** , 0.5, 0.3)	**First**	0.8290	0.8232
	Second	0.5968	0.5944
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.6334	0.6350
	Second	0.4330	0.4350
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6356	0.6350
	Second	0.4234	0.4230
(0.2 , 0.5 , **0.5** , 0.2 ,0.0)	**First**	0.4820	0.4826
	Second	0.3158	0.3222
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.8690	0.8738
	Second	0.6578	0.6490
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.6312	0.6310
	Second	0.4234	0.4266
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.4658	**0.4892**
	Second	0.3076	**0.3230**
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2536	0.2452
	Second	0.1744	0.1712
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2432	0.2592
	Second	0.1684	0.1804
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2562	0.2512
	Second	0.1836	0.1782

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Table 5.61. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2556	0.2498
	Second	0.1714	0.1792
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2550	0.2582
	Second	0.1742	0.1788
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2546	0.2558
	Second	0.1738	0.1794

Table 5.62. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.4922	0.4886
	Second	0.3328	0.3286
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.3554	0.3606
	Second	0.2324	0.2362
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.3598	**0.3706**
	Second	0.2430	0.2464
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.4890	**0.5018**
	Second	0.3258	0.3222
(0.0 , 0.4 , **0.7** , 0.2, 0.0)	**First**	0.7228	0.7312
	Second	0.5044	0.5152
(0.0 , 0.4 , **0.7** , 0.4, 0.2)	**First**	0.6172	0.6276
	Second	0.4244	0.4310
(0.2 , 0.4 , **0.7** , 0.4, 0.0)	**First**	0.6152	0.6136
	Second	0.4106	0.4150
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.6766	0.6822
	Second	0.4514	0.4512
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4958	0.5074
	Second	0.3186	0.3376
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4782	**0.4968**
	Second	0.3260	0.3346
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3724	0.3698
	Second	0.2470	0.2482
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.7202	0.7260
	Second	0.4972	0.4978
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4940	0.4842
	Second	0.3290	0.3244
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.3624	0.3648
	Second	0.2518	0.2406
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	**0.2108**	0.1934
	Second	0.1532	0.1398
(0.5 , 0.5 ,**0.5** , 0.5 , 0.0)	**First**	0.1900	0.1932
	Second	0.1410	0.1446
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1926	0.1998
	Second	0.1412	0.1448

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Table 5.62. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 degrees of freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$. (Continued)

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1950	0.2044
	Second	0.1452	0.1476
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1938	0.1996
	Second	0.1402	0.1494
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1964	0.1948
	Second	0.1366	0.1412

Table 5.63. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.9140	0.9222
	Second	0.7000	0.7084
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.7884	0.7924
	Second	0.5456	0.5398
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.7906	0.7844
	Second	0.5356	0.5334
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.9352	0.9234
	Second	0.7448	0.7226
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9952	0.9918
	Second	0.9218	0.9064
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9812	0.9738
	Second	0.8466	0.8254
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.9766	0.9692
	Second	0.8410	0.8220
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9918	0.9856
	Second	0.8874	0.8750
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.9118	0.9140
	Second	0.6968	0.6952
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.9164	0.9084
	Second	0.7118	0.6986
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.7944	0.7902
	Second	0.5696	0.5528
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9900	0.9886
	Second	0.8880	0.8922
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.8874	**0.9126**
	Second	0.6744	**0.6970**
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.7546	**0.7794**
	Second	0.5244	**0.5418**
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.3964	0.4074
	Second	0.2630	0.2626
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3970	**0.4150**
	Second	0.2544	0.2604
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4014	0.4034
	Second	0.2570	0.2514

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Table 5.63. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$. (Continued)

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.3936	0.4054
	Second	0.2466	0.2648
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	**0.4346**	0.4094
	Second	**0.2800**	0.2526
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	**0.4344**	0.4156
	Second	**0.2782**	0.2598

5.4. Three Treatments at Unknown Peak

This section presents the estimated results of the significant level and the power for the proposed test statistics, once there are three treatments, and the peak is unknown.

5.4.1. Estimated Significant Level α

Tables 5.64 through 5.66 show the results of the estimated significant level. The results are based on the consideration of the type of underlying distribution. We consider symmetric (normal, student's t), and skewed (exponential) underlying distribution:

1. **Symmetric underlying distribution:** Tables 5.64 through 5.65 present the estimated significant level once we hold the sample size for the (CRD) portion at $n = 10$ and increase the number of blocks for the (RCBD) portion as $b = 5, 10, 20$. As a result of the simulation study, the estimated significant level α is around 0.05 for the proposed test statistics (3.69), (3.70),(3.71), (3.72) introduced in Chapter 3.

Table 5.64. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and Increase The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
10	5	**First**	0.0488	0.0480
		Second	0.0530	0.0522
10	10	**First**	0.0528	0.0520
		Second	0.0522	0.0504
10	20	**First**	0.0468	0.0518
		Second	0.0522	0.0442

Table 5.65. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and Increase The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
10	5	**First**	0.0498	0.0508
		Second	0.0516	0.0538
10	10	**First**	0.0474	0.0530
		Second	0.0528	0.0472
10	20	**First**	0.0544	0.0522
		Second	0.0514	0.0484

2. **Skewed underlying distribution:** Table 5.66 presents the estimated significant level once we hold the sample size for the (CRD) portion at $n = 10$ and increase the number of blocks for the (RCBD) portion as $b = 5, 10, 20$. The results show that the estimated significant level α is around 0.05 for all proposed test statistics (3.69), (3.70),(3.71), (3.72) introduced in Chapter 3.

Table 5.66. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and Increase The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
10	5	**First**	0.0522	0.0472
		Second	0.0446	0.0544
10	10	**First**	0.0474	0.0506
		Second	0.0480	0.0496
10	20	**First**	0.0484	0.0492
		Second	0.0446	0.0470

5.4.2. Estimated Power

Tables 5.67 through 5.75 show the results of the estimated power for proposed test statistics (3.69), (3.70),(3.71), (3.72). The results are based on two types of underlying distribution:

1. **Symmetric underlying distribution:** Tables 5.67 through 5.72 present the estimated power once we hold the sample size for the (CRD) portion at $n = 10$ and increase the number of blocks for the (RCBD) portion as $b = 5, 10, 20$. The simulation study shows the estimated power of the proposed test statistics with a squared distance modification, as given in (3.71), (3.72), varies slightly from the estimated power of the proposed test statistics without modification, as given in (3.69), (3.70), in most different configurations of the umbrella hypothesis. In either case of combining the two test statistics for the (CRD) and (RCBD) designs, the estimated power by using the first method to propose the test statistics, as given in (3.71) and (3.69), is better than the second method, as given in (3.72) and (3.70).

Table 5.67. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(0.7 , 0.5 , 0.0)	**First**	**0.3628**	0.3478
	Second	**0.3110**	0.2948
(0.0 , 0.5 , 1.0)	**First**	0.6144	0.6152
	Second	0.5256	0.5382
(0.0 , 0.5 , 0.5)	**First**	0.2454	0.2408
	Second	0.2062	0.2138
(0.0 , 0.7 , 0.0)	**First**	0.4530	0.4480
	Second	0.3860	0.3982
(0.0 , 0.7 , 0.5)	**First**	**0.3414**	0.3250
	Second	0.2794	0.2892

Table 5.68. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.7 , 0.5 , 0.0)	**First**	0.4592	0.4674
	Second	0.3392	0.3354
(0.0 , 0.5 , 1.0)	**First**	0.7394	0.7506
	Second	0.5732	0.5800
(0.0 , 0.5 , 0.5)	**First**	0.3024	0.3018
	Second	0.2286	0.2242
(0.0 , 0.7 , 0.0)	**First**	0.5724	0.5756
	Second	0.4164	0.4056
(0.0 , 0.7 , 0.5)	**First**	**0.4262**	0.4074
	Second	0.3026	0.2924

Table 5.69. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Squared Distance
(0.7 , 0.5 , 0.0)	**First**	0.6046	0.6044
	Second	0.3808	0.3814
(0.0 , 0.5 , 1.0)	**First**	0.8866	0.8982
	Second	0.6480	0.6574
(0.0 , 0.5 , 0.5)	**First**	0.4034	0.4146
	Second	0.2510	0.2576
(0.0 , 0.7 , 0.0)	**First**	0.7352	0.7462
	Second	0.4792	0.4748
(0.0 , 0.7 , 0.5)	**First**	0.5602	0.5638
	Second	0.3522	0.3434

Table 5.70. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(0.7 , 0.5 , 0.0)	**First**	0.2532	0.2516
	Second	0.2100	0.2208
(0.0 , 0.5 , 1.0)	**First**	0.4266	0.4364
	Second	0.3642	0.3882
(0.0 , 0.5 , 0.5)	**First**	0.1664	0.1702
	Second	0.1524	0.1600
(0.0 , 0.7 , 0.0)	**First**	0.3020	0.3130
	Second	0.2690	0.2692
(0.0 , 0.7 , 0.5)	**First**	0.2268	**0.2484**
	Second	0.1954	**0.2116**

Table 5.71. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.7 , 0.5 , 0.0)	**First**	0.3120	0.3154
	Second	0.2282	0.2248
(0.0 , 0.5 , 1.0)	**First**	0.5468	0.5470
	Second	0.3996	0.3854
(0.0 , 0.5 , 0.5)	**First**	0.2122	0.2234
	Second	0.1606	0.1798
(0.0 , 0.7 , 0.0)	**First**	0.4018	0.4062
	Second	0.2874	0.2810
(0.0 , 0.7 , 0.5)	**First**	0.2904	0.2914
	Second	0.2278	0.2110

Table 5.72. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.7 , 0.5 , 0.0)	**First**	0.4584	0.4398
	Second	0.2732	0.2704
(0.0 , 0.5 , 1.0)	**First**	0.7152	0.7122
	Second	0.4610	0.4634
(0.0 , 0.5 , 0.5)	**First**	0.3050	0.2918
	Second	0.1870	0.1948
(0.0 , 0.7 , 0.0)	**First**	0.5486	0.5322
	Second	0.3338	0.3240
(0.0 , 0.7 , 0.5)	**First**	0.4038	0.4060
	Second	0.2546	0.2572

2. **Skewed underlying distribution:** We assume an exponential underlying distribution. The results show a similarity with the case of the symmetric underlying distribution. Where the variation of the estimated power for the test statistics without modification, as given in (3.69), (3.70), and the estimated power for a squared distance modification, case (3.71), (3.72) is small in most cases of the umbrella hypothesis, as is shown in Tables 5.73 through 5.75. Also, the first method of combining the test statistics for (CRD) and (RCBD) designs

to propose the test statistics, as given in (3.71) and (3.69), is better than the second method, as given in (3.72) and (3.70).

Table 5.73. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(0.7 , 0.5 , 0.0)	**First**	**0.6360**	0.6014
	Second	**0.5516**	0.5294
(0.0 , 0.5 , 1.0)	**First**	**0.8654**	0.8338
	Second	**0.7972**	0.7590
(0.0 , 0.5 , 0.5)	**First**	0.4224	0.4192
	Second	0.3568	0.3680
(0.0 , 0.7 , 0.0)	**First**	0.7308	0.7400
	Second	0.6346	0.6570
(0.0 , 0.7 , 0.5)	**First**	0.5520	0.5470
	Second	0.4706	0.4720

Table 5.74. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.7 , 0.5 , 0.0)	**First**	**0.7524**	0.7370
	Second	**0.5970**	0.5652
(0.0 , 0.5 , 1.0)	**First**	0.9452	0.9314
	Second	0.8454	0.8026
(0.0 , 0.5 , 0.5)	**First**	0.5490	0.5426
	Second	0.4016	0.4018
(0.0 , 0.7 , 0.0)	**First**	0.8570	0.8570
	Second	0.6846	0.6824
(0.0 , 0.7 , 0.5)	**First**	0.6768	0.6844
	Second	0.5056	0.5018

Table 5.75. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 3 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Nonmodification	Modification
(0.7 , 0.5 , 0.0)	**First**	0.8862	0.8742
	Second	0.6514	0.6442
(0.0 , 0.5 , 1.0)	**First**	0.9932	0.9838
	Second	0.8950	0.8706
(0.0 , 0.5 , 0.5)	**First**	0.6768	0.6756
	Second	0.4492	0.4514
(0.0 , 0.7 , 0.0)	**First**	0.9610	0.9606
	Second	0.7744	0.7712
(0.0 , 0.7 , 0.5)	**First**	0.8160	**0.8320**
	Second	0.5668	0.5774

5.5. Four Treatments at Unknown Peak

In this section, we present the results from the simulation study of estimating the significant level and the power for the proposed test statistics, once there are four treatments, and the peak is unknown.

5.5.1. Estimated Significant Level α

Tables 5.76 through 5.78 show the results of the estimated significant level. We consider two types of underlying distributions, symmetric and skewed:

1. **Symmetric underlying distribution:** Tables 5.76 through 5.77 present the estimated significant level based on assuming underlying normal and student's t distribution. We hold the sample size for the (CRD) portion at $n = 10$ and increase the number of blocks for the (RCBD) portion as $b = 5, 10, 20$. As a result of the simulation study, the estimated significant level α is around 0.05 for the proposed test statistics (3.69), (3.70),(3.71), and (3.72), introduced in Chapter 3 .

Table 5.76. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and Increase The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
10	5	**First**	0.0502	0.0494
		Second	0.0540	0.0552
10	10	**First**	0.0552	0.0508
		Second	0.0620	0.0542
10	20	**First**	0.0462	0.0500
		Second	0.0538	0.0558

Table 5.77. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and Increase The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
10	5	**First**	0.0512	0.0432
		Second	0.0556	0.0548
10	10	**First**	0.0472	0.0468
		Second	0.0508	0.0482
10	20	**First**	0.0504	0.0494
		Second	0.0576	0.0580

2. **Skewed underlying distribution:** Table 5.78 presents the estimated significant level within having an exponential underlying distribution. We hold the sample size for the (CRD) portion at $n = 10$ and increase the number of blocks for the (RCBD) portion as $b = 5, 10, 20$. The results of a simulation study show that the estimated significant level α is around 0.05 for all proposed test statistics (3.69), (3.70), (3.71), (3.72).

Table 5.78. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and Increase The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
10	5	**First**	0.0460	0.0456
		Second	0.0552	0.0530
10	10	**First**	0.0468	0.0448
		Second	0.0560	0.0430
10	20	**First**	0.0422	0.0520
		Second	0.0442	0.0582

5.5.2. Estimated Power

In this section, we present the results of the estimated power for proposed test statistics (3.69), (3.70), (3.71), (3.72), as seen in Tables 5.79 through 5.87. The results are based on symmetric and skewed underlying distributions:

1. **Symmetric underlying distribution:** Tables 5.79 through 5.84 show the results of the estimated power once we hold the sample size for the (CRD) portion at $n = 10$ and increase the number of blocks for the (RCBD) portion as $b = 5, 10, 20$. Generally, the estimated power for the proposed test statistics without modification (3.69), (3.70) is slightly different from the proposed test statistics with a squared distance modification (3.71), (3.72). However, there are a few cases that show the estimated power for the test statistics with a square distance modification is better than the test statistics without modification.

   It is good to mention the first method of combining the two test statistics to propose the test statistics for a mixed design, as given in (3.71) and (3.69), is better than the second method, as given in (3.72) and (3.70).

