PROPOSED NONPARAMETRIC TESTS FOR EQUALITY OF LOCATION AND SCALE AGAINST ORDERED ALTERNATIVES

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ABSTRACT

Ordered alternatives tests are sometimes used in life-testing experiments and drug-screening studies. An ordered alternative test is sometimes used to gain power if the researcher thinks parameters will be ordered in a certain way if they are different. This research proposal focuses on developing new nonparametric tests for the nondecreasing ordered alternative problem for k ($k \geq 3$) populations when testing for differences in both location and scale.

Six nonparametric tests are proposed for the nondecreasing ordered alternative when testing for a difference in either location or scale. The six tests are various combinations of a well-known ordered alternatives test for location and a test based on the Moses test technique for testing differences in scale. A simulation study is conducted to determine how well the proposed tests maintain their significance levels. Powers are estimated for the proposed tests under a variety of conditions for three, four and five populations. Several types of variable parameters are considered: when the location parameters are different and the scale parameters are equal; when the location parameters are equal and the scale parameters are different; when the location and scale parameters are both different. Equal and unequal samples sizes of 18 and 30 are considered. Subgroup sizes of 3 and 6 are both used when applying the Moses test technique. Recommendations are given for which test should be used for various situations.

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CHAPTER 1. GENERAL INTRODUCTION

Ordered alternatives tests are used in life-testing experiments and drug-screening studies. An ordered alternative test is used to gain power if the researcher thinks parameters will be ordered in a certain way if they are different. Nonparametric tests are more suitable when there are weaker assumptions about the underlying populations and the requirements for the measurement scales. We could test the life expectancy and instability for the nations with increased capital income to determine whether the average life length and the variance of the life were different and nondecreasing with income level. In our research, we will propose several nonparametric test statistics to test the equality of location and scale parameters for the nondecreasing ordered alternative hypothesis for k $(k \geq 3)$ populations.

Let $X_{i1}, X_{i2}, \dots, X_{in_i}, i = 1,2, \dots, k$ $(k \geq 3)$ be random independent samples each of size n_i from k populations, where $-\infty < \mu_i < +\infty$ and $\sigma_i > 0$ are location and scale parameters, respectively. The null hypothesis can be expressed as

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \ and \ \sigma_1 = \sigma_2 = \cdots = \sigma_k.$$

The ordered alternative states that the distributions are stochastically ordered,

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_k \ and \ \sigma_1 \leq \sigma_2 \leq \cdots \leq \sigma_k, \text{ at least one of the inequalities is strict}$$

Several test statistics have been proposed to test location or scale only for $k \geq 2$ populations. For example, the Mann-Whitney test only tests location parameters and the Ansari-Bradley test only tests scale parameters. We are proposing some tests for testing both location and scale at the same time of nondecreasing ordered alternative problem for $k \geq 3$ populations.

Chapter Two will present a review of the literature regarding some nonparametric statistical tests for the location or scale based on nondecreasing ordered alternatives. Chapter Three will introduce the proposed tests, and Chapter Four outlines the simulation study for this research. In Chapter Five, the results obtained from the simulation study will be illustrated using tables, Chapter Six will state the conclusions concerning the proposed tests.

CHAPTER 2. REVIEW OF LITERATURE

2.1. Mann-Whitney Test

The Mann-Whitney test is a standard test statistic for examining the null hypothesis of equal population location parameters (Mann and Whitney, 1947). The null hypothesis and alternative hypothesis are

$$H_0: \mu_1 = \mu_2$$

$$H_{\alpha1}: \mu_1 \neq \mu_2,\ \ H_{\alpha2}:\ \mu_1 < \mu_2,\ H_{\alpha3}:\ \mu_1 > \mu_2$$

Assume that n_1 and n_2 are the sample sizes of population 1 and population 2, respectively. Combine samples from population 1 and population 2 and order the observations from smallest to largest. Ranks are then assigned to the ordered measurements. S_j will be the rank of jth observation in sample 2, within the set of ranks. The test statistic MW is the sum of the ranks of all observations in sample 2.

$$\text{MW} = \sum S_j$$

The standardized version of Mann-Whitney test is given by:

$$MW^* = \frac{MW - E_0(MW)}{\sqrt{var_0(MW)}}$$

$$E_0(MW) = \frac{n_2(N+1)}{2}$$

$$var_0(MW) = \frac{n_1 n_2 (N+1)}{12}$$

where $N = n_1 + n_2$.

