NONPARAMETRIC TESTS FOR THE UMBRELLA ALTERNATIVE IN A MIXED DESIGN FOR A KNOWN PEAK

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Nonparametric Test for the Umbrella Alternative in a Mixed Design for a Known Peak

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ABSTRACT

When an assumption from a parametric test cannot be verified, a nonparametric test provides a simple way of conducting a test on populations. The motivation behind conducting a test of the hypothesis is to examine the effect of a treatment or multiple treatments against one another.

For example, in dose-response studies, monkeys are assigned to k groups corresponding to k doses of an experimental drug. The effect of the drug on these monkeys is likely to increase or decrease with increasing and decreasing doses. The drug's effect on these monkeys may be an increasing function of dosage to a certain level, and then its effect decreases with further increasing doses. An umbrella alternative, in this case, is considered the most appropriate hypothesis for these kinds of studies.

Tests statistics are proposed to test for the umbrella alternative in mixed designs consisting of combinations of a Completely Randomized Design (CRD), a Randomized Complete Block Design (RCBD), an Incomplete Block Design (IBD) and a Balanced Incomplete Block Design (BIBD). Powers obtained were based on a variety of cases. Different proportions of blocks to different sample sizes of a Completely Randomized Design portion were considered.

In all treatments, equal sample sizes for the Completely Randomized Design were considered. Furthermore, an equal number of blocks of a randomized complete block design to an Incomplete Block Design and Balanced Incomplete blocks were considered.

Studies in a Monte Carlo simulation were conducted using SAS to vary the design and to estimate the test statistic powers to each other. The underlying distributions considered were normal, t and exponential.

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CHAPTER 1. INTRODUCTION

There are several hypotheses a researcher may want to consider when conducting a proper statistical procedure. One such scenario is where the set of hypotheses has an alternative known as the umbrella alternative. When considering multiple treatments, the most inherent scenario is testing the null hypothesis that the effects of the different treatments are all equivalent, with an alternative that the immensity of treatment effects is non-decreasing until reaching a specific treatment known as the peak and is non-increasing for all treatments afterward.

For example, in dose-response studies, animals are assigned to k groups corresponding to k doses of an experimental drug. The effect of the drug on animals is likely to increase or decrease with increasing doses. In this scenario, an ordered alternative model is appropriate. However, in some such studies, the drug's effect on the animals might be an increasing function of dosage to a certain level and then its effect decreases with further increasing doses. In such a scenario, the umbrella alternative is the most appropriate model. The test of hypotheses for the umbrella alternative is given by

1.1. Null Hypothesis

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \text{ treatments effects assumed to be equal} \tag{1.1}$$

Versus

1.2. Alternative Hypothesis

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \cdots \geq \mu_k \text{ with at least one strict inequality} \tag{1.2}$$

where p is the peak of the umbrella and $\mu_1, \ldots, \mu_k$ are the location parameters of the i^{th} sample.

Mack – Wolfe (1981) were the first researchers to propose nonparametric tests for umbrella alternatives based on simple random samples. The Mack – Wolfe (1981) test uses Mann – Whitney (1947) counts for its test statistic. Mack - Wolfe (1981) is an extension of the Jonckheere -Terpstra test to test for a nondecreasing alternative.

A researcher at times may wish to test for umbrella alternatives when a blocking factor is introduced. The researcher may use a Randomized Complete Block Design (RCBD) instead of a Completely Randomized Design (CRD). For example, in testing the effectiveness of a drug, a blocking factor could be the location and the experimental units are the patients. Blocking is a technique where homogeneous subjects are grouped forming a block and effects are compared within that block. Kim-Kim (1992) proposed a nonparametric procedure for testing umbrella alternatives for a randomized block design which is an extension of Mack-Wolfe (1981).

In performing an experiment and conducting a test of the hypothesis, the researcher needs to decide on the design structure of the test to be used. Some researchers normally may begin with one experimental design. While the experiment is being implemented, it is realistic for the researcher to expect that some conditions around the experiment may change. Though the researcher may have started with one experimental design, the design may change before the completion of the experiment due to a variety of reasons.

One such reasons is that a situation may arise when it may not be possible to continue the experiment using a full RCBD. When a researcher after a point realizes that the design is too expensive to continue or ends up with incomplete blocks due to participants dropping out, the researcher at this point may switch to a completely randomized design. In the example of the effectiveness of the drug above, the researcher has the option to switch from applying the drug in each location in the facility to randomly assigning drugs to patients. The data obtained will then be organized in what is known as a mixture of RCBD and CRD designs.

For example, we have five locations that have five patients each, as in the first part which formed the RCBD portion. In this part, we randomly and equally assign the treatments on the experimental units within each block. The second part formed the CRD portion. In this part, we randomly and equally assign the treatments on the experimental units. In this example, we have two patients for each drug. This is illustrated in the figure below. In Figure 1, P denotes Patient and X denotes no observation is taken.

	Drug1	Drug2	Drug3	Drug4	Drug5	
Location1	P	P	P	P	P	RCBD Portion No replication
Location2	P	P	P	P	P	
Location3	P	P	P	P	P	
Location4	P	P	P	P	P	
Location5	P	P	P	P	P	
Location6	P	X	X	X	X	CRD Portion
Location7	X	X	X	P	X	
Location8	X	P	X	X	X	
Location9	X	X	X	X	P	
Location10	X	X	P	X	X	
Location11	X	X	X	X	P	
Location12	P	X	X	X	X	
Location13	X	X	P	X	X	
Location14	X	X	X	P	X	
Location15	X	P	X	X	X	

Figure 1. Layout for RCBD and CRD.

Another scenario is where a researcher initially conducts an experiment intending to employ a full RCBD, but issues may arise while data are being collected. The cost of obtaining subjects may perhaps be higher than projected or the number of subjects responding to a questionnaire of the experiments may be lower than expected for certain blocks. It may be impractical for the researcher to continue using RCBD. The researcher has the option at this point to continue conducting the remainder of the experiment by implementing the BIBD so that every treatment does not need to be applied to all observations within each block. The data obtained will then be organized in what is known as a mixture of RCBD and BIBD designs. For example, we have five locations that have three patients each, as in the first part which formed the BIBD portion. In this part, we assign the same number of missing observations for each location (block). The second part formed the CRD portion. In this part, we randomly assign the drug to patients. In this example, we have two patients for each drug. This is illustrated in the figure below. In Figure 2, P denotes Patient and X denotes no observation is taken.

	Drug1	Drug2	Drug3	Drug4	Drug5	
Location1	P	X	P	X	P	BIBD Portion
Location2	X	P	P	P	X	
Location3	P	P	X	X	P	
Location4	X	X	P	P	P	
Location5	P	P	X	P	X	
Location6	P	X	X	X	X	CRD Portion
Location7	X	X	X	P	X	
Location8	X	P	X	X	X	
Location9	X	X	X	X	P	
Location10	X	X	P	X	X	
Location11	X	X	X	X	P	
Location12	P	X	X	X	X	
Location13	X	X	P	X	X	
Location14	X	X	X	P	X	
Location15	X	P	X	X	X	

Figure 2. Layout for BIBD and CRD.

In the example of the effectiveness of the drug, the researcher again may switch from applying the drug in each location in the facility to randomly assigning drugs to patients or may continue conducting the remainder of the experiment by implementing the BIBD so that every treatment does not need to be applied to all observations within each block. The data obtained will then be organized in what is known as a mixture of RCBD, BIBD and CRD designs. For example, we have five locations that have five patients each, as in the first part which formed the RCBD portion. In the second part, we have five locations that have three patients each, which formed the BIBD portion. The third part has two patients for each drug. In Figure 3 below, P denotes Patient and X denotes no observation is taken.

	Drug1	Drug2	Drug3	Drug4	Drug5	
Location1	P	P	P	P	P	RCBD Portion
Location2	P	P	P	P	P	
Location3	P	P	P	P	P	
Location4	P	P	P	P	P	No replication
Location5	P	P	P	P	P	
Location6	P	X	P	X	P	BIBD Portion
Location7	X	P	P	P	X	
Location8	P	P	X	X	P	
Location9	X	X	P	P	P	
Location10	P	P	X	P	X	
Location11	P	X	X	X	X	CRD Portion
Location12	X	X	X	P	X	
Location13	X	P	X	X	X	
Location14	X	X	X	X	P	
Location15	X	X	P	X	X	
Location16	X	X	X	X	P	
Location17	P	X	X	X	X	
Location18	X	X	P	X	X	
Location19	X	X	X	P	X	
Location20	X	P	X	X	X	

Figure 3. Layout for RCBD, BIBD and CRD.

Neuhauser et al. (1998) developed a test for ordered alternatives based on a differential weighting of the Mann-Whitney statistic in Jonckheere (1954) and Terpstra (1992)'s statistic in a CRD. They showed that their test performed better than Jonckheere (1954) and Terpstra's (1992)'s statistic. Motivated by the results of Neuhauser et al. (1998), Esra and Fikri (2016) expanded their work and developed a test for both known and unknown umbrella peaks in the CRD.

Mack -Wolfe (1981) developed a test for the unknown peak in the umbrella alternative as well as the known peak. Hettmansperger and Norton (1987) and Shi (1998) also developed tests for the unknown peak in the CRD. They estimated the peak to be at the population that maximizes their test statistics.

Dubnicka and Hettmansperger (2002) proposed a test in the case of a mixed two-sample design. Their test is a mixture of the independent sample Mann-Whitney test with paired observations. Magel, Tersptra, Jeff, Katrina and Canonizado (2010), Magel and Hemmer (2012), and Magel and Ndungu (2011) proposed tests statistics for the umbrella alternative when the peak is known for various types of mixed designs. These designs include mixtures of a Completely Randomized Design, Randomized Complete Block Design, and Balanced Incomplete Block Design; a mixture of Completely Randomized Design, Randomized Complete Block Design, and Incomplete Block Design. Some of the test statistics include weights proposed by Nehauser et al. (1998) and Esra and Fikri (2016).

In this paper, additional tests will be proposed for the umbrella alternative in various mixed designs consisting of combinations of a CRD, an RCBD, a BIBD, and an IBD. The tests will all be compared based on estimated powers under a variety of situations. Recommendations will be made.

CHAPTER 2. LITERATURE REVIEW

When an assumption from a parametric test cannot be verified, the nonparametric test provides a simple way of conducting a test on populations. The test assumes that the underlying distributions are of equal type and only differ in location. This chapter seeks to look at nonparametric test procedures for a variety of experimental designs and their alternative hypotheses. Experiment design is how the treatments will be applied. The treatment is a product or substance of interest to the researcher.

The test procedure is dependent on the experiment design and the alternative hypothesis. The general form of the hypothesis for the umbrella alternatives are given below:

2.1. Null Hypothesis

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \text{ treatments effects assumed to be equal} \tag{2.1}$$

Versus

2.2. Alternative Hypothesis

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \cdots \geq \mu_k \text{ with at least one strict inequality} \tag{2.2}$$

where p is the peak of the umbrella and $\mu_1, \ldots, \mu_k$ are the location parameters of the i^{th} sample. Below is a detailed look at several relevant nonparametric test statistics.

2.3. Completely Randomized Design (CRD)

In a CRD subjects are randomly assigned to treatments. Each subject is exposed to only one treatment. The number of subjects assigned to each treatment does not have to be the same for all treatments. The treatment effect is then the difference among the groups formed by the treatments. The subject is an individual or object to which the treatment is applied. The effect is the difference in the outcome attributed to the treatment.

Consider a case of dose-response studies, where monkeys are assigned to k groups corresponding to k doses of an experimental drug. The effect of the drug on these monkeys is likely to increase or decrease with increasing and decreasing doses. The drug effect on these monkeys may be an increasing function of dosage to a certain level, and then its effect decreases with further increasing doses. An umbrella alternative, in this case, is considered the most appropriate model for this test. The general forms of the hypothesis for the umbrella alternatives are given below:

2.3.1. Null Hypothesis

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \text{ treatments effects assumed to be equal} \qquad (2.3)$$

Versus

2.3.2. Alternative Hypothesis

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \cdots \geq \mu_k \text{ with at least one strict} \qquad (2.4)$$

inequality where p is the peak of the umbrella and $\mu_1, \dots, \mu_k$ are the location parameters of the i^{th} sample.

2.3.3. Mack – Wolfe (1981)

Mack and Wolfe were the first researchers to propose nonparametric tests for umbrella alternatives based on simple random samples. Two versions of the test were developed. One of the versions is for when the peak, p, is known, and the other version is for an unknown peak p. In this research, we will consider the peak known cases.

When the peak is known, the Mack-Wolfe test statistic A_p given in Eq. (2.5) is the sum of the Mann-Whitney counts which are on the left side of the peak U_{uv} and the reverse Mann-Whitney counts which are on the right side of the peak U_{uv} (the sum of Jonckheere – Terpstra test statistics given in Eq (2.13).

Therefore, the test statistic A_p, has the following form

$$A_p = \sum_{u=1}^{v-1} \sum_{v=2}^{p} U_{uv} + \sum_{u=1}^{v-1} \sum_{v=p+1}^{k} U_{vu} \tag{2.5}$$

at a significance level of α, H_0 is rejected if $A_p \geq A_{p,\alpha}$; otherwise it is not rejected. The Mack - Wolfe (1981) test is asymptotically standard normal as the sample size tends to infinity. The Mack-Wolfe test statistics A_p is no longer distribution free when the k samples implicate unequal variances. Under the H_0 that all population means are equal, the expected value $E(A_p)$ and variance of (A_p) are respectively given below.

$$E_0(A_p) = \frac{N_1^2 + N_2^2 - \sum_{i=1}^{t} n_i^2 - n_p^2}{4} \tag{2.6}$$

$$\begin{aligned} var_0(A_p) = \tfrac{1}{72}\{&2(N_1^2 + N_2^2) + 3(N_1^2 + N_2^2) - \textstyle\sum_{i=1}^{k} n_i^2(n_i + 3) - \\ &n_p(2n_p + 3)) + 12n_pN_1N_2 - 12n_p^2N\} \end{aligned} \tag{2.7}$$

N_1: total number of subjects to the left of the peak, inclusive

N_2: total number of subjects to the right of the peak, inclusive

n_p: number of subjects in the peak treatment.

Mack and Wolfe (1981) used the standardized test statistic A_p^* of the form

$$A_p^* = \frac{A_p - E(A_p)}{\sqrt{Var(A_p)}} \tag{2.8}$$

When H_0 is true, and the sample sizes becomes large, the distribution of A_p^* is an asymptotically standard normal distribution. At a significance level of α, H_0 is rejected if $A_p^* \geq z_\alpha$ where z_α is the critical value of the upper tail probability $(1 - \alpha)\%$ of the standard normal distribution; otherwise we do not reject it.

2.3.4. Jonckheere (1954) -Terpstra (1952)

Jonckheere (1954) and Terpstra (1952) were among the first to develop a distribution-free test for a non-decreasing ordered alternative for location parameters. Their test is appropriate for testing differences in more than two treatment effects. The Jonckheere - Terpstra test is to test for directional differences. Like the Mann – Whitney (1947) test, the JT is not rank based. The JT test statistic is based on a comparison of pairs of treatments using their corresponding values. The Jonckheere - Terpstra test is a special case of the umbrella alternative with known peak p.

The main purpose is to consider a procedure for testing the hypotheses

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \text{ treatments effects assumed to be equal} \tag{2.9}$$

against a prior ordered alternative

$$H_a: \mu_1 \leq \cdots \leq \mu_k \text{ with at least one strict inequality} \tag{2.10}$$

where $\mu_1, \ldots, \mu_k$ are the location parameters of the i^{th} sample. Here treatments are labeled so that they are in the expected order associated with the alternative H_a. Labeling must be done prior to data collection. All calculated $\frac{k(k-1)}{2}$ Mann-Whitney counts U_{uv} are summed up to compute the Jonckheere-Terpstra Statistic, JT, which is given by,

$$U_{uv} = \sum_{i=1}^{n_u} \sum_{j=1}^{n_v} \emptyset(X_{iu}, X_{jv}), 1 \leq u \leq v \leq k \tag{2.11}$$

where U_{uv} is the number sample u before v precede and

$$\emptyset(X_{iu}, X_{jv}) = \begin{cases} 1 & if\ X_{iu} < X_{jv} \\ 0 & otherwise \end{cases} \tag{2.12}$$

Jonckheere – Terpstra test statistic, JT, therefore is the sum of these the $\frac{k(k-1)}{2}$ Mann-Whitney counts which is given by

$$JT = \sum_{u=1}^{v-1} \sum_{v=2}^{k} U_{uv} \tag{2.13}$$

For a non-decreasing alternative, the null hypothesis is rejected for large values and vice versa.

Under $H_0: \mu_1 = \mu_2 = \cdots = \mu_k$, the expected value and variance of J are given by

$$E_0(J) = \frac{N^2 - \sum_{j=1}^{t} n_j^2}{4} \tag{2.14}$$

And

$$var_0(J) = \frac{N^2(2N+3) - \sum_{j=1}^{t} n_j^2(2n_j + 3)}{72} \tag{2.15}$$

The standardized JT is given by

$$Z_{JT} = \frac{JT - [(N^2 - \sum_{j=1}^{t} n_j^2)/4]}{\sqrt{[N^2(2N+3) - \sum_{j=1}^{t} n_j^2(2n_j + 3)]/72}} \tag{2.16}$$

Z_J has been shown to have an asymptotically standard normal distribution. The null hypothesis is rejected if $JT \geq J_{\propto}$

2.4. Randomized Complete Block Design (RCBD)

An RCBD introduces blocking to the CRD. Instead of randomly assigning subjects to treatments, an RCBD exposes each subject to all treatments. The order in which the treatment is exposed can be random though. Treatment effects are then measured within each block. An RCBD is sometimes expensive since there cannot be missing observations.

The Kim-Kim (1992) test was designed for the umbrella alternative in a Randomized Complete Block Design with cases of a known peak. The test was rank based and uses the same technique as in Friedman's (1940) and Page's (1963). Kim-Kim (1992) test statistics assume no interaction between blocks. Kim-Kim (1992) applied the Mack – Wolfe (1981) test in each block and then summed the counts. The test of the hypotheses is given by

2.4.1. Null Hypothesis

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \text{ treatment effects assumed to be equal} \tag{2.17}$$

Versus

2.4.2. Alternative Hypothesis

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \cdots \geq \mu_k \text{ with at least one strict inequality} \tag{2.18}$$

where p is the peak of the umbrella and $\mu_1, \ldots, \mu_k$ are the location parameters of the i^{th} sample.

The test requires that Mack-Wolfe (1981) test statistics must first be calculated within each block and then sums all the Mack-Wolfe statistics across all blocks. It is given by

$$KK = \sum_{j=1}^{b} A_{jp} \tag{2.19}$$

And

$$A_{jp} = \sum_{j=1}^{b} \left\{ \sum_{u=1}^{v-1} \sum_{v=2}^{p} U_{juv} + \sum_{u=p}^{v-1} \sum_{v=p+1}^{k} U_{jvu} \right\} \tag{2.20}$$

where A_{jp} is the Mack and Wolfe (1981) test statistic of the j^{th} block, b is the number of blocks, p is the known peak, k is the number of treatments, U_{juv} and U_{jvu} are the U statistics associated with the j^{th} block where U_{juv} counts the number of times the observation receiving treatment u is less than the observation receiving treatment v summed up over the appropriate treatment ranges . U_{jvu} counts the number of times the observation receiving treatment v is less than the observation receiving treatment u over the appropriate range within block b.

Kim-Kim (1992) test statistic follows an asymptotic normal distribution when H_0 is true with the mean and variance given by;

$$E_0(KK) = \sum_{j=1}^{b} \left\{ \frac{N_1^2 + N_2^2 - \sum_{i=1}^{t} n_i^2 - n_p^2}{4} \right\} \tag{2.21}$$

$$var_0(KK) = \frac{1}{72} \sum_{j=1}^{b} \left\{ 2\left(N_{j1}^3 + N_{j2}^3\right) + 3\left(N_{j1}^2 + N_{j2}^2\right) - \sum_{i=1}^{k} n_{ij}^2\left(2n_{ij} + 3\right) \right.$$
$$\left. - n_{jp}^2\left(2n_{jp} + 3\right) + 12n_{jp}N_{j1}N_{j2} - 12n_{jp}^2 N_j \right\} \tag{2.22}$$

where

$N = N_1 + N_2 - n_p$ $N_1 = \sum_{i=1}^{p} n_i$, $N_2 = \sum_{i=p}^{k} n_i$

$b =$ the number of blocks

$k =$ the number of treatments

$p =$ the known peak

When $n_i = 1$, the expected value and the variance are reduced to the form given by

$$E_0(KK) = \frac{b(p^2 + (k - p + 1)^2 - k - 1)}{4} \tag{2.23}$$

And

$$var_0(KK) = \frac{b}{72}[2(p^3 + (k - p + 1)^3) + 3(p^2 + (k - p + 1)^2)$$
$$- 5k - 5 + 12p(k - p + 1) - 12k] \tag{2.24}$$

when H_0 is true, the standardized version of Kim-Kim test has an asymptotic standard normal distribution and is given by

$$A^* = \frac{KK - E_0(KK)}{\sqrt{var_0(KK)}} \tag{2.25}$$

The null hypothesis is rejected when $A^* \geq Z_\alpha$

2.5. Balanced Incomplete Block Design (BIBD)

A Balanced Incomplete Block Design is a form of Incomplete Block Design where missing observations form a balanced pattern. It introduces a fair comparison of treatments in the presence of missing observations. The Durbin (1951) test was developed for testing a significant difference between treatment effects.

In a BIBD, Durbin test was developed to test the existence of missing observations. Page's (1963) test like Durbin's (1951) test, was rank based but assigned a rank of zero to the missing observations. Magel and Ndungu's (2011) tests were developed to test for the direction of treatment differences, especially with non – decreasing differences. Magel and Ndungu's (2011) test also assigned a rank of zero to the missing observations. Ndungu's (2011) test statistic is related to Page's (1963) test since ranked observations within blocks are summed up.

2.6. Incomplete Block Design (IBD)

This is where missing observations are selected at random and therefore a balanced pattern is not formed. In other words, an IBD does not allocate all treatments to every block. Alvo and Cabilio (1995) proposed a test that addressed the randomness of missing observations. The test was an extension of Page's (1963) statistic which was used for the non – decreasing alternative. It, therefore, had the same data requirements and followed a similar ranking procedure as that of Page's (1963) test statistic. Alvo and Cabilio (1995) in their proposed test, assigned an average of the ranks appearing in a block to deal with missing observations. They further assigned a weight to a block with missing observations using the following formula:

$$\sum_{j=1}^{t} \frac{(t+1)}{(k+1)} \mu_{ij} \tag{2.26}$$

where $\boldsymbol{\mu_{ij}}$ is the rank of the observation or the average of the ranks if missing. The test statistic was then the sum of the above quantity over all $\boldsymbol{b}$ blocks. Like Magel and Ndungu (2011), Alvo and Cabilio's (1995) variance was dependent on the order of missing observations within a block. The variance of each block was given by

$$\sigma^2(i) = \frac{k(t+1)}{12(k+1)}\sum_{j=1}^{k}(O_{ij} - \bar{O}_i)^2 \tag{2.27}$$

where O_{ij} is the order of the observation's treatment in the block and $\bar{O}_i$ is the average of the treatment order numbers appearing in the block.

2.6.1. Mungai Test Statistic

The Mungai statistic for the umbrella alternative was proposed by Magel and Ndungu (2011). The test statistic is applicable under complete and incomplete blocks design. Magel and Ndungu (2011) gave the test statistic as follows:

$$M = \sum_{b=1}^{n} M_b \tag{2.28}$$

where

$$M_b = \sum_{i<j}\sum_{j+1>p} U_{ijb} \tag{2.29}$$

U_{ijb} is similar to the Mack – Wolfe. It is the number of pairs of observations (x, y) in block *b* for which

$x < y$ if x & $y < peak$. y can be the peak

$x > y$ if x & $y > peak$. x can be the peak

Comparisons are restricted to only treatments on the same side of the peak.

The following criterion is used for the values of U_{ijb} if there are missing observations. Without loss of generality, assume a pair $i < j \leq peak$ where j can also be the peak:

$U_{ijb} = 0$ if i and j are greater than the peak p .

0.5 if i and j are missing

$1 - \frac{r_i}{k+1}$ if j is missing and i is not, where r_i is the rank of i within the block and k is the number of treatments appearing in the block.

$\frac{r_j}{k+1}$ if i is missing and j is not, where r_j is the rank of j within the block and k is the number of treatments appearing in the block.

The expected value for the Mungai Test statistic is dependent on the number of treatments and the position of the peak treatment. The general formula is given by

$$E(M) = \sum_{i=1}^{n} E(M_i) \qquad (2.30)$$

where $E(M_i)$ is the expected value of block i and n is the total number of blocks in the IBD.

Similarly, to the expected value the variance also varies from block to block. The general formula is given by

$$Var(M) = \sum_{i=1}^{n} \sigma_i^2 \qquad (2.31)$$

where σ_i^2 is the variance of block i. The values of σ_i^2 will vary depending on the pattern of missing observations, the number of treatments and the position of the peak.

The following is an illustration of how the expected value and variance are computed. Arrangements that represent a block where only one treatment observation occurs are not considered. There are four general steps involved:

List all the possible $c = k!$ arrangements of ranks and missing observations for a block where k is the number of treatments appearing in the block

For each combination, calculate the statistic U_{ijb}.

- Calculate the average of the statistics.
- Calculate the variance of the statistics.

The standardized Mungai statistic is then given by

$$Z_M = \frac{M - E(M)}{\sqrt{Var(M)}} \tag{2.32}$$

The following is a detailed illustration of how the variance is calculated. Consider a case with three treatments and the second treatment is the peak. There are four possible scenarios for any given block:

- No missing observations.
- The peak treatment is missing.
- The first treatment is missing.
- The last treatment is missing.

Each scenario is further explored in detail next. For the case of no missing observations, for example, if treatments =3 and number of treatments appearing in the block k=3, then there are six possible arrangements of ranks (3! =6). The first column of Table 1 lists the combinations (step 1), the second column calculates Mungai Test statistic for each combination (step 2), the third column calculates the arithmetic average of the values from step two (step 3) and the final column calculates the variance of step two’s values (step 4).

Table 1. t = 3, k=3, & Peak = 2& No Observation Missing

Step 1			Step 2	Step 3	Step 4
1	2	3	1	1	2/3
1	3	2			
2	1	3			
2	3	1			
3	1	2			
3	2	1			

The following are the detailed step calculations.

Step 2: considering the first arrangement the following pairs are formed and their values of U_{ija}:

(1, 2) → $U_{12a} = 1$; (2, 3) → $U_{23a} = 0$.

$$\sum U_{ija} = 1 + 0 = 1.$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$$
$$= \frac{1+2+0+2+0+1}{6} = 1.$$

- Step 4

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$
$$= \frac{(1-1)^2 + (2-1)^2 + (0-1)^2 + (2-1)^2 + (0-1)^2 + (1-1)^2}{6} = \frac{2}{3}.$$

In the second scenario where the peak is missing there are two possible ways in which the order of magnitude can vary. Table 2. lists steps one to three.

Table 2. t = 3, k=2, Peak = 2 & Observation at Peak Missing

Step 1			Step 2	Step 3	Step 4
1	-	2	1	1	0
2	-	1	1		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$(1, \_) \rightarrow U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; $(,\_ ,2) \rightarrow U_{23a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{2}{3} + \frac{1}{3} = 1.$$

$(2, \_) \rightarrow U_{12a} = 1 - \frac{2}{3} = \frac{1}{3}$; $(\_ , 1) \rightarrow U_{23a} = 1 - \frac{1}{3} = \frac{2}{3}$

  - $\sum U_{ija} = \frac{1}{3} + \frac{2}{3} = 1.$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{1+1}{2} = 1.$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{(1-1)^2 + (1-1)^2}{2} = 0\,.$$

In the third scenario where the first observation is missing, there are two possible ways in which the order of magnitude can vary. Table 3. lists steps one to three.

Table 3. t = 3, k=2, Peak = 2 & First Observation is Missing

Step 1			Step 2	Step 3	Step 4
-	1	2	1/3	1	4/9
-	2	1	5/3		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$(\_ ,1) \rightarrow U_{12a} = \frac{1}{3}$; $(1,2) \rightarrow U_{23a} = 0$

$$\sum U_{ija} = \frac{1}{3} + 0 = \frac{1}{3}.$$

$(\_ ,2) \rightarrow U_{12a} = \frac{2}{3}$; $(2,1) \rightarrow U_{23a} = 1$

$$\sum U_{ija} = 1 + \frac{2}{3} = \frac{5}{3}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{1}{3} + \frac{5}{3}}{2} = 1.$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c}$$

$$= \frac{(\frac{1}{3} - 1)^2 + (\frac{5}{3} - 1)^2}{2} = \frac{4}{9}.$$

Similarly, where the third observation is missing, table 4. Shows the calculation on step by step basis.

Table 4. t = 3, k=2, Peak = 2 & First Observation is Missing

	Step 1		Step 2	Step 3	Step 4
1	2	-	1/3	1	4/9
2	1	-	5/3		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$(1, 2) \rightarrow U_{12a} = 1$; $(\_ ,1) \rightarrow U_{23a} = \frac{2}{3}$

$$\sum U_{ija} = 1 + \frac{2}{3} = \frac{5}{3}.$$

$(2, 1) \rightarrow U_{12a} = 0$; $(1, \_) \rightarrow U_{23a} = \frac{1}{3}$

$$\sum U_{ija} = 0 + \frac{1}{3} = \frac{1}{3}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{1}{3}+\frac{5}{3}}{2} = 1.$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{(\frac{5}{3}-1)^2 + (\frac{1}{3}-1)^2}{2} = \frac{4}{9}.$$

2.7. Mixed Designs

There have been several research studies that investigating the combination of experimental designs. Each combination investigated brings a challenging framework observed in real life data.

2.7.1. Esra and Fikri (2016)

Esra and Fikri (2016) proposed tests for testing the umbrella alternatives. When the peak is known, the modified Mack-Wolfe test statistic mA_p given in Eq. (2.33) is the sum of the weighted Mann- Whitney counts which are on the left side of the peak $(v-u)U_{uv}$ and the reverse weighted Mann-Whitney counts which are on the right side of the peak $(u-v)U_{uv}$. In general, the Mann-Whitney counts U_{uv} and U_{vu} get the weights $(v-u)$ and $(u-v)$ in their respective form. The proposed statistic is the sum of two modified Jonckheere statistics as investigated by Neuhauser et al. (1988) for ordered alternatives. The test statistic of Neuhauster et al. (1988) is a member of the class of weighted nonparametric statistics investigated by Tryon and Hettmansperger (1973). Thus the modified Mack-Wolfe test statistic mA_p is given by

$$mA_\mathrm{p} = \sum_{\mathrm{u}=1}^{\mathrm{v}-1}\sum_{\mathrm{v}=2}^{p}(\mathrm{v}-\mathrm{u})U_{uv} + \sum_{\mathrm{u}=p}^{\mathrm{v}-1}\sum_{\mathrm{p}=i+1}^{k}(\mathrm{u}-\mathrm{v})U_{vu} \qquad (2.33)$$

At a significance level of α, H_0 is rejected if $mA_p \geq mA_{p,\alpha}$; otherwise it is not rejected. Under the H_0 that all population means are equal, the expected value and variance of (A_p) are respectively given below.

$$E_0(mA_{\mathrm{p}}) = \frac{n^2}{2}\left[\binom{p+1}{3} + \binom{k-p+2}{3}\right] \tag{2.34}$$

And

$$var_0(mA_{\mathrm{p}}) = \frac{n^2p^2(p^2-1)(np+1) + n^2(k-p+1)((k-p+1)^2-1)(n(k-p+1)+1)}{144} + \frac{n^3p(p-1)(k-p)(k-p+1)}{24} \tag{2.35}$$

Under H_0 when the sample size gets large, the standard version mA_p^*, has an asymptotic standard normal distribution given by

$$mA_p^* = \frac{mA_{\mathrm{p}} - E_0(mA_{\mathrm{p}})}{\sqrt{var_0(mA_{\mathrm{p}})}} \tag{2.36}$$

When H_0 is true, and the sample sizes becomes large, the distribution of mA_p^* is an asymptotically standard normal distribution. At a significance level of α, H_0 is rejected if $mA_p^* \geq z_\alpha$ where the z_α is the critical value of the upper tail probability $(1-\alpha)\%$ of the standard normal distribution; otherwise we do not reject it.

2.7.2. Magel and Mathisen (2011)

Magel and Mathisen looked at a combination of RCBD and a BIBD. This was shown in an example where funds were not enough to complete the experiment in an RCBD. Magel and Mathisen (2011) used Page's (1963), and Magel and Ndungu's (2011) test statistics to form two new statistics:

Standardizing first:

$$T1 = \frac{Z_{Page} + Z_{Ndungu}}{\sqrt{2}} \tag{2.37}$$

And

Standardizing last:

$$T2 = \frac{L + M - [E(L) + E(M)]}{\sqrt{Var(L) + Var(M)}} \tag{2.38}$$

Results obtained from their simulation studies showed that, when the ratio of complete blocks to balanced incomplete blocks was 1:1, Standardized Last was better when there were fewer observations in the incomplete blocks. The reverse was true when there were more observations in the incomplete blocks. Standardized last again was also better when there were more complete blocks compared to the incomplete blocks, for instance where the ratio was 2:1. When the ratio of complete to incomplete was 1:2 there was no significant difference between the two methods.

2.7.3. Dubnicka, Blair and Hettmansperger (2002)

Dubnicka, Blair and Hettmansperger (2002) proposed a Mixed Design which was comprised of a two sample design which is a mixture of paired and independent observations. The test applied the Wilcoxon (1945) signed rank test to the paired data and the Mann – Whitney (1947) to the independent samples. The combination of both tests' statistic values and standardizing the quantity formed their new test. Dubnicka et al (2002) recommended using weights unless the two independent samples combined were less than the paired data sample.

Their proposed test, T^{+}, is given by

$$T^{+} = S^{+} + U^{+} \tag{2.39}$$

where S^+ is the Wilcoxon signed -rank test statistic and U^+ is the Mann-Whitney test statistic. Under H_0, they showed that the expected value and the variance of T^+ are given below

$$E_0(T^+) = \frac{n(n+1)}{4} + \frac{n_1 n_2}{2} \tag{2.40}$$

And

$$v_0(T^+) = \frac{n(n+1)(2n+1)}{24} + \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} \tag{2.41}$$

The standardized version of Dubnicka et al.'s (2002) test statistic is given below. The test has an asymptotic standard normal distribution under H_0. The null hypothesis is rejected in favor of the one-sided alternative when $T^* \geq Z_\alpha$, where Z_α is the critical value of the upper tail probability of the standard normal distribution.

$$T^* = \frac{T^+ - E_0(T^+)}{\sqrt{var_0(T^+)}} \tag{2.42}$$

2.7.4. Magel, Tersptra and Katrina (2010)

Magel, Tersptra Jeff, Canonizado and Katrina (2010) proposed two tests for the Mixed Design under the umbrella alternative. The Mixed Design consists of a Completely Randomized Design and randomized complete block design. The test of the hypotheses is given by

2.7.5. Null Hypothesis

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \text{ treatments effects assumed to be equal} \tag{2.43}$$

Versus

2.7.6. Alternative Hypothesis

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \cdots \geq \mu_k \text{ with at least one strict inequality} \tag{2.44}$$

where $\mu_1, \ldots, \mu_k$ are the location parameters of the i^{th} sample.

Their first proposed A_p^{**} test is given by

$$A_p^{**} = A_p^* + KK^* \tag{2.45}$$

where A_p^* is the standardized version of the usual Mack-Wolfe test for CRD and A^* is the standard version of the Kim-Kim (1992) test tfor RCBD. Under H_0, both A_p^* and KK^* have an asymptotic standard normal distribution, thus the expected value and variance of A_p^{**} are given by

$$E_0\left(A_p^{**}\right) = E_0\left(A_p^*\right) + E_0(KK^*) = 0 \tag{2.46}$$

And

$$var_0\left(A_p^{**}\right) = var_0\left(A_p^*\right) + var_0(KK^*) = 1 + 1 = 2 \tag{2.47}$$

The standardized version of the first proposed test is given by

$$A^{**} = \frac{A_p^{**} - E_0\left(A_p^{**}\right)}{\sqrt{var_0\left(A_p^{**}\right)}} = \frac{A_p^{**} - 0}{\sqrt{2}} \tag{2.48}$$

Under H_0, A^{**}has an asymptotic standard normal distribution. The null hypothesis is rejected for large values.

The second proposed A_p^{***} test is given by

$$A_p^{***} = A_P + KK \tag{2.49}$$

where A_P is the usual Mack-Wolfe (1981) test for CRD and A is the Kim-Kim (1992) for RCBD. Under H_0, the expected value and variance of A_p^{***} are the sum of the means and variances for the Mack-Wolfe and Kim-Kim tests. They are given by

$$E_0\left(A_p^{***}\right) = E_0(A_P) + E_0(KK) \tag{2.50}$$

And

$$var_0\left(A_p^{***}\right) = var_0(A_P) + var_0(KK) \tag{2.51}$$

where $E_0(A_P)$, $E_0(A), var_0(A_P)\ and\ var_0(KK)$ are the expected values and variances of the usual Mack-Wolfe (1981) test for CRD and A is the Kim-Kim (1992) for RCBD respectively.

The standardized version of the second proposed test is given by

$$A^{***} = \frac{A_p^{***} - E_0\left(A_p^{***}\right)}{\sqrt{var_0\left(A_p^{***}\right)}} \tag{2.52}$$

Under H_0, A^{**}has asymptotic standard normal distribution. The null hypothesis is rejected for large values.

CHAPTER 3. PRELIMINARY PROPOSED TEST STATISTICS

In this chapter, we will introduce the preliminary tests for each portion of the design. The test statistics are developed for the umbrella alternative with known peak p, but for various designs. The designs considered are the completely randomized design (CRD), randomized complete block design (RCBD) and incomplete block designs (IBD). The test of hypotheses for the Umbrella Alternative with a known peak is given below in Eq.(3.1) and Eq.(3.2).

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \text{ treatments effects assumed to be equal} \tag{3.1}$$

Versus

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_{p-1} \leq \mu_p \geq \mu_{p+1} \geq \cdots \geq \mu_k \text{ with at least one strict inequality} \tag{3.2}$$

where p is the peak of the umbrella and $\mu_1, \ldots, \mu_k$ are the location parameters of the i^{th} sample.

3.1. Randomized Complete Block Design (RCBD)

In Chapter Two, we wrote down the formula for the Kim-Kim (1992) test which was designed for the umbrella alternative in a Randomized Complete Block Design with cases of the known peak p. The Kim-Kim (1992) test statistic was shown to follow an asymptotic normal distribution when H_0 is true with the mean and variance given in 2.21 and 2.22 respectively. When H_0 is true, the standardized version of the Kim-Kim (1992) test has an asymptotic standard normal distribution which is given in equation 2.25.

We propose two test statistics which are modification to the Kim-Kim (1992) test for the RCBD. One modification considers the distances between each of the populations when comparing two of the k populations. The second modification considers the distances squared when comparing two of the k populations. The first modified test is given in 3.3 and 3.4.

$$KA_p = \sum_{i=1}^{b} mKA_{ip} \tag{3.3}$$

$$mKA_{ip} = \sum_{i=1}^{b} \left\{ \sum_{u=1}^{\mathrm{v}-1} \sum_{\mathrm{v}=2}^{p} (\mathrm{v} - u) U_{iuv} + \sum_{u=p}^{\mathrm{v}-1} \sum_{\mathrm{v}=p+1}^{k} (u - \mathrm{v}) U_{ivu} \right\} \tag{3.4}$$

where mKA_{ip} is the modified Mack and Wolfe (1981) test statistic of the i^{th} block, b is the number of blocks, p is the known peak, k is the number of treatments, U_{iuv} and U_{ivu} are the U statistics associated with the i^{th} block where U_{iuv} counts the number of times the observation receiving treatment u is less than the observation receiving treatment v summed up over the appropriate treatment ranges. U_{ivu} counts the number of times the observation receiving treatment v is less than the observation receiving treatment u over the appropriate range within block b.

Neuhauser et al. (1998) added this weight modification $(v - u)$ or $(u - v)$ to the Jonckheere – Terpstra statistic for the ordered alternatives. We are extending this weighted modification to the umbrella alternative. Under H_0 (null hypothesis), the test statistic KA_p^* will have an asymptotic normal distribution (follows from Kim and Kim (1992)). The value of KA_p^* will be large whenever the alternative hypothesis is true. Therefore at the significance level of α, H_0 is rejected for a large value of KA_p^*. Under H_0 the expected value and variance of KA_p^* for various populations at different peaks being derived in Appendix G and written as follows:

Note: We are considering only one observation appearing for each treatment in each block

The exact mean and variance of the distance modification of the Kim- Kim (1992) test in 3.3 for three treatments at peaks one or three are respectively given in Appendix G.1.3 and G.14 by

$$E_0(KA_1) = E_0(KA_3) = 2b \tag{3.5}$$

And

$$var_0(KA_1) = var_0(KA_3) = 2b \tag{3.6}$$

The exact mean and variance of the distance modification of the Kim- Kim (1992) test in 3.3 for three treatments at peak two are respectively given in Appendix G.2.3 and G.2.4 by

$$E_0(KA_2) = b \tag{3.7}$$

And

$$var_0(KA_2) = \frac{2b}{3} \tag{3.8}$$

The exact mean and variance of the distance modification of the Kim- Kim (1992) test in 3.3 for four treatments at peaks one or four are respectively given by

$$E_0(KA_1) = E_0(KA_4) = 5b \tag{3.9}$$

And

$$var_0(KA_1) = var_0(KA_4) = \frac{25b}{3} \tag{3.10}$$

The exact mean and variance of the distance modification of the Kim- Kim (1992) test in 3.3 for four treatments at peaks two or three are respectively given in Appendix G.3.3 and G.3.4 by

$$E_0(KA_3) = E_0(KA_2) = \frac{5b}{2} \tag{3.11}$$

And

$$var_0(KA_3) = var_0(KA_2) = \frac{11b}{4} \quad (3.12)$$

The exact mean and variance of the distance modification of the Kim- Kim (1992) test in 3.3 for five treatments at peaks two or four are respectively given in G.6.3 and G.6.4 by

$$E_0(KA_4) = E_0(KKA_2) = \frac{11b}{2} \quad . \quad (3.13)$$

And

$$var_0(KA_4) = var_0(KA_2) = \frac{115b}{12} \quad (3.14)$$

The exact mean and variance of the distance modification of the Kim- Kim (1992) test in 3.3 for five treatments at peak three is given in Appendix G.7.3 and G.7.4 by

$$E_0(KA_3) = 4b \quad (3.15)$$

And

$$var_0(KA_3) = \frac{11\text{b}}{2} \quad (3.16)$$

The exact mean and variance of the distance modification of the Kim- Kim (1992) test in 3.3 for five treatments at peaks one or five is given in Appendix G.9.3 and G 9.4 by

$$E_0(KA_1) = E_0(KA_5) = 10b \quad (3.17)$$

And

$$var_0(KA_1) = var_0(KA_5) = 25b \quad (3.18)$$

Therefore, the standardized KA_m^* has an asymptotic standard normal distributed where

$$KA_m^* = \frac{KA_p - E_0(KA_p)}{\sqrt{var_0(KA_p)}} \quad (3.19)$$

The null hypothesis is rejected when $KA_m^* \geq z_\alpha$

We propose a distance squared modification of the Kim-Kim (1992) test for the randomized complete block design as follows:

$$KA_p^* = \sum_{i=1}^{b} KA_{ip}^{**} \tag{3.20}$$

$$KA_{ip}^{**} = \sum_{i=1}^{b} \left\{ \sum_{u=1}^{\mathrm{v}-1} \sum_{\mathrm{v}=2}^{p} (\mathrm{v}-u)^2 U_{iuv} + \sum_{u=\mathrm{p}}^{\mathrm{v}-1} \sum_{\mathrm{v}=\mathrm{p}+1}^{k} (u-\mathrm{v})^2 U_{ivu} \right\} \tag{3.21}$$

where KA_{ip}^{**} is the modified Mack-Wolfe (1981) test statistic of the i^{th} block, b is the number of blocks, p is the known peak, k is the number of treatments, U_{iuv} and U_{ivu} are the U statistics in the i^{th} block as defined earlier in Eq.(3.4)

The exact mean and variance of the distance squared modification test in 3.21 for three treatments at peaks one or three is given in Appendix G.1.7 and G.1.8 by

$$E_0(KA_1^*) = E_0(KA_3^*) = 3b \tag{3.22}$$

And

$$var_0(KA_1^*) = var_0(KA_3^*) = \frac{17b}{3} \tag{3.23}$$

The exact mean and variance of the distance squared modification of the Kim- Kim (1992) test in 3.21 for three treatments at peak two is given in Appendix G.2.3 and G.2.4 by

$$E_0(KA_2^*) = b \tag{3.24}$$

And

$$var_0(KA_2^*) = \frac{2b}{3} \tag{3.25}$$

The exact mean and variance of the distance squared modification of the Kim- Kim (1992) test in 3.21 for four treatments at peak one or four is given in Appendix G.3.7 and G.3.8 by

$$E_0(KA_1^*) = E_0(KA_4^*) = 10\mathrm{b} \tag{3.26}$$

And

$$var_0(KA_1^*) = var_0(KA_4^*) = \frac{135b}{3} \tag{3.27}$$

The exact mean and variance of the distance squared modification of the Kim- Kim (1992) test in 3.21 for four treatments at peak two or three is given in Appendix G.4.7 and G.4.8 by

$$E_0(KA_2^*) = E_0(KA_3^*) = \frac{7b}{2} \tag{3.28}$$

And

$$var_0(KA_2^*) = var_0 KA_3^* = \frac{27b}{4} \tag{3.29}$$

The exact mean and variance of the distance squared modification of the Kim- Kim (1992) test in 3.21 for five treatments at peak two or four is given in Appendix G.6.7 and G.6.8 by

$$E_0(KA_4^*) = E_0(KA_4^*) = \frac{21b}{2} \tag{3.30}$$

And

$$var_0(KA_4^*) = var_0(KA_4^*) = \frac{571b}{12} \tag{3.31}$$

The exact mean and variance of the distance squared modification of the Kim- Kim (1992) test in 3.21 for five treatments at peak three is given in Appendix G.7.7 and G.7.8 by

$$E_0(KA_3^*) = 6b \tag{3.32}$$

And

$$var_0(KA_3^*) = 62b \tag{3.33}$$

The exact mean and variance of the distance squared modification of the Kim- Kim (1992) test in 3.21 for five treatments at peak one or five is given in Appendix G.9.7 and G.9.8 by

$$E_0(KA_1^*) = E_0(KA_5^*) = 25b \tag{3.34}$$

And

$$var_0(KA_1^*) = var_0(KA_5^*) = \frac{652b}{3} \tag{3.35}$$

Therefore, the standardized version of KA_m^{**} has an asymptotic standard normal distributed where

$$KA_m^{**} = \frac{KA_p^* - E_0(A_P^*)}{\sqrt{var_0(A_P^*)}} \tag{3.36}$$

The null hypothesis is rejected when $KA_m^{**} \geq z_\alpha$

3.2. Completely Randomized Block Design (CRD)

Esra and Fikri (2016) have already proposed a distance modification to the Mack- Wolfe (1982) test statistic for CRD. Their test statistic was given in Chapter Two along with the mean and variance given in Eqs.(2.34) and (2.35) respectively.

Following the work of Esra and Fikri (2016), we propose a new modification test called the distance squared modification of modified Mack - Wolfe (1981) test as follows:

$$sA_p = \sum_{u=1}^{\mathrm{v}-1}\sum_{\mathrm{v}=2}^{p}(\mathrm{v}-u)^2 U_{uv} + \sum_{u=p}^{\mathrm{v}-1}\sum_{\mathrm{v}=p+1}^{k}(u-\mathrm{v})^2 U_{vu} \tag{3.37}$$

where sA_p is the squared distance modification of the Mack -Wolfe (1981) test statistic, p is the known peak, k is the number of treatments, where U_{uv} and U_{vu} are the U statistics as defined in Eq.(3.4). Under H_0, the population sample sizes are all equal. The expected value and variance of sA_p for various populations at different peaks being derived in Appendix G written as follows:

The exact mean and variance of the distance squared modification of the Mack - Wolfe (1981) test for three treatments at peak one or three are given in Appendix G.1.5 and G.1.6 by

$$E_0(sA_2) = 3n^2 \tag{3.38}$$

And

$$var_0(sA_2) = \frac{n^2}{6}[25n + 9] \tag{3.39}$$

where $n = n_1 = n_2 = n_3 = \cdots = n_k$ and k is the number of treatments in the part of the completely randomized design.

The exact mean and variance of the distance squared modification of the Mack - Wolfe (1981) test for three treatments at peak two is given in Appendix G.2.1 and G.2.2 by

$$E_0(sA_2) = n^2 \tag{3.40}$$

And

$$var_0(sA_2) = \frac{n^2}{6}[3n + 1] \tag{3.41}$$

where $n = n_1 = n_2 = n_3 = \cdots = n_k$ and k is the number of treatments in the part of the completely randomized design.

The exact mean and variance of the distance squared modification of the Mack - Wolfe (1981) test for four treatments at peak one or four is given in Appendix G.3.5 and G.3.6 by

$$E_0(sA_2) = E_0(sA_3) = 10n^2 \tag{3.42}$$

And

$$var_0(sA_2) = var_0(sA_3) = \frac{n^2}{3}[106n + 29] \tag{3.43}$$

where $n = n_1 = n_2 = n_3 = \cdots = n_k$ and k is the number of treatments in the part of the completely randomized design.

The exact mean and variance of the distance squared modification of the Mack - Wolfe (1981) test for four treatments at peak two or three is given in Appendix G.4.5 and G.4.6 by

$$E_0(sA_2) = E_0(sA_3) = \frac{7n^2}{2} \tag{3.44}$$

And

$$var_0(sA_2) = var_0(sA_3) = \frac{n^2}{12}[62n + 19] \tag{3.45}$$

where $n = n_1 = n_2 = n_3 = \cdots = n_k$ and k is the number of treatments in the part of the completely randomized design.

The exact mean and variance of the distance squared modification of the Mack - Wolfe (1981) test for five treatments at peak two or four is given in Appendix G.6.5 and G.6.6 by

$$E_0(sA_2) = E_0(sA_4) = \frac{21n^2}{2} \tag{3.46}$$

And

$$var_0(sA_2) = var_0(sA_4) = \frac{n^2}{12}[454n + 117] \tag{3.47}$$

where $n = n_1 = n_2 = n_3 = \cdots = n_k$ and k is the number of treatments in the part of the completely randomized design.

The exact mean and variance of the distance squared modification of the Mack - Wolfe (1981) test for five treatments at peak three is given in Appendix G.7.5 and G.7.6 by

$$E_0(sA_3) = 6n^2 \tag{3.48}$$

And

$$var_0(sA_3) = \frac{n^2}{12}[36(2n + 1) + 66n] \tag{3.49}$$

where $n = n_1 = n_2 = n_3 = \cdots = n_k$ and k is the number of treatments in the part of the completely randomized design.

The exact mean and variance of the distance squared modification of the Mack - Wolfe (1981) test for five treatments at peak one or five is given in Appendix G.9.5 and G.9.6 by

$$E_0(sA_3) = 25n^2 \tag{3.50}$$

And

$$var_0(sA_3) = \frac{n^2}{6}(1096n + 235) \tag{3.51}$$

where $n = n_1 = n_2 = n_3 = \cdots = n_k$ and k is the number of treatments in the part of the completely randomized design.

Therefore, the standardized version of sA_m^* has an asymptotic standard normal distributed where

$$sA_m^* = \frac{sA_p - E_0(sA_p)}{\sqrt{var_0(sA_p)}} \tag{3.52}$$

The null hypothesis is rejected when $sA_m^* \geq z_\alpha$

3.3. Incomplete Block Design

Mungai statistic for the umbrella alternative was proposed by Magel and Ndungu (2011). The test statistic is applicable under complete and incomplete block design. The Magel and Ndungu (2011) test statistic was shown to follow an asymptotic normal distribution when H_0 is true with the mean and variance given in 2.30 to 2.31 respectively. When H_0 is true, the standardized version of the Magel and Ndungu (2011) test has an asymptotic standard normal distribution given in equation 2.32.

Inspired by the work of the Magel and Ndungu (2011) test for the complete and incomplete blocks design, we propose two additional test statistics which are modification to this test. The two modification tests are distance modification and distance squared modification tests.

3.3.1. Distance Modification for Magel and Ndungu (2011) Test

Under the distance modification test of the Magel and Ndungu (2011) test, the modified test givesthe weight $(j - i)$ to U_{ijb} in Eq (3.54) in two forms: where there are no missing observations and where there are mising observations. Magel and Ndungu (2011) gave the test statistic as follows:

$$M = \sum_{b=1}^{n} M_b \qquad (3.53)$$

where

$$M_b = \sum_{i<j} \sum_{j+1>p} U_{ijb} \qquad (3.54)$$

U_{ijb} is the number of pairs of observations (x, y) in block b for which

$x < y$ if x & $y < peak$. y can be the peak

$x > y$ if x & $y > peak$. x can be the peak

Comparisons are restricted to only treatments on the same side of the peak. Without a loss of generality, assume a pair $i < j \leq peak$ where j can also be the peak:

If there are no missing observations, then

$U_{ijb} = 0$ if i and j are greater than the peak p

$(j-i)$ if i and j are less than the peak and according to the positions of i^{th} and j^{th} in the populations. The further i and j are from the peak p (j can be the peak), the greater the weight.

If there are missing observations, then

$\frac{1}{2} * (j-i)$ if i and j are missing

$(j-i) * \left[1 - \frac{r_i}{k+1}\right]$ if j is missing and i is not, where r_i is the rank of i within the block and k is the number of treatments appearing in the block

$(j-i) * \frac{r_j}{k+1}$ if i is missing and j is not.

We denote the Distance Modification Mungai Test statistic as

$$M_p^* = \sum_{i=1}^{n} M_i^* \tag{3.55}$$

Under H_0, the expected value and variance are given by

$$E_0(M_p^*) = \sum_{i=1}^{n} E_0(M_i^*) \tag{3.56}$$

where $E_0(M_i^*)$ is the expected value of block i and n is the total number of blocks in the IBD. And

$$Var(M_p^*) = \sum_{i=1}^{n} \sigma_i^2 \tag{3.57}$$

where σ_i^2 is the variance of block i. The value of σ_i^2 will vary depending on the parttern of missing observations, number of treatments and the position of the peak.

The standardized version of the Distance Modification Mungai is written by

$$M_m^* = \frac{M_p^* - E(M_p^*)}{\sqrt{Var(M_p^*)}} \tag{3.58}$$

The null hypothesis is rejected when $M_m^* \geq Z_\alpha$

For example, consider the case with four treatments and the second treatment is the peak. There are three possible scenarios for any given block: with several other variations in the second and third.

- No missing observations.
- One observation is missing.
- Two observations are missing.

For example, consider the case of no missing observations for example, treatments =4 and number of treatments appearing in the block k=4, then there are 24 different arrangements of ranks (4! =24). Table 5. Shows the calculation on step by step basis.

Table 5. t = 4, k = 4 & Peak =2 & No Observation Missing

Step 1				Step 2	Step 3	Step 4
1	2	3	4	1	5/2	11/4
1	2	4	3	2		
1	3	2	4	2		
1	3	4	2	4		
1	4	2	3	4		
1	4	3	2	5		
2	1	3	4	0		
2	2	4	3	1		
2	3	1	4	2		
2	3	4	1	4		
2	4	1	3	4		
2	4	3	1	5		
3	1	2	4	0		
3	1	4	2	1		
3	2	1	4	1		
3	2	4	1	3		
3	4	1	2	4		
3	4	2	1	5		
4	1	2	3	0		
4	1	3	2	1		
4	2	1	3	1		
4	2	3	1	3		
4	3	1	2	3		
4	3	2	1	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$$(1, 2) \rightarrow U_{12a} = 1; (2, 3) \rightarrow U_{23a} = 0; (2, 4) \rightarrow U_{24a} = 0; (3, 4) \rightarrow U_{34a} = 0.$$

$$\sum U_{ija} = 1 + 0 + 0 + 0 = 1.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{1 + 2 + 2 + \cdots + 3 + 4}{24} = \frac{60}{24} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(1 - 2.5)^2 + (2 - 2.5)^2 + \cdots + (4 - 2)^2}{24} = 2.75$$

Now consider the second scenario where only one treatment observation is missing**.**

There are six different arrangements for the treatment ranks. Table 6. to Table 9. show the calculations involved in computing the expected value and variance in the presence of missing observations.

Table 6. t = 4, k=3, peak =2 & Fourth Observation Missing

Step 1				Step 2	Step 3	Step 4
1	2	3	-	2.75	2.5	31/24
1	3	2	-	4		
2	1	3	-	1.25		
2	3	1	-	3.75		
3	1	2	-	1		
3	2	1	-	2.25		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

  $(1, 2) \rightarrow U_{12a} = 1; (2, 3) \rightarrow U_{23a} = 0; (2, \_) \rightarrow U_{24a} = \frac{2}{4} * 2 = 1;$

  $(3, \_) \rightarrow U_{34a} = \frac{3}{4}.$

$$\sum U_{ija} = 1 + 1 + \frac{3}{4} = 2.75.$$

- Step 3: $E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$

$$= \frac{2.75 + 4 + 1.25 + 3.75 + 1 + 2.25}{6} = \frac{15}{6} = 2.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(2.75-2.5)^2+(4-2.5)^2+\cdots+(2.25-2.5)^2}{6} = \frac{31}{24}$

Table 7. t = 4, k=3, peak =2 & Third Observation Missing

Step 1				Step 2	Step 3	Step 4
1	2	-	3	7/2	2.5	61/24
1	3	-	2	17/4		
2	1	-	3	1/2		
2	3	-	1	18/4		
3	1	-	2	3/4		
3	2	-	1	13/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1, 2) \rightarrow U_{12a} = 1; (2, \_) \rightarrow U_{23a} = \frac{2}{4}; (2, 3) \rightarrow U_{24a} = \frac{2}{4} * 2 = 0;$

$(3, \_) \rightarrow U_{34a} = 1 - \frac{3}{4} = \frac{1}{4}.$

$$\sum U_{ija} = 1 + \frac{2}{4} + \frac{1}{4} = \frac{7}{4}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$$

$$= \frac{\frac{7}{4} + \frac{17}{4} + \cdots + 13/4}{6} = \frac{15}{6} = 2.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{\left(\frac{7}{4} - 2.5\right)^2 + \left(\frac{17}{4} - 2.5\right)^2 + \cdots + \left(\frac{13}{4} - 2.5\right)^2}{6} = \frac{61}{24}$$

Table 8. t = 4, k=3, peak =2 & Second Observation Missing

	Step 1			Step 2	Step 3	Step 4
1	-	2	3	7/4	2.5	61/24
1	-	3	2	3		
2	-	1	3	7/4		
2	-	3	1	13/4		
3	-	1	2	2		
3	-	2	1	13/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1, \_) \rightarrow U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}; (\_, 2) \rightarrow U_{23a} = 1 - \frac{2}{4} = \frac{2}{4};$

$(\_, 3) \rightarrow U_{24a} = 2 * \left(1 - \frac{3}{4}\right) = \frac{2}{4}; (2, 3) \rightarrow U_{34a} = 0.$

$$\sum U_{ija} = \frac{3}{4} + \frac{2}{4} + \frac{2}{4} = \frac{7}{4}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{7}{4} + 3 + \cdots + \frac{13}{4}}{6} = \frac{15}{6} = 2.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{\left(\frac{7}{4} - 2.5\right)^2 + (3 - 2.5)^2 + \cdots + \left(\frac{13}{4} - 2.5\right)^2}{6} = \frac{11}{24}$$

Table 9. t = 4, k=3, Peak =2 & First Observation Missing

Step 1				Step 2	Step 3	Step 4
-	1	2	3	1/4	2.5	61/24
-	1	3	2	5/4		
-	2	1	3	6/4		
-	2	3	1	14/4		
-	3	1	2	15/4		
-	3	2	1	19/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{4}$; (1 , 2) → $U_{23a} = 0$; (1 , 3) → $U_{24a} = 0$; (2 , 3) → $U_{34a} = 0$.

$$\sum U_{ija} = \frac{1}{4} + 0 + 0 + 0 = \frac{1}{4}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$$

$$= \frac{\frac{1}{4} + \frac{5}{4} + \cdots + \frac{19}{4}}{6} = \frac{15}{6} = 2.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{\left(\frac{1}{4} - 2.5\right)^2 + \left(\frac{5}{4} - 2.5\right)^2 + \cdots + \left(\frac{19}{4} - 2.5\right)^2}{6} = \frac{61}{24}$$

Again, consider the scenario where two observations are missing. There are two different arrangements for the treatment ranks. Table 10 shows the arrangements, expected value and variance.

Table 10. t = 4, k=2, Peak =2 & Third & Fourth Observations Missing

Step 1				Step 2	Step 3	Step 4
1	2	-	-	7/2	2.5	1
2	1	-	-	3/2		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$$(1\ ,2) \rightarrow U_{12a} = 1; (2,\_) \rightarrow U_{23a} = \frac{2}{3}; (2,\_) \rightarrow U_{24a} = \frac{2}{3} * 2 = \frac{4}{3}; (\_\ ,\_) \rightarrow U_{34a} = \frac{1}{2}.$$

$$\sum U_{ija} = 1 + \frac{2}{3} + \frac{4}{3} + \frac{1}{2} = \frac{7}{2}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{7}{2} + \frac{3}{2}}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{(\frac{7}{2} - 2.5)^2 + (\frac{3}{2} - 2.5)^2}{2} = 1$$

Table 11. t = 4, k=2, Peak =2 & First & Fourth Observations Missing

Step 1				Step 2	Step 3	Step 4
-	1	2	-	5/3	2.5	25/36
-	2	1	-	10/3		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$(\_ , 1) \rightarrow U_{12a} = \frac{1}{3}$; $(1, 2) \rightarrow U_{23a} = 0$; $(1, \_) \rightarrow U_{24a} = \frac{1}{3} * 2 = \frac{2}{3}$; $(2 , \_) \rightarrow U_{34a} = \frac{2}{3}$.

$$\sum U_{ija} = \frac{1}{3} + 0 + \frac{2}{3} + \frac{2}{3} = \frac{5}{3}$$

- Step 3 :$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$

$$= \frac{\frac{5}{3} + \frac{10}{3}}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{(\frac{5}{3} - 2.5)^2 + (\frac{10}{3} - 2.5)^2}{2} = \frac{25}{36}$$

Table 12. t = 4, k=2, Peak =2 & First & Second Observations Missing

Step 1				Step 2	Step 3	Step 4
-	-	1	2	11/6	2.5	4/9
-	-	2	1	19/6		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$(\_ , \_) \rightarrow U_{12a} = \frac{1}{2}$; $(\_ , 1) \rightarrow U_{23a} = 1 - \frac{1}{3} = \frac{2}{3}$; $(\_ , 2) \rightarrow U_{24a} = \left(1 - \frac{2}{3}\right) * 2 = \frac{2}{3}$;

$(1 , 2) \rightarrow U_{34a} = 0$.

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{3} + \frac{2}{3} + 0 = \frac{11}{6}$$

- Step 3 :$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$

$$= \frac{\frac{11}{6} + \frac{19}{6}}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{(\frac{11}{6} - 2.5)^2 + (\frac{19}{6} - 2.5)^2}{2} = \frac{4}{9}$$

Table 13. t = 4, k=2, Peak =2 & Second & Third Observations Missing

Step 1				Step 2	Step 3	Step 4
1	-	-	2	13/6	2.5	1/9
2	-	-	1	17/6		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$(1, \_) \rightarrow U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; $(\_ , \_) \rightarrow U_{23a} = \frac{1}{2}$; $(\_ , 2) \rightarrow U_{24a} = \left(1 - \frac{2}{3}\right) * 2 = \frac{2}{3}$;

$(\_ , 2) \rightarrow U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$.

$$\sum U_{ija} = \frac{2}{3} + \frac{1}{2} + \frac{2}{3} + \frac{1}{3} = \frac{13}{6}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{13}{6} + \frac{17}{6}}{2} = \frac{5}{2} = 2.5$$

Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{(\frac{13}{6} - 2.5)^2 + (\frac{17}{6} - 2.5)^2}{2} = \frac{1}{9}$$

Table 14. t = 4, k=2, Peak =2 & Second & Fourth Observations Missing

Step 1				Step 2	Step 3	Step 4
1	-	2	-	8/3	2.5	1/36
2	-	1	-	7/3		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$(1, \_) \rightarrow U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; $(\_ , 2) \rightarrow U_{23a} = 1 - \frac{2}{3} = \frac{1}{3}$; $(\_ , \_) \rightarrow U_{24a} = \left(\frac{1}{2}\right) * 2 = 1$;

$(2, \_) \rightarrow U_{34a} = \frac{2}{3}$.

$$\sum U_{ija} = \frac{2}{3} + \frac{1}{3} + 1 + \frac{2}{3} = \frac{8}{3}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{8}{3} + \frac{7}{3}}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c}$$

$$= \frac{(\frac{8}{3} - 2.5)^2 + \left(\frac{7}{3} - 2.5\right)^2}{2} = \frac{1}{36}$$

Table 15. t = 4, k=2, Peak =2 & First & Third Observations Missing

Step 1				Step 2	Step 3	Step 4
-	1	-	2	1	2.5	9/4
-	2	-	2	4		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$(\_ , 1) \rightarrow U_{12a} = \frac{1}{3}$; $(1, \_) \rightarrow U_{23a} = \frac{1}{3}$; $(1, 2) \rightarrow U_{24a} = 0$; $(\_ , 2) \rightarrow U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$.

$$\sum U_{ija} = \frac{1}{3} + \frac{1}{3} + 0 + \frac{1}{3} + \frac{2}{3} = 1$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$$

$$= \frac{1+4}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{(1-2.5)^2 + (4-2.5)^2}{2} = \frac{9}{4}$$

We now consider the case with four treatments and the third treatment is the peak. There are three possible scenarios for any given block: with several other variations in the second and third.

- No missing observations.
- One observation is missing.
- Two observations are missing.

For the case of no missing observations for example, treatments =4 and number of treatments appearing in the block k=4, then there are 24 different arrangements of ranks (4! =24). Tables 16 shows the calculation on step by step basis.

Table 16. t = 4, k = 4 & Peak =3 & No Observation Missing

Step 1				Step 2	Step 3	Step 4
1	2	3	4	4	5/2	11/4
1	2	4	3	5		
1	3	2	4	3		
1	3	4	2	5		
1	4	2	3	3		
1	4	3	2	4		
2	1	3	4	3		
2	1	4	3	4		
2	3	1	4	1		
2	3	4	1	5		
2	4	1	3	1		
2	4	3	1	4		
3	1	2	4	1		
3	1	4	2	4		
3	2	1	4	0		
3	2	4	1	4		
3	4	1	2	1		
3	4	2	1	2		
4	1	2	3	1		
4	1	3	2	2		
4	2	1	3	0		
4	2	3	1	2		
4	3	1	2	0		
4	3	2	1	1		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$$(1, 2) \rightarrow U_{12a} = 1; (2, 3) \rightarrow U_{13a} = 2; (2, 3) \rightarrow U_{23a} = 1; (3, 4) \rightarrow U_{34a} = 0.$$

$$\sum U_{ija} = 1 + 2 + 1 + 0 = 4$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$$

$$= \frac{4 + 5 + 3 \ldots + 0 + 1}{24} = \frac{60}{24} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c}$$

$$= \frac{(4-2.5)^2 + (5-2.5)^2 + \cdots + (1-2.5)^2}{24} = \frac{11}{4}$$

Now consider the second scenario where only one treatment observation is missing. There are six different arrangements for the treatment ranks.Table 17. to Table 20. show the calculations involved in computing the expected value and variance in the presence of missing observations.

Table 17. t = 4, k=3, Peak =3 & Fourth Observation Missing

	Step 1			Step 2	Step 3	Step 4
1	2	3	-	19/4	2.5	61/24
1	3	2	-	14/4		
2	1	3	-	15/4		
2	3	1	-	5/4		
3	1	2	-	6/4		
3	2	1	-	1/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

  $(1, 2) \rightarrow U_{12a} = 1; (1, 3) \rightarrow U_{13a} = 2; (2, 3) \rightarrow U_{23a} = 1;$

  $(3, \_) \rightarrow U_{34a} = \frac{3}{4}.$

$$\sum U_{ija} = 1 + 2 + 1 + \frac{3}{4} = \frac{19}{4}$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$$

$$= \frac{\frac{19}{4} + \frac{14}{4} + \cdots + \frac{1}{4}}{6} = \frac{60}{24} = 2.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{(\frac{19}{4} - 2.5)^2 + \left(\frac{14}{4} - 2.5\right)^2 + \cdots + \left(\frac{1}{4} - 2.5\right)^2}{6} = \frac{61}{24}$$

Table 18. t = 4, k=3, Peak =3 & Third Observation Missing

Step 1				Step 2	Step 3	Step 4
1	2	-	3	13/4	2.5	11/24
1	3	-	2	13/4		
2	1	-	3	2		
2	3	-	1	3		
3	1	-	2	7/4		
3	2	-	1	7/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (1, \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{4}\right) = \frac{6}{4}; (2 , \_) \rightarrow U_{23a} = 1 - \frac{2}{4} = \frac{2}{4};$

$(\_ , 3) \rightarrow U_{34a} = 1 - \frac{3}{4} = \frac{1}{4}.$

$$\sum U_{ija} = 1 + \frac{6}{4} + \frac{2}{4} + \frac{1}{4} = \frac{13}{4}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$$

$$= \frac{\frac{13}{4} + \frac{13}{4} + \cdots + \frac{7}{4}}{6} = \frac{15}{6} = 2.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{2 * \left(\frac{13}{4} - 2.5\right)^2 + \cdots + 2 * \left(\frac{7}{4} - 2.5\right)^2}{6} = \frac{11}{24}$$

Table 19. t = 4, k=3, Peak = 3 & Second Observation Missing

Step 1				Step 2	Step 3	Step 4
1	-	2	3	13/4	2.5	61/24
1	-	3	2	18/4		
2	-	1	3	3/4		
2	-	3	1	17/4		
3	-	1	2	2/4		
3	-	2	1	7/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1, _)→ $U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}$; (1 , 2) → $U_{13a} = 2 * 1 = 2$, (_, 2) → $U_{23a} = \frac{2}{4}$;

(2 , 3) → $U_{34a} = 0$.

$$\sum U_{ija} = \frac{3}{4} + 2 + \frac{2}{4} + 0 = \frac{13}{4}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{13}{4} + \frac{18}{4} \dots + \frac{7}{4}}{6} = \frac{15}{6} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{13}{4} - 2.5\right)^2 + \left(\frac{18}{4} - 2.5\right)^2 + \dots + \left(\frac{7}{4} - 2.5\right)^2}{6} = \frac{61}{24}$$

Table 20. t = 4, k=3, Peak =3 & First Observation Missing

Step 1				Step 2	Step 3	Step 4
-	1	2	3	9/4	2.5	31/24
-	1	3	2	15/4		
-	2	1	3	1		
-	2	3	1	16/4		
-	3	1	2	5/4		
-	3	2	1	11/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{4}$; (_ , 2)→ $U_{13a} = 2 * \frac{2}{4} = 1$ (1 , 2) → $U_{23a} = 1$; (2 , 3) → $U_{34a} = 0$;

$$\sum U_{ija} = \frac{1}{4} + 1 + 1 + 0 = \frac{9}{4}.$$

- Step 3: $E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$

$$= \frac{\frac{9}{4} + \frac{15}{4} + \cdots + \frac{11}{4}}{6} = \frac{15}{6} = 2.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{\left(\frac{9}{4} - 2.5\right)^2 + \left(\frac{15}{4} - 2.5\right)^2 + \cdots + \left(\frac{11}{4} - 2.5\right)^2}{6} = \frac{61}{24}$$

Now consider the third scenario where two treatments observations are missing. Table 21 to 26 show the calculations involved in computing the expected value and variance in the presence of missing observations.

Table 21. t = 4, k=2, Peak =3 & Third & Fourth Observations Missing

Step 1				Step 2	Step 3	Step 4
1	2	-	-	19/6	2.5	4/9
2	1	-	-	11/6		

The following are the detailed step calculations.

- Step 2: using the first arrangement

$(1, 2) \rightarrow U_{12a} = 1$; $(1, \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$; $(2, \_) \rightarrow U_{23a} = 1 - \frac{2}{3} = \frac{1}{3}$;

$(\_, \_) \rightarrow U_{34a} = \frac{1}{2}$.

$$\sum U_{ija} = 1 + \frac{4}{3} + \frac{1}{3} + \frac{1}{2} = \frac{19}{6}$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{19}{6} + \frac{11}{6}}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{\left(\frac{19}{6} - 2.5\right)^2 + \left(\frac{11}{6} - 2.5\right)^2}{2} = \frac{4}{9}$$

Table 22. t = 4, k=2, Peak =3 & First & Fourth Observations Missing

Step 1				Step 2	Step 3	Step 4
-	1	2	-	10/3	2.5	25/36
-	2	1	-	5/3		

The following are the detailed step calculations.

- Step 2: using the first arrangement

(_ , 1) → $U_{12a} = \frac{1}{3}$; (_ , 2) → $U_{13a} = \frac{2}{3} * 2 = \frac{4}{3}$; (1, 2) → $U_{23a} = 1$;

(2 , _) → $U_{34a} = \frac{2}{3}$.

$$\sum U_{ija} = \frac{1}{3} + \frac{4}{3} + 1 + \frac{2}{3} = \frac{10}{3}$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{10}{3} + \frac{5}{3}}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{10}{3} - 2.5\right)^2 + \left(\frac{5}{3} - 2.5\right)^2}{2} = \frac{25}{36}$$

Table 23. t = 4, k=2, Peak =3 & First & Second Observations Missing

Step 1				Step 2	Step 3	Step 4
-	-	1	2	3/2	2.5	1
-	-	2	1	7/2		

The following are the detailed step calculations.

Step 2: using the first arrangement

(_ , _) → $U_{12a} = \frac{1}{2}$; (_ , 1) → $U_{13a} = \frac{1}{3} * 2 = \frac{4}{3}$; (_ , 1) → $U_{23a} = \frac{1}{3}$, (1 , 2)→ $U_{34a} = 0$.

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{3} + \frac{1}{3} + 0 = \frac{3}{2}$$

Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{3}{2} + \frac{7}{2}}{2} = \frac{5}{2} = 2.5$$

Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{(\frac{3}{2} - 2.5)^2 + (\frac{7}{2} - 2.5)^2}{2} = 1$$

Table 24. t = 4, k=2, Peak =3 & Second & Third Observations Missing

	Step 1			Step 2	Step 3	Step 4
1	-	-	2	17/6	2.5	1/9
2	-	-	1	13/6		

The following are the detailed step calculations.

- Step 2: using the first arrangement

(1, _) → $U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; (1 , _) → $U_{13a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$;

(_ , _) → $U_{23a} = \frac{1}{2}$; (_ , 2) → $U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{2}{3} + \frac{4}{3} + \frac{1}{2} + \frac{1}{3} = \frac{17}{6}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{17}{6} + \frac{13}{6}}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{(\frac{17}{6} - 2.5)^2 + (\frac{13}{6} - 2.5)^2}{2} = \frac{1}{9}$$

Table 25. t = 4, k=2, Peak =3 & Second & Fourth Observations Missing

Step 1				Step 2	Step 3	Step 4
1	-	2	-	4	2.5	9/4
2	-	1	-	1		

The following are the detailed step calculations.

- Step 2: using the first arrangement

(1, _) → $U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; (1 , 2) → $U_{13a} = 2 * 1 = 2$; (_ , 2)→ $U_{23a} = \frac{2}{3}$;

(2, _) → $U_{34a} = \frac{2}{3}$.

$$\sum U_{ija} = \frac{2}{3} + 2 + \frac{2}{3} + \frac{2}{3} = 4.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{4+1}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(4-2.5)^2 + (1-2.5)^2}{2} = \frac{9}{4}$$

Table 26. t = 4, k=2, Peak =3 & First & Third Observations Missing

Step 1				Step 2	Step 3	Step 4
-	1	-	2	7/3	2.5	1/36
-	2	-	2	8/3		

The following are the detailed step calculations.

- Step 2: using the first arrangement

(_ , 1) → $U_{12a} = \frac{1}{3}$; (_ , _) → $U_{13a} = 2 * \frac{1}{2} = 1$; (1 , _) → $U_{23a} = 1 - \frac{1}{3} = \frac{2}{3}$;

(_ , 2) → $U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$.

$$\sum U_{ija} = \frac{1}{3} + 2 + \frac{2}{3} + \frac{1}{3} = \frac{7}{3}$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{7}{3} + \frac{8}{3}}{2} = \frac{5}{2} = 2.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{7}{3} - 2.5\right)^2 + \left(\frac{8}{3} - 2.5\right)^2}{2} = \frac{1}{36}$$

We now consider Five Treatments in the design. There are four main scenarios for any given block in the design:

No Missing Observations.

One treatment is missing.

Two treatments are missing.

Three treatments are missing.

There are 120 different arrangements of the ranks. Table 27 shows the overall expected value and the variance under Step 3 and Step 4. Tables 28 to 41 give the rest of the 120 combinations and the contribution to the statistic (Step 2).

Table 27. t = 5, k = 5 & Peak =2 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	4	5	1	11/2	49/4
1	2	3	5	4	2		
1	2	4	3	5	2		
1	2	4	5	3	4		
1	2	5	3	4	4		
1	2	5	4	3	5		
1	3	2	4	5	2		
1	3	2	5	4	3		
1	3	4	2	5	4		
1	3	4	5	2	7		
1	3	5	2	4	6		
1	3	5	4	2	8		
1	4	2	3	5	4		
1	4	2	5	3	6		
1	4	3	2	5	5		
1	4	3	5	2	8		
1	4	5	2	3	9		
1	4	5	3	2	10		
1	5	2	3	4	7		
1	5	2	4	3	8		
1	5	3	2	4	8		
1	5	3	4	2	10		
1	5	4	2	3	10		
1	5	4	3	2	11		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1, 2) → $U_{12a} = 1$; (2, 3) → $U_{23a} = 0$; (2, 4) → $U_{24a} = 2 * 0 = 0$;

(2, 5) → $U_{34a} = 3 * 0 = 0.$, (3, 4) → $U_{34a} = 0$, (3, 5) → $U_{35a} = 2 * 0 = 0.$

(4, 5) → $U_{34a} = 0.$

$$\sum U_{ija} = 1 + 0 + 0 + 0 + 0 + 0 + 0 = 1.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 1 + 2 + 2 + \cdots + 10 + 11 = 144$$

Table 28. t = 5, k = 5 & Peak =2 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
2	1	3	4	5	0		
2	1	3	5	4	1		
2	1	4	3	5	1		
2	1	4	5	3	3		
2	1	5	3	4	3		
2	1	5	4	3	4		
2	3	1	4	5	2		
2	3	1	5	4	3		
2	3	4	1	5	4		
2	3	4	5	1	7		
2	3	5	2	4	6		
2	3	5	4	1	8		
2	4	1	3	5	4		
2	4	1	5	3	6		
2	4	3	1	5	5		
2	4	3	5	1	8		
2	4	5	2	3	9		
2	4	5	3	1	10		
2	5	1	3	4	7		
2	5	1	4	3	8		
2	5	3	1	4	8		
2	5	3	4	1	10		
2	5	4	1	3	10		
2	5	4	3	1	11		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(2, 1) → $U_{12a} = 0$; (1, 3) → $U_{23a} = 0$; (1, 4) → $U_{24a} = 0$;

(1, 5) → $U_{25a} = 0$., (3, 4) → $U_{34a} = 0$, (3, 5) → $U_{35a} = 0 = 0$.

(4, 5) → $U_{45a} = 0$.

$$\sum U_{ija} = 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 0 + 1 + 1 + \cdots + 10 + 11 = 138$$

Table 29. t = 5, k = 5 & Peak =2 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
3	1	2	4	5	0		
3	1	2	5	4	1		
3	1	4	2	5	1		
3	1	4	5	2	3		
3	1	5	2	4	3		
3	1	5	4	2	4		
3	2	1	4	5	1		
3	2	1	5	4	2		
3	2	4	1	5	3		
3	2	4	5	1	6		
3	2	5	2	4	5		
3	2	5	4	1	7		
3	4	1	2	5	4		
3	4	1	5	2	6		
3	4	2	1	5	5		
3	4	2	5	1	8		
3	4	5	2	2	9		
3	4	5	2	1	10		
3	5	1	2	4	7		
3	5	1	4	2	8		
3	5	2	1	4	8		
3	5	2	4	1	10		
3	5	4	1	2	10		
3	5	4	2	1	11		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(3, 1) → $U_{12a} = 0$; (1, 2) → $U_{23a} = 0$; (1, 4) → $U_{24a} = 0$;

(1, 5) → $U_{25a} = 0$., (2, 4) → $U_{34a} = 0$, (2, 5) → $U_{35a} = 0 = 0$.

(4, 5) → $U_{45a} = 0$.

$$\sum U_{ija} = 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 0 + 1 + 1 + 3 + 3 + \cdots + 10 + 11 = 132$$

Table 30. t = 5, k = 5 & Peak =2 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
4	1	2	3	5	0		
4	1	2	5	3	1		
4	1	3	2	5	1		
4	1	3	5	2	3		
4	1	5	2	3	3		
4	1	5	3	2	4		
4	2	1	3	5	1		
4	2	1	5	3	2		
4	2	3	1	5	3		
4	2	3	5	1	6		
4	2	5	2	3	5		
4	2	5	3	1	7		
4	3	1	2	5	3		
4	3	1	5	2	5		
4	3	2	1	5	4		
4	3	2	5	1	7		
4	3	5	2	2	8		
4	3	5	2	1	9		
4	5	1	2	3	7		
4	5	1	3	2	8		
4	5	2	1	3	8		
4	5	2	3	1	10		
4	5	3	1	2	10		
4	5	3	2	1	11		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(4, 1) → $U_{12a} = 0$; (1, 2) → $U_{23a} = 0$; (1, 3) → $U_{24a} = 0$;

(1, 5) → $U_{25a} = 0$., (2, 3) → $U_{34a} = 0$, (2, 5) → $U_{35a} = 0 = 0.$

(3, 5) → $U_{45a} = 0.$

$$\sum U_{ija} = 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 0 + 1 + 1 + 3 + \cdots + 10 + 11 = 126$$

Table 31. t = 5, k = 5 & Peak =2 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
5	1	2	3	4	0		
5	1	2	4	3	1		
5	1	3	2	4	1		
5	1	3	4	2	3		
5	1	4	2	3	3		
5	1	4	3	2	4		
5	2	1	4	3	2		
5	2	1	3	4	1		
5	2	3	1	4	3		
5	2	3	4	1	6		
5	2	4	1	3	5		
5	2	4	3	1	7		
5	3	1	2	4	3		
5	3	1	4	2	5		
5	3	2	1	4	4		
5	3	2	4	1	7		
5	3	4	1	2	8		
5	3	4	2	1	9		
5	4	1	2	3	6		
5	4	1	3	2	7		
5	4	2	1	3	7		
5	4	2	3	1	9		
5	4	3	1	2	9		
5	4	3	2	1	10		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(5, 1) → $U_{12a} = 0$; (1, 2) → $U_{23a} = 0$; (1, 3) → $U_{24a} = 0$;

(1, 4) → $U_{25a} = 0.$, (2, 3) → $U_{34a} = 0$, (2, 4) → $U_{35a} = 0 = 0.$

(3, 4) → $U_{45a} = 0.$

$$\sum U_{ija} = 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 0 + 1 + 1 + 3 + \cdots + 9 + 10 = 120$$

Therefore

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} \frac{144 + 138 + 132 + 126 + 120}{120} = \frac{11}{2} = 5.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{(0-5.5)^2 + (1-5.5)^2 + \cdots + (9-5.5)^2 + (9-5.5)^2 + (10-5.5)^2}{6} = \frac{49}{4}$$

Table 32. t = 5, k = 5 & Peak =3 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	4	5	4	4	11/4
1	2	3	5	4	5		
1	2	4	3	5	5		
1	2	4	5	3	7		
1	2	5	3	4	7		
1	2	5	4	3	8		
1	3	2	4	5	3		
1	3	2	5	4	4		
1	3	4	2	5	5		
1	3	4	5	2	7		
1	3	5	2	4	7		
1	3	5	4	2	8		
1	4	2	3	5	3		
1	4	2	5	3	4		
1	4	3	2	5	4		
1	4	3	5	2	6		
1	4	5	2	3	7		
1	4	5	3	2	8		
1	5	2	3	4	3		
1	5	2	4	3	4		
1	5	3	2	4	4		
1	5	3	4	2	6		
1	5	4	2	3	6		
1	5	4	3	2	7		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1, 2) → $U_{12a} = 1$; (1, 3) → $U_{13a} = 2$; (2, 3) → $U_{23a} = 1$;

(3, 4) → $U_{34a} = 0$., (3, 5) → $U_{35a} = 0$, (3, 5) → $U_{35a} = 0$.

(4, 5) → $U_{45a} = 0$.

$$\sum U_{ija} = 1 + 2 + 1 + 0 + 0 + 0 = 4.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 4 + 5 + 5 + \cdots + 6 + 7 = 132$$

Table 33. t = 5, k = 5 & Peak =3 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
2	1	3	4	5	3		
2	1	3	5	4	4		
2	1	4	3	5	4		
2	1	4	5	3	6		
2	1	5	3	4	6		
2	1	5	4	3	7		
2	3	1	4	5	1		
2	3	1	5	4	2		
2	3	4	1	5	5		
2	3	4	5	1	7		
2	3	5	2	4	7		
2	3	5	4	1	8		
2	4	1	3	5	1		
2	4	1	5	3	2		
2	4	3	1	5	4		
2	4	3	5	1	6		
2	4	5	2	3	7		
2	4	5	3	1	8		
2	5	1	3	4	1		
2	5	1	4	3	2		
2	5	3	1	4	4		
2	5	3	4	1	6		
2	5	4	1	3	6		
2	5	4	3	1	7		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(2, 1) → $U_{12a} = 0$; (2 , 3) → $U_{13a} = 2$; (1, 3) → $U_{23a} = 1$;

(3, 4) → $U_{34a} = 0.$, (3, 5) → $U_{34a} = 0$, (4, 5) → $U_{45a} = 0.$

$$\sum U_{ija} = 0 + 2 + 1 + 0 + 0 + 0 = 3.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 3 + 4 + 4 + \cdots + 6 + 7 = 114$$

Table 34. t = 5, k = 5 & Peak =3 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
3	1	2	4	5	1		
3	1	2	5	4	2		
3	1	4	2	5	4		
3	1	4	5	2	6		
3	1	5	2	4	6		
3	1	5	4	2	7		
3	2	1	4	5	0		
3	2	1	5	4	1		
3	2	4	1	5	4		
3	2	4	5	1	6		
3	2	5	2	4	6		
3	2	5	4	1	7		
3	4	1	2	5	1		
3	4	1	5	2	2		
3	4	2	1	5	2		
3	4	2	5	1	4		
3	4	5	2	2	7		
3	4	5	2	1	8		
3	5	1	2	4	1		
3	5	1	4	2	2		
3	5	2	1	4	2		
3	5	2	4	1	4		
3	5	4	1	2	6		
3	5	4	2	1	7		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(3, 1) → $U_{12a} = 0$; (3 , 2) → $U_{13a} = 0$; (1, 2) → $U_{23a} = 1$;

(2 , 4) → $U_{34} = 0.$, (2, 5) → $U_{35a} = 0$, (4, 5) → $U_{45a} = 0.$

$$\sum U_{ija} = 0 + 0 + 1 + 0 + 0 + 0 = 1.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 1 + 2 + 4 + \cdots + 4 + 6 + 7 = 96$$

Table 35. t = 5, k = 5 & Peak =3 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
4	1	2	3	5	1		
4	1	2	5	3	2		
4	1	3	2	5	2		
4	1	3	5	2	4		
4	1	5	2	3	6		
4	1	5	3	2	7		
4	2	1	3	5	0		
4	2	1	5	3	1		
4	2	3	1	5	2		
4	2	3	5	1	4		
4	2	5	2	3	6		
4	2	5	3	1	7		
4	3	1	2	5	0		
4	3	1	5	2	1		
4	3	2	1	5	1		
4	3	2	5	1	3		
4	3	5	2	2	6		
4	3	5	2	1	7		
4	5	1	2	3	1		
4	5	1	3	2	2		
4	5	2	1	3	2		
4	5	2	3	1	4		
4	5	3	1	2	4		
4	5	3	2	1	5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(4, 1) → $U_{12a} = 0$; (1, 2) → $U_{23a} = 1$; (4 , 2) → $U_{13a} = 0$;

(2, 3) → $U_{34a} = 0.$, (2, 5) → $U_{35a} = 0$, (3, 5) → $U_{45a} = 0$.

$$\sum U_{ija} = 0 + 1 + 0 + 0 + 0 + 0 + 0 = 1.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 1 + 2 + 2 + \cdots + 4 + 4 + 5 = 78$$

Table 36. t = 5, k = 5 & Peak =3 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
5	1	2	3	4	1		
5	1	2	4	3	2		
5	1	3	2	4	2		
5	1	3	4	2	4		
5	1	4	2	3	4		
5	1	4	3	2	5		
5	2	1	4	3	1		
5	2	1	3	4	0		
5	2	3	1	4	2		
5	2	3	4	1	4		
5	2	4	1	3	4		
5	2	4	3	1	5		
5	3	1	2	4	0		
5	3	1	4	2	1		
5	3	2	1	4	1		
5	3	2	4	1	3		
5	3	4	1	2	4		
5	3	4	2	1	5		
5	4	1	2	3	0		
5	4	1	3	2	1		
5	4	2	1	3	1		
5	4	2	3	1	3		
5	4	3	1	2	3		
5	4	3	2	1	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(5, 1) → $U_{12a} = 0$; (1, 2) → $U_{23a} = 1$; (5 , 2) → $U_{13a} = 0$;

(2, 3) → $U_{24a} = 0$., (2, 4) → $U_{35a} = 0$, (3, 4) → $U_{45a} = 0 = 0$.

$$\sum U_{ija} = 0 + 1 + 0 + 0 + 0 + 0 = 1.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 1 + 2 + 2 + \cdots + 3 + 4 = 60$$

Therefore

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c}$$

$$\frac{132 + 114 + 96 + 78 + 60}{120} = 4$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{(1-4)^2 + (2-4)^2 + \cdots + (9-4)^2 + (9-4)^2 + (10-4)^2}{6} = \frac{115}{12}$$

Table 37. t = 5, k = 5 & Peak =4 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	4	5	10	11/2	115/12
1	2	3	5	4	11		
1	2	4	3	5	9		
1	2	4	5	3	11		
1	2	5	3	4	9		
1	2	5	4	3	10		
1	3	2	4	5	9		
1	3	2	5	4	10		
1	3	4	2	5	7		
1	3	4	5	2	11		
1	3	5	2	4	7		
1	3	5	4	2	10		
1	4	2	3	5	7		
1	4	2	5	3	10		
1	4	3	2	5	6		
1	4	3	5	2	10		
1	4	5	2	3	7		
1	4	5	3	2	8		
1	5	2	3	4	7		
1	5	2	4	3	8		
1	5	3	2	4	6		
1	5	3	4	2	8		
1	5	4	2	3	6		
1	5	4	3	2	7		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , 2) → $U_{12a} = 1$; (1, 3) → $U_{13a} = 2$; (1 , 4) → $U_{14a} = 3$;

(2 , 3) → $U_{23a} = 1$., (2,4) → $U_{24a} = 2$, (3, 4) → $U_{34a} = 1$.

(4, 5) → $U_{45a} = 0$.

$$\sum U_{ija} = 1 + 2 + 3 + 1 + 2 + 1 + 0 = 10.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 10 + 11 + \cdots + 6 + 7 = 204$$

Table 38. t = 5, k = 5 & Peak =4 and No Observation Missing

	Step 1				Step 2	Step 3	Step 4
2	1	3	4	5	9		
2	1	3	5	4	10		
2	1	4	3	5	8		
2	1	4	5	3	10		
2	1	5	3	4	8		
2	1	5	4	3	9		
2	3	1	4	5	7		
2	3	1	5	4	8		
2	3	4	1	5	4		
2	3	4	5	1	11		
2	3	5	2	4	4		
2	3	5	4	1	10		
2	4	1	3	5	5		
2	4	1	5	3	8		
2	4	3	1	5	3		
2	4	3	5	1	10		
2	4	5	2	3	4		
2	4	5	3	1	8		
2	5	1	3	4	5		
2	5	1	4	3	6		
2	5	3	1	4	3		
2	5	3	4	1	8		
2	5	4	1	3	3		
2	5	4	3	1	7		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(2, 1) → $U_{12a} = 0$; (2 , 3) → $U_{13a} = 2$; (2 , 4) → $U_{14a} = 3$;

(1 , 3) → $U_{23a} = 1$., (1, 4) → $U_{34a} = 2$, (3, 4) → $U_{34a} = 1$.

(4 , 5) → $U_{45a} = 0$

$$\sum U_{ija} = 0 + 2 + 3 + 1 + 2 + 1 + 0 = 9.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 9 + 10 + 8 + \cdots + 8 + 3 + 7 = 168$$

Table 39. t = 5, k = 5 & Peak =4 and No Observation Missing

		Step 1			Step 2	Step 3	Step 4
3	1	2	4	5	7		
3	1	2	5	4	8		
3	1	4	2	5	5		
3	1	4	5	2	10		
3	1	5	2	4	5		
3	1	5	4	2	9		
3	2	1	4	5	6		
3	2	1	5	4	7		
3	2	4	1	5	3		
3	2	4	5	1	10		
3	2	5	2	4	3		
3	2	5	4	1	9		
3	4	1	2	5	2		
3	4	1	5	2	8		
3	4	2	1	5	1		
3	4	2	5	1	8		
3	4	5	2	2	4		
3	4	5	2	1	5		
3	5	1	2	4	2		
3	5	1	4	2	6		
3	5	2	1	4	1		
3	5	2	4	1	6		
3	5	4	1	2	3		
3	5	4	2	1	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(3, 1) → $U_{12a} = 0$; (3 , 2) → $U_{13a} = 0$; (3, 4) → $U_{14a} = 3$;

(1 , 2) → $U_{23} = 1.$, (1 , 4) → $U_{24a} = 2$, (2, 4) → $U_{34a} = 1.$

(4 , 5) → $U_{45a} = 0.$

$$\sum U_{ija} = 0 + 0 + 3 + 1 + 2 + 1 + 0 = 7.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 7 + 8 + 5 + \cdots + 6 + 3 + 4 = 132$$

Table 40. t = 5, k = 5 & Peak =4 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
4	1	2	3	5	1		
4	1	2	5	3	2		
4	1	3	2	5	2		
4	1	3	5	2	4		
4	1	5	2	3	6		
4	1	5	3	2	7		
4	2	1	3	5	0		
4	2	1	5	3	1		
4	2	3	1	5	2		
4	2	3	5	1	4		
4	2	5	2	3	6		
4	2	5	3	1	7		
4	3	1	2	5	0		
4	3	1	5	2	1		
4	3	2	1	5	1		
4	3	2	5	1	3		
4	3	5	2	2	6		
4	3	5	2	1	7		
4	5	1	2	3	1		
4	5	1	3	2	2		
4	5	2	1	3	2		
4	5	2	3	1	4		
4	5	3	1	2	4		
4	5	3	2	1	5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(4, 1) → $U_{12a} = 0$; (4, 2) → $U_{13a} = 0$; (4 , 3) → $U_{14a} = 0$;

(1 , 2) → $U_{23a} = 1.$, (1, 3) → $U_{24a} = 2$, (2 , 3) → $U_{34a} = 1$.

(3 , 5) → $U_{45a} = 0$

$$\sum U_{ija} = 0 + 0 + 0 + 1 + 2 + 1 + 0 = 4.$$

$$(M_i) = \sum_{a=1}^{c} M_a = 4 + 8 + 3 + \cdots + 3 + 1 + 2 = 96$$

Table 41. t = 5, k = 5 & Peak =4 and No Observation Missing

Step 1					Step 2	Step 3	Step 4
5	1	2	3	4	4		
5	1	2	4	3	5		
5	1	3	2	4	3		
5	1	3	4	2	5		
5	1	4	2	3	3		
5	1	4	3	2	4		
5	2	1	4	3	4		
5	2	1	3	4	3		
5	2	3	1	4	1		
5	2	3	4	1	5		
5	2	4	1	3	1		
5	2	4	3	1	4		
5	3	1	2	4	1		
5	3	1	4	2	4		
5	3	2	1	4	0		
5	3	2	4	1	4		
5	3	4	1	2	1		
5	3	4	2	1	2		
5	4	1	2	3	1		
5	4	1	3	2	2		
5	4	2	1	3	0		
5	4	2	3	1	2		
5	4	3	1	2	0		
5	4	3	2	1	1		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(5, 1) → $U_{12a} = 0$; (5, 2) → $U_{13a} = 0$; (5 , 3) → $U_{14a} = 0$;

(1, 2) → $U_{23a} = 1$., (1 , 3) → $U_{24a} = 2$, (2, 3) → $U_{34a} = 1$.

(3 , 4) → $U_{34a} = 0$

$$\sum U_{ija} = 0 + 0 + 0 + 1 + 2 + 1 + 0 = 4$$

- $(M_i) = \sum_{a=1}^{c} M_a = 4 + 5 + 3 + \cdots + 2 + 0 + 1 = 60$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} \frac{204 + 168 + 132 + 96 + 60}{120} = \frac{11}{2} = 5.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c}$$

$$= \frac{(10-5.5)^2 + (11-5.5)^2 + \cdots + (2-5.5)^2 + (0-5.5)^2 + (1-5.5)^2}{120} = \frac{115}{12}$$

We now consider the second scenario where only one observation is missing, and peak is at two. There are 24 possible arrangements of treatment ranks (k! = 4! = 24). Tables 42 to 116 list the arrangements, the expected values and the corresponding variances.

Table 42. t = 5, k = 4 & Peak =2 and First Observation Missing

Step 1					Step 2	Step 3	Step 4
-	1	2	3	4	1/5	$11/5$	$563/60$
-	1	2	4	3	6/5		
-	1	3	2	4	6/5		
-	1	3	4	2	16/5		
-	1	4	2	3	16/5		
-	1	4	3	2	21/5		
-	2	1	3	4	7/5		
-	2	1	4	3	12/5		
-	2	3	1	4	17/5		
-	2	3	4	1	32/5		
-	2	4	1	3	27/5		
-	2	4	3	1	37/5		
-	3	1	2	4	18/5		
-	3	1	4	2	28/5		
-	3	2	1	4	23/5		
-	3	2	4	1	38/5		
-	3	4	1	2	43/5		
-	3	4	2	1	48/5		
-	4	1	2	3	345		
-	4	1	3	2	39/5		
-	4	2	1	3	39/5		
-	4	2	3	1	49/5		
-	4	3	1	2	49/5		
-	4	3	2	1	54/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1) → $U_{12a} = \frac{1}{5}$; (1 , 2) → $U_{23a} = 0$; (1, 3) → $U_{24a} = 0$;

(1 , 4) → $U_{25a} = 0.$, (2 , 3) → $U_{34a} = 0$, (2 , 4) → $U_{35a} = 0$,

(3 , 4) → $U_{45a} = 0$.

$$\sum U_{ija} = \frac{1}{5} + 0 + 0 + 0 + 0 + 0 + 0 = \frac{1}{5}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{1}{5}+\frac{6}{5}+\cdots+\frac{49}{5}+\frac{54}{5}}{24} = \frac{11}{2} = 5.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{\left(\frac{1}{5}-5.5\right)^2+\left(\frac{6}{5}-5.5\right)^2+\cdots+\left(\frac{49}{5}-5.5\right)^2+\left(\frac{54}{5}-5.5\right)^2}{24} = \frac{563}{60}$$

Table 43. t = 5, k = 4 & Peak =2 and Second Observation Missing

		Step 1			Step 2	Step 3	Step 4
1	-	2	3	4	14/5	11/2	191/60
1	-	2	4	3	4		
1	-	3	2	4	4		
1	-	3	4	2	32/5		
1	-	4	2	3	32/5		
1	-	4	3	2	38/5		
2	-	1	3	4	14/5		
2	-	1	4	3	4		
2	-	3	1	4	21/5		
2	-	3	4	1	34/5		
2	-	4	1	3	33/5		
2	-	4	3	1	8		
3	-	1	2	4	3		
3	-	1	4	2	22/5		
3	-	2	1	4	21/5		
3	-	2	4	1	34/5		
3	-	4	1	2	7		
3	-	4	2	1	41/5		
4	-	1	2	3	17/5		
4	-	1	3	2	23/5		
4	-	2	1	3	23/5		
4	-	2	3	1	7		
4	-	3	1	2	7		
4	-	3	2	1	41/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , _) → $U_{12a} = 1 - \frac{1}{5} = \frac{4}{5}$; (_ , 2) → $U_{23a} = 1 - \frac{2}{5} = \frac{3}{5}$; (_ , 3) → $U_{24a} = 2 * \left(1 - \frac{3}{5}\right) = \frac{4}{5}$; (- , 4) → $U_{25a} = 3 * \left(1 - \frac{4}{5}\right) = \frac{3}{5}$., (2 , 3) → $U_{34a} = 0$, (2 , 4) → $U_{35a} = 0$,

(3 , 4) → $U_{45a} = 0$.

$$\sum U_{ija} = \frac{4}{5} + \frac{3}{5} + \frac{4}{5} + \frac{3}{5} + 0 + 0 + 0 = \frac{14}{5}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{14}{5} + 4 + \cdots + 7 + \frac{41}{5}}{24} = \frac{11}{2} = 5.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{14}{5} - 5.5\right)^2 + (4 - 5.5)^2 + \cdots + (7 - 5.5)^2 + \left(\frac{41}{5} - 5.5\right)^2}{24}$$

$$= \frac{191}{60}$$

Table 44. t = 5, k = 4 & Peak =2 and Third Observation Missing

		Step 1			Step 2	Step 3	Step 4
1	2	-	3	4	11/5	$11/2$	$103/12$
1	2	-	4	3	17/5		
1	3	-	2	4	23/5		
1	3	-	4	2	35/5		
1	4	-	2	3	41/5		
1	4	-	3	2	47/5		
2	1	-	3	4	5/5		
2	1	-	4	3	11/5		
2	3	-	1	4	24/5		
2	3	-	4	1	37/5		
2	4	-	1	3	42/5		
2	4	-	3	1	49/5		
3	1	-	2	4	6/5		
3	1	-	4	2	13/5		
3	2	-	1	4	18/5		
3	2	-	4	1	31/5		
3	4	-	1	2	44/5		
3	4	-	2	1	50/5		
4	1	-	2	3	8/5		
4	1	-	3	2	14/5		
4	2	-	1	3	20/5		
4	2	-	3	1	32/5		
4	3	-	1	2	38/5		
4	3	-	2	1	44/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , 2) → $U_{12a} = 1$; (2 , _) → $U_{23a} = \frac{2}{5}$; (2, 3) → $U_{24a} = 0$; (2 , 4) → $U_{25a} = 0$.,

$$(\_ , 3) \rightarrow U_{34a} = \tfrac{2}{5}, (\_ , 4) \rightarrow U_{35a} = 2 * \left(1 - \tfrac{4}{5}\right) = \tfrac{2}{5},\ (3 , 4) \rightarrow U_{45a} = 0.$$

$$\sum U_{ija} = 1 + \frac{2}{5} + 0 + 0 + \frac{2}{5} + \frac{2}{5} + 0 = \frac{11}{5}.$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{11}{5} + \frac{17}{5} + \cdots + \frac{38}{5} + \frac{44}{5}}{24} = \frac{11}{2} = 5.5$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{11}{5}-5.5\right)^2 + \left(\frac{17}{5}-5.5\right)^2 + \cdots + \left(\frac{38}{5}-5.5\right)^2 + \left(\frac{44}{5}-5.5\right)^2}{24} = \frac{103}{12}$

Table 45. t = 5, k = 4 & Peak =2 and Fourth Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	-	4	13/5	11/2	103/60
1	2	4	-	3	5		
1	3	2	-	4	19/5		
1	3	4	-	2	43/5		
1	4	2	-	3	37/5		
1	4	3	-	2	49/5		
2	1	3	-	4	6/5		
2	1	4	-	3	18/5		
2	3	1	-	4	18/5		
2	3	4	-	1	44/5		
2	4	1	-	3	36/5		
2	4	3	-	1	50/5		
3	1	2	-	4	5/5		
3	1	4	-	2	19/5		
3	2	1	-	4	11/5		
3	2	4	-	1	37/5		
3	4	1	-	2	37/5		
3	4	2	-	1	49/5		
4	1	2	-	3	6/5		
4	1	3	-	2	18/5		
4	2	1	-	3	12/5		
4	2	3	-	1	36/5		
4	3	1	-	2	6		
4	3	2	-	1	42/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (2 , 3) \rightarrow U_{23a} = 0; (2, \_) \rightarrow U_{24a} = 2 * \left(\frac{2}{5}\right) = \frac{4}{5};$

$(2 , 4) \rightarrow U_{25a} = 0., (3 , \_) \rightarrow U_{34a} = \frac{3}{5}, (3 , 4) \rightarrow U_{35a} = 0.$

$(\_ , 4) \rightarrow U_{45a} = 1 - \frac{4}{5} = \frac{1}{5}. \sum U_{ija} = 1 + 0 + \frac{4}{5} + 0 + \frac{3}{5} + 0 + \frac{1}{5} = \frac{13}{5}.$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{13}{5} + 5 + \cdots + 6 + \frac{42}{5}}{24} = \frac{11}{2} = 5.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{\left(\frac{13}{5} - 5.5\right)^2 + (5 - 5.5)^2 + \cdots + (6 - 5.5)^2 + \left(\frac{42}{5} - 5.5\right)^2}{24}$$
$$= \frac{103}{60}$$

Table 46. t = 5, k = 4 & Peak =2 and Fifth Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	4	-	21/5	5/2	11/4
1	2	4	3	-	27/5		
1	3	2	4	-	27/5		
1	3	4	2	-	39/5		
1	4	2	3	-	39/5		
1	4	3	2	-	9		
2	1	3	4	-	13/5		
2	1	4	3	-	19/5		
2	3	1	4	-	5		
2	3	4	1	-	38/5		
2	4	1	3	-	37/5		
2	4	3	1	-	44/5		
3	1	2	4	-	11/5		
3	1	4	2	-	18/5		
3	2	1	4	-	17/5		
3	2	4	1	-	6		
3	4	1	2	-	36/5		
3	4	2	1	-	42/5		
4	1	2	3	-	2		
4	1	3	2	-	16/5		
4	2	1	3	-	16/5		
4	2	3	1	-	28/5		
4	3	1	2	-	28/5		
4	3	2	1	-	34/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (2 , 3) \rightarrow U_{23a} = 0; (2 , 4) \rightarrow U_{24a} = 0;$

$(2 , 5) \rightarrow U_{25a} = 3 * \frac{2}{5} = \frac{6}{5}., (3 , 4) \rightarrow U_{34a} = 0, (3 , \_) \rightarrow U_{35a} = 2 * \frac{3}{5} = \frac{6}{5}.$

$(4 , \_) \rightarrow U_{45a} = \frac{4}{5}.$

$$\sum U_{ija} = 1 + 0 + 0 + \frac{6}{5} + 0 + \frac{6}{5} + \frac{4}{5} = \frac{21}{5}.$$

Therefore

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} \frac{\frac{21}{5} + \frac{27}{5} + \cdots + \frac{28}{5} + \frac{34}{5}}{120} = \frac{11}{2} = 5.5$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c}$$

$$= \frac{\left(\frac{21}{5} - 5.5\right)^2 + \left(\frac{27}{5} - 5.5\right)^2 + \cdots + \left(\frac{28}{5} - 5.5\right)^2 + \left(\frac{34}{5} - 5.5\right)^2}{120} = \frac{55}{12}$$

Table 47. t = 5, k=3, Peak =2 & Fourth & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	-	-	25/4	11/2	21/8
1	3	2	-	-	31/4		
2	1	3	-	-	16/4		
2	3	1	-	-	28/4		
3	1	2	-	-	13/4		
3	2	1	-	-	19/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (2 , 3) \rightarrow U_{23a} = 0; (2 , \_) \rightarrow U_{24a} = \frac{4}{4} = 1;$

$(2 , \_) \rightarrow U_{25a} = 3 * \left(\frac{2}{4}\right) = \frac{6}{4}; (3 , \_) \rightarrow U_{34a} = \frac{3}{4}, (3 , \_) \rightarrow U_{35a} = 2 * \frac{3}{4} = \frac{6}{4}$

$(\_ , \_) \rightarrow U_{34a} = \frac{1}{2}.$

$$\sum U_{ija} = 1 + 0 + 1 + \frac{6}{4} + \frac{3}{4} + \frac{6}{4} + \frac{1}{2} = \frac{25}{4}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{25}{4} + \frac{31}{4} + \cdots + \frac{13}{4} + \frac{19}{4}}{6} = \frac{11}{2}$$

- Step 4: $\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$

$$= \frac{(\frac{25}{4} - 5.5)^2 + \left(\frac{31}{4} - 5.5\right)^2 + \cdots + \left(\frac{19}{4} - 5.5\right)^2}{6} = \frac{21}{8}$$

Table 48. t = 5, k=3, Peak =2 & Third & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	2	-	3	-	20/4	11/2	17/4
1	3	-	2	-	32/4		
2	1	-	3	-	12/4		
2	3	-	1	-	32/4		
3	1	-	2	-	12/4		
3	2	-	1	-	24/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , 2) → $U_{12a} = 1$; (2 , _) → $U_{23a} = \frac{2}{4}$; (2 , 3) → $U_{24a} = 0$;

(2 , _) → $U_{25a} = 3 * \left(\frac{2}{4}\right) = \frac{6}{4}$; (_ , 3) → $U_{34a} = \frac{1}{4}$;

(_ , _) → $U_{35a} = 2 * \frac{1}{2} = 1$;(3 , _) → $U_{45a} = \frac{3}{4}$.

$$\sum U_{ija} = 1 + \frac{2}{4} + 0 + \frac{6}{4} + \frac{1}{4} + 1 + \frac{3}{4} = \frac{20}{4}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{20}{4} + \frac{32}{4} + \cdots + \frac{12}{4} + \frac{24}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{(\frac{20}{4} - 5.5)^2 + \left(\frac{32}{4} - 5.5\right)^2 + \cdots + \left(\frac{24}{4} - 5.5\right)^2}{6} = \frac{17}{4}$$

Table 49. t = 5, k=3, Peak =2 & Second & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	-	2	3	-	20/4	11/2	3/4
1	-	3	2	-	26/4		
2	-	1	3	-	18/4		
2	-	3	1	-	26/4		
3	-	1	2	-	18/4		
3	-	2	1	-	24/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}$; $(\_ , 2) \rightarrow U_{23a} = 1 - \frac{2}{4} = \frac{2}{4}$;

$(\_ , 3) \rightarrow U_{24a} = 2\left(1 - \frac{3}{4}\right) = \frac{2}{4}$;

$(\_ , \_) \rightarrow U_{25a} = 3 * \frac{1}{2} = \frac{3}{2}$; $(2 , 3) \rightarrow U_{34a} = 0$; $(2 , \_) \rightarrow U_{34a} = 2 * \left(\frac{2}{4}\right) = 1$;

$(3 , \_) \rightarrow U_{45a} = \frac{3}{4}$.

- Step 3:

$$\sum U_{ija} = \frac{3}{4} + \frac{2}{4} + \frac{2}{4} + \frac{3}{2} + 0 + 1 + \frac{3}{4} = \frac{20}{4}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{20}{4} + \frac{26}{4} + \cdots + \frac{18}{4} + \frac{24}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{20}{4} - 5.5)^2 + \left(\frac{26}{4} - 5.5\right)^2 + \cdots + \left(\frac{24}{4} - 5.5\right)^2}{6} = \frac{3}{4}$$

Table 50. t = 5, k=3, Peak =2 & Third & Fourth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	2	-	-	3	15/4	11/2	61/8
1	3	-	-	2	33/4		
2	1	-	-	3	8/4		
2	3	-	-	1	36/4		
3	1	-	-	2	11/4		
3	2	-	-	1	29/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (2 , \_) \rightarrow U_{23a} = \frac{2}{4}; (2 , \_) \rightarrow U_{24a} = 2 * \frac{2}{4} = 1;$

$(2 , 3) \rightarrow U_{25a} = 0; (\_ , \_) \rightarrow U_{34a} = \frac{1}{2};\ (\_ , 3) \rightarrow U_{35a} = 2 * \left(1 - \frac{3}{4}\right) = \frac{2}{4};$

$(\_ , 3) \rightarrow U_{45a} = 1 - \frac{3}{4} = \frac{1}{4}.$

- Step 3:

$$\sum U_{ija} = 1 + \frac{2}{4} + 1 + 0 + \frac{1}{2} + \frac{2}{4} + \frac{1}{4} = \frac{15}{4}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{15}{4} + \frac{33}{4} + \cdots + \frac{29}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{(\frac{15}{4} - 5.5)^2 + \left(\frac{33}{4} - 5.5\right)^2 + \cdots + \left(\frac{29}{4} - 5.5\right)^2}{6} = \frac{61}{8}$$

Table 51. t = 5, k=3, Peak =2 & Second & Third Observations Missing

Step 1					Step 2	Step 3	Step 4
1	-	-	2	3	19/4	11/2	13/12
1	-	-	3	2	14/4		
2	-	-	1	3	15/4		
2	-	-	3	1	5/4		
3	-	-	1	2	6/4		
3	-	-	2	1	1/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \to U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}$; $(\_ , \_) \to U_{23a} = \frac{1}{2}$; $(\_ , 2) \to U_{24a} = 2\left(1 - \frac{2}{4}\right) = 1$;

$(\_ , 3) \to U_{25a} = 3 * \left(1 - \frac{3}{4}\right) = \frac{3}{4}$; $(\_ , 3) \to U_{34a} = 2 * \left(1 - \frac{3}{4}\right) = \frac{2}{4}$;

$(2 , 3) \to U_{45a} = 0$;

- Step 3:

$$\sum U_{ija} = \frac{3}{4} + \frac{1}{2} + 1 + \frac{3}{4} + \frac{2}{4} + \frac{2}{4} = \frac{16}{4} = 4$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{16}{4} + \frac{22}{4} + \cdots + \frac{28}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{16}{4} - 5.5)^2 + \cdots + \left(\frac{28}{4} - 5.5\right)^2}{6} = \frac{13}{12}$$

Table 52. t = 5, k=3, Peak =2 & Second Fourth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	-	2	-	3	15/4	11/2	23/8
1	-	3	-	2	27/4		
2	-	1	-	3	14/4		
2	-	3	-	1	30/4		
3	-	1	-	2	17/4		
3	-	2	-	1	29/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \to U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}$; $(\_ , 2) \to U_{23a} = \frac{2}{4}$; $(\_ , \_) \to U_{24a} = 2 * \frac{1}{2} = 1$;

$(\_ , 3) \to U_{25a} = 3 * \left(1 - \frac{3}{4}\right) = \frac{3}{4}$; $(2 , \_) \to U_{34a} = \frac{2}{4} = \frac{2}{4}$; $(2 , 3) \to U_{35a} = 0$;

$(\_ , 3) \to U_{45a} = 1 - \frac{3}{4} = \frac{1}{4}$

- Step 3:

$$\sum U_{ija} = \frac{3}{4} + \frac{2}{4} + 1 + \frac{3}{4} + \frac{2}{4} + 0 + \frac{1}{4} = \frac{15}{4}{=}4$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{15}{4} + \cdots + \frac{29}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{15}{4} - 5.5)^2 + \cdots + \left(\frac{29}{4} - 5.5\right)^2}{6} = \frac{23}{8}$$

Table 53. t = 5, k=3, Peak =2 & First & Third Observations Missing

Step 1					Step 2	Step 3	Step 4
-	1	-	2	3	6/4	11/2	67/8
-	1	-	3	2	11/4		
-	2	-	1	3	17/4		
-	2	-	3	1	27/4		
-	3	-	1	2	33/4		
-	3	-	2	1	38/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{4}$; (1 , _) → $U_{23a} = \frac{1}{4}$, (1 , 2) → $U_{23a} = 0$; (1 , 3) → $U_{25a} = 0$.

(_ , 2)→ $U_{34a} = \frac{2}{4}$; (- , 3) → $U_{35a} = 2 * \frac{1}{4} = \frac{2}{4}$, (2 , 3) → $U_{45a} = 0$;

$$\sum U_{ija} = \frac{1}{4} + \frac{1}{4} + 0 + 0 + \frac{2}{4} + \frac{2}{4} = \frac{6}{4}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{6}{4} + \cdots + \frac{38}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{6}{4} - 5.5\right)^2 + \cdots + \left(\frac{38}{4} - 5.5\right)^2}{6} = \frac{67}{8}$$

Table 54. t = 5, k=3, Peak =2 & First & Fourth Observations Missing

Step 1					Step 2	Step 3	Step 4
-	1	2	-	3	6/4	11/2	49/6
-	1	3	-	2	16/4		
-	2	1	-	3	12/4		
-	2	3	-	1	32/4		
-	3	1	-	2	28/4		
-	3	2	-	1	38/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{4}$; (1 , 2)→ $U_{23a} = 0$; (1 , _) → $U_{24a} = 2 * \frac{1}{4} = \frac{2}{4}$; (1 , 3) → $U_{23a} = 0$;

(2 , _)→ $U_{34a} = \frac{2}{4}$; (2 , 3)→ $U_{35a} = 0$; (_ , 3) → $U_{45a} = \frac{1}{4}$

$$\sum U_{ija} = \frac{1}{4} + 0 + \frac{2}{4} + 0 + \frac{2}{4} + 0 + \frac{1}{4} = \frac{6}{4}.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{6}{4} + + \cdots + \frac{38}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{6}{4} - 5.5\right)^2 + \cdots + \left(\frac{38}{4} - 5.5\right)^2}{6} = \frac{49}{6}$$

Table 55. t = 5, k=3, Peak =2 & First & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
-	1	2	3	-	11/4	11/2	19/24
-	1	3	2	-	16/4		
-	2	1	3	-	17/4		
-	2	3	1	-	27/4		
-	3	1	2	-	28/4		
-	3	2	1	-	33/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ , 1) \rightarrow U_{12a} = \frac{1}{4}$; $(1 , 2) \rightarrow U_{23a} = 0$; $(1 , 3) \rightarrow U_{24a} = 0$; $(1 , \_) \rightarrow U_{25a} = 3 * \frac{1}{4} = \frac{3}{4}$;

$(2 , 3) \rightarrow U_{34a} = 0$; $(2 , \_) \rightarrow U_{35a} = 2 * \frac{2}{4} = 1$; $(3 , \_) \rightarrow U_{45a} = \frac{3}{4}$

$$\sum U_{ija} = \frac{1}{4} + 0 + 0 + \frac{3}{4} + 0 + 1 + \frac{3}{4} = \frac{11}{4}.$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{11}{4} + \cdots + \frac{33}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{11}{4} - 5.5\right)^2 + \cdots + \left(\frac{33}{4} - 5.5\right)^2}{6} = \frac{19}{24}$$

Table 56. t = 5, k=3, Peak =2 & First & Second Observations Missing

Step 1					Step 2	Step 3	Step 4
-	-	1	2	3	12/4	11/2	25/8
-	-	1	3	2	17/4		
-	-	2	1	3	17/4		
-	-	2	3	1	27/4		
-	-	3	1	2	27/4		
-	-	3	2	1	32/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ , \_) \rightarrow U_{12a} = \frac{1}{2}$; $(\_ , 1) \rightarrow U_{23a} = \frac{3}{4}$; $(\_ , 2) \rightarrow U_{24a} = 2 * \frac{2}{4} = 1$;

$(\_ , 3) \rightarrow U_{25a} = 3 * \frac{1}{4} = \frac{3}{4}$; $(1 , 2) \rightarrow U_{34a} = 0$; $(1 , 3) \rightarrow U_{35a} = 0$;

$(2 , 3) \rightarrow U_{45a} = 0$

$$\sum U_{ija} = \frac{1}{2} + \frac{3}{4} + \frac{4}{4} + \frac{3}{4} + 0 + 0 = \frac{12}{4}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{12}{4} + \cdots + \frac{32}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{12}{4} - 5.5\right)^2 + \cdots + \left(\frac{32}{4} - 5.5\right)^2}{6} = \frac{25}{8}$$

Table 57. t = 5, k=2, Peak = 2 & Third, Fourth & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	2	-	-	-	7	11/2	9/4
2	1	-	-	-	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , 2)→ $U_{12a} = 1$; (2 , _)→ $U_{23a} = \frac{2}{3}$; (2 , _) → $U_{24a} = 2 * \frac{2}{3} = \frac{4}{3}$;

(2 , _) → $U_{25a} = 3 * \frac{2}{3} = \frac{6}{3}$; (_ , _)→ $U_{34a} = \frac{1}{2}$; (_ , _)→ $U_{35a} = 2 * \frac{1}{2} = 1$;

(_ , _) → $U_{45a} = \frac{1}{2}$

$$\sum U_{ija} = 1 + \frac{2}{3} + \frac{4}{3} + \frac{6}{3} + \frac{1}{2} + 1 + \frac{1}{2} = 7$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{7 + 4}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(7 - 5.5)^2 + (4 - 5.5)^2}{2} = \frac{9}{4}$$

Table 58. t = 5, k=2, Peak = 2 & Second, Fourth & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	-	2	-	-	6	11/2	1/4
2	-	1	-	-	5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , _)→ $U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; (_ , 2)→ $U_{23a} = 1 - \frac{2}{3} = \frac{1}{3}$; (_ , _) → $U_{24a} = 2 * \frac{1}{2} = 1$;

(_, _) → $U_{25a} = 3 * \frac{1}{2} = \frac{3}{2}$; (2 , _)→ $U_{34a} = \frac{2}{3}$; (2 , _)→ $U_{35a} = 2 * \frac{2}{3} = \frac{4}{3}$;

(_ , _) → $U_{45a} = \frac{1}{2}$

$$\sum U_{ija} = \frac{2}{3} + \frac{1}{3} + 1 + \frac{3}{2} + \frac{2}{3} + \frac{4}{3} + \frac{1}{2} = 6$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{6+5}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(6-5.5)^2 + (5-5.5)^2}{2} = \frac{1}{4}$$

Table 59. t = 5, k=2, Peak = 2 & Second, Third & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	-	-	2	-	16/3	11/2	1/36
2	-	-	1	-	17/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , _)→ $U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; (_ , _)→ $U_{23a} = \frac{1}{2}$; (_ , 2) → $U_{24a} = 2 * \left(1 - \frac{2}{3}\right) = \frac{2}{3}$;

(_, _) → $U_{25a} = 3 * \frac{1}{2} = \frac{3}{2}$; (_ , 2)→ $U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$; (_ , _)→ $U_{35a} = 2 * \frac{1}{2} = 1$;

(2 , _) → $U_{45a} = \frac{2}{3}$

$$\sum U_{ija} = \frac{2}{3} + \frac{1}{2} + \frac{2}{3} + \frac{3}{2} + \frac{1}{3} + 1 + \frac{2}{3} = \frac{16}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{16}{3} + \frac{17}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{16}{3} - 5.5\right)^2 + \left(\frac{17}{3} - 5.5\right)^2}{2} = \frac{1}{36}$$

Table 60. t = 5, k=2, Peak = 2 & Second, Third & Fourth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	-	-	-	2	14/3	11/2	25/36
2	-	-	-	1	19/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , _)→ $U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; (_ , _)→ $U_{23a} = \frac{1}{2}$; (_ , _) → $U_{24a} = 2 * \frac{1}{2} = 1$;

(_, 2) → $U_{25a} = 3 * \left(1 - \frac{2}{3}\right) = 1$; (_ , _)→ $U_{34a} = \frac{1}{2}$;

(_ , 2)→ $U_{35a} = 2 * \left(1 - \frac{2}{3}\right) = \frac{2}{3}$; (_ , 2) → $U_{45a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{2}{3} + \frac{1}{2} + 1 + 1 + \frac{1}{2} + \frac{2}{3} + \frac{1}{3} = \frac{14}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{14}{3} + \frac{19}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{14}{3} - 5.5\right)^2 + \left(\frac{19}{3} - 5.5\right)^2}{2} = \frac{25}{36}$$

Table 61. t = 5, k=2, Peak = 2 & First, Fourth & Fifth Observations Missing

Step 1			Step 2		Step 2	Step 3	Step 4
-	1	2	-	-	9/2	11/2	1
-	2	1	-	-	13/2		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{3}$; (1 , 2)→ $U_{23a} = 0$; (1 , _) → $U_{24a} = 2 * \frac{1}{3} = \frac{2}{3}$;

(1 , _) → $U_{25a} = 3 * \frac{1}{3} = 1$; (2 , _)→ $U_{34a} = \frac{2}{3}$;

(2 , _)→ $U_{35a} = 2 * \frac{2}{3} = \frac{4}{3}$; (_ , _) → $U_{45a} = \frac{1}{2}$

$$\sum U_{ija} = \frac{1}{3} + 0 + \frac{2}{3} + 1 + \frac{2}{3} + \frac{4}{3} + \frac{1}{2} = \frac{9}{2}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{9}{2} + \frac{13}{2}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{9}{2} - 5.5\right)^2 + \left(\frac{13}{2} - 5.5\right)^2}{2} = 1$$

Table 62. t = 5, k=2, Peak = 2 & First, Third & Fifth Observations Missing

Step 1			Step 2		Step 2	Step 3	Step 4
-	1	-	2	-	11/3	11/2	121/36
-	2	-	1	-	22/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{3}$; (1 , _)→ $U_{23a} = \frac{1}{3}$; (1 , 2) → $U_{24a} = 0$

(1 , _) → $U_{25a} = 3 * \frac{1}{3} = 1$; (_ , 2)→ $U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$;

(_ , _)→ $U_{35a} = 2 * \frac{1}{2} = 1$; (2 , _) → $U_{45a} = \frac{2}{3}$

$$\sum U_{ija} = \frac{1}{3} + \frac{1}{3} + 0 + 1 + \frac{1}{3} + 1 + \frac{2}{3} = \frac{11}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{11}{3} + \frac{22}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{11}{3} - 5.5\right)^2 + \left(\frac{22}{2} - 5.5\right)^2}{2} = \frac{121}{36}$$

Table 63. t = 5, k=2, Peak = 2 & First, Third & Fourth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	1	-	-	2	17/6	11/2	64/9
-	2	-	-	1	49/6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{3}$; (1 , _)→ $U_{23a} = \frac{1}{3}$; (1 , _) → $U_{24a} = 2 * \frac{1}{3} = \frac{2}{3}$

(1 , 2) → $U_{25a} = 0$; (_ , _)→ $U_{34a} = \frac{1}{2}$; (_ , 2)→ $U_{35a} = 2 * \left(1 - \frac{2}{3}\right) = \frac{2}{3}$;

(_ , 2) → $U_{45a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{1}{3} + \frac{1}{3} + \frac{2}{3} + 0 + \frac{1}{2} + \frac{2}{3} + \frac{1}{3} = \frac{17}{6}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{17}{6} + \frac{49}{6}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{17}{6} - 5.5\right)^2 + \left(\frac{49}{6} - 5.5\right)^2}{2} = \frac{64}{9}$$

Table 64. t = 5, k=2, Peak = 2 & First, Second, & Fifth Observations Missing

Step 1		Step 2			Step 2	Step 3	Step 4
-	-	1	2	-	14/3	11/2	25/36
-	-	2	1	-	19/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , _)→ $U_{12a} = \frac{1}{2}$; (_ , 1)→ $U_{23a} = 1 - \frac{1}{3} = \frac{2}{3}$; (_ , 2) → $U_{24a} = 2 * \left(1 - \frac{2}{3}\right) = \frac{2}{3}$

(_ , _) → $U_{25a} = 3 * \frac{1}{2} = \frac{3}{2}$; (1 , 2)→ $U_{34a} = 0$; (1 , _)→ $U_{35a} = 2 * \frac{1}{3} = \frac{2}{3}$;

(2 , _) → $U_{45a} = \frac{2}{3}$

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{3} + \frac{2}{3} + \frac{3}{2} + 0 + \frac{2}{3} + \frac{2}{3} = \frac{14}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{14}{3} + \frac{19}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{14}{3} - 5.5\right)^2 + \left(\frac{19}{2} - 5.5\right)^2}{2} = \frac{25}{36}$$

Table 65. t = 5, k=2, Peak = 2 & First, Second, & Fourth Observations Missing

Step 1		Step 2			Step 2	Step 3	Step 4
-	-	1	-	2	33/6	11/2	0
-	-	2	-	1	33/6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ , \_) \rightarrow U_{12a} = \frac{1}{2}; (\_ , 1) \rightarrow U_{23a} = 1 - \frac{1}{3} = \frac{2}{3}; (\_ , \_) \rightarrow U_{24a} = 2 * \frac{1}{2} = 1$

$(\_ , 2) \rightarrow U_{25a} = 3 * \left(1 - \frac{2}{3}\right) = 1; (1 , \_) \rightarrow U_{34a} = \frac{1}{3}; (1 , 2) \rightarrow U_{35a} = 0;$

$(\_ , 2) \rightarrow U_{45a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{3} + 1 + 1 + \frac{1}{3} + 0 + \frac{1}{3} = \frac{33}{6}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{33}{6} + \frac{33}{6}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{33}{6} - 5.5\right)^2 + \left(\frac{33}{6} - 5.5\right)^2}{2} = 0$$

Table 66. t = 5, k=2, Peak = 2 & First, Second & Third Observations Missing

Step 1			Step 2		Step 2	Step 3	Step 4
-	-	-	1	2	14/3	11/2	25/36
-	-	-	2	1	19/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ ; \_) \rightarrow U_{12a} = \frac{1}{2}; (\_ , \_) \rightarrow U_{23a} = \frac{1}{2}; (\_ , 1) \rightarrow U_{24a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$

$(\_ , 2) \rightarrow U_{25a} = 3 * \left(1 - \frac{2}{3}\right) = 1; (\_ , 1) \rightarrow U_{34a} = 1 - \frac{1}{3} = \frac{2}{3};$

$(\_ , 2) \rightarrow U_{35a} = 2 * \left(1 - \frac{2}{3}\right) = \frac{2}{3}; (1 , 2) \rightarrow U_{45a} = 0$

$$\sum U_{ija} = \frac{1}{2} + \frac{1}{2} + \frac{4}{3} + 1 + \frac{2}{3} + \frac{2}{3} + 0 = \frac{14}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{14}{3} + \frac{19}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{14}{3} - 5.5\right)^2 + \left(\frac{19}{2} - 5.5\right)^2}{2} = \frac{25}{36}$$

We now consider the second scenario where only one observation is missing, and peak is at three.

