

LIMITING FINANCIAL RISK FROM CATASTROPHIC EVENTS IN PROJECT MANAGEMENT

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Limiting the Financial Risk of Catastrophic Events in Project Management

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ABSTRACT

This dissertation develops a mixed integer linear program to establish the upper and lower bounds of the Alhorn of Uncertainty. For a project manager, planning for uncertainty is a staple of their jobs and education. But the uncertainty associated with a catastrophic event presents difficulties not easily controlled with traditional methods of risk management. This dissertation brings and modifies the concept of a project schedule as a bounded “Alhorn of Uncertainty” to the problem of how to reduce the risk of a catastrophic event wreaking havoc on a project and, by extension, the company participating in that project. The dissertation presents new mathematical models underpinning the methods proposed to reduce risk as well as simulations to demonstrate the accuracy of those models. The dissertation further assesses the complexity of the models and thus their practical application. Finally, the dissertation presents strategies to reduce the risk to a project of a catastrophic event using the upper bound of the Alhorn as the measure of risk.

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DEDICATION

This work is dedicated to my wife. For your inspiration on this and so many other things, great and small.

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LIST OF ABBREVIATIONS

AoU	Alphorn of Uncertainty.
ILP	Integer Linear Program
L.B.	Lower Bound of Alphorn of Uncertainty
LP	Linear Program
MCS	Monte Carlo Simulation. Method of modeling incorporating random variables predicting outcomes under uncertainty by providing a range of possible outcomes.
NPV	Net Present value. The current value of a stream of future cash flows discounted at a predetermined interest rate.
U.B.	Upper Bound of the Alphorn of Uncertainty.

1. INTRODUCTION

This dissertation advances the research of risk-planning for projects by presenting the first mixed integer linear program (MILP) to establish the maximum and minimum bounds of the “Alphorn of Uncertainty” (AoU) as first described by Elmaghraby and Zhang (2011). The MILP can be solved in a reasonable amount of time for large problems, computing a 98 node network with complexity of 2.1 in an average time of seventy-five seconds. Further, the MILP is used to demonstrate the effectiveness of using a rail schedule vs. a roadrunner schedule thus giving a project manager tools to significantly reduce the risk of a project at risk of a catastrophic event or events. For project managers, planning for uncertainty is a staple of their jobs and education (Jacobs, 2010; Pinedo, 2012). But the uncertainty associated with a catastrophic event presents difficulties not easily controlled with traditional methods of risk management (Hällgren & Wilson, 2008). The bounds established by the MILP developed here show a project manager the maximum and minimum costs associated with each point in time during project execution giving him or her insights not previously available. Finally, the MILP is more efficient (with certain caveats) than either a simulation or the methods presented in the literature to date for developing these bounds.

A situation known by the author presents a compelling example of the need for analysis of and measures to protect against a catastrophic event. A developer of light industrial machinery (COMPANY) was selected to manage an entire project to develop a group of light factories for a manufacturing park in a depressed area of a developing nation. This project included equipment produced by COMPANY and a significant amount of equipment, buildings, infrastructure, and management technology the provision of which was to be provided by COMPANY. The government overseeing and paying for this project provided limited legal protection for commerce in the area of development, was not subject to democratic constraints, and was otherwise unstable but firmly in control of the country at the time of the project. Further, the area being developed was adjacent to a low level, but brutal, insurgency and violence had crossed into this area resulting in the deaths of associates of COMPANY.

The risks to COMPANY included:

- (1) the loss of government payment due to overthrow or change in temperament of the existing government;
- (2) Terrorist activity destroying significant parts—or all—of the project before completion and payment;
- (3) The killing of key personnel in country making completion of the project impossible; and
- (4) The impossibility of acquiring insurance against any of the above risks.

These risks are difficult to quantify using standard probabilities and even if they are, the probability of any event or combination of events would be so small as to render normal methods of limited value. The company would have benefited from an understanding of the maximum financial risk at each point of the project and a plan to minimize that risk. Using the AoU would have added meaningful insight to the risk profile of this project by establishing the maximum cost possibly incurred at any point in the project.

We will introduce a second, smaller application to another real-life example later in the paper.

The dissertation is comprised of seven parts. After this introduction the second section is a literature review and the third section is a review of the AoU and a presentation of a new mathematical approach to solving the bounds of that AoU. The fourth section is a simulation to support the accuracy of the models and confirm the validity of the new model. The fifth section is an examination of two broad methods for modifying and using the mathematical approach to reduce catastrophic risk. The sixth section is an evaluation of the methods discussed in section four to compare the risk profiles of different projects and the seventh section is a summary of the work including areas of future study.

2. LITERATURE REVIEW

Managing projects under uncertainty has a long pedigree in research. Much work has been done on disruptions that do not force the cancelation of a project. Klastorin and Mitchell (2013) discuss delays of a project where overhead, indirect costs and penalties continue to mount, and the delay analyzed is singular and of unknown duration. Their method of addressing the delay is to compress the duration of activities only after the project has started or until after the delay occurs. Likewise, Hallgren and Wilson (2008) discuss a guerrilla attack that does not result in the cancelation of the project. Further, although the catastrophes used in this dissertation resemble those presented by the “force majeure” risks discussed in Sanchez-Cazorla et al. (2017), those risks do not result in the cancelation of the projects.

Activity failure, where the failure of an activity results in the failure of the project as a whole, has been well studied. Catastrophic failure was first studied by De Reyck and Leus (2008) in the context of R&D projects and has since been extended to consider modular projects (Coolen et al. , 2014, Huysmans et al. , 2015), choices regarding alternative-technologies (Ranjbar , 2013, Ranjbar and Davari , 2013, Creemers et al. , 2015), and make-or-buy decisions (Zhao and Chen , 2010). An unusual type of disruption occurs when an outside entity actively attempts to end the project after it discovers the project and determines it is a threat. Pinker et al. (2013) modeled the response to this risk to minimize exposer time—the time between when an adversary detects the project and when the project is scheduled to be completed. Pinker et al. (2014) further developed understanding the complexity of minimizing exposed time. Gutin et al. (2015) considered the type of adversary who can delay project activities, whereas Hermans et al. (2019) model intelligence gathering for an adversary trying to detect a project, consider it a zero-sum game. None of these risks from a rival necessarily result in the cancelation of the projects.

Disruption of a project can occur from within the project—endogenous—or from outside the project—exogenous. A further third type of disruption occurs when a mix of endogenous and exogenous forces combine to threaten the project such as those situations

discussed by Pinker et al. (2013, 2014). Here, the goals of crashing the project to finish as quickly as possible and the goal of preventing disruption to the project from an outside source which may require lengthening the project can come into conflict. The tools presented here can be used to account for stoppage attributed to either type of disruption, but the examples used are exogenous.

The tool used to measure a project and assess its vulnerability to failure due to a catastrophic event is the AoU first presented by Zhang and Elmaghraby (2014). The AoU for a project is the maximum and minimum costs (negative costs can represent cash inflows) of a project at all times possible during project execution. This creates an upper bound (U.B.) and a lower bound (L.B.) on the cost of the project at each time period. Although Elmaghraby theorizes the complexity of developing the U.B. and L.B. we present models to develop these bounds using a MILP. MILPs have a long history of use in project management (Szmerekovsky, J. G., & Venkateshan, P., 2012; Blazewicz J., Dror, M., & Iglarz, J., 1991).

Two types of scheduling are presented here: roadrunner scheduling and rail (or railroad) scheduling. Zhang and Elmaghraby (2014) present only a roadrunner start schedule and we introduce the rail schedule as a method of reducing risk from a catastrophic event. Rail scheduling was introduced by Herroelen and Leus (2004). This can limit the risk of non-catastrophic delay in scheduling but can increase the total duration of the project. As we will show, using a rail start schedule to protect against catastrophic event failure has the same results. Rail starts have also been used with priority lists or resource flow networks by Tian and Demuelemeester (2013), critical chain and buffer management methodology by Goldratt (1997), and project monitoring by Martens and Vanhoucke (2017).

After the U.B. and L.B. are presented the U.B. is subjected to an NPV review. NPV has a long history in project management (Sung, C. S., & Lim, S. K., 1994). This approach is also used to compare different AoUs using the same roadrunner start schedule. A recent review of project scheduling with the NPV objective is provided by Wiesemann and Kuhn (2015) who argue that uncertainty should be accounted for explicitly and explain complexities involved in doing so. In the mirror image of the goals of our work, Zheng et al. (2018) study a resource

constrained project scheduling problem where the objective is to maximize NPV given random activity durations.

Thus, this dissertation combines two major themes of project management, the NPV of outgoing cash flows in projects and project disruption using a relatively new tool in the AoU.

3. THE AOU

Introduced by Elmaghraby and Zhang (2011) the AoU represents the possible costs associated with a project and consists of an U.B. and a L.B. that are monotone, non-decreasing. These bounds will represent the “best” and “worst” case scenarios for any point of time during the project. An example from nature is the forecast of the possible paths a hurricane might take as it approaches shore: the further out from shore—the beginning of the AoU—the wider the mouth of the AoU is as it predicts where the storm will make landfall. The authors recognized that this approach has been used extensively to plan for *duration* of projects but add the insight that if durations of activities are random variables, *costs and payments* are also random variables. This should be of great interest to project managers attempting to measure the desirability of any given project. For the work of this paper, only costs are considered.

The principle addition made Elmaghraby and Zhang (2011), in addition to furthering the attention on cost, is the use of random variables—based on Monte Carlo Simulation (MCS)—to represent duration, cost, and income. Further, Elmaghraby and Zhang (2011) uses two modes of income: “...one that is based on a schedule of payments realized over time, and another that is based on the realization of certain milestone events”. Elmaghraby and Zhang (2011) postulates that the mathematical computations needed to determine the AoU are very difficult to compute and notes two “intractable” issues: first, which activities contribute to the cumulative costs and, second, how to estimate the costs of those activities. Because of the intractability noted, Elmaghraby and Zhang (2011) proposes a Monte Carlo Simulation (MCS) to mitigate these problems.

While an MCS is a legitimate method for obtaining the U.B. and L.B.—and indeed is used to support the accuracy of the LP introduced here—Elmaghraby and Zhang (2011) overstates the complexity of developing a sufficient mathematical model to establish the U.B. and L.B. of the AoU. Although they rely on an MCS, they also develop a convolutional integral with complex “regions” described by different density functions. The complexity of developing a model to show the range of outcomes for any given t (the reason Elmaghraby and Zhang

(2011)'s computations are so complicated) is unnecessary since all we are interested in are the two bounds. The full distribution is not relevant.

3.1. Mathematical Programming to Determine the AoU

Mathematical programming formulations for solving scheduling problems have a long history (Blazewicz, Dror, & Iglarz, 1991; Klemmt, Horn, Iigert, & Wolter, 2009; Pinedo, 2012). Since, "Many scheduling problems can be formulated as integer programs" (Pinedo, 2012), it is reasonable to expect that the bounds of the AoU problem can also be solved using integer programming.

For the AoU, the U.B. and L.B. are established by using two linear programs to develop a model that can provide these bounds at any given time during the project. This research demonstrates that the bounds have successfully been established in spite of the difficulties discussed by Elmaghraby (2011). Further, Elmaghraby and Zhang (2011)'s calculations result in a model, "...that the MCS cumulative distribution is sometimes smaller than the theoretical value *but mostly exceeds* it" (2011) (*italic added*). The model developed here shows that the MILPs' solutions either equal or exceed the results of the MCS as would be expected of a robust model. For the model of Elmaghraby and Zhang (2011) the differences between the calculations and the MCS are small whereas the difference between the solutions of the MILPs developed here and the MCS are meaningful.

3.1.1. Notation

There are N activities with a set of precedence constraints $(i, j) \in E$. Each activity j takes a random time to complete on the interval $[a_j, b_j]$ and incurs a cost of c_j per unit of time assuming a roadrunner start schedule is used (each activity must start as soon as its last predecessor finishes). Alternate start scheduling possibilities are discussed in section 4. We wish to find the maximum and minimum cost (the U.B. and L.B, respectively) that may accrue by time T . The likelihood and timing of disruption to the project is independent of the project schedule. If disruption indeed happens, the project is completely cancelled.