When H_0 is true, the test statistic MW^* has approximately a standard normal distribution. H_0 will be rejected for the two sided alternative when $|MW^*| \geq Z_{\alpha/2}$ at the α level of significance where $Z_{\alpha/2}$ is the (1- $\alpha/2$) 100% percentile of the standard normal distribution.

For very large samples, the power-efficiency of the Mann-Whitney test approaches $\frac{3}{\pi} \approx 95.5\%$ when the underlying populations are normally distributed (Daniel, 1990).

2.2. Jonckheere-Terpstra (JT) Test

The Jonckheere-Terpstra(JT) test is used for ordered alternatives and was proposed by Terpstra(1952) and Jonckheere(1954). For populations $X_{i1}, X_{i2}, \dots, X_{in}, i = 1,2, \dots, k$ $(k \geq 3)$, the null hypothesis is

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_i \ \ (i = 1,2, \dots, k),$$

and the hypothesis is

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_i \text{ and } \mu_1 < \mu_i$$

The test statistic JT corresponds to the sum of the $k(k-1)/2$ Mann-Whitney statistics

$$JT = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} U_{ij}$$

where U_{ij} is the number of pairs of observations (a, b) for which x_{ia} is less than x_{jb}, $x_{ia}(or\ x_{jb})$ denotes the $ath(or\ bth)$ sample observation for $ith(or\ jth)$ population $X_i(or\ X_j)$.

For large sample size, JT is approximately normal distributed under H_0. The mean and variance of this statistic are

$$E_0(JT) = \frac{N^2 - \sum_{i=1}^{k} n_i^2}{4}$$

and

$$var_0(JT) = \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)}{72}$$

where $N = n_1 + n_2 + \cdots + n_k$.

The standardized version of JT test is:

$$JT^* = \frac{JT - E_0(JT)}{\sqrt{var_0(JT)}}$$

The asymptotic null distribution of JT^* is the standard normal distribution.

For the power of the JT test for ordered alternatives, Potter and Sturm (1981) show how the maximum achievable power of the test can be computed for some sample alternatives. They found that under certain shift alternatives and sample sizes, the power of the test was significantly different from one.

2.3. Modified JT Test

Neuhauser, Liu, and Hothorn (1998) proposed the modified JT statistic to test the null hypothesis against the ordered alternatives,

$$MJT = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)U_{ij}.$$

This statistic has a normal distribution under H_0, and its mean and variance are

$$E(MJT) = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)E(U_{ij})$$

$$var(MJT) = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)^2 var(U_{ij}) + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{l=j+1}^{k} Cov(U_{ij}, U_{il})$$

$$E_0(U_{ij}) = \frac{1}{2} n_i n_j, \forall i \neq j$$

$$var_0(U_{ij}) = \frac{1}{12} n_i n_j (n_i + n_j + 1), \forall i \neq j$$

$$Cov(U_{ij}, U_{il}) = Cov(U_{ji}, U_{li}) = \frac{1}{12} n_i n_j n_l, if\ all\ i, j, and\ l\ are\ different$$

$$Cov(U_{ij}, U_{li}) = Cov(U_{ji}, U_{il}) = -\frac{1}{12} n_i n_j n_l, if\ all\ i, j, and\ l\ are\ different$$

$$Cov(U_{ij}, U_{lm}) = 0, if\ all\ i, j, and\ l\ are\ different.$$

Neuhauser, Liu, and Hothorn (1998) found that if the exact permutation distribution is used for inference, MJT test had more type I error, but MJT test had substantially more powerful than the common JT test. If the asymptotic normality was used for inference, the MJT test was slightly more powerful.