Table 67. t = 5, k = 4 & Peak =3 & First Observation Missing

Step 1					Step 2	Step 3	Step 4
-	1	2	3	4	2	4	41/10
-	1	2	4	3	3		
-	1	3	2	4	17/5		
-	1	3	4	2	27/5		
-	1	4	2	3	29/5		
-	1	4	3	2	34/5		
-	2	1	3	4	4/5		
-	2	1	4	3	9/5		
-	2	3	1	4	18/5		
-	2	3	4	1	28/5		
-	2	4	1	3	6		
-	2	4	3	1	7		
-	3	1	2	4	1		
-	3	1	4	2	2		
-	3	2	1	4	12/5		
-	3	2	4	1	22/5		
-	3	4	1	2	31/5		
-	3	4	2	1	36/5		
-	4	1	2	3	6/5		
-	4	1	3	2	11/5		
-	4	2	1	3	13/5		
-	4	2	3	1	23/5		
-	4	3	1	2	5		
-	4	3	2	1	6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1) → $U_{12a} = \frac{1}{5}$; (_ , 2) → $U_{13a} = 2 * \frac{2}{5} = \frac{4}{5}$; (1, 2) → $U_{23a} = 1$;

(2 , 3) → $U_{34a} = 0.$, (2 , 5) → $U_{35a} = 0$, (3 , 4) → $U_{45a} = 0$,

$(3, 4) \rightarrow U_{45a} = 0.$

$$\sum U_{ija} = \frac{1}{5} + \frac{4}{5} + 1 + 0 + 0 + 0 = 2$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{2 + 3 + \cdots + 5 + 6}{24} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c}$$

$$= \frac{(2-4)^2 + (3-4)^2 + \cdots + \left(\frac{23}{5} - 4\right)^2 + (5-4)^2 + (6-4)^2}{24} = \frac{41}{10}$$

Table 68. t = 5, k = 4 & Peak =3 & Second Observation Missing

	Step 1				Step 2	Step 3	Step 4
1	-	2	3	4	16/5	4	89/30
1	-	2	4	3	19/5		
1	-	3	2	4	18/5		
1	-	3	4	2	28/5		
1	-	4	2	3	27/5		
1	-	4	3	2	32/5		
2	-	1	3	4	6/5		
2	-	1	4	3	11/5		
2	-	3	1	4	19/5		
2	-	3	4	1	29/5		
2	-	4	1	3	28/5		
2	-	4	3	1	33/5		
3	-	1	2	4	7/5		
3	-	1	4	2	12/5		
3	-	2	1	4	11/5		
3	-	2	4	1	21/5		
3	-	4	1	2	29/5		
3	-	4	2	1	34/5		
4	-	1	2	3	8/5		
4	-	1	3	2	13/5		
4	-	2	1	3	12/5		
4	-	2	3	1	22/5		
4	-	3	1	2	21/5		
4	-	3	2	1	26/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{5} = \frac{4}{5}; (1 , 2) \rightarrow U_{13a} = 2;$

$(\_ , 2) \rightarrow U_{23a} = \frac{2}{5}; (2 , 3) \rightarrow U_{34a} = 0.,$

$(2 , 4) \rightarrow U_{35a} = 0, (3 , 4) \rightarrow U_{45a} = 0,$

$$\sum U_{ija} = \frac{4}{5} + 2 + \frac{2}{5} + 0 + 0 + 0 = \frac{16}{5}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{16}{5} + \frac{19}{5} + \cdots + \frac{21}{5} + \frac{26}{5}}{24} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c}$$

$$= \frac{\left(\frac{16}{5} - 4\right)^2 + \left(\frac{19}{5} - 4\right)^2 + \cdots + \left(\frac{21}{5} - 4\right)^2 + \left(\frac{26}{5} - 4\right)^2}{24} = \frac{89}{30}$$

Table 69. t = 5, k = 4 & Peak =3 & Third Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	-	3	4	4	4	9/10
1	2	-	4	3	26/5		
1	3	-	2	4	4		
1	3	-	4	2	27/5		
1	4	-	2	3	21/5		
1	4	-	3	2	27/5		
2	1	-	3	4	14/5		
2	1	-	4	3	4		
2	3	-	1	4	19/5		
2	3	-	4	1	27/5		
2	4	-	1	3	4		
2	4	-	3	1	27/5		
3	1	-	2	4	13/5		
3	1	-	4	2	4		
3	2	-	1	4	13/5		
3	2	-	4	1	21/5		
3	4	-	1	2	4		
3	4	-	2	1	26/5		
4	1	-	2	3	13/5		
4	1	-	3	2	19/5		
4	2	-	1	3	13/5		
4	2	-	3	1	4		
4	3	-	1	2	14/5		
4	3	-	2	1	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$$(1 , 2) \rightarrow U_{12a} = 1; (1 , \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{5}\right) = \frac{8}{5}; (2, \_) \rightarrow U_{23a} = \left(1 - \frac{2}{5}\right) = \frac{3}{5};$$

$$(\_ , 3) \rightarrow U_{34a} = 1 - \frac{3}{5} = \frac{2}{5}., (\_ , 4) \rightarrow U_{35a} = 2 * \left(1 - \frac{4}{5}\right) = \frac{2}{5}, (3 , 4) \rightarrow U_{45a} = 0.$$

$$\sum U_{ija} = 1 + \frac{8}{5} + \frac{3}{5} + \frac{2}{5} + \frac{2}{5} + 0 = 4.$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{11}{5} + \frac{26}{5} + \cdots + \frac{14}{5} + 4}{24} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{(4-4)^2 + \left(\frac{26}{5} - 4\right)^2 + \cdots + \left(\frac{14}{5} - 4\right)^2 + (4-4)^2}{24} = \frac{9}{10}$$

Table 70. t = 5, k = 4 & Peak =3 & Fourth Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	-	4	24/5	4	53/10
1	2	4	-	3	36/5		
1	3	2	-	4	18/5		
1	3	4	-	2	37/5		
1	4	2	-	3	19/5		
1	4	3	-	2	31/5		
2	1	3	-	4	19/5		
2	1	4	-	3	31/5		
2	3	1	-	4	7/5		
2	3	4	-	1	38/5		
2	4	1	-	3	8/5		
2	4	3	-	1	32/5		
3	1	2	-	4	8/5		
3	1	4	-	2	32/5		
3	2	1	-	4	2/5		
3	2	4	-	1	33/5		
3	4	1	-	2	9/5		
3	4	2	-	1	21/5		
4	1	2	-	3	9/5		
4	1	3	-	2	21/5		
4	2	1	-	3	3/5		
4	2	3	-	1	22/5		
4	3	1	-	2	4/5		
4	3	2	-	1	16/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1, 2) \rightarrow U_{12a} = 1; (1, 3) \rightarrow U_{13a} = 2; (2, 3) \rightarrow U_{23a} = 1;$

$(3, \_) \rightarrow U_{34a} = \frac{3}{5}., (3, 4) \rightarrow U_{35a} = 0, (\_, 4) \rightarrow U_{45a} = 1 - \frac{4}{5} = \frac{1}{5}.$

$$\sum U_{ija} = 1 + 2 + 1 + \frac{3}{5} + 0 + \frac{1}{5} = \frac{24}{5}.$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{24}{5} + \frac{36}{5} + \cdots + \frac{4}{5} + \frac{16}{5}}{24} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{24}{5} - 4\right)^2 + \left(\frac{36}{5} - 4\right)^2 + \cdots + \left(\frac{4}{5} - 4\right)^2 + \left(\frac{16}{5} - 4\right)^2}{24} = \frac{53}{10}$$

Table 71. t = 5, k = 4 & Peak =3 and Fifth Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	4	-	6	4	41/10
1	2	4	3	-	36/5		
1	3	2	4	-	23/5		
1	3	4	2	-	7		
1	4	2	3	-	22/5		
1	4	3	2	-	28/5		
2	1	3	4	-	5		
2	1	4	3	-	31/5		
2	3	1	4	-	11/5		
2	3	4	1	-	34/5		
2	4	1	3	-	2		
2	4	3	1	-	27/5		
3	1	2	4	-	13/5		
3	1	4	2	-	6		
3	2	1	4	-	6/5		
3	2	4	1	-	29/5		
3	4	1	2	-	9/5		
3	4	2	1	-	3		
4	1	2	3	-	12/5		
4	1	3	2	-	18/5		
4	2	1	3	-	1		
4	2	3	1	-	17/5		
4	3	1	2	-	4/5		
4	3	2	1	-	2		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (1 , 3) \rightarrow U_{13a} = 2; (2 , 3) \rightarrow U_{23a} = 1;$

$(3 , 4) \rightarrow U_{34a} = 0., (3 , \_) \rightarrow U_{35a} = 2 * \frac{3}{5} = \frac{6}{5}, (4 , \_) \rightarrow U_{45a} = \frac{4}{5}.$

$$\sum U_{ija} = 1 + 2 + 1 + 0 + \frac{6}{5} + \frac{4}{5} = 6.$$

Therefore

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} \frac{6 + \frac{36}{5} + \cdots + \frac{4}{5} + 2}{24} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(6-4)^2 + \left(\frac{36}{5} - 4\right)^2 + \cdots + \left(\frac{4}{5} - 4\right)^2 + (2-4)^2}{24} = \frac{41}{10}$$

Table 72. t = 5, k=3, Peak =3 & Fourth & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	-	-	27/4	4	31/8
1	3	2	-	-	20/4		
2	1	3	-	-	23/4		
2	3	1	-	-	9/4		
3	1	2	-	-	12/4		
3	2	1	-	-	5/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (1 , 3) \rightarrow U_{13a} = 2; (2 , 3) \rightarrow U_{23a} = 1;$

$(3 , \_) \rightarrow U_{34a} = \frac{3}{4}; (3 , \_) \rightarrow U_{35a} = \frac{6}{4}, (\_ , \_) \rightarrow U_{45a} = \frac{2}{4}.$

$$\sum U_{ija} = 1 + 2 + 1 + \frac{3}{4} + \frac{6}{4} + \frac{2}{4} = \frac{27}{4}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{27}{4} + \cdots + \frac{5}{4}}{6} = 4$$

- Step 4

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{27}{4} - 4)^2 + + \cdots + \left(\frac{5}{4} - 4\right)^2}{6} = \frac{31}{8}$$

Table 73. t = 5, k=3, Peak =3 & Third & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	2	-	3	-	20/4	4	13/24
1	3	-	2	-	19/4		
2	1	-	3	-	15/4		
2	3	-	1	-	17/4		
3	1	-	2	-	13/4		
3	2	-	1	-	12/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , 2) → $U_{12a} = 1$; (1 , _) → $U_{13a} = 2 * \left(1 - \frac{1}{4}\right) = \frac{6}{4}$; (2 , _) → $U_{23a} = 1 - \frac{2}{4} = \frac{2}{4}$;

(_ , 3) → $U_{34a} = 1 - \frac{3}{4} = \frac{1}{4}$; (_ , _) → $U_{35a} = 2 * \frac{1}{2} = 1$;

;(3 , _) → $U_{45a} = \frac{3}{4}$.

$$\sum U_{ija} = 1 + \frac{6}{4} + \frac{2}{4} + \frac{1}{4} + 1 + \frac{3}{4} = \frac{20}{4}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{20}{4} + \frac{19}{4} + \cdots + \frac{12}{4}}{6} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{20}{4} - 4)^2 + \cdots + \left(\frac{12}{4} - 4\right)^2}{6} = \frac{13}{24}$$

Table 74. t = 5, k=3, Peak =3 & Second & Fifth Observations Missing

		Step 1			Step 2	Step 3	Step 4
1	-	2	3	-	5	4	15/4
1	-	3	2	-	26		
2	-	1	3	-	2		
2	-	3	1	-	6/4		
3	-	1	2	-	6/4		
3	-	2	1	-	3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}; (1 , 2) \rightarrow U_{13a} = 2; (\_ , 2) \rightarrow U_{23a} = \frac{2}{4};$

$(2 , 3) \rightarrow U_{34a} = 0; (2 , \_) \rightarrow U_{35a} = 2 * \frac{2}{4} = 1; \; (3 , \_) \rightarrow U_{45a} = \frac{3}{4};$

- Step 3:

$$\sum U_{ija} = \frac{3}{4} + 2 + \frac{2}{4} + 0 + 1 + \frac{3}{4} = 5$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{5 + \cdots + 3}{6} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{(5-4)^2 + \cdots + (3-4)^2}{6} = \frac{15}{4}$$

Table 75. t = 5, k=3, Peak =3 & Third & Fourth Observations Missing

		Step 1			Step 2	Step 3	Step 4
1	2	-	-	3	17/4	4	13/24
1	3	-	-	2	19/4		
2	1	-	-	3	12/4		
2	3	-	-	1	20/4		
3	1	-	-	2	13/4		
3	2	-	-	1	15/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (1 , \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{4}\right) = \frac{6}{4}; (2 , \_) \rightarrow U_{23a} = 1 - \frac{2}{4} = \frac{2}{4};$

$(\_ , \_) \rightarrow U_{34a} = \frac{1}{2}$; $(\_ , 3) \rightarrow U_{35a} = 2 * \left(1 - \frac{3}{4}\right) = \frac{2}{4}$;

$(\_ , 3) \rightarrow U_{45a} = 1 - \frac{3}{4} = \frac{1}{4}$.

- Step 3:

$$\sum U_{ija} = 1 + \frac{6}{4} + \frac{2}{4} + \frac{1}{2} + \frac{2}{4} + \frac{1}{4} = \frac{17}{4}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{17}{4} + \frac{19}{4} + \cdots + \frac{15}{4}}{6} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{17}{4} - 4)^2 + \left(\frac{19}{4} - 4\right)^2 + \cdots + \left(\frac{15}{4} - 4\right)^2}{6} = \frac{13}{24}$$

Table 76. t = 5, k=3, Peak =3 & Second & Third Observations Missing

Step 1					Step 2	Step 3	Step 4
1	-	-	2	3	15/4	4	13/24
1	-	-	3	2	5		
2	-	-	1	3	13/4		
2	-	-	3	1	19/4		
3	-	-	1	2	3		
3	-	-	2	1	17/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}$; $(1 , \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{4}\right) = \frac{6}{4}$; $(\_ , \_) \rightarrow U_{23a} = \frac{1}{2}$;

$(\_ , 2) \rightarrow U_{34a} = 1 - \frac{2}{4} = \frac{2}{4}$; $(\_ , 3) \rightarrow U_{35a} = 2 * \left(1 - \frac{3}{4}\right) = \frac{2}{4}$; $(2 ,\ 3) \rightarrow U_{45a} = 0$;

- Step 3:

$$\sum U_{ija} = \frac{3}{4} + \frac{6}{4} + \frac{1}{2} + \frac{2}{4} + \frac{2}{4} + 0 = \frac{15}{4}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{15}{4} + 5 + \cdots + \frac{17}{4}}{6} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{15}{4} - 4)^2 + \cdots + \left(\frac{17}{4} - 4\right)^2}{6} = \frac{13}{12}$$

Table 77. t = 5, k=3, Peak =3 & Second Fourth Observations Missing

	Step 1				Step 2	Step 3	Step 4
1	-	2	-	3	4	4	121/24
1	-	3	-	2	27/4		
2	-	1	-	3	5/4		
2	-	3	-	1	27/4		
3	-	1	-	2	5/4		
3	-	2	-	1	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , _) → $U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}$; (1 , 2) → $U_{13a} = 2$; (_ , 2) → $U_{23a} = \frac{2}{4}$;

(2 , _) → $U_{34a} = \frac{2}{4}$; (2 , 3) → $U_{35a} = 0$; (_ , 3) → $U_{45a} = 1 - \frac{3}{4} = \frac{1}{4}$;

(_ , 3) → $U_{45a} = 1 - \frac{3}{4} = \frac{1}{4}$

- Step 3:

$$\sum U_{ija} = \frac{3}{4} + 2 + \frac{2}{4} + \frac{2}{4} + 0 + \frac{1}{4} = 4$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{4 + \frac{27}{4} + \cdots + 4}{6} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(4-4)^2 + (\frac{27}{4} - 4)^2 \ldots + (4-4)^2}{6} = \frac{121}{24}$$

Table 78. t = 5, k=3, Peak =3 & First & Third Observations Missing

Step 1					Step 2	Step 3	Step 4
-	1	-	2	3	3	4	13/24
-	1	-	3	2	17/4		
-	2	-	1	3	13/4		
-	2	-	3	1	19/4		
-	3	-	1	2	15/4		
-	3	-	2	1	5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ , 1) \rightarrow U_{12a} = \frac{1}{4}; (\_ , \_) \rightarrow U_{13a} = 2 * \frac{1}{2} = 1 , (\_ , 1) \rightarrow U_{23a} = \frac{3}{4};$

$(\_ , 2) \rightarrow U_{34a} = \frac{2}{4}; (\_ , 3) \rightarrow U_{35a} = 2 * \left(1 - \frac{3}{4}\right) = \frac{2}{4}; (2 , 3) \rightarrow U_{45a} = 0$

$$\sum U_{ija} = \frac{1}{4} + 1 + \frac{3}{4} + \frac{2}{4} + \frac{2}{4} + 0 = 3.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{3 + \cdots + 5}{6} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{(3-4)^2 + \cdots + (5-4)^2}{6} = \frac{13}{24}$$

Table 79. t = 5, k=3, Peak =3 & First & Fourth Observations Missing

Step 1					Step 2	Step 3	Step 4
-	1	2	-	3	3	4	15/4
-	1	3	-	2	6		
-	2	1	-	3	6/4		
-	2	3	-	1	26/4		
-	3	1	-	2	2		
-	3	2	-	1	5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ , 1) \rightarrow U_{12a} = \frac{1}{4}; (\_ , 2) \rightarrow U_{13a} = 2 * \frac{2}{4} = 1; (1 , 2) \rightarrow U_{23a} = 1; (2 , \_) \rightarrow U_{34a} = \frac{2}{4};$

(2 , 3)→ $U_{35a} = 0$; (_ , 3)→ $U_{45a} = 1 - \frac{3}{4} = \frac{1}{4}$

$$\sum U_{ija} = \frac{1}{4} + 1 + 1 + \frac{2}{4} + 0 + \frac{1}{4} = 3.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{3 + \cdots + 5}{6} = 4$$

- Step 4

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(3-4)^2 + \cdots + (5-4)^2}{6} = \frac{15}{4}$$

Table 80. t = 5, k=3, Peak =3 & First & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
-	1	2	3	-	4	4	49/24
-	1	3	2	-	23/4		
-	2	1	3	-	9/4		
-	2	3	1	-	23/4		
-	3	1	2	-	9/4		
-	3	2	1	-	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{4}$; (_ , 2)→ $U_{13a} = 2 * \frac{2}{4} = 1$; (1 , 2) → $U_{23a} = 1$;

(2 , 3) → $U_{34a} = 0$;(2 , _)→ $U_{35a} = 2 * \frac{2}{4} = 1$; (3 , _)→ $U_{45a} = \frac{3}{4}$

$$\sum U_{ija} = \frac{1}{4} + 1 + 1 + 0 + 1 + \frac{3}{4} = 4.$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{4 + \frac{23}{4} + \cdots + 4}{6} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(4-4)^2 + \left(\frac{23}{4}-4\right)^2 + \cdots + (4-4)^2}{6} = \frac{49}{24}$$

Table 81. t = 5, k=3, Peak =3 & First & Second Observations Missing

		Step 1			Step 2	Step 3	Step 4
-	-	1	2	3	5/4	4	31/8
-	-	1	3	2	9/4		
-	-	2	1	3	3		
-	-	2	3	1	5		
-	-	3	1	2	23/4		
-	-	3	2	1	27/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , _)→ $U_{12a} = \frac{1}{2}$; (_ , 1)→ $U_{13a} = 2 * \frac{1}{4} = \frac{2}{4}$; (_ , 1) → $U_{23a} = \frac{1}{4}$;

(1 , 2) → $U_{34a} = 0$; (1 , 3)→ $U_{35a} = 0$; (2 , 3)→ $U_{45a} = 0$;

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{4} + \frac{1}{4} + 0 + 0 + 0 = \frac{5}{4}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{5}{4} + \cdots + \frac{27}{4}}{6} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{5}{4}-4\right)^2 + \cdots + \left(\frac{27}{4}-4\right)^2}{6} = \frac{31}{8}$$

Table 82. t = 5, k=2, Peak = 3 & Third, Fourth & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	2	-	-	-	14/3	4	4/9
2	1	-	-	-	10/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (1 , \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}; (2 , \_) \rightarrow U_{24a} = 1 - \frac{2}{3} = \frac{1}{3};$

$(\_ , \_) \rightarrow U_{34a} = \frac{1}{2}; (\_ , \_) \rightarrow U_{35a} = 2 * \frac{1}{2} = 1; (\_ , \_) \rightarrow U_{45a} = \frac{1}{2};$

$$\sum U_{ija} = 1 + \frac{4}{3} + \frac{1}{3} + \frac{1}{2} + 1 + \frac{1}{2} = \frac{14}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{14}{3} + \frac{10}{3}}{2} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{\left(\frac{14}{3} - 4\right)^2 + \left(\frac{10}{3} - 4\right)^2}{2} = \frac{4}{9}$$

Table 83. t = 5, k=2, Peak = 3 & Second, Fourth & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	-	2	-	-	35/6	4	125/72
2	-	1	-	-	13/6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}; (1 , 2) \rightarrow U_{13a} = 2; (\_ , 2) \rightarrow U_{23a} = \frac{2}{3};$

$(2 , \_) \rightarrow U_{34a} = \frac{2}{3}; (2 , \_) \rightarrow U_{35a} = 2 * \frac{2}{3} = \frac{4}{3}; (\_ , \_) \rightarrow U_{45a} = \frac{1}{2}$

$$\sum U_{ija} = \frac{2}{3} + 2 + \frac{2}{3} + \frac{2}{3} + \frac{4}{3} + \frac{1}{2} = \frac{35}{6}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{35}{6} + \frac{13}{6}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{35}{6} - 4\right)^2 + \left(\frac{13}{6} - 4\right)^2}{2} = \frac{125}{72}$$

Table 84. t = 5, k=2, Peak = 3 & Second, Third & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	-	-	2	-	9/2	4	1/4
2	-	-	1	-	7/2		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , _)→ $U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; (1 , _)→ $U_{13a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$;

(_ , _) → $U_{23a} = \frac{1}{2}$; (_, 2) → $U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$; (_ , _)→ $U_{35a} = 1$;

(2_ , _)→ $U_{45a} = \frac{2}{3}$;

$$\sum U_{ija} = \frac{2}{3} + \frac{4}{3} + \frac{1}{2} + \frac{1}{3} + 1 + \frac{2}{3} = \frac{9}{2}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{9}{2} + \frac{7}{2}}{2} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{9}{2} - 4\right)^2 + \left(\frac{7}{2} - 4\right)^2}{2} = \frac{1}{4}$$

Table 85. t = 5, k=2, Peak = 3 & Second, Third & Fourth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	-	-	-	2	4	4	0
2	-	-	-	1	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , _)→ $U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; (1, _)→ $U_{13a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$;

(_ , _) → $U_{23a} = \frac{1}{2}$; (_, _) → $U_{34a} = \frac{1}{2}$; (_ , 2)→ $U_{35a} = 2 * \left(1 - \frac{2}{3}\right) = \frac{2}{3}$;

(_ , 2)→ $U_{45a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{2}{3} + \frac{4}{3} + \frac{1}{2} + \frac{1}{2} + \frac{2}{3} + \frac{1}{3} = 4$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{4+4}{2} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(4-4)^2 + (4-4)^2}{2} = 0$$

Table 86. t = 5, k=2, Peak = 3 & First, Fourth & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	1	2	-	-	31/6	4	1/3
-	2	1	-	-	17/6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{3}$; (_ , 2)→ $U_{13a} = 2 * \frac{2}{3} = \frac{4}{3}$; (1 , 2) → $U_{23a} = 1$;

(2 , _) → $U_{34a} = \frac{2}{3}$; (2 , _)→ $U_{35a} = 2 * \frac{2}{3} = \frac{4}{3}$; (_ , _)→ $U_{45a} = \frac{1}{2}$

$$\sum U_{ija} = \frac{1}{3} + \frac{4}{3} + 1 + \frac{2}{3} + \frac{4}{3} + \frac{1}{2} = \frac{31}{6}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{31}{6} + \frac{17}{6}}{2} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{\left(\frac{31}{6} - 4\right)^2 + \left(\frac{17}{6} - 4\right)^2}{2} = \frac{1}{3}$$

Table 87. t = 5, k=2, Peak = 3 & First, Third & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	1	-	2	-	4	4	0
-	2	-	1	-	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{3}$; (_ , _)→ $U_{13a} = 2 * \frac{1}{2} = 1$; (1 , _) → $U_{23a} = 1 - \frac{1}{3} = \frac{2}{3}$

(_ , 2) → $U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$; (_ , _)→ $U_{35a} = 2 * \frac{1}{2} = 1$; (2 , _) → $U_{45a} = \frac{2}{3}$

$$\sum U_{ija} = \frac{1}{3} + 1 + \frac{2}{3} + \frac{1}{3} + 1 + \frac{2}{3} = 4$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{4 + 4}{2} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{(4-4)^2 + (4-4)^2}{2} = 0$$

Table 88. t = 5, k=2, Peak = 3 & First, Third & Fourth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	1	-	-	2	7/2	4	1/4
-	2	-	-	1	9/2		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ , 1) \rightarrow U_{12a} = \frac{1}{3}; (\_ , \_) \rightarrow U_{13a} = 2 * \frac{1}{2} = 1; (1 , \_) \rightarrow U_{23a} = 1 - \frac{1}{3} = \frac{2}{3}$

$(\_ , \_) \rightarrow U_{34a} = \frac{1}{2}; (\_ , 2) \rightarrow U_{35a} = 2 * \left(1 - \frac{2}{3}\right) = \frac{2}{3}; (\_ , 2) \rightarrow U_{45a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{1}{3} + 1 + \frac{2}{3} + \frac{1}{2} + \frac{2}{3} + \frac{1}{3} = \frac{7}{2}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{7}{2} + \frac{9}{2}}{2} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{7}{2} - 4\right)^2 + \left(\frac{9}{2} - 4\right)^2}{2} = \frac{1}{4}$$

Table 89. t = 5, k=2, Peak = 3 & First, Second, & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	-	1	2	-	17/6	4	1/3
-	-	2	1	-	31/6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ , \_) \rightarrow U_{12a} = \frac{1}{2}; (\_ , 1) \rightarrow U_{13a} = 2 * \frac{1}{3} = \frac{2}{3}; (\_ , 1) \rightarrow U_{23a} = \frac{1}{3}$

$(1 , 2) \rightarrow U_{34a} = 0; (1 , \_) \rightarrow U_{35a} = 2 * \frac{1}{3} = \frac{2}{3};\ (2 , \_) \rightarrow U_{45a} = \frac{2}{3}$

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{3} + \frac{1}{3} + 0 + \frac{2}{3} + \frac{2}{3} = \frac{17}{6}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{17}{6} + \frac{31}{6}}{2} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{17}{6} - 4\right)^2 + \left(\frac{31}{6} - 4\right)^2}{2} = \frac{1}{3}$$

Table 90. t = 5, k=2, Peak = 3 & First, Second, & Fourth Observations Missing

Step 1		Step 2			Step 2	Step 3	Step 4
-	-	1	-	2	13/6	4	125/72
-	-	2	-	1	35/6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , _)→ $U_{12a} = \frac{1}{2}$; (_ , 1)→ $U_{13a} = 2 * \frac{1}{3} = \frac{2}{3}$; (_ , 1) → $U_{23a} = \frac{1}{3}$

(1 , _)→ $U_{34a} = \frac{1}{3}$; (1 , 2)→ $U_{35a} = 0$; (_ , 2) → $U_{45a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{3} + \frac{1}{3} + \frac{1}{3} + 0 + \frac{1}{3} = \frac{13}{6}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{13}{6} + \frac{35}{6}}{2} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{13}{6} - 4\right)^2 + \left(\frac{35}{6} - 4\right)^2}{2} = \frac{125}{72}$$

Table 91. t = 5, k=2, Peak = 3 & First, Second & Third Observations Missing

Step 1		Step 2			Step 2	Step 3	Step 4
-	-	-	1	2	10/3	4	4/9
-	-	-	2	1	14/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ ; \_) \rightarrow U_{12a} = \frac{1}{2}; (\_ , \_) \rightarrow U_{13a} = 2 * \frac{1}{2} = 1; (\_ , \_) \rightarrow U_{23a} = \frac{1}{2}$

$(\_ , 1) \rightarrow U_{34a} = 1 - \frac{1}{3} = \frac{2}{3}; (\_ , 2) \rightarrow U_{35a} = 2 * \left(1 - \frac{2}{3}\right) = \frac{2}{3}; (1 , 2) \rightarrow U_{45a} = 0$

$$\sum U_{ija} = \frac{1}{2} + 1 + \frac{1}{2} + \frac{2}{3} + \frac{2}{3} + 0 = \frac{10}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{10}{3} + \frac{14}{3}}{2} = 4$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{10}{3} - 4\right)^2 + \left(\frac{14}{3} - 4\right)^2}{2} = \frac{4}{9}$$

Now consider the scenario where only one observation is missing, and peak is at four

Table 92. t = 5, k = 4 & Peak =4 & First Observation Missing

Step 1					Step 2	Step 3	Step 4
-	1	2	3	4	34/5	11/2	25/4
-	1	2	4	3	42/5		
-	1	3	2	4	28/5		
-	1	3	4	2	44/5		
-	1	4	2	3	6		
-	1	4	3	2	38/5		
-	2	1	3	4	28/5		
-	2	1	4	3	36/5		
-	2	3	1	4	16/5		
-	2	3	4	1	9		
-	2	4	1	3	18/5		
-	2	4	3	1	39/5		
-	3	1	2	4	16/5		
-	3	1	4	2	37/5		
-	3	2	1	4	2		
-	3	2	4	1	39/5		
-	3	4	1	2	19/5		
-	3	4	2	1	27/5		
-	4	1	2	3	17/5		
-	4	1	3	2	5		
-	4	2	1	3	11/5		
-	4	2	3	1	27/5		
-	4	3	1	2	13/5		
-	4	3	2	1	21/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_ , 1) \rightarrow U_{12a} = \frac{1}{5}$; $(\_ , 2) \rightarrow U_{13a} = 2 * \frac{2}{5} = \frac{4}{5}$; $(\_ , 3) \rightarrow U_{14a} = 3 * \frac{3}{5} = \frac{9}{5}$;

$(1 , 2) \rightarrow U_{23a} = 1.$, $(1 , 3) \rightarrow U_{24a} = 2$, $(2 , 3) \rightarrow U_{34a} = 1$,

$(3 , 4) \rightarrow U_{45a} = 0.$

$$\sum U_{ija} = \frac{1}{5} + \frac{4}{5} + \frac{9}{5} + 1 + 2 + 1 + 0 = \frac{34}{5}$$

- Step 3 :

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{34}{5} + \frac{42}{5} + \cdots + \frac{13}{5} + \frac{21}{5}}{24} = \frac{11}{2}$$

- Step 4 :

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c}$$

$$= \frac{\left(\frac{34}{5} - \frac{11}{2}\right)^2 + \left(\frac{42}{5} - \frac{11}{2}\right)^2 + \cdots + \left(\frac{13}{5} - \frac{11}{2}\right)^2 + \left(\frac{21}{5} - \frac{11}{2}\right)^2}{24} = \frac{25}{4}$$

Table 93. t = 5, k = 4 & Peak =4 & Second Observation Missing

Step 1					Step 2	Step 3	Step 4
1	-	2	3	4	42/5	11/2	103/12
1	-	2	4	3	49/5		
1	-	3	2	4	36/5		
1	-	3	4	2	10		
1	-	4	2	3	37/5		
1	-	4	3	2	44/5		
2	-	1	3	4	6		
2	-	1	4	3	37/5		
2	-	3	1	4	18/5		
2	-	3	4	1	49/5		
2	-	4	1	3	19/5		
2	-	4	3	1	43/5		
3	-	1	2	4	12/5		
3	-	1	4	2	36/5		
3	-	2	1	4	6/5		
3	-	2	4	1	37/5		
3	-	4	1	2	18/5		
3	-	4	2	1	5		
4	-	1	2	3	11/5		
4	-	1	3	2	18/5		
4	-	2	1	3	1		
4	-	2	3	1	19/5		
4	-	3	1	2	6/5		
4	-	3	2	1	13/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{5} = \frac{4}{5}$; $(1 , 2) \rightarrow U_{13a} = 2$;$(1 , 3) \rightarrow U_{14a} = 3$;

$(\_ , 2) \rightarrow U_{23a} = \frac{2}{5}$., $(\_ , 3) \rightarrow U_{24a} = 2 * \frac{3}{5} = \frac{6}{5}$; $(2 , 3) \rightarrow U_{34a} = 1$;

$(3 , 4) \rightarrow U_{45a} = 0$., $\sum U_{ija} = \frac{4}{5} + 2 + 3 + \frac{2}{5} + \frac{6}{5} + 1 + 0 = \frac{42}{5}$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{42}{5} + \frac{49}{5} + \cdots + \frac{6}{5} + \frac{13}{5}}{24} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{42}{5}-\frac{11}{2}\right)^2 + \left(\frac{49}{5}-\frac{11}{2}\right)^2 + \cdots + \left(\frac{6}{5}-\frac{11}{2}\right)^2 + \left(\frac{13}{5}-\frac{11}{2}\right)^2}{24}$$

$$= \frac{103}{12}$$

Table 94. t = 5, k = 4 & Peak =4 & Third Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	-	3	4	44/5	4	9/10
1	2	-	4	3	10		
1	3	-	2	4	32/5		
1	3	-	4	2	49/5		
1	4	-	2	3	31/5		
1	4	-	3	2	37/5		
2	1	-	3	4	38/5		
2	1	-	4	3	44/5		
2	3	-	1	4	14/5		
2	3	-	4	1	47/5		
2	4	-	1	3	13/5		
2	4	-	3	1	8		
3	1	-	2	4	3		
3	1	-	4	2	42/5		
3	2	-	1	4	8/5		
3	2	-	4	1	41/5		
3	4	-	1	2	11/5		
3	4	-	2	1	17/5		
4	1	-	2	3	18/5		
4	1	-	3	2	24/5		
4	2	-	1	3	6/5		
4	2	-	3	1	23/5		
4	3	-	1	2	1		
4	3	-	2	1	11/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1$; $(1 , \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{5}\right) = \frac{8}{5}$; $(1, 3) \rightarrow U_{23a} = 3$;

$(2, \_) \rightarrow U_{23a} = \left(1 - \frac{2}{5}\right) = \frac{3}{5}$; $(2 , 3) \rightarrow U_{24a} = 2.$,

$(\_ , 3) \rightarrow U_{34a} = \frac{3}{5}$, $(3 , 4) \rightarrow U_{45a} = 0.$

$$\sum U_{ija} = 1 + \frac{8}{5} + 3 + \frac{3}{5} + 2 + \frac{3}{5} + 0 = \frac{44}{5}.$$

- Step 3 :

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{44}{5} + 10 + \cdots + 1 + \frac{11}{5}}{24} = \frac{11}{2}$$

- Step 4 :

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{44}{5} - \frac{11}{2}\right)^2 + \left(10 - \frac{11}{2}\right)^2 + \cdots + \left(1 - \frac{11}{2}\right)^2 + \left(\frac{11}{5} - \frac{11}{2}\right)^2}{24}$$

$$= \frac{107}{12}$$

Table 95. t = 5, k = 4 & Peak =4 & Fourth Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	-	4	41/5	11/2	191/60
1	2	4	-	3	41/5		
1	3	2	-	4	7		
1	3	4	-	2	8		
1	4	2	-	3	34/5		
1	4	3	-	2	34/5		
2	1	3	-	4	7		
2	1	4	-	3	7		
2	3	1	-	4	23/5		
2	3	4	-	1	38/5		
2	4	1	-	3	22/5		
2	4	3	-	1	32/5		
3	1	2	-	4	23/5		
3	1	4	-	2	33/5		
3	2	1	-	4	17/5		
3	2	4	-	1	32/5		
3	4	1	-	2	4		
3	4	2	-	1	4		
4	1	2	-	3	21/5		
4	1	3	-	2	21/5		
4	2	1	-	3	3		
4	2	3	-	1	4		
4	3	1	-	2	14/5		
4	3	2	-	1	14/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (1 , 3) \rightarrow U_{13a} = 2; (1 , \_) \rightarrow U_{14a} = 3 * \left(1 - \frac{1}{5}\right) = \frac{12}{5};$

$(2 , 3) \rightarrow U_{23a} = 1; (2 , \_) \rightarrow U_{24a} = 2 * \left(1 - \frac{2}{5}\right) = \frac{6}{5},$

$(3 , \_) \rightarrow U_{34a} = 1 - \frac{3}{5} = \frac{2}{5}.; (\_ , 4) \rightarrow U_{45a} = 1 - \frac{4}{5} = \frac{1}{5}.$

$$\sum U_{ija} = 1 + 2 + \frac{12}{5} + 1 + \frac{6}{5} + \frac{2}{5} + \frac{1}{5} = \frac{41}{5}.$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{41}{5} + \frac{41}{5} + \cdots + \frac{14}{5} + \frac{14}{5}}{24} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{41}{5} - \frac{11}{2}\right)^2 + \left(\frac{41}{5} - \frac{11}{2}\right)^2 + \cdots + \left(\frac{14}{5} - \frac{11}{2}\right)^2 + \left(\frac{14}{5} - \frac{11}{2}\right)^2}{24}$$

$$= \frac{191}{60}$$

Table 96. t = 5, k = 4 & Peak =4 and Fifth Observation Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	4	-	54/5	11/2	563/60
1	2	4	3	-	48/5		
1	3	2	4	-	49/5		
1	3	4	2	-	37/5		
1	4	2	3	-	38/5		
1	4	3	2	-	32/5		
2	1	3	4	-	49/5		
2	1	4	3	-	43/5		
2	3	1	4	-	39/5		
2	3	4	1	-	21/5		
2	4	1	3	-	28/5		
2	4	3	1	-	39/5		
3	1	2	4	-	27/5		
3	1	4	2	-	34/5		
3	2	1	4	-	34/5		
3	2	4	1	-	16/5		
3	4	1	2	-	12/5		
3	4	2	1	-	6/5		
4	1	2	3	-	23/5		
4	1	3	2	-	17/5		
4	2	1	3	-	18/5		
4	2	3	1	-	6/5		
4	3	1	2	-	7/5		
4	3	2	1	-	1/5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (1 , 3) \rightarrow U_{13a} = 2; (1 , 4) \rightarrow U_{14a} = 3;$

$(2 , 3) \rightarrow U_{23a} = 1., (2 , 4) \rightarrow U_{24a} = 2, (3 , 4) \rightarrow U_{34a} = 1;$

$(4 , \_) \rightarrow U_{45a} = \frac{4}{5}$

$$\sum U_{ija} = 1 + 2 + 3 + 1 + 2 + 1 + \frac{4}{5} = \frac{54}{5}.$$

Therefore

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} \frac{\frac{54}{5} + \frac{48}{5} + \cdots + \frac{7}{5} + \frac{1}{5}}{24} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{\left(\frac{54}{5} - \frac{11}{2}\right)^2 + \left(\frac{48}{5} - \frac{11}{2}\right)^2 + \cdots + \left(\frac{7}{5} - \frac{11}{2}\right)^2 + \left(\frac{1}{5} - \frac{11}{2}\right)^2}{24}$$

$$= \frac{563}{60}$$

Table 97. t = 5, k=3, Peak =4 & Fourth & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	2	3	-	-	8	11/2	25/8
1	3	2	-	-	27/4		
2	1	3	-	-	27/4		
2	3	1	-	-	17/4		
3	1	2	-	-	17/4		
3	2	1	-	-	3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (1 , 3) \rightarrow U_{13a} = 2; (1 , \_) \rightarrow U_{14a} = 3 * \left(1 - \frac{1}{4}\right) = \frac{9}{4};$

$(2 , 3) \rightarrow U_{23a} = 1; (2 , \_) \rightarrow U_{24a} = 2 * \left(1 - \frac{2}{4}\right) = 1;$

(3 , _) $\rightarrow U_{34a} = 1 - \frac{3}{4} = \frac{1}{4}$;(_ , _) $\rightarrow U_{45a} = \frac{1}{2}$.

$$\sum U_{ija} = 1 + 2 + \frac{9}{4} + 1 + 1 + \frac{1}{4} + \frac{1}{2} = 8$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{8 + \cdots + 3}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(8-5.5)^2 + \cdots + (3-5.5)^2}{6} = \frac{25}{8}$$

Table 98. t = 5, k=3, Peak =4 & Third & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	2	-	3	-	38/4	11/2	67/8
1	3	-	2	-	27/4		
2	1	-	3	-	33/4		
2	3	-	1	-	11/4		
3	1	-	2	-	17/4		
3	2	-	1	-	6/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , 2) $\rightarrow U_{12a} = 1$; (1 , _) $\rightarrow U_{13a} = 2 * \left(1 - \frac{1}{4}\right) = \frac{6}{4}$; (1 , 3) $\rightarrow U_{14a} = 3$;

(2 , _) $\rightarrow U_{23a} = 1 - \frac{2}{4} = \frac{2}{4}$; (2 , 3) $\rightarrow U_{24a} = 2$; (_ , 3) $\rightarrow U_{34a} = \frac{3}{4}$;

(3 , _) $\rightarrow U_{45a} = \frac{3}{4}$.

$$\sum U_{ija} = 1 + \frac{6}{4} + 3 + \frac{2}{4} + 2 + \frac{3}{4} + \frac{3}{4} = \frac{38}{4}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{38}{4} + \cdots + \frac{6}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{38}{4} - 5.5)^2 + \cdots + \left(\frac{6}{4} - 5.5\right)^2}{6} = \frac{67}{8}$$

Table 99. t = 5, k=3, Peak =4 & Second & Fifth Observations Missing

		Step 1			Step 2	Step 3	Step 4
1	-	2	3	-	38/4	11/2	49/6
1	-	3	2	-	8		
2	-	1	3	-	7		
2	-	3	1	-	4		
3	-	1	2	-	3		
3	-	2	1	-	6/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

  (1 , _) → $U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}$; (1 , 2) → $U_{13a} = 2$; (1 , 3) → $U_{14a} = 3$;

  (_ , 2) → $U_{23a} = \frac{2}{4}$; (_ , 3) → $U_{24a} = 2 * \frac{3}{4} = \frac{6}{4}$; (2 , 3) → $U_{34a} = 1$;

  (3 , _) → $U_{45a} = \frac{3}{4}$.

- Step 3:

$$\sum U_{ija} = \frac{3}{4} + 2 + 3 + \frac{2}{4} + \frac{6}{4} + 1 + \frac{3}{4} = \frac{38}{4}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{38}{4} + \cdots + \frac{6}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{38}{4} - 5.5)^2 + \cdots + \left(\frac{6}{4} - 5.5\right)^2}{6} = \frac{49}{6}$$

Table 100. t = 5, k=3, Peak =4 & Third & Fourth Observations Missing

Step 1					Step 2	Step 3	Step 4
1	2	-	-	3	7	11/2	13/12
1	3	-	-	2	26/4		
2	1	-	-	3	22/4		
2	3	-	-	1	22/4		
3	1	-	-	2	18/4		
3	2	-	-	1	4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , 2) \rightarrow U_{12a} = 1; (1 , \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{4}\right) = \frac{6}{4};$

$(1 , \_) \rightarrow U_{14a} = 3 * \left(1 - \frac{1}{4}\right) = \frac{9}{4}; (2 , \_) \rightarrow U_{23a} = 1 - \frac{2}{4} = \frac{2}{4};$

$(2 , \_) \rightarrow U_{24a} = 2 * \left(1 - \frac{2}{4}\right) = 1; (\_ , \_) \rightarrow U_{34a} = \frac{2}{4} ;(\_ , 3) \rightarrow U_{45a} = 1 - \frac{3}{4} = \frac{1}{4}$

- Step 3:

$$\sum U_{ija} = 1 + \frac{6}{4} + \frac{9}{4} + \frac{2}{4} + 1 + \frac{2}{4} + \frac{1}{4} = 7$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{7 + \cdots + 4}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(7 - 5.5)^2 + \cdots + (4 - 5.5)^2}{6} = \frac{13}{12}$$

Table 101. t = 5, k=3, Peak =4 & Second & Third Observations Missing

Step 1					Step 2	Step 3	Step 4
1	-	-	2	3	29/4	11/2	61/8
1	-	-	3	2	9		
2	-	-	1	3	11/4		
2	-	-	3	1	33/4		
3	-	-	1	2	2		
3	-	-	2	1	15/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}$; $(1 , \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{4}\right) = \frac{6}{4}$; $(1 , 2) \rightarrow U_{14a} = 3$;

$(\_ , \_) \rightarrow U_{23a} = \frac{1}{2}$; $(\_ , 2) \rightarrow U_{24a} = 2 * \frac{2}{4} = 1$; $(\_ , 2) \rightarrow U_{34a} = \frac{2}{4}$;

$(2 , 3) \rightarrow U_{45a} = 0$

- Step 3:

$$\sum U_{ija} = \frac{3}{4} + \frac{6}{4} + 3 + \frac{1}{2} + 1 + \frac{2}{4} + 0 = \frac{29}{4}{=}4$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{29}{4} + \cdots + \frac{15}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c} (M_a - E(M_i))^2}{c} = \frac{(\frac{29}{4} - 5.5)^2 + \cdots + \left(\frac{15}{4} - 5.5\right)^2}{6} = \frac{61}{8}$$

Table 102. t = 5, k=3, Peak =4 & Second & Fourth Observations Missing

		Step 1			Step 2	Step 3	Step 4
1	-	2	-	3	29/4	11/2	23/8
1	-	3	-	2	30/4		
2	-	1	-	3	17/4		
2	-	3	-	1	27/4		
3	-	1	-	2	14/4		
3	-	2	-	1	15/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{4} = \frac{3}{4}$; $(1 , 2) \rightarrow U_{13a} = 2$; $(1 , \_) \rightarrow U_{14a} = 3 * \left(1 - \frac{1}{4}\right) = \frac{9}{4}$;

$(\_ , 2) \rightarrow U_{23a} = \frac{2}{4}$; $(\_ , \_) \rightarrow U_{24a} = 2 * \frac{1}{2} = 1$; $(2 , \_) \rightarrow U_{34a} = 1 - \frac{2}{4} = \frac{2}{4}$

$(\_ , 3) \rightarrow U_{45a} = 1 - \frac{3}{4} = \frac{1}{4}$

- Step 3:

$$\sum U_{ija} = \frac{3}{4} + 2 + \frac{9}{4} + \frac{2}{4} + 1 + \frac{2}{4} + \frac{1}{4} = \frac{29}{4}$$

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{29}{4} + \cdots + \frac{15}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(\frac{29}{4} - 5.5)^2 + \cdots + \left(\frac{15}{4} - 5.5\right)^2}{6} = \frac{23}{8}$$

Table 103. t = 5, k=3, Peak =4 & First & Third Observations Missing

Step 1					Step 2	Step 3	Step 4
-	1	-	2	3	6	11/2	17/4
-	1	-	3	2	8		
-	2	-	1	3	3		
-	2	-	3	1	8		
-	3	-	1	2	3		
-	3	-	2	1	5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{4}$; (_ , _) → $U_{13a} = 2 * \frac{1}{2} = 1$, (_ , 2) → $U_{14a} = 3 * \frac{2}{4} = \frac{6}{4}$;

(1 , _) → $U_{23a} = 1 - \frac{1}{4} = \frac{3}{4}$; (1 , 2)→ $U_{24a} = 2$; (- , 2) → $U_{34a} = \frac{2}{4}$,

(2 , 3) → $U_{45a} = 0$;

$$\sum U_{ija} = \frac{1}{4} + 1 + \frac{6}{4} + \frac{3}{4} + 2 + \frac{2}{4} + 0 = 6.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{6 + \cdots + 5}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(6-5.5)^2 + \cdots + (5-5.5)^2}{6} = \frac{17}{4}$$

Table 104. t = 5, k=3, Peak =4 & First & Fourth Observations Missing

Step 1					Step 2	Step 3	Step 4
-	1	2	-	3	6	11/2	3/4
-	1	3	-	2	26/4		
-	2	1	-	3	18/4		
-	2	3	-	1	26/4		
-	3	1	-	2	18/4		
-	3	2	-	1	5		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{4}$; (_ , 2)→ $U_{13a} = 2 * \frac{2}{4} = 1$; (_ , _) → $U_{14a} = 3 * \frac{1}{2} = \frac{3}{2}$;

(1 , 2) → $U_{23a} = 1$; (1 , _) → $U_{24a} = 2 * \left(1 - \frac{1}{4}\right) = \frac{6}{4}$; (2 , _)→ $U_{34a} = 1 - \frac{2}{4} = \frac{2}{4}$;

(_ , 3) → $U_{45a} = 1 - \frac{3}{4} = \frac{1}{4}$

$$\sum U_{ija} = \frac{1}{4} + 1 + \frac{3}{2} + 1 + \frac{6}{4} + \frac{2}{4} + \frac{1}{4} = 6.$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{6 + \cdots + 5}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(6-5.5)^2 + \cdots + (5-5.5)^2}{6} = \frac{3}{4}$$

Table 105. t = 5, k=3, Peak =4 & First & Fifth Observations Missing

Step 1					Step 2	Step 3	Step 4
-	1	2	3	-	33/4	11/2	19/24
-	1	3	2	-	27/4		
-	2	1	3	-	7		
-	2	3	1	-	4		
-	3	1	2	-	27/4		
-	3	2	1	-	11/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{4}$; (_ , 2)→ $U_{13a} = 2 * \frac{2}{4} = 1$; (_ , 3) → $U_{14a} = 3 * \frac{3}{4} = \frac{9}{4}$;

(1 , 2) → $U_{23a} = 1$; (1 , 3)→ $U_{24a} = 2$; (2 , 3)→ $U_{34a} = 1$; (3 , _) → $U_{45a} = \frac{3}{4}$

$$\sum U_{ija} = \frac{1}{4} + 1 + \frac{9}{4} + 1 + 2 + 1 + \frac{3}{4} = \frac{33}{4}.$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{33}{4} + \cdots + \frac{11}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{33}{4} - 5.5\right)^2 + \cdots + \left(\frac{11}{4} - 5.5\right)^2}{6} = \frac{19}{24}$$

Table 106. t = 5, k=3, Peak =4 & First & Second Observations Missing

Step 1					Step 2	Step 3	Step 4
-	-	1	2	3	19/4	11/2	21/8
-	-	1	3	2	7		
-	-	2	1	3	13/4		
-	-	2	3	1	31/4		
-	-	3	1	2	4		
-	-	3	2	1	25/4		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , _)→ $U_{12a} = \frac{1}{2}$; (_ , 1)→ $U_{12a} = 2 * \frac{1}{4} = \frac{2}{4}$; (_ , 2) → $U_{14a} = 3 * \frac{2}{4} = \frac{6}{4}$;

(_ , 1) → $U_{23a} = \frac{1}{4}$; (_ , 2)→ $U_{24a} = 2 * \frac{2}{4} = 1$; (1 , 2)→ $U_{34a} = 1$;

(2 , 3) → $U_{45a} = 0$

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{4} + \frac{6}{4} + \frac{1}{4} + 1 + 1 + 0 = \frac{19}{4}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{19}{4} + \cdots + \frac{25}{4}}{6} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{19}{4} - 5.5\right)^2 + \cdots + \left(\frac{25}{4} - 5.5\right)^2}{6} = \frac{21}{8}$$

Table 107. t = 5, k=2, Peak = 4 & Third, Fourth & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	2	-	-	-	19/3	11/2	25/36
2	1	-	-	-	14/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , 2)→ $U_{12a} = 1$; (1 , _)→ $U_{13a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$;

(1 , _) → $U_{14a} = 3 * \left(1 - \frac{1}{3}\right) = \frac{6}{3}$; (2 , _) → $U_{23a} = 1 - \frac{2}{3} = \frac{1}{3}$;

(2 , _)→ $U_{24a} = 2 * \left(1 - \frac{2}{3}\right) = \frac{2}{3}$; (_ , _)→ $U_{34a} = \frac{1}{2}$;

(_ , _) → $U_{45a} = \frac{1}{2}$

$$\sum U_{ija} = 1 + \frac{4}{3} + \frac{6}{3} + \frac{1}{3} + \frac{2}{3} + \frac{1}{2} + \frac{1}{2} = \frac{19}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{19}{3} + \frac{14}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{19}{3} - 5.5\right)^2 + \left(\frac{14}{3} - 5.5\right)^2}{2} = \frac{25}{36}$$

Table 108. t = 5, k=2, Peak = 4 & Second, Fourth & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	-	2	-	-	43/6	11/2	25/9
2	-	1	-	-	23/6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(1 , _)→ $U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; (1 , 2)→ $U_{13a} = 2$; (1 , _) → $U_{14a} = 3 * \left(1 - \frac{1}{3}\right) = \frac{6}{3}$;

(_, 2) → $U_{23a} = \frac{2}{3}$; (_ , _)→ $U_{24a} = 2 * \frac{1}{2} = 1$; (2 , _)→ $U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$;

(_ , _) → $U_{45a} = \frac{1}{2}$

$$\sum U_{ija} = \frac{2}{3} + 2 + \frac{6}{3} + \frac{2}{3} + 1 + \frac{1}{3} + \frac{1}{2} = \frac{43}{6}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{43}{6} + \frac{23}{6}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{43}{6} - 5.5\right)^2 + \left(\frac{23}{6} - 5.5\right)^2}{2} = \frac{25}{9}$$

Table 109. t = 5, k=2, Peak = 4 & Second, Third & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	-	-	2	-	49/6	11/2	64/9
2	-	-	1	-	17/6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; $(1 , \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$; $(1 , 2) \rightarrow U_{14a} = 3$;

$(\_ , \_) \rightarrow U_{23a} = \frac{1}{2}$; $(\_ , 2) \rightarrow U_{24a} = 2 * \frac{2}{3} = \frac{4}{3}$; $(\_ , 2) \rightarrow U_{34a} = \frac{2}{3}$;

$(2 , \_) \rightarrow U_{45a} = \frac{2}{3}$

$$\sum U_{ija} = \frac{2}{3} + \frac{4}{3} + 3 + \frac{1}{2} + \frac{4}{3} + \frac{2}{3} + \frac{2}{3} = \frac{49}{6}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{49}{6} + \frac{17}{6}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{49}{6} - 5.5\right)^2 + \left(\frac{17}{6} - 5.5\right)^2}{2} = \frac{64}{9}$$

Table 110. t = 5, k=2, Peak = 4 & Second, Third & Fourth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
1	-	-	-	2	19/3	11/2	25/36
2	-	-	-	1	14/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(1 , \_) \rightarrow U_{12a} = 1 - \frac{1}{3} = \frac{2}{3}$; $(1 , \_) \rightarrow U_{13a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$;

$(1 , \_) \rightarrow U_{14a} = 3 * \left(1 - \frac{1}{3}\right) = \frac{6}{3}$; $(\_ , \_) \rightarrow U_{23a} = \frac{1}{2}$; $(\_ , \_) \rightarrow U_{24a} = 2 * \frac{1}{2} = 1$;

$(\_,\_) \rightarrow U_{34a} = \frac{1}{2}$; $(\_,2) \rightarrow U_{45a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{2}{3} + \frac{4}{3} + \frac{6}{3} + \frac{1}{2} + 1 + \frac{1}{2} + \frac{1}{3} = \frac{19}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{19}{3} + \frac{14}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{19}{3} - 5.5\right)^2 + \left(\frac{14}{3} - 5.5\right)^2}{2} = \frac{25}{36}$$

Table 111. t = 5, k=2, Peak = 4 & First, Fourth & Fifth Observations Missing

Step 1			Step 2		Step 2	Step 3	Step 4
-	1	2	-	-	19/3	11/2	25/36
-	2	1	-	-	14/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

$(\_,1) \rightarrow U_{12a} = \frac{1}{3}$; $(\_,2) \rightarrow U_{13a} = \frac{4}{3}$; $(\_,\_) \rightarrow U_{14a} = 3 * \frac{1}{2} = \frac{3}{2}$;

$(1,2) \rightarrow U_{23a} = 1$; $(1,\_) \rightarrow U_{24a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$; $(2,\_) \rightarrow U_{34a} = 1 - \frac{2}{3} = \frac{1}{3}$;

$(\_,\_) \rightarrow U_{45a} = \frac{1}{2}$;

$$\sum U_{ija} = \frac{1}{3} + \frac{4}{3} + \frac{3}{2} + 1 + \frac{4}{3} + \frac{1}{3} + \frac{1}{2} = \frac{19}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{19}{3} + \frac{14}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{19}{3} - 5.5\right)^2 + \left(\frac{14}{3} - 5.5\right)^2}{2} = \frac{25}{36}$$

Table 112. t = 5, k=2, Peak = 4 & First, Third & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	1	-	2	-	22/3	11/2	121/36
-	2	-	1	-	11/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement (_ , 1)→ $U_{12a} = \frac{1}{3}$; (_ , _)→ $U_{13a} = 2 * \frac{1}{2} = 1$;
- (_ , 2) → $U_{14a} = 3 * \frac{2}{3} = \frac{6}{3}$, (1 , _) → $U_{23a} = 1 - \frac{1}{3} = \frac{2}{3}$; (1 , 2)→ $U_{24a} = 2$;
- (_ , 2)→ $U_{34a} = \frac{2}{3}$; (2 , _) → $U_{45a} = \frac{2}{3}$

$$\sum U_{ija} = \frac{1}{3} + 1 + \frac{6}{3} + \frac{2}{3} + 2 + \frac{2}{3} + \frac{2}{3} = \frac{22}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{22}{3} + \frac{11}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{22}{3} - 5.5\right)^2 + \left(\frac{11}{2} - 5.5\right)^2}{2} = \frac{121}{36}$$

Table 113. t = 5, k=2, Peak = 4 & First, Third & Fourth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	1	-	-	2	17/3	11/2	1/36
-	2	-	-	1	16/3		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , 1)→ $U_{12a} = \frac{1}{3}$; (_ , _)→ $U_{13a} = 2 * \frac{1}{2} = 1$; (_ , _) → $U_{14a} = 3 * \frac{1}{2} = \frac{3}{2}$

(1 , _) → $U_{23a} = 1 - \frac{1}{3} = \frac{2}{3}$; (1 , _)→ $U_{24a} = 2 * \left(1 - \frac{1}{3}\right) = \frac{4}{3}$;

(_ , _)→ $U_{34a} = \frac{1}{2}$; (_ , 2) → $U_{45a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{1}{3} + 1 + \frac{3}{2} + \frac{2}{3} + \frac{4}{3} + \frac{1}{2} + \frac{1}{3} = \frac{17}{3}$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{17}{3} + \frac{16}{3}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{17}{3} - 5.5\right)^2 + \left(\frac{16}{3} - 5.5\right)^2}{2} = \frac{1}{36}$$

Table 114. t = 5, k=2, Peak = 4 & First, Second, & Fifth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	-	1	2	-	39/6	11/2	1
-	-	2	1	-	27/6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement (_ , _)→ $U_{12a} = \frac{1}{2}$; (_ , 1)→ $U_{13a} = 2 * \frac{1}{3} = \frac{2}{3}$;

(_ , 2) → $U_{14a} = 3 * \frac{2}{3} = \frac{6}{3}$; (_ , 1) → $U_{23a} = \frac{1}{3}$;(_ , 2)→ $U_{24a} = 2 * \frac{2}{3} = \frac{4}{3}$;

- (1 , 2)→ $U_{34a} = 1$; (2 , _) → $U_{45a} = \frac{2}{3}$

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{3} + \frac{6}{3} + \frac{1}{3} + \frac{4}{3} + 1 + \frac{2}{3} = \frac{39}{6}$$

- Step 3:

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{\frac{39}{6} + \frac{27}{6}}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{\left(\frac{39}{6} - 5.5\right)^2 + \left(\frac{27}{6} - 5.5\right)^2}{2} = 1$$

Table 115. t = 5, k=2, Peak = 4 & First, Second, & Fourth Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	-	1	-	2	5	11/2	1/4
-	-	2	-	1	6		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement

(_ , _)→ $U_{12a} = \frac{1}{2}$; (_ , 1)→ $U_{13a} = 2 * \frac{1}{3} = \frac{2}{3}$; (_ , _) → $U_{14a} = 3 * \frac{1}{2} = \frac{3}{2}$;

(_ , 1) → $U_{23a} = \frac{1}{3}$; (_ , _)→ $U_{24a} = 2 * \frac{1}{2} = 1$; (1 , _)→ $U_{34a} = 1 - \frac{1}{3} = \frac{2}{3}$;

(_ , 2) → $U_{45a} = 1 - \frac{2}{3} = \frac{1}{3}$

$$\sum U_{ija} = \frac{1}{2} + \frac{2}{3} + \frac{3}{2} + \frac{1}{3} + 1 + \frac{2}{3} + \frac{1}{3} = 5$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{5 + 6}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(5 - 5.5)^2 + (6 - 5.5)^2}{2} = \frac{1}{4}$$

Table 116. t = 5, k=2, Peak = 4 & First, Second & Third Observations Missing

	Step 1		Step 2		Step 2	Step 3	Step 4
-	-	-	1	2	4	11/2	9/4
-	-	-	2	1	7		

The following are the steps for calculating the mean and variance.

- Step 2: using the first arrangement (_ , _)→ $U_{12a} = \frac{1}{2}$; (_ , _)→ $U_{13a} = 2 * \frac{1}{2} = 1$;

$(\_ , 1) \rightarrow U_{14a} = 3 * \frac{1}{3} = 1$,$(\_ , \_) \rightarrow U_{23a} = \frac{1}{2}$; $(\_ , 1) \rightarrow U_{24a} = 2 * \frac{1}{3} = \frac{2}{3}$;

$(\_ , 1) \rightarrow U_{34a} = \frac{1}{3}$; $(1 , 2) \rightarrow U_{45a} = 0$

$$\sum U_{ija} = \frac{1}{2} + 1 + 1 + \frac{1}{2} + \frac{2}{3} + \frac{1}{3} + 0 = 4$$

- Step 3

$$E(M_i) = \frac{\sum_{a=1}^{c} M_a}{c} = \frac{4+7}{2} = \frac{11}{2}$$

- Step 4:

$$\sigma_i^2 = \frac{\sum_{a=1}^{c}(M_a - E(M_i))^2}{c} = \frac{(4-5.5)^2 + (7-5.5)^2}{2} = \frac{9}{4}$$

The List of the Expected Values and Variances for all the Cases considered in the Mungai test.

Table 117. Expected Value and Variance When t = 3 and Peak = 2

Case Number (i)	Pattern of Missing	Expected Value	Variance (σ_i^2)
1	None Missing	1	2/3
2	First or Third Missing	1	4/9
3	Second (Peak) Missing	1	0

The expected value is given by $E(M_{32}) = \sum_1^n 1 = n$

The variance is given by $Var(M_{32}) = \sum_1^n n_i\sigma_i^2 = \frac{2}{3}n_1 + \frac{4}{9}n_2 + 0 * n_3$ where $n_i$is the total number of blocks like case $i = 1, 2, 3$ and n is the total number of blocks in the design.

Table.118. Expected Value and Variance When t = 4 and Peak = 2

Case Number (i)	Pattern of Missing	Expected Value	Variance (σ_i^2)
1	None Missing	5/2	11/4
2	First Missing	5/2	61/24
3	Second Missing	5/2	11/24
4	Third Missing	5/2	61/24
5	Fourth Missing	5/2	31/24
6	First and Second Missing	5/2	4/9
7	First and Third Missing	5/2	9/4
8	First and Fourth Missing	5/2	25/36
9	Second and Third Missing	5/2	1/9
10	Second and Fourth Missing	5/2	1/36
11	Third and Third Missing	5/2	1

The expected value is given by $E(M_{42}) = \sum_1^n \frac{5}{2} = \frac{5}{2}n$

The variance is given by

$$Var(M_{42}) = \sum_1^n n_i\sigma_i^2$$
$$= \frac{11}{4}n_1 + \frac{11}{24}n_3 + \frac{61}{24}(n_2 + n_4) + \frac{31}{24}n_5 + \frac{4}{9}n_6 + \frac{9}{4}n_7 + \frac{25}{36}n_8 + \frac{1}{9}n_9 + \frac{1}{36}n_{10} + n_{11}$$

where $n_i$is the total number of blocks like case $i = 1, 2, 3, \ldots, n_{11}$ and n is the total number of blocks in the design.

Table 119. Expected Value and Variance When t = 4 and Peak = 3

Case Number (i)	Pattern of Missing	Expected Value	Variance (σ_i^2)
1	None Missing	5/2	11/4
2	First Missing	5/2	13/12
3	Second Missing	5/2	61/24
4	Third Missing	5/2	11/24
5	Fourth Missing	5/2	61/24
6	First and Second Missing	5/2	1
7	First and Third Missing	5/2	1/36
8	First and Fourth Missing	5/2	25/36
9	Second and Third Missing	5/2	1/9
10	Second and Fourth Missing	5/2	9/4
11	Third and Third Missing	5/2	4/9

The expected value is given by

$$E(M_{43}) = \sum_{1}^{n} \frac{5}{2} = \frac{5}{2}n$$

The variance is given by

$$Var(M_{43}) = \sum_{1}^{n} n_i \sigma_i^2$$
$$= \frac{11}{4}n_1 + \frac{13}{12}n_2 + \frac{11}{24}n_3 + \frac{61}{24}(n_3 + n_5) + \frac{11}{24}n_4 + n_6 + \frac{1}{36}n_7 + \frac{25}{36}n_8 + \frac{1}{9}n_9 + \frac{9}{4}n_{10} + \frac{4}{9}n_{11}$$

Where $n_i$is the total number of blocks like case $i = 1, 2, 3, \ldots, n_{11}$ and n is the total number of blocks in the design

Table 120. Expected Value and Variance When t = 5 and Peak = 2

Case Number (i)	Pattern of Missing	Expected Value	Variance (σ_i^2)
1	None Missing	11/2	49/4
2	First Missing	11/2	563/60
3	Second Missing	11/2	191/60
4	Third Missing	11/2	103/12
5	Fourth Missing	11/2	103/12
6	Fifth Missing	11/2	55/12
7	First and Second Missing	11/2	25/8
8	First and Third Missing	11/2	67/8
9	First and Fourth Missing	11/2	49/6
10	First and Fifth Missing	11/2	19/24
11	Second and Third Missing	11/2	13/12
12	Second and Fourth Missing	11/2	23/8
13	Second and Fifth Missing	11/2	3/4
14	Third and Fourth Missing	11/2	61/8
15	Third and Fifth Missing	11/2	17/4
16	Fourth and Fifth Missing	11/2	21/8
17	First, Second and Third Missing	11/2	25/36
18	First, Second and Fourth Missing	11/2	0
19	First, Second and Fifth Missing	11/2	25/36
20	First, third and Fourth Missing	11/2	64/9
21	First, third and Fifth Missing	11/2	121/36
22	First, Fourth and Fifth Missing	11/2	1
23	Second, Third and Fourth Missing	11/2	25/36
24	Second, Third and Fifth Missing	11/2	1/36
25	Second, Fourth and Fifth Missing	11/2	1/4
26	Third , Fourth and Fifth Missing	11/2	9/4

The expected value is given by

$$E(M_{52}) = \sum_{1}^{n} \frac{11}{2} = \frac{11}{2} n$$

The variance is given by

$$Var(M_{52}) = \sum_{1}^{n} n_i \sigma_i^2$$
$$= \frac{49}{4} n_1 + \frac{563}{60} n_2 + \frac{191}{60} n_3 + \frac{103}{12}(n_4 + n_5) + \frac{55}{12} n_6 + \frac{25}{8} n_7 + \frac{67}{8} n_8 + \frac{49}{6} n_9$$
$$+ \frac{19}{24} n_{10} + \frac{13}{12} n_{11} + \frac{23}{8} n_{12} + \frac{3}{4} n_{13} + \frac{61}{8} n_{14} + \frac{17}{4} n_{15} + \frac{21}{8} n_{16}$$
$$+ \frac{25}{36}(n_{17} + n_{19} + n_{23}) + 0 * n_{18} + \frac{64}{9} n_{20} + \frac{121}{36} n_{21} + n_{22} + \frac{1}{36} n_{24} + \frac{1}{4} n_{25}$$
$$+ \frac{9}{4} n_{26}$$

Where $n_i$is the total number of blocks like the case $i = 1, 2, 3, \dots, n_{26}$ and n is the total number of blocks in the design.

Table 121. Expected Value and Variance When t = 5 and Peak = 3

Case Number (i)	Pattern of Missing	Expected Value	Variance (σ_i^2)
1	None Missing	4	11/2
2	First Missing	4	41/10
3	Second Missing	4	89/30
4	Third Missing	4	9/10
5	Fourth Missing	4	53/10
6	Fifth Missing	4	41/10
7	First and Second Missing	4	31/8
8	First and Third Missing	4	13/24
9	First and Fourth Missing	4	15/4
10	First and Fifth Missing	4	49/4
11	Second and Third Missing	4	13/24
12	Second and Fourth Missing	4	121/24
13	Second and Fifth Missing	4	15/4
14	Third and Fourth Missing	4	13/24
15	Third and Fifth Missing	4	13/24
16	Fourth and Fifth Missing	4	31/8
17	First, Second and Third Missing	4	4/9
18	First, Second and Fourth Missing	4	125/72
19	First, Second and Fifth Missing	4	1/3
20	First, third and Fourth Missing	4	1/4
21	First, third and Fifth Missing	4	0
22	First, Fourth and Fifth Missing	4	1/3
23	Second, Third and Fourth Missing	4	0
24	Second, Third and Fifth Missing	4	1/4
25	Second, Fourth and Fifth Missing	4	125/72
26	Third, Fourth and Fifth Missing	4	4/9

The expected value is given by

$$E(M_{32}) = \sum_{1}^{n} \frac{11}{2} = \frac{11}{2} n$$

The variance is given by

$$\begin{aligned} Var(M_{32}) &= \sum_{1}^{n} n_i \sigma_i^2 \\ &= \frac{11}{4} n_1 + \frac{41}{10}(n_2 + n_6) + \frac{89}{30} n_3 + \frac{9}{10} n_4 + \frac{53}{10} n_5 + \frac{31}{8}(n_7 + n_{16}) \\ &+ \frac{13}{24}(n_8 + n_{11} + n_{14} + n_{15}) + \frac{121}{24} n_{12} + \frac{15}{4}(n_9 + n_{13}) + \frac{4}{9}(n_{17} + n_{26}) \\ &+ \frac{125}{72}(n_{18} + n_{25}) + \frac{1}{3}(n_{19} + n_{22}) + \frac{1}{4}(n_{20} + n_{24}) + 0(n_{21} + n_{23}) \end{aligned}$$

Where n_i is the total number of blocks like the case $i = 1, 2, 3, \dots, n_{26}$ and n is the total number of blocks in the design.

Table 122. Expected Value and Variance When t = 5 and Peak = 4

Case Number (i)	Pattern of Missing	Expected Value	Variance (σ_i^2)
1	None Missing	11/2	115/12
2	First Missing	11/2	25/4
3	Second Missing	11/2	103/12
4	Third Missing	11/2	107/12
5	Fourth Missing	11/2	191/60
6	Fifth Missing	11/2	563/60
7	First and Second Missing	11/2	21/8
8	First and Third Missing	11/2	17/8
9	First and Fourth Missing	11/2	3/4
10	First and Fifth Missing	11/2	19/24
11	Second and Third Missing	11/2	61/8
12	Second and Fourth Missing	11/2	23/8
13	Second and Fifth Missing	11/2	49/6
14	Third and Fourth Missing	11/2	13/12
15	Third and Fifth Missing	11/2	67/8
16	Fourth and Fifth Missing	11/2	25/8
17	First, Second and Third Missing	11/2	9/4
18	First, Second and Fourth Missing	11/2	1/4
19	First, Second and Fifth Missing	11/2	1
20	First, third and Fourth Missing	11/2	1/36
21	First, third and Fifth Missing	11/2	121/36
22	First, Fourth and Fifth Missing	11/2	26/36
23	Second, Third and Fourth Missing	11/2	25/36
24	Second, Third and Fifth Missing	11/2	64/9
25	Second, Fourth and Fifth Missing	11/2	25/9
26	Third, Fourth and Fifth Missing	11/2	25/36

The expected value is given by

$$E(M_{52}) = \sum_{1}^{n} \frac{11}{2} = \frac{11}{2} n$$

The variance is given by

$$Var(M_{54}) = \sum_{1}^{n} n_i \sigma_i^2$$
$$= \frac{115}{12}n_1 + \frac{25}{4}n_2 + \frac{103}{12}n_3 + \frac{107}{12}n_4 + \frac{191}{60}n_5 + \frac{563}{60}n_6 + \frac{21}{8}n_7 + \frac{17}{4}n_8$$
$$+ \frac{3}{4}n_9 + \frac{19}{24}n_{10} + \frac{61}{8}n_{11} + \frac{23}{8}n_{12} + \frac{49}{6}n_{13} + \frac{13}{12}n_{14} + \frac{67}{8}n_{15} + \frac{25}{8}n_{16}$$
$$+ \frac{9}{4}n_{17} + \frac{1}{4}n_{18} + n_{19} + \frac{1}{36}n_{20} + \frac{121}{36}n_{21} + \frac{25}{36}(n_{22} + n_{23} + n_{26}) + \frac{64}{9}n_{24}$$
$$+ \frac{25}{9}n_{25}$$

3.3.2. Distance Squared Modification for Magel and Ndungu (2011) Test

Similarly, under the Distance Squared-Modification test of the Magel and Ndungu (2011) test for complete and incomplete blocks design, the modified test gives the weight $(j-i)^2$ to U_{ijb} in Eq (3.54) in two forms: where there are no missing observations and where there are mising observations.

If there are no missing observations, then

$U_{ijb} = 0$ if i and j are greater than the peak p

$(j-i)^2$ if i and j are less than the peak p and according to the positions of i^{th} and j^{th}in the populations. The further i and j are from p, the greater the weight.

If there are missing observations, then

$\frac{1}{2} * (j-i)^2$ if i and j are missing

$(j-i)^2 * \left[1 - \frac{r_i}{k+1}\right]$ if j is missing and i is not, then r_i is the rank of i within the block and k is the number of treatments appearing in the block

$(j-i)^2 * \frac{r_j}{k+1}$ if i is missing and j is not. The standardized Modified Squared Mungai

statistic is given by

$$M_p^{**} = \sum_{i=1}^{n} M_i^{**} \tag{3.59}$$

Under H_0, the expected value and variance are given by

$$E_0(M_p^{**}) = \sum_{\mathrm{i}=1}^{n} E_0(M_i^{**}) \tag{3.60}$$

Where $E_0(M_i^{**})$ is the expected value of block i and n is the total number of blocks in the IBD.

And

$$Var(M_p^{*}) = \sum_{i=1}^{n} \sigma_i^2 \tag{3.61}$$

Where σ_i^2 is the variance of block i. The value of σ_i^2 will vary depending on the pattern of missing observations, number of treatments and the position of the peak.

The standardized version of the modified Mungai is written by

$$M_m^{**} = \frac{M_p^{**} - E(M_p^{**})}{\sqrt{Var(M_p^{**})}} \tag{3.62}$$

The null hypothesis is rejected when $M_m^{**} \geq Z_\alpha$

CHAPTER 4. PROPOSED TESTS FOR MIXED DESIGNS

In this chapter, we consider various mixed designs for the umbrella alternative when the peak p is known. The first case consists of a Completely Randomized Design (CRD), Randomized Complete Block Design (RCBD) and Balanced Incomplete Block Design (BIBD).

4.1. Proposed Test for a Mixed Design of CRD, RCBD and BIBD

We consider a combination of three tests cases namely the Mack-Wolfe (1981) test for CRD, the Kim-Kim (1992) test for the RCBD and the Magel and Ndungu (2011) test for BIBD. Three situations are considered for this test: a combination without applying any modification called Non-Modification; a combination by applying Distance-Modification and a combination by applying Distance Squared- Modification.

For each modification test, we propose standardized versions of the test statistics. These are standardizing the sum of the standardized test statistics called standardized first and standardizing the sum of the unstandardized test statistics called standardized last.

In the first test (Standardized first), the Mack-Wolfe, Kim-Kim and Magel-Ndungu test statistics are standardized first, added together and then re-standardized. In the second test (Standardized last), the unstandardized versions of the Mack-Wolfe, Kim-Kim and Magel-Ndungu test statistics are added together first and then standardized. Below are the proposed standardized versions of the test.

4.1.1. Non-Modification for Three Mixed Designs: Mack-Wolfe (1981) for CRD, Kim-Kim (1992) for RCBD and Magel and Ndungu (2011) for (BIBD)

Standardized First: The first proposed KA_{mp} test is given by

$$KA_{mp} = A_p^* + KA_n^* + M_n^* \tag{4.1}$$

where A_p^*, is the standardized version of the usual Mack-Wolfe (1981) test for CRD, $KA_n^*$is the standard version of the Kim-Kim (1992) test for RCBD and M_n^* is the standardized version of the Magel and Ndungu (2011) test for (BIBD). Under H_0 $A_p^*,\ KA_n^*$, and M_n^* have an asymptotic standard normal distribution, thus the expected value and variance of KA_{mp} are given by

$$E_0(KA_{mp}) = E_0(A_p^*) + E_0(KA_n^*) + E_0(M_n^*) = 0 \quad (4.2)$$

And

$$var_0(KA_{mp}) = var_0(A_p^*) + var_0(KA_n^*) + var_0(M_n^*) = 1 + 1 + 1 = 3 \quad (4.3)$$

The standardized version of the first proposed test is given by

$$KA_{pm}^* = \frac{KA_{mp} - E_0(KA_{mp})}{\sqrt{var_0(KA_{mp})}} = \frac{A_{mp} - 0}{\sqrt{3}} \quad (4.4)$$

.

Under H_0, KA_{pm} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized Last: The second proposed KA_{mp}^* test is given by

$$KA_{mp}^* = A_P + KA_n + M_n \quad (4.5)$$

where A_P is the usual Mack-Wolfe (1981) test for CRD, KA_n is the Kim-Kim (1992) test for RCBD and M_n is the Magel and Ndungu (2011) test for BIBD. Under H_0, the expected value and variance of KA_{mp}^* are the sum of the means and variances for the Mack-Wolfe (1981), Kim- Kim (1992) and Magel and Ndungu tests. They are given by

$$E_0(KA_{mp}^*) = E_0(A_P) + E_0(KA_n) + E_0(M_n) \quad (4.6)$$

And

$$var_0(A_{mp}^*) = var_0(A_P) + var_0(KA_n) + var_0(M_n) \quad (4.7)$$

where $E_0(A_P)$, $E_0(KA_n), E_0(M_n), var_0(A_P), var_0(KA_n)$ and $var_0(M_n)$ are the expected values and variances of the usual Mack-Wolfe (1981) test for CRD, Kim-Kim (1992) test for RCBD, and the Magel and Ndungu for BIBD respectively.

The standardized version of the second proposed test is given by

$$KA_{pm}^* = \frac{KA_{mp}^* - E_0\left(KA_{mp}^*\right)}{\sqrt{var_0\left(KA_{mp}^*\right)}} \tag{4.8}$$

Under H_0, KA_{pm}^* has an asymptotic standard normal distribution. The null is rejected for large values.

4.1.2. Distance - Modification for Three Designs Mack-Wolfe (1981) for CRD, Kim-Kim (1992) for RCBD and Magel and Ndungu (2011) for BIBD:

Standardized First: The third proposed test, KA_{mp} is given by

$$KA_{mp} = A_p^* + KA_d{}^* + M_d^* \tag{4.9}$$

where A_p^* is the standardized version of the Modified Distance Mack-Wolfe (1981) test for CRD, $KA_d{}^*$ is the standard version of the Modified Distance Kim-Kim (1992) test for RCBD and the standardized version of the Modified Distance Magel and Ndungu (2011) test for BIBD is M_d^*. Under H_0, $A_p^*, KA_d{}^*$ and M_d^* have an asymptotic standard normal distribution, thus the expected value and variance of KA_{mp} are given by

$$E_0\left(KA_{mp}\right) = E_0\left(A_p^*\right) + E_0(KA_d{}^*) + E_0(M_d^*) = 0 \tag{4.10}$$

And

$$var_0\left(KA_{mp}\right) = var_0\left(A_p^*\right) + var_0(KA_d{}^*) + var_0(M_d^*) = 1 + 1 + 1 = 3 \tag{4.11}$$

The standardized version of the third proposed test is given by

$$KA_{pk} = \frac{KA_{mp} - E_0(KA_{mp})}{\sqrt{var_0(KA_{mp})}} = \frac{A_{mp} - 0}{\sqrt{3}} \tag{4.12}$$

Under H_0, KA_{pk} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized last: The fourth proposed KA^*_{mp} test is given by

$$KA^*_{mp} = A_{mm} + KA_d + M_d \tag{4.13}$$

where A_{mm} is the Modified Distance Mack-Wolfe (1981) test for CRD, KA_d is the Modified Distance Kim-Kim (1992) test for RCBD and M_d is the Modified Distance Magel and Ndungu (2011) test for BIBD. Under H_0, the expected value and variance of KA^*_{mp} are the sum of the means and variances for the Mack-Wolfe, Kim- Kim and Magel and Ndungu tests. They are given by

$$E_0(KA^*_{mp}) = E_0(A_{mm}) + E_0(KA_d) + E_0(M_d) \tag{4.14}$$

And

$$var_0(KA^*_{mp}) = var_0(A_{mm}) + var_0(KA_d) + var_0(M_d) \tag{4.15}$$

where $E_0(A_{mm})$, $E_0(KA_d), E_0(M_d), var_0(A_P), var_0(KA_d)$ and $var_0(M_d)$ are the expected values and variances of the Modified Distance Mack-Wolfe (1981) test for CRD, the Modified Distance Kim-Kim (1992) test for RCBD, and the Modified Distance Magel and Ndungu (2011) test for BIBD respectively.

The standardized version of the fourth proposed test is given by

$$KA^*_{pm} = \frac{A^*_{mp} - E_0(A^*_{mp})}{\sqrt{var_0(A^*_{mp})}} \tag{4.16}$$

Under H_0, KA_{pm}^{*} has an asymptotic standard normal distribution. The null is rejected for large values.

4.1.3. Distance Squared Modification for three designs (Mack-Wolfe (1981) for CRD, Kim-Kim (1992) for RCBD and Magel and Ndungu (2011) for BIBD)

Standardized First: The fifth proposed test $A_{mp,}^{**}$ is given by

$$kA_{mp}^{**} = pA^{*} + KA_{s}^{*} + M_{s}^{*} \tag{4.17}$$

where pA^{*} is the standardized version of the Distance Squared-Modification of the Mack-Wolfe (1981) test for CRD, KA_{s}^{*} is the standard version of the Distance Squared-Modification of Kim-Kim (1992) test for RCBD and M_{s}^{*} is the standardized version of the Distance Squared-Modification of Magel and Ndungu (2011). Under H_0, $pA^{*}, KA_{s}^{*} and\ M_{s}^{*}$ have an asymptotic standard normal distribution, thus the expected value and variance of A_{mp}^{**} are given by

$$E_0\left(KA_{mp}^{**}\right) = E_0(pA^{*}) + E_0(KA_{s}^{*}) + E_0(M_{s}^{*}) = 0 \tag{4.18}$$

And

$$var_0\left(KA_{mp}^{**}\right) = var_0(pA^{*}) + var_0(KA_{s}^{*}) + var_0(M_{s}^{*}) = 1 + 1 + 1 = 3 \tag{4.19}$$

The standardized version of the fifth proposed test is given by

$$KA_{pm}^{**} = \frac{A_{mp}^{**} - E_0\left(A_{mp}^{**}\right)}{\sqrt{var_0\left(A_{mp}^{**}\right)}} = \frac{A_{mp}^{**} - 0}{\sqrt{3}} \tag{4.20}$$

Under H_0, KA_{pm}^{**} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized last: The sixth proposed A_{mp}^{**} test is given by

$$KA_{mp}^{**} = pA_{m} + KA_{s} + M_{s} \tag{4.21}$$

where $pA_m$is the Distance Squared-Modification of Mack-Wolfe (1981) test for CRD, KA_s is the Distance Squared-Modification of Kim-Kim (1992) for RCBD and M_s is the Distance Squared-Modification of Magel and Ndungu (2011) test for (BIBD). Under H_0, the expected value and variance of KA_{mp}^{***} are the sum of the means and variances for the Distance Squared-Modification of the Mack-Wolfe (1981), the Distance Squared-Modification of the Kim-Kim (1992) and the Distance Squared-Modification the Magel and Ndungu (2011) for (BIBD) tests. They are given by

$$E_0\left(KA_{mp}^{***}\right) = E_0(pA_m) + E_0(KA_s) + E_0(M_s) \tag{4.22}$$

And

$$var_0\left(KA_{mp}^{***}\right) = var_0(pA_m) + var_0(KA_s) + var_0(M_s) \tag{4.23}$$

where $E_0(pA_m), E_0(KK_s), E_0(M_s), var_0(pA_m),\ var_0(KA_s)\ and\ var_0(M_s)$ are the expected values and variances of the the Distance Squared-Modification of the Mack-Wolfe (1981) test for CRD, the Distance Squared-Modification of the Kim-Kim(1992) test for RCBD and the Distance Squared-Modification of the Magel and Ndungu (2011) test for (BIBD) respectively.

The standardized version of the sixth proposed test is given by

$$KA_{pm}^{***} = \frac{A_{mp}^{***} - E_0\left(A_{mp}^{***}\right)}{\sqrt{var_0\left(A_{mp}^{***}\right)}} \tag{4.24}$$

Under H_0, KA_{pm}^{***} has an asymptotic standard normal distribution. The null is rejected for large values.

4.2. Proposed Test for a Mixed Design of CRD and BIBD

We consider a combination of two tests cases namely the Mack-Wolfe (1982) test for the CRD and the Magel - Ndungu (2011) test for BIBD. Three situations are considered for this case: a combination without applying any modification called Non-Modification; a combination by applying Distance-Modification and a combination by applying Distance Squared-Modification.

For each modification test, we propose standardized versions of the test statistics. These are standardizing the sum of the standardized test statistics called standardized first and standardizing the sum of the unstandardized test statistics called standardized last.

In the first test (Standardized first), the Mack-Wolfe and Magel-Ndungu test statistics are standardized first, added together and then re-standardized. In the second test (Standardized last), the unstandardized versions of the Mack-Wolfe and Magel-Ndungu test statistics are added together first and then standardized. Below are the proposed standardized versions of the test.

4.2.1. Non-Modification for Two Mixed Designs (Mack-Wolfe test for CRD and Magel and Ndungu (2011) test for (BIBD)

Standardized First: The seventh proposed A_{mp} test is given by

$$pA_{mm} = A_p^* + M_n^* \tag{4.25}$$

where A_p^* is the standardized version of the usual Mack-Wolfe (1981) test for CRD and M_n^* the standardized version of the Magel and Ndungu (2011) test for (BIBD). Under H_0, A_p^* and M_n^* have an asymptotic standard normal distribution, thus the expected value and variance of pA_{mm} are given by

$$E_0(pA_{mm}) = E_0\left(A_p^*\right) + E_0(M_n^*) = 0 \tag{4.26}$$

And

$$var_0(pA_{mm}) = var_0\left(A_p^*\right) + var_0(M_n^*) = 1 + 1 = 2 \tag{4.27}$$

The standardized version of the seventh proposed test is given by

$$pA_{pm} = \frac{pA_{mm} - E_0(pA_{mm})}{\sqrt{var_0(pA_{mm})}} = \frac{A_{mp} - 0}{\sqrt{2}} \tag{4.28}$$

Under H_0, pA_{pm} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized Last: The eighth proposed pA^*_{mm} test is given by

$$pA^*_{mp} = A_P + M_n \tag{4.29}$$

where A_P is the usual Mack-Wolfe (1981) test for CRD and M is the Magel and Ndungu (2011) test for (BIBD). Under H_0, the expected value and variance of pA^*_{mp} are the sum of the means and variances for the Mack-Wolfe (1981) and Magel and Ndungu (2011) test for (BIBD). They are given by

$$E_0(pA^*_{mp}) = E_0(A_P) + (M_n) \tag{4.30}$$

And

$$var_0(pA^*_{mp}) = var_0(A_P) + var_0(M_n) \tag{4.31}$$

where $E_0(A_P), E_0(M_n), var_0(A_P)$ and $var_0(M_n)$ are the expected values and variances of the usual Mack-Wolfe (1981) test for CRD and the Magel and Ndungu (2011) test for (BIBD) respectively. The standardized version of the eighth proposed test is given by

$$pA^*_{mm} = \frac{pA^*_{mp} - E_0(pA^*_{mp})}{\sqrt{var_0(pA^*_{mp})}} \tag{4.32}$$

Under H_0, pA^*_{mm} has an asymptotic standard normal distribution. The null is rejected for large values.

4.2.2. Distance-Modification for Two Designs: (Mack-Wolfe test for CRD and Magel and Ndungu (2011) test for (BIBD)

Standardized First: The ninth proposed test, pA_{mp}^{**} is given by

$$pA_{mp}^{**} = A_p^* + M_d^* \tag{4.33}$$

where A_p^* is the standardized version of the Modified Distance Mack-Wolfe (1981) test for CRD and M_d^* is the standardized version of Modified Distance Magel and Ndungu (2011) test for (BIBD). Under H_0, A_p^* and M_d^* have an asymptotic standard normal distribution, thus the expected value and variance of pA_{mp}^{**} are given by

$$E_0\left(pA_{mp}^{**}\right) = E_0\left(A_p^*\right) + E_0(M_d^*) = 0 \tag{4.34}$$

And

$$var_0\left(pA_{mp}^{**}\right) = var_0\left(A_p^*\right) + (M_d^*) = 1 + 1 = 2 \tag{4.35}$$

The standardized version of the ninth proposed test is given by

$$pA_{mm}^{**} = \frac{pA_{mp}^{**} - E_0\left(pA_{mp}^{**}\right)}{\sqrt{var_0\left(pA_{mp}^{**}\right)}} = \frac{pA_{mp}^{**} - 0}{2} \tag{4.36}$$

Under H_0, pA_{mm}^{**} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized last: The tenth proposed test pA_{mp}^{***} is given by

$$pA_{mp}^{***} = A_{mm} + M_d \tag{4.37}$$

where A_{mm} is the Modified Distance Mack-Wolfe (1981) test for CRD and M_d is Modified Distance Magel and Ndungu (2011) test for (BIBD). Under H_0, the expected value and variance of pA_{mp}^{***} are the sum of the means and variances for the Modified Distance Mack-Wolfe and Modified Distance Magel and Ndungu (2011) test for (BIBD). They are given by

$$E_0\left(pA_{mp}^{***}\right) = E_0(A_{mm}) + E_0(M_d) \tag{4.38}$$

And

$$var_0\left(pA_{mp}^{***}\right) = var_0(A_{mm}) + (M_d) \tag{4.39}$$

where $E_0(A_{mm}),\ E_0(M_d), var_0(A_P)$ and $var_0(M_d)$ are the expected values and variances of the Modified Distance Mack-Wolfe (1981) test for CRD and the Modified Distance Magel and Ndungu (2011) test for (BIBD) respectively.

The standardized version of the tenth proposed test is given by

$$pA_{mm}^{***} = \frac{pA_{mp}^{***} - E_0\left(pA_{mp}^{***}\right)}{\sqrt{var_0\left(pA_{mp}^{***}\right)}} \tag{4.40}$$

Under H_0, pA_{mp}^{***} has an asymptotic standard normal distribution. The null is rejected for large values.

4.2.3. Distance Squared Modification for Two Designs (Mack-Wolfe test for CRD and Magel and Ndungu (2011) test for (BIBD)

Standardized First: The eleventh proposed test pA_{mp}^{****} is given by

$$pA_{mp}^{****} = pA^{*} + M_s^{*} \tag{4.41}$$

where pA^{*} is the standardized version of the Distance Squared-Modification of Mack-Wolfe (1981) test for CRD and the standardized version of the Distance Squared-Modification of the Magel and Ndungu (2011) test for (BIBD). Under H_0, pA^{*} and M_s^{*} have an asymptotic standard normal distribution, thus the expected value and variance of pA_{mp}^{****} are given by

$$E_0\left(pA_{mp}^{****}\right) = E_0(pA^{*}) + E_0(M_s^{*}) = 0 \tag{4.42}$$

And

$$var_0\left(pA_{mp}^{****}\right) = var_0(pA^{*}) + var_0(M_s^{*}) = 1 + 1 = 0 \tag{4.43}$$

The standardized version of the eleventh proposed test is given by

$$pA_{mm}^{****} = \frac{pA_{mp}^{****} - E_0\left(pA_{mp}^{****}\right)}{\sqrt{var_0\left(pA_{mp}^{****}\right)}} = \frac{pA_{mp}^{****} - 0}{\sqrt{2}} \tag{4.44}$$

Under H_0, pA_{mm}^{****} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized last: The twelfth proposed pA_{mp}^{v} test is given by

$$pA_{mp}^{v} = pA_m + M_s \tag{4.45}$$

where $pA_m$is the Distance Squared-Modification of Mack-Wolfe (1981) test for CRD, and M_s is the Distance Squared-Modification of Magel and Ndungu (2011) test for BIBD. Under H_0, the expected value and variance of pA_{mp}^{*****} are the sum of the means and variances for the Mack-Wolfe (1981) and the Distance Squared-Modification of the Magel and Ndungu (2011) test for (BIBD) tests. They are given by

$$E_0\left(pA_{mp}^{v}\right) = E_0(pA_m) + E_0(M_s) \tag{4.46}$$

And

$$var_0\left(pA_{mp}^{v}\right) = var_0(pA_m) + var_0(M_s) \tag{4.47}$$

where $E_0(pA_m), E_0(M_s), var_0(pA_m)\ and\ var_0(M_s)$ are the expected values and variances of the Distance Squared-Modification of Mack-Wolfe (1981) test for CRD and the Distance Squared-Modification of Magel and Ndungu (2011) test for BIBD respectively.

The standardized version of the twelfth proposed test is given by

$$pA_{mm}^{v} = \frac{pA_{mp}^{v} - E_0\left(pA_{mp}^{v}\right)}{\sqrt{var_0\left(pA_{mp}^{v}\right)}} \tag{4.48}$$

Under H_0, pA_{mm}^{v} has an asymptotic standard normal distribution. The null is rejected for large values.

4.3. Proposed Test for a Mixed Design of CRD and RCBD

We consider a combination of two tests cases, namely Mack-Wolfe (1982) for the CRD and the Kim-Kim (1992) test for RCBD. Three situations are considered for this case: A combination without applying any modification called Non-Modification; a combination by applying Distance-Modification and a combination by applying Distance Squared- Modification.

For each modification test, we propose standardized versions of the test statistics. These are standardizing the sum of the standardized test statistics called standardized first and standardizing the sum of the unstandardized test statistics called standardized last.

In the first test (Standardized first), the Mack-Wolfe and Kim-Kim test statistics are standardized first, added together and then re-standardized. In the second test (Standardized last), the unstandardized versions of the Mack-Wolfe and Kim-Kim test statistics are added together first and then standardized Below are the proposed standardized versions of the test.

4.3.1. Non-Modification for Two Mixed Designs: (Mack-Wolfe (1981) for CRD, Kim-Kim (1992) for RCBD)

Standardized First: The thirteenth proposed KA_{mp} test is given by

$$KA_{mp} = A_p^* + KA_n^* \tag{4.49}$$

where A_p^* is the standardized version of the usual Mack-Wolfe (1981) test for CRD and KA_n^* is the standard version of the Kim-Kim (1992) test for RCBD Under H_0, A_p^* and KA_n^* have an asymptotic standard normal distribution, thus the expected value and variance of KA_{mp} are given by

$$E_0\left(KA_{mp}\right) = E_0\left(A_p^*\right) + E_0(KA_n^*) = 0 \tag{4.50}$$

And

$$var_0\left(KA_{mp}\right) = var_0\left(A_p^*\right) + var_0(KA_n^*) = 1 + 1 = 2 \tag{4.51}$$

The standardized version of the thirteenth proposed test is given by

$$KA_{pp} = \frac{KA_{mp} - E_0\left(KA_{mp}\right)}{\sqrt{var_0\left(KA_{mp}\right)}} = \frac{KA_{mp} - 0}{2} \tag{4.52}$$

Under H_0, KA_{pp} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized Last: The fourteenth proposed KA_{mp}^* test is given by

$$KA_{mp}^* = A_P + KA_n \tag{4.53}$$

where A_P is the usual Mack-Wolfe (1981) test for CRD and KA_n is the Kim - Kim (1992) test for RCBD. Under H_0, the expected value and variance of KA_{mp}^* are the sum of the means and variances for the Mack-Wolfe, Kim- Kim and Mungai Tests. They are given by

$$E_0\left(KA_{mp}^*\right) = E_0(A_P) + E_0(KA_n) \tag{4.54}$$

And

$$var_0\left(KA_{mp}^*\right) = var_0(A_P) + var_0(KA_n) \tag{4.55}$$

where $E_0(A_P),\ E_0(KK_n), var_0(A_P)\ and\ var_0(KA_n)$ are the expected values and variances of the usual Mack-Wolfe (1981) test for CRD and Kim-Kim (1992) for RCBD respectively.

The standardized version of the fourteenth proposed test is given by

$$KA_{pp}^* = \frac{KA_{mp}^* - E_0\left(KA_{mp}^*\right)}{\sqrt{var_0\left(KA_{mp}^*\right)}} \tag{4.56}$$

Under H_0, KA_{pp}^{*} has an asymptotic standard normal distribution. The null is rejected for large values.

4.3.2. Distance-Modification for Two Designs: (Mack-Wolfe (1981) for CRD, Kim-Kim (1992) for RCBD)

Standardized First: The fifteenth proposed test, KA_{mp}^{**} is given by

$$KA_{mp}^{**} = A_p^{*} + \mathrm{KA}_d^{*} \tag{4.57}$$

where A_p^{*} is the standardized version of the Modified Distance Mack-Wolfe (1981) test for CRD and KK^{**} is the standard version of the Modified Distance Kim - Kim (1992) test for RCBD. Under H_0, A_p^{*} and $\mathrm{K}A_d^{*}$ have an asymptotic standard normal distribution, thus the expected value and variance of KA_{mp}^{**} are given by

$$E_0\left(KA_{mp}^{**}\right) = E_0\left(A_p^{*}\right) + E_0(\mathrm{KA}_d^{*}) = 0 \tag{4.58}$$

And

$$var_0\left(KA_{mp}^{**}\right) = var_0\left(A_p^{*}\right) + var_0(\mathrm{KA}_d^{*}) = 1 + 1 = 2 \tag{4.59}$$

The standardized version of the fifteenth proposed test is given by

$$KA_{pp}^{**} = \frac{KA_{mp}^{**} - E_0\left(KA_{mp}^{**}\right)}{\sqrt{var_0\left(KA_{mp}^{**}\right)}} = \frac{KA_{mp}^{**} - 0}{\sqrt{2}} \tag{4.60}$$

Under H_0, KA_{pp}^{**} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized last: The sixteenth proposed KA_{mp}^{***} test is given by

The fourth proposed test KA_{mp}^{***}, is given by

$$KA_{mp}^{***} = A_{mm} + \mathrm{KA}_d \tag{4.61}$$

. .

where A_{mm} is the Modified Distance Mack-Wolfe (1981) test for CRD and $\mathrm{K}A_d$ is the Modified Distance Kim - Kim (1992) test for RCBD. Under H_0, the expected value and variance of A_{mp}^* are the sum of the means and variances for the Modified Distance Mack-Wolfe and the Modified Distance Kim- Kim tests. They are given by

$$E_0\left(KA_{mp}^{***}\right) = E_0(A_{mm}) + E_0(\mathrm{KA}_d) \tag{4.62}$$

And

$$var_0\left(KA_{mp}^{***}\right) = var_0(A_{mm}) + var_0(\mathrm{KA}_d) \tag{4.63}$$

where $E_0(A_{mm}),\ E_0(\mathrm{K}A_d), var_0(A_P)$ and $var_0(\mathrm{K}A_d)$ are the expected values and variances of the Modified Distance Mack-Wolfe (1981) test for CRD and the Modified Distance Kim-Kim (1992) test for RCBD respectively.

The standardized version of the sixteenth proposed test is given by

$$KA_{pp}^{***} = \frac{KA_{mp}^{***} - E_0\left(KA_{mp}^{***}\right)}{\sqrt{var_0\left(KA_{mp}^{***}\right)}} \tag{4.64}$$

Under H_0, KA_{pp}^{***} has an asymptotic standard normal distribution. The null is rejected for large values.

4.3.3.Distance Squared Modification for Two Designs: (Mack-Wolfe (1981) for CRD, Kim-Kim (1992) for RCBD)

Standardized First: The seventeenth proposed test $KA_{mp,}^{****}$ is given by

$$KA_{mp}^{****} = pA^* + \mathrm{KA}_s^* \tag{4.65}$$

where pA^* is the standardized version of the Distance Squared-Modification of the Mack-Wolfe (1981) test for CRD, the standard version of the Distance Squared-Modification of Kim-Kim

(1992) for RCBD. Under H_0, pA^* and KA_s^* have an asymptotic standard normal distribution, thus the expected value and variance of kA_{mp}^{****} are given by

$$E_0\left(KA_{mp}^{****}\right) = E_0(pA^*) + E_0(\mathrm{KA}_s^*) = 0 \tag{4.66}$$

And

$$var_0\left(KA_{mp}^{****}\right) = var_0(pA^*) + var_0(\mathrm{KA}_s^*) = 1 + 1 = 2 \tag{4.67}$$

The standardized version of the seventeenth proposed test is given by

$$KA_{pp}^{****} = \frac{KA_{mp}^{****} - E_0\left(KA_{mp}^{****}\right)}{\sqrt{var_0\left(KA_{mp}^{****}\right)}} = \frac{KA_{mp}^{****} - 0}{\sqrt{2}} \tag{4.68}$$

Under H_0, KA_{pp}^{****} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized last: The eighteenth proposed kA_{mp}^{v} test is given by

$$KA_{mp}^{v} = pA_m + \mathrm{KA}_s \tag{4.69}$$

where $pA_m$is the Distance Squared-Modification of the Mack-Wolfe (1981) test for CRD and KA_s is the Distance Squared-Modification of Kim -Kim (1992) for RCBD. Under H_0, the expected value and variance of kA_{mp}^{v} are the sum of the means and variances for the Distance Squared-Modification of Mack-Wolfe (1981) and the Distance Squared-Modification of Kim-Kim (1992). They are given by

$$E_0\left(KA_{mp}^{v}\right) = E_0(pA_m) + E_0(\mathrm{KA}_s) \tag{4.70}$$

And

$$var_0\left(KA_{mp}^{v}\right) = var_0(pA_m) + var_0(\mathrm{KA}_s) \tag{4.71}$$

where $E_0(pA_m), E_0(\mathrm{K}A_s), var_0(pA_m)$ and $var_0(\mathrm{K}A_s)$ are the expected values and variances of the Distance Squared-Modification of the Mack-Wolfe (1981) test for CRD and the Distance Squared-Modification of the Kim-Kim(1992) test for RCBD respectively.

The standardized version of the eighteenth proposed test is given by

$$KA_{pp}^{v} = \frac{KA_{mp}^{v} - E_0\left(KA_{mp}^{v}\right)}{\sqrt{var_0\left(KA_{mp}^{v}\right)}} \tag{4.72}$$

Under H_0, KA_{pp}^{v} has an asymptotic standard normal distribution.

4.4. Proposed Test for a Mixed Design of CRD, IBD and RCBD

We consider a combination of three tests cases namely Mack-Wolfe (1982) for the CRD Magel – Ndungu(2011) for IBD and Kim-Kim (1992) test for RCBD. Two situations are considered for this case: A combination without applying any modification called Non-Modification, and a combination by applying Distance-Modification

For each modification test, we propose standardized versions of the test statistics. These are standardizing the sum of the standardized test statistics called standardized first and standardizing the sum of the unstandardized test statistics called standardized last.

In the first test (Standardized first), the Mack-Wolfe , Magel -Ndungu and Kim-Kim test statistics are standardized first, added together and then re-standardized. In the second test (Standardized last), the unstandardized versions of the the Mack-Wolfe , Magel -Ndungu and Kim-Kim test statistics are added together first and then standardized Below are the proposed standardized versions of the test.

4.4.1. Non-Modification for Three Mixed Designs: Magel and Ndungu (2011) test for IBD, (Mack-Wolfe (1981) for CRD and Kim-Kim (1992) for RCBD

Standardized First: The nineteenth proposed A_{mpp} test is given by

$$KA_{mpp} = A_{mI} + \mathrm{KA}_n^* + \mathrm{M}_n^* \tag{4.73}$$

where A_p^* is the standardized version of the usual Mack-Wolfe test for CRD, KA_n^* is the standard version of the Kim-Kim (1992) test for RCBD and M_n^* the standardized version of the Magel and Ndungu (2011) test for IBD. Under H_0, A_p^*, KA_n^* and M_n^* have an asymptotic standard normal distribution, thus the expected value and variance of KA_{mp} are given by

$$E_0(KA_{mpp}) = E_0(A_p^*) + E_0(\mathrm{KA}_n^*) + E_0(\mathrm{M}_n^*) = 0 \tag{4.74}$$

And

$$var_0(KA_{mpp}) = var_0(A_p^*) + var_0(\mathrm{KA}_n^*) + var_0(\mathrm{M}_n^*) = 1 + 1 + 1 = 3 \tag{4.75}$$

The standardized version of the first proposed test is given by

$$KA_{pmI} = \frac{KA_{mpp} - E_0(KA_{mpp})}{\sqrt{var_0(KA_{mpp})}} = \frac{A_{mp} - 0}{\sqrt{3}} \tag{4.76}$$

Under H_0, KA_{pmI} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized Last: The twentieth proposed KA_{mpI}^* test is given by

$$KA_{mpI}^* = A_P + KA_n + M_n \tag{4.77}$$

where A_P is the usual Mack-Wolfe (1981) test for CRD, KA_n is the Kim-Kim (1992) test for RCBD and M_n is the Magel and Ndungu (2011) test for IBD. Under H_0, the expected value and variance of KA_{mpI}^* are the sum of the means and variances for the Mack-Wolfe, the Kim- Kim and Mungai tests. They are given by

$$E_0\left(KA^*_{mpI}\right) = E_0(A_P) + E_0(KA_n) + E_0(M_n) \tag{4.78}$$

And

$$var_0\left(KA^*_{mpI}\right) = var_0(A_P) + var_0(KA_n) + var_0(M_n) \tag{4.79}$$

where $E_0(A_P)$, $E_0(KA_n), E_0(M_n), var_0(A_P), var_0(KA_n)$ and $var_0(M_n)$ are the expected values and variances of the usual Mack-Wolfe (1981) test for CRD, the Kim-Kim (1992) for RCBD, and the Magel and Ndungu (2011) test for IBD respectively.

The standardized version of the twentieth proposed test is given by

$$KA^*_{pmI} = \frac{KA^*_{mpI} - E_0\left(KA^*_{mpI}\right)}{\sqrt{var_0\left(KA^*_{mpI}\right)}} \tag{4.80}$$

Under H_0, KA^*_{pm} has an asymptotic standard normal distribution. The null is rejected for large values.

4.4.2. Distance - Modification for Three Designs: Magel and Ndungu (2011) test for IBD, (Mack-Wolfe (1981) for CRD and Kim-Kim (1992) for RCBD

Standardized First: The twenty-first proposed test, A_{mp} is given by

$$KA_{mpI} = A^*_p + \mathrm{KA}^*_d + M^*_{Id} \tag{4.81}$$

where A^*_p is the standardized version of the Modified Distance Mack-Wolfe test for CRD, KA^*_d is the standard version of the Modified Distance Kim-Kim (1992) test for RCBD and M^*_{Id} is the standardized version of the Modified Distance Magel and Ndungu (2011) test for IBD.

Under H_0, A^*_p, KA^*_d and M^*_{Id} have an asymptotic standard normal distribution, thus the expected value and variance of kA_{mpI} are given by

$$E_0\left(KA_{mpI}\right) = E_0\left(A^*_p\right) + E_0(\mathrm{KA}^*_d) + E_0(M^*_{Id}) = 0 \tag{4.82}$$

And

$$var_0(KA_{mpI}) = var_0(A_p^*) + var_0(\mathrm{KA}_d^*) + var_0(M_{Id}^*) = 1 + 1 + 1 = 3 \quad (4.83)$$

The standardized version of the twenty-first proposed test is given by

$$KA_{pkI} = \frac{KA_{mpI} - E_0(KA_{mpI})}{\sqrt{var_0(KA_{mpI})}} = \frac{A_{mp} - 0}{\sqrt{3}} \quad (4.84)$$

Under H_0, KA_{pkI} has an asymptotic standard normal distribution. The null is rejected for large values.

Standardized last: The twenty-second proposed A_{mp}^* test is given by

$$KA_{mpI}^* = A_{mm} + KA_d + M_{Id} \quad (4.85)$$

where A_{mm} is the Modified Distance Mack-Wolfe (1981) test for CRD, KA_d is the Modified Distance Kim-Kim (1992) test for RCBD and MI^* is the Modified Distance Magel and Ndungu (2011) test for IBD. Under H_0, the expected value and variance of KA_{mpI}^* are the sum of the means and variances for the Modified Distance Mack-Wolfe (1981), the Modified Distance Kim-Kim (1992) and the Modified Distance Magel and Ndungu (2011) test. They are given by

$$E_0(KA_{mpI}^*) = E_0(A_{mm}) + E_0(KA_d) + E_0(M_{Id}) \quad (4.86)$$

And

$$var_0(KA_{mpI}^*) = var_0(A_{mm}) + var_0(KA_d) + var_0(M_{Id}) \quad (4.87)$$

where $E_0(A_{mm})$, $E_0(KA_d), E_0(M_{Id}), var_0(A_P), var_0(KA_d)$ and $var_0(M_{Id})$ are the expected values and variances of the Modified Distance Mack-Wolfe (1981) test for CRD, the Modified Distance Kim-Kim (1992) test for RCBD, and the Distance Modified Magel and Ndungu (2011) test for IBD respectively.

The standardized version of the twenty-second proposed test is given by

$$KA_{pmI}^{*} = \frac{KA_{mpI}^{*} - E_0\left(KA_{mpI}^{*}\right)}{\sqrt{var_0\left(KA_{mpI}^{*}\right)}} \tag{4.88}$$

Under H_0, KA_{pmI}^{*} has an asymptotic standard normal distribution. The null is rejected for large values**.**

CHAPTER 5. SIMULATION STUDY

This chapter describes the procedure used in the simulation study. It presents an outline of the general structure of the language used in the programming, the logic behind the simulation data, details design variation and the calculation of the power of the test. The source of information for the mixed design overview was referenced from Magel and Ndungu (2011).

5.1. Overview of Mixed Design Simulation

The main statistical software used in the research study was SAS®. The function RAND found in the software was used to simulate observations. The RAND function requires that the starting point (known as the seed) be defined by users. It uses the Call Streaminit function before the RAND function is called.

Call Streaminit (Seed).

The seed used in this research was zero (0). This instructs the RAND function to use the internal clock time. The function RAND again requires the distribution to be defined. It is done by using the call function

RAND (Distribution).

Depending on the type of distribution, the function may require additional input of parameters. Missing observations were created using the Uniform distribution. Simulated observations were individually assigned a probability $\boldsymbol{p}$ of missing by using the call function

RAND ('Uniform').

The Uniform distribution produced a random number between zero and one. The probability $\boldsymbol{p}$ of an observation missing was then given by the following If statement:

*If RAND ('Uniform') < (insert probability **p**) then .else RAND ('Distribution');*

SAS® uses a semicolon (;) to mark the end of a command and reads a period (.) as a missing observation. Blocks with less than two observations were excluded from the analysis. There were no missing observations simulated in the CRD and RCBD. This would be incapable of producing any useful results since there was no blocking in the CRD design and therefore the ratio of treatment sample would be irrelevant.

Observations were generated either column wise for the Completely Randomized Design (CRD) or row wise for the Incomplete Block Design (IBD) until the intended number was reached. The process enlisted a series of Do loops until the sample size was attained. Generating observations for a design involves the following processes.

Do (until given number of blocks/ observations per treatment)

Generate random row of numbers

End (loop).

For the CRD, one number per row was generated by the loop until the column (which represents a treatment) had attained the number of observations.

When the mixed design was simulated, the appropriate tests were applied and the decision to reject the null hypothesis was registered using a counter variable. The counter variable then computed the number of times the null hypothesis was rejected by adding one every time that criterion was met. This was carried out using an If conditional statement given by

If test > 1.645 then Counter + 1;

By approximating the power of the test statistic, the proportion of simulation was calculated thereby rejecting the null hypothesis.

For each mixed design generated, a given set of parameters was repeated 5000 times. Thus, an experiment was simulated repeatedly. The power of the test statistic, therefore, was found by dividing the final value of Counter by 5000. The power approximation was calculated for every test statistic for all combinations of variations caused by the following features:

- Number of treatments (three, four and five treatments in the design)
- Underlying distribution
- Probability of an observation missing
- Combination of treatment sample size to the number of blocks
- Position of the peak treatment

The number of treatments considered in the research study was three, four and five. Three underlying distributions were considered under this research. These are

- The Normal distribution
- The Exponential distribution
- The T, with three degrees of freedom, distribution

The call function for the normal distribution was given by

RAND (‘Normal’, μ, α)

where μ was the mean and α was the standard deviation. Zero and one were the default value functions for the mean and variance respectively. *RAND (‘Normal’, 0.5, 1)* thus, generated a single observation from a normal distribution with a mean of 0.5 and a standard deviation of one.

The exponential distribution uses a call function given by

RAND (‘Exponential’).

The function generated a random number from an exponential distribution with a mean and variance of one. Thus, the transformation of the variable was imperative to obtaining a desired distribution.

The T distribution with three degrees of freedom used the following call function given by

RAND ('T', 3).

Three degrees of freedom were selected under the T-distribution to simulate heavier tails in the distribution than normal. The location parameter was shifted to the preferred value by adding the treatment's mean to the generated value.

Each observation had a probability of missing being controlled by a probability $\boldsymbol{p}$ that followed a Uniform distribution. Three probabilities were considered. These are 0.1, 0.2 and 0.4. The power of a test under a mixed design with a given set of parameters was therefore simulated 5000 times for each probability for a total of 5000 simulations.

In all the proposed mixed designs, an equal number of complete blocks to incomplete blocks was considered. For populations three and four, the following combinations of the sample size to the number of blocks were considered:

5.2. Mixed Design of BIBD, CRD and RCBD

For a mixed design of BIBD, CRD and RCBD, the following combinations of sample size to the number of blocks were considered for populations three and four.

- **When the sample size is equal to the number of blocks.**

6: 6: 6 (that's 6 blocks (BIBD), 6 sample size (CRD) and 6 blocks (RCBD)

12: 12: 12 (that's 12 blocks (BIBD), 12 sample size (CRD) and 12 blocks (RCBD)

18: 18: 18 (that's 18 blocks (BIBD), 18 sample size (CRD) and 18 blocks (RCBD)

- **When the sample size is greater than the number of blocks.**

6: 12: 6 (that's 6 blocks (BIBD), 12 sample size (CRD) and 6 blocks (RCBD)

6: 18: 6 (that's 6 blocks (BIBD), 18 sample size (CRD) and 6 blocks (RCBD)

12: 18: 12 (that's 12 blocks (BIBD), 18 sample size (CRD) and 12 blocks (RCBD)

- **When the sample size is less than the number of blocks.**

12: 6: 12 (that's 12 blocks (BIBD), 6 sample size (CRD) and 12 blocks (RCBD)

18: 6: 18 (that's 18 blocks (BIBD), 6 sample size (CRD) and 18 blocks (RCBD)

18: 12: 18 (that's 18 blocks (BIBD), 12 sample size (CRD) and 18 blocks (RCBD)

For a population of five, the following combinations of sample size to the number of blocks were considered for a mixed design of (BIBD, CRD and RCBD)

- **When the sample size is equal to the number of blocks.**

5: 5: 5 (that's 5 blocks (BIBD), 5-sample size (CRD) and 5 blocks (RCBD)

10: 10: 10 (that's 10 blocks (BIBD), 10 sample size (CRD) and 10 blocks (RCBD)

15: 15: 15 (that's 15 blocks (BIBD), 15 sample size (CRD) and 15 blocks (RCBD)

- **When the sample size is greater than the number of blocks.**

5: 10: 5 (that's 5 blocks (BIBD), 10 sample size (CRD) and 5 blocks (RCBD)

5: 15: 5 (that's 5 blocks (BIBD), 15 sample size (CRD) and 5 blocks (RCBD)

10: 15: 10 (that's 10 blocks (BIBD), 15 sample size (CRD) and 10 blocks (RCBD)

- **When the sample size is less than the number of blocks.**

10: 5: 10 (that's 10 blocks (BIBD), 5 sample size (CRD) and 10 blocks (RCBD)

15: 5: 15 (that's 15 blocks (BIBD), 5 sample size (CRD) and 15 blocks (RCBD)

15: 10: 15 (that's 15 blocks (BIBD), 10 sample size (CRD) and 15 blocks (RCBD)

5.3. Mixed Design of BIBD and CRD

For a mixed design of BIBD and CRD, the following combinations of sample size to the number of blocks were considered for populations three and four.

- **When the sample size is equal to the number of blocks.**

6: 6 (that's 6 blocks (BIBD), 6 sample size (CRD))

12: 12 (that's 12 blocks (BIBD), 12 sample size (CRD))

18: 18 (that's 18 blocks (BIBD), 18 sample size (CRD))

- **When the sample size is greater than the number of blocks.**

6: 12 (that's 6 blocks (BIBD), 12 sample size (CRD))

6: 18 (that's 6 blocks (BIBD), 18 sample size (CRD))

12: 18 (that's 12 blocks (BIBD), 18 sample size (CRD))

- **When the sample size is less than the number of blocks.**

12: 6 (that's 12 blocks (BIBD), 6 sample size (CRD))

18: 6 (that's 18 blocks (BIBD), 6 sample size (CRD))

18: 12 (that's 18 blocks (BIBD), 12 sample size (CRD))

For a population of five, the following combinations of sample size to the number of blocks were considered for a mixed design of (BIBD, CRD and RCBD)

- **When the sample size is equal to the number of blocks.**

5: 5 (that's 5 blocks (BIBD), 5-sample size (CRD))

10: 10 (that's 10 blocks (BIBD), 10 sample size (CRD))

15: 15 (that's 15 blocks (BIBD), 15 sample size (CRD))

- **When the sample size is greater than the number of blocks.**

5: 10 (that's 5 blocks (BIBD), 10 sample size (CRD))

5: 15 (that's 5 blocks (BIBD), 15 sample size (CRD))

10: 15 (that's 10 blocks (BIBD), 15 sample size (CRD))

- **When the sample size is less than the number of blocks.**

10: 5 (that's 10 blocks (BIBD), 5 sample size (CRD))

15: 5 (that's 15 blocks (BIBD), 5 sample size (CRD))

15: 10 (that's 15 blocks (BIBD), 10 sample size (CRD)

5.4. Mixed Design of CRD and RCBD

For a mixed design of CRD and RCBD, the following combinations of sample size to the number of blocks were considered for populations three and four.

- **When the sample size is equal to the number of blocks.**

6: 6 (that's 6-sample size (CRD) and 6 blocks (RCBD)

12: 12 (that's 12-sample size (CRD) and 12 blocks (RCBD)

18: 18 (that's 18-sample size (CRD) and 18 blocks (RCBD)

- **When the sample size is greater than the number of blocks.**

12: 6 (that's 12-sample size (CRD) and 6 blocks (RCBD)

18: 6 (that's 18-sample size (CRD) and 6 blocks (RCBD)

18: 12 (that's 18-sample size (CRD) and 12 blocks (RCBD)

- **When the sample size is less than the number of blocks.**

6: 12 (that's 6-sample size (CRD) and 12 blocks (RCBD)

6: 18 (that's 6-sample size (CRD) and 18 blocks (RCBD)

12: 18 (that's 12-sample size (CRD) and 18 blocks (RCBD)

For a population of five, the following combinations of sample size to the number of blocks were considered for a mixed design of (BIBD, CRD and RCBD)

- **When the sample size is equal to the number of blocks.**

5: 5 (that's 5-sample size (CRD) and 5 blocks (RCBD)

10: 10 (that's 10-sample size (CRD) and 10 blocks (RCBD)

15: 15 (that's 15-sample size (CRD) and 15 blocks (RCBD)

- **When the sample size is greater than the number of blocks.**

10: 5 (that's 10-sample size (CRD) and 5 blocks (RCBD)

15: 5 (that's 15-sample size (CRD) and 5 blocks (RCBD)

15: 10 (that's 15-sample size (CRD) and 10 blocks (RCBD)

- **When the sample size is less than the number of blocks.**

5: 10 (that's 5-sample size (CRD) and 10 blocks (RCBD)

5: 15 (that's 5-sample size (CRD) and 15 blocks (RCBD)

10: 15 (that's 10-sample size (CRD) and 15 blocks (RCBD)

5.5. Mixed Design of IBD, CRD and RCBD

For a mixed design of IBD, CRD and RCBD, the following combinations of sample size to the number of blocks were considered for a population four.

- **When the sample size is equal to the number of blocks.**

6: 6: 6 (that's 6 blocks (IBD), 6 sample size (CRD) and 6 blocks (RCBD)

12: 12: 12 (that's 12 blocks (IBD), 12 sample size (CRD) and 12 blocks (RCBD)

18: 18: 18 (that's 18 blocks (IBD), 18 sample size (CRD) and 18 blocks (RCBD)

- **When the sample size is greater than the number of blocks.**

6: 12: 6 (that's 6 blocks (IBD), 12 sample size (CRD) and 6 blocks (RCBD)

6: 18: 6 (that's 6 blocks (IBD), 18 sample size (CRD) and 6 blocks (RCBD)

12: 18: 12 (that's 12 blocks (IBD), 18 sample size (CRD) and 12 blocks (RCBD)

- **When the sample size is less than the number of blocks.**

12: 6: 12 (that's 12 blocks (IBD), 6 sample size (CRD) and 12 blocks (RCBD)

18: 6: 18 (that's 18 blocks (IBD), 6 sample size (CRD) and 18 blocks (RCBD)

18: 12: 18 (that's 18 blocks (IBD), 12 sample size (CRD) and 18 blocks (RCBD)

For a population of five, the following combinations of sample size to the number of blocks were considered for a mixed design of (IBD, CRD and RCBD)

- **When the sample size is equal to the number of blocks.**

5: 5: 5 (that's 5 blocks (IBD), 5-sample size (CRD) and 5 blocks (RCBD)

10: 10: 10 (that's 10 blocks (IBD), 10 sample size (CRD) and 10 blocks (RCBD)

15: 15: 15 (that's 15 blocks (IBD), 15 sample size (CRD) and 15 blocks (RCBD)

- **When the sample size is greater than the number of blocks.**

5: 10: 5 (that's 5 blocks (IBD), 10 sample size (CRD) and 5 blocks (RCBD)

5: 15: 5 (that's 5 blocks (IBD), 15 sample size (CRD) and 5 blocks (RCBD)

10: 15: 10 (that's 10 blocks (IBD), 15 sample size (CRD) and 10 blocks (RCBD)

- **When the sample size is less than the number of blocks.**

10: 5: 10 (that's 10 blocks (IBD), 5 sample size (CRD) and 10 blocks (RCBD)

15: 5: 15 (that's 15 blocks (IBD), 5 sample size (CRD) and 15 blocks (RCBD)

15: 10: 15 (that's 15 blocks (IBD), 10 sample size (CRD) and 15 blocks (RCBD)

5.6. Mixed Design of IBD, CD and RCBD

For a mixed design of IBD, CD and RCBD, the following combinations of blocks were considered for a population four.

- **Equal number of blocks.**

6: 6: 6 (that's 6 blocks (IBD), 6 blocks(CD) and 6 blocks (RCBD)

12: 12: 12 (that's 12 blocks (IBD), 12 blocks(CD) and 12 blocks (RCBD)

18: 18: 18 (that's 18 blocks (IBD), 18 blocks(CD) and 18 blocks (RCBD)

For a population of five, the following combinations of blocks were considered for a mixed design of (IBD, CD and RCBD)

- **Equal number of blocks.**

5: 5: 5 (that's 5 blocks (IBD), 5-blocks(CD) and 5 blocks (RCBD)

10: 10: 10 (that's 10 blocks (IBD), 10 blocks(CD) and 10 blocks (RCBD)

15: 15: 15 (that's 15 blocks (IBD), 15 blocks(CD) and 15 blocks (RCBD)

Under the umbrella alternative, the peak treatment was assumed to have the largest effect, but this did not occur at either end (the first or last treatments were not considered). The combinations of peak and number of treatments considered were

- Three treatments with the peak at two
- Four treatments with the peak at two
- Four treatments with the peak at three
- Five treatments with the peak at two
- Five treatments with the peak at three
- Five treatments with the peak at four

Regardless of number of treatments or the peak, two cases were looked at: evaluation of type I error and under a violation of the umbrella alternative. The treatment means were all set to zero. Thus, the power of the test below was the approximation of its type 1 error.

The following is a look at how the means were shifted by the number of treatments in the design. With three treatments, only one case was considered.

5.7. Three Populations with Peak at 2

For three populations with a peak at 2, powers were estimated in the following cases:

1. The peak is distinct and there is equal spacing between parameters, for example $(0.0, 0.5, 0.0)$
2. One additional parameter equals the peak, for example $(0.5, 0.5, 0.0)$
3. The peak is distinct and there is unequal spacing between parameters, for example $(0.0, 1.0, 0.6)$

With four treatments, the following cases were considered

5.8. Four Populations with Peak at 2

For four populations with a peak at 2, powers were estimated in the following cases:

1. There is equal spacing between parameters, for example $(0.5, 1.0, 0.5, 0.0)$
2. One additional parameter is equal to the peak and the other parameters are different from the peak, but are equal, for example $(0.5, 0.5, 0.0, 0.0)$
3. There is unequal spacing between parameters, for example $(0.0, 0.7, 0.5, 0.2)$
4. Two population parameters are the same, but different from the peak, for example $(0.0, 1.0, 0.2, 0.2)$
5. Two population parameters are equal to the peak, for example $(1.0, 1.0, 1.0, 0.0)$

5.9. Four Populations with Peak at 3

For four populations with a peak at 3, powers were estimated in the following cases:

1. There is equal spacing between parameters, for example $(0.0, 0.5, 1.0, 0.5)$
2. One additional parameter is equal to the peak and the other parameters are different from the peak, but are equal, for example $(0.0, 0.5, 0.5, 0.0)$
3. One additional parameter is equal to the peak and the other parameters are different from the peak, and different from each other, for example $(0.0, 1.0, 1.0, 0.2)$
4. There is unequal spacing between parameters, for example $(0.0, 0.2, 0.7, 0.5)$
5. Two population parameters are the same, but different from the peak, for example $(0.0, 1.0, 1.0, 0.2)$
6. Two additional parameters are equal to the peak, for example $(1.0, 1.0, 1.0, 0.0)$

With treatments five, the following cases were considered

5.10. Five Populations with Peak at 2

For five populations with a peak at 2, powers were estimated in the following cases:

1. One additional parameter is equal to the peak and the other parameters are different from the peak, but are equal, for example $(1.0, 1.0, 0.5, 0.5, 0.5)$
2. One additional parameter is equal to the peak and the other parameters are different from the peak, and different from each other, for example $(1.0, 1.0, 0.5, 0.2, 0.0)$
3. Two additional parameters are equal to the peak and the other parameters are different from the peak, and different from each other, for example $(0.5, 1.0, 1.0, 1.0, 0.7)$
4. Three parameters are different from the peak, but are equal to each other, for example $(0.7, 1.0, 0.7, 0.7, 0.5)$
5. One additional parameter is equal to the peak, two other parameters are equal are to each other, but not the peak, and one parameter different, for example $(1.0, 1.0, 0.5, 0.5, 0.2)$
6. Three additional parameters are equal to the peak, for example $(1.0, 1.0, 1.0, 1.0,0)$
7. There is unequal spacing between parameters, for example $(0.0, 1.6, 0.8, 0.4,0.2)$
8. There is equal spacing between parameters, for example $(0.75, 1.0, 0.75, 0.5,0.5, 0.25)$

5.11. Five Populations with Peak at 3

For five populations with a peak at 3, powers were estimated in the following cases:

1. One additional parameter is equal to the peak and the other parameters are different from the peak, but are equal to each other, for example $(0.0, 1.0, 1.0, 0.0, 0.0)$
2. One additional parameter is equal to the peak and the other parameters are different from the peak, and different from each other, for example $(0.0, 0.5, 1.0, 1.0, 0.7)$
3. Two additional parameters are equal to the peak and the other parameters are different from the peak, and different from each other, for example $(1.0, 1.0, 1.0, 0.5, 0.0)$

4. Two parameters are different from the peak, but are equal to each other, the other two additional parameters are also different from the peak, but are equal to each other, for example $(0.7, 0.7, 1.0, 0.5, 0.5)$
5. There is equal spacing between parameters, for example $(0.0, 0.5, 1.0, 0.5, 0.0)$
6. Three additional parameters are equal to the peak, for example $(1.0, 1.0, 1.0, 1.0, 0)$

5.12. Five Populations with Peak at 4

For five populations with a peak at 4, powers were estimated in the following cases:

1. One additional parameter is equal to the peak and the other parameters are different from the peak, but are equal to each other, for example $(0.5, 0.5, 0.5, 1.0, 1.0)$
2. One additional parameter is equal to the peak and the other parameters are different from the peak, and different from each other, for example $(0.0, 0.5, 1.0, 1.0, 0.7)$
3. Two additional parameters are equal to the peak and the other parameters are different from the peak, and different from each other, for example $(0.0, 0.5, 1.0, 1.0, 1.0)$
4. Two additional parameters are equal to the peak and the other parameters are also different from the peak, but are equal to each other, for example $(0.0, 0.0, 1.0, 1.0, 1.0)$
5. Two parameters are different from the peak, but are equal to each other, the other two additional parameters are also different from the peak, but are equal to each other, for example $(0.5, 0.5, 0.7, 1.0, 0.7)$
6. Two parameters are different from the peak, but are equal to each other, the other two additional parameters are also different from the peak and different from each other, for example $(0.0, 0.25, 0.5, 1.0, 0.25)$
7. Three additional parameters are equal to the peak, for example $(1.0, 1.0, 1.0, 1.0, 0.0)$
8. There is unequal spacing between parameters, for example $(0.0, 0.4, 0.8, 1.6, 0.2)$

CHAPTER 6. RESULTS

In this chapter, we investigate the results of the simulation study. Three modification tests were developed for the umbrella alternative with a known peak under the various mixed designs in Chapter Four. These are the Non-Modification, Distance Modification, and Distance Squared Modification tests.