3.2. Upper Bound of the AoU

For the U.B. of the AoU with regular time-cost tradeoffs, the notation for the MILP is as follows:

$$M_1 = \text{Undisrupted project completion time when } d_j = b_j \quad \forall j$$

$$M_2 = \text{A sufficiently large number (ex. the maximum cost that can be incurred by the project, } \sum_j c_j b_j)$$

3.2.1. Variables

$$d_j = \text{duration of activity } j$$

$$s_j = \text{start time of activity } j$$

$$X_j = 1 \text{ if activity } j \text{ starts by time } T, 0 \text{ otherwise,}$$

$$Y_j = 1 \text{ if activity } j \text{ finishes by time } T, 0 \text{ otherwise,}$$

$$Z_j = \text{total cost accrued from activity } j \text{ by time } T.$$

$$F_{ij} = 1 \text{ if } i \text{ is a predecessor of } j \text{ and } j \text{ starts immediately after } i \text{ finishes.}$$

$$l_j = \text{Predefined start time for } j \text{ if rail scheduling is used as discussed below.}$$

$$F_j^l = 1 \text{ if } l_j \text{ is the start time of } j, 0 \text{ otherwise.}$$

3.2.2. Objective Function

$$\text{Max } \sum_{j=1}^n z_j \quad (1)$$

3.2.3. Constraints

$$s_j \leq T + M_1(1 - X_j) \quad \forall j = 1, \dots, N \quad (2)$$

$$s_j \geq T - M_1 X_j \quad \forall j = 1, \dots, N \quad (3)$$

$$s_j + d_j \leq T + M_1(1 - Y_j) \quad \forall j = 1, \dots, N \quad (4)$$

$$s_j + d_j \geq T - M_1 Y_j \quad \forall j = 1, \dots, N \quad (5)$$

$$Z_j \leq c_j (T - s_j) + M_2(1 - X_j + Y_j) \quad \forall j = 1, \dots, N \quad (6)$$

$$Z_j \leq c_j d_j + M_2(1 - Y_j) \quad \forall j = 1, \dots, N \quad (7)$$

$$Z_j \leq M_2 X_j \quad \forall j = 1, \dots, N \quad (8)$$

$$s_j \geq s_i + d_i \quad \forall (i, j) \in E \quad (9)$$

$$\sum_{(i,j) \in E} F_{ij} + F_j^l = 1 \quad \forall j = 1, \dots, N \quad (10)$$

$$s_j \leq s_i + d_i + M_1(1 - F_{ij}) \quad (i, j) \in E \quad (11)$$

$$s_j \geq s_i + d_i - M_1(1 - F_{ij}) \quad (i, j) \in E \quad (12)$$

$$s_j \leq l_j + M_1(1 - F_j^l) \quad \forall j = 1, \dots, N \quad (13)$$

$$s_j \geq l_j - M_1(1 - F_j^l) \quad \forall j = 1, \dots, N \quad (14)$$

$$s_1 = 0 \quad (15)$$

$$a_j \leq d_j \leq b_j \quad \forall j = 1, \dots, N \quad (16)$$

$$X_j, Y_j, F_{ij}, F_{jl} \in \{0, 1\} \quad \forall j = 1, \dots, N, (i, j) \in E \quad (17)$$

3.2.4. Explanation

To determine U.B. for the project at time T it is necessary to determine which activities have started (X_j) at time T. If j has started, the total cost for the duration of j completed before T (Z_j) is added to the objective function (1). This cost is the lesser of the partial cost for j , $c_j * (T - s_j)$, and the total cost for j , ($c_j * d_j$). Inequalities (2) and (3) ensure that j starts by time T if its starting time does not exceed T. Inequalities (4) and (5) ensure the same for finish times of activities and variable Y_j . Inequalities (6), (7), and (8) ensure that activity j contributes to the total cost at T only if it has started by T. Further they ensure that the cost contributed to T by j does not exceed its cost for duration d_j at T. Constraint (9) ensures that the precedent requirements of the network are enforced. Constraints (9) through (14) ensure that the roadrunner start schedule is used and constraint (15) sets the start time of activity 1 to 0. Equation (16) sets the duration of j between the lower (a_j) and upper (b_j) duration bounds from which the MILP is allowed to choose. Equation (17) makes the appropriate variables binary.

3.3. Lower Bound of the AoU

For the lower bound (L.B.) of the AoU with regular time-cost tradeoffs, the linear program notation is as follows:

3.3.1. Objective Function

$$\text{Min } \sum_{j=1}^n z_j \quad (18)$$

3.3.2. Constraints

$$Z_j \geq c_j (T - s_j) - M2(1 - X_j + Y_j) \quad \forall j = 1, \dots, N \quad (19)$$

$$Z_j \geq c_j d_j - M2(1 - Y_j) \quad \forall j = 1, \dots, N \quad (20)$$

$$Z_j \geq -M2X_j \quad \forall j = 1, \dots, N \quad (21)$$

$$s_j \geq s_i + d_i \quad \forall (i, j) \in E \quad (22)$$

3.3.3. Explanation

To determine the L.B. of the AoU the MIN objective function should delay the start of all activities as late as possible. Like the MAX LP the MIN LP must determine when activities have started (s_j) and how much each activity has contributed to the total cost (Z_j). Inequalities 19, 20 and 21 ensure that the MILP will assess costs appropriately by making sure the contribution of j equals the cost corresponding to its duration. Further, they ensure that it contributes only if it has started by time T . All other constraints serve the same function as they do in the MILP for the U.B. Inequality 22 forces every activity to start after its predecessor finishes.

3.4. Project Generation

Unless otherwise indicated, all problem instances were generated based on the following scheme. All projects are represented by an activity-on-node (AON) network N with n activities as described by Kolosch (1996) using ProGen. We generated networks of $n=20$, $n=50$, $n=98$ with network densities of 1.5, 1.8, and 2.1. Network density is defined as the ratio of the number of arcs in a project network to the total number of arcs possible where potential connections = $n*(n-1)/2$. Thus Network Density = Actual Connections/Potential Connections (Szmerekovsky, 2012). The density values are between 15% and 25% for $n=20$, 10% and 11% for $n=50$, and 3% and 4% for $n=98$. Network complexity is measured as the average number of non-redundant arcs per node including the dummy activities. The number of arcs actually incorporated into the network ActArcs is controlled by ϵ_{NET} , the network complexity deviation tolerance defined by (Kolisch, 1995) :

J = the number of non-dummy activities in the project

ϵ_{NET} = network complexity deviation tolerance

$$\text{ActArcs} \in [J^* \text{NC} * (1 - \epsilon_{\text{NET}})];$$

$$J^* \text{NC} * (1 - \epsilon_{\text{NET}})]$$

All random numbers for minimum (maximum) duration d^{\min} (d^{\max}) were generated using the Mersenne Twister pseudorandom number generator on which Microsoft Excel's RAND function is based (Mélard, 2014) where $1 \leq d^{\min} \leq 10$ and $d^{\min} < d^{\max} \leq d^{\min} + 10$. The durations for the AON—and thus s_j and f_j —are established by the MILP where $d^{\min} \leq d_j \leq d^{\max}$. Activity costs per unit time are uniform [1,5].

Each activity has predecessor set p_i where $0 \leq i \leq n$ resulting in Network N .

3.5. Calculating the AoU - Solution Times

U.B.s and L.B.s were calculated for all integer times using CPLEX 12.7 and a 3.6GHz processor with 16 GB of RAM. The programs were written in IBM ILOG CPLEX Optimization Studio version 12.8.0.0. The results for the U.B. are in Table 1 and the results for the L.B. are in Table 2. All times are for projects using a roadrunner schedule and are in seconds.

Table 1. Solution times for U. B. (seconds)

Nodes		Complexity		
		1.5	1.8	2.1
20	Min	3.94	2.93	2.48
	Max	5.79	6.10	7.28
	Average	5.54	4.54	4.16
	stdev	0.98	0.94	0.98
50	Min	9.89	8.19	11.61
	Max	16.95	15.45	19.08
	Average	13.56	12.15	14.64
	stdev	2.18	2.15	2.47
98	Min	52.62	65.82	67.22
	Max	60.16	73.46	75.16
	Average	56.31	69.58	71.39
	stdev	2.58	2.57	2.57

Table 2. Solution times for L.B. (seconds)

Nodes		Complexity		
		1.5	1.8	2.1
20	Min	3.73	2.62	2.79
	Max	6.08	5.81	7.06
	Average	5.32	4.28	4.42
	stdev	0.99	0.93	0.99
50	Min	11.59	10.09	6.09
	Max	18.45	16.75	21.48
	Average	14.97	13.73	12.45
	stdev	2.24	2.04	4.37
98	Min	39.62	52.22	55.02
	Max	46.96	61.46	61.21
	Average	43.34	56.54	58.35
	stdev	2.65	2.85	2.08

3.6. Results

Figure 1 shows the AoU for an example where $n=98$ and complexity is 2.1.

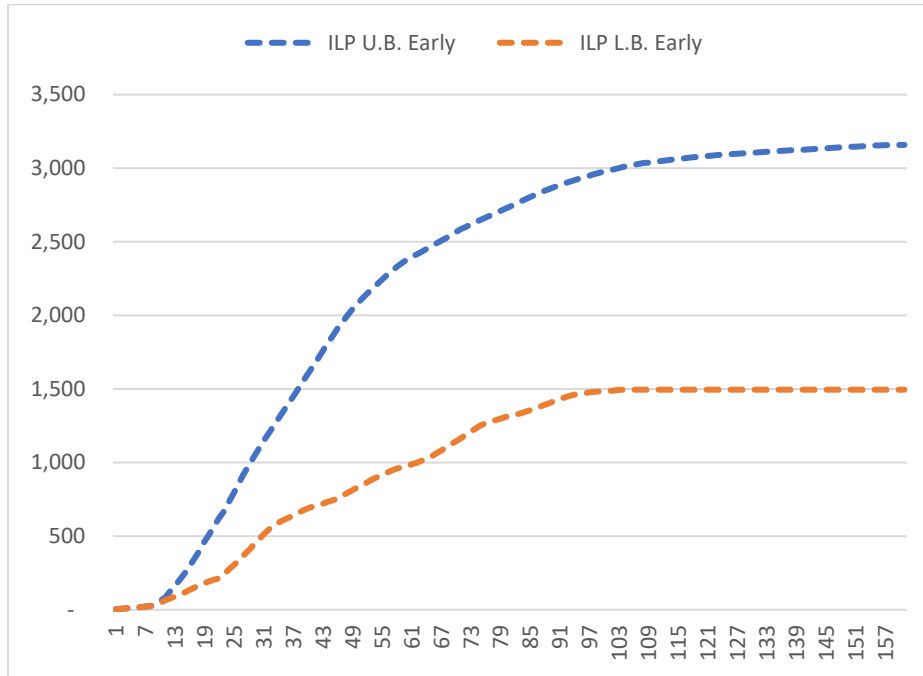


Figure 1. Upper and L.B.s for an AoU

4. SIMULATION VS. LP

The simulation was designed in Microsoft Excel using VBA programming. The appropriate screen shots and VBA code are included in Appendix B. Figure 2 shows a representative example of an AoU simulation compared to the MILP shown in Figure 1. For every T the MILP exceeds the simulation as would be required of a properly constructed MILP. Figure 3 shows the percentage variation of the U.B. for each T of this same AoU.

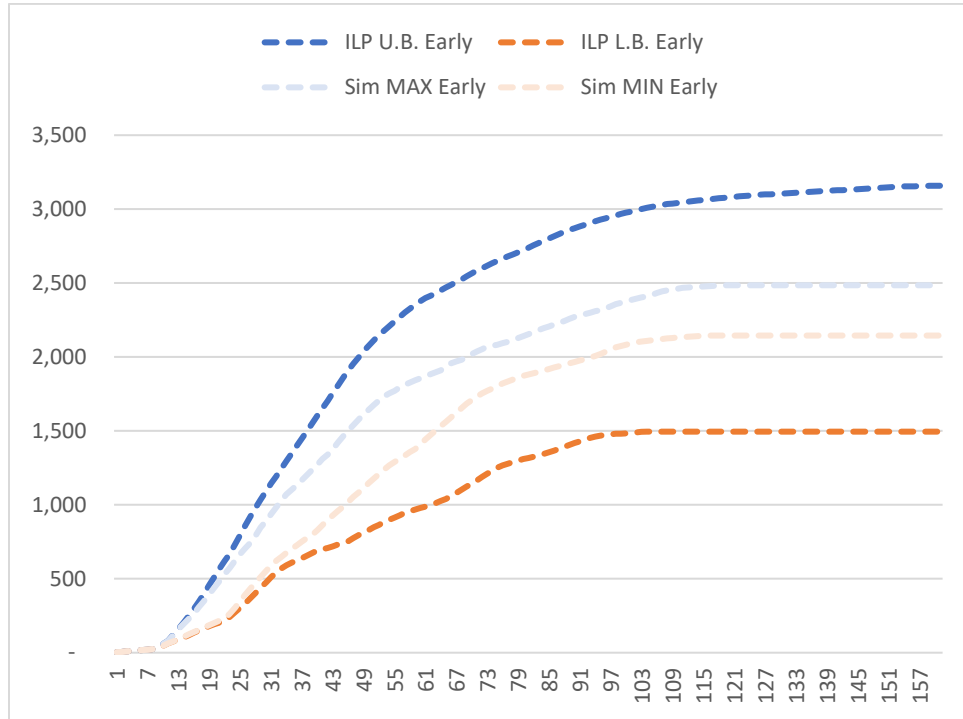


Figure 2. AoU ILP vs Simulation

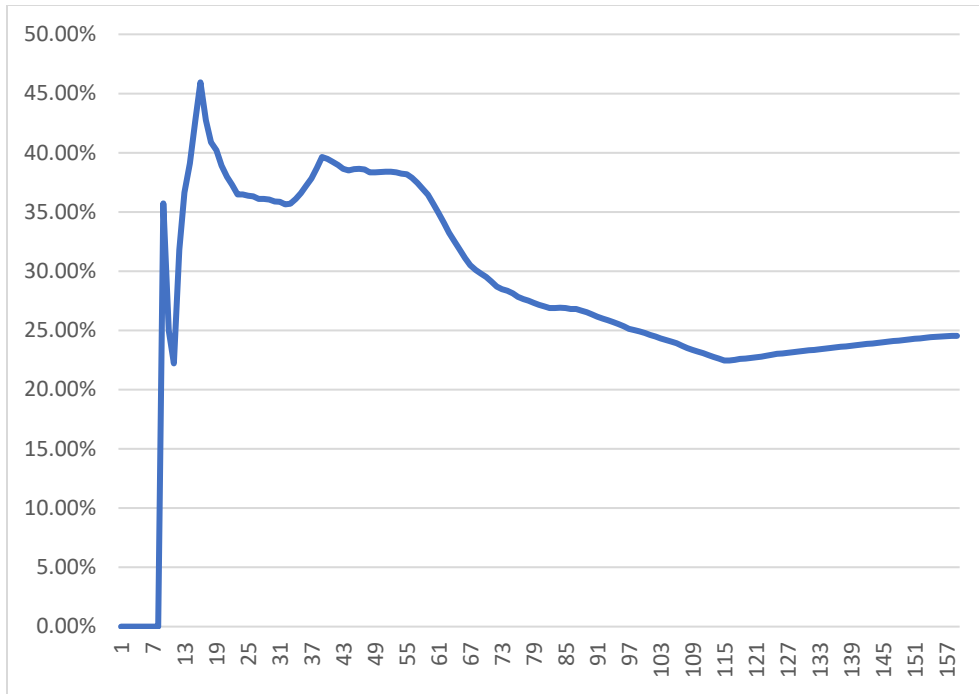


Figure 3. % ILP Exceeds Simulation Based on 1000 Runs.