2.4. Shan Test

Guogen Shan (2014) proposed a new nonparametric rank test for different location alternatives under the ordered alternative which is named as Shan test. This test captures not only the sign of the difference between observations, but also the value of the difference. The null hypothesis and alternative hypothesis are given below:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k$$

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_k \text{ and } \mu_1 < \mu_k$$

For comparing two groups, the new rank based nonparametric test by incorporating the actual differences is given as

$$S = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} D_{ij},$$

where$D_{ij} = \sum_{l=1}^{n_i} \sum_{m=1}^{n_j} Z_{ijlm}$, $Z_{ijlm} = (R_{jm} - R_{il}) I(X_{jm} > X_{il})$ and $R_{il}(R_{jm})$ denotes the rank of the observation $X_{il}(X_{jm})$ in the combined data. The exact mean and variance of the null sampling distribution are given as

$$E_0(S) = \frac{N+1}{6} \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} n_i n_j \ ,$$

and

$$var_0(S) = \left(\frac{N^2+N}{12} - \frac{(N+1)^2}{36}\right) \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} n_i n_j + 2\left[\sum_{i=1}^{k-1} n_i \binom{\sum_{j=i+1}^{k} n_j}{2} + \sum_{i=2}^{k} n_i \binom{\sum_{j=1}^{i-1} n_j}{2}\right] CovA +$$

$$2\left(\sum_{i=1}^{k-2} \sum_{j=i+1}^{k-1} \sum_{l=j+1}^{k} n_i n_j n_l\right) CovB,$$

where $CovA = \frac{2N^2+N-1}{90}$, and $CovB = \frac{-7N^2-11N-4}{360}$.

The standardized version of Shan test is:

$$S^* = \frac{S - E_0(S)}{\sqrt{var_0(S)}}$$

The asymptotic null distribution of S^* is the standard normal distribution.

Shan (2014) showed that in the power comparison between the Shan test and other existing tests shows that the Shan test is generally more powerful than the other tests for various distributions (normal, t, exponential and mixed distribution).

2.5. Fligner-Wolfe Test

Often in biological sciences, it is necessary to investigate the response of treatments compared to a control. Situations in which this often occurs are clinical trials, pharmacology experiments and agricultural experiments (Olet, 2014). The Fligner-Wolfe test statistic is designed for use in this type of situation (Fligner and Wolfe, 1982).

The Fligner-Wolfe test statistic compares the median of the control group, to the medians of a number of other treatment groups simultaneously (Fligner and Wolfe, 1982). There are k samples with $i = 1$ denoting the control sample and the remaining i $(2 \leq i \leq k)$ indicating treatment samples. Assume that the means in the treatment populations are at least as large as the mean of the control population. The null hypothesis and alternative hypothesis are

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k,$$

$H_\alpha: \mu_1 \leq [\mu_2, \ldots, \mu_k]$ with at least one strict inequality.

To calculate the Fligner-Wolfe test statistic, it is useful to visualize two populations. One population is the control ($i = 1$) and the other $k - 1$ populations are the combined treatment population. Assume that n_1 and n_2 are the sample sizes of the control population and the combined treatment population, respectively. All the observations of the control sample and treatment sample are merged and subsequently ranked from smallest to largest. Let the rank r_{ij} $(i = 1,2$ and $j = 1,2,3,\ldots,n_i)$ indicate the rank of the j^{th} observation in the i^{th} sample (if it is the control sample, $i = 1$, and if it is the combined treatment sample, $i = 2$).

$$T_1 = FW = \sum_{\substack{2 \le i \le k \\ 1 < j < n_i}} r_{ij}$$

Where k is the number of treatments, n_i is the number of observations in treatment i and r_{ij} is the rank of the observation in the jth group subjected to the ith treatment.

The expectation and variance of FW under the null distribution are

$$E(T_1) = E_0(FW) = \frac{n_2(N+1)}{2}$$

$$var(T_1) = var_0(FW) = \left\{\frac{n_1 n_2(N+1)}{12}\right\}$$

where $N = n_1 + n_2$.

The standardized version of Fligner-Wolfe test FW^* is stated below.

$$FW^* = \frac{FW - E_0(FW)}{\sqrt{var_0(FW)}}$$

The H_0 is rejected when $FW^* \ge Z_\alpha$ at the α level of significance where Z_α is the (1- α) 100% percentile of the standard normal distribution.