The Non-Modification test is a combination of the usual Mack-Wolfe (1981) test, Kim-Kim (1992) test, and Magel -Ndungu (2011) test. The Distance Modification test is a combination of the Distance Modification of the Mack-Wolfe (1981) test, the Distance Modification of the Kim-Kim (1992) test and the Distance Modification of the Magel -Ndungu (2011) test. The Distance Squared Modification test is also a combination of the Distance Squared Modification of the Mack-Wolfe (1981) test, the Distance Squared Modification of the Kim-Kim (1992) test and the Distance Squared Modification of the Magel -Ndungu (2011) test.

Each modification test consists of the proposed standardized versions of the test statistics. These are standardized first and standardized last. Standardized first standardized the test statistics first, and summed them before re-standardizing them. Standardized last combined the unstandardized test statistic values and then standardized the sum.

Simulated results showed a common trend when the powers of the underlying distributions are studied. The exponential distribution reported the highest powers. It was followed by the normal and then the T with three degrees of freedom. Type I error stayed proportionately around 5% for all test statistics.

Results for each mixed design followed a common trend under normal, exponential and T-distributions. Therefore, only a few results are selected for the analysis and the rest are kept under Appendix.

6.1. Mixed Design of BIBD, CRD, and RCBD

Table 123. Treatments= 3, BIBD =6, CRD=6, RCBD=6 Peak=2 (One Missing Observations)

Distribution	Location			Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	First	Last	First	Last	First	Last
Normal	0	0	0	0.0494	0.0480	0.0494	0.0480	0.0494	0.0480
	0	0.5	0.5	**0.1698**	0.1446	**0.1698**	0.1446	**0.1698**	0.1446
	0.5	0.5	0	**0.1752**	0.1468	**0.1752**	0.1468	**0.1752**	0.1468
	0.2	0.6	0.2	**0.3024**	0.2406	**0.3024**	0.2406	**0.3024**	0.2406
	0	0.5	0	**0.4066**	0.3110	**0.4066**	0.3110	**0.4066**	0.3110
	0	1	0.6	**0.6014**	0.4732	**0.6014**	0.4732	**0.6014**	0.4732
	0.6	1	0	**0.6084**	0.4668	**0.6084**	0.4668	**0.6084**	0.4668
Exponential	0	0	0	0.0548	0.0534	0.0548	0.0534	0.0548	0.0534
	0	0.5	0.5	**0.2416**	0.1866	**0.2416**	0.1866	**0.2416**	0.1866
	0.5	0.5	0	**0.2444**	0.1890	**0.2444**	0.1890	**0.2444**	0.1890
	0.2	0.6	0.2	**0.5104**	0.3874	**0.5104**	0.3874	**0.5104**	0.3874
	0	0.5	0	**0.6520**	0.5092	**0.6520**	0.5092	**0.6520**	0.5092
	0	1	0.6	**0.8242**	0.6810	**0.8242**	0.6810	**0.8242**	0.6810
	0.6	1	0	**0.8214**	0.6810	**0.8214**	0.6810	**0.8214**	0.6810
T with 3 degrees of freedom	0	0	0	0.0514	0.0472	0.0514	0.0472	0.0514	0.0472
	0	0.5	0.5	**0.1354**	0.1142	**0.1354**	0.1142	**0.1354**	0.1142
	0.5	0.5	0	**0.1338**	0.1156	**0.1338**	0.1156	**0.1338**	0.1156
	0.2	0.6	0.2	**0.2276**	0.1822	**0.2276**	0.1822	**0.2276**	0.1822
	0	0.5	0	**0.3046**	0.2340	**0.3046**	0.2340	**0.3046**	0.2340
	0	1	0.6	**0.4488**	0.3468	**0.4488**	0.3468	**0.4488**	0.3468
	0.6	1	0	**0.4480**	0.3516	**0.4480**	0.3516	**0.4480**	0.3516

6.1.1. Three Treatment at Peak Two

In the case of three treatments at peak 2, we found that the results for the distinct modification tests for the mixed design of BIBD, CRD, and RCBD were the same. This was expected since the distinct modification tests have an equal distance weight of one. Overall, the standardized first version of the test provides estimated powers slightly higher than the standardized last version of the test regardless of the underlying distribution and the proportion of BIBD to CRD and BIBD to RCBD. This means that standardizing the sum of the standardized test statistics of Magel - Ndungu (2011), Mack-Wolfe (1981), and Kim-Kim (1992) was better than standardizing the sum of the unstandardized test statistics. Table 123 (Refer to

Table A.1. in the Appendix) and Figure 4 show the probabilities of all the three modification tests.

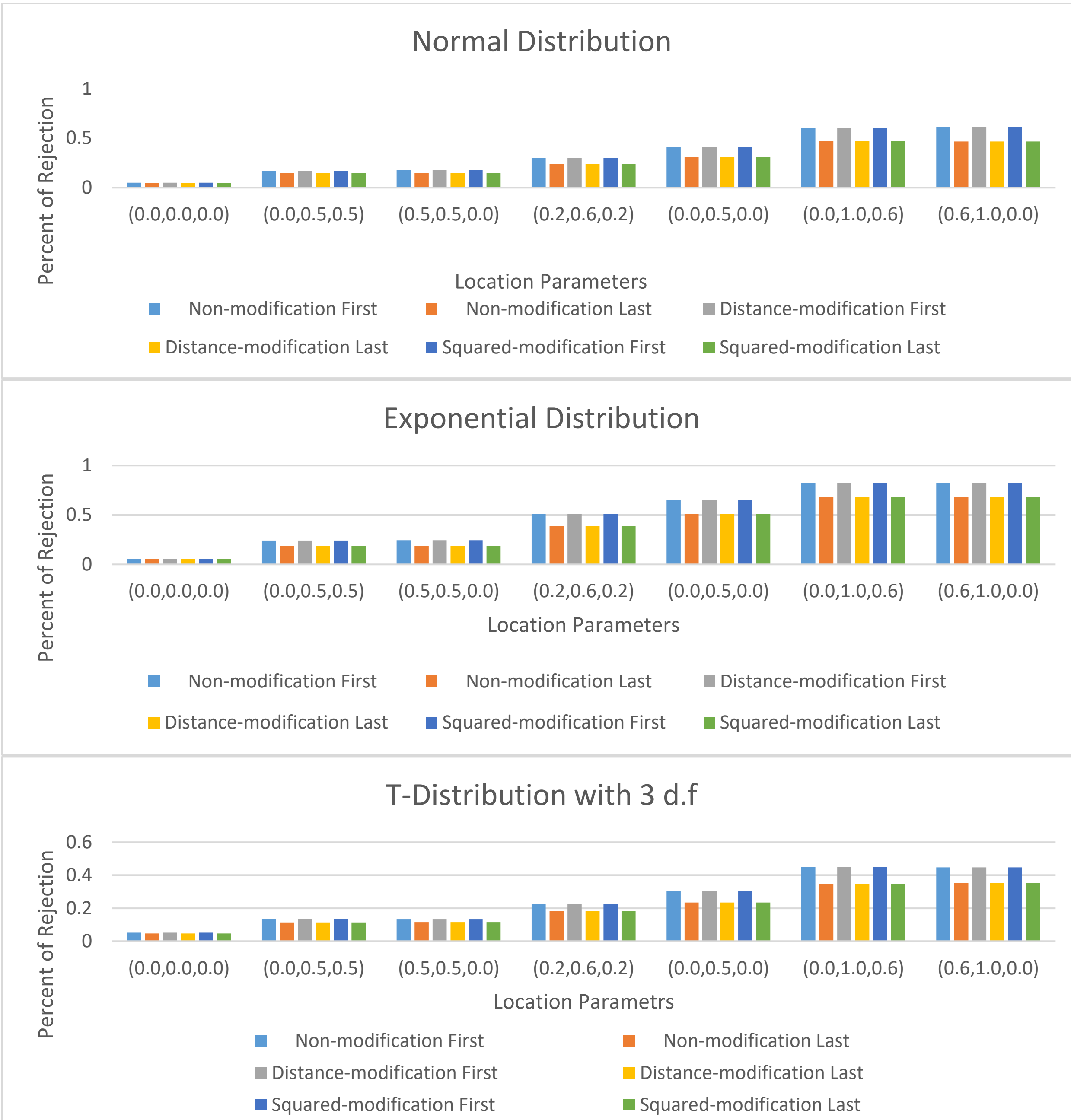

Figure 4. Treatments= 3, BIBD =6, CRD=6, RCBD=6 Peak=2 (One Missing Observations)

Table 124. Treatments=4, BIBD = 6, CRD=6, RCBD = 6 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0556	0.0486	0.0428	0.0528	0.0528	0.0494
	0.5	0.5	0	0	0.2132	0.1728	0.2384	0.1890	**0.2396**	0.2092
	0	1	0.2	0.2	**0.7358**	0.6090	0.7246	0.6326	0.7180	0.6620
	1	1	0	0	0.5178	0.3904	0.5564	0.4340	**0.5708**	0.4790
	0	0.7	0.2	0	0.5870	0.4756	**0.5902**	0.4898	0.5732	0.5214
	0.5	1	0.5	0	**0.7312**	0.5982	0.7282	0.6156	0.7310	0.6846
Exponential	0	0	0	0	0.0534	0.0472	0.0532	0.0470	0.0474	0.0528
	0.5	0.5	0	0	0.3402	0.2544	0.3670	0.2698	**0.3816**	0.3102
	0	1	0.2	0.2	**0.9320**	0.8280	0.9204	0.8486	0.9168	0.8748
	1	1	0	0	0.6822	0.5432	0.6984	0.5794	**0.7280**	0.6332
	0	0.7	0.2	0	0.8462	0.7402	**0.8636**	0.7582	0.8394	0.7912
	0.5	1	0.5	0	**0.9394**	0.8396	0.9336	0.8530	0.9276	0.8884
T with 3 degrees of freedom	0	0	0	0	0.0504	0.0530	0.0484	0.0470	0.0522	0.0504
	0.5	0.5	0	0	0.1754	0.1476	0.1896	0.1554	**0.1902**	0.1732
	0	1	0.2	0.2	**0.5876**	0.4766	0.5684	0.4692	0.5556	0.5010
	1	1	0	0	0.4024	0.2948	0.4126	0.3224	**0.4318**	0.3706
	0	0.7	0.2	0	0.4400	0.3498	**0.4642**	0.3770	0.4396	0.3822
	0.5	1	0.5	0	**0.5742**	0.4306	0.5722	0.4638	0.5584	0.5044

6.1.2. Four Treatment at Peak Two

In the case of four treatments at peak two, we observed that the standardized first versions of the test provides estimated powers slightly higher than the standardized last versions of the test regardless of the underlying distribution and the proportion of BIBD to CRD and BIBD to RCBD. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If the distance between population parameters before the peak and the peak is greater than the distance between parameters after the peak and the peak, such as $(0.0,1.0,0.2,0.2)$, the non-modification first performed better. Where one additional parameter is equal to the peak and the other parameters are different from the peak, but are equal, such as $(1.0,1.0,0.0,0.0)$, the distance squared modification first performed better. Furthermore, where two population parameters are the same, but different from the peak, such as (0.0,0.7,0.2,0.0), the distance modification first performed better. Table 124 (Tables B.1. to B.9. in the Appendix) and Figure 5 show the percentage of rejection of all the three modification tests.

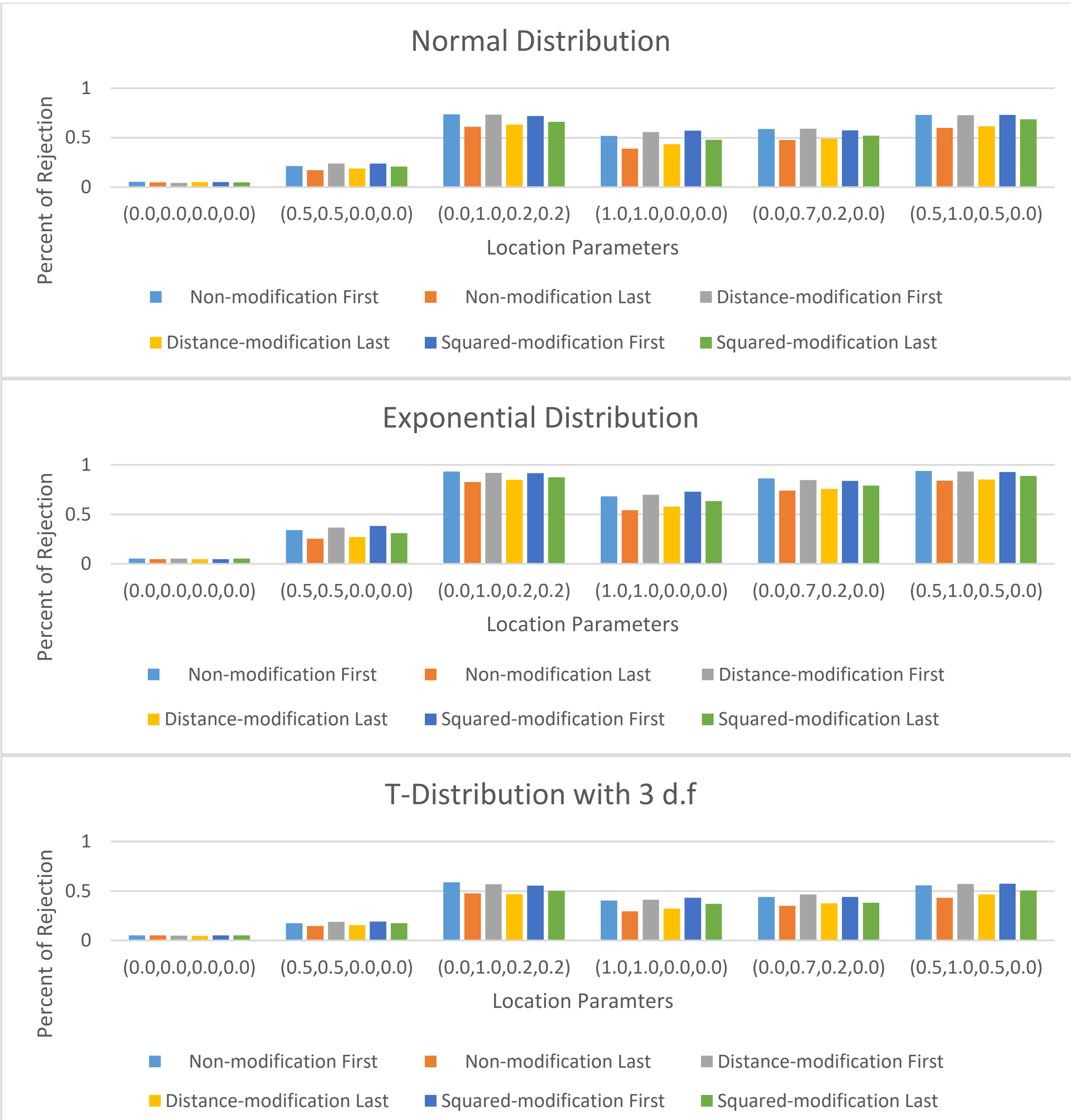

Figure 5. Treatments=4, BIBD = 6, CRD=6, RCBD = 6 Peak=2 (Two Missing Observations)

Table 125. Treatments=4, BIBD = 6, CRD=6, RCBD = 6 Peak=3 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0478	0.0494	0.0488	0.0552	0.0464	0.0494
	0	0.5	0.5	0	0.3732	0.2984	**0.4488**	0.3530	0.3762	0.3430
	0	1	1	0.2	0.8062	0.6678	**0.8700**	0.7296	0.7842	0.7204
	0	0	1	0.2	0.9854	0.9200	**0.9988**	0.9948	0.9840	0.9428
	1	1	1	0	0.2018	0.1572	**0.2030**	0.1796	0.1636	0.1448
	0	0.2	0.7	0.5	0.4206	0.3294	**0.5016**	0.3856	0.4274	0.3692
	0	0.5	1	0.5	0.7224	0.5848	**0.8182**	0.6756	0.7362	0.6550
Exponential	0	0	0	0	0.0504	0.0542	0.0560	0.0490	0.0530	0.0476
	0	0.5	0.5	0	0.6096	0.4790	**0.7058**	0.5718	0.6144	0.5366
	0	1	1	0.2	0.9358	0.8440	**0.9698**	0.9006	0.9256	0.8928
	0	0	1	0.2	0.9524	0.8696	**0.9804**	0.9262	0.9442	0.9146
	1	1	1	0	0.2474	0.1898	**0.2468**	0.2112	0.1990	0.1726
	0	0.2	0.7	0.5	0.6666	0.5332	**0.7756**	0.6316	0.6808	0.6046
	0	0.5	1	0.5	0.9326	0.8298	**0.9744**	0.9020	0.9274	0.8906
T with 3 degrees of freedom	0	0	0	0	0.0510	0.0520	0.0490	0.0478	0.0530	0.0480
	0	0.5	0.5	0	0.2944	0.2392	**0.3236**	0.2674	0.2784	0.2578
	0	1	1	0.2	0.6338	0.5132	**0.7140**	0.5860	0.6230	0.5560
	0	0	1	0.2	0.6236	0.5150	**0.7186**	0.5880	0.6202	0.5798
	1	1	1	0	0.1638	0.1408	**0.1658**	0.1540	0.1292	0.1282
	0	0.2	0.7	0.5	0.3230	0.2536	**0.3756**	0.3014	0.3174	0.2892
	0	0.5	1	0.5	0.5656	0.4536	**0.6518**	0.5104	0.5808	0.4928

6.1.3. Four Treatments at Peak Three

The results in Table 125 showed that the standardized first versions of the test provides estimated powers slightly higher than the standardized last versions of the test regardless of the underlying distribution and the proportion of BIBD to CRD and BIBD to RCBD. It is important to note that, among the three distinct tests, the distance modification first provides the highest values of the estimated powers test statistics in all the population parameters. This was expected since the means and variances for four treatments at peaks other than peak three were equal. Refer to Tables B.28. to B.36. in the Appendix.

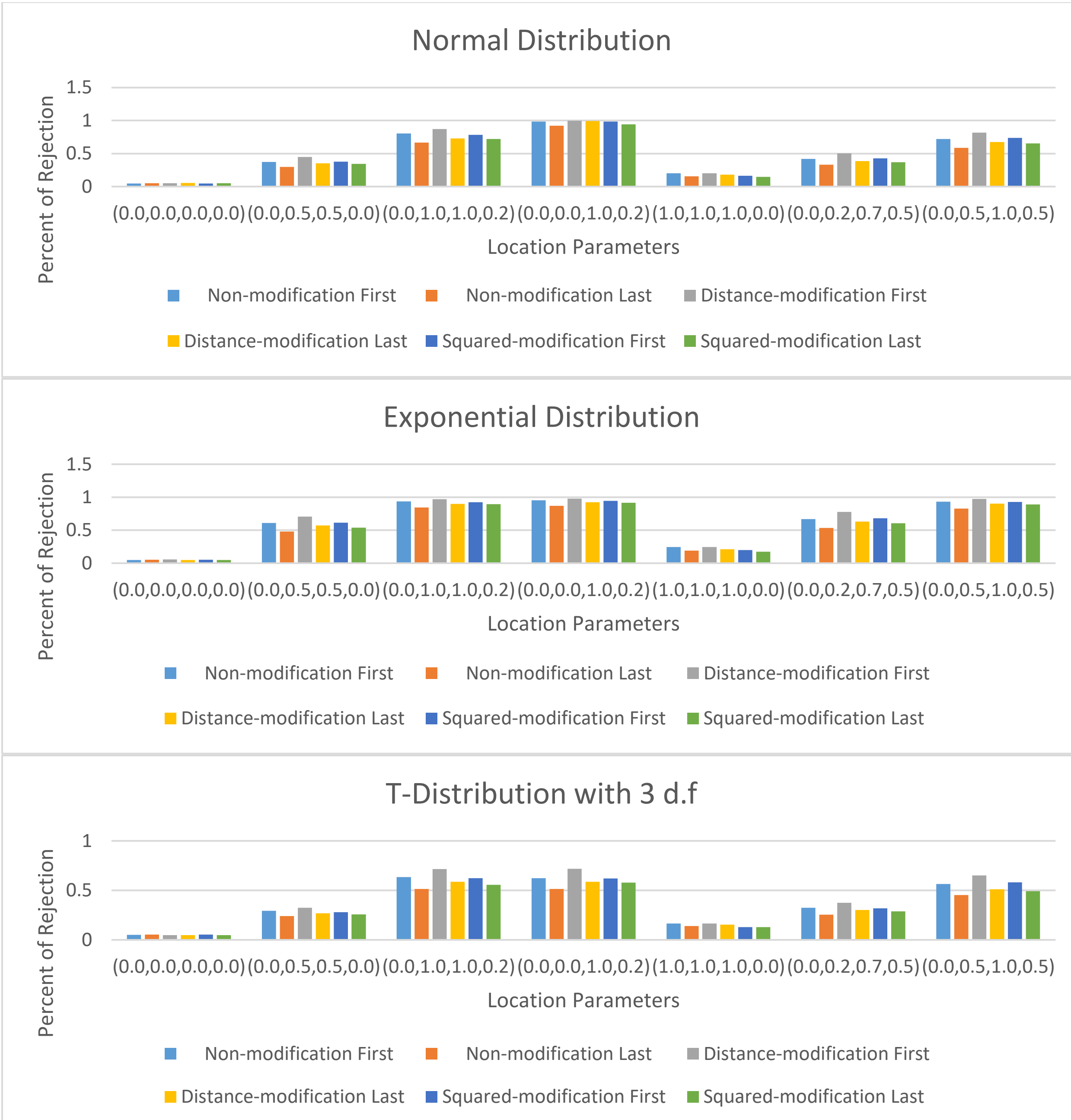

Figure 6. Treatments=4, BIBD = 6, CRD=6, RCBD = 6 Peak=3 (Two Missing Observations)

Table 126. Treatments=5, BIBD =5, CRD=5, RCBD=5 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0486	0.0528	0.0512	0.0520	0.0438	0.0474
	1	1	0.5	0.5	0.5	0.2062	0.1712	**0.2240**	0.1994	0.2228	0.2116
	0.5	1	1	1	0.7	**0.1960**	0.1652	0.1870	0.1820	0.1832	0.1818
	0.7	1	0.7	0.7	0.5	0.2742	0.2252	**0.2760**	0.2428	0.2656	0.2550
	1	1	0.5	0.5	0.2	0.3874	0.3358	**0.4112**	0.3672	0.4032	0.3896
	0.75	1	0.75	0.5	0.25	0.4752	0.3956	**0.4762**	0.4296	0.4722	0.4566
	1	1	1	1	0	0.4994	0.4078	0.5164	0.4608	**0.5238**	0.4914
	1	1	0.5	0.2	0	0.6054	0.4848	**0.6222**	0.5658	0.6090	0.5932
	0	1.6	0.8	0.4	0.2	**0.9654**	0.9174	0.9636	0.9442	0.9502	0.9430
Exponential	0	0	0	0	0	0.0430	0.0480	0.0496	0.0540	0.0540	0.0502
	1	1	0.5	0.5	0.5	0.3316	0.2604	**0.3544**	0.2946	0.3308	0.3244
	0.5	1	1	1	0.7	**0.3554**	0.2790	0.3200	0.2968	0.2958	0.2862
	0.7	1	0.7	0.7	0.5	0.4746	0.3894	**0.4874**	0.4350	0.4530	0.4510
	1	1	0.5	0.5	0.2	0.6136	0.5110	0.**6540**	0.5770	0.6350	0.6166
	0.75	1	0.75	0.5	0.25	0.7622	0.6540	**0.7640**	0.7024	0.7322	0.7276
	1	1	1	1	0	0.6400	0.5310	0.6770	0.5942	**0.6802**	0.6620
	1	1	0.5	0.2	0	0.8340	0.7248	**0.8490**	0.7950	0.8360	0.8118
	0	1.6	0.8	0.4	0.2	**0.9984**	0.9870	0.9950	0.9944	0.9936	0.9890
T with 3 degrees of freedom	0	0	0	0	0	0.0514	0.0506	0.0486	0.0524	0.0510	0.0540
	1	1	0.5	0.5	0.5	0.1614	0.1452	0.1730	0.1640	**0.1798**	0.1794
	0.5	1	1	1	0.7	**0.1660**	0.1494	0.1590	0.1538	0.1450	0.1428
	0.7	1	0.7	0.7	0.5	**0.2116**	0.1848	0.2036	0.1866	0.2112	0.2004
	1	1	0.5	0.5	0.2	0.2952	0.2426	**0.3116**	0.2742	0.2956	0.3080
	0.75	1	0.75	0.5	0.25	0.3580	0.2964	**0.3730**	0.3274	0.3454	0.3610
	1	1	1	1	0	0.3736	0.3156	0.3876	0.3438	**0.3998**	0.3816
	1	1	0.5	0.2	0	0.4530	0.3654	**0.4788**	0.4046	0.4706	0.4498
	0	1.6	0.8	0.4	0.2	**0.8676**	0.7668	0.8642	0.8140	0.8392	0.8178

6.1.4. Five Treatment at Peaks Two

With five treatment at peak two, the standardized first was slightly higher than the standardized last versions of the test regardless of the underlying distribution. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If the distance between population parameters before the peak and the peak is greater than the distance between parameters after the peak and the peak, such as (0.5,1.0,1.0,1.0,0.7), the non-modification first performed better. Where one additional parameter is equal to the peak and the other parameters are different from the peak, but are equal, such as (1.0,1.0,0.5,0.2,0.0), the distance modification first performed better. Moreover, where three population parameters are equal to the peak and the other parameter is different from the peak, such as (1.0,1.0,1.0,1.0,0.0), the distance squared modification first performed slightly better.

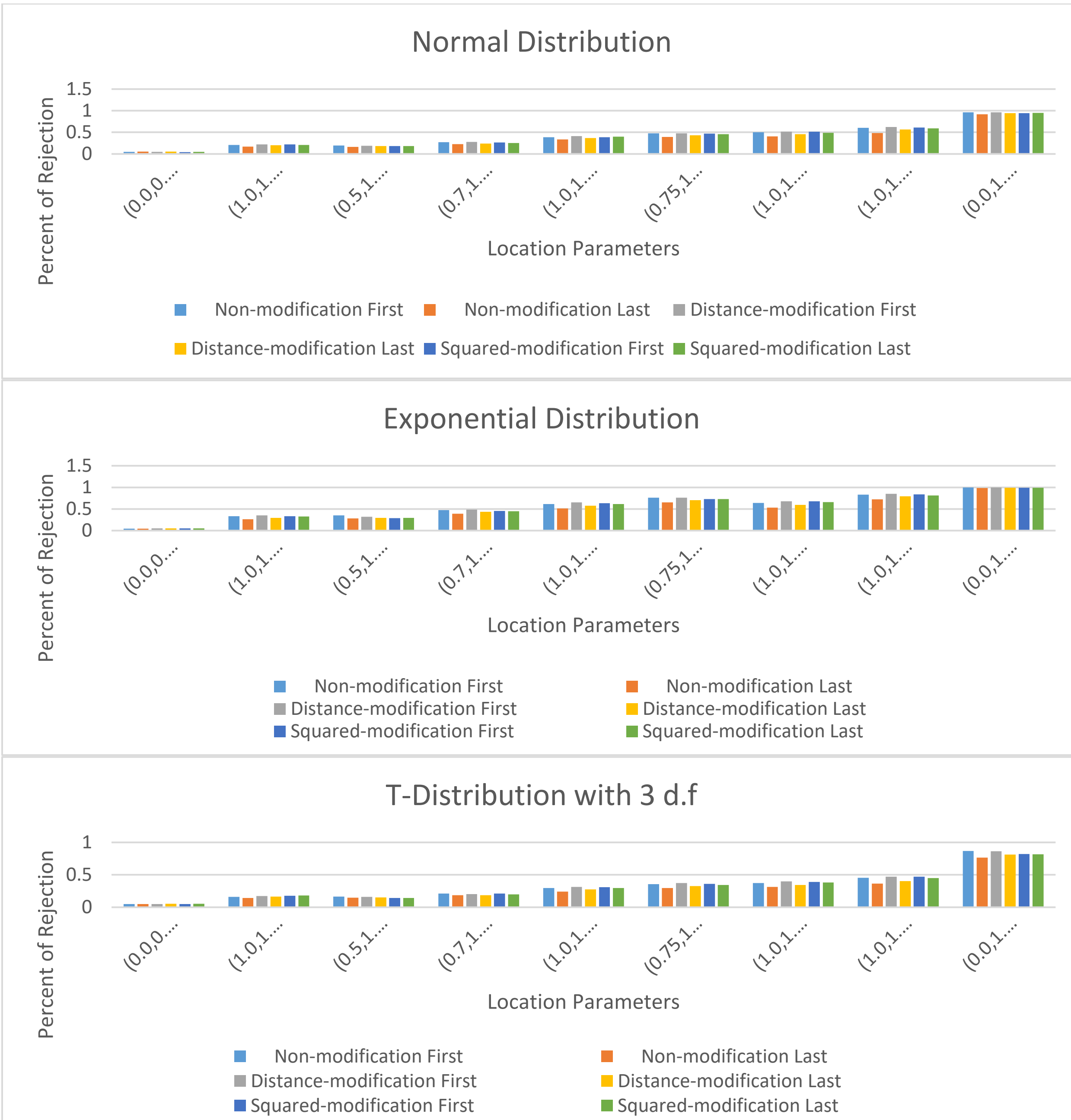

Figure 7. Treatments=5, BIBD =5, CRD=5, RCBD=5 Peak=2

Table 127. Treatments=5, BIBD =5, CRD=5, RCBD=5 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0484	0.0460	0.0534	0.0548	0.0496	0.0530
	0	0.5	1	1	0.7	0.3054	0.2974	0.4894	0.4130	**0.4920**	0.4434
	0.7	0.7	1	0.5	0.5	0.2096	0.2024	0.2540	0.2224	**0.2758**	0.2398
	0	0.5	1	0.5	0	0.6800	0.6400	0.8018	0.7194	**0.8022**	0.7852
	0	1	1	0	0	**0.8204**	0.6860	0.7912	0.7132	0.7958	0.7508
	1	1	1	0.5	0	**0.4340**	0.3078	0.3324	0.2760	0.3276	0.3106
	1	1	1	1	0	**0.3750**	0.2698	0.3222	0.2618	0.3310	0.2836
Exponential	0	0	0	0	0	0.0492	0.0474	0.0480	0.0484	0.0590	0.0490
	0	0.5	1	1	0.7	0.4712	0.4648	0.7276	0.6454	**0.7284**	0.6894
	0.7	0.7	1	0.5	0.5	0.3476	0.3078	0.4336	0.3558	**0.4346**	0.4000
	0	0.5	1	0.5	0	0.8858	0.8746	0.9686	0.9312	**0.9696**	0.9576
	0	1	1	0	0	**0.9476**	0.8636	0.9374	0.8798	0.9366	0.9152
	1	1	1	0.5	0	**0.6302**	0.4398	0.4662	0.3994	0.4758	0.4424
	1	1	1	1	0	**0.4982**	0.3672	0.4078	0.3468	0.4030	0.3838
T with 3 degrees of freedom	0	0	0	0	0	0.0484	0.0476	0.0490	0.0462	0.0496	0.0520
	0	0.5	1	1	0.7	0.2374	0.2290	0.3552	0.3132	**0.3734**	0.3420
	0.7	0.7	1	0.5	0.5	0.1702	0.1634	0.1974	0.1830	**0.2082**	0.2020
	0	0.5	1	0.5	0	0.5194	0.4778	0.6476	0.5590	**0.6590**	0.6060
	0	1	1	0	0	**0.6678**	0.5358	0.6204	0.5382	0.6386	0.5944
	1	1	1	0.5	0	**0.3340**	0.2354	0.2600	0.2250	0.2564	0.2258
	1	1	1	1	0	**0.2864**	0.2202	0.2496	0.2154	0.2560	0.2270

6.1.5. Five Treatments at Peak Three

With five treatment at peak 3, the standardized first was slightly higher than the standardized last versions of the test for the mixed design of BIBD, CRD, and RCBD, regardless of the underlying distribution and the proportion of BIBD to CRD and RCBD. In particular, with a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If one additional parameter is equal to the peak and the other parameters are equal, but different from the peak, such as (0.0,1.0,1.0,0.0,0.0), the non-modification first performed better. On the other hand, where there is equal spacing among population parameters, such as

(0.0,0.5,1.0,0.5, 0.0), the distance squared modification first performed better. Refer to Tables C.28. to C.36. in the Appendix.

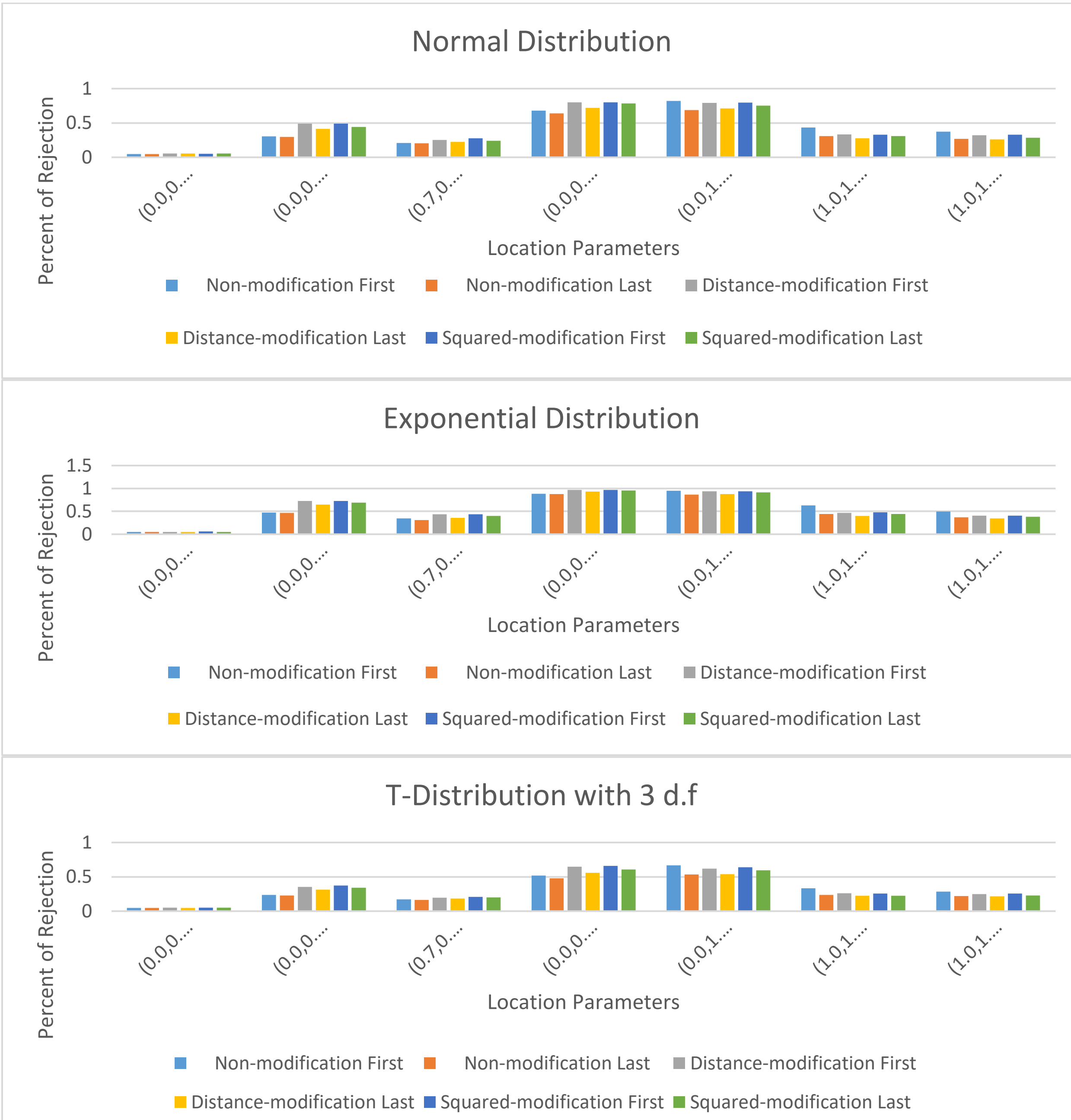

Figure 8. Treatments=5, BIBD =5, CRD=5, RCBD=5 Peak=3

Table 128. Treatments=5, BIBD =5, CRD=5, RCBD=5 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0480	0.0460	0.0468	0.0466	0.0582	0.0508
	0.5	0.5	0.5	1	1	0.2180	0.1708	**0.2218**	0.2030	0.1962	0.1818
	0	0.5	1	1	0.7	0.6946	0.5878	**0.7120**	0.6682	0.6322	0.6070
	0	0.5	1	1	1	0.6336	0.5296	**0.6508**	0.5892	0.5352	0.4922
	0	0	1	1	1	0.7312	0.5974	**0.7564**	0.6858	0.5868	0.5676
	0.5	0.5	0.7	1	0.7	0.3164	0.2630	**0.3190**	0.2722	0.2064	0.2118
	1	1	1	1	0	0.1234	0.1074	0.1252	0.1062	**0.1900**	0.1630
	1	0.4	0.8	1.6	0.4	**0.9794**	0.9272	0.9748	0.9518	0.8832	0.8786
	0	0.25	0.5	1	0.25	**0.7516**	0.6282	0.7532	0.6914	0.5578	0.5476
Exponential	0	0	0	0	0	0.0514	0.0528	0.0494	0.0486	0.0534	0.0492
	0.5	0.5	0.5	1	1	0.3274	0.2462	**0.3490**	0.2902	0.2768	0.2572
	0	0.5	1	1	0.7	0.9028	0.8004	0.9126	0.8454	0.8506	0.8172
	0	0.5	1	1	1	0.8334	0.7356	**0.8606**	0.8018	0.7280	0.7082
	0	0	1	1	1	0.8724	0.7714	**0.8950**	0.8440	0.7504	0.7244
	0.5	0.5	0.7	1	0.7	**0.5502**	0.4398	0.5446	0.4832	0.3424	0.3524
	1	1	1	1	0	0.1474	0.1172	0.1284	0.1070	**0.2288**	0.1784
	1	0.4	0.8	1.6	0.4	**0.9982**	0.9922	0.9976	0.9964	0.9800	0.9832
	0	0.25	0.5	1	0.25	**0.9448**	0.8806	0.9334	0.9144	0.8206	0.8152
T with 3 degrees of freedom	0	0	0	0	0	0.0478	0.0468	0.0468	0.0466	0.0560	0.0516
	0.5	0.5	0.5	1	1	0.1736	0.1462	**0.1758**	0.1686	0.1676	0.1488
	0	0.5	1	1	0.7	0.5372	0.4406	**0.5608**	0.4822	0.4856	0.4520
	0	0.5	1	1	1	0.4792	0.3844	**0.5080**	0.4420	0.4074	0.3848
	0	0	1	1	1	0.5612	0.4540	**0.5756**	0.5076	0.4482	0.4282
	0.5	0.5	0.7	1	0.7	0.2476	0.2050	**0.2570**	0.2140	0.1614	0.1606
	1	1	1	1	0	0.1096	0.1050	0.1022	0.1018	**0.1600**	0.1340
	1	0.4	0.8	1.6	0.4	0.8950	0.7962	0.8948	0.8414	0.7384	0.7324
	0	0.25	0.5	1	0.25	**0.5954**	0.4818	0.5868	0.5222	0.4384	0.4282

6.1.6. Five Treatments at Peak Four

With five treatments at peak four, the standardized first provides estimated powers slightly higher than the standardized last versions of the test for the mixed design of BIBD, CRD, and RCBD, regardless of the underlying distribution and the proportion of BIBD to CRD and RCBD. With a significant difference of 0.01, between the results of the three modification tests, the distance modification first in most cases reported the highest values of the estimated powers test statistics, except where two population parameters are equal and different from the peak and the other parameters are different from each other, such as (0.0,0.25,0.5,1.0,0.25). In this case, the non-modification first performed better. Where three population parameters are equal to the peak and the other parameter different from the peak, such as (1.0,1.0,1.0,1.0,0.0), the distance squared modification first performed better. Refer to Tables C.54. to C.62. in the Appendix.

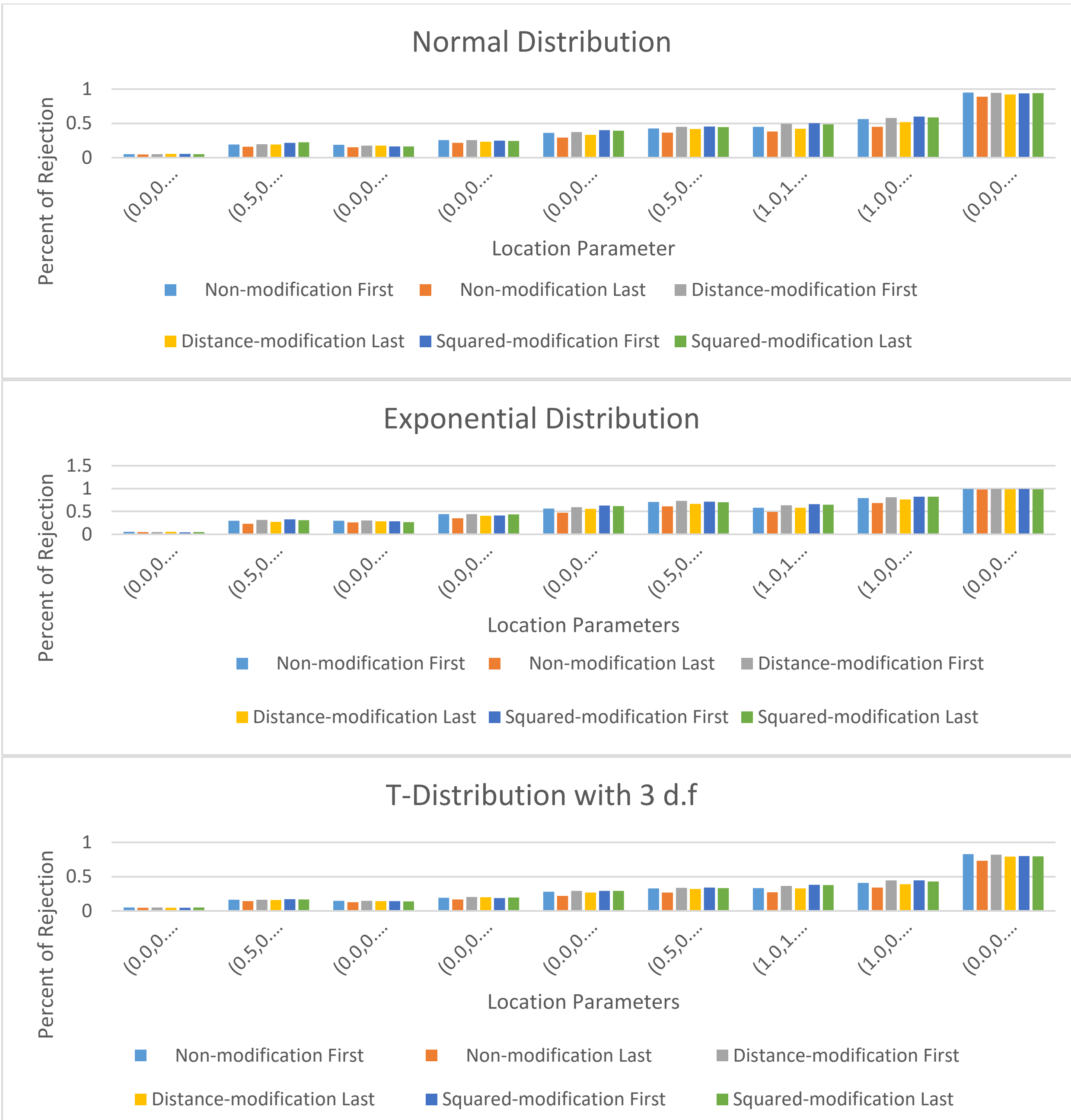

Figure 9. Treatments=5, BIBD =5, CRD=5, RCBD=5 Peak=4

6.2. Mixed Design of BIBD and CRD

Table 129. Treatments =3, BIBD =6, CRD=6 Peak=2 (One Missing Observation)

Distribution	Location			Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	First	Last	First	Last	First	Last
Normal	0	0	0	0.0534	0.0522	0.0534	0.0522	0.0534	0.0522
	0	0.5	0.5	**0.1310**	0.1212	**0.1310**	0.1212	**0.1310**	0.1212
	0.5	0.5	0	**0.1328**	0.1228	**0.1328**	0.1228	**0.1328**	0.1228
	0.2	0.6	0.2	**0.2184**	0.2014	**0.2184**	0.2014	**0.2184**	0.2014
	0	0.5	0	**0.2818**	0.2654	**0.2818**	0.2654	**0.2818**	0.2654
	0	1	0.6	**0.4202**	0.3932	**0.4202**	0.3932	**0.4202**	0.3932
	0.6	1	0	**0.4172**	0.3796	**0.4172**	0.3796	**0.4172**	0.3796
Exponential	0	0	0	0.0518	0.0532	0.0518	0.0532	0.0518	0.0532
	0	0.5	0.5	**0.1780**	0.1572	**0.1780**	0.1572	**0.1780**	0.1572
	0.5	0.5	0	**0.1776**	0.1582	**0.1776**	0.1582	**0.1776**	0.1582
	0.2	0.6	0.2	**0.3436**	0.3148	**0.3436**	0.3148	**0.3436**	0.3148
	0	0.5	0	**0.4456**	0.4124	**0.4456**	0.4124	**0.4456**	0.4124
	0	1	0.6	**0.5936**	0.5678	**0.5936**	0.5678	**0.5936**	0.5678
	0.6	1	0	**0.5918**	0.5620	**0.5918**	0.5620	**0.5918**	0.5620
T with 3 degrees of freedom	0	0	0	0.0508	0.0458	0.0508	0.0458	0.0508	0.0458
	0	0.5	0.5	**0.1054**	0.1038	**0.1054**	0.1038	**0.1054**	0.1038
	0.5	0.5	0	**0.1064**	0.1028	**0.1064**	0.1028	**0.1064**	0.1028
	0.2	0.6	0.2	**0.1654**	0.1574	**0.1654**	0.1574	**0.1654**	0.1574
	0	0.5	0	**0.2102**	0.1994	**0.2102**	0.1994	**0.2102**	0.1994
	0	1	0.6	**0.3154**	0.2908	**0.3154**	0.2908	**0.3154**	0.2908
	0.6	1	0	**0.3094**	0.2878	**0.3094**	0.2878	**0.3094**	0.2878

6.2.1. Three Treatment at Peak Two

In the case of three treatments at peak two, the standardized first was slightly higher than the standardized last versions of the test for the mixed design of BIBD, and CRD except where the ratio of BIBD to CRD was 1:2 or 1:3 or 2:3. Again, we found that the results for all distinct tests were the same. The reason was that all the distinct tests have an equal distance weight of one. Table 129 above (Refer to Tables A.2. to A.5. in the Appendix) and Figure 10 below show the percentage of rejection of all the three modification tests.

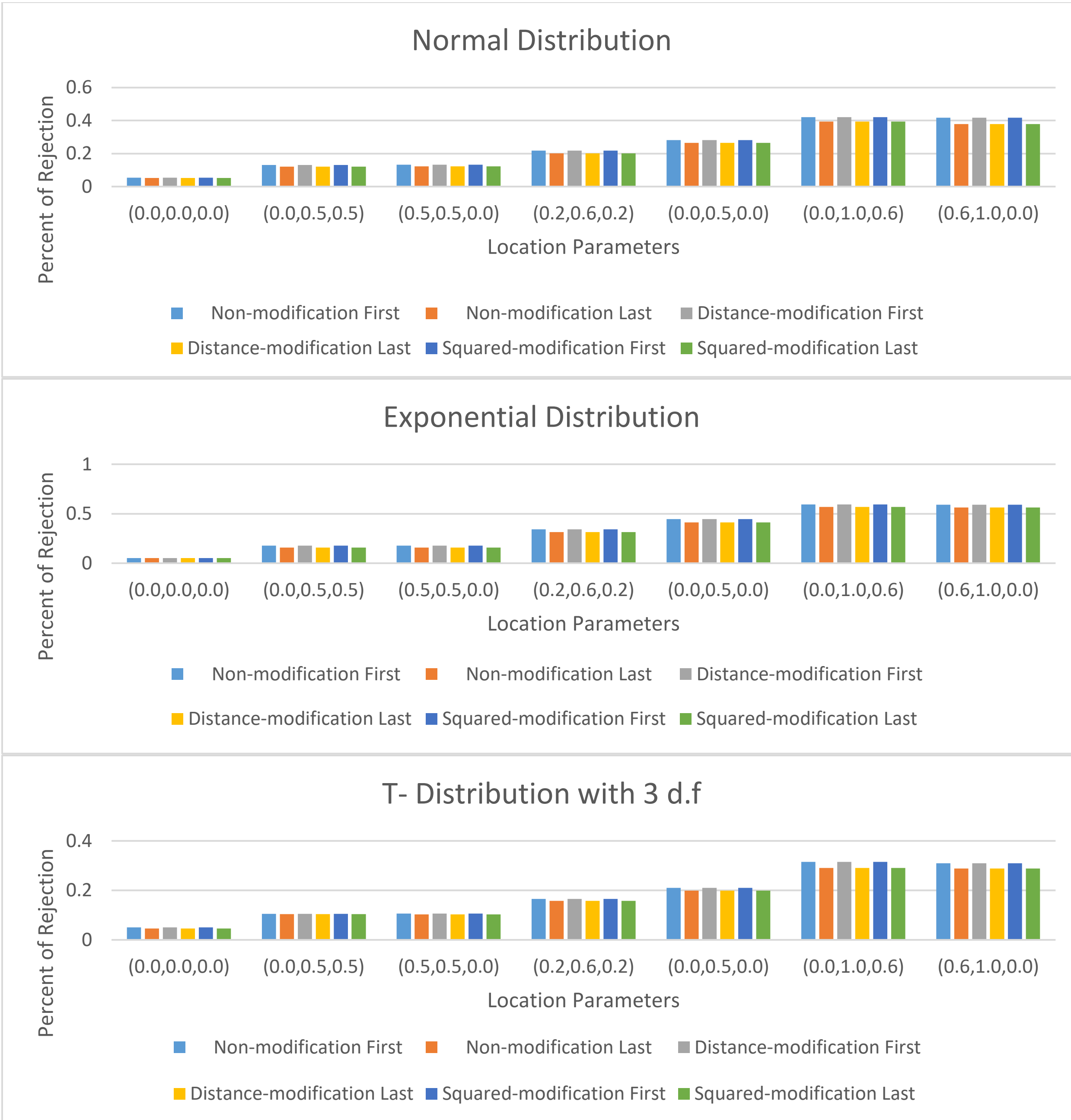

Figure 10. Treatments =3, BIBD =6, CRD=6 Peak=2 (One Missing Observation)

Table 130. Treatments=3, BIBD =6, CRD=18 Peak=2 (One Missing Observation)

Distribution	Location			Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	First	Last	First	Last	First	Last
Normal	0	0	0	0.0520	0.0510	0.0520	0.0510	0.0520	0.0510
	0	0.5	0.5	0.1896	**0.2028**	0.1896	**0.2028**	0.1896	**0.2028**
	0.5	0.5	0	0.1962	**0.2004**	0.1962	**0.2004**	0.1962	**0.2004**
	0.2	0.6	0.2	0.3516	**0.3720**	0.3516	**0.3720**	0.3516	**0.3720**
	0	0.5	0	0.4656	**0.5008**	0.4656	**0.5008**	0.4656	**0.5008**
	0	1	0.6	0.6914	**0.7476**	0.6914	**0.7476**	0.6914	**0.7476**
	0.6	1	0	0.6926	**0.7536**	0.6926	**0.7536**	0.6926	**0.7536**
Exponential	0	0	0	0.0472	0.0556	0.0472	0.0556	0.0472	0.0556
	0	0.5	0.5	0.2986	**0.3120**	0.2986	**0.3120**	0.2986	**0.3120**
	0.5	0.5	0	0.2882	**0.3082**	0.2882	**0.3082**	0.2882	**0.3082**
	0.2	0.6	0.2	0.5966	**0.6576**	0.5966	**0.6576**	0.5966	**0.6576**
	0	0.5	0	0.7382	**0.8038**	0.7382	**0.8038**	0.7382	**0.8038**
	0	1	0.6	0.8880	**0.9426**	0.8880	**0.9426**	0.8880	**0.9426**
	0.6	1	0	0.8870	**0.9408**	0.8870	**0.9408**	0.8870	**0.9408**
T with 3 degrees of freedom	0	0	0	0.0508	0.0520	0.0508	0.0520	0.0508	0.0520
	0	0.5	0.5	0.1626	**0.1724**	0.1626	**0.1724**	0.1626	**0.1724**
	0.5	0.5	0	0.1578	**0.1682**	0.1578	**0.1682**	0.1578	**0.1682**
	0.2	0.6	0.2	0.2728	**0.2990**	0.2728	**0.2990**	0.2728	**0.2990**
	0	0.5	0	0.3658	**0.3880**	0.3658	**0.3880**	0.3658	**0.3880**
	0	1	0.6	0.5368	**0.5806**	0.5368	**0.5806**	0.5368	**0.5806**
	0.6	1	0	0.5382	**0.5854**	0.5382	**0.5854**	0.5382	**0.5854**

Similarly, with three treatments at peak two, the standardized first was slightly higher than the standardized last versions of the test for the mixed design of BIBD, and CRD except where the ratio of BIBD to CRD was 1:2 or 1:3 or 2:3. Again, we found that the results for all distinct tests were the same. The reason was that all the distinct tests have an equal distance weight of one. Table 130 above (Refer Table A.2. to A.5. in the Appendix) and Figures 22-24 below show the probabilities of all three modification tests.

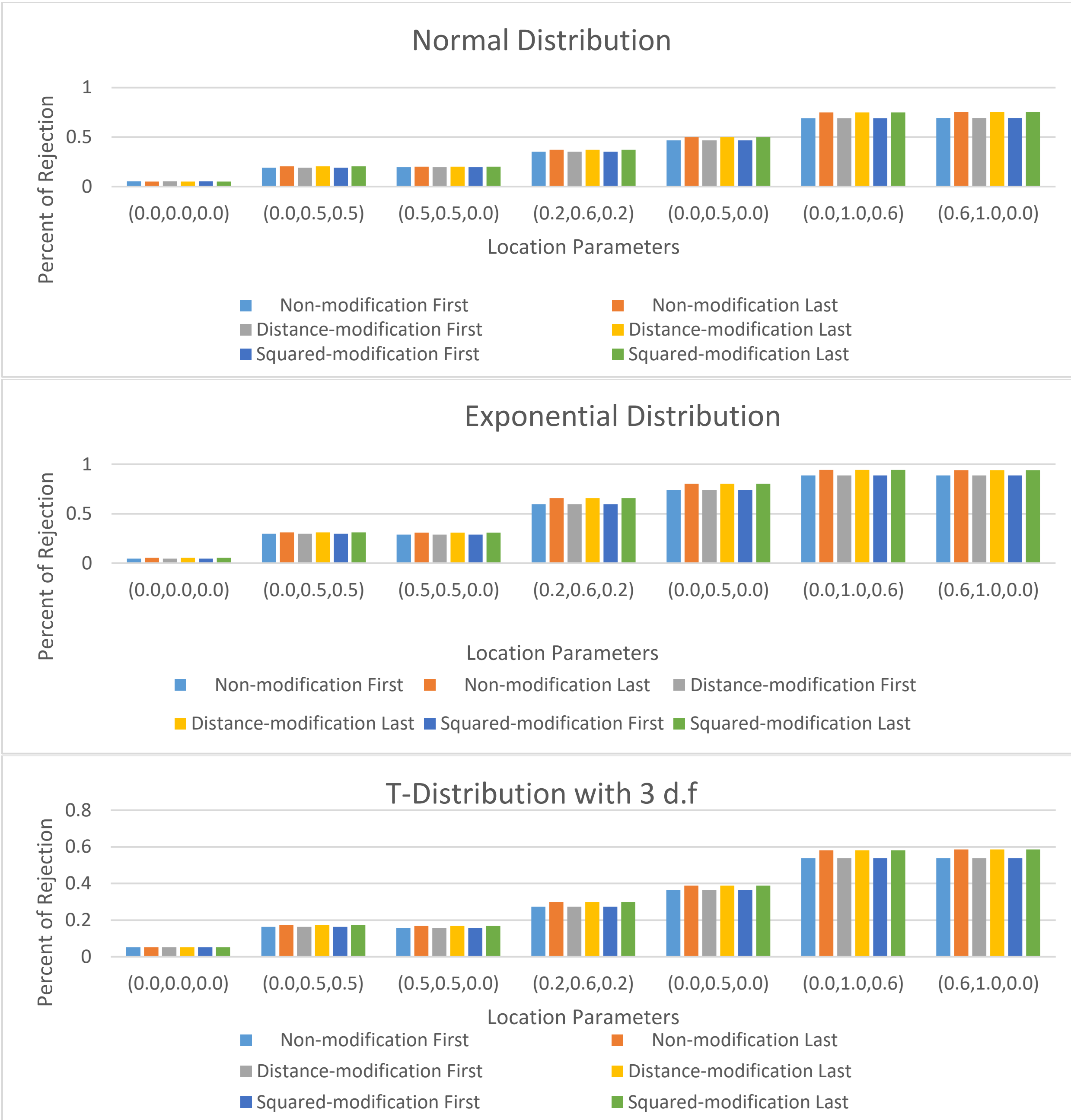

Figure 11. Treatments=3, BIBD =6, CRD=18 Peak=2 (One Missing Observation)

Table 131. Treatments=4, BIBD = 6, CRD=6 Peak=2 (Two missing observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0456	0.0476	0.0498	0.0504	0.0468	0.0454
	0.5	0.5	0	0	**0.1656**	0.1498	0.1638	0.1542	0.1610	0.1478
	0	1	0.2	0.2	**0.5344**	0.5096	0.5274	0.5036	0.5102	0.5002
	1	1	0	0	0.3450	0.3210	**0.3638**	0.3192	0.3472	0.3262
	0	0.7	0.2	0	0.4080	0.3844	0.3982	0.3712	**0.4122**	0.3858
	0.5	1	0.5	0	0.5134	0.4896	0.5134	0.4820	**0.5168**	0.4838
Exponential	0	0	0	0	0.0512	0.0518	0.0496	0.0508	0.0518	0.0520
	0.5	0.5	0	0	**0.2426**	0.2016	0.2392	0.2114	0.2282	0.1990
	0	1	0.2	0.2	**0.7156**	0.7132	0.7088	0.7118	0.7140	0.7074
	1	1	0	0	0.4538	0.4602	0.4682	0.4360	**0.4638**	0.4336
	0	0.7	0.2	0	0.6298	0.6214	0.6276	0.6118	**0.6436**	0.6146
	0.5	1	0.5	0	0.7420	0.7414	0.7410	0.7344	**0.7442**	0.7276
T with 3 degrees of freedom	0	0	0	0	0.0446	0.0544	0.0486	0.0478	0.0536	0.0472
	0.5	0.5	0	0	**0.1342**	0.1212	0.1338	0.1294	0.1274	0.1314
	0	1	0.2	0.2	**0.3926**	0.3726	0.3904	0.3746	0.3886	0.3718
	1	1	0	0	0.2668	0.2530	**0.2742**	0.2490	**0.2742**	0.2532
	0	0.7	0.2	0	0.3052	0.2932	0.3088	0.2858	**0.3152**	0.2982
	0.5	1	0.5	0	0.3868	0.3632	0.3732	0.3700	**0.3878**	0.3636

6.2.2. Four Treatment at Peak Two

With four treatments at peak two, the standard first versions of the test provide estimated powers slightly higher than the standardized last versions of the test for the mixed design of BIBD and CRD. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If one additional parameter is equal to the peak and the other parameters are different from the peak, but are equal, such as $(1.0,1.0,0.0,0.0)$, the distance modification first performed better. Where three population parameters are equal to the peak and the other parameter is different from the peak, such as $(0.0,0.7,0.2,0.0)$, the distance squared modification first performed better. Table 131 above (Refer to Tables B.19. to B.27. in the Appendix) and Figure 12 below show the probabilities of all the three modification tests.

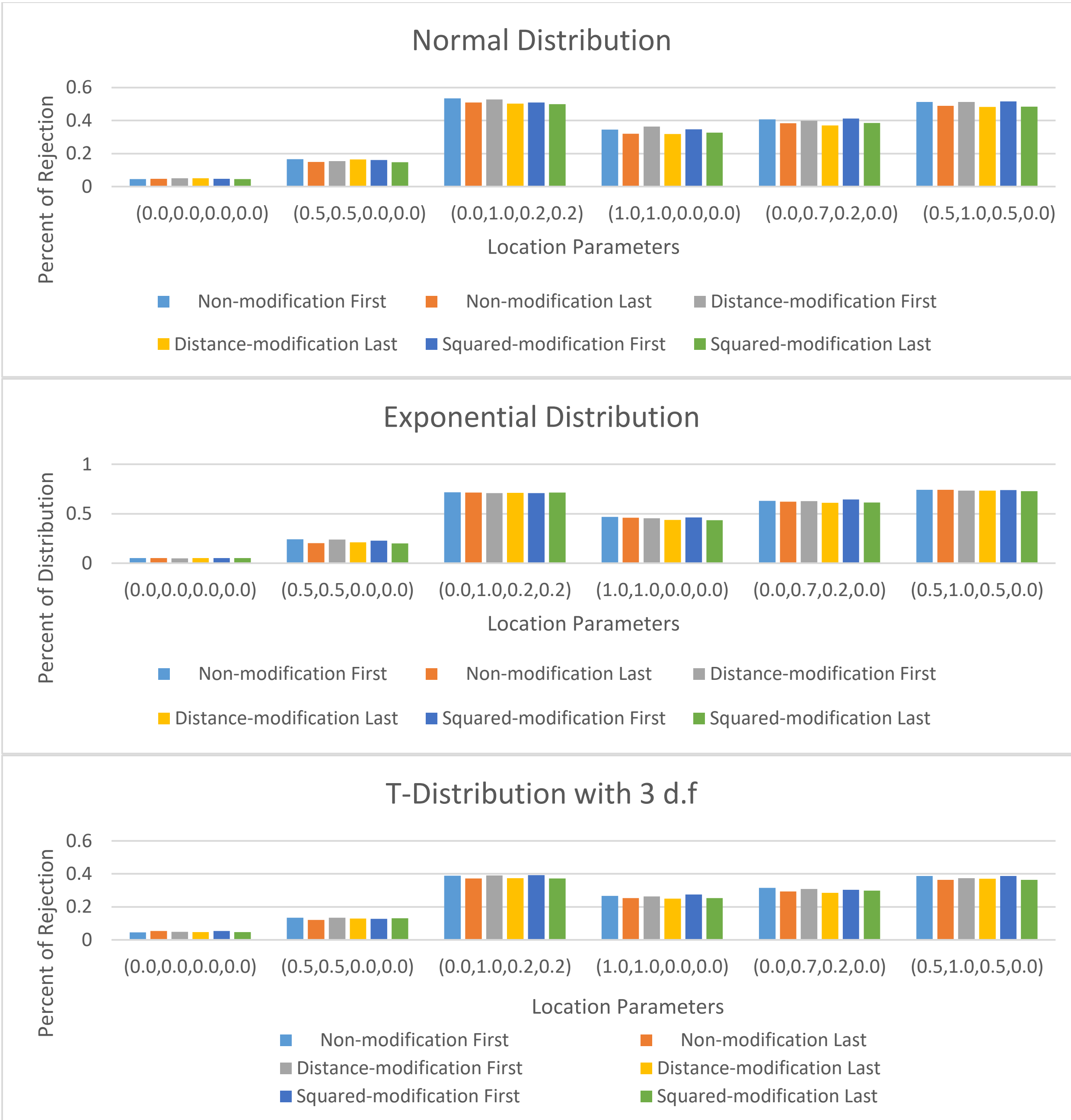

Figure 12. Treatments=4, BIBD = 6, CRD=6 Peak=2 (Two missing observations)

Table 132. Treatments=4, BIBD = 6, CRD=18 Peak=2 (Two missing observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0592	0.0504	0.0524	0.0462	0.0494	0.0452
	0.5	0.5	0	0	0.2672	0.2722	0.2582	**0.2816**	0.2512	0.2882
	0	1	0.2	0.2	0.8202	0.8756	0.8142	**0.8842**	0.8230	0.8796
	1	1	0	0	0.6142	**0.6808**	0.5950	0.6694	0.5956	0.6692
	0	0.7	0.2	0	0.6880	**0.7562**	0.6754	0.7532	0.6862	0.7524
	0.5	1	0.5	0	0.7890	**0.8690**	0.8040	0.8694	0.8084	0.8572
Exponential	0	0	0	0	0.0468	0.0498	0.0478	0.0514	0.0524	0.0524
	0.5	0.5	0	0	0.4000	**0.4656**	0.3988	0.4558	0.4088	0.4600
	0	1	0.2	0.2	0.9554	**0.9918**	0.9600	0.9906	0.9586	0.9892
	1	1	0	0	0.7544	0.8216	0.7462	**0.8294**	0.7468	0.8240
	0	0.7	0.2	0	0.9156	**0.9664**	0.9104	0.9612	0.9108	0.9602
	0.5	1	0.5	0	0.9634	0.9862	0.9646	**0.9902**	0.9666	0.9900
T with 3 degrees of freedom	0	0	0	0	0.0462	0.0508	0.0570	0.0498	0.0516	0.0502
	0.5	0.5	0	0	0.2052	0.2262	0.1958	**0.2278**	0.1996	0.2288
	0	1	0.2	0.2	0.6556	0.7332	0.6802	**0.7468**	0.6668	0.7314
	1	1	0	0	0.4630	**0.5202**	0.4610	0.5136	0.4576	0.5166
	0	0.7	0.2	0	0.5318	**0.5888**	0.5168	0.5862	0.5150	0.5766
	0.5	1	0.5	0	0.6400	0.7080	0.6454	**0.7092**	0.6384	0.7094

Similarly, with four treatments at peak two, the standardized first was slightly higher than the standardized last versions of the test for the mixed design of BIBD, and CRD except where the ratio of BIBD to CRD was 1:2 or 1:3 or 2:3. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If the distance between population parameters before the peak and the peak is greater than the distance between parameters after the peak and the peak, such as $(0.0,1.0,0.2,0.2)$, the distance modification last performed better. Where one additional parameter is equal to the peak and the other parameters are different from the peak, but are equal, such as $(1.0,1.0,0.0,0.0)$, the non-modification last performed better. Refer to Table 132 and Figure 13 below shows the probabilities of all the three modification tests.

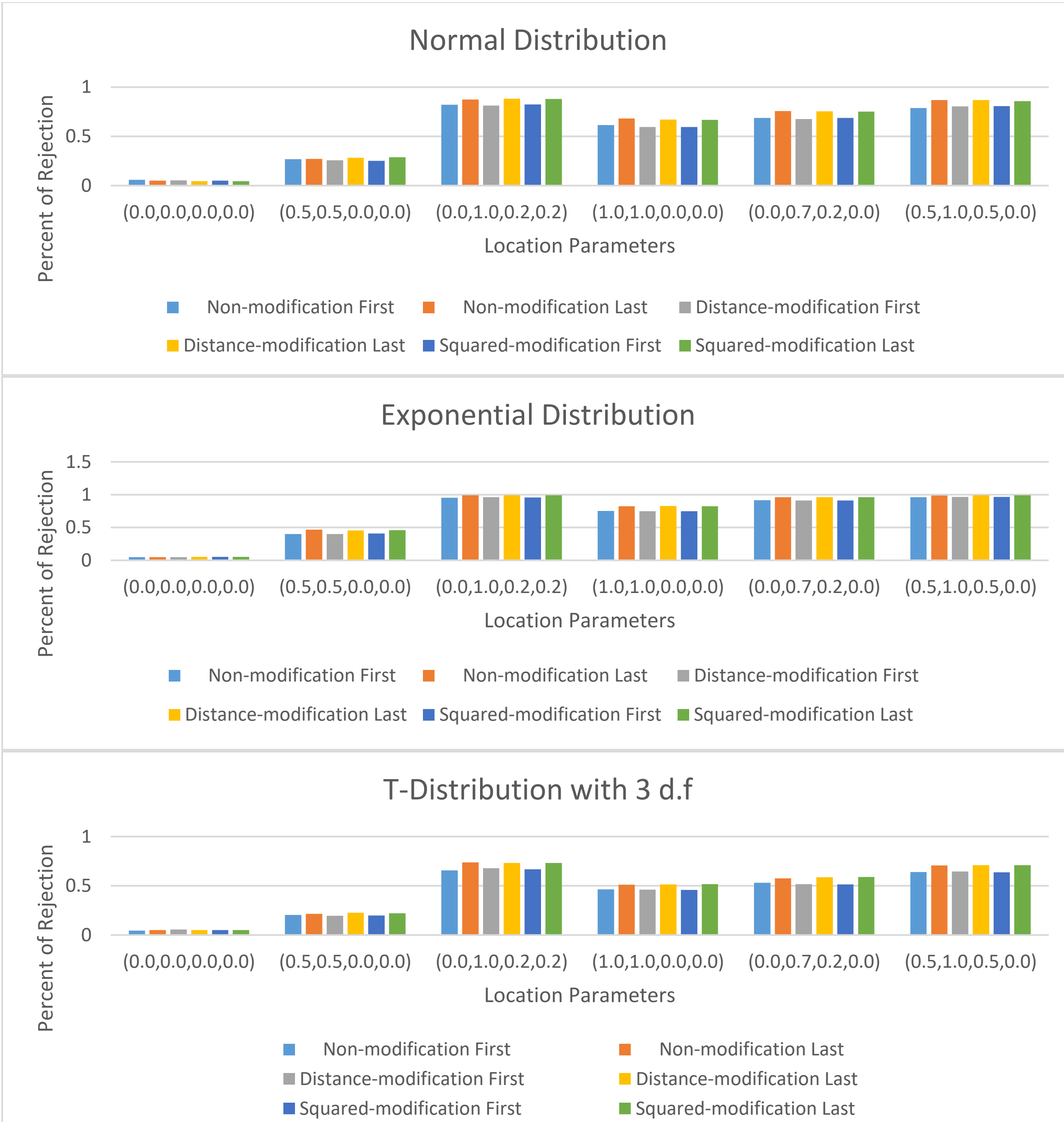

Figure 13. Treatments=4, BIBD = 6, CRD=18 Peak=2 (Two Missing Observation)

Table 133. Treatments=4, BIBD = 6, CRD=6 Peak=3 (Two missing observations)

Distribution	Location				Non-modification		Distance		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0466	0.0486	0.0544	0.0508	0.0514	0.0454
	0	0.5	0.5	0	0.2516	0.2284	0.2480	0.2502	**0.2582**	0.2492
	0	1	1	0.2	**0.5742**	0.5458	0.5650	0.5430	0.5688	0.5390
	0	0	1	0.2	0.9468	0.8286	0.9476	0.8186	**0.9450**	0.8180
	1	1	1	0	0.1432	0.1326	**0.1518**	0.1376	0.1482	0.1382
	0	0.2	0.7	0.5	0.2792	0.2634	0.2738	0.2740	0**.2800**	0.2684
	0	0.5	1	0.5	0.4904	0.4714	**0.5040**	0.4952	0.4956	0.4834
Exponential	0	0	0	0	0.0510	0.0494	0.0512	0.0566	0.0514	0.0460
	0	0.5	0.5	0	0.4160	0.4026	**0.4182**	0.4106	0.4140	0.3994
	0	1	1	0.2	**0.7616**	0.7274	0.7514	0.7302	0.7552	0.7336
	0	0	1	0.2	**0.7636**	0.7608	0.7594	0.7598	0.7546	0.7604
	1	1	1	0	0.1610	0.1552	**0.1720**	0.1500	0.1708	0.1486
	0	0.2	0.7	0.5	0.4460	0.4300	0.4436	0.4378	**0.4510**	0.4366
	0	0.5	1	0.5	**0.7442**	0.7400	0.7496	0.7362	0.7320	0.7254
T with 3 degrees of freedom	0	0	0	0	0.0528	0.0472	0.0538	0.0486	0.0518	0.0512
	0	0.5	0.5	0	0.1942	0.1876	**0.2020**	0.2006	0.1986	0.1908
	0	1	1	0.2	**0.4378**	0.4108	0.4280	0.4158	0.4298	0.4142
	0	0	1	0.2	0.4286	0.4238	**0.4302**	0.4242	0.4276	0.4060
	1	1	1	0	**0.1304**	0.1182	0.1210	0.1150	0.1130	0.1248
	0	0.2	0.7	0.5	0.2190	0.2122	0.2120	0.2150	**0.2210**	0.2032
	0	0.5	1	0.5	0.3796	0.3576	0.3822	0.3784	**0.3878**	0.3678

6.2.3. Four Treatment at Peak Three

In the case of four treatment at peak three, the standardized first provides estimated powers slightly higher than the standardized last versions of the test for the mixed design of BIBD and CRD. Again, the Mixed design of BIBD and CRD obtained the highest percentage of rejection when the ratio was 1:1. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

Where one additional parameter is equal to the peak and the other two parameters are different from each other, such as (0.0,1.0,1.0,0.2), the non-modification last performed better. Where one additional parameter is equal to the peak and the other parameters are different from the peak, but are equal, such as (0.0,0.2,0.7,0.5), the distance squared modification first performed better. Moreover, where two population parameters are equal but different from the peak and the other parameter different from the peak, such as (0.0,0.0,1.0,0.2), the distance modification first performed better. Table 133 above (Refer to Tables B.37. to 45. in the Appendix) and Figure 14 below shows the probabilities of all the three modification tests.

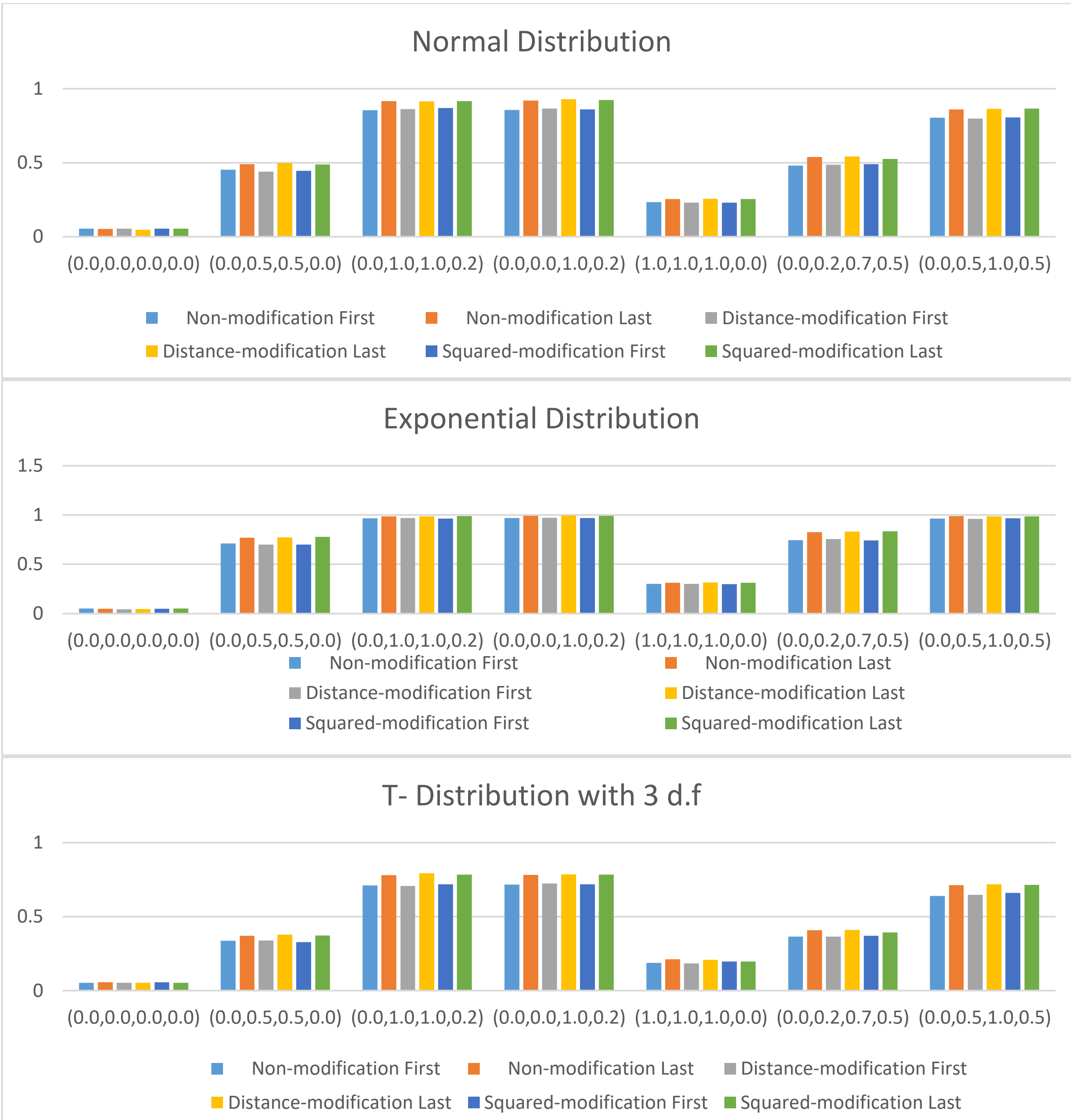

Figure 14. Treatments=4, BIBD = 6, CRD=6 Peak=3 (Two missing observation)

Table 134. Treatments=4, BIBD = 6, CRD=18 Peak=3 (Two missing observations)

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0554	0.0522	0.0550	0.0476	0.0548	0.0542
	0	0.5	0.5	0	0.4536	0.4902	0.4396	**0.4984**	0.4458	0.4894
	0	1	1	0.2	0.8558	**0.9160**	0.8616	0.9150	0.8694	0.9172
	0	0	**1**	0.2	0.8562	0.9214	0.8656	**0.9294**	0.8608	0.9242
	1	1	1	0	0.2346	**0.2558**	0.2296	0.2538	0.2304	0.2348
	0	0.2	0.7	0.5	0.4802	0.5398	0.4872	**0.5434**	0.4912	0.5258
	0	0.5	1	0.5	0.8054	0.8610	0.7996	**0.8642**	0.8060	0.8660
Exponential	0	0	0	0	0.0526	0.0504	0.0442	0.0454	0.0506	0.0518
	0	0.5	0.5	0	0.7120	0.7702	0.6994	**0.7790**	0.7014	0.7734
	0	1	1	0.2	0.9670	0.9866	0.9684	0.9870	0.9650	**0.9890**
	0	0	1	0.2	0.9702	0.9936	0.9734	**0.9950**	0.9702	0.9934
	1	1	1	0	0.3022	**0.3166**	0.3030	0.3140	0.2986	0.3122
	0	0.2	0.7	0.5	0.7462	0.8268	0.7556	**0.8330**	0.7428	0.8362
	0	0.5	1	0.5	0.9634	**0.9902**	0.9624	0.9864	0.9680	0.9880
T with 3 degrees of freedom	0	0	0	0	0.0530	0.0562	0.0536	0.0522	0.0560	0.0536
	0	0.5	0.5	0	0.3366	0.3710	0.3396	**0.3772**	0.3278	0.3728
	0	1	1	0.2	0.7102	0.7796	0.7066	**0.7832**	0.7176	0.7926
	0	0	1	0.2	0.7158	0.7812	0.7244	**0.7852**	0.7188	0.7848
	1	1	1	0	0.1880	**0.2114**	0.1844	0.2078	0.1962	0.1974
	0	0.2	0.7	0.5	0.3652	0.4076	0.3642	**0.4108**	0.3714	0.3934
	0	0.5	1	0.5	0.6398	0.7136	0.6476	**0.7194**	0.6612	0.7142

Similarly, with four treatments at peak three, the standardized first was slightly higher than the standardized last versions of the test for the mixed design of BIBD, and CRD except where the ratio of BIBD to CRD was 1:2 or 1:3 or 2:3 where standardized last performed better. The three distinct tests provide the highest values of the estimated powers test statistics in the following cases:

Where two additional parameters are equal to the peak and the other parameter is different from the peak, such as (1.0,1.0,1.0,0.0), the non- modification last performed better. Where one additional parameter is equal to the peak and the other parameters are different from the peak, but are equal, such as (0.0,0.2,0.7,0.5), the distance modification first performed better.

Table 134 above (Refer to Tables B.37. to 45. in the Appendix) and figure 15 below show the probabilities of all the three modification tests.

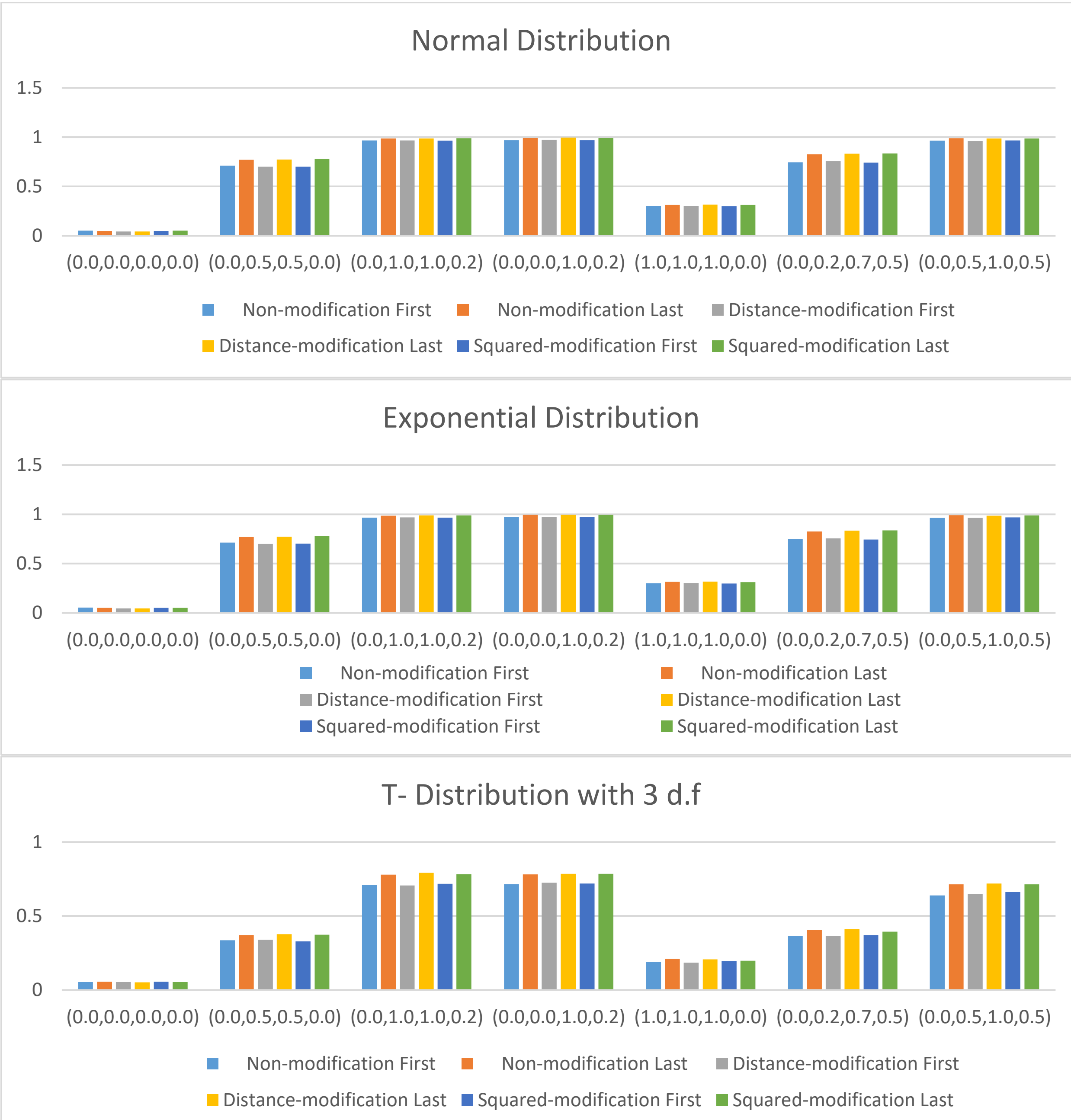

Figure 15. Treatments=4, BIBD = 6, CRD=18 Peak=3 (Two missing observation)

Table 135. Treatments=5, BIBD =5, CRD=5 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0492	0.0438	0.0498	0.0482	0.0500	0.0510
	1	1	0.5	0.5	0.5	**0.1540**	0.1436	0.1492	0.1482	0.1530	0.1520
	0.5	1	1	1	0.7	0.1416	0.1414	0.1476	0.1408	**0.1478**	0.1296
	0.7	1	0.7	0.7	0.5	0.1812	0.1910	0.1896	0.1860	**0.1930**	0.1902
	1	1	0.5	0.5	0.2	0.2504	0.2442	0.2446	0.2500	**0.2572**	0.2458
	0.75	1	0.75	0.5	0.25	0.3140	0.3020	0.3118	0.3110	**0.3142**	0.3066
	1	1	1	1	0	0.3168	0.3062	**0.3218**	0.3126	0.3128	0.3042
	1	1	0.5	0.2	0	0.3888	0.3852	0.3734	0.3698	**0.3912**	0.3766
	0	1.6	0.8	0.4	0.2	**0.8182**	0.7976	0.8084	0.7980	0.8036	0.8020
Exponential	0	0	0	0	0	0.0496	0.0464	0.0506	0.0512	0.0502	0.0544
	1	1	0.5	0.5	0.5	0.1976	0.1912	0.2032	0.1910	**0.2124**	0.1886
	0.5	1	1	1	0.7	0.2262	0.2160	0.2236	0.2136	**0.2306**	0.2184
	0.7	1	0.7	0.7	0.5	0.3062	0.3000	0.3044	0.2980	**0.3076**	0.3010
	1	1	0.5	0.5	0.2	**0.4014**	0.3960	0.3944	0.3978	0.4010	0.3870
	0.75	1	0.75	0.5	0.25	0.5152	0.5126	**0.5166**	0.5040	0.5182	0.5106
	1	1	1	1	0	0.4214	0.4172	**0.4372**	0.4182	0.4290	0.4168
	1	1	0.5	0.2	0	0.5882	0.5754	0.5744	0.5900	**0.5944**	0.5822
	0	1.6	0.8	0.4	0.2	0.9426	0.9258	**0.9496**	0.9228	0.9436	0.9312
T with 3 degrees of freedom	0	0	0	0	0	0.0506	0.0560	0.0558	0.0538	0.0512	0.0526
	1	1	0.5	0.5	0.5	**0.1318**	0.1312	0.1304	0.1278	0.1116	0.1242
	0.5	1	1	1	0.7	**0.1258**	0.1176	0.1246	0.1228	0.1094	0.1210
	0.7	1	0.7	0.7	0.5	0.1460	0.1422	0.1498	0.1452	**0.1594**	0.1526
	1	1	0.5	0.5	0.2	0.2006	0.1882	0.1942	0.1882	**0.2020**	0.1954
	0.75	1	0.75	0.5	0.25	0.2486	0.2406	**0.2534**	0.2414	0.2428	0.2392
	1	1	1	1	0	0.2454	0.2444	0.2474	0.2460	**0.2488**	0.2448
	1	1	0.5	0.2	0	0.2968	0.2854	0.2920	0.2864	**0.3024**	0.2878
	0	1.6	0.8	0.4	0.2	**0.6438**	0.6404	0.6374	0.6422	0.6424	0.6392

6.2.4. Five Treatments at Peak Two

With five treatment at peak two, the standardized first versions of the test performed better than standardized last versions of the test. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

Where two additional parameters are equal to the peak and the other parameter is different from the peak, such as (1.0,1.0,1.0,1.0,0.0), the distance modification first performed better. On the other hand, where one population parameter is equal to the peak and the other parameters are different from each other, such as (1.0,1.0,0.5,0.2,0.0), distance squared modification first slightly performed better. Table 135 above (Refer to Tables C.10. to C.18. in the Appendix) and figure 16 below show the probabilities of all the three modification tests.

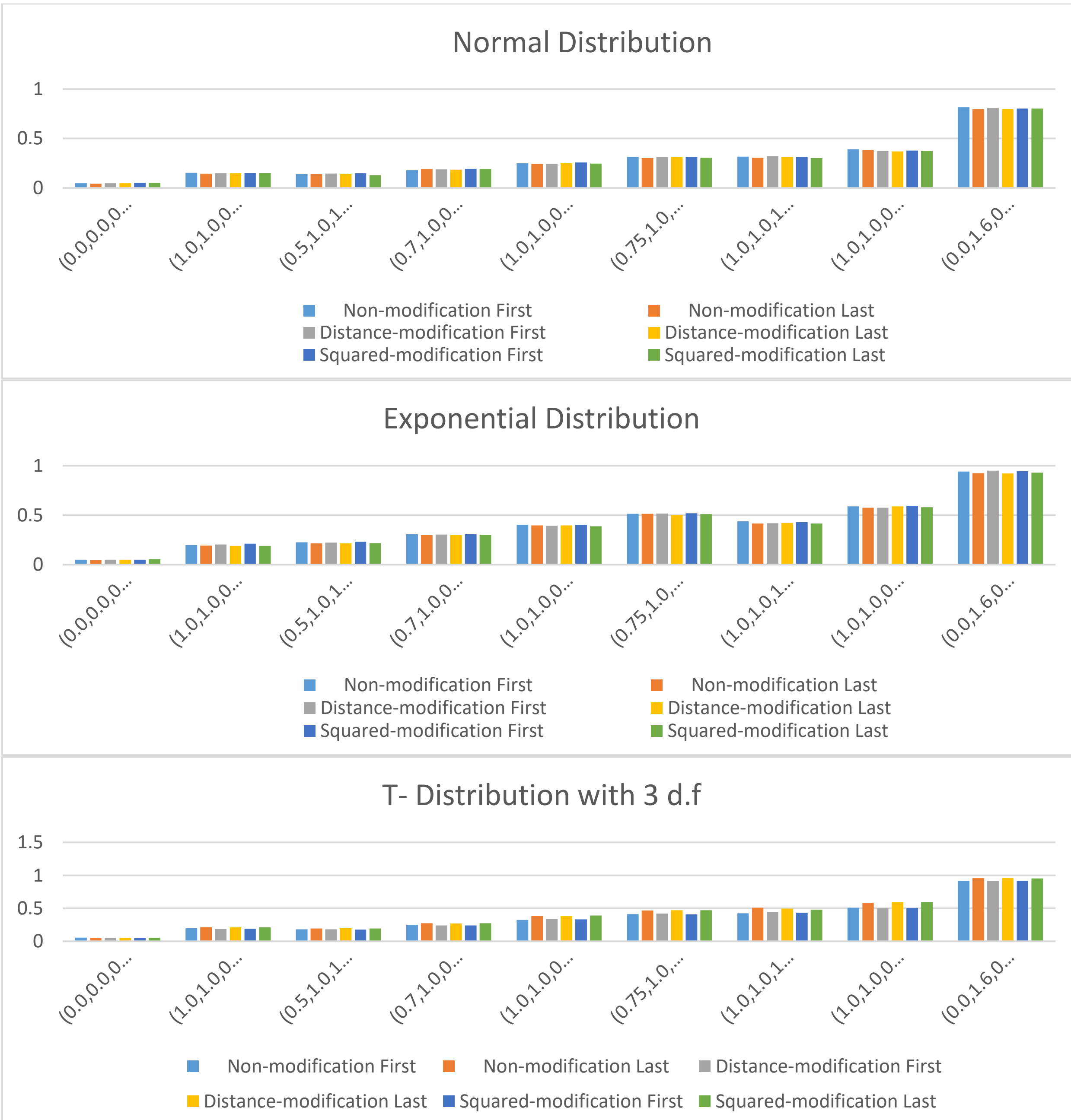

Figure 16. Treatments=5, BIBD =5, CRD=5 Peak=2

Table 136. Treatments=5, BIBD =5, CRD=15 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0518	0.0502	0.0588	0.0490	0.0500	0.0540
	1	1	0.5	0.5	0.5	0.2306	0.2680	0.2460	0.2800	0.2396	**0.2810**
	0.5	1	1	1	0.7	0.2188	0.2540	0.2422	0.2558	0.2224	**0.2690**
	0.7	1	0.7	0.7	0.5	0.3042	0.3518	0.3162	**0.3636**	0.3016	0.3564
	1	1	0.5	0.5	0.2	0.4360	0.5070	0.4628	0.5030	0.4400	**0.5096**
	0.75	1	0.75	0.5	0.25	0.5444	0.6192	0.5302	**0.6274**	0.5452	0.6112
	1	1	1	1	0	0.5544	**0.6552**	0.5558	0.6398	0.5548	0.6412
	1	1	0.5	0.2	0	0.6632	0.7536	0.6632	0.7476	0.6538	**0.7566**
	0	1.6	0.8	0.4	0.2	0.9800	0.9972	0.9838	0.9972	0.9780	**0.9982**
Exponential	0	0	0	0	0	0.0512	0.0486	0.0516	0.0492	0.0488	0.0496
	1	1	0.5	0.5	0.5	0.3782	0.4374	0.3788	0.4164	0.3834	**0.4388**
	0.5	1	1	1	0.7	0.3676	0.4424	0.3806	0.4412	0.3822	**0.4464**
	0.7	1	0.7	0.7	0.5	0.5612	0.6152	0.5366	0.6326	0.5358	**0.6422**
	1	1	0.5	0.5	0.2	0.6984	0.7832	0.6972	0.7936	0.6872	**0.7980**
	0.75	1	0.75	0.5	0.25	0.8172	0.8952	0.8250	**0.8980**	0.8192	0.8914
	1	1	1	1	0	0.7204	**0.7970**	0.7144	0.7890	0.7138	0.7846
	1	1	0.5	0.2	0	0.8824	0.9496	0.8786	**0.9498**	0.8808	0.9492
	0	1.6	0.8	0.4	0.2	0.9998	**1.0000**	0.9988	**1.0000**	0.9998	**1.0000**
T with 3 degrees of freedom	0	0	0	0	0	0.0560	0.0492	0.0522	0.0536	0.0484	0.0516
	1	1	0.5	0.5	0.5	0.1994	**0.2162**	0.1866	0.2102	0.1920	0.2102
	0.5	1	1	1	0.7	0.1832	0.1926	0.1824	**0.1976**	0.1764	0.1944
	0.7	1	0.7	0.7	0.5	0.2480	**0.2760**	0.2410	0.2716	0.2400	0.2722
	1	1	0.5	0.5	0.2	0.3230	0.3818	0.3418	0.3850	0.3334	**0.3900**
	0.75	1	0.75	0.5	0.25	0.4110	0.4680	0.4218	**0.4716**	0.4076	0.4698
	1	1	1	1	0	0.4242	**0.5070**	0.4440	0.4980	0.4312	0.4786
	1	1	0.5	0.2	0	0.5106	0.5862	0.4994	0.5922	0.5030	**0.5962**
	0	1.6	0.8	0.4	0.2	0.9156	0.9574	0.9162	0.9614	0.9150	**0.9550**

Similarly, with five treatments at peak two, the standardized first was slightly higher than the standardized last versions of the test for the mixed design of BIBD, and CRD except where the ratio of BIBD to CRD was 1:2 or 1:3 or 2:3. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

Where three additional parameters equal to the peak and the other parameter is different from the peak, such (1.0,1.0,1.0,1.0,0.0), as the non-modification last performed better. Where there is equal spacing among parameters, such (0.75,1.0,0.75,0.5,0.25), t as the distance modification last performed better. Moreover, where one population parameter is equal to the peak and the other parameters different from each other, such (1.0,1.0,0.5,0.5, 0.2), as distance squared modification last performed better.

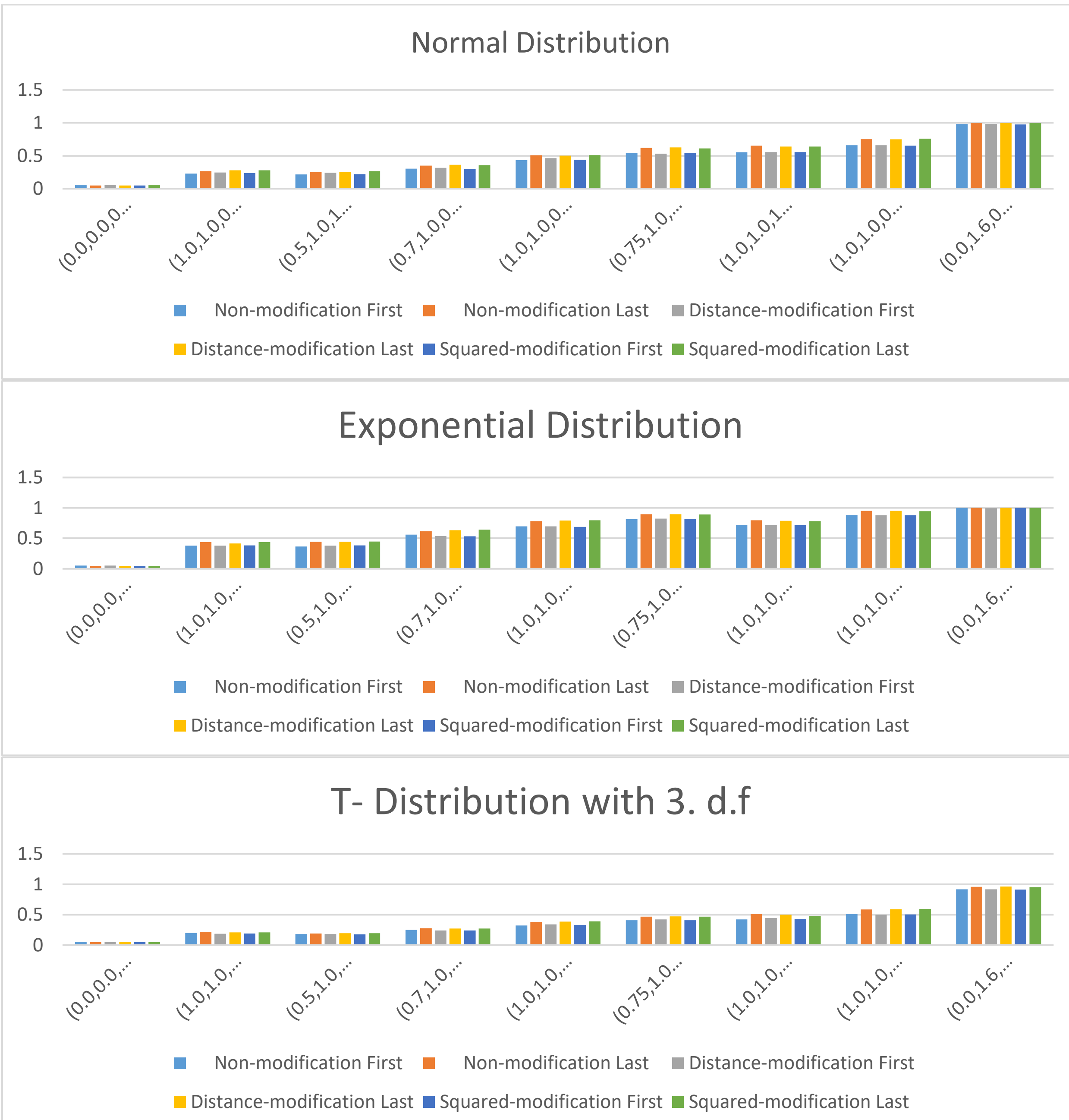

Figure 17. Treatments=5, BIBD =5, CRD=15 Peak=2

Table 137. Treatments=5, BIBD =5, CRD=5 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	**0**	**0**	**0**	**0**	**0**	0.0456	0.0500	0.0506	0.0520	0.0530	0.0490
	0	0.5	1	1	0.7	0.3064	0.3062	**0.3126**	0.3026	0.3094	0.3078
	0.7	0.7	1	0.5	0.5	0.1746	0.1712	0.1778	0.1748	**0.1800**	0.1752
	0	0.5	1	0.5	0	0.5722	0.5648	**0.5752**	0.5630	0. 5670	0.5540
	0	1	1	0	0	**0.5426**	0.5420	0.5318	0.5380	0.5382	0.5356
	1	1	1	0.5	0	0.2134	0.2012	**0.2170**	0.2146	0.2118	0.2138
	1	1	1	1	0	**0.2084**	0.2034	0.2008	0.2030	0.2070	0.2030
Exponential	0	0	0	0	0	0.0526	0.0524	0.0534	0.0486	0.0470	0.0526
	0	0.5	1	1	0.7	0.4840	0.4760	0.4762	0.4710	**0.4802**	0.4754
	0.7	0.7	1	0.5	0.5	0.2632	0.2442	0.2724	0.2532	**0.2736**	0.2480
	0	0.5	1	0.5	0	0.8054	0.7998	0.7906	0.8128	**0.8172**	0.7772
	0	1	1	0	0	**0.7322**	0.7098	0.7022	0.7090	0.7204	0.6960
	1	1	1	0.5	0	0.2970	0.2938	0.2936	0.3054	**0.3068**	0.3064
	1	1	1	1	0	0.2680	0.2544	0.2600	0.2518	**0.2758**	0.2350
T with 3 degrees of freedom	0	0	0	0	0	0.0528	0.0540	0.0550	0.0496	0.0464	0.0464
	0	0.5	1	1	0.7	0.2398	0.2212	0.2404	0.2380	**0.2456**	0.2452
	0.7	0.7	1	0.5	0.5	0.1486	0.1472	0.1436	0.1324	**0.1504**	0.1390
	0	0.5	1	0.5	0	0.4182	0.4142	0.4204	0.4196	**0.4306**	0.4154
	0	1	1	0	0	**0.4068**	0.4026	0.4038	0.3976	0.4062	0.4060
	1	1	1	0.5	0	**0.1784**	0.1682	0.1756	0.1730	0. 1708	0.1594
	1	1	1	1	0	0.1686	0.1678	**0.1834**	0.1696	0.1704	0.1640

6.2.5. Five Treatments at Peak Three

In the case of five treatments at peak three, the standardized first performed better than the standardized last versions of the test with an equal proportion of BIBD and CRD. With a significant difference of 0.01, between the results of the three modification tests, the highest value of the estimated powers test statistics was reported in the following case:

If the distance between population parameters before the peak and the peak is less than the distance between population parameters after the peak and the peak, such as $(0.7, 0.7, 1.0, 0.5, 0.5)$, the distance squared modification first performed better. Table 137 above (Refer to Tables C.37. to C.45. in the Appendix) and Figure 18 below show the probabilities of all three modification tests.

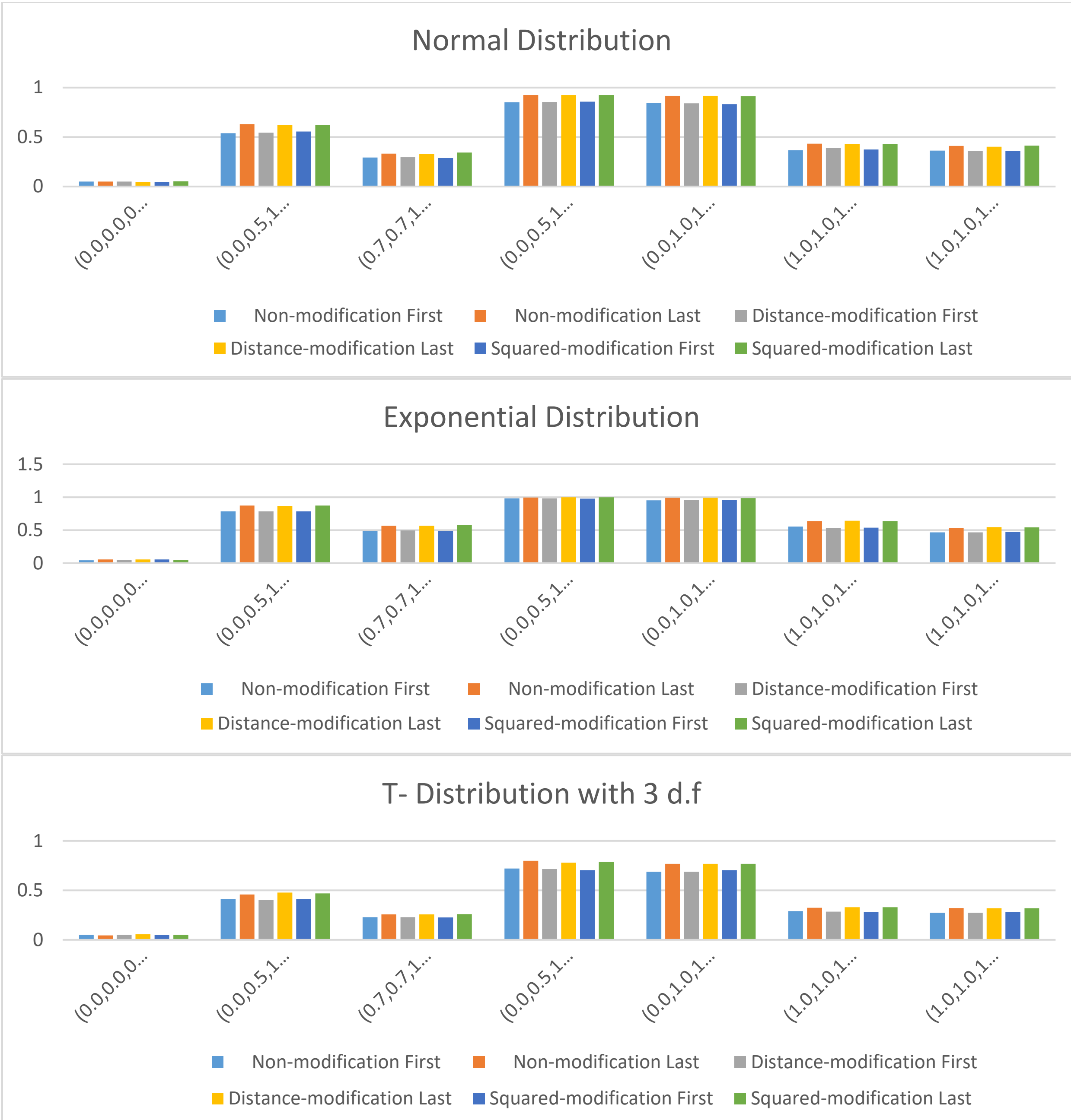

Figure 18. Treatments=5, BIBD =5, CRD=5 Peak=3

Table 138. Treatments=5, BIBD =5, CRD=15 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0506	0.0514	0.0492	0.0438	0.0474	0.0522
	0	0.5	1	1	0.7	0.5382	**0.6322**	0.5446	0.6236	0.5560	0.6226
	0.7	0.7	1	0.5	0.5	0.2946	0.3330	0.2960	0.3292	0.2886	**0.3444**
	0	0.5	1	0.5	0	0.8524	0.9242	0.8544	0.9248	0.8576	**0.9254**
	0	1	1	0	0	0.8430	**0.9162**	0.8406	0.9156	0.8316	0.9140
	1	1	1	0.5	0	0.3654	0.4340	0.3888	**0.4302**	0.3736	0.4272
	1	1	1	1	0	0.3626	0.4106	0.3612	0.4032	0.3590	**0.4134**
Exponential	0	0	0	0	0	0.0454	0.0550	0.0494	0.0560	0.0558	0.0486
	0	0.5	1	1	0.7	0.7844	**0.8734**	0.7868	0.8702	0.7844	0.8724
	0.7	0.7	1	0.5	0.5	0.4878	0.5696	0.4944	0.5692	0.4848	**0.5764**
	0	0.5	1	0.5	0	0.9826	0.9970	0.9812	0.9974	0.9782	**0.9978**
	0	1	1	0	0	0.9536	0.9898	0.9588	**0.9900**	0.9558	0.9876
	1	1	1	0.5	0	0.5536	0.6374	0.5336	**0.6436**	0.5368	0.6370
	1	1	1	1	0	0.4678	0.5306	0.4652	**0.5466**	0.4764	0.5444
T with 3 degrees of freedom	0	0	0	0	0	0.0496	0.0446	0.0512	0.0554	0.0480	0.0496
	0	0.5	1	1	0.7	0.4138	0.4684	0.4036	**0.4776**	0.4114	0.4684
	0.7	0.7	1	0.5	0.5	0.2292	0.2578	0.2300	0.2578	0.2268	**0.2602**
	0	0.5	1	0.5	0	0.7206	**0.7984**	0.7152	0.7806	0.7054	0.7878
	0	1	1	0	0	0.6878	0.7676	0.6866	0.7694	0.7048	**0.7696**
	1	1	1	0.5	0	0.2892	0.3250	0.2858	0.3284	0.2782	**0.3296**
	1	1	1	1	0	0.2728	**0.3212**	0.2744	0.3190	0.2794	0.3180

When the sample size is twice or thrice or two thirds the BIDB portion to the CRD portion, the standardized last performed better than standardized first versions of the test. With a significant difference of 0.01, between the results of the three modification tests, the highest value of the estimated powers test statistics was reported in the following case:

If the distance between population parameters before the peak and the peak is less than the distance between population parameters after the peak and the peak, such as $(0.7, 0.7, 1.0, 0.5, 0.5)$, the distance squared modification last performed better. Table 138 above (Refer to Tables C.37. to C.45. in the Appendix) and Figure 19 below shows the probabilities of all three modification tests.

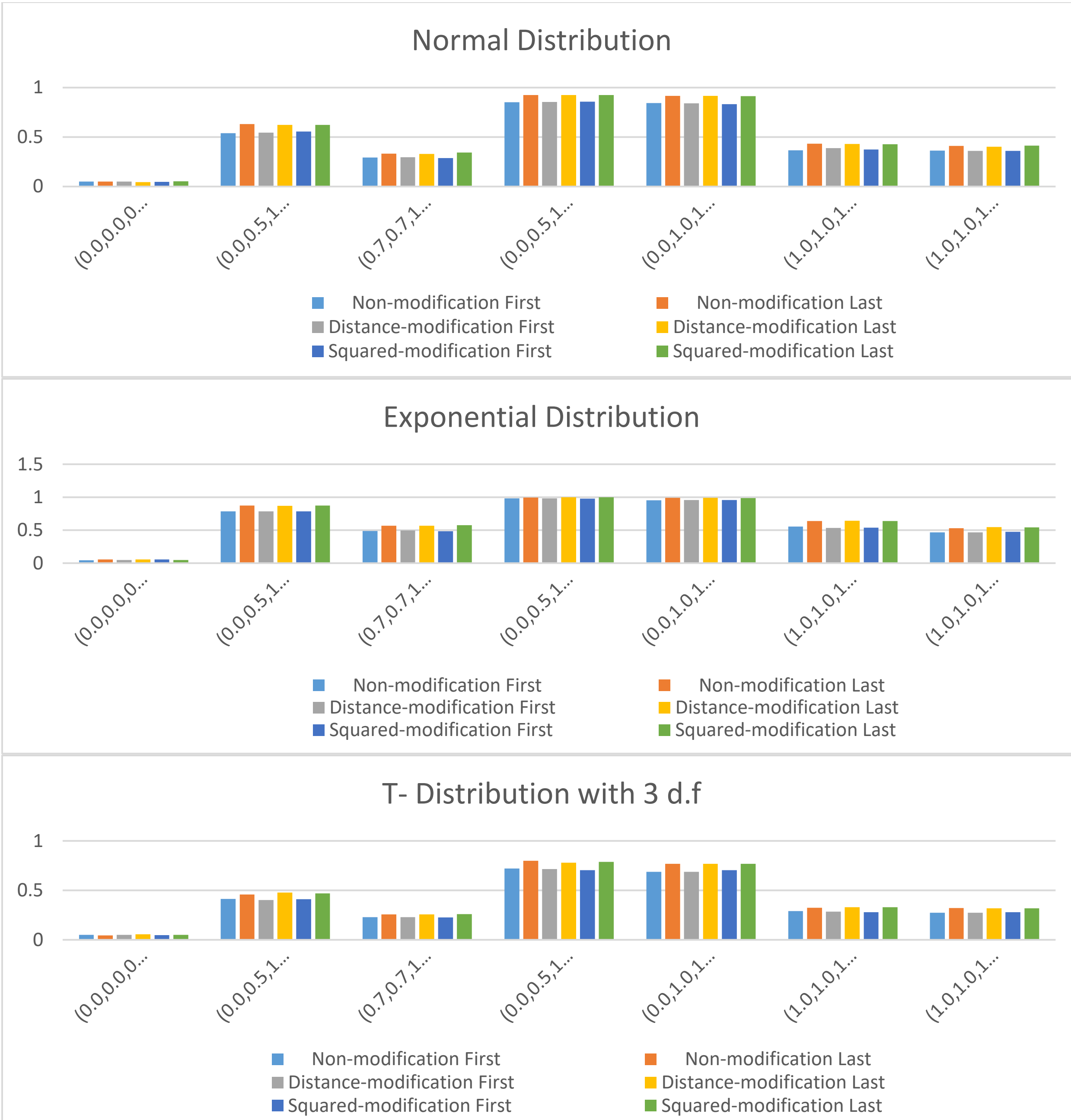

Figure 19. Treatments=5, BIBD =5, CRD=15 Peak=3

Table 139. Treatments=5, BIBD =5, CRD=5 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0480	0.0544	0.0518	0.0500	0.0548	0.0462
	0.5	0.5	0.5	1	1	0.1492	0.1446	**0.1552**	0.1516	0.1552	0.1470
	0	0.5	1	1	0.7	0.4762	0.4730	**0.4852**	0.4610	0.4670	0.4620
	0	0.5	1	1	1	0.4180	0.4120	**0.4228**	0.4122	0.4116	0.4194
	0	0	1	1	1	0.4900	0.4746	0.4830	0.4812	**0.4942**	0.4686
	0.5	0.5	0.7	1	0.7	**0.2114**	0.2022	0.2088	0.2070	0.2112	0.2072
	1	1	1	1	0	0.1056	0.1016	**0.1088**	0.1070	0.1052	0.1016
	1	0.4	0.8	1.6	0.4	**0.8408**	0.8340	0.8370	0.8326	0.8360	0.8286
	0	0.25	0.5	1	0.25	**0.5198**	0.5040	0.5168	0.5056	0.5036	0.5000
Exponential	0	0	0	0	0	0.0440	0.0472	0.0486	0.0512	0.0510	0.0534
	0.5	0.5	0.5	1	1	0.2134	0.2000	0.2118	0.1920	**0.2202**	0.1880
	0	0.5	1	1	0.7	0.6724	0.6708	**0.6822**	0.6732	0.6730	0.6690
	0	0.5	1	1	1	**0.6050**	0.5894	0.5840	0.5984	0.6012	0.5968
	0	0	1	1	1	0.6332	0.6306	0.6268	0.6258	**0.6460**	0.6270
	0.5	0.5	0.7	1	0.7	**0.3508**	0.3416	0.3476	0.3252	0.3450	0.3240
	1	1	1	1	0	0.1090	0.1010	**0.1148**	0.1054	0.1082	0.1066
	1	0.4	0.8	1.6	0.4	0.9586	0.9490	0.9626	0.9528	**0.9668**	0.9474
	0	0.25	0.5	1	0.25	**0.7668**	0.7406	0.7530	0.7430	0.7662	0.7506
T with 3 degrees of freedom	0	0	0	0	0	0.0474	0.0496	0.0508	0.0490	0.0542	0.0500
	0.5	0.5	0.5	1	1	0.1178	0.1130	0.1242	0.1202	**0.1334**	0.1186
	0	0.5	1	1	0.7	0.3536	0.3448	**0.3712**	0.3408	0.3638	0.3494
	0	0.5	1	1	1	0.3118	0.3038	0.3124	0.3106	**0.3230**	0.3048
	0	0	1	1	1	0.3532	0.3496	0.3566	0.3454	**0.3572**	0.3546
	0.5	0.5	0.7	1	0.7	**0.1728**	0.1602	0.1622	0.1658	0.1762	0.1720
	1	1	1	1	0	**0.1050**	0.1016	0.1040	0.1014	0.1016	0.1012
	1	0.4	0.8	1.6	0.4	**0.6852**	0.6704	0.6718	0.6656	0.6752	0.6652
	0	0.25	0.5	1	0.25	0.3794	0.3778	0.3820	0.3670	**0.3834**	0.3806

6.2.6. Five Treatments at Peak Four

The results from Table 139 showed that the standardized first versions performed better than standardized last versions of the test with an equal portion of BIBD to CRD. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

Where two additional parameters are equal to the peak and the other parameters are equal but different from the peak, such as (0.0,0.0,1.0,1.0,1.0), the distance squared modification first performed better. Where one additional parameter is equal to the peak and the other parameters are different from each other, such as (0.0,0.5,1.0,1.0,0.7), the distance modification first performed better. If two population parameters are different from the peak and the additional parameters are equal, such as (0.5,0.5,0.7,1.0,0.7), the non-modification first performed better.

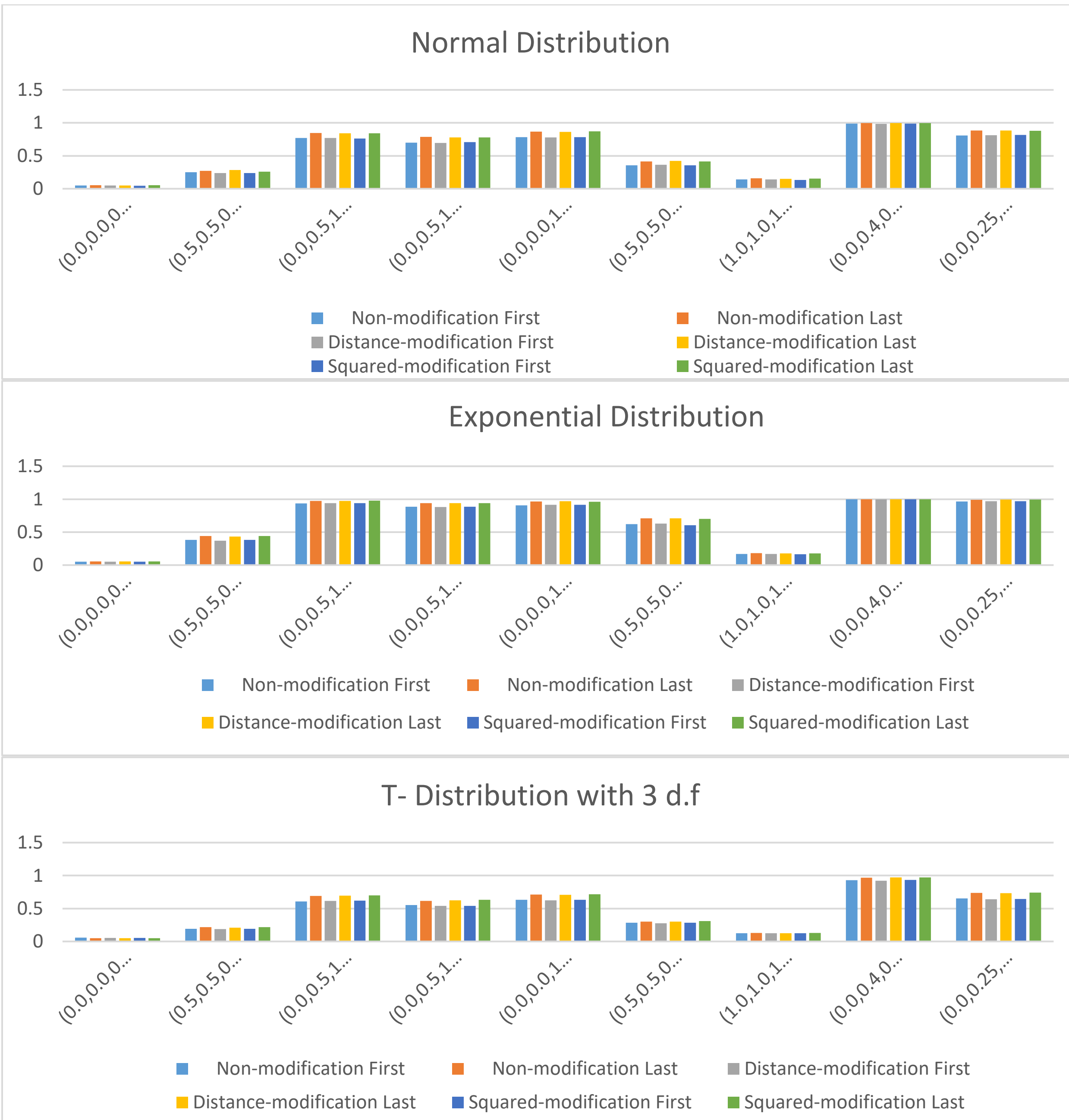

Figure 20. Treatments=5, BIBD =5, CRD=15 Peak=2

Table 140. Treatments=5, BIBD =5, CRD=15 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0482	0.0522	0.0496	0.0476	0.0460	0.0522
	0.5	0.5	0.5	1	1	0.2526	0.2720	0.2402	**0.2848**	0.2402	0.2588
	0	0.5	1	1	0.7	0.7716	**0.8440**	0.7702	0.8418	0.7618	0.8418
	0	0.5	1	1	1	0.7004	**0.7858**	0.6958	0.7800	0.7062	0.7776
	0	0	1	1	1	0.7812	0.8690	0.7792	0.8628	0.7852	**0.8716**
	0.5	0.5	0.7	1	0.7	0.3556	0.4156	0.3638	**0.4226**	0.3550	0.4148
	1	1	1	1	0	0.1412	**0.1588**	0.1432	0.1522	0.1354	0.1544
	1	0.4	0.8	1.6	0.4	0.9902	**0.9986**	0.9864	0.9982	0.9874	0.9972
	0	0.25	0.5	1	0.25	0.8094	0.8826	0.8132	**0.8834**	0.8158	0.8804
Exponential	0	0	0	0	0	0.0534	0.0550	0.0512	0.0562	0.0516	0.0564
	0.5	0.5	0.5	1	1	0.3830	0.4414	0.3702	**0.4548**	0.3818	0.4302
	0	0.5	1	1	0.7	0.9370	0.9748	0.9412	0.9758	0.9402	**0.9762**
	0	0.5	1	1	1	0.8840	0.9392	0.8828	**0.9422**	0.8842	0.9396
	0	0	1	1	1	0.9088	0.9662	0.9154	0.9686	0.9156	**0.9694**
	0.5	0.5	0.7	1	0.7	0.6208	**0.7108**	0.6308	0.7086	0.6046	0.6998
	1	1	1	1	0	0.1696	**0.1798**	0.1694	0.1756	0.1656	0.1792
	1	0.4	0.8	1.6	0.4	0.9992	**1.0000**	0.9998	**1.0000**	0.9996	**1.0000**
	0	0.25	0.5	1	0.25	0.9670	0.9914	0.9696	0.9944	0.9690	**0.9942**
T with 3 degrees of freedom	0	0	0	0	0	0.0566	0.0482	0.0540	0.0514	0.0530	0.0482
	0.5	0.5	0.5	1	1	0.1926	0.2158	0.1876	**0.2282**	0.1904	0.2170
	0	0.5	1	1	0.7	0.6088	0.6904	0.6148	0.6968	0.6206	**0.6974**
	0	0.5	1	1	1	0.5526	0.6158	0.5392	0.6240	0.5384	**0.6322**
	0	0	1	1	1	0.6336	0.7132	0.6246	0.7092	0.6316	**0.7178**
	0.5	0.5	0.7	1	0.7	0.2848	0.2994	0.2764	**0.3016**	0.2844	0.3078
	1	1	1	1	0	0.1240	**0.1290**	0.1232	0.1266	0.1236	0.1272
	1	0.4	0.8	1.6	0.4	0.9294	0.9694	0.9234	**0.9722**	0.9338	0.9726
	0	0.25	0.5	1	0.25	0.6512	**0.7376**	0.6390	0.7342	0.6434	0.7412

Similarly, the results in table 140 showed that where the sample was twice or thrice or two thirds the number of blocks, the standardized last versions performed better than standardized first versions of the test with an equal portion of BIBD and CRD. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If the distance between population parameters before the peak and at the peak is greater than the distance between parameter after the peak and the peak, such as (0.5,0.5,0.5,1.0,1.0), the distance modification last performed better. Where three population parameters are equal to the peak, such as (1.0,1.0,1.0,1.0,0.0), the non-modification last performed better. Refer to Tables C.63. to C.71. in the Appendix).

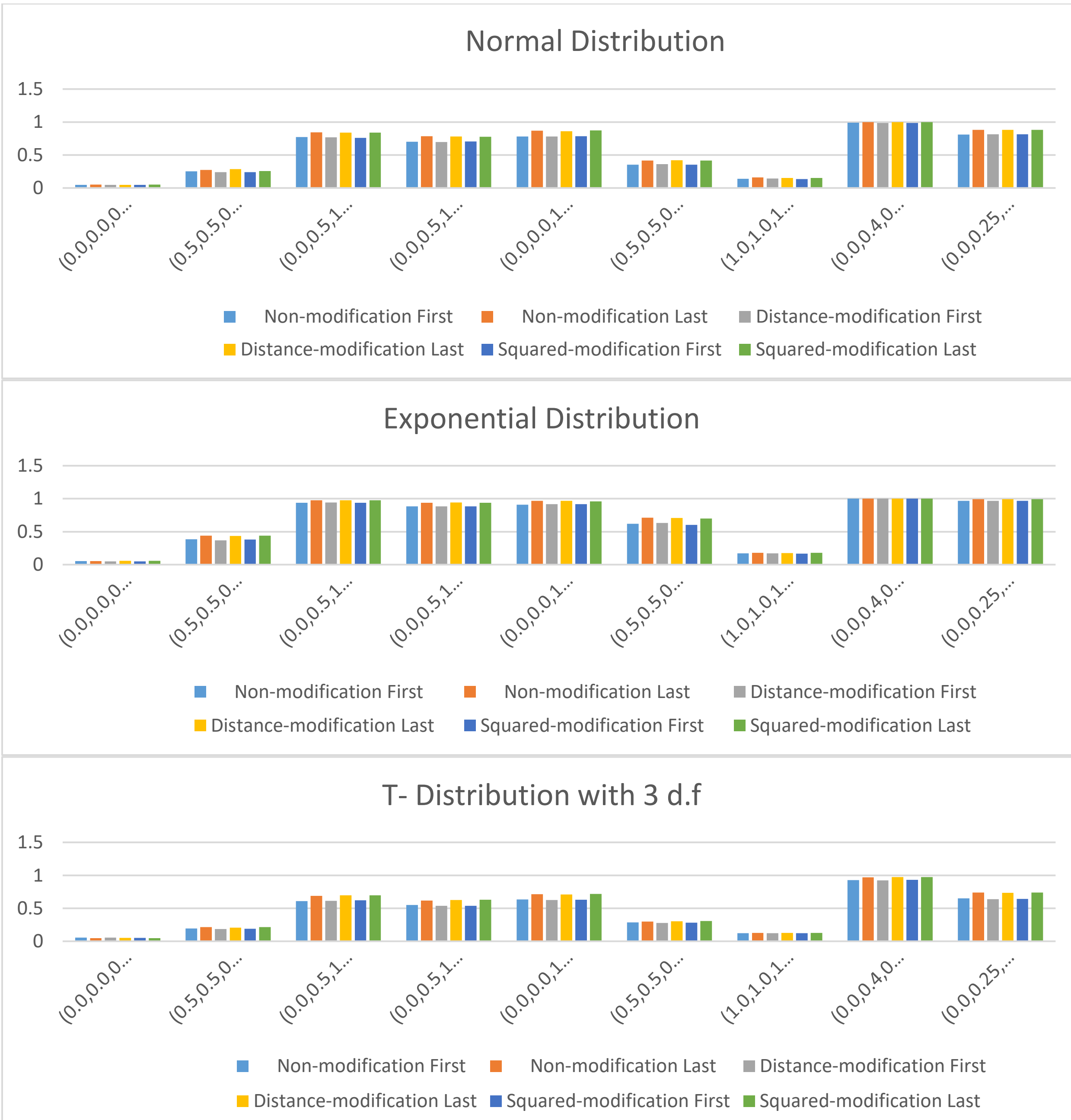

Figure 21. Treatments=5, BIBD =5, CRD=15 Peak=4

6.3. Mixed Design of CRD and RCBD

Table 141. Treatments =3, CRD=6, RCBD=6 Peak=2

Distribution	Location			Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	First	Last	First	Last	First	Last
Normal	0	0	0	0.0514	0.0540	0.0514	0.0540	0.0514	0.0540
	0	0.5	0.5	**0.1538**	0.1434	**0.1538**	0.1434	**0.1538**	0.1434
	0.5	0.5	0	**0.1482**	0.1400	**0.1482**	0.1400	**0.1482**	0.1400
	0.2	0.6	0.2	**0.2656**	0.2322	**0.2656**	0.2322	**0.2656**	0.2322
	0	0.5	0	**0.3528**	0.3016	**0.3528**	0.3016	**0.3528**	0.3016
	0	1	0.6	**0.5348**	0.4460	**0.5348**	0.4460	**0.5348**	0.4460
	0.6	1	0	**0.5394**	0.4400	**0.5394**	0.4400	**0.5394**	0.4400
Exponential	0	0	0	0.0530	0.0556	0.0530	0.0556	0.0530	0.0556
	0	0.5	0.5	**0.2144**	0.1798	**0.2144**	0.1798	**0.2144**	0.1798
	0.5	0.5	0	**0.2168**	0.1840	**0.2168**	0.1840	**0.2168**	0.1840
	0.2	0.6	0.2	**0.4486**	0.3690	**0.4486**	0.3690	**0.4486**	0.3690
	0	0.5	0	**0.5706**	0.4774	**0.5706**	0.4774	**0.5706**	0.4774
	0	1	0.6	**0.7508**	0.6388	**0.7508**	0.6388	**0.7508**	0.6388
	0.6	1	0	**0.7532**	0.6394	**0.7532**	0.6394	**0.7532**	0.6394
T with 3 degrees of freedom	0	0	0	0.0534	0.0544	0.0534	0.0544	0.0534	0.0544
	0	0.5	0.5	**0.1236**	0.1180	**0.1236**	0.1180	**0.1236**	0.1180
	0.5	0.5	0	**0.1290**	0.1192	**0.1290**	0.1192	**0.1290**	0.1192
	0.2	0.6	0.2	**0.2062**	0.1830	**0.2062**	0.1830	**0.2062**	0.1830
	0	0.5	0	**0.2670**	0.2298	**0.2670**	0.2298	**0.2670**	0.2298
	0	1	0.6	**0.3988**	0.3310	**0.3988**	0.3310	**0.3988**	0.3310
	0.6	1	0	**0.3886**	0.3334	**0.3886**	0.3334	**0.3886**	0.3334

6.3.1. Three Treatment at Peak Two

With three treatment at peak two, we found that the three modification tests were the same. The reason being that all the distinct tests have an equal distance weight of one. Overall, the standardized first version provides estimated powers slightly higher than the standardized last version of the test regardless of the underlying distribution and the proportion of CRD to RCBD. Table 141 above (Refer to Table A.5. in the Appendix) and Figure 22 below shows the probabilities of all the three modification tests.