Table 5.79. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0)	**First**	0.4752	**0.5308**
	Second	0.4040	**0.4476**
(1.0 , 0.75 , 0.5 , 0.25)	**First**	0.4124	0.4182
	Second	0.3480	0.3692
(0.0 , 0.0 , 0.25 , 1.0)	**First**	0.5860	0.5840
	Second	0.5282	0.5110
(0.75 , 0.75 , 0.5 , 0.0)	**First**	0.4662	**0.4844**
	Second	0.4246	0.4304
(0.5 , 0.5 , 0.5 , 0.0)	**First**	0.2318	0.2388
	Second	0.2238	0.2256
(0.0 , 0.0 , 0.75 , 0.75)	**First**	0.5658	0.5750
	Second	0.5138	0.4990
(0.5 , 1.0 , 0.2 , 0.2)	**First**	0.4324	**0.4548**
	Second	0.3914	0.3926
(0.0 , 0.25 , 0.5 , 0.25)	**First**	0.1946	0.2056
	Second	0.1960	0.1938
(0.2 , 0.75 , 1.0 , 0.75)	**First**	0.4264	**0.4620**
	Second	0.3862	**0.4054**
(0.8 , 1.0 , 0.75 , 0.2)	**First**	0.4170	**0.4614**
	Second	0.3936	0.4056

Table 5.80. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0)	**First**	0.6252	**0.6714**
	Second	0.4544	**0.4798**
(1.0 , 0.75 , 0.5 , 0.25)	**First**	0.5316	0.5344
	Second	0.3766	0.3896
(0.0 , 0.0 , 0.25 , 1.0)	**First**	0.7334	**0.7520**
	Second	0.5540	0.5686
(0.75 , 0.75 , 0.5 , 0.0)	**First**	0.5880	**0.6212**
	Second	0.4444	**0.4746**
(0.5 , 0.5 , 0.5 , 0.0)	**First**	0.2826	**0.3030**
	Second	0.2178	0.2264
(0.0 , 0.0 , 0.75 , 0.75)	**First**	0.7098	**0.7212**
	Second	0.5492	0.5470
(0.5 , 1.0 , 0.2 , 0.2)	**First**	0.5616	**0.5804**
	Second	0.4098	**0.4214**
(0.0 , 0.25 , 0.5 , 0.25)	**First**	0.2552	0.2564
	Second	0.1966	0.2038
(0.2 , 0.75 , 1.0 , 0.75)	**First**	0.5380	**0.5766**
	Second	0.4066	**0.4328**
(0.8 , 1.0 , 0.75 , 0.2)	**First**	0.5404	**0.5826**
	Second	0.4102	**0.4366**

Table 5.81. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0)	**First**	0.8200	**0.8422**
	Second	0.5338	**0.5670**
(1.0 , 0.75 , 0.5 , 0.25)	**First**	0.6974	0.6990
	Second	0.4446	0.4554
(0.0 , 0.0 , 0.25 , 1.0)	**First**	0.8884	0.8970
	Second	0.6352	0.6600
(0.75 , 0.75 , 0.5 , 0.0)	**First**	0.7438	**0.7828**
	Second	0.5016	**0.5350**
(0.5 , 0.5 , 0.5 , 0.0)	**First**	0.3900	**0.4224**
	Second	0.2522	**0.2720**
(0.0 , 0.0 , 0.75 , 0.75)	**First**	0.8598	0.8608
	Second	0.6116	0.6168
(0.5 , 1.0 , 0.2 , 0.2)	**First**	0.7342	0.7342
	Second	0.4788	0.4780
(0.0 , 0.25 , 0.5 , 0.25)	**First**	0.3442	0.3484
	Second	0.2196	0.2382
(0.2 , 0.75 , 1.0 , 0.75)	**First**	0.7040	**0.7372**
	Second	0.4878	0.4990
(0.8 , 1.0 , 0.75 , 0.2)	**First**	0.7022	**0.7400**
	Second	0.4838	**0.5042**

Table 5.82. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	**Method**	**Nonmodification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0)	**First**	0.3370	0.3478
	Second	0.2872	0.2876
(1.0 , 0.75 , 0.5 , 0.25)	**First**	0.2704	**0.2970**
	Second	0.2436	**0.2610**
(0.0 , 0.0 , 0.25 , 1.0)	**First**	0.3882	**0.4118**
	Second	0.3482	0.3556
(0.75 , 0.75 , 0.5 , 0.0)	**First**	0.3308	**0.3542**
	Second	0.2974	**0.3208**
(0.5 , 0.5 , 0.5 , 0.0)	**First**	0.1712	0.1762
	Second	0.1696	0.1716
(0.0 , 0.0 , 0.75 , 0.75)	**First**	0.4060	0.4114
	Second	0.3648	0.3666
(0.5 , 1.0 , 0.2 , 0.2)	**First**	0.3128	0.3060
	Second	0.2782	0.2692
(0.0 , 0.25 , 0.5 , 0.25)	**First**	0.1498	0.1546
	Second	0.1502	0.1582
(0.2 , 0.75 , 1.0 , 0.75)	**First**	0.3018	0.3148
	Second	0.2814	0.2916
(0.8 , 1.0 , 0.75 , 0.2)	**First**	0.3022	0.3162
	Second	0.2854	0.2988

Table 5.83. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degree of Freedom for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Nonmodification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0)	**First**	0.4242	**0.4626**
	Second	0.3032	0.3126
(1.0 , 0.75 , 0.5 , 0.25)	**First**	0.3584	**0.3708**
	Second	0.2636	0.2692
(0.0 , 0.0 , 0.25 , 1.0)	**First**	0.5422	0.5382
	Second	0.3850	0.3794
(0.75 , 0.75 , 0.5 , 0.0)	**First**	0.4280	**0.4574**
	Second	0.3086	**0.3282**
(0.5 , 0.5 , 0.5 , 0.0)	**First**	0.2018	0.2162
	Second	0.1652	0.1746
(0.0 , 0.0 , 0.75 , 0.75)	**First**	0.5232	0.5350
	Second	0.3908	0.3998
(0.5 , 1.0 , 0.2 , 0.2)	**First**	0.4040	0.3964
	Second	0.3060	0.3036
(0.0 , 0.25 , 0.5 , 0.25)	**First**	0.1842	0.1920
	Second	0.1626	0.1582
(0.2 , 0.75 , 1.0 , 0.75)	**First**	0.3946	0.4088
	Second	0.2908	0.3038
(0.8 , 1.0 , 0.75 , 0.2)	**First**	0.4026	0.4150
	Second	0.2968	0.3016

Table 5.84. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	**Method**	**Nonmodification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0)	**First**	0.5980	**0.6340**
	Second	0.3620	**0.3824**
(1.0 , 0.75 , 0.5 , 0.25)	**First**	0.4942	**0.5146**
	Second	0.3040	0.3118
(0.0 , 0.0 , 0.25 , 1.0)	**First**	0.7200	0.7308
	Second	0.4482	0.4694
(0.75 , 0.75 , 0.5 , 0.0)	**First**	0.5694	**0.5850**
	Second	0.3696	0.3656
(0.5 , 0.5 , 0.5 , 0.0)	**First**	0.2844	0.2990
	Second	0.2006	0.1978
(0.0 , 0.0 , 0.75 , 0.75)	**First**	0.6718	**0.6930**
	Second	0.4452	0.4466
(0.5 , 1.0 , 0.2 , 0.2)	**First**	0.5422	0.5466
	Second	0.3436	0.3386
(0.0 , 0.25 , 0.5 , 0.25)	**First**	0.2488	0.2484
	Second	0.1740	0.1720
(0.2 , 0.75 , 1.0 , 0.75)	**First**	0.5106	**0.5450**
	Second	0.3422	0.3502
(0.8 , 1.0 , 0.75 , 0.2)	**First**	0.5240	**0.5762**
	Second	0.3472	**0.3696**

2. **Skewed underlying distribution:** There is an unclear pattern of the estimated power for the test statistics (3.69), (3.70), (3.71), (3.72) with assuming an exponential underlying distribution, as it is shown in Tables 5.85 through 5.87. Also, the first method of combining the two test statistics for (CRD) and (RCBD) designs to propose the test statistics, as given in (3.71) and (3.69), is better than the second method, as given in (3.72) and (3.70).

Table 5.85. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0)	**First**	0.7008	**0.7448**
	Second	0.6202	**0.6630**
(1.0 , 0.75 , 0.5 , 0.25)	**First**	**0.7316**	0.6920
	Second	**0.6532**	0.6154
(0.0 , 0.0 , 0.25 , 1.0)	**First**	**0.8668**	0.8448
	Second	**0.7962**	0.7550
(0.75 , 0.75 , 0.5 , 0.0)	**First**	**0.7410**	0.7230
	Second	**0.6692**	0.6476
(0.5 , 0.5 , 0.5 , 0.0)	**First**	0.4024	0.4046
	Second	0.3788	0.3610
(0.0 , 0.0 , 0.75 , 0.75)	**First**	0.8078	0.8138
	Second	0.7378	0.7402
(0.5 , 1.0 , 0.2 , 0.2)	**First**	0.7058	0.6988
	Second	0.6332	0.6168
(0.0 , 0.25 , 0.5 , 0.25)	**First**	0.3998	0.4002
	Second	0.3556	0.3510
(0.2 , 0.75 , 1.0 , 0.75)	**First**	0.6988	0.6882
	Second	0.6458	0.6176
(0.8 , 1.0 , 0.75 , 0.2)	**First**	**0.7036**	0.6844
	Second	**0.6368**	0.6116

Table 5.86. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0)	**First**	0.8488	**0.8778**
	Second	0.6650	**0.6990**
(1.0 , 0.75 , 0.5 , 0.25)	**First**	0.8522	0.8432
	Second	0.6994	0.6660
(0.0 , 0.0 , 0.25 , 1.0)	**First**	0.9508	0.9472
	Second	0.8388	0.8158
(0.75 , 0.75 , 0.5 , 0.0)	**First**	0.8522	0.8482
	Second	0.6990	0.6940
(0.5 , 0.5 , 0.5 , 0.0)	**First**	0.5040	**0.5278**
	Second	0.3756	**0.3928**
(0.0 , 0.0 , 0.75 , 0.75)	**First**	0.9206	0.9168
	Second	0.7906	0.7826
(0.5 , 1.0 , 0.2 , 0.2)	**First**	**0.8486**	0.8266
	Second	**0.6768**	0.6596
(0.0 , 0.25 , 0.5 , 0.25)	**First**	0.5030	0.5008
	Second	0.3790	0.3736
(0.2 , 0.75 , 1.0 , 0.75)	**First**	0.8194	0.8142
	Second	0.6626	0.6578
(0.8 , 1.0 , 0.75 , 0.2)	**First**	0.8274	0.8280
	Second	0.6806	0.6640

Table 5.87. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(1.0 , 0.0 , 0.0 , 0.0)	**First**	0.9660	0.9686
	Second	0.7578	0.7764
(1.0 , 0.75 , 0.5 , 0.25)	**First**	0.9542	0.9424
	Second	0.7662	0.7334
(0.0 , 0.0 , 0.25 , 1.0)	**First**	0.9926	0.9900
	Second	0.8908	0.8686
(0.75 , 0.75 , 0.5 , 0.0)	**First**	0.9576	0.9488
	Second	0.7738	0.7578
(0.5 , 0.5 , 0.5 , 0.0)	**First**	0.6504	**0.6878**
	Second	0.4536	0.4602
(0.0 , 0.0 , 0.75 , 0.75)	**First**	0.9830	0.9840
	Second	0.8552	0.8404
(0.5 , 1.0 , 0.2 , 0.2)	**First**	**0.9616**	0.9496
	Second	**0.7542**	0.7366
(0.0 , 0.25 , 0.5 , 0.25)	**First**	0.6612	0.6532
	Second	0.4380	0.4184
(0.2 , 0.75 , 1.0 , 0.75)	**First**	0.9350	0.9282
	Second	0.7420	0.7264
(0.8 , 1.0 , 0.75 , 0.2)	**First**	0.9304	0.9366
	Second	0.7326	0.7328

5.6. Five Treatments at Unknown Peak

This section presents the results of estimating the significant level and the power for proposed test statistics in terms of having five treatments, and the peak is unknown.

5.6.1. Estimated Significant Level α

Tables 5.88 through 5.90 show the results of the estimated significant level based on the consideration of the type of underlying distribution:

1. **Symmetric underlying distribution:** Tables 5.88 through 5.89 present the estimated significant level for a normal and a student's t underlying distribution. Besides that, we hold the sample size for the (CRD) portion at $n = 10$ and increase the number of blocks for the (RCBD) portion as $b = 5, 10, 20$. The results show that the estimated significant level α for

all proposed test statistics (3.69), (3.70), (3.71), (3.72), introduced in Chapter 3, is around 0.05.

Table 5.88. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and Increase The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
10	5	**First**	0.0472	0.0442
		Second	0.0512	0.0486
10	10	**First**	0.0458	0.0488
		Second	0.0446	0.0480
10	20	**First**	0.0486	0.0482
		Second	0.0490	0.0512

Table 5.89. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and Increase The Number of Blocks for The RCBD Portion.

n	**b**	**Method**	**Non Modification**	**Modification**
10	5	**First**	0.0544	0.0456
		Second	0.0526	0.0446
10	10	**First**	0.0518	0.0482
		Second	0.0442	0.0482
10	20	**First**	0.0478	0.0504
		Second	0.0498	0.0508

2. **Skewed underlying distribution:** Table 5.90 presents the estimated significant level for the underlying exponential distribution. We hold the sample size for the (CRD) portion at $n = 10$ and increase the number of blocks for the (RCBD) portion as $b = 5, 10, 20$. As a result of the simulation study, the estimated significant level α is around 0.05 for proposed test statistics (3.69), (3.70), (3.71), (3.72).

Table 5.90. Estimated Significant Level α for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and Increase The Number of Blocks for The RCBD Portion.

n	b	Method	Non Modification	Modification
10	5	**First**	0.0504	0.0492
		Second	0.0462	0.0502
10	10	**First**	0.0520	0.0450
		Second	0.0544	0.0510
10	20	**First**	0.0566	0.0488
		Second	0.0498	0.0478

5.6.2. Estimated Power

Tables 5.91 through 5.99 present the results of the estimated power for proposed test statistics (3.69), (3.70), (3.71), (3.72) once the peak is unknown. The results are based on symmetric and skewed underlying distributions:

1. **Symmetric underlying distribution:** Tables 5.91 through 5.96 present the estimated power once we hold the sample size for the (CRD) portion at $n = 10$ and increase the number of blocks for the (RCBD) portion as $b = 5, 10, 20$. In general, we can note that the estimated power for the proposed test statistics without modification (3.69), (3.70) varies slightly from the proposed test statistics with modification(3.71), (3.72). However, we can notice some cases that show the estimated power for the test statistics with a square distance modification is better than the test statistics without modification. In either case of combining the test statistics for (CRD) and (RCBD), the estimated power by using the first method to propose the test statistics for a mixed design, as given in (3.71) and (3.69), is better than the second method, as given in (3.72) and (3.70).