2.6. Mack-Wolfe

The Mack-Wolfe test statistic is designed to test the umbrella alternative that is based on simple random samples (Mack and Wolfe, 1981). The Mack-Wolfe test statistic has two versions. The first version is used when the peak is known, and the second version is used when the peak is unknown. The null hypothesis and alternative hypothesis are

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k,$$

$$H_\alpha: \mu_1 \leq \mu_2 \leq \ \ldots \leq \mu_{\mathrm{p}} \geq \mu_{\mathrm{p+1}} \geq \ \ldots \geq \mu_k \text{ with at least one strict inequality,}$$

where μ_i is the median of the i^{th} sample.

When the peak p is known, the Mack-Wolfe test statistic is

$$A_p = \sum_{u=1}^{v-1} \sum_{v=2}^{p} U_{uv} + \sum_{u=p}^{v-1} \sum_{v=p+1}^{k} U_{uv},$$

the first U_{uv} is the Mann-Whitney counts for every pair of treatments with outcomes less than or equal to the hypothesized peak $1 \leq u \leq v \leq p$, and. the second U_{uv} is the reverse Mann-Whitney and counts every pair of treatments with outcomes greater than or equal to the hypothesized peak $p \leq u \leq v \leq k$(Daniel, 1990). The null hypothesis of this test is used for large values of the test statistic.

2.7. Ansari-Bradley Test

The Ansari-Bradley test is a nonparametric test designed to test for equality of variances based on independent samples from 2 populations (Ansari and Bradley, 1960). The null hypothesis and alternative hypothesis are

$$H_0: \sigma_1 = \sigma_2$$

$$H_{\alpha 1}: \sigma_1 \neq \sigma_2, \quad H_{\alpha 2}: \sigma_1 < \sigma_2, \quad H_{\alpha 3}: \sigma_1 > \sigma_2$$

where σ_1 and σ_2 are the dispersion parameters for populations 1 and 2 respectively.

To obtain the Ansari-Bradley test statistic, all the observations from the two samples should be combined in order from smallest to largest. The ranks will be assigned to the ordered observations as follows: The smallest observation and the largest observation will each be given a rank of 1; the second smallest observation and the second largest observation will each be given a rank of 2; and continue in this manner until all measurements have been assigned a rank.

Let R_i be the rank of i^{th} observation in the first sample in the set of ranks. The test statistic Ansari-Bradley (T) is the sum of the ranks of all observations in the first sample:

$$T = \sum R_i$$

If N= n_1 + n_2 is an even number

$$E_0(T) = \frac{n_1(N+2)}{4}$$

$$var_0(T) = \left\{\frac{n_1 n_2(N+2)(N-2)}{48(N-1)}\right\}$$

If N= n_1 + n_2 is an odd number

$$E_0(T) = \frac{n_1(N+1)^2}{4N}$$

$$var_0(T) = \left\{\frac{n_1 n_2(N+1)(3+N^2)}{48N^2}\right\}$$

The standardized version of Ansari-Bradley test is:

$$T^* = \frac{AB - E_0(AB)}{\sqrt{var_0(AB)}}$$

The asymptotic null distribution of T^* is the standard normal distribution. If population 1 has the greater amount of dispersion, T^* will tend to be small.

Ansari and Bradley (1960) state that the relative efficiency of their statistic when compared with the parametric F test is $\frac{6}{\pi^2}$ when sampling is from normally distributed populations. They also note that the statistic is less efficient asymptotically than some other dispersion tests but easier to apply.