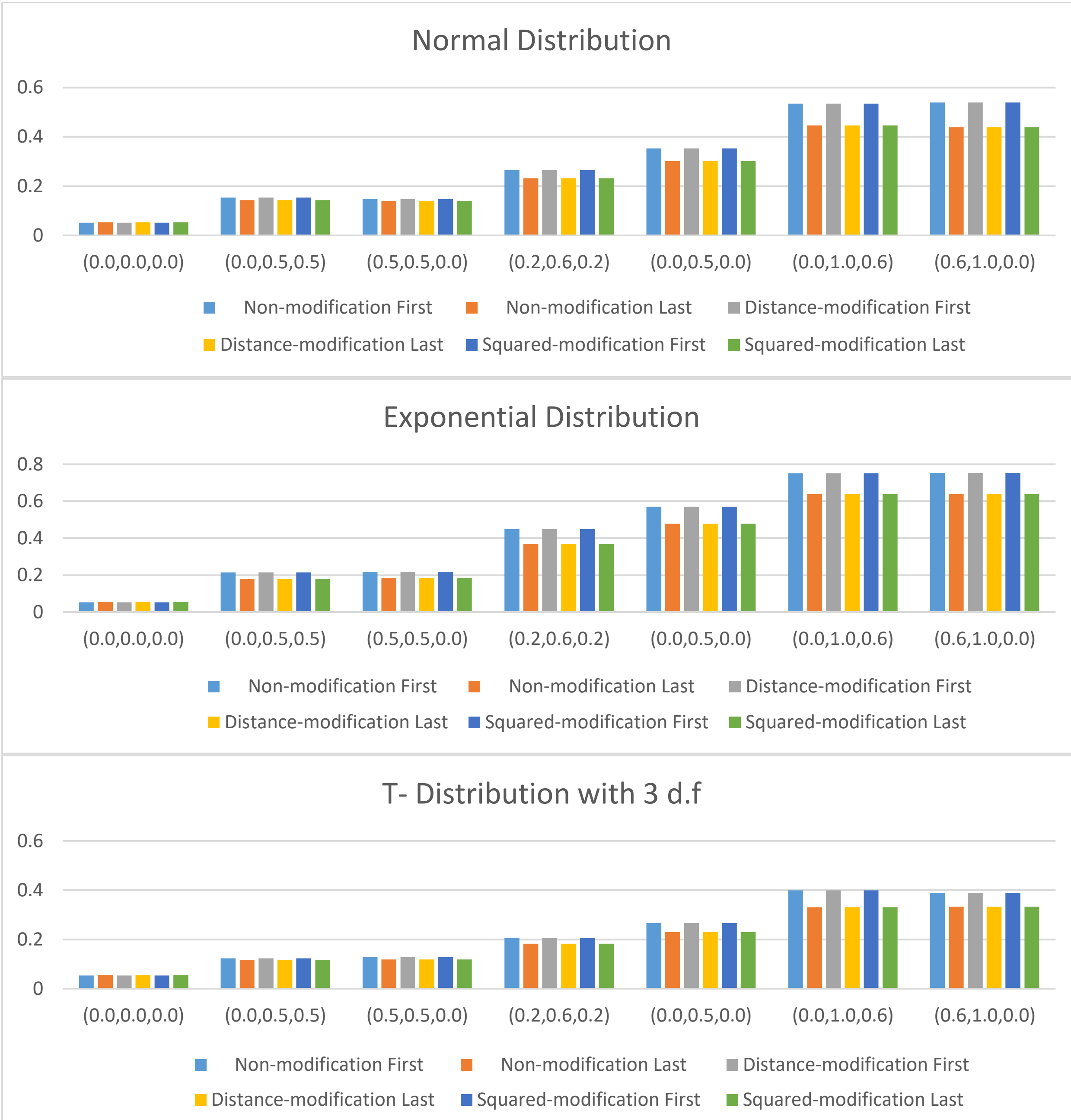

Figure 22. Treatments =3, CRD=6, RCBD=6 Peak=2

Table 142. Treatments=4, CRD = 18, RCBD=6 Peak=2

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0496	0.0516	0.0580	0.0468	0.0470	0.0458
	0.5	0.5	0	0	0.3128	0.3030	0.3264	0.2934	**0.3470**	0.2936
	0	1	0.2	0.2	**0.9110**	0.8934	0.9086	0.8986	0.8916	0.8986
	1	1	0	0	0.7184	0.6768	0.7542	0.6714	**0.7606**	0.6960
	0	0.7	0.2	0	**0.7860**	0.7620	0.7796	0.7566	0.7800	0.7520
	0.5	1	0.5	0	0.8984	0.8710	0.9020	0.8784	**0.9036**	0.8892
Exponential	0	0	0	0	0.0458	0.0500	0.0498	0.0544	0.0524	0.0482
	0.5	0.5	0	0	0.5012	0.4752	0.5360	0.4712	**0.5468**	0.4920
	0	1	0.2	0.2	**0.9968**	0.9898	0.9926	0.9910	0.9932	0.9942
	1	1	0	0	0.8640	0.8316	0.8820	0.8318	**0.9050**	0.8434
	0	0.7	0.2	0	**0.9782**	0.9670	0.9702	0.9692	0.9624	0.9682
	0.5	1	0.5	0	**0.9944**	0.9878	0.9942	0.9908	0.9930	0.9918
T with 3 degrees of freedom	0	0	0	0	0.0464	0.0498	0.0492	0.0474	0.0488	0.0478
	0.5	0.5	0	0	0.2438	0.2256	0.2464	0.2314	**0.2540**	0.2258
	0	1	0.2	0.2	**0.7634**	0.7454	0.7616	0.7478	0.7520	0.7476
	1	1	0	0	0.5580	0.5202	0.5950	0.5320	**0.6096**	0.5364
	0	0.7	0.2	0	**0.6204**	0.5926	0.6182	0.6060	0.6174	0.5958
	0.5	1	0.5	0	0.7454	0.7170	**0.7752**	0.7346	0.7664	0.7396

6.3.2. Four Treatment at Peak Two

In the case of four treatments at peak two, the standardized first versions of the test performed better than standardized last versions of the test. With a significant difference of 0.01, between the results of the thee modification tests, the distance squared modification test reported the highest values of the estimated powers test statistics in the following cases:

If one additional parameter is equal to the peak and the other parameters are equal, but are different from the peak, such as (1.0,1.0,0.0,0.0) and (0.5,0.5,0.0,0.0), the distance squared modification does better. Table 142 above (Refer to Tables B.10. to B.18. in the Appendix) and Figure 23 below show the probabilities of all three modification tests.

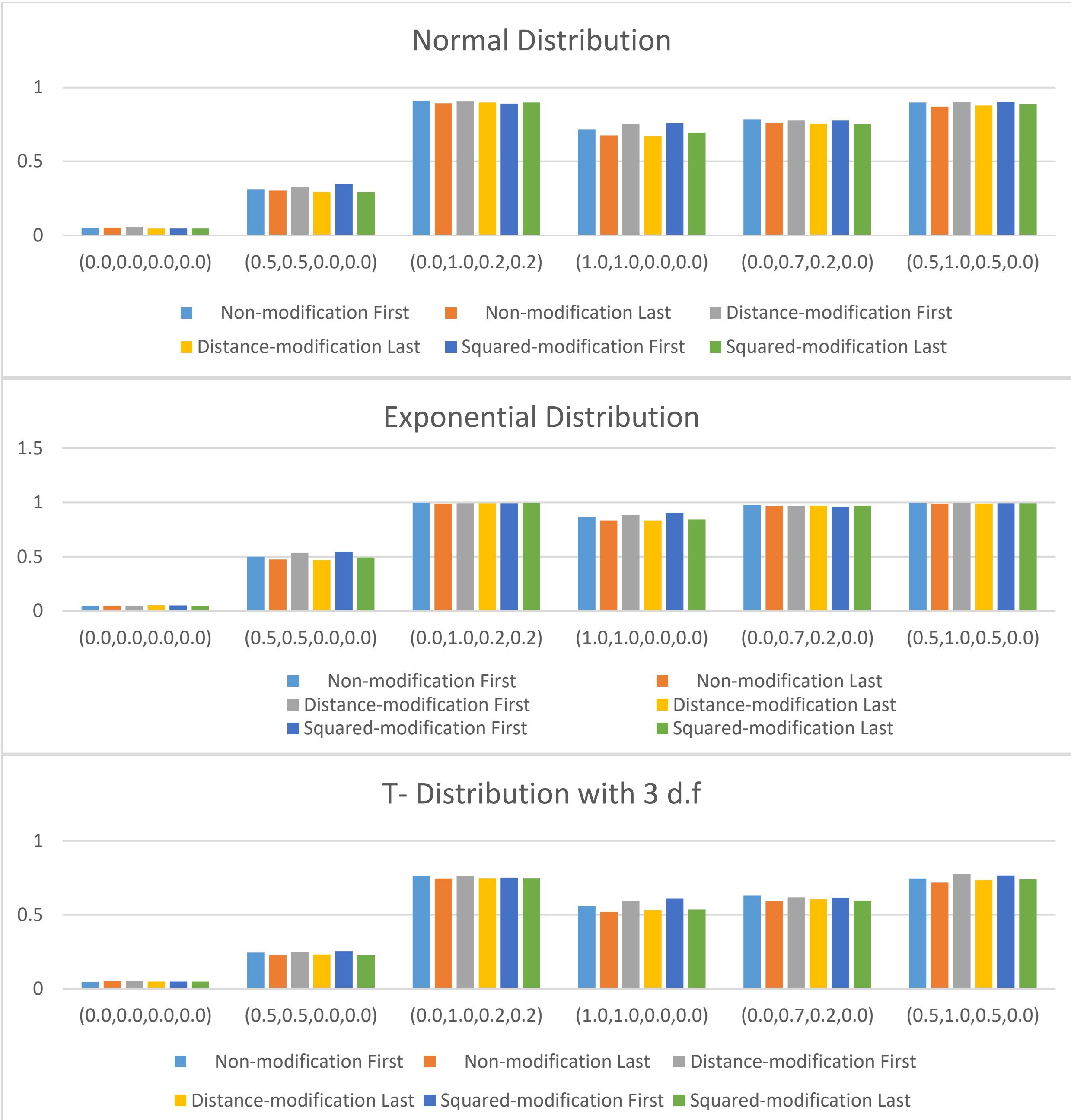

Figure 23. Treatments=4, CRD = 18, RCBD=6 Peak=2

Table 143. Treatments =4, CRD = 6, RCBD=6 Peak=3

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0492	0.0516	0.0492	0.0558	0.0468	0.0532
	0	0.5	0.5	0	**0.3502**	0.2674	0.3470	0.2920	0.3326	0.3172
	0	1	1	0.2	0.7288	0.6082	**0.7346**	0.6454	0.7304	0.6852
	0	0	1	0.2	**0.7458**	0.6120	0.7434	0.6522	0.7324	0.6776
	1	1	1	0	**0.1780**	0.1422	0.1536	0.1496	0.1410	0.1348
	0	0.2	0.7	0.5	0.3770	0.3012	0.3828	0.3214	**0.3868**	0.3574
	0	0.5	1	0.5	0.6626	0.5470	**0.6856**	0.5772	0.6740	0.6278
Exponential	0	0	0	0	0.0512	0.0498	0.0496	0.0486	0.0492	0.0528
	0	0.5	0.5	0	0.5504	0.4432	**0.5610**	0.4836	0.5406	0.5056
	0	1	1	0.2	0.8970	0.7928	**0.8976**	0.8262	0.8896	0.8614
	0	0	1	0.2	**0.9320**	0.8236	0.9224	0.8572	0.9152	0.8842
	1	1	1	0	**0.2234**	0.1772	0.1880	0.1596	0.1630	0.1708
	0	0.2	0.7	0.5	0.6094	0.4888	0.6314	0.5348	**0.6340**	0.5748
	0	0.5	1	0.5	0.9010	0.7920	**0.9090**	0.8310	0.8912	0.8686
T with 3 degrees of freedom	0	0	0	0	0.0518	0.0468	0.0526	0.0478	0.0522	0.0480
	0	0.5	0.5	0	0.2560	0.2108	**0.2724**	0.2344	0.2582	0.2388
	0	1	1	0.2	0.5722	0.4540	**0.5792**	0.4984	0.5604	0.5342
	0	0	1	0.2	**0.5900**	0.4706	0.5674	0.4902	0.5620	0.5222
	1	1	1	0	**0.1482**	0.1172	0.1348	0.1284	0.1240	0.1186
	0	0.2	0.7	0.5	0.2868	0.2278	0.3086	0.2442	**0.3096**	0.2756
	0	0.5	1	0.5	0.5050	0.4090	0.5262	0.4446	**0.5306**	0.4638

6.3.3. Four Treatments at Peak Three

In the case of four treatments at peak three, the standardized first versions of the test performed better than standardized last versions of the test. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If one additional parameter is equal to the peak and the other parameters are different from each other, such as (0.0,1.0,1.0,0.2), the non modification first performed better. On the other hand, if the distance between parameters before the peak and the peak is greater than the distance between parameters after the peak and the peak, such as (0.0,0.2,0.7,0.5), the distance squared modification first performed better. Where one population parameter is equal to the peak and the other two parameters are equal but different from the peak, such as (0.0,0.5,0.5,0.0), the distance modification first performed better. Table143 above (Refer to Tables B.46. to B.54. in the Appendix).and Figure 24 below shows the probabilities of all three modification tests.

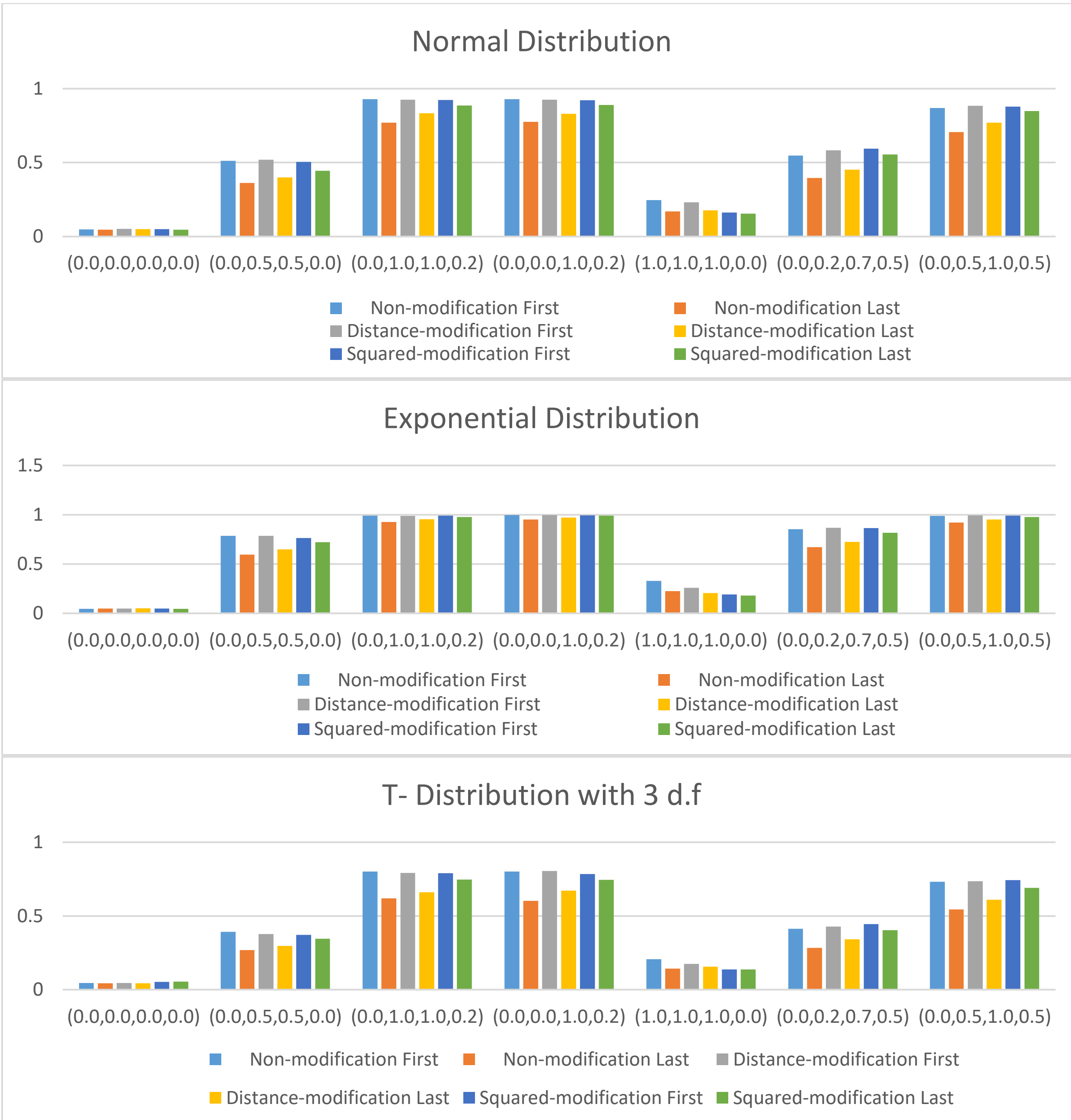

Figure 24. Treatments =4, CRD = 6, RCBD=6 Peak=3

Table 144. Treatments=4, CRD = 18, RCBD=6, Peak=3

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0486	0.0456	0.0522	0.0494	0.0500	0.0464
	0	0.5	0.5	0	**0.5188**	0.3622	0.5120	0.3996	0.5046	0.4446
	0	1	1	0.2	**0.9292**	0.7698	0.9248	0.8334	0.9230	0.8864
	0	0	1	0.2	**0.9288**	0.7760	0.9244	0.8296	0.9218	0.8902
	1	1	1	0	**0.2470**	0.1696	0.2314	0.1768	0.1626	0.1540
	0	0.2	0.7	0.5	0.5474	0.3952	0.5838	0.4522	**0.5948**	0.5558
	0	0.5	1	0.5	0.8688	0.7072	**0.8844**	0.7706	0.8790	0.8490
Exponential	0	0	0	0	0.0452	0.0496	0.0484	0.0516	0.0484	0.0448
	0	0.5	0.5	0	**0.7860**	0.5946	0.7854	0.6492	0.7640	0.7218
	0	1	1	0.2	**0.9912**	0.9260	0.9892	0.9552	0.9906	0.9762
	0	0	1	0.2	**0.9964**	0.9522	0.9966	0.9724	0.9950	0.9898
	1	1	1	0	**0.3276**	0.2264	0.2578	0.2044	0.1920	0.1808
	0	0.2	0.7	0.5	0.8544	0.6724	0.8662	0.7256	**0.8778**	0.8162
	0	0.5	1	0.5	0.9894	0.9202	**0.9926**	0.9512	0.9904	0.9758
T with 3 degrees of freedom	0	0	0	0	0.0454	0.0426	0.0458	0.0436	0.0518	0.0538
	0	0.5	0.5	0	**0.3920**	0.2688	0.3766	0.2966	0.3720	0.3456
	0	1	1	0.2	**0.8008**	0.6198	0.7926	0.6606	0.7908	0.7462
	0	0	1	0.2	0.8014	0.6016	**0.8054**	0.6722	0.7850	0.7454
	1	1	1	0	**0.2062**	0.1436	0.1738	0.1560	0.1374	0.1366
	0	0.2	0.7	0.5	0.4130	0.2826	0.4276	0.3424	**0.4440**	0.4030
	0	0.5	1	0.5	0.7320	0.5434	0.7362	0.6090	**0.7424**	0.6900

In the case of four treatments at peak three, the standardized first versions of the test performed better than standardized last versions of the test. The results from the distance squared modification first and the non -modification first provided the highest values of the estimated powers test statistics in the following cases:

If one additional parameter is equal to the peak and the other parameters are equal, but are different from the peak, such as $(0.0,0.5,0.5,0.0)$, the non-modification does better. Where the distance between parameters before the peak and the peak is greater than the distance between the parameter after the peak and the peak, such as $(0.0,0.2,0.7,0.5)$, the distance squared modification first performed better. Table 144 above and Figure 25 below show the probabilities of all the three modification tests.

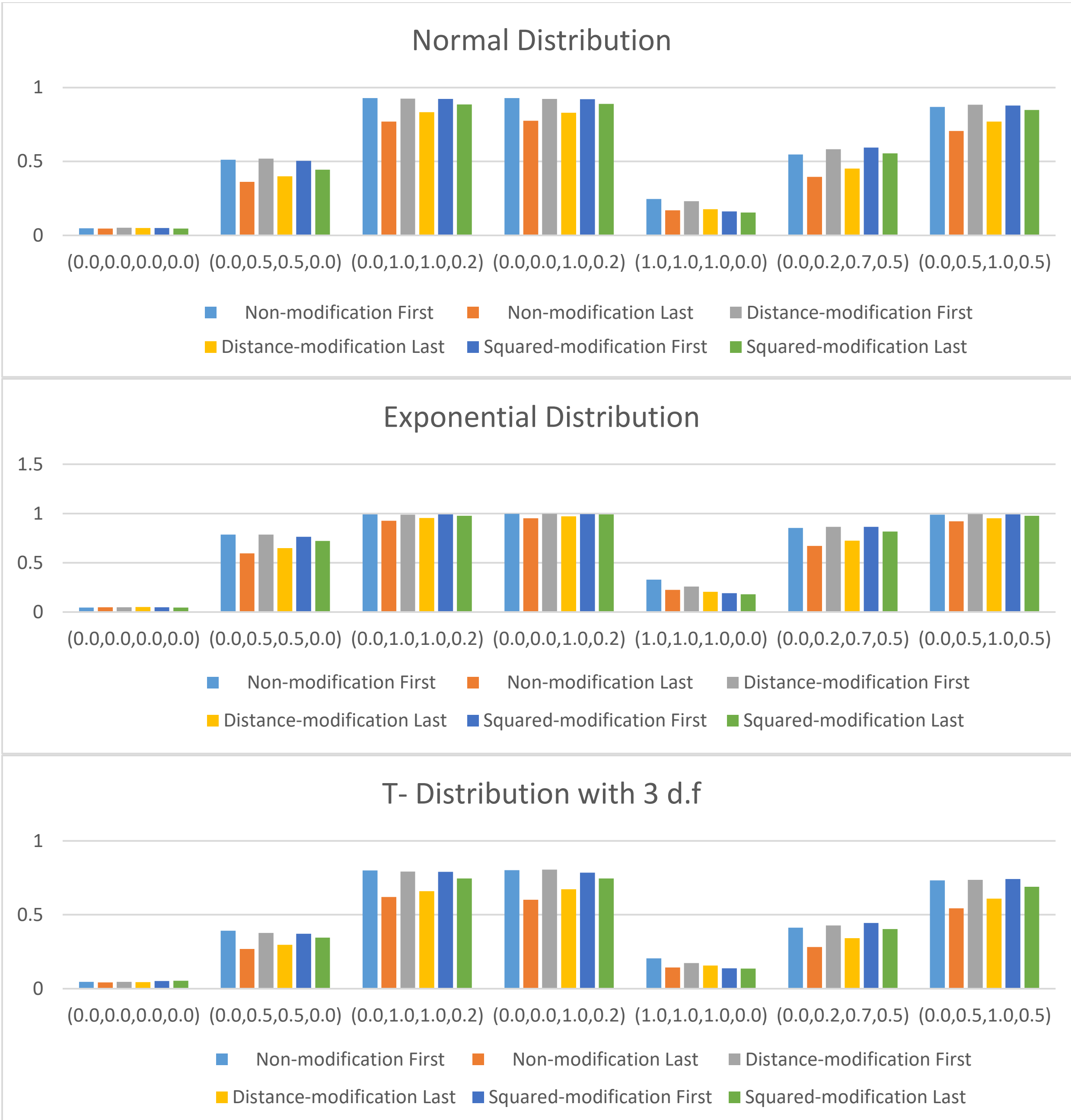

Figure 25. Treatments=4, CRD = 18, RCBD=6, Peak=3

Table 145. Treatments=5, CRD=5, RCBD=5 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0494	0.0480	0.0528	0.0550	0.0540	0.0508
	1	1	0.5	0.5	0.5	0.1936	0.1600	0.1962	0.1922	**0.2236**	0.2180
	0.5	1	1	1	0.7	**0.1878**	0.1504	0.1762	0.1744	0.1656	0.1650
	0.7	1	0.7	0.7	0.5	0.2568	0.2156	**0.2576**	0.2314	0.2492	0.2446
	1	1	0.5	0.5	0.2	0.3632	0.2936	0.3726	0.3328	**0.3998**	0.3946
	0.75	1	0.75	0.5	0.25	0.4240	0.3666	0.4512	0.4194	**0.4542**	0.4468
	1	1	1	1	0	0.4500	0.3816	0.4954	0.4198	**0.5038**	0.4868
	1	1	0.5	0.2	0	0.5616	0.4494	0.5780	0.5180	**0.5894**	0.5850
	0	1.6	0.8	0.4	0.2	**0.9508**	0.8908	0.9474	0.9200	0.9400	0.9386
Exponential	0	0	0	0	0	0.0534	0.0488	0.0500	0.0524	0.0426	0.0482
	1	1	0.5	0.5	0.5	0.2966	0.2286	0.3180	0.2730	**0.3286**	0.3108
	0.5	1	1	1	0.7	0.2960	0.2598	**0.3032**	0.2820	0.2828	0.2646
	0.7	1	0.7	0.7	0.5	0.4312	0.3526	**0.4438**	0.4028	0.4344	0.4148
	1	1	0.5	0.5	0.2	0.5618	0.4752	0.5950	0.5564	**0.6280**	0.6172
	0.75	1	0.75	0.5	0.25	0.7092	0.6100	0.7118	0.6638	**0.7312**	0.6994
	1	1	1	1	0	0.5822	0.4878	0.6330	0.5812	**0.6586**	0.6506
	1	1	0.5	0.2	0	0.7928	0.6862	0.8126	0.7614	**0.8258**	0.8246
	0	1.6	0.8	0.4	0.2	**0.9944**	0.9814	0.9930	0.9890	0.9908	0.9888
T with 3 degrees of freedom	0	0	0	0	0	0.0502	0.0470	0.0498	0.0474	0.0472	0.0528
	1	1	0.5	0.5	0.5	0.1648	0.1416	0.1644	0.1616	**0.1700**	0.1684
	0.5	1	1	1	0.7	0.1464	0.1260	**0.1472**	0.1452	0.1434	0.1404
	0.7	1	0.7	0.7	0.5	0.1936	0.1668	**0.2056**	0.1996	0.1950	0.1896
	1	1	0.5	0.5	0.2	0.2812	0.2214	0.2920	0.2686	**0.2982**	0.2622
	0.75	1	0.75	0.5	0.25	0.3282	0.2668	0.3354	0.3210	**0.3420**	0.3336
	1	1	1	1	0	0.3320	0.2718	0.3652	0.3298	**0.3806**	0.3764
	1	1	0.5	0.2	0	0.4094	0.3404	0.4454	0.3906	**0.4482**	0.4300
	0	1.6	0.8	0.4	0.2	0.8270	0.7322	0.8198	0.7916	0.8002	0.7976

6.3.4. Five Treatments at Peaks Two

In the case of four treatments at peak two, the standardized first versions of the test performed better than standardized last versions of the test. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If the distance between parameters before the peak and the peak is less than the distance after the peak and the peak, such as $(1.0,1.0,0.5,0.5,0.2)$, the distance squared modification first performed better. Where three population parameters are equal, but are different from the peak, and the additional parameter is different from the peak, such as (0.7,1.0,0.7,0.7,0.5), the distance modification first performed better.

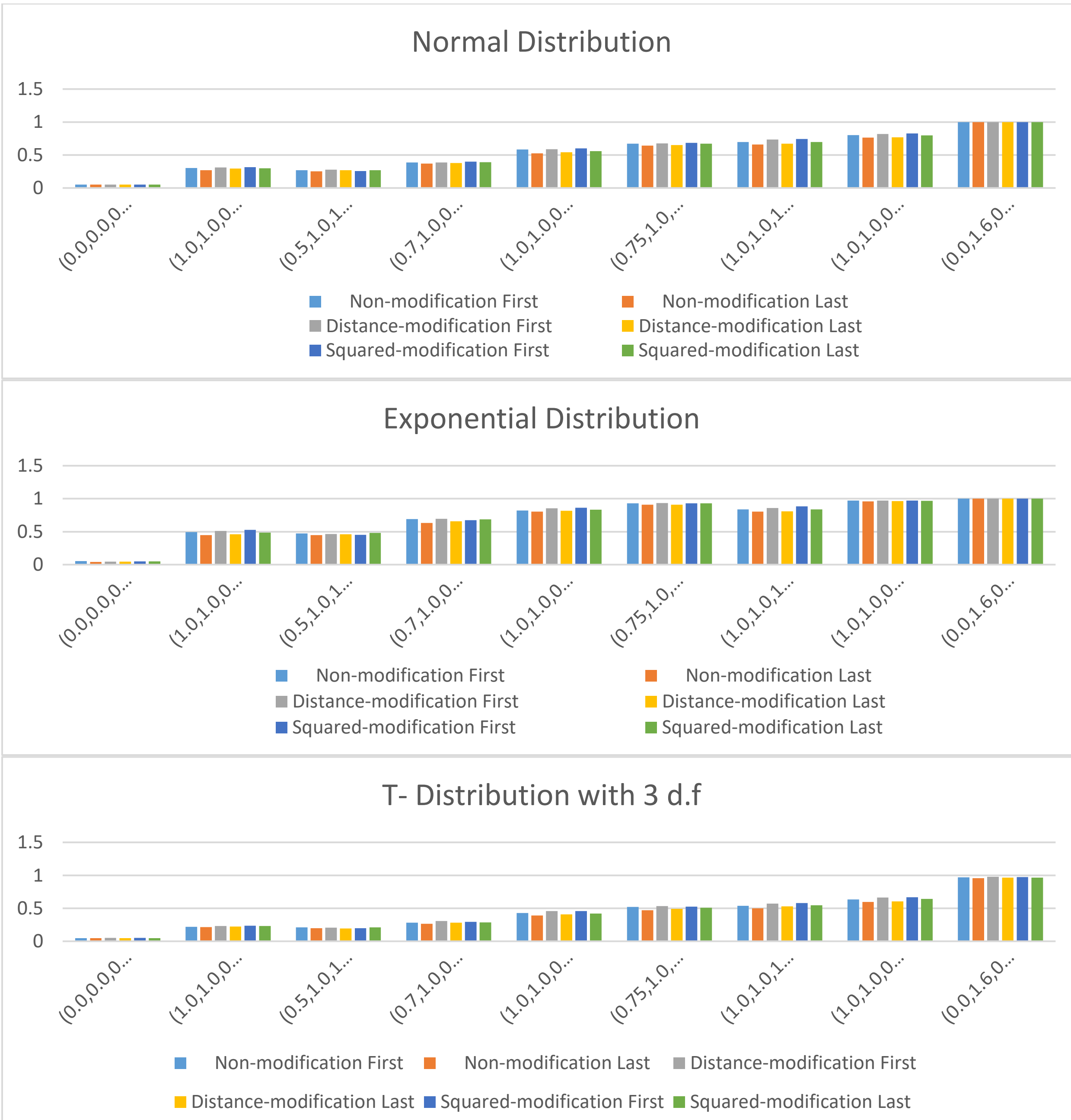

Figure 26. Treatments=5, CRD=5, RCBD=5 Peak=2

Table 146. Treatments=5, CRD=5, RCBD=5 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0544	0.0514	0.0522	0.0458	0.0568	0.0474
	0	0.5	1	1	0.7	0.2806	0.1734	0.4362	0.3800	**0.4468**	0.4034
	0.7	0.7	1	0.5	0.5	0.1962	0.1882	0.2412	0.1972	**0.2504**	0.2220
	0	0.5	1	0.5	0	0.6006	0.5756	0.7582	0.6864	**0.7638**	0.7250
	0	1	1	0	0	**0.7852**	0.6558	0.7410	0.6614	0.7486	0.7118
	1	1	1	0.5	0	**0.4188**	0.2898	0.2866	0.2596	0.2952	0.2802
	1	1	1	1	0	**0.3474**	0.2666	0.2808	0.2296	0.2886	0.2608
Exponential	0	0	0	0	0	0.0518	0.0588	0.0504	0.0450	0.0528	0.0492
	0	0.5	1	1	0.7	0.4474	0.2512	0.6772	0.5824	**0.6870**	0.6462
	0.7	0.7	1	0.5	0.5	0.2854	0.2810	0.3864	0.2984	**0.4008**	0.3506
	0	0.5	1	0.5	0	0.8428	0.8046	**0.9486**	0.8994	0.9460	0.9370
	0	1	1	0	0	**0.9348**	0.8376	0.9126	0.8476	0.9136	0.8840
	1	1	1	0.5	0	**0.6258**	0.4156	0.4342	0.3658	0.4294	0.4006
	1	1	1	1	0	**0.4614**	0.3414	0.3760	0.3088	0.3746	0.3440
T with 3 degrees of freedom	0	0	0	0	0	0.0504	0.0542	0.0536	0.0532	0.0464	0.0500
	0	0.5	1	1	0.7	0.2144	0.1338	0.3274	0.2766	**0.3418**	0.3204
	0.7	0.7	1	0.5	0.5	0.1554	0.1422	0.1902	0.1554	**0.1960**	0.1754
	0	0.5	1	0.5	0	0.4578	0.4398	**0.5990**	0.5004	0.5980	0.5634
	0	1	1	0	0	**0.6264**	0.5124	0.5696	0.4946	0.5864	0.5462
	1	1	1	0.5	0	**0.3212**	0.2252	0.2328	0.1924	0.2378	0.2174
	1	1	1	1	0	**0.2770**	0.2094	0.2220	0.1942	0.2362	0.2022

6.3.5. Five Treatments at Peaks Three

In the case of four treatment at peak three, the standardized first versions of the test performed better than standardized last versions of the test. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If one the population parameter equals the peak and the other parameters are different from the peak and different from each other such as (0.0,0.5,1.0,1.0,0.7), the distance squared modification first performed better. Where the distance between parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as (1.0,1.0,1.0,0.5,0.0), the non-modification first performed better. Also where two additional population parameters are equal to each other , but are different from the peak such as

(0.0,0.5,1.0,0.5,0.0), the distance modification first performed better. Table 146 above (Refer to Tables C.46. to C.53. in the Appendix) and Figure 27 below show the probabilities of all the three modification tests.

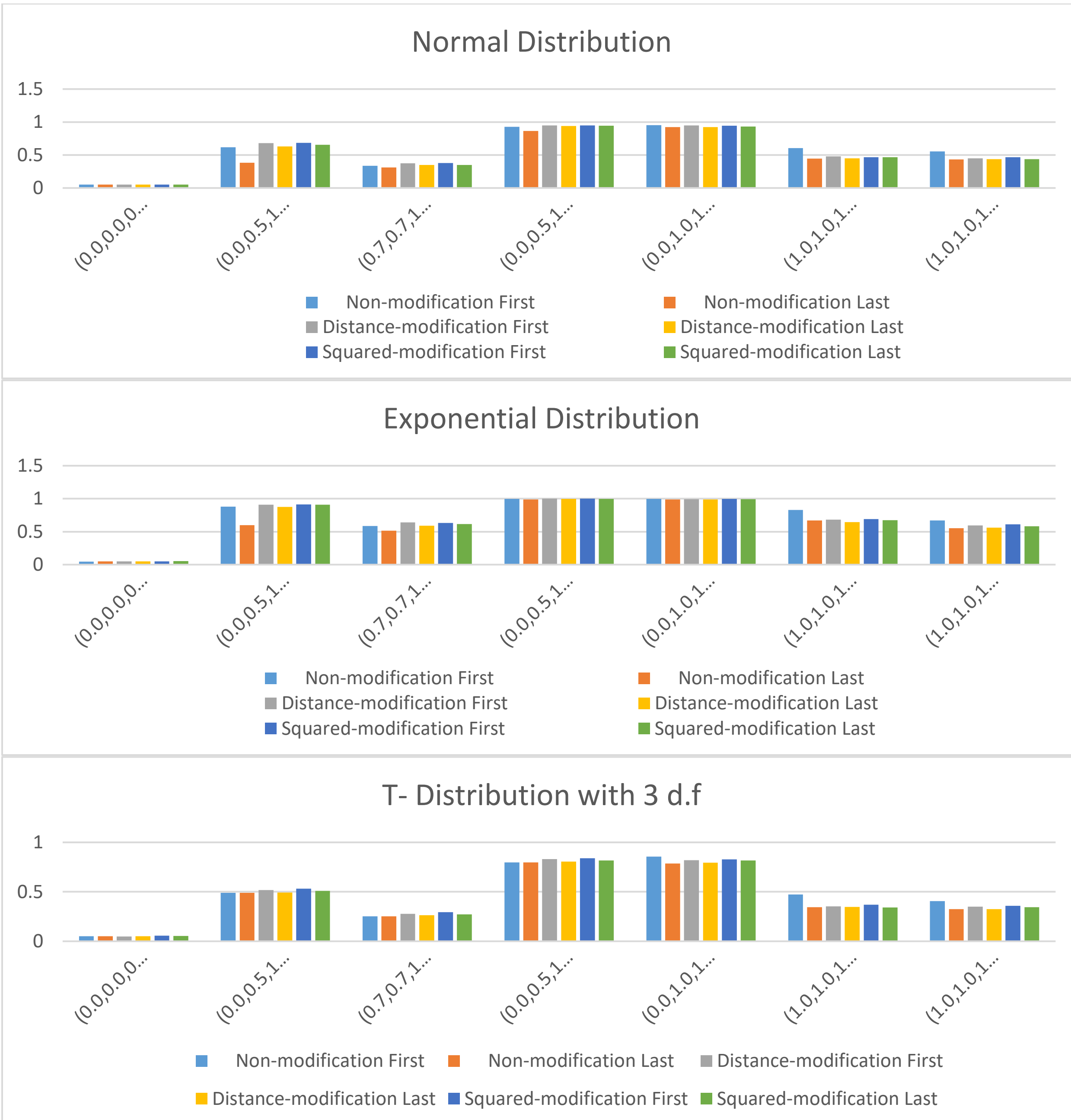

Figure 27. Treatments=5, CRD=5, RCBD=5 Peak=3

Table 147. Treatments=5, CRD=15, RCBD=5 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0524	0.0504	0.0524	0.0510	0.0532	0.0500
	0	0.5	1	1	0.7	0.6166	0.3834	0.6782	0.6318	**0.6854**	0.6568
	0.7	0.7	1	0.5	0.5	0.3356	0.3122	0.3728	0.3480	**0.3770**	0.3496
	0	0.5	1	0.5	0	0.9270	0.8654	0.9492	0.9388	**0.9494**	0.9428
	0	1	1	0	0	**0.9550**	0.9230	0.9506	0.9252	0.9434	0.9308
	1	1	1	0.5	0	**0.6054**	0.4476	0.4794	0.4502	0.4680	0.4686
	1	1	1	1	0	**0.5568**	0.4316	0.4500	0.4386	0.4650	0.4388
Exponential	0	0	0	0	0	0.0476	0.0508	0.0490	0.0496	0.0512	0.0546
	0	0.5	1	1	0.7	0.8778	0.5992	0.9102	0.8772	**0.9150**	0.9096
	0.7	0.7	1	0.5	0.5	0.5870	0.5170	**0.6416**	0.5896	0.6306	0.6142
	0	0.5	1	0.5	0	0.9958	0.9866	**0.9996**	0.9982	0.9994	0.9980
	0	1	1	0	0	**0.9980**	0.9906	0.9946	0.9902	0.9954	0.9916
	1	1	1	0.5	0	**0.8308**	0.6692	0.6836	0.6462	0.6918	0.6744
	1	1	1	1	0	**0.6702**	0.5508	0.5958	0.5624	0.6118	0.5800
T with 3 degrees of freedom	0	0	0	0	0	0.0522	0.0524	0.0486	0.0504	0.0566	0.0538
	0	0.5	1	1	0.7	0.4888	0.4888	0.5166	0.4926	**0.5318**	0.5088
	0.7	0.7	1	0.5	0.5	0.2524	0.2524	0.2788	0.2626	**0.2946**	0.2722
	0	0.5	1	0.5	0	0.7984	0.7984	0.8308	0.8054	**0.8402**	0.8180
	0	1	1	0	0	**0.8554**	0.7862	0.8190	0.7960	0.8284	0.8184
	1	1	1	0.5	0	**0.4744**	0.3444	0.3536	0.3476	0.3708	0.3422
	1	1	1	1	0	**0.4058**	0.3240	0.3502	0.3238	0.3578	0.3446

With four treatments at peak three, the standardized first versions of the test performed better than standardized last versions of the test. The results from the distance squared modification first and the non-modification first provide the highest values of the estimated power test statistics in the following cases:

If one of the population parameter equals the peak and the other parameters are different from the peak and different from each other, such as (0.0,0.5,1.0,1.0,0.7), the distance squared modification first performed better. On the other hand, if the distance between parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as (1.0,1.0,1.0,0.5,0.0), the non-modification first performed better. Table 147 above

(Refer to Tables C.46. to C.53. in the Appendix) and Figure 28 below show probabilities of all the three modification tests.

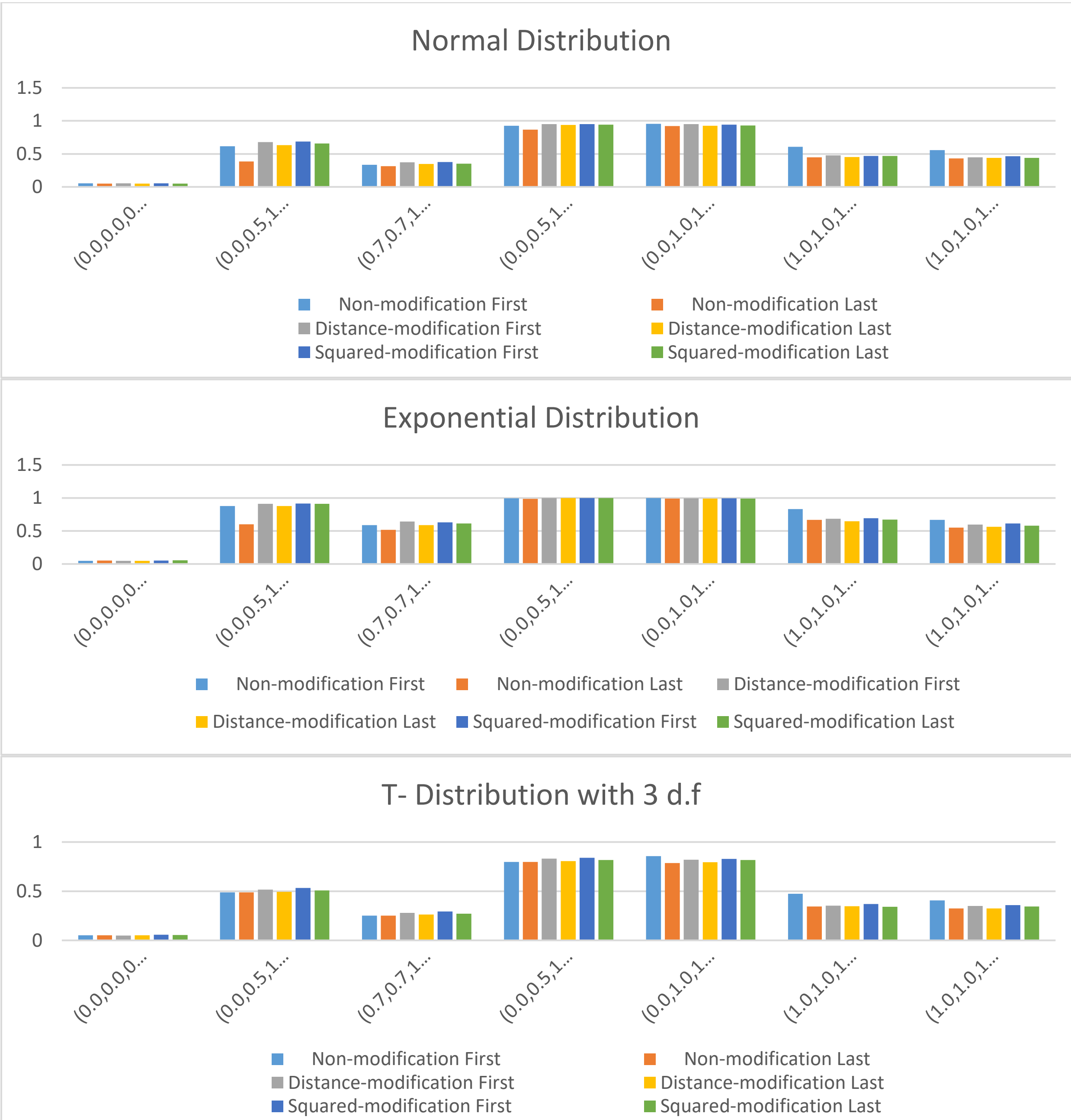

Figure 28. Treatments=5, CRD=15, RCBD=5 Peak=3

Table 148. Treatments=5, CRD=5, RCBD=5 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0480	0.0494	0.0490	0.0512	0.0520	0.0496
	0.5	0.5	0.5	1	1	0.2002	0.1700	**0.2064**	0.1970	0.1714	0.1696
	0	0.5	1	1	0.7	0.6426	0.5420	**0.6692**	0.6134	0.5500	0.5496
	0	0.5	1	1	1	0.5770	0.4730	**0.6146**	0.5594	0. 4694	0.4480
	0	0	1	1	1	0.6576	0.5562	**0.7136**	0.6432	0. 5174	0.4480
	0.5	0.5	0.7	1	0.7	**0.2892**	0.2480	0.2796	0.2734	0. 1984	0.1756
	1	1	1	1	0	0.1256	0.1010	0.1020	0.1114	**0.1990**	0.1534
	1	0.4	0.8	1.6	0.4	**0.9664**	0.9026	0.9622	0.9396	0. 8472	0.7770
	0	0.25	0.5	1	0.25	0.6862	0.5820	**0.6950**	0.6564	0. 5168	0.4580
Exponential	0	0	0	0	0	0.0442	0.0466	0.0520	0.0442	0.0500	0.0484
	0.5	0.5	0.5	1	1	0.2894	0.2374	**0.2974**	0.2796	0.2520	0.2358
	0	0.5	1	1	0.7	0.8602	0.7568	**0.8704**	0.8372	0.8010	0.7886
	0	0.5	1	1	1	0.7884	0.6804	**0.8178**	0.7538	0.6510	0.6388
	0	0	1	1	1	0.8230	0.7306	**0.8632**	0.8150	0.6914	0.6484
	0.5	0.5	0.7	1	0.7	**0.5060**	0.3916	0.4832	0.4636	0.3240	0.2600
	1	1	1	1	0	0.1320	0.1106	0.1104	0.1054	**0.2308**	0.1770
	1	0.4	0.8	1.6	0.4	**0.9970**	0.9852	0.9968	0.9950	0.9666	0.9396
	0	0.25	0.5	1	0.25	**0.9274**	0.8436	0.9170	0.8878	0.7716	0.7042
T with 3 degrees of freedom	0	0	0	0	0	0.0502	0.0550	0.0540	0.0506	0.0498	0.0540
	0.5	0.5	0.5	1	1	0.1570	0.1344	**0.1682**	0.1498	0.1462	0.1458
	0	0.5	1	1	0.7	0.4960	0.4038	**0.5198**	0.4782	0.4452	0.4244
	0	0.5	1	1	1	0.4340	0.3688	**0.4808**	0.4334	0.3548	0.3442
	0	0	1	1	1	0.5100	0.4150	**0.5454**	0.5054	0.3866	0.3602
	0.5	0.5	0.7	1	0.7	**0.2154**	0.1940	0.2290	0.2084	0.1540	0.1418
	1	1	1	1	0	0.1046	0.1018	0.1058	0.1060	**0.1600**	0.1344
	1	0.4	0.8	1.6	0.4	0.8436	0.7644	**0.8494**	0.8222	0.6894	0.6162
	0	0.25	0.5	1	0.25	**0.5464**	0.4538	0.5300	0.4994	0.3900	0.3506

6.3.6. Five Treatments at Peaks Four

In all three modification tests, the standardized first performed better than the standardized last versions of the test. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If the distance between parameters before the peak and the peak is less than the distance between parameter after the peak and the peak, such as (1.0,1.0,1.0,1.0,0.0), the distance squared modification first performed better. Where two population parameters are equal to the peak and the other two are different from each other, such as (0.0,0.5,1.0,1.0,1.0), the distance modification first performed better. Where two additional population parameters are equal to each other but, different from the peak such as (0.5,0.5,07.0,1.0,0.7), the non-modification first performed better.

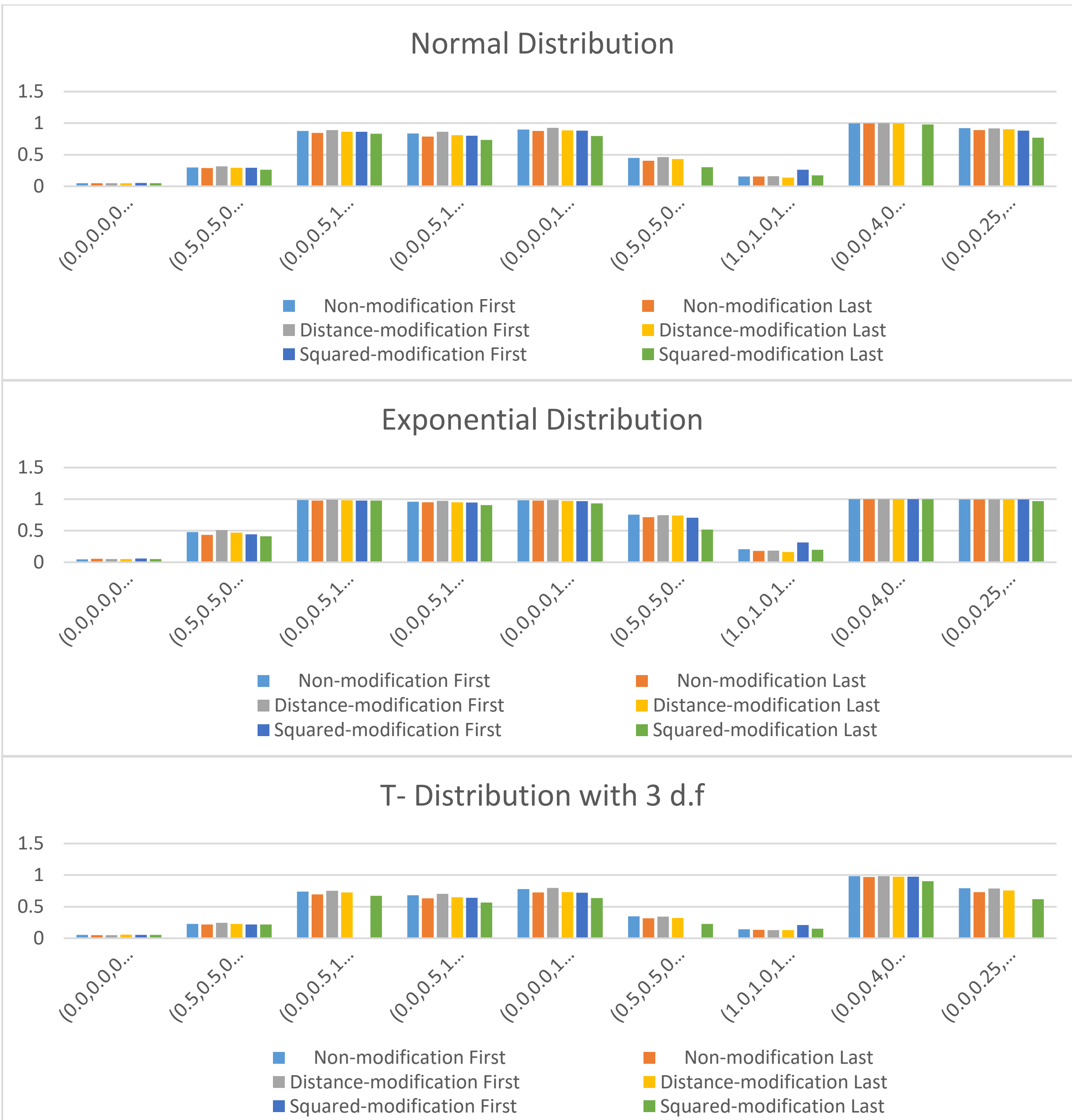

Figure 29. Treatments=5, CRD=5, RCBD=5 Peak=4

Table 149. Treatments=5, CRD=15, RCBD=5 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0508	0.0470	0.0486	0.0484	0.0532	0.0480
	0.5	0.5	0.5	1	1	0.2992	0.2914	**0.3154**	0.2932	0.2920	0.2630
	0	0.5	1	1	0.7	**0.8906**	0.8434	0.8756	0.8644	0.8642	0.8336
	0	0.5	1	1	1	0.8382	0.7890	**0.8630**	0.8090	0.7994	0.7328
	0	0	1	1	1	0.9006	0.8746	**0.9272**	0.8872	0.8794	0.7952
	0.5	0.5	0.7	1	0.7	**0.4628**	0.4064	0.4490	0.4324	0. 4112	0.3012
	1	1	1	1	0	0.1570	0.1572	0.1594	0.1384	**0.2610**	0.1744
	1	0.4	0.8	1.6	0.4	0.9988	0.9970	**0.9992**	0.9990	0. 9982	0.9812
	0	0.25	0.5	1	0.25	**0.9204**	0.8902	0.9174	0.9054	0.8820	0.7704
Exponential	0	0	0	0	0	0.0474	0.0534	0.0508	0.0514	0.0584	0.0502
	0.5	0.5	0.5	1	1	0.4780	0.4358	**0.5106**	0.4684	0.4428	0.4098
	0	0.5	1	1	0.7	**0.9860**	0.9786	0.9882	0.9804	0.9780	0.9748
	0	0.5	1	1	1	0.9606	0.9480	**0.9736**	0.9510	0.9466	0.9076
	0	0	1	1	1	0.9832	0.9752	**0.9866**	0.9744	0.9698	0.9318
	0.5	0.5	0.7	1	0.7	**0.7554**	0.7158	0.7436	0.7388	0.7052	0.5188
	1	1	1	1	0	0.2054	0.1814	0.1836	0.1626	**0.3140**	0.1996
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	0.9996
	0	0.25	0.5	1	0.25	0.9960	0.9934	0.9962	0.9950	0.9944	0.9694
T with 3 degrees of freedom	0	0	0	0	0	0.0536	0.0494	0.0480	0.0556	0.0542	0.0548
	0.5	0.5	0.5	1	1	0.2266	0.2158	**0.2462**	0.2280	0.2194	0.2164
	0	0.5	1	1	0.7	**0.7518**	0.6950	0.7406	0.7252	0. 7044	0.6726
	0	0.5	1	1	1	0.6824	0.6330	**0.7022**	0.6494	0.6394	0.5644
	0	0	1	1	1	0.7768	0.7276	**0.7984**	0.7302	0.7192	0.6350
	0.5	0.5	0.7	1	0.7	**0.3482**	0.3156	0.3448	0.3208	0. 3130	0.2254
	1	1	1	1	0	0.1428	0.1322	0.1294	0.1280	**0.2076**	0.1508
	1	0.4	0.8	1.6	0.4	**0.9836**	0.9706	0.9828	0.9760	0.9746	0.9042
	0	0.25	0.5	1	0.25	**0.7910**	0.7310	0.7868	0.7558	0. 7434	0.6208

In all the three modification tests, the standardized first performed better than standardized last versions of the test. With a significant difference of 0.01, between the results of the three modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If the distance between parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as (1.0,1.0,1.0,1.0,0.0), and the distance squared modification first performed better. Where two population parameters are equal to the peak and the other parameters are different from each other, such as (0.0,0.5,1.0,1.0,1.0), the distance modification first performed slightly better. If one population parameter equals the peak and the other parameters are different from each other, such as (0.0,0.5,1.0,1.0,0.7), the non-modification performed first slightly better.

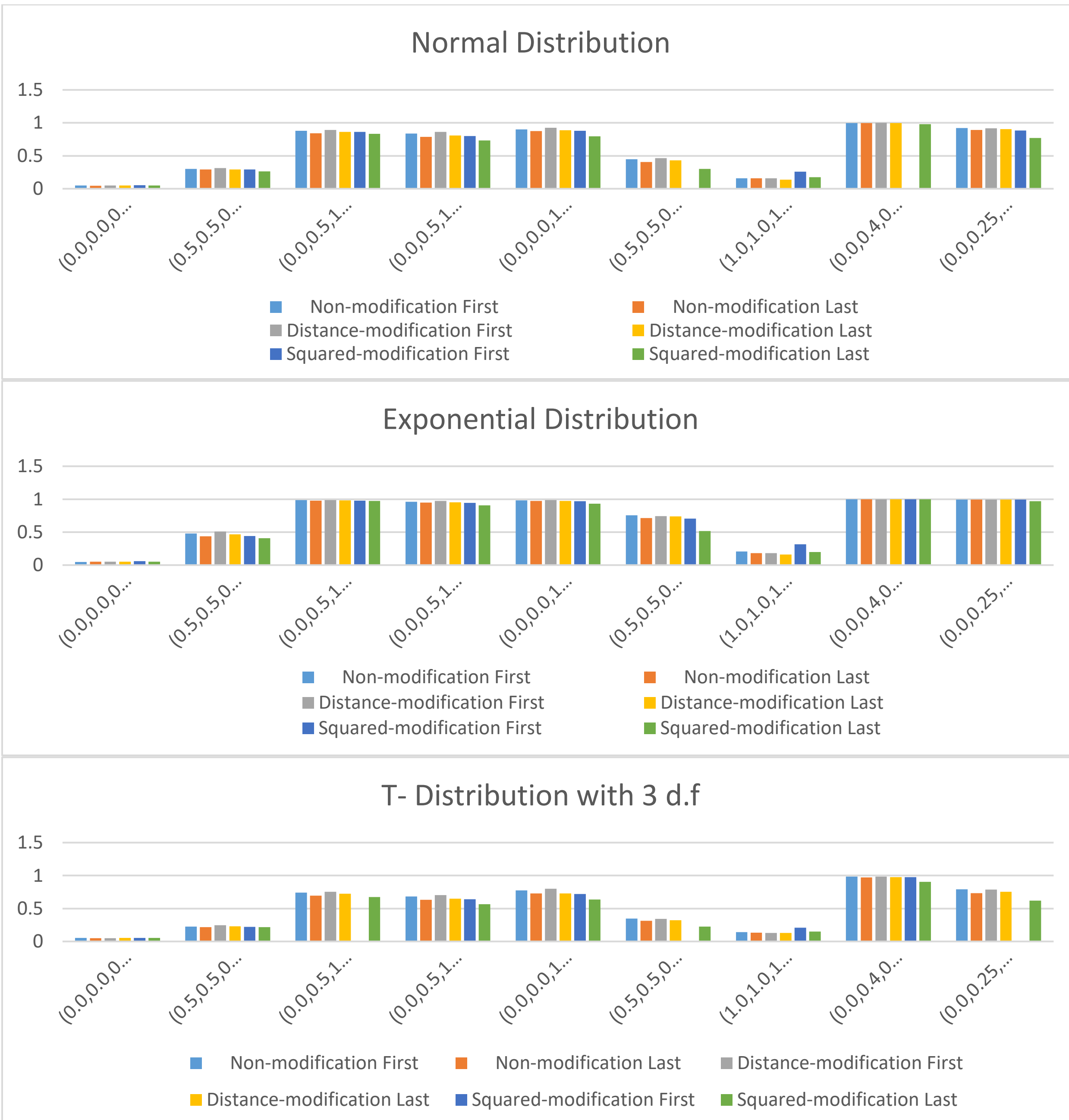

Figure 30. Treatments=5, CRD=15, RCBD=5 Peak=4

6.4. Mixed Design of IBD, CRD and RCBD

Table 150. Treatments =4, IBD = 6, CRD = 6, RCBD = 6, Peak = 2, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0518	0.0516	0.0500	0.0492
	0.5	0.5	0	0	0.2486	0.1852	**0.2584**	0.2064
	0	1	0.2	0.2	**0.7994**	0.6408	0.7976	0.6662
	1	1	0	0	0.5682	0.4162	**0.6100**	0.4620
	0	0.7	0.2	0	**0.6654**	0.5120	0.6424	0.5330
	0.5	1	0.5	0	0.7832	0.6210	**0.7890**	0.6538
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0514	0.0470	0.0508	0.0466
	0.5	0.5	0	0	0.3844	0.2738	**0.4136**	0.2936
	0	1	0.2	0.2	**0.9638**	0.8724	0.9598	0.8830
	1	1	0	0	0.7304	0.5710	**0.7718**	0.6104
	0	0.7	0.2	0	**0.9096**	0.7744	0.9024	0.7916
	0.5	1	0.5	0	0.9678	0.8670	**0.9696**	0.8920
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0496	0.0540	0.0500	0.0534
	0.5	0.5	0	0	0.1858	0.1470	**0.2000**	0.1672
	0	1	0.2	0.2	0.6388	0.4874	**0.6470**	0.5260
	1	1	0	0	0.4330	0.3292	**0.4618**	0.3412
	0	0.7	0.2	0	**0.5082**	0.3784	0.4976	0.4010
	0.5	1	0.5	0	0.6276	0.4778	**0.6324**	0.5002

6.4.1. Four Treatments at Peak Two

With the mixed design of IBD, CRD, and RCBD, the powers of the standardized first were slightly higher than the standardized last versions of the test, regardless of the probabilities $(p = 0.1, 0.2, 0.4)$ associated with the IBD. With a significant difference of 0.01, between the results of the two modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If the distance between parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as $(1.0,1.0,0.0,0.0)$ and $(0.5, 0.5, 0.0, 0.0)$ the distance modification first performed better. Where two population parameters are equal, but different from the peak, such as $(0.0,1.0,0.2,0.2)$, the distance modification performed slightly better than the non-modification test. Table 150 above (Refer to Tables D.1. to D.18. in the Appendix) and Figure 31 below show the probabilities of all the three modification tests.

Figure 31. Treatment = 4, IBD = 4, CRD = 6 RCBD =6, Peak = 2, p=0.1

Table 151. Treatments=4, IBD = 12, CRD=12, RCBD=12, Peak=2, p = 0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0522	0.0476	0.0530	0.0512
	0.5	0.5	0	0	0.3812	0.2652	**0.3930**	0.2650
	0	1	0.2	0.2	**0.9674**	0.8242	0.9640	0.8430
	1	1	0	0	0.8228	0.6050	**0.8572**	0.6400
	0	0.7	0.2	0	0.8824	0.6840	**0.8848**	0.7026
	0.5	1	0.5	0	0.9576	0.8030	**0.9592**	0.8256
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0560	0.0516	0.0504	0.0514
	0.5	0.5	0	0	0.6190	0.4028	**0.6478**	0.4124
	0	1	0.2	0.2	**0.9996**	0.9750	0.9994	0.9798
	1	1	0	0	0.9410	0.7528	**0.9506**	0.7866
	0	0.7	0.2	0	**0.9962**	0.9238	0.9936	0.9394
	0.5	1	0.5	0	**0.9996**	0.9720	0.9986	0.9750
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0522	0.0514	0.0474	0.0482
	0.5	0.5	0	0	0.2900	0.2008	**0.3074**	0.2076
	0	1	0.2	0.2	**0.8712**	0.6702	0.8678	0.6888
	1	1	0	0	0.6780	0.4536	**0.7082**	0.4784
	0	0.7	0.2	0	**0.7336**	0.5306	0.7332	0.5368
	0.5	1	0.5	0	**0.8552**	0.6552	0.8500	0.6598

With the mixed design of IBD, CRD, and RCBD, the powers of the standardized first were slightly higher than the standardized last versions of the test regardless of the probabilities $(p = 0.1, 0.2, 0.4)$ associated with the IBD. With a significant difference of 0.01, between the results of the two modification tests, the highest values of the estimated powers test statistics were reported in the following cases:

If the distance between parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as $(1.0,1.0,0.0,0.0)$ and $(0.5, 0.5, 0.0, 0.0)$, the distance modification first performed better. Where two population parameters are equal, but are different from the peak, such as $(0.0,1.0,0.2,0.2)$, the distance modification performed slightly better than the non-modification test. Table 151 above (Refer to tables D.1. to D.18. in the Appendix) and Figure 32 below show the probabilities of all the three modification tests.

Figure 32. Treatments=4, IBD = 12, CRD=12, RCBD=12, Peak=2, p=0.2

Table 152. Treatments=4, IBD = 6, CRD=6, RCBD=6, Peak=3, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0490	0.0480	0.0590	0.0478
	0	0.5	0.5	0	**0.4232**	0.3262	0.4120	0.3344
	0	1	1	0.2	0.8464	0.6948	**0.8550**	0.7300
	0	0	1	0.2	0.9972	0.9870	**0.9984**	0.9884
	1	1	1	0	**0.2208**	0.1686	0.2058	0.1516
	0	0.2	0.7	0.5	0.4714	0.3434	**0.4764**	0.3554
	0	0.5	1	0.5	0.7782	0.6172	**0.7942**	0.6474
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0484	0.0540	0.0562	0.0516
	0	0.5	0.5	0	**0.6866**	0.5264	0.6844	0.5480
	0	1	1	0.2	**0.9676**	0.8730	0.9662	0.8910
	0	0	1	0.2	**0.9794**	0.9004	0.9752	0.9162
	1	1	1	0	**0.2818**	0.1982	0.2488	0.1878
	0	0.2	0.7	0.5	0.7462	0.5728	**0.7564**	0.5868
	0	0.5	1	0.5	0.9622	0.8644	**0.9648**	0.8856
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T Distribution with 3 degrees of freedom	0	0	0	0	0.0500	0.0538	0.0496	0.0506
	0	0.5	0.5	0	**0.3238**	0.2400	0.3126	0.2500
	0	1	1	0.2	**0.6946**	0.5364	0.6882	0.5746
	0	0	1	0.2	0.6982	0.5352	**0.7026**	0.5454
	1	1	1	0	**0.1730**	0.1402	0.1620	0.1378
	0	0.2	0.7	0.5	0.3466	0.2554	**0.3688**	0.2794
	0	0.5	1	0.5	0.6320	0.4768	**0.6356**	0.4986

6.4.2. Four Treatments at Peak Three

With a probability of 0.2 associated with the IBD portion, the Mixed design of IBD, CRD, and RCBD had the highest percentage of rejection when the ratio of IBD to CRD to RCBD was 1:1:1 (18:18:18). Again, the results showed that powers of the standardized first were slightly higher than the standardized last test regardless of the probabilities 0.2, 0.3, and 0.4 associated with the IBD. The results from the distance square modification first and the non-modification first reported the highest values of the estimated powers test statistics in the following cases:

If the distance between population parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as $(1.0, 1.0, 1.0, 0.0)$, the non-modification first performed better. Where population parameters are different from each other, such as (0.0,0.2,0.7,0.5), the distance modification first performed slightly better than the non-modification first. Table 152 above (Refer to Tables D.19. to D.36. in the Appendix) and Figure 33 below shows the probabilities of all the three modification tests.

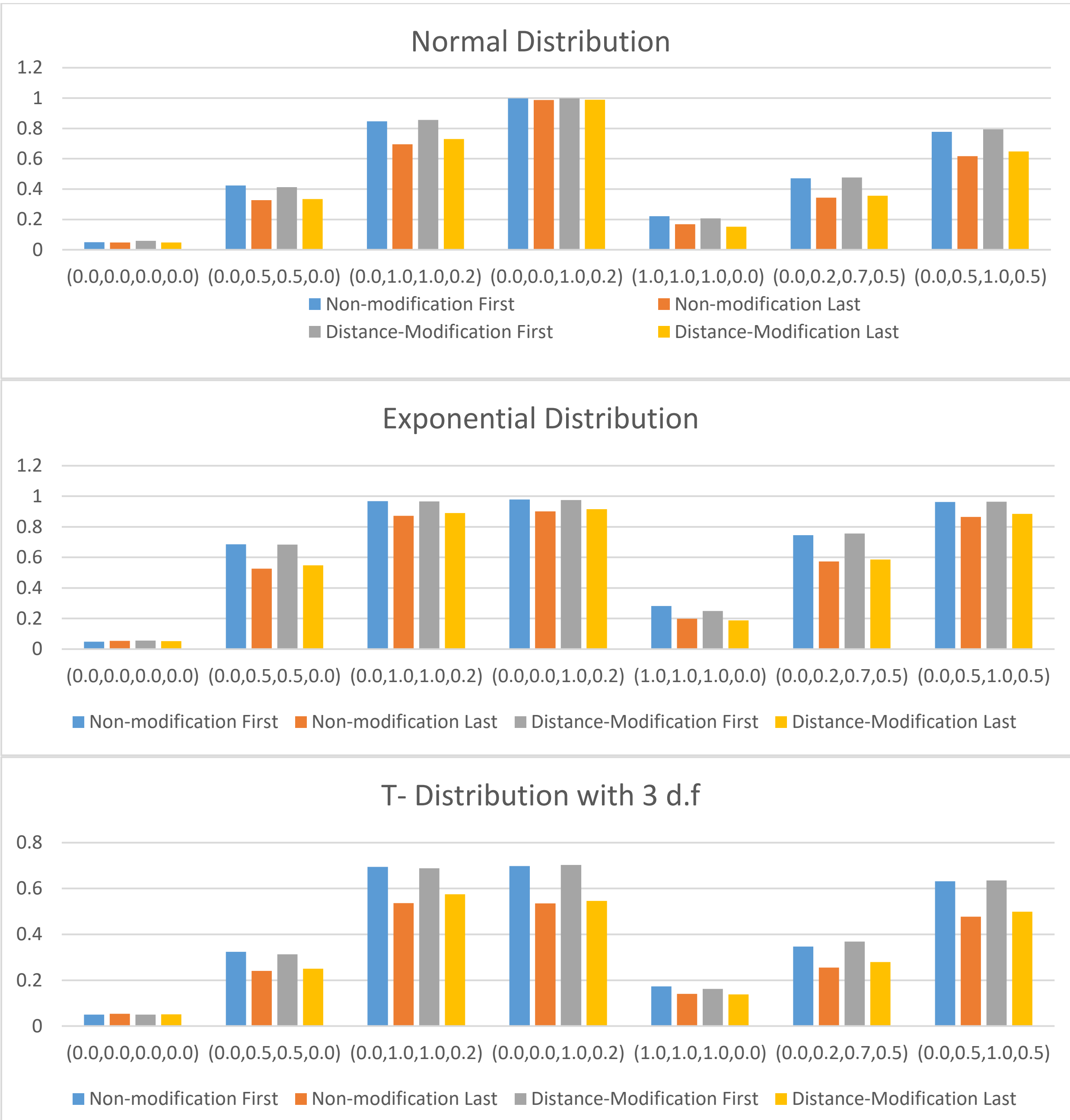

Figure 33. Treatments=4, IBD = 6, CRD=6, RCBD=6, Peak=3, p=0.1

Table 153. Treatments=4, IBD = 12, CRD=12, RCBD=12, Peak=3, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0506	0.0502	0.0504	0.0492
	0.5	0.5	0	0	0.6544	0.4456	0.6574	0.4522
Normal	0	1	0.2	0.2	**0.9826**	0.8752	0.9794	0.8824
	1	1	0	0	**0.9834**	0.8786	0.9832	0.8910
	0	0.7	0.2	0	**0.3416**	0.2340	0.3054	0.2332
	0.5	1	0.5	0	**0.7250**	0.4728	0.7118	0.5072
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0514	0.0440	0.0552	0.0516
Exponential	0.5	0.5	0	0	**0.9006**	0.7144	0.9004	0.7182
	0	1	0.2	0.2	**0.9998**	0.9706	0.9992	0.9748
	1	1	0	0	**0.9998**	0.9880	0.9996	0.9878
	0	0.7	0.2	0	**0.4504**	0.2874	0.3994	0.2752
	0.5	1	0.5	0	**0.9466**	0.7702	0.9420	0.7926
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0462	0.0508	0.0520	0.0484
of freedom	0.5	0.5	0	0	0.5064	0.3306	**0.5154**	0.3428
	0	1	0.2	0.2	0.9084	0.7178	**0.9130**	0.7344
	1	1	0	0	**0.9280**	0.7184	0.9180	0.7280
	0	0.7	0.2	0	**0.2656**	0.1870	0.2338	0.1798
	0.5	1	0.5	0	0.5548	0.3644	**0.5666**	0.3788

The results showed that powers of the standardized first were slightly higher than the standardized last test regardless of the probabilities 0.2, 0.3, and 0.4 associated with the IBD portion. The results from the distance square modification first and the non-modification first reported the highest values of the estimated power test statistics in the following cases:

If the distance between parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as $(1.0,1.0,0.0,0.0)$, the non-modification first performed better. If one population parameter equals the peak and the other additional parameters are different from the peak but are equal, such as $(0.5,0.5,0.0,0.0)$, the distance modification first performed better. Table 153 above (Refer to Tables D.19. to D.36. in the Appendix) and Figure 34 below show the probabilities of all the three modification tests.

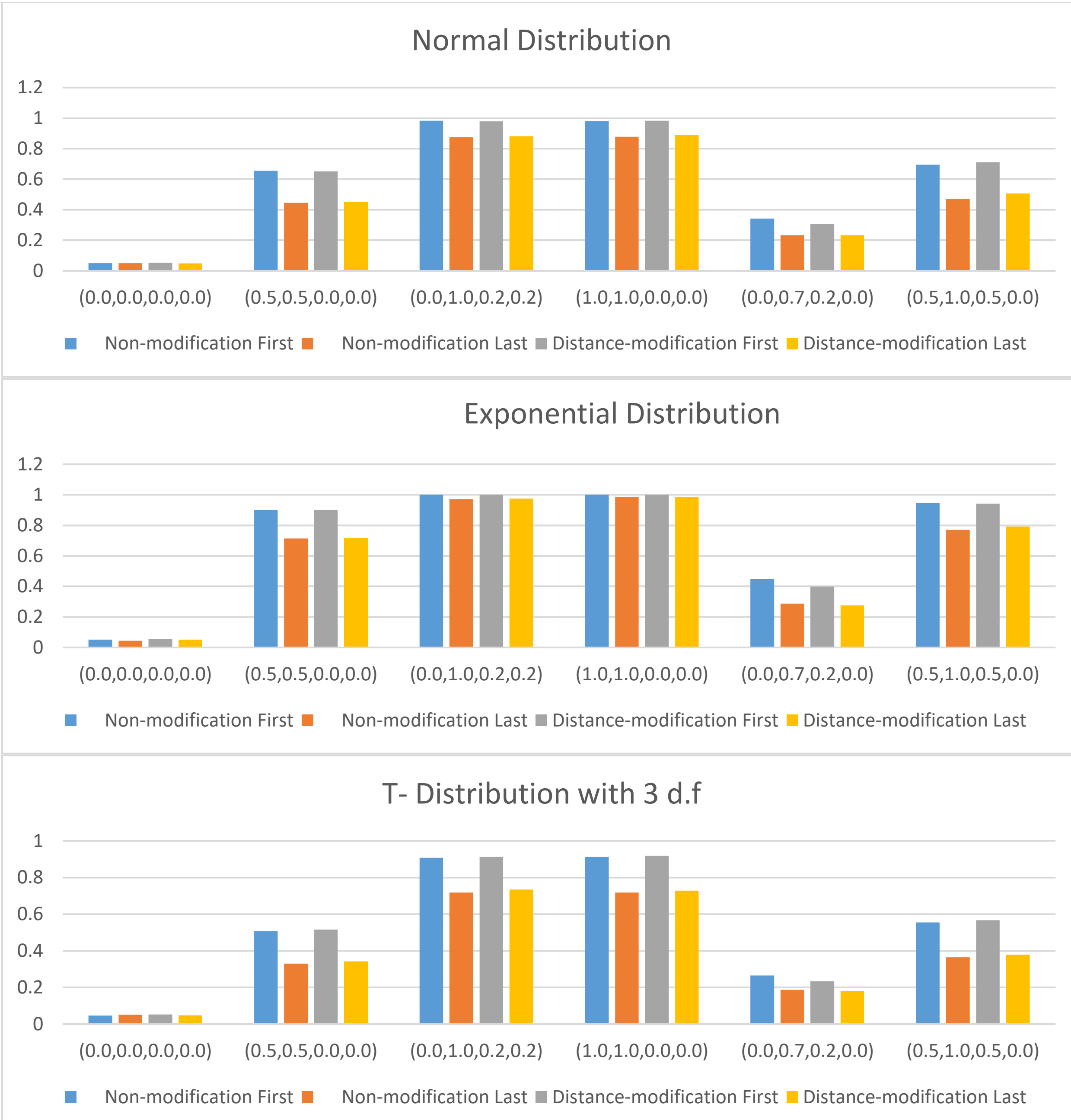

Figure 34. Treatments=4, IBD = 12, CRD=12, RCBD=12, Peak=3, p=0.2

Table 154. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=2, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0514	0.0464	0.0446	0.0480
	1	1	0.5	0.5	0.5	0.2260	0.1850	**0.2460**	0.2172
	0.5	1	1	1	0.7	0.2282	0.1808	**0.2368**	0.1886
	0.7	1	0.7	0.7	0.5	0.2946	0.2456	**0.3104**	0.2590
	1	1	0.5	0.5	0.2	0.4428	0.3424	**0.4678**	0.3774
	0.75	1	0.75	0.5	0.25	0.5452	0.4150	**0.5584**	0.4772
	1	1	1	1	0	**0.5922**	0.4322	0.5848	0.4844
	1	1	0.5	0.2	0	**0.6918**	0.5232	0.6888	0.5838
	0	1.6	0.8	0.4	0.2	0.9822	0.9324	0.9834	0.9514

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0554	0.0536	0.0502	0.0464
	1	1	0.5	0.5	0.5	0.3682	0.2726	**0.4010**	0.3102
	0.5	1	1	1	0.7	0.3768	0.2950	**0.3836**	0.3100
	0.7	1	0.7	0.7	0.5	0.5448	0.4172	**0.5558**	0.4574
	1	1	0.5	0.5	0.2	0.7088	0.5600	**0.7292**	0.6002
	0.75	1	0.75	0.5	0.25	0.8340	0.6920	**0.8446**	0.7368
	1	1	1	1	0	0.7180	0.5688	**0.7446**	0.6232
	1	1	0.5	0.2	0	0.9040	0.7738	**0.9214**	0.8248
	0	1.6	0.8	0.4	0.2	0.9980	0.9946	**0.9992**	0.9960

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0488	0.0532	0.0480	0.0528
	1	1	0.5	0.5	0.5	0.1884	0.1602	**0.1946**	0.1720
	0.5	1	1	1	0.7	0.1874	0.1538	**0.1898**	0.1464
	0.7	1	0.7	0.7	0.5	0.2380	0.1846	**0.2396**	0.1986
	1	1	0.5	0.5	0.2	0.3310	0.2658	**0.3604**	0.2864
	0.75	1	0.75	0.5	0.25	0.4066	0.3244	**0.4332**	0.3540
	1	1	1	1	0	**0.4510**	0.3208	0.4414	0.3716
	1	1	0.5	0.2	0	0.5012	0.3944	**0.5342**	0.4426
	0	1.6	0.8	0.4	0.2	**0.9204**	0.8068	0.9158	0.8340

6.4.3. Five Treatments at Peak Two

In the case of the Mixed design of IBD, CRD, and RCBD powers of the standardized first were slightly higher than the standardized last versions of the tests, regardless of the probabilities 0.1,0.2, and 0.4 associated with the IBD portion. It is important to note that results for five treatments at peak two were consistent when the probabilities associated with the IBD portion were 0.1 and 0.4. Again, the distance modification first and the non-modification first were consistent in the following cases:

If the distance between parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as (1.0,1.0,0.0,0.0), and the non-modification first performed better. Where two population parameters are equal to the peak and the other additional parameters are different from each other, such as (0.5,0.5,0.0,0.0), the distance modification first performed slightly better than the non-modification first.

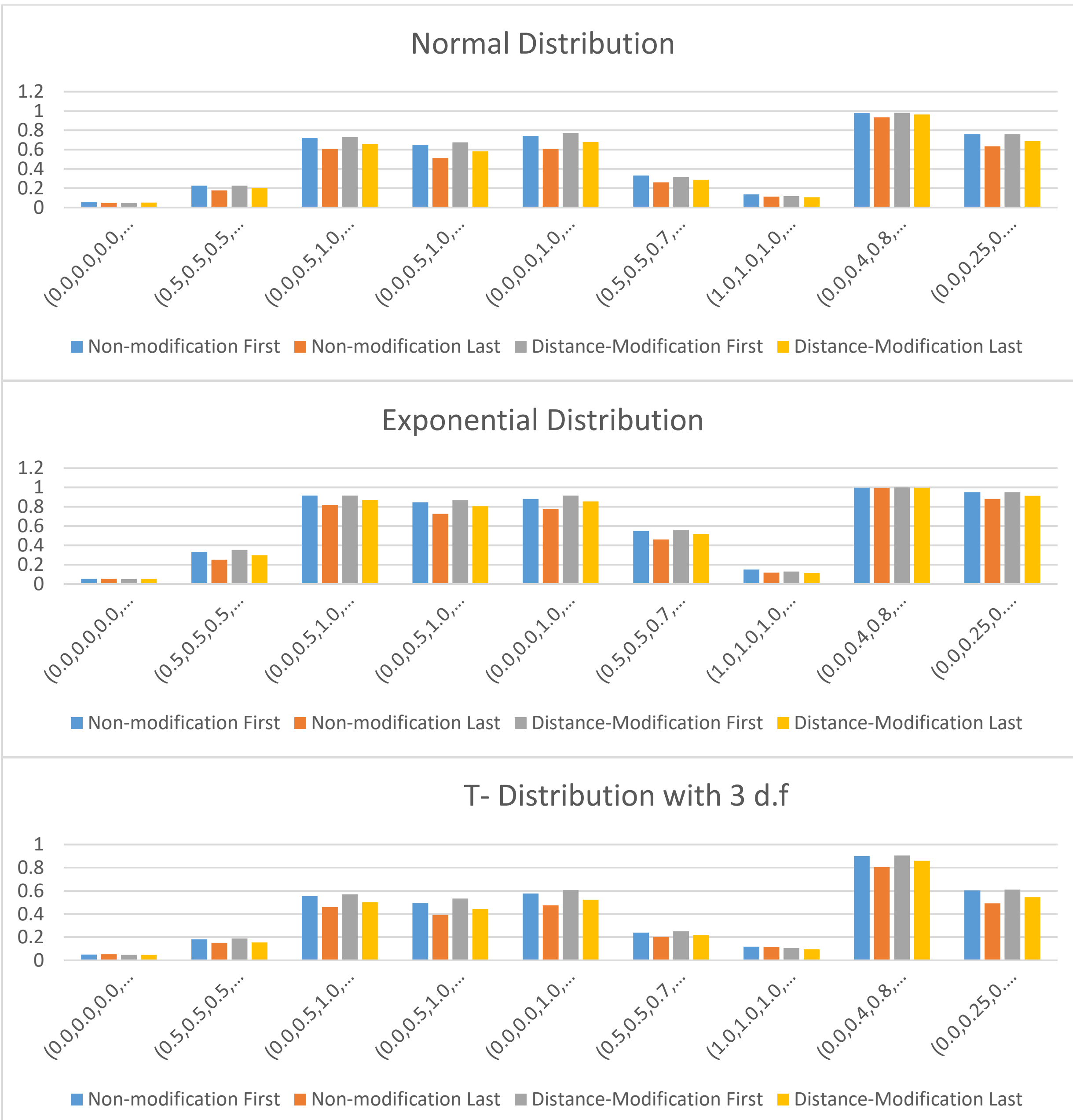

Figure 35. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=2, p=0.1

Table 155. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=2, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0512	0.0520	0.0528	0.0502
	1	1	0.5	0.5	0.5	0.2158	0.1794	**0.2322**	0.2052
	0.5	1	1	1	0.7	0.2054	0.1756	**0.2244**	0.1792
	0.7	1	0.7	0.7	0.5	0.2872	0.2314	**0.2896**	0.2558
	1	1	0.5	0.5	0.2	0.4074	0.3310	**0.4314**	0.3626
	0.75	1	0.75	0.5	0.25	0.4874	0.3936	**0.5054**	0.4472
	1	1	1	1	0	**0.5358**	0.4002	0.5202	0.4856
	1	1	0.5	0.2	0	0.6052	0.4990	**0.6246**	0.5636
	0	1.6	0.8	0.4	0.2	0.9710	0.9246	0.9708	0.9456

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0516	0.0470	0.0552	0.0440
	1	1	0.5	0.5	0.5	0.3298	0.2526	**0.3686**	0.2980
	0.5	1	1	1	0.7	0.3470	0.2748	**0.3598**	0.2892
	0.7	1	0.7	0.7	0.5	0.4912	0.3850	**0.4984**	0.4298
	1	1	0.5	0.5	0.2	0.6424	0.5180	**0.6676**	0.5838
	0.75	1	0.75	0.5	0.25	0.7746	0.6560	**0.7876**	0.7118
	1	1	1	1	0	0.6592	0.5468	**0.6890**	0.6064
	1	1	0.5	0.2	0	0.8396	0.7330	**0.8550**	0.7990
	0	1.6	0.8	0.4	0.2	0.9994	0.9880	0.9972	0.9936

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0548	0.0490	0.0516	0.0504
	1	1	0.5	0.5	0.5	0.1688	0.1556	**0.1776**	0.1598
	0.5	1	1	1	0.7	0.1626	0.1386	**0.1688**	0.1502
	0.7	1	0.7	0.7	0.5	0.2126	0.1776	**0.2408**	0.1958
	1	1	0.5	0.5	0.2	0.3130	0.2546	**0.3322**	0.2730
	0.75	1	0.75	0.5	0.25	0.3740	0.2988	0.3796	0.3340
	1	1	1	1	0	0.3776	0.3110	**0.4132**	0.3548
	1	1	0.5	0.2	0	0.4580	0.3688	**0.4770**	0.4232
	0	1.6	0.8	0.4	0.2	**0.8804**	0.7848	0.8726	0.8186

With the Mixed design of IBD, CR D, and RCBD, the powers of the standardized first were slightly higher than the standardized last versions of the test, regardless of the probabilities 0.1, 0.2, and 0.4 associated with the IBD portion. Moreover, the distance modification first and the non-modification first were consistent in the following cases:

If the distance between parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as $(1.0,1.0,1.0,1.0,0.0)$ and $(1.0, 1.0, 0.5, 0.2, 0.0)$, the non-modification first performed better. Where two population parameters are equal to the peak and the other additional parameters are different from each other, such as (0.5,1.0,1.0,1.0,0.7), the distance modification first performed slightly better than non-modification first. Table 155 above (Refer to Tables E.1. to E.18. in the Appendix) and Figure 36 below shows the probabilities of all the three modification tests.

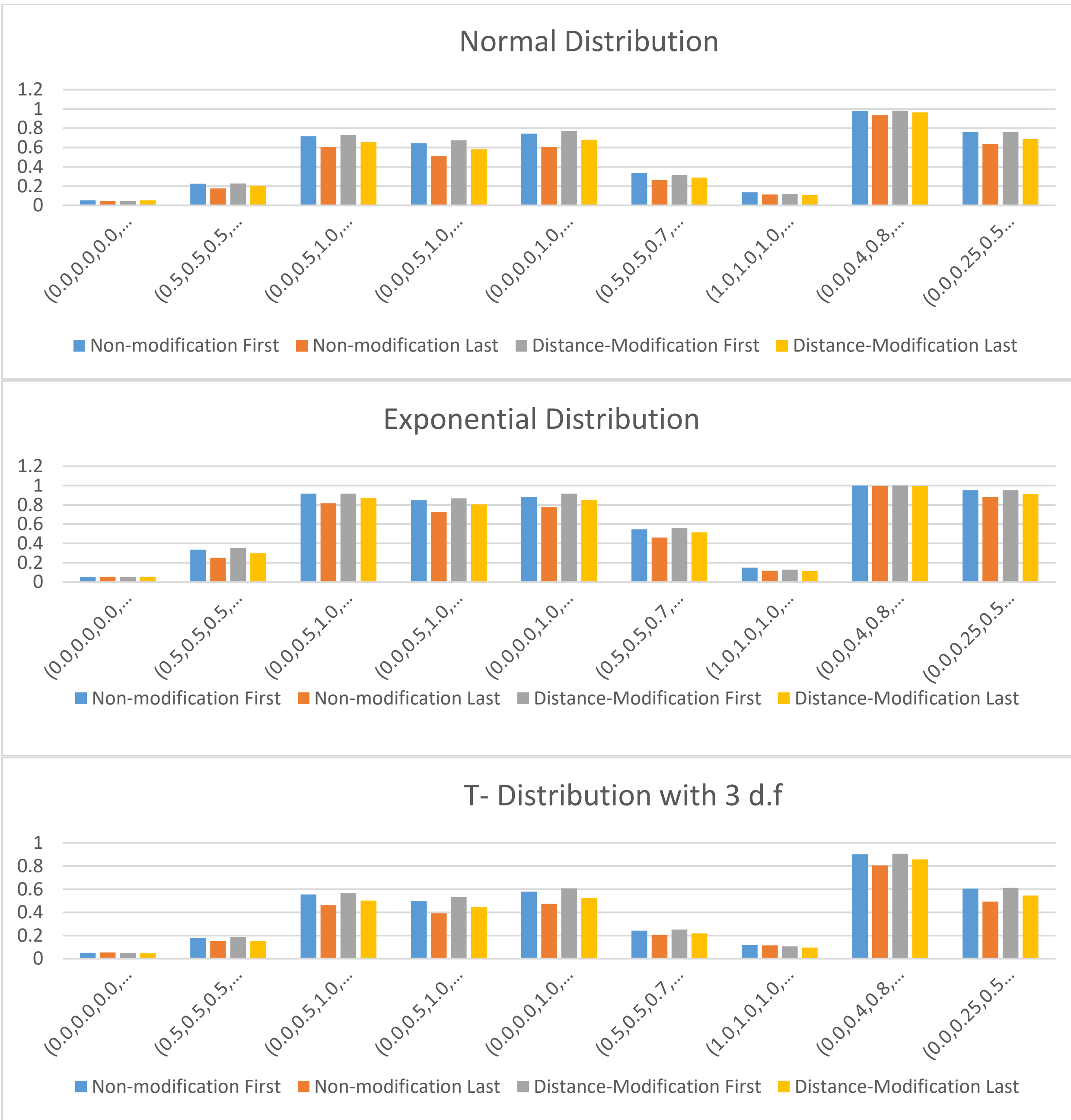

Figure 36. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=2, p=0.4

Table 156. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=3, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0540	0.0470	0.0506	0.0568
	0	0.5	1	1	0.7	0.2972	0.3218	**0.5386**	0.4446
	0.7	0.7	1	0.5	0.5	0.2524	0.2154	**0.2980**	0.2476
	0	0.5	1	0.5	0	0.7594	0.6820	**0.8626**	0.7620
Normal	0	1	1	0	0	**0.8858**	0.7440	0.8558	0.7598
	1	1	1	0.5	0	**0.4828**	0.3160	0.3718	0.3132
	1	1	1	1	0	**0.4310**	0.2918	0.3568	0.2768

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0402	0.0552	0.0498	0.0468
	0	0.5	1	1	0.7	0.4690	0.5206	**0.8170**	0.681
	0.7	0.7	1	0.5	0.5	0.4066	0.3380	**0.4922**	0.3884
Exponential	0	0.5	1	0.5	0	0.9436	0.9090	**0.9866**	0.9558
	0	1	1	0	0	**0.9804**	0.8978	0.9716	0.9094
	1	1	1	0.5	0	**0.7012**	0.4788	0.5430	0.438
	1	1	1	1	0	**0.5606**	0.3996	0.4644	0.3838

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0488	0.0516	0.0504	0.0508
	0	0.5	1	1	0.7	0.2256	0.2658	**0.4218**	0.3386
T	0.7	0.7	1	0.5	0.5	0.1928	0.1736	**0.2300**	0.185
distribution	0	0.5	1	0.5	0	0.5996	0.5194	**0.7208**	0.6054
with 3	0	1	1	0	0	**0.7274**	0.5636	0.6974	0.5814
degrees of	1	1	1	0.5	0	**0.3656**	0.2554	0.2848	0.2406
freedom	1	1	1	1	0	**0.3224**	0.2298	0.2686	0.2158

6.4.4. Five Treatments at Peak Three

With the Mixed design of IBD, CRD, and RCBD powers of the standardized first were slightly higher than the standardized last versions of the test, regardless of the probabilities 0.1, 0.2, and 0.4 associated with the IBD. Again, the results for five treatments at peak three were consistent when the probabilities associated with the IBD portion were 0.1 and 0.4. The distance squared modification first and the non-modification first had the highest values of the estimated powers test statistics in the following cases:

If the distance between parameters before the peak and the peak is less than or equal to the distance between parameters after the peak and the peak, such as $(0.0,1.0,1.0,0.0,0.0)$ and

$(1.0, 1.0, 1.0, 0.5, 0.0)$, the non-modification first performed better. Where two population parameters are different from the peak but are equal and the other additional parameters are different from the peak, but equal such as $(0.7,0.7,1.0,0.5,0.5)$, the distance modification first performed slightly better than the non-modification first. Table 156 above (Refer to Tables E.28 to E.36. in the Appendix) and Figure 37 below shows the probabilities of all the three modification tests.

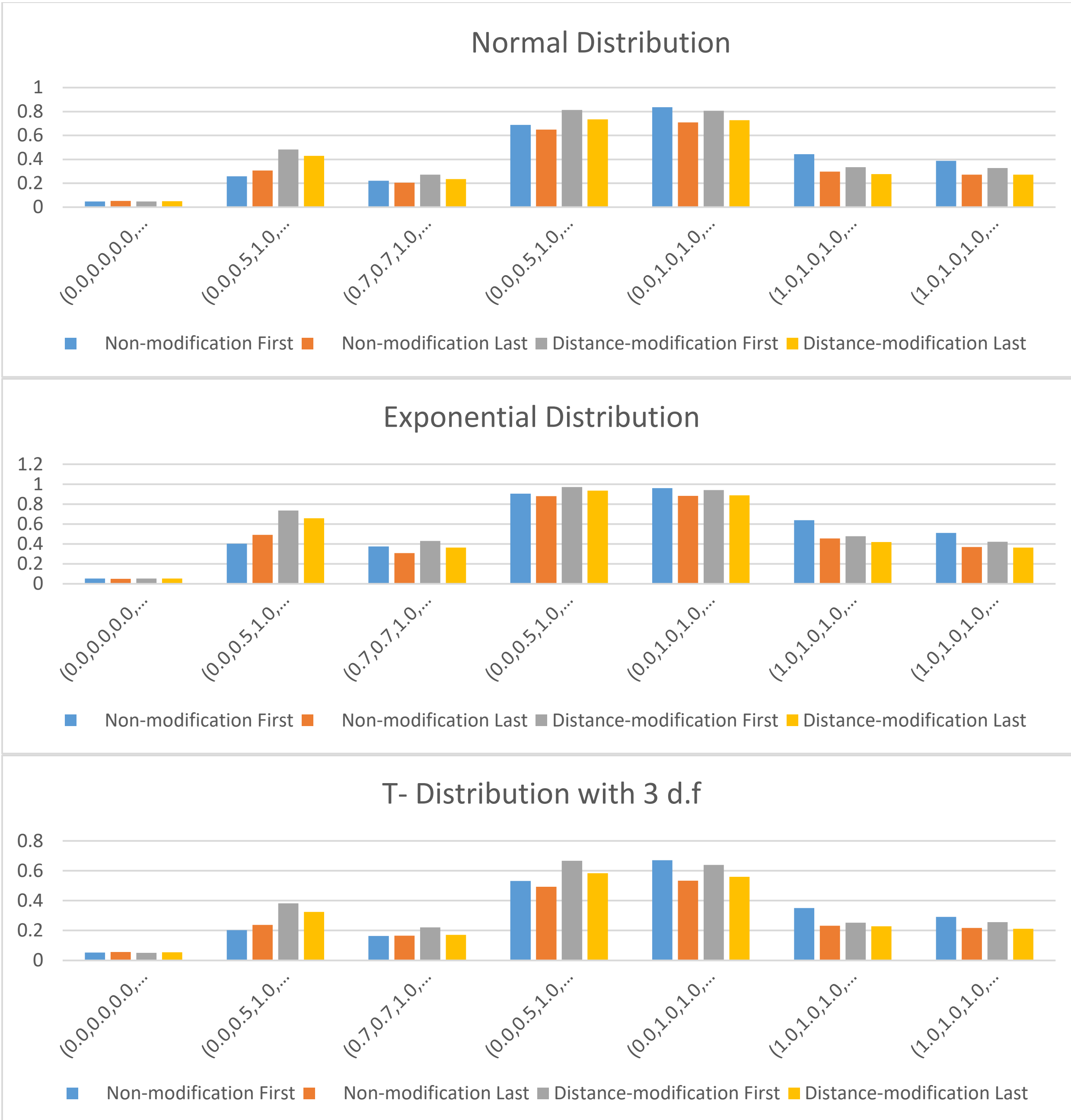

Figure 37. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=3, p=0.1

Table 157. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=3, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0482	0.0522	0.0478	0.0488
	0	0.5	1	1	0.7	0.2576	0.3068	**0.4812**	0.4294
	0.7	0.7	1	0.5	0.5	0.2206	0.2056	**0.2712**	0.2344
	0	0.5	1	0.5	0	0.6892	0.6480	**0.8122**	0.7346
Normal	0	1	1	0	0	**0.8356**	0.7100	0.8062	0.7276
	1	1	1	0.5	0	**0.4430**	0.2970	0.3354	0.2774
	1	1	1	1	0	**0.3884**	0.2730	0.3264	0.2730

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0540	0.0488	0.0536	0.0532
	0	0.5	1	1	0.7	0.4014	0.4908	**0.7364**	0.6570
	0.7	0.7	1	0.5	0.5	0.3754	0.3070	**0.4312**	0.3624
Exponential	0	0.5	1	0.5	0	0.9040	0.8798	**0.9716**	0.9362
	0	1	1	0	0	**0.9616**	0.8814	0.9408	0.8868
	1	1	1	0.5	0	**0.6390**	0.4556	0.4770	0.4186
	1	1	1	1	0	**0.5106**	0.3690	0.4208	0.3646

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0526	0.0558	0.0514	0.0538
	0	0.5	1	1	0.7	0.2024	0.2372	**0.3824**	0.3242
T	0.7	0.7	1	0.5	0.5	0.1634	0.1662	**0.2218**	0.1716
distribution	0	0.5	1	0.5	0	0.5322	0.4922	**0.6672**	0.5834
with 3	0	1	1	0	0	**0.6702**	0.5338	0.6400	0.5590
degrees of	1	1	1	0.5	0	**0.3502**	0.2314	0.2524	0.2290
freedom	1	1	1	1	0	**0.2910**	0.2176	0.2562	0.2128

With the fourth mixed design of IBD, CRD, and RCBD, powers of the standardized first were slightly higher than the standardized last versions of the test, regardless of the probabilities 0.1 0.2, and 0.4 associated with the IBD. The results from the distance squared modification first and the non-modification first provide the highest values of the estimated powers test statistics in the following cases:

If the distance between parameters before the peak and the peak is less than or equal to the distance between parameters after the peak and the peak, such as $(0.0,1.0,1.0,0.0,0.0)$ and $(1.0, 1.0, 1.0, 0.5, 0.0)$, the non-modification first performed better. Where the distance between the population parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as (0.7, 0.7, 1.0, 0.5, 0.5), the distance modification first performed slightly better than the non-modification first. Table 157 above (Refer to Tables E.28 to E.45. in the Appendix) and Figure 38 below show the probabilities of all the three modification tests.

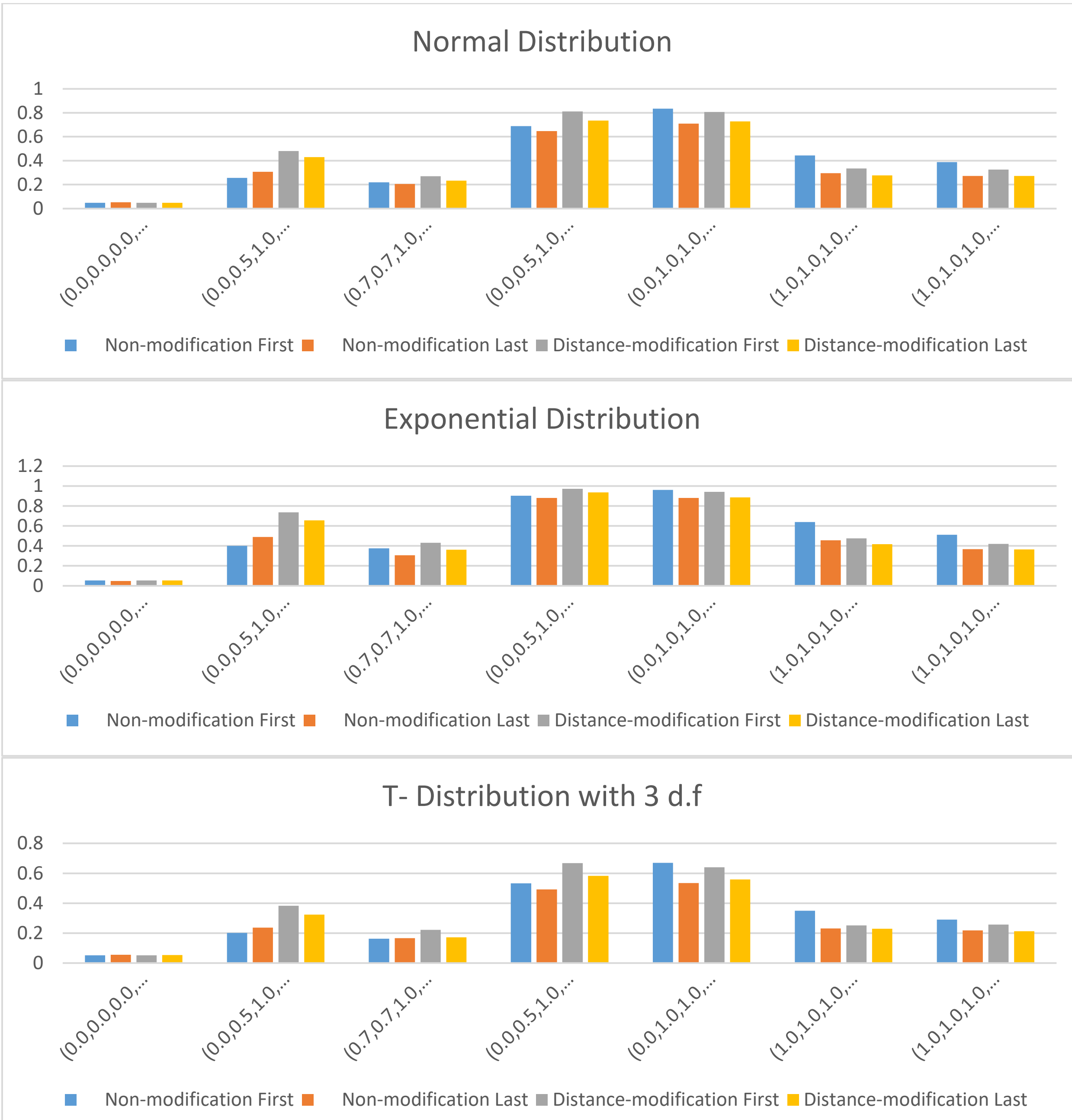

Figure 38. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=3, p=0.4

Table 158. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=4, p=0.2

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0520	0.0516	0.0518	0.0494
	0.5	0.5	0.5	1	1	0.2326	0.1846	**0.2394**	0.2114
	0	0.5	1	1	0.7	0.7542	0.6322	**0.7614**	0.6742
	0	0.5	1	1	1	0.6884	0.5386	**0.7130**	0.6128
	0	0	1	1	1	0.7772	0.6300	**0.8056**	0.7082
	0.5	0.5	0.7	1	0.7	0.3566	0.2598	**0.3576**	0.2936
	1	1	1	1	0	**0.1326**	0.1186	0.1112	0.1120
	0	0.4	0.8	1.6	0.4	0.9882	0.9520	**0.9898**	0.9666
	0	0.25	0.5	1	0.25	0.7968	0.6632	**0.8064**	0.7008
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0484	0.0532	0.0496	0.0518
	0.5	0.5	0.5	1	1	0.3556	0.2638	**0.3668**	0.3192
	0	0.5	1	1	0.7	0.9354	0.8376	**0.9326**	0.8834
	0	0.5	1	1	1	0.8812	0.7472	**0.8976**	0.8162
	0	0	1	1	1	0.9160	0.8036	**0.9300**	0.8538
	0.5	0.5	0.7	1	0.7	0.5942	0.4644	**0.6016**	0.4988
	1	1	1	1	0	**0.1508**	0.1240	0.1270	0.1176
	0	0.4	0.8	1.6	0.4	0.9880	0.9936	**0.9996**	0.9972
	0	0.25	0.5	1	0.25	0.9754	0.9022	**0.9632**	0.9286
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0510	0.0464	0.0488	0.0538
	0.5	0.5	0.5	1	1	0.1790	0.1476	**0.1896**	0.1746
	0	0.5	1	1	0.7	0.5960	0.4752	**0.6080**	0.5256
	0	0.5	1	1	1	0.5322	0.4118	**0.5424**	0.4620
	0	0	1	1	1	0.6226	0.4632	**0.6420**	0.5366
	0.5	0.5	0.7	1	0.7	0.2558	0.2218	**0.2664**	0.2284
	1	1	1	1	0	**0.1280**	0.1204	0.1064	0.1012
	0	0.4	0.8	1.6	0.4	0.9242	0.8212	**0.9272**	0.8566
	0	0.25	0.5	1	0.25	0.6240	0.5020	**0.6258**	0.5466

6.4.5. Five Treatments at Peak Four

In the case of five treatments at peak four, the mixed design of IBD, CRD, and RCBD provide estimated powers of the standardized first slightly higher than the standardized last versions of the test regardless of the probabilities 0.1 0.2, and 0.4 associated with the IBD portion. It is important to note that the results for five treatments at peak four were consistent when the probabilities associated with the IBD portions were 0.2 and 0.4. The distance modification first in most cases had estimated powers slightly higher than the non-modification first, except where the distance between the parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as (1.0,1.0,1.0,1.0,0.0), when the non-modification first does better. Refer to Tables F.18. to F.45. in the Appendix.

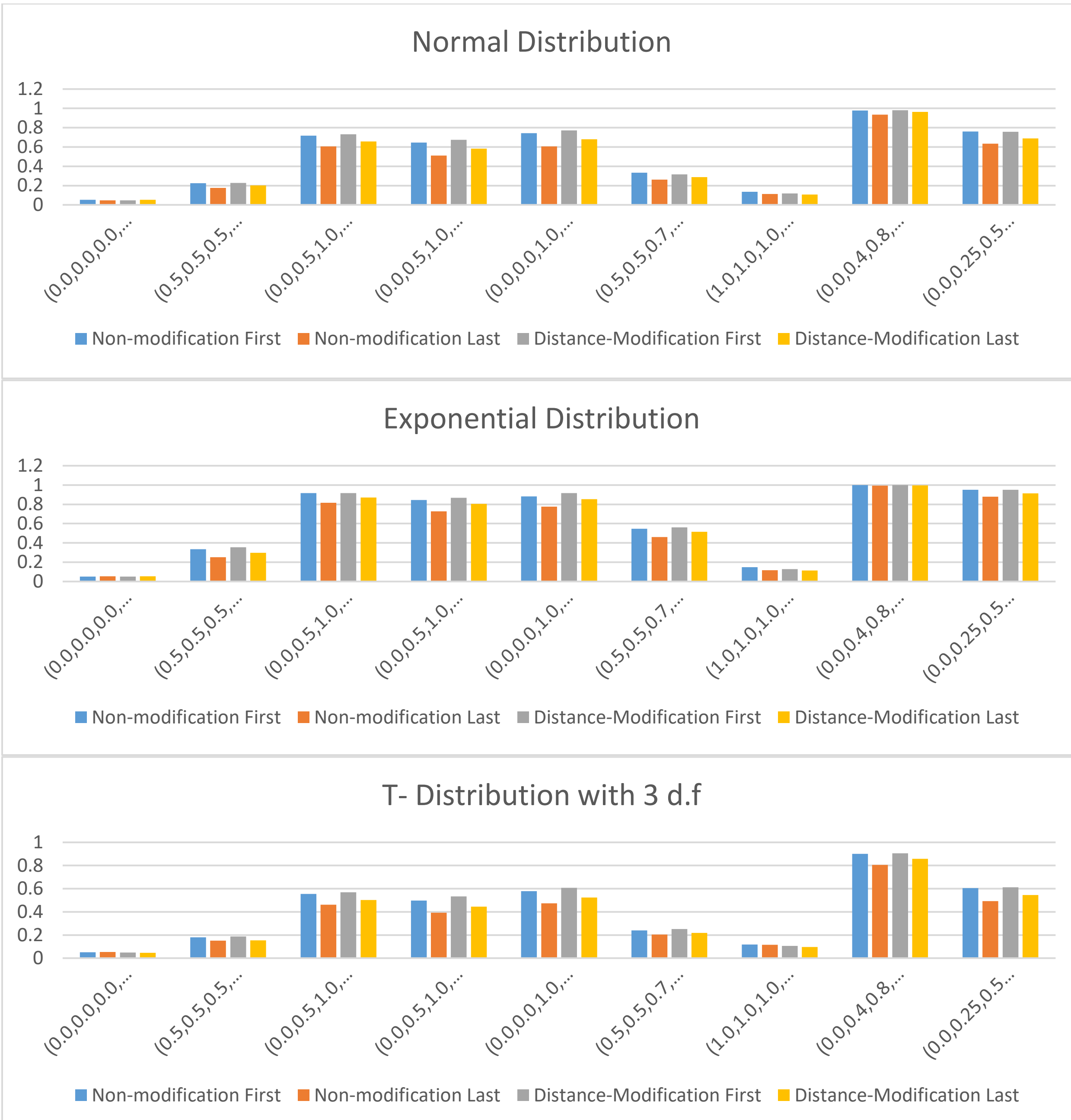

Figure 39. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=4, p=0.2

Table 159. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=4, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0540	0.0474	0.0478	0.0520
	0.5	0.5	0.5	1	1	0.2252	0.1772	**0.2264**	0.2024
	0	0.5	1	1	0.7	0.7178	0.6044	**0.7318**	0.6578
	0	0.5	1	1	1	0.6450	0.5118	**0.6750**	0.5822
	0	0	1	1	1	0.7418	0.6050	**0.7710**	0.6792
	0.5	0.5	0.7	1	0.7	**0.3324**	0.2616	0.3166	0.2884
	1	1	1	1	0	**0.1370**	0.1138	0.1178	0.1082
	0	0.4	0.8	1.6	0.4	0.9778	0.9358	**0.9800**	0.9624
	0	0.25	0.5	1	0.25	0.7510	0.6356	**0.7588**	0.6896

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0518	0.0542	0.0502	0.0532
	0.5	0.5	0.5	1	1	0.3338	0.2518	**0.3540**	0.2974
	0	0.5	1	1	0.7	0.9152	0.8154	**0.9162**	0.8694
	0	0.5	1	1	1	0.8462	0.7272	**0.8680**	0.8036
	0	0	1	1	1	0.8816	0.7750	**0.9154**	0.8532
	0.5	0.5	0.7	1	0.7	0.5470	0.4612	**0.5598**	0.5152
	1	1	1	1	0	**0.1482**	0.1184	0.1296	0.1142
	0	0.4	0.8	1.6	0.4	0.9982	0.9950	**0.9988**	0.9966
	0	0.25	0.5	1	0.25	0.94512	0.8806	**0.9494**	0.9138

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0506	0.0526	0.0484	0.0470
	0.5	0.5	0.5	1	1	0.1806	0.1524	**0.1878**	0.1540
	0	0.5	1	1	0.7	0.5542	0.4606	**0.5702**	0.5018
	0	0.5	1	1	1	0.4968	0.3934	**0.5340**	0.4440
	0	0	1	1	1	0.5776	0.4742	**0.6062**	0.5240
	0.5	0.5	0.7	1	0.7	0.2408	0.2030	**0.2508**	0.2172
	1	1	1	1	0	**0.1288**	0.1150	0.1050	0.1052
	0	0.4	0.8	1.6	0.4	0.9004	0.8044	**0.9060**	0.8586
	0	0.25	0.5	1	0.25	0.6042	0.4926	**0.6120**	0.5462

With the Mixed design of IBD, CRD, and RCBD, the powers of the standardized first were slightly higher than the standardized last versions of the test regardless of the probabilities 0.1, 0.2, and 0.4 associated with the IBD. The results showed that the distance modification first in most cases had the estimated powers slightly higher than the non-modification first, except where the distance between the parameters before the peak and the peak is less than the distance between parameters after the peak and the peak, such as (1.0,1.0,1.0,1.0,0.0) when the non-modification first does better. Refer to Tables F.18. to F.45. in the Appendix.

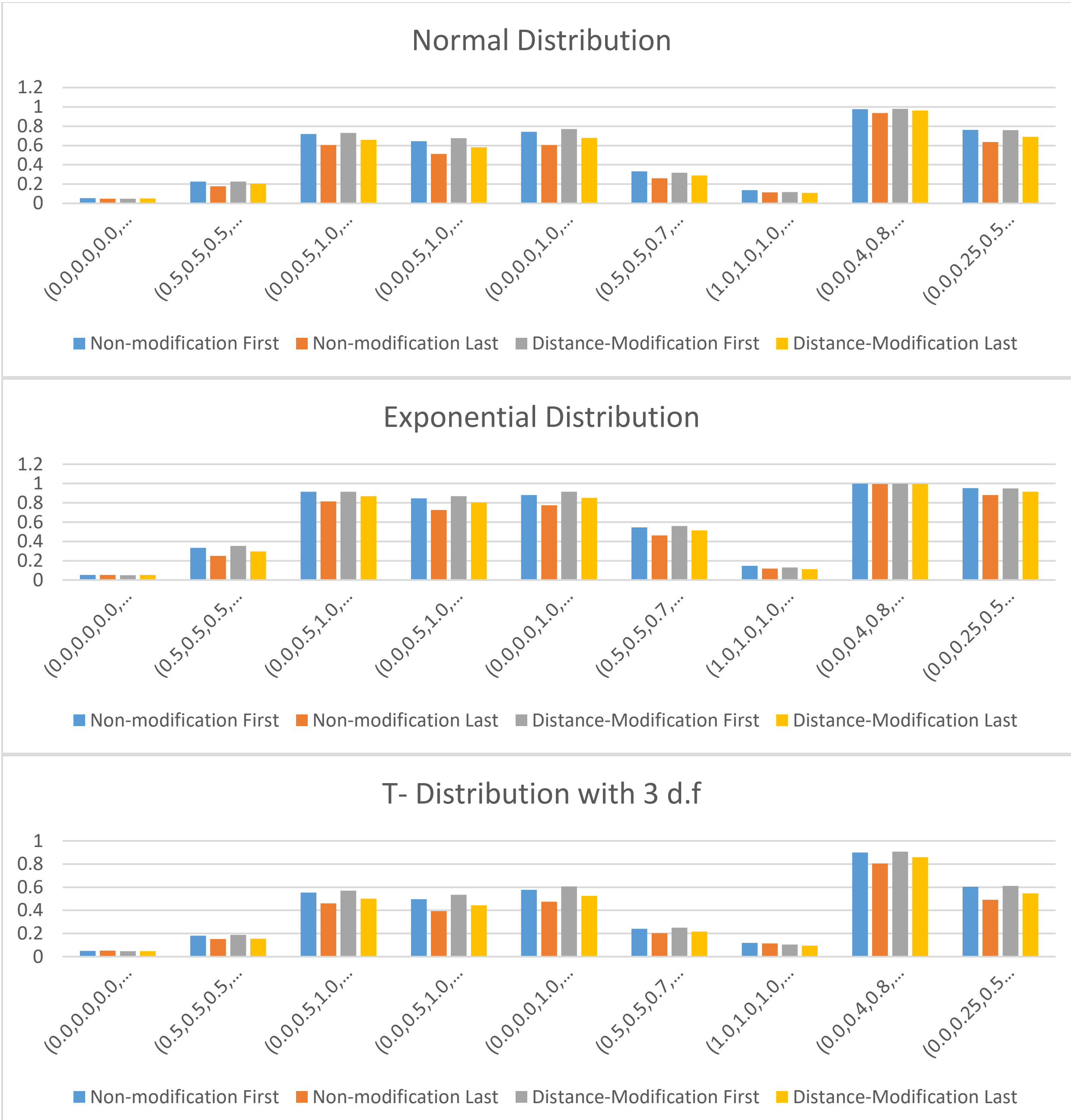

Figure 40. Treatments=5, IBD =5, CRD=5, RCBD=5 Peak=4, $p = 0.4$.

CHAPTER 7. CONCLUSIONS

This study proposed four Mixed Design under the umbrella alternative consisting of Magel and Ndungu (2011) for Balanced Incomplete Block Design, Mack-Wolfe (1981) for Completely Randomized Design and Kim-Kim (1992) for the Randomized Complete Block Design, Magel and Ndungu (2011) for Balanced Incomplete Block Design and Kim-Kim (1992) for the Randomized Complete Block Design; Mack-Wolfe (1981) for Completely Randomized Design and Kim-Kim for Randomized Complete Block Design; and a Mixed Design consisting of Magel and Ndungu (2011) for Incomplete Block Design (IBD), Kim-Kim (1992) for Randomized Complete Block Design (RCBD) and Mack-Wolfe (1981) for Completely Randomized Design.

Three situations were considered for each mixed deisgn: a combination without applying any modification called Non-Modification; a combination by applying Distance-Modification and a combination by applying Distance Squared- Modification.

For each modification test, we propose standardized versions of the test statistics. These are standardizing the sum of the standardized test statistics called standardized first and standardizing the sum of the unstandardized test statistics called standardized last.

A significance level of 0.05 was considered for all the proposed tests based on the asymptotic standard normal distribution of the test statistics under the null hypothesis. When all the population location parameters were zero, the rejection percentages were obtained as a check to see if the tests maintained their significance level at $\alpha = 0.05$. A variety of location parameters configurations were considered. There were variations among the three modification tests which influence the analysis.

7.1. Mixed Design of BIBD, CRD and RCBD Results

With three treatments at peak two, the results showed that regardless of the treatments and peak levels and regardless of the underlying distribution and the proportion of BIBD to CRD and BIBD to RCBD, the standardized first performed better than standardized last. This means that standardizing the sum of the standardized test statistics of Magel-Ndungu (2011), Mack-Wolfe (1981), and Kim-Kim (1992) was better than standardizing the sum of the unstandardized test statistics. Again, the results for the distinct modification tests were the same. The reason was that all the modification tests have equal distance weight of one.

In the case of four treatments at peaks two and three, the results showed that the standardized first versions of the test had estimated powers slightly higher than the standardized last versions of the test regardless of the underlying distribution and the proportion of BIBD to CRD and BIBD to RCBD. Generally, the results among the three distinct modification tests for four treatments at peaks two and three, vary from one configuration to the other and from one distribution to the other. It was, therefore, difficult to indicate which modification test provided the highest values of the estimated powers test statistics.

Similarly, with five treatments at peaks two, three, and four, the results showed that the standardized first versions of the test provided estimated powers slightly higher than the standardized last versions of the test regardless of the underlying distribution and the proportion of BIBD to CRD and BIBD to RCBD. As found in treatments four at peaks two and three, the results for the three modification tests for five treatments at peaks two, three, and four were not different as the results vary from one configuration to the other and from one distribution to the other. Consequently, it was difficult to indicate which modification test had the highest values of the estimated powers test statistics.

7.2. Mixed Design of BIBD and CRD Results

In the case of the mixed design of BIBD and CRD, regardless of the treatments and the peaks levels, the results showed that the standardized first versions of the test had estimated powers slightly higher than the standardized last versions of the test when the number of blocks was greater than the sample sizes. However, when the sample sizes were twice, thrice, or two-thirds of the BIDB portions, the standardized last had better results than standardized first irrespective of the treatments and peaks levels. This shows that standardizing the sum of the unstandardized test statistics of Magel-Ndungu (2011) and Mack-Wolfe (1981 had better results than standardizing the sum of the standardized test statistics.

7.3. Mixed Design of CRD and RCBD Results

With the mixed design of BIBD and CRD, regardless of the treatments and the peaks levels, the results showed that the standardized first versions of the test had estimated powers slightly higher than the standardized last versions of the test. This means that standardizing the sum of the standardized test statistics of Mack-Wolfe (1981) and Kim-Kim (1992) was better than standardizing the sum of the unstandardized.

The analysis, however, indicated that with four treatment at peak three, the mixed design of CRD and RCBD reported the highest powers when the proportion of RCD to RCBD was 18:18. The results for the modification tests, in general, vary from one configuration to the other and from one distribution to the other due to the variations in the three modification tests. It was, therefore, difficult to indicate which modification test provided the highest values of the estimated powers test statistics.

7.4. Mixed Design of IBD and CRD Results

The results under four treatments with peak at three showed that the mixed design of IBD, CRD, and RCBD reported the highest powers when the mixed design had a ratio of 1:1:1(18:18:18) with a probability of 0.2 assciated with the IBD portion. In the case of the five treatments at peak four, the results showed that the standardized first was more powerful than standardized last versions of the test regardless of the probabilitie 0.1, 0.2, and 0.4 associated with the IBD portion. It is important to note that with the distinct modification tests for five treatments at peaks four, three, and two, results were consistent when the probabilities associated with the IBD portions were (0.2 and 0.4) , (0.1 and 0.4), and (0.1 and 0.4) respectively.

Overall, in all the mixed designs except the mixed design of BIDB and CRD, it is recommended to use the standardized first since it had the highest powers for the test statistics. In the case of the mixed design of BIDB and CRD, it is recommended to use the standardized last when the sample size is n times higher than the number of blocks b in the BIBD portion since it provided the highest powers test statistics, where $n \geq 2$.

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APPENDIX A. MIXED DESIGN OF BIBD, CRD AND RCBD FOR TREATMENTS 3 AT PEAK 2; MIXED DESIGN OF BIBD AND CRD FOR TREATMENTS 3 AT PEAK 2; MIXED DESIGN OF CRD AND RCBD FOR TREATMENTS 3 AT PEAK 2

Table A.1. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Three Treatments at Peak 2. Treatments=3, BIBD =6, CRD=12 Peak=2 (One Missing Observation).

Distribution	Location			Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	First	Last	First	Last	First	Last
Normal	0	0	0	0.0464	0.0468	0.0464	0.0468	0.0464	0.0468
	0	0.5	0.5	0.1550	**0.1650**	0.1550	**0.1650**	0.1550	**0.1650**
	0.5	0.5	0	0.1570	**0.1652**	0.1570	**0.1652**	0.1570	**0.1652**
	0.2	0.6	0.2	0.2838	**0.2880**	0.2838	**0.2880**	0.2838	**0.2880**
	0	0.5	0	0.3804	**0.3908**	0.3804	**0.3908**	0.3804	**0.3908**
	0	1	0.6	0.5802	**0.5912**	0.5802	**0.5912**	0.5802	**0.5912**
	0.6	1	0	0.5736	**0.5910**	0.5736	**0.5910**	0.5736	**0.5910**
Exponential	0	0	0	0.0516	0.0518	0.0516	0.0518	0.0516	0.0518
	0	0.5	0.5	0.2434	**0.2442**	0.2434	**0.2442**	0.2434	**0.2442**
	0.5	0.5	0	0.2404	**0.2442**	0.2404	**0.2442**	0.2404	**0.2442**
	0.2	0.6	0.2	0.4906	**0.5094**	0.4906	**0.5094**	0.4906	**0.5094**
	0	0.5	0	0.6226	**0.6486**	0.6226	**0.6486**	0.6226	**0.6486**
	0	1	0.6	0.7914	**0.8244**	0.7914	**0.8244**	0.7914	**0.8244**
	0.6	1	0	0.7892	**0.8170**	0.7892	**0.8170**	0.7892	**0.8170**
T with 3 degrees of freedom	0	0	0	0.0502	0.0530	0.0502	0.0530	0.0502	0.0530
	0	0.5	0.5	0.1364	**0.1380**	0.1364	**0.1380**	0.1364	**0.1380**
	0.5	0.5	0	0.1378	**0.1416**	0.1378	**0.1416**	0.1378	**0.1416**
	0.2	0.6	0.2	0.2292	**0.2320**	0.2292	**0.2320**	0.2292	**0.2320**
	0	0.5	0	0.2980	**0.3068**	0.2980	**0.3068**	0.2980	**0.3068**
	0	1	0.6	0.4360	**0.4448**	0.4360	**0.4448**	0.4360	**0.4448**
	0.6	1	0	0.4366	**0.4470**	0.4366	**0.4470**	0.4366	**0.4470**

Selected results for treatments 3 at Peak 2 for a Mixed Design of BIBD and CRD. With the sample size being twice the number of blocks, standardized last performed better in all distinct tests than standardized last. Results as far as how the distinct tests performed relative to one another were similar for all distributions. The same reason as before could be a factor.