Table 5.91. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0 , 0.0)	**First**	0.3990	**0.4572**
	Second	0.3162	**0.3728**
(0.0 , 0.0 , 0.0 , 0.5 , 1.0)	**First**	0.6582	0.6676
	Second	0.5582	0.5824
(1.0 , 0.2 , 0.2 , 0.2 , 0.0)	**First**	0.4270	**0.4800**
	Second	0.3476	**0.3976**
(0.8 , 0.8 , 0.5 , 0.2 , 0.0)	**First**	0.6060	**0.6342**
	Second	0.5252	**0.5428**
(0.5 , 0.5 , 0.2 , 0.2 , 0.0)	**First**	0.2458	0.2542
	Second	0.2042	0.2164
(0.5 , 0.5 , 0.2 , 0.0 , 0.0)	**First**	0.3146	0.3110
	Second	0.2606	0.2724
(0.5 , 0.5 , 0.5 , 0.0 , 0.0)	**First**	0.3594	0.3580
	Second	0.3018	0.3110
(0.0 , 0.0 , 0.0 , 0.5 , 0.5)	**First**	0.2818	0.2958
	Second	0.2356	0.2484
(0.0 , 0.0 , 0.5 , 0.5 , 0.5)	**First**	0.3608	0.3656
	Second	0.3070	0.3090
(0.2 , 0.2 , 0.5 , 0.0 , 0.0)	**First**	0.1938	0.1946
	Second	0.1630	0.1782

Table 5.92. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0 , 0.0)	**First**	0.5438	**0.6056**
	Second	0.3538	**0.4146**
(1.0 , 0.2 , 0.2 , 0.2 , 0.0)	**First**	0.5922	**0.6308**
	Second	0.3918	**0.4502**
(0.0 , 0.0 , 0.0 , 0.5 , 1.0)	**First**	0.8062	0.8160
	Second	0.6086	0.6296
(0.8 , 0.8 , 0.5 , 0.2 , 0.0)	**First**	0.7548	0.7670
	Second	0.5650	0.5966
(0.5 , 0.5 , 0.2 , 0.2 , 0.0)	**First**	0.3200	0.3286
	Second	0.2138	0.2352
(0.5 , 0.5 , 0.2 , 0.0 , 0.0)	**First**	0.4078	0.4186
	Second	0.2676	0.2992
(0.5 , 0.5 , 0.5 , 0.0 , 0.0)	**First**	0.4620	0.4706
	Second	0.3188	0.3306
(0.0 , 0.0 , 0.0 , 0.5 , 0.5)	**First**	0.3844	0.3834
	Second	0.2542	0.2672
(0.0 , 0.0 , 0.5 , 0.5 , 0.5)	**First**	0.4680	0.4650
	Second	0.3158	0.3416
(0.2 , 0.2 , 0.5 , 0.0 , 0.0)	**First**	0.2504	0.2568
	Second	0.1716	0.1878

Table 5.93. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0 , 0.0)	**First**	0.7416	**0.7892**
	Second	0.4290	**0.4884**
(1.0 , 0.2 , 0.2 , 0.2 , 0.0)	**First**	0.7758	**0.8030**
	Second	0.4732	**0.5076**
(0.0 , 0.0 , 0.0 , 0.5 , 1.0)	**First**	0.9380	0.9414
	Second	0.6902	0.7052
(0.8 , 0.8 , 0.5 , 0.2 , 0.0)	**First**	0.8968	0.8972
	Second	0.6418	0.6664
(0.5 , 0.5 , 0.2 , 0.2 , 0.0)	**First**	0.4592	0.4586
	Second	0.2688	0.2762
(0.5 , 0.5 , 0.2 , 0.0 , 0.0)	**First**	0.5490	**0.5648**
	Second	0.3210	**0.3416**
(0.5 , 0.5 , 0.5 , 0.0 , 0.0)	**First**	0.6172	0.6220
	Second	0.3788	0.3904
(0.0 , 0.0 , 0.0 , 0.5 , 0.5)	**First**	0.5492	0.5442
	Second	0.3140	0.3114
(0.0 , 0.0 , 0.5 , 0.5 , 0.5)	**First**	0.6162	0.6268
	Second	0.3728	0.3964
(0.2 , 0.2 , 0.5 , 0.0 , 0.0)	**First**	0.3272	**0.3428**
	Second	0.1978	**0.2154**

Table 5.94. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0 , 0.0)	**First**	0.2738	**0.2976**
	Second	0.2292	**0.2452**
(1.0 , 0.2 , 0.2 , 0.2 , 0.0)	**First**	0.2932	0.3022
	Second	0.2440	0.2480
(0.0 , 0.0 , 0.0 , 0.5 , 1.0)	**First**	0.4792	0.4612
	Second	0.3958	0.3892
(0.8 , 0.8 , 0.5 , 0.2 , 0.0)	**First**	0.4456	0.4538
	Second	0.3768	0.3910
(0.5 , 0.5 , 0.2 , 0.2 , 0.0)	**First**	0.1824	0.1722
	Second	0.1454	0.1656
(0.5 , 0.5 , 0.2 , 0.0 , 0.0)	**First**	0.2098	0.2140
	Second	0.1850	0.1950
(0.5 , 0.5 , 0.5 , 0.0 , 0.0)	**First**	0.2540	0.2482
	Second	0.2142	0.2292
(0.0 , 0.0 , 0.0 , 0.5 , 0.5)	**First**	0.2052	0.2012
	Second	0.1636	0.1828
(0.0 , 0.0 , 0.5 , 0.5 , 0.5)	**First**	0.2526	0.2586
	Second	0.2058	0.2412
(0.2 , 0.2 , 0.5 , 0.0 , 0.0)	**First**	0.1508	0.1506
	Second	0.1302	0.1396

Table 5.95. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0 , 0.0)	**First**	0.3750	**0.4068**
	Second	0.2450	**0.2718**
(1.0 , 0.2 , 0.2 , 0.2 , 0.0)	**First**	0.3996	**0.4180**
	Second	0.2482	**0.2866**
(0.0 , 0.0 , 0.0 , 0.5 , 1.0)	**First**	0.6130	0.6040
	Second	0.4118	0.4314
(0.8 , 0.8 , 0.5 , 0.2 , 0.0)	**First**	0.5642	0.5742
	Second	0.3940	0.4216
(0.5 , 0.5 , 0.2 , 0.2 , 0.0)	**First**	0.2228	0.2284
	Second	0.1538	0.1666
(0.5 , 0.5 , 0.2 , 0.0 , 0.0)	**First**	0.2842	0.2856
	Second	0.1980	0.2058
(0.5 , 0.5 , 0.5 , 0.0 , 0.0)	**First**	0.3326	0.3278
	Second	0.2216	0.2404
(0.0 , 0.0 , 0.0 , 0.5 , 0.5)	**First**	0.2730	0.2656
	Second	0.1866	0.1838
(0.0 , 0.0 , 0.5 , 0.5 , 0.5)	**First**	0.3266	0.3384
	Second	0.2284	0.2348
(0.2 , 0.2 , 0.5 , 0.0 , 0.0)	**First**	0.1772	0.1776
	Second	0.1316	0.1462

Table 5.96. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(1.0 , 0.0 , 0.0 , 0.0 , 0.0)	**First**	0.5438	**0.5856**
	Second	0.2904	**0.3148**
(1.0 , 0.2 , 0.2 , 0.2 , 0.0)	**First**	0.5656	**0.6030**
	Second	0.3004	**0.3482**
(0.0 , 0.0 , 0.0 , 0.5 , 1.0)	**First**	0.7766	**0.7902**
	Second	0.4816	**0.5050**
(0.8 , 0.8 , 0.5 , 0.2 , 0.0)	**First**	0.7308	0.7458
	Second	0.4634	0.4970
(0.5 , 0.5 , 0.2 , 0.2 , 0.0)	**First**	0.3014	**0.3284**
	Second	0.1832	**0.2042**
(0.5 , 0.5 , 0.2 , 0.0 , 0.0)	**First**	0.3974	0.4012
	Second	0.2320	0.2464
(0.5 , 0.5 , 0.5 , 0.0 , 0.0)	**First**	0.4402	0.4554
	Second	0.2610	0.2796
(0.0 , 0.0 , 0.0 , 0.5 , 0.5)	**First**	0.3726	0.3812
	Second	0.2246	0.2204
(0.0 , 0.0 , 0.5 , 0.5 , 0.5)	**First**	0.4424	0.4436
	Second	0.2718	0.2770
(0.2 , 0.2 , 0.5 , 0.0 , 0.0)	**First**	0.2422	0.2458
	Second	0.1462	0.1620

2. **Skewed underlying distribution:** The estimated power for the test statistics (3.69), (3.70), (3.71), (3.72) has an unclear pattern for an exponential underlying distribution, as is shown in Tables 5.97 through 5.99. Besides that, the first method of combining the two test statistics to propose the test statistics for a mixed design, as given in (3.71) and (3.69), is better than the second method, as given in (3.72) and (3.70).

Table 5.97. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 5$.

Location Parameter	Method	Non Modification	Modification
(1.0 , 0.0 , 0.0 , 0.0 , 0.0)	**First**	0.6118	**0.6742**
	Second	0.4994	**0.5664**
(1.0 , 0.2 , 0.2 , 0.2 , 0.0)	**First**	0.7294	0.7356
	Second	0.6214	0.6354
(0.0 , 0.0 , 0.0 , 0.5 , 1.0)	**First**	0.9228	0.8998
	Second	0.8510	0.8286
(0.8 , 0.8 , 0.5 , 0.2 , 0.0)	**First**	**0.8892**	0.8680
	Second	**0.8158**	0.7960
(0.5 , 0.5 , 0.2 , 0.2 , 0.0)	**First**	**0.4932**	0.4770
	Second	**0.4254**	0.4070
(0.5 , 0.5 , 0.2 , 0.0 , 0.0)	**First**	**0.6140**	0.5740
	Second	**0.5100**	0.4996
(0.5 , 0.5 , 0.5 , 0.0 , 0.0)	**First**	**0.6284**	0.6094
	Second	0.5332	0.5344
(0.0 , 0.0 , 0.0 , 0.5 , 0.5)	**First**	0.5564	0.5472
	Second	0.4642	0.4588
(0.0 , 0.0 , 0.5 , 0.5 , 0.5)	**First**	**0.6406**	0.6038
	Second	0.5288	0.5352
(0.2 , 0.2 , 0.5 , 0.0 , 0.0)	**First**	0.3332	**0.3594**
	Second	0.2720	**0.3018**

Table 5.98. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(1.0 , 0.0 , 0.0 , 0.0 , 0.0)	**First**	0.7814	**0.8210**
	Second	0.5366	**0.6034**
(1.0 , 0.2 , 0.2 , 0.2 , 0.0)	**First**	0.8680	**0.8806**
	Second	0.6546	**0.6986**
(0.0 , 0.0 , 0.0 , 0.5 , 1.0)	**First**	0.9808	0.9722
	Second	0.8836	0.8726
(0.8 , 0.8 , 0.5 , 0.2 , 0.0)	**First**	**0.9668**	0.9494
	Second	**0.8532**	0.8254
(0.5 , 0.5 , 0.2 , 0.2 , 0.0)	**First**	**0.6434**	0.6220
	Second	0.4474	0.4470
(0.5 , 0.5 , 0.2 , 0.0 , 0.0)	**First**	**0.7564**	0.7196
	Second	**0.5494**	0.5270
(0.5 , 0.5 , 0.5 , 0.0 , 0.0)	**First**	0.7494	0.7588
	Second	0.5570	0.5704
(0.0 , 0.0 , 0.0 , 0.5 , 0.5)	**First**	0.7080	0.6978
	Second	0.4996	0.5072
(0.0 , 0.0 , 0.5 , 0.5 , 0.5)	**First**	**0.7620**	0.7470
	Second	0.5682	0.5548
(0.2 , 0.2 , 0.5 , 0.0 , 0.0)	**First**	0.4568	**0.4794**
	Second	0.3148	**0.3420**

Table 5.99. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Unknown Peak; The Sample Size for The CRD Portion is $n = 10$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(1.0 , 0.0 , 0.0 , 0.0 , 0.0)	**First**	0.9264	**0.9514**
	Second	0.6516	**0.7072**
(1.0 , 0.2 , 0.2 , 0.2 , 0.0)	**First**	0.9698	**0.9722**
	Second	0.7602	**0.7824**
(0.0 , 0.0 , 0.0 , 0.5 , 1.0)	**First**	0.9994	0.9968
	Second	0.9414	0.9174
(0.8 , 0.8 , 0.5 , 0.2 , 0.0)	**First**	0.9938	0.9906
	Second	0.9064	0.8852
(0.5 , 0.5 , 0.2 , 0.2 , 0.0)	**First**	**0.8098**	0.7842
	Second	**0.5268**	0.5098
(0.5 , 0.5 , 0.2 , 0.0 , 0.0)	**First**	**0.8936**	0.8736
	Second	**0.6262**	0.6096
(0.5 , 0.5 , 0.5 , 0.0 , 0.0)	**First**	0.8958	0.8858
	Second	0.6334	0.6476
(0.0 , 0.0 , 0.0 , 0.5 , 0.5)	**First**	0.8688	0.8572
	Second	0.5806	0.5842
(0.0 , 0.0 , 0.5 , 0.5 , 0.5)	**First**	**0.9002**	0.8884
	Second	0.6526	0.6502
(0.2 , 0.2 , 0.5 , 0.0 , 0.0)	**First**	0.6280	0.6396
	Second	0.3600	0.3882

5.7. Asymptotic Critical Values for an Unknown Peak Case

In this section, we will discuss the asymptotic critical values for the proposed test statistics for a mixed design under the umbrella hypothesis once the peak is unknown. It is presented earlier in Section 3.3.2, Chapter 3. We propose the $90th$, $95th$, and $99th$ percentile for the asymptotic null distribution of the test statistics in terms of a nonmodification case and modification case.

5.7.1. Non-Modification Case

Table 3.3 in Section 3.3.2, Chapter 3, shows the asymptotic critical values for the test statistics A^*_{maxI} and A^*_{maxII} for both methods of combining Mack-Wolfe and Kim-Kim test statistics in terms of a nonmodification case. The results are based on the empirical cumulative distribution of A^*_{maxI} and A^*_{maxII} by sampling 10,000 samples. We hold the sample size $n = 10$ for the (CRD) portion and increase the number of blocks $b = 5, 10, 20$ for each different number of treatments

$k = 3, 4, 5$. As a result of the simulation study, for both methods of combining the test statistics the critical values differ slightly once the number of blocks is increased within the same number of treatments. Besides that, the critical values increase as the number of treatments increase.

5.7.2. Modification Case

We presented in Section 3.3.2, Chapter 3, the asymptotic critical values for the test statistics MA^*_{maxI} and MA^*_{maxII} for the two methods of combining the modified version of Mack-Wolfe and Kim-Kim test statistics in Table 3.4. We sample 10,000 samples in terms of simulating the empirical cumulative distribution of MA^*_{maxI} and MA^*_{maxII}. We hold the sample size $n = 10$ for the (CRD) portion and increase the number of blocks $b = 5, 10, 20$ for different numbers of treatments $k = 3, 4, 5$. The simulation study shows that the critical values vary slightly once the number of blocks is increased within the same number of treatments. Besides that, there is an increase in the critical values as the number of treatments increase.

6. CONCLUSION AND FUTURE WORK

We propose test statistics for a mixed design consisting of a completely randomized design (CRD) and a randomized complete block design (RCBD) under an umbrella alternative hypothesis as given in (6.1)

$$
\begin{aligned}
&H_0 : \mu_1 = \mu_2 = \ldots = \mu_k \\
&H_1 : \mu_1 \leq \mu_2 \leq \ldots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \ldots \geq \mu_k \qquad \textit{with at least one strict inequality}
\end{aligned} \tag{6.1}
$$

where μ_i is a location parameter for the i^{th} population; $i = 1, 2, \ldots, p, \ldots, k$, and p is the peak of the umbrella alternative.

Either case of the peak of the umbrella hypothesis is considered in this research. We propose test statistics for a mixed design with a case of a known and unknown peak. In general, the proposed test statistics are based on combining the test statistic of the (CRD) portion and the test statistic of the (RCBD) portion, in terms of using a square distance modification. Besides that, we apply two different methods of combining the test statistics for the (CRD) portion and the (RCBD) portion.

6.1. Peak Known

In this section, we conclude the results of the performance of the estimated power for the proposed test statistic once the peak is known. The simulation study shows that the first method of combining the two statistics of the (CRD) and the (RCBD) is generally better than the second method regardless of the underlying distribution, number of treatments, and the peak. We can distinguish some cases that show the square distance modification results in improvement with an estimated power.

- ***Four treatments at peak 2:*** Regardless of the underlying distribution, and the relationship between a sample size of the (CRD) and block's number of the (RCBD), the estimated power of proposed test statistics with a square distance modification (3.58), (3.68) for a mixed design in the case of the ***distinct*** peak is better than the estimated power of test statistics without modification (2.16), (2.20) introduced by Magel et al. (2010) under the umbrella hypothesis. As long as, the location parameter on the left side of the umbrella hypothesis (*upside*) is

greater than all the different location parameters on the right side of the umbrella hypothesis (*downside*), such as (0.8 , **1.0** , 0.75 , 0.2), as well as, the case of the location parameter of the *upside* is equal to the first one and greater than the last one of the *downside*, such as, (0.75 , **1**, 0.75 , 0.2). On the other hand, in the case of the ***indistinct*** peak with the first location parameter, the estimated power for test statistics (3.58), (3.68) is still higher than the estimated power for (2.16), (2.20) for any configuration on the right side of the umbrella hypothesis, for instance:

  - (0.5 , **0.5** , 0.2 , 0.0)
  - (0.5 , **0.5** , 0.0 , 0.0)
  - (0.5 , **0.5** , 0.5 , 0.0)

- ***Five treatments at peak 2:*** The estimated power of the proposed test statistics with a square distance modification (3.58), (3.68) for a mixed design in the case of a ***distinct*** peak is better than the estimated power of test statistics without modification (2.16), (2.20) introduced by Magel et al. (2010) under the umbrella hypothesis, regardless of the underlying distribution, the relationship between a sample size of the (CRD) and block's number of the (RCBD), as long as, the location parameter on left side of the umbrella hypothesis (*upside*) is greater than all the different location parameters on the right side of the umbrella hypothesis (*downside*), such as (0.75 , **0.8** , 0.6 , 0.4 , 0.2). On the other hand, in the case of the ***indistinct*** peak with first location parameter the estimated power for test statistics (3.58), (3.68) is still higher than the estimated power for (2.16), (2.20) for any configuration on the right side of the umbrella hypothesis, for instance:

  - (0.8 , **0.8** , 0.5 , 0.2 , 0.0)
  - (0.5 , **0.5** , 0.2 , 0.0 , 0.0)
  - (0.5 , **0.5** , 0.2 , 0.2 , 0.0)
  - (0.5 , **0.5** , 0.0 , 0.0 , 0.0)
  - (0.5 , **0.5** , 0.5 , 0.2 , 0.0)
  - (0.5 ,**0.5** , 0.5 , 0.0 , 0.0)

– (0.5 , **0.5** , 0.5 , 0.5 , 0.0).