Since each AoU has a distinct shape and duration getting an “average” difference between the LP and the simulation is problematic. Shown below is comparison of ten AoUs depicting the maximum, minimum, and average % difference between the LP and simulation for every t of that AoU. There is a wide variation across and within projects. The average difference between the simulations and the ILPs for each time t for the U.B.s of 10 AoUs shown in Figure 5 below is 28%.

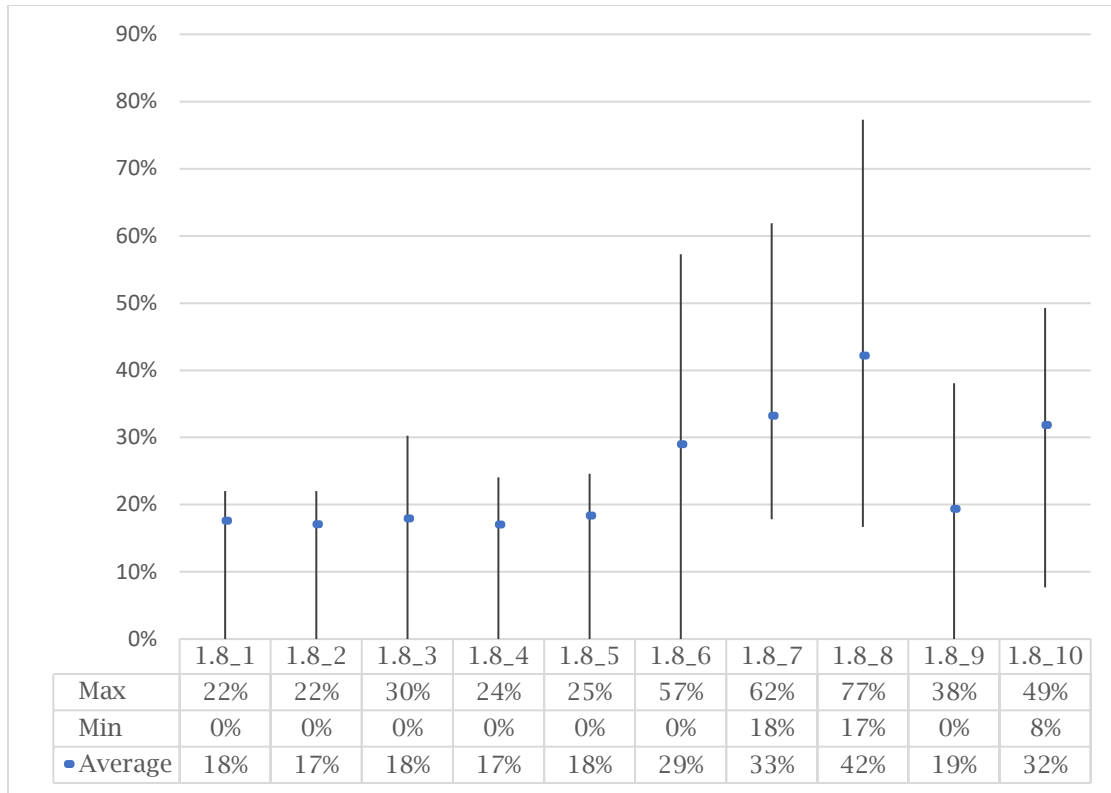


Figure 4. % Difference Between LP and Simulation

The fact that the MILP exceeds the simulation by a significant extent is verified across a selection of projects at 25%, 50%, 75% and 100% of completion (Table 3).

Over many AoUs the difference between the ILP and the simulation remains considerable.

There is only a very modest increase in accuracy gained by increasing the runs of the simulation. Increasing the number of simulations runs to 100,000 only reduces the disparity to 23% and the time needed to run a 100,000-run simulation with 98 nodes and a complexity of 1.8 is prohibitive (Figure 5).

Table 3. Average % the ILP Exceeds Simulation (based on 1000 runs)

	25%	50%	75%	100%
1.8_1	21%	21%	19%	21%
1.8_2	18%	22%	18%	20%
1.8_3	25%	19%	19%	20%
1.8_4	23%	19%	18%	18%
1.8_5	22%	20%	23%	21%
1.8_6	52%	27%	19%	19%
1.8_7	41%	20%	18%	20%
1.8_8	61%	30%	21%	21%
1.8_9	21%	18%	20%	22%
1.8_10	47%	32%	22%	21%
min	18%	18%	18%	18%
max	61%	32%	23%	22%
average	33%	23%	20%	20%
stdev	0.15	0.05	0.02	0.01

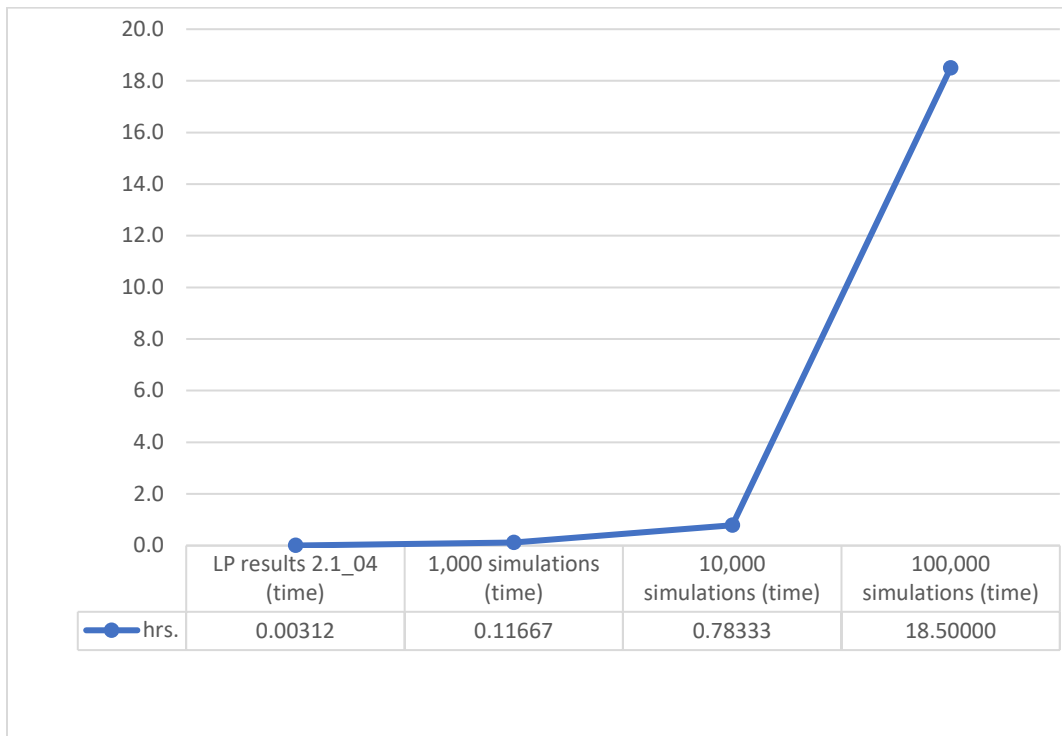


Figure 5. Simulation Run Time Based on Number of Runs.

All told, an MILP accurately models the AoU, U. B. and L.B. Note that the MILP's consistently higher numbers—combined with the ineffectiveness of increasing the runs of the simulation to meaningfully reduce the gap—speaks to the usefulness of the simulation as a potentially more “practical” number for planning purposes. It is very rare, based on the simulation results, for a project to reach the theoretical bounds revealed by the MILP. Planning

for such an extreme and unlikely eventuality may be an inappropriate use of resources. Planners may plan for the worst based on an MILP that in fact shows a theoretical maximum for the U.B. that would rarely ever be realized during an actual project. A simulation with a small number of runs can execute quickly and provide results similar to a simulation with a larger number of runs that could not be run in a reasonable amount of time.

5. REDUCING THE RISK FROM A CATASTROPHIC EVENT

Catastrophic events are very rare and thus unpredictable; therefore, they are often not accounted for in the planning process. Further, they allow for very little recourse by the actors after they occur. “As its name suggests, unforeseen uncertainty can’t be identified during project planning. There is no Plan B. The team either is unaware of the event’s possibility or considers it unlikely and doesn’t bother creating contingencies” (De Meyer, Loch, & Pich, 2002; Geraldi, Lee-Kelley, & Kutsch, 2010). However, catastrophic events are arguably inevitable given enough projects (Hällgren & Wilson, 2008). A catastrophic event can devastate an entity should it occur during a project. A natural disaster, political crisis, extended labor unrest, or war can permanently terminate a project leaving the harmed party without recourse to recover losses from the responsible party. For instance, if a government contracting with a company is overthrown, the new government might not recognize debts incurred or obligations born by the defunct government.

But the situation causing a catastrophic event might be far more pedestrian than revolution. In another example known to the author, a subcontractor on a large government security renovation was responsible for installing blast resistant glass in several federal buildings. The sub-contractor was given the go-ahead by the general contractor after the concerned government agency gave the project the green light. The subcontractor purchased the highly specialized glass on credit from his regular supplier and, because the scale of the project would keep his company busy for well over a year, turned down other projects available in the then booming commercial market. Subsequently, the government agency placed an unexpected hold on the project pending the results of a departmental audit. The glass subcontractor was not able to cancel the glass order and—even though the glass supplier was willing to wait on payment—could not secure further credit for any new work. To make matters worse, a financial crisis came on, eliminating most new commercial construction in the subcontractor’s primary market even had additional credit been available. Both the suspension of the government project and the severe economic downturn are the kind of catastrophic events difficult to account for in a model. Although there may have been contractual or

statutory remedies for breach of contract, these proceedings are difficult and time consuming in the best structured agreements. Many agreements—particularly when dealing with foreign and authoritarian governments—can prove practically impossible to enforce.

In determining the risk of a catastrophic event, the “worst case” outcomes will be used. Thus, the analysis will be limited to adjustments to and comparisons of the U.B. of various AoUs for a project. We will examine two approaches to scheduling to see how best to reduce risk: roadrunner schedule start times and rail schedule start times.

5.1. Reducing Risk: Roadrunner Start Times

Since a key premise of this analysis is that catastrophic events are unpredictable, they can occur at any time of the project with equal probability. Thus, lengthening the project could add to the risk of occurrence. Hence, all else being equal, shorter is better. The basic MI LP established in Section 2 requires each activity to start at the end of its latest finishing predecessor but allows the program to select the best or worst duration (d_j) as needed to establish the U.B. and L.B. Thus the model follows roadrunner start time scheduling. Starting each activity at its earliest possible time will inherently minimize the total time of the project thereby minimizing the risk associated with the length of time to complete the project but could increase the total cost associated with most discrete times. So, the model “as is” will accomplish this method of scheduling a project to minimize the risk associated with a catastrophic event by limiting the total time a project is underway. In this approach $s_j = (s_i + d_i)$ for some i where $(i, j) \in E$.

5.2. Reducing Risk: Rail Schedule Start Times

Artificially delaying the start times of an activity reduces the U.B but can extend the duration of the project. We present one method to delay the start times: scheduled start time (rail scheduling). Then we present three methods to compare the U.B.s of a project using this method of delay.

In this approach, where S_j is the scheduled start time of activity j , either $s_j = (s_i + d_i)$ for some i where $(i, j) \in E$ or $s_j = S_j$.