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APPENDIX B. MIXED DESIGN OF BIBD, CRD AND RCBD AT PEAKS 2 AND 3; MIXED DESIGN OF BIBD AND CRD AT PEAKS 2 AND 3; MIXED DESIGN OF CRD AND RCBD FOR TREATMENTS 4 AT PEAKS 2 AND 3

Table B.1. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, BIBD = 12, CRD=6, RCBD = 12 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
Normal	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
	0	0	0	0	0.0546	0.0442	0.0464	0.0496	0.0538	0.0518
	0.5	0.5	0	0	0.2430	0.1928	0.2650	0.2044	**0.2658**	0.2260
	0	1	0.2	0.2	**0.8932**	0.7378	0.8786	0.7546	0.8670	0.7810
	1	1	0	0	0.6878	0.5022	0.7222	0.5712	**0.7528**	0.6538
	0	0.7	0.2	0	**0.7762**	0.5672	0.7570	0.6202	0.7402	0.6586
	0.5	1	0.5	0	0.8734	0.7060	**0.8884**	0.7482	0.8824	0.8020
	0	0	0	0	0.0532	0.0464	0.0500	0.0530	0.0504	0.0504
	0.5	0.5	0	0	0.4756	0.3252	0.5152	0.3656	**0.5242**	0.4390
	0	1	0.2	0.2	0.9872	0.9264	**0.9880**	0.9420	0.9836	0.9598
Exponential	1	1	0	0	0.8476	0.6574	0.8664	0.7214	**0.8940**	0.8004
	0	0.7	0.2	0	0.9590	0.8536	**0.9650**	0.8708	0.9508	0.8968
	0.5	1	0.5	0	0.9892	0.9292	0.9888	0.9502	**0.9896**	0.9642
	0	0	0	0	0.0466	0.0470	0.0468	0.0520	0.0464	0.0502
	0.5	0.5	0	0	0.2330	0.1810	0.2360	0.1984	**0.2594**	0.2180
T with 3	0	1	0.2	0.2	**0.7576**	0.5652	0.7388	0.6068	0.7158	0.6380
degrees	1	1	0	0	0.5328	0.3702	0.5760	0.4428	**0.5932**	0.4946
of freedom	0	0.7	0.2	0	**0.6050**	0.4404	0.5818	0.4618	0.5914	0.5074
	0.5	1	0.5	0	0.7396	0.5510	**0.7442**	0.6080	0.7384	0.6566

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD, RCBD and CRD). Results show that with a combination 12 BIBD, 6 CRD and 12 RCBD, Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.2. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, BIBD = 18, CRD=6, RCBD = 18 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0472	0.0490	0.0524	0.0518	0.0480	0.0452
	0.5	0.5	0	0	0.3624	0.2642	0.3866	0.2782	**0.3882**	0.3412
	0	1	0.2	0.2	**0.9536**	0.8160	0.9524	0.8544	0.9406	0.8986
	1	1	0	0	0.7990	0.6166	0.8336	0.6942	**0.8596**	0.7912
	0	0.7	0.2	0	**0.8596**	0.6746	**0.8540**	0.7222	0.8440	0.7662
	0.5	1	0.5	0	0.9484	0.7936	**0.9522**	0.8488	0.8988	0.8968
Exponential	0	0	0	0	0.0506	0.0512	0.0492	0.0510	0.0510	0.0546
	0.5	0.5	0	0	0.5956	0.4060	0.6248	0.4724	**0.6434**	0.5600
	0	1	0.2	0.2	**0.9990**	0.9698	0.9982	0.9836	0.9978	0.9896
	1	1	0	0	0.9244	0.7630	0.9406	0.8280	**0.9554**	0.9078
	0	0.7	0.2	0	0.9924	0.9228	0.9890	0.9428	0.9856	0.9638
	0.5	1	0.5	0	0.9984	0.9698	0.9980	0.9814	**0.9986**	0.9928
T with 3 degrees of freedom	0	0	0	0	0.0480	0.0560	0.0482	0.0528	0.0554	0.0552
	0.5	0.5	0	0	0.2778	0.2106	0.2922	0.2350	**0.2988**	0.2738
	0	1	0.2	0.2	**0.8464**	0.6670	0.8446	0.7036	0.8304	0.7400
	1	1	0	0	0.6538	0.4692	0.6802	0.5226	**0.6902**	0.6242
	0	0.7	0.2	0	0.7120	0.5286	**0.7126**	0.5656	0.6900	0.6146
	0.5	1	0.5	0	0.8326	0.6484	0.8422	0.7046	**0.8456**	0.7614

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD, RCBD and CRD). Results show that with a combination 18 BIBD, 6 CRD and 18 RCBD, Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.3. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, BIBD = 6, CRD=12, RCBD = 6 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0520	0.0514	0.0500	0.0486	0.0514	0.0538
	0.5	0.5	0	0	0.2780	0.2388	0.2846	0.2396	**0.2904**	0.2548
	0	1	0.2	0.2	**0.8540**	0.7682	0.8476	0.7838	0.8296	0.7972
	1	1	0	0	0.6398	0.5538	0.6774	0.5672	**0.6920**	0.5842
	0	0.7	0.2	0	**0.7124**	0.6268	0.7118	0.6338	0.6926	0.6556
	0.5	1	0.5	0	0.8358	0.7504	**0.8442**	0.7680	0.8382	0.7826
	0	0	0	0	0.0444	0.0426	0.0502	0.0524	0.0474	0.0550
	0.5	0.5	0	0	0.4164	0.3620	0.4676	0.3718	**0.4718**	0.3754
	0	1	0.2	0.2	**0.9784**	0.9582	0.9750	0.9606	0.9694	0.9648
Exponential	1	1	0	0	0.7886	0.7128	0.8260	0.7264	**0.8342**	0.7372
	0	0.7	0.2	0	**0.9438**	0.8880	0.9386	0.8950	0.9288	0.8996
	0.5	1	0.5	0	0.9814	0.9536	**0.9838**	0.9544	0.9804	0.9584
	0	0	0	0	0.0498	0.0510	0.0520	0.0492	0.0520	0.0480
	0.5	0.5	0	0	0.2086	0.1800	0.2254	0.1966	**0.2274**	0.1964
T with 3	0	1	.2	0.2	0.6886	0.6118	**0.6972**	0.6204	0.6790	0.6306
degrees	1	1	0	0	0.4868	0.4170	0.5102	0.4048	**0.5184**	0.4306
of freedom	0	0.7	.2	0	**0.5552**	0.4848	0.5378	0.4950	0.5474	0.4926
	0.5	1	.5	0	0.6708	0.5958	**0.6898**	0.6134	0.6808	0.6270

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD, RCBD and CRD). Results show that with a combination 6 BIBD, 12 CRD and 6 RCBD, Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.4. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, BIBD = 12, CRD=12, RCBD = 12 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
	0	0	0	0	0.0480	0.0496	0.0460	0.0518	0.0530	0.0474
Normal	0.5	.5	0	0	0.3616	0.2514	0.3630	0.2592	**0.3778**	0.2774
	0	1	.2	0.2	0.9452	0.8144	**0.9472**	0.8380	0.9376	0.8498
	1	1	0	0	0.7842	0.5988	0.8088	0.6138	**0.8386**	0.6612
	0	.7	.2	0	**0.8538**	0.6830	0.8410	0.6908	0.8344	0.7114
	0.5	1	.5	0	0.9380	0.7922	**0.9388**	0.8100	0.9382	0.8370
	0	0	0	0	0.0550	0.0482	0.0514	0.0458	0.0532	0.0532
	0.5	.5	0	0	0.5744	0.4012	0.6036	0.4088	**0.6272**	0.4518
Exponential	0	1	.2	0.2	0.9970	0.9730	**0.9978**	0.9750	0.9954	0.9832
	1	1	0	0	0.9118	0.7492	0.9342	0.7752	**0.9536**	0.8164
	0	.7	.2	0	**0.9878**	0.9206	**0.9878**	**0.9304**	**0.9876**	0.9390
	0.5	1	.5	0	0.9976	0.9708	**0.9978**	0.9730	0.9974	0.9808
	0	0	0	0	0.0472	0.0504	0.0512	0.0496	0.0448	0.0550
	0.5	.5	0	0	0.2764	0.1966	0.2860	0.1912	**0.2936**	0.2194
T with 3	0	1	.2	0.2	**0.8370**	0.6516	0.8276	0.6724	0.8200	0.7050
degrees	1	1	0	0	0.6272	0.4542	0.6610	0.4618	**0.6752**	0.5014
of freedom	0	.7	.2	0	0.6958	0.5054	**0.6980**	0.5246	0.6832	0.5492
	0.5	1	.5	0	**0.8302**	0.6550	0.8234	0.6420	0.8254	0.6808

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD, RCBD and CRD). Results show that with a combination 12 BIBD, 12 CRD and 12 RCBD, Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.5. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, BIBD = 18, CRD=12, RCBD = 18 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0536	0.0496	0.0480	0.0458	0.0452	0.0484
	0.5	.5	0	0	0.4226	0.2772	0.4234	0.2780	**0.4644**	0.3062
	0	1	.2	0.2	**0.9786**	0.8458	0.9796	0.8678	0.9748	0.8938
	1	1	0	0	0.8718	0.6248	0.8910	0.6726	**0.9064**	0.7298
	0	.7	.2	0	0.9182	0.7154	**0.9192**	0.7248	0.9114	0.7662
	0.5	1	.5	0	0.9766	0.8242	**0.9798**	0.8472	0.9778	0.8850
Exponential	0	0	0	0	0.0492	0.0476	0.0446	0.0466	0.0524	0.0496
	0.5	.5	0	0	0.6678	0.4140	0.7006	0.4562	**0.7320**	0.5196
	0	1	.2	0.2	0.9996	0.9814	**0.9998**	0.9858	0.9996	0.9900
	1	1	0	0	0.9578	0.7770	0.9782	0.8200	**0.9810**	0.8802
	0	.7	.2	0	0.9990	0.9458	0.9978	0.9534	0.9970	0.9666
	0.5	1	.5	0	0.9996	0.9800	0.9994	0.9870	0.9996	0.9908
T with 3 degrees of freedom	0	0	0	0	0.0556	0.0530	0.0534	0.0436	0.0478	0.0488
	0.5	.5	0	0	0.3196	0.2086	0.3392	0.2288	**0.3574**	0.2436
	0	1	.2	0.2	0.9144	0.6924	0.9122	0.7094	**0.8914**	0.7510
	1	1	0	0	0.7350	0.4806	0.7662	0.5112	**0.7910**	0.5688
	0	.7	.2	0	**0.7928**	0.5470	0.7906	0.5690	0.7822	0.6054
	0.5	1	.5	0	0.8908	0.6754	0.9008	0.7086	**0.9032**	0.7382

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD, RCBD and CRD). Results show that with a combination 18 BIBD, 12 CRD and 18 RCBD, Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.6. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, BIBD = 12, CRD=18, RCBD = 12 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0546	0.0452	0.0422	0.0494	0.0536	0.0508
	0.5	.5	0	0	0.3994	0.2926	0.4074	0.3124	**0.4280**	0.3318
	0	1	.2	0.2	0.9756	0.9038	**0.9764**	0.9094	0.9662	0.9228
	1	1	0	0	0.8458	0.7068	0.8738	0.7206	**0.8876**	0.7388
	0	.7	.2	0	0.8948	0.7738	**0.8960**	0.7934	0.8880	0.7948
	0.5	1	.5	0	0.9708	0.8924	**0.9720**	0.9010	0.9698	0.9044
Exponential	0	0	0	0	0.0480	0.0496	0.0494	0.0506	0.0456	0.0508
	0.5	.5	0	0	0.6484	0.5018	0.6740	0.5012	**0.6972**	0.5264
	0	1	.2	0.2	**0.9996**	0.9912	0.9994	0.9940	0.9990	0.9952
	1	1	0	0	0.9492	0.8510	0.9628	0.8634	**0.9716**	0.8818
	0	.7	.2	0	**0.9972**	0.9710	0.9962	0.9754	0.9934	0.9798
	0.5	1	.5	0	**0.9998**	0.9938	0.9996	0.9934	0.9994	0.9952
T with 3 degrees of freedom	0	0	0	0	0.0594	0.0494	0.0456	0.0468	0.0536	0.0528
	0.5	.5	0	0	0.2956	0.2354	0.3186	0.2460	**0.3306**	0.2470
	0	1	.2	0.2	0.8896	0.7794	**0.8906**	0.7718	0.8682	0.7880
	1	1	0	0	0.7026	0.5458	0.7312	0.5698	**0.7476**	0.5824
	0	.7	.2	0	0.7630	0.6256	**0.7662**	0.6252	0.7582	0.6300
	0.5	1	.5	0	0.8796	0.7460	**0.8944**	0.7552	0.8862	0.7684

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD, RCBD and CRD). Results show that with a combination 12 BIBD, 18 CRD and 12 RCBD, Standardized First test still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.7. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, BIBD = 18, CRD=18, RCBD = 18 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0468	0.0552	0.0542	0.0442	0.0502	0.0510
	0.5	.5	0	0	0.4542	0.3202	0.5008	0.3220	**0.5026**	0.3418
	0	1	.2	0.2	**0.9800**	0.9090	**0.9828**	0.9140	**0.9744**	0.9254
	1	1	0	0	0.9170	0.7292	0.9364	0.7434	**0.9428**	0.7718
	0	.7	.2	0	**0.9548**	0.8076	0.9510	0.8022	0.9448	0.8234
	0.5	1	.5	0	0.9872	0.8988	0.9926	0.9142	0.9894	0.9310
Exponential	0	0	0	0	0.0472	0.0518	0.0478	0.0474	0.0512	0.0530
	0.5	.5	0	0	0.7394	0.5086	0.7670	0.5328	**1.0000**	0.5716
	0	1	.2	0.2	**0.9998**	0.9956	**0.9998**	0.9966	0.9888	0.9962
	1	1	0	0	0.9810	0.8680	0.9872	0.8850	**0.9996**	0.9072
	0	.7	.2	0	0.9994	0.9808	0.9990	0.9786	**1.0000**	0.9866
	0.5	1	.5	0	0.9998	0.9942	**1.0000**	0.9964	0.9928	0.9966
T with 3 degrees of freedom	0	0	0	0	0.0506	0.0482	0.0472	0.0478	0.0420	0.0534
	0.5	.5	0	0	0.3516	0.2286	**0.3738**	0.2512	0.3316	0.2598
	0	1	.2	0.2	**0.9430**	0.7800	0.9398	0.7924	0.8376	0.8136
	1	1	0	0	0.7946	0.5762	0.8078	0.5816	**0.9434**	0.6174
	0	.7	.2	0	**0.8610**	0.6290	0.8450	0.6422	0.8502	0.6798
	0.5	1	.5	0	0.9322	0.7540	**0.9342**	0.7792	0.5026	0.7988

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD, RCBD and CRD). Results show that with a combination 18 BIBD, 18 CRD and 18 RCBD, Standardized First test still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.8. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, CRD=6, RCBD = 12 Peak=2

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0510	0.0490	0.0504	0.0482	0.0490	0.0578
	0.5	0.5	0	0	0.1986	0.1600	0.2248	0.1764	**0.2894**	0.2552
	0	1	0.2	0.2	**0.8244**	0.6716	0.8042	0.6786	0.7842	0.7358
	1	1	0	0	0.5962	0.4370	0.6364	0.5004	**0.6786**	0.6066
	0	0.7	0.2	0	**0.6646**	0.5196	0.6580	0.5458	0.6570	0.5998
	0.5	1	0.5	0	0.7980	0.6326	0.8056	0.6866	**0.8134**	0.7562
Exponential	0	0	0	0	0.0490	0.0564	0.0522	0.0458	0.0490	0.0498
	0.5	0.5	0	0	0.4008	0.2726	0.4360	0.3138	**0.4660**	0.4100
	0	1	0.2	0.2	**0.9700**	0.8760	0.9630	0.8946	0.9530	0.9312
	1	1	0	0	0.7604	0.5836	0.7970	0.6578	**0.8278**	0.7610
	0	0.7	0.2	0	**0.9222**	0.7770	0.9062	0.8136	0.8972	0.8658
	0.5	1	0.5	0	**0.9728**	0.8802	0.9726	0.9092	0.9684	0.9476
T with 3 degrees of freedom	0	0	0	0	0.0504	0.0486	0.0460	0.0484	0.0524	0.0512
	0.5	0.5	0	0	0.2064	0.1534	0.2120	0.1688	**0.2134**	0.2002
	0	1	0.2	0.2	**0.6556**	0.5092	0.6404	0.5336	0.6220	0.5736
	1	1	0	0	0.4546	0.3430	0.4966	0.3738	**0.5182**	0.4550
	0	0.7	0.2	0	**0.5220**	0.3902	0.5042	0.4200	0.4892	0.4534
	0.5	1	0.5	0	0.6382	0.4874	0.6468	0.5204	**0.6530**	0.5856

Selected results for Treatments 4 at Peak 2 for mixed Designs of (RCBD and CRD). Results show that Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.9. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, CRD = 6, RCBD=18 Peak=2

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0506	0.0528	0.0440	0.0428	0.0480	0.0442
	0.5	.5	0	0	0.2944	0.2098	0.3326	0.2472	**0.3414**	0.2938
	0	1	.2	0.2	**0.8860**	0.7198	0.8734	0.7690	0.8674	0.8230
	1	1	0	0	0.6852	0.4942	0.7498	0.5804	**0.7794**	0.7232
	0	.7	.2	0	0.7544	0.5688	**0.7662**	0.6240	0.7324	0.6870
	0.5	1	.5	0	0.8778	0.7010	0.8886	0.7682	**0.8898**	0.8420
Exponential	0	0	0	0	0.0490	0.0512	0.0456	0.0468	0.0504	0.0458
	0.5	.5	0	0	**0.4918**	0.3262	**0.5264**	0.3750	**0.5582**	0.4928
	0	1	.2	0.2	**0.9914**	0.9246	0.9886	0.9558	0.9822	0.9730
	1	1	0	0	0.8444	0.6512	0.8786	0.7490	**0.9176**	0.8662
	0	.7	.2	0	**0.9626**	0.8506	0.9612	0.8926	0.9454	0.9304
	0.5	1	.5	0	0.9900	0.9208	0.9916	0.9524	**0.9926**	0.9788
T with 3 degrees of freedom	0	0	0	0	0.0444	0.0476	0.0494	0.0460	0.0504	0.0464
	0.5	.5	0	0	0.2228	0.1574	0.2416	0.1902	**0.2544**	0.2344
	0	1	2	0.2	**0.7540**	0.5568	0.7256	0.6162	0.7052	0.6660
	1	1	0	0	0.5316	0.3832	0.5842	0.4616	**0.6208**	0.5498
	0	0.7	0.2	0	**0.6086**	0.4312	**0.6086**	0.4806	0.5854	0.5398
	0.5	1	0.5	0	**0.7400**	0.5334	0.7256	0.6070	0.7386	0.6884

Selected results for Treatments 4 at Peak 2 for mixed Designs of (RCBD and CRD). Results show that Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.10. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, CRD = 12, RCBD=6 Peak=2

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0476	0.0454	0.0476	0.0474	0.0468	0.0426
	0.5	0.5	0	0	0.2518	0.2374	0.2780	0.2356	**0.2888**	0.2396
	0	1	.2	0.2	**0.8322**	0.7726	0.8194	0.7754	0.8174	0.7974
	1	1	0	0	0.6030	0.5340	0.6484	0.5396	**0.6762**	0.5748
	0	0.7	.2	0	0.6924	0.6140	**0.7018**	0.6150	0.6822	0.6346
		1	.5	0	0.8172	0.7396	0.8336	0.7504	**0.8376**	0.7682
Exponential	0	0	0	0	0.0480	0.0522	0.0528	0.0460	0.0512	0.0534
	0.5	0.5	0	0	0.4278	0.3528	0.4534	0.3750	**0.4544**	0.3934
	0	1	.2	0.2	**0.9782**	0.9508	0.9740	0.9522	0.9614	0.9622
	1	1	0	0	0.7740	0.6982	0.8114	0.7054	**0.8314**	0.7254
	0	0.7	.2	0	**0.9324**	0.8746	0.9280	0.8820	0.9142	0.8956
	0.5	1	.5	0	**0.9758**	0.9462	0.9742	0.9510	**0.9758**	0.9570
T with 3 degrees of freedom	0	0	0	0	0.0514	0.0492	0.0534	0.0480	0.0492	0.0534
	0.5	0.5	0	0	0.2032	0.1814	0.2178	0.1760	**0.2284**	0.1902
	0	1	.2	0.2	0.6652	0.6052	**0.6812**	0.5954	0.6598	0.6168
	1	1	0	0	0.4658	0.4058	0.4962	0.4146	**0.5164**	0.4402
	0	0.7	.2	0	**0.5444**	0.4796	0.5270	0.4738	0.5210	0.4956
	0.5	1	.5	0	**0.6724**	0.5940	0.6674	0.5864	0.6720	0.6050

Selected results for Treatments 4 at Peak 2 for mixed Designs of (RCBD and CRD). Results show that Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.11. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, CRD = 12, RCBD=12 Peak=2

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
	0	0	0	0	0.0496	0.0514	0.0506	0.0474	0.0502	0.0484
	0.5	.5	0	0	0.3160	0.2386	0.3444	0.2512	**0.3514**	0.2692
Normal	0	1	.2	0.2	**0.9206**	0.7792	**0.9206**	0.8096	0.8960	0.8300
	1	1	0	0	0.7320	0.5516	0.7752	0.5936	**0.8072**	0.6324
	0	.7	.2	0	0.7870	0.6314	**0.7988**	0.6560	0.7896	0.6928
	0.5	1	.5	0	0.9044	0.7740	**0.9156**	0.7890	0.9144	0.8152
	0	0	0	0	0.0474	0.0476	0.0450	0.0458	0.0456	0.0484
	0.5	.5	0	0	0.5020	0.3726	0.5670	0.4012	**0.5890**	0.4360
	0	1	.2	0.2	**0.9964**	0.9572	0.9928	0.9680	0.9916	0.9770
Exponential	1	1	0	0	0.8748	0.7270	0.9116	0.7412	**0.9312**	0.7818
	0	0.7	0.2	0	**0.9744**	0.8954	0.9736	0.9154	0.9730	0.9224
	0.5	1	0.5	0	0.9942	0.9566	**0.9970**	0.9650	0.9944	0.9736
	0	0	0	0	0.0528	0.0470	0.0490	0.0488	0.0500	0.0532
	0.5	0.5	0	0	0.2460	0.1810	0.2514	0.2014	**0.2804**	0.2018
T with 3	0	1	0.2	0.2	**0.7916**	0.6236	0.7844	0.6414	0.7436	0.6624
degrees	1	1	0	0	0.5664	0.4256	0.6120	0.4422	**0.6468**	0.4908
of freedom	0	0.7	0.2	0	**0.6514**	0.4784	0.6412	0.5086	0.6144	0.5302
	0.5	1	0.5	0	0.7694	0.5950	0.7784	0.6198	**0.7844**	0.6676

Selected results for Treatments 4 at Peak 2 for mixed Designs of (RCBD and CRD). Results show that Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.12. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, CRD = 12, RCBD=18 Peak=2

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0494	0.0490	0.0470	0.0510	0.0472	0.0546
	0.5	0.5	0	0	0.3696	0.2472	0.4074	0.2634	**0.4138**	0.3104
	0	1	0.2	0.2	**0.9612**	0.8194	0.9512	0.8098	0.9494	0.8750
	1	1	0	0	0.8142	0.5894	0.8482	0.8376	**0.8664**	0.7030
	0	0.7	0.2	0	**0.8672**	0.6680	0.8620	0.6392	0.8476	0.7342
	0.5	1	0.5	0	0.9496	0.7938	**0.9586**	0.7014	0.9504	0.8714
Exponential	0	0	0	0	0.0482	0.0468	0.0514	0.0560	0.0538	0.0520
	0.5	0.5	0	0	0.5994	0.3952	0.6466	0.4250	**0.6700**	0.4950
	0	1	0.2	0.2	**0.9994**	0.9684	0.9984	0.9568	0.9978	0.9878
	1	1	0	0	0.9242	0.7440	0.9470	0.9822	**0.9666**	0.8358
	0	0.7	0.2	0	0.9920	0.9146	0.9920	0.7808	0.9888	0.9496
	0.5	1	0.5	0	0.9986	0.9688	0.9994	0.9364	0.9982	0.9870
T with 3 degrees of freedom	0	0	0	0	0.0494	0.0492	0.0478	0.0474	0.0516	0.0496
	0.5	0.5	0	0	0.2862	0.2008	0.2988	0.2186	**0.3046**	0.2354
	0	1	0.2	0.2	**0.8596**	0.6560	0.8506	0.6484	0.8260	0.7164
	1	1	0	0	0.6260	0.4528	0.6978	0.6772	**0.7260**	0.5356
	0	0.7	0.2	0	**0.7348**	0.4984	0.7170	0.4800	0.7018	0.5728
	0.5	1	0.5	0	0.8454	0.6230	**0.8582**	0.5408	0.8502	0.7156

Selected results for Treatments 4 at Peak 2 for mixed Designs of (RCBD and CRD). Results show that Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.13. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, CRD = 18, RCBD=12 Peak=2

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0486	0.0480	0.0452	0.0498	0.0482	0.0492
	0.5	0.5	0	0	0.3824	0.2894	0.3890	0.2990	**0.4164**	0.3140
	0	1	0.2	0.2	**0.9636**	0.9036	0.9586	0.8988	0.9504	0.9066
	1	1	0	0	0.8292	0.7056	0.8392	0.6940	**0.8744**	0.7330
	0	0.7	0.2	0	0.8722	0.7710	**0.8746**	0.7644	0.8720	0.8000
	0.5	1	0.5	0	0.9528	0.8816	**0.9576**	0.8866	0.9532	0.9014
Exponential	0	0	0	0	0.0502	0.0552	0.0442	0.0442	0.0448	0.0500
	0.5	0.5	0	0	0.5948	0.4862	0.6548	0.4928	**0.6692**	0.5214
	0	1	0.2	0.2	**0.9998**	0.9916	0.9984	0.9954	0.9992	0.9942
	1	1	0	0	0.9396	0.8402	0.9544	0.8570	**0.9660**	0.8722
	0	0.7	0.2	0	**0.9924**	**0.**9682	**0.9924**	0.9696	0.9908	0.9770
	0.5	1	0.5	0	0.9990	0.9938	0.9992	0.9922	**0.9996**	0.9936
T with 3 degrees of freedom	0	0	0	0	0.0508	0.0556	0.0562	0.0558	0.0538	0.0480
	0.5	0.5	0	0	0.2796	0.2378	0.2988	0.2364	**0.3036**	0.2368
	0	1	0.2	0.2	**0.8746**	0.7522	0.8578	0.7526	0.8446	0.7660
	1	1	0	0	0.6634	0.5532	0.6974	0.5436	**0.7124**	0.5828
	0	0.7	0.2	0	**0.7344**	0.6184	0.7246	0.6262	0.6980	0.6350
	0.5	1	0.5	0	0.8418	0.7330	0.8556	0.7482	**0.8564**	0.7690

Selected results for Treatments 4 at Peak 2 for mixed Designs of (RCBD and CRD). Results show that Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.14. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Four Treatments at Peak 2. Treatments=4, CRD = 18, RCBD=18 Peak=2

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0502	0.0522	0.0436	0.0586	0.0504	0.0548
	0.5	0.5	0	0	0.4162	0.2890	0.4454	0.3116	**0.4650**	0.3396
	0	1	0.2	0.2	**0.9662**	0.8972	0.9590	0.9098	0.9512	0.9196
	1	1	0	0	0.8740	0.7110	0.9168	0.7260	**0.9240**	0.7564
	0	0.7	0.2	0	**0.9214**	0.7708	0.9162	0.7776	0.9142	0.8112
	0.5	1	0.5	0	0.9778	0.8942	0.9834	0.9008	**0.9854**	0.9234
Exponential	0	0	0	0	0.0546	0.0468	0.0516	0.0460	0.0526	0.0478
	0.5	0.5	0	0	0.6808	0.4866	0.7164	0.5102	**0.7618**	0.5614
	0	1	0.2	0.2	0.9998	0.9948	**1.0000**	0.9952	**1.0000**	0.9972
	1	1	0	0	0.9574	0.8576	0.9782	0.8606	**0.9858**	0.8924
	0	0.7	0.2	0	**0.9986**	0.9726	0.9984	0.9774	0.9970	0.9824
	0.5	1	0.5	0	0.9998	0.9926	0.9998	0.9936	**1.0000**	0.9958
T with 3 degrees of freedom	0	0	0	0	0.0502	0.0566	0.0486	0.0522	0.0454	0.0542
	0.5	0.5	0	0	0.3208	0.2364	0.3434	0.2400	**0.3538**	0.2596
	0	1	0.2	0.2	**0.9106**	0.7714	0.9070	0.7784	0.8932	0.7970
	1	1	0	0	0.7444	0.5480	0.7706	0.5766	**0.7896**	0.5968
	0	0.7	0.2	0	**0.7988**	0.6098	0.7844	0.6304	0.7768	0.6488
	0.5	1	0.5	0	0.9006	0.7466	0.9144	0.7670	**0.9172**	0.7924

Selected results for Treatments 4 at Peak 2 for mixed Designs of (RCBD and CRD). Results show that Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.15. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Four Treatments at Peak 2. Treatments=4, BIBD = 12, CRD=6 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0520	0.0518	0.0484	0.0490	0.0548	0.0474
	0.5	0.5	0	0	0.1864	0.1542	0.1752	0.1678	**0.1918**	0.1638
	0	1	0.2	0.2	**0.6232**	0.5302	0.6218	0.5410	0.6212	0.5336
	1	1	0	0	0.4200	0.3622	**0.4240**	0.3506	0.4224	0.3558
	0	0.7	0.2	0	0.4754	0.4076	0.4760	0.4096	**0.4784**	0.4240
	0.5	1	0.5	0	0.6082	0.5134	**0.6144**	0.5144	0.5914	0.5104
Exponential	0	0	0	0	0.0432	0.0482	0.0520	0.0522	0.0510	0.0534
	0.5	0.5	0	0	**0.2822**	0.2160	0.2746	0.2182	0.2800	0.2250
	0	1	0.2	0.2	0.8304	0.7490	0.8302	0.7430	**0.8310**	0.7640
	1	1	0	0	**0.5604**	0.4670	0.5554	0.4604	0.5418	0.4634
	0	0.7	0.2	0	0.7302	0.6508	0.7374	0.6520	**0.7388**	0.6526
	0.5	1	.5	0	0.8412	0.7678	0.8418	0.7622	**0.8432**	0.7752
T with 3 degrees of freedom	0	0	0	0	0.0512	0.0506	0.0518	0.0508	0.0548	0.0466
	0.5	.5	0	0	**0.1498**	0.1262	0.1440	0.1342	0.1432	0.1266
	0	1	.2	0.2	0.4588	0.4054	**0.4814**	0.3994	0.4790	0.3992
	1	1	0	0	0.3262	0.2794	**0.3300**	0.2708	0.3178	0.2830
	0	0.7	.2	0	0.3616	0.3066	**0.3738**	0.3162	0.3620	0.3198
	0.5	1	0.5	0	0.4526	0.3982	**0.4608**	0.3956	0.4464	0.3826

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD and CRD). Results show that with a combination 12 BIBD and 6 CRD, Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.16. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Four Treatments at Peak 2. Treatments=4, BIBD = 18, CRD=6 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0494	0.0520	0.0438	0.0546	0.0508	0.0458
	0.5	0.5	0	0	**0.2070**	0.1748	**0.2048**	0.1616	**0.2112**	0.1690
	0	1	0.2	0.2	0.6914	0.5794	**0.6942**	0.5736	0.6936	0.5684
	1	1	0	0	0.4888	0.3770	**0.4890**	0.3808	0.4692	0.3690
	0	0.7	0.2	0	0.5334	0.4326	**0.5428**	0.4358	0.5412	0.4322
	0.5	1	0.5	0	0.6684	0.5548	**0.6752**	0.5502	0.6672	0.5538
Exponential	0	0	0	0	0.0486	0.0476	0.0448	0.0496	0.0532	0.0502
	0.5	0.5	0	0	0.3124	0.2326	**0.3182**	0.2474	0.3000	0.2354
	0	1	0.2	0.2	**0.8980**	0.7934	0.8792	0.7830	0.8876	0.7962
	1	1	0	0	0.6258	0.5132	**0.6284**	0.5068	0.6186	0.5004
	0	0.7	0.2	0	0.8110	0.6960	**0.8128**	0.6876	0.8112	0.6948
	0.5	1	0.5	0	0.8974	0.8086	0.8990	0.8000	**0.8994**	0.7930
T with 3 degrees of freedom	0	0	0	0	0.0508	0.0516	0.0488	0.0478	0.0582	0.0502
	0.5	0.5	0	0	0.1600	0.1406	**0.1644**	0.1388	0.1596	0.1446
	0	1	0.2	0.2	0.5182	0.4270	**0.5340**	0.4300	0.5310	0.4342
	1	1	0	0	0.3506	0.2888	**0.3712**	0.2914	0.3622	0.2852
	0	0.7	0.2	0	0.4090	0.3332	0.4182	0.3396	**0.4190**	0.3408
	0.5	1	0.5	0	**0.5220**	0.4224	0.5174	0.4218	0.5218	0.4168

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD and CRD). Results show that with a combination 18 BIBD and 6 CRD, Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.17. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Four Treatments at Peak 2. Treatments=4, BIBD = 6, CRD=12 Peak=2 (Two missing observation)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0502	0.0506	0.0500	0.0492	0.0518	0.0594
	0.5	0.5	0	0	0.2138	0.2140	0.2226	**0.2224**	0.2168	0.2150
	0	1	0.2	0.2	0.6972	**0.7498**	0.7188	**0.7498**	0.6946	0.7446
	1	1	0	0	0.5048	0.5146	0.4910	0.5036	0.4936	**0.5190**
	0	0.7	0.2	0	0.5660	0.5946	0.5594	0.5880	0.5476	**0.6038**
	0.5	1	0.5	0	0.6752	0.7306	0.6854	0.7308	0.6850	**0.7398**
Exponential	0	0	0	0	0.0472	0.0558	0.0490	0.0484	0.0490	0.0532
	0.5	0.5	0	0	0.3320	**0.3422**	0.3226	0.3338	0.3376	0.3310
	0	1	0.2	0.2	0.8918	0.9424	0.8960	**0.9426**	0.8916	0.9398
	1	1	0	0	0.6428	0.6732	0.6434	0.6732	0.6286	**0.6752**
	0	0.7	0.2	0	0.8158	**0.8702**	0.8274	0.8666	0.8126	0.8682
	0.5	1	0.5	0	0.9032	**0.9388**	0.9130	0.9360	0.8994	0.9380
T with 3 degrees of freedom	0	0	0	0	0.0538	0.0528	0.0522	0.0510	0.0508	0.0538
	0.5	0.5	0	0	0.1736	**0.1824**	0.1702	0.1816	0.1722	0.1680
	0	1	0.2	0.2	0.5534	**0.5874**	0.5452	0.5822	0.5354	0.5862
	1	1	0	0	0.3778	0.3842	0.3568	0.3920	0.3804	**0.3930**
	0	0.7	0.2	0	0.4330	0.4588	0.4208	**0.4496**	0.4268	0.4502
	0.5	1	0.5	0	0.5332	0.5548	0.5182	**0.5690**	0.5302	0.5628

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD and CRD). Results show that with a combination 6 BIBD and 12 CRD, Standardized Last test still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.18. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Four Treatments at Peak 2. Treatments=4, BIBD = 12, CRD=12 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0496	0.0500	0.0510	0.0522	0.0496	0.0448
	0.5	0.5	0	0	0.2388	0.2282	0.2404	0.2332	**0.2414**	0.2208
	0	1	0.2	0.2	0.7848	0.7538	0.7882	0.7566	**0.7992**	0.7688
	1	1	0	0	0.5700	0.5394	0.5562	0.5206	**0.5740**	0.5360
	0	0.7	0.2	0	0.6322	0.6150	0.6444	0.6022	**0.6506**	0.6048
	0.5	1	0.5	0	0.7616	0.7344	0.7640	0.7390	**0.7726**	0.7422
Exponential	0	0	0	0	0.0522	0.0464	0.0536	0.0492	0.0476	0.0514
	0.5	0.5	0	0	0.3728	0.3424	0.3712	0.3500	**0.3746**	0.3390
	0	1	0.2	0.2	0.9430	0.9428	**0.9500**	0.9446	0.9464	0.9460
	1	1	0	0	0.7086	0.6776	**0.7244**	0.6778	0.7130	0.6876
	0	0.7	0.2	0	0.8938	0.8810	**0.8956**	0.8722	0.8946	0.8692
	0.5	1	0.5	0	0.9514	0.9418	**0.9534**	0.9456	0.9510	0.9416
T with 3 degrees of freedom	0	0	0	0	0.0486	0.0572	0.0506	0.0538	0.0472	0.0480
	0.5	0.5	0	0	**0.2030**	0.1822	0.1986	0.1764	0.1966	0.1792
	0	1	0.2	0.2	0.6200	0.5906	0.6248	0.5998	**0.6264**	0.5876
	1	1	0	0	0.4248	0.4020	**0.4306**	0.4084	0.4282	0.3990
	0	0.7	0.2	0	0.4916	0.4480	**0.5014**	0.4490	0.4904	0.4672
	0.5	1	0.5	0	0.6078	0.5708	**0.6174**	0.5744	0.6126	0.5652

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD and CRD). Results show that with a combination 12 BIBD and 12 CRD, Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.19. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Four Treatments at Peak 2. Treatments=4, BIBD = 18, CRD=12 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0472	0.0538	0.0516	0.0504	0.0472	0.0514
	0.5	0.5	0	0	**0.2698**	0.2280	0.2626	0.2322	0.2600	0.2268
	0	1	0.2	0.2	0.8348	0.7660	**0.8440**	0.7718	0.8438	0.7734
	1	1	0	0	0.6212	0.5448	0.6224	0.5384	**0.6274**	0.5396
	0	0.7	0.2	0	0.6926	0.6132	**0.7022**	0.6212	0.6922	0.6028
	0.5	1	0.5	0	**0.8324**	0.7450	0.8112	0.7504	0.8250	0.7444
Exponential	0	0	0	0	0.0468	0.0540	0.0536	0.0528	0.0508	0.0484
	0.5	0.5	0	0	**0.4324**	0.3528	0.4216	0.3448	0.4224	0.3576
	0	1	0.2	0.2	**0.9726**	0.9524	0.9684	0.9520	0.9696	0.9444
	1	1	0	0	0.7788	0.6960	**0.7858**	0.6824	0.7732	0.6790
	0	0.7	0.2	0	0.9258	0.8862	0.9286	0.8802	**0.9320**	0.8896
	0.5	1	0.5	0	0.9730	0.9486	**0.9766**	0.9474	0.9738	0.9498
T with 3 degrees of freedom	0	0	0	0	0.0492	0.0438	0.0510	0.0508	0.0460	0.0480
	0.5	0.5	0	0	0.1970	0.1912	0.2022	0.1772	**0.2106**	0.1754
	0	1	0.2	0.2	**0.6938**	0.6072	0.6732	0.5862	0.6842	0.6102
	1	1	0	0	0.4854	0.4086	0.4630	0.4090	**0.4698**	0.4130
	0	0.7	0.2	0	**0.5444**	0.4730	0.5388	0.4648	0.5420	0.4662
	0.5	1	0.5	0	0.6600	0.5896	0.6694	0.5610	**0.6696**	0.5886

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD and CRD). Results show that with a combination 18 BIBD and 12 CRD, Standardized First still performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.20. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Four Treatments at Peak 2. Treatments=4, BIBD = 12, CRD=18 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0550	0.0524	0.0538	0.0476	0.0540	0.0476
	0.5	0.5	0	0	0.2858	**0.2936**	0.2940	0.2858	0.2754	0.2876
	0	1	0.2	0.2	0.8708	0.8800	0.8778	0.8852	0.8806	**0.8854**
	1	1	0	0	0.6702	**0.6810**	0.6738	0.6720	0.6684	0.6786
	0	0.7	0.2	0	0.7444	**0.7584**	0.7390	0.7534	0.7426	0.7420
	0.5	1	0.5	0	0.8608	0.8660	0.8740	0.8662	0.8612	**0.8710**
Exponential	0	0	0	0	0.0518	0.0480	0.0464	0.0554	0.0404	0.0496
	0.5	0.5	0	0	0.4662	**0.4738**	0.4712	0.4632	0.4686	0.4640
	0	1	0.2	0.2	0.9846	**0.9916**	0.9786	0.9904	0.9818	0.9896
	1	1	0	0	0.8222	**0.8326**	0.8212	**0.8322**	0.8206	**0.8296**
	0	0.7	0.2	0	0.9528	0.9600	0.9536	**0.9612**	0.9490	0.9610
	0.5	1	0.5	0	0.9846	0.9884	0.9838	**0.9922**	0.9828	0.9912
T with 3 degrees of freedom	0	0	0	0	0.0440	0.0520	0.0538	0.0460	0.0512	0.0506
	0.5	0.5	0	0	0.2184	**0.2234**	0.2296	0.2194	0.2220	0.2194
	0	1	0.2	0.2	0.7358	**0.7370**	0.7294	0.7326	0.7340	0.7324
	1	1	0	0	0.5114	0.5160	0.5240	**0.5226**	0.5250	0.5168
	0	0.7	0.2	0	0.5794	0.5822	0.5840	**0.5938**	0.5884	0.5870
	0.5	1	0.5	0	0.7088	0.7184	0.7270	**0.7222**	0.7114	0.7160

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD and CRD). Results show that with a combination 12 BIBD and 18 CRD, Standardized Last test performed better than Standardized First in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.21. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Four Treatments at Peak 2. Treatments=4, BIBD = 18, CRD=18 Peak=2 (Two Missing Observations)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0504	0.0458	0.0490	0.0532	0.0520	0.0504
	0.5	0.5	0	0	0.2894	**0.3204**	0.2858	0.3098	0.2900	0.2894
	0	1	0.2	0.2	0.8864	**0.9158**	0.8888	0.9102	0.8844	0.8864
	1	1	0	0	0.6942	**0.7302**	0.6866	0.7218	0.6884	0.6942
	0	0.7	0.2	0	0.7534	**0.8036**	0.7496	0.7912	0.7476	0.7534
	0.5	1	0.5	0	0.8752	**0.9054**	0.8640	0.9002	0.8680	0.8752
Exponential	0	0	0	0	0.0506	0.0526	0.0520	0.0442	0.0476	0.0506
	0.5	0.5	0	0	0.4786	0.5108	0.4556	**0.5110**	0.4668	0.4786
	0	1	0.2	0.2	0.9908	**0.9934**	0.9932	0.9920	0.9918	0.9908
	1	1	0	0	0.8366	**0.8680**	0.8314	0.8650	0.8280	0.8366
	0	0.7	0.2	0	0.9652	**0.9746**	0.9602	0.9680	0.9648	0.9652
	0.5	1	0.5	0	0.9882	0.9940	0.9904	**0.9942**	0.9908	0.9882
T with 3 degrees of freedom	0	0	0	0	0.0524	0.0474	0.0458	0.0538	0.0526	0.0524
	0.5	0.5	0	0	0.2266	0.2364	0.2318	**0.2464**	0.2244	0.2266
	0	1	0.2	0.2	0.7476	0.7896	0.7376	**0.7934**	0.7398	0.7476
	1	1	0	0	0.5256	0.5628	0.5306	**0.5678**	0.5230	0.5256
	0	0.7	0.2	0	0.5950	**0.6396**	0.6028	0.6330	0.5832	0.5950
	0.5	1	0.5	0	0.7208	0.7634	0.7156	**0.7604**	0.7292	0.7208

Selected results for Treatments 4 at Peak 2 for mixed Designs of (BIBD and CRD). Results show that with a combination 18 BIBD and 18 CRD, Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.22. Mixed Design for Balanced Incomplete Block Design, Randomized Complete Block Design and Completely Randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 12, CRD=6, RCBD=12, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0524	0.0530	0.0494	0.0446	0.0528	0.0468
	0	0.5	0.5	0	0.5150	0.3754	**0.6088**	0.4300	0.4914	0.4178
	0	1	1	0.2	0.9324	0.7826	**0.9726**	0.8678	0.9202	0.8630
	0	0	1	0.2	0.9208	0.7782	**0.9716**	0.8680	0.9218	0.8540
	1	1	1	0	0.2594	0.1954	**0.2696**	0.2186	0.1980	0.1768
	0	0.2	0.7	0.5	0.5520	0.3940	**0.6608**	0.4846	0.5980	0.4988
	0	0.5	1	0.5	0.8818	0.6988	**0.9400**	0.8076	0.8852	0.7986
Exponential	0	0	0	0	0.0500	0.0468	0.0502	0.0488	0.0570	0.0526
	0	0.5	0.5	0	0.7766	0.6060	**0.8602**	0.7130	0.7582	0.6838
	0	1	1	0.2	0.9870	0.9214	**0.9974**	0.9698	0.9892	0.9692
	0	0	1	0.2	0.9942	0.9538	**0.9988**	0.9856	0.9898	0.9832
	1	1	1	0	0.3538	0.2396	**0.3462**	0.2504	0.2514	0.2132
	0	0.2	0.7	0.5	0.8462	0.6626	**0.9176**	0.7740	0.8524	0.7650
	0	0.5	1	0.5	0.9898	0.9274	**0.9986**	0.9746	0.9920	0.9716
T with 3 degrees of freedom	0	0	0	0	0.0496	0.0484	0.0468	0.0486	0.0476	0.0468
	0	0.5	0.5	0	0.3928	0.2934	**0.4504**	0.3472	0.3742	0.3276
	0	1	1	0.2	0.8038	0.6224	**0.8810**	0.7072	0.8082	0.7130
	0	0	1	0.2	0.7990	0.6306	**0.8898**	0.7240	0.7886	0.7052
	1	1	1	0	0.2124	0.1586	**0.2218**	0.1700	0.1634	0.1492
	0	0.2	0.7	0.5	0.4282	0.3044	**0.5190**	0.3662	0.4368	0.3736
	0	0.5	1	0.5	0.7216	0.5378	**0.8264**	0.6440	0.7470	0.6404

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD, RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.23. Mixed Design for Balanced Incomplete Block Design, Randomized Complete Block Design and Completely Randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 18, CRD=6, RCBD=18, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0494	0.0528	0.0536	0.0524	0.0476	0.0510
	0	0.5	0.5	0	0.6230	0.4478	**0.7068**	0.5532	0.6000	0.5088
	0	1	1	0.2	0.9692	0.8666	**0.9924**	0.9540	0.9724	0.9304
	0	0	1	0.2	0.9790	0.8706	**0.9946**	0.9482	0.9706	0.9362
	1	1	1	0	0.3336	0.2270	**0.3488**	0.2536	0.2216	0.1972
	0	0.2	0.7	0.5	0.6704	0.4830	**0.7766**	0.6182	0.6962	0.5990
	0	0.5	1	0.5	0.9458	0.8032	**0.9820**	0.9128	0.9508	0.9028
Exponential	0	0	0	0	0.0516	0.0452	0.0546	0.0402	0.0452	0.0518
	0	0.5	0.5	0	0.8824	0.7100	**0.9436**	0.8088	0.8688	0.7946
	0	1	1	0.2	0.9986	0.9638	**1.0000**	0.9916	0.9978	0.9888
	0	0	1	0.2	0.9996	0.9818	**1.0000**	0.9990	0.9992	0.9948
	1	1	1	0	**0.4344**	0.2882	**0.4344**	0.3200	0.2910	0.2372
	0	0.2	0.7	0.5	0.9208	0.7732	**0.9750**	0.8910	0.9314	0.8742
	0	0.5	1	0.5	0.9988	0.9686	**1.0000**	0.9942	0.9988	0.9912
T with 3 degrees of freedom	0	0	0	0	0.0492	0.0492	0.0484	0.0494	0.0498	0.0464
	0	0.5	0.5	0	0.4756	0.3420	**0.5632**	0.4114	0.4590	0.4008
	0	1	1	0.2	0.8966	0.7164	**0.9530**	0.8432	0.8738	0.8126
	0	0	1	0.2	0.8906	0.7162	**0.9510**	0.8428	0.8874	0.8126
	1	1	1	0	**0.2624**	0.1788	0.2582	0.2030	0.1906	0.1552
	0	0.2	0.7	0.5	0.5024	0.3732	**0.6168**	0.4604	0.5374	0.4736
	0	0.5	1	0.5	0.8306	0.6394	**0.9064**	0.7788	0.8378	0.7674

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD, RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.24. Mixed Design for Balanced Incomplete Block Design, Randomized Complete Block Design and Completely Randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 6, CRD=12, RCBD=6, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0528	0.0488	0.0504	0.0510	0.0532	0.0518
	0	0.5	0.5	0	0.4680	0.3976	**0.5382**	0.4246	0.4578	0.4118
	0	1	1	0.2	0.8954	0.8250	**0.9402**	0.8462	0.8936	0.8472
	0	0	1	0.2	0.8926	0.8272	**0.9412**	0.8548	0.8902	0.8542
	1	1	1	0	0.2474	0.2144	**0.2454**	0.2110	0.1998	0.1954
	0	0.2	0.7	0.5	0.4976	0.4314	**0.5886**	0.4540	0.5374	0.4710
	0	0.5	1	0.5	0.8366	0.7668	**0.9002**	0.7734	0.8494	0.7840
Exponential	0	0	0	0	0.0498	0.0482	0.0504	0.0490	0.0492	0.0464
	0	0.5	0.5	0	0.7336	0.6520	**0.8042**	0.6826	0.7184	0.6648
	0	1	1	0.2	0.9820	0.9510	**0.9896**	0.9628	0.9774	0.9672
	0	0	1	0.2	0.9894	0.9716	**0.9962**	0.9822	0.9842	0.9792
	1	1	1	0	0.3240	0.2558	**0.3204**	0.2586	0.2530	0.2468
	0	0.2	0.7	0.5	0.8016	0.7270	**0.8694**	0.7398	0.8078	0.7552
	0	0.5	1	0.5	0.9806	0.9572	**0.9948**	0.9640	0.9792	0.9628
T with 3 degrees of freedom	0	0	0	0	0.0436	0.0490	0.0492	0.0538	0.0510	0.0542
	0	0.5	0.5	0	0.3526	0.3150	**0.4030**	0.3172	0.3556	0.3132
	0	1	1	0.2	0.7478	0.6588	**0.8152**	0.6844	0.7520	0.6912
	0	0	1	0.2	0.7452	0.6464	**0.8230**	0.6752	0.7466	0.6914
	1	1	1.	0	0.1964	0.1740	**0.1996**	0.1736	0.1638	0.1718
	0	0.2	0.7	0.5	0.3832	0.3320	**0.4336**	0.3366	0.4030	0.3412
	0	0.5	1	0.5	0.6724	0.5912	**0.7518**	0.6208	0.6914	0.6134

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD, RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.25. Mixed Design for Balanced Incomplete Block Design, Randomized Complete Block Design and Completely Randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 12, CRD=12, RCBD=12, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0470	0.0496	0.0542	0.0522	0.0512	0.0496
	0	0.5	0.5	0	0.6126	0.4184	**0.6798**	0.4760	0.6114	0.4670
	0	1	1	0.2	0.9654	0.8560	**0.9902**	0.8980	0.9664	0.8944
	0	0	1	0.2	0.9678	0.8646	**0.9904**	0.8978	0.9692	0.8956
	1	1	1	0	0.3098	0.2204	**0.3214**	0.2260	0.2440	0.2208
	0	0.2	0.7	0.5	0.6590	0.4724	**0.7560**	0.5180	0.6778	0.5250
	0	0.5	1	0.5	0.9344	0.7978	**0.9748**	0.8364	0.9438	0.8366
Exponential	0	0	0	0	0.0452	0.0460	0.0514	0.0534	0.0518	0.0508
	0	0.5	0.5	0	0.8782	0.6890	**0.9300**	0.7304	0.8732	0.7392
	0	1	1	0.2	0.9976	0.9702	**0.9998**	0.9798	0.9964	0.9818
	0	0	1	0.2	0.9996	0.9846	**1.0000**	0.9904	0.9982	0.9928
	1	1	1	0	0.4046	0.2762	**0.4344**	0.2828	0.3048	0.2678
	0	0.2	0.7	0.5	0.9242	0.7608	**0.9700**	0.8094	0.9310	0.8116
	0	0.5	1	0.5	0.9984	0.9690	**0.9996**	0.9840	0.9982	0.9826
T with 3 degrees of freedom	0	0	0	0	0.0536	0.0494	0.0502	0.0554	0.0522	0.0494
	0	0.5	0.5	0	0.4622	0.3180	**0.5298**	0.3556	0.4434	0.3662
	0	1	1	0.2	0.8802	0.7106	**0.9374**	0.7488	0.8740	0.7492
	0	0	1	0.2	0.8880	0.7192	**0.9326**	0.7548	0.8810	0.7560
	1	1	1	0	0.2488	0.1782	**0.2508**	0.1852	0.1908	0.1778
	0	0.2	0.7	0.5	0.5026	0.3644	**0.5846**	0.3852	0.5290	0.3968
	0	0.5	1	0.5	0.8128	0.6372	**0.8982**	0.6788	0.8308	0.6822

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD, RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.26. Mixed Design for Balanced Incomplete Block Design, Randomized Complete Block Design and Completely Randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 18, CRD=12, RCBD=18, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0486	0.0558	**0.0534**	0.0530	0.0530	0.0476
	0	0.5	0.5	0	0.7142	0.4802	**0.7870**	0.5064	0.6886	0.5136
	0	1	1	0.2	0.9928	0.8876	**0.9970**	0.9236	0.9914	0.9328
	0	0	1	0.2	0.9924	0.8916	**0.9984**	0.9236	0.9910	0.9400
	1	1	1	0	0.3900	0.2458	**0.3916**	0.2406	0.2800	0.2350
	0	0.2	0.7	0.5	0.7580	0.4980	**0.8456**	0.5554	0.7714	0.5888
	0	0.5	1	0.5	0.9748	0.8262	**0.9908**	0.8842	0.9800	0.8874
Exponential	0	0	0	0	0.0540	0.0494	0.0522	0.0466	0.0516	0.0496
	0	0.5	0.5	0	0.9356	0.7334	**0.9708**	0.7832	0.9248	0.7884
	0	1	1	0.2	0.9992	0.9766	0.9996	0.9914	**1.0000**	0.9900
	0	0	1	0.2	**1.0000**	0.9898	**1.0000**	0.9960	**1.0000**	0.9970
	1	1	1	0	0.5034	0.2990	**0.5140**	0.3238	0.3586	0.2892
	0	0.2	0.7	0.5	0.9618	0.7888	**0.9918**	0.8528	0.9696	0.8640
	0	0.5	1	0.5	0.9998	0.9806	**1.0000**	0.9896	**1.0000**	0.9926
T with 3 degrees of freedom	0	0	0	0	0.0492	0.0492	0.0526	0.0504	0.0472	0.0480
	0	0.5	0.5	0	0.5554	0.3592	**0.6292**	0.3836	0.5348	0.3848
	0	1	1	0.2	0.9462	0.7372	**0.9784**	0.7884	0.9414	0.8032
	0	0	1	0.2	0.9412	0.7538	**0.9774**	0.8098	0.9372	0.8044
	1	1	1	0	0.2950	0.1960	**0.2968**	0.1958	0.2224	0.1880
	0	0.2	0.7	0.5	0.5910	0.3736	**0.7028**	0.4216	0.6168	0.4414
	0	0.5	1	0.5	0.8956	0.6698	**0.9516**	0.7364	0.8958	0.7450

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD, RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.27. Mixed Design for Balanced Incomplete Block Design, Randomized Complete Block Design and Completely Randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 6, CRD=18, RCBD=6, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0542	0.0494	0.0494	0.0512	0.0478	0.0484
	0	0.5	0.5	0	0.5440	0.5192	**0.6018**	0.5190	0.5372	0.5196
	0	1	1	0.2	0.9432	0.9238	**0.9704**	0.9332	0.9426	0.9294
	0	0	1	0.2	0.9418	0.9258	**0.9722**	0.9292	0.9352	0.9318
	1	1	1	0	**0.3030**	0.2606	0.2904	0.2618	0.2370	0.2714
	0	0.2	0.7	0.5	0.5644	0.5510	**0.6644**	0.5696	0.6076	0.5668
	0	0.5	1	0.5	0.9026	0.8772	**0.9406**	0.8698	0.9050	0.8836
Exponential	0	0	0	0	0.0516	0.0466	0.0512	0.0460	0.0444	0.0542
	0	0.5	0.5	0	0.8094	0.7728	**0.8694**	0.7990	0.8078	0.7880
	0	1	1	0.2	0.9922	0.9876	**0.9984**	0.9882	0.9918	0.9920
	0	0	1	0.2	0.9958	0.9962	**0.9988**	0.9970	0.9946	0.9976
	1	1	1	0	0.3678	0.3308	**0.3658**	0.3308	0.2860	0.3426
	0	0.2	0.7	0.5	0.8688	0.8556	**0.9276**	0.8626	0.8720	0.8642
	0	0.5	1	0.5	0.9930	0.9898	**0.9988**	0.9914	0.9912	0.9922
T with 3 degrees of freedom	0	0	0	0	0.0484	0.0552	0.0508	0.0526	0.0512	0.0484
	0	0.5	0.5	0	0.4110	0.3984	**0.4664**	0.3770	0.3964	0.3930
	0	1	1	0.2	0.8270	0.7926	**0.8844**	0.8058	0.8234	0.7930
	0	0	1	0.2	0.8268	0.7980	**0.8798**	0.8086	0.8260	0.8102
	1	1	1	0	**0.2238**	0.2112	0.2232	0.2122	0.1862	0.2078
	0	0.2	0.7	0.5	0.4510	0.4200	**0.5048**	0.4256	0.4614	0.4388
	0	0.5	1	0.5	0.7610	0.7316	**0.8226**	0.7424	0.7696	0.7268

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD, RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.28. Mixed Design for Balanced Incomplete Block Design, Randomized Complete Block Design and Completely Randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 12, CRD=18, RCBD=12, Peak=3 (Tow missing observation)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0524	0.0502	0.0520	0.0442	0.0478	0.0546
	0	0.5	0.5	0	0.6748	0.5238	**0.7550**	0.5570	**0.6536**	0.5506
	0	1	1	0.2	0.9860	0.9436	**0.9960**	0.9452	**0.9876**	0.9512
	0	0	1	0.2	0.9840	0.9366	**0.9974**	0.9474	**0.9870**	0.9476
	1	1	1	0	0.3668	0.2706	**0.3716**	0.2770	**0.2918**	0.2768
	0	0.2	0.7	0.5	0.7226	0.5650	**0.8098**	0.5908	**0.7454**	0.6024
	0	0.5	1	0.5	0.9682	0.8920	**0.9910**	0.9118	**0.9724**	0.9096
Exponential	0	0	0	0	0.0492	0.0506	0.0464	0.0462	0.0494	0.0554
	0	0.5	0.5	0	0.9152	0.8006	**0.9578**	0.8252	0.9196	0.8264
	0	1	1	0.2	0.9996	0.9924	**1.0000**	0.9958	0.9994	0.9946
	0	0	1	0.2	**1.0000**	0.9978	**1.0000**	0.9976	**1.0000**	0.9988
	1	1	1	0	0.4788	0.3578	**0.4852**	0.3560	0.3598	0.3398
	0	0.2	0.7	0.5	0.9538	0.8706	**0.9862**	0.8876	0.9572	0.8822
	0	0.5	1	0.5	0.9992	0.9942	**1.0000**	0.9950	0.9994	0.9950
T with 3 degrees of freedom	0	0	0	0	0.0512	0.0404	0.0500	0.0510	0.0482	0.0488
	0	0.5	0.5	0	0.5316	0.4034	**0.5888**	0.4264	0.5086	0.4198
	0	1	1	0.2	0.9280	0.8154	**0.9596**	0.8280	0.9256	0.8360
	0	0	1	0.2	0.9208	0.8154	**0.9646**	0.8306	0.9222	0.8372
	1	1	1	0	0.2732	0.2176	**0.2822**	0.2168	0.2228	0.2064
	0	0.2	0.7	0.5	0.5644	0.4316	**0.6460**	0.4498	0.5850	0.4758
	0	0.5	1	0.5	0.8748	0.7506	**0.9340**	0.7620	0.8802	0.7686

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD, RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.29. Mixed Design for Balanced Incomplete Block Design, Randomized Complete Block Design and Completely Randomized Design for Treatment Four at peak 3. Treatments=4, BIBD =18, CRD=18, RCBD=18, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance		Squared Distance	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0504	0.0506	0.0548	0.0484	0.0498	0.0466
	0	0.5	0.5	0	0.7616	0.5414	**0.8212**	0.5770	0.7540	0.5802
	0	1	1	0.2	0.9966	0.9468	**0.9990**	0.9632	0.9964	0.9586
	0	0	1	0.2	0.9954	0.9504	**0.9992**	0.9628	0.9938	0.9622
	1	1	1	0	0.4280	0.2790	**0.4474**	0.2938	0.3262	0.2626
	0	0.2	0.7	0.5	0.8130	0.5854	**0.8876**	0.6324	0.8314	0.6414
	0	0.5	1	0.5	0.9896	0.8992	**0.9972**	0.9314	0.9910	0.9286
Exponential	0	0	0	0	0.0548	0.0596	0.0524	0.0502	0.0504	0.0494
	0	0.5	0.5	0	0.9604	0.8244	**0.9826**	0.8570	0.9564	0.8600
	0	1	1	0.2	**1.0000**	0.9964	**1.0000**	0.9970	**1.0000**	0.9968
	0	0	1	0.2	**1.0000**	0.9988	1.0000	0.9996	**1.0000**	0.9994
	1	1	1	0	0.5712	0.3724	**0.5668**	0.3852	0.4196	0.3596
	0	0.2	0.7	0.5	0.9808	0.8846	**0.9958**	0.9062	0.9868	0.9134
	0	0.5	1	0.5	**1.0000**	0.9944	**1.0000**	0.9972	**1.0000**	0.9974
T with 3 degrees of freedom	0	0	0	0	0.0506	0.0536	0.0524	0.0476	0.0470	0.0474
	0	0.5	0.5	0	0.6084	0.4124	**0.6738**	0.4390	0.5796	0.4370
	0	1	1	0.2	0.9622	0.8338	**0.9868**	0.8538	0.9652	0.8582
	0	0	1	0.2	0.9656	0.8382	**0.9870**	0.8650	0.9648	0.8552
	1	1	1	0	**0.3386**	0.2120	0.3312	0.2290	0.2446	0.2152
	0	0.2	0.7	0.5	0.6418	0.4466	**0.7430**	0.4856	0.6766	0.4700
	0	0.5	1	0.5	0.9340	0.7638	**0.9728**	0.8050	0.9394	0.7946

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD, RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.30. Mixed Design for Balanced Incomplete Block Design and Complete randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 12, CRD=6, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance-modification		Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0472	0.0524	0.0510	0.0512	0.0474	0.0490
	0	0.5	0.5	0	0.3084	0.2682	**0.3152**	0.2670	0.3052	0.2614
	0	1	1	0.2	**0.6824**	0.5944	0.6732	0.5810	0.6790	0.5818
	0	0	1	0.2	**0.6824**	0.5856	0.6774	0.5836	0.6682	0.5938
	1	1	1	0	**0.1720**	0.1446	0.1704	0.1382	0.1674	0.1550
	0	0.2	0.7	0.5	**0.3448**	0.2898	0.3272	0.2820	0.3370	0.2756
	0	0.5	1	0.5	**0.6054**	0.5330	0.6024	0.5270	0.5944	0.5168
Exponential	0	0	0	0	0.0490	0.0494	0.0492	0.0450	0.0460	0.0464
	0	0.5	0.5	0	**0.5136**	0.4274	0.5082	0.4510	0.5036	0.4218
	0	1	1	0.2	**0.8454**	0.7752	**0.8574**	0.7740	0.8508	0.7630
	0	0	1	0.2	**0.8644**	0.8004	0.8570	0.7994	0.8594	0.7974
	1	1	1	0	0.2020	0.1722	**0.2072**	0.1662	0.2044	0.1674
	0	0.2	0.7	0.5	0.5460	0.4646	**0.5598**	0.4692	0.5554	0.4674
	0	0.5	1	0.5	**0.8346**	0.7686	**0.8366**	0.7678	0.8362	0.7806
T with 3 degrees of freedom	0	0	0	0	0.0494	0.0472	0.0500	0.0526	0.0440	0.0504
	0	0.5	0.5	0	0.2324	0.2102	**0.2472**	0.2022	0.2408	0.2108
	0	1	1	**0.2**	0.5166	0.4434	0.5124	0.4460	**0.5258**	0.4346
	0	0	1	0.2	**0.5238**	0.4472	0.5110	0.4344	0.5022	0.4400
	1	1	1	0	**0.1478**	0.1152	0.1456	0.1302	0.1396	0.1166
	0	0.2	0.7	0.5	**0.2628**	0.2240	0.2512	0.2148	0.2582	0.2306
	0	0.5	1	0.5	**0.4616**	0.3910	0.4574	0.3828	0.4552	0.3896

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.31. Mixed Design for Balanced Incomplete Block Design and Complete randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 18, CRD=6, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0456	0.0480	0.0534	0.0492	0.0492	0.0474
	0	0.5	0.5	0	0.3416	0.2834	0.3464	0.2860	**0.3482**	0.2846
	0	1	1	0.2	**0.7518**	0.6072	0.7494	0.6176	0.7370	0.6264
	0	0	1	0.2	0.7420	0.6268	0.7400	0.6294	**0.7470**	0.6270
	1	1	1	0	0.1954	0.1468	0.1824	0.1480	**0.1988**	0.1524
	0	0.2	0.7	0.5	**0.3730**	0.3052	0.3766	0.3110	0.3712	0.2900
	0	0.5	1	0.5	**0.6840**	0.5596	0.6610	0.5404	0.6756	0.5506
Exponential	0	0	0	0	0.0428	0.0454	0.0504	0.0526	0.0484	0.0498
	0	0.5	0.5	0	0.5614	0.4586	0.5542	0.4526	**0.5700**	0.4502
	0	1	1	0.2	**0.9050**	0.7970	0.8954	0.7996	0.8994	0.8080
	0	0	1	0.2	**0.9196**	0.8252	0.9094	0.8330	0.9104	0.8404
	1	1	1	0	**0.2404**	0.1648	0.2244	0.1682	0.2402	0.1716
	0	0.2	0.7	0.5	**0.6224**	0.5040	0.6208	0.4892	0.6198	0.4994
	0	0.5	1	0.5	0.8960	0.7946	**0.9016**	0.8136	0.8932	0.8068
T with 3 degrees of freedom	0	0	0	0	0.0480	0.0474	0.0494	0.0472	0.0512	0.0554
	0	0.5	0.5	0	**0.2690**	0.2280	**0.2732**	0.2076	**0.2538**	0.2204
	0	1	1	0.2	**0.5786**	0.4676	0.5836	0.4634	**0.5844**	0.4592
	0	0	1	0.2	**0.5868**	0.4686	0.5810	0.4732	0.5842	0.4746
	1	1	1	0	**0.1560**	0.1214	0.1498	0.1276	0.1508	0.1272
	0	0.2	0.7	0.5	0.2746	0.2296	0.2886	0.2238	**0.2912**	0.2268
	0	0.5	1	0.5	**0.5202**	0.4324	0.5144	0.4176	0.5066	0.4140

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.32. Mixed Design for Balanced Incomplete Block Design and Complete randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 6, CRD=12, Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0536	0.0488	0.0468	0.0502	0.0526	0.0520
	0	0.5	0.5	0	0.3628	0.3696	0.3640	0.3786	0.3584	**0.3844**
	0	1	1	0.2	0.7652	0.7928	0.7596	0.7876	0.7580	**0.7988**
	0	0	1	0.2	0.7570	**0.7972**	0.7530	0.7946	0.7558	0.7940
	1	1	1	0	0.1886	0.2000	0.2012	0.2022	0.1962	**0.2032**
	0	0.2	0.7	0.5	0.3894	0.4148	0.3984	**0.4216**	0.3814	0.4208
	0	0.5	1	0.5	0.6894	0.7164	0.6818	0.7150	0.6828	**0.7210**
Exponential	0	0	0	0	0.0496	0.0538	0.0480	0.0536	0.0476	0.0482
	0	0.5	0.5	0	0.5750	**0.6242**	0.5812	0.6094	0.5828	0.6236
	0	1	1	0.2	0.9088	0.9360	0.9080	0.9364	0.9056	**0.9380**
	0	0	1	0.2	0.9212	0.9634	0.9232	**0.9658**	0.9136	0.9610
	1	1	1	0	0.2298	0.2316	0.2258	**0.2396**	0.2278	0.2348
	0	0.2	0.7	0.5	0.6218	**0.6940**	0.6430	0.6822	0.6342	0.6776
	0	0.5	1	0.5	0.9064	0.9374	0.9116	**0.9396**	0.9030	0.9388
T with 3 degrees of freedom	0	0	0	0	0.0472	0.0564	0.0496	0.0516	0.0518	0.0530
	0	0.5	0.5	0	0.2770	0.2856	0.2740	**0.2916**	0.2632	0.2854
	0	1	1	0.2	0.6098	0.6240	0.5956	0.6278	0.6004	**0.6312**
	0	0	1	0.2	0.5982	0.6366	0.5912	0.6240	0.6056	0.6332
	1	1	1	0	0.1556	0.1644	0.1534	0.1612	0.1632	**0.1706**
	0	0.2	0.7	0.5	0.2906	0.3042	0.2964	0.3140	0.3002	**0.3158**
	0	0.5	1	0.5	0.5302	**0.5674**	0.5314	0.5538	0.5372	0.5514

Selected results for Treatments 4 at Peak 2 for a Mixed Design of BIBD and CRD. With the sample size being twice the number of Blocks, Standardized Last performed better in all distinct tests (Non-modification, Distance-Modification modification and squared modification) thank standardized last.

Table B.33. Mixed Design for Balanced Incomplete Block Design and Complete randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 12, CRD=12 Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0496	0.0536	0.0446	0.0442	0.0450	0.0458
	0	0.5	0.5	0	**0.4190**	0.4000	**0.4246**	0.3914	**0.4232**	0.3924
	0	1	1	0.2	**0.8404**	0.7994	**0.8288**	0.8012	**0.8336**	0.8122
	0	0	1	0.2	0.8402	0.7990	0.8340	0.8128	0.8312	0.8036
	1	1	1	0	**0.2150**	0.1982	**0.2148**	0.1976	**0.2184**	0.1890
	0	0.2	0.7	0.5	**0.4414**	0.4270	**0.4440**	0.4136	**0.4464**	0.4124
	0	0.5	1	0.5	**0.7648**	0.7380	**0.7674**	0.7336	**0.7730**	0.7402
Exponential	0	0	0	0	0.0480	0.0502	0.0576	0.0492	0.0558	0.0514
	0	0.5	0.5	0	**0.6704**	0.6248	**0.6778**	0.6314	**0.6632**	0.6274
	0	1	1	0.2	**0.9510**	0.9390	**0.9570**	0.9378	**0.9542**	0.9372
	0	0	1	0.2	0.9630	0.9642	0.9612	0.9700	0.9632	0.9670
	1	1	1	0	**0.2738**	0.2534	**0.2806**	0.2548	**0.2812**	0.2408
	0	0.2	0.7	0.5	**0.7238**	0.6938	**0.7220**	0.6908	**0.7282**	0.6982
	0	0.5	1	0.5	**0.9564**	0.9466	**0.9530**	0.9444	**0.9506**	0.9430
T with 3 degrees of freedom	0	0	0	0	0.0462	0.0488	0.0522	0.0526	0.0502	0.0478
	0	0.5	0.5	0	**0.3216**	0.2910	**0.3102**	0.2900	**0.3018**	0.3064
	0	1	1	0.2	**0.6776**	0.6332	0.6772	0.6432	**0.6776**	0.6332
	0	0	1	0.2	0.6786	0.6454	0.6758	0.6374	**0.6788**	0.6464
	1	1	1	**0**	**0.1816**	0.1618	0.1758	0.1700	0.1760	0.1638
	0	0.2	0.7	0.5	0.3222	0.3110	**0.3442**	0.3242	0.3368	0.3152
	0	0.5	1	0.5	0.6130	0.5620	**0.6178**	0.5756	0.6034	0.5726

Selected results for Treatments 4 at Peak 2 for a Mixed Design of BIBD and CRD. With the sample size being twice the number of Blocks, Standardized Last performed better in all distinct tests (Non-modification, Distance-Modification modification and squared modification) thank standardized last. Results as far as how the distinct tests performed relative to one another were similar for all distributions.

Table B.34. Mixed Design for Balanced Incomplete Block Design and Complete randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 18, CRD=12 Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0520	0.0542	0.0428	0.0560	0.0464	0.0524
	0	0.5	0.5	0	0.4372	0.3902	0.4558	0.3896	**0.4678**	0.3926
	0	1	1	0.2	0.8834	0.8086	**0.8914**	0.8090	0.8834	0.8132
	0	0	1	0.2	0.8852	0.8166	0.8778	0.8236	0.8756	0.8174
	1	1	1	0	0.2414	0.2020	0.2436	0.2078	**0.2490**	0.2136
	0	0.2	0.7	0.5	**0.5080**	0.4266	0.5018	0.4230	0.4970	0.4304
	0	0.5	1	0.5	0.8190	0.7412	**0.8306**	0.7398	0.8176	0.7296
Exponential	0	0	0	0	0.0502	0.0510	0.0512	0.0534	0.0498	0.0482
	0	0.5	0.5	0	0.7188	0.6364	0.7180	0.6390	**0.7194**	0.6414
	0	1	1	0.2	**0.9742**	0.9454	**0.9764**	0.9478	0.9736	0.9452
	0	0	1	0.2	0.9814	0.9680	0.9802	0.9676	0.9832	0.9646
	1	1	1	0	0.3062	0.2388	**0.3130**	0.2474	0.3044	0.2498
	0	0.2	0.7	0.5	0.7780	0.7046	**0.7854**	0.6948	0.7752	0.6918
	0	0.5	1	0.5	0.9718	0.9470	0.9718	0.9516	**0.9778**	0.9440
T with 3 degrees of freedom	0	0	0	0	0.0482	0.0468	0.0522	0.0534	0.0454	0.0514
	0	0.5	0.5	0	0.3454	0.3110	**0.3466**	0.2908	0.3418	0.3012
	0	1	1	0.2	0.7300	0.6552	0.7482	0.6524	**0.7490**	0.6550
	0	0	1	0.2	0.7322	0.6498	0.7380	0.6648	0.7452	0.6556
	1	1	1	0	0.1866	0.1688	0.1880	0.1684	**0.1928**	0.1620
	0	0.2	0.7	0.5	**0.3852**	0.3188	0.3786	0.3280	0.3780	0.3268
	0	0.5	1	0.5	**0.6678**	0.5830	0.6506	0.5860	0.6514	0.5744

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD, RCBD and CRD). Results show Standardized Last test performed better than Standardized First in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.35. Mixed Design for Balanced Incomplete Block Design and Complete randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 12, CRD=18 Peak=3 (Two missing observation)

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0504	0.0518	0.0498	0.0496	0.0510	0.0510
	0	0.5	0.5	0	0.4920	**0.5128**	0.4964	0.5112	0.5044	0.5074
	0	1	1	0.2	0.9206	**0.9244**	0.9150	0.9162	0.9204	0.9224
	0	0	1	0.2	0.9188	**0.9274**	0.9132	0.9256	0.9126	0.9202
	1	1	1	0	0.2616	0.2616	0.2606	**0.2684**	0.2574	0.2594
	0	0.2	0.7	0.5	0.5384	0.5404	0.5252	0.5534	0.5442	**0.5580**
	0	0.5	1	0.5	0.8596	**0.8734**	0.8594	0.8672	0.8686	0.8696
Exponential	0	0	0	0	0.0512	0.0492	0.0442	0.0550	0.0484	0.0546
	0	0.5	0.5	0	0.7726	**0.7754**	0.7688	0.7708	0.7632	0.7658
	0	1	1	0.2	0.9818	**0.9900**	0.9868	0.9886	0.9858	0.9866
	0	0	1	0.2	0.9926	0.9948	0.9892	0.9958	0.9888	**0.9950**
	1	1	1	0	0.3362	**0.3406**	0.3228	0.3388	0.3296	0.3374
	0	0.2	0.7	0.5	0.8176	**0.8418**	0.8254	0.8338	0.8316	0.8318
	0	0.5	1	0.5	0.9826	**0.9922**	0.9874	0.9890	0.9852	0.9902
T with 3 degrees of freedom	0	0	0	0	0.0460	0.0546	0.0498	0.0552	0.0542	0.0458
	0	0.5	0.5	0	0.3622	0.3806	0.3762	**0.3902**	0.3752	0.3766
	0	1	1	0.2	0.7846	**0.7940**	0.7742	0.7816	0.7800	0.7874
	0	0	1	0.2	0.7804	0.7824	0.7816	0.7840	0.7788	0.7816
	1	1	1	0	0.2002	0.2008	0.1994	**0.2098**	0.1972	0.2008
	0	0.2	0.7	0.5	0.4110	**0.4256**	0.4080	0.4080	0.4108	0.4162
	0	0.5	1	0.5	0.7006	0.7122	0.7114	**0.7200**	0.7120	0.7160

Selected results for Treatments 4 at Peak 2 for a Mixed Design of BIBD and CRD. When the ratio of the number of Blocks is $\frac{2}{3}$ the sample size, Standardized Last performed better in all distinct tests (Non-modification, Distance-Modification modification and squared modification) than Standardized First.

Table B.36. Mixed Design for Balanced Incomplete Block Design and Complete randomized Design for Treatment Four at peak 3. Treatments=4, BIBD = 18, CRD=18 Peak=3 (Two missing observation)

Distribution			Location		Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0468	0.0532	0.0586	0.0474	0.0514	0.0530
	0	0.5	0.5	0	0.5398	0.5098	0.5454	0.4938	**0.5460**	0.5062
	0	1	1	0.2	0.9380	0.9212	0.9426	0.9214	**0.9502**	0.9214
	0	0	1	0.2	0.9424	0.9190	**0.9450**	0.9310	0.9420	0.9184
	1	1	1	0	**0.2952**	0.2640	0.2880	0.2678	0.2950	0.2684
	0	0.2	0.7	0.5	0.5888	0.5426	**0.5946**	0.5596	0.5942	0.5624
	0	0.5	1	0.5	0.8974	0.8808	0.9030	0.8742	**0.9038**	0.8698
Exponential	0	0	0	0	0.0566	0.0508	0.0502	0.0474	0.0564	0.0532
	0	0.5	0.5	0	0.8170	0.7812	0.8164	0.7770	**0.8200**	0.7730
	0	1	1	0.2	0.9922	0.9880	**0.9942**	0.9884	0.9934	0.9872
	0	0	1	0.2	0.9958	0.9948	0.9954	0.9956	**0.9966**	0.9946
	1	1	1	0	0.3638	0.3382	0.3752	0.3300	**0.3810**	0.3394
	0	0.2	0.7	0.5	0.8654	0.8476	0.8658	0.8432	**0.8660**	0.8438
	0	0.5	1	0.5	0.9914	0.9888	0.9934	0.9872	**0.9962**	0.9914
T with 3 degrees of freedom	0	0	0	0	0.0542	0.0518	0.0454	0.0510	0.0480	0.0516
	0	0.5	0.5	0	**0.3960**	0.3810	**0.4284**	0.3650	0.4262	0.3914
	0	1	1	0.2	**0.8294**	0.7910	0.8222	0.7778	0.8190	0.7906
	0	0	1	0.2	**0.8386**	0.7928	0.8320	0.7924	0.8364	0.7850
	1	1	1	0	**0.2266**	0.2032	0.2222	0.2026	0.2254	0.2054
	0	0.2	0.7	0.5	**0.4646**	0.4162	0.4418	0.4204	0.4604	0.4214
	0	0.5	1	0.5	0.7634	0.7186	**0.7640**	0.7244	0.7636	0.7262

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.37. Mixed Design for Completely Randomized Design (CRD) and Randomized Complete Block Design for Treatment Four at peak 3. Treatments=4, RCBD = 12, CRD=6, Peak=3

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0488	0.0468	0.0490	0.0490	0.0478	0.0464
	0	0.5	0.5	0	**0.4360**	0.3250	0.4350	0.3456	0.4208	0.3812
	0	1	1	0.2	0.8612	0.7026	**0.8668**	0.7520	0.8580	0.8100
	0	0	1	0.2	0.8620	0.7202	**0.8662**	0.7534	0.8526	0.8068
	1	1	1	0	**0.2264**	0.1720	0.1808	0.1636	0.1560	0.1480
	0	0.2	0.7	0.5	0.4712	0.3682	0.4966	0.3846	**0.5208**	0.4602
	0	0.5	1	0.5	0.8112	0.6296	0.8072	0.6912	**0.8138**	0.7460
Exponential	0	0	0	0	0.0506	0.0508	0.0482	0.0466	0.0510	0.0500
	0	0.5	0.5	0	**0.7032**	0.5314	0.6776	0.5754	0.6662	0.6106
	0	1	1	0.2	**0.9718**	0.8660	0.9668	0.9054	0.9668	0.9394
	0	0	1	0.2	**0.9830**	0.9090	**0.9830**	0.9328	0.9816	0.9674
	1	1	1	0	**0.2872**	0.2052	0.2262	0.1996	0.1804	0.1836
	0	0.2	0.7	0.5	0.7650	0.5896	**0.7840**	0.6400	0.7826	0.7086
	0	0.5	1	0.5	0.9708	0.8764	**0.9710**	0.9116	0.9652	0.9452
T with 3 degrees of freedom	0	0	0	0	0.0480	0.0540	0.0522	0.0492	0.0508	0.0504
	0	0.5	0.5	0	**0.3346**	0.2562	0.3214	0.2632	0.3158	0.2920
	0	1	1	0.2	0.7066	0.5520	**0.7110**	0.5858	0.6978	0.6462
	0	0	1	0.2	0.7042	0.5510	0.7048	0.5778	0.6912	0.6586
	1	1	1	0	**0.1876**	0.1422	0.1466	0.1394	0.1322	0.1380
	0	0.2	0.7	0.5	0.3586	0.2760	0.3760	0.2902	**0.3890**	0.3354
	0	0.5	1	0.5	0.6452	0.4796	0.6472	0.5182	**0.6488**	0.5942

Selected results for Treatments 4 at peak 3 for mixed Designs of (RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.38. Mixed Design for Completely Randomized Design (CRD) and Randomized Complete Block Design for Treatment Four at peak 3. Treatments=4, RCBD = 18, CRD=6, Peak=3

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0486	0.0456	0.0522	0.0494	0.0500	0.0464
	0	0.5	0.5	0	0.5120	0.3622	**0.5188**	0.3996	0.5046	0.4446
	0	1	1	0.2	**0.9292**	0.7698	0.9248	0.8334	0.9230	0.8864
	0	0	1	0.2	**0.9288**	0.7760	0.9244	0.8296	0.9218	0.8902
	1	1	1	0	**0.2470**	0.1696	0.2314	0.1768	0.1626	0.1540
	0	0.2	0.7	0.5	0.5474	0.3952	0.5838	0.4522	**0.5948**	0.5558
	0	0.5	1	0.5	0.8688	0.7072	**0.8844**	0.7706	**0**.8790	0.8490
Exponential	0	0	0	0	0.0452	0.0496	0.0484	0.0516	0.0484	0.0448
	0	0.5	0.5	0	**0.7860**	0.5946	0.7854	0.6492	0.7640	0.7218
	0	1	1	0.2	**0.9912**	0.9260	0.9892	0.9552	0.9906	0.9762
	0	0	1	0.2	0.9964	0.9522	0.9966	0.9724	0.9950	0.9898
	1	1	1	0	**0.3276**	0.2264	0.2578	0.2044	0.1920	0.1808
	0	0.2	0.7	0.5	0.8544	0.6724	**0.8662**	0.7256	0.8648	0.8162
	0	0.5	1	0.5	0.9894	0.9202	**0.9926**	0.9512	0.9904	0.9758
T with 3 degrees of freedom	0	0	0	0	0.0454	0.0426	0.0458	0.0436	0.0518	0.0538
	0	0.5	0.5	0	**0.3920**	0.2688	0.3766	0.2966	0.3720	0.3456
	0	1	1	0.2	**0.8008**	0.6198	0.7926	0.6606	0.7908	0.7462
	0	0	1	0.2	0.8014	0.6016	0.8054	0.6722	0.7850	0.7454
	1	1	1	0	**0.2062**	0.1436	0.1738	0.1560	0.1374	0.1366
	0	0.2	0.7	0.5	0.4130	0.2826	0.4276	0.3424	**0.4440**	0.4030
	0	0.5	1	0.5	0.7320	0.5434	0.7362	0.6090	**0.7424**	0.6900

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.39. Mixed Design for Completely Randomized Design (CRD) and Randomized Complete Block Design for Treatment Four at peak 3. Treatments=4, RCBD= 18, CRD=12, Peak=3

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0454	0.0472	0.0536	0.0548	0.0500	0.0510
	0	0.5	0.5	0	**0.4538**	0.4114	0.4388	0.3750	0.4396	0.4112
	0	1	1	0.2	**0.8812**	0.8220	0.8758	0.8212	0.8760	0.8264
	0	0	1	0.2	**0.8808**	0.8186	0.8782	0.8234	0.8710	0.8302
	1	1	1	0	**0.2294**	0.1914	0.2170	0.2032	0.1852	0.1954
	0	0.2	0.7	0.5	0.4888	0.4166	0.5066	0.4444	**0.5266**	0.4564
	0	0.5	1	0.5	0.8068	0.7486	0.8208	0.7484	**0.8290**	0.7712
Exponential	0	0	0	0	0.0492	0.0534	0.0504	0.0514	0.0546	0.0504
	0	0.5	0.5	0	**0.7128**	0.6464	0.7012	0.6578	0.6954	0.6488
	0	1	1	0.2	**0.9774**	0.9460	0.9764	0.9526	0.9730	0.9568
	0	0	1	0.2	0.9848	0.9726	**0.9852**	0.9694	0.9832	0.9760
	1	1	1	0	**0.2742**	0.2464	0.2420	0.2368	0.2166	0.2466
	0	0.2	0.7	0.5	0.7708	0.6974	**0.7878**	0.6994	0.7846	0.7280
	0	0.5	1	0.5	0.9764	0.9496	0.9762	0.9488	**0.9792**	0.9620
T with 3 degrees of freedom	0	0	0	0	0.0538	0.0530	0.0532	0.0510	0.0518	0.0530
	0	0.5	0.5	0	0.3408	0.3016	0.3416	0.2910	**0.3450**	0.3054
	0	1	1	0.2	0.7274	0.6578	0.7318	0.6634	**0.7342**	0.6788
	0	0	1	0.2	**0.7282**	0.6580	0.7270	0.6644	0.7266	0.6906
	1	1	1	0	**0.1870**	0.1580	0.1762	0.1612	0.1518	0.1610
	0	0.2	0.7	0.5	0.3790	0.3212	0.3666	0.3220	**0.3860**	0.3446
	0	0.5	1	0.5	0.6546	0.5848	**0.6742**	0.5848	0.6730	0.6214

Selected results for Treatments 4 at peak 3 for mixed Designs of (BIBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table B.40. Mixed Design for Completely Randomized Design (CRD) and Randomized Complete Block Design for Treatment Four at peak 3. Treatments=4, RCBD = 12, CRD=18 Peak=3

Distribution	Location				Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0510	0.0548	0.0510	0.0466	0.0484	0.0450
	0	0.5	0.5	0	**0.6330**	0.5144	0.6282	0.5236	0.6210	0.5464
	0	1	1	0.2	0.9748	0.9326	**0.9802**	0.9302	0.9768	0.9490
	0	0	1	0.2	0.9806	0.9354	0.9796	0.9326	0.9794	0.9462
	1	1	1	0	**0.3348**	0.2672	0.2886	0.2624	0.2414	0.2640
	0	0.2	0.7	0.5	0.6836	0.5724	0.7134	0.5688	**0.7200**	0.6042
	0	0.5	1	0.5	**0.9536**	0.8850	0.9618	0.8860	**0.9570**	0.9004
Exponential	0	0	0	0	0.0500	0.0474	0.0522	0.0468	0.0498	0.0590
	0	0.5	0.5	0	**0.8916**	0.7928	0.8900	0.7964	0.8840	0.8030
	0	1	1	0.2	**0.9992**	0.9894	0.9988	0.9898	0.9990	0.9956
	0	0	1	0.2	**1.0000**	0.9976	0.9998	0.9978	0.9992	0.9978
	1	1	1	0	**0.4360**	0.3342	0.3682	0.3352	0.3060	0.3400
	0	0.2	0.7	0.5	0.9360	0.8550	**0.9466**	0.8660	0.9394	0.8742
	0	0.5	1	0.5	0.9992	0.9914	**0.9996**	0.9900	0.9982	0.9932
T with 3 degrees of freedom	0	0	0	0	0.0448	0.0524	0.0536	0.0492	0.0552	0.0496
	0	0.5	0.5	0	**0.4832**	0.3864	0.4752	0.4040	0.4674	0.4034
	0	1	1	0.2	**0.9050**	0.8068	0.9006	0.8088	0.9016	0.8182
	0	0	1	0.2	0.9160	0.8104	0.9094	0.8094	0.8972	0.8262
	1	1	1	0	**0.2556**	0.2068	0.2212	0.2098	0.1986	0.2050
	0	0.2	0.7	0.5	0.5292	0.4292	0.5414	0.4326	**0.5616**	0.4536
	0	0.5	1	0.5	0.8468	0.7470	**0.8644**	0.7452	0.8538	0.7712

Selected results for Treatments 4 at peak 3 for mixed Designs of (RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

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Table B.41. Mixed Design for Completely Randomized Design (CRD) and Randomized Complete Block Design for Treatment Four at peak 3. Treatments=4, RCBD = 18, CRD=18 Peak=3

Distribution	Location				modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last	First	Last
Normal	0	0	0	0	0.0458	0.0518	0.0462	0.0490	0.0472	0.0516
	0	0.5	0.5	0	**0.6982**	0.5264	0.6908	0.5356	0.6452	0.5600
	0	1	1	0.2	**0.9926**	0.9358	0.9920	0.9414	0.9904	0.9528
	0	0	1	0.2	**0.9942**	0.9396	0.9930	0.9488	0.9912	0.9564
	1	1	1	0	**0.3756**	0.2718	0.3146	0.2782	0.2518	0.2714
	0	0.2	0.7	0.5	0.7530	0.5626	0.7744	0.5772	**0.7846**	0.6142
	0	0.5	1	0.5	0.9760	0.8944	**0.9858**	0.8974	0.9820	0.9144
Exponential	0	0	0	0	0.0482	0.0510	0.0534	0.0448	0.0472	0.0528
	0	0.5	0.5	0	**0.9396**	0.7994	0.9356	0.8154	0.9200	0.8362
	0	1	1	0.2	0.9996	0.9896	0.9998	0.9916	**1.0000**	0.9962
	0	0	1	0.2	**1.0000**	0.9978	0.9998	0.9978	0.9998	0.9994
	1	1	1	0	**0.5116**	0.3606	0.4124	0.3432	0.3244	0.3536
	0	0.2	0.7	0.5	0.9704	0.8622	**0.9756**	0.8774	0.9708	0.8904
	0	0.5	1	0.5	**0.9998**	0.9934	**0.9998**	0.9966	**0.9998**	0.9950
T with 3 degrees of freedom	0	0	0	0	0.0510	0.0502	0.0544	0.0496	0.0554	0.0524
	0	0.5	0.5	0	**0.5438**	0.4054	0.5480	0.4074	0.5380	0.4238
	0	1	1	0.2	0.9460	0.8088	**0.9468**	0.8118	0.9416	0.8462
	0	0	1	0.2	0.9420	0.8068	**0.9440**	0.8208	0.9358	0.8522
	1	1	1	0	**0.2952**	0.2106	0.2508	0.2098	0.1968	0.2152
	0	0.2	0.7	0.5	0.6020	0.4348	0.6240	0.4596	**0.6318**	0.4728
	0	0.5	1	0.5	0.8980	0.7334	0.9048	0.7610	**0.9074**	0.7804

Selected results for Treatments 4 at peak 3 for mixed Designs of (RCBD and CRD). Results show Standardized First performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

APPENDIX C. MIXED DESIGN OF BIBD, CRD AND RCBD FOR TREATMENTS 5 AT PEAKS 2, 3 AND 4; MIXED DESIGN OF BIBD AND CRD TREATMENTS 5 AT PEAKS 2, 3 AND 4; MIXED DESIGN OF CRD AND RCBD FOR TREATMENTS 5 AT PEAKS 2, 3 AND 4

Table C.1. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, BIBD =5, CRD=10, RCBD=5 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0460	0.0480	0.0520	0.0540	0.0506	0.0494
	1	1	0.5	0.5	0.5	0.2500	0.2306	**0.2736**	0.2400	0.2636	0.2594
	0.5	1	1	1	0.7	**0.2416**	0.2172	0.2402	0.2242	0.2164	0.2364
	0.7	1	0.7	0.7	0.5	0.3266	0.2938	**0.3384**	0.3012	0.3234	0.3344
	1	1	0.5	0.5	0.2	0.4812	0.4178	**0.4974**	0.4462	0.4908	0.4876
	0.75	1	0.75	0.5	0.25	0.5904	0.5214	**0.6088**	0.5416	0.5704	0.5800
	1	1	1	1	0	0.6172	0.5436	**0.6492**	0.5646	0.6174	0.6124
	1	1	0.5	0.2	0	0.7112	0.6298	**0.7258**	0.6658	0.7116	0.7232
	0	1.6	0.8	0.4	0.2	0.9924	0.9848	**0.9918**	0.9856	0.9820	0.9910
Exponential	0	0	0	0	0	0.0516	0.0458	0.0534	0.0492	0.0510	0.0560
	1	1	0.5	0.5	0.5	0.4034	0.3384	**0.4360**	0.3822	0.4290	0.4168
	0.5	1	1	1	0.7	0.4180	0.3660	0.3998	0.3620	0.3710	0.4066
	0.7	1	0.7	0.7	0.5	0.5916	0.5092	**0.6012**	0.5456	0.5512	0.5772
	1	1	0.5	0.5	0.2	0.7424	0.6632	**0.7518**	0.7066	0.7494	0.7488
	0.75	1	0.75	0.5	0.25	0.8614	0.7980	**0.8706**	0.8226	0.8436	0.8582
	1	1	1	1	0	0.7690	0.6766	**0.7780**	0.7130	0.7820	0.7754
	1	1	0.5	0.2	0	0.9322	0.8836	**0.9352**	0.8994	0.9074	0.9288
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9990	0.9994	0.9990	0.9978	0.9998
T with 3 degrees of freedom	0	0	0	0	0	0.0524	0.0516	0.0468	0.0480	0.0478	0.0528
	1	1	0.5	0.5	0.5	0.2030	0.1802	**0.2162**	0.1906	0.2102	0.1972
	0.5	1	1	1	0.7	0.1844	0.1688	**0.1852**	0.1718	0.1722	0.1764
	0.7	1	0.7	0.7	0.5	0.2566	0.2358	**0.2602**	0.2392	0.2406	0.2482
	1	1	0.5	0.5	0.2	0.3706	0.3158	**0.3848**	0.3236	0.3846	0.3672
	0.75	1	0.75	0.5	0.25	0.4574	0.4026	**0.4608**	0.4034	0.4472	0.4370
	1	1	1	1	0	0.4700	0.3932	**0.4894**	0.4180	0.4770	0.4790
	1	1	0.5	0.2	0	0.5366	0.4864	**0.5892**	0.5132	0.5522	0.5582
	0	1.6	0.8	0.4	0.2	**0.9400**	0.8978	0.9388	0.9138	0.9204	0.9332

Results show Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.2. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, BIBD =5, CRD=15, RCBD=5 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0520	0.0558	0.0530	0.0534	0.0496	0.0492
	1	1	0.5	0.5	0.5	0.2928	0.2766	**0.3118**	0.2928	0.3008	0.3000
	0.5	1	1	1	0.7	**0.2828**	0.2538	0.2620	0.2544	**0.2828**	0.2470
	0.7	1	0.7	0.7	0.5	0.3906	0.3580	**0.3934**	0.3868	0.3864	0.3686
	1	1	0.5	0.5	0.2	0.5706	0.5314	**0.5762**	0.5438	0.5670	0.5624
	0.75	1	0.75	0.5	0.25	0.6652	0.6338	**0.6904**	0.6374	0.6780	0.6590
	1	1	1	1	0	0.6930	0.6682	**0.7170**	0.6866	0.7132	0.7004
	1	1	0.5	0.2	0	0.7822	0.7650	0.8060	0.7860	**0.8064**	0.7926
	0	1.6	0.8	0.4	0.2	0.9976	0.9968	0.9974	0.9976	**0.9984**	0.9956
Exponential	0	0	0	0	0	0.0508	0.0484	0.0466	0.0466	0.0524	0.0536
	1	1	0.5	0.5	0.5	0.4698	0.4574	**0.5014**	0.4680	0.5030	0.4880
	0.5	1	1	1	0.7	**0.4892**	0.4418	0.4602	0.4534	0.4854	0.4292
	0.7	1	0.7	0.7	0.5	**0.6850**	0.6484	0.6784	0.6614	0.6744	0.6364
	1	1	0.5	0.5	0.2	0.8318	0.7968	**0.8454**	0.8228	0.8482	0.8248
	0.75	1	0.75	0.5	0.25	0.9216	0.9092	**0.9256**	0.9136	0.9230	0.9078
	1	1	1	1	0	0.8282	0.8078	**0.8544**	0.8218	0.8462	0.8500
	1	1	0.5	0.2	0	0.9672	0.9588	0.9642	0.9588	**0.9746**	0.9538
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	0.9998
T with 3 degrees of freedom	0	0	0	0	0	0.0572	0.0482	0.0496	0.0480	0.0514	0.0500
	1	1	0.5	0.5	0.5	0.2244	0.2206	0.2328	0.2276	**0.2354**	0.2368
	0.5	1	1	1	0.7	0.1974	0.2030	**0.2160**	0.2040	0.2114	0.2100
	0.7	1	0.7	0.7	0.5	**0.3038**	0.2750	0.2974	0.3008	0.2964	0.2754
	1	1	0.5	0.5	0.2	0.4214	0.3830	**0.4546**	0.4134	0.4410	0.4194
	0.75	1	0.75	0.5	0.25	0.4964	0.4822	**0.5224**	0.5014	0.5136	0.5000
	1	1	1	1	0	0.5320	0.5150	0.5542	0.5296	**0.5598**	0.5346
	1	1	0.5	0.2	0	0.6342	0.6062	**0.6644**	0.6194	0.6576	0.6336
	0	1.6	0.8	0.4	0.2	0.9728	0.9640	**0.9780**	0.9736	0.9620	0.9620

Results show Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.3. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, BIBD =10, CRD=5, RCBD=10 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0526	0.0530	0.0454	0.0462	0.0456	0.0442
	1	1	0.5	0.5	0.5	0.2830	0.2248	**0.3040**	0.2600	0.2790	0.2924
	0.5	1	1	1	0.7	**0.2576**	0.1970	0.2508	0.2178	0.2242	0.2070
	0.7	1	0.7	0.7	0.5	**0.3678**	0.2798	0.3668	0.3208	0.3482	0.3410
	1	1	0.5	0.5	0.2	0.5334	0.4100	**0.5654**	0.4864	0.5366	0.5506
	0.75	1	0.75	0.5	0.25	0.6322	0.5054	**0.6510**	0.5760	0.6238	0.6054
	1	1	1	1	0	0.6532	0.4988	**0.7040**	0.6244	0.6776	0.6832
	1	1	0.5	0.2	0	0.7682	0.5912	**0.7930**	0.7124	0.7628	0.7776
	0	1.6	0.8	0.4	0.2	0.9960	0.9686	**0.9968**	0.9860	0.9926	0.9872
Exponential	0	0	0	0	0	0.0462	0.0478	0.0486	0.0482	0.0562	0.0500
	1	1	0.5	0.5	0.5	0.4518	0.3272	**0.4758**	0.4158	0.4704	0.4648
	0.5	1	1	1	0.7	**0.4554**	0.3476	**0.4374**	0.3838	0.3840	0.3706
	0.7	1	0.7	0.7	0.5	**0.6480**	0.4828	0.6468	0.5564	0.5972	0.5884
	1	1	0.5	0.5	0.2	0.7948	0.6386	**0.8206**	0.7502	0.7838	0.8080
	0.75	1	0.75	0.5	0.25	0.9002	0.7782	**0.9030**	0.8480	0.8776	0.8722
	1	1	1	1	0	0.8188	0.6594	**0.8454**	0.7508	0.8254	0.8336
	1	1	0.5	0.2	0	0.9508	0.8496	**0.9584**	0.9228	0.9398	0.9490
	0	1.6	0.8	0.4	0.2	0.9996	0.9992	**0.9998**	0.9990	0.9990	0.9996
T with 3 degrees of freedom	0	0	0	0	0	0.0508	0.0480	0.0468	0.0498	0.0492	0.0532
	1	1	0.5	0.5	0.5	0.2116	0.1742	**0.2270**	0.1990	0.2236	0.2234
	0.5	1	1	1	0.7	0.2020	0.1758	**0.2024**	0.1774	0.1794	0.1786
	0.7	1	0.7	0.7	0.5	0.2798	0.2126	**0.2860**	0.2498	0.2560	0.2636
	1	1	0.5	0.5	0.2	0.4000	0.3010	**0.4236**	0.3688	0.4068	0.4104
	0.75	1	0.75	0.5	0.25	0.4796	0.3756	**0.5102**	0.4314	0.4714	0.4680
	1	1	1	1	0	0.5202	0.3772	**0.5458**	0.4616	0.5274	0.5380
	1	1	0.5	0.2	0	0.5976	0.4584	**0.6368**	0.5578	0.6058	0.6088
	0	1.6	0.8	0.4	0.2	**0.9710**	0.8780	0.9628	0.9236	0.9320	0.9258

Results show Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.4. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, BIBD =10, CRD=10, RCBD=10 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0510	0.0538	0.0464	0.0514	0.0546	0.0512
	1	1	0.5	0.5	0.5	0.3378	0.2384	**0.3534**	0.2640	0.3370	0.3118
	0.5	1	1	1	0.7	**0.3060**	0.2244	0.3042	0.2430	0.2798	0.2532
	0.7	1	0.7	0.7	0.5	**0.4410**	0.3130	**0.4322**	0.3330	0.4226	0.3882
	1	1	0.5	0.5	0.2	0.6284	0.4706	**0.6642**	0.4986	0.6196	0.5780
	0.75	1	0.75	0.5	0.25	0.7346	0.5524	**0.7468**	0.6016	0.7084	0.6504
	1	1	1	1	0	0.7602	0.5776	**0.7812**	0.6488	0.7696	0.7230
	1	1	0.5	0.2	0	0.8498	0.6754	**0.8754**	0.7338	0.8492	0.8078
	0	1.6	0.8	0.4	0.2	**0.9992**	0.9862	**0.9992**	0.9952	0.9986	0.9976
Exponential	0	0	0	0	0	0.0500	0.0456	0.0506	0.0516	0.0520	0.0526
	1	1	0.5	0.5	0.5	0.5458	0.3836	**0.5700**	0.4256	0.5488	0.4996
	0.5	1	1	1	0.7	**0.5398**	0.3954	0.5238	0.4114	0.4762	0.4422
	0.7	1	0.7	0.7	0.5	0.7452	0.5656	**0.7506**	0.6134	0.7030	0.6786
	1	1	0.5	0.5	0.2	0.8870	0.7236	**0.9018**	0.7720	0.8796	0.8510
	0.75	1	0.75	0.5	0.25	0.9610	0.8470	**0.9586**	0.8780	0.9390	0.9282
	1	1	1	1	0	0.8858	0.7278	**0.9066**	0.7862	0.8962	0.8644
	1	1	0.5	0.2	0	0.9822	0.9108	**0.9876**	0.9418	0.9776	0.9678
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9998	**1.0000**	0.9996	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0534	0.0508	0.0480	0.0526	0.0520	0.0486
	1	1	0.5	0.5	0.5	0.2656	0.1898	0.2738	0.2098	**0.2742**	0.2414
	0.5	1	1	1	0.7	**0.2334**	0.1704	0.2238	0.1836	0.2100	0.2046
	0.7	1	0.7	0.7	0.5	**0.3300**	0.2514	0.3288	0.2516	0.3264	0.2928
	1	1	0.5	0.5	0.2	0.4894	0.3464	**0.4976**	0.3778	0.4772	0.4506
	0.75	1	0.75	0.5	0.25	0.5724	0.4146	**0.5894**	0.4566	0.5424	0.5132
	1	1	1	1	0	0.5994	0.4318	**0.6182**	0.4852	0.6196	0.5584
	1	1	0.5	0.2	0	0.7104	0.5322	**0.7338**	0.5724	0.7016	0.6562
	0	1.6	0.8	0.4	0.2	0.9866	0.9262	**0.9884**	0.9476	0.9774	0.9656

Results show Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.5. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, BIBD =10, CRD=15, RCBD=10 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0530	0.0522	0.0504	0.0518	0.0516	0.0520
	1	1	0.5	0.5	0.5	0.3792	0.3012	**0.3956**	0.3070	0.3852	0.3374
	0.5	1	1	1	0.7	**0.3534**	0.2712	0.3356	0.2828	0.3024	0.2926
	0.7	1	0.7	0.7	0.5	**0.4958**	0.3654	0.4952	0.3990	0.4762	0.4320
	1	1	0.5	0.5	0.2	0.6832	0.5614	**0.7160**	0.5770	0.6934	0.6360
	0.75	1	0.75	0.5	0.25	0.7986	0.6582	**0.8106**	0.6740	0.7866	0.7342
	1	1	1	1	0	0.8210	0.6932	**0.8504**	0.7052	0.8356	0.7656
	1	1	0.5	0.2	0	0.9060	0.7800	**0.9170**	0.8152	0.9036	0.8644
	0	1.6	0.8	0.4	0.2	0.9998	0.9988	**1.0000**	0.9998	**1.0000**	0.9994
Exponential	0	0	0	0	0	0.0580	0.0536	0.0514	0.0486	0.0454	0.0506
	1	1	0.5	0.5	0.5	0.6192	0.4840	**0.6366**	0.4868	0.6288	0.5642
	0.5	1	1	1	0.7	**0.6036**	0.4634	**0.5878**	0.4870	0.5538	0.5112
	0.7	1	0.7	0.7	0.5	0.8136	0.6690	**0.8144**	0.7034	0.7794	0.7334
	1	1	0.5	0.5	0.2	0.9306	0.8322	**0.9328**	0.8522	0.9238	0.8902
	0.75	1	0.75	0.5	0.25	0.9776	0.9298	**0.9816**	0.9360	0.9740	0.9576
	1	1	1	1	0	0.9324	0.8146	**0.9442**	0.8590	0.9400	0.8924
	1	1	0.5	0.2	0	0.9948	0.9636	**0.9952**	0.9750	0.9924	0.9862
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0454	0.0570	0.0412	0.0474	0.0478	0.0530
	1	1	0.5	0.5	0.5	0.2836	0.2244	**0.3088**	0.2398	0.2938	0.2610
	0.5	1	1	1	0.7	0.2650	0.2122	**0.2698**	0.2202	0.2346	0.2272
	0.7	1	0.7	0.7	0.5	0.3660	0.2948	**0.3818**	0.3072	0.3488	0.3240
	1	1	0.5	0.5	0.2	0.5498	0.4236	**0.5538**	0.4454	0.5520	0.4868
	0.75	1	0.75	0.5	0.25	0.6314	0.5086	**0.6462**	0.5198	0.6252	0.5804
	1	1	1	1	0	0.6666	0.5450	**0.7058**	0.5588	0.6838	0.6054
	1	1	0.5	0.2	0	0.7590	0.6298	**0.7970**	0.6442	0.7752	0.7058
	0	1.6	0.8	0.4	0.2	**0.9964**	0.9724	**0.9964**	0.9794	0.9898	0.9880

Results show Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.6. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, BIBD =15, CRD=5, RCBD=15 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1		μ_2	μ_3	μ_4	μ_5 First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0472	0.0486	0.0552	0.0496	0.0494	0.0514
	1	1	0.5	0.5	0.5	0.3346	0.2496	0.3620	0.3296	**0.3634**	0.3532
	0.5	1	1	1	0.7	**0.3200**	0.2486	0.3012	0.2606	0.2610	0.2470
	0.7	1	0.7	0.7	0.5	**0.4538**	0.3396	0.4446	0.3956	0.4198	0.4026
	1	1	0.5	0.5	0.2	0.6356	0.4870	**0.6696**	0.6022	0.6520	0.6518
	0.75	1	0.75	0.5	0.25	0.7396	0.5868	**0.7658**	0.6884	0.7200	0.7214
	1	1	1	1	0	0.7730	0.6226	**0.8040**	0.7346	0.7814	0.8104
	1	1	0.5	0.2	0	0.8604	0.7056	**0.8856**	0.8372	0.8744	0.8704
	0	1.6	0.8	0.4	0.2	**0.9998**	0.9924	0.9992	0.9984	0.9982	0.9980
Exponential	0	0	0	0	0	0.0508	0.0468	0.0486	0.0502	0.0496	0.0492
	1	1	0.5	0.5	0.5	0.5694	0.3964	0.5846	0.5348	**0.5892**	0.5642
	0.5	1	1	1	0.7	**0.5518**	0.4030	0.5274	0.4618	0.4580	0.4374
	0.7	1	0.7	0.7	0.5	0.7566	0.6026	**0.7620**	0.6712	0.7032	0.7086
	1	1	0.5	0.5	0.2	0.8866	0.7428	**0.9112**	0.8578	0.9016	0.8836
	0.75	1	0.75	0.5	0.25	0.9616	0.8696	**0.9638**	0.9332	0.9414	0.9436
	1	1	1	1	0	0.8898	0.7560	**0.9262**	0.8646	0.9262	0.9118
	1	1	0.5	0.2	0	0.9850	0.9312	**0.9912**	0.9722	0.9886	0.9796
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0508	0.0564	0.0540	0.0442	0.0516	0.0438
	1	1	0.5	0.5	0.5	**0**.2654	0.2052	**0.2784**	0.2410	0.2764	0.2696
	0.5	1	1	1	0.7	**0.2352**	0.1874	0.2302	0.2020	0.2100	0.1946
	0.7	1	0.7	0.7	0.5	**0.3508**	0.2540	0.3348	0.3132	0.3338	0.2930
	1	1	0.5	0.5	0.2	0.4896	0.3700	**0.5058**	0.4498	0.5048	0.5008
	0.75	1	0.75	0.5	0.25	0.5954	0.4492	**0.5976**	0.5340	0.5662	0.5522
	1	1	1	1	0	0.6194	0.4734	**0.6552**	0.5806	0.6360	0.6202
	1	1	0.5	0.2	0	0.7052	0.5476	**0.7486**	0.6744	0.7266	0.7142
	0	1.6	0.8	0.4	0.2	**0.9910**	0.9446	0.9882	0.9704	0.9794	0.9696

Results show Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.7. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, BIBD =15, CRD=10, RCBD=15 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0510	0.0504	0.0462	0.0504	0.0510	0.0448
	1	1	0.5	0.5	0.5	0.4014	0.2556	**0.4204**	0.2964	0.4042	0.3620
	0.5	1	1	1	0.7	**0.3662**	0.2380	0.3480	0.2698	0.3174	0.2794
	0.7	1	0.7	0.7	0.5	0.5228	0.3458	**0.5242**	0.3778	0.4840	0.4528
	1	1	0.5	0.5	0.2	0.7194	0.4866	**0.7448**	0.5724	0.7424	0.6612
	0.75	1	0.75	0.5	0.25	0.8160	0.5960	**0.8338**	0.6582	0.8010	0.7556
	1	1	1	1	0	0.8522	0.6122	**0.8718**	0.6952	0.8668	0.8166
	1	1	0.5	0.2	0	0.9244	0.7290	**0.9344**	0.7932	0.9200	0.8858
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9952	0.9998	0.9986	0.9996	0.9990
Exponential	0	0	0	0	0	0.0490	0.0478	0.0438	0.0558	0.0532	0.0502
	1	1	0.5	0.5	0.5	0.6406	0.4128	**0.6838**	0.4808	0.6576	0.5890
	0.5	1	1	1	0.7	**0.6342**	0.4242	0.6172	0.4568	0.5388	0.4976
	0.7	1	0.7	0.7	0.5	0.8428	0.6084	**0.8466**	0.6698	0.7970	0.7384
	1	1	0.5	0.5	0.2	0.9438	0.7600	**0.9506**	0.8302	0.9406	0.9030
	0.75	1	0.75	0.5	0.25	**0.9888**	0.8810	0.9840	0.9224	0.9768	0.9646
	1	1	1	1	0	0.9496	0.7686	**0.9588**	0.8364	0.9544	0.9174
	1	1	0.5	0.2	0	0.9952	0.9358	**0.9962**	0.9642	0.9944	0.9906
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9992	**1.0000**	1.0000	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0538	0.0462	0.0460	0.0472	0.0478	0.0476
	1	1	0.5	0.5	0.5	0.3020	0.1954	**0.3210**	0.2354	0.3108	0.2860
	0.5	1	1	1	0.7	0.2688	0.1842	**0.2744**	0.2034	0.2392	0.2334
	0.7	1	0.7	0.7	0.5	**0.3990**	0.2670	**0.3932**	0.2930	0.3694	0.3296
	1	1	0.5	0.5	0.2	0.5624	0.3654	**0.5830**	0.4196	0.5812	0.5054
	0.75	1	0.75	0.5	0.25	0.6724	0.4554	**0.6932**	0.5082	0.6562	0.5918
	1	1	1	1	0	0.6900	0.4764	**0.7304**	0.5314	0.7154	0.6518
	1	1	0.5	0.2	0	0.7958	0.5788	**0.8070**	0.6416	0.8060	0.7472
	0	1.6	0.8	0.4	0.2	**0.9976**	0.9556	0.9964	0.9678	0.9938	0.9876

Results show Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.8. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, BIBD =15, CRD=15, RCBD=15 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0512	0.0478	0.0534	0.0456	0.0514	0.0544
	1	1	0.5	0.5	0.5	0.4438	0.2976	**0.4678**	0.3268	0.4552	0.3770
	0.5	1	1	1	0.7	**0.4028**	0.2868	0.3926	0.2908	0.3482	0.3232
	0.7	1	0.7	0.7	0.5	0.5734	0.3970	**0.5742**	0.4142	0.5510	0.4732
	1	1	0.5	0.5	0.2	0.7732	0.5806	**0.8018**	0.6120	0.7760	0.6978
	0.75	1	0.75	0.5	0.25	0.8712	0.6752	**0.8792**	0.7156	0.8594	0.7782
	1	1	1	1	0	0.8938	0.7120	**0.9194**	0.7552	0.9084	0.8216
	1	1	0.5	0.2	0	0.9494	0.8094	**0.9626**	0.8406	0.9550	0.8974
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9980	**1.0000**	0.9994	**1.0000**	1.0000
Exponential	0	0	0	0	0	0.0554	0.0512	0.0470	0.0444	0.0462	0.0494
	1	1	0.5	0.5	0.5	0.7182	0.4842	**0.7346**	**0.5290**	**0.7088**	0.6198
	0.5	1	1	1	0.7	**0.6838**	0.4988	0.6666	0.5148	**0.6158**	0.5496
	0.7	1	0.7	0.7	0.5	**0.8908**	0.6998	0.8856	0.7318	0.8594	0.7904
	1	1	0.5	0.5	0.2	0.9682	0.8360	**0.9740**	0.8718	0.9724	0.9180
	0.75	1	0.75	0.5	0.25	**0.9940**	0.9452	**0.9940**	0.9476	0.9902	0.9704
	1	1	1	1	0	0.9678	0.8472	0.9748	0.8690	**0.9758**	0.9280
	1	1	0.5	0.2	0	0.9980	0.9716	**0.9990**	0.9824	0.9964	0.9918
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0488	0.0494	0.0486	0.0502	0.0522	0.0494
	1	1	0.5	0.5	0.5	0.3258	0.2436	**0.3542**	0.2508	0.3394	0.2864
	0.5	1	1	1	0.7	**0.3170**	0.2238	0.2938	0.2386	0.2710	0.2400
	0.7	1	0.7	0.7	0.5	0.4270	0.3042	**0.4368**	0.3220	0.4096	0.3670
	1	1	0.5	0.5	0.2	0.6112	0.4344	**0.6686**	0.4684	0.6424	0.5318
	0.75	1	0.75	0.5	0.25	0.7434	0.5284	**0.7472**	0.5514	0.7258	0.6248
	1	1	1	1	0	0.7674	0.5518	**0.7866**	0.5848	0.7732	0.6722
	1	1	0.5	0.2	0	0.8456	0.6528	**0.8724**	0.6978	0.8544	0.7644
	0	1.6	0.8	0.4	0.2	**0.9994**	0.9822	0.9992	0.9840	0.9986	0.9920

Results show Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.9. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 2. Treatments=5, BIBD =5, CRD=10 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0526	0.0478	0.0420	0.0526	0.0490	0.0462
	1	1	0.5	0.5	0.5	0.1962	0.2098	0.2034	0.2082	0.1880	0.2164
	0.5	1	1	1	0.7	0.1930	0.1952	0.1868	0.2012	0.1888	**0.2048**
	0.7	1	0.7	0.7	0.5	0.2654	0.2758	0.2492	0.2758	0.2524	**0.2798**
	1	1	0.5	0.5	0.2	0.3614	**0.3992**	0.3576	0.3900	0.3624	0.3794
	0.75	1	0.75	0.5	0.25	0.4476	0.4712	0.4422	**0.4960**	0.4432	0.4702
	1	1	1	1	0	0.4532	0.4914	0.4438	0.4884	0.4518	**0.4946**
	1	1	0.5	0.2	0	0.5480	0.5984	0.5432	0.5984	0.5308	**0.5986**
	0	1.6	0.8	0.4	0.2	0.9388	0.9724	0.9394	**0.9726**	0.9404	0.9686
Exponential	0	0	0	0	0	0.0496	0.0530	0.0436	0.0476	0.0470	0.0498
	1	1	0.5	0.5	0.5	0.2938	0.3236	0.3030	0.3264	0.2844	**0.3288**
	0.5	1	1	1	0.7	0.3088	**0.3366**	0.3074	0.3332	0.2996	0.3234
	0.7	1	0.7	0.7	0.5	0.4370	**0.4876**	0.4464	0.4842	0.4402	0.4708
	1	1	0.5	0.5	0.2	0.5784	**0.6320**	0.5766	0.6120	0.5726	0.6264
	0.75	1	0.75	0.5	0.25	0.7054	0.7542	0.6982	**0.7732**	0.6932	0.7662
	1	1	1	1	0	0.5894	0.6370	0.5962	0.6334	0.5942	**0.6442**
	1	1	0.5	0.2	0	0.7792	**0.8472**	0.7816	0.8428	0.7798	0.8426
	0	1.6	0.8	0.4	0.2	0.9880	**0.9996**	0.9904	0.9984	0.9924	0.9988
T with 3 degrees of freedom	0	0	0	0	0	0.0498	0.0532	0.0536	0.0462	0.0516	0.0450
	1	1	0.5	0.5	0.5	0.1574	0.1704	0.1570	0.1702	0.1708	0.1636
	0.5	1	1	1	0.7	0.1510	**0.1706**	0.1480	0.1566	0.1562	0.1638
	0.7	1	0.7	0.7	0.5	0.1984	**0.2266**	0.2034	0.2120	0.1996	0.2218
	1	1	0.5	0.5	0.2	0.2672	0.2942	0.2844	0.2970	0.2706	**0.3044**
	0.75	1	0.75	0.5	0.25	0.3220	0.3524	0.3284	**0.3628**	0.3242	0.3576
	1	1	1	1	0	0.3402	0.3684	0.3462	0.3736	0.3424	**0.3780**
	1	1	0.5	0.2	0	0.4054	0.4484	0.4158	0.4554	0.4142	**0.4566**
	0	1.6	0.8	0.4	0.2	0.8214	0.8700	0.8280	0.8742	0.8246	**0.8786**

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (BIBD and CRD). Results show that with a combination 5 BIBD and 10 CRD, Standardized Last test performed better than Standardized First in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.10. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 2. Treatments=5, BIBD =10, CRD=5 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0508	0.0498	0.0550	0.0502	0.0452	0.0538
	1	1	0.5	0.5	0.5	0.1764	0.1516	0.1672	0.1478	0.1830	0.1628
	0.5	1	1	1	0.7	0.1608	0.1538	0.1638	0.1488	**0.1656**	0.1466
	0.7	1	0.7	0.7	0.5	**0.2224**	0.1906	0.2144	0.1928	0.2016	0.1942
	1	1	0.5	0.5	0.2	0.2920	0.2734	**0.3126**	0.2578	0.2944	0.2576
	0.75	1	0.75	0.5	0.25	**0.3664**	0.3196	0.3610	0.3276	**0.3664**	0.3240
	1	1	1	1	0	0.3752	0.3334	**0.3852**	0.3350	0.3740	0.3248
	1	1	0.5	0.2	0	**0.4600**	0.4126	0.4586	0.4074	0.4538	0.4146
	0	1.6	0.8	0.4	0.2	0.8816	0.8422	0.8804	0.8388	**0.8840**	0.8340
Exponential	0	0	0	0	0	0.0510	0.0510	0.0504	0.0554	0.0508	0.0488
	1	1	0.5	0.5	0.5	0.2552	0.2064	0.2520	0.2108	0.2494	0.2122
	0.5	1	1	1	0.7	0.2560	0.2370	0.2576	0.2370	**0.2600**	0.2410
	0.7	1	0.7	0.7	0.5	**0.3734**	0.3320	0.3720	0.3254	0.3682	0.3264
	1	1	0.5	0.5	0.2	**0.4744**	0.4244	0.4722	0.4158	0.4704	0.4258
	0.75	1	0.75	0.5	0.25	0.6166	0.5556	0.6026	0.5596	**0.6168**	0.5602
	1	1	1	1	0	**0.5106**	0.4490	0.5084	0.4452	0.5000	0.4402
	1	1	0.5	0.2	0	**0.6810**	0.6224	0.6736	0.6260	0.6680	0.6334
	0	1.6	0.8	0.4	0.2	**0.9734**	0.9618	0.9686	0.9600	0.9726	0.9598
T with 3 degrees of freedom	0	0	0	0	0	0.0494	0.0500	0.0458	0.0466	0.0502	0.0478
	1	1	0.5	0.5	0.5	0.1512	0.1278	0.1454	0.1252	0.1336	0.1340
	0.5	1	1	1	0.7	0.1298	0.1276	**0.1472**	0.1340	0.1410	0.1252
	0.7	1	0.7	0.7	0.5	0.1678	0.1548	0.1698	0.1624	**0.1784**	0.1572
	1	1	0.5	0.5	0.2	0.2278	0.2002	0.2294	0.1990	**0.2338**	0.2124
	0.75	1	0.75	0.5	0.25	0.2876	0.2432	**0.2884**	0.2598	0.2812	0.2444
	1	1	1	1	0	**0.2946**	0.2654	0.2912	0.2582	0.2798	0.2620
	1	1	0.5	0.2	0	0.3484	0.3144	0.3496	0.3074	**0.3506**	0.3102
	0	1.6	0.8	0.4	0.2	0.7308	0.6688	0.7304	0.6720	**0.7314**	0.6670

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (BIBD and CRD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.11. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 2. Treatments=5, BIBD =10, CRD=10 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0472	0.0482	0.0516	0.0530	0.0488	0.0466
	1	1	0.5	0.5	0.5	**0.2280**	0.2206	0.2254	0.2210	0.2214	0.2050
	0.5	1	1	1	0.7	0.2126	0.2092	**0.2146**	0.1860	0.2046	0.2040
	0.7	1	0.7	0.7	0.5	**0.2950**	0.2780	0.2790	0.2752	0.2712	0.2906
	1	1	0.5	0.5	0.2	0.4176	0.3984	**0.4184**	0.3900	0.4068	0.3886
	0.75	1	0.75	0.5	0.25	**0.4974**	0.4770	0.4968	0.4910	0.4872	0.5004
	1	1	1	1	0	0.5258	0.5138	0.5054	0.5044	**0.5316**	0.5046
	1	1	0.5	0.2	0	**0.6180**	0.6002	**0.6148**	0.6014	**0.6056**	0.6114
	0	1.6	0.8	0.4	0.2	0.9750	0.9758	**0.9762**	0.9770	0.9716	0.9732
Exponential	0	0	0	0	0	0.0500	0.0492	0.0520	0.0542	0.0492	0.0514
	1	1	0.5	0.5	0.5	0.3392	0.3336	0.3450	0.3148	0.3560	0.3234
	0.5	1	1	1	0.7	**0.3576**	0.3406	0.3556	0.3360	0.3474	0.3308
	0.7	1	0.7	0.7	0.5	0.5004	0.4962	0.4928	0.4900	**0.5182**	0.4998
	1	1	0.5	0.5	0.2	**0.6570**	0.6424	0.6398	0.6376	0.6444	0.6328
	0.75	1	0.75	0.5	0.25	0.7704	0.7684	0.7826	0.7756	**0.7896**	0.7712
	1	1	1	1	0	0.6618	0.6580	0.6652	0.6626	**0.6722**	0.6610
	1	1	0.5	0.2	0	0.8514	0.8512	0.8432	0.8372	**0.8522**	0.8494
	0	1.6	0.8	0.4	0.2	0.9986	0.9982	0.9972	0.9976	**0.9988**	0.9988
T with 3 degrees of freedom	0	0	0	0	0	0.0518	0.0530	0.0498	0.0572	0.0486	0.0476
	1	1	0.5	0.5	0.5	0.1798	0.1776	**0.1908**	0.1674	0.1794	0.1700
	0.5	1	1	1	0.7	**0.1724**	0.1566	0.1676	0.1594	0.1604	0.1570
	0.7	1	0.7	0.7	0.5	**0.2306**	0.2166	0.2190	0.2146	0.2142	0.2286
	1	1	0.5	0.5	0.2	0.3154	0.2952	**0.3168**	0.3008	0.3060	0.3070
	0.75	1	0.75	0.5	0.25	0.3706	0.3604	0.3804	0.3648	**0.3844**	0.3694
	1	1	1	1	0	0.4056	0.3838	0.4020	0.3784	**0.4058**	0.3824
	1	1	0.5	0.2	0	0.4676	0.4686	**0.4910**	0.4518	0.4632	0.4580
	0	1.6	0.8	0.4	0.2	**0.8908**	0.8764	0.8894	0.8818	0.8840	0.8738

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (BIBD and CRD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.12. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 2. Treatments=5, BIBD =10, CRD=15, Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0512	0.0492	0.0474	0.0536	0.0480	0.0498
	1	1	0.5	0.5	0.5	0.2642	0.2736	0.2630	0.2846	0.2602	**0.2904**
	0.5	1	1	1	0.7	0.2496	0.2570	0.2494	0.2718	0.2490	**0.2730**
	0.7	1	0.7	0.7	0.5	0.3594	0.3536	0.3480	0.3560	0.3450	**0.3628**
	1	1	0.5	0.5	0.2	0.4896	0.5136	0.5050	0.5154	0.4986	**0.5240**
	0.75	1	0.75	0.5	0.25	0.6038	0.6244	0.6018	0.6270	0.6066	**0.6312**
	1	1	1	1	0	0.6292	0.6524	0.6120	0.6564	0.6416	**0.6738**
	1	1	0.5	0.2	0	0.7284	0.7542	0.7278	0.7470	0.7216	**0.7696**
	0	1.6	0.8	0.4	0.2	0.9940	**0.9978**	0.9950	0.9956	0.9942	0.9974
Exponential	0	0	0	0	0	0.0518	0.0488	0.0540	0.0416	0.0506	0.0494
	1	1	0.5	0.5	0.5	0.4236	0.4346	0.4290	0.4342	0.4110	**0.4562**
	0.5	1	1	1	0.7	0.4252	0.4274	0.4230	0.4376	0.4242	**0.4400**
	0.7	1	0.7	0.7	0.5	0.6090	0.6458	0.6092	0.6382	0.6056	**0.6520**
	1	1	0.5	0.5	0.2	0.7528	0.7958	0.7552	0.7898	0.7676	**0.8064**
	0.75	1	0.75	0.5	0.25	0.8858	0.9002	0.8724	0.9052	0.8770	**0.9106**
	1	1	1	1	0	0.7718	0.7920	0.7744	0.7968	0.7794	**0.8142**
	1	1	0.5	0.2	0	0.9264	0.9524	0.9264	0.9524	0.9258	**0.9540**
	0	1.6	0.8	0.4	0.2	0.9998	**1.0000**	0.9998	**1.0000**	0.9998	**1.0000**
T with 3 degrees of freedom	0	0	0	0	0	0.0516	0.0510	0.0544	0.0526	0.0494	0.0472
	1	1	0.5	0.5	0.5	0.2140	0.2110	0.2194	**0.2146**	0.1994	0.2122
	0.5	1	1	1	0.7	0.2034	0.1964	0.1982	0.2044	0.1906	**0.2064**
	0.7	1	0.7	0.7	0.5	0.2724	0.2650	0.2704	0.2766	0.2686	**0.2814**
	1	1	0.5	0.5	0.2	0.3740	**0.3940**	0.4004	0.3852	0.3808	0.3890
	0.75	1	0.75	0.5	0.25	0.4732	0.4618	0.4498	**0.4862**	0.4640	0.4816
	1	1	1	1	0	0.4760	**0.4982**	0.4740	0.4942	0.4734	0.4972
	1	1	0.5	0.2	0	0.5656	0.5906	0.5784	0.5930	0.5714	**0.6108**
	0	1.6	0.8	0.4	0.2	0.9454	0.9598	0.9480	0.9632	0.9486	**0.9674**

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (BIBD and CRD). Results show that with a combination 10 BIBD and 15 CRD, Standardized Last test performed better than Standardized First in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.13. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 2. Treatments=5, BIBD =15, CRD=5, Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0552	0.0482	0.0466	0.0536	0.0532	0.0468
	1	1	0.5	0.5	0.5	**0.1884**	0.1650	0.1798	0.1700	0.1880	0.1672
	0.5	1	1	1	0.7	0.1732	0.1538	**0.1820**	0.1604	0.1736	0.1508
	0.7	1	0.7	0.7	0.5	**0.2404**	0.2042	0.2372	0.2038	0.2398	0.2020
	1	1	0.5	0.5	0.2	0.3428	0.2772	**0.3452**	0.2940	0.3434	0.2792
	0.75	1	0.75	0.5	0.25	0.4126	0.3484	**0.4140**	0.3280	0.4124	0.3402
	1	1	1	1	0	**0.4414**	0.3588	0.4256	0.3616	0.4324	0.3664
	1	1	0.5	0.2	0	**0.5190**	0.4308	0.5186	0.4320	0.5146	0.4280
	0	1.6	0.8	0.4	0.2	0.9248	0.8686	**0.9274**	0.8690	0.9252	0.8600
Exponential	0	0	0	0	0	0.0506	0.0554	0.0488	0.0520	0.0566	0.0554
	1	1	0.5	0.5	0.5	0.2756	0.2260	0.2826	0.2128	**0.2830**	0.2236
	0.5	1	1	1	0.7	**0.2912**	0.2530	0.2810	0.2412	0.2894	0.2506
	0.7	1	0.7	0.7	0.5	**0.4204**	0.3436	0.4130	0.3378	0.4192	0.3428
	1	1	0.5	0.5	0.2	**0.5514**	0.4552	0.5314	0.4420	0.5410	0.4456
	0.75	1	0.75	0.5	0.25	0.6718	0.5806	**0.6846**	0.5892	0.6730	0.5788
	1	1	1	1	0	0.5606	0.4872	0.5604	0.4686	**0.5626**	0.4654
	1	1	0.5	0.2	0	0.7406	0.6586	0.7422	0.6586	**0.7450**	0.6580
	0	1.6	0.8	0.4	0.2	0.9882	0.9716	0.9876	0.9732	**0.9886**	0.9724
T with 3 degrees of freedom	0	0	0	0	0	0.0470	0.0502	0.0518	0.0496	0.0500	0.0508
	1	1	0.5	0.5	0.5	0.1610	0.1354	0.1614	0.1346	**0.1654**	0.1336
	0.5	1	1	1	0.7	0.1434	0.1268	**0.1470**	**0.1258**	**0.1442**	0.1254
	0.7	1	0.7	0.7	0.5	0.1888	0.1608	**0.1954**	0.1672	0.1928	0.1584
	1	1	0.5	0.5	0.2	0.2606	0.2176	0.2530	0.2190	**0.2648**	0.2206
	0.75	1	0.75	0.5	0.25	0.3068	0.2722	**0.3174**	0.2530	0.3066	0.2678
	1	1	1	1	0	0.3288	0.2774	**0.3350**	0.2758	0.3220	0.2730
	1	1	0.5	0.2	0	0.3788	0.3308	0.3828	0.3194	**0.3906**	0.3288
	0	1.6	0.8	0.4	0.2	0.8036	0.7120	**0.8098**	0.7094	0.7994	0.7044