- ***Five treatments at peak 3:*** The estimated power of the proposed test statistics for a mixed design differs based on the underlying distribution and the sample size of the (CRD). The results show that the estimated power of proposed test statistics with a square distance modification (3.58), (3.68) is slightly different from the estimated power for test statistics (2.16), (2.20) introduced by Magel et al. (2010) in most of the location parameters configurations of the umbrella hypothesis. However, there are a few cases of the umbrella hypothesis once the peak is ***distinct*** that show the estimated power for proposed test statistics with a square distance modification (3.58), (3.68) is better than the estimated power in (2.16) and (2.20), as long as, the underlying distribution is symmetric distribution (normal, student's t).

6.2. Peak Unknown

In this section, we conclude the results of the performance of the estimated power for the proposed test statistic once the peak is unknown. The simulation study shows that the first method of combining the two statistics of the (CRD) and the (RCBD) is generally better than the second method, regardless of the underlying distribution and the number of treatments. We can distinguish some cases that show the square distance modification results in improvement with an estimated power.

- ***Three treatments:*** In general, the estimated power for the proposed test statistics with a square distance modification (3.71), (3.72) is slightly different from the estimated power of test statistic without modification (3.69), (3.70) in two cases of symmetric and skewed underlying distributions.

- ***Four treatments:*** We can conclude that once the symmetric underlying distribution, there is no big difference in the estimated power between the test statistics without modification (3.69), (3.70), and a square distance modification (3.71), (3.72). However, there are a few cases that show the estimated power for the test statistics with a square distance modification is better than the test statistics without modification. On the other hand, there is an unclear pattern of the estimated power for the test statistics (3.69), (3.70), (3.71), (3.72) for the case of having a skewed underlying distribution.

- ***Five treatments:*** The conclusion for five treatments is similar to the conclusion for four treatments at an unknown peak. The estimated power for the test statistics without modification (3.69), (3.70) differs slightly from a squared distance modification once (3.71), (3.72) for the symmetric underlying distribution. However, the estimated power for the test statistic with a square distance modification is much better than the test statistics without modification for some cases. For the case of having a skewed underlying distribution, there is an unclear pattern of the estimated power for the test statistics (3.69), (3.70), (3.71), (3.72).

For future study, the exact mean and variance of the modified test statistics of Mack-Wolfe and Kim-Kim will be obtained for an unbalanced design in terms of using them to find the exact mean and variance for a mixed design.

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APPENDIX A. FOUR TREATMENTS AT PEAK 2

The sample size of all treatments for (CRD) portion is *twice* the number of blocks for (RCBD) portion:

Table A.1. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.5794	0.5202
	Second	0.5406	0.4578
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.3460	0.3960
	Second	0.3212	0.3562
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1770	0.2148
	Second	0.1766	0.1886
(0 , **1.0** , 0.5 , 0.5)	**First**	0.4096	0.3122
	Second	0.3780	0.2854
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.2808	0.2646
	Second	0.2680	0.2280
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.4230	0.4390
	Second	0.4002	0.4048
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1728	0.2168
	Second	0.1624	0.1928
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.0920	0.0716
	Second	0.0944	0.0706
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1678	0.2082
	Second	0.1638	0.1834
(0.75 , **1** , 0.75 , 0.2)	**First**	0.3724	0.4022
	Second	0.3580	0.3722

Table A.2. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 8$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9272	0.8692
	Second	0.8746	0.8070
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.6884	0.7560
	Second	0.6056	0.6836
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3106	0.4046
	Second	0.2890	0.3518
(0 , **1.0** , 0.5 , 0.5)	**First**	0.7536	0.6184
	Second	0.6684	0.5422
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.5538	0.5034
	Second	0.4750	0.4392
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.7972	0.8030
	Second	0.7132	0.7194
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3152	0.4012
	Second	0.2710	0.3438
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1378	0.0820
	Second	0.1272	0.0768
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3092	0.4064
	Second	0.2688	0.3502
(0.75 , 1.0 , 0.75 , 0.2)	**First**	0.7140	0.7748
	Second	0.6364	0.6810

Table A.3. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9590	0.9278
	Second	0.9172	0.8706
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.7598	0.8310
	Second	0.6704	0.7500
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3634	0.4654
	Second	0.3176	0.3918
(0 , **1.0** , 0.5 , 0.5)	**First**	0.8298	0.6930
	Second	0.7514	0.5970
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.6386	0.5994
	Second	0.5522	0.5080
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.8744	0.8796
	Second	0.7914	0.8062
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3738	0.4660
	Second	0.3146	0.3950
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1540	0.0918
	Second	0.1346	0.0916
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3700	0.4634
	Second	0.3226	0.3956
(0.75 , 1.0 , 0.75 , 0.2)	**First**	0.7834	0.8408
	Second	0.6994	0.7696

Table A.4. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.4252	0.3812
	Second	0.4038	0.3452
(0.8 , 1 , 0.75 , 0.2)	**First**	0.2806	0.2994
	Second	0.2626	0.2708
(0.5 , 0.5 , 0.2 , 0.0)	**First**	0.1372	0.1740
	Second	0.1338	0.1516
(0 , **1.0** , 0.5 , 0.5)	**First**	0.2976	0.2438
	Second	0.2906	0.2196
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.2208	0.2068
	Second	0.2120	0.2042
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.3270	0.3356
	Second	0.3098	0.3078
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1414	0.1692
	Second	0.1330	0.1542
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.0852	0.0682
	Second	0.0836	0.0634
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1458	0.1658
	Second	0.1456	0.1506
(0.75 , 1.0 , 0.75 , 0.2)	**First**	0.2818	0.3146
	Second	0.2700	0.2904

Table A.5. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 8$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.7912	0.7202
	Second	0.7070	0.6302
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.5382	0.5928
	Second	0.4598	0.5218
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2458	0.3126
	Second	0.2076	0.2644
(0 , **1.0** , 0.5 , 0.5)	**First**	0.5814	0.4676
	Second	0.4980	0.4094
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.4180	0.3942
	Second	0.3580	0.3476
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.6428	0.6468
	Second	0.5596	0.5648
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2394	0.3014
	Second	0.2104	0.2626
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1228	0.0748
	Second	0.1124	0.0712
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2492	0.3068
	Second	0.2150	0.2620
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.5480	0.6054
	Second	0.4758	0.5292

Table A.6. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.8696	0.8042
	Second	0.7908	0.7272
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.6246	0.6742
	Second	0.5324	0.5874
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2852	0.3506
	Second	0.2460	0.3028
(0 , **1.0** , 0.5 , 0.5)	**First**	0.6848	0.5418
	Second	0.5992	0.4636
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.4856	0.4586
	Second	0.4074	0.3990
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.7192	0.7296
	Second	0.6360	0.6442
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2772	0.3472
	Second	0.2428	0.3004
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1320	0.0864
	Second	0.1202	0.0760
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2752	0.3522
	Second	0.2444	0.2974
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.6194	0.6928
	Second	0.5258	0.6122

Table A.7. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.8108	0.7350
	Second	0.7606	0.6682
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.5596	0.6124
	Second	0.5210	0.5488
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2734	0.3382
	Second	0.2670	0.3050
(0 , **1.0** , 0.5 , 0.5)	**First**	0.5914	0.4932
	Second	0.5576	0.4348
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.4488	0.4410
	Second	0.4106	0.3868
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.6484	0.6374
	Second	0.5908	0.5882
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2382	0.3096
	Second	0.2306	0.2922
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1112	0.0832
	Second	0.1114	0.0714
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2630	0.3322
	Second	0.2372	0.3012
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.6018	0.6320
	Second	0.5476	0.5770

Table A.8. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 8$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9946	0.9830
	Second	0.9816	0.9586
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.9212	0.9448
	Second	0.8586	0.8938
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5596	0.6606
	Second	0.4804	0.5776
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9614	0.8654
	Second	0.9070	0.8006
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.8424	0.7886
	Second	0.7690	0.7120
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9740	0.9640
	Second	0.9386	0.9250
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.5110	0.6568
	Second	0.4448	0.5746
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1968	0.0982
	Second	0.1708	0.0924
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5160	0.6484
	Second	0.4380	0.5694
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.9362	0.9560
	Second	0.8838	0.9096

Table A.9. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9992	0.9938
	Second	0.9960	0.9808
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.9642	0.9758
	Second	0.9224	0.9416
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6564	0.7474
	Second	0.5576	0.6648
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9842	0.9270
	Second	0.9570	0.8702
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.9178	0.8780
	Second	0.8530	0.8018
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9908	0.9890
	Second	0.9722	0.9638
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.6054	0.7194
	Second	0.5062	0.6416
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2162	0.1102
	Second	0.1922	0.1054
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5896	0.7278
	Second	0.5060	0.6418
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.9702	0.9808
	Second	0.9270	0.9508

The sample size of all treatments for (CRD) portion is *equal* the number of blocks for (RCBD) portion:

Table A.10. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.6854	0.6132
	Second	0.5656	0.4936
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.4380	0.4898
	Second	0.3518	0.3856
(0.5 ,**0.5** , 0.2 , 0.0)	**First**	0.2080	0.2518
	Second	0.1712	0.2072
(0 , **1.0** , 0.5 , 0.5)	**First**	0.4994	0.3830
	Second	0.3854	0.3168
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.3470	0.3218
	Second	0.2782	0.2648
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.5346	0.5508
	Second	0.4202	0.4400
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1934	0.2550
	Second	0.1680	0.2122
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.0984	0.0746
	Second	0.0812	0.0716
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2014	0.2560
	Second	0.1690	0.2042
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.4634	0.4946
	Second	0.3668	0.3992

Table A.11. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 16$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9724	0.9452
	Second	0.8802	0.8234
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.7936	0.8620
	Second	0.6232	0.6922
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3904	0.4928
	Second	0.2794	0.3656
(0 , **1.0** , 0.5 , 0.5)	**First**	0.8442	0.7320
	Second	0.6888	0.5638
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.6630	0.6248
	Second	0.5016	0.4608
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.8922	0.8918
	Second	0.7354	0.7470
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3848	0.4806
	Second	0.2784	0.3488
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1670	0.0950
	Second	0.1412	0.0890
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3890	0.4870
	Second	0.2868	0.3590
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.8090	0.8686
	Second	0.6458	0.7082

Table A.12. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9920	0.9766
	Second	0.9340	0.8918
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.8592	0.9258
	Second	0.6892	0.7832
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4566	0.5612
	Second	0.3268	0.4150
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9146	0.7962
	Second	0.7620	0.6224
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.7424	0.6940
	Second	0.5546	0.5318
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9456	0.9440
	Second	0.8128	0.8214
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4494	0.5688
	Second	0.3260	0.4156
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1880	0.0964
	Second	0.1430	0.0804
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4662	0.5720
	Second	0.3234	0.4212
(0.75 , 1.0 , 0.75 , 0.2)	**First**	0.8884	0.9232
	Second	0.7302	0.7856

Table A.13. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The number of Blocks for The RCBD Portion is $b = 6$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.5286	0.4720
	Second	0.4192	0.3830
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.3286	0.3610
	Second	0.2586	0.2918
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1714	0.1946
	Second	0.1440	0.1654
(0 , **1.0** , 0.5 , 0.5)	**First**	0.3660	0.2998
	Second	0.2996	0.2434
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.2668	0.2472
	Second	0.2132	0.1972
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.3978	0.4138
	Second	0.3142	0.3314
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1694	0.1942
	Second	0.1380	0.1646
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.0904	0.0688
	Second	0.0840	0.0662
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1636	0.2022
	Second	0.1440	0.1684
(0.75 , 1 , 0.75 , 0.2)	**First**	0.3448	0.3752
	Second	0.2734	0.3098

Table A.14. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 16$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.8842	0.8354
	Second	0.7356	0.6712
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.6380	0.7100
	Second	0.4876	0.5412
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2992	0.3578
	Second	0.2226	0.2718
(0, **1.0** , 0.5 , 0.5)	**First**	0.7024	0.5710
	Second	0.5326	0.4304
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.5128	0.4772
	Second	0.3716	0.3564
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.7380	0.7596
	Second	0.5704	0.5828
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2876	0.3842
	Second	0.2222	0.2898
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1420	0.0796
	Second	0.1204	0.0746
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2964	0.3630
	Second	0.2300	0.2724
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.6606	0.7220
	Second	0.4998	0.5398

Table A.15. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9424	0.8944
	Second	0.8104	0.7294
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.7046	0.7838
	Second	0.5168	0.6108
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3458	0.4388
	Second	0.2544	0.3234
(0 , **1.0** , 0.5 , 0.5)	**First**	0.7998	0.6584
	Second	0.6052	0.4920
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.5940	0.5492
	Second	0.4380	0.4074
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.8290	0.8314
	Second	0.6498	0.6616
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3362	0.4240
	Second	0.2354	0.3008
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1452	0.0872
	Second	0.1190	0.0760
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3442	0.4248
	Second	0.2436	0.3064
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.7420	0.7978
	Second	0.5598	0.6320

Table A.16. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9006	0.8436
	Second	0.7938	0.7312
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.6848	0.7238
	Second	0.5546	0.6118
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3472	0.4104
	Second	0.2646	0.3346
(0 , **1.0** , 0.5 , 0.5)	**First**	0.7364	0.6114
	Second	0.5806	0.4888
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.5700	0.5384
	Second	0.4276	0.4302
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.7798	0.7756
	Second	0.6426	0.6512
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3020	0.4110
	Second	0.2310	0.3270
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1280	0.0804
	Second	0.0984	0.0726
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3180	0.4024
	Second	0.2650	0.3280
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.7072	0.7380
	Second	0.5786	0.6214

Table A.17. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 16$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9998	0.9972
	Second	0.9872	0.9724
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.9706	0.9834
	Second	0.8824	0.9084
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6728	0.7824
	Second	0.5024	0.6038
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9908	0.9470
	Second	0.9260	0.8234
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.9306	0.8966
	Second	0.7936	0.7340
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9948	0.9910
	Second	0.9454	0.9362
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.6246	0.7680
	Second	0.4564	0.5910
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2336	0.1098
	Second	0.1828	0.0898
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.6204	0.7420
	Second	0.4576	0.5872
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.9790	0.9850
	Second	0.9008	0.9230

Table A.18. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	1.0000	0.9994
	Second	0.9944	0.9864
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.9900	0.9954
	Second	0.9332	0.9442
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.7542	0.8634
	Second	0.5694	0.6954
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9972	0.9758
	Second	0.9654	0.8848
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.9666	0.9486
	Second	0.8620	0.8146
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9988	0.9984
	Second	0.9798	0.9732
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.7188	0.8516
	Second	0.5334	0.6668
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2678	0.1262
	Second	0.2034	0.1006
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.6892	0.8422
	Second	0.5170	0.6728
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.9924	0.9960
	Second	0.9418	0.9536

The sample size of all treatments for (CRD) portion is *half* the number of blocks for (RCBD) portion:

Table A.19. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.8280	0.7618
	Second	0.6686	0.5912
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.5498	0.6192
	Second	0.4240	0.4684
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2552	0.3216
	Second	0.1942	0.2396
(0 , **1.0** , 0.5 , 0.5)	**First**	0.6154	0.4960
	Second	0.4680	0.3736
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.4476	0.4026
	Second	0.3450	0.3154
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.6792	0.6676
	Second	0.5118	0.5100
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2584	0.3152
	Second	0.1996	0.2272
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1206	0.0824
	Second	0.1050	0.0734
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2602	0.3234
	Second	0.2010	0.2374
(0.75 , 1.0 , 0.75 , 0.2)	**First**	0.5942	0.6306
	Second	0.4446	0.4742