The methods used to compare the different AoUs of a project are also useful in comparing the risks represented by the AoUs of different projects. To determine if rail scheduling is superior to roadrunner scheduling we attempt to answer three questions:

- (1) Does delaying the start times of activities reduce the U.B. for any individual time;
- (2) Does the increase in start times increase the length of time to finish the project; and;
- (3) Is the benefit in reduction of the U.B. for any given period or all periods outweigh the increased length of the project?

5.3. Effects of Delaying Start Times on Costs of the Network

The results of the two methods (roadrunner start and rail start) on the U.B. of the AoU for our representative 98 node network are shown in Figure 6.

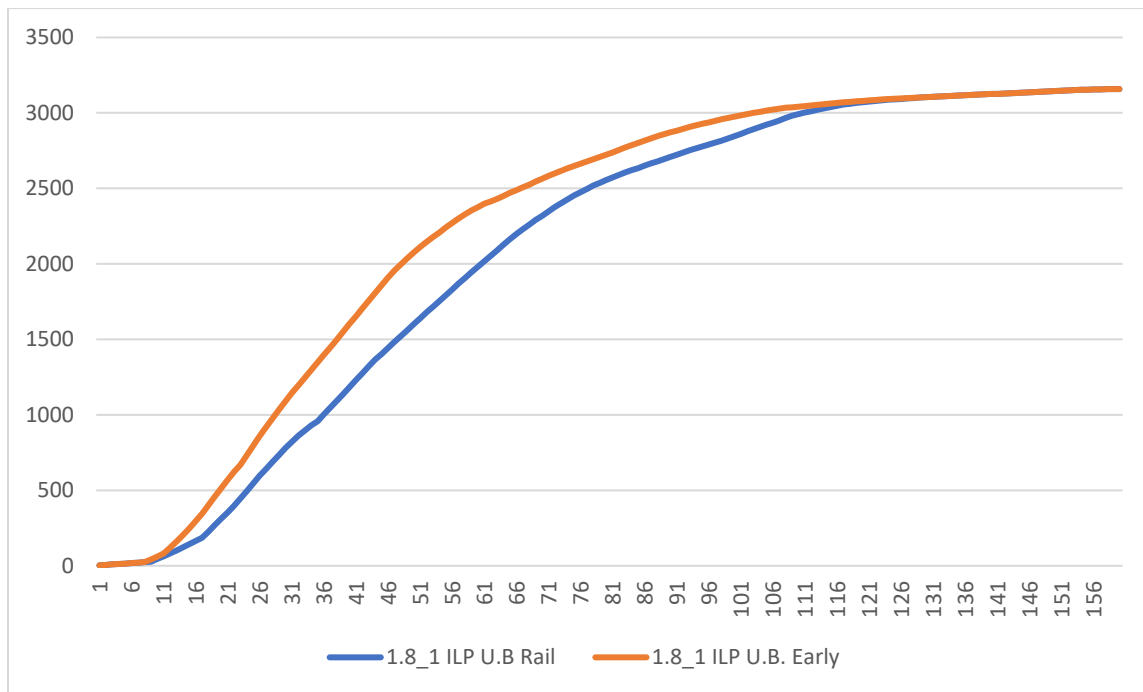


Figure 6. Roadrunner vs. Rail Schedule U.B.

After the very first few periods of a project where the AoUs are the same, forcing each activity to start immediately after its last finishing predecessor—roadrunner scheduling—increases the U.B. vs. rail scheduling and this holds true and increases over the first half of the AoU and then remains higher until the last periods of the project when the AoUs again converge. This pattern is consistent across all size networks and all complexities. A project's

rail schedule is set by using the average start time for each activity derived from the simulation for that project.

5.3.1. Comparison of Costs at Different Completion Time

The primary rationale for setting a start schedule with $s_{j,Rail} \geq s_{j,Roadrunner}$ is to lower the cost at any given point of the AoU for as long as possible thus lowering the total risk at many points of the project. The U.B. for the rail schedule compared to the U.B. for the roadrunner schedule for the AoUs is shown in Figure 6 at given percentages of the completion time.

Table 4. Sums of Costs for Roadrunner and Rail Scheduling

At % of Completion Time	$\sum_{j=1}^{n_s} z_j$		Average reduction in total cost with Rail Schedule
	ILP U.B Rail	ILP U.B. Roadrunner	
10%	123	202	-39.1%
25%	1094	1498	-27.0%
50%	2497	2679	-6.8%
75%	3055	3070	-0.5%
90%	3125	3125	0.0%
100%	3158	3158	0.0%

Compared to roadrunner scheduling, at the earlier stages of the project there is a considerable reduction in risk using rail scheduling. The reduction in roadrunner cost using rail scheduling is pronounced. The clear benefit of using rail scheduling is manifest in Table 4. Using rail schedule reduces risk for the first 50% of the project with an average reduction in risk of 25%. This finding is consistent across projects.

However, using expected completion times from the simulation, rail scheduling extends the time to complete the project (Figure 7).

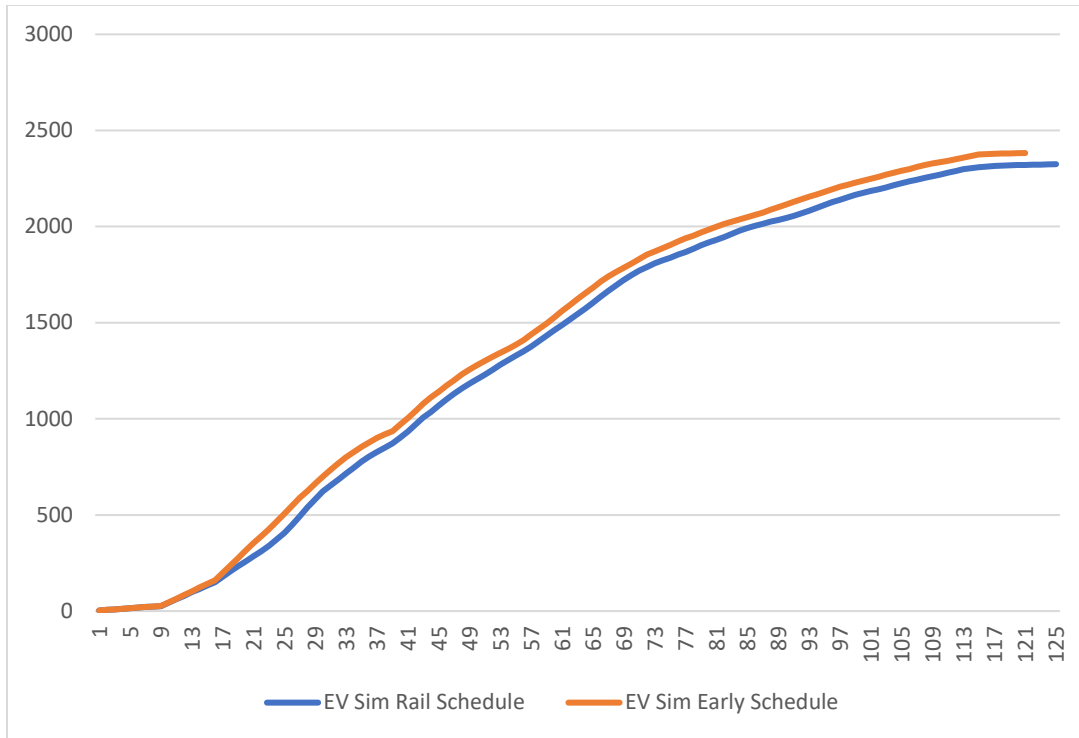


Figure 7. Rail start vs. Roadrunner Start Simulations

So, there is a tradeoff in how the schedule chosen affects the project: lower earlier costs reducing risk, longer project completion times at greater total costs later in the project.

Since roadrunner scheduling will always have an activity start as soon as possible and start adding costs as soon as possible we would expect the total cost for each t to be higher than when using rail scheduling. Likewise, by delaying the start of some activities rail scheduling pushes back the expected finish time of the entire project. Although the difference is small (Table 5), We need to ascertain whether the reduction in risk during the bulk project is worth the increased risk due to the lengthening of the time a catastrophic event can occur.

Table 5. Completion Time Rail vs. Roadrunner Scheduling

	Sim Rail Schedule	Sim Roadrunner Schedule	% increase of Rail
final t	125	121	3.31%

5.3.2. Simple NPV Comparison

Using a simple NPV analysis where we discount the marginal cost at each period, we can normalize the AoUs to account for the change in duration of the project vs. the cost reduction for roadrunner scheduling. Below are the cost schedules for the two start options under consideration with the addition of the marginal cost schedules of the simulation.

Table 6. Total and Marginal Costs for Upper Bound

t (weeks)	EV Sim Rail Schedule Total Cost	EV Sim Roadrunner Schedule Total Cost	EV Sim Rail Schedule Marginal Cost	EV Sim Roadrunner Schedule Marginal Cost
1	3	3	3	3
2	6	6	3	3
3	9	9	3	3
4	12	12	3	3
5	15	15	3	3
6	18	18	3	3
7	21	21	3	3
8	24	24	3	3
9	26	27	2	3
10	44	45	18	18
11	62	63	18	18
12	78	83	16	20
13	98	102	20	20
14	115	123	18	21
15	133	142	18	19
16	150	160	18	19
17	178	198	28	38
18	206	237	28	40
19	234	275	28	38
20	259	314	25	39
113	2298	2359	9	8
114	2305	2367	7	8
115	2309	2375	4	8
116	2313	2378	4	3
117	2316	2379	3	1
118	2318	2380	2	1
119	2319	2381	1	1
120	2320	2382	1	1
121	2321	2383	1	1
122	2322	2383	1	
123	2323	2383	1	
124	2324	2383	1	
125	2325	2383	1	

The rail schedule yields a lower total cost for the project but has a longer expected finish time based on the EV of the simulation.

The two options for discounting the U.B. cost schedule are to either discount the marginal costs or to discount the total costs at each time t. Discounting the marginal costs shows the incremental risk associated with an additional day of delay. Discounting only the marginal costs adds only a very small additional cost to the project and does not reflect the total additional risk of extending the completion of the project. When this increase is discounted using NPV its additional risk is not just negligible, the overall NPV and thus risk is *less* using rail scheduling since the NPV of the roadrunner periods overwhelms the extra costs for the extra period to complete the project as shown in Table 7.

Table 7. NPV of Total Costs at 10%

Discount Rate (week)	roadrunner start (LP)	rail start (LP)	additional cost of rail start
0.192%	2381.8579	2323.8572	-2%

This method of analysis may then underestimate the risk of extending the length of the project.

One method to address this problem is to sum all maximum potential costs for all times along the alphorn. What the “sum of all sums” does is reflect the potentially significant increase in risk that could result in lengthening the project. In our example above, the lengthening of the project due to the rail schedule is only five days, or less than 7% of the project length. But those five days are five days of *maximum risk for the project* since they are seven days of maximum costs. Using the NPV of the marginal costs, may not reflect this reality. Indeed, as demonstrated below, marginal cost analysis shows little or no increase in risk (in this case the reduction in risk is not consistent across all AoUs tested) for a delayed start time. Using a marginal cost analysis would only add the additional risk of the slight increase in costs from each increased T added by the rail schedule instead of the fact the entire cost of the project is “on the line” for the extra days of delay.

Using a simple NPV for the sum of all costs shows very little penalty for using rail scheduling and thus delaying the finish time for the project. Discounting the total costs for each T of the AoU yields the following NPVs for the sum of total costs (10% annual discount rate, period unit = weeks):

Table 8. NPV Sum of Total Costs at 10%

Discount Rate (week)	roadrunner start (LP)	rail start (LP)	additional cost of rail start
0.192%	144,706	145,697	1%

Even though the sum of all costs shows a slight increase in risk for using rail scheduling the risk is very small. But an inherent disadvantage of either simple NPV analysis is that it does not account for the reality of a failure ending the cost stream to the project. It compares the total NPV of the completed project. To better assess the risks of different methods of delaying start times, some accounting must be made for the effects on the project of a catastrophic event occurring during the project.