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (BIBD and CRD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.14. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 2. Treatments=5, BIBD =15, CRD=10, Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0538	0.0466	0.0448	0.0464	0.0498	0.0498
	1	1	0.5	0.5	0.5	**0.2506**	0.2078	0.2430	0.2182	0.2402	0.2240
	0.5	1	1	1	0.7	0.2210	0.1984	0.2256	0.2042	**0.2282**	0.2112
	0.7	1	0.7	0.7	0.5	0.3262	0.2782	0.3070	0.2782	**0.3266**	0.2874
	1	1	0.5	0.5	0.2	0.4452	0.4038	**0.4594**	0.4042	0.4516	0.4012
	0.75	1	0.75	0.5	0.25	0.5480	0.5104	0.5568	0.4950	**0.5582**	0.5022
	1	1	1	1	0	0.5682	0.5146	0.5680	0.5086	**0.5698**	0.5256
	1	1	0.5	0.2	0	0.6688	0.6232	0.6588	0.6170	**0.6724**	0.6032
	0	1.6	0.8	0.4	0.2	**0.9874**	0.9768	0.9872	0.9746	0.9868	0.9802
Exponential	0	0	0	0	0	0.0468	0.0484	0.0488	0.0488	0.0532	0.0494
	1	1	0.5	0.5	0.5	**0.3794**	0.3168	0.3712	0.3212	0.3790	0.3316
	0.5	1	1	1	0.7	**0.3886**	0.3416	0.3802	0.3532	0.3794	0.3514
	0.7	1	0.7	0.7	0.5	0.5624	0.5004	0.5562	0.5058	**0.5668**	0.5094
	1	1	0.5	0.5	0.2	0.6958	0.6562	**0.7008**	0.6390	0.6964	0.6596
	0.75	1	0.75	0.5	0.25	**0.8300**	0.7790	0.8268	0.7830	0.8292	0.7876
	1	1	1	1	0	**0.7232**	0.6704	**0.7232**	0.6568	0.7144	0.6574
	1	1	0.5	0.2	0	0.8926	0.8656	**0.8928**	0.8602	0.8906	0.8634
	0	1.6	0.8	0.4	0.2	0.9986	0.9992	0.9988	0.9986	**0.9994**	0.9988
T with 3 degrees of freedom	0	0	0	0	0	0.0490	0.0502	0.0486	0.0550	0.0496	0.0510
	1	1	0.5	0.5	0.5	**0.1976**	0.1668	0.1818	0.1686	0.1960	0.1714
	0.5	1	1	1	0.7	0.1802	0.1614	**0.1834**	0.1670	0.1814	0.1566
	0.7	1	0.7	0.7	0.5	0.2354	0.2188	0.2370	0.2276	**0.2406**	0.2118
	1	1	0.5	0.5	0.2	**0.3500**	0.3060	0.3470	0.3012	0.3308	0.3154
	0.75	1	0.75	0.5	0.25	0.4138	0.3674	0.4138	0.3778	**0.4150**	0.3802
	1	1	1	1	0	**0.4316**	0.3862	0.4202	0.3880	0.4252	0.3796
	1	1	0.5	0.2	0	**0.5270**	0.4706	0.5184	0.4662	0.5218	0.4614
	0	1.6	0.8	0.4	0.2	0.9182	0.8886	0.9250	0.8902	**0.9260**	0.8882

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (BIBD and CRD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.15. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 2. Treatments=5, BIBD =15, CRD=15, Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0452	0.0476	0.0482	0.0516	0.0460	0.0516
	1	1	0.5	0.5	0.5	**0.2976**	0.2728	0.2920	0.2842	0.2794	0.2748
	0.5	1	1	1	0.7	**0.2854**	0.2662	0.2772	0.2578	0.2578	0.2498
	0.7	1	0.7	0.7	0.5	0.3676	0.3546	**0.3728**	0.3638	**0.3690**	0.3724
	1	1	0.5	0.5	0.2	0.5440	0.5210	0.5398	0.5266	**0.5470**	0.5346
	0.75	1	0.75	0.5	0.25	0.6394	0.6228	0.6388	0.6370	**0.6482**	0.6234
	1	1	1	1	0	**0.6852**	0.6492	0.6830	0.6502	0.6778	0.6456
	1	1	0.5	0.2	0	0.7678	0.7582	**0.7802**	0.7652	0.7620	0.7604
	0	1.6	0.8	0.4	0.2	0.9976	0.9994	**0.9994**	0.9984	0.9982	0.9974
Exponential	0	0	0	0	0	0.0460	0.0494	0.0476	0.0492	0.0558	0.0530
	1	1	0.5	0.5	0.5	**0.4696**	0.4528	0.4600	0.4410	0.4672	0.4420
	0.5	1	1	1	0.7	0.4642	0.4394	0.4662	0.4374	**0.4702**	0.4386
	0.7	1	0.7	0.7	0.5	0.6584	0.6308	0.6612	0.6322	**0.6622**	0.6376
	1	1	0.5	0.5	0.2	0.7936	0.7916	0.7980	0.7896	**0.8056**	0.7916
	0.75	1	0.75	0.5	0.25	0.9148	0.9032	**0.9210**	0.9040	0.9100	0.9068
	1	1	1	1	0	0.8124	0.7926	0.8194	0.7896	**0.8208**	0.8002
	1	1	0.5	0.2	0	0.9524	0.9556	0.9530	0.9560	**0.9532**	0.9504
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0496	0.0506	0.0448	0.0480	0.0484	0.0536
	1	1	0.5	0.5	0.5	0.2220	0.2168	**0.2240**	0.2228	0.2182	0.2180
	0.5	1	1	1	0.7	0.2066	0.1988	**0.2130**	0.2044	0.2082	0.2034
	0.7	1	0.7	0.7	0.5	0.2862	0.2778	**0.2976**	0.2752	0.2926	0.2754
	1	1	0.5	0.5	0.2	**0.4180**	0.3912	0.4080	0.3884	0.4010	0.4004
	0.75	1	0.75	0.5	0.25	**0.4954**	0.4712	0.4930	0.4804	0.4926	0.4864
	1	1	1	1	0	**0.5372**	0.4952	0.5206	0.5070	0.5264	0.5012
	1	1	0.5	0.2	0	**0.6204**	0.5900	0.6172	0.5958	0.6040	0.5950
	0	1.6	0.8	0.4	0.2	0.9688	0.9656	0.9700	0.9634	**0.9708**	0.9636

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (BIBD and CRD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.16. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, CRD=10, RCBD=5 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0566	0.0478	0.0550	0.0512	0.0430	0.0546
	1	1	0.5	0.5	0.5	0.2562	0.2242	0.2664	0.2192	**0.2682**	0.2454
	0.5	1	1	1	0.7	**0.2326**	0.2088	0.2242	0.2032	0.2188	0.2306
	0.7	1	0.7	0.7	0.5	0.3218	0.2824	**0.3326**	0.2938	0.3248	0.3306
	1	1	0.5	0.5	0.2	0.4740	0.4024	0.4924	0.4440	**0.5100**	0.4742
	0.75	1	0.75	0.5	0.25	0.5740	0.4956	0.5884	0.5222	**0.5944**	0.5804
	1	1	1	1	0	0.6048	0.5106	0.6440	0.5508	**0.6454**	0.6058
	1	1	0.5	0.2	0	0.7036	0.6100	0.7290	0.6618	**0.7392**	0.7154
	0	1.6	0.8	0.4	0.2	0.9916	0.9772	**0.9920**	0.9840	0.9908	0.9866
Exponential	0	0	0	0	0	0.0528	0.0494	0.0468	0.0522	0.0510	0.0528
	1	1	0.5	0.5	0.5	0.4022	0.3294	0.4152	0.3662	**0.4502**	0.3958
	0.5	1	1	1	0.7	0.3904	0.3408	**0.3954**	0.3600	0.3732	0.3852
	0.7	1	0.7	0.7	0.5	**0.5836**	0.5042	0.5758	0.5310	0.5678	0.5856
	1	1	0.5	0.5	0.2	0.7412	0.6626	**0.7682**	0.6822	0.7656	0.7330
	0.75	1	0.75	0.5	0.25	**0.8562**	0.7908	0.8634	0.8166	0.8658	0.8486
	1	1	1	1	0	0.7460	0.6682	0.7878	0.6926	**0.8028**	0.7536
	1	1	0.5	0.2	0	0.9264	0.8674	0.9360	0.8906	**0.9362**	0.9256
	0	1.6	0.8	0.4	0.2	0.9998	0.9994	0.9996	0.9994	**1.0000**	0.9998
T with 3 degrees of freedom	0	0	0	0	0	0.0502	0.0486	0.0522	0.0516	0.0532	0.0472
	1	1	0.5	0.5	0.5	0.2060	0.1762	0.2046	0.1954	**0.2128**	0.2062
	0.5	1	1	1	0.7	**0.1850**	0.1654	0.1754	0.1760	0.1718	0.1798
	0.7	1	0.7	0.7	0.5	0.2448	0.2054	**0.2522**	0.2340	0.2470	0.2456
	1	1	0.5	0.5	0.2	0.3514	0.3034	0.3892	0.3254	**0.3956**	0.3706
	0.75	1	0.75	0.5	0.25	0.4388	0.3796	0.4462	0.3922	**0.4494**	0.4276
	1	1	1	1	0	0.4436	0.3876	0.4792	0.4200	**0.4844**	0.4688
	1	1	0.5	0.2	0	0.5516	0.4732	0.5676	0.4886	**0.5712**	0.5640
	0	1.6	0.8	0.4	0.2	0.9406	0.8910	**0.9346**	0.9018	0.9310	0.9260

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (CRD and RCBD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.17. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, CRD=5, RCBD=10 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0496	0.0530	0.0468	0.0484	0.0522	0.0516
	1	1	0.5	0.5	0.5	0.2394	0.1928	0.2686	0.2332	**0.2730**	0.2748
	0.5	1	1	1	0.7	**0.2304**	0.1778	0.2120	0.2020	0.2106	0.2074
	0.7	1	0.7	0.7	0.5	**0.3230**	0.2454	0.3184	0.2936	0.3228	0.3126
	1	1	0.5	0.5	0.2	0.4570	0.3464	0.4892	0.4370	**0.5228**	0.5082
	0.75	1	0.75	0.5	0.25	0.5498	0.4308	0.5796	0.5184	**0.5834**	0.5644
	1	1	1	1	0	0.5678	0.4508	0.6278	0.5496	**0.6522**	0.6446
	1	1	0.5	0.2	0	0.6780	0.5326	0.7238	0.6542	**0.7488**	0.7392
	0	1.6	0.8	0.4	0.2	**0.9900**	0.9464	0.9896	0.9728	0.9806	0.9790
Exponential	0	0	0	0	0	0.0458	0.0454	0.0472	0.0474	0.0526	0.0544
	1	1	0.5	0.5	0.5	0.3776	0.2836	0.4108	0.3678	**0.4460**	0.4420
	0.5	1	1	1	0.7	**0.3878**	0.3036	0.3616	0.3212	0.3446	0.3480
	0.7	1	0.7	0.7	0.5	**0.5658**	0.4448	0.5604	0.5158	**0.5568**	0.5662
	1	1	0.5	0.5	0.2	0.7248	0.5734	0.7500	0.6876	**0.7796**	0.7692
	0.75	1	0.75	0.5	0.25	0.8410	0.7082	**0.8478**	0.8006	0.8442	0.8464
	1	1	1	1	0	0.7280	0.5770	0.7764	0.7114	**0.8130**	0.8048
	1	1	0.5	0.2	0	0.9114	0.7918	0.9240	0.8914	**0.9350**	0.9308
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9966	0.9998	0.9982	0.9988	0.9988
T with 3 degrees of freedom	0	0	0	0	0	0.0534	0.0504	0.0526	0.0548	0.0480	0.0486
	1	1	0.5	0.5	0.5	0.1974	0.1540	0.2074	0.1808	**0.2180**	0.2134
	0.5	1	1	1	0.7	**0.1838**	0.1436	0.1692	0.1568	0.1624	0.1608
	0.7	1	0.7	0.7	0.5	0.2418	0.1952	**0.2534**	0.2292	0.2408	0.2470
	1	1	0.5	0.5	0.2	0.3448	0.2610	0.3752	0.3302	**0.3990**	0.3808
	0.75	1	0.75	0.5	0.25	0.4132	0.3258	0.4336	0.3932	**0.4420**	0.4332
	1	1	1	1	0	0.4328	0.3316	0.4732	0.4258	**0.5126**	0.5086
	1	1	0.5	0.2	0	0.5132	0.4204	0.5652	0.4958	**0.5924**	0.5726
	0	1.6	0.8	0.4	0.2	**0.9318**	0.8176	0.9200	0.8876	0.9068	0.9060

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (CRD and RCBD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.18. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, CRD=10, RCBD=10 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0506	0.0528	0.0530	0.0544	0.0514	0.0450
	1	1	0.5	0.5	0.5	0.3056	0.2400	0.3214	0.2464	**0.3338**	0.3074
	0.5	1	1	1	0.7	**0.2722**	0.2208	0.2700	0.2290	0.2524	0.2546
	0.7	1	0.7	0.7	0.5	**0.4114**	0.3062	0.4106	0.3194	0.3976	0.3712
	1	1	0.5	0.5	0.2	0.5594	0.4318	0.6112	0.4672	**0.6364**	0.5654
	0.75	1	0.75	0.5	0.25	0.6866	0.5470	**0.7042**	0.5738	0.7038	0.6516
	1	1	1	1	0	0.7100	0.5526	0.7540	0.5974	**0.7696**	0.6968
	1	1	0.5	0.2	0	0.8056	0.6544	0.8480	0.6934	**0.8496**	0.7978
	0	1.6	0.8	0.4	0.2	**0.9994**	0.9836	0.9992	0.9894	0.9986	0.9970
Exponential	0	0	0	0	0	0.0504	0.0474	0.0482	0.0508	0.0582	0.0446
	1	1	0.5	0.5	0.5	0.4920	0.3672	0.5348	0.3974	**0.5414**	0.4760
	0.5	1	1	1	0.7	**0.4898**	0.3738	0.4746	0.3920	0.4366	0.4324
	0.7	1	0.7	0.7	0.5	0.7030	0.5502	**0.7102**	0.5778	0.6898	0.6566
	1	1	0.5	0.5	0.2	0.8404	0.6894	0.8736	0.7432	**0.8744**	0.8384
	0.75	1	0.75	0.5	0.25	0.9374	0.8208	**0.9434**	0.8570	0.9334	0.9096
	1	1	1	1	0	0.8478	0.7022	0.8846	0.7544	**0.9036**	0.8474
	1	1	0.5	0.2	0	0.9752	0.8936	**0.9820**	0.9172	0.9796	0.9698
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9998	**1.0000**	1.0000	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0524	0.0556	0.0504	0.0486	0.0502	0.0462
	1	1	0.5	0.5	0.5	0.2358	0.1956	**0.2564**	0.2082	0.2544	0.2402
	0.5	1	1	1	0.7	**0.2270**	0.1794	0.2040	0.1768	0.2020	0.1836
	0.7	1	0.7	0.7	0.5	**0.3094**	0.2486	0.3058	0.2452	0.2962	0.2806
	1	1	0.5	0.5	0.2	0.4340	0.3294	0.4614	0.3680	**0.4836**	0.4208
	0.75	1	0.75	0.5	0.25	0.5352	0.4066	**0.5504**	0.4302	0.5456	0.5012
	1	1	1	1	0	0.5760	0.4210	0.5952	0.4704	**0.5996**	0.5494
	1	1	0.5	0.2	0	0.6504	0.5032	0.6890	0.5426	**0.6992**	0.6422
	0	1.6	0.8	0.4	0.2	**0.9808**	0.9170	0.9792	0.9344	0.9738	0.9640

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (CRD and RCBD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.19. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, CRD=15, RCBD=10 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0534	0.0478	0.0512	0.0480	0.0466	0.0484
	1	1	0.5	0.5	0.5	0.3642	0.2780	**0.3912**	0.3046	0.3886	0.3344
	0.5	1	1	1	0.7	**0.3272**	0.2560	0.3112	0.2744	0.3014	0.2758
	0.7	1	0.7	0.7	0.5	0.4626	0.3648	0.4598	0.4032	**0.4704**	0.4260
	1	1	0.5	0.5	0.2	0.6580	0.5360	0.7036	0.5782	**0.7078**	0.6242
	0.75	1	0.75	0.5	0.25	0.7726	0.6568	0.7808	0.6696	**0.7890**	0.7166
	1	1	1	1	0	0.8020	0.6670	0.8232	0.6920	**0.8490**	0.7700
	1	1	0.5	0.2	0	0.8748	0.7794	0.9074	0.8084	**0.9106**	0.8546
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9976	0.9996	0.9988	0.9994	0.9988
Exponential	0	0	0	0	0	0.0500	0.0506	0.0528	0.0500	0.0518	0.0486
	1	1	0.5	0.5	0.5	0.5806	0.4676	0.6186	0.4834	**0.6378**	0.5354
	0.5	1	1	1	0.7	**0.5726**	0.4728	0.5502	0.4716	0.5206	0.4984
	0.7	1	0.7	0.7	0.5	**0.7956**	0.6684	0.7892	0.6944	0.7788	0.7402
	1	1	0.5	0.5	0.2	0.9084	0.8220	0.9314	0.8444	**0.9338**	0.8786
	0.75	1	0.75	0.5	0.25	0.9734	0.9164	**0.9736**	0.9340	0.9712	0.9498
	1	1	1	1	0	0.9162	0.8044	0.9406	0.8466	**0.9450**	0.8902
	1	1	0.5	0.2	0	0.9928	0.9634	0.9944	0.9734	**0.9954**	0.9854
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0520	0.0476	0.0488	0.0516	0.0514	0.0442
	1	1	0.5	0.5	0.5	0.2780	0.2316	**0.2988**	0.2338	0.2888	0.2542
	0.5	1	1	1	0.7	**0.2536**	0.2078	0.2448	0.2136	0.2298	0.2192
	0.7	1	0.7	0.7	0.5	0.3304	0.2832	0.3408	0.3030	**0.3606**	0.3174
	1	1	0.5	0.5	0.2	0.5042	0.4102	0.5454	0.4338	**0.5560**	0.4680
	0.75	1	0.75	0.5	0.25	0.6026	0.4872	0.6364	0.5218	**0.6368**	0.5774
	1	1	1	1	0	0.6386	0.5202	0.6766	0.5432	**0.6980**	0.6056
	1	1	0.5	0.2	0	0.7424	0.6176	0.7736	0.6466	**0.7820**	0.7000
	0	1.6	0.8	0.4	0.2	**0.9946**	0.9698	**0.9946**	0.9782	0.9902	0.9860

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (CRD and RCBD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.20. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, CRD=5, RCBD=15 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0498	0.0530	0.0512	0.0440	0.0500	0.0534
	1	1	0.5	0.5	0.5	0.2870	0.2094	0.3188	0.2878	**0.3192**	0.3480
	0.5	1	1	1	0.7	**0.2626**	0.2080	0.2570	0.2362	0.2424	0.2256
	0.7	1	0.7	0.7	0.5	0.3620	0.2976	0.3718	0.3578	**0.3778**	0.3938
	1	1	0.5	0.5	0.2	0.5476	0.4062	0.5788	0.5384	**0.6056**	0.6364
	0.75	1	0.75	0.5	0.25	0.6284	0.5014	0.6626	0.6178	**0.6710**	0.6936
	1	1	1	1	0	0.6760	0.5276	0.7310	0.6714	**0.7406**	0.7710
	1	1	0.5	0.2	0	0.7680	0.6244	0.8126	0.7686	**0.8228**	0.8550
	0	1.6	0.8	0.4	0.2	**0.9972**	0.9774	0.9968	0.9928	0.9940	0.9964
Exponential	0	0	0	0	0	0.0582	0.0550	0.0474	0.0508	0.0524	0.0512
	1	1	0.5	0.5	0.5	0.4634	0.3434	0.5090	0.4526	**0.5530**	0.5222
	0.5	1	1	1	0.7	**0.4580**	0.3630	0.4352	0.4140	0.4006	0.4032
	0.7	1	0.7	0.7	0.5	0.6542	0.5090	**0.6618**	0.6058	0.6572	0.6378
	1	1	0.5	0.5	0.2	0.8100	0.6620	0.8468	0.8068	**0.8740**	0.8560
	0.75	1	0.75	0.5	0.25	0.9162	0.7910	0.9158	0.8818	**0.9194**	0.9080
	1	1	1	1	0	0.8114	0.6646	0.8672	0.8164	**0.8810**	0.9094
	1	1	0.5	0.2	0	0.9518	0.8786	0.9678	0.9532	**0.9756**	0.9708
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9988	0.9998	1.0000	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0506	0.0514	0.0514	0.0450	0.0514	0.0442
	1	1	0.5	0.5	0.5	0.2296	0.1790	0.2292	0.2394	**0.2568**	0.2546
	0.5	1	1	1	0.7	**0.1992**	0.1804	0.1896	0.1806	0.1918	0.1868
	0.7	1	0.7	0.7	0.5	0.2814	0.2254	0.2824	0.2714	**0.2936**	0.2776
	1	1	0.5	0.5	0.2	0.4044	0.3146	0.4342	0.4106	**0.4910**	0.4606
	0.75	1	0.75	0.5	0.25	0.4936	0.3812	0.5176	0.4966	**0.5404**	0.5234
	1	1	1	1	0	0.5118	0.3894	0.5560	0.5208	**0.6112**	0.5840
	1	1	0.5	0.2	0	0.6170	0.4704	0.6612	0.6180	**0.6974**	0.6696
	0	1.6	0.8	0.4	0.2	**0.9712**	0.8960	0.9650	0.9476	0.9548	0.9526

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (CRD and RCBD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.21. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, CRD=10, RCBD=15 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0470	0.0510	0.0454	0.0478	0.0518	0.0458
	1	1	0.5	0.5	0.5	0.3494	0.2420	0.3724	0.2908	**0.4046**	0.3456
	0.5	1	1	1	0.7	**0.3312**	0.2402	0.3106	0.2460	0.2844	0.2806
	0.7	1	0.7	0.7	0.5	0.4546	0.3134	**0.4694**	0.3512	0.4554	0.4352
	1	1	0.5	0.5	0.2	0.6426	0.4672	0.6918	0.5166	**0.7152**	0.6408
	0.75	1	0.75	0.5	0.25	0.7692	0.5540	0.7804	0.6288	**0.7836**	0.7302
	1	1	1	1	0	0.7956	0.5784	0.8304	0.6630	**0.8500**	0.7982
	1	1	0.5	0.2	0	0.8740	0.6910	0.9006	0.7436	**0.9104**	0.8758
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9908	**1.0000**	0.9958	0.9996	0.9994
Exponential	0	0	0	0	0	0.0460	0.0506	0.0450	0.0480	0.0552	0.0474
	1	1	0.5	0.5	0.5	0.5834	0.3968	0.6116	0.4444	**0.6516**	0.5698
	0.5	1	1	1	0.7	**0.5540**	0.4072	0.5346	0.4180	0.5042	0.4692
	0.7	1	0.7	0.7	0.5	0.7754	0.5774	**0.7800**	0.6216	0.7634	0.7196
	1	1	0.5	0.5	0.2	0.9060	0.7284	0.9232	0.8038	**0.9328**	0.8970
	0.75	1	0.75	0.5	0.25	**0.9738**	0.8544	0.9698	0.9024	0.9688	0.9512
	1	1	1	1	0	0.8996	0.7390	0.9334	0.8064	**0.9516**	0.9066
	1	1	0.5	0.2	0	0.9916	0.9182	0.9920	0.9550	0.9938	0.9866
	0	1.6	0.8	0.4	0.2	1.0000	0.9994	**1.0000**	0.9998	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0502	0.0526	0.0502	0.0500	0.0510	0.0560
	1	1	0.5	0.5	0.5	0.2656	0.2060	0.2722	0.2210	**0.2992**	0.2574
	0.5	1	1	1	0.7	**0.2446**	0.1850	0.2312	0.1874	0.2240	0.2084
	0.7	1	0.7	0.7	0.5	**0.3432**	0.2526	0.3422	0.2842	0.3374	0.3244
	1	1	0.5	0.5	0.2	0.4982	0.3558	0.5388	0.3974	**0.5554**	0.4884
	0.75	1	0.75	0.5	0.25	0.5956	0.4296	**0.6280**	0.4694	0.6242	0.5702
	1	1	1	1	0	0.6264	0.4442	0.6736	0.5080	**0.6980**	0.6316
	1	1	0.5	0.2	0	0.7350	0.5288	0.7656	0.5896	**0.7708**	0.7154
	0	1.6	0.8	0.4	0.2	**0.9948**	0.9300	0.9918	0.9558	0.9918	0.9832

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (CRD and RCBD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.22. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2. Treatments=5, CRD=15, RCBD=15 Peak=2

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0500	0.0512	0.0548	0.0498	0.0522	0.0474
	1	1	0.5	0.5	0.5	0.4044	0.2918	0.4202	0.3112	**0.4460**	0.3802
	0.5	1	1	1	0.7	**0.3684**	0.2772	0.3452	0.2800	0.3510	0.3034
	0.7	1	0.7	0.7	0.5	0.5254	0.3838	0.5134	0.4020	**0.5428**	0.4622
	1	1	0.5	0.5	0.2	0.7316	0.5522	0.7646	0.6040	**0.7798**	0.6738
	0.75	1	0.75	0.5	0.25	0.8284	0.6600	0.8480	0.6962	**0.8484**	0.7758
	1	1	1	1	0	0.8648	0.6910	0.8962	0.7328	**0.9040**	0.8208
	1	1	0.5	0.2	0	0.9268	0.7950	0.9506	0.8264	**0.9532**	0.8834
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9984	**1.0000**	0.9992	**1.0000**	0.9996
Exponential	0	0	0	0	0	0.0510	0.0486	0.0486	0.0512	0.0512	0.0428
	1	1	0.5	0.5	0.5	0.6566	0.4706	0.6996	0.5114	**0.7244**	0.6106
	0.5	1	1	1	0.7	**0.6302**	0.4732	0.6144	0.4926	0.5890	0.5488
	0.7	1	0.7	0.7	0.5	0.8458	0.6738	**0.8522**	0.7166	0.8354	0.7784
	1	1	0.5	0.5	0.2	0.9544	0.8204	0.9614	0.8558	**0.9674**	0.9278
	0.75	1	0.75	0.5	0.25	0.9856	0.9240	**0.9892**	0.9458	0.9880	0.9728
	1	1	1	1	0	0.9490	0.8286	0.9654	0.8682	**0.9740**	0.9282
	1	1	0.5	0.2	0	**0.9976**	0.9658	0.9974	0.9788	0.9972	0.9922
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
T with 3 degrees of freedom	0	0	0	0	0	0.0494	0.0538	0.0486	0.0472	0.0458	0.0496
	1	1	0.5	0.5	0.5	0.3054	0.2262	0.3294	0.2408	**0.3482**	0.2818
	0.5	1	1	1	0.7	**0.2836**	0.2030	0.2716	0.2142	0.2652	0.2440
	0.7	1	0.7	0.7	0.5	0.3896	0.2954	0.3804	0.3066	**0.4012**	0.3450
	1	1	0.5	0.5	0.2	0.5880	0.4246	0.6104	0.4618	**0.6198**	0.5132
	0.75	1	0.75	0.5	0.25	0.6870	0.5026	0.6926	0.5524	**0.7002**	0.5964
	1	1	1	1	0	0.7046	0.5286	0.7530	0.5792	**0.7632**	0.6564
	1	1	0.5	0.2	0	0.8046	0.6330	0.8404	0.6704	**0.8412**	0.7516
	0	1.6	0.8	0.4	0.2	**0.9986**	0.9752	0.9974	0.9834	0.9980	0.9928

Selected results for Treatments 4 at Peak 2 for Mixed Design s of (CRD and RCBD). Results show that with Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.23. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, BIBD =5, CRD=10, RCBD=5 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0498	0.0486	0.0542	0.0486	0.0508	0.0502
	0	0.5	1	1	0.7	0.3378	0.4846	0.5902	0.5354	0.5980	0.5514
	0.7	0.7	1	0.5	0.5	0.2700	0.2676	**0.3306**	0.2840	0.3122	0.2840
	0	0.5	1	0.5	0	0.8388	0.8106	0.8930	0.8520	**0.9058**	0.8734
	0	1	1	0	0	**0.9198**	0.8272	0.9022	0.8364	0.8952	0.8618
	1	1	1	0.5	0	**0.5270**	0.3624	0.4098	0.3646	0.4004	0.3748
	1	1	1	1	0	**0.4644**	0.3536	0.3912	0.3506	0.3956	0.3656
Exponential	0	0	0	0	0	0.0462	0.0500	0.0400	0.0458	0.0520	0.0508
	0	0.5	1	1	0.7	0.7334	0.5494	**0.8522**	0.7926	0.8422	0.8110
	0.7	0.7	1	0.5	0.5	0.4510	0.4336	**0.5472**	0.4710	0.5426	0.5038
	0	0.5	1	0.5	0	0.9796	0.9656	**0.9956**	0.9840	0.9920	0.9898
	0	1	1	0	0	**0.9864**	0.9638	0.9788	0.9666	0.9804	0.9714
	1	1	1	0.5	0	**0.7256**	0.5546	0.5866	0.5340	0.5894	0.5508
	1	1	1	1	0	**0.5884**	0.4520	0.5052	0.4572	0.5206	0.4728
T with 3 degrees of freedom	0	0	0	0	0	0.0464	0.0540	0.0508	0.0520	0.0504	0.0476
	0	0.5	1	1	0.7	0.3700	0.2538	0.4474	0.3900	**0.4662**	0.4174
	0.7	0.7	1	0.5	0.5	0.2134	0.2088	**0.2558**	0.2134	0.2396	0.2344
	0	0.5	1	0.5	0	0.6656	0.6470	0.7556	0.6882	**0.7624**	0.7100
	0	1	1	0	0	**0.7762**	0.6744	0.7488	0.6880	0.7576	0.6986
	1	1	1	0.5	0	**0.3966**	0.2868	0.3058	0.2694	0.3124	0.2766
	1	1	1	1	0	**0.3656**	0.2644	0.2944	0.2626	0.2994	0.2680

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.24. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, BIBD =5, CRD=15, RCBD=5 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0510	0.0516	0.0480	0.0458	0.0456	0.0524
	0	0.5	1	1	0.7	0.6208	0.4246	0.6570	0.6536	**0.6738**	0.6716
	0.7	0.7	1	0.5	0.5	0.3398	0.3202	0.3642	0.3618	**0.3752**	0.3598
	0	0.5	1	0.5	0	0.9292	0.8848	0.9470	0.9354	**0.9552**	0.9432
	0	1	1	0	0	**0.9488**	0.9306	0.9318	0.9234	0.9392	0.9382
	1	1	1	0.5	0	**0.5812**	0.4558	0.4672	0.4522	0.4740	0.4528
	1	1	1	1	0	**0.5236**	0.4388	0.4494	0.4368	0.4540	0.4408
Exponential	0	0	0	0	0	0.0506	0.0498	0.0534	0.0522	0.0498	0.0466
	0	0.5	1	1	0.7	0.8708	0.6454	**0.9082**	0.8970	0.9014	0.9012
	0.7	0.7	1	0.5	0.5	0.5756	0.5310	0.6218	0.6010	**0.6268**	0.6290
	0	0.5	1	0.5	0	0.9976	0.9874	**0.9988**	0.9962	0.9974	0.9986
	0	1	1	0	0	**0.9960**	0.9920	0.9938	0.9922	0.9940	0.9932
	1	1	1	0.5	0	**0.7986**	0.6680	0.6752	0.6634	0.6742	0.6722
	1	1	1	1	0	**0.6594**	0.5638	0.5916	0.5614	0.6000	0.5780
T with 3 degrees of freedom	0	0	0	0	0	0.0566	0.0508	0.0520	0.0496	0.0506	0.0502
	0	0.5	1	1	0.7	0.4824	0.3358	**0.5228**	0.4978	0.5220	0.5104
	0.7	0.7	1	0.5	0.5	0.2624	0.2348	0.2752	0.2678	**0.2888**	0.2666
	0	0.5	1	0.5	0	0.7998	0.7476	0.8304	0.8176	**0.8348**	0.8240
	0	1	1	0	0	**0.8568**	0.7946	0.8250	0.8006	0.8218	0.8234
	1	1	1	0.5	0	**0.4500**	0.3468	0.3528	0.3320	0.3570	0.3448
	1	1	1	1	0	**0.3988**	0.3272	0.3408	0.3248	0.3502	0.3458

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.25. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, BIBD =10, CRD=5, RCBD=10 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0488	0.0526	0.0508	0.0466	0.0548	0.0510
	0	0.5	1	1	0.7	0.3184	0.3000	0.6370	0.5350	**0.6578**	0.5922
	0.7	0.7	1	0.5	0.5	0.2790	0.2372	0.3468	0.2890	**0.3474**	0.3144
	0	0.5	1	0.5	0	0.8130	0.7366	0.9336	0.8628	**0.9338**	0.9092
	0	1	1	0	0	**0.9482**	0.8428	0.9200	0.8450	0.9242	0.8976
	1	1	1	0.5	0	**0.6008**	0.4206	0.4442	0.3560	0.4648	0.4060
	1	1	1	1	0	**0.5176**	0.3696	0.4208	0.3478	0.4128	0.3900
Exponential	0	0	0	0	0	0.0504	0.0490	0.0492	0.0480	0.0536	0.0554
	0	0.5	1	1	0.7	0.4956	0.4604	0.8842	0.7964	**0.8868**	0.8368
	0.7	0.7	1	0.5	0.5	0.4628	0.3798	0.5800	0.4852	**0.6006**	0.5390
	0	0.5	1	0.5	0	0.9666	0.9376	**0.9962**	0.9836	0.9954	0.9938
	0	1	1	0	0	**0.9948**	0.9610	0.9900	0.9640	0.9918	0.9860
	1	1	1	0.5	0	**0.8234**	0.6226	0.6416	0.5142	0.6406	0.5902
	1	1	1	1	0	**0.6782**	0.4872	0.5558	0.4676	0.5680	0.5148
T with 3 degrees of freedom	0	0	0	0	0	0.0526	0.0582	0.0488	0.0516	0.0552	0.0584
	0	0.5	1	1	0.7	0.2518	0.2396	0.4908	0.4148	**0.5102**	0.4428
	0.7	0.7	1	0.5	0.5	0.2218	0.1868	0.2730	0.2300	**0.2866**	0.2462
	0	0.5	1	0.5	0	0.6594	0.5684	**0.8144**	0.7002	0.8070	0.7542
	0	1	1	0	0	**0.8352**	0.6778	0.7946	0.6884	0.7980	0.7558
	1	1	1	0.5	0	**0.4600**	0.3314	0.3388	0.2796	0.3476	0.3110
	1	1	1	1	0	**0.3914**	0.2812	0.3362	0.2636	0.3276	0.2986

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.26. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, BIBD =10, CRD=10, RCBD=10 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0552	0.0524	0.0544	0.0514	0.0458	0.0500
	0	0.5	1	1	0.7	0.4936	0.3940	0.7300	0.5760	**0.7460**	0.6364
	0.7	0.7	1	0.5	0.5	0.3394	0.2970	**0.4210**	0.3082	0.4076	0.3348
	0	0.5	1	0.5	0	0.9204	0.8580	**0.9744**	0.8834	0.9690	0.9328
	0	1	1	0	0	**0.9800**	0.8750	0.9698	0.8806	0.9776	0.9196
	1	1	1	0.5	0	**0.6734**	0.4028	0.5292	0.3956	0.5382	0.4356
	1	1	1	1	0	**0.6060**	0.3934	0.5102	0.3672	0.5082	0.4154
Exponential	0	0	0	0	0	0.0570	0.0482	0.0476	0.0454	0.0514	0.0498
	0	0.5	1	1	0.7	0.7554	0.6222	0.9422	0.8444	**0.9434**	0.8790
	0.7	0.7	1	0.5	0.5	0.5786	0.4854	0.7090	0.5394	**0.7096**	0.5706
	0	0.5	1	0.5	0	0.9946	0.9864	**0.9998**	0.9946	0.9994	0.9964
	0	1	1	0	0	**0.9992**	0.9782	0.9988	0.9826	0.9988	0.9874
	1	1	1	0.5	0	**0.8832**	0.6078	0.7494	0.5844	0.7420	0.6338
	1	1	1	1	0	0.7688	0.5184	0.6644	0.4890	**0.6788**	0.5486
T with 3 degrees of freedom	0	0	0	0	0	0.0448	0.0528	0.0472	0.0466	0.0518	0.0514
	0	0.5	1	1	0.7	0.3646	0.3050	0.5908	0.4486	**0.5958**	0.4904
	0.7	0.7	1	0.5	0.5	0.2526	0.2306	**0.3124**	0.2444	0.3092	0.2652
	0	0.5	1	0.5	0	0.7832	0.6958	0.8886	0.7630	**0.8980**	0.7948
	0	1	1	0	0	**0.9018**	0.7332	0.8750	0.7372	0.8828	0.7878
	1	1	1	0.5	0	**0.5318**	0.3210	0.4074	0.2980	0.4040	0.3342
	1	1	1	1	0	**0.4744**	0.2844	0.3758	0.2960	0.3916	0.3158

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.27. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, BIBD =10, CRD=15, RCBD=10 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0460	0.0550	0.0510	0.0502	0.0460	0.0496
	0	0.5	1	1	0.7	0.6292	0.4920	0.8082	0.6922	**0.8122**	0.7088
	0.7	0.7	1	0.5	0.5	0.3884	0.3620	0.4634	0.3760	**0.4726**	0.4024
	0	0.5	1	0.5	0	0.9536	0.9308	**0.9896**	0.9536	0.9884	0.9564
	0	1	1	0	0	**0.9932**	0.9362	0.9852	0.9452	0.9832	0.9546
	1	1	1	0.5	0	**0.7506**	0.4732	0.5948	0.4868	0.6066	0.4968
	1	1	1	1	0	**0.6606**	0.4654	0.5842	0.4580	0.5750	0.4778
Exponential	0	0	0	0	0	0.0548	0.0534	0.0516	0.0514	0.0572	0.0506
	0	0.5	1	1	0.7	0.8850	0.7258	0.9712	0.9152	**0.9738**	0.9318
	0.7	0.7	1	0.5	0.5	0.6460	0.5966	**0.7740**	0.6320	0.7656	0.6724
	0	0.5	1	0.5	0	0.9986	0.9984	**1.0000**	0.9990	**1.0000**	0.9988
	0	1	1	0	0	**0.9998**	0.9942	**0.9998**	0.9958	0.9992	0.9974
	1	1	1	0.5	0	**0.9296**	0.7038	0.8162	0.6886	0.8188	0.7172
	1	1	1	1	0	**0.8192**	0.5936	0.7434	0.5850	0.7226	0.6164
T with 3 degrees of freedom	0	0	0	0	0	0.0504	0.0512	0.0502	0.0478	0.0488	0.0478
	0	0.5	1	1	0.7	0.4794	0.3672	**0.6548**	0.5346	0.6516	0.5514
	0.7	0.7	1	0.5	0.5	0.2940	0.2702	0.3532	0.2732	**0.3676**	0.2928
	0	0.5	1	0.5	0	0.8566	0.8192	0.9258	0.8362	**0.9310**	0.8622
	0	1	1	0	0	**0.9374**	0.8198	0.9210	0.8210	0.9252	0.8492
	1	1	1	0.5	0	**0.5822**	0.3576	0.4476	0.3598	0.4658	0.3768
	1	1	1	1	0	**0.5180**	0.3620	0.4492	0.3518	0.4554	0.3622

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.28. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, BIBD =15, CRD=5, RCBD=15 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0500	0.0498	0.0470	0.0428	0.0526	0.0494
	0	0.5	1	1	0.7	0.3410	0.3220	**0.7516**	0.6584	0.7438	0.7092
	0.7	0.7	1	0.5	0.5	0.3328	0.2692	**0.4328**	0.3504	0.4304	0.3968
	0	0.5	1	0.5	0	**0.9938**	0.8140	0.9766	0.9354	0.9762	0.9612
	0	1	1	0	0	**0.9864**	0.9228	0.9716	0.9258	0.9744	0.9598
	1	1	1	0.5	0	**0.7246**	0.5468	0.5442	0.4480	0.5456	0.5114
	1	1	1	1	0	**0.6268**	0.4844	0.5250	0.4296	0.5184	0.5060
Exponential	0	0	0	0	0	0.0494	0.0546	0.0492	0.0506	0.0506	0.0506
	0	0.5	1	1	0.7	0.5216	0.5126	**0.9520**	0.8898	0.9498	0.9270
	0.7	0.7	1	0.5	0.5	0.5592	0.4556	0.7086	0.5942	**0.7212**	0.6664
	0	0.5	1	0.5	0	0.9914	0.9710	0.9996	0.9976	**1.0000**	0.9992
	0	1	1	0	0	**0.9998**	0.9898	0.9988	0.9898	0.9988	0.9980
	1	1	1	0.5	0	**0.9158**	0.7602	0.7558	0.6426	0.7426	0.7058
	1	1	1	1	0	**0.7980**	0.6008	0.6726	0.5668	0.6860	0.6426
T with 3 degrees of freedom	0	0	0	0	0	0.0436	0.0532	0.0508	0.0464	0.0544	0.0546
	0	0.5	1	1	0.7	0.2572	0.2440	0.5944	0.4964	**0.6084**	0.5566
	0.7	0.7	1	0.5	0.5	0.2586	0.2056	0.3060	0.2712	**0.3254**	0.3002
	0	0.5	1	0.5	0	0.7696	0.6538	**0.8964**	0.8138	0.8948	0.8646
	0	1	1	0	0	**0.9200**	0.7932	0.8852	0.7944	0.8804	0.8650
	1	1	1	0.5	0	**0.5674**	0.4134	0.4038	0.3444	0.4098	0.3790
	1	1	1	1	0	**0.5026**	0.3662	0.3890	0.3304	0.4116	0.3940

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.29. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, BIBD =15, CRD=10, RCBD=15 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0538	0.0516	0.0528	0.0592	0.0538	0.0546
	0	0.5	1	1	0.7	0.5182	0.4586	**0.8302**	0.6356	0.8300	0.7010
	0.7	0.7	1	0.5	0.5	0.3968	0.3020	0.4888	0.3426	**0.5076**	0.3858
	0	0.5	1	0.5	0	0.9940	0.8844	**0.9946**	0.9352	0.9926	0.9572
	0	**1**	**1**	**0**	**0**	**0.9950**	0.9118	0.9920	0.9158	0.9926	0.9552
	1	1	1	0.5	0	**0.7904**	0.4680	0.6260	0.4408	0.6288	0.4906
	1	1	1	1	0	**0.7184**	0.4248	0.6166	0.4264	0.6102	0.4704
Exponential	0	0	0	0	0	0.0446	0.0474	0.0476	0.0490	0.0482	0.0488
	0	0.5	1	1	0.7	0.7486	0.6828	0.9748	0.8894	**0.9802**	0.9226
	0.7	0.7	1	0.5	0.5	0.6630	0.5062	0.7922	0.5948	**0.7938**	0.6534
	0	0.5	1	0.5	0	0.9974	0.9904	**1.0000**	0.9978	**1.0000**	0.9984
	0	1	1	0	0	**1.0000**	0.9856	0.9990	0.9908	0.9998	0.9976
	1	1	1	0.5	0	**0.9498**	0.6730	0.8370	0.6428	0.8348	0.7056
	1	1	1	1	0	**0.8550**	0.5580	0.7536	0.5572	0.7760	0.6164
T with 3 degrees of freedom	0	0	0	0	0	0.0498	0.0512	0.0484	0.0502	0.0444	0.0470
	0	0.5	1	1	0.7	0.3824	0.3364	**0.6792**	0.4902	0.6748	0.5348
	0.7	0.7	1	0.5	0.5	0.3068	0.2316	0.3746	0.2588	**0.3754**	0.2954
	0	0.5	1	0.5	0	**0.8474**	0.7282	0.9480	0.8010	0.9466	0.8492
	0	1	1	0	0	**0.9594**	0.7720	0.9392	0.7790	0.9444	0.8410
	1	1	1	0.5	0	**0.6376**	0.3574	0.4784	0.3390	0.4822	0.3802
	1	1	1	1	0	**0.5644**	0.3176	0.4720	0.3314	0.4724	0.3588

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.30. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, BIBD =15, CRD=15, RCBD=15 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0478	0.0526	0.0470	0.0486	0.0516	0.0510
	0	0.5	1	1	0.7	0.6304	0.5322	**0.8858**	0.7014	0.8876	0.7466
	0.7	0.7	1	0.5	0.5	0.4382	0.3656	0.5396	0.3730	**0.5508**	0.4180
	0	0.5	1	0.5	0	0.9796	0.9510	**0.9980**	0.9604	0.9978	0.9722
	0	1	1	0	0	0.9976	0.9536	0.9962	0.9558	0.9968	0.9682
	1	1	1	0.5	0	**0.8382**	0.5014	0.6756	0.5028	0.6936	0.5222
	1	1	1	1	0	**0.7522**	0.4794	0.6432	0.4932	0.6780	0.5064
Exponential	0	0	0	0	0	0.0480	0.0524	0.0496	0.0496	0.0514	0.0540
	0	0.5	1	1	0.7	0.8836	0.7808	0.9896	0.9356	**0.9910**	0.9488
	0.7	0.7	1	0.5	0.5	0.7284	0.6290	0.8474	0.6674	**0.8594**	0.7014
	0	0.5	1	0.5	0	0.9998	0.9982	**1.0000**	0.9988	**1.0000**	0.9994
	0	1	1	0	0	**1.0000**	0.9948	**1.0000**	0.9974	**1.0000**	0.9986
	1	1	1	0.5	0	**0.9706**	0.7522	0.8978	0.7264	0.8840	0.7448
	1	1	1	1	0	**0.8962**	0.6246	0.8168	0.6206	0.8308	0.6540
T with 3 degrees of freedom	0	0	0	0	0	0.0494	0.0512	0.0524	0.0440	0.0518	0.0432
	0	0.5	1	1	0.7	0.4986	0.4030	**0.7374**	0.5492	0.7370	0.5724
	0.7	0.7	1	0.5	0.5	0.3334	0.2740	**0.4280**	0.3008	0.4210	0.3142
	0	0.5	1	0.5	0	0.9072	0.8302	**0.9706**	0.8626	0.9686	0.8832
	0	1	1	0	0	**0.9760**	0.8386	0.9680	0.8438	0.9646	0.8680
	1	1	1	0.5	0	**0.6792**	0.3968	0.5424	0.3830	0.5298	0.4128
	1	1	1	1	0	**0.6076**	0.3596	0.5124	0.3582	0.5246	0.3886

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.31. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 3. Treatments=5, BIBD =5, CRD=10 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0494	0.0534	0.0504	0.0548	0.0458	0.0438
	0	0.5	1	1	0.7	0.4318	0.4788	0.4386	0.4712	0.4368	**0.4906**
	0.7	0.7	1	0.5	0.5	0.2292	0.2522	0.2334	0.2608	0.2362	**0.2610**
	0	0.5	1	0.5	0	0.7560	0.8004	0.7450	0.8086	0.7552	**0.8102**
	0	1	1	0	0	0.7372	0.7786	0.7324	0.7886	0.7326	**0.7970**
	1	1	1	0.5	0	0.3004	0.3256	0.3084	0.3262	0.3010	**0.3282**
	1	1	1	1	0	0.2852	**0.3216**	0.2970	0.3088	0.2916	0.3036
Exponential	0	0	0	0	0	0.0494	0.0494	0.0524	0.0510	0.0512	0.0464
	0	0.5	1	1	0.7	0.6720	0.7362	0.6728	0.7432	0.6724	**0.7480**
	0.7	0.7	1	0.5	0.5	0.3798	0.4214	0.3840	0.4128	0.3786	**0.4220**
	0	0.5	1	0.5	0	0.9394	0.9682	0.9378	0.9718	0.9368	**0.9736**
	0	1	1	0	0	0.8880	0.9322	0.8902	**0.9444**	0.8810	0.9360
	1	1	1	0.5	0	0.4354	0.4644	0.4414	0.4802	0.4442	**0.4834**
	1	1	1	1	0	0.3830	0.4104	0.3746	**0.4134**	0.3718	0.4062
T with 3 degrees of freedom	0	0	0	0	0	0.0486	0.0526	0.0522	0.0506	0.0520	0.0506
	0	0.5	1	1	0.7	0.3304	**0.3666**	0.3342	0.3534	0.3302	0.3514
	0.7	0.7	1	0.5	0.5	0.1972	0.1982	0.1848	**0.2018**	0.1848	0.1986
	0	0.5	1	0.5	0	0.5966	**0.6424**	0.5870	**0.6280**	0.5866	**0.6348**
	0	1	1	0	0	0.5646	**0.6212**	0.5668	**0.6210**	0.5670	**0.6286**
	1	1	1	0.5	0	0.2382	**0.2542**	0.2262	**0.2530**	0.2472	**0.2496**
	1	1	1	1	0	0.2230	0.2388	0.2310	**0.2440**	0.2192	0.2400

Results show that Standardized Last test performed better than Standardized First in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.32. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 3. Treatments=5, BIBD =10, CRD=5 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0498	0.0462	0.0496	0.0530	0.0516	0.0524
	0	0.5	1	1	0.7	**0.3690**	0.3266	0.3634	0.3226	0.3636	0.3256
	0.7	0.7	1	0.5	0.5	0.1992	0.1852	0.1976	0.1966	**0.2122**	0.1762
	0	0.5	1	0.5	0	0.6544	0.6006	**0.6588**	0.5920	0.6510	0.5974
	0	1	1	0	0	**0.6442**	0.5730	0.6392	0.5722	0.6340	0.5682
	1	1	1	0.5	0	0.2476	0.2168	**0.2522**	0.2220	0.2508	0.2218
	1	1	1	1	0	0.2464	0.2176	0.2414	0.2166	**0.2486**	0.2074
Exponential	0	0	0	0	0	0.0524	0.0528	0.0554	0.0510	0.0524	0.0530
	0	0.5	1	1	0.7	0.5700	0.5040	0.5694	0.5162	**0.5726**	0.5104
	0.7	0.7	1	0.5	0.5	**0.3256**	0.2670	**0.3142**	0.2622	0.3238	0.2616
	0	0.5	1	0.5	0	0.8796	0.8406	**0.8860**	0.8404	0.8804	0.8396
	0	1	1	0	0	0.8086	0.7510	**0.8150**	0.7506	0.8082	0.7666
	1	1	1	0.5	0	0.3456	0.3110	**0.3564**	0.3060	0.3470	0.3194
	1	1	1	1	0	0.3176	0.2722	**0.3308**	0.2812	0.3152	0.2692
T with 3 degrees of freedom	0	0	0	0	0	0.0444	0.0500	0.0482	0.0514	0.0512	0.0474
	0	0.5	1	1	0.7	**0.2824**	0.2574	0.2750	0.2476	0.2762	0.2448
	0.7	0.7	1	0.5	0.5	0.1602	0.1510	**0.1678**	0.1422	0.1596	0.1478
	0	0.5	1	0.5	0	0.5024	0.4492	**0.5036**	0.4548	0.5020	0.4466
	0	1	1	0	0	**0.4914**	0.4260	0.4872	0.4382	0.4726	0.4308
	1	1	1	0.5	0	0.1960	0.1770	0.1902	0.1816	**0.2084**	0.1798
	1	1	1	1	0	0.2006	0.1776	0.1802	0.1716	0.1916	0.1728

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.33. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 3. Treatments=5, BIBD =10, CRD=10 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0520	0.0478	0.0510	0.0566	0.0528	0.0576
	0	0.5	1	1	0.7	0.5094	0.4986	**0.5144**	0.4838	0.5028	0.4894
	0.7	0.7	1	0.5	0.5	**0.2746**	0.2718	0.2634	0.2616	0.2704	0.2560
	0	0.5	1	0.5	0	**0.8290**	0.8168	0.8274	0.8164	0.8208	0.8204
	0	1	1	0	0	0.8030	0.7982	0.8008	0.7956	**0.8126**	0.8012
	1	1	1	0.5	0	**0.3502**	0.3372	**0.3454**	0.3322	0.3340	0.3350
	1	1	1	1	0	0.3342	0.3234	**0.3410**	0.3184	0.3214	0.3186
Exponential	0	0	0	0	0	0.0498	0.0464	0.0516	0.0512	0.0504	0.0530
	0	0.5	1	1	0.7	0.7340	0.7340	**0.7448**	0.7400	0.7418	0.7380
	0.7	0.7	1	0.5	0.5	0.4482	0.4376	**0.4512**	0.4452	0.4444	0.4292
	0	0.5	1	0.5	0	**0.9770**	0.9734	**0.9762**	0.9718	0.9750	0.9762
	0	1	1	0	0	**0.9462**	0.9416	0.9348	0.9436	0.9364	0.9446
	1	1	1	0.5	0	0.5016	0.4864	0.4986	0.4914	**0.5018**	0.4804
	1	1	1	1	0	**0.4380**	0.4206	0.4360	0.4248	0.4374	0.4166
T with 3 degrees of freedom	0	0	0	0	0	0.0494	0.0530	0.0524	0.0514	0.0474	0.0508
	0	0.5	1	1	0.7	0.3718	0.3662	0.3776	0.3646	**0.3876**	0.3670
	0.7	0.7	1	0.5	0.5	0.2098	0.2072	0.2040	0.2008	**0.2084**	0.1966
	0	0.5	1	0.5	0	0.6676	0.6454	0.6584	0.6464	**0.6748**	0.6558
	0	1	1	0	0	0.6404	0.6338	0.6390	0.6364	**0.6466**	0.6376
	1	1	1	0.5	0	0.2610	0.2480	0.2602	0.2476	**0.2638**	0.2530
	1	1	1	1	0	0.2450	0.2400	**0.2508**	0.2488	0.2502	0.2414

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.34. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 3. Treatments=5, BIBD =10, CRD=15, Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0444	0.0508	0.0470	0.0512	0.0468	0.0566
	0	0.5	1	1	0.7	0.5938	0.6286	0.6036	0.6316	0.6030	**0.6356**
	0.7	0.7	1	0.5	0.5	0.3246	0.3360	0.3174	**0.3424**	0.3226	0.3376
	0	0.5	1	0.5	0	0.9126	0.9202	0.9136	**0.9242**	0.9054	0.9186
	0	1	1	0	0	0.8992	0.9132	0.8986	0.9144	0.8928	**0.9174**
	1	1	1	0.5	0	0.4140	0.4392	0.4218	**0.4404**	0.4272	0.4400
	1	1	1	1	0	0.4006	0.4052	0.3892	**0.4188**	0.3978	0.4122
Exponential	0	0	0	0	0	0.0462	0.0578	0.0514	0.0568	0.0480	0.0488
	0	0.5	1	1	0.7	0.8436	0.8800	0.8506	0.8746	0.8532	**0.8852**
	0.7	0.7	1	0.5	0.5	0.5520	0.5764	0.5532	0.5538	0.5522	**0.5796**
	0	0.5	1	0.5	0	0.9946	0.9970	0.9946	0.9972	0.9946	**0.9978**
	0	1	1	0	0	0.9796	**0.9890**	0.9788	**0.9890**	0.9780	0.9882
	1	1	1	0.5	0	0.6014	0.6364	0.6046	**0.6476**	0.6104	0.6438
	1	1	1	1	0	0.5206	**0.5408**	0.5228	**0.5408**	0.55292	0.5330
T with 3 degrees of freedom	0	0	0	0	0	0.0538	0.0472	0.0524	0.0506	0.0490	0.0484
	0	0.5	1	1	0.7	0.4604	0.4848	0.4662	**0.4942**	0.4808	0.4846
	0.7	0.7	1	0.5	0.5	0.2438	0.2514	0.2622	**0.2664**	0.2470	0.2540
	0	0.5	1	0.5	0	0.7744	**0.8004**	0.7622	0.7970	0.7758	0.7870
	0	1	1	0	0	0.7548	0.7836	0.7538	0.7872	0.7562	**0.7886**
	1	1	1	0.5	0	0.3102	0.3268	0.3224	0.3244	0.3156	**0.3356**
	1	1	1	1	0	0.3216	0.3104	0.3098	**0.3146**	0.3016	0.3020

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.35. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 3. Treatments=5, BIBD =15, CRD=5, Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0454	0.0456	0.0450	0.0496	0.0522	0.0540
	0	0.5	1	1	0.7	**0.4162**	0.3452	0.4076	0.3414	0.4142	0.3592
	0.7	0.7	1	0.5	0.5	0.2168	0.1910	0.2302	0.1926	**0.2310**	0.1940
	0	0.5	1	0.5	0	0.7252	0.6312	0.7240	0.6296	**0.7300**	0.6176
	0	1	1	0	0	**0.7068**	0.5996	0.6880	0.6134	0.6998	0.6216
	1	1	1	0.5	0	0.2864	0.2440	0.2758	0.2296	**0.2878**	0.2360
	1	1	1	1	0	0.2580	0.2214	**0.2736**	0.2240	0.2694	0.2318
Exponential	0	0	0	0	0	0.0486	0.0442	0.0508	0.0504	0.0548	0.0512
	0	0.5	1	1	0.7	0.6152	0.5510	**0.6476**	0.5470	0.6276	0.5272
	0.7	0.7	1	0.5	0.5	**0.3690**	0.2924	0.3566	0.2962	0.3640	0.2804
	0	0.5	1	0.5	0	**0.9350**	0.8658	0.9250	0.8648	0.9278	0.8654
	0	1	1	0	0	**0.8676**	0.7900	0.8630	0.7820	0.8656	0.7906
	1	1	1	0.5	0	0.4018	0.3270	0.4068	0.3306	**0.4076**	0.3326
	1	1	1	1	0	0.3614	0.2958	0.3502	0.2982	**0.3640**	0.2838
T with 3 degrees of freedom	0	0	0	0	0	0.0526	0.0542	0.0516	0.0498	0.0514	0.0478
	0	0.5	1	1	0.7	0.3090	0.2696	**0.3152**	0.2702	0.3046	0.2644
	0.7	0.7	1	0.5	0.5	0.1740	0.1614	0.1816	0.1590	**0.1834**	0.1580
	0	0.5	1	0.5	0	**0.5756**	0.4764	0.5724	0.4836	0.5614	0.4778
	0	1	1	0	0	0.5424	0.4592	0.5466	0.4612	**0.5480**	0.4494
	1	1	1	0.5	0	0.2244	0.1906	0.2208	0.1890	**0.2256**	0.1888
	1	1	1	1	0	0.2066	0.1760	0.2116	0.1758	**0.2162**	0.1750

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.36. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 3. Treatments=5, BIBD =15, CRD=10, Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0482	0.0486	0.0468	0.0452	0.0494	0.0564
	0	0.5	1	1	0.7	**0.5464**	0.4916	0.5414	0.4886	0.5402	0.4972
	0.7	0.7	1	0.5	0.5	0.2860	0.2498	**0.3080**	0.2618	0.2952	0.2614
	0	0.5	1	0.5	0	0.8720	0.8324	0.8752	0.8276	**0.8792**	0.8212
	0	1	1	0	0	0.8558	0.8158	0.8542	0.8140	**0.8614**	0.8098
	1	1	1	0.5	0	**0.3844**	0.3314	0.3778	0.3364	0.3784	0.3384
	1	1	1	1	0	0.3538	0.3224	**0.3610**	0.3318	0.3592	0.3150
Exponential	0	0	0	0	0	0.0422	0.0466	0.0530	0.0494	0.0492	0.0484
	0	0.5	1	1	0.7	0.7922	0.7384	0.7950	0.7442	**0.8002**	0.7546
	0.7	0.7	1	0.5	0.5	0.4968	0.4346	0.5020	0.4328	**0.5130**	0.4376
	0	0.5	1	0.5	0	**0.9878**	0.9772	0.9844	0.9800	0.9876	0.9796
	0	1	1	0	0	0.9662	0.9486	**0.9674**	0.9452	0.9598	0.9488
	1	1	1	0.5	0	0.5480	0.4950	**0.5498**	0.4888	0.5370	0.5106
	1	1	1	1	0	**0.4886**	0.4284	0.4752	0.4152	0.4748	0.4184
T with 3 degrees of freedom	0	0	0	0	0	0.0474	0.0478	0.0444	0.0534	0.0556	0.0518
	0	0.5	1	1	0.7	**0.4210**	0.3772	0.4040	0.3770	0.4124	0.3768
	0.7	0.7	1	0.5	0.5	0.2300	0.2082	**0.2302**	0.2126	0.2220	0.2002
	0	0.5	1	0.5	0	**0.7248**	0.6642	0.7190	0.6634	0.7142	0.6666
	0	1	1	0	0	0.7018	0.6432	**0.7044**	0.6364	0.6868	0.6418
	1	1	1	0.5	0	0.2864	0.2536	**0.2886**	0.2538	**0.2886**	0.2608
	1	1	1	1	0	0.2728	0.2616	**0.2820**	0.2520	0.2692	0.2458

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.37. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 3. Treatments=5, BIBD =15, CRD=15, Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0492	0.0530	0.0508	0.0526	0.0518	0.0476
	0	0.5	1	1	0.7	**0.6514**	0.6286	**0.6514**	0.6260	0.6506	0.6348
	0.7	0.7	1	0.5	0.5	**0.3582**	0.3324	0.3546	0.3332	0.3550	0.3398
	0	0.5	1	0.5	0	0.9346	0.9274	**0.9394**	0.9286	0.9372	0.9296
	0	1	1	0	0	**0.9326**	0.9138	0.9250	0.9160	0.9318	0.9108
	1	1	1	0.5	0	**0.4622**	0.4342	0.4578	0.4302	0.4596	0.4424
	1	1	1	1	0	0.4256	0.4202	0.4366	0.4180	**0.4386**	0.4122
Exponential	0	0	0	0	0	0.0518	0.0460	0.0462	0.0488	0.0512	0.0518
	0	0.5	1	1	0.7	0.8860	0.8742	**0.8890**	0.8846	0.8774	0.8832
	0.7	0.7	1	0.5	0.5	0.6008	0.5900	0.5970	0.5958	**0.6014**	0.5846
	0	0.5	1	0.5	0	0.9980	0.9982	0.9978	0.9974	**0.9986**	0.9976
	0	1	1	0	0	0.9914	0.9912	**0.9930**	0.9920	0.9898	0.9924
	1	1	1	0.5	0	**0.6642**	0.6408	0.6530	0.6604	0.6428	0.6372
	1	1	1	1	0	**0.5674**	0.5408	0.5750	0.5422	0.5814	0.5420
T with 3 degrees of freedom	0	0	0	0	0	0.0504	0.0466	0.0466	0.0480	0.0524	0.0502
	0	0.5	1	1	0.7	0.5118	0.4790	0.5022	0.4814	**0.5202**	0.4828
	0.7	0.7	1	0.5	0.5	0.2732	0.2654	**0.2782**	0.2598	0.2726	0.2564
	0	0.5	1	0.5	0	**0.8302**	0.8030	0.8146	0.7954	0.8098	0.7922
	0	1	1	0	0	**0.8096**	0.7844	0.8048	0.7934	0.8006	0.7872
	1	1	1	0.5	0	0.3418	0.3372	0.3464	0.3302	**0.3496**	0.3264
	1	1	1	1	0	**0.3368**	0.3268	0.3324	0.3260	0.3378	0.3124

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.38. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, CRD=10, RCBD=5 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0508	0.0514	0.0500	0.0502	0.0492	0.0558
	0	0.5	1	1	0.7	0.4820	0.2892	**0.5912**	0.5174	0.5770	0.5378
	0.7	0.7	1	0.5	0.5	0.2660	0.2524	0.3020	0.2780	**0.3266**	0.2794
	0	0.5	1	0.5	0	0.7740	0.8128	0.8864	0.8474	**0.8902**	0.8694
	0	1	1	0	0	**0.9112**	0.8176	0.8752	0.8364	0.8846	0.8454
	1	1	1	0.5	0	**0.5234**	0.3484	0.3714	0.3378	0.4056	0.3698
	1	1	1	1	0	**0.4628**	0.3406	0.3834	0.3380	0.3878	0.3480
Exponential	0	0	0	0	0	0.0498	0.0518	0.0524	0.0552	0.0526	0.0482
	0	0.5	1	1	0.7	0.4428	0.7174	0.8364	0.7736	**0.8400**	0.8020
	0.7	0.7	1	0.5	0.5	0.4136	0.4278	0.5134	0.4538	**0.5372**	0.4896
	0	0.5	1	0.5	0	0.9474	0.9704	**0.9926**	0.9830	0.9908	0.9868
	0	1	1	0	0	**0.9878**	0.9522	0.9814	0.9616	0.9814	0.9680
	1	1	1	0.5	0	**0.7490**	0.5256	0.5800	0.5242	0.5716	0.5492
	1	1	1	1	0	**0.5894**	0.4242	0.4990	0.4510	0.5206	0.4668
T with 3 degrees of freedom	0	0	0	0	0	0.0496	0.0478	0.0490	0.0502	0.0538	0.0454
	0	0.5	1	1	0.7	0.3506	0.2152	0.4384	0.3972	**0.4464**	0.4084
	0.7	0.7	1	0.5	0.5	0.2012	0.1974	0.2362	0.2154	**0.2504**	0.2208
	0	0.5	1	0.5	0	0.6574	0.6128	0.7436	0.6824	**0.7606**	0.7152
	0	1	1	0	0	**0.7690**	0.6540	0.7306	0.6718	0.7378	0.6866
	1	1	1	0.5	0	**0.4002**	0.2824	0.2992	0.2698	0.3084	0.2844
	1	1	1	1	0	**0.3414**	0.2490	0.2974	0.2582	0.3058	0.2634

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.39. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, CRD=5, RCBD=10 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0532	0.0512	0.0494	0.0538	0.0564	0.0516
	0	0.5	1	1	0.7	0.2512	0.1774	0.5476	0.4856	**0.5648**	0.5388
	0.7	0.7	1	0.5	0.5	0.2222	0.1962	0.2890	0.2590	**0.3074**	0.3010
	0	0.5	1	0.5	0	0.6866	0.6388	**0.8782**	0.7954	0.8718	0.8642
	0	1	1	0	0	**0.9206**	0.7532	0.8680	0.7918	0.8718	0.8526
	1	1	1	0.5	0	**0.5802**	0.3720	0.3680	0.3184	0.3962	0.3708
	1	1	1	1	0	**0.5004**	0.3152	0.3642	0.2904	0.3648	0.3434
Exponential	0	0	0	0	0	0.0512	0.0424	0.0520	0.0468	0.0502	0.0562
	0	0.5	1	1	0.7	0.3950	0.2594	0.8118	0.7264	**0.8172**	0.7930
	0.7	0.7	1	0.5	0.5	0.3666	0.3100	0.5130	0.4140	**0.5228**	0.5040
	0	0.5	1	0.5	0	0.8980	0.8828	0.9854	0.9704	**0.9916**	0.9870
	0	1	1	0	0	**0.9896**	0.9202	0.9772	0.9390	0.9758	0.9698
	1	1	1	0.5	0	**0.7932**	0.5648	0.5514	0.4642	0.5366	0.5172
	1	1	1	1	0	**0.6364**	0.4240	0.4800	0.4126	0.4974	0.4682
T with 3 degrees of freedom	0	0	0	0	0	0.0506	0.0456	0.0432	0.0486	0.0516	0.0480
	0	**0.5**	**1**	**1**	**0.7**	**0.1912**	0.1386	**0.4344**	0.3764	0.4332	0.4054
	0.7	0.7	1	0.5	0.5	0.1704	0.1606	0.2332	0.1960	**0.2388**	0.2242
	0	0.5	1	0.5	0	0.5232	0.4972	0.7296	0.6434	**0.7320**	0.7036
	0	1	1	0	0	**0.7818**	0.5998	0.7160	0.6188	0.7334	0.6850
	1	1	1	0.5	0	**0.4434**	0.2786	0.2836	0.2482	0.3034	0.2706
	1	1	1	1	0	**0.3700**	0.2488	0.2756	0.2364	0.2948	0.2680

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.40. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, CRD=10, RCBD=10 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0482	0.0526	0.0538	0.0532	0.0508	0.0450
	0	0.5	1	1	0.7	0.4652	0.2636	0.6908	0.5764	**0.7124**	0.6110
	0.7	0.7	1	0.5	0.5	0.2986	0.2614	**0.3948**	0.2910	0.3776	0.3218
	0	0.5	1	0.5	0	0.8490	0.8346	**0.9596**	0.8700	0.9558	0.9066
	0	1	1	0	0	**0.9706**	0.8506	0.9476	0.8624	0.9500	0.8988
	1	1	1	0.5	0	**0.6666**	0.3942	0.4838	0.3700	0.4966	0.3974
	1	1	1	1	0	**0.5802**	0.3612	0.4812	0.3682	0.4800	0.3936
Exponential	0	0	0	0	0	0.0466	0.0470	0.0444	0.0506	0.0510	0.0470
	0	0.5	1	1	0.7	0.7046	0.4360	**0.9242**	0.8212	0.9196	0.8558
	0.7	0.7	1	0.5	0.5	0.4722	0.4458	**0.6624**	0.5094	0.6572	0.5474
	0	0.5	1	0.5	0	**0.9998**	0.9740	0.9980	0.9904	0.9988	0.9944
	0	1	1	0	0	**0.9980**	0.9690	0.9964	0.9708	0.9974	0.9844
	1	1	1	0.5	0	**0.8798**	0.5946	0.6828	0.5570	0.6964	0.6074
	1	1	1	1	0	**0.7308**	0.4796	0.6202	0.4670	0.6308	0.5126
T with 3 degrees of freedom	0	0	0	0	0	0.0498	0.0552	0.0488	0.0432	0.0550	0.0510
	0	0.5	1	1	0.7	0.3612	0.2206	0.5332	0.4220	**0.5366**	0.4608
	0.7	0.7	1	0.5	0.5	0.2234	0.2082	0.2880	0.2272	**0.2998**	0.2438
	0	0.5	1	0.5	0	0.6846	0.6710	**0.8558**	0.7186	0.8464	0.7662
	0	1	1	0	0	**0.8846**	0.6944	0.8380	0.7106	0.8404	0.7602
	1	1	1	0.5	0	**0.5222**	0.2900	0.3548	0.2856	0.3776	0.3106
	1	1	1	1	0	**0.4454**	0.2774	0.3542	0.2826	0.3620	0.3100

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.41. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, CRD=15, RCBD=10 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0500	0.0512	0.0516	0.0522	0.0546	0.0490
	0	0.5	1	1	0.7	0.6130	0.3210	**0.7836**	0.6744	0.7820	0.6972
	0.7	0.7	1	0.5	0.5	0.3470	0.3448	0.4470	0.3636	**0.4498**	0.3784
	0	0.5	1	0.5	0	0.9262	0.9250	**0.9840**	0.9456	0.9826	0.9538
	0	1	1	0	0	**0.9900**	0.9278	0.9814	0.9416	0.9826	0.9538
	1	1	1	0.5	0	**0.7528**	0.4654	0.5758	0.4740	0.5736	0.4844
	1	1	1	1	0	**0.6512**	0.4396	0.5478	0.4340	0.5518	0.4670
Exponential	0	0	0	0	0	0.0538	0.0516	0.0524	0.0464	0.0560	0.0512
	0	0.5	1	1	0.7	0.8734	0.5726	0.9638	0.9084	**0.9702**	0.9190
	0.7	0.7	1	0.5	0.5	0.5912	0.5896	**0.7448**	0.6214	0.7426	0.6562
	0	0.5	1	0.5	0	0.9982	0.9944	**1.0000**	0.9984	**1.0000**	1.0000
	0	1	1	0	0	0.9996	0.9942	0.9996	0.9940	0.9994	0.9964
	1	1	1	0.5	0	**0.9330**	0.6988	0.7806	0.6722	0.7836	0.6942
	1	1	1	1	0	**0.8178**	0.5840	0.6984	0.6002	0.7102	0.6024
T with 3 degrees of freedom	0	0	0	0	0	0.0490	0.0494	0.0548	0.0514	0.0550	0.0452
	0	0.5	1	1	0.7	0.4572	0.2808	**0.6318**	0.5138	0.6258	0.5356
	0.7	0.7	1	0.5	0.5	0.2628	0.2608	0.3320	0.2732	**0.3382**	0.2930
	0	0.5	1	0.5	0	0.8142	0.7812	0.9140	0.8282	**0.9200**	0.8492
	0	1	1	0	0	**0.9314**	0.8056	0.9006	0.8256	0.9140	0.8380
	1	1	1	0.5	0	**0.5830**	0.3586	0.4300	0.3506	0.4258	0.3660
	1	1	1	1	0	**0.5140**	0.3302	0.4090	0.3422	0.4182	0.3602

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.42. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, CRD=5, RCBD=15 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0494	0.0490	0.0480	0.0542	0.0526	0.0474
	0	0.5	1	1	0.7	0.6406	0.1570	0.6422	0.5778	**0.6498**	0.6426
	0.7	0.7	1	0.5	0.5	0.2514	0.2200	0.3560	0.3116	**0.3680**	0.3472
	0	0.5	1	0.5	0	0.7560	0.6988	0.9362	0.8826	**0.9374**	0.9302
	0	1	1	0	0	**0.9610**	0.8564	0.9292	0.8808	0.9366	0.9292
	1	1	1	0.5	0	**0.6876**	0.4634	0.4628	0.3866	0.4510	0.4410
	1	1	1	1	0	**0.5796**	0.3996	0.4430	0.3770	0.4482	0.4228
Exponential	0	0	0	0	0	0.0502	0.0464	0.0550	0.0544	0.0492	0.0524
	0	0.5	1	1	0.7	0.8718	0.2490	0.8902	0.8290	**0.8974**	0.8832
	0.7	0.7	1	0.5	0.5	0.4124	0.3388	0.5956	0.5186	**0.6094**	0.6024
	0	0.5	1	0.5	0	0.9444	0.9214	**0.9990**	0.9902	0.9976	0.9966
	0	1	1	0	0	**0.9980**	0.9694	0.9940	0.9812	0.9926	0.9932
	1	1	1	0.5	0	**0.8818**	0.6824	0.6496	0.5654	0.6534	0.6400
	1	1	1	1	0	**0.7354**	0.5272	0.5732	0.4882	0.5810	0.5606
T with 3 degrees of freedom	0	0	0	0	0	0.0502	0.0472	0.0506	0.0566	0.0504	0.0466
	0	0.5	1	1	0.7	**0.5166**	0.1518	0.5094	0.4274	0.5054	0.5060
	0.7	0.7	1	0.5	0.5	0.1940	0.1762	0.2686	0.2396	**0.2826**	0.2756
	0	0.5	1	0.5	0	0.5926	0.5448	**0.8206**	0.7494	0.8136	0.8178
	0	1	1	0	0	**0.8636**	0.7024	0.8102	0.7268	0.8140	0.7918
	1	1	1	0.5	0	**0.5370**	0.3558	0.3408	0.2954	0.3534	0.3232
	1	1	1	1	0	**0.4404**	0.3070	0.3232	0.2900	0.3448	0.3212

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.43. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, CRD=10, RCBD=15 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0464	0.0578	0.0520	0.0494	0.0564	0.0468
	0	0.5	1	1	0.7	0.4762	0.2748	0.7566	0.6064	**0.7684**	0.6552
	0.7	0.7	1	0.5	0.5	0.3284	0.2892	0.4346	0.3314	**0.4424**	0.3590
	0	0.5	1	0.5	0	0.8958	0.8448	0.9792	0.9092	**0.9808**	0.9412
	0	1	1	0	0	**0.9870**	0.8826	0.9792	0.9020	0.9818	0.9370
	1	1	1	0.5	0	**0.7824**	0.4352	0.5468	0.4210	0.5542	0.4646
	1	1	1	1	0	**0.6684**	0.3936	0.5376	0.3964	0.5578	0.4442
Exponential	0	0	0	0	0	0.0476	0.0454	0.0576	0.0498	0.0502	0.0530
	0	0.5	1	1	0.7	0.7196	0.4222	**0.9652**	0.8602	0.9616	0.8938
	0.7	0.7	1	0.5	0.5	0.5382	0.4766	**0.7282**	0.5582	0.7216	0.6168
	0	0.5	1	0.5	0	0.9902	0.9864	**1.0000**	0.9954	**1.0000**	0.9976
	0	1	1	0	0	**0.9998**	0.9824	0.9994	0.9822	0.9988	0.9932
	1	1	1	0.5	0	**0.9466**	0.6458	0.7728	0.6076	0.7848	0.6730
	1	1	1	1	0	**0.8182**	0.5208	0.6906	0.5240	0.7104	0.5730
T with 3 degrees of freedom	0	0	0	0	0	0.0500	0.0526	0.0476	0.0514	0.0550	0.0520
	0	0.5	1	1	0.7	0.3650	0.2048	0.6112	0.4696	**0.6214**	0.5000
	0.7	0.7	1	0.5	0.5	0.2488	0.2278	0.3316	0.2462	**0.3450**	0.2750
	0	0.5	1	0.5	0	0.7306	0.6964	0.9062	0.7686	**0.9106**	0.8240
	0	1	1	0	0	**0.9372**	0.7470	0.8920	0.7660	0.9096	0.8120
	1	1	1	0.5	0	**0.6276**	0.3304	0.4220	0.3138	0.4292	0.3524
	1	1	1	1	0	**0.5236**	0.3064	0.4102	0.3084	0.4112	0.3338

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.44. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 3. Treatments=5, CRD=15, RCBD=15 Peak=3

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0488	0.0490	0.0506	0.0466	0.0472	0.0412
	0	0.5	1	1	0.7	0.6138	0.3564	0.8388	0.6934	**0.8410**	0.7168
	0.7	0.7	1	0.5	0.5	0.3888	0.3544	0.4836	0.3716	**0.5160**	0.3920
	0	0.5	1	0.5	0	0.9538	0.9396	**0.9938**	0.9546	0.9912	0.9672
	0	1	1	0	0	**0.9974**	0.9446	0.9924	0.9450	0.9928	0.9668
	1	1	1	0.5	0	**0.8274**	0.4900	0.6306	0.4758	0.6386	0.5126
	1	1	1	1	0	**0.7424**	0.4512	0.6110	0.4610	0.6140	0.4916
Exponential	0	0	0	0	0	0.0534	0.0456	0.0522	0.0472	0.0538	0.0488
	0	0.5	1	1	0.7	0.8734	0.5844	**0.9850**	0.9260	0.9820	0.9410
	0.7	0.7	1	0.5	0.5	0.6376	0.6068	0.8100	0.6422	**0.8164**	0.6846
	0	0.5	1	0.5	0	0.9976	0.9974	**1.0000**	0.9992	**1.0000**	0.9994
	0	1	1	0	0	0.9998	0.9950	0.9998	0.9974	**1.0000**	0.9980
	1	1	1	0.5	0	**0.9680**	0.7110	0.8558	0.6944	0.8610	0.7366
	1	1	1	1	0	**0.8856**	0.6062	0.7852	0.5966	0.7908	0.6538
T with 3 degrees of freedom	0	0	0	0	0	0.0522	0.0420	0.0496	0.0460	0.0490	0.0492
	0	0.5	1	1	0.7	0.4764	0.2820	**0.6932**	0.5234	0.6882	0.5622
	0.7	0.7	1	0.5	0.5	0.2856	0.2706	**0.3830**	0.2798	**0.3924**	0.3000
	0	0.5	1	0.5	0	0.8402	0.8106	**0.9508**	0.8412	0.9504	0.8706
	0	1	1	0	0	**0.9686**	0.8226	0.9462	0.8308	0.9450	0.8564
	1	1	1	0.5	0	**0.6804**	0.3748	0.4944	0.3574	0.4878	0.4062
	1	1	1	1	0	**0.5996**	0.3564	0.4772	0.3586	0.4838	0.3718

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.45. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, BIBD =5, CRD=10, RCBD=5 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0480	0.0444	0.0444	0.0484	0.0516	0.0480
	0.5	0.5	0.5	1	1	0.2560	0.2310	**0.2710**	0.2466	0.2358	0.2328
	0	0.5	1	1	0.7	0.8208	0.7476	**0.8354**	0.7598	0.7574	0.7522
	0	0.5	1	1	1	0.7610	0.6754	**0.7814**	0.6910	0.6758	0.6608
	0	0	1	1	1	0.8338	0.7498	**0.8540**	0.7840	0.7680	0.7296
	0.5	0.5	0.7	1	0.7	0.3794	0.3310	**0.4010**	0.3628	0.3230	0.2640
	1	1	1	1	0	0.1472	0.1364	0.1330	0.1254	**0.2140**	0.1594
	1	0.4	0.8	1.6	0.4	**0.9952**	0.9856	0.9946	0.9904	0.9870	0.9654
	0	0.25	0.5	1	0.25	**0.8562**	0.7978	0.8554	0.7996	0.7712	0.7116
Exponential	0	0	0	0	0	0.0444	0.0526	0.0472	0.0498	0.0512	0.0498
	0.5	0.5	0.5	1	1	0.4166	0.3318	**0.4250**	0.3664	0.3616	0.3496
	0	0.5	1	1	0.7	0.9644	0.9256	**0.9672**	0.9388	0.9378	0.9374
	0	0.5	1	1	1	0.9184	0.8720	**0.9384**	0.8886	0.8706	0.8490
	0	0	1	1	1	0.9512	0.9112	**0.9638**	0.9228	0.9082	0.8806
	0.5	0.5	0.7	1	0.7	**0.6748**	0.5968	0.6670	0.6148	0.5652	0.4704
	1	1	1	1	0	0.1764	0.1484	0.1468	0.1540	**0.2828**	0.1880
	1	0.4	0.8	1.6	0.4	0.9998	1.0000	**1.0000**	0.9998	**1.0000**	0.9980
	0	0.25	0.5	1	0.25	**0.9848**	0.9680	0.9818	0.9764	0.9672	0.9276
T with 3 degrees of freedom	0	0	0	0	0	0.0502	0.0480	0.0510	0.0538	0.0500	0.0530
	0.5	0.5	0.5	1	1	**0.1992**	0.1788	0.1972	0.1874	0.1886	0.1790
	0	0.5	1	1	0.7	0.6592	0.5786	**0.6810**	0.6170	0.5992	0.5986
	0	0.5	1	1	1	0.5868	0.5270	**0.6228**	0.5460	0.5144	0.4984
	0	0	1	1	1	0.6848	0.6004	**0.7118**	0.6320	0.5984	0.5732
	0.5	0.5	0.7	1	0.7	0.2974	0.2572	**0.2982**	0.2656	0.2468	0.2130
	1	1	1	1	0	0.1200	0.1172	0.1130	0.1190	**0.1636**	0.1328
	1	0.4	0.8	1.6	0.4	**0.9614**	0.9200	0.9606	0.9336	0.9168	0.8642
	0	0.25	0.5	1	0.25	**0.7086**	0.6184	0.6978	0.6512	0.6134	0.5576

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.46. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, BIBD =5, CRD=15, RCBD=5 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0458	0.0562	0.0574	0.0478	0.0486	0.0486
	0.5	0.5	0.5	1	1	**0.3096**	0.2862	0.2984	0.2950	0.2910	0.2794
	0	0.5	1	1	0.7	0.8772	0.8640	**0.8906**	0.8726	0.8734	0.8292
	0	0.5	1	1	1	0.8276	0.7960	**0.8518**	0.8216	0.8136	0.7478
	0	0	1	1	1	0.8876	0.8818	**0.9158**	0.8896	0.8784	0.8202
	0.5	0.5	0.7	1	0.7	**0.4502**	0.4244	0.4388	0.4320	0.4104	0.3178
	1	1	1	1	0	0.1642	0.1610	0.1584	0.1464	**0.2408**	0.1734
	1	0.4	0.8	1.6	0.4	0.9984	0.9986	0.9988	0.9988	**0.9990**	0.9856
	0	0.25	0.5	1	0.25	0.9106	0.8972	**0.9162**	0.9042	0.8926	0.8078
Exponential	0	0	0	0	0	0.0534	0.0520	0.0498	0.0476	0.0520	0.0492
	0.5	0.5	0.5	1	1	0.4788	0.4574	**0.5128**	0.4752	0.4668	0.4288
	0	0.5	1	1	0.7	0.9836	0.9750	**0.9854**	0.9786	0.9772	0.9764
	0	0.5	1	1	1	0.9584	0.9504	**0.9678**	0.9592	0.9514	0.9168
	0	0	1	1	1	0.9770	0.9778	**0.9828**	0.9774	0.9686	0.9414
	0.5	0.5	0.7	1	0.7	0.7430	0.7326	**0.7504**	0.7400	0.7278	0.5662
	1	1	1	1	0	0.1924	0.1852	0.1840	0.1682	**0.2894**	0.2032
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	0.9996
	0	0.25	0.5	1	0.25	**0.9950**	0.9954	0.9946	0.9942	**0.9950**	0.9694
T with 3 degrees of freedom	0	0	0	0	0	0.0462	0.0504	0.0452	0.0528	0.0566	0.0484
	0.5	0.5	0.5	1	1	0.2284	0.2242	**0.2452**	0.2164	0.2166	0.2154
	0	0.5	1	1	0.7	0.7368	0.7088	**0.7450**	0.7294	0.7150	0.6916
	0	0.5	1	1	1	0.6760	0.6354	**0.7046**	0.6510	0.6354	0.5850
	0	0	1	1	1	0.7640	0.7402	**0.7916**	0.7508	0.7296	0.6562
	0.5	0.5	0.7	1	0.7	0.3358	0.3150	**0.3484**	0.3274	0.3120	0.2494
	1	1	1	1	0	0.1348	0.1276	0.1346	0.1260	**0.1964**	0.1438
	1	0.4	0.8	1.6	0.4	**0.9840**	0.9746	0.9808	0.9804	0.9764	0.9376
	0	0.25	0.5	1	0.25	0.7778	0.7530	**0.7882**	0.7634	0.6388	0.7570

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.47. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, BIBD =10, CRD=5, RCBD=10 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0542	0.0486	0.0506	0.0494	0.0500	0.0584
	0.5	0.5	0.5	1	1	0.2882	0.2132	**0.2994**	0.2508	0.2512	0.2338
	0	0.5	1	1	0.7	0.8566	0.7088	**0.8722**	0.8082	0.7872	0.7236
	0	0.5	1	1	1	0.7982	0.6546	**0.8218**	0.7418	0.6662	0.6124
	0	0	1	1	1	0.8688	0.7264	**0.9014**	0.8358	0.7338	0.6656
	0.5	0.5	0.7	1	0.7	0.4288	0.3200	**0.4300**	0.3700	0.2644	0.2260
	1	1	1	1	0	0.1614	0.1338	0.1418	0.1164	**0.2646**	0.2370
	1	0.4	0.8	1.6	0.4	**0.9994**	0.9816	0.9982	0.9950	0.9660	0.9362
	0	0.25	0.5	1	0.25	**0.8914**	0.7652	**0.8914**	0.8370	0.7200	0.6398
Exponential	0	0	0	0	0	0.0542	0.0568	0.0492	0.0554	0.0550	0.0508
	0.5	0.5	0.5	1	1	0.4546	0.3348	**0.4750**	0.4028	0.3822	0.3402
	0	0.5	1	1	0.7	0.9780	0.9036	**0.9808**	0.9548	0.9530	0.9260
	0	0.5	1	1	1	0.9482	0.8450	**0.9614**	0.9140	0.8632	0.8180
	0	0	1	1	1	0.9698	0.8884	**0.9792**	0.9518	0.8882	0.8238
	0.5	0.5	0.7	1	0.7	**0.8146**	0.6330	0.8136	0.6898	0.5758	0.5892
	1	1	1	1	0	0.1892	0.1556	0.1436	0.1354	**0.3312**	0.3072
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9990	**1.0000**	1.0000	**0.9966**	0.9950
	0	0.25	0.5	1	0.25	**0.9946**	0.9552	**0.9922**	0.9808	**0.9296**	0.8878
T with 3 degrees of freedom	0	0	0	0	0	0.0490	0.0510	0.0478	0.0478	0.0492	0.0482
	0.5	0.5	0.5	1	1	0.2190	0.1770	**0.2390**	0.2070	0.1990	0.1788
	0	0.5	1	1	0.7	0.7138	0.5598	**0.7246**	0.6374	0.6376	0.5728
	0	0.5	1	1	1	0.6602	0.4924	**0.6836**	0.5904	0.5180	0.4618
	0	0	1	1	1	0.7174	0.5738	**0.7752**	0.6826	0.5724	0.5102
	0.5	0.5	0.7	1	0.7	**0.3256**	0.2458	0.3200	0.2814	0.1982	0.1818
	1	1	1	1	0	0.1306	0.1038	0.1146	0.1028	**0.2066**	0.1974
	1	0.4	0.8	1.6	0.4	**0.9754**	0.9078	0.9734	0.9416	0.8736	0.8046
	0	0.25	0.5	1	0.25	0.7490	0.5914	**0.7498**	0.6722	0.5452	0.4766

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.48. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, BIBD =10, CRD=10, RCBD=10 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0504	0.0550	0.0540	0.0518	0.0566	0.0536
	0.5	0.5	0.5	1	1	0.3444	0.2360	**0.3484**	0.2718	0.2914	0.2474
	0	0.5	1	1	0.7	0.9226	0.7934	**0.9370**	0.8250	0.8720	0.8158
	0	0.5	1	1	1	0.8836	0.7240	**0.9090**	0.7698	0.7896	0.7290
	0	0	1	1	1	0.9364	0.8062	**0.9574**	0.8604	0.8458	0.8084
	0.5	0.5	0.7	1	0.7	**0.5028**	0.3676	0.5006	0.3918	0.3302	0.3262
	1	1	1	1	0	0.1794	0.1474	0.1508	0.1446	**0.2886**	0.1962
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9942	0.9998	0.9956	0.9930	0.9918
	0	0.25	0.5	1	0.25	**0.9530**	0.8298	0.9476	0.8682	0.8328	0.8070
Exponential	0	0	0	0	0	0.0424	0.0496	0.0438	0.0484	0.0526	0.0514
	0.5	0.5	0.5	1	1	**0.5476**	0.3742	**0.5788**	0.4308	**0.4804**	0.4016
	0	0.5	1	1	0.7	**0.9964**	0.9482	**0.9950**	0.9674	**0.9830**	0.9640
	0	0.5	1	1	1	**0.9794**	0.8966	**0.9864**	0.9224	**0.9430**	0.9028
	0	0	1	1	1	**0.9902**	0.9382	**0.9936**	0.9624	**0.9574**	0.9322
	0.5	0.5	0.7	1	0.7	**0.8148**	0.6322	**0.8112**	0.6868	**0.5824**	0.5842
	1	1	1	1	0	**0.2232**	0.1570	**0.1850**	0.1632	**0.3854**	0.2298
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9998	**1.0000**	0.9998	**0.9998**	0.9998
	0	0.25	0.5	1	0.25	**0.9988**	0.9816	**0.9986**	0.9880	**0.9770**	0.9766
T with 3 degrees of freedom	0	0	0	0	0	0.0546	0.0482	0.0512	0.0460	0.0542	0.0554
	0.5	0.5	0.5	1	1	**0.2488**	0.2008	**0.2758**	0.2078	**0.2232**	0.1932
	0	0.5	1	1	0.7	**0.8070**	0.6262	**0.8116**	0.6684	**0.7312**	0.6506
	0	0.5	1	1	1	**0.7372**	0.5682	**0.7638**	0.6066	**0.6300**	0.5702
	0	0	1	1	1	**0.8096**	0.6498	**0.8570**	0.7034	**0.6826**	0.6440
	0.5	0.5	0.7	1	0.7	**0.3682**	0.2884	**0.3792**	0.3114	**0.2468**	0.2464
	1	1	1	1	0	**0.1558**	0.1216	**0.1264**	0.1196	**0.2294**	0.1432
	1	0.4	0.8	1.6	0.4	**0.9946**	0.9424	**0.9934**	0.9630	**0.9440**	0.9360
	0	0.25	0.5	1	0.25	**0.8380**	0.6724	**0.8466**	0.7100	**0.6686**	0.6518

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.49. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, BIBD =10, CRD=15, RCBD=10 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0466	0.0494	0.0556	0.0520	0.0486	0.0514
	0.5	0.5	0.5	1	1	0.3744	0.2934	**0.3822**	0.3016	0.3544	0.3020
	0	0.5	1	1	0.7	0.9592	0.8730	**0.9616**	0.8906	0.9318	0.8832
	0	0.5	1	1	1	0.9190	0.8172	**0.9388**	0.8544	0.8532	0.8258
	0	0	1	1	1	0.9664	0.8952	**0.9768**	0.9104	0.9022	0.9012
	0.5	0.5	0.7	1	0.7	0.5568	0.4400	**0.5584**	0.4604	0.4180	0.3812
	1	1	1	1	0	0.2068	0.1598	0.1704	0.1676	**0.3188**	0.1996
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9982	**1.0000**	0.9996	0.9982	0.9992
	0	0.25	0.5	1	0.25	**0.9748**	0.9028	0.9744	0.9228	0.8936	0.9142
Exponential	0	0	0	0	0	0.0476	0.0550	0.0480	0.0500	0.0560	0.0502
	0.5	0.5	0.5	1	1	**0.6168**	0.4754	**0.6572**	0.4920	**0.5406**	0.4890
	0	0.5	1	1	0.7	**0.9974**	0.9832	**0.9990**	0.9872	**0.9938**	0.9854
	0	0.5	1	1	1	0.9914	0.9588	0.9936	0.9646	0.9716	0.9622
	0	0	1	1	1	**0.9966**	0.9780	**0.9982**	0.9876	**0.9826**	0.9768
	0.5	0.5	0.7	1	0.7	**0.8760**	0.7440	**0.8840**	0.7558	**0.7336**	0.6546
	1	1	1	1	0	**0.2486**	0.1966	**0.2064**	0.1916	**0.4098**	0.2372
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**0.9996**	0.9958	**0.9990**	0.9968	**0.9914**	0.9964
T with 3 degrees of freedom	0	0	0	0	0	0.0486	0.0518	0.0500	0.0444	0.0498	0.0494
	0.5	0.5	0.5	1	1	0.2850	0.2338	**0.3124**	0.2342	0.2632	0.2330
	0	0.5	1	1	0.7	0.8624	0.7356	**0.8796**	0.7518	0.8016	0.7498
	0	0.5	1	1	1	0.8110	0.6770	**0.8306**	0.6914	0.7104	0.6854
	0	0	1	1	1	0.8786	0.7556	**0.9092**	0.7884	0.7696	0.7548
	0.5	0.5	0.7	1	0.7	0.4306	0.3394	**0.4390**	0.3480	0.3292	0.2890
	1	1	1	1	0	0.1674	0.1330	0.1422	0.1334	**0.2518**	0.1516
	1	0.4	0.8	1.6	0.4	**0.9974**	0.9820	**0.9972**	0.9860	0.9796	0.9746
	0	0.25	0.5	1	0.25	**0.9040**	0.7740	0.8920	0.8078	0.7584	0.7802

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.50. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, BIBD =15, CRD=5, RCBD=15 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0450	0.0486	0.0558	0.0478	0.0494	0.0530
	0.5	0.5	0.5	1	1	0.3430	0.2538	**0.3640**	0.3162	0.2962	0.2560
	0	0.5	1	1	0.7	0.9350	0.8130	**0.9474**	0.8980	0.8714	0.8188
	0	0.5	1	1	1	0.8926	0.7434	**0.9082**	0.8656	0.7704	0.7050
	0	0	1	1	1	0.9434	0.8286	**0.9646**	0.9248	0.8324	0.7574
	0.5	0.5	0.7	1	0.7	0.5104	0.3850	**0.5246**	0.4596	0.3078	0.2380
	1	1	1	1	0	0.1946	0.1486	0.1548	0.1264	**0.3310**	0.3156
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9944	0.9998	0.9990	0.9870	0.9616
	0	0.25	0.5	1	0.25	**0.9554**	0.8486	0.9524	0.9192	0.7876	0.6980
Exponential	0	0	0	0	0	0.0494	0.0506	0.0496	0.0480	0.0550	0.0558
	0.5	0.5	0.5	1	1	0.5668	0.4046	**0.5930**	0.5254	0.4666	0.4146
	0	0.5	1	1	0.7	0.9948	0.9600	**0.9958**	0.9888	0.9852	0.9642
	0	0.5	1	1	1	0.9824	0.9236	**0.9864**	0.9706	0.9332	0.8802
	0	0	1	1	1	0.9920	0.9516	**0.9936**	0.9910	0.9468	0.8984
	0.5	0.5	0.7	1	0.7	0.8202	0.6644	**0.8246**	0.7616	0.5356	0.4082
	1	1	1	1	0	0.2342	0.1734	0.1702	0.1516	**0.4286**	0.4152
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9998	**1.0000**	1.0000	0.9998	0.9976
	0	0.25	0.5	1	0.25	**0.9990**	0.9846	**0.9990**	0.9960	0.9730	0.9368
T with 3 degrees of freedom	0	0	0	0	0	0.0540	0.0482	0.0490	0.0496	0.0594	0.0530
	0.5	0.5	0.5	1	1	**0.2574**	0.1958	0.2750	0.2344	0.2290	0.2148
	0	0.5	1	1	0.7	0.8100	0.6528	**0.8286**	0.7636	0.7328	0.6636
	0	0.5	1	1	1	0.7484	0.5788	**0.8022**	0.7156	0.6164	0.5324
	0	0	1	1	1	0.8318	0.6756	**0.8656**	0.8056	0.6812	0.5790
	0.5	0.5	0.7	1	0.7	0.3948	0.2874	**0.3988**	0.3482	0.2476	0.1890
	1	1	1	1	0	0.1450	0.1222	0.1232	0.1114	**0.2630**	0.2506
	1	0.4	0.8	1.6	0.4	0.9936	0.9596	**0.9946**	0.9852	0.9298	0.8670
	0	0.25	0.5	1	0.25	**0.8556**	**0.7098**	**0.8460**	0.7852	0.6388	0.5552

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.51. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, BIBD =15, CRD=10, RCBD=15 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0550	0.0478	0.0490	0.0470	0.0478	0.0482
	0.5	0.5	0.5	1	1	0.3894	0.2586	**0.4280**	0.3008	0.3378	0.2788
	0	0.5	1	1	0.7	0.9702	0.8186	**0.9710**	0.8806	0.9428	0.8514
	0	0.5	1	1	1	0.9468	0.7650	**0.9500**	0.8298	0.8714	0.7746
	0	0	1	1	1	0.9752	0.8444	**0.9856**	0.9076	0.9192	0.8444
	0.5	0.5	0.7	1	0.7	**0.5956**	0.3924	0.5930	0.4454	0.3712	0.3398
	1	1	1	1	0	0.2230	0.1406	0.1796	0.1420	**0.3656**	0.2252
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9976	**1.0000**	0.9988	0.9974	0.9950
	0	0.25	0.5	1	0.25	0.9792	0.8610	**0.9806**	0.9060	0.8946	0.8292
Exponential	0	0	0	0	0	0.0504	0.0578	**0.0466**	0.0494	0.0582	0.0504
	0.5	0.5	0.5	1	1	0.6334	0.4182	**0.6756**	0.4752	0.5536	0.4646
	0	0.5	1	1	0.7	0.9984	0.9644	**0.9988**	0.9844	0.9978	0.9788
	0	0.5	1	1	1	0.9952	0.9222	**0.9958**	0.9648	0.9748	0.9370
	0	0	1	1	1	0.9982	0.9598	**0.9986**	0.9786	0.9820	0.9522
	0.5	0.5	0.7	1	0.7	**0.9054**	0.6892	0.8948	0.7344	0.6286	0.6086
	1	1	1	1	0	0.2710	0.1710	0.2092	0.1766	**0.4808**	0.2912
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000	1.0000	1.0000
	0	0.25	0.5	1	0.25	**1.0000**	0.9908	**1.0000**	0.9954	0.9942	0.9834
T with 3 degrees of freedom	0	0	0	0	0	0.0478	0.0500	0.0532	0.0524	0.0468	0.0456
	0.5	0.5	0.5	1	1	0.2992	0.2110	**0.3300**	0.2406	0.2588	0.2218
	0	0.5	1	1	0.7	0.8810	0.6616	**0.8982**	0.7356	0.8242	0.7144
	0	0.5	1	1	1	0.8208	0.5888	**0.8518**	0.6828	0.7188	0.6152
	0	0	1	1	1	0.8984	0.6880	**0.9198**	0.7680	0.7700	0.6902
	0.5	0.5	0.7	1	0.7	0.4556	0.3016	**0.4720**	0.3326	0.2892	0.2476
	1	1	1	1	0	0.1712	0.1272	0.1426	0.1144	**0.2878**	0.1818
	1	0.4	0.8	1.6	0.4	**0.9990**	0.9572	0.9982	0.9808	0.9794	0.9518
	0	0.25	0.5	1	0.25	**0.9128**	0.7132	**0.9148**	0.7704	0.7460	0.6782

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.52. Mixed Design for Balanced Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4.. Treatments=5, BIBD =15, CRD=15, RCBD=15 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0518	0.0522	0.0522	0.0492	0.0510	0.0468
	0.5	0.5	0.5	1	1	0.4280	0.3080	**0.4658**	0.3288	0.3868	0.3108
	0	0.5	1	1	0.7	0.9864	0.8938	**0.9884**	0.9182	0.9644	0.9060
	0	0.5	1	1	1	0.9670	0.8386	**0.9760**	0.8708	0.9122	0.8468
	0	0	1	1	1	0.9884	0.9052	**0.9946**	0.9350	0.9506	0.9168
	0.5	0.5	0.7	1	0.7	0.6452	0.4664	**0.6552**	0.4876	0.4410	0.4254
	1	1	1	1	0	0.2344	0.1604	0.1970	0.1648	**0.3910**	0.1992
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9996	**1.0000**	0.9998	0.9996	0.9992
	0	0.25	0.5	1	0.25	0.9910	0.9276	**0.9924**	0.9430	0.9326	0.9130
Exponential	0	0	0	0	0	0.0476	0.0526	0.0466	0.0522	0.0548	0.0428
	0.5	0.5	0.5	1	1	0.6982	0.4908	**0.7518**	0.5394	0.6204	0.5200
	0	0.5	1	1	0.7	**0.9994**	0.9868	**0.9996**	0.9910	0.9992	0.9914
	0	0.5	1	1	1	0.9988	0.9644	**0.9994**	0.9774	0.9906	0.9726
	0	0	1	1	1	0.9994	0.9826	**1.0000**	0.9894	0.9942	0.9836
	0.5	0.5	0.7	1	0.7	**0.9368**	0.7700	0.9344	0.8134	0.7356	0.7234
	1	1	1	1	0	0.3008	0.2032	0.2274	0.1964	**0.5036**	0.2608
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**1.0000**	0.9966	**1.0000**	0.9986	0.9992	0.9972
T with 3 degrees of freedom	0	0	0	0	0	0.0534	0.0536	0.0536	0.0538	0.0488	0.0470
	0.5	0.5	0.5	1	1	**0.3238**	0.2258	**0.3566**	0.2568	**0.3024**	0.2424
	0	0.5	1	1	0.7	**0.9250**	0.7582	**0.9338**	0.7928	**0.8818**	0.7752
	0	0.5	1	1	1	**0.8778**	0.6854	**0.8962**	0.7322	**0.7894**	0.6812
	0	0	1	1	1	**0.9328**	0.7756	**0.9502**	0.8090	**0.8526**	0.7870
	0.5	0.5	0.7	1	0.7	**0.5066**	0.3424	**0.5086**	0.3742	**0.3306**	0.3272
	1	1	1	1	0	**0.1850**	0.1394	**0.1494**	0.1446	**0.2948**	0.1744
	1	0.4	0.8	1.6	0.4	**0.9998**	0.9818	**0.9998**	0.9898	**0.9934**	0.9824
	0	0.25	0.5	1	0.25	**0.9458**	0.7946	**0.9508**	0.8202	**0.8234**	0.7748

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.53. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 4. Treatments=5, BIBD =5, CRD=10 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0518	0.0512	0.0524	0.0502	0.0518	0.0484
	0.5	0.5	0.5	1	1	0.2066	**0.2164**	0.1984	0.2134	0.1980	0.2104
	0	0.5	1	1	0.7	0.6476	0.6988	0.6550	**0.7048**	0.6492	0.6972
	0	0.5	1	1	1	0.5906	0.6296	0.5884	0.6314	0.5716	**0.6326**
	0	0	1	1	1	0.6752	0.7162	0.6666	0.7226	0.6566	**0.7236**
	0.5	0.5	0.7	1	0.7	0.2864	**0.3118**	0.2818	0.3080	0.2744	0.2970
	1	1	1	1	0	0.1270	**0.1284**	0.1218	0.1246	0.1234	0.1252
	1	0.4	0.8	1.6	0.4	0.9476	**0.9808**	0.9554	0.9776	0.9610	0.9800
	0	0.25	0.5	1	0.25	0.6836	0.7412	0.6878	**0.7500**	0.6822	0.7462
Exponential	0	0	0	0	0	0.0488	0.0450	0.0506	0.0516	0.0510	0.0500
	0.5	0.5	0.5	1	1	0.3010	0.3170	0.3092	**0.3182**	0.3004	0.3168
	0	0.5	1	1	0.7	0.8614	0.8940	0.8572	0.9052	0.8552	**0.9054**
	0	0.5	1	1	1	0.7834	0.8334	0.7814	**0.8394**	0.7782	0.8260
	0	0	1	1	1	0.8136	0.8770	0.8258	0.8786	0.8212	**0.8818**
	0.5	0.5	0.7	1	0.7	0.4974	0.5518	0.4912	**0.5548**	0.4990	0.5478
	1	1	1	1	0	0.1468	**0.1498**	0.1296	0.1338	0.1504	0.1344
	1	0.4	0.8	1.6	0.4	0.9944	**0.9998**	0.9936	0.9992	0.9940	0.9996
	0	0.25	0.5	1	0.25	0.9072	0.9492	0.9066	**0.9518**	0.9072	0.9500
T with 3 degrees of freedom	0	0	0	0	0	0.0518	0.0508	0.0518	0.0460	0.0512	0.0550
	0.5	0.5	0.5	1	1	0.1544	0.1712	0.1632	0.1600	0.1544	**0.1760**
	0	0.5	1	1	0.7	0.5038	0.5450	0.4992	**0.5486**	0.5048	0.5424
	0	0.5	1	1	1	0.4570	0.4784	0.4402	0.4648	0.4360	**0.4828**
	0	0	1	1	1	0.5262	**0.5702**	0.5132	0.5584	0.5166	0.5490
	0.5	0.5	0.7	1	0.7	0.2250	0.2326	0.2156	**0.2434**	0.2218	0.2338
	1	1	1	1	0	0.1076	0.1078	0.1120	**0.1230**	0.1088	0.1120
	1	0.4	0.8	1.6	0.4	0.8494	**0.8960**	0.8532	0.8904	0.8406	0.8896
	0	0.25	0.5	1	0.25	0.5474	0.5832	0.5460	**0.5934**	0.5292	0.5810

Results show that Standardized Last test performed better than Standardized First in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.54. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 4. Treatments=5, BIBD =5, CRD=15 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0482	0.0522	0.0496	0.0476	0.0460	0.0522
	0.5	0.5	0.5	1	1	0.2526	0.2720	0.2402	**0.2848**	0.2402	0.2588
	0	0.5	1	1	0.7	0.7716	**0.8440**	0.7702	0.8418	0.7618	0.8418
	0	0.5	1	1	1	0.7004	**0.7858**	0.6958	0.7800	0.7062	0.7776
	0	0	1	1	1	0.7812	0.8690	0.7792	**0.8628**	0.7852	**0.8716**
	0.5	0.5	0.7	1	0.7	0.3556	0.4156	0.3638	**0.4226**	0.3550	0.4148
	1	1	1	1	0	0.1412	**0.1588**	0.1432	0.1522	0.1354	0.1544
	1	0.4	0.8	1.6	0.4	0.9902	**0.9986**	0.9864	0.9982	0.9874	0.9972
	0	0.25	0.5	1	0.25	0.8094	0.8826	0.8132	**0.8834**	0.8158	0.8804
Exponential	0	0	0	0	0	0.0534	0.0550	0.0512	0.0562	0.0516	0.0564
	0.5	0.5	0.5	1	1	0.3830	**0.4414**	0.3702	0.4348	0.3818	0.4402
	0	0.5	1	1	0.7	0.9370	0.9748	0.9412	0.9758	0.9402	**0.9762**
	0	0.5	1	1	1	0.8840	0.9392	0.8828	**0.9422**	0.8842	0.9396
	0	0	1	1	1	0.9088	0.9662	0.9154	**0.9686**	0.9156	0.9614
	0.5	0.5	0.7	1	0.7	0.6208	**0.7108**	0.6308	0.7086	0.6046	0.6998
	1	1	1	1	0	0.1696	**0.1798**	0.1694	0.1756	0.1656	0.1792
	1	0.4	0.8	1.6	0.4	0.9992	**1.0000**	0.9998	**1.0000**	0.9996	**1.0000**
	0	0.25	0.5	1	0.25	0.9670	0.9914	0.9696	**0.9944**	0.9690	0.9942
T with 3 degrees of freedom	0	0	0	0	0	0.0566	0.0482	0.0540	0.0514	0.0530	0.0482
	0.5	0.5	0.5	1	1	0.1926	0.2158	0.1876	0.2082	0.1904	**0.2170**
	0	0.5	1	1	0.7	0.6088	0.6904	0.6148	0.6968	0.6206	**0.6974**
	0	0.5	1	1	1	0.5526	0.6158	0.5392	0.6240	0.5384	**0.6322**
	0	0	1	1	1	0.6336	0.7132	0.6246	0.7092	0.6316	**0.7178**
	0.5	0.5	0.7	1	0.7	0.2848	0.2994	0.2764	0.3016	0.2844	**0.3078**
	1	1	1	1	0	0.1240	0.1272	0.1232	0.1266	0.1236	**0.1290**
	1	0.4	0.8	1.6	0.4	0.9294	0.9694	0.9234	0.9722	0.9338	**0.9726**
	0	0.25	0.5	1	0.25	0.6512	0.7376	0.6390	0.7342	0.6434	**0.7412**

Results show that Standardized Last test performed better than Standardized First in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.55. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 4. Treatments=5, BIBD =10, CRD=5 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0502	0.0506	0.0468	0.0506	0.0504	0.0464
	0.5	0.5	0.5	1	1	0.1680	0.1482	0.1704	0.1516	**0.1766**	0.1732
	0	0.5	1	1	0.7	0.5590	0.4776	0.5520	0.5012	**0.5642**	0.4812
	0	0.5	1	1	1	0.4884	0.4448	**0.4906**	0.4362	0.4882	0.4344
	0	0	1	1	1	0.5692	0.5150	0.5708	0.5004	**0.5814**	0.5118
	0.5	0.5	0.7	1	0.7	**0.2464**	0.2206	0.2428	0.2262	0.2366	0.2202
	1	1	1	1	0	**0.1214**	0.1010	0.1132	0.0866	0.1158	0.1022
	1	0.4	0.8	1.6	0.4	**0.9100**	0.8672	**0.9074**	0.8684	0.9094	0.8574
	0	0.25	0.5	1	0.25	0.6028	0.5478	**0.6100**	0.5444	0.5930	0.5306
Exponential	0	0	0	0	0	0.0468	0.0496	0.0522	0.0494	0.0558	0.0526
	0.5	0.5	0.5	1	1	**0.2444**	0.2100	0.2398	0.2092	0.2430	0.2102
	0	0.5	1	1	0.7	**0.7834**	0.7134	0.7792	0.7162	0.7698	0.7102
	0	0.5	1	1	1	**0.6976**	0.6354	0.6888	0.6224	0.6884	0.6342
	0	0	1	1	1	0.7378	0.6752	**0.7414**	0.6630	0.7360	0.6558
	0.5	0.5	0.7	1	0.7	0.5672	0.5548	**0.5748**	0.5558	0.5598	0.5470
	1	1	1	1	0	0.1260	0.1014	**0.1302**	0.1138	0.1228	0.1066
	1	0.4	0.8	1.6	0.4	**0.9852**	0.9730	0.9846	0.9714	0.9842	0.9708
	0	0.25	0.5	1	0.25	**0.8502**	0.7914	0.8366	0.7792	0.8486	0.7960
T with 3 degrees of freedom	0	0	0	0	0	0.0524	0.0508	0.0466	0.0482	0.0420	0.0504
	0.5	0.5	0.5	1	1	0.1336	0.1262	0.1338	0.1228	**0.1508**	0.1284
	0	0.5	1	1	0.7	0.4114	0.3926	0.4058	0.3782	**0.4242**	0.3632
	0	0.5	1	1	1	0.3634	0.3242	**0.3780**	0.3242	0.3772	0.3308
	0	0	1	1	1	**0.4422**	0.3840	0.4384	0.3870	0.4214	0.3858
	0.5	0.5	0.7	1	0.7	0.1896	0.1844	**0.1916**	0.1748	0.1898	0.1760
	1	1	1	1	0	0.1088	0.1008	0.1026	0.1010	**0.1090**	0.1016
	1	0.4	0.8	1.6	0.4	0.7612	0.7084	**0.7686**	0.7178	0.7574	0.7084
	0	0.25	0.5	1	0.25	**0.4452**	0.4122	**0.4480**	0.3994	**0.4498**	0.4094

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.56. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 4. Treatments=5, BIBD =10, CRD=10 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0512	0.0586	0.0526	0.0466	0.0494	0.0536
	0.5	0.5	0.5	1	1	0.2262	0.2228	0.2176	0.2190	**0.2278**	0.2096
	0	0.5	1	1	0.7	**0.7324**	0.7092	0.7244	0.7182	0.7230	0.7232
	0	0.5	1	1	1	0.6548	0.6380	**0.6618**	0.6428	0.6580	0.6496
	0	0	1	1	1	0.7440	0.7378	0.7420	0.7282	**0.7536**	0.7244
	0.5	0.5	0.7	1	0.7	0.3216	0.3160	**0.3290**	0.3138	0.3278	0.3174
	1	1	1	1	0	**0.1390**	0.1320	0.1376	0.1254	0.1354	0.1244
	1	0.4	0.8	1.6	0.4	0.9842	0.9806	0.9798	0.9820	**0.9856**	0.9796
	0	0.25	0.5	1	0.25	**0.7684**	0.7554	**0.7672**	0.7584	**0.7670**	0.7632
Exponential	0	0	0	0	0	0.0492	0.0490	0.0502	0.0532	0.0510	0.0514
	0.5	0.5	0.5	1	1	0.3538	0.3276	**0.3554**	0.3216	0.3538	0.3226
	0	0.5	1	1	0.7	0.9166	0.9146	0.9140	0.9120	**0.9190**	0.9028
	0	0.5	1	1	1	**0.8566**	0.8494	0.8424	0.8464	0.8462	0.8424
	0	0	1	1	1	0.8940	0.8844	0.8936	0.8832	**0.8948**	0.8766
	0.5	0.5	0.7	1	0.7	0.5676	0.5654	0.5600	0.5620	**0.5710**	0.5686
	1	1	1	1	0	**0.1626**	0.1480	**0.1626**	0.1486	0.1582	0.1440
	1	0.4	0.8	1.6	0.4	0.9992	0.9988	**0.9992**	0.9992	**0.9994**	0.9992
	0	0.25	0.5	1	0.25	**0.9532**	0.9544	**0.9542**	0.9528	0.9504	0.9570
T with 3 degrees of freedom	0	0	0	0	0	0.0454	0.0534	0.0512	0.0454	0.0492	0.0482
	0.5	0.5	0.5	1	1	**0.1830**	0.1750	0.1666	0.1654	0.1818	0.1702
	0	0.5	1	1	0.7	**0.5764**	0.5486	0.5556	0.5392	0.5546	0.5440
	0	0.5	1	1	1	0.5118	0.4854	0.5102	0.4940	**0.5218**	0.5020
	0	0	1	1	1	0.5752	0.5730	0.5914	0.5742	**0.5964**	0.5692
	0.5	0.5	0.7	1	0.7	0.2420	0.2408	**0.2478**	0.2466	0.2462	0.2404
	1	1	1	1	0	**0.1140**	0.1120	0.1136	0.1126	**0.1204**	0.1104
	1	0.4	0.8	1.6	0.4	**0.9080**	0.9072	0.9062	0.8978	0.9056	0.8966
	0	0.25	0.5	1	0.25	**0.6126**	0.5926	0.6010	0.5906	0.6032	0.5910

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.57. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 4. Treatments=5, BIBD =10, CRD=15, Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0490	0.0504	0.0500	0.0520	0.0530	0.0500
	0.5	0.5	0.5	1	1	0.2690	0.2732	0.2560	0.2772	0.2712	**0.2750**
	0	0.5	1	1	0.7	0.8326	0.8522	0.8254	0.8504	0.8282	**0.8562**
	0	0.5	1	1	1	0.7648	**0.7942**	0.7688	0.7910	0.7784	0.7900
	0	0	1	1	1	0.8500	0.8680	0.8568	**0.8746**	0.8444	0.8670
	0.5	0.5	0.7	1	0.7	0.4030	0.4042	0.3930	**0.4246**	0.4010	0.4112
	1	1	1	1	0	0.1502	0.1538	0.1602	**0.1750**	0.1432	0.1566
	1	0.4	0.8	1.6	0.4	0.9962	**0.9986**	0.9956	0.9978	0.9964	0.9980
	0	0.25	0.5	1	0.25	0.8630	0.8932	0.8600	**0.8956**	0.8672	0.8868
Exponential	0	0	0	0	0	0.0514	0.0466	0.0548	0.0548	0.0458	0.0498
	0.5	0.5	0.5	1	1	0.4358	0.4412	0.4274	0.4374	0.4274	**0.4438**
	0	0.5	1	1	0.7	0.9682	**0.9768**	0.9638	0.9720	0.9656	0.9718
	0	0.5	1	1	1	0.9248	0.9414	0.9226	**0.9490**	0.9248	0.9398
	0	0	1	1	1	0.9558	0.9604	0.9564	0.9670	0.9514	**0.9682**
	0.5	0.5	0.7	1	0.7	0.6820	0.7024	0.6738	0.7010	0.6786	**0.7140**
	1	1	1	1	0	0.1778	0.1758	0.1890	0.1898	0.1708	**0.1978**
	1	0.4	0.8	1.6	0.4	1.0000	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**
	0	0.25	0.5	1	0.25	0.9870	0.9906	0.9882	**0.9926**	0.9878	0.9924
T with 3 degrees of freedom	0	0	0	0	0	0.0526	0.0454	0.0480	0.0496	0.0512	0.0528
	0.5	0.5	0.5	1	1	0.2216	0.2116	0.2038	**0.2308**	0.2078	0.2158
	0	0.5	1	1	0.7	0.6784	**0.6978**	0.6950	0.6930	0.6800	0.6972
	0	0.5	1	1	1	0.6226	0.6286	0.6132	0.6264	0.6200	**0.6302**
	0	0	1	1	1	0.7050	0.7176	0.6888	0.7174	0.6974	**0.7252**
	0.5	0.5	0.7	1	0.7	0.3092	0.3086	0.3118	**0.3154**	0.3066	0.3142
	1	1	1	1	0	0.1282	**0.1312**	0.1264	0.1252	0.1288	**0.1330**
	1	0.4	0.8	1.6	0.4	0.9664	**0.9724**	0.9642	0.9712	0.9564	**0.9748**
	0	0.25	0.5	1	0.25	0.7130	0.7356	0.7208	**0.7458**	0.7204	0.7366

Results show that Standardized Last test performed better than Standardized First in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.58. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 4. Treatments=5, BIBD =15, CRD=5, Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0512	0.0484	0.0502	0.0506	0.0486	0.0530
	0.5	0.5	0.5	1	1	0.1884	0.1588	0.1862	0.1622	**0.1928**	0.1716
	0	0.5	1	1	0.7	0.6172	0.5244	0.6152	0.5226	**0.6188**	0.5198
	0	0.5	1	1	1	0.5488	0.4756	0.5518	0.4638	**0.5616**	0.4524
	0	0	1	1	1	**0.6480**	0.5444	0.6318	0.5500	0.6346	0.5406
	0.5	0.5	0.7	1	0.7	0.2738	0.2390	**0.2814**	0.2294	0.2660	0.2318
	1	1	1	1	0	0.1226	0.1070	**0.1266**	0.1020	**0.1240**	0.1046
	1	0.4	0.8	1.6	0.4	0.9472	0.8926	**0.9514**	0.8902	0.9488	0.8948
	0	0.25	0.5	1	0.25	**0.6732**	0.5852	0.6588	0.5628	0.6670	0.5724
Exponential	0	0	0	0	0	0.0550	0.0492	0.0460	0.0552	0.0482	0.0506
	0.5	0.5	0.5	1	1	0.2718	0.2180	**0.2894**	0.2188	0.2798	0.2216
	0	0.5	1	1	0.7	**0.8414**	0.7470	0.8378	0.7516	0.8350	0.7540
	0	0.5	1	1	1	0.7496	0.6612	**0.7504**	0.6620	0.7496	0.6644
	0	0	1	1	1	0.8034	0.7196	**0.8080**	0.7048	0.7942	0.7110
	0.5	0.5	0.7	1	0.7	0.4636	0.3812	**0.4694**	0.3816	0.4586	0.3798
	1	1	1	1	0	**0.1422**	0.1106	0.1400	0.1136	0.1336	0.1110
	1	0.4	0.8	1.6	0.4	**0.9936**	0.9802	**0.9930**	0.9812	0.9928	0.9790
	0	0.25	0.5	1	0.25	**0.8940**	0.8222	0.8884	0.8196	**0.8940**	0.8204
T with 3 degrees of freedom	0	0	0	0	0	0.0524	0.0534	0.0458	0.0504	0.0518	0.0516
	0.5	0.5	0.5	1	1	**0.1634**	0.1436	0.1542	0.1406	0.1592	0.1366
	0	0.5	1	1	0.7	0.4756	0.4028	0.4718	0.3876	**0.4770**	0.4036
	0	0.5	1	1	1	0.4258	0.3368	**0.4276**	0.3442	0.4108	0.3534
	0	0	1	1	1	0.4862	0.4210	**0.4866**	0.4002	0.4838	0.3976
	0.5	0.5	0.7	1	0.7	0.2126	0.1852	0.2166	0.1788	**0.2184**	0.1842
	1	1	1	1	0	**0.1022**	0.0888	**0.1044**	0.1032	**0.1034**	0.1006
	1	0.4	0.8	1.6	0.4	0.8220	0.7430	0.8248	0.7394	**0.8264**	0.7372
	0	0.25	0.5	1	0.25	0.5014	0.4260	0.5124	0.4216	**0.5126**	0.4364

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.59. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 4. Treatments=5, BIBD =15, CRD=10, Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0508	0.0472	0.0498	0.0528	0.0514	0.0476
	0.5	0.5	0.5	1	1	0.2444	0.2208	0.2420	0.2128	**0.2470**	0.2132
	0	0.5	1	1	0.7	0.7796	0.7190	0.7760	0.7316	**0.7798**	0.7308
	0	0.5	1	1	1	**0.7210**	0.6416	0.7108	0.6474	0.7012	0.6650
	0	0	1	1	1	**0.8016**	0.7472	0.7946	0.7422	0.7932	0.7404
	0.5	0.5	0.7	1	0.7	**0.3642**	0.3216	0.3602	0.3220	0.3550	0.3290
	1	1	1	1	0	0.1428	0.1336	0.1430	0.1308	**0.1468**	0.1332
	1	0.4	0.8	1.6	0.4	**0.9916**	0.9828	0.9896	0.9836	0.9906	0.9860
	0	0.25	0.5	1	0.25	**0.8180**	0.7692	0.8130	0.7712	**0.8180**	0.7584
Exponential	0	0	0	0	0	0.0488	0.0504	0.0538	0.0538	0.0524	0.0480
	0.5	0.5	0.5	1	1	0.3880	0.3286	**0.3996**	0.3380	0.3840	0.3348
	0	0.5	1	1	0.7	**0.9506**	0.9124	0.9452	0.9148	0.9468	0.9064
	0	0.5	1	1	1	0.8962	0.8462	**0.8896**	0.8546	0.8828	0.8530
	0	0	1	1	1	**0.9262**	0.8966	0.9244	0.8968	0.9224	0.8942
	0.5	0.5	0.7	1	0.7	0.6254	0.5610	0.6282	0.5722	**0.6330**	0.5716
	1	1	1	1	0	0.1732	0.1410	**0.1754**	0.1532	0.1684	0.1386
	1	0.4	0.8	1.6	0.4	**0.9998**	0.9998	**0.9998**	0.9994	**0.9998**	0.9998
	0	0.25	0.5	1	0.25	0.9738	0.9580	0.9746	0.9628	**0.9766**	0.9568
T with 3 degrees of freedom	0	0	0	0	0	0.0546	0.0524	0.0500	0.0516	0.0496	0.0514
	0.5	0.5	0.5	1	1	**0.1902**	0.1680	0.1900	0.1714	0.1864	0.1862
	0	0.5	1	1	0.7	0.6128	0.5568	0.6066	0.5638	**0.6284**	0.5522
	0	0.5	1	1	1	0.5494	0.5020	0.5496	0.4886	**0.5634**	0.4992
	0	0	1	1	1	**0.6376**	0.5768	0.6368	0.5818	0.6352	0.5764
	0.5	0.5	0.7	1	0.7	0.2772	0.2394	**0.2802**	0.2540	0.2748	0.2422
	1	1	1	1	0	0.1228	0.1102	0.1204	0.1062	**0.1278**	0.1216
	1	0.4	0.8	1.6	0.4	**0.9412**	0.9092	0.9400	0.9010	0.9388	0.9044
	0	0.25	0.5	1	0.25	0.6542	0.5994	**0.6690**	0.5994	0.6658	0.6120

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.60. Mixed Design for Balanced Incomplete Block Design and Completely Randomized Design for Treatment Five at Peak 4. Treatments=5, BIBD =15, CRD=15, Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0506	0.0504	0.0508	0.0470	0.0526	0.0490
	0.5	0.5	0.5	1	1	0.2918	0.2730	0.2806	0.2658	**0.2932**	0.2818
	0	0.5	1	1	0.7	**0.8750**	0.8540	0.8674	0.8460	0.8670	0.8572
	0	0.5	1	1	1	0.8050	0.8038	**0.8138**	0.8026	0.8092	0.8024
	0	0	1	1	1	0.8862	0.8810	0.8872	0.8752	**0.8908**	0.8718
	0.5	0.5	0.7	1	0.7	**0.4374**	0.4134	0.4258	0.4208	0.4236	0.4122
	1	1	1	1	0	**0.1748**	0.1596	0.1658	0.1546	0.1672	0.1564
	1	0.4	0.8	1.6	0.4	0.9980	0.9974	**0.9994**	0.9974	0.9984	0.9980
	0	0.25	0.5	1	0.25	0.9012	0.8880	**0.9018**	0.8976	0.9004	0.8918
Exponential	0	0	0	0	0	0.0528	0.0518	0.0524	0.0530	0.0588	0.0464
	0.5	0.5	0.5	1	1	0.4644	0.4366	0.4640	0.4368	**0.4724**	0.4364
	0	0.5	1	1	0.7	0.9776	0.9740	0.9782	0.9746	**0.9812**	0.9788
	0	0.5	1	1	1	**0.9508**	0.9512	0.9506	0.9418	0.9500	0.9390
	0	0	1	1	1	**0.9734**	0.9686	0.9682	0.9658	0.9702	0.9664
	0.5	0.5	0.7	1	0.7	**0.7322**	0.7018	0.7312	0.7242	0.7240	0.7202
	1	1	1	1	0	**0.2052**	0.1816	0.1972	0.1724	0.1982	0.1770
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000	1.0000	1.0000
	0	0.25	0.5	1	0.25	0.9922	0.9936	**0.9940**	0.9934	0.9936	0.9926
T with 3 degrees of freedom	0	0	0	0	0	0.0502	0.0510	0.0524	0.0520	0.0470	0.0484
	0.5	0.5	0.5	1	1	0.2164	0.2086	**0.2240**	0.2082	0.2236	0.2132
	0	0.5	1	1	0.7	0.7202	0.6970	**0.7278**	0.6918	0.7248	0.6986
	0	0.5	1	1	1	0.6484	0.6446	**0.6520**	0.6244	0.6472	0.6330
	0	0	1	1	1	0.7458	0.7360	**0.7472**	0.7178	0.7436	0.7264
	0.5	0.5	0.7	1	0.7	0.3360	0.3086	0.3290	0.3254	**0.3442**	0.3158
	1	1	1	1	0	**0.1416**	0.1246	**0.1416**	0.1246	0.1328	0.1382
	1	0.4	0.8	1.6	0.4	**0.9798**	0.9740	**0.9794**	0.9784	0.9780	0.9706
	0	0.25	0.5	1	0.25	0.7612	0.7468	**0.7670**	0.7510	0.7646	0.7296

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.61. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, CRD=10, RCBD=5 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0504	0.0458	0.0434	0.0520	0.0546	0.0526
	0.5	0.5	0.5	1	1	0.2552	0.2228	**0.2560**	0.2294	0.2240	0.2240
	0	0.5	1	1	0.7	0.7990	0.7256	**0.8194**	0.7580	0.7476	0.7232
	0	0.5	1	1	1	0.7500	0.6504	**0.7624**	0.6846	0.6674	0.6092
	0	0	1	1	1	0.8308	0.7626	**0.8542**	0.7820	0. 7492	0.6698
	0.5	0.5	0.7	1	0.7	0.3628	0.3240	**0.3740**	0.3424	0. 3148	0.2368
	1	1	1	1	0	0.1438	0.1206	0.1360	0.1248	**0.2240**	0.1518
	1	0.4	0.8	1.6	0.4	**0.9948**	0.9860	**0.9934**	0.9908	**0.9852**	0.9360
	0	0.25	0.5	1	0.25	**0.8422**	0.7748	**0.8370**	0.7918	**0. 7692**	0.6520
Exponential	0	0	0	0	0	0.0442	0.0450	0.0518	0.0532	0.0468	0.0498
	0.5	0.5	0.5	1	1	0.3982	0.3320	**0.4214**	0.3650	0.3558	0.3354
	0	0.5	1	1	0.7	0.9602	0.9108	**0.9532**	0.9358	0.9312	0.9258
	0	0.5	1	1	1	0.9158	0.8542	**0.9288**	0.8748	0.8864	0.8314
	0	0	1	1	1	0.9358	0.8976	**0.9634**	0.9174	0.8984	0.8394
	0.5	0.5	0.7	1	0.7	**0.6564**	0.5722	**0.6424**	0.5964	0.5414	0.3944
	1	1	1	1	0	0.1662	0.1482	**0.1598**	0.1448	**0.2834**	0.1858
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9998	**1.0000**	0.9996	0.9992	0.9960
	0	0.25	0.5	1	0.25	0.9828	0.9656	**0.9840**	0.9678	0.9604	0.8972
T with 3 degrees of freedom	0	0	0	0	0	0.0496	0.0502	0.0488	0.0498	0.0524	0.0448
	0.5	0.5	0.5	1	1	0.2088	0.1710	**0.2142**	0.1890	0.1728	0.1712
	0	0.5	1	1	0.7	0.6462	0.5624	**0.6634**	0.6036	0.5782	0.5686
	0	0.5	1	1	1	0.5774	0.5052	**0.5908**	0.5360	0.5116	0.4678
	0	0	1	1	1	0.6736	0.5850	**0.6890**	0.6060	0.7776	0.5142
	0.5	0.5	0.7	1	0.7	0.2762	0.2496	**0.2912**	0.2678	**0.2334**	0.1994
	1	1	1	1	0	0.1198	0.1092	0.1136	0.1126	0.1828	0.1302
	1	0.4	0.8	1.6	0.4	**0.9550**	0.9098	0.9496	0.9250	0.9124	0.8264
	0	0.25	0.5	1	0.25	0.6804	0.6156	**0.6898**	0.6328	0.5892	0.5072

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.62. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, CRD=5, RCBD=10 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0510	0.0536	0.0532	0.0486	0.0560	0.0476
	0.5	0.5	0.5	1	1	0.2410	0.1862	**0.2580**	0.2348	0.2008	0.1980
	0	0.5	1	1	0.7	0.7800	0.6314	**0.7994**	0.7458	0.6730	0.6634
	0	0.5	1	1	1	0.7198	0.5764	**0.7488**	0.6952	0.5320	0.5308
	0	0	1	1	1	0.8108	0.6624	**0.8530**	0.7836	0.5820	0.5752
	0.5	0.5	0.7	1	0.7	0.3712	0.2848	**0.3722**	0.3242	0.1932	0.1880
	1	1	1	1	0	0.1406	0.1164	0.1102	0.1034	**0.2672**	0.2320
	1	0.4	0.8	1.6	0.4	**0.9930**	0.9600	**0.9930**	0.9824	0.8808	0.8552
	0	0.25	0.5	1	0.25	0.8210	0.6812	**0.8266**	0.7780	0.5648	0.5280
Exponential	0	0	0	0	0	0.0480	0.0498	0.0550	0.0508	0.0566	0.0544
	0.5	0.5	0.5	1	1	0.3774	0.2806	**0.4132**	0.3720	0.3058	0.2950
	0	0.5	1	1	0.7	0.9464	0.8524	**0.9530**	0.9178	0.8974	0.8774
	0	0.5	1	1	1	0.9010	0.7722	**0.9254**	0.8850	0.7326	0.7310
	0	0	1	1	1	0.9378	0.8236	**0.9598**	0.9186	0.7648	0.7504
	0.5	0.5	0.7	1	0.7	0.7752	0.6132	**0.7784**	0.6490	0. 5620	0.4200
	1	1	1	1	0	0.1628	0.1262	0.1246	0.1184	**0.3440**	0.2866
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9972	0.9998	0.9992	0.9856	0.9774
	0	0.25	0.5	1	0.25	**0.9828**	0.9156	0.9796	0.9542	0. 8218	0.7956
T with 3 degrees of freedom	0	0	0	0	0	0.0488	0.0526	0.0554	0.0500	0.0546	0.0508
	0.5	0.5	0.5	1	1	0.1830	0.1612	**0.2050**	0.1876	0.1660	0.1578
	0	0.5	1	1	0.7	0.6144	0.4946	**0.6328**	0.5908	0.5296	0.5258
	0	0.5	1	1	1	0.5566	0.4354	**0.5866**	0.5340	0.4052	0.3972
	0	0	1	1	1	0.6524	0.5142	**0.6888**	0.6184	0.4566	0.4428
	0.5	0.5	0.7	1	0.7	**0.2880**	0.2320	0.2784	0.2568	0.1558	0.1494
	1	1	1	1	0	0.1204	0.1120	0.1094	0.1064	**0.2116**	0.1784
	1	0.4	0.8	1.6	0.4	**0.9436**	0.8504	0.9432	0.9086	0.7316	0.7070
	0	0.25	0.5	1	0.25	0.6712	0.5286	**0.6760**	0.6138	0.4146	0.4048

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.63. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, CRD=10, RCBD=10 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0482	0.0506	0.0474	0.0506	0.0518	0.0548
	0.5	0.5	0.5	1	1	0.2976	0.2302	**0.3236**	0.2524	0.2658	0.2418
	0	0.5	1	1	0.7	0.8944	0.7578	**0.9042**	0.8042	0.8202	0.7848
	0	0.5	1	1	1	0.8400	0.6816	**0.8776**	0.7462	0.6976	0.6908
	0	0	1	1	1	0.9120	0.7812	**0.9382**	0.8380	0.7814	0.7544
	0.5	0.5	0.7	1	0.7	0.4676	0.3534	**0.4716**	0.3778	0. 3262	0.2612
	1	1	1	1	0	0.1652	0.1400	0.1378	0.1350	**0.3154**	0.1834
	1	0.4	0.8	1.6	0.4	**0.9996**	0.9914	0.9992	0.9956	0.9866	0.9670
	0	0.25	0.5	1	0.25	0.9178	0.8026	**0.9318**	0.8326	0.7864	0.7026
Exponential	0	0	0	0	0	0.0482	0.0488	0.0526	0.0496	0.0504	0.0498
	0.5	0.5	0.5	1	1	0.4922	0.3636	**0.5332**	0.4044	0.4006	0.3890
	0	0.5	1	1	0.7	0.9904	0.9350	**0.9890**	0.9570	0.9656	0.9500
	0	0.5	1	1	1	0.9690	0.8824	**0.9772**	0.9120	0.8848	0.8828
	0	0	1	1	1	0.9830	0.9188	**0.9914**	0.9454	0.9290	0.9176
	0.5	0.5	0.7	1	0.7	**0.7822**	0.6080	0.7806	0.6580	0. 5546	0.4324
	1	1	1	1	0	0.1866	0.1494	0.1604	0.1566	**0.3840**	0.2228
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000	0.9996	0.9986
	0	0.25	0.5	1	0.25	**0.9974**	0.9764	**0.9974**	0.9812	0.9734	0.9444
T with 3 degrees of freedom	0	0	0	0	0	0.0452	0.0576	0.0566	0.0496	0.0528	0.0462
	0.5	0.5	0.5	1	1	0.2440	0.1938	**0.2526**	0.1896	0.2132	0.1966
	0	0.5	1	1	0.7	0.7560	0.6096	**0.7890**	0.6412	0.6602	0.6248
	0	0.5	1	1	1	0.6922	0.5406	**0.7312**	0.5750	0.5508	0.5470
	0	0	1	1	1	0.7816	0.6172	**0.8208**	0.6682	0.6136	0.6020
	0.5	0.5	0.7	1	0.7	0.3446	0.2788	**0.3528**	0.2800	0.2526	0.1982
	1	1	1	1	0	0.1376	0.1158	0.1146	0.1084	0.2418	0.1492
	1	0.4	0.8	1.6	0.4	0.9844	0.9386	**0.9874**	0.9552	0. 9260	0.8694
	0	0.25	0.5	1	0.25	**0.7998**	0.6552	0.7976	0.6826	0. 6296	0.5634

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.64. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, CRD=15, RCBD=10 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0458	0.0500	0.0530	0.0518	0.0546	0.0458
	0.5	0.5	0.5	1	1	0.3596	0.2982	**0.3854**	0.3086	0.3200	0.2864
	0	0.5	1	1	0.7	0.9462	0.8740	**0.9546**	0.8808	0.8922	0.8830
	0	0.5	1	1	1	0.9146	0.8028	**0.9262**	0.8252	0.8202	0.8004
	0	0	1	1	1	0.9566	0.8802	**0.9716**	0.9114	0.8878	0.8454
	0.5	0.5	0.7	1	0.7	0.5326	0.4448	**0.5546**	0.4408	0. 4168	0.3202
	1	1	1	1	0	0.1970	0.1684	0.1588	0.1538	**0.3384**	0.1838
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9986	**1.0000**	0.9994	0.9992	0.9918
	0	0.25	0.5	1	0.25	**0.9690**	0.9040	**0.9666**	0.9136	0.8988	0.8190
Exponential	0	0	0	0	0	0.0532	0.0466	0.0528	0.0468	0.0534	0.0512
	0.5	0.5	0.5	1	1	0.5896	0.4526	**0.6210**	0.4868	0.4906	0.4694
	0	0.5	1	1	0.7	0.9972	0.9814	**0.9976**	0.9848	0.9912	0.9846
	0	0.5	1	1	1	0.9906	0.9514	**0.9934**	0.9660	0.9544	0.9508
	0	0	1	1	1	0.9966	0.9752	**0.9982**	0.9834	0.9738	0.9634
	0.5	0.5	0.7	1	0.7	**0.8558**	0.7310	0.8512	0.7546	0. 7072	0.5546
	1	1	1	1	0	0.2352	0.1920	0.1888	0.1880	**0.4330**	0.2368
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	0.9998
	0	0.25	0.5	1	0.25	1.0000	0.9954	0.9996	0.9980	0.9952	0.9838
T with 3 degrees of freedom	0	0	0	0	0	0.0462	0.0522	0.0556	0.0476	0.0522	0.0534
	0.5	0.5	0.5	1	1	0.2636	0.2284	**0.2828**	0.2372	0.2458	0.2242
	0	0.5	1	1	0.7	0.8286	0.7228	**0.8544**	0.7438	0.7558	0.7302
	0	0.5	1	1	1	0.7828	0.6512	**0.7990**	0.6864	0.6622	0.6332
	0	0	1	1	1	0.8590	0.7394	**0.8740**	0.7656	0.7442	0.6938
	0.5	0.5	0.7	1	0.7	**0.4232**	0.3244	0.4226	0.3488	0. 3148	0.2440
	1	1	1	1	0	0.1648	0.1400	0.1474	0.1244	**0.2538**	0.1560
	1	0.4	0.8	1.6	0.4	0.9964	0.9792	**0.9978**	0.9858	0.9754	0.9438
	0	0.25	0.5	1	0.25	0.8728	0.7694	**0.8732**	0.7824	0. 7462	0.6650

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.65. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, CRD=5, RCBD=15 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0536	0.0456	0.0420	0.0514	0.0544	0.0458
	0.5	0.5	0.5	1	1	0.2806	0.2242	**0.3106**	0.2910	0.2242	0.2204
	0	0.5	1	1	0.7	0.8668	0.7390	**0.8834**	0.8420	0.7476	0.7408
	0	0.5	1	1	1	0.8058	0.6588	**0.8516**	0.8002	0.6082	0.6010
	0	0	1	1	1	0.8840	0.7528	**0.9164**	0.8874	0.6536	0.6476
	0.5	0.5	0.7	1	0.7	0.4274	0.3280	**0.4228**	0.4150	0. 1984	0.1880
	1	1	1	1	0	0.1646	0.1328	0.1244	0.1164	**0.3214**	0.2934
	1	0.4	0.8	1.6	0.4	**0.9992**	0.9868	**0.9984**	0.9966	0.9106	0.9104
	0	0.25	0.5	1	0.25	**0.8976**	0.7692	**0.8930**	0.8654	0.5950	0.5776
Exponential	0	0	0	0	0	0.0498	0.0454	0.0448	0.0546	0.0458	0.0496
	0.5	0.5	0.5	1	1	0.4378	0.3428	**0.4982**	0.4598	0.3836	0.3402
	0	0.5	1	1	0.7	0.9786	0.9128	**0.9818**	0.9724	0.9420	0.9324
	0	0.5	1	1	1	0.9530	0.8580	**0.9628**	0.9530	0.8088	0.7992
	0	0	1	1	1	0.9724	0.9032	**0.9864**	0.9722	0.8286	0.8126
	0.5	0.5	0.7	1	0.7	0.7296	0.5878	**0.7384**	0.6972	0. 3106	0.2962
	1	1	1	1	0	0.1822	0.1526	0.1350	0.1316	**0.4170**	0.3926
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9992	**1.0000**	1.0000	0.9914	0.9902
	0	0.25	0.5	1	0.25	0.9932	0.9626	0.9922	0.9860	0.8636	0.8594
T with 3 degrees of freedom	0	0	0	0	0	0.0522	0.0510	0.0522	0.0548	0.0536	0.0490
	0.5	0.5	0.5	1	1	0.2186	0.1850	**0.2318**	0.2224	0.1820	0.1818
	0	0.5	1	1	0.7	0.7162	0.5828	**0.7554**	0.6948	0.5946	0.5790
	0	0.5	1	1	1	0.6378	0.5118	**0.6930**	0.6502	0.4728	0.4504
	0	0	1	1	1	0.7290	0.5876	**0.7842**	0.7464	0.4974	0.4898
	0.5	0.5	0.7	1	0.7	0.3226	0.2586	**0.3284**	0.3194	0.1634	0.1566
	1	1	1	1	0	0.1356	0.1150	0.1102	0.1006	**0.2484**	0.2196
	1	0.4	0.8	1.6	0.4	0.9774	0.9112	**0.9776**	0.9654	0.7640	0.7610
	0	0.25	0.5	1	0.25	0.7474	0.6186	**0.7616**	0.7204	0.4548	0.4518

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.66. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, CRD=10, RCBD=15 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0472	0.0512	0.0472	0.0512	0.0544	0.0464
	0.5	0.5	0.5	1	1	0.3700	0.2566	**0.3832**	0.2864	0.3052	0.2678
	0	0.5	1	1	0.7	0.9348	0.7818	**0.9516**	0.8566	0.8794	0.8210
	0	0.5	1	1	1	0.9036	0.7248	**0.9228**	0.7960	0.7552	0.7314
	0	0	1	1	1	0.9518	0.8086	**0.9734**	0.8780	0.8010	0.8076
	0.5	0.5	0.7	1	0.7	**0.5256**	0.3648	0.5256	0.4108	0. 3072	0.2670
	1	1	1	1	0	0.1868	0.1372	0.1418	0.1356	**0.3640**	0.2274
	1	0.4	0.8	1.6	0.4	**0.9998**	0.9934	**1.0000**	0.9984	**0.9902**	0.9816
	0	0.25	0.5	1	0.25	0.9598	0.8462	0.9652	0.8794	0.7918	0.7584
Exponential	0	0	0	0	0	0.0506	0.0486	0.0472	0.0546	0.0556	0.0530
	0.5	0.5	0.5	1	1	0.5766	0.3904	**0.6118**	0.4548	0.4652	0.4048
	0	0.5	1	1	0.7	0.9962	0.9548	**0.9964**	0.9752	0.9856	0.9626
	0	0.5	1	1	1	0.9854	0.8998	**0.9924**	0.9430	0.9320	0.9114
	0	0	1	1	1	0.9956	0.9416	**0.9982**	0.9736	0.9414	0.9368
	0.5	0.5	0.7	1	0.7	**0.8480**	0.6442	0.8472	0.7012	0. 5732	0.4416
	1	1	1	1	0	0.2336	0.1594	0.1684	0.1602	**0.4902**	0.2716
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	0.9998	0.9996	0.9994
	0	0.25	0.5	1	0.25	**0.9996**	0.9834	0.9990	0.9914	0.9754	0.9598
T with 3 degrees of freedom	0	0	0	0	0	0.0460	0.0478	0.0466	0.0480	0.0564	0.0482
	0.5	0.5	0.5	1	1	0.2662	0.1960	**0.2968**	0.2208	0.2276	0.1992
	0	0.5	1	1	0.7	0.8208	0.6354	**0.8400**	0.6946	0.7288	0.6696
	0	0.5	1	1	1	0.7626	0.5656	**0.8128**	0.6408	0.5910	0.5730
	0	0	1	1	1	0.8460	0.6494	**0.8848**	0.7298	0.6532	0.6372
	0.5	0.5	0.7	1	0.7	**0.4048**	0.2930	0.4036	0.3044	0.2490	0.2000
	1	1	1	1	0	0.1510	0.1188	0.1268	0.1204	**0.2852**	0.1714
	1	0.4	0.8	1.6	0.4	**0.9958**	0.9448	0.9944	0.9708	0.9346	0.9104
	0	0.25	0.5	1	0.25	0.8640	0.6694	**0.8664**	0.7292	0.6342	0.6078

Results show that Standardized First test performed better than Standardized Last in all distinct tests. In general, Distance Squared-Modification relatively performed better than the other two tests.