Table A.20. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 32$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9958	0.9858
	Second	0.9146	0.8638
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.9064	0.9460
	Second	0.6524	0.7344
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4914	0.6244
	Second	0.2962	0.3930
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9444	0.8476
	Second	0.7356	0.5978
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.7914	0.7568
	Second	0.5192	0.5010
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9580	0.9670
	Second	0.7704	0.7878
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4968	0.6260
	Second	0.3032	0.3950
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2014	0.0966
	Second	0.1444	0.0814
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4870	0.6070
	Second	0.2998	0.3886
(0.75 , 1.0 , 0.75 , 0.2)	**First**	0.9184	0.9504
	Second	0.6794	0.7582

Table A.21. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 40$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9992	0.9954
	Second	0.9462	0.9068
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.9518	0.9774
	Second	0.7434	0.8040
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5750	0.7000
	Second	0.3364	0.4458
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9740	0.9190
	Second	0.8030	0.6556
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.8656	0.8352
	Second	0.5980	0.5626
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9828	0.9882
	Second	0.8294	0.8502
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.5744	0.7004
	Second	0.3400	0.4264
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2254	0.1098
	Second	0.1460	0.0928
(0.5 ,**0.5** , 0.5 , 0.0)	**First**	0.5672	0.7048
	Second	0.3456	0.4418
(0.75 , 1.0 , 0.75 , 0.2)	**First**	0.9658	0.9798
	Second	0.7492	0.8084

Table A.22. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.6702	0.6122
	Second	0.5148	0.4612
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.4248	0.4634
	Second	0.3136	0.3552
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1992	0.2378
	Second	0.1610	0.1874
(0 , **1.0** , 0.5 , 0.5)	**First**	0.4776	0.3694
	Second	0.3538	0.2784
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.3348	0.3164
	Second	0.2508	0.2372
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.5180	0.5364
	Second	0.3836	0.3874
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1976	0.2376
	Second	0.1626	0.1818
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.0982	0.0746
	Second	0.0846	0.0724
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1898	0.2484
	Second	0.1558	0.1898
(0.75 , 1 , 0.75 , 0.2)	**First**	0.4406	0.4892
	Second	0.3290	0.3722

Table A.23. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degree of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The number of Blocks for The RCBD Portion is $b = 32$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9598	0.9376
	Second	0.7690	0.7152
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.7640	0.8392
	Second	0.5176	0.5856
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3724	0.4788
	Second	0.2384	0.3020
(0 , **1.0** , 0.5 , 0.5)	**First**	0.8318	0.7012
	Second	0.5606	0.4528
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.6350	0.5990
	Second	0.3926	0.3700
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.8638	0.8808
	Second	0.6044	0.6232
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3748	0.4750
	Second	0.2378	0.3008
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1620	0.0896
	Second	0.1134	0.0720
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3700	0.4600
	Second	0.2332	0.2980
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.7868	0.8462
	Second	0.5162	0.5854

Table A.24. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degree of Freedom for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 40$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9876	0.9632
	Second	0.8338	0.7692
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.8408	0.8960
	Second	0.5580	0.6448
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4524	0.5524
	Second	0.2700	0.3360
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9010	0.7760
	Second	0.6476	0.5144
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.7266	0.6854
	Second	0.4546	0.4288
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9228	0.9310
	Second	0.6810	0.6900
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4266	0.5468
	Second	0.2596	0.3338
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1870	0.0910
	Second	0.1348	0.0742
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4402	0.5462
	Second	0.2684	0.3316
(0.75 , 1 , 0.75 , 0.2)	**First**	0.8678	0.9054
	Second	0.5954	0.6462

Table A.25. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	0.9708	0.9330
	Second	0.8834	0.8046
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.8200	0.8636
	Second	0.6580	0.7126
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4438	0.5312
	Second	0.3300	0.3906
(0 , **1.0** , 0.5 , 0.5)	**First**	0.8700	0.7422
	Second	0.7018	0.5708
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.7060	0.6588
	Second	0.5180	0.4910
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.8970	0.9010
	Second	0.7352	0.7502
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3964	0.5260
	Second	0.2872	0.3912
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1516	0.0856
	Second	0.1186	0.0784
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4124	0.5202
	Second	0.3106	0.3842
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.8472	0.8632
	Second	0.6796	0.7084

Table A.26. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 32$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	1.0000	0.9998
	Second	0.9926	0.9780
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.9950	0.9982
	Second	0.9068	0.9342
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.8124	0.8868
	Second	0.5400	0.6386
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9986	0.9866
	Second	0.9494	0.8534
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.9856	0.9684
	Second	0.8308	0.7874
(0.5 , **1.0** , 0.2 , 0.2)	**First**	0.9996	0.9998
	Second	0.9632	0.9576
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.7818	0.8734
	Second	0.4890	0.6254
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2972	0.1358
	Second	0.1816	0.1078
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.7554	0.8734
	Second	0.5010	0.6212
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.9970	0.9992
	Second	0.9216	0.9456

Table A.27. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 4 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 40$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **1.0** , 0.75 , 0.2)	**First**	1.0000	1.0000
	Second	0.9982	0.9922
(0.8 , **1.0** , 0.75 , 0.2)	**First**	0.9990	0.9998
	Second	0.9486	0.9680
(0.5 , **0.5** , 0.2 , 0.0)	**First**	0.8692	0.9516
	Second	0.6024	0.7248
(0 , **1.0** , 0.5 , 0.5)	**First**	0.9998	0.9970
	Second	0.9764	0.9070
(0.5 , **1.0** , 0.5 , 0.5)	**First**	0.9962	0.9862
	Second	0.8984	0.8444
(0.5 , **1.0** , 0.2 , 0.2)	**First**	1.0000	1.0000
	Second	0.9868	0.9778
(0.5 , **0.5** , 0.0 , 0.0)	**First**	0.8460	0.9414
	Second	0.5494	0.7064
(0.0 , **0.5** , 0.5 , 0.5)	**First**	0.3528	0.1444
	Second	0.2038	0.0994
(0.5 , **0.5** , 0.5 , 0.0)	**First**	0.8376	0.9342
	Second	0.5744	0.6950
(0.75 , **1.0** , 0.75 , 0.2)	**First**	0.9996	0.9998
	Second	0.9602	0.9738

APPENDIX B. FIVE TREATMENTS AT PEAK 2

The sample size of all treatments for (CRD) portion is *twice* the number of blocks for (RCBD) portion:

Table B.1. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.4236	0.3470
	Second	0.3710	0.3184
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.3936	0.4742
	Second	0.3650	0.4280
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.4030	0.4756
	Second	0.3598	0.4274
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.3350	0.2710
	Second	0.3028	0.2514
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.5710	0.6064
	Second	0.5244	0.5460
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.2136	0.2490
	Second	0.1922	0.2316
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.2376	0.1896
	Second	0.2070	0.1806
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.5472	0.5576
	Second	0.4842	0.4988
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.1840	0.2236
	Second	0.1740	0.2100
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	0.2280	0.1886
	Second	0.2028	0.1768
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.2728	0.2460
	Second	0.2478	0.2200
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.2266	0.2436
	Second	0.2114	0.2214
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.1724	0.2290
	Second	0.1648	0.1922
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.4370	0.4726
	Second	0.3946	0.4210
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.2298	0.2650
	Second	0.2126	0.2454
(0.2 , **0.7** , 0.2 , 0.0 ,0.0)	**First**	0.4054	0.3964
	Second	0.3630	0.3690
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.2592	0.2978
	Second	0.2338	0.2734
(0.2 , **0.5** , 0.2 , 0.2, 0.0)	**First**	0.2282	0.2344
	Second	0.2118	0.2150
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.1858	0.2386
	Second	0.1676	0.2198
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.0734	0.0526
	Second	0.0700	0.0528

Table B.2. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 8$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.7868	0.7072
	Second	0.6676	0.5878
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.7590	0.8224
	Second	0.6538	0.7472
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.7370	0.8274
	Second	0.6540	0.7442
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.6468	0.5340
	Second	0.5534	0.4584
(0.5 , **1.0** , 0.2, 0.0, 0.0)	**First**	0.9212	0.9318
	Second	0.8576	0.8812
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.4134	0.4958
	Second	0.3596	0.4246
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.5976	0.4958
	Second	0.3908	0.3148
(0.5 , **1.0** , 0.2, 0.2, 0.0)	**First**	0.9010	0.9152
	Second	0.8296	0.8566
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3642	0.4426
	Second	0.3150	0.3740
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.6002	0.5032
	Second	0.3908	0.3196
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.5184	0.4592
	Second	0.4430	0.3860
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.4436	0.4464
	Second	0.3872	0.3798
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3334	0.4304
	Second	0.2966	0.3646
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.8108	0.8226
	Second	0.7204	0.7414
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.4606	0.5258
	Second	0.3942	0.4602
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0. 7518	0.7376
	Second	0.6696	0.6578
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.5248	0.5998
	Second	0.4408	0.5220
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4464	0.4662
	Second	0.3826	0.3980
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.3404	0.4150
	Second	0.3034	0.3622
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1090	0.0730
	Second	0.0934	0.0612

Table B.3. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.8452	0.7552
	Second	0.7488	0.6664
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8348	0.8974
	Second	0.7444	0.8158
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8162	0.8904
	Second	0.7252	0.8256
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.7402	0.6358
	Second	0.6476	0.5460
(0.5 , **1.0** , 0.2, 0.0, 0.0)	**First**	0.9696	0.9692
	Second	0.9168	0.9338
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.4624	0.5742
	Second	0.4008	0.4972
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.5186	0.4158
	Second	0.4410	0.3610
(0.5 , **1.0** , 0.2, 0.2, 0.0)	**First**	0.9504	0.9544
	Second	0.8962	0.9104
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4216	0.5072
	Second	0.3658	0.4306
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.5140	0.4266
	Second	0.4220	0.3614
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.5932	0.5416
	Second	0.4930	0.4554
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.5214	0.5214
	Second	0.4366	0.4484
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3992	0.4884
	Second	0.3312	0.4208
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.8846	0.8950
	Second	0.8028	0.8228
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.5184	0.6194
	Second	0.4386	0.5350
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.8474	0.8300
	Second	0.7552	0.7330
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.5928	0.6824
	Second	0.5178	0.5902
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.5198	0.5224
	Second	0.4436	0.4432
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.3998	0.5038
	Second	0.3438	0.4336
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1234	0.0564
	Second	0.1088	0.0592

Table B.4. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.3126	0.2674
	Second	0.2868	0.2402
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.2998	0.3532
	Second	0.2722	0.3250
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.3010	0.3378
	Second	0.2762	0.3172
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.2554	0.2166
	Second	0.2290	0.2078
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.4296	0.4394
	Second	0.3816	0.4020
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.1734	0.1922
	Second	0.1692	0.1774
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.1832	0.1466
	Second	0.1704	0.1412
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.4092	0.4344
	Second	0.3690	0.3872
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.1470	0.1784
	Second	0.1428	0.1628
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.1854	0.1466
	Second	0.1602	0.1384
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.2064	0.1930
	Second	0.1780	0.1780
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.1742	0.1782
	Second	0.1632	0.1644
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.1478	0.1800
	Second	0.1424	0.1594
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.3408	0.3604
	Second	0.3042	0.3256
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.1846	0.2044
	Second	0.1652	0.1904
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.3046	0.3042
	Second	0.2744	0.2726
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.2034	0.2352
	Second	0.1888	0.2210
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.1856	0.1912
	Second	0.1662	0.1860
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.1498	0.1752
	Second	0.1354	0.1666
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.0780	0.0548
	Second	0.0726	0.0550

Table B.5. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degree of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 8$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.6286	0.5468
	Second	0.5212	0.4582
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.5896	0.6760
	Second	0.5068	0.5840
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.5676	0.6724
	Second	0.4792	0.5852
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.4856	0.4254
	Second	0.4276	0.3644
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.7866	0.8184
	Second	0.6994	0.7316
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.3030	0.3890
	Second	0.2610	0.3252
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.4662	0.3780
	Second	0.2980	0.2376
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.3964	0.7794
	Second	0.6420	0.6910
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2686	0.3370
	Second	0.2254	0.2908
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.4404	0.3780
	Second	0.2902	0.2490
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.3872	0.3528
	Second	0.3382	0.3034
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.3304	0.3404
	Second	0.2844	0.2972
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2636	0.3252
	Second	0.2264	0.2782
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.6558	0.6732
	Second	0.5676	0.5828
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.3410	0.3976
	Second	0.2840	0.3412
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.6158	0.5842
	Second	0.5234	0.5072
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.3858	0.4524
	Second	0.3184	0.3890
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3402	0.3438
	Second	0.2994	0.2960
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.2654	0.3278
	Second	0.2262	0.2780
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1020	0.0550
	Second	0.0944	0.0586

Table B.6. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degree of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.6828	0.5888
	Second	0.6056	0.5044
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6654	0.7578
	Second	0.5736	0.6696
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6638	0.7522
	Second	0.5596	0.6654
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.5700	0.4814
	Second	0.4874	0.4104
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.8776	0.8870
	Second	0.7910	0.8026
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.3650	0.4454
	Second	0.3134	0.3688
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.3970	0.3208
	Second	0.3326	0.2692
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.8380	0.8468
	Second	0.7442	0.7742
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3212	0.3690
	Second	0.2682	0.2996
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.3974	0.3272
	Second	0.3334	0.2750
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.4564	0.4086
	Second	0.3898	0.3448
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.3844	0.3966
	Second	0.3236	0.3360
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3006	0.3878
	Second	0.2606	0.3188
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.7358	0.7620
	Second	0.6508	0.6738
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.3844	0.4526
	Second	0.3348	0.3930
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.6904	0.6696
	Second	0.5926	0.5826
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.4442	0.5182
	Second	0.3826	0.4374
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4068	0.3988
	Second	0.3360	0.3440
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.3032	0.3792
	Second	0.2590	0.3250
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1038	0.0590
	Second	0.0944	0.0636

Table B.7. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 3$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.6656	0.5792
	Second	0.5990	0.5282
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6522	0.7044
	Second	0.5930	0.6462
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6238	0.7034
	Second	0.5682	0.6514
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.5624	0.4640
	Second	0.5006	0.4116
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.8212	0.7880
	Second	0.7624	0.7316
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.3470	0.4204
	Second	0.3118	0.3676
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.3986	0.3178
	Second	0.3468	0.2958
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.7798	0.7694
	Second	0.7260	0.7150
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3054	0.3642
	Second	0.2732	0.3342
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.3624	0.3126
	Second	0.3046	0.2776
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.4110	0.3940
	Second	0.3598	0.3478
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.3564	0.3596
	Second	0.3082	0.3214
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2622	0.3472
	Second	0.2274	0.3038
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.7166	0.7016
	Second	0.6538	0.6412
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.3932	0.4640
	Second	0.3602	0.4150
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.6604	0.6140
	Second	0.5924	0.5546
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.4166	0.5020
	Second	0.3674	0.4624
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4064	0.4040
	Second	0.3612	0.3708
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.2992	0.3652
	Second	0.2610	0.3234
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.0948	0.0634
	Second	0.0858	0.0544

Table B.8. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 8$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9722	0.9304
	Second	0.9298	0.8500
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9586	0.9736
	Second	0.9112	0.9418
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9524	0.9740
	Second	0.9002	0.9368
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.9186	0.8398
	Second	0.8572	0.7588
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9974	0.9946
	Second	0.9868	0.9800
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.6936	0.7834
	Second	0.5986	0.6896
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.8824	0.7808
	Second	0.6654	0.5560
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9952	0.9918
	Second	0.9800	0.9710
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.6212	0.7110
	Second	0.5336	0.6186
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.8648	0.7702
	Second	0.6496	0.5418
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.7972	0.7380
	Second	0.7134	0.6498
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.7230	0.7290
	Second	0.6246	0.6364
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.5574	0.6820
	Second	0.4726	0.5962
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9802	0.9722
	Second	0.9526	0.9416
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.7190	0.8096
	Second	0.6388	0.7200
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.9744	0.9450
	Second	0.9362	0.8946
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.7816	0.8516
	Second	0.6900	0.7712
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.7616	0.7512
	Second	0.6748	0.6662
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.5656	0.6678
	Second	0.4710	0.5904
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1306	0.0688
	Second	0.1264	0.0638