5.3.3. NPV Comparison Assuming Probability of a Catastrophic Event

A related method involves expected value of costs realizing some probability of catastrophic failure during the project. Although catastrophic events as we are using the concept here are exceedingly rare and therefore do not lend themselves to normal probability evaluation, if we assign a probability even though small, we can get the NPV of the expected costs considering some chance of failure at each step of the project. Assuming that catastrophic disruptions occur at a rate of 0.01 each period we can model the likelihood of a disruption by time t as:

$$1 - e^{\left(\frac{-t}{\beta}\right)}$$

Where t = time period and:

$$\beta = -\frac{1}{\ln(1 - p)}$$

Where p = Daily rate of failure

With this addition to the NPV analysis we put some consideration in the calculations that a cost may not occur at all due to a catastrophic failure. The NPV of expected costs for each t in the project after figuring the probability of failure is (discount rate 10%, daily rate 0.0274%):

Table 9. Cost Schedule with Probability of Failure

t (week)	P(failure/day)	Beta	Exp. Cost	roadrunner start (simulation)	rail start (simulation)
1	0.010%	9999.5000	0.9999	2.9997	2.9997
2	0.010%	9999.5000	0.9998	5.9988	5.9988
3	0.010%	9999.5000	0.9997	8.9973	8.9973
4	0.010%	9999.5000	0.9996	11.9952	11.9952
5	0.010%	9999.5000	0.9995	14.9925	14.9925
6	0.010%	9999.5000	0.9994	17.9892	17.9892
7	0.010%	9999.5000	0.9993	20.9853	20.9853
8	0.010%	9999.5000	0.9992	23.9808	23.9808
9	0.010%	9999.5000	0.9991	26.9757	25.4771
10	0.010%	9999.5000	0.9990	44.9550	43.4565
100	0.010%	9999.5000	0.9900	2216.2254	2154.3474
101	0.010%	9999.5000	0.9900	2226.3983	2163.5365
102	0.010%	9999.5000	0.9899	2236.5691	2171.7338
103	0.010%	9999.5000	0.9898	2246.7378	2181.9091
104	0.010%	9999.5000	0.9897	2256.9045	2193.0719
105	0.010%	9999.5000	0.9896	2266.0796	2202.2533
106	0.010%	9999.5000	0.9895	2274.7581	2211.4329
107	0.010%	9999.5000	0.9894	2285.4135	2220.1160
108	0.010%	9999.5000	0.9893	2295.5722	2228.7973
109	0.010%	9999.5000	0.9892	2303.2559	2237.4769
110	0.010%	9999.5000	0.9891	2309.9490	2246.1547
111	0.010%	9999.5000	0.9890	2316.6407	2254.8307
112	0.010%	9999.5000	0.9889	2324.3200	2263.5050
113	0.010%	9999.5000	0.9888	2331.9976	2272.1775
114	0.010%	9999.5000	0.9887	2339.6738	2278.3766
115	0.010%	9999.5000	0.9886	2347.3483	2282.1030
116	0.010%	9999.5000	0.9885	2350.0790	2285.8287
117	0.010%	9999.5000	0.9884	2350.8323	2288.5652
118	0.010%	9999.5000	0.9883	2351.5855	2290.3129
119	0.010%	9999.5000	0.9882	2352.3385	2291.0720
120	0.010%	9999.5000	0.9881	2353.0914	2291.8310
121	0.010%	9999.5000	0.9880	2353.8440	2292.5898
122	0.010%	9999.5000	0.9879		2293.3484
123	0.010%	9999.5000	0.9878		2294.1068
124	0.010%	9999.5000	0.9877		2294.8651
125	0.010%	9999.5000	0.9876		2295.6232

This results in the total NPV of costs at all t of the network, accounting for the risk of failure for each t:

Table 10. Project Completion Time: Rail vs. Roadrunner Scheduling

98 Nodes	Rail Start (LP)	Roadrunner Start (LP)	Additional Cost of Rail Start
1.8_1	130	128	-1.54%
1.8_2	134	132	-1.49%
1.8_3	148	143	-3.38%
1.8_4	123	119	-3.25%
1.8_5	164	159	-3.05%
1.8_6	139	138	-0.72%
1.8_7	154	153	-0.65%
1.8_8	167	163	-2.40%
1.8_9	133	129	-3.01%
1.8_10	160	154	-3.75%
Average			2.23%
stdv			1.08%

Using this method of analysis, rail scheduling increases the risk of the project. Further, there is a tradeoff between the different start protocols for longer projects: lower total roadrunner cost for the entire project vs. lower rail schedule costs for discrete times during most of the project. This increase in finish times for projects using rail scheduling is also consistent across multiple projects (Table 11). Neither the additional time nor the additional cost is great, but it is present and needs to be considered by project managers as they make their scheduling decisions.

Table 11. Project Cost: Rail vs. Roadrunner (98 Node)

98 Nodes	Discount Rate (10%/52 weeks)	Rail Start (LP)	Roadrunner Start (LP)	Additional Cost of Rail Start
1.8_1	10%	163,178	162,994	0.11%
1.8_2	10%	163,237	164,697	-0.88%
1.8_3	10%	192,604	189,901	1.42%
1.8_4	10%	153,478	145,428	5.67%
1.8_5	10%	211,349	207,666	1.77%
1.8_6	10%	199,133	190,885	4.32%
1.8_7	10%	179,372	175,356	2.29%
1.8_8	10%	196,453	191,299	2.69%
1.8_9	10%	160,779	157,744	1.92%
1.8_10	10%	179,127	174,013	2.94%
Average				2.23%
stdv				1.79%

However, for smaller networks there was no advantage in total cost for roadrunner scheduling vs. rail scheduling. This is also true across multiple projects. This would intuitively make sense since there is less time needed to complete the entire project and thus less time for the failure to occur using either method of scheduling.

Table 12. Project Cost: Rail vs. Roadrunner (20 Node)

20 Nodes	Discount Rate (10%/52 weeks)	Rail Start (LP)	Roadrunner Start (LP)	Additional Cost of Rail Start
1.5_1	10%	18,564	19,974	7.60%
1.5_2	10%	24,801	24,725	-0.31%
1.5_3	10%	23,741	23,298	-1.87%
1.5_4	10%	18,882	18,467	-2.20%
1.5_5	10%	16,207	15,594	-3.78%
1.5_6	10%	30,330	30,765	1.43%
1.5_7	10%	16,968	15,693	-7.51%
1.5_8	10%	18,718	17,369	-7.21%
1.5_9	10%	24,993	26,708	6.86%
1.5_10	10%	16,858	18,375	9.00%
Average stdv				0.20% 5.64%

5.4. Preferred Risk Reduction Strategy

Both methods for scheduling projects—roadrunner scheduling and rail scheduling—have attributes that make them attractive to a project manager. Roadrunner scheduling has the advantage of simplicity as few additional computations are needed once the AoU is established. Further, road runner scheduling is intuitively pleasing and thus easy to explain to and be understood by the parties involved. In the large development project detailed above, the principles were uninterested in reducing risk any way except, “start everything right away, get finished, get paid, and get out.” Further, for smaller projects, the entire duration of the project is short and the risk reduction during the early part of the project is less important.

Rail scheduling has the significant advantage of reducing the maximum risk for many stages of the project without unacceptably offsetting those reductions by extending the length of the entire project. In addition, it gives any subcontractors on the project a “no earlier than” start date for all activities of the project.

Using the two real-world examples described above, the large development project, rail scheduling is recommended because the complexity of the organization running the project allows a more sophisticated scheduling approach and the savings are greater due to the much longer duration of the project. The smaller “glass” project would benefit from U.B. planning but would further benefit from the simpler roadrunner scheduling. However, an even simpler method may have been available to this smaller and simpler project: a down payment from the contractor to the glass sub-contractor to cover some or all the cost of the glass. In either case using the U.B. of the AoU would benefit planners by establishing the maximum risk presented by the projects.

6. COMPARING UPPER BOUNDS FOR DIFFERENT PROJECTS

When comparing U.B.s of different projects, the same methods can be used to compare risk that we used to compare risks between different U.B.s created for the same project using different start times. If one of the U.B.s is below for every time period and thus lower cost for the entirety of the project, we may consider this AoU superior (i.e. less risk) than the other for the purpose of minimizing maximum risk. We may consider it akin to stochastic dominance as used to compare probability distributions (Rothschild & Stiglitz, 1970). Where the difficulty lies--as in all stochastic dominance issues—is in comparing AoU U.B.s that do not clearly dominate one another but rather cross or “twist”.

As an example, the following two crossing AoU U.B.s are both 98 node projects with a complexity of 1.8 using roadrunner scheduling. It is not immediately clear which project has less risk. Earlier in the life of both projects project A has lower potential maximum cost. But later in the lives of the projects, project A has both higher maximum potential cost and a longer project life.

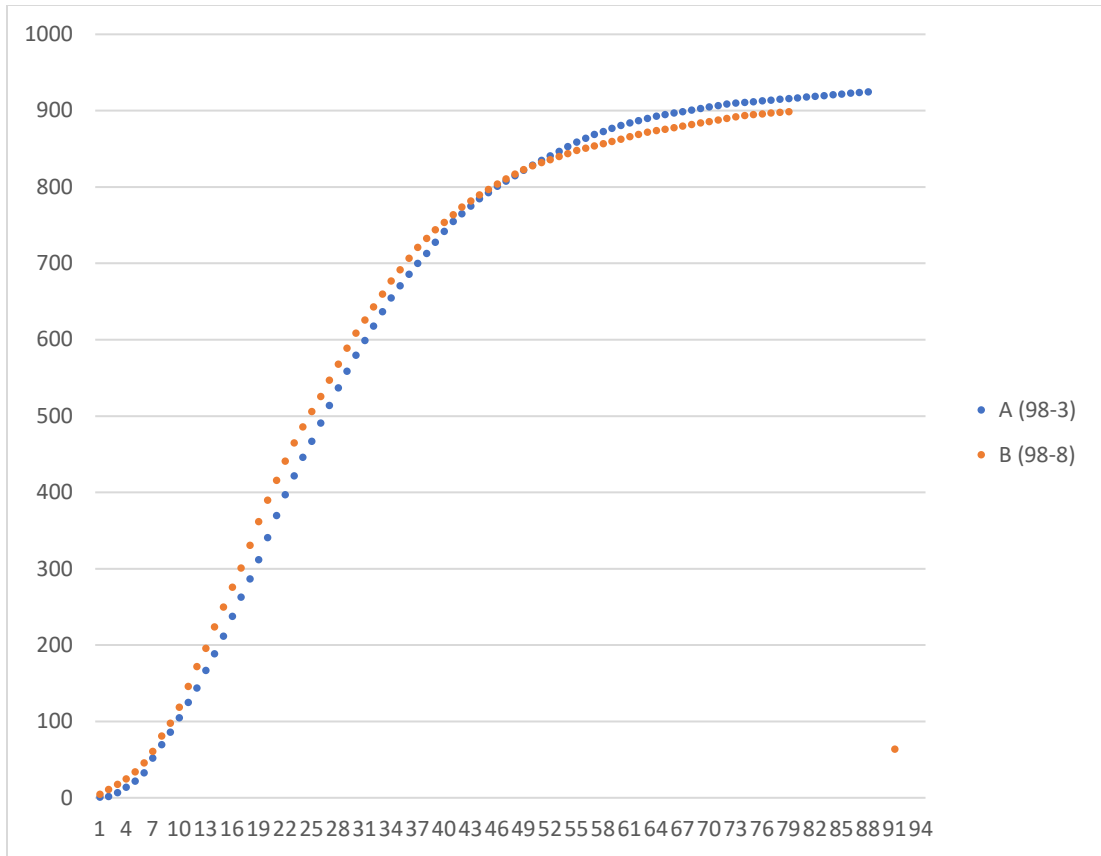


Figure 8. “Twisted” AoUs

6.1. The NPV of the Sum of Total Costs Method

Using the total cost with probability of failure is the preferred method for comparing twisted AoUs. Table 13 shows the schedule of expected costs and Table 14 the NPVs associated with that schedule. Here the discount rate is 8% per year with each period representing one day.

Table 13. Costs with Probability of Failure for Different Projects

t (week)	P(failure/day)	Beta	Exp. Cost	Expected Cost A	Expected Cost B
1	0.010%	9999.5000	0.9999	1.000	5.000
2	0.010%	9999.5000	0.9998	2.000	10.998
3	0.010%	9999.5000	0.9997	6.998	17.995
4	0.010%	9999.5000	0.9996	13.994	24.990
5	0.010%	9999.5000	0.9995	21.989	33.893
6	0.010%	9999.5000	0.9994	32.980	45.972
7	0.010%	9999.5000	0.9993	51.964	60.957
8	0.010%	9999.5000	0.9992	69.994	80.935
72	0.010%	9999.5000	0.9928	902.478	883.615
73	0.010%	9999.5000	0.9927	903.381	885.512
74	0.010%	9999.5000	0.9826	904.283	887.408
75	0.010%	9999.5000	0.9825	905.185	888.312
76	0.010%	9999.5000	0.9824	906.087	889.216
77	0.010%	9999.5000	0.9823	906.989	890.119
78	0.010%	9999.5000	0.9822	907.890	891.022
79	0.010%	9999.5000	0.9821	908.792	891.926
80	0.010%	9999.5000	0.9820	909.693	
81	0.010%	9999.5000	0.9819	910.594	
82	0.010%	9999.5000	0.9818	911.495	
83	0.010%	9999.5000	0.9817	912.395	
84	0.010%	9999.5000	0.9816	913.296	
85	0.010%	9999.5000	0.9815	914.196	
86	0.010%	9999.5000	0.9814	915.096	
87	0.010%	9999.5000	0.9813	915.996	
88	0.010%	9999.5000	0.9812	916.895	
89	0.010%	9999.5000	0.9811	917.795	
90	0.010%	9999.5000	0.9810	918.694	

Table 14. NPV of Total Costs for sample “Twisted” Projects

Discount Rate (day)	NPV A	NPV B
0.03%	57,722	48,303

Project B has a significantly lower risk profile than Project A. The roadrunner cost savings of A are overcome with the higher cost later in the project and the longer duration of A. Thus, using an LP to determine the U.B. of the AoU and applying the NPV adjustment with the estimated probability of a catastrophic event can assist project managers in deciding between different projects where one project is not clearly riskier than the other.