Table C.67. Mixed Design for Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4. Treatments=5, CRD=15, RCBD=15 Peak=4

Distribution	Location					Non-modification		Distance-Modification		Distance Squared-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last	First	Last
Normal	0	0	0	0	0	0.0478	0.0490	0.0536	0.0486	0.0540	0.0488
	0.5	0.5	0.5	1	1	0.4012	0.3016	**0.4202**	0.3322	0.3414	0.3216
	0	0.5	1	1	0.7	**0.9774**	0.8746	0.9742	0.9048	0.9308	0.9008
	0	0.5	1	1	1	0.9490	0.8242	**0.9654**	0.8512	0.8514	0.8312
	0	0	1	1	1	0.9812	0.9006	**0.9890**	0.9266	0.9014	0.8918
	0.5	0.5	0.7	1	0.7	0.5982	0.4404	**0.6068**	0.4712	0.4178	0.3172
	1	1	1	1	0	0.2158	0.1668	0.1600	0.1576	**0.3950**	0.2126
	1	0.4	0.8	1.6	0.4	**1.0000**	0.9990	**1.0000**	0.9998	0.9990	0.9964
	0	0.25	0.5	1	0.25	0.9838	0.9090	0.9842	0.9318	0.8992	0.8600
Exponential	0	0	0	0	0	0.0532	0.0528	0.0504	0.0478	0.0506	0.0482
	0.5	0.5	0.5	1	1	0.6464	0.4758	**0.6964**	0.4980	0.5486	0.5026
	0	0.5	1	1	0.7	0.9996	0.9832	**0.9998**	0.9894	0.9956	0.9902
	0	0.5	1	1	1	0.9964	0.9606	**0.9976**	0.9730	0.9736	0.9592
	0	0	1	1	1	0.9986	0.9794	**0.9994**	0.9864	0.9792	0.9768
	0.5	0.5	0.7	1	0.7	0.9028	0.7574	**0.9064**	0.7884	0. 7140	0.5712
	1	1	1	1	0	0.2548	0.1890	0.2000	0.1928	**0.5228**	0.2522
	1	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	0.9998	0.9966	**1.0000**	0.9970	0.9936	0.9882
T with 3 degrees of freedom	0	0	0	0	0	0.0538	0.0554	0.0436	0.0516	0.0546	0.0494
	0.5	0.5	0.5	1	1	0.3028	0.2324	**0.3378**	0.2482	0.2620	0.2476
	0	0.5	1	1	0.7	0.8900	0.7294	**0.9014**	0.7674	0.8136	0.7658
	0	0.5	1	1	1	0.8502	0.6730	**0.8634**	0.7106	0.6980	0.6662
	0	0	1	1	1	0.9106	0.7606	**0.9242**	0.7918	0.7640	0.7498
	0.5	0.5	0.7	1	0.7	**0.4754**	0.3232	0.4636	0.3542	0. 3198	0.2540
	1	1	1	1	0	0.1702	0.1324	0.1366	0.1332	**0.3136**	0.1638
	1	0.4	0.8	1.6	0.4	**0.9988**	0.9818	0.9992	0.9884	0.9794	0.9604
	0	0.25	0.5	1	0.25	**0.9236**	0.7700	**0.9236**	0.8132	0.7634	0.7156

Results show that Standardized First performed better than Standardized Last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

APPENDIX D. MIXED DESIGN OF IBD, CRD AND RCBD WITH PROBABILITIES OF 0.1, 0.2 AND 0.4 FOR TREATMENTS 4 AT PEAKS 2 AND 3.

Table D.1. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.1. Treatments=4, IBD = 12, CRD=6, RCBD=12, Peak=2, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0492	0.0444	0.0502	0.0536
	0.5	0.5	0	0	0.3360	0.2436	**0.3452**	0.2588
Normal	0	1	0.2	0.2	0.9366	0.7884	**0.9376**	0.8054
	1	1	0	0	0.7578	0.5598	**0.7936**	0.623
	0	0.7	0.2	0	**0.8320**	0.6396	0.8184	0.6762
	0.5	1	0.5	0	0.9288	0.7670	**0.9314**	0.8104
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0506	0.0496	0.0508	0.0464
Exponential	0.5	0.5	0	0	0.5438	0.3750	**0.5634**	0.4164
	0	1	0.2	0.2	**0.9976**	0.9540	0.9962	0.9666
	1	1	0	0	0.8952	0.7068	**0.9186**	0.7620
	0	0.7	0.2	0	0.9850	0.8936	0.9816	0.9124
	0.5	1	0.5	0	0.9962	0.9520	0.9974	0.9718
Distribution	Location				**Non-modification**		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	**First**	Last	First	Last
T with 3 degrees	0	0	0	0	0.0478	0.0450	0.0502	0.0526
of freedom	0.5	0.5	0	0	0.2640	0.1970	**0.2686**	0.1964
	0	1	0.2	0.2	**0.8172**	0.6266	0.8086	0.6494
	1	1	0	0	0.6038	0.4288	**0.6382**	0.4736
	0	0.7	0.2	0	**0.6730**	0.4802	0.6576	0.5108
	0.5	1	0.5	0	0.7870	0.5948	**0.8060**	0.6364

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.2. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.1. Treatments=4, IBD = 18, CRD=6, RCBD=18, Peak=2, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0536	0.0528	0.0500	0.0528
	0.5	0.5	0	0	0.4082	0.2888	**0.4450**	0.3134
Normal	0	1	0.2	0.2	0.9770	0.8768	**0.9776**	0.9088
	1	1	0	0	**0.8628**	0.6780	**0.8940**	0.7432
	0	0.7	0.2	0	**0.9176**	0.7506	0.9112	0.7774
	0.5	1	0.5	0	**0.9772**	0.8668	0.9750	0.8956
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0494	0.0508	0.0454	0.0544
Exponential	0.5	0.5	0	0	0.6654	0.4560	**0.6968**	0.5162
	0	1	0.2	0.2	**1.0000**	0.9878	**1.0000**	0.9918
	1	1	0	0	0.9596	0.8282	**0.9730**	0.8748
	0	0.7	0.2	0	**0.9978**	0.9546	0.9976	0.9704
	0.5	1	0.5	0	**0.9998**	0.9898	**0.9998**	0.9926
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0478	0.0488	0.0502	0.0520
of freedom	0.5	0.5	0	0	0.2954	0.2038	**0.3418**	0.2548
	0	1	0.2	0.2	**0.9032**	0.7316	**0.8986**	0.7480
	1	1	0	0	0.7246	0.5296	**0.7488**	0.5842
	0	0.7	0.2	0	0.7810	0.5962	**0.7876**	0.6226
	0.5	1	0.5	0	0.8854	0.7132	**0.8936**	0.7452

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.3. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.1. Treatments=4, IBD = 6, CRD=12, RCBD=6, Peak=2, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0498	0.0498	0.0460	0.0516
	0.5	0.5	0	0	0.3110	0.2450	**0.3250**	0.2322
	0	1	0.2	0.2	**0.9004**	0.7904	0.8886	0.8042
	1	1	0	0	0.6958	0.5594	**0.7204**	0.5700
	0	0.7	0.2	0	0.7626	0.6312	**0.7680**	0.6438
	0.5	1	0.5	0	0.8748	0.7676	**0.8856**	0.7744
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0458	0.0438	0.0538	0.0506
	0.5	0.5	0	0	**0.4800**	0.3596	0.4538	0.3732
	0	1	0.2	0.2	**0.9880**	0.9598	0.5022	0.9634
	1	1	0	0	0.8484	0.7202	**0.9894**	0.7280
	0	0.7	0.2	0	**0.9700**	0.9006	0.8680	0.9060
	0.5	1	0.5	0	**0.9914**	0.9596	0.9664	0.9638
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0494	0.0482	0.0478	0.0500
	0.5	0.5	0	0	0.2238	0.1888	**0.2468**	0.1954
	0	1	0.2	0.2	**0.7512**	0.6324	0.7468	0.6302
	1	1	0	0	0.5426	0.4116	**0.5588**	0.4310
	0	0.7	0.2	0	**0.6100**	0.4922	0.6056	0.4796
	0.5	1	0.5	0	0.7356	0.5984	**0.7398**	0.6128

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.4. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.1. Treatments=4, IBD = 12, CRD=12, RCBD=12, Peak=2, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0594	0.0530	0.0468	0.0474
	0.5	0.5	0	0	0.3846	0.2508	**0.4160**	0.2648
Normal	0	1	0.2	0.2	**0.9736**	0.8280	0.9676	0.8478
	1	1	0	0	0.8524	0.6212	**0.8740**	0.6286
	0	0.7	0.2	0	**0.8898**	0.6798	**0.8998**	0.7060
	0.5	1	0.5	0	0.9652	0.8236	**0.9688**	0.8252
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0480	0.0496	0.0456	0.0544
Exponential	0.5	0.5	0	0	0.6494	0.4200	**0.6602**	0.4208
	0	1	0.2	0.2	**1.0000**	0.9776	0.9996	0.9812
	1	1	0	0	0.9502	0.7688	**0.9600**	0.7966
	0	0.7	0.2	0	0.9960	0.9304	**0.9962**	0.9390
	0.5	1	0.5	0	0.9996	0.9766	**1.0000**	0.9780
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0456	0.0472	0.0474	0.0538
of freedom	0.5	0.5	0	0	0.2962	0.1974	**0.3036**	0.2136
	0	1	0.2	0.2	0.8846	0.6652	**0.8856**	0.6990
	1	1	0	0	0.6968	0.4624	**0.7250**	0.4884
	0	0.7	0.2	0	0.7638	0.5406	**0.7642**	0.5484
	0.5	1	0.5	0	**0.8874**	0.6644	0.8862	0.6750

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.5. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.1. Treatments=4, IBD = 18, CRD=12, RCBD=18, Peak=2, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0504	0.0520	0.0578	0.0508
	0.5	0.5	0	0	0.4708	0.2738	**0.4984**	0.2992
	0	1	0.2	0.2	**0.9936**	0.8676	0.9916	0.8792
	1	1	0	0	**0.9224**	0.6628	**0.9362**	0.6794
	0	0.7	0.2	0	**0.9626**	0.7400	0.9544	0.7586
	0.5	1	0.5	0	0.9908	0.8534	**0.9922**	0.8698
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0464	0.0492	0.0506	0.0506
	0.5	0.5	0	0	0.7512	0.4418	**0.7806**	0.4872
	0	1	0.2	0.2	**1.0000**	0.9842	**1.0000**	0.9898
	1	1	0	0	0.9842	0.8154	**0.9918**	0.8476
	0	0.7	0.2	0	0.9990	0.9602	**0.9996**	0.9652
	0.5	1	0.5	0	**1.0000**	0.9848	**1.0000**	0.9898
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0520	0.0548	0.0468	0.0542
	0.5	0.5	0	0	0.3710	0.2162	**0.3796**	0.2258
	0	1	0.2	0.2	**0.9502**	0.7248	0.9426	0.7458
	1	1	0	0	0.8062	0.5044	**0.8172**	0.5400
	0	0.7	0.2	0	0.8502	0.5718	**0.8564**	0.5898
	0.5	1	0.5	0	**0.9430**	0.6952	0.9378	0.7310

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.6. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.1. Treatments=4, IBD =6, CRD=18, RCBD=6, Peak=2, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0508	0.0488	0.0472	0.0538
	0.5	0.5	0	0	0.3510	0.3008	**0.3588**	0.2972
Normal	0	1	0.2	0.2	**0.9454**	0.8944	0.9420	0.8942
	1	1	0	0	0.7894	0.6992	**0.7904**	0.7004
	0	0.7	0.2	0	**0.8272**	0.7618	0.8264	0.7558
	0.5	1	0.5	0	0.9246	0.8742	**0.9360**	0.8868
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0466	0.0542	0.0546	0.0500
Exponential	0.5	0.5	0	0	0.5580	0.4748	**0.5800**	0.4934
	0	1	0.2	0.2	0.9976	0.9950	**0.9984**	0.9932
	1	1	0	0	0.8996	0.8378	**0.9170**	0.8478
	0	0.7	0.2	0	**0.9882**	0.9720	0.9852	0.9662
	0.5	1	0.5	0	0.9974	0.9916	0**.9976**	0.9948
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0492	0.0470	0.0490	0.0494
of freedom	0.5	0.5	0	0	**0.2630**	0.2294	0.2606	0.2386
	0	1	0.2	0.2	0.8196	0.7428	**0.8276**	0.7616
	1	1	0	0	0.6082	0.5316	**0.6376**	0.5386
	0	0.7	0.2	0	0.6778	0.6052	**0.6782**	0.6112
	0.5	1	0.5	0	0.8070	0.7296	**0.8134**	0.7298

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.7. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.1. Treatments=4, IBD = 12, CRD=18, RCBD=12, Peak=2, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0444	0.0522	0.0478	0.0504
	0.5	0.5	0	0	0.4436	0.3100	**0.4658**	0.2946
Normal	0	1	0.2	0.2	**0.9890**	0.9064	0.9836	0.9184
	1	1	0	0	0.8990	0.7264	**0.9140**	0.7266
	0	0.7	0.2	0	0.9298	0.7908	**0.9372**	0.7972
	0.5	1	0.5	0	**0.9840**	0.8990	0.9824	0.9014
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0582	0.0548	0.0432	0.0562
Exponential	0.5	0.5	0	0	0.7038	0.4972	**0.7366**	0.5226
	0	1	0.2	0.2	**1.0000**	0.9948	**1.0000**	0.9944
	1	1	0	0	**0.9788**	0.8648	0.9764	0.8710
	0	0.7	0.2	0	0.9990	0.9750	**0.9992**	0.9782
	0.5	1	0.5	0	**0.9998**	0.9952	**0.9998**	0.9970
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0446	0.0446	0.0486	0.0478
of freedom	0.5	0.5	0	0	0.3414	0.2412	**0.3488**	0.2350
	0	1	0.2	0.2	**0.9296**	0.7668	0.9266	0.7920
	1	1	0	0	0.7716	0.5602	**0.7826**	0.5704
	0	0.7	0.2	0	**0.8262**	0.6238	0.8168	0.6330
	0.5	1	0.5	0	**0**.9162	0.7550	**0.9210**	0.7612

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.8. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.1. Treatments=4, IBD = 18, CRD=18, RCBD=18, Peak=2, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0438	0.0468	0.0454	0.0506
	0.5	0.5	0	0	0.5162	0.3162	**0.5382**	0.3084
	0	1	0.2	0.2	**0.9952**	0.9232	0.9934	0.9246
	1	1	0	0	0.9492	0.7392	**0.9634**	0.7742
	0	0.7	0.2	0	**0.9770**	0.9770	0.9736	0.8150
	0.5	1	0.5	0	**0.9976**	0.9162	0.9968	0.9104
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0476	0.0508	0.0456	0.0478
	0.5	0.5	0	0	0.8072	0.5168	**0.8310**	0.5504
	0	1	0.2	0.2	**1.0000**	0.9956	**1.0000**	0.9980
	1	1	0	0	0.9908	0.8828	**0.9940**	0.8844
	0	0.7	0.2	0	**0.9998**	0.9842	**0.9998**	0.9834
	0.5	1	0.5	0	**1.0000**	0.9950	**1.0000**	0.9974
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0456	0.0508	0.0526	0.0480
	0.5	0.5	0	0	0.3820	0.2472	**0.4118**	0.2536
	0	1	0.2	0.2	**0.9692**	0.7958	**0.9692**	0.8176
	1	1	0	0	0.8448	0.5850	**0.8682**	0.5966
	0	0.7	0.2	0	0.8984	0.6606	**0.8908**	0.6696
	0.5	1	0.5	0	0.9616	0.7676	**0.9630**	0.7858

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.9. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.2. IBD=6, CRD=6, RCBD=6, Peak=2, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0470	0.0532	0.0500	0.0512
	0.5	0.5	0	0	0.2404	0.1806	**0.2472**	0.2024
	0	1	0.2	0.2	**0.7876**	0.6340	0.7754	0.6566
	1	1	0	0	0.5668	0.4180	**0.5802**	0.4402
	0	0.7	0.2	0	0.6318	0.4974	**0.6368**	0.5178
	0.5	1	0.5	0	0.7698	0.6128	**0.7790**	0.6516
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0462	0.0526	0.0502	0.0490
	0.5	0.5	0	0	0.3752	0.2776	**0.3900**	0.2928
	0	1	0.2	0.2	**0.9534**	0.8474	0.9506	0.8694
	1	1	0	0	0.7300	0.5664	**0.7428**	0.5960
	0	0.7	0.2	0	**0.8982**	0.7508	0.8914	0.7738
	0.5	1	0.5	0	0.9540	0.8552	**0.9584**	0.8752
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0428	0.0472	0.0526	0.0532
	0.5	0.5	0	0	0.1824	0.1562	**0.1888**	0.1620
	0	1	0.2	0.2	**0.6190**	0.4784	0.6100	0.4990
	1	1	0	0	0.4350	0.3202	**0.4546**	0.3530
	0	0.7	0.2	0	**0.4864**	0.3744	0.4806	0.3830
	0.5	1	0.5	0	0.6040	0.4572	**0.6176**	0.4848

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.10. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.2. Treatments=4, IBD = 12, CRD=6, RCBD=12, Peak=2, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0502	0.0540	0.0512	0.0496
	0.5	0.5	0	0	0.3208	0.2288	**0.3460**	0.2548
	0	1	0.2	0.2	**0.9252**	0.7700	0.9196	0.7922
	1	1	0	0	0.7342	0.5390	**0.7676**	0.5832
	0	0.7	0.2	0	**0.8056**	0.6234	**0.8032**	0.6472
	0.5	1	0.5	0	0.9146	0.7394	**0.9214**	0.7844
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0494	0.0480	0.0496	0.0486
	0.5	0.5	0	0	0.5272	0.3514	**0.5586**	0.3992
	0	1	0.2	0.2	**0.9960**	0.9492	0.9950	0.9630
	1	1	0	0	0.8850	0.6974	**0.9106**	0.7540
	0	0.7	0.2	0	**0.9800**	0.8806	0.9776	0.9042
	0.5	1	0.5	0	0.9970	0.9434	**0.9978**	0.9586
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0548	0.0532	0.0496	0.0480
	0.5	0.5	0	0	**0.2538**	0.1870	**0.2572**	0.2044
	0	1	0.2	0.2	**0.7992**	0.6010	0.7856	0.6338
	1	1	0	0	0.5858	0.4188	**0.6242**	0.4566
	0	0.7	0.2	0	0.6368	0.4686	**0.6458**	0.4912
	0.5	1	0.5	0	0.7710	0.5886	**0.7914**	0.6284

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.11. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.2. Treatments=4, IBD = 18, CRD=6, RCBD=18, Peak=2, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0448	0.0520	0.0492	0.0468
	0.5	0.5	0	0	0.3934	0.2688	**0.4196**	0.3010
Normal	0	1	0.2	0.2	**0.9788**	0.8610	0.9724	0.8858
	1	1	0	0	0.8448	0.6350	**0.8808**	0.7270
	0	0.7	0.2	0	**0.8998**	0.7250	0.8856	0.7510
	0.5	1	0.5	0	**0.9646**	0.8450	0.9638	0.8786
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0544	0.0498	0.0400	0.0498
Exponential	0.5	0.5	0	0	**0.6358**	0.4368	**0.6732**	0.5062
	0	1	0.2	0.2	**0.9996**	0.9870	0.9992	0.9906
	1	1	0	0	0.9484	0.7976	**0.9622**	0.8620
	0	0.7	0.2	0	0.9974	0.9474	0.9960	0.9628
	0.5	1	0.5	0	0.9996	0.9844	0.9998	0.9890
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0486	0.0512	0.0512	0.0550
of freedom	0.5	0.5	0	0	0.3042	0.2200	**0.3174**	0.2348
	0	1	0.2	0.2	**0.8906**	0.7098	0.8796	0.7516
	1	1	0	0	**0.6948**	0.4806	**0.7310**	0.5574
	0	0.7	0.2	0	**0.7680**	0.5738	0.7744	0.6026
	0.5	1	0.5	0	**0.8776**	0.6826	**0.8776**	0.7414

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.12. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.2. Treatments=4, IBD = 6, CRD=12, RCBD=6, Peak=2, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0544	0.0516	0.0560	0.0462
	0.5	0.5	0	0	0.2820	0.2432	**0.3104**	0.2360
Normal	0	1	0.2	0.2	**0.8826**	0.7836	0.8762	0.8048
	1	1	0	0	0.6982	0.5748	**0.7038**	0.5736
	0	0.7	0.2	0	0.7500	0.6384	**0.7598**	0.6476
	0.5	1	0.5	0	0.8706	0.7626	**0.8744**	0.7708
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0488	0.0502	0.0500	0.0516
Exponential	0.5	0.5	0	0	0.4646	0.3644	**0.4880**	0.3846
	0	1	0.2	0.2	**0.9910**	0.9514	0.9886	0.9668
	1	1	0	0	0.8184	0.7088	**0.8540**	0.7278
	0	0.7	0.2	0	0.9606	0.9038	**0.9636**	0.9094
	0.5	1	0.5	0	**0.9880**	0.9504	0.9886	0.9616
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0506	0.0494	0.0500	0.0516
of freedom	0.5	0.5	0	0	0.2300	0.1804	**0.2350**	0.1944
	0	1	0.2	0.2	**0.7394**	0.6228	0.7330	0.6210
	1	1	0	0	0.5206	0.4202	**0.5414**	0.4412
	0	0.7	0.2	0	**0.6030**	0.4790	0.5770	0.4906
	0.5	1	0.5	0	0.7198	0.5974	**0.7326**	0.6134

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.13. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.2. Treatments=4, IBD = 18, CRD=12, RCBD=18, Peak=2, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0540	0.0506	0.0564	0.0490
	0.5	0.5	0	0	0.4510	0.2696	**0.4726**	0.2926
Normal	0	1	0.2	0.2	**0.9896**	0.8704	**0.9872**	0.8824
	1	1	0	0	0.9098	0.6496	**0.9306**	0.6952
	0	0.7	0.2	0	0.9420	0.7144	**0.9450**	0.7348
	0.5	1	0.5	0	0.9862	0.8454	**0.9876**	0.8638
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0452	0.0482	0.0518	0.0470
Exponential	0.5	0.5	0	0	0.7244	0.4294	**0.7582**	0.4746
	0	1	0.2	0.2	0.9998	0.9872	**1.0000**	0.9882
	1	1	0	0	0.9792	0.8068	**0.9844**	0.8396
	0	0.7	0.2	0	**0.9998**	0.9564	0.9996	0.9606
	0.5	1	0.5	0	**1.0000**	0.9858	**1.0000**	0.9900
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0560	0.0520	0.0490	0.0542
of freedom	0.5	0.5	0	0	**0.3396**	0.2226	0.3652	0.2238
	0	1	0.2	0.2	**0.9400**	0.7068	**0.9400**	0.7274
	1	1	0	0	0.7786	0.5112	**0.8048**	0.5346
	0	0.7	0.2	0	0.8380	0.5628	**0.8390**	0.5830
	0.5	1	0.5	0	0.9248	0.6954	**0.9304**	0.7234

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.14. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.2. Treatments=4, IBD =6, CRD=18, RCBD=6, Peak=2, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0514	0.0510	0.0480	0.0556
	0.5	0.5	0	0	0.3272	0.2858	**0.3374**	0.3018
Normal	0	1	0.2	0.2	0.9282	0.8940	**0.9272**	0.9014
	1	1	0	0	0.7546	0.6986	**0.7886**	0.7040
	0	0.7	0.2	0	**0.8236**	0.7684	0.8186	0.7660
	0.5	1	0.5	0	0.9190	0.8790	**0.9264**	0.8770
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0556	0.0496	0.0512	0.0566
Exponential	0.5	0.5	0	0	**0.5520**	0.4842	**0.5672**	0.4850
	0	1	0.2	0.2	**0.9976**	0.9940	0.9970	0.9934
	1	1	0	0	**0.8874**	0.8398	**0.9128**	0.8446
	0	0.7	0.2	0	**0.9868**	0.9658	0.9820	0.9688
	0.5	1	0.5	0	**0.9966**	0.9912	0.9956	0.9924
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0482	0.0496	0.0492	0.0516
of freedom	0.5	0.5	0	0	0.2484	0.2346	**0.2680**	0.2290
	0	1	0.2	0.2	0.8084	0.7526	**0.8094**	0.7528
	1	1	0	0	0.5928	0.5366	**0.6252**	0.5466
	0	0.7	0.2	0	**0.6778**	0.5870	0.6530	0.6056
	0.5	1	0.5	**0**	**0.7936**	0.7268	0.7908	0.7268

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.15. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.2. Treatments=4, IBD = 12, CRD=18, RCBD=12, Peak=2, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0492	0.0446	0.0552	0.0530
	0.5	0.5	0	0	0.4200	0.3184	**0.4478**	0.3288
	0	1	0.2	0.2	0.9818	0.9060	**0.9856**	0.9140
	1	1	0	0	0.8836	0.7198	**0.9156**	0.7240
	0	0.7	0.2	0	**0.9336**	0.7724	0.9264	0.7916
	0.5	1	0.5	0	0.9790	0.8960	**0.9810**	0.8932
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0502	0.0476	0.0502	0.0504
	0.5	0.5	0	0	0.6834	0.4858	**0.6930**	0.5172
	0	1	0.2	0.2	**1.0000**	0.9948	**1.0000**	0.9958
	1	1	0	0	0.9654	0.8562	**0.9770**	0.8678
	0	0.7	0.2	0	**0.9976**	0.9772	0.9974	0.9790
	0.5	1	0.5	0	**0.9996**	0.9940	**0.9996**	0.9938
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0500	0.0528	0.0492	0.0456
	0.5	0.5	0	0	0.3304	0.2374	**0.3380**	0.2478
	0	1	0.2	0.2	**0.9194**	0.7802	0.9136	0.7806
	1	1	0	0	0.7362	0.5552	**0.7674**	0.5560
	0	0.7	0.2	0	0.8004	0.6164	**0.8120**	0.6338
	0.5	1	0.5	0	0.8978	0.7564	**0.9062**	0.7552

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.16. Mixed Design for IBD, CRD and RCBD for Treatment Four at Peak 2 with Probability of 0.2. Treatments=4, IBD = 18, CRD=18, RCBD=18, Peak=2, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0506	0.0500	0.0462	0.0506
	0.5	0.5	0	0	0.5058	0.3242	**0.5112**	0.3322
	0	1	0.2	0.2	**0.9920**	0.9186	0.9906	0.9194
	1	1	0	0	**0.9446**	0.7440	**0.9544**	0.7478
	0	0.7	0.2	0	**0.9678**	0.8072	0.9674	0.8096
	0.5	1	0.5	0	**0.9950**	0.9062	0.9948	0.9174
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0544	0.0528	0.0480	0.0502
	0.5	0.5	0	0	0.7824	0.5266	**0.8236**	0.5370
	0	1	0.2	0.2	**1.0000**	0.9976	**1.0000**	0.9986
	1	1	0	0	0.9892	0.8832	**0.9934**	0.8888
	0	0.7	0.2	0	0.9998	0.9804	**1.0000**	0.9816
	0.5	1	0.5	0	**1.0000**	0.9966	**1.0000**	0.9972
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0470	0.0482	0.0520	0.0532
	0.5	0.5	0	0	**0.3780**	0.2386	**0.4022**	0.2540
	0	1	0.2	0.2	0.9618	0.7836	**0.9620**	0.8082
	1	1	0	0	0.8254	0.5938	**0.8606**	0.5882
	0	0.7	0.2	0	0.8822	0.6470	**0.8828**	0.6484
	0.5	1	0.5	0	0.9550	0.7608	**0.9640**	0.7914

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.17. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.1. Treatments=4, IBD = 12, CRD=6, RCBD=12, Peak=3, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0476	0.0514	0.0436	0.0528
	0	0.5	0.5	0	**0.5774**	0.4076	**0.5862**	0.4224
	0	1	1	0.2	**0.9616**	0.8310	0.9608	0.8578
	0	0	1	0.2	**0.9626**	0.8424	0.9622	0.8638
	1	1	1	0	**0.3062**	0.2152	0.2606	0.2088
	0	0.2	0.7	0.5	**0.6298**	0.4570	**0.6370**	0.4820
	0	0.5	1	0.5	**0.9290**	0.7716	0.9248	0.7930
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0596	0.0508	0.0458	0.0524
	0	0.5	0.5	0	**0.8596**	0.6572	0.8420	0.6886
	0	1	1	0.2	**0.9968**	0.9506	0.9966	0.9664
	0	0	1	0.2	0.9982	0.9686	**0.9986**	0.9822
	1	1	1	0	**0.3956**	0.2616	0.3328	0.2536
	0	0.2	0.7	0.5	0.9034	0.7188	**0.9084**	0.7694
	0	0.5	1	0.5	0.9968	0.9562	**0.9978**	0.9702
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T Distribution with 3 degrees of freedom	0	0	0	0	0.0498	0.0498	0.0500	0.0500
	0	0.5	0.5	0	**0.4480**	0.3170	0.4470	0.3272
	0	1	1	0.2	**0.8606**	0.6780	**0.8678**	0.6962
	0	0	1	0.2	**0.8706**	0.6652	0.8652	0.7090
	1	1	1	0	**0.2374**	0.1760	0.2094	0.1580
	0	0.2	0.7	0.5	0.4844	0.3330	**0.4922**	0.3562
	0	0.5	1	0.5	0.7978	0.5890	**0.8020**	0.6304

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.18. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.1. Treatments=4, IBD = 18, CRD=6, RCBD=18, Peak=3, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0530	0.0504	0.0540	0.0516
	0	0.5	0.5	0	**0.6932**	0.4890	0.6810	0.5230
	0	1	1	0.2	**0.9894**	0.9100	0.9850	0.9338
	0	0	1	0.2	**0.9896**	0.9086	0.9884	0.9334
	1	1	1	0	**0.3796**	0.2596	0.3262	0.2456
	0	0.2	0.7	0.5	0.7412	0.5486	**0.7602**	0.5920
	0	0.5	1	0.5	0.9766	0.8622	**0.9794**	0.8976
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0482	0.0512	0.0494	0.0562
	0	0.5	0.5	0	**0.9342**	0.7680	0.9290	0.8126
	0	1	1	0.2	**0.9998**	0.9866	**0.9998**	0.9940
	0	0	1	0.2	**1.0000**	0.9926	**1.0000**	0.9964
	1	1	1	0	**0.4938**	0.3264	0.4208	0.3166
	0	0.2	0.7	0.5	0.9710	0.8404	**0.9716**	0.8736
	0	0.5	1	0.5	0.9996	0.9878	**1.0000**	0.9944
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T Distribution with 3 degrees of freedom	0	0	0	0	0.0494	0.0558	0.0472	0.0574
	0	0.5	0.5	0	0.5378	0.3906	**0.5418**	0.4124
	0	1	1	0.2	0.9376	0.7788	**0.9400**	0.8178
	0	0	1	0.2	0.9398	0.7848	**0.9408**	0.8204
	1	1	1	0	**0.2774**	0.2024	0.2456	0.1978
	0	0.2	0.7	0.5	0.5858	0.4246	**0.6100**	0.4504
	0	0.5	1	0.5	0.8964	0.7188	**0.8992**	0.7624

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.19. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.1. Treatments=4, IBD = 6, CRD=12, RCBD=6, Peak=3, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0546	0.0554	0.0518	0.0546
	0	0.5	0.5	0	**0.5092**	0.4060	0.5076	0.4208
	0	1	1	0.2	0.9266	0.8334	**0.9368**	0.8412
	0	0	1	0.2	**0.9282**	0.8332	0.9280	0.8492
	1	1	1	0	**0.2692**	0.2068	0.2456	0.2184
	0	0.2	0.7	0.5	0.5636	0.4508	**0.5808**	0.4580
	0	0.5	1	0.5	0.8734	0.7694	**0.8920**	0.7722
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0484	0.0504	0.0542	0.0552
	0	0.5	0.5	0	**0.7962**	0.6634	0.7960	0.6642
	0	1	1	0.2	**0.9908**	0.9552	0.9902	0.9630
	0	0	1	0.2	0.9954	0.9700	**0.9970**	0.9792
	1	1	1	0	**0.3420**	0.2710	0.3056	0.2608
	0	0.2	0.7	0.5	**0.8470**	0.7352	**0.8574**	0.7492
	0	0.5	1	0.5	**0.9932**	0.9532	0.9918	0.9642
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T Distribution with 3 degrees of freedom	0	0	0	0	0.0488	0.0490	0.0482	0.0426
	0	0.5	0.5	0	**0.3916**	0.3156	0.3854	0.3014
	0	1	1	0.2	**0.7928**	0.6710	**0.8092**	0.6854
	0	0	1	0.2	**0.8100**	0.6768	0.8024	0.6726
	1	1	1	0	**0.2090**	0.1642	0.1958	0.1754
	0	0.2	0.7	0.5	0.4338	0.3332	**0.4362**	0.3448
	0	0.5	1	0.5	0.7264	0.6040	**0.7432**	0.6146

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.20. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.1. Treatments=4, IBD = 12, CRD=12, RCBD=12, Peak=3**,** p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0538	0.0452	0.0556	0.0534
	0	0.5	0.5	0	**0.6660**	0.4590	0.6518	0.4496
Normal	0	1	1	0.2	0.9878	0.8770	**0.9880**	0.8838
	0	0	1	0.2	**0.9872**	0.8780	0.9862	0.8998
	1	1	1	0	**0.3652**	0.2222	0.3238	0.2272
	0	0.2	0.7	0.5	0.7158	0.4812	**0.7318**	0.5124
	0	0.5	1	0.5	**0.9650**	0.8174	**0.9656**	0.8396
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0522	0.0496	0.0488	0.0524
Exponential	0	0.5	0.5	0	**0.9184**	0.7002	0.9156	0.7292
	0	1	1	0.2	0.9996	0.9710	**0.9998**	0.9754
	0	0	1	0.2	0.9998	0.9866	**1.0000**	0.9906
	1	1	1	0	**0.4770**	0.2916	0.3986	0.2894
	0	0.2	0.7	0.5	**0.9576**	0.7814	**0.9616**	0.7850
	0	0.5	1	0.5	**0.9998**	0.9764	0.9994	0.9796
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0528	0.0464	0.0520	0.0504
T Distribution	0	0.5	0.5	0	**0.5182**	0.3312	0.5134	0.3420
with 3 degrees of	0	1	1	0.2	**0.9342**	0.7268	0.9250	0.7382
freedom	0	0	1	0.2	0.9174	0.7322	**0.9246**	0.7426
	1	1	1	0	**0.2748**	0.1806	0.2426	0.1868
	0	0.2	0.7	0.5	0.5728	0.3660	**0.5838**	0.3804
	0	0.5	1	0.5	0.8808	0.6562	**0.8872**	0.6738

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.21. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.1. Treatments=4, IBD = 18, CRD=12, RCBD=18, Peak=3, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0520	0.0520	0.0518	0.0450
	0	0.5	0.5	0	**0.7776**	0.4860	0.7604	0.5096
	0	1	1	0.2	**0.9974**	0.9122	0.9968	0.9208
	0	0	1	0.2	**0.9970**	0.9072	0.9968	0.9230
	1	1	1	0	**0.4356**	0.2496	0.3756	0.2444
	0	0.2	0.7	0.5	0.8156	0.5210	**0.8232**	0.5448
	0	0.5	1	0.5	**0.9912**	0.8608	0.9908	0.8704
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0504	0.0512	0.0514	0.0508
	0	0.5	0.5	0	**0.9680**	0.7692	0.9662	0.7720
	0	1	1	0.2	**1.0000**	0.9854	**1.0000**	0.9882
	0	0	1	0.2	**1.0000**	0.9944	**1.0000**	0.9946
	1	1	1	0	**0.5606**	0.3132	0.5004	0.3220
	0	0.2	0.7	0.5	0.9862	0.8162	**0.9880**	0.8422
	0	0.5	1	0.5	1.0000	0.9858	1.0000	0.9884
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T Distribution with 3 degrees of freedom	0	0	0	0	0.0516	0.0506	0.0464	0.0516
	0	0.5	0.5	0	**0.6232**	0.3680	0.6064	0.3882
	0	1	1	0.2	0.9668	0.7712	**0.9674**	0.7900
	0	0	1	0.2	**0.9748**	0.7714	0.9686	0.7948
	1	1	1	0	**0.3404**	0.1964	0.2836	0.1944
	0	0.2	0.7	0.5	0.6504	0.4020	**0.6874**	0.4258
	0	0.5	1	0.5	**0.9446**	0.6972	0.9442	0.7230

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.22. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.1. Treatments=4, IBD = 6, CRD=18, RCBD=6, Peak=3, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0492	0.0486	0.0488	0.0496
	0.5	0.5	0	0	**0.5942**	0.5048	0.5836	0.5178
	0	1	0.2	0.2	0.9664	0.9334	**0.9694**	0.9318
	1	1	0	0	0.9628	0.9308	**0.9668**	0.9322
	0	0.7	0.2	0	**0.3048**	0.2592	0.2860	0.2612
	0.5	1	0.5	0	0.6340	0.5738	**0.6468**	0.5688
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0476	0.0482	0.0554	0.0486
	0.5	0.5	0	0	**0.8564**	0.7878	**0.8564**	0.8058
	0	1	0.2	0.2	0.9968	0.9888	**0.9976**	0.9912
	1	1	0	0	**0.9998**	0.9970	0.9990	0.9972
	0	0.7	0.2	0	**0.4124**	0.3504	0.3610	0.3288
	0.5	1	0.5	0	**0.9104**	0.8506	0.9082	0.8604
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0510	0.0516	0.0566	0.0514
	0.5	0.5	0	0	**0.4558**	0.3958	0.4542	0.3976
	0	1	0.2	0.2	**0.8726**	0.7922	0.8700	0.7910
	1	1	0	0	0.8642	0.8054	**0.8722**	0.8072
	0	0.7	0.2	0	**0.2316**	0.2056	0.2194	0.2128
	0.5	1	0.5	0	0.4880	0.4208	**0.5004**	0.4252

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.23. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.1. Treatments=4, IBD = 12, CRD=18, RCBD=12, Peak=3, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0470	0.0472	0.0484	0.0484
	0.5	0.5	0	0	**0.7340**	0.5422	0.7280	0.5434
Normal	0	1	0.2	0.2	0.9944	0.9464	**0.9954**	0.9476
	1	1	0	0	0.9942	0.9394	**0.9952**	0.9516
	0	0.7	0.2	0	**0.4048**	0.2790	0.3520	0.2732
	0.5	1	0.5	0	0.7856	0.5906	**0.7974**	0.5952

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0482	0.0480	0.0502	0.0500
Exponential	0.5	0.5	0	0	**0.9480**	0.8162	0.9468	0.8108
	0	1	0.2	0.2	**1.0000**	0.9930	**1.0000**	0.9952
	1	1	0	0	**1.0000**	0.9988	**1.0000**	0.9980
	0	0.7	0.2	0	**0.5286**	0.3550	0.4720	0.3512
	0.5	1	0.5	0	0.9772	0.8716	**0.9818**	0.8790

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0458	0.0472	0.0458	0.0468
of freedom	0.5	0.5	0	0	0.5626	0.4018	**0.5748**	0.4092
	0	1	0.2	0.2	**0.9560**	0.8220	0.9550	0.8340
	1	1	0	0	0.9538	0.8238	**0.9544**	0.8332
	0	0.7	0.2	0	**0.2920**	0.2238	0.2762	0.2184
	0.5	1	0.5	0	0.6210	0.4412	**0.6384**	0.4582

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.24. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.1. Treatments=4, IBD = 18, CRD=18, RCBD=18, Peak=3, p=0.1

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0492	0.0514	0.0490	0.0496
	0.5	0.5	0	0	**0.8266**	0.5548	0.8210	0.5716
Normal	0	1	0.2	0.2	**0.9990**	0.9550	0.9986	0.9626
	1	1	0	0	**0.9992**	0.9566	0.9990	0.9606
	0	0.7	0.2	0	**0.4904**	0.2900	0.4268	0.2926
	0.5	1	0.5	0	0.8662	0.6082	**0.8736**	0.6144
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0510	0.0472	0.0514	0.0494
Exponential	0.5	0.5	0	0	0.9798	0.8384	**0.9814**	0.8426
	0	1	0.2	0.2	**1.0000**	0.9948	**1.0000**	0.9944
	1	1	0	0	**1.0000**	0.9992	**1.0000**	0.9988
	0	0.7	0.2	0	**0.6248**	0.3764	0.5526	0.3760
	0.5	1	0.5	0	**0.9942**	0.8866	0.9938	0.8972
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0494	0.0494	0.0498	0.0528
of freedom	0.5	0.5	0	0	**0.6716**	0.4182	0.6522	0.4364
	0	1	0.2	0.2	0.9806	0.8420	**0.9850**	0.8472
	1	1	0	0	0.9844	0.8422	**0.9880**	0.8600
	0	0.7	0.2	0	**0.3684**	0.2202	0.3218	0.2338
	0.5	1	0.5	0	0.7210	0.4504	**0.7156**	0.4652

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.25. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.2. Treatments=4, IBD = 6, CRD=6, RCBD=6, Peak=3, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0488	0.0490	0.0456	0.0508
	0.5	0.5	0	0	**0.4120**	0.3132	0.4112	0.3168
Normal	0	1	0.2	0.2	0.8276	0.6944	**0.8404**	0.7058
	1	1	0	0	0.9960	0.9768	**0.9968**	0.9824
	0	0.7	0.2	0	**0.2136**	0.1728	0.1978	0.1658
	0.5	1	0.5	0	0.4364	0.3380	**0.4644**	0.3656
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0474	0.0482	0.0490	0.0472
Exponential	0.5	0.5	0	0	0.6652	0.5078	**0.6666**	0.5332
	0	1	0.2	0.2	**0.9594**	0.8614	0.9558	0.8758
	1	1	0	0	0.9708	0.8948	**0.9712**	0.9038
	0	0.7	0.2	0	**0.2650**	0.2020	0.2290	0.2010
	0.5	1	0.5	0	0.7182	0.5716	**0.7304**	0.5970
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0530	0.0542	0.0488	0.0522
of freedom	0.5	0.5	0	0	0.3090	0.2456	**0.3132**	0.2600
	0	1	0.2	0.2	0.6790	0.5214	**0.6910**	0.5430
	1	1	0	0	**0.6892**	0.5192	0.6738	0.5534
	0	0.7	0.2	0	**0.1818**	0.1398	0.1588	0.1306
	0.5	1	0.5	0	0.3438	0.2606	**0.3508**	0.2712

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.26. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.2. Treatments=4, IBD = 12, CRD=6, RCBD=12, Peak=3, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0514	0.0500	0.0574	0.0528
	0.5	0.5	0	0	0.5726	0.3888	**0.5574**	0.4170
Normal	0	1	0.2	0.2	0.9536	0.8174	**0.9564**	0.5518
	1	1	0	0	0.9492	0.8242	**0.9564**	0.8564
	0	0.7	0.2	0	**0.2942**	0.2066	0.2552	0.1998
	0.5	1	0.5	0	**0.6150**	0.4392	0.2526	0.4712
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0462	0.0492	0.0538	0.0518
Exponential	0.5	0.5	0	0	**0.8324**	0.6342	0.8312	0.6694
	0	1	0.2	0.2	0.9956	0.9428	**0.9958**	0.9586
	1	1	0	0	**0.9988**	0.9638	0.9982	0.9800
	0	0.7	0.2	0	**0.3732**	0.2562	0.3314	0.2482
	0.5	1	0.5	0	**0.8890**	0.7014	**0.8988**	0.7502
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0544	0.0512	0.0458	0.0546
of freedom	0.5	0.5	0	0	**0.4404**	0.2958	**0.4208**	0.3260
	0	1	0.2	0.2	**0.8476**	0.6450	0.8430	0.6862
	1	1	0	0	**0.8436**	0.6534	0.8410	0.6984
	0	0.7	0.2	0	**0.2240**	0.1672	0.2058	0.1688
	0.5	1	0.5	0	0.4728	0.3304	**0.4904**	0.3554

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.27. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.2. Treatments=4, IBD = 18, CRD=6, RCBD=18, Peak=3, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0518	0.0486	0.0498	0.0512
	0.5	0.5	0	0	**0.6744**	0.4880	0.6638	0.5086
Normal	0	1	0.2	0.2	**0.9880**	0.9006	0.9854	0.9204
	1	1	0	0	**0.9898**	0.8992	0.9868	0.9246
	0	0.7	0.2	0	**0.3736**	0.2486	0.3062	0.2420
	0.5	1	0.5	0	0.7220	0.5274	**0.7448**	0.5710
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0502	0.0544	0.0534	0.0510
Exponential	0.5	0.5	0	0	0.9196	0.7496	**0.9218**	0.7868
	0	1	0.2	0.2	0.9992	0.9790	**0.9998**	0.9894
	1	1	0	0	**1.0000**	0.9906	0.9998	0.9942
	0	0.7	0.2	0	**0.4864**	0.3058	0.4032	0.2876
	0.5	1	0.5	0	0.9466	0.8212	**0.9612**	0.8638
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees	0	0	0	0	0.0542	0.0462	0.0490	0.0476
of freedom	0.5	0.5	0	0	0.5214	0.3726	**0.5174**	0.3888
	0	1	0.2	0.2	0.9228	0.7560	**0.9200**	0.8038
	1	1	0	0	**0.9268**	0.7622	0.9262	0.8026
	0	0.7	0.2	0	**0.2670**	0.2002	0.2310	0.1868
	0.5	1	0.5	0	0.5598	0.4046	**0.5808**	0.4356

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.28. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.2. Treatments=4, IBD = 6, CRD=12, RCBD=6, Peak=3**,** p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0502	0.0496	0.0512	0.0482
	0.5	0.5	0	0	**0.5068**	0.4158	0.4918	0.4034
	0	1	0.2	0.2	**0.9238**	0.8282	0.9172	0.8428
	1	1	0	0	0.9178	0.8336	**0.9210**	0.8398
	0	0.7	0.2	0	**0.2574**	0.2038	0.2364	0.2162
	0.5	1	0.5	0	0.5414	0.4416	**0.5534**	0.4624
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0526	0.0474	0.0498	0.0482
	0.5	0.5	0	0	**0.7784**	0.6584	0.7670	0.6604
	0	1	0.2	0.2	**0.9898**	0.9580	0.9878	0.9582
	1	1	0	0	**0.9960**	0.9744	0.9938	0.9758
	0	0.7	0.2	0	**0.3410**	0.2570	0.2982	0.2684
	0.5	1	0.5	0	**0.8466**	0.7242	0.8428	0.7360
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T with 3 degrees of freedom	0	0	0	0	0.0474	0.0476	0.0520	0.0556
	0.5	0.5	0	0	**0.3766**	0.3122	0.3724	0.3176
	0	1	0.2	0.2	**0.7864**	0.6732	0.7770	0.6870
	1	1	0	0	**0.7942**	0.6662	0.7832	0.6962
	0	0.7	0.2	0	**0.1928**	0.1746	0.1850	0.1718
	0.5	1	0.5	0	0.4116	0.3304	**0.4190**	0.3458

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.29. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.2. Treatments=4, IBD = 18, CRD=12, RCBD=18, Peak=3, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0516	0.0552	0.0470	0.0472
	0	0.5	0.5	0	**0.7488**	0.4916	0.7538	0.4952
	0	1	1	0.2	**0.9964**	0.9120	0.9954	0.9134
	0	0	1	0.2	0.9956	0.9054	**0.9966**	0.9142
	1	1	1	0	**0.4234**	0.2364	0.3708	0.2510
	0	0.2	0.7	0.5	0.7946	0.5108	**0.8168**	0.5466
	0	0.5	1	0.5	**0.9894**	0.8424	0.9892	0.8606
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0468	0.0506	0.0486	0.0506
	0	0.5	0.5	0	**0.9578**	0.7550	0.9576	0.7722
	0	1	1	0.2	**1.0000**	0.9828	0.9998	0.9880
	0	0	1	0.2	**1.0000**	0.9924	**1.0000**	0.9954
	1	1	1	0	**0.5458**	0.3116	0.4654	0.2986
	0	0.2	0.7	0.5	0.9834	0.8140	**0.9850**	0.8384
	0	0.5	1	0.5	**1.0000**	0.9832	**1.0000**	0.9886
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T Distribution with 3 degrees of freedom	0	0	0	0	0.0424	0.0456	0.0530	0.0384
	0	0.5	0.5	0	0.5848	0.3582	**0.5864**	0.3850
	0	1	1	0.2	**0.9664**	0.7622	0.9636	0.7760
	0	0	1	0.2	0.9584	0.7692	**0.9590**	0.7852
	1	1	1	0	**0.3096**	0.1980	0.2746	0.1948
	0	0.2	0.7	0.5	**0.6520**	0.3926	0.6506	0.4196
	0	0.5	1	0.5	0.9268	0.7018	**0.9318**	0.7112

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.30. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.2. Treatments=4, IBD = 6, CRD=18, RCBD=6, Peak=3, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0510	0.0494	0.0538	0.0494
	0	0.5	0.5	0	**0.5916**	0.5190	0.5652	0.5148
	0	1	1	0.2	**0.9648**	0.9246	0.9594	0.9270
	0	0	1	0.2	0.9616	0.9314	0.9602	0.9390
	1	1	1	0	**0.3028**	0.2564	0.2748	0.2596
	0	0.2	0.7	0.5	0.6220	0.5594	**0.6306**	0.5764
	0	0.5	1	0.5	0.9158	0.8810	**0.9286**	0.8820
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0454	0.0446	0.0466	0.0530
	0	0.5	0.5	0	0.8426	0.7842	**0.8458**	0.7936
	0	1	1	0.2	**0.9976**	0.9914	0.9964	0.9906
	0	0	1	0.2	0.9984	0.9970	**0.9990**	0.9954
	1	1	1	0	**0.4014**	0.3372	0.3474	0.3254
	0	0.2	0.7	0.5	0.9028	0.8504	**0.9054**	0.8646
	0	0.5	1	0.5	0.9964	0.9922	**0.9970**	0.9920
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T Distribution with 3 degrees of freedom	0	0	0	0	0.0492	0.0488	0.0536	0.0490
	0	0.5	0.5	0	**0.4436**	0.3900	0.4294	0.3888
	0	1	1	0.2	**0.8592**	0.8080	0.8572	0.8060
	0	0	1	0.2	0.8460	0.7976	**0.8614**	0.8090
	1	1	1	0	**0.2420**	0.2144	0.2196	0.2060
	0	0.2	0.7	0.5	0.4642	0.4348	**0.4872**	0.4314
	0	0.5	1	0.5	0.7858	0.7164	**0.7922**	0.7320

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.31. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.2. Treatments=4, IBD = 12, CRD=18, RCBD=12, Peak=3, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0486	0.0476	0.0514	0.0516
	0	0.5	0.5	0	**0.7076**	0.5308	0.7050	0.5462
	0	1	1	0.2	0.9924	0.9414	**0.9938**	0.9464
Normal	0	0	1	0.2	**0.9940**	0.9444	0.9926	0.9398
	1	1	1	0	**0.3940**	0.2718	0.3388	0.2636
	0	0.2	0.7	0.5	0.7688	0.5792	**0.7696**	0.5866
	0	0.5	1	0.5	0.9760	0.8984	**0.9842**	0.8994
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0476	0.0472	0.0456	0.0546
	0	0.5	0.5	0	0.9400	0.8130	**0.9456**	0.8080
	0	1	1	0.2	0.9988	0.9924	**0.9998**	0.9942
Exponential	0	0	1	0.2	**1.0000**	0.9986	**1.0000**	0.9982
	1	1	1	0	**0.5006**	0.3638	0.4592	0.3542
	0	0.2	0.7	0.5	0.9716	0.8672	**0.9762**	0.8694
	0	0.5	1	0.5	**1.0000**	0.9928	0.9994	0.9928
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
	0	0	0	0	0.0498	0.0486	0.0488	0.0522
	0	0.5	0.5	0	**0.5650**	0.4060	0.5484	0.4046
T Distribution with 3 degrees of freedom	0	1	1	0.2	**0.9440**	0.8124	**0.9440**	0.8314
	0	0	1	0.2	**0.9456**	0.8290	0.9454	0.8346
	1	1	1	0	**0.3046**	0.2222	0.2732	0.2216
	0	0.2	0.7	0.5	0.6004	0.4474	**0.6302**	0.4324
	0	0.5	1	0.5	0.9044	0.7576	**0.9134**	0.7532

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table D.32. Mixed Design for IBD, CRD and RCBD for Treatment four at peak 3 with Probability of 0.2. Treatments=4, IBD = 18, CRD=18, RCBD=18, Peak=3, p=0.2

Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Normal	0	0	0	0	0.0506	0.0498	0.0490	0.0518
	0	0.5	0.5	0	**0.8002**	0.5476	0.7892	0.5608
	0	1	1	0.2	**0.9972**	0.9516	**0.9972**	0.9586
	0	0	1	0.2	**0.9978**	0.9536	0.9974	0.9588
	1	1	1	0	**0.4590**	0.2924	0.4048	0.2952
	0	0.2	0.7	0.5	**0.8456**	0.6174	0.8558	0.6158
	0	0.5	1	0.5	**0.9944**	0.9072	0.9934	0.9150
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
Exponential	0	0	0	0	0.0464	0.0472	0.0470	0.0492
	0	0.5	0.5	0	0.9656	0.8296	**0.9756**	0.8372
	0	1	1	0.2	**1.0000**	0.9944	**1.0000**	0.9960
	0	0	1	0.2	**1.0000**	0.9982	**1.0000**	0.9994
	1	1	1	0	**0.5964**	0.3648	0.5326	0.3592
	0	0.2	0.7	0.5	0.9908	0.8820	**0.9938**	0.8956
	0	0.5	1	0.5	**1.0000**	0.9950	**1.0000**	0.9968
Distribution	Location				Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	First	Last	First	Last
T Distribution with 3 degrees of freedom	0	0	0	0	0.0526	0.0526	0.0516	0.0548
	0	0.5	0.5	0	0.6564	0.4272	**0.6504**	0.4348
	0	1	1	0.2	**0.9834**	0.8374	0.9808	0.8592
	0	0	1	0.2	0.9824	0.8434	**0.9830**	0.8554
	1	1	1	0	**0.3518**	0.2372	0.3062	0.2222
	0	0.2	0.7	0.5	0.6942	0.4592	**0.7072**	0.4774
	0	0.5	1	0.5	0.9550	0.7754	**0.9622**	0.7920

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

APPENDIX E. MIXED DESIGN OF IBD, CRD AND RCBD WITH PROBABILITIES OF 0.1 AND 0.4 FOR TREATMENTS 5 AT PEAKS 2 AND 3

Table E.1. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.1
Treatments=5, IBD =5, CRD=10, RCBD=5 Peak=2, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0532	0.0484	0.0458	0.0470
	1	1	0.5	0.5	0.5	0.2846	0.2238	**0.2974**	0.2572
	0.5	1	1	1	0.7	**0.2640**	0.2112	0.2618	0.2248
	0.7	1	0.7	0.7	0.5	0.3646	0.3040	**0.3794**	0.3180
	1	1	0.5	0.5	0.2	0.5440	0.4382	**0.5740**	0.4580
	0.75	1	0.75	0.5	0.25	0.6426	0.5218	**0.6642**	0.5446
	1	1	1	1	0	0.6694	0.5540	**0.7030**	0.5720
	1	1	0.5	0.2	0	0.7754	0.6510	**0.8016**	0.6718
	0	1.6	0.8	0.4	0.2	**0.9988**	0.9852	0.9968	0.9876

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0538	0.0468	0.0516	0.0478
	1	1	0.5	0.5	0.5	0.4698	0.3556	**0.4936**	0.3660
	0.5	1	1	1	0.7	**0.4604**	0.3696	0.4528	0.3772
	0.7	1	0.7	0.7	0.5	**0.6662**	0.5318	0.6586	0.5496
	1	1	0.5	0.5	0.2	0.8140	0.6886	**0.8326**	0.7134
	0.75	1	0.75	0.5	0.25	0.9154	0.8168	**0.9216**	0.8420
	1	1	1	1	0	0.8152	0.6958	**0.8490**	0.7338
	1	1	0.5	0.2	0	0.9592	0.8946	**0.9676**	0.9116
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9998	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0486	0.0534	0.0566	0.0496
	1	1	0.5	0.5	0.5	0.2276	0.1868	**0.2410**	0.1870
	0.5	1	1	1	0.7	**0.2162**	0.1800	0.1952	0.1822
	0.7	1	0.7	0.7	0.5	0.2804	0.2294	**0.2892**	0.2318
	1	1	0.5	0.5	0.2	0.4072	0.3294	**0.4306**	0.3344
	0.75	1	0.75	0.5	0.25	0.4836	0.4070	**0.5100**	0.4142
	1	1	1	1	0	0.5170	0.4090	**0.5432**	0.4452
	1	1	0.5	0.2	0	0.6306	0.5076	**0.6348**	0.5244
	0	1.6	0.8	0.4	0.2	0.9684	0.9088	**0.9686**	0.9200

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.2. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.1
Treatments=5, IBD =5, CRD=15, RCBD=5 Peak=2, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0442	0.0518	0.0560	0.0518
	1	1	0.5	0.5	0.5	0.3336	0.2848	**0.3354**	0.2846
	0.5	1	1	1	0.7	**0.3072**	0.2574	0.2922	0.2718
	0.7	1	0.7	0.7	0.5	0.4250	0.3694	**0.4406**	0.3820
Normal	1	1	0.5	0.5	0.2	0.6124	0.5480	**0.6402**	0.5512
	0.75	1	0.75	0.5	0.25	0.7142	0.6440	**0.7450**	0.6586
	1	1	1	1	0	0.7648	0.6786	**0.7898**	0.6930
	1	1	0.5	0.2	0	0.8448	0.7780	**0.8580**	0.7882
	0	1.6	0.8	0.4	0.2	**0.9998**	0.9980	0.9992	0.9974

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0468	0.0532	0.0526	0.0500
	1	1	0.5	0.5	0.5	0.5294	0.4496	**0.5416**	0.4688
	0.5	1	1	1	0.7	**0.5386**	0.4560	0.5198	0.4636
Exponential	0.7	1	0.7	0.7	0.5	0.7338	0.6596	**0.7414**	0.6788
	1	1	0.5	0.5	0.2	0.8806	0.8016	**0.8942**	0.8276
	0.75	1	0.75	0.5	0.25	0.9536	0.9164	**0.9628**	0.9206
	1	1	1	1	0	0.8804	0.8074	**0.8936**	0.8252
	1	1	0.5	0.2	0	0.9842	0.9556	**0.9864**	0.9630
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0494	0.0486	0.0508	0.0524
	1	1	0.5	0.5	0.5	0.2644	0.2318	**0.2674**	0.2348
T distribution	0.5	1	1	1	0.7	**0.2430**	0.2044	0.2278	0.2094
with 3 degrees	0.7	1	0.7	0.7	0.5	0.3202	0.2732	**0.3218**	0.2924
of freedom	1	1	0.5	0.5	0.2	0.4712	0.3972	**0.4932**	0.4128
	0.75	1	0.75	0.5	0.25	0.5564	0.4986	**0.5738**	0.5026
	1	1	1	1	0	0.6052	0.5170	**0.6096**	0.5228
	1	1	0.5	0.2	0	0.6956	0.6140	**0.7230**	0.6220
	0	1.6	0.8	0.4	0.2	**0.9886**	0.9700	0.9872	0.9728

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.3. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.1
Treatments=5, IBD =10, CRD=5, RCBD=10 Peak=2, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0482	0.0556	0.0432	0.0498
	1	1	0.5	0.5	0.5	0.3332	0.2444	**0.3500**	0.2942
	0.5	1	1	1	0.7	**0.3082**	0.2294	0.2758	0.2414
	0.7	1	0.7	0.7	0.5	**0.4340**	0.3146	0.4220	0.3452
Normal	1	1	0.5	0.5	0.2	0.6056	0.4630	**0.6356**	0.5294
	0.75	1	0.75	0.5	0.25	0.7264	0.5624	**0.7308**	0.6290
	1	1	1	1	0	0.7352	0.5820	**0.7680**	0.6620
	1	1	0.5	0.2	0	0.8412	0.6838	**0.8694**	0.7712
	0	1.6	0.8	0.4	0.2	**0.9994**	0.9886	0.9986	0.9942

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0488	0.0524	0.0504	0.0488
	1	1	0.5	0.5	0.5	0.5314	0.3756	**0.5574**	0.4492
	0.5	1	1	1	0.7	**0.5102**	0.3868	0.4920	0.4244
Exponential	0.7	1	0.7	0.7	0.5	**0.7316**	0.5528	0.7294	0.6222
	1	1	0.5	0.5	0.2	0.8708	0.7088	**0.8906**	0.8036
	0.75	1	0.75	0.5	0.25	0.9556	0.8248	**0.9568**	0.8958
	1	1	1	1	0	0.8742	0.7196	**0.9024**	0.8094
	1	1	0.5	0.2	0	0.9810	0.8994	**0.9860**	0.9490
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9998	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0492	0.0506	0.0516	0.0472
	1	1	0.5	0.5	0.5	0.2430	0.1866	**0.2650**	0.2260
T distribution	0.5	1	1	1	0.7	**0.2302**	0.1730	0.2206	0.1880
with 3 degrees	0.7	1	0.7	0.7	0.5	0.3132	0.2434	**0.3264**	0.2750
of freedom	1	1	0.5	0.5	0.2	0.4622	0.3528	**0.4934**	0.3942
	0.75	1	0.75	0.5	0.25	0.5554	0.4254	**0.5682**	0.4776
	1	1	1	1	0	0.5852	0.4324	**0.6202**	0.5100
	1	1	0.5	0.2	0	0.6860	0.5234	**0.6994**	0.6094
	0	1.6	0.8	0.4	0.2	**0.9888**	0.9358	0.9856	0.0472

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.4. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.1
Treatments=5, IBD =10, CRD=10, RCBD=10 Peak=2, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0440	0.0572	0.0526	0.9510
	1	1	0.5	0.5	0.5	0.3724	0.2456	**0.3960**	0.0524
	0.5	1	1	1	0.7	**0.3418**	0.2346	0.3404	0.2812
	0.7	1	0.7	0.7	0.5	**0.5000**	0.3332	0.4912	0.2552
	1	1	0.5	0.5	0.2	0.6980	0.4866	**0.7272**	0.3494
	0.75	1	0.75	0.5	0.25	0.7968	0.5990	**0.8122**	0.5114
	1	1	1	1	0	0.8308	0.6132	**0.8534**	0.6328
	1	1	0.5	0.2	0	0.9072	0.7048	**0.9254**	0.6540
	0	1.6	0.8	0.4	0.2	0.9996	0.9928	**1.0000**	0.7544

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0430	0.0466	0.0506	0.0424
	1	1	0.5	0.5	0.5	0.6272	0.3944	**0.6518**	0.4502
	0.5	1	1	1	0.7	**0.6072**	0.4032	0.5832	0.4358
	0.7	1	0.7	0.7	0.5	**0.8188**	0.5978	0.8164	0.6252
	1	1	0.5	0.5	0.2	0.9320	0.7562	**0.9424**	0.7932
	0.75	1	0.75	0.5	0.25	0.9818	0.8676	**0.9820**	0.9008
	1	1	1	1	0	0.9336	0.7524	**0.9532**	0.8014
	1	1	0.5	0.2	0	**0.9958**	0.9148	0.9952	0.9514
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0540	0.0460	0.0540	0.0480
	1	1	0.5	0.5	0.5	0.2918	0.2004	**0.3076**	0.2114
	0.5	1	1	1	0.7	0.2662	0.1942	**0.2688**	0.1956
	0.7	1	0.7	0.7	0.5	0.3778	0.2526	**0.3800**	0.2744
	1	1	0.5	0.5	0.2	0.5430	0.3682	**0.5748**	0.4040
	0.75	1	0.75	0.5	0.25	0.6494	0.4498	**0.6688**	0.4770
	1	1	1	1	0	0.6752	0.4676	**0.7080**	0.5088
	1	1	0.5	0.2	0	0.7606	0.5390	**0.7974**	0.5926
	0	1.6	0.8	0.4	0.2	**0.9958**	0.9404	0.9956	0.9564

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.5. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.1
Treatments=5, IBD =10, CRD=15, RCBD=10 Peak=2, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0490	0.0506	0.0498	0.0508
	1	1	0.5	0.5	0.5	0.4192	0.2890	**0.4504**	0.3166
	0.5	1	1	1	0.7	**0.3924**	0.2736	0.3778	0.2824
	0.7	1	0.7	0.7	0.5	0.5516	0.3890	**0.5622**	0.3976
Normal	1	1	0.5	0.5	0.2	0.7552	0.5696	**0.7766**	0.5920
	0.75	1	0.75	0.5	0.25	0.8532	0.6716	**0.8638**	0.6968
	1	1	1	1	0	0.8906	0.6992	**0.8990**	0.7332
	1	1	0.5	0.2	0	0.9412	0.8022	**0.9536**	0.8250
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9986	**1.0000**	0.9982

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0494	0.0474	0.0474	0.0508
	1	1	0.5	0.5	0.5	0.6842	0.4894	**0.7194**	0.5134
	0.5	1	1	1	0.7	**0.6650**	0.4756	0.6614	0.4922
Exponential	0.7	1	0.7	0.7	0.5	**0.8734**	0.6798	0.8572	0.7088
	1	1	0.5	0.5	0.2	0.9594	0.8392	**0.9664**	0.8554
	0.75	1	0.75	0.5	0.25	0.9904	0.9342	**0.9926**	0.9466
	1	1	1	1	0	0.9672	0.8300	**0.9722**	0.8618
	1	1	0.5	0.2	0	0.9980	0.9726	**0.9984**	0.9778
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0478	0.0474	0.0512	0.0500
	1	1	0.5	0.5	0.5	0.3152	0.2428	**0.3382**	0.2422
T distribution	0.5	1	1	1	0.7	**0.3010**	0.2142	0.2906	0.2244
with 3	0.7	1	0.7	0.7	0.5	0.4136	0.2982	**0.4194**	0.3050
degrees of	1	1	0.5	0.5	0.2	0.5964	0.4276	**0.6322**	0.4636
freedom	0.75	1	0.75	0.5	0.25	0.6978	0.5196	**0.7188**	0.5328
	1	1	1	1	0	0.7392	0.5388	**0.7500**	0.5736
	1	1	0.5	0.2	0	0.8248	0.6494	**0.8546**	0.6714
	0	1.6	0.8	0.4	0.2	**0.9988**	0.9782	**0.9988**	0.9820

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.6. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.1
Treatments=5, IBD =15, CRD=5, RCBD=15 Peak=2, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0544	0.0504	0.0500	0.0546
	1	1	0.5	0.5	0.5	0.3970	0.2890	**0.4288**	0.3618
	0.5	1	1	1	0.7	**0.3684**	0.2836	0.3550	0.3024
	0.7	1	0.7	0.7	0.5	0.5076	0.3838	**0.5190**	0.4366
	1	1	0.5	0.5	0.2	0.7288	0.5522	**0.7424**	0.6488
	0.75	1	0.75	0.5	0.25	0.8198	0.6810	**0.8394**	0.7474
	1	1	1	1	0	0.8508	0.6860	**0.8720**	0.7944
	1	1	0.5	0.2	0	0.9272	0.7968	**0.9418**	0.8780
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9988	**1.0000**	0.9992

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0512	0.0518	0.0488	0.0420
	1	1	0.5	0.5	0.5	0.6588	0.4806	**0.6868**	0.5902
	0.5	1	1	1	0.7	**0.6334**	0.4692	0.6070	0.5176
	0.7	1	0.7	0.7	0.5	0.8378	0.6722	**0.8470**	0.7540
	1	1	0.5	0.5	0.2	0.9480	0.8356	**0.9538**	0.9026
	0.75	1	0.75	0.5	0.25	0.9864	0.9258	**0.9892**	0.9570
	1	1	1	1	0	0.9460	0.8306	**0.9636**	0.9118
	1	1	0.5	0.2	0	0.9956	0.9670	**0.9980**	0.9898
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0490	0.0456	0.0512	0.0508
	1	1	0.5	0.5	0.5	0.3038	0.2168	**0.3166**	0.2796
	0.5	1	1	1	0.7	**0.2886**	0.2060	0.2548	0.2318
	0.7	1	0.7	0.7	0.5	0.3818	0.2996	**0.4062**	0.3394
	1	1	0.5	0.5	0.2	0.5612	0.4312	**0.5944**	0.5172
	0.75	1	0.75	0.5	0.25	0.6846	0.5110	**0.6920**	0.5910
	1	1	1	1	0	0.7012	0.5326	**0.7348**	0.6348
	1	1	0.5	0.2	0	0.7950	0.6334	**0.8268**	0.7260
	0	1.6	0.8	0.4	0.2	**0.9972**	0.9678	0.9962	0.9856

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.7. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.1
Treatments=5, IBD =15, CRD=10, RCBD=15 Peak=2, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0514	0.0530	0.0486	0.0512
	1	1	0.5	0.5	0.5	0.4560	0.2756	**0.4818**	0.3108
	0.5	1	1	1	0.7	**0.4384**	0.2636	0.4002	0.2814
	0.7	1	0.7	0.7	0.5	0.5896	0.3626	**0.6008**	0.4010
Normal	1	1	0.5	0.5	0.2	0.8004	0.5414	**0.8230**	0.5974
	0.75	1	0.75	0.5	0.25	0.8880	0.6364	**0.9028**	0.6958
	1	1	1	1	0	0.9040	0.6518	**0.9288**	0.7334
	1	1	0.5	0.2	0	0.9610	0.7614	**0.9668**	0.8208
	0	1.6	0.8	0.4	0.2	1.0000	0.9980	1.0000	0.9990

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0486	0.0530	0.0456	0.0464
	1	1	0.5	0.5	0.5	0.7374	0.4462	**0.7494**	0.5076
	0.5	1	1	1	0.7	**0.7096**	0.4558	0.6856	0.4892
Exponential	0.7	1	0.7	0.7	0.5	0.9102	0.6372	**0.9016**	0.7054
	1	1	0.5	0.5	0.2	0.9768	0.7924	**0.9798**	0.8542
	0.75	1	0.75	0.5	0.25	0.9964	0.9070	0.9960	0.9340
	1	1	1	1	0	0.9758	0.8002	**0.9854**	0.8628
	1	1	0.5	0.2	0	0.9992	0.9528	**0.9994**	0.9764
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0504	0.0558	0.0526	0.0482
	1	1	0.5	0.5	0.5	0.3430	0.2126	**0.3598**	0.2510
T distribution with 3 degrees of freedom	0.5	1	1	1	0.7	**0.3150**	0.1868	0.3078	0.2206
	0.7	1	0.7	0.7	0.5	**0.4470**	0.2758	0.4544	0.3066
	1	1	0.5	0.5	0.2	0.6556	0.3982	**0.6676**	0.4604
	0.75	1	0.75	0.5	0.25	0.7482	0.4812	**0.7590**	0.5414
	1	1	1	1	0	0.7808	0.4992	**0.8008**	0.5868
	1	1	0.5	0.2	0	0.8596	0.6084	**0.8872**	0.6634
	0	1.6	0.8	0.4	0.2	**0.9992**	0.9642	0.9998	0.9774

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.8. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.1
Treatments=5, IBD =15, CRD=15, RCBD=15 Peak=2, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0466	0.0550	0.0524	0.0462
	1	1	0.5	0.5	0.5	0.4940	0.3122	**0.5182**	0.3442
	0.5	1	1	1	0.7	**0.4624**	0.2968	0.4422	0.3040
	0.7	1	0.7	0.7	0.5	**0.6532**	0.4066	0.6376	0.4216
	1	1	0.5	0.5	0.2	0.8396	0.5976	**0.8600**	0.6312
	0.75	1	0.75	0.5	0.25	0.9164	0.7026	**0.9384**	0.7344
	1	1	1	1	0	0.9444	0.7320	**0.9524**	0.7622
	1	1	0.5	0.2	0	0.9764	0.8214	**0.9828**	0.8618
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9994	**1.0000**	0.9998

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0454	0.0488	0.0478	0.0484
	1	1	0.5	0.5	0.5	0.7918	0.5136	**0.8102**	0.5472
	0.5	1	1	1	0.7	**0.7630**	0.4946	**0.7396**	0.5314
	0.7	1	0.7	0.7	0.5	**0.9360**	0.7168	0.9348	0.7502
	1	1	0.5	0.5	0.2	0.9876	0.8596	**0.9920**	0.8898
	0.75	1	0.75	0.5	0.25	0.9980	0.9464	**0.9992**	0.9614
	1	1	1	1	0	0.9874	0.8652	**0.9900**	0.8898
	1	1	0.5	0.2	0	**1.0000**	0.9790	**1.0000**	0.9852
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0520	0.0502	0.0474	0.0484
	1	1	0.5	0.5	0.5	0.3880	0.2394	**0.4048**	0.2494
	0.5	1	1	1	0.7	**0.3524**	0.2302	0.3462	0.2398
	0.7	1	0.7	0.7	0.5	**0.5088**	0.3054	**0.5020**	0.3390
	1	1	0.5	0.5	0.2	0.7036	0.4468	**0.7200**	0.4720
	0.75	1	0.75	0.5	0.25	0.8044	0.5412	**0.8082**	0.5872
	1	1	1	1	0	0.8258	0.5654	**0.8502**	0.6052
	1	1	0.5	0.2	0	0.9016	0.6610	**0.9198**	0.7050
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9808	**1.0000**	0.9882

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.9. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.4
Treatments=5, IBD =5, CRD=10, RCBD=5 Peak=2, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0500	0.0518	0.0562	0.0460
	1	1	0.5	0.5	0.5	0.2600	0.2288	**0.2754**	0.2392
	0.5	1	1	1	0.7	**0.2548**	0.2126	**0.2430**	0.2274
	0.7	1	0.7	0.7	0.5	0.3346	0.2888	**0.3560**	0.3084
Normal	1	1	0.5	0.5	0.2	0.4976	0.4214	**0.5162**	0.4480
	0.75	1	0.75	0.5	0.25	0.6126	0.5168	**0.6124**	0.5434
	1	1	1	1	0	0.6274	0.5278	**0.6556**	0.5524
	1	1	0.5	0.2	0	0.7334	0.6370	**0.7516**	0.6668
	0	1.6	0.8	0.4	0.2	**0.9964**	0.9818	0.9914	0.9854

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0468	0.0424	0.0502	0.0494
	1	1	0.5	0.5	0.5	0.4418	0.3428	**0.4514**	0.3548
	0.5	1	1	1	0.7	**0.4126**	0.3702	0.4086	0.3742
Exponential	0.7	1	0.7	0.7	0.5	**0.6164**	0.5152	0.6008	0.5274
	1	1	0.5	0.5	0.2	0.7696	0.6766	**0.7764**	0.6970
	0.75	1	0.75	0.5	0.25	0.8774	0.7996	**0.8932**	0.8238
	1	1	1	1	0	0.7654	0.6902	**0.7968**	0.7142
	1	1	0.5	0.2	0	0.9378	0.8812	**0.9440**	0.8934
	0	1.6	0.8	0.4	0.2	0.9996	0.9994	**0.9998**	0.9998

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0508	0.0522	0.0468	0.0512
	1	1	0.5	0.5	0.5	0.2016	0.1764	**0.2076**	0.1986
T distribution with 3 degrees of freedom	0.5	1	1	1	0.7	**0.1982**	0.1750	0.1878	0.1706
	0.7	1	0.7	0.7	0.5	**0.2738**	0.2282	0.2554	0.2412
	1	1	0.5	0.5	0.2	0.3764	0.3158	**0.3920**	0.3496
	0.75	1	0.75	0.5	0.25	0.4444	0.3938	**0.4676**	0.4030
	1	1	1	1	0	0.4746	0.4142	**0.5074**	0.4306
	1	1	0.5	0.2	0	0.5810	0.4976	**0.6022**	0.5036
	0	1.6	0.8	0.4	0.2	**0.9572**	0.9048	0.9474	0.9096

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.10. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.4
Treatments=5, IBD =5, CRD=15, RCBD=5 Peak=2, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0474	0.0496	0.0512	0.0494
	1	1	0.5	0.5	0.5	0.3130	0.2866	**0.3158**	0.2904
	0.5	1	1	1	0.7	**0.2870**	0.2714	0.2778	0.2688
	0.7	1	0.7	0.7	0.5	0.4030	0.3642	**0.4112**	0.3768
	1	1	0.5	0.5	0.2	0.5684	0.5332	**0.5936**	0.5556
	0.75	1	0.75	0.5	0.25	0.6856	0.6322	**0.6862**	0.6628
	1	1	1	1	0	0.7032	0.6778	**0.7328**	0.6726
	1	1	0.5	0.2	0	**0.8068**	0.7622	**0.8068**	0.7714
	0	1.6	0.8	0.4	0.2	0.9980	0.9964	0.9988	0.9982

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0506	0.0488	0.0492	0.0504
	1	1	0.5	0.5	0.5	0.4924	0.4504	**0.5200**	0.4558
	0.5	1	1	1	0.7	**0.4920**	0.4430	0.4768	0.4652
	0.7	1	0.7	0.7	0.5	0.6960	0.6524	**0.6896**	0.6692
	1	1	0.5	0.5	0.2	0.8404	0.7974	**0.8450**	0.8270
	0.75	1	0.75	0.5	0.25	0.9322	0.9130	**0.9358**	0.9154
	1	1	1	1	0	0.8402	0.8162	**0.8754**	0.8334
	1	1	0.5	0.2	0	0.9674	0.9586	**0.9702**	0.9612
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0496	0.0486	0.0566	0.0484
	1	1	0.5	0.5	0.5	0.2288	0.2188	**0.2394**	0.2252
	0.5	1	1	1	0.7	**0.2254**	0.2006	0.2154	0.2080
	0.7	1	0.7	0.7	0.5	0.3100	0.2722	**0.3150**	0.2946
	1	1	0.5	0.5	0.2	0.4498	0.4066	**0.4560**	0.4098
	0.75	1	0.75	0.5	0.25	0.5212	0.4858	**0.5402**	0.5066
	1	1	1	1	0	0.5526	0.5058	**0.5768**	0.5316
	1	1	0.5	0.2	0	0.6484	0.6106	**0.6722**	0.6226
	0	1.6	0.8	0.4	0.2	**0.9780**	0.9694	0.9758	0.9716

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.11. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.4. Treatments=5, IBD =10, CRD=5, RCBD=10 Peak=2, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0526	0.0498	0.0496	0.0482
	1	1	0.5	0.5	0.5	0.2836	0.2250	**0.3176**	0.2580
	0.5	1	1	1	0.7	**0.2708**	0.2058	0.2654	0.2382
	0.7	1	0.7	0.7	0.5	0.3726	0.2776	**0.3834**	0.3194
	1	1	0.5	0.5	0.2	0.5478	0.4052	**0.5750**	0.4960
	0.75	1	0.75	0.5	0.25	0.6616	0.4974	**0.6780**	0.5720
	1	1	1	1	0	0.6802	0.5094	**0.7274**	0.6114
	1	1	0.5	0.2	0	0.7764	0.6260	**0.8122**	0.7322
	0	1.6	0.8	0.4	0.2	0.9982	0.9790	0.9970	0.9894
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0500	0.0500	0.0520	0.0542
	1	1	0.5	0.5	0.5	0.4584	0.3408	**0.5086**	0.4106
	0.5	1	1	1	0.7	**0.4620**	0.3564	0.4540	0.3832
	0.7	1	0.7	0.7	0.5	**0.6730**	0.5142	**0.6696**	0.5766
	1	1	0.5	0.5	0.2	0.8112	0.6642	**0.8364**	0.7462
	0.75	1	0.75	0.5	0.25	0.9192	0.7884	**0.9268**	0.8648
	1	1	1	1	0	0.8298	0.6664	**0.8588**	0.7736
	1	1	0.5	0.2	0	0.9560	0.8626	**0.9666**	0.9276
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9990	**1.0000**	0.9998
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0468	0.0478	0.0468	0.0542
	1	1	0.5	0.5	0.5	0.2142	0.1786	**0.2514**	0.2106
	0.5	1	1	1	0.7	**0.2168**	0.1668	0.1932	0.1784
	0.7	1	0.7	0.7	0.5	0.2772	0.2182	**0.3012**	0.2592
	1	1	0.5	0.5	0.2	0.4188	0.3174	**0.4436**	0.3660
	0.75	1	0.75	0.5	0.25	0.5078	0.3938	**0.5182**	0.4468
	1	1	1	1	0	0.5214	0.3910	**0.5670**	0.4668
	1	1	0.5	0.2	0	0.6152	0.4714	**0.6582**	0.5576
	0	1.6	0.8	0.4	0.2	**0.9712**	0.8910	0.9692	0.9334

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.12. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.4
Treatments=5, IBD =10, CRD=10, RCBD=10 Peak=2, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0526	0.0550	0.0468	0.0490
	1	1	0.5	0.5	0.5	0.3412	0.2410	**0.3642**	0.2684
	0.5	1	1	1	0.7	**0.3190**	0.2324	0.3012	0.2340
	0.7	1	0.7	0.7	0.5	**0.4574**	0.3032	0.4540	0.3438
	1	1	0.5	0.5	0.2	0.6418	0.4514	**0.6666**	0.4920
	0.75	1	0.75	0.5	0.25	0.7504	0.5678	**0.7746**	0.6068
	1	1	1	1	0	0.7730	0.5756	**0.7972**	0.6258
	1	1	0.5	0.2	0	0.8614	0.6922	**0.8874**	0.7368
	0	1.6	0.8	0.4	0.2	0.9998	0.9920	0.9998	0.9944

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0492	0.0488	0.0498	0.0496
	1	1	0.5	0.5	0.5	0.5494	0.3880	**0.5974**	0.4292
	0.5	1	1	1	0.7	**0.5440**	0.3936	0.5280	0.4092
	0.7	1	0.7	0.7	0.5	**0.7666**	0.5650	**0.7550**	0.6178
	1	1	0.5	0.5	0.2	0.8934	0.7084	**0.9126**	0.7774
	0.75	1	0.75	0.5	0.25	**0.9682**	0.8526	**0.9682**	0.8774
	1	1	1	1	0	0.9048	0.7308	**0.9230**	0.7792
	1	1	0.5	0.2	0	0.9870	0.9112	**0.9908**	0.9438
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9998	**1.0000**	0.9998

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0506	0.0498	0.0502	0.0516
	1	1	0.5	0.5	0.5	0.2548	0.1980	**0.2668**	0.2126
	0.5	1	1	1	0.7	**0.2378**	0.1904	0.2294	0.1888
	0.7	1	0.7	0.7	0.5	**0.3422**	0.2416	0.3398	0.2646
	1	1	0.5	0.5	0.2	0.4816	0.3562	**0.5216**	0.3824
	0.75	1	0.75	0.5	0.25	0.5998	0.4334	**0.6332**	0.4752
	1	1	1	1	0	0.6136	0.4504	**0.6530**	0.4728
	1	1	0.5	0.2	0	0.7232	0.5430	**0.7470**	0.5710
	0	1.6	0.8	0.4	0.2	**0.9916**	0.9282	0.9906	0.9428

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.13. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.4
Treatments=5, IBD =10, CRD=15, RCBD=10 Peak=2, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0512	0.0508	0.0538	0.0464
	1	1	0.5	0.5	0.5	0.3868	0.2966	**0.4178**	0.3142
	0.5	1	1	1	0.7	**0.3492**	0.2780	0.3420	0.2780
	0.7	1	0.7	0.7	0.5	**0.5148**	0.3742	0.5138	0.4000
	1	1	0.5	0.5	0.2	0.7108	0.5612	**0.7304**	0.5894
	0.75	1	0.75	0.5	0.25	0.8046	0.6506	**0.8276**	0.6864
	1	1	1	1	0	0.8322	0.6868	**0.8726**	0.7058
	1	1	0.5	0.2	0	0.9128	0.7920	**0.9308**	0.8162
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9976	**1.0000**	0.9992

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0486	0.0506	0.0498	0.0530
	1	1	0.5	0.5	0.5	0.6150	0.4858	**0.6616**	0.4986
	0.5	1	1	1	0.7	**0.6270**	0.4874	0.6076	0.5044
	0.7	1	0.7	0.7	0.5	**0.8274**	0.6786	**0.8210**	0.6986
	1	1	0.5	0.5	0.2	0.9402	0.8278	**0.9446**	0.8554
	0.75	1	0.75	0.5	0.25	**0.9840**	0.9252	0.9828	0.9382
	1	1	1	1	0	0.9448	0.8242	**0.9556**	0.8528
	1	1	0.5	0.2	0	0.9946	0.9688	**0.9968**	0.9732
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0496	0.0492	0.0530	0.0470
	1	1	0.5	0.5	0.5	0.2878	0.2228	**0.3014**	0.2418
	0.5	1	1	1	0.7	**0.2654**	0.2058	0.2642	0.2158
	0.7	1	0.7	0.7	0.5	0.3790	0.2928	**0.3868**	0.3182
	1	1	0.5	0.5	0.2	0.5548	0.4172	**0.5658**	0.4386
	0.75	1	0.75	0.5	0.25	0.6446	0.4954	**0.6562**	0.5258
	1	1	1	1	0	0.6808	0.5328	**0.7122**	0.5614
	1	1	0.5	0.2	0	0.7786	0.6200	**0.8034**	0.6452
	0	1.6	0.8	0.4	0.2	0.9964	0.9756	**0.9968**	0.9812

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.14. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.4
Treatments=5, IBD =15, CRD=5, RCBD=15 Peak=2, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0518	0.0518	0.0500	0.0474
	1	1	0.5	0.5	0.5	0.3514	0.2654	**0.3732**	0.3184
	0.5	1	1	1	0.7	**0.3168**	0.2428	0.3096	0.2784
	0.7	1	0.7	0.7	0.5	0.4544	0.3460	**0.4698**	0.4096
	1	1	0.5	0.5	0.2	0.6620	0.5048	**0.6838**	0.6062
	0.75	1	0.75	0.5	0.25	0.7578	0.5914	**0.7824**	0.6914
	1	1	1	1	0	0.7926	0.6226	**0.8272**	0.7564
	1	1	0.5	0.2	0	0.8820	0.7470	**0.9026**	0.8364
	0	1.6	0.8	0.4	0.2	**1.0000**	0.9942	0.9996	0.9988

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0530	0.0514	0.0414	0.0462
	1	1	0.5	0.5	0.5	0.5900	0.4156	**0.6120**	0.5274
	0.5	1	1	1	0.7	**0.5602**	0.4304	0.5306	0.4572
	0.7	1	0.7	0.7	0.5	0.7798	0.6058	**0.7694**	0.6970
	1	1	0.5	0.5	0.2	0.9084	0.7634	**0.9220**	0.8702
	0.75	1	0.75	0.5	0.25	0.9708	0.8858	**0.9710**	0.9352
	1	1	1	1	0	0.9070	0.7894	**0.9366**	0.8762
	1	1	0.5	0.2	0	0.9886	0.9346	**0.9918**	0.9762
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0464	0.0452	0.0486	0.0544
	1	1	0.5	0.5	0.5	0.2700	0.2194	**0.2902**	0.2644
	0.5	1	1	1	0.7	**0.2526**	0.1956	0.2484	0.2044
	0.7	1	0.7	0.7	0.5	0.3546	0.2676	**0.3448**	0.3126
	1	1	0.5	0.5	0.2	0.5026	0.3930	**0.5380**	0.4558
	0.75	1	0.75	0.5	0.25	0.6170	0.4576	**0.6302**	0.5496
	1	1	1	1	0	0.6078	0.4728	**0.6762**	0.5818
	1	1	0.5	0.2	0	0.7330	0.5766	**0.7684**	0.6916
	0	1.6	0.8	0.4	0.2	0.9922	0.9506	**0.9928**	0.9742

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.15. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.4
Treatments=5, IBD =15, CRD=10, RCBD=15 Peak=2, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0500	0.0472	0.0484	0.0550
	1	1	0.5	0.5	0.5	0.4012	0.2700	**0.4452**	0.2988
	0.5	1	1	1	0.7	**0.3722**	0.2488	0.3716	0.2720
	0.7	1	0.7	0.7	0.5	0.5324	0.3456	**0.5372**	0.3824
Normal	1	1	0.5	0.5	0.2	0.7430	0.5072	**0.7668**	0.5798
	0.75	1	0.75	0.5	0.25	0.8456	0.6060	**0.8668**	0.6726
	1	1	1	1	0	0.8658	0.6236	**0.8950**	0.6990
	1	1	0.5	0.2	0	0.9344	0.7394	**0.9504**	0.7996
	0	1.6	0.8	0.4	0.2	1.0000	0.9944	1.0000	0.9984

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0430	0.0508	0.0510	0.0524
	1	1	0.5	0.5	0.5	0.6620	0.4176	**0.7016**	0.4936
	0.5	1	1	1	0.7	**0.6426**	0.4296	0.6252	0.4612
Exponential	0.7	1	0.7	0.7	0.5	**0.8516**	0.6146	0.8512	0.6724
	1	1	0.5	0.5	0.2	0.9524	0.7782	**0.9658**	0.8402
	0.75	1	0.75	0.5	0.25	0.9894	0.8874	**0.9914**	0.9176
	1	1	1	1	0	0.9558	0.7852	**0.9666**	0.8384
	1	1	0.5	0.2	0	**0.9978**	0.9402	0.9966	0.9712
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	**1.0000**	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0454	0.0542	0.0464	0.0508
	1	1	0.5	0.5	0.5	0.3124	0.2036	**0.3256**	0.2376
T distribution with 3 degrees of freedom	0.5	1	1	1	0.7	**0.2868**	0.1958	0.2628	0.2036
	0.7	1	0.7	0.7	0.5	0.4044	0.2652	**0.4196**	0.2746
	1	1	0.5	0.5	0.2	0.5802	0.3772	**0.6124**	0.4352
	0.75	1	0.75	0.5	0.25	0.6722	0.4638	**0.7020**	0.5176
	1	1	1	1	0	0.7046	0.4788	**0.7584**	0.5558
	1	1	0.5	0.2	0	0.8158	0.5612	**0.8406**	0.6442
	0	1.6	0.8	0.4	0.2	**0.9988**	0.9550	0.9974	0.9742

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.16. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 2 with probability of 0.4
Treatments=5, IBD =15, CRD=15, RCBD=15 Peak=2, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0514	0.0484	0.0522	0.0514
	1	1	0.5	0.5	0.5	0.4540	0.3006	**0.4788**	0.3226
	0.5	1	1	1	0.7	**0.4278**	0.2868	0.4024	0.2984
	0.7	1	0.7	0.7	0.5	0.5894	0.4006	**0.5940**	0.4296
	1	1	0.5	0.5	0.2	0.7910	0.5732	**0.8218**	0.6272
	0.75	1	0.75	0.5	0.25	0.8776	0.6862	**0.8878**	0.7224
	1	1	1	1	0	0.9122	0.7060	**0.9350**	0.7512
	1	1	0.5	0.2	0	0.9552	0.8106	**0.9744**	0.8494
	0	1.6	0.8	0.4	0.2	1.0000	0.9988	**1.0000**	0.9992

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0582	0.0522	0.0480	0.0534
	1	1	0.5	0.5	0.5	0.7430	0.4904	**0.7520**	0.5262
	0.5	1	1	1	0.7	**0.7060**	0.4846	0.6746	0.5246
	0.7	1	0.7	0.7	0.5	**0.8974**	0.6976	0.8948	0.7192
	1	1	0.5	0.5	0.2	0.9732	0.8418	**0.9812**	0.8868
	0.75	1	0.75	0.5	0.25	0.9926	0.9420	**0.9956**	0.9472
	1	1	1	1	0	0.9750	0.8538	**0.9808**	0.8852
	1	1	0.5	0.2	0	**0.9996**	0.9800	0.9992	0.9844
	0	1.6	0.8	0.4	0.2	**1.0000**	1.0000	1.0000	1.0000

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0498	0.0504	0.0534	0.0512
	1	1	0.5	0.5	0.5	0.3352	0.2330	**0.3586**	0.2526
	0.5	1	1	1	0.7	**0.3092**	0.2258	**0.3040**	0.2378
	0.7	1	0.7	0.7	0.5	0.4496	0.2910	**0.4552**	0.3264
	1	1	0.5	0.5	0.2	0.6384	0.4260	**0.6654**	0.4602
	0.75	1	0.75	0.5	0.25	0.7438	0.5404	**0.7692**	0.5726
	1	1	1	1	0	0.7848	0.5522	**0.7964**	0.5970
	1	1	0.5	0.2	0	0.8548	0.6556	**0.8782**	0.6942
	0	1.6	0.8	0.4	0.2	**0.9988**	0.9816	**0.9994**	0.9830

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.17. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.1
Treatments=5, IBD =5, CRD=10, RCBD=5 Peak=3, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0486	0.0514	0.0496	0.0516
	0	0.5	1	1	0.7	0.4058	0.4946	**0.6444**	0.547
	0.7	0.7	1	0.5	0.5	0.3058	0.2774	**0.3518**	0.2784
	0	0.5	1	0.5	0	0.8688	0.8346	**0.9362**	0.8674
Normal	0	1	1	0	0	**0.9478**	0.8274	0.9310	0.8478
	1	1	1	0.5	0	**0.5642**	0.3724	0.4598	0.3488
	1	1	1	1	0	**0.4996**	0.3574	0.4352	0.3602

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0534	0.0450	0.0484	0.0528
	0	0.5	1	1	0.7	0.6186	0.7564	**0.8952**	0.793
	0.7	0.7	1	0.5	0.5	0.5138	0.4594	**0.6076**	0.474
Exponential	0	0.5	1	0.5	0	0.9854	0.9826	**0.9974**	0.9872
	0	1	1	0	0	**0.9962**	0.9652	0.9954	0.9682
	1	1	1	0.5	0	**0.7908**	0.5570	0.6584	0.5512
	1	1	1	1	0	**0.6574**	0.4772	0.5770	0.466

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0576	0.0512	0.0546	0.0516
	0	0.5	1	1	0.7	0.3030	0.3776	**0.5092**	0.4102
T distribution	0.7	0.7	1	0.5	0.5	0.2258	0.2066	**0.2672**	0.2254
with 3 degrees	0	0.5	1	0.5	0	0.7434	0.6856	**0.8250**	0.7072
of freedom	0	1	1	0	0	**0.8358**	0.6884	0.8088	0.693
	1	1	1	0.5	0	**0.4382**	0.2824	0.4420	0.2736
	1	1	1	1	0	**0.3768**	0.2836	0.3452	0.279

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.18. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.1. Treatments=5, IBD =5, CRD=15, RCBD=5 Peak=3, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0486	0.0530	0.0512	0.0444
	0	0.5	1	1	0.7	0.4918	0.6266	**0.7282**	0.6518
	0.7	0.7	1	0.5	0.5	0.3552	0.3388	**0.4008**	0.36
	0	0.5	1	0.5	0	0.9284	0.9312	**0.9734**	0.9418
Normal	0	1	1	0	0	**0.9750**	0.9288	0.9692	0.9276
	1	1	1	0.5	0	**0.6400**	0.4600	0.5240	0.4508
	1	1	1	1	0	**0.5618**	0.4318	0.5130	0.4278

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0482	0.0506	0.0516	0.0496
	0	0.5	1	1	0.7	0.7358	0.8792	**0.9452**	0.9058
	0.7	0.7	1	0.5	0.5	0.6000	0.5856	**0.6832**	0.6176
Exponential	0	0.5	1	0.5	0	0.9972	0.9974	**0.9998**	0.9990
	0	1	1	0	0	0.9996	0.9918	**0.9982**	0.9936
	1	1	1	0.5	0	**0.8390**	0.6718	0.7516	0.6750
	1	1	1	1	0	**0.7148**	0.5720	0.6488	0.5638

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0506	0.0486	0.0504	0.0562
	0	0.5	1	1	0.7	0.3764	0.4708	**0.5738**	0.5096
T distribution	0.7	0.7	1	0.5	0.5	0.2688	0.2490	**0.3114**	0.2724
with 3	0	0.5	1	0.5	0	0.8100	0.7954	**0.8828**	0.8226
degrees of	0	1	1	0	0	**0.8834**	0.8054	0.8738	0.7998
freedom	1	1	1	0.5	0	**0.4994**	0.3440	0.3896	0.3502
	1	1	1	1	0	**0.4384**	0.3400	0.3614	0.3316

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.19. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.1. Treatments=5, IBD =10, CRD=5, RCBD=10 Peak=3, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	**0**	**0**	**0**	**0**	0.0468	0.0478	0.0496	0.052
	0	0.5	1	1	0.7	0.3812	0.3708	**0.7306**	0.606
	0.7	0.7	1	0.5	0.5	0.3284	0.2646	**0.4040**	0.3228
	0	0.5	1	0.5	0	0.9618	0.8156	0.9678	0.9032
Normal	0	1	1	0	0	**0.9768**	0.8790	0.9656	0.8928
	1	1	1	0.5	0	**0.6634**	0.4700	0.5214	0.3978
	1	1	1	1	0	**0.5942**	0.4198	0.5014	0.3914

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	**0**	**0**	**0**	**0**	0.0510	0.0456	0.0506	0.0482
	0	0.5	1	1	0.7	0.5942	0.5834	**0.9388**	0.8522
	0.7	0.7	1	0.5	0.5	0.5410	0.4354	**0.6844**	0.55
Exponential	0	0.5	1	0.5	0	0.9926	0.9714	**0.9988**	0.9946
	0	1	1	0	0	**0.9986**	0.9808	**0.9986**	0.985
	1	1	1	0.5	0	**0.8850**	0.6698	0.7314	0.6004
	1	1	1	1	0	**0.7546**	0.5394	0.6476	0.5112

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	**0**	**0**	**0**	**0**	0.0564	0.0468	0.0538	0.0514
	0	0.5	1	1	0.7	0.2946	0.2882	**0.5722**	0.452
T distribution	0.7	0.7	1	0.5	0.5	0.2548	0.2214	**0.2972**	0.2438
with 3	0	0.5	1	0.5	0	0.7660	0.6488	**0.8762**	0.762
degrees of	0	1	1	0	0	**0.9046**	0.7396	0.8724	0.7452
freedom	1	1	1	0.5	0	**0.5228**	0.3474	0.3852	0.313
	1	1	1	1	0	**0.4538**	0.3118	0.3882	0.3018

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.20. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.1. Treatments=5, IBD =10, CRD=10, RCBD=10 Peak=3, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0478	0.0512	0.0514	0.0530
	0	0.5	1	1	0.7	0.4924	0.5054	**0.8128**	0.6048
	0.7	0.7	1	0.5	0.5	0.4024	0.3032	**0.4650**	0.3224
	0	0.5	1	0.5	0	**0.9974**	0.8786	0.9886	0.9094
Normal	0	1	1	0	0	**0.9926**	0.8900	0.9846	0.9044
	1	1	1	0.5	0	**0.7532**	0.4468	0.6142	0.4058
	1	1	1	1	0	**0.6714**	0.4090	0.5904	0.3974
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0526	0.0526	0.0516	0.0500
	0	0.5	1	1	0.7	0.7528	0.7708	**0.9738**	0.8634
	0.7	0.7	1	0.5	0.5	0.6602	0.4986	**0.7804**	0.5664
Exponential	0	0.5	1	0.5	0	0.9978	0.9922	**1.0000**	0.9952
	0	1	1	0	0	**1.0000**	0.9864	**1.0000**	0.9872
	1	1	1	0.5	0	**0.9350**	0.6422	0.8254	0.6048
	1	1	1	1	0	**0.8226**	0.5170	0.7408	0.5250
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0592	0.0508	0.0508	0.0468
	0	0.5	1	1	0.7	0.3668	0.3890	**0.6544**	0.4716
T distribution	0.7	0.7	1	0.5	0.5	0.3040	0.2340	**0.3500**	0.2410
with 3 degrees	0	0.5	1	0.5	0	0.8658	0.7280	**0.9394**	0.7812
of freedom	0	1	1	0	0	**0.9456**	0.7546	**0.9232**	0.7728
	1	1	1	0.5	0	**0.5800**	0.3362	**0.4622**	0.3224
	1	1	1	1	0	**0.5192**	0.3134	**0.4464**	0.3108

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.21. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.1. Treatments=5, IBD =10, CRD=15, RCBD=10 Peak=3, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0492	0.0520	0.0480	0.0524
	0	0.5	1	1	0.7	0.5860	0.6260	**0.8548**	0.6874
	0.7	0.7	1	0.5	0.5	0.4262	0.3632	**0.5242**	0.3872
	0	0.5	1	0.5	0	**0.9968**	0.9488	0.9954	0.9568
Normal	0	1	1	0	0	**0.9962**	0.9452	0.9952	0.9498
	1	1	1	0.5	0	**0.8040**	0.5010	0.6658	0.469
	1	1	1	1	0	**0.7440**	0.4658	0.6444	0.4618

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0502	0.0516	0.0534	0.048
	0	0.5	1	1	0.7	0.8276	0.8920	**0.9884**	0.9164
	0.7	0.7	1	0.5	0.5	0.7286	0.6118	**0.8384**	0.649
Exponential	0	0.5	1	0.5	0	0.9994	0.9990	**1.0000**	0.9994
	0	1	1	0	0	**1.0000**	0.9956	0.9998	0.9984
	1	1	1	0.5	0	**0.9566**	0.7164	0.8774	0.7054
	1	1	1	1	0	**0.8756**	0.6158	0.7976	0.6172

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0520	0.0516	0.0500	0.0468
	0	0.5	1	1	0.7	0.4436	0.5036	**0.7154**	0.5256
T distribution	0.7	0.7	1	0.5	0.5	0.3324	0.2714	**0.3974**	0.2862
with 3 degrees	0	0.5	1	0.5	0	0.9052	0.8250	**0.9620**	0.849
of freedom	0	1	1	0	0	**0.9666**	0.8366	0.9514	0.8458
	1	1	1	0.5	0	**0.6354**	0.3612	0.5076	0.3756
	1	1	1	1	0	**0.5654**	0.3550	0.4964	0.3516

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.22. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.1. Treatments=5, IBD =15, CRD=5, RCBD=15 Peak=3, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0518	0.0542	0.0446	0.0490
	0	0.5	1	1	0.7	0.4506	0.4232	**0.8408**	0.7220
	0.7	0.7	1	0.5	0.5	0.3866	0.3244	**0.4992**	0.4052
	0	0.5	1	0.5	0	0.9576	0.8800	**0.9924**	0.9708
	0	1	1	0	0	**0.9962**	0.9550	0.9902	0.9630
	1	1	1	0.5	0	**0.7908**	0.5940	0.6224	0.5160
	1	1	1	1	0	**0.7288**	0.5286	0.6058	0.4980

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0500	0.0534	0.0500	0.0482
	0	0.5	1	1	0.7	0.6994	0.6488	**0.9776**	0.9370
	0.7	0.7	1	0.5	0.5	0.6650	0.5340	**0.8018**	0.6850
	0	0.5	1	0.5	0	0.9986	0.9920	**1.0000**	0.9996
	0	1	1	0	0	**1.0000**	0.9964	0.9998	0.9984
	1	1	1	0.5	0	**0.9536**	0.8210	0.8434	0.7288
	1	1	1	1	0	**0.8618**	0.6756	0.7608	0.6482

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0464	0.0488	0.0492	0.0468
	0	0.5	1	1	0.7	0.3404	0.3108	**0.6714**	0.5700
	0.7	0.7	1	0.5	0.5	0.3034	0.2444	**0.3668**	0.3044
	0	0.5	1	0.5	0	0.8498	0.7512	**0.9416**	0.8662
	0	1	1	0	0	**0.9578**	0.8552	0.9424	0.8660
	1	1	1	0.5	0	**0.6260**	0.4630	0.4830	0.3844
	1	1	1	1	0	**0.5608**	0.4086	0.4676	0.3696

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.23. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.1. Treatments=5, IBD =15, CRD=10, RCBD=15 Peak=3, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0508	0.0568	0.0488	0.0496
	0	0.5	1	1	0.7	0.5656	0.5326	**0.8940**	0.6656
	0.7	0.7	1	0.5	0.5	0.4702	0.3300	**0.5586**	0.3666
	0	0.5	1	0.5	0	0.9824	0.9082	**0.9988**	0.9482
	0	1	1	0	0	**0.9998**	0.9334	0.9968	0.9392
	1	1	1	0.5	0	**0.8614**	0.4870	0.7034	0.4742
	1	1	1	1	0	**0.7818**	0.4478	0.6706	0.4536

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0504	0.0510	0.0494	0.0488
	0	0.5	1	1	0.7	0.8148	0.7994	**0.9938**	0.9102
	0.7	0.7	1	0.5	0.5	0.7578	0.5358	**0.8782**	0.6212
	0	0.5	1	0.5	0	0.9998	0.9946	**1.0000**	0.9986
	0	1	1	0	0	**1.0000**	0.9946	**1.0000**	0.9944
	1	1	1	0.5	0	**0.9800**	0.7112	0.9038	0.6872
	1	1	1	1	0	**0.9110**	0.5860	0.8316	0.5810

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0536	0.0490	0.0436	0.0512
	0	0.5	1	1	0.7	0.4370	0.4126	**0.7518**	0.5086
	0.7	0.7	1	0.5	0.5	0.3494	0.2548	**0.4390**	0.2716
	0	0.5	1	0.5	0	0.9236	0.7748	**0.9702**	0.8318
	0	1	1	0	0	**0.9812**	0.8066	0.9740	0.8244
	1	1	1	0.5	0	**0.6936**	0.3692	0.5476	0.3534
	1	1	1	1	0	**0.6158**	0.3434	0.5354	0.3474

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.24. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.1. Treatments=5, IBD =15, CRD=15, RCBD=15 Peak=3, p=0.1

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0458	0.0444	0.0516	0.0454
	0	0.5	1	1	0.7	0.6582	0.6436	**0.9240**	0.7308
	0.7	0.7	1	0.5	0.5	0.5192	0.3824	**0.6100**	0.4094
	0	0.5	1	0.5	0	0.9922	0.9518	**0.9990**	0.9698
Normal	0	1	1	0	0	0.9998	0.9594	0.9978	0.9652
	1	1	1	0.5	0	**0.8796**	0.5384	0.7594	0.5102
	1	1	1	1	0	**0.8234**	0.4792	0.7390	0.4844

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0554	0.0510	0.0462	0.0494
	0	0.5	1	1	0.7	0.8820	0.8994	**0.9976**	0.9428
	0.7	0.7	1	0.5	0.5	0.8264	0.6344	**0.9148**	0.682
Exponential	0	0.5	1	0.5	0	**1.0000**	0.9990	**1.0000**	0.9996
	0	1	1	0	0	**1.0000**	0.9972	**1.0000**	0.9972
	1	1	1	0.5	0	**0.9872**	0.7664	0.9430	0.752
	1	1	1	1	0	**0.9408**	0.6450	0.8806	0.6442

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0472	0.0494	0.0466	0.0472
	0	0.5	1	1	0.7	0.4960	0.4990	**0.8050**	0.5644
T distribution	0.7	0.7	1	0.5	0.5	0.4018	0.2912	**0.4660**	0.3032
with 3 degrees	0	0.5	1	0.5	0	0.9566	0.8490	**0.9890**	0.8688
of freedom	0	1	1	0	0	**0.9902**	0.8688	0.9858	0.8582
	1	1	1	0.5	0	**0.7452**	0.3920	0.5882	0.396
	1	1	1	1	0	**0.6814**	0.3746	0.5846	0.3774

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.25. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.4. Treatments=5, IBD =5, CRD=10, RCBD=5 Peak=3, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0510	0.0508	0.0524	0.0498
	0	0.5	1	1	0.7	0.3518	0.4792	**0.6136**	0.5452
	0.7	0.7	1	0.5	0.5	0.2634	0.2680	**0.3256**	0.2798
	0	0.5	1	0.5	0	0.8294	0.8302	**0.9052**	0.8518
Normal	0	1	1	0	0	**0.9268**	0.8340	0.9040	0.8392
	1	1	1	0.5	0	**0.5250**	0.3722	0.4152	0.3602
	1	1	1	1	0	**0.4680**	0.3410	0.3950	0.3462

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0480	0.0494	0.0492	0.0460
	0	0.5	1	1	0.7	0.7244	0.6290	**0.8534**	0.7882
	0.7	0.7	1	0.5	0.5	0.4666	0.4490	**0.5626**	0.4660
Exponential	0	0.5	1	0.5	0	0.9770	0.9808	**0.9950**	0.9874
	0	1	1	0	0	**0.9874**	0.9650	0.9844	0.9650
	1	1	1	0.5	0	**0.7436**	0.5462	0.6140	0.5368
	1	1	1	1	0	**0.6164**	0.4468	0.5254	0.4450

Distribution	Location					Non-modification		**Distance-Modification**	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	**First**	Last
	0	0	0	0	0	0.0500	0.0464	**0.0500**	0.0500
	0	0.5	1	1	0.7	0.3590	0.2596	**0.4662**	0.3996
T distribution	0.7	0.7	1	0.5	0.5	0.2234	0.2088	**0.2468**	0.2254
with 3 degrees	0	0.5	1	0.5	0	0.6798	0.6640	**0.7874**	0.6984
of freedom	0	1	1	0	0	**0.7868**	0.6652	0.7606	0.6816
	1	1	1	0.5	0	**0.3984**	0.2764	0.3172	0.2802
	1	1	1	1	0	**0.3532**	0.2638	0.3040	0.2558

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.26. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.4. Treatments=5, IBD =5, CRD=15, RCBD=5 Peak=3, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0570	0.0566	0.0534	0.0510
	0	0.5	1	1	0.7	0.4458	0.6136	**0.6824**	0.6510
	0.7	0.7	1	0.5	0.5	0.3120	0.3428	**0.3862**	0.3482
	0	0.5	1	0.5	0	0.8988	0.9294	**0.9506**	0.9356
Normal	0	1	1	0	0	**0.9604**	0.9272	0.9444	0.9332
	1	1	1	0.5	0	**0.5966**	0.4416	0.4858	0.4510
	1	1	1	1	0	**0.5190**	0.4316	0.4666	0.4350

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0512	0.0496	0.0512	0.0448
	0	0.5	1	1	0.7	**0.6856**	0.8768	**0.9130**	0.8964
	0.7	0.7	1	0.5	0.5	**0.5428**	0.5806	**0.6388**	0.5950
Exponential	0	0.5	1	0.5	0	0.9890	0.9978	**0.9992**	0.9980
	0	1	1	0	0	**0.9964**	0.9918	0.9946	0.9924
	1	1	1	0.5	0	**0.8218**	0.6706	0.7020	0.6650
	1	1	1	1	0	**0.6828**	0.5612	0.6042	0.5752

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0472	0.0490	0.0508	0.0538
	0	0.5	1	1	0.7	0.3416	0.4790	**0.5332**	0.4952
T distribution	0.7	0.7	1	0.5	0.5	0.2454	0.2548	**0.2884**	0.2726
with 3 degrees	0	0.5	1	0.5	0	0.7630	0.7982	**0.8444**	0.8076
of freedom	0	1	1	0	0	**0.8558**	0.8004	0.8302	0.8018
	1	1	1	0.5	0	**0.4486**	0.3456	0.3700	0.3344
	1	1	1	1	0	**0.4090**	0.3132	0.3536	0.3384

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.27. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.4. Treatments=5, IBD =10, CRD=5, RCBD=10 Peak=3, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0470	0.0520	0.0542	0.0492
	0	0.5	1	1	0.7	0.3356	0.3202	**0.6636**	0.5502
	0.7	0.7	1	0.5	0.5	0.2868	0.2390	**0.3562**	0.2900
	0	0.5	1	0.5	0	0.8384	0.7456	**0.9492**	0.8642
	0	1	1	0	0	**0.9502**	0.8526	0.9326	0.8484
	1	1	1	0.5	0	**0.6332**	0.4366	0.4696	0.3846
	1	1	1	1	0	**0.5348**	0.3694	0.4354	0.3638

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0530	0.0540	0.0488	0.0476
	0	0.5	1	1	0.7	0.4812	0.4990	**0.8862**	0.8082
	0.7	0.7	1	0.5	0.5	0.4818	0.3978	**0.6190**	0.5048
	0	0.5	1	0.5	0	0.9778	0.9492	**0.9974**	0.9866
	0	1	1	0	0	**0.9960**	0.9664	0.9950	0.9734
	1	1	1	0.5	0	**0.8380**	0.6342	0.6660	0.5512
	1	1	1	1	0	**0.6888**	0.5052	0.5904	0.4786

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0520	0.0478	0.0504	0.0490
	0	0.5	1	1	0.7	0.2520	0.2568	**0.5212**	0.4234
	0.7	0.7	1	0.5	0.5	0.2236	0.1894	**0.2722**	0.2316
	0	0.5	1	0.5	0	0.6832	0.5928	**0.8248**	0.7132
	0	1	1	0	0	**0.8556**	0.6866	0.8276	0.7054
	1	1	1	0.5	0	**0.4610**	0.3304	0.3590	0.2780
	1	1	1	1	0	**0.4220**	0.2922	0.3428	0.2758

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.28. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.4. Treatments=5, IBD =10, CRD=10, RCBD=10 Peak=3, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0480	0.0526	0.0480	0.0478
	0	0.5	1	1	0.7	0.4222	0.4866	**0.7566**	0.5788
	0.7	0.7	1	0.5	0.5	0.3500	0.2906	**0.4292**	0.3158
	0	0.5	1	0.5	0	0.9172	0.8566	**0.9740**	0.8974
	0	1	1	0	0	**0.9856**	0.8816	0.9734	0.8892
	1	1	1	0.5	0	**0.6940**	0.4210	0.5408	0.4092
	1	1	1	1	0	**0.6320**	0.3816	0.5390	0.3900

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0512	0.0494	0.0498	0.0472
	0	0.5	1	1	0.7	**0.6400**	0.7482	**0.9580**	0.8458
	0.7	0.7	1	0.5	0.5	**0.6046**	0.4906	**0.7204**	0.5530
	0	0.5	1	0.5	0	0.9954	0.9872	0.9998	0.9928
	0	1	1	0	0	0.9996	0.9800	0.9990	0.9816
	1	1	1	0.5	0	**0.9020**	0.6278	**0.7624**	0.6054
	1	1	1	1	0	**0.7752**	0.4998	**0.6774**	0.5096

Distribution	Location					**Non-modification**		**Distance-Modification**	
	μ_1	μ_2	μ_3	μ_4	μ_5	**First**	Last	**First**	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0534	0.0512	0.0526	0.0494
	0	0.5	1	1	0.7	0.3212	0.3776	**0.6052**	0.4508
	0.7	0.7	1	0.5	0.5	0.2716	0.2276	**0.3092**	0.2386
	0	0.5	1	0.5	0	0.7984	0.7176	**0.9004**	0.7546
	0	1	1	0	0	**0.9140**	0.7300	0.8922	0.7336
	1	1	1	0.5	0	**0.5558**	0.3204	0.4156	0.3076
	1	1	1	1	0	**0.4842**	0.2936	0.4036	0.2882

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.29. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.4. Treatments=5, IBD =10, CRD=15, RCBD=10 Peak=3, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0500	0.0508	0.0512	0.0442
	0	0.5	1	1	0.7	0.6216	0.5278	**0.8268**	0.6778
	0.7	0.7	1	0.5	0.5	0.3940	0.3416	**0.4768**	0.3768
	0	0.5	1	0.5	0	0.9576	0.9416	**0.9930**	0.9534
Normal	0	1	1	0	0	**0.9942**	0.9432	0.9898	0.9466
	1	1	1	0.5	0	**0.7580**	0.4896	0.6128	0.4824
	1	1	1	1	0	**0.6812**	0.4652	0.5864	0.4522

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0526	0.0506	0.0510	0.0498
	0	0.5	1	1	0.7	0.7466	0.8848	**0.9762**	0.9166
	0.7	0.7	1	0.5	0.5	0.6732	0.5924	**0.7834**	0.6264
Exponential	0	0.5	1	0.5	0	0.9990	0.9972	**0.9998**	0.9984
	0	1	1	0	0	0.9998	0.9946	**1.0000**	0.9948
	1	1	1	0.5	0	**0.9252**	0.7088	0.8242	0.6928
	1	1	1	1	0	**0.8354**	0.5916	0.7448	0.5982

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0506	0.0512	0.0524	0.0518
	0	0.5	1	1	0.7	0.3820	0.4858	**0.6486**	0.5264
T distribution	0.7	0.7	1	0.5	0.5	0.2992	0.2684	**0.3714**	0.2754
with 3	0	0.5	1	0.5	0	0.8646	0.8204	**0.9382**	0.8366
degrees of	0	1	1	0	0	**0.9496**	0.8214	0.9292	0.8284
freedom	1	1	1	0.5	0	**0.5972**	0.3666	0.4588	0.3780
	1	1	1	1	0	**0.5430**	0.3458	0.4564	0.3500

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test

Table E.30. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.4. Treatments=5, IBD =15, CRD=5, RCBD=15 Peak=3, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0512	0.0512	0.0478	0.0462
	0	0.5	1	1	0.7	0.3702	0.3408	**0.7750**	0.6604
Normal	0.7	0.7	1	0.5	0.5	0.3446	0.2770	**0.4274**	0.3560
	0	0.5	1	0.5	0	0.9108	0.8408	**0.9798**	0.9408
	0	1	1	0	0	**0.9874**	0.9336	0.9772	0.9320
	1	1	1	0.5	0	**0.7552**	0.5480	0.5488	0.4596
	1	1	1	1	0	**0.6554**	0.4804	0.5486	0.4482

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0486	0.0508	0.0516	0.0494
	0	0.5	1	1	0.7	**0.5690**	0.5468	**0.9590**	0.9034
	0.7	0.7	1	0.5	0.5	**0.5860**	0.4606	**0.7306**	0.6216
Exponential	0	0.5	1	0.5	0	0.9950	0.9756	1.0000	0.9988
	0	1	1	0	0	1.0000	0.9910	0.9992	0.9942
	1	1	1	0.5	0	**0.9182**	0.7742	**0.7730**	0.6638
	1	1	1	1	0	**0.8014**	0.6316	**0.7008**	0.5808

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0450	0.0444	0.0540	0.0468
	0	0.5	1	1	0.7	0.2806	0.2624	**0.6134**	0.5206
T distribution	0.7	0.7	1	0.5	0.5	0.2616	0.2184	**0.3394**	0.2886
with 3	0	0.5	1	0.5	0	0.7822	0.6688	**0.9126**	0.8216
degrees of	0	1	1	0	0	**0.9248**	0.8122	0.8978	0.8164
freedom	1	1	1	0.5	0	**0.5746**	0.4384	0.4214	0.3484
	1	1	1	1	0	**0.5106**	0.3702	0.4162	0.3486

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.31. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.4. Treatments=5, IBD =15, CRD=10, RCBD=15 Peak=3, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0510	0.0462	0.0492	0.0496
	0	0.5	1	1	0.7	**0.4638**	0.4974	**0.8452**	0.6496
	0.7	0.7	1	0.5	0.5	**0.4038**	0.3174	**0.5016**	0.3446
	0	0.5	1	0.5	0	**0.9608**	0.8844	**0.9950**	0.9332
Normal	0	1	1	0	0	**0.9968**	0.9174	**0.9932**	0.9228
	1	1	1	0.5	0	**0.8044**	0.4636	**0.6476**	0.4538
	1	1	1	1	0	**0.7340**	0.4360	**0.6312**	0.4392

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
	0	0	0	0	0	0.0482	0.0480	0.0500	0.0510
	0	0.5	1	1	0.7	**0.7676**	0.7008	**0.9788**	0.8866
	0.7	0.7	1	0.5	0.5	**0.6892**	0.5276	**0.8212**	0.5878
Exponential	0	0.5	1	0.5	0	0.9994	0.9918	1.0000	0.9978
	0	1	1	0	0	0.9998	0.9900	1.0000	0.9916
	1	1	1	0.5	0	**0.9548**	0.6892	**0.8582**	0.6576
	1	1	1	1	0	**0.8708**	0.5838	**0.7776**	0.5588

Distribution	Location					**Non-modification**		**Distance-Modification**	
	μ_1	μ_2	μ_3	μ_4	μ_5	**First**	Last	**First**	Last
	0	0	0	0	0	**0.0520**	0.0498	**0.0512**	0.0450
	0	0.5	1	1	0.7	0.3604	0.3730	**0.7036**	0.4964
T distribution	0.7	0.7	1	0.5	0.5	0.3130	0.2350	**0.3828**	0.2654
with 3 degrees	0	0.5	1	0.5	0	0.8738	0.7352	**0.9534**	0.8120
of freedom	0	1	1	0	0	**0.9656**	0.7822	0.9504	0.7950
	1	1	1	0.5	0	**0.6410**	0.3626	0.5032	0.3362
	1	1	1	1	0	**0.5830**	0.3250	0.4844	0.3330

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table E.32. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for treatments five at peak 3 with probability of 0.4. Treatments=5, IBD =15, CRD=15, RCBD=15 Peak=3, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0480	0.0508	0.0574	0.0468
	0	0.5	1	1	0.7	0.6450	0.5462	**0.8910**	0.7048
	0.7	0.7	1	0.5	0.5	0.4638	0.3570	**0.5648**	0.3870
	0	0.5	1	0.5	0	0.9862	0.9476	**0.9974**	0.9600
	0	1	1	0	0	**0.9980**	0.9546	0.9978	0.9558
	1	1	1	0.5	0	**0.8406**	0.5096	0.7026	0.4994
	1	1	1	1	0	**0.7834**	0.4902	0.6802	0.4830

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0478	0.0480	0.0508	0.0526
	0	0.5	1	1	0.7	0.8080	0.8844	**0.9938**	0.9310
	0.7	0.7	1	0.5	0.5	0.7474	0.6268	**0.8724**	0.6654
	0	0.5	1	0.5	0	0.9998	0.9996	**1.0000**	0.9992
	0	1	1	0	0	**1.0000**	0.9974	**1.0000**	0.9976
	1	1	1	0.5	0	**0.9708**	0.7430	0.9096	0.7200
	1	1	1	1	0	**0.9000**	0.6250	0.8328	0.6126

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0526	0.0516	0.0528	0.0462
	0	0.5	1	1	0.7	0.4966	0.4336	**0.7670**	0.5582
	0.7	0.7	1	0.5	0.5	0.3520	0.2766	**0.4268**	0.2848
	0	0.5	1	0.5	0	0.9270	0.8428	**0.9700**	0.8674
	0	1	1	0	0	**0.9796**	0.8306	0.9682	0.8606
	1	1	1	0.5	0	**0.6978**	0.3826	0.5578	0.3840
	1	1	1	1	0	**0.6202**	0.3666	0.5364	0.3658

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

APPENDIX F. MIXED DESIGN OF IBD, CRD AND RCBD WITH PROBABILITIES OF 0.2 AND 0.4 FOR TREATMENTS 5 AT PEAK 4.