Table B.9. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9886	0.9550
	Second	0.9654	0.9058
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9844	0.9920
	Second	0.9576	0.9716
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9818	0.9922
	Second	0.9548	0.9732
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.9624	0.9100
	Second	0.9134	0.8286
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9998	0.9990
	Second	0.9966	0.9928
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.7868	0.8556
	Second	0.6960	0.7794
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.8320	0.7234
	Second	0.7312	0.6322
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9990	0.9972
	Second	0.9930	0.9902
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.7066	0.7920
	Second	0.6276	0.6966
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.8180	0.7078
	Second	0.7266	0.6022
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.8792	0.8318
	Second	0.7944	0.7368
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.8258	0.8136
	Second	0.7306	0.7218
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.6630	0.7706
	Second	0.5510	0.6836
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9952	0.9888
	Second	0.9814	0.9734
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.8094	0.8794
	Second	0.7166	0.8056
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.9902	0.9780
	Second	0.9688	0.9428
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.8642	0.9206
	Second	0.7840	0.8580
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.8526	0.8240
	Second	0.7682	0.7448
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.6342	0.7622
	Second	0.5430	0.6694
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1586	0.0722
	Second	0.1276	0.0708

The sample size of all treatments for (CRD) portion is *equal* the number of blocks for (RCBD) portion:

Table B.10. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.4994	0.4330
	Second	0.3974	0.3534
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.4912	0.5768
	Second	0.3806	0.4726
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.4758	0.5840
	Second	0.3754	0.4652
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.4136	0.3422
	Second	0.3266	0.2666
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.7026	0.7202
	Second	0.5740	0.6028
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.2588	0.3082
	Second	0.2080	0.2518
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.2820	0.2288
	Second	0.2306	0.1930
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.6532	0.6760
	Second	0.5282	0.5598
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2292	0.2624
	Second	0.1844	0.2186
(0.0 , **0.6** , 0.2 , 0.2 , 0.2)	**First**	0.2836	0.2322
	Second	0.2332	0.1944
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.3150	0.2928
	Second	0.2568	0.2406
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.2704	0.2794
	Second	0.2240	0.2348
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2156	0.2692
	Second	0.1746	0.2264
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.5512	0.5716
	Second	0.4334	0.4700
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.2864	0.3340
	Second	0.2320	0.2618
(0.2 , **0.7** , 0.2 , 0.0 ,0.0)	**First**	0.5088	0.4960
	Second	0.4126	0.4094
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.3190	0.3758
	Second	0.2520	0.3022
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2862	0.2924
	Second	0.2216	0.2394
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.2196	0.2610
	Second	0.1744	0.2204
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.0826	0.0614
	Second	0.0716	0.0534

Table B.11. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 16$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.8660	0.7842
	Second	0.6994	0.6068
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8524	0.9112
	Second	0.6780	0.7660
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8438	0.9124
	Second	0.6646	0.7702
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.7638	0.6598
	Second	0.5808	0.4870
(0.5 , **1.0** , 0.2, 0.0, 0.0)	**First**	0.9728	0.9796
	Second	0.8798	0.9014
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.5154	0.5888
	Second	0.3766	0.4252
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.5526	0.4492
	Second	0.3996	0.3248
(0.5 , **1.0** , 0.2, 0.2, 0.0)	**First**	0.9596	0.9690
	Second	0.8506	0.8736
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4180	0.5190
	Second	0.2958	0.3678
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.5340	0.4386
	Second	0.3892	0.3246
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.6222	0.5718
	Second	0.4496	0.4136
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.5622	0.5394
	Second	0.4044	0.3948
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.4228	0.5224
	Second	0.3060	0.3806
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9004	0.9134
	Second	0.7474	0.7628
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.5418	0.6506
	Second	0.3984	0.4764
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.8684	0.8550
	Second	0.7050	0.6854
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.6184	0.7022
	Second	0.4546	0.5264
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.5520	0.5580
	Second	0.4028	0.4060
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.4172	0.5170
	Second	0.3062	0.3796
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1238	0.0632
	Second	0.1124	0.0570

Table B.12. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9220	0.8552
	Second	0.7710	0.6816
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9108	0.9580
	Second	0.7630	0.8380
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9056	0.9582
	Second	0.7364	0.8398
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.8360	0.7530
	Second	0.6492	0.5664
(0.5 , **1.0** , 0.2, 0.0, 0.0)	**First**	0.9912	0.9942
	Second	0.9342	0.9466
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.5746	0.6892
	Second	0.4212	0.5154
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.6234	0.5190
	Second	0.4500	0.3806
(0.5 , **1.0** , 0.2, 0.2, 0.0)	**First**	0.9840	0.9886
	Second	0.9048	0.9228
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4940	0.5900
	Second	0.3562	0.4352
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.6356	0.5122
	Second	0.4630	0.3686
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.7074	0.6504
	Second	0.5186	0.4822
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.6150	0.6480
	Second	0.4530	0.4652
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.4944	0.5986
	Second	0.3500	0.4356
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9496	0.9578
	Second	0.8282	0.8366
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.6322	0.7138
	Second	0.4478	0.5294
(0.2 ,**0.7** , 0.2 , 0.0 , 0.0)	**First**	0.9158	0.9150
	Second	0.7672	0.7604
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.7102	0.7948
	Second	0.5264	0.6088
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.6224	0.6344
	Second	0.4516	0.4706
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.4966	0.5882
	Second	0.3474	0.4252
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1414	0.0620
	Second	0.1142	0.0572

Table B.13. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.3794	0.3228
	Second	0.3058	0.2512
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.3778	0.4210
	Second	0.2970	0.3440
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.3560	0.4278
	Second	0.2854	0.3428
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.3128	0.2576
	Second	0.2472	0.2062
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.5452	0.5560
	Second	0.4206	0.4564
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.2004	0.2356
	Second	0.1720	0.2086
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.2174	0.1786
	Second	0.1758	0.1540
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.5008	0.5230
	Second	0.4124	0.4188
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.1750	0.1956
	Second	0.1440	0.1600
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.2164	0.1854
	Second	0.1698	0.1530
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.2502	0.2302
	Second	0.2030	0.1814
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.2082	0.2190
	Second	0.1752	0.1794
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.1810	0.2040
	Second	0.1500	0.1726
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.4230	0.4314
	Second	0.3316	0.3496
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.2190	0.2568
	Second	0.1802	0.2116
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.3776	0.3826
	Second	0.3008	0.3016
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.2484	0.2906
	Second	0.1968	0.2250
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2210	0.2192
	Second	0.1726	0.1826
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.1786	0.2100
	Second	0.1520	0.1730
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.0748	0.0560
	Second	0.0686	0.0490

Table B.14. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 16$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.7216	0.6232
	Second	0.5500	0.4574
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6876	0.7820
	Second	0.5166	0.6082
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6842	0.7822
	Second	0.5204	0.6018
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.5914	0.5098
	Second	0.4378	0.3730
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.8870	0.9004
	Second	0.7340	0.7380
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.3872	0.4654
	Second	0.2746	0.3340
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.4054	0.3334
	Second	0.2900	0.2492
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.8680	0.8810
	Second	0.6966	0.7138
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3260	0.4016
	Second	0.2486	0.2878
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.4162	0.3356
	Second	0.3064	0.2462
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.4876	0.4236
	Second	0.3472	0.3024
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.4144	0.4154
	Second	0.2912	0.3058
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3052	0.3896
	Second	0.2242	0.2784
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.7696	0.7700
	Second	0.5950	0.6102
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.4188	0.4974
	Second	0.3098	0.3656
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.7046	0.6962
	Second	0.5326	0.5192
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.4852	0.5450
	Second	0.3424	0.3962
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4040	0.4100
	Second	0.2944	0.2982
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.3162	0.4054
	Second	0.2330	0.2918
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1042	0.0612
	Second	0.0910	0.0576

Table B.15. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.7910	0.7146
	Second	0.6062	0.5390
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.7948	0.8624
	Second	0.6106	0.6842
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.7798	0.8566
	Second	0.5818	0.6860
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.6860	0.5874
	Second	0.5052	0.4238
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9474	0.9526
	Second	0.8090	0.8180
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.4460	0.5336
	Second	0.3182	0.3842
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.4818	0.3944
	Second	0.3418	0.2772
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9206	0.9276
	Second	0.7652	0.7892
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3676	0.4650
	Second	0.2654	0.3258
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.4804	0.3886
	Second	0.3452	0.2794
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.5374	0.4946
	Second	0.3896	0.3568
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.4716	0.4770
	Second	0.3426	0.3546
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3676	0.4636
	Second	0.2654	0.3290
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.8380	0.8596
	Second	0.6598	0.6688
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.4926	0.5698
	Second	0.3520	0.4214
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.7908	0.7770
	Second	0.6202	0.5986
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.5516	0.6466
	Second	0.3902	0.4754
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4776	0.4954
	Second	0.3446	0.3456
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.3758	0.4598
	Second	0.2718	0.3224
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1078	0.0630
	Second	0.0928	0.0592

Table B.16. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 6$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.7858	0.6856
	Second	0.6570	0.5674
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.7660	0.8038
	Second	0.6244	0.6938
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.7476	0.8156
	Second	0.6198	0.6914
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.6852	0.5870
	Second	0.5538	0.4816
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9198	0.9016
	Second	0.8084	0.7892
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.4546	0.5184
	Second	0.3604	0.4200
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.4762	0.4040
	Second	0.3700	0.3176
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.8888	0.8858
	Second	0.7774	0.7716
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3870	0.4564
	Second	0.2916	0.3668
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.4432	0.3830
	Second	0.3302	0.3068
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.5226	0.4666
	Second	0.4006	0.3692
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.4490	0.4556
	Second	0.3452	0.3600
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.3202	0.4402
	Second	0.2544	0.3384
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.8466	0.8178
	Second	0.7264	0.6992
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.4836	0.5518
	Second	0.3818	0.4562
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.7868	0.7366
	Second	0.6514	0.6192
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.5306	0.6038
	Second	0.4206	0.4974
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4886	0.4902
	Second	0.3828	0.3888
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.3486	0.4226
	Second	0.2738	0.3426
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.0980	0.0550
	Second	0.0902	0.0540

Table B.17. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 16$.

Location Parameter	**Method**	**Non Modification**	**Modification**
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9914	0.9712
	Second	0.9410	0.8754
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9878	0.9958
	Second	0.9314	0.9552
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9860	0.9954
	Second	0.9214	0.9462
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.9772	0.9210
	Second	0.8796	0.7788
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9998	0.9996
	Second	0.9930	0.9866
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.8228	0.8818
	Second	0.6340	0.7176
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.8670	0.7482
	Second	0.6876	0.5692
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9992	0.9990
	Second	0.9846	0.9788
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.7276	0.8150
	Second	0.5500	0.6364
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.8492	0.7386
	Second	0.6668	0.5394
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.9042	0.8488
	Second	0.7266	0.6722
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.8428	0.8304
	Second	0.6622	0.6506
(0.5 ,**0.5** , 0.0 , 0.0 , 0.0)	**First**	0.6840	0.8044
	Second	0.4968	0.6264
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9976	0.9964
	Second	0.9676	0.9530
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.8314	0.9058
	Second	0.6706	0.7508
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.9934	0.9868
	Second	0.9452	0.9146
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.8878	0.9384
	Second	0.7218	0.8040
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.8714	0.8516
	Second	0.7064	0.6696
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.6676	0.7796
	Second	0.4990	0.6100
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1670	0.0708
	Second	0.1280	0.0640

Table B.18. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9984	0.9910
	Second	0.9712	0.9282
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9980	0.9982
	Second	0.9656	0.9732
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9962	0.9988
	Second	0.9542	0.9784
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.9916	0.9646
	Second	0.9278	0.8554
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	1.0000	1.0000
	Second	0.9984	0.9952
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.8826	0.9372
	Second	0.7126	0.7894
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.9184	0.8260
	Second	0.7636	0.6532
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	1.0000	0.9996
	Second	0.9948	0.9910
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.8204	0.8870
	Second	0.6198	0.7284
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.9150	0.8146
	Second	0.7466	0.6188
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.9570	0.9100
	Second	0.8278	0.7464
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.9140	0.8994
	Second	0.7414	0.7276
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.7758	0.8792
	Second	0.5784	0.7004
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9994	0.9992
	Second	0.9852	0.9796
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.9090	0.9464
	Second	0.7456	0.8184
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.9994	0.9976
	Second	0.9792	0.9542
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.9372	0.9712
	Second	0.7886	0.8708
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.9286	0.9076
	Second	0.7838	0.7532
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.7526	0.8544
	Second	0.5646	0.6786
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1802	0.0794
	Second	0.1358	0.0686

The sample size of all treatments for (CRD) portion is *half* the number of blocks for (RCBD) portion:

Table B.19. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.6348	0.5520
	Second	0.4822	0.4186
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6180	0.7086
	Second	0.4606	0.5460
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.6060	0.7094
	Second	0.4528	0.5392
(0.0 , **0.7** , 0.4 , 0.2, 0.2)	**First**	0.5290	0.4354
	Second	0.3988	0.3256
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.8196	0.8468
	Second	0.6564	0.6908
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.3284	0.3882
	Second	0.2484	0.2870
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.3556	0.2926
	Second	0.2796	0.2214
(0.5 , **1.0** , 0.2, 0.2, 0.0)	**First**	0.7980	0.8238
	Second	0.6358	0.6600
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2802	0.3340
	Second	0.2170	0.2582
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.3582	0.2968
	Second	0.2666	0.2236
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.4078	0.3594
	Second	0.3076	0.2702
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.3458	0.3624
	Second	0.2582	0.2728
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2736	0.3324
	Second	0.2112	0.2464
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.6942	0.7062
	Second	0.5376	0.5336
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.3610	0.4254
	Second	0.2736	0.3206
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.6152	0.6236
	Second	0.4724	0.4730
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.4096	0.4758
	Second	0.3088	0.3536
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.3584	0.3726
	Second	0.2692	0.2726
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.2690	0.3288
	Second	0.2100	0.2474
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.0970	0.0584
	Second	0.0822	0.0590

Table B.20. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 32$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9500	0.9000
	Second	0.7356	0.6594
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9366	0.9732
	Second	0.7086	0.8076
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9424	0.9680
	Second	0.7168	0.8000
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.8702	0.7912
	Second	0.6114	0.5244
(0.5 , **1.0** , 0.2, 0.0, 0.0)	**First**	0.9956	0.9978
	Second	0.9082	0.9192
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.6288	0.7344
	Second	0.3910	0.4816
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.6902	0.5664
	Second	0.4376	0.3438
(0.5 , **1.0** , 0.2, 0.2, 0.0)	**First**	0.9938	0.9948
	Second	0.8854	0.8970
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.5394	0.6374
	Second	0.3294	0.3976
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.6804	0.5600
	Second	0.4214	0.3422
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.7556	0.7062
	Second	0.4978	0.4450
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.6658	0.6792
	Second	0.4316	0.4332
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.5400	0.6490
	Second	0.3400	0.4114
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9698	0.9768
	Second	0.7776	0.8014
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.6800	0.7778
	Second	0.4438	0.5072
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.9476	0.9438
	Second	0.7412	0.7316
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.7612	0.8276
	Second	0.4962	0.5682
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.6698	0.6866
	Second	0.4280	0.4348
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.5340	0.6554
	Second	0.3162	0.4194
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1412	0.0708
	Second	0.1160	0.0626

Table B.21. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 40$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9814	0.9466
	Second	0.8106	0.7188
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9736	0.9926
	Second	0.7974	0.8742
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9702	0.9908
	Second	0.7796	0.8596
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.9396	0.8652
	Second	0.6950	0.5974
(0.5 , **1.0** , 0.2, 0.0, 0.0)	**First**	0.9992	0.9998
	Second	0.9488	0.9534
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.7212	0.8220
	Second	0.4556	0.5472
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.7598	0.6350
	Second	0.4800	0.4048
(0.5 , **1.0** , 0.2, 0.2, 0.0)	**First**	0.9976	0.9994
	Second	0.9288	0.9430
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.6208	0.7320
	Second	0.3750	0.4634
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.7584	0.6490
	Second	0.4922	0.3976
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.8358	0.7836
	Second	0.5548	0.5046
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.7566	0.7626
	Second	0.4864	0.4876
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0. 6112	0.7460
	Second	0.3842	0.4688
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9926	0.9906
	Second	0.8532	0.8610
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.7622	0.8452
	Second	0.5020	0.5672
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.9826	0.9792
	Second	0.8122	0.7984
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.8384	0.8988
	Second	0.5680	0.6388
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.7592	0.7576
	Second	0.4850	0.4820
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.6200	0.7246
	Second	0.3840	0.4570
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1592	0.0650
	Second	0.1128	0.0600