7. SUMMARY

It is both necessary and possible to assess the risk of a project failing due to a catastrophic event even if that event is extremely unlikely. Using the U.B. of the AoU, the maximum cost possible at each time period of the project can assist in identifying the maximum risk of a project. Further, different methods of scheduling can reduce the risk during execution of a project. Using rail scheduling to delay some start times reduces the total risk of a project compared to using roadrunner scheduling. Using NPV methods can also make the AoU useful for comparing the total risk from catastrophic events for different projects.

We demonstrated that both the U.B. and L.B. of the AoU can be established using a MILP. The legitimacy of the MILP was verified by using a simulation and comparing the results with the MILP. In every case, the bounds established by the simulation were contained within the bounds of the MILP. However, the likelihood of costs reaching an U.B. at any time during project execution is so small that a project manager may decide the simulation is sufficient for planning.

8. AREAS FOR FUTURE RESEARCH

The following areas lend themselves naturally to further research:

- (1) Since the numerical experiments assume a lump sum payment at the end for the project, what are the effects on both solvability of the MILPs and on methods to extend start times if payments are made upon completion of milestones or a set calendar schedule?
- (2) Is the presence of a “trip-wire” cost amount below which the catastrophic failure of a project would not be catastrophic to the overall health of the entity engaging in the project and above which the failure of the project would be catastrophic change the decisions managers make about the project?
- (3) Once actual costs for periods are known as the project progresses, does re-running the MILPs change the risk profiles enough to change the decisions made by the project managers (ex. crashing certain activities of the project).
- (4) Computational improvements for the MILPs.
- (5) Finding the optimal project schedule.
- (6) Alternative measures of project risk and utility.
- (7) Incorporating resource constraints.
- (8) The ability to reduce buffer time for an activity if rail scheduling is used, the schedule itself is a form of buffer.
- (9) The impact of roadrunner and rail scheduling on the disbursement schedule.

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APPENDIX A: CPLEX CODE FOR THE UPPER BOUND

```
// -----  
//  
//   Model for Upper Bound  
//  
//  
//read in contents from .dat  
int N = ...;    //total number of activities (indexed by j)    //<--- greater equal to 1  
int t = ...;    //time period being calculated  
string outputFile= ...;  
//  
range Act = 1 .. N;    //Activities  
//  
int c[Act] = ...; //c_j = cost per period  
int a[Act] = ...; //a_j = minimum duration for activity j  
int b[Act] = ...; //b_j = maximum duration for activity j  
int r[Act] = ...; //r_j = rail schedule for activity j (rail schedule)  
  
//model with activities on the arcs of a Figure  
int p[Act][Act] = ...; // {0,1} predecessor matrix: p[from][to]=1 iff there (from, to) is an arc in the  
Figure  
  
//calculated values  
int SUMB = sum(j in Act) b[j];  
int numPre[j in Act] = sum(f in Act)(p[f][j]); //counts the number of predecessors for each  
activity  
int M1 = SUMB * 10;
```



```

int M2 = SUMB * 10;

//decision variables

//    d_j = duration of activity j, in [min,max] time of act. j (e2)(e3)
dvar float+    d[j in Act] in a[j]..b[j];

dvar float+ s[j in Act] in r[j] .. SUMB; //s_j = start time of activity j //vin: improved u.b. (rail
schedule)

dvar int+    x[Act] in 0 .. 1; //binary, to establish if act. j has started
dvar int+    y[Act] in 0 .. 1; //binary, to establish if act. j has finished

//Added as per Dr. Joe
dvar int+    F[Act][Act] in 0 .. 1; //binary to establish if i is a predecessor of j and j starts
immediately after i finishes.
dvar int+    G[Act] in 0 .. 1; //binary to establish if rail schedule is used

//    total cost of act j through time    range of z is between 0 and c[j]*b[j] (e8)
dvar float+    z[j in Act] in 0 .. c[j]*b[j];    //vin: improved u.b.

maximize sum(j in Act) z[j]; //(e1)

subject to{
    forall (j in Act){
        e2:    s[j] <= t + M1*(1-x[j]);
        e3: s[j] >= t - M1*x[j];
        e4: s[j] + d[j] <= t + M1*(1-y[j]);
        e5: s[j] + d[j] >= t - M1*y[j];
        e6: z[j] <= c[j]*(t - s[j]) + M2*(1 - x[j] + y[j]);
        e7: z[j] <= c[j]*d[j]+ M2*(1 - y[j]);
    }
}

```

```

        e8: z[j] <= M2*x[j];
    }

//for activities with predecessors
    forall(j in Act : numPre[j] > 0){          //forces j to start
        forall(i in Act : p[i][j]==1){
            e9: s[j] >= s[i] + d[i];
            e11: s[j] <= s[i] + d[i] + M1*(1 - F[i][j]);
            e12: s[j] >= s[i] + d[i] - M1*(1 - F[i][j]);
        }
    }

    forall (j in Act){
        e13: s[j] <= r[j] + M1*(1 - G[j]);
        e14: s[j] >= r[j] - M1*(1 - G[j]);
    }

    forall (j in Act){
        e10: sum(i in Act : p[i][j]==1)
            F[i][j] + G[j] == 1;
    }

//for activities with no predecessors
    forall(j in Act : numPre[j]==0){
        e14b: s[j] == 0;
    }
};

```

APPENDIX B: CPLEX CODE FOR THE LOWER BOUND

```
// -----  
//  
// Model for Lower Bound  
//  
//  
//read in contents from .dat  
int N = ...; //total number of activities (indexed by j) //<--- greater equal to 1  
int t = ...; //time period being calculated  
string outputFile= ...;  
//  
range Act = 1 .. N; //Activities  
//  
int c[Act] = ...; //c_j = cost per period  
int a[Act] = ...; //a_j = minimum duration for activity j  
int b[Act] = ...; //b_j = maximum duration for activity j  
int r[Act] = ...; //r_j = rail schedule for activity j (rail schedule)  
  
//model with activities on the arcs of a Figure  
int p[Act][Act] = ...; // {0,1} predecessors' matrix: p[from][to]=1 iff there (from, to) is an arc in  
the Figure  
  
//calculated values  
int SUMB = sum(j in Act) b[j];  
int numPre[j in Act] = sum(f in Act)(p[f][j]); //counts the number of predecessors for each  
activity  
int M1 = SUMB * 10;
```

```

int M2 = SUMB * 10;

//decision variables
//    d_j = duration of activity j, in [min,max] time of act. j (e2)(e3)
dvar float+    d[j in Act] in a[j]..b[j];
dvar float+    s[j in Act] in r[j] .. SUMB; //s_j = start time of activity j //vin: improved u.b. (rail
schedule)
dvar int+      x[Act] in 0 .. 1; //binary, to establish if act. j has started
dvar int+      y[Act] in 0 .. 1; //binary, to establish if act. j has finished

//Added as per Dr. Joe
dvar int+      F[Act][Act] in 0 .. 1; //binary to establish if i is a predecessor of j and j starts
immediately after i finishes.
dvar int+      G[Act] in 0 .. 1; //binary to establish if rail schedule is used

//    total cost of act j through time    range of z is between 0 and c[j]*b[j] (e8)
dvar float+    z[j in Act] in 0 .. c[j]*b[j]; //vin: improved u.b.

minimize sum(j in Act) z[j]; //e17

subject to{
    forall (j in Act){
        e2: s[j] <= t + M1*(1-x[j]);
        e3: s[j] >= t - M1*x[j];
        e4: s[j] + d[j] <= t + M1*(1-y[j]);
        e5: s[j] + d[j] >= t - M1*y[j];
        e18: z[j] >= c[j]*(t-s[j]) - M2*(1-x[j]+y[j]);
        e19: z[j] >= c[j]*d[j] - M2*(1-y[j]);
    }
}

```

```

    e20: z[j] >= -M2*x[j];
}

//for activities with predecessors
forall(j in Act : numPre[j] > 0){      //forces j to start
    forall(i in Act : p[i][j]==1){
        e9: s[j] >= s[i] + d[i];
        e11: s[j] <= s[i] + d[i] + M1*(1 - F[i][j]);
        e12: s[j] >= s[i] + d[i] - M1*(1 - F[i][j]);
    }
}

forall (j in Act){
    e13: s[j] <= r[j] + M1*(1 - G[j]);
    e14: s[j] >= r[j] - M1*(1 - G[j]);
}

forall (j in Act){
    e10: sum(i in Act : p[i][j]==1)
        F[i][j] + G[j] == 1;
}

//for activities with no predecessors
forall(j in Act : numPre[j]==0){
    e14b: s[j] == 0;
}
};

```

APPENDIX C: CONTROL CODE TO RUN ALL T'S IN A NETWORK

```
//  
//  
//  
//time period to loop upon t=[minT..maxT]  
int maxT=...;  
int minT=...;  
range Time=minT..maxT;  
  
main {  
    //var filename="Alphorn20.dat";//Change this for the data file being used  
    var filename="Data_2.1_98/Alphorn2.1_1_11";//Change this for the data file being used  
    var modelname="AlphornSolution_UB.mod";  
  
    var source = new IloOplModelSource(modelname);  
    var def = new IloOplModelDefinition(source);  
    var cplex = new IloCplex();  
    var data0= new IloOplDataElements();  
    var data = new IloOplDataSource(filename);  
  
    data0.outputFile="output.txt"  
    //////////////////////////////////////  
    //clean output file  
    var ofile = new IloOplOutputFile(data0.outputFile,false); //in overwrite mode  
    ofile.writeln("period;u.b.");  
    ofile.close();  
    thisOplModel.generate();
```

```

var time0 = new Date();

//loop over all t
for(var t in thisOplModel.Time){

    data0.t=t

    var opl = new IloOplModel(def,cplex);

    opl.addDataSource(data0);

    opl.addDataSource(data);

    opl.generate();

    if (cplex.solve()) {

        writeln(cplex.getObjValue()); //Obj Value only. Best for exporting
to spreadsheet for additional work
//          writeln("Period:\t " + t);
//          writeln(" Obj Value:\t" + cplex.getObjValue());
        opl.postProcess();

    }

    else {

        writeln("No solution");

    }

}

var time1 = new Date();

writeln("solve time is ", time1-time0);

opl.end();

data.end();

data0.end();

```

```
def.end();  
cplex.end();  
source.end();  
}
```


APPENDIX D: CONTROL CODE TO SET NUMBER OF RUNS FOR CPLEX CODE

```
/******
```

```
* OPL 12.5 Data
```

```
* Author: pds
```

```
* Creation Date: Nov 11, 2015 at 2:31:35 PM
```

```
*****/
```

```
minT = 1;
```

```
maxT = 50;
```

APPENDIX E: COMPLETE CODE AND SOLUTION SET FOR 11 NODE PROJECT

Activity	1	2	3	4	5	6	7	8	9	10	11
Period: 1 Obj Value:	3										
Cost c_j :	3	3	4	5	5	5	5	4	3	5	1
duration d_j :	14	1	4	5	2	2	2	3	4	6	5
start time s_j :	0	88	14	18	23	23	14	14	16	23	14
schedule time r_j :	0	0	10	17	21	21	10	10	13	21	10
G_j :	1	1	0	0	0	0	0	0	0	0	0
F_{ij} :	0	0	1	0	0	0	1	1	0	0	1
	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	1	1	0	0	0	1	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
z_{cj} :	3	0	0	0	0	0	0	0	0	0	0
finish time:	14	89	18	23	25	25	16	17	20	29	19
e2: $s[j] \leq t + M1*(1-x[j])$:	1	89	89	89	89	89	89	89	89	89	89
e3: $s[j] \geq t - M1*x[j]$:	-87	1	1	1	1	1	1	1	1	1	1
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	89	89	89	89	89	89	89	89	89	89	89
e5: $s[j] + d[j] \geq t - M1*y[j]$:	1	1	1	1	1	1	1	1	1	1	1
e9: $s[j] \geq s[i] + d[i]$:	14	89	18	23	25	25	16	17	20	29	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	177	106	111	113	113	104	105	108	117	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	1	-70	-65	-63	-63	-72	-71	-68	-59	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	101	101	-101	101	101	101	101	101	101	101	101
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	0	0	0	0	0	0	0	0	0	0	0
$F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0