Table F.1. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.2. Treatments=5, IBD =5, CRD=10, RCBD=5 Peak=4, p=0.2

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0550	0.0520	0.0496	0.0490
	0.5	0.5	0.5	1	1	0.2784	0.2382	**0.2994**	0.2414
	0	0.5	1	1	0.7	0.8600	0.7464	**0.8662**	0.7764
	0	0.5	1	1	1	0.7944	0.6880	**0.8270**	0.7116
	0	0	1	1	1	0.8844	0.7696	**0.8958**	0.8060
	0.5	0.5	0.7	1	0.7	0.4234	0.3440	**0.4352**	0.3510
	1	1	1	1	0	**0.1584**	0.1378	0.1414	0.1326
	0	0.4	0.8	1.6	0.4	0.9978	0.9874	0.9982	0.9898
	0	0.25	0.5	1	0.25	**0.8918**	0.7930	**0.8896**	0.8052

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0510	0.0490	0.0514	0.0490
	0.5	0.5	0.5	1	1	0.4500	0.3524	**0.4742**	0.3786
	0	0.5	1	1	0.7	0.9760	0.9246	**0.9810**	0.9418
	0	0.5	1	1	1	0.9474	0.8760	**0.9600**	0.8946
	0	0	1	1	1	0.9690	0.9222	**0.9798**	0.9252
	0.5	0.5	0.7	1	0.7	0.7188	0.6022	**0.7224**	0.6194
	1	1	1	1	0	**0.1886**	0.1442	0.1696	0.1494
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9996	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	0.9938	0.9716	**0.9940**	0.9814

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0476	0.0518	0.0450	0.0452
	0.5	0.5	0.5	1	1	0.2166	0.1898	**0.2244**	0.1956
	0	0.5	1	1	0.7	0.7140	0.5948	**0.7260**	0.6168
	0	0.5	1	1	1	0.6342	0.5158	**0.6602**	0.5542
	0	0	1	1	1	0.7228	0.6016	**0.7468**	0.6238
	0.5	0.5	0.7	1	0.7	**0.3204**	0.2606	0.3158	0.2666
	1	1	1	1	0	**0.1412**	0.1162	0.1186	0.1188
	0	0.4	0.8	1.6	0.4	**0.9766**	0.9308	0.9752	0.9364
	0	0.25	0.5	1	0.25	0.7512	0.6310	**0.7514**	0.6678

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.2. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.2. Treatments=5, IBD =5, CRD=15, RCBD=5 Peak=4, p=0.2

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0452	0.0478	0.0442	0.0484
	0.5	0.5	0.5	1	1	0.3038	0.2798	**0.3392**	0.3038
	0	0.5	1	1	0.7	0.9136	0.8620	**0.9214**	0.8806
	0	0.5	1	1	1	0.8618	0.8012	**0.8826**	0.8194
	0	0	1	1	1	0.9296	0.8888	**0.9376**	0.8956
	0.5	0.5	0.7	1	0.7	**0.4842**	0.4346	**0.4810**	0.4374
	1	1	1	1	0	**0.1780**	0.1566	**0.1550**	0.1632
	0	0.4	0.8	1.6	0.4	**0.9994**	0.9986	**0.9992**	0.9992
	0	0.25	0.5	1	0.25	**0.9396**	0.9052	**0.9318**	0.9074

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0496	0.0488	0.0488	0.0490
	0.5	0.5	0.5	1	1	0.5186	0.4500	**0.5444**	0.4652
	0	0.5	1	1	0.7	0.9920	0.9802	**0.9934**	0.9824
	0	0.5	1	1	1	0.9786	0.9484	**0.9812**	0.9582
	0	0	1	1	1	0.9888	0.9722	0.9932	0.9764
	0.5	0.5	0.7	1	0.7	0.7974	0.7412	**0.7986**	0.7498
	1	1	1	1	0	**0.2128**	0.1874	0.1880	0.1834
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	0.9976	0.9946	0.9982	0.9952

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0496	0.0500	0.0520	0.0532
	0.5	0.5	0.5	1	1	0.2424	0.2234	**0.2534**	0.2222
	0	0.5	1	1	0.7	0.7866	0.7074	**0.7886**	0.7232
	0	0.5	1	1	1	0.7126	0.6422	**0.7382**	0.6620
	0	0	1	1	1	0.8146	0.7312	**0.8278**	0.7548
	0.5	0.5	0.7	1	0.7	0.3658	0.3286	**0.3706**	0.3346
	1	1	1	1	0	**0.1502**	0.1380	**0.1204**	0.1288
	0	0.4	0.8	1.6	0.4	0.9906	0.9734	**0.9922**	0.9786
	0	0.25	0.5	1	0.25	0.8192	0.7548	**0.8304**	0.7732

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.3. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.2. Treatments=5, IBD =10, CRD=5, RCBD=10 Peak=4, p=0.2

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0438	0.0476	0.0538	0.0498
	0.5	0.5	0.5	1	1	0.3138	0.2322	**0.3318**	0.2794
	0	0.5	1	1	0.7	0.9058	0.7552	**0.9184**	0.8456
	0	0.5	1	1	1	0.8576	0.6952	**0.8832**	0.7934
	0	0	1	1	1	0.9186	0.7784	**0.9474**	0.8690
	0.5	0.5	0.7	1	0.7	0.4766	0.3422	**0.4924**	0.4016
	1	1	1	1	0	**0.1772**	0.1394	0.1450	0.1202
	0	0.4	0.8	1.6	0.4	**0.9996**	0.9888	0.9990	0.9964
	0	0.25	0.5	1	0.25	0.9312	0.8094	**0.9338**	0.8708

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0522	0.0536	0.0552	0.0492
	0.5	0.5	0.5	1	1	0.4944	0.3622	**0.5568**	0.4388
	0	0.5	1	1	0.7	0.9892	0.9354	**0.9922**	0.9676
	0	0.5	1	1	1	0.9758	0.8818	**0.9772**	0.9338
	0	0	1	1	1	0.9880	0.9182	**0.9926**	0.9718
	0.5	0.5	0.7	1	0.7	**0.8688**	0.6686	0.8636	0.7038
	1	1	1	1	0	**0.2056**	0.1612	0.1692	0.1440
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9994	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	0.9966	0.9704	**0.9980**	0.9894

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0486	0.0544	0.0528	0.0516
	0.5	0.5	0.5	1	1	**0.2498**	0.1920	0.2486	0.2104
	0	0.5	1	1	0.7	0.7686	0.5930	**0.7740**	0.6742
	0	0.5	1	1	1	0.7072	0.5182	**0.7342**	0.6300
	0	0	1	1	1	0.7932	0.6114	**0.8172**	0.7262
	0.5	0.5	0.7	1	0.7	**0.3674**	0.2650	0.3660	0.2892
	1	1	1	1	0	**0.1480**	0.1214	0.1182	0.1104
	0	0.4	0.8	1.6	0.4	**0.9890**	0.9364	0.9880	0.9588
	0	0.25	0.5	1	0.25	**0.8100**	0.6554	0.8078	0.7088

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.4. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.2. Treatments=5, IBD =10, CRD=10, RCBD=10 Peak=4, p=0.2

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0530	0.0524	0.0498	0.0496
	0.5	0.5	0.5	1	1	0.3742	0.2550	**0.3844**	0.2820
	0	0.5	1	1	0.7	0.9542	0.7986	**0.9626**	0.8492
	0	0.5	1	1	1	0.9158	0.7502	**0.9294**	0.7754
	0	0	1	1	1	0.9692	0.8232	**0.9786**	0.8664
	0.5	0.5	0.7	1	0.7	0.5496	0.3690	**0.5654**	0.3994
	1	1	1	1	0	0.2066	0.1478	0.1622	0.1418
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9948	**1.0000**	0.9976
	0	0.25	0.5	1	0.25	0.9678	0.8418	**0.9706**	0.8694
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0480	0.0476	0.0498	0.0542
	0.5	0.5	0.5	1	1	0.6118	0.3986	**0.6394**	0.4428
	0	0.5	1	1	0.7	**0.9990**	0.9586	0.9978	0.9696
	0	0.5	1	1	1	0.9896	0.9212	**0.9942**	0.9370
	0	0	1	1	1	0.9976	0.9410	**0.9994**	0.9674
	0.5	0.5	0.7	1	0.7	**0.8770**	0.6508	0.8662	0.6984
	1	1	1	1	0	**0.2470**	0.1564	0.2100	0.1656
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	0.9994	0.9862	**0.9998**	0.9892
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0452	0.0446	0.0492	0.0542
	0.5	0.5	0.5	1	1	0.2826	0.1952	**0.2872**	0.2134
	0	0.5	1	1	0.7	0.8448	0.6366	**0.8568**	0.6808
	0	0.5	1	1	1	0.7938	0.5720	**0.8238**	0.6212
	0	0	1	1	1	0.8678	0.6648	**0.8908**	0.7210
	0.5	0.5	0.7	1	0.7	0.4106	0.2794	**0.4304**	0.3052
	1	1	1	1	0	**0.1536**	0.1224	0.1418	0.1138
	0	0.4	0.8	1.6	0.4	**0.9974**	0.9526	**0.9974**	0.9690
	0	0.25	0.5	1	0.25	0.8822	0.6950	**0.8908**	0.7266

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.5. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.2. Treatments=5, IBD =10, CRD=15, RCBD=10 Peak=4, p=0.2

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0500	0.0564	0.0550	0.0524
	0.5	0.5	0.5	1	1	0.4086	0.2956	**0.4352**	0.3176
	0	0.5	1	1	0.7	0.9814	0.8782	**0.9842**	0.8986
	0	0.5	1	1	1	0.9522	0.8334	**0.9668**	0.8510
	0	0	1	1	1	0.9824	0.9038	**0.9882**	0.9164
	0.5	0.5	0.7	1	0.7	0.6008	0.4328	**0.6198**	0.4582
	1	1	1	1	0	**0.2236**	0.1606	0.1850	0.1614
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9990	**1.0000**	0.9992
	0	0.25	0.5	1	0.25	**0.9880**	0.9124	0.9856	0.9210

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0570	0.0444	0.0570	0.0456
	0.5	0.5	0.5	1	1	0.6682	0.4706	**0.6972**	0.5150
	0	0.5	1	1	0.7	**0.9994**	0.9848	**0.9994**	0.9892
	0	0.5	1	1	1	0.9962	0.9638	**0.9974**	0.9736
	0	0	1	1	1	**0.9996**	0.9786	0.9994	0.9862
	0.5	0.5	0.7	1	0.7	**0.9134**	0.7560	0.9088	0.7826
	1	1	1	1	0	**0.2840**	0.1894	0.2162	0.1992
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**1.0000**	0.9966	**1.0000**	0.9982

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0468	0.0544	0.0482	0.0500
	0.5	0.5	0.5	1	1	0.3156	0.2268	**0.3296**	0.2382
	0	0.5	1	1	0.7	0.8980	0.7294	**0.9126**	0.7638
	0	0.5	1	1	1	0.8480	0.6708	**0.8536**	0.6978
	0	0	1	1	1	0.9094	0.7580	**0.9294**	0.7910
	0.5	0.5	0.7	1	0.7	**0.4774**	0.3360	0.4740	0.3440
	1	1	1	1	0	**0.1840**	0.1308	0.1454	0.1424
	0	0.4	0.8	1.6	0.4	**0.9990**	0.9834	**0.9998**	0.9888
	0	0.25	0.5	1	0.25	**0.9278**	0.7860	0.9234	0.8068

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.6. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.2. Treatments=5, IBD =15, CRD=5, RCBD=15 Peak=4, p=0.2

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0508	0.0490	0.0534	0.0544
	0.5	0.5	0.5	1	1	0.3912	0.2850	**0.4024**	0.3506
	0	0.5	1	1	0.7	0.9598	0.8648	**0.9700**	0.9218
	0	0.5	1	1	1	0.9294	0.8064	**0.9414**	0.8922
	0	0	1	1	1	0.9724	0.8800	**0.9828**	0.9490
	0.5	0.5	0.7	1	0.7	**0.5782**	0.4270	0.5716	0.4966
	1	1	1	1	0	**0.2050**	0.1630	0.1658	0.1358
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9974	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**0.9790**	0.9006	0.9718	0.9450

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0542	0.0524	0.0474	0.0506
	0.5	0.5	0.5	1	1	0.6194	0.4506	**0.6610**	0.5732
	0	0.5	1	1	0.7	0.9984	0.9778	**0.9994**	0.9946
	0	0.5	1	1	1	0.9936	0.9534	**0.9952**	0.9852
	0	0	1	1	1	0.9986	0.9740	**0.9988**	0.9932
	0.5	0.5	0.7	1	0.7	0.8742	0.7358	**0.8928**	0.8104
	1	1	1	1	0	0.2606	0.1898	**0.1936**	0.1646
	0	0.4	0.8	1.6	0.4	1.0000	1.0000	1.0000	1.0000
	0	0.25	0.5	1	0.25	1.0000	0.9968	0.9998	0.9988

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0448	0.0490	0.0458	0.0516
	0.5	0.5	0.5	1	1	0.2806	0.2226	**0.3126**	0.2620
	0	0.5	1	1	0.7	0.8718	0.7182	**0.8880**	0.8028
	0	0.5	1	1	1	0.8140	0.6380	**0.8528**	0.7534
	0	0	1	1	1	0.8876	0.7426	**0.8988**	0.8388
	0.5	0.5	0.7	1	0.7	0.4304	0.3208	**0.4320**	0.3854
	1	1	1	1	0	**0.1724**	0.1244	0.1360	0.1266
	0	0.4	0.8	1.6	0.4	**0.9986**	0.9764	0.9976	0.9906
	0	0.25	0.5	1	0.25	**0.9118**	0.7534	0.8978	0.8254

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.7. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.2. Treatments=5, IBD =15, CRD=10, RCBD=15 Peak=4, p=0.2

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0538	0.0540	0.0468	0.0462
	0.5	0.5	0.5	1	1	0.4402	0.2830	**0.4568**	0.3076
	0	0.5	1	1	0.7	0.9854	0.8470	**0.9882**	0.8894
	0	0.5	1	1	1	**0.9748**	0.7954	0.9726	0.8450
	0	0	1	1	1	0.9886	0.8690	**0.9944**	0.9130
	0.5	0.5	0.7	1	0.7	0.6446	0.4128	**0.6652**	0.4518
	1	1	1	1	0	**0.2332**	0.1604	0.1922	0.1542
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9974	**1.0000**	0.9994
	0	0.25	0.5	1	0.25	0.9910	0.8834	**0.9918**	0.9148
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	**First**	Last	First	Last
Exponential	0	0	0	0	0	0.0474	0.0500	0.0532	0.0486
	0.5	0.5	0.5	1	1	0.7022	0.4444	**0.7274**	0.5008
	0	0.5	1	1	0.7	0.9994	0.9746	**0.9998**	0.9890
	0	0.5	1	1	1	0.9984	0.9360	**0.9990**	0.9718
	0	0	1	1	1	**0.9996**	0.9686	**0.9996**	0.9844
	0.5	0.5	0.7	1	0.7	**0.9362**	0.7148	**0.9342**	0.7696
	1	1	1	1	0	**0.2996**	0.1822	0.2370	0.1788
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**1.0000**	0.9922	**1.0000**	0.9972
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0534	0.0536	0.0564	0.0536
	0.5	0.5	0.5	1	1	0.3400	0.2208	**0.3582**	0.2382
	0	0.5	1	1	0.7	0.9216	0.6972	**0.9252**	0.7564
	0	0.5	1	1	1	0.8818	0.6300	**0.9044**	0.6902
	0	0	1	1	1	0.9388	0.7074	**0.9472**	0.7748
	0.5	0.5	0.7	1	0.7	0.5062	0.3130	**0.5140**	0.3416
	1	1	1	1	0	**0.1912**	0.1332	0.1558	0.1286
	0	0.4	0.8	1.6	0.4	**0.9998**	0.9736	0.9990	0.9808
	0	0.25	0.5	1	0.25	0.9458	0.7306	**0.9474**	0.7866

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.8. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.2. Treatments=5, IBD =15, CRD=15, RCBD=15 Peak=4, p=0.2

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0478	0.0444	0.0566	0.0444
	0.5	0.5	0.5	1	1	0.4840	0.3142	**0.5122**	0.3228
	0	0.5	1	1	0.7	0.9926	0.9084	**0.9946**	0.9210
	0	0.5	1	1	1	0.9834	0.8564	**0.9870**	0.8836
	0	0	1	1	1	0.9958	0.9200	**0.9976**	0.9438
	0.5	0.5	0.7	1	0.7	**0.7056**	0.4722	**0.7098**	0.5026
	1	1	1	1	0	**0.2530**	0.1738	**0.2112**	0.1662
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	0.9944	0.9318	**0.9954**	0.9448

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0450	0.0510	0.0518	0.0454
	0.5	0.5	0.5	1	1	0.7754	0.4980	**0.7984**	0.5534
	0	0.5	1	1	0.7	**1.0000**	0.9904	**1.0000**	0.9922
	0	0.5	1	1	1	0.9988	0.9718	0.9994	0.9812
	0	0	1	1	1	0.9998	0.9860	**1.0000**	0.9946
	0.5	0.5	0.7	1	0.7	**0.9626**	0.7882	0.9578	0.8134
	1	1	1	1	0	**0.3180**	0.2086	0.2598	0.2036
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**1.0000**	0.9978	**1.0000**	0.9984

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0554	0.0484	0.0524	0.0492
	0.5	0.5	0.5	1	1	0.3718	0.2466	**0.3812**	0.2504
	0	0.5	1	1	0.7	0.9562	0.7602	**0.9588**	0.7942
	0	0.5	1	1	1	0.9276	0.7028	**0.9348**	0.7412
	0	0	1	1	1	0.9602	0.7894	**0.9682**	0.8200
	0.5	0.5	0.7	1	0.7	0.5514	0.3684	**0.5552**	0.3788
	1	1	1	1	0	**0.2010**	0.1392	0.1698	0.1442
	0	0.4	0.8	1.6	0.4	0.9996	0.9908	**0.9998**	0.9904
	0	0.25	0.5	1	0.25	**0.9672**	0.8034	**0.9690**	0.8388

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.9. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.4. Treatments=5, IBD =5, CRD=10, RCBD=5 Peak=4, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0558	0.0466	0.0502	0.0510
	0.5	0.5	0.5	1	1	0.2660	0.2294	**0.2798**	0.2328
	0	0.5	1	1	0.7	0.8252	0.7364	**0.8412**	0.7650
	0	0.5	1	1	1	0.7712	0.6654	**0.7982**	0.6990
	0	0	1	1	1	0.8574	0.7480	**0.8670**	0.7912
	0.5	0.5	0.7	1	0.7	0.3952	0.3434	**0.4144**	0.3448
	1	1	1	1	0	**0.1520**	0.1382	0.1346	0.1362
	0	0.4	0.8	1.6	0.4	0.9960	0.9890	**0.9974**	0.9924
	0	0.25	0.5	1	0.25	**0.8734**	0.7824	0.8696	0.8172

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0492	0.0498	0.0500	0.0532
	0.5	0.5	0.5	1	1	0.4288	0.3676	**0.4476**	0.3682
	0	0.5	1	1	0.7	0.9648	0.9250	**0.9668**	0.9414
	0	0.5	1	1	1	0.9324	0.8694	**0.9386**	0.8946
	0	0	1	1	1	0.9518	0.9082	**0.9666**	0.9244
	0.5	0.5	0.7	1	0.7	**0.6896**	0.5848	0.6762	0.6086
	1	1	1	1	0	**0.1914**	0.1426	0.1474	0.1462
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9998	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	0.9852	0.9680	**0.9866**	0.9758

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0528	0.0556	0.0544	0.0540
	0.5	0.5	0.5	1	1	0.2044	0.1826	**0.2148**	0.1890
	0	0.5	1	1	0.7	0.6670	0.5758	**0.6898**	0.6074
	0	0.5	1	1	1	0.5990	0.5252	**0.6380**	0.5454
	0	0	1	1	1	0.6824	0.5930	**0.7264**	0.6304
	0.5	0.5	0.7	1	0.7	0.3018	0.2648	0.2956	0.2730
	1	1	1	1	0	**0.1276**	0.1148	0.1186	0.1078
	0	0.4	0.8	1.6	0.4	**0.9610**	0.9220	0.9604	0.9362
	0	0.25	0.5	1	0.25	**0.7172**	0.6204	0.7116	0.6488

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.10. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.4.
Treatments=5, IBD =5, CRD=15, RCBD=5 Peak=4, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0502	0.0498	0.0494	0.0498
	0.5	0.5	0.5	1	1	0.3066	0.2940	**0.3232**	0.2916
	0	0.5	1	1	0.7	0.8804	0.8592	**0.8962**	0.8684
	0	0.5	1	1	1	0.8418	0.8064	**0.8576**	0.8130
	0	0	1	1	1	0.9084	0.8770	**0.9216**	0.8788
	0.5	0.5	0.7	1	0.7	0.4612	0.4120	**0.4460**	0.4260
	1	1	1	1	0	**0.1646**	0.1610	0.1466	0.1540
	0	0.4	0.8	1.6	0.4	**0.9996**	0.9994	0.9994	0.9982
	0	0.25	0.5	1	0.25	**0.9164**	0.8956	0.9160	0.9010

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0518	0.0480	0.0550	0.0476
	0.5	0.5	0.5	1	1	0.4704	0.4590	**0.5138**	0.4702
	0	0.5	1	1	0.7	0.9850	0.9800	**0.9882**	0.9858
	0	0.5	1	1	1	0.9634	0.9482	**0.9680**	0.9596
	0	0	1	1	1	0.9790	0.9702	**0.9876**	0.9768
	0.5	0.5	0.7	1	0.7	0.7652	0.7276	**0.7654**	0.7480
	1	1	1	1	0	**0.2052**	0.1952	0.1756	0.1748
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	0.9998
	0	0.25	0.5	1	0.25	0.9962	0.9964	0.9956	0.9976

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0516	0.0478	0.0504	0.0500
	0.5	0.5	0.5	1	1	**0.2438**	0.2166	0.2376	0.2204
	0	0.5	1	1	0.7	0.7570	0.7088	**0.7798**	0.7210
	0	0.5	1	1	1	0.6864	0.6502	**0.6990**	0.6552
	0	0	1	1	1	**0.7912**	0.7390	**0.7912**	0.7546
	0.5	0.5	0.7	1	0.7	**0.3472**	0.3326	0.3444	0.3350
	1	1	1	1	0	**0.1482**	0.1374	0.1320	0.1350
	0	0.4	0.8	1.6	0.4	**0.9862**	0.9766	**0.9832**	0.9782
	0	0.25	0.5	1	0.25	0.7924	0.7494	**0.7954**	0.7710

Results show that standardized first performed better than standardized last in all distinct tests.
In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.11. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.4. Treatments=5, IBD =10, CRD=5, RCBD=10 Peak=4, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0470	0.0506	0.0476	0.0520
	0.5	0.5	0.5	1	1	0.2928	0.2252	**0.2946**	0.2604
	0	0.5	1	1	0.7	0.8722	0.7252	**0.8840**	0.8226
	0	0.5	1	1	1	0.8170	0.6516	**0.8448**	0.7590
	0	0	1	1	1	0.8910	0.7456	**0.9204**	0.8498
	0.5	0.5	0.7	1	0.7	0.4368	0.3238	**0.4522**	0.3738
	1	1	1	1	0	**0.1636**	0.1386	0.1402	0.1272
	0	0.4	0.8	1.6	0.4	0.9976	0.9824	**0.9986**	0.9940
	0	0.25	0.5	1	0.25	0.9068	0.7776	**0.9110**	0.8344
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0524	0.0482	0.0544	0.0510
	0.5	0.5	0.5	1	1	0.4698	0.3270	**0.4850**	0.4108
	0	0.5	1	1	0.7	0.9832	0.9146	**0.9854**	0.9598
	0	0.5	1	1	1	0.9556	0.8558	**0.9676**	0.9218
	0	0	1	1	1	0.9772	0.9028	**0.9842**	0.9614
	0.5	0.5	0.7	1	0.7	**0.8338**	0.6366	0.8310	0.6884
	1	1	1	1	0	**0.1948**	0.1456	0.1534	0.1336
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9992	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	0.9936	0.9542	0.9954	0.9804
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0492	0.0512	0.0490	0.0524
	0.5	0.5	0.5	1	1	0.2188	0.1796	**0.2382**	0.2040
	0	0.5	1	1	0.7	0.7302	0.5730	**0.7364**	0.6614
	0	0.5	1	1	1	0.6814	0.5000	**0.6926**	0.6072
	0	0	1	1	1	0.7446	0.6010	**0.7708**	0.6966
	0.5	0.5	0.7	1	0.7	**0.3394**	0.2584	0.3218	0.2868
	1	1	1	1	0	**0.1334**	0.1180	0.1222	0.1058
	0	0.4	0.8	1.6	0.4	**0.9848**	0.9156	0.9792	0.9482
	0	0.25	0.5	1	0.25	**0.7732**	0.6116	0.7708	0.6786

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.12. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.4. Treatments=5, IBD =10, CRD=10, RCBD=10 Peak=4, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0470	0.0550	0.0558	0.0546
	0.5	0.5	0.5	1	1	0.3408	0.2518	**0.3548**	0.2658
	0	0.5	1	1	0.7	0.9408	0.7834	**0.9502**	0.8358
	0	0.5	1	1	1	0.8940	0.7246	**0.9142**	0.7764
	0	0	1	1	1	0.9526	0.8120	**0.9626**	0.8598
	0.5	0.5	0.7	1	0.7	**0.5204**	0.3654	0.5038	0.4076
	1	1	1	1	0	**0.1868**	0.1418	0.1628	0.1396
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9946	0.9996	0.9968
	0	0.25	0.5	1	0.25	**0.9610**	0.8320	0.9500	0.8726
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0488	0.0440	0.0486	0.0484
	0.5	0.5	0.5	1	1	0.5508	0.3894	**0.5898**	0.4168
	0	0.5	1	1	0.7	0.9962	0.9534	**0.9978**	0.9656
	0	0.5	1	1	1	0.9840	0.9032	**0.9884**	0.9340
	0	0	1	1	1	0.9940	0.9296	**0.9964**	0.9636
	0.5	0.5	0.7	1	0.7	**0.8392**	0.6488	0.8324	0.6968
	1	1	1	1	0	**0.2292**	0.1576	0.1898	0.1700
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**0.9994**	0.9806	0.9990	0.9900
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0496	0.0478	0.0466	0.0514
	0.5	0.5	0.5	1	1	0.2628	0.1916	**0.2816**	0.2076
	0	0.5	1	1	0.7	0.8114	0.6238	**0.8350**	0.6600
	0	0.5	1	1	1	0.7706	0.5644	**0.7894**	0.6056
	0	0	1	1	1	0.8452	0.6602	**0.8592**	0.7026
	0.5	0.5	0.7	1	0.7	**0.4072**	0.2866	0.3846	0.2940
	1	1	1	1	0	**0.1504**	0.1184	0.1252	0.1172
	0	0.4	0.8	1.6	0.4	0.9946	0.9474	**0.9956**	0.9636
	0	0.25	0.5	1	0.25	**0.8628**	0.6768	**0.8578**	0.7094

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.13. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.4.
Treatments=5, IBD =10, CRD=15, RCBD=10 Peak=4, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0498	0.0518	0.0516	0.0512
	0.5	0.5	0.5	1	1	0.3864	0.2858	**0.4058**	0.3008
	0	0.5	1	1	0.7	0.9636	0.8904	**0.9688**	0.8982
	0	0.5	1	1	1	0.9336	0.8290	**0.9440**	0.8534
	0	0	1	1	1	0.9730	0.8976	**0.9842**	0.9182
	0.5	0.5	0.7	1	0.7	0.5846	0.4444	**0.5928**	0.4580
	1	1	1	1	0	**0.2092**	0.1652	0.1780	0.1644
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9998	**1.0000**	0.9992
	0	0.25	0.5	1	0.25	0.9782	0.9148	**0.9814**	0.9256
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0530	0.0536	0.0480	0.0560
	0.5	0.5	0.5	1	1	0.6238	0.4792	**0.6548**	0.5060
	0	0.5	1	1	0.7	0.9984	0.9854	**0.9994**	0.9892
	0	0.5	1	1	1	0.9936	0.9592	**0.9944**	0.9692
	0	0	1	1	1	0.9980	0.9778	**0.9984**	0.9852
	0.5	0.5	0.7	1	0.7	0.8816	0.7662	**0.8870**	0.7856
	1	1	1	1	0	**0.2526**	0.1882	0.2104	0.1942
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9998	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	0.9996	0.9976	**1.0000**	0.9980
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0540	0.0512	0.0538	0.0518
	0.5	0.5	0.5	1	1	0.3018	0.2258	**0.3226**	0.2464
	0	0.5	1	1	0.7	0.8736	0.7314	**0.8852**	0.7606
	0	0.5	1	1	1	0.8212	0.6586	**0.8424**	0.6944
	0	0	1	1	1	0.8840	0.7552	**0.9110**	0.7886
	0.5	0.5	0.7	1	0.7	**0.4420**	0.3320	**0.4368**	0.3522
	1	1	1	1	0	**0.1662**	0.1316	0.1442	0.1320
	0	0.4	0.8	1.6	0.4	**0.9988**	0.9786	0.9984	0.9860
	0	0.25	0.5	1	0.25	**0.9082**	0.7856	0.8990	0.7882

Results show that standardized first performed better than standardized last in all distinct tests.
In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.14. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.4. Treatments=5, IBD =15, CRD=5, RCBD=15 Peak=4, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0484	0.0464	0.0494	0.0516
	0.5	0.5	0.5	1	1	0.3530	0.2768	**0.3766**	0.3184
	0	0.5	1	1	0.7	0.9480	0.8192	**0.9530**	0.9044
	0	0.5	1	1	1	0.8988	0.7726	**0.9288**	0.8610
	0	0	1	1	1	0.9622	0.8518	**0.9718**	0.9354
	0.5	0.5	0.7	1	0.7	0.5324	0.4050	**0.5424**	0.4658
	1	1	1	1	0	**0.1948**	0.1530	0.1516	0.1312
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9972	1.0000	0.9992
	0	0.25	0.5	1	0.25	**0.9666**	0.8702	0.9580	0.9226
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0534	0.0490	0.0514	0.0546
	0.5	0.5	0.5	1	1	0.5766	0.4264	**0.6218**	0.5426
	0	0.5	1	1	0.7	0.9960	0.9614	**0.9974**	0.9872
	0	0.5	1	1	1	0.9870	0.9244	**0.9896**	0.9762
	0	0	1	1	1	0.9950	0.9582	**0.9974**	0.9876
	0.5	0.5	0.7	1	0.7	0.8424	0.6932	**0.8492**	0.7662
	1	1	1	1	0	**0.2444**	0.1856	0.1732	0.1552
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**0.9996**	0.9852	0.9984	0.9972
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0438	0.0478	0.0518	0.0474
	0.5	0.5	0.5	1	1	**0.2846**	0.2052	**0.2846**	0.2592
	0	0.5	1	1	0.7	0.8308	0.6854	**0.8502**	0.7690
	0	0.5	1	1	1	0.7692	0.6152	**0.7968**	0.7236
	0	0	1	1	1	0.8492	0.7022	**0.8818**	0.8194
	0.5	0.5	0.7	1	0.7	0.4026	0.3100	**0.4108**	0.3450
	1	1	1	1	0	**0.1594**	0.1254	0.1328	0.1142
	0	0.4	0.8	1.6	0.4	**0.9952**	0.9602	**0.9952**	0.9830
	0	0.25	0.5	1	0.25	0.8648	0.7090	**0.8772**	0.7996

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.15. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.4. Treatments=5, IBD =15, CRD=10, RCBD=15 Peak=4, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0452	0.0478	0.0526	0.0496
	0.5	0.5	0.5	1	1	0.4148	0.2758	**0.4328**	0.3022
	0	0.5	1	1	0.7	0.9766	0.8290	**0.9796**	0.8828
	0	0.5	1	1	1	0.9486	0.7702	**0.9628**	0.8364
	0	0	1	1	1	0.9822	0.8418	**0.9858**	0.9082
	0.5	0.5	0.7	1	0.7	0.6046	0.4112	**0.6118**	0.4392
	1	1	1	1	0	**0.2274**	0.1574	**0.1858**	0.1450
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9974	**1.0000**	0.9986
	0	0.25	0.5	1	0.25	**0.9868**	0.8708	0.9854	0.9074
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0490	0.0492	0.0464	0.0460
	0.5	0.5	0.5	1	1	0.6778	0.4380	**0.7100**	0.4954
	0	0.5	1	1	0.7	**0.9996**	0.9690	**0.9996**	0.9852
	0	0.5	1	1	1	0.9950	0.9326	**0.9982**	0.9594
	0	0	1	1	1	0.9988	0.9610	**0.9998**	0.9800
	0.5	0.5	0.7	1	0.7	**0.9126**	0.6978	0.9068	0.7506
	1	1	1	1	0	0.2714	0.1720	**0.2196**	0.1614
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**1.0000**	0.9894	**1.0000**	0.9964
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0488	0.0476	0.0470	0.0482
	0.5	0.5	0.5	1	1	0.3088	0.2070	**0.3338**	0.2330
	0	0.5	1	1	0.7	0.8844	0.6788	**0.9018**	0.7354
	0	0.5	1	1	1	0.8560	0.5902	**0.8696**	0.6866
	0	0	1	1	1	0.9150	0.6974	**0.9368**	0.7696
	0.5	0.5	0.7	1	0.7	0.4644	0.3002	**0.4730**	0.3492
	1	1	1	1	0	**0.1750**	0.1214	0.1458	0.1230
	0	0.4	0.8	1.6	0.4	0.9988	0.9630	**0.9994**	0.9830
	0	0.25	0.5	1	0.25	0.9212	0.7194	**0.9242**	0.7764

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

Table F.16. Mixed Design for Incomplete Block Design, Completely Randomized Design and Randomized Complete Block Design for Treatment Five at Peak 4 with probability of 0.4. Treatments=5, IBD =15, CRD=15, RCBD=15 Peak=4, p=0.4

Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Normal	0	0	0	0	0	0.0490	0.0524	0.0478	0.0534
	0.5	0.5	0.5	1	1	0.4582	0.3092	**0.4690**	0.3338
	0	0.5	1	1	0.7	0.9856	0.9046	**0.9898**	0.9244
	0	0.5	1	1	1	0.9758	0.8436	**0.9772**	0.8790
	0	0	1	1	1	0.9918	0.9112	**0.9960**	0.9406
	0.5	0.5	0.7	1	0.7	0.6734	0.4572	**0.6782**	0.4878
	1	1	1	1	0	**0.2488**	0.1694	0.2052	0.1662
	0	0.4	0.8	1.6	0.4	**1.0000**	0.9996	**1.0000**	0.9998
	0	0.25	0.5	1	0.25	**0.9940**	0.9260	0.9900	0.9392
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
Exponential	0	0	0	0	0	0.0488	0.0536	0.0502	0.0494
	0.5	0.5	0.5	1	1	0.7222	0.4966	**0.7630**	0.5288
	0	0.5	1	1	0.7	**1.0000**	0.9886	0.9998	0.9950
	0	0.5	1	1	1	0.9986	0.9664	**0.9992**	0.9784
	0	0	1	1	1	**1.0000**	0.9858	**1.0000**	0.9916
	0.5	0.5	0.7	1	0.7	0.9452	0.7860	**0.9512**	0.8096
	1	1	1	1	0	**0.3024**	0.2028	0.2324	0.1918
	0	0.4	0.8	1.6	0.4	**1.0000**	1.0000	**1.0000**	1.0000
	0	0.25	0.5	1	0.25	**1.0000**	0.9968	**1.0000**	0.9984
Distribution	Location					Non-modification		Distance-Modification	
	μ_1	μ_2	μ_3	μ_4	μ_5	First	Last	First	Last
T distribution with 3 degrees of freedom	0	0	0	0	0	0.0546	0.0472	0.0504	0.0474
	0.5	0.5	0.5	1	1	**0.3532**	0.2334	**0.3616**	0.2460
	0	0.5	1	1	0.7	**0.9304**	0.7506	**0.9428**	0.7988
	0	0.5	1	1	1	**0.8882**	0.6878	**0.9076**	0.7442
	0	0	1	1	1	**0.9472**	0.7732	**0.9584**	0.8214
	0.5	0.5	0.7	1	0.7	**0.5244**	0.3498	**0.5326**	0.3708
	1	1	1	1	0	**0.1914**	0.1422	**0.1630**	0.1394
	0	0.4	0.8	1.6	0.4	**0.9990**	0.9866	**1.0000**	0.9910
	0	0.25	0.5	1	0.25	**0.9540**	0.7886	**0.9546**	0.8308

Results show that standardized first performed better than standardized last in all distinct tests. In general, Distance-Modification test relatively performed better than the Non-modification test.

APPENDIX G. DERIVATION OF THE EXPECTED VALUES AND VARIANCES FOR MODIFIED MACK-WOLFE

Here we looked at the derivation of the general formulas for the exact means and variances for the Modified Mack-Wolfe (1981) for CRD, Modified Kim-Kim (1992) for RCBD and modified Magel and Ndungu (2011) (2011) for Complete and Incomplete blocks. The general formulas for the exact means and variances for the modified Mack-Wolfe (1981) test were derived by Esra and Fikri and are given in equations (2.34) to (2.37). Derivation of the general formulas for Distance Squared Modification of Mack-Wolfe (1981) for CRD and Distance Modification and Distance Squared Modification of Kim-Kim (1992) for RCBD are shown below. These formulas are in general form and are therefore used when building the simulation code.

In calculating the mean and variance for a test statistic for real data, it becomes more tedious and cumbersome to physically calculate the mean and variance. Thus, the need to derive a formula that will extricate the exact mean and variance for modified Mack-Wolfe (1981) for CRD, Modified Kim-Kim (1992) for RCBD, for populations in three at peak 2, populations in four at peaks 2 and 3 and populations in five peaks at 2,3 and 4. Two assumptions were made in deriving the formulas:

All sample sizes are equal. That is $n = n_1 = n_2 = n_3 = \cdots = n_k$ where k is the number of treatments in the part of the completely randomized design.

Considering the portion of randomized complete block design, $n_{ij} = 1$, for $i = 1,2,3, \ldots, b$ and $j = 1,2,3, \ldots, k$.

After deriving the formula, we saw that, treatments three with peaks at one and three have the same means and variances. Similarly, with treatments four, peaks at one and four have the same means and variances. Again, with four treatments, peaks at two and three have the same means and variances. Also, with treatments five, peaks at two and four have the same means and variances. Furthermore, with treatment five, peaks at one and five also have the same means and variances.

From the results of Tryon and Hettmansperger (1973), for $U_{ij} = \#(X_{iu} < X_{jv})$ under H_0, the null values are given by

$$E_0(U_{ij}) = \frac{n_i n_j}{2} \quad \forall\, i \neq j \tag{G.1}$$

$$Var_0(U_{ij}) = \frac{n_i n_j(n_i + n_j + 1)}{12} \quad \forall\, i \neq j \tag{G.2}$$

$$Cov_0(U_{ij,}U_{il}) = Cov_0(U_{ji,}U_{li}) = \frac{n_i n_j n_l}{12} \;\; if \;\; i, j, l \text{ are all different,} \tag{G.3}$$

$$Cov_0(U_{ij,}U_{li}) = Cov_0(U_{ji,}U_{il}) = -\frac{n_i n_j n_l}{12} \;\; if \;\; i, j, l \text{ are all different,} \tag{G.4}$$

$$Cov_0(U_{ij,}U_{lm}) = 0 \;\; if \;\; i, j, l, m \text{ are all different,} \tag{G.5}$$

Using the above, we derive the following formulas.

G.1. Mean and Variance Formula for Three Treatments with Peaks at 1 or 3

Modified Mack-Wolfe: The exact mean and variance of the modified Mack-Wolfe for Three treatments with peaks at one or three for equal sample size is given by

$H_0: \mu_1 = \mu_2 = \mu_3$

Versus alternative

$H_a: \mu_1 \leq \mu_2 \leq \mu_3$

Case 1 and Case 2 are the same:

Distance-Modification Modification

Expectation of Case 1

Case: $A_1 = U_{12} + 2U_{13} + U_{23}$

Squaring the coefficient of case 1, we obtain case 2

Case: $A_2 = U_{12} + 4U_{13} + U_{23}$

Taking expectation of either case 1, we obtain

$$E_0(A_1) = E_0(U_{12} + U_{13} + U_{23}) = E_0(U_{12}) + 2E_0(U_{13}) + E_0(U_{23})$$

$$= \frac{n_1 n_2}{2} + \frac{2n_1 n_3}{2} + \frac{n_2 n_3}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$E_0(A_1) = \frac{n * n}{2} + \frac{2n * n}{2} + \frac{n * n}{2} = \frac{4n^2}{2} = 2n^2 \qquad \text{(G.1.1)}$$

$$\begin{aligned} Var_0(A_1) = Var_0(U_{12} + 2U_{13} + U_{23}) \\ = Var_0(U_{12}) + 4Var_0(U_{13}) + Var_0(U_{23}) + 4Cov_0(U_{12}, U_{13}) + 2Cov_0(U_{12}, U_{23}) \\ + 4Cov_0(U_{13}, U_{23}) \end{aligned}$$

$$= \frac{n_1 n_2(n_1 + n_2 + 1)}{12} + \frac{4n_1 n_3(n_1 + n_3 + 1)}{12} + \frac{n_2 n_3(n_2 + n_3 + 1)}{12} + \frac{4n_1 n_2 n_3}{12} - \frac{2n_1 n_2 n_3}{12} + \frac{4n_1 n_2 n_3}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$Var_0(A_1) = \frac{6n^2(2n + 1)}{12} + \frac{6n^3}{12} = \frac{n^2}{2}(3n + 1) \qquad \text{(G.1.2)}$$

Modified Kim -Kim: The exact mean and variance of the distance modification of Kim - Kim for Three treatments with peaks at one or three for equal sample size is given by

$$E_0(KK_1) = E_0(KK_3) = 2bn^2 \qquad \text{(G.1.3)}$$

$$Var_0(KK_1) = Var_0(KK_3) = \frac{n^2 b}{2}(3n + 1) \qquad \text{(G.1.4)}$$

Distance Squared Modification

Expectation of Case 2

Case: $A_2 = U_{12} + 4U_{13} + U_{23}$

Taking expectation of either case 2, we obtain,

$E_0(A_1) = E_0(U_{12} + 4U_{13} + U_{23}) = E_0(U_{12}) + 4E_0(U_{13}) + E_0(U_{23})$

$= \frac{n_1 n_2}{2} + \frac{4n_1 n_3}{2} + \frac{n_2 n_3}{2}$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$E_0(A_1) = \frac{n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} = \frac{6n^2}{2} = 3n^2 \qquad \text{(G.1.5)}$$

$Var_0(A_1) = Var_0(U_{12} + 4U_{13} + U_{23})$

$= Var_0(U_{12}) + 16Var_0(U_{13}) + Var_0(U_{23}) + 8Cov_0(U_{12}, U_{13})$

$+ 2Cov_0(U_{12}, U_{23}) + 8Cov_0(U_{13}, U_{23})$

$$= \frac{n_1 n_2(n_1 + n_2 + 1)}{12} + \frac{16n_1 n_3(n_1 + n_3 + 1)}{12} + \frac{n_2 n_3(n_2 + n_3 + 1)}{12} + \frac{8n_1 n_2 n_3}{12} - \frac{2n_1 n_2 n_3}{12} + \frac{8n_1 n_2 n_3}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$Var_0(A_1) = \frac{18n^2(2n + 1)}{12} + \frac{14n^3}{12} = \frac{n^2}{6}(25n + 9) \qquad \text{(G.1.6)}$$

Modified Kim -Kim: The exact mean and variance of the distance squared modification of Kim - Kim Kim for Three treatments with peaks at one or three for equal sample size is given by

$$E_0(KK_1) = E_0(KK_3) = 3bn^2 \tag{G.1.7}$$

$$Var_0(KK_1) = Var_0(KK_3) = \frac{n^2 b}{6}(25n + 9) \tag{G.1.8}$$

G.2. Mean and Variance Formula for Three Treatments with Peak at 2

Modified Mack-Wolfe: The exact mean and variance of the modified Mack-Wolfe for Three treatments with peak at two for equal sample size is given by

$$H_0: \mu_1 = \mu_2 = \mu_3$$

Versus alternative

$$H_a: \mu_1 \leq \mu_2 \geq \mu_3$$

Case 1 and Case 2 are the same:

Case. $A_1 = U_{12} + U_{32}$

Squaring the coefficient of Case 1, we obtain case 2

Case 2: $A_1 = U_{12} + U_{32}$

Taking expectation of either case 1 or case 2, we obtain,

$$E_0(A_2) = E_0(U_{12} + U_{32}) = E_0(U_{12}) + E_0(U_{32}) = \frac{n_1 n_2}{2} + \frac{n_3 n_2}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$E_0(A_2) = \frac{n_1 n_2}{2} + \frac{n_3 n_2}{2} = \frac{n * n}{2} + \frac{n * n}{2} = \frac{2n^2}{2} = n^2 \tag{G.2.1}$$

$$Var_0(A_2) = Var_0(U_{12} + U_{32}) = Var_0(U_{12}) + Var_0(U_{32}) + 2Cov_0(U_{12}, U_{32})$$

$$= \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} + \frac{n_3 n_2 (n_3 + n_2 + 1)}{12} + \frac{2 n_1 n_2 n_3}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$Var_0(A_2) = \frac{n_1 n_2(n_1 + n_2 + 1)}{12} + \frac{n_3 n_2(n_3 + n_2 + 1)}{12} + \frac{2n_1 n_2 n_3}{12}$$
$$= \frac{n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{2n^3}{12} = \frac{2n^2(2n + 1)}{12} + \frac{2n^3}{12} = \frac{n^2(3n + 1)}{6}$$

$$Var_0(A_2) = \frac{n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{2n^3}{12} = \frac{2n^2(2n + 1)}{12} + \frac{2n^3}{12} \qquad \text{(G.2.2)}$$

$$= \frac{n^2(3n + 1)}{6}$$

Modified Kim -Kim: The exact mean and variance of the modified Kim - Kim for Three treatments with peak at two for equal sample size is given by

$$E_0(KK_2) = n^2 b \qquad \text{(G.2.3)}$$

$$Var_0(KK_2) = \frac{n^2 b(3n + 1)}{6} \qquad \text{(G.2.4)}$$

G.3 . Mean and Variance Formula for Four Treatments with Peak at 1 OR 4

Modified Mack-Wolfe: The exact mean and variance of the modified Mack-Wolfe for Four treatments with peaks at one or four equal sample size is given by

H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$

Versus alternative

H_a: $\mu_1 \leq \mu_2 \leq \mu_3 \leq \mu_4$

Case 1 and Case 2 are the same:

Case: $A_1 = U_{12} + 2U_{13} + 3U_{14} + U_{23} + 2U_{24} + U_{34}$

Squaring the coefficient of Case 1, we obtain case 2

Case: $A_2 = U_{12} + 4U_{13} + 9U_{14} + U_{23} + 4U_{24} + U_{34}$

Distance-Modification Modification

Expectation of Case 1

Taking expectation of either case 1, we obtain,

$$E_0(A_1) = E_0(U_{12} + 2U_{13} + 3U_{14} + U_{23} + 2U_{24} + U_{34})$$
$$= E_0(U_{12}) + 2E_0(U_{13}) + 3E_0(U_{14}) + E_0(U_{23}) + 2E_0(U_{24}) + E_0(U_{34})$$

$$= \frac{n_1 n_2}{2} + \frac{2n_1 n_3}{2} + \frac{3n_1 n_4}{2} + \frac{n_2 n_3}{2} + \frac{2n_2 n_4}{2} + \frac{n_3 n_4}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$E_0(A_1) = \frac{n*n}{2} + \frac{2n*n}{2} + \frac{3n*n}{2} + \frac{n*n}{2} + \frac{2n*n}{2} + \frac{n*n}{2} = 5n^2 \qquad \text{(G.3.1)}$$

$$Var_0(A_1) = Var_0(U_{12} + 2U_{13} + 3U_{14} + U_{23} + 2U_{24} + U_{34})$$

$$= Var_0(U_{12}) + 4Var_0(U_{13}) + 9Var_0(U_{14}) + Var_0(U_{23}) + 4Var_0(U_{24})$$

$$+ Var_0(U_{34}) + 4Cov_0(U_{12}, U_{13}) + 6Cov_0(U_{12}, U_{14}) + 2Cov_0(U_{12}, U_{23})$$

$$+ 4Cov_0(U_{12}, U_{24}) + 2Cov_0(U_{12}, U_{34}) + 12Cov_0(U_{13}, U_{14}) + 4Cov_0(U_{13}, U_{23})$$

$$+ 8Cov_0(U_{13}, U_{24}) + 4Cov_0(U_{13}, U_{34}) + 6Cov_0(U_{14}, U_{23}) + 12Cov_0(U_{14}, U_{24})$$

$$+ 6Cov_0(U_{14}, U_{34}) + 4Cov_0(U_{23}, U_{24}) + 2Cov_0(U_{23}, U_{34}) + 4Cov_0(U_{24}, U_{34})$$

$$= \frac{n_1 n_2(n_1 + n_2 + 1)}{12} + \frac{4n_1 n_3(n_1 + n_3 + 1)}{12} + \frac{9n_1 n_4(n_1 + n_4 + 1)}{12} + \frac{n_2 n_3(n_2 + n_3 + 1)}{12}$$
$$+ \frac{4n_2 n_4(n_2 + n_4 + 1)}{12} + \frac{n_3 n_4(n_3 + n_4 + 1)}{12} + \frac{4n_1 n_2 n_3}{12} + \frac{6n_1 n_2 n_4}{12} - \frac{2n_1 n_2 n_3}{12}$$
$$- \frac{4n_1 n_2 n_4}{12} + \frac{12n_1 n_3 n_4}{12} + \frac{4n_1 n_2 n_3}{12} - \frac{4n_1 n_3 n_4}{12} + \frac{12n_1 n_2 n_4}{12} + \frac{6n_1 n_3 n_4}{12}$$
$$+ \frac{4n_2 n_3 n_4}{12} - \frac{2n_2 n_3 n_4}{12} + \frac{4n_2 n_3 n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$Var_0(A_1) = \frac{n^2(2n+1)}{12} + \frac{4n^2(2n+1)}{12} + \frac{9n^2(2n+1)}{12} + \frac{n^2(2n+1)}{12} + \frac{4n^2(2n+1)}{12} + \frac{n^2(2n+1)}{12} + \frac{4n^3}{12} + \frac{6n^3}{12} - \frac{2n^3}{12} - \frac{4n^3}{12} + \frac{12n^3}{12} + \frac{4n^3}{12} - \frac{4n^3}{12} + \frac{12n^3}{12} + \frac{6n^3}{12} + \frac{4n^3}{12} - \frac{2n^3}{12} + \frac{4n^3}{12} = \frac{n^2(20n+5)}{3} \quad (G.3.2)$$

Modified Kim -Kim: The exact mean and variance of the distance modification of Kim - Kim for Four treatments with peaks at one or four for equal sample size is given by

$$E_0(KK_1) = E_0(KK_4) = 5bn^2 \quad (G.3.3)$$

$$Var_0(KK_1) = Var_0(KK_4) = \frac{n^2 b(20n+5)}{3} \quad (G.3.4)$$

Distance Squared Modification

Expectation of Case 2

Case: $A_2 = U_{12} + 4U_{13} + U_{23}$

Taking expectation of either case 2, we obtain,

$E_0(A_2) = E_0(U_{12} + 4U_{13} + 9U_{14} + U_{23} + 4U_{24} + U_{34})$

$$= E_0(U_{12}) + 4E_0(U_{13}) + 9E_0(U_{14}) + E_0(U_{23}) + 4E_0(U_{24}) + E_0(U_{34})$$

$$= \frac{n_1 n_2}{2} + \frac{4n_1 n_3}{2} + \frac{9n_1 n_4}{2} + \frac{n_2 n_3}{2} + \frac{4n_2 n_4}{2} + \frac{n_3 n_4}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$E_0(A_2) = \frac{n*n}{2} + \frac{4n*n}{2} + \frac{9n*n}{2} + \frac{n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} = 10n^2 \quad (G.3.5)$$

$$Var_0(A_2) = Var_0(U_{12} + 4U_{13} + 9U_{14} + U_{23} + 4U_{24} + U_{34})$$

$$= Var_0(U_{12}) + 16Var_0(U_{13}) + 81Var_0(U_{14}) + Var_0(U_{23}) + 16Var_0(U_{24})$$

$$+ Var_0(U_{34}) + 8Cov_0(U_{12}, U_{13}) + 18Cov_0(U_{12}, U_{14}) + 2Cov_0(U_{12}, U_{23})$$

$$+ 8Cov_0(U_{12}, U_{24}) + 2Cov_0(U_{12}, U_{34}) + 72Cov_0(U_{13}, U_{14}) + 8Cov_0(U_{13}, U_{23})$$

$$+ 32Cov_0(U_{13}, U_{24}) + 8Cov_0(U_{13}, U_{34}) + 18Cov_0(U_{14}, U_{23})$$

$$+ 72Cov_0(U_{14}, U_{24}) + 18Cov_0(U_{14}, U_{34}) + 8Cov_0(U_{23}, U_{24})$$

$$+ 2Cov_0(U_{23}, U_{34}) + 8Cov_0(U_{24}, U_{34})$$

$$= \frac{n_1n_2(n_1+n_2+1)}{12} + \frac{16n_1n_3(n_1+n_3+1)}{12} + \frac{81n_1n_4(n_1+n_4+1)}{12} + \frac{n_2n_3(n_2+n_3+1)}{12}$$
$$+ \frac{16n_2n_4(n_2+n_4+1)}{12} + \frac{n_3n_4(n_3+n_4+1)}{12} + \frac{8n_1n_2n_3}{12} + \frac{18n_1n_2n_4}{12}$$
$$- \frac{2n_1n_2n_3}{12} - \frac{8n_1n_2n_4}{12} + \frac{72n_1n_3n_4}{12} + \frac{8n_1n_2n_3}{12} - \frac{8n_1n_3n_4}{12} + \frac{72n_1n_2n_4}{12}$$
$$+ \frac{18n_1n_3n_4}{12} + \frac{8n_2n_3n_4}{12} - \frac{2n_2n_3n_4}{12} + \frac{8n_2n_3n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n$

$$Var_0(A_2) = \frac{n^2(2n+1)}{12} + \frac{16n^2(2n+1)}{12} + \frac{81n^2(2n+1)}{12} + \frac{n^2(2n+1)}{12} \quad \text{(G.3.6)}$$
$$+ \frac{16n^2(2n+1)}{12} + \frac{n^2(2n+1)}{12} + \frac{8n^3}{12} + \frac{18n^3}{12} - \frac{2n^3}{12} - \frac{8n^3}{12}$$
$$+ \frac{72n^3}{12} + \frac{8n^3}{12} - \frac{8n^3}{12} + \frac{72n^3}{12} + \frac{18n^3}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} + \frac{8n^3}{12}$$
$$= \frac{n^2(106n+29)}{3}$$

Modified Kim -Kim: The exact mean and variance of the distance squared modification of Kim - Kim for Four treatments with peaks at one or four for equal sample size is given by

$$E_0(KK_1) = E_0(KK_4) = 10bn^2 \quad \text{(G.3.7)}$$

$$Var_0(KK_1) = Var_0(KK_4) = \frac{n^2b(106n + 29)}{3} \quad \text{(G.3.8)}$$

G.4. Mean and Variance Formula for Four Treatments with Peak at 2

Modified Mack-Wolfe: The exact mean and variance of the modified Mack-Wolfe for four treatments with peak at two for equal sample size is given by

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$

Versus alternative

$H_a: \mu_1 \leq \mu_2 \geq \mu_3 \geq \mu_4$

Case1: $A_2 = U_{12} + U_{32} + 2U_{42} + U_{43}$

Distance-Modification Modification

Expectation of Case 1

$$E_0(A_2) = E_0(U_{12}) + E_0(U_{32}) + 2E_0(U_{42}) + E_0(U_{43}) = \frac{n_1n_2}{2} + \frac{n_3n_2}{2} + \frac{2n_4n_2}{2} + \frac{n_4n_3}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n$

$$E_0(A_2) = \frac{n_1n_2}{2} + \frac{n_3n_2}{2} + \frac{2n_4n_2}{2} + \frac{n_4n_3}{2} = \frac{n*n}{2} + \frac{n*n}{2} + \frac{2n*n}{2} + \frac{n*n}{2} = \frac{5n^2}{2} \quad \text{(G.4.1)}$$

Variance of Case 1

Case1: $A_2 = U_{12} + U_{32} + 2U_{42} + U_{43}$

$$var_0(A_2) = var_0(U_{12} + U_{32} + 2U_{42} + U_{43})$$

$$var_0(A_2) = var_0(U_{12}) + var_0(U_{32}) + 4var_0(U_{42}) + var_0(U_{43}) + 2Cov_0(U_{12}, U_{32}) + 4Cov_0(U_{12}, U_{42}) + 2Cov_0(U_{12}, U_{43}) + 4Cov_0(U_{32}, U_{42}) + 2Cov_0(U_{32}, U_{43}) + 4Cov_0(U_{42}, U_{43})$$

$$= \frac{n_1 n_2(n_1 + n_2 + 1)}{12} + \frac{n_3 n_2(n_3 + n_2 + 1)}{12} + \frac{4n_4 n_2(n_4 + n_2 + 1)}{12} + \frac{n_4 n_3(n_4 + n_3 + 1)}{12} + \frac{2n_1 n_2 n_3}{12} + \frac{4n_1 n_2 n_4}{12} + \frac{4n_2 n_3 n_4}{12} - \frac{2n_2 n_3 n_4}{12} + \frac{4n_2 n_3 n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n$

$$Var_0(A_1) = \frac{n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{4n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{2n^3}{12} + \frac{4n^3}{12} + \frac{4n^3}{12} - \frac{2n^3}{12} + \frac{4n^3}{12} = \frac{n^2(26n+7)}{12} \quad \text{(G.4.2)}$$

Note: $Cov_0(U_{12}, U_{43}) = 0$

Modified Kim -Kim: The exact mean and variance of the distance modification of Kim -Kim for Four treatments with peak at two for equal sample size is given by

$$E_0(KK_2) = \frac{5}{2}bn^2 \quad \text{(G.4.3)}$$

$$Var_0(KK_2) = \frac{n^2 b(26n+7)}{12} \quad \text{(G.4.4)}$$

Distance Squared Modification

Expectation of Case 2

Case 2. $A_2 = U_{12} + U_{32} + 4U_{42} + U_{43}$

$$E_0(A_2) = E_0(U_{12}) + E_0(U_{32}) + 4E_0(U_{42}) + E_0(U_{43}) = \frac{n_1 n_2}{2} + \frac{n_3 n_2}{2} + \frac{4n_4 n_2}{2} + \frac{n_4 n_3}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n$

$$E_0(A_2) = \frac{n_1 n_2}{2} + \frac{n_3 n_2}{2} + \frac{4n_4 n_2}{2} + \frac{n_4 n_3}{2} = \frac{n*n}{2} + \frac{n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} = \frac{7n^2}{2} \quad \text{(G.4.5)}$$

Variance of Case 2

Case1: $A_2 = U_{12} + U_{32} + 4U_{42} + U_{43}$

$var_0(A_2) = var_0(U_{12} + U_{32} + 4U_{42} + U_{43})$

$$var_0(A_2) = var_0(U_{12}) + var_0(U_{32}) + 16var_0(U_{42}) + var_0(U_{43}) + 2Cov_0(U_{12}, U_{32}) + 8Cov_0(U_{12}, U_{42}) + 2Cov_0(U_{12}, U_{43}) + 8Cov_0(U_{32}, U_{42}) + 2Cov_0(U_{32}, U_{43}) + 8Cov_0(U_{42}, U_{43})$$

$$= \frac{n_1n_2(n_1 + n_2 + 1)}{12} + \frac{n_3n_2(n_3 + n_2 + 1)}{12} + \frac{16n_4n_2(n_4 + n_2 + 1)}{12} + \frac{n_4n_3(n_4 + n_3 + 1)}{12} + \frac{2n_1n_2n_3}{12} + \frac{8n_1n_2n_4}{12} + \frac{8n_2n_3n_4}{12} - \frac{2n_2n_3n_4}{12} + \frac{8n_2n_3n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n$

$$Var_0(A_1) = \frac{n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{16n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{2n^3}{12} + \frac{8n^3}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} + \frac{8n^3}{12} = \frac{n^2(62n + 19)}{12} \quad \text{(G.4.6)}$$

Note: $Cov_0(U_{12}, U_{43}) = 0$

Modified Kim -Kim: The exact mean and variance of the distance squared modification of Kim - Kim for Four treatments with peak at two for equal sample size is given by

$$E_0(KK_2) = \frac{7}{2}bn^2 \quad \text{(G.4.7)}$$

$$Var_0(KK_2) = \frac{n^2b(62n + 19)}{12} \quad \text{(G.4.8)}$$

G.5. Mean and Variance Formula for Four Treatments with Peak at 3

Modified Mack-Wolfe: The exact mean and variance of the modified Mack-Wolfe for four treatments with peak at three for equal sample size is given by

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4$$

Versus alternative

$$H_a: \mu_1 \leq \mu_2 \leq \mu_3 \geq \mu_4$$

Distance-Modification Modification

Expectation of Case 1

Case1: $A_3 = U_{12} + 2U_{13} + U_{23} + U_{43}$

$$E_0(A_3) = E_0(U_{12}) + 2E_0(U_{13}) + E_0(U_{23}) + E_0(U_{43}) = \frac{n_1 n_2}{2} + \frac{2n_1 n_3}{2} + \frac{n_2 n_3}{2} + \frac{n_4 n_3}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n$

$$E_0(A_3) = \frac{n_1 n_2}{2} + \frac{2n_1 n_3}{2} + \frac{n_2 n_3}{2} + \frac{n_4 n_3}{2} = \frac{n * n}{2} + \frac{2n * n}{2} + \frac{n * n}{2} + \frac{n * n}{2} = \frac{5n^2}{2} \qquad \text{(G.5.1)}$$

Variance of Case 1

Case1: $A_3 = U_{12} + 2U_{13} + U_{23} + U_{43}$

$$var_0(A_3) = var_0(U_{12} + 2U_{13} + U_{23} + U_{43})$$

$$var_0(A_3) = var_0(U_{12}) + 4var_0(U_{13}) + var_0(U_{23}) + var_0(U_{43}) + 4Cov_0(U_{12}, U_{13}) + 2Cov_0(U_{12}, U_{23}) + 2Cov_0(U_{12}, U_{43}) + 4Cov_0(U_{13}, U_{23}) + 4Cov_0(U_{13}, U_{43}) + 2Cov_0(U_{23}, U_{43})$$

$$= \frac{n_1 n_2(n_1 + n_2 + 1)}{12} + \frac{4n_1 n_3(n_1 + n_3 + 1)}{12} + \frac{n_3 n_2(n_3 + n_2 + 1)}{12} + \frac{n_4 n_3(n_4 + n_3 + 1)}{12} + \frac{4n_1 n_2 n_3}{12} - \frac{2n_1 n_2 n_3}{12} + \frac{4n_1 n_2 n_3}{12} + \frac{4n_1 n_4 n_3}{12} + \frac{2n_2 n_3 n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n$

$$Var_0(A_3) = \frac{n^2(n+n+1)}{12} + \frac{4n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{4n^3}{12} - \frac{2n^3}{12} + \frac{4n^3}{12} + \frac{4n^3}{12} + \frac{2n^3}{12} = \frac{n^2(26n+7)}{12} \quad \text{(G.5.2)}$$

Note: $Cov_0(U_{12}, U_{43}) = 0$

Modified Kim -Kim: The exact mean and variance of the distance modification of Kim - Kim for Four treatments with peak at three for equal sample size is given by

$$E_0(KK_3) = \frac{5}{2}bn^2 \quad \text{(G.5.3)}$$

$$Var_0(KK_3) = \frac{n^2b(26n+7)}{12} \quad \text{(G.5.4)}$$

Distance Squared Modification

Expectation of Case 2

Case 2: $A_3 = U_{12} + 4U_{13} + U_{23} + U_{43}$

$$E_0(A_3) = E_0(U_{12}) + 4E_0(U_{13}) + E_0(U_{23}) + E_0(U_{43}) = \frac{n_1n_2}{2} + \frac{4n_1n_3}{2} + \frac{n_2n_3}{2} + \frac{n_4n_3}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n$

$$E_0(A_3) = \frac{n_1n_2}{2} + \frac{4n_1n_3}{2} + \frac{n_2n_3}{2} + \frac{n_4n_3}{2} = \frac{n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} + \frac{n*n}{2} = \frac{7n^2}{2} \quad \text{(G.5.5)}$$

Variance of Case 2

Case1: $A_3 = U_{12} + 4U_{13} + U_{23} + U_{43}$

$var_0(A_3) = var_0(U_{12} + 4U_{13} + U_{23} + U_{43})$

$$var_0(A_3) = var_0(U_{12}) + 16var_0(U_{13}) + var_0(U_{23}) + var_0(U_{43}) + 8Cov_0(U_{12}, U_{13}) + 2Cov_0(U_{12}, U_{23}) + 2Cov_0(U_{12}, U_{43}) + 8Cov_0(U_{13}, U_{23}) + 8Cov_0(U_{13}, U_{43}) + 2Cov_0(U_{23}, U_{43})$$

$$= \frac{n_1n_2(n_1 + n_2 + 1)}{12} + \frac{16n_1n_3(n_1 + n_3 + 1)}{12} + \frac{n_3n_2(n_3 + n_2 + 1)}{12} + \frac{n_4n_3(n_4 + n_3 + 1)}{12} + \frac{8n_1n_2n_3}{12} - \frac{2n_1n_2n_3}{12} + \frac{8n_1n_2n_3}{12} + \frac{8n_1n_4n_3}{12} + \frac{2n_2n_3n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n$

$$Var_0(A_3) = \frac{n^2(n + n + 1)}{12} + \frac{16n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} + \frac{8n^3}{12} + \frac{8n^3}{12} + \frac{2n^3}{12} = \frac{n^2(62n + 19)}{12} \quad \text{(G.5.6)}$$

Note: $Cov_0(U_{12}, U_{43}) = 0$

Modified Kim -Kim: The exact mean and variance of the distance squared modification of Kim - Kim for Four treatments with peak at three for equal sample size is given by

$$E_0(KK_3) = \frac{7}{2}bn^2 \quad \text{(G.5.7)}$$

$$Var_0(KK_3) = \frac{n^2(62n + 19)}{12} \quad \text{(G.5.8)}$$

G.6. Mean and Variance Formula for Five Treatments with Peak at 2

Modified Mack-Wolfe: The exact mean and variance of the modified Mack-Wolfe for Five Treatments with peak at two for equal sample size is given by

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

Versus alternative

$H_a: \mu_1 \leq \mu_2 \geq \mu_3 \geq \mu_4 \geq \mu_5$

Case1: $A_2 = U_{12} + U_{32} + 2U_{42} + U_{43} + 3U_{52} + 2U_{53} + U_{54}$

Case 2: $A_2 = U_{12} + U_{32} + 4U_{42} + U_{43} + 9U_{52} + 4U_{53} + U_{54}$

Distance-Modification Modification

Expectation of Case 1

$$E_0(A_2) = E_0(U_{12}) + E_0(U_{32}) + 2E_0(U_{42}) + E_0(U_{43}) + 3E_0(U_{52}) + 2E_0(U_{53}) + E_0(U_{54})$$

$$= \frac{n_1 n_2}{2} + \frac{n_3 n_2}{2} + \frac{2n_4 n_2}{2} + \frac{n_4 n_3}{2} + \frac{3n_5 n_2}{2} + \frac{2n_5 n_3}{2} + \frac{n_5 n_4}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$E_0(A_2) = \frac{n_1 n_2}{2} + \frac{n_3 n_2}{2} + \frac{2n_4 n_2}{2} + \frac{n_4 n_3}{2} + \frac{3n_5 n_2}{2} + \frac{2n_5 n_3}{2} + \frac{n_5 n_4}{2} \qquad \text{(G.6.1)}$$

$$= \frac{n*n}{2} + \frac{n*n}{2} + \frac{2n*n}{2} + \frac{n*n}{2} + \frac{3n*n}{2} + \frac{2n*n}{2} + \frac{n*n}{2}$$

$$= \frac{11n^2}{2}$$

Variance of Case 1

Case1: $A_2 = U_{12} + U_{32} + 2U_{42} + U_{43} + 3U_{52} + 2U_{53} + U_{54}$

$$var_0(A_2) = var_0(U_{12} + U_{32} + 2U_{42} + U_{43} + 3U_{52} + 2U_{53} + U_{54})$$

$$var_0(A_2) = var_0(U_{12}) + var_0(U_{32}) + 4var_0(U_{42}) + var_0(U_{43}) + 9var_0(U_{52}) + 4var_0(U_{53}) + var_0(U_{54}) + 2cov_0(U_{12}, U_{32}) + 4cov_0(U_{12}, U_{42}) + 2cov_0(U_{12}, U_{43}) + 6cov_0(U_{12}, U_{52}) + 4cov_0(U_{12}, U_{53}) + 2cov_0(U_{12}, U_{54}) + 4cov_0(U_{32}, U_{42}) + 2cov_0(U_{32}, U_{43}) + 6cov_0(U_{32}, U_{52}) + 4cov_0(U_{32}, U_{53}) + 2cov_0(U_{32}, U_{54}) + 4cov_0(U_{42}, U_{43}) + 12cov_0(U_{42}, U_{52}) + 8cov_0(U_{42}, U_{53}) + 4cov_0(U_{42}, U_{54}) + 6cov_0(U_{43}, U_{52}) + 4cov_0(U_{43}, U_{53}) + 2cov_0(U_{43}, U_{54}) + 12cov_0(U_{52}, U_{53}) + 6cov_0(U_{52}, U_{54}) + 4cov_0(U_{53}, U_{54})$$

$$= \frac{n_1n_2(n_1 + n_2 + 1)}{12} + \frac{n_3n_2(n_3 + n_2 + 1)}{12} + \frac{4n_4n_2(n_4 + n_2 + 1)}{12} + \frac{9n_2n_5(n_2 + n_5 + 1)}{12} + \frac{4n_3n_5(n_3 + n_5 + 1)}{12} + \frac{n_3n_4(n_3 + n_4 + 1)}{12} + \frac{n_5n_4(n_5 + n_4 + 1)}{12} + \frac{2n_1n_2n_3}{12} + \frac{4n_1n_2n_4}{12} + \frac{6n_1n_2n_5}{12} + \frac{4n_2n_3n_4}{12} - \frac{2n_2n_3n_4}{12} + \frac{6n_2n_3n_5}{12} - \frac{4n_2n_3n_5}{12} + \frac{4n_2n_3n_4}{12} + \frac{12n_2n_4n_5}{12} - \frac{4n_2n_4n_5}{12} + \frac{4n_3n_4n_5}{12} - \frac{2n_5n_3n_4}{12} + \frac{12n_2n_3n_5}{12} + \frac{6n_2n_4n_5}{12} + \frac{4n_5n_3n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$Var_0(A_2) = \frac{n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{4n^2(n + n + 1)}{12} + \frac{9n^2(n + n + 1)}{12} + \frac{4n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{2n^3}{12} + \frac{4n^3}{12} + \frac{6n^3}{12} + \frac{4n^3}{12} - \frac{2n^3}{12} + \frac{6n^3}{12} - \frac{4n^3}{12} + \frac{4n^3}{12} + \frac{12n^3}{12} - \frac{4n^3}{12} + \frac{4n^3}{12} - \frac{2n^3}{12} + \frac{12n^3}{12} + \frac{6n^3}{12} + \frac{4n^3}{12} = \frac{n^2(94n + 21)}{12} \qquad \text{(G.6.2)}$$

Note: $cov_0(U_{12}, U_{43}) = cov_0(U_{12}, U_{53}) = cov_0(U_{12}, U_{54}) = 0$

$cov_0(U_{32}, U_{54}) = cov_0(U_{42}, U_{53}) = cov_0(U_{43}, U_{52}) = 0$

Modified Kim -Kim: The exact mean and variance of the distance modification of Kim - Kim for Five treatments with peak at two for equal sample size is given by

$$E_0(KK_3) = \frac{11}{2} bn^2 \quad \text{(G.6.3)}$$

$$Var_0(KK_3) = \frac{n^2 b(94n + 21)}{12} \quad \text{(G.6.4)}$$

Distance Squared Modification

Expectation of Case 2

Case 2: $A_2 = U_{12} + U_{32} + 4U_{42} + U_{43} + 9U_{52} + 4U_{53} + U_{54}$

$E_0(A_2) = E_0(U_{12}) + E_0(U_{32}) + 4E_0(U_{42}) + E_0(U_{43}) + 9E_0(U_{52}) + 4E_0(U_{53}) + E_0(U_{54})$

$= \frac{n_1 n_2}{2} + \frac{n_3 n_2}{2} + \frac{4n_2 n_4}{2} + \frac{n_4 n_3}{2} + \frac{9n_5 n_2}{2} + \frac{4n_5 n_3}{2} + \frac{n_5 n_4}{2}$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$\begin{aligned} E_0(A_2) &= \frac{n_1 n_2}{2} + \frac{n_3 n_2}{2} + \frac{4n_2 n_4}{2} + \frac{n_4 n_3}{2} + \frac{9n_5 n_2}{2} + \frac{4n_5 n_3}{2} + \frac{n_5 n_4}{2} \\ &= \frac{n*n}{2} + \frac{n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} + \frac{9n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} \\ &= \frac{21n^2}{2} \end{aligned} \quad \text{(G.6.5)}$$

Variance of Case 2

Case1: $A_2 = U_{12} + U_{32} + 4U_{42} + U_{43} + 9U_{52} + 4U_{53} + U_{54}$

$var_0(A_2) = var_0(U_{12} + U_{32} + 4U_{42} + U_{43} + 9U_{52} + 4U_{53} + U_{54})$

$$var_0(A_2) = var_0(U_{12}) + var_0(U_{32}) + 16var_0(U_{42}) + var_0(U_{43}) + 81var_0(U_{52})$$

$$+ 16var_0(U_{53}) + var_0(U_{54}) + 2cov_0(U_{12}, U_{32}) + 8cov_0(U_{12}, U_{42})$$

$$+ 2cov_0(U_{12}, U_{43}) + 18cov_0(U_{12}, U_{52}) + 8cov_0(U_{12}, U_{53}) + 2cov_0(U_{12}, U_{53})$$

$$+ 8cov_0(U_{32}, U_{42}) + 2cov_0(U_{32}, U_{43}) + 18cov_0(U_{32}, U_{52}) + 8cov_0(U_{32}, U_{53})$$

$$+ 2cov_0(U_{32}, U_{54}) + 8cov_0(U_{42}, U_{43}) + 72cov_0(U_{42}, U_{52}) + 32cov_0(U_{42}, U_{53})$$

$$+ 8cov_0(U_{42}, U_{54}) + 18cov_0(U_{43}, U_{52}) + 8cov_0(U_{43}, U_{53}) + 2cov_0(U_{43}, U_{54})$$

$$+ 72cov_0(U_{52}, U_{53}) + 18cov_0(U_{52}, U_{54}) + 8cov_0(U_{53}, U_{54})$$

$$= \frac{n_1 n_2(n_1 + n_2 + 1)}{12} + \frac{n_3 n_2(n_3 + n_2 + 1)}{12} + \frac{16n_4 n_2(n_4 + n_2 + 1)}{12} + \frac{n_3 n_4(n_3 + n_4 + 1)}{12}$$
$$+ \frac{81n_2 n_5(n_2 + n_5 + 1)}{12} + \frac{16n_3 n_5(n_3 + n_5 + 1)}{12} + \frac{n_5 n_4(n_5 + n_4 + 1)}{12}$$
$$+ \frac{2n_1 n_2 n_3}{12} + \frac{8n_1 n_2 n_4}{12} + \frac{18n_1 n_2 n_5}{12} + \frac{8n_2 n_3 n_4}{12} - \frac{2n_2 n_3 n_4}{12} + \frac{18n_2 n_3 n_5}{12}$$
$$- \frac{8n_2 n_3 n_5}{12} + \frac{8n_2 n_3 n_4}{12} + \frac{72n_2 n_4 n_5}{12} - \frac{8n_2 n_4 n_5}{12} + \frac{8n_3 n_4 n_5}{12} - \frac{2n_5 n_3 n_4}{12}$$
$$+ \frac{72n_2 n_3 n_5}{12} + \frac{18n_2 n_4 n_5}{12} + \frac{8n_5 n_3 n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$Var_0(A_2) = \frac{n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{16n^2(n + n + 1)}{12} \quad \text{(G.6.6)}$$
$$+ \frac{n^2(n + n + 1)}{12} + \frac{81n^2(n + n + 1)}{12} + \frac{16n^2(n + n + 1)}{12}$$
$$+ \frac{n^2(n + n + 1)}{12} + \frac{2n^3}{12} + \frac{8n^3}{12} + \frac{18n^3}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} + \frac{18n^3}{12}$$
$$- \frac{8n^3}{12} + \frac{8n^3}{12} + \frac{72n^3}{12} - \frac{8n^3}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} + \frac{72n^3}{12} + \frac{18n^3}{12} + \frac{8n^3}{12}$$
$$= \frac{n^2(454n + 117)}{12}$$

Note: $cov_0(U_{12}, U_{43}) = cov_0(U_{12}, U_{53}) = cov_0(U_{12}, U_{54}) = cov_0(U_{32}, U_{54}) =$

$cov_0(U_{42}, U_{53}) = 0$

$cov_0(U_{43}, U_{52}) = 0$

Modified Kim -Kim: The exact mean and variance of the distance squared modification of Kim - Kim for Five treatments with peak at two for equal sample size is given by

$$E_0(KK_2) = \frac{21}{2}bn^2 \tag{G.6.7}$$

$$Var_0(KK_2) = \frac{n^2 b(454n + 117)}{12} \tag{G.6.8}$$

G.7. Mean and Variance Formula for Five Treatments with Peaks at 3

Modified Mack-Wolfe: The exact mean and variance of the modified Mack-Wolfe for Five Treatments with peak at three for equal sample size is given by

$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$

Versus alternative

$H_a: \mu_1 \leq \mu_2 \leq \mu_3 \geq \mu_4 \geq \mu_5$

Case1: $A_3 = U_{12} + 2U_{13} + U_{23} + U_{43} + 2U_{53} + U_{54}$

Case 2: $A_3 = U_{12} + 4U_{13} + U_{23} + U_{43} + 4U_{53} + U_{54}$

Distance-Modification Modification

Expectation of Case 1

$$A_3 = U_{12} + 2U_{13} + U_{23} + U_{43} + 2U_{53} + U_{54}$$

$$E_0(A_3) = E_0(U_{12}) + 2E_0(U_{13}) + E_0(U_{23}) + E_0(U_{43}) + 2E_0(U_{53}) + E_0(U_{54})$$
$$= \frac{n_1 n_2}{2} + \frac{2n_1 n_3}{2} + \frac{n_2 n_3}{2} + \frac{n_4 n_3}{2} + \frac{2n_5 n_3}{2} + \frac{n_5 n_4}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$E_0(A_3) = \frac{n_1 n_2}{2} + \frac{2n_1 n_3}{2} + \frac{n_2 n_3}{2} + \frac{n_4 n_3}{2} + \frac{2n_5 n_3}{2} + \frac{n_5 n_4}{2} \quad \text{(G.7.1)}$$

$$= \frac{n * n}{2} + \frac{2n * n}{2} + \frac{n * n}{2} + \frac{n * n}{2} + \frac{2n * n}{2} + \frac{n * n}{2} = 4n^2$$

Variance of Case 1

Case1: $A_3 = U_{12} + 2U_{13} + U_{23} + U_{43} + 2U_{53} + U_{54}$

$$var_0(A_3) = var_0(U_{12} + 2U_{13} + U_{23} + U_{43} + 2U_{53} + U_{54})$$

$$var_0(A_3) = var_0(U_{12}) + 4var_0(U_{13}) + var_0(U_{23}) + var_0(U_{43}) + 4var_0(U_{53}) + var_0(U_{54})$$

$$+ 4cov_0(U_{12}, U_{13}) + 2cov_0(U_{12}, U_{23}) + 2cov_0(U_{12}, U_{43}) + 4cov_0(U_{12}, U_{53})$$

$$+ 2cov_0(U_{12}, U_{54}) + 4cov_0(U_{13}, U_{23}) + 4cov_0(U_{13}, U_{43}) + 8cov_0(U_{13}, U_{53})$$

$$+ 4cov_0(U_{13}, U_{54}) + 2cov_0(U_{23}, U_{43}) + 4cov_0(U_{23}, U_{53}) + 2cov_0(U_{23}, U_{54})$$

$$+ 4cov_0(U_{43}, U_{53}) + 2cov_0(U_{43}, U_{54}) + 4cov_0(U_{53}, U_{54})$$

$$= \frac{n_1 n_2 (n_1 + n_2 + 1)}{12} + \frac{4n_3 n_1 (n_3 + n_1 + 1)}{12} + \frac{n_3 n_2 (n_3 + n_2 + 1)}{12} + \frac{n_4 n_3 (n_4 + n_3 + 1)}{12}$$
$$+ \frac{4n_3 n_5 (n_3 + n_5 + 1)}{12} + \frac{n_5 n_4 (n_5 + n_4 + 1)}{12} + \frac{4n_1 n_2 n_3}{12} - \frac{2n_1 n_2 n_3}{12} + \frac{4n_1 n_2 n_3}{12}$$
$$+ \frac{4n_1 n_3 n_4}{12} + \frac{8n_1 n_3 n_5}{12} + \frac{2n_2 n_3 n_4}{12} + \frac{4n_2 n_3 n_5}{12} + \frac{4n_4 n_3 n_5}{12} - \frac{2n_3 n_4 n_5}{12}$$
$$+ \frac{4n_3 n_4 n_5}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$Var_0(A_3) = \frac{n^2(n + n + 1)}{12} + \frac{4n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} \quad \text{(G.7.2)}$$
$$+ \frac{n^2(n + n + 1)}{12} + \frac{4n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12} + \frac{4n^3}{12}$$
$$- \frac{2n^3}{12} + \frac{4n^3}{12} + \frac{4n^3}{12} + \frac{8n^3}{12} + \frac{2n^3}{12} + \frac{4n^3}{12} + \frac{4n^3}{12} - \frac{2n^3}{12} + \frac{4n^3}{12}$$
$$= \frac{n^2(9n + 2)}{2}$$

Note: $cov_0(U_{12}, U_{43}) = cov_0(U_{12}, U_{53}) = cov_0(U_{12}, U_{54}) = cov_0(U_{32}, U_{54}) =$

$cov_0(U_{42}, U_{53}) = 0$

$$cov_0(U_{12}, U_{54}) = cov_0(U_{43}, U_{52}) = cov_0(U_{23}, U_{54}) = 0$$

Modified Kim -Kim: The exact mean and variance of the distance modification of Kim - Kim for Five treatments with peak at three for equal sample size is given by

$$E_0(KK_3) = 4bn^2 \quad \text{(G.7.3)}$$

$$Var_0(KK_3) = \frac{n^2 b(9n+2)}{2} \quad \text{(G.7.4)}$$

Distance Squared Modification

Expectation of Case 2

Case 2:$A_3 = U_{12} + 4U_{13} + U_{23} + U_{43} + 4U_{53} + U_{54}$

$$E_0(A_3) = E_0(U_{12}) + 4E_0(U_{13}) + E_0(U_{23}) + E_0(U_{43}) + 4E_0(U_{53}) + E_0(U_{54})$$
$$= \frac{n_1 n_2}{2} + \frac{4n_1 n_3}{2} + \frac{n_2 n_3}{2} + \frac{n_4 n_3}{2} + \frac{4n_5 n_3}{2} + \frac{n_5 n_4}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$E_0(A_3) = \frac{n_1 n_2}{2} + \frac{4n_1 n_3}{2} + \frac{n_2 n_3}{2} + \frac{n_4 n_3}{2} + \frac{4n_5 n_3}{2} + \frac{n_5 n_4}{2} \quad \text{(G.7.5)}$$
$$= \frac{n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} + \frac{n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} = 6n^2$$

Variance of Case 2

Case1: $A_3 = U_{12} + 4U_{13} + U_{23} + U_{43} + 4U_{53} + U_{54}$

$$var_0(A_3) = var_0(U_{12} + 4U_{13} + U_{23} + U_{43} + 4U_{53} + U_{54})$$

$$var_0(A_3) = var_0(U_{12}) + 16var_0(U_{13}) + var_0(U_{23}) + var_0(U_{43}) + 16var_0(U_{53}) + var_0(U_{54})$$
$$+ 8cov_0(U_{12}, U_{13}) + 2cov_0(U_{12}, U_{23}) + 2cov_0(U_{12}, U_{43}) + 8cov_0(U_{12}, U_{53})$$
$$+ 2cov_0(U_{12}, U_{54}) + 8cov_0(U_{13}, U_{23}) + 8cov_0(U_{13}, U_{43}) + 32cov_0(U_{13}, U_{53})$$
$$+ 8cov_0(U_{13}, U_{54}) + 2cov_0(U_{23}, U_{43}) + 8cov_0(U_{23}, U_{53}) + 2cov_0(U_{23}, U_{54})$$
$$+ 8cov_0(U_{43}, U_{53}) + 2cov_0(U_{43}, U_{54}) + 8cov_0(U_{53}, U_{54})$$

$$= \frac{n_1n_2(n_1+n_2+1)}{12} + \frac{16n_3n_1(n_3+n_1+1)}{12} + \frac{n_3n_2(n_3+n_2+1)}{12} + \frac{n_4n_3(n_4+n_3+1)}{12} + \frac{16n_3n_5(n_3+n_5+1)}{12} + \frac{n_5n_4(n_5+n_4+1)}{12} + \frac{8n_1n_2n_3}{12} - \frac{2n_1n_2n_3}{12} + \frac{8n_1n_2n_3}{12} + \frac{8n_1n_3n_4}{12} + \frac{32n_1n_3n_5}{12} + \frac{2n_2n_3n_4}{12} + \frac{8n_2n_3n_5}{12} + \frac{8n_4n_3n_5}{12} - \frac{2n_3n_4n_5}{12} + \frac{8n_3n_4n_5}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$Var_0(A_3) = \frac{n^2(n+n+1)}{12} + \frac{16n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{16n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} + \frac{8n^3}{12} + \frac{8n^3}{12} + \frac{32n^3}{12} + \frac{2n^3}{12} + \frac{8n^3}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} + \frac{8n^3}{12} = \frac{n^2(150n+36)}{3} \qquad \text{(G.7.6)}$$

Note: $cov_0(U_{12}, U_{43}) = cov_0(U_{12}, U_{53}) = cov_0(U_{12}, U_{54}) = cov_0(U_{32}, U_{54}) =$ $cov_0(U_{42}, U_{53}) = 0$

$$cov_0(U_{12}, U_{54}) = cov_0(U_{43}, U_{52}) = cov_0(U_{23}, U_{54}) = 0$$

Modified Kim -Kim: The exact mean and variance of the distance squared modification of Kim - Kim for Five treatments with peak at three for equal sample size is given by

$$E_0(KK_3) = 6bn^2 \qquad \text{(G.7.7)}$$

$$Var_0(KK_3) = \frac{n^2 b(150n + 36)}{3} \qquad \text{(G.7.8)}$$

G.8. Mean and Variance Formula for Five Treatments with Peak at 4

Modified Mack-Wolfe: The exact mean and variance of the modified Mack-Wolfe for Five Treatments with peak at four for equal sample size is given by

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

Versus alternative

$$H_a: \mu_1 \le \mu_2 \le \mu_3 \le \mu_4 \ge \mu_5$$

Case1: $A_4 = U_{12} + 2U_{13} + 3U_{14} + U_{23} + 2U_{24} + U_{34} + U_{54}$

Case 2: $A_4 = U_{12} + 4U_{13} + 9U_{14} + U_{23} + 4U_{24} + U_{34} + U_{54}$

Distance-Modification Modification

Expectation of Case 1

$$E_0(A_4) = E_0(U_{12}) + 2E_0(U_{13}) + 3E_0(U_{14}) + E_0(U_{23}) + 2E_0(U_{24}) + E_0(U_{34}) + E_0(U_{54})$$
$$= \frac{n_1 n_2}{2} + \frac{2n_3 n_1}{2} + \frac{3n_4 n_1}{2} + \frac{n_2 n_3}{2} + \frac{2n_2 n_4}{2} + \frac{n_4 n_3}{2} + \frac{n_5 n_4}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$E_0(A_4) = \frac{n_1 n_2}{2} + \frac{2n_3 n_1}{2} + \frac{3n_4 n_1}{2} + \frac{n_2 n_3}{2} + \frac{2n_2 n_4}{2} + \frac{n_4 n_3}{2} + \frac{n_5 n_4}{2} \qquad \text{(G.8.1)}$$
$$= \frac{n*n}{2} + \frac{2n*n}{2} + \frac{3n*n}{2} + \frac{n*n}{2} + \frac{2n*n}{2} + \frac{n*n}{2} + \frac{n*n}{2}$$
$$= \frac{11n^2}{2}$$

Variance of Case 1

Case1: $A_4 = U_{12} + 2U_{13} + 3U_{14} + U_{23} + 2U_{24} + U_{34} + U_{54}$

$$var_0(A_4) = var_0(U_{12} + 2U_{13} + 3U_{14} + U_{23} + 2U_{24} + U_{34} + U_{54})$$

$$var_0(A_4) = var_0(U_{12}) + 4var_0(U_{13}) + 9var_0(U_{14}) + var_0(U_{23}) + 4var_0(U_{24}) + var_0(U_{34})$$

$$+ var_0(U_{54}) + 4cov_0(U_{12}, U_{13}) + 6cov_0(U_{12}, U_{14}) + 2cov_0(U_{12}, U_{23})$$

$$+ 4cov_0(U_{12}, U_{24}) + 2cov_0(U_{12}, U_{34}) + 2cov_0(U_{12}, U_{54}) + 12cov_0(U_{13}, U_{14})$$

$$+ 4cov_0(U_{13}, U_{23}) + 8cov_0(U_{13}, U_{24}) + 4cov_0(U_{13}, U_{34}) + 2cov_0(U_{13}, U_{54})$$

$$+ 6cov_0(U_{14}, U_{23}) + 12cov_0(U_{14}, U_{24}) + 6cov_0(U_{14}, U_{34}) + 6cov_0(U_{14}, U_{54})$$

$$+ 4cov_0(U_{23}, U_{24}) + 2cov_0(U_{23}, U_{34}) + 2cov_0(U_{23}, U_{54}) + 4cov_0(U_{24}, U_{34})$$

$$+ 4cov_0(U_{24}, U_{54}) + 2cov_0(U_{34}, U_{54})$$

$$= \frac{n_1 n_2(n_1 + n_2 + 1)}{12} + \frac{4n_3 n_1(n_3 + n_1 + 1)}{12} + \frac{9n_4 n_1(n_4 + n_1 + 1)}{12} + \frac{n_2 n_3(n_2 + n_3 + 1)}{12}$$
$$+ \frac{4n_2 n_4(n_2 + n_4 + 1)}{12} + \frac{n_3 n_4(n_3 + n_4 + 1)}{12} + \frac{n_5 n_4(n_5 + n_4 + 1)}{12} + \frac{4n_1 n_2 n_3}{12}$$
$$+ \frac{6n_1 n_2 n_4}{12} - \frac{2n_1 n_2 n_3}{12} - \frac{4n_1 n_2 n_4}{12} + \frac{12n_1 n_3 n_4}{12} + \frac{4n_1 n_2 n_3}{12} - \frac{4n_1 n_3 n_4}{12}$$
$$+ \frac{12n_1 n_2 n_4}{12} + \frac{6n_1 n_3 n_4}{12} + \frac{6n_1 n_4 n_5}{12} + \frac{4n_2 n_3 n_4}{12} - \frac{2n_2 n_3 n_4}{12} + \frac{4n_2 n_3 n_4}{12}$$
$$+ \frac{4n_2 n_4 n_5}{12} + \frac{2n_5 n_3 n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$Var_0(A_4) = \frac{n^2(n + n + 1)}{12} + \frac{4n^2(n + n + 1)}{12} + \frac{9n^2(n + n + 1)}{12} \quad \text{(G.8.2)}$$
$$+ \frac{n^2(n + n + 1)}{12} + \frac{4n^2(n + n + 1)}{12} + \frac{n^2(n + n + 1)}{12}$$
$$+ \frac{n^2(n + n + 1)}{12} + \frac{4n^3}{12} + \frac{6n^3}{12} - \frac{2n^3}{12} - \frac{4n^3}{12} + \frac{12n^3}{12} + \frac{4n^3}{12}$$
$$- \frac{4n^3}{12} + \frac{12n^3}{12} + \frac{6n^3}{12} + \frac{6n^3}{12} + \frac{4n^3}{12} - \frac{2n^3}{12} + \frac{4n^3}{12} + \frac{4n^3}{12} + \frac{2n^3}{12}$$
$$= \frac{n^2(94n + 21)}{12}$$

Note: $cov_0(U_{12}, U_{34}) = cov_0(U_{12}, U_{54}) = cov_0(U_{13}, U_{24}) = cov_0(U_{13}, U_{54}) = 0$

$$cov_0(U_{14}, U_{23}) = cov_0(U_{23}, U_{54}) = 0$$

Modified Kim -Kim: The exact mean and variance of the distance modification of Kim - Kim for Five treatments with peak at four for equal sample size is given by

$$E_0(KK_4) = \frac{11bn^2}{2} \tag{G.8.3}$$

$$Var_0(KK_4) = \frac{n^2b(94n+21)}{12} \tag{G.8.4}$$

Distance Squared Modification

Expectation of Case 2

Case 2: $A_4 = U_{12} + 4U_{13} + 9U_{14} + U_{23} + 4U_{24} + U_{34} + U_{54}$

$$E_0(A_4) = E_0(U_{12}) + 2E_0(U_{13}) + 9E_0(U_{14}) + E_0(U_{23}) + 4E_0(U_{24}) + E_0(U_{34}) + E_0(U_{54})$$
$$= \frac{n_1n_2}{2} + \frac{4n_3n_1}{2} + \frac{9n_4n_1}{2} + \frac{n_2n_3}{2} + \frac{4n_2n_4}{2} + \frac{n_4n_3}{2} + \frac{n_5n_4}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$E_0(A_4) = \frac{n_1n_2}{2} + \frac{4n_3n_1}{2} + \frac{9n_4n_1}{2} + \frac{n_2n_3}{2} + \frac{4n_2n_4}{2} + \frac{n_4n_3}{2} + \frac{n_5n_4}{2} \tag{G.8.5}$$
$$= \frac{n*n}{2} + \frac{4n*n}{2} + \frac{9n*n}{2} + \frac{n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} + \frac{n*n}{2}$$
$$= \frac{21n^2}{2}$$

Variance of Case 2

Case1: $A_4 = U_{12} + 4U_{13} + 9U_{14} + U_{23} + 4U_{24} + U_{34} + U_{54}$

$$var_0(A_4) = var_0(U_{12} + 4U_{13} + 9U_{14} + U_{23} + 4U_{24} + U_{34} + U_{54})$$

$$var_0(A_4) = var_0(U_{12}) + 16var_0(U_{13}) + 81var_0(U_{14}) + var_0(U_{23}) + 16var_0(U_{24})$$

$$+ var_0(U_{34}) + var_0(U_{54}) + 8cov_0(U_{12}, U_{13}) + 18cov_0(U_{12}, U_{14})$$

$$+ 2cov_0(U_{12}, U_{23}) + 8cov_0(U_{12}, U_{24}) + 2cov_0(U_{12}, U_{34}) + 2cov_0(U_{12}, U_{54})$$

$$+ 72cov_0(U_{13}, U_{14}) + 8cov_0(U_{13}, U_{23}) + 32cov_0(U_{13}, U_{24}) + 8cov_0(U_{13}, U_{34})$$

$$+ 2cov_0(U_{13}, U_{54}) + 18cov_0(U_{14}, U_{23}) + 72cov_0(U_{14}, U_{24}) + 18cov_0(U_{14}, U_{34})$$

$$+ 18cov_0(U_{14}, U_{54}) + 8cov_0(U_{23}, U_{24}) + 2cov_0(U_{23}, U_{34}) + 2cov_0(U_{23}, U_{54})$$

$$+ 8cov_0(U_{24}, U_{34}) + 8cov_0(U_{24}, U_{54}) + 2cov_0(U_{34}, U_{54})$$

$$= \frac{n_1n_2(n_1+n_2+1)}{12} + \frac{16n_3n_1(n_3+n_1+1)}{12} + \frac{81n_4n_1(n_4+n_1+1)}{12} + \frac{n_2n_3(n_2+n_3+1)}{12}$$
$$+ \frac{16n_2n_4(n_2+n_4+1)}{12} + \frac{n_3n_4(n_3+n_4+1)}{12} + \frac{n_5n_4(n_5+n_4+1)}{12} + \frac{8n_1n_2n_3}{12}$$
$$+ \frac{18n_1n_2n_4}{12} - \frac{2n_1n_2n_3}{12} - \frac{8n_1n_2n_4}{12} + \frac{72n_1n_3n_4}{12} + \frac{8n_1n_2n_3}{12} - \frac{8n_1n_3n_4}{12}$$
$$+ \frac{72n_1n_2n_4}{12} + \frac{18n_1n_3n_4}{12} + \frac{18n_1n_4n_5}{12} + \frac{8n_2n_3n_4}{12} - \frac{2n_2n_3n_4}{12} + \frac{8n_2n_3n_4}{12}$$
$$+ \frac{8n_2n_4n_5}{12} + \frac{2n_5n_3n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$Var_0(A_4) = \frac{n^2(n+n+1)}{12} + \frac{16n^2(n+n+1)}{12} + \frac{81n^2(n+n+1)}{12} \qquad \text{(G.8.6)}$$

$$+ \frac{n^2(n+n+1)}{12} + \frac{16n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12}$$

$$+ \frac{n^2(n+n+1)}{12} + \frac{8n^3}{12} + \frac{18n^3}{12} - \frac{2n^3}{12} - \frac{8n^3}{12} + \frac{72n^3}{12} + \frac{8n^3}{12}$$

$$- \frac{8n^3}{12} + \frac{72n^3}{12} + \frac{18n^3}{12} + \frac{18n^3}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} + \frac{8n^3}{12} + \frac{8n^3}{12} + \frac{2n^3}{12}$$

$$= \frac{n^2(454n+117)}{12}$$

Note: $cov_0(U_{12}, U_{34}) = cov_0(U_{12}, U_{54}) = cov_0(U_{13}, U_{24}) = cov_0(U_{13}, U_{54}) = 0$

$$cov_0(U_{14}, U_{23}) = cov_0(U_{23}, U_{54}) = 0$$

Modified Kim -Kim: The exact mean and variance of the distance squared modification of Kim - Kim for Five treatments with peak at four for equal sample size is given by

$$E_0(KK_4) = \frac{21bn^2}{2} \quad \text{(G.8.7)}$$

$$Var_0(KK_4) = \frac{n^2b(454n + 117)}{12} \quad \text{(G.8.8)}$$

G.9. Mean and Variance Formula for Five Treatments with Peaks at one or five

Modified Mack-Wolfe: The exact mean and variance of the modified Mack-Wolfe for Five Treatments with peak at one or five for equal sample size is given by

$$H_0: \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$$

Versus alternative

$$H_a: \mu_1 \leq \mu_2 \leq \mu_3 \leq \mu_4 \leq \mu_5$$

Case1: $A_1 = A_5 = U_{12} + 2U_{13} + 3U_{14} + 4U_{15} + U_{23} + 2U_{24} + 3U_{25} + U_{34} + 2U_{35} + U_{45}$

Case 2: $A_1 = A_5 = U_{12} + 4U_{13} + 9U_{14} + 16U_{15} + U_{23} + 4U_{24} + 9U_{25} + U_{34} + 4U_{35} + U_{45}$

Distance-Modification Modification

Expectation of Case 1

$$\begin{aligned} E_0(A_1) = E_0(A_5) \\ &= E_0(U_{12}) + 2E_0(U_{13}) + 3E_0(U_{14}) + 4E_0(U_{15}) + E_0(U_{23}) + 2E_0(U_{24}) \\ &+ 3E_0(U_{25}) + E_0(U_{34}) + 2E_0(U_{35}) + E_0(U_{45}) \\ &= \frac{n_1n_2}{2} + \frac{2n_1n_3}{2} + \frac{3n_1n_4}{2} + \frac{4n_1n_5}{2} + \frac{n_2n_3}{2} + \frac{2n_2n_4}{2} + \frac{3n_2n_5}{2} + \frac{n_4n_3}{2} + \frac{2n_3n_5}{2} \\ &+ \frac{n_4n_5}{2} \end{aligned}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$E_0(A_5) = \frac{n_1n_2}{2} + \frac{2n_1n_3}{2} + \frac{3n_1n_4}{2} + \frac{4n_1n_5}{2} + \frac{n_2n_3}{2} + \frac{2n_2n_4}{2} + \frac{3n_2n_5}{2} + \frac{n_4n_3}{2} + \frac{2n_3n_5}{2} + \frac{n_4n_5}{2} = \frac{n*n}{2} + \frac{2n*n}{2} + \frac{3n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} + \frac{2n*n}{2} + \frac{3n*n}{2} + \frac{n*n}{2} + \frac{2n*n}{2} + \frac{n*n}{2} = 10n^2 \quad \text{(G.9.1)}$$

Variance of Case 1

Case1: $A_1 = A_5 = U_{12} + 2U_{13} + 3U_{14} + 4U_{15} + U_{23} + 2U_{24} + 3U_{25} + U_{34} + 2U_{35} + U_{45}$

$$A_1 = A_5 = var_0(U_{12} + 2U_{13} + 3U_{14} + 4U_{15} + U_{23} + 2U_{24} + 3U_{25} + U_{34} + 2U_{35} + U_{45})$$

$$\begin{aligned} var_0(A_5) = {} & var_0(U_{12}) + 4var_0(U_{13}) + 9var_0(U_{14}) + 16var_0(U_{15}) + var_0(U_{23}) \\ & + 4var_0(U_{24}) + 9var_0(U_{25}) + var_0(U_{34}) + 4var_0(U_{35}) + var_0(U_{45}) \\ & + 4cov_0(U_{12}, U_{13}) + 6cov_0(U_{12}, U_{14}) + 8cov_0(U_{12}, U_{15}) + 2cov_0(U_{12}, U_{23}) \\ & + 4cov_0(U_{12}, U_{24}) + 6cov_0(U_{12}, U_{25}) + 2cov_0(U_{12}, U_{34}) + 4cov_0(U_{12}, U_{35}) \\ & + 2cov_0(U_{12}, U_{45}) + 12cov_0(U_{13}, U_{14}) + 16cov_0(U_{13}, U_{15}) + 4cov_0(U_{13}, U_{23}) \\ & + 8cov_0(U_{13}, U_{24}) + 12cov_0(U_{13}, U_{25}) + 4cov_0(U_{13}, U_{34}) + 8cov_0(U_{13}, U_{35}) \\ & + 4cov_0(U_{13}, U_{45}) + 24cov_0(U_{14}, U_{15}) + 6cov_0(U_{14}, U_{23}) + 12cov_0(U_{14}, U_{24}) \\ & + 18cov_0(U_{14}, U_{25}) + 6cov_0(U_{14}, U_{34}) + 12cov_0(U_{14}, U_{35}) + 6cov_0(U_{14}, U_{45}) \\ & + 8cov_0(U_{15}, U_{23}) + 16cov_0(U_{15}, U_{24}) + 24cov_0(U_{15}, U_{25}) + 8cov_0(U_{15}, U_{34}) \\ & + 16cov_0(U_{15}, U_{35}) + 8cov_0(U_{15}, U_{45}) + 4cov_0(U_{23}, U_{24}) + 6cov_0(U_{23}, U_{25}) \\ & + 2cov_0(U_{23}, U_{34}) + 4cov_0(U_{23}, U_{35}) + 2cov_0(U_{23}, U_{45}) + +12cov_0(U_{24}, U_{25}) \\ & + 4cov_0(U_{24}, U_{34}) + 8cov_0(U_{24}, U_{35}) + 4cov_0(U_{24}, U_{45}) + 6cov_0(U_{25}, U_{34}) \\ & + 12cov_0(U_{25}, U_{35}) + 6cov_0(U_{25}, U_{45}) + 4cov_0(U_{34}, U_{35}) + 2cov_0(U_{34}, U_{45}) \\ & + 4cov_0(U_{35}, U_{45}) \end{aligned}$$

$$= \frac{n_1 n_2(n_1+n_2+1)}{12} + \frac{4n_3 n_1(n_3+n_1+1)}{12} + \frac{9n_4 n_1(n_4+n_1+1)}{12} + \frac{16n_1 n_5(n_1+n_5+1)}{12}$$
$$+ \frac{n_2 n_3(n_2+n_3+1)}{12} + \frac{4n_2 n_4(n_2+n_4+1)}{12} + \frac{9n_2 n_5(n_2+n_5+1)}{12}$$
$$+ \frac{n_3 n_4(n_3+n_4+1)}{12} + \frac{4n_3 n_5(n_3+n_5+1)}{12} + \frac{n_4 n_5(n_4+n_5+1)}{12} + \frac{4n_1 n_2 n_3}{12}$$
$$+ \frac{6n_1 n_2 n_4}{12} + \frac{8n_1 n_2 n_5}{12} - \frac{2n_1 n_2 n_3}{12} - \frac{4n_1 n_2 n_4}{12} - \frac{6n_1 n_2 n_5}{12} + \frac{12n_1 n_3 n_4}{12}$$
$$+ \frac{16n_1 n_3 n_5}{12} + \frac{4n_1 n_2 n_3}{12} - \frac{4n_1 n_3 n_4}{12} - \frac{8n_1 n_3 n_5}{12} + \frac{24n_1 n_4 n_5}{12} + \frac{12n_1 n_2 n_4}{12}$$
$$+ \frac{6n_1 n_3 n_4}{12} - \frac{6n_1 n_4 n_5}{12} + \frac{24n_1 n_2 n_5}{12} + \frac{16n_1 n_3 n_5}{12} + \frac{8n_1 n_4 n_5}{12} + \frac{4n_2 n_3 n_4}{12}$$
$$+ \frac{6n_2 n_3 n_5}{12} - \frac{2n_2 n_3 n_4}{12} - \frac{4n_2 n_3 n_5}{12} + \frac{12n_2 n_4 n_5}{12} + \frac{4n_3 n_2 n_4}{12} - \frac{4n_2 n_5 n_4}{12}$$
$$+ \frac{12n_2 n_3 n_5}{12} + \frac{6n_4 n_5 n_2}{12} + \frac{4n_5 n_3 n_4}{12} - \frac{2n_4 n_5 n_3}{12} + \frac{4n_5 n_3 n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$Var_0(A_4) = \frac{n^2(n+n+1)}{12} + \frac{4n^2(n+n+1)}{12} + \frac{9n^2(n+n+1)}{12} \tag{G.9.2}$$
$$+ \frac{16n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{9n^2(n+n+1)}{12}$$
$$+ \frac{4n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{4n^2(n+n+1)}{12}$$
$$+ \frac{n^2(n+n+1)}{12} + \frac{4n^3}{12} + \frac{6n^3}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} - \frac{4n^3}{12} - \frac{6n^3}{12}$$
$$+ \frac{12n^3}{12} + \frac{16n^3}{12} + \frac{4n^3}{12} - \frac{4n^3}{12} - \frac{8n^3}{12} + \frac{24n^3}{12} + \frac{12n^3}{12} + \frac{6n^3}{12}$$
$$- \frac{6n^3}{12} + \frac{24n^3}{12} + \frac{16n^3}{12} + \frac{8n^3}{12} + \frac{4n^3}{12} + \frac{6n^3}{12} - \frac{2n^3}{12} - \frac{4n^3}{12} + \frac{12n^3}{12}$$
$$+ \frac{4n^3}{12} - \frac{4n^3}{12} + \frac{12n^3}{12} + \frac{6n^3}{12} + \frac{4n^3}{12} - \frac{2n^3}{12} + \frac{4n^3}{12}$$
$$= \frac{25n^2(5n+1)}{6}$$

Note:

$cov_0(U_{12}, U_{34}) = cov_0(U_{12}, U_{35}) = cov_0(U_{12}, U_{45}) = 0$

$cov_0(U_{13}, U_{24}) = cov_0(U_{13}, U_{25}) = cov_0(U_{13}, U_{45}) = 0$

$cov_0(U_{14}, U_{23}) = cov_0(U_{14}, U_{25}) = cov_0(U_{14}, U_{35}) = 0$

$cov_0(U_{15}, U_{23}) = cov_0(U_{15}, U_{24}) = cov_0(U_{15}, U_{34}) = 0$

$cov_0(U_{23}, U_{45}) = cov_0(U_{24}, U_{35}) = cov_0(U_{25}, U_{34}) = 0$

Modified Kim -Kim: The exact mean and variance of the distance modification of Kim - Kim for Five treatments with peak at one or five for equal sample size is given by

$$E_0(KK_5) = 10bn^2 \quad \text{(G.9.3)}$$

$$Var_0(KK_5) = \frac{25n^2 b(5n+1)}{6} \quad \text{(G.9.4)}$$

Distance Squared Modification

Expectation of Case 2

Case 2: $A_1 = A_5 = U_{12} + 4U_{13} + 9U_{14} + 16U_{15} + U_{23} + 4U_{24} + 9U_{25} + U_{34} + 4U_{35} + U_{45}$

$$E_0(A_1) = E_0(A_5) = E_0(U_{12} + 4U_{13} + 9U_{14} + 16U_{15} + U_{23} + 4U_{24} + 9U_{25} + U_{34} + 4U_{35} + U_{45})$$

$$= E_0(U_{12}) + 4E_0(U_{13}) + 9E_0(U_{14}) + 16E_0(U_{15}) + E_0(U_{23}) + 4E_0(U_{24}) + 9E_0(U_{25}) + E_0(U_{34}) + 4E_0(U_{35}) + E_0(U_{45})$$

$$= \frac{n_1n_2}{2} + \frac{4n_1n_3}{2} + \frac{9n_1n_4}{2} + \frac{16n_1n_5}{2} + \frac{n_2n_3}{2} + \frac{4n_2n_4}{2} + \frac{9n_2n_5}{2} + \frac{n_4n_3}{2} + \frac{4n_3n_5}{2} + \frac{n_4n_5}{2}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$E_0(A_1) = \frac{n_1n_2}{2} + \frac{4n_1n_3}{2} + \frac{9n_1n_4}{2} + \frac{16n_1n_5}{2} + \frac{n_2n_3}{2} + \frac{4n_2n_4}{2} + \frac{9n_2n_5}{2} + \frac{n_4n_3}{2} + \frac{4n_3n_5}{2} + \frac{n_4n_5}{2} = \frac{n*n}{2} + \frac{4n*n}{2} + \frac{9n*n}{2} + \frac{16n*n}{2} + \frac{n*n}{2} + \frac{4n*n}{2} + \frac{9n*n}{2} + \frac{n*n}{2} + \frac{4n*n}{2} + \frac{n*n}{2} = 25n^2 \quad \text{(G.9.5)}$$

Variance of Case 2

Case2: $A_1 = A_5 = U_{12} + 4U_{13} + 9U_{14} + 16U_{15} + U_{23} + 4U_{24} + 9U_{25} + U_{34} + 4U_{35} + U_{45}$

$$A_1 = A_5 = var_0(U_{12} + 4U_{13} + 9U_{14} + 16U_{15} + U_{23} + 4U_{24} + 9U_{25} + U_{34} + 4U_{35} + U_{45})$$

$$var_0(A_5) = var_0(U_{12}) + 16var_0(U_{13}) + 81var_0(U_{14}) + 256(U_{15}) + var_0(U_{23}) + 16var_0(U_{24}) + 81var_0(U_{25}) + var_0(U_{34}) + 16var_0(U_{35}) + var_0(U_{45}) + 8cov_0(U_{12}, U_{13}) + 18cov_0(U_{12}, U_{14}) + 32cov_0(U_{12}, U_{15}) + 2cov_0(U_{12}, U_{23}) + 8cov_0(U_{12}, U_{24}) + 18cov_0(U_{12}, U_{25}) + 2cov_0(U_{12}, U_{34}) + 8cov_0(U_{12}, U_{35}) + 2cov_0(U_{12}, U_{45}) + 72cov_0(U_{13}, U_{14}) + 128cov_0(U_{13}, U_{15}) + 8cov_0(U_{13}, U_{23}) + 32cov_0(U_{13}, U_{24}) + 72cov_0(U_{13}, U_{25}) + 8cov_0(U_{13}, U_{34}) + 32cov_0(U_{13}, U_{35}) + 8cov_0(U_{13}, U_{45}) + 288cov_0(U_{14}, U_{15}) + 18cov_0(U_{14}, U_{23}) + 72cov_0(U_{14}, U_{24}) + 162cov_0(U_{14}, U_{25}) + 18cov_0(U_{14}, U_{34}) + 72cov_0(U_{14}, U_{35}) + 18cov_0(U_{14}, U_{45}) + 32cov_0(U_{15}, U_{23}) + 128cov_0(U_{15}, U_{24}) + 288cov_0(U_{15}, U_{25}) + 32cov_0(U_{15}, U_{34}) + 128cov_0(U_{15}, U_{35}) + 32cov_0(U_{15}, U_{45}) + 8cov_0(U_{23}, U_{24}) + 18cov_0(U_{23}, U_{25}) + 2cov_0(U_{23}, U_{34}) + 8cov_0(U_{23}, U_{35}) + 2cov_0(U_{23}, U_{45}) + 72cov_0(U_{24}, U_{25}) + 8cov_0(U_{24}, U_{34}) + 32cov_0(U_{24}, U_{35}) + 8cov_0(U_{24}, U_{45}) + 18cov_0(U_{25}, U_{34}) + 72cov_0(U_{25}, U_{35}) + 18cov_0(U_{25}, U_{45}) + 8cov_0(U_{34}, U_{35}) + 2cov_0(U_{34}, U_{45}) + 8cov_0(U_{35}, U_{45})$$

$$= \frac{n_1 n_2(n_1+n_2+1)}{12} + \frac{16n_3 n_1(n_3+n_1+1)}{12} + \frac{81n_4 n_1(n_4+n_1+1)}{12} + \frac{256n_1 n_5(n_1+n_5+1)}{12} + \frac{n_2 n_3(n_2+n_3+1)}{12} + \frac{16n_2 n_4(n_2+n_4+1)}{12} + \frac{81n_2 n_5(n_2+n_5+1)}{12} + \frac{n_3 n_4(n_3+n_4+1)}{12} + \frac{16n_3 n_5(n_3+n_5+1)}{12} + \frac{n_4 n_5(n_4+n_5+1)}{12} + \frac{8n_1 n_2 n_3}{12} + \frac{18n_1 n_2 n_4}{12} + \frac{32n_1 n_2 n_5}{12} - \frac{2n_1 n_2 n_3}{12} - \frac{8n_1 n_2 n_4}{12} - \frac{18n_1 n_2 n_5}{12} + \frac{72n_1 n_3 n_4}{12} + \frac{128n_1 n_3 n_5}{12} + \frac{8n_1 n_2 n_3}{12} - \frac{8n_1 n_3 n_4}{12} - \frac{32n_1 n_3 n_5}{12} + \frac{288n_1 n_4 n_5}{12} + \frac{72n_1 n_2 n_4}{12} + \frac{18n_1 n_3 n_4}{12} - \frac{18n_1 n_4 n_5}{12} + \frac{288n_1 n_2 n_5}{12} + \frac{128n_1 n_3 n_5}{12} + \frac{32n_1 n_4 n_5}{12} + \frac{8n_2 n_3 n_4}{12} + \frac{18n_2 n_3 n_5}{12} - \frac{2n_2 n_3 n_4}{12} - \frac{8n_2 n_3 n_5}{12} + \frac{72n_2 n_4 n_5}{12} + \frac{8n_3 n_2 n_4}{12} - \frac{8n_2 n_5 n_4}{12} + \frac{72n_2 n_3 n_5}{12} + \frac{18n_4 n_5 n_2}{12} + \frac{8n_5 n_3 n_4}{12} - \frac{2n_4 n_5 n_3}{12} + \frac{8n_5 n_3 n_4}{12}$$

For equal sample sizes, $n_1 = n_2 = n_3 = n_4 = n_5 = n$

$$Var_0(A_5) = \frac{n^2(n+n+1)}{12} + \frac{16n^2(n+n+1)}{12} + \frac{81n^2(n+n+1)}{12} + \frac{256n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{81n^2(n+n+1)}{12} + \frac{16n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{16n^2(n+n+1)}{12} + \frac{n^2(n+n+1)}{12} + \frac{8n^3}{12} + \frac{18n^3}{12} + \frac{32n^3}{12} - \frac{2n^3}{12} - \frac{8n^3}{12} - \frac{18n^3}{12} + \frac{72n^3}{12} + \frac{128n^3}{12} + \frac{8n^3}{12} - \frac{8n^3}{12} - \frac{32n^3}{12} + \frac{288n^3}{12} + \frac{72n^3}{12} + \frac{18n^3}{12} - \frac{18n^3}{12} + \frac{288n^3}{12} + \frac{128n^3}{12} + \frac{32n^3}{12} + \frac{8n^3}{12} + \frac{18n^3}{12} - \frac{2n^3}{12} - \frac{8n^3}{12} + \frac{72n^3}{12} + \frac{8n^3}{12} - \frac{8n^3}{12} + \frac{72n^3}{12} + \frac{18n^3}{12} + \frac{8n^3}{12} - \frac{2n^3}{12} + \frac{8n^3}{12} = \frac{n^2(1069n+235)}{6} \quad \text{(G.9.6)}$$

Note:

$cov_0(U_{12}, U_{34}) = cov_0(U_{12}, U_{35}) = cov_0(U_{12}, U_{45}) = 0$

$cov_0(U_{13}, U_{24}) = cov_0(U_{13}, U_{25}) = cov_0(U_{13}, U_{45}) = 0$

$cov_0(U_{14}, U_{23}) = cov_0(U_{14}, U_{25}) = cov_0(U_{14}, U_{35}) = 0$

$cov_0(U_{15}, U_{23}) = cov_0(U_{15}, U_{24}) = cov_0(U_{15}, U_{34}) = 0$

$cov_0(U_{23}, U_{45}) = cov_0(U_{24}, U_{35}) = cov_0(U_{25}, U_{34}) = 0$

Modified Kim -Kim: The exact mean and variance of the distance squared modification of Kim - Kim for Five treatments with peak at one or five for equal sample size is given by

$$E_0(KK_5) = 25bn^2 \tag{G.9.7}$$

$$Var_0(KK_5) = \frac{n^2 b(1069n + 235)}{6} \tag{G.9.8}$$