Table B.22. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.4934	0.4296
	Second	0.3634	0.3142
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.4740	0.5648
	Second	0.3604	0.4214
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.4504	0.5446
	Second	0.3300	0.4030
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.4024	0.3372
	Second	0.3096	0.2546
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.6646	0.6922
	Second	0.5112	0.5298
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.2448	0.3022
	Second	0.1952	0.2300
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.2856	0.2132
	Second	0.2240	0.1682
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.6416	0.6410
	Second	0.4888	0.4828
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2218	0.2660
	Second	0.1782	0.2066
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.2650	0.2278
	Second	0.2082	0.1790
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.3062	0.2904
	Second	0.2358	0.2192
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.2692	0.2626
	Second	0.2178	0.2148
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.2044	0.2548
	Second	0.1710	0.2004
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.5428	0.5492
	Second	0.4100	0.4188
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.2704	0.3154
	Second	0.2038	0.2394
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.4854	0.4718
	Second	0.3626	0.3472
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.3252	0.3718
	Second	0.2496	0.2756
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.2662	0.2848
	Second	0.2096	0.2202
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.2170	0.2526
	Second	0.1806	0.1996
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.0850	0.0586
	Second	0.0826	0.0572

Table B.23. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 32$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.8348	0.7544
	Second	0.5786	0.4946
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8292	0.9030
	Second	0.5666	0.6516
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8154	0.8894
	Second	0.5480	0.6522
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.7284	0.6352
	Second	0.4730	0.4006
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9626	0.9732
	Second	0.7708	0.7946
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.4916	0.5770
	Second	0.3102	0.3564
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.5316	0.4240
	Second	0.3370	0.2638
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9472	0.9560
	Second	0.7348	0.7508
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4066	0.5074
	Second	0.2432	0.3062
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.5296	0.4262
	Second	0.3220	0.2602
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.6058	0.5488
	Second	0.3700	0.3378
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.5232	0.5302
	Second	0.3340	0.3262
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.4012	0.4888
	Second	0.2488	0.2994
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.8878	0.8930
	Second	0.6298	0.6434
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.5220	0.6192
	Second	0.3192	0.3880
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.8392	0.8252
	Second	0.5836	0.5566
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.5984	0.6824
	Second	0.3714	0.4338
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.5204	0.5360
	Second	0.3136	0.3314
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.4100	0.4932
	Second	0.2528	0.3098
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1162	0.0628
	Second	0.0906	0.0614

Table B.24. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degree of Freedom for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 40$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9102	0.8414
	Second	0.6512	0.5692
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8982	0.9444
	Second	0.6350	0.7106
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8926	0.9482
	Second	0.6226	0.7178
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.8284	0.7262
	Second	0.5416	0.4702
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9858	0.9886
	Second	0.8446	0.8492
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.5598	0.6654
	Second	0.3414	0.4100
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.6192	0.5086
	Second	0.3728	0.3140
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9780	0.9850
	Second	0.8072	0.8210
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4616	0.5808
	Second	0.2858	0.3576
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.6038	0.4920
	Second	0.3642	0.2960
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.6818	0.6268
	Second	0.4170	0.3864
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.5956	0.6128
	Second	0.3534	0.3748
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.4814	0.5724
	Second	0.2848	0.3364
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9316	0.9420
	Second	0.6974	0.7104
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.6064	0.7034
	Second	0.3636	0.4396
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.8978	0.8932
	Second	0.6296	0.6442
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.6910	0.7722
	Second	0.4350	0.5062
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.6112	0.6170
	Second	0.3830	0.3784
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.4732	0.5782
	Second	0.2820	0.3422
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1298	0.0680
	Second	0.0930	0.0664

Table B.25. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 6$ and The Number of Blocks for The RCBD Portion is $b = 12$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9008	0.8202
	Second	0.7688	0.6542
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8890	0.9246
	Second	0.7376	0.7860
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.8778	0.9198
	Second	0.7292	0.7814
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.8230	0.7186
	Second	0.6576	0.5614
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	0.9792	0.9654
	Second	0.9042	0.8680
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.5660	0.6406
	Second	0.4242	0.4778
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.6160	0.5062
	Second	0.4646	0.3870
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	0.9662	0.9564
	Second	0.8662	0.8540
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.4886	0.5764
	Second	0.3608	0.4412
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.5900	0.4910
	Second	0.4348	0.3518
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.6678	0.6146
	Second	0.4904	0.4534
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.5982	0.5854
	Second	0.4338	0.4432
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.4308	0.5438
	Second	0.3134	0.4044
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	0.9418	0.9162
	Second	0.8206	0.7888
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.6084	0.6798
	Second	0.4598	0.5220
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.9056	0.8640
	Second	0.7546	0.7086
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.6502	0.7506
	Second	0.4886	0.5808
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.6348	0.6168
	Second	0.4776	0.4624
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.4476	0.5494
	Second	0.3354	0.4084
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1076	0.0616
	Second	0.1004	0.0574

Table B.26. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 32$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	0.9990	0.9948
	Second	0.9630	0.8974
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9988	1.0000
	Second	0.9534	0.9662
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	0.9982	0.9998
	Second	0.9404	0.9638
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.9956	0.9788
	Second	0.9112	0.8148
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	1.0000	1.0000
	Second	0.9960	0.9896
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.9244	0.9556
	Second	0.6804	0.7498
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.9528	0.8680
	Second	0.7314	0.6198
(0.5 , **1.0** , 0.2 , 0.2 , 0.0)	**First**	1.0000	0.9998
	Second	0.9924	0.9870
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.8558	0.9232
	Second	0.5854	0.6784
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.9474	0.8584
	Second	0.7160	0.5916
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.9746	0.9448
	Second	0.7854	0.7160
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.9402	0.9322
	Second	0.6996	0.6966
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.8374	0.9090
	Second	0.5528	0.6624
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	1.0000	0.9996
	Second	0.9790	0.9684
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.9330	0.9692
	Second	0.7060	0.7926
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	0.9998	0.9978
	Second	0.9618	0.9306
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.9578	0.9838
	Second	0.7668	0.8330
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.9540	0.9394
	Second	0.7442	0.7206
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.7972	0.8904
	Second	0.5332	0.6534
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.1928	0.0682
	Second	0.1290	0.0648

Table B.27. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 2; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 40$.

Location Parameter	Method	Non Modification	Modification
(0.0 , **0.8** , 0.6 , 0.4 , 0.2)	**First**	1.0000	0.9990
	Second	0.9804	0.9422
(0.75 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	1.0000	1.0000
	Second	0.9750	0.9816
(0.8 , **0.8** , 0.5 , 0.2 , 0.0)	**First**	1.0000	1.0000
	Second	0.9664	0.9830
(0.0 , **0.7** , 0.4, 0.2, 0.2)	**First**	0.9998	0.9916
	Second	0.9498	0.8768
(0.5 , **1.0** , 0.2 , 0.0 , 0.0)	**First**	1.0000	1.0000
	Second	0.9992	0.9968
(0.5 , **0.5** , 0.2 , 0.0 , 0.0)	**First**	0.9622	0.9856
	Second	0.7586	0.8142
(0.0 , **0.6** , 0.4, 0.4, 0.2)	**First**	0.9812	0.9318
	Second	0.7978	0.6958
(0.5 , 1 , 0.2 , 0.2 , 0.0)	**First**	1.0000	1.0000
	Second	0.9972	0.9966
(0.5 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.9240	0.9616
	Second	0.6700	0.7590
(0.0 , **0.6** , 0.2 , 0.2, 0.2)	**First**	0.9762	0.9206
	Second	0.7812	0.6584
(0.0 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.9910	0.9780
	Second	0.8558	0.7862
(0.2 , **0.5** , 0.0, 0.0, 0.0)	**First**	0.9778	0.9668
	Second	0.7864	0.7672
(0.5 , **0.5** , 0.0 , 0.0 , 0.0)	**First**	0.8940	0.9566
	Second	0.6104	0.7326
(0.4 , **0.8** , 0.4 , 0.2 , 0.0)	**First**	1.0000	1.0000
	Second	0.9910	0.9864
(0.5 , **0.5** , 0.5 , 0.2 , 0.0)	**First**	0.9718	0.9880
	Second	0.7750	0.8488
(0.2 , **0.7** , 0.2 , 0.0 , 0.0)	**First**	1.0000	0.9998
	Second	0.9840	0.9622
(0.5 , **0.5** , 0.5 , 0.0 , 0.0)	**First**	0.9818	0.9964
	Second	0.8276	0.8972
(0.2 , **0.5** , 0.2 , 0.2 , 0.0)	**First**	0.9844	0.9762
	Second	0.8074	0.8024
(0.5 , **0.5** , 0.5 , 0.5 , 0.0)	**First**	0.8626	0.9406
	Second	0.5926	0.7182
(0.0 , **0.5** , 0.5 , 0.5 , 0.5)	**First**	0.2226	0.0714
	Second	0.1424	0.0658

APPENDIX C. FIVE TREATMENTS AT PEAK 3

The sample size of all treatments for (CRD) portion is *twice* the number of blocks for (RCBD) portion:

Table C.1. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 8$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.5720	0.5742
	Second	0.4952	0.4876
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.4186	0.4192
	Second	0.3524	0.3556
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.4200	0.4284
	Second	0.3508	0.3672
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.5746	0.5676
	Second	0.4976	0.4792
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.8156	0.8256
	Second	0.7432	0.7358
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.7138	0.7050
	Second	0.6328	0.6114
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.7078	0.7086
	Second	0.6234	0.6150
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.7590	0.7638
	Second	0.6802	0.6734
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.5674	0.5764
	Second	0.4948	0.4930
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5772	0.5570
	Second	0.4958	0.4880
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4338	0.4156
	Second	0.3706	0.3582
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.8196	0.8118
	Second	0.7128	0.7206
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5590	0.5710
	Second	0.4960	0.4990
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.4232	0.4234
	Second	0.3620	0.3718
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2194	0.2286
	Second	0.1976	0.2054
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2274	0.2314
	Second	0.1956	0.2034
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2108	0.2300
	Second	0.1886	0.1956
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2166	0.2164
	Second	0.2016	0.1914
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2298	0.2242
	Second	0.2002	0.2060
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2280	0.2332
	Second	0.1902	0.1932

Table C.2. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.6496	0.6374
	Second	0.5704	0.5584
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.5094	0.4890
	Second	0.4218	0.4178
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.4890	0.4824
	Second	0.4144	0.4084
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.6500	0.6562
	Second	0.5538	0.5696
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.8884	0.8888
	Second	0.8204	0.8154
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.7904	0.7918
	Second	0.6922	0.6938
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.7926	0.7908
	Second	0.7080	0.6994
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.8478	0.8392
	Second	0.7638	0.7492
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.6418	0.6486
	Second	0.5470	0.5576
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6520	0.6596
	Second	0.5674	0.5730
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4920	0.4932
	Second	0.4186	0.4230
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.8842	0.8862
	Second	0.7962	0.8200
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.6482	0.6474
	Second	0.5662	0.5632
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.4942	0.4870
	Second	0.4234	0.4294
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2572	0.2540
	Second	0.2212	0.2148
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2644	0.2614
	Second	0.2366	0.2274
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2650	0.2552
	Second	0.2158	0.2198
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2540	0.2672
	Second	0.2234	0.2276
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2670	0.2568
	Second	0.2318	0.2194
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2570	0.2538
	Second	0.2216	0.2248

Table C.3. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 8$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.4370	0.4342
	Second	0.3738	0.3626
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.3154	0.3240
	Second	0.2792	0.2726
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.3276	0.3164
	Second	0.2804	0.2732
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.4404	0.4316
	Second	0.3720	0.3692
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.6576	0.6660
	Second	0.5826	0.5782
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.5490	0.5470
	Second	0.4668	0.4578
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.5572	0.5570
	Second	0.4762	0.4688
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.5970	0.5910
	Second	0.5166	0.5108
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4336	0.4284
	Second	0.3674	0.3692
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4222	0.4258
	Second	0.3686	0.3750
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3150	0.3356
	Second	0.2676	0.2816
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.6432	0.6524
	Second	0.5548	0.5650
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4308	0.4424
	Second	0.3644	0.3730
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.3272	0.3114
	Second	0.2738	0.2758
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.1828	0.1732
	Second	0.1640	0.1644
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1840	0.1748
	Second	0.1656	0.1516
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.1706	0.1814
	Second	0.1614	0.1614
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.1754	0.1836
	Second	0.1562	0.1572
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.1856	0.1804
	Second	0.1646	0.1584
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1818	0.1740
	Second	0.1572	0.1584

Table C.4. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degree of Freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.4938	0.4980
	Second	0.4344	0.4192
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.3708	0.3776
	Second	0.3148	0.3178
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.3830	0.3790
	Second	0.3282	0.3228
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.5052	0.5120
	Second	0.4370	0.4374
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.7466	0.7510
	Second	0.6494	0.6674
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.6286	0.6324
	Second	0.5490	0.5444
(0.2 , 0.4 , **0.7** , 0.4, 0.0)	**First**	0.6274	0.6390
	Second	0.5452	0.5506
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.6872	0.6838
	Second	0.5912	0.5896
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4986	0.4992
	Second	0.4270	0.4188
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5036	0.4978
	Second	0.4192	0.4132
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3700	0.3774
	Second	0.3230	0.3144
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.7476	0.7480
	Second	0.6586	0.6392
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5042	0.5064
	Second	0.4178	0.4216
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.3696	0.3796
	Second	0.3136	0.3168
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2048	0.1922
	Second	0.1814	0.1774
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.1910	0.2034
	Second	0.1780	0.1778
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2068	0.1970
	Second	0.1764	0.1738
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2044	0.2078
	Second	0.1774	0.1776
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2090	0.2152
	Second	0.1816	0.1758
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.1968	0.2164
	Second	0.1712	0.1828

Table C.5. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 8$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.8718	0.8704
	Second	0.7928	0.7820
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.7218	0.7164
	Second	0.6178	0.6238
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.7186	0.7284
	Second	0.6104	0.6318
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.8932	0.8780
	Second	0.8118	0.7954
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9866	0.9826
	Second	0.9612	0.9488
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9506	0.9382
	Second	0.9074	0.8854
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.9582	0.9454
	Second	0.9018	0.8958
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9708	0.9648
	Second	0.9356	0.9252
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.8614	0.8596
	Second	0.7702	0.7832
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.8650	0.8630
	Second	0.7854	0.7802
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.7206	0.7082
	Second	0.6214	0.6178
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9730	0.9730
	Second	0.9292	0.9432
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.8288	0.8570
	Second	0.7416	0.7762
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.6968	0.7072
	Second	0.6058	0.6270
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.3600	0.3644
	Second	0.2990	0.3086
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3536	0.3522
	Second	0.3032	0.2940
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3360	0.3520
	Second	0.2706	0.3020
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.3462	0.3496
	Second	0.2818	0.2912
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3856	0.3636
	Second	0.3216	0.3072
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.3704	0.3564
	Second	0.3206	0.3054

Table C.6. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 10$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.9308	0.9328
	Second	0.8634	0.8650
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.8138	0.7960
	Second	0.7102	0.7030
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.8100	0.8046
	Second	0.7158	0.7188
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.9422	0.9362
	Second	0.8836	0.8794
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9960	0.9928
	Second	0.9838	0.9800
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9814	0.9790
	Second	0.9482	0.9450
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.9814	0.9754
	Second	0.9546	0.9368
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9902	0.9886
	Second	0.9694	0.9594
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.9100	0.9148
	Second	0.8346	0.8446
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.9224	0.9248
	Second	0.8544	0.8536
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.8150	0.8026
	Second	0.7160	0.7172
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9922	0.9936
	Second	0.9702	0.9722
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.8990	0.9162
	Second	0.8220	0.8394
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.7724	0.8030
	Second	0.6852	0.7066
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.4234	0.4082
	Second	0.3510	0.3402
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4226	0.4144
	Second	0.3444	0.3430
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4056	0.4228
	Second	0.3280	0.3474
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.4154	0.4144
	Second	0.3296	0.3388
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4420	0.4292
	Second	0.3766	0.3528
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.4452	0.4220
	Second	0.3776	0.3476