Gj:	1	1	0	0	0	0	0	0	0	0	0
	0	0	1	0	0	0	1	1	0	0	1
	0	0	0	1	0	0	0	0	0	0	0
	0	0	0	0	1	0	0	0	0	1	0
	0	0	0	0	0	1	0	0	0	0	0
Fij:	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
zcj:	9	0	0	0	0	0	0	0	0	0	0
finish time:	14	6	25	30	32	32	19	17	23	36	19
e2: $s[j] \leq t + M1*(1-x[j])$:	3	3	91	91	91	91	91	91	91	91	91
e3: $s[j] \geq t - M1*x[j]$:	-85	-85	3	3	3	3	3	3	3	3	3
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	91	91	91	91	91	91	91	91	91	91	91
e5: $s[j] + d[j] \geq t - M1*y[j]$:	3	3	3	3	3	3	3	3	3	3	3
e9: $s[j] \geq s[i] + d[i]$:	14	6	25	30	32	32	19	17	23	36	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	94	113	118	120	120	107	105	111	124	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-82	-63	-58	-56	-56	-69	-71	-65	-52	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99	-99
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	0	0	0	0	0	0	0	0	0	0	0
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 4 Obj Value:	12										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	14	4	11	5	2	2	5	3	4	6	5
start time sj:	0	4	14	25	30	30	14	14	19	30	14
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	0	0	0	0	0	0	0

Fij:	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
zcj:	15	0	0	0	0	0	0	0	0	0	0
finish time:	14	10	25	30	32	32	19	17	23	36	19
e2: $s[j] \leq t + M1*(1-x[j])$:	5	5	93	93	93	93	93	93	93	93	93
e3: $s[j] \geq t - M1*x[j]$:	-83	-83	5	5	5	5	5	5	5	5	5
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	93	93	93	93	93	93	93	93	93	93	93
e5: $s[j] + d[j] \geq t - M1*y[j]$:	5	5	5	5	5	5	5	5	5	5	5
e9: $s[j] \geq s[i] + d[i]$:	14	10	25	30	32	32	19	17	23	36	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	98	113	118	120	120	107	105	111	124	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-78	-63	-58	-56	-56	-69	-71	-65	-52	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97	-97
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	0	0	0	0	0	0	0	0	0	0	0
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 6 Obj Value:	18										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	14	6	11	5	2	2	5	3	4	6	5
start time sj:	0	6	14	25	30	30	14	14	19	30	14
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	0	0	0	0	0	0	0

Fij:	0	0	1	0	0	0	1	1	0	0	1
zcj:	18	0	0	0	0	0	0	0	0	0	0
finish time:	14	12	25	30	32	32	19	17	23	36	19
e2: $s[j] \leq t + M1*(1-x[j])$:	6	6	94	94	94	94	94	94	94	94	94
e3: $s[j] \geq t - M1*x[j]$:	-82	-82	6	6	6	6	6	6	6	6	6
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	94	94	94	94	94	94	94	94	94	94	94
e5: $s[j] + d[j] \geq t - M1*y[j]$:	6	6	6	6	6	6	6	6	6	6	6
e9: $s[j] \geq s[i] + d[i]$:	14	12	25	30	32	32	19	17	23	36	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	100	113	118	120	120	107	105	111	124	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-76	-63	-58	-56	-56	-69	-71	-65	-52	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96	-96
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	0	0	0	0	0	0	0	0	0	0	0
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 7 Obj Value:	18										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	6	1	4	3	2	2	2	3	4	6	5
start time sj:	0	7	10	17	21	21	10	10	13	21	10
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	1	1	1	1	1	1	1	1	1

	0	0	0	0	0	0	0	0	0	0	0
	000	000	000	000	000	000	000	000	000	000	000
	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000
Fij:	000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	000000
	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000
	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000
	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000
	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000
zcj:	18	0	0	0	0	0	0	0	0	0	0
finish time:	6	8	14	20	23	23	12	13	17	27	15
e2: $s[j] \leq t + M1*(1-x[j])$:	7	7	95	95	95	95	95	95	95	95	95
e3: $s[j] \geq t - M1*x[j]$:	-81	-81	7	7	7	7	7	7	7	7	7
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	7	95	95	95	95	95	95	95	95	95	95
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-81	7	7	7	7	7	7	7	7	7	7
e9: $s[j] \geq s[i] + d[i]$:	6	8	14	20	23	23	12	13	17	27	15
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	94	96	102	108	111	111	100	101	105	115	103
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	-82	-80	-74	-68	-65	-65	-76	-75	-71	-61	-73
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91	-91
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	0	0	0	0	0	0	0	0	0	0	0
e?: $F[i][j] + G[j] == 1$:	1	1	1	1	1	1	1	1	1	1	1
Period: 8 Obj Value:	18										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	6	1	4	3	2	2	2	3	4	6	5
start time sj:	0	8	10	17	21	21	10	10	13	21	10
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	1	1	1	1	1	1	1	1	1

	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
Fij:	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
zcj:	18	0	0	0	0	0	0	0	0	0	0
finish time:	6	9	14	20	23	23	12	13	17	27	15
e2: $s[j] \leq t + M1*(1-x[j])$:	8	8	96	96	96	96	96	96	96	96	96
e3: $s[j] \geq t - M1*x[j]$:	-80	-80	8	8	8	8	8	8	8	8	8
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	8	96	96	96	96	96	96	96	96	96	96
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-80	8	8	8	8	8	8	8	8	8	8
e9: $s[j] \geq s[i] + d[i]$:	6	9	14	20	23	23	12	13	17	27	15
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	94	97	102	108	111	111	100	101	105	115	103
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	-82	-79	-74	-68	-65	-65	-76	-75	-71	-61	-73
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-90	-90	-90	-90	-90	-90	-90	-90	-90	-90	-90
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	0	0	0	0	0	0	0	0	0	0	0
e?: $F[i][j] + G[j] == 1$:	1	1	1	1	1	1	1	1	1	1	1
Period: 9 Obj Value:	18										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	6	1	4	3	2	2	2	3	4	6	5
start time sj:	0	9	10	17	21	21	10	10	13	21	10
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	1	1	1	1	1	1	1	1	1

	0	0	1	0	0	0	1	1	0	0	1
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
Fij:	0	0	0	0	0	0	0	0	0	0	0
zcj:	36	0	0	0	0	0	0	0	0	0	0
finish time:	14	21	18	23	25	25	19	17	23	29	19
e2: $s[j] \leq t + M1*(1-x[j])$:	12	12	100	100	100	100	100	100	100	100	100
e3: $s[j] \geq t - M1*x[j]$:	-76	-76	12	12	12	12	12	12	12	12	12
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	100	100	100	100	100	100	100	100	100	100	100
e5: $s[j] + d[j] \geq t - M1*y[j]$:	12	12	12	12	12	12	12	12	12	12	12
e9: $s[j] \geq s[i] + d[i]$:	14	21	18	23	25	25	19	17	23	29	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	109	106	111	113	113	107	105	111	117	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-67	-70	-65	-63	-63	-69	-71	-65	-59	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-90	-90	-90	-90	-90	-90	-90	-90	-90	-90	-90
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	0	0	0	0	0	0	0	0	0	0	0
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 13 Obj Value:	39										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	13	9	4	5	2	2	5	3	4	6	5
start time sj:	0	13	13	17	22	22	13	13	18	22	13
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	0	0	0	0	0	0	0

Fij:	0	0	1	0	0	0	1	1	0	0	1
zcj:	39	0	0	0	0	0	0	0	0	0	0
finish time:	13	22	17	22	24	24	18	16	22	28	18
e2: $s[j] \leq t + M1*(1-x[j])$:	13	13	101	101	101	101	101	101	101	101	101
e3: $s[j] \geq t - M1*x[j]$:	-75	-75	13	13	13	13	13	13	13	13	13
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	101	101	101	101	101	101	101	101	101	101	101
e5: $s[j] + d[j] \geq t - M1*y[j]$:	13	13	13	13	13	13	13	13	13	13	13
e9: $s[j] \geq s[i] + d[i]$:	13	22	17	22	24	24	18	16	22	28	18
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	13	110	105	110	112	112	106	104	110	116	106
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	13	-66	###	-66	-64	-64	-70	-72	-66	-60	0
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	0	0	0	0	0	0	0	0	0	0	0
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 14 Obj Value:	42										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	14	9	11	5	2	2	5	4	4	6	5
start time sj:	0	14	14	25	30	30	14	14	19	30	14
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	0	0	0	0	0	0	0

Fij:	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
zcj:	42	0	0	0	0	0	0	0	0	0	0
finish time:	14	23	25	30	32	32	19	18	23	36	19
e2: $s[j] \leq t + M1*(1-x[j])$:	14	14	14	102	102	102	14	14	102	102	14
e3: $s[j] \geq t - M1*x[j]$:	-74	-74	-74	14	14	14	-74	-74	14	14	-74
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	14	102	102	102	102	102	102	102	102	102	102
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-74	14	14	14	14	14	14	14	14	14	14
e9: $s[j] \geq s[i] + d[i]$:	14	23	25	30	32	32	19	18	23	36	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	111	113	118	120	120	107	106	111	124	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-65	-63	-58	-56	-56	-69	-70	-65	-52	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	0	0	0	0	0	0	0	0	0	0	0
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 15 Obj Value:	56										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	14	9	11	5	2	2	5	5	4	6	5
start time sj:	0	15	14	25	30	30	14	14	19	30	14
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	0	0	0	0	0	0	0

	0	0	1	0	0	0	1	1	0	0	1
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0
Fij:	0	0	0	0	0	0	0	0	0	0	0
zcj:	42	0	4	0	0	0	5	4	0	0	1
finish time:	14	24	25	30	32	32	19	19	23	36	19
e2: $s[j] \leq t + M1*(1-x[j])$:	15	15	15	103	103	103	15	15	103	103	15
e3: $s[j] \geq t - M1*x[j]$:	-73	-73	-73	15	15	15	-73	-73	15	15	-73
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	15	103	103	103	103	103	103	103	103	103	103
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-73	15	15	15	15	15	15	15	15	15	15
e9: $s[j] \geq s[i] + d[i]$:	14	24	25	30	32	32	19	19	23	36	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	112	113	118	120	120	107	107	111	124	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-64	-63	-58	-56	-56	-69	-69	-65	-52	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	1	1	1	1	1	1	1	1	1	1	1
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83	-83
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 16 Obj Value:	70										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	14	9	11	5	2	2	5	6	4	6	6
start time sj:	0	16	14	25	30	30	14	14	19	30	14
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	0	0	0	0	0	0	0

Fij:	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
	0	0	1	0	0	0	1	1	0	0	1
zcj:	42	0	8	0	0	0	10	8	0	0	2
finish time:	14	25	25	30	32	32	19	20	23	36	20
e2: $s[j] \leq t + M1*(1-x[j])$:	16	16	16	104	104	104	16	16	104	104	16
e3: $s[j] \geq t - M1*x[j]$:	-72	-72	-72	16	16	16	-72	-72	16	16	-72
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	16	104	104	104	104	104	104	104	104	104	104
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-72	16	16	16	16	16	16	16	16	16	16
e9: $s[j] \geq s[i] + d[i]$:	14	25	25	30	32	32	19	20	23	36	20
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	113	113	118	120	120	107	108	111	124	108
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-63	-63	-58	-56	-56	-69	-68	-65	-52	-68
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	2	2	2	2	2	2	2	2	2	2	2
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82	-82
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 17 Obj Value:	73										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	6	1	4	3	2	2	2	3	4	6	5
start time sj:	0	17	10	17	21	21	10	10	13	21	10
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	1	1	1	1	1	1	1	1	1

	0	0	0	0	0	0	0	0	0	0	0
	000	000	000	000	000	000	000	000	000	000	000
	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000
Fij:	000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	000000
	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000
	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000
	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000
	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000
zcj:	18	0	16	5	0	0	10	12	12	0	5
finish time:	6	19	14	20	23	23	12	13	17	27	15
e2: $s[j] \leq t + M1*(1-x[j])$:	18	18	18	18	106	106	18	18	18	106	18
e3: $s[j] \geq t - M1*x[j]$:	-70	-70	-70	-70	18	18	-70	-70	-70	18	-70
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	18	106	18	106	106	106	18	18	18	106	18
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-70	18	-70	18	18	18	-70	-70	-70	18	-70
e9: $s[j] \geq s[i] + d[i]$:	6	19	14	20	23	23	12	13	17	27	15
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	94	107	102	108	111	111	100	101	105	115	103
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	-82	-69	-74	-68	-65	-65	-76	-75	-71	-61	-73
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80	-80
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	5	5	5	5	5	5	5	5	5	5	5
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	1	1	1	1	1	1	1	1	1	1
Period: 19 Obj Value:	83										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	6	1	4	3	2	2	2	3	4	6	5
start time sj:	0	19	10	17	21	21	10	10	13	21	10
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	1	1	1	1	1	1	1	1	1