The sample size of all treatments for (CRD) portion is *equal* the number of blocks for (RCBD) portion:

Table C.7. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 16$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.6748	0.6724
	Second	0.5208	0.5142
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.5074	0.5244
	Second	0.3548	0.3710
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.5252	0.5392
	Second	0.3812	0.3962
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.6828	0.6840
	Second	0.5200	0.5040
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9034	0.9052
	Second	0.7516	0.7564
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.8148	0.8052
	Second	0.6302	0.6432
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.8166	0.8148
	Second	0.6458	0.6378
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.8620	0.8638
	Second	0.7022	0.6952
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.6764	0.6874
	Second	0.4902	0.5150
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6760	0.6804
	Second	0.5048	0.5124
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5198	0.5352
	Second	0.3770	0.3850
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9000	0.9082
	Second	0.7476	0.7458
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.6746	0.6858
	Second	0.5150	0.5098
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.5034	0.5232
	Second	0.3624	0.3870
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2880	0.2642
	Second	0.2116	0.2058
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2784	0.2754
	Second	0.2100	0.2016
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2726	0.2710
	Second	0.2018	0.2070
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2858	0.2852
	Second	0.2068	0.2192
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2686	0.2724
	Second	0.2054	0.1984
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2882	0.2778
	Second	0.2192	0.2094

Table C.8. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.7630	0.7558
	Second	0.5940	0.5620
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.5866	0.5854
	Second	0.4256	0.4272
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.5906	0.6018
	Second	0.4226	0.4360
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.7712	0.7726
	Second	0.5814	0.5856
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9562	0.9530
	Second	0.8356	0.8188
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.8778	0.8868
	Second	0.7172	0.7142
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.8924	0.8866
	Second	0.7208	0.7176
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9270	0.9196
	Second	0.7812	0.7670
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.7676	0.7614
	Second	0.5880	0.5898
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.7698	0.7642
	Second	0.5788	0.5854
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5940	0.5960
	Second	0.4296	0.4246
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9450	0.9594
	Second	0.8122	0.8284
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.7588	0.7744
	Second	0.5880	0.5816
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.5878	0.5972
	Second	0.4260	0.4264
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.3176	0.3124
	Second	0.2314	0.2166
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3210	0.3156
	Second	0.2462	0.2274
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3100	0.3066
	Second	0.2248	0.2196
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.3196	0.3224
	Second	0.2356	0.2212
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3226	0.3356
	Second	0.2378	0.2386
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.3084	0.3218
	Second	0.2284	0.2252

Table C.9. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 16$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.5292	0.5304
	Second	0.3770	0.3846
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.3882	0.3864
	Second	0.2938	0.2776
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.3836	0.3958
	Second	0.2886	0.2890
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.5322	0.5366
	Second	0.3872	0.3918
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.7694	0.7698
	Second	0.6036	0.5980
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.6516	0.6568
	Second	0.4888	0.4934
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.6696	0.6568
	Second	0.4926	0.4794
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.7154	0.7308
	Second	0.5390	0.5506
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.5236	0.5294
	Second	0.3824	0.3838
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5380	0.5274
	Second	0.3900	0.3914
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3944	0.3982
	Second	0.2872	0.2842
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.7718	0.7728
	Second	0.5886	0.5974
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5358	0.5192
	Second	0.3754	0.3720
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.3944	0.3880
	Second	0.2804	0.2854
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2118	0.2180
	Second	0.1562	0.1686
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2042	0.2056
	Second	0.1632	0.1690
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2130	0.2172
	Second	0.1594	0.1612
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2182	0.2114
	Second	0.1656	0.1710
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2182	0.2124
	Second	0.1702	0.1604
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2100	0.2176
	Second	0.1620	0.1706

Table C.10. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degree of Freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.6002	0.6064
	Second	0.4424	0.4314
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.4550	0.4494
	Second	0.3284	0.3152
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.4594	0.4590
	Second	0.3270	0.3278
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.6132	0.6026
	Second	0.4418	0.4390
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.8464	0.8468
	Second	0.6640	0.6598
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.7422	0.7430
	Second	0.5710	0.5630
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.7526	0.7528
	Second	0.5642	0.5612
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.8120	0.7984
	Second	0.6276	0.6074
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.6090	0.6066
	Second	0.4386	0.4408
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6138	0.6014
	Second	0.4528	0.4462
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4586	0.4542
	Second	0.3240	0.3278
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.8334	0.8440
	Second	0.6504	0.6596
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.6120	0.6124
	Second	0.4488	0.4352
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.4506	0.4554
	Second	0.3226	0.3290
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2450	0.2448
	Second	0.1778	0.1828
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2508	0.2412
	Second	0.1786	0.1832
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2540	0.2454
	Second	0.1890	0.1808
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2414	0.2400
	Second	0.1842	0.1822
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2438	0.2366
	Second	0.1872	0.1784
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2418	0.2398
	Second	0.1846	0.1758

Table C.11. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 16$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.9388	0.9332
	Second	0.8100	0.8126
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.8356	0.8280
	Second	0.6494	0.6398
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.8354	0.8276
	Second	0.6412	0.6508
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.9532	0.9510
	Second	0.8274	0.8188
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9984	0.9968
	Second	0.9682	0.9656
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9874	0.9844
	Second	0.9176	0.9088
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.9902	0.9848
	Second	0.9216	0.9066
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9942	0.9924
	Second	0.9422	0.9390
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.9366	0.9430
	Second	0.7870	0.8054
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.9366	0.9386
	Second	0.8024	0.8040
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.8412	0.8358
	Second	0.6574	0.6548
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9944	0.9944
	Second	0.9436	0.9514
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.9138	0.9390
	Second	0.7694	0.7980
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.8112	0.8126
	Second	0.6340	0.6374
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.4342	0.4310
	Second	0.3156	0.3130
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.4452	0.4320
	Second	0.3160	0.3050
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4396	0.4134
	Second	0.3038	0.2918
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.4234	0.4320
	Second	0.2924	0.3000
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4550	0.4484
	Second	0.3192	0.3164
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.4670	0.4524
	Second	0.3272	0.3166

Table C.12. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 20$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.9806	0.9784
	Second	0.8888	0.8766
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.9048	0.8988
	Second	0.7278	0.7306
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.9064	0.8970
	Second	0.7300	0.7276
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.9802	0.9770
	Second	0.8978	0.8864
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9996	0.9996
	Second	0.9868	0.9802
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9978	0.9958
	Second	0.9612	0.9546
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.9972	0.9954
	Second	0.9602	0.9548
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9992	0.9978
	Second	0.9786	0.9704
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.9730	0.9772
	Second	0.8652	0.8746
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.9710	0.9702
	Second	0.8772	0.8692
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.9098	0.9026
	Second	0.7382	0.7296
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9982	0.9994
	Second	0.9714	0.9764
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.9574	0.9688
	Second	0.8382	0.8630
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.8686	0.8960
	Second	0.7030	0.7172
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.5140	0.5174
	Second	0.3652	0.3650
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5018	0.5142
	Second	0.3594	0.3588
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.5108	0.5132
	Second	0.3442	0.3540
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.5030	0.5144
	Second	0.3552	0.3538
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5490	0.5224
	Second	0.3840	0.3758
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.5470	0.5186
	Second	0.3836	0.3596

The sample size of all treatments for (CRD) portion is *half* the number of blocks for (RCBD) portion:

Table C.13. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 32$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.8122	0.8098
	Second	0.5446	0.5348
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.6454	0.6586
	Second	0.4074	0.4042
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.6418	0.6432
	Second	0.4020	0.4008
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.8128	0.8176
	Second	0.5520	0.5586
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9732	0.9764
	Second	0.7954	0.8048
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9192	0.9146
	Second	0.6720	0.6762
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.9134	0.9160
	Second	0.6802	0.6768
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9514	0.9526
	Second	0.7476	0.7388
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.8070	0.8160
	Second	0.5476	0.5464
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.8216	0.8048
	Second	0.5534	0.5372
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6554	0.6602
	Second	0.4070	0.4068
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9692	0.9686
	Second	0.7934	0.7882
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.8156	0.8088
	Second	0.5486	0.5342
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.6400	0.6386
	Second	0.4052	0.3978
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.3406	0.3394
	Second	0.2184	0.2152
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3516	0.3542
	Second	0.2180	0.2170
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3398	0.3286
	Second	0.2108	0.2074
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.3370	0.3514
	Second	0.2032	0.2184
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3506	0.3512
	Second	0.2224	0.2302
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.3446	0.3490
	Second	0.2120	0.2164

Table C.14. Estimated Powers for a Test Statistic of a Mixed Design Under The Normal Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 40$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.8868	0.8820
	Second	0.6164	0.6100
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.7362	0.7362
	Second	0.4674	0.4574
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.7244	0.7358
	Second	0.4514	0.4598
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.8864	0.8840
	Second	0.6152	0.6154
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9888	0.9920
	Second	0.8578	0.8598
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9580	0.9600
	Second	0.7528	0.7588
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.9658	0.9642
	Second	0.7474	0.7422
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9754	0.9820
	Second	0.8012	0.8090
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.8800	0.8842
	Second	0.6104	0.6118
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.8860	0.8884
	Second	0.6160	0.6194
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.7412	0.7348
	Second	0.4674	0.4580
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9856	0.9912
	Second	0.8502	0.8518
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.8756	0.8810
	Second	0.5930	0.6100
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.7462	0.7390
	Second	0.4666	0.4690
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.4044	0.3982
	Second	0.2308	0.2358
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3936	0.4056
	Second	0.2394	0.2446
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.4092	0.4034
	Second	0.2516	0.2300
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.4028	0.4086
	Second	0.2426	0.2462
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4022	0.4054
	Second	0.2414	0.2414
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.4092	0.4060
	Second	0.2550	0.2430

Table C.15. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degrees of Freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 32$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.6634	0.6694
	Second	0.4202	0.4172
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.5006	0.5004
	Second	0.3112	0.3004
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.5032	0.4952
	Second	0.3172	0.3084
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.6688	0.6560
	Second	0.4254	0.4176
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.8870	0.8842
	Second	0.6404	0.6396
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.7814	0.7946
	Second	0.5256	0.5160
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.7936	0.7832
	Second	0.5214	0.5150
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.8410	0.8380
	Second	0.5752	0.5802
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.6646	0.6538
	Second	0.4190	0.4154
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6440	0.6566
	Second	0.4052	0.4086
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.4926	0.5038
	Second	0.3126	0.3166
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.8808	0.8926
	Second	0.6150	0.6376
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.6614	0.6630
	Second	0.4228	0.4168
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.4960	0.4924
	Second	0.3090	0.3070
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2690	0.2664
	Second	0.1730	0.1664
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.2720	0.2686
	Second	0.1758	0.1878
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.2606	0.2650
	Second	0.1762	0.1776
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.2606	0.2634
	Second	0.1716	0.1694
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.2562	0.2708
	Second	0.1800	0.1696
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.2632	0.2640
	Second	0.1658	0.1730

Table C.16. Estimated Powers for a Test Statistic of a Mixed Design Under The Student's t Distribution with 3 Degree of Freedom for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 40$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.7516	0.7424
	Second	0.4740	0.4638
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.5616	0.5854
	Second	0.3400	0.3590
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.5660	0.5820
	Second	0.3468	0.3452
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.7468	0.7506
	Second	0.4794	0.4848
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	0.9406	0.9398
	Second	0.7044	0.7068
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.8660	0.8656
	Second	0.6046	0.5988
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.8730	0.8702
	Second	0.5874	0.5984
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9092	0.9114
	Second	0.6536	0.6520
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.7380	0.7346
	Second	0.4598	0.4582
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.7502	0.7460
	Second	0.4804	0.4822
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.5828	0.5732
	Second	0.3456	0.3460
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9420	0.9406
	Second	0.6994	0.6932
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.7424	0.7390
	Second	0.4674	0.4792
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.5752	0.5754
	Second	0.3468	0.3602
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.2954	0.2966
	Second	0.1754	0.1812
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.3010	0.2986
	Second	0.1940	0.1890
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.3072	0.3154
	Second	0.1884	0.1880
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.3064	0.3036
	Second	0.1866	0.1846
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.3072	0.3114
	Second	0.1942	0.1908
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.3044	0.3004
	Second	0.1960	0.1868

Table C.17. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 16$ and The Number of Blocks for The RCBD Portion is $b = 32$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.9874	0.9880
	Second	0.8372	0.8542
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.9368	0.9360
	Second	0.6952	0.6928
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.9408	0.9296
	Second	0.6940	0.6880
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.9894	0.9896
	Second	0.8718	0.8534
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	1.0000	1.0000
	Second	0.9806	0.9762
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9992	0.9980
	Second	0.9440	0.9318
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.9988	0.9990
	Second	0.9370	0.9346
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	0.9998	0.9994
	Second	0.9680	0.9606
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.9878	0.9868
	Second	0.8288	0.8360
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.9868	0.9894
	Second	0.8392	0.8474
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.9322	0.9332
	Second	0.6978	0.6974
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	0.9996	0.9998
	Second	0.9656	0.9662
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.9756	0.9812
	Second	0.8188	0.8256
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.9058	0.9168
	Second	0.6646	0.6710
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.5570	0.5554
	Second	0.3440	0.3266
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.5606	0.5646
	Second	0.3394	0.3384
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.5702	0.5564
	Second	0.3424	0.3234
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.5506	0.5548
	Second	0.3156	0.3330
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6040	0.5844
	Second	0.3582	0.3506
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.6088	0.5812
	Second	0.3594	0.3448

Table C.18. Estimated Powers for a Test Statistic of a Mixed Design Under The Exponential Distribution for 5 Treatments at Peak 3; The Sample Size for The CRD Portion is $n = 20$ and The Number of Blocks for The RCBD Portion is $b = 40$.

Location Parameter	Method	Non Modification	Modification
(0.0 , 0.0 , **0.5** , 0.0 , 0.0)	**First**	0.9976	0.9968
	Second	0.9080	0.8948
(0.0 , 0.0 , **0.5** , 0.2 , 0.2)	**First**	0.9766	0.9746
	Second	0.7726	0.7744
(0.2 , 0.2 , **0.5** , 0.0 , 0.0)	**First**	0.9678	0.9728
	Second	0.7572	0.7644
(0.0 , 0.2 , **0.5** , 0.2 , 0.0)	**First**	0.9988	0.9966
	Second	0.9234	0.9110
(0.0 , 0.4 , **0.7** , 0.2 , 0.0)	**First**	1.0000	1.0000
	Second	0.9932	0.9914
(0.0 , 0.4 , **0.7** , 0.4 , 0.2)	**First**	0.9998	0.9998
	Second	0.9774	0.9688
(0.2 , 0.4 , **0.7** , 0.4 , 0.0)	**First**	0.9996	1.0000
	Second	0.9736	0.9688
(0.0 , 0.2 , **0.8** , 0.5 , 0.3)	**First**	1.0000	0.9998
	Second	0.9852	0.9788
(0.0 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.9936	0.9956
	Second	0.8866	0.8956
(0.0 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.9972	0.9970
	Second	0.9020	0.9028
(0.2 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.9694	0.9722
	Second	0.7754	0.7756
(0.0 , 0.8 , **0.8** , 0.5 , 0.2)	**First**	1.0000	1.0000
	Second	0.9814	0.9836
(0.0 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.9912	0.9952
	Second	0.8722	0.8864
(0.0 , 0.5 , **0.5** , 0.5 , 0.2)	**First**	0.9520	0.9650
	Second	0.7312	0.7552
(0.0 , 0.5 , **0.5** , 0.5 , 0.5)	**First**	0.6528	0.6480
	Second	0.3902	0.3862
(0.5 , 0.5 , **0.5** , 0.5 , 0.0)	**First**	0.6388	0.6318
	Second	0.3846	0.3752
(0.5 , 0.5 , **0.5** , 0.0 , 0.0)	**First**	0.6528	0.6532
	Second	0.3782	0.3810
(0.0 , 0.0 , **0.5** , 0.5 , 0.5)	**First**	0.6510	0.6510
	Second	0.3776	0.3852
(0.5 , 0.5 , **0.5** , 0.2 , 0.0)	**First**	0.6858	0.6592
	Second	0.4130	0.3870
(0.0 , 0.2 , **0.5** , 0.5 , 0.5)	**First**	0.6952	0.6646
	Second	0.4054	0.3912