	0	0	0	0	0	0	0	0	0	0	0
	0000	0000	0000	001000	00010000	00010000	00000000	0000	0000	00010000	00000000
Fij:	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000	000100000000	000000000000
	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000	000000000000
zcj:	18	0	44	25	0	0	10	12	12	0	5
finish time:	6	27	21	26	28	28	12	13	17	32	15
e2: $s[j] \leq t + M1*(1-x[j])$:	26	26	26	26	114	114	26	26	26	114	26
e3: $s[j] \geq t - M1*x[j]$:	-62	-62	-62	-62	26	26	-62	-62	-62	26	-62
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	26	114	26	114	114	114	26	26	26	114	26
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-62	26	-62	26	26	26	-62	-62	-62	26	-62
e9: $s[j] \geq s[i] + d[i]$:	6	27	21	26	28	28	12	13	17	32	15
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	94	115	109	114	116	116	100	101	105	120	103
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	-82	-61	-67	-62	-60	-60	-76	-75	-71	-56	-73
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72	-72
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	5	5	5	5	5	5	5	5	5	5	5
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	1	1	1	1	1	1	1	1	1	1
Period: 27 Obj Value:	135										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	14	9	11	5	2	2	2	3	4	6	5
start time sj:	0	27	14	25	30	30	14	14	16	30	14
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	0	0	0	0	0	0	0

Fij:	0	0	1	0	0	0	1	1	0	0	1
	0	0	0	1	0	1	0	0	0	1	0
	0	0	0	0	0	0	0	0	1	0	0
	0	0	0	0	0	0	0	0	0	0	0
zcj:	42	0	44	10	0	0	10	12	12	0	5
finish time:	14	36	25	30	32	32	16	17	20	36	19
e2: $s[j] \leq t + M1*(1-x[j])$:	27	27	27	27	115	115	27	27	27	115	27
e3: $s[j] \geq t - M1*x[j]$:	-61	-61	-61	-61	27	27	-61	-61	-61	27	-61
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	27	115	27	115	115	115	27	27	27	115	27
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-61	27	-61	27	27	27	-61	-61	-61	27	-61
e9: $s[j] \geq s[i] + d[i]$:	14	36	25	30	32	32	16	17	20	36	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	124	113	118	120	120	104	105	108	124	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-52	-63	-58	-56	-56	-72	-71	-68	-52	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75	-75
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	5	5	5	5	5	5	5	5	5	5	5
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 28 Obj Value:	138										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	6	1	4	3	2	2	2	3	4	6	5
start time sj:	0	28	10	17	21	21	10	10	13	21	10
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	1	1	1	1	1	1	1	1	1

	0	0	0	0	0	0	0	0	0	0	0
	000	000	000	000	000	000	000	000	000	000	000
	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000	00000
Fij:	000000	000000	000000	000000	000000	000000	000000	000000	000000	000000	000000
	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000	0000000
	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000	00000000
	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000	000000000
	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000
zcj:	18	0	16	15	10	10	12	12	12	30	5
finish time:	6	31	14	20	23	23	12	13	17	27	15
e2: $s[j] \leq t + M1*(1-x[j])$:	30	30	30	30	30	30	30	30	30	30	30
e3: $s[j] \geq t - M1*x[j]$:	-58	-58	-58	-58	-58	-58	-58	-58	-58	-58	-58
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	30	118	30	30	30	30	30	30	30	30	30
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-58	30	-58	-58	-58	-58	-58	-58	-58	-58	-58
e9: $s[j] \geq s[i] + d[i]$:	6	31	14	20	23	23	12	13	17	27	15
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	94	119	102	108	111	111	100	101	105	115	103
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	-82	-57	-74	-68	-65	-65	-76	-75	-71	-61	-73
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	10	10	10	10	10	10	10	10	10	10	10
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-68	-68	-68	-68	-68	-68	-68	-68	-68	-68	-68
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	5	5	5	5	5	5	5	5	5	5	5
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	1	1	1	1	1	1	1	1	1	1
Period: 31 Obj Value:	145										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	6	1	4	3	2	2	2	3	4	6	5
start time sj:	0	31	10	17	21	21	10	10	13	21	10
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	1	1	1	1	1	1	1	1	1

	0	0	1	0	0	0	1	1	0	0	1
	0000000000	0000000000	1000000000	0010000000	0000000000	0000000000	1000000000	1000000000	0000001000	0000000000	1000000000
Fij:	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000	0000000000
zcj:	40	0	16	15	10	10	10	12	12	30	5
finish time:	13	34	17	20	23	23	15	16	19	27	18
e2: $s[j] \leq t + M1*(1-x[j])$:	33	33	33	33	33	33	33	33	33	33	33
e3: $s[j] \geq t - M1*x[j]$:	-55	-55	-55	-55	-55	-55	-55	-55	-55	-55	-55
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	33	121	33	33	33	33	33	33	33	33	33
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-55	33	-55	-55	-55	-55	-55	-55	-55	-55	-55
e9: $s[j] \geq s[i] + d[i]$:	13	34	17	20	23	23	15	16	19	27	18
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	13	122	105	108	111	111	103	104	107	115	106
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	13	-54	-71	-68	-65	-65	-73	-72	-69	-61	-70
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-68	-68	-68	-68	-68	-68	-68	-68	-68	-68	-68
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	5	5	5	5	5	5	5	5	5	5	5
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 34 Obj Value:	164										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	14	1	4	3	2	2	2.4	3	4	6	5
start time sj:	0	34	14	18	21	21	14	14	16	21	14
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	0	1	0	0	0	1	0

	0	0	1	0	0	0	1	1	0	0	1
	0000	0000	1000	0100	0001	0000	1000	1000	0000	0000	1000
	0000	0000	0000	0000	0001	0000	0000	0000	0000	0000	0000
Fij:	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000	0000
zcj:	42	0	16	15	10	10	12	12	12	30	5
finish time:	14	35	18	21	23	23	16	17	20	27	19
e2: $s[j] \leq t + M1*(1-x[j])$:	34	34	34	34	34	34	34	34	34	34	34
e3: $s[j] \geq t - M1*x[j]$:	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54	-54
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	34	122	34	34	34	34	34	34	34	34	34
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-54	34	-54	-54	-54	-54	-54	-54	-54	-54	-54
e9: $s[j] \geq s[i] + d[i]$:	14	35	18	21	23	23	16	17	20	27	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	123	106	109	111	111	104	105	108	115	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-53	-70	-67	-65	-65	-72	-71	-68	-61	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-68	-68	-68	-68	-68	-68	-68	-68	-68	-68	-68
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	5	5	5	5	5	5	5	5	5	5	5
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 35 Obj Value:	169										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	14	1	4	3	2	2	3.4	3	4	6	5
start time sj:	0	35	14	18	21	21	14	14	17	21	14
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	0	0	0	0	0	0	0

Fij:	0	0	1	0	0	0	1	1	0	0	1
zcj:	42	0	16	15	10	10	17	12	12	30	5
finish time:	14	36	18	21	23	23	17	17	21	27	19
e2: $s[j] \leq t + M1*(1-x[j])$:	35	35	35	35	35	35	35	35	35	35	35
e3: $s[j] \geq t - M1*x[j]$:	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53	-53
e4: $s[j] + d[j] \leq t + M1*(1-y[j])$:	35	123	35	35	35	35	35	35	35	35	35
e5: $s[j] + d[j] \geq t - M1*y[j]$:	-53	35	-53	-53	-53	-53	-53	-53	-53	-53	-53
e9: $s[j] \geq s[i] + d[i]$:	14	36	18	21	23	23	17	17	21	27	19
e11: $s[j] \leq s[i] + d[i] + M1*(1 - F[i][j])$:	14	124	106	109	111	111	105	105	109	115	107
e12: $s[j] \geq s[i] + d[i] - M1*(1 - F[i][j])$:	14	-52	-70	-67	-65	-65	-71	-71	-67	-61	-69
e13: $s[j] \leq r[j] + M1*(1 - G[j])$:	98	98	98	98	98	98	98	98	98	98	98
e14: $s[j] \geq r[j] - M1*(1 - G[j])$:	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78	-78
e18: $z[j] \geq c[j]*(t-s[j]) - M2*(1-x[j]+y[j])$:	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67	-67
e19: $z[j] \geq c[j]*d[j] - M2*(1-y[j])$:	5	5	5	5	5	5	5	5	5	5	5
e20: $z[j] \geq -M2*x[j]$:	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88	-88
e?: $F[i][j] + G[j] == 1$:	1	0	0	0	0	0	0	0	0	0	0
Period: 36 Obj Value:	174										
Cost cj:	3	3	4	5	5	5	5	4	3	5	1
duration dj:	14	1	4	3	2	2	4.4	3	4	6	5
start time sj:	0	36	14	18	21	21	14	14	18	21	14
schedule time rj:	0	0	10	17	21	21	10	10	13	21	10
Gj:	1	1	0	0	1	1	0	0	0	1	0

APPENDIX F: SCREEN SHOT AND VBA CODE FOR UPPER BOUND SIMULATION

	A	B	C	D
1	a	4	3	8
2	b	11	6	9
3	c	3	1	1
4	t	155		
5	Σ	=SUM(B9:G9)		
6	d	5	5	9
7	s			6
8	f	=B7+B6	=C7+C6	=D7+D6
9	c	=IF(B7>\$B\$4,0,IF(B8<=\$B\$4,(B8-\$B\$4)/(\$B\$4-B7)*\$B\$3))	=IF(C7>\$B\$4,0,IF(C8<=\$B\$4,(C8-\$B\$4)/(\$B\$4-C7)*\$B\$3))	=IF(D7>\$B\$4,0,IF(D8<=\$B\$4,(D8-\$B\$4)/(\$B\$4-D7)*\$B\$3))
10	controlling precedent			
11	activity j	1	2	3
212				
213	finish time	Average finish time =		=AVERAGE(GT215:GT218)
214	activity j	1	2	
1217	start time	0	0	=AVERAGE(D2020:D2219)
1218	activity j	0	0	=MIN(D2020:D2219)+AVERAGE(D1,D2)
1219	activity j	1	2	
2020	s	0	0	4
2021	s			6
2022	s			4
2023	s			10
2024	s			7
2025	s			7

Option Explicit

Sub RunAll()

Dim precedents(155, 155) As Integer 'two-dimensional array holding all precedents

Dim a As Integer 'activity on the node

Dim t As Integer

Dim tTotal As Integer 'time

Dim p As Integer 'precedent

Dim s As Integer

Dim tempP As Variant 'temporary holder to compare finish times of precedents

Dim costsActivAtTime(155) As Integer 'array that holds total cost at time t

Dim costsTotalAtTime(155) As Integer

Dim simulation As Double

Dim runs As Double

```
Sheets("simulation (ub)").Select
```

```
Application.ScreenUpdating = False
```

```
runs = Range("D4").Value
```

```
Calculate
```

```
Range("A1").Select
```

```
'establishes an array and stores the precedents array
```

```
For a = 0 To 97
```

```
    For p = 0 To 97
```

```
        precedents(a, p) = ActiveCell.Offset(11 + p, 1 + a).Range("A1").Value
```

```
    Next p
```

```
Next a
```

```
a = 0
```

```
'Clears any old information for results table
```

```
Range("B112:B211").Select
```

```
    Selection.ClearContents
```

```
For simulation = 1 To runs
```

```
    'sets random duration for each activity
```

```
    Range("B6").Select
```

```
    ActiveCell.FormulaR1C1 = "=RANDBETWEEN(R[-5]C,R[-4]C)"
```

```
    Range("B6").Select
```

```
    Selection.AutoFill Destination:=Range("B6:CU6"), Type:=xlFillDefault
```

```
    Range("B6:CU6").Select
```

```

Selection.Copy
Range("B6").Select
Selection.PasteSpecial Paste:=xlPasteValues, Operation:=xlNone, SkipBlanks _
:=False, Transpose:=False
Application.CutCopyMode = False
'clears any old start times
Range("B7:CU7").Select
Selection.ClearContents
Range("A1").Select

```

```

For a = 0 To 97

```

```

'checks each dominant predecessor for an activity and sets correct predecessor's
finish time

```

```

For p = 0 To 97

```

```

tempP = precedents(a, p)

```

```

If tempP = 1 Then

```

```

'use for road runner start

```

```

tempP = ActiveCell.Offset(11 + p, 0).Range("A1").Value

```

```

tempP = ActiveCell.Offset(7, tempP).Range("A1").Value

```

```

'use for rail schedule

```

```

'tempP = ActiveCell.Offset(6, tempP).Range("A1").Value + ActiveCell.Offset(6,
tempP).Range("A1").Value'rail

```

```

'tempP = ActiveCell.Offset(7, tempP).Range("A1").Value + 1'delay 1

```

```

End If

```

```

If tempP > ActiveCell.Offset(6, 1 + a).Range("A1").Value Then

```

```

ActiveCell.Offset(6, 1 + a).Range("A1").Value = tempP

```

```

End If

```



```

    Next p
Next a
'sets correct start time for an activity
For s = 0 To 154
    Range("B4").Value = s + 1
    costsActivAtTime(s) = Range("B5").Value
Next s

For t = 0 To 154
    costsTotalAtTime(t) = ActiveCell.Offset(111 + t, 1).Range("A1").Value
Next t

For tTotal = 0 To 154
    If costsActivAtTime(tTotal) > costsTotalAtTime(tTotal) Then
        costsTotalAtTime(tTotal) = costsActivAtTime(tTotal)
        ActiveCell.Offset(111 + tTotal, 1).Range("A1").Value = costsTotalAtTime(tTotal)
    End If
Next tTotal
Next simulation
Application.ScreenUpdating = True
'Sheets("lp vs. Sim").Select

End Sub

```