2.8. Moses Test

The Moses test is a classic nonparametric test for equality of dispersion parameters and was proposed by Moses (1963). The null hypothesis and alternative hypothesis are

$$H_0: \sigma_1 = \sigma_2$$

$$H_a: \sigma_1 \neq \sigma_2 \ (> or <).$$

The data consist of 2 random samples $X_1,\ X_2,\ \dots,\ X_{n_1}$ and $Y_1,\ Y_2,\ \dots,\ Y_{n_2}$ from populations 1 and 2 respectively. Divide the X observations randomly into m_1 sub-samples of equal size l. Divide the Y observations randomly into m_2 sub-samples of equal size l. For each sub-sample, obtain the sum of the squared deviations of observations from their mean. The numerator has the form $\sum(X_i - \bar{X})^2$ or $\sum(Y_i - \bar{Y})^2$, $X_i(or\ Y_i)$ denotes the ith sample observation for population $1(or\ 2)$ and $\bar{X}(or\ \bar{Y})$ denotes the mean of the samples for population $1\ (or\ 2)$. Arrange the sum of the squares in ascending order and assign ranks.

The test statistic is then

$$M = S - m_1(m_1 + 1)/2$$

where S is equal to the sum of the ranks assigned to the sums of squares (SS) computed from the sub-samples of X's.

The standardized version of Moses test is given by:

$$M^* = \frac{M - E_0(M)}{\sqrt{var_0(M)}}$$

where $E_0(M) = m_2(m_1 + m_2\ + 1)/2$

$$var_0(M) = \ m_1 m_2(m_1 + m_2\ + 1)/12$$

The asymptotic null distribution of M^* is the standard normal distribution. If population 1 has the greater amount of dispersion, M^* will tend to be small.

In order to obtain meaningful results from the application of the location test, Shorack (1969) recommends that the number of the subgroups should be as large as possible, but not larger than 10, so subgroup sample size can be large enough to test the scale parameters.

The Moses test does not assume equality of location parameters, so it has wider applicability than Ansari-Bradley test(Daniel, 1990).

2.9. Lepage Test

The Lepage test is based on two linear rank tests, one for location and one for scale (Lepage, 1971).

The classical test of Lepage is a combination of the Wilcoxon test (W) for location and the Ansari-Bradley test (AB) for scale alternatives.

Let m and n be the size of two random samples $X_1,\ X_2,\ \ldots,\ X_m$ and $Y_1,\ Y_2,\ \ldots,\ Y_n$, N=m+n. We assume that the elements within each sample are independent and identically distributed, and we assume independence between the two samples. The null hypothesis to be tested is that both samples come from the same population

$$H_0: \mu_1 = \mu_2 \ and \ \sigma_1 = \sigma_2$$

$$H_a: \mu_1 \neq \mu_2 \ and \ \sigma_1 \neq \sigma_2$$

The Lepage statistic is defined as

$$LP = \frac{[W - E(W)]^2}{Var(W)} + \frac{[AB - E(AB)]^2}{Var(AB)}$$

The Wilcoxon statistic is defined as

$$W = \sum_{j=1}^{N} jV_j,$$

where V_j=1 when the jth smallest of the N observations are from the X sample and V_j=0 otherwise.

Under H_0, we have

$$E_0(W) = \frac{m(N+1)}{2}$$

$$var_0(W) = \frac{mn(N+1)}{12}$$

The Ansari-Bradley statistic is defined as

$$AB = \frac{1}{2}m(N+1) - \sum_{j=1}^{N} \left| j - \frac{1}{2}(N+1) \right| V_j$$

Under H_0, we have

when N is even

$$E_0(AB) = \frac{m(N+2)}{4}$$

$$var_0(AB) = \frac{mn(N+2)(N-2)}{48(N-1)}$$

when N is odd, same as above

$$E_0(AB) = \frac{m(N+1)^2}{4}$$

$$var_0(AB) = \frac{mn(N+1)(N^2+3)}{48N^2}$$

The Lepage test has a chi-square distribution with two degrees of freedom when the null hypothesis is true.

For the two-sample location-scale problem, several other researchers designed nonparametric tests similar to Lepage(1971). Marozzi (2013) found that there is not a clear winner for all the situations. Some tests have good power, but no single test dominates.

2.10. Fligner-Wolfe & Ansari-Bradley Test

Alsubie and Magel (2020) proposed two tests L_1 and L_2 for the simple tree alternative for location and scale testing. These tests are a combination of the Fligner-Wolfe test for detecting location changes and the modified Ansari-Bradley test for detecting scale changes. The hypotheses are

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k, \text{ and } \sigma_1 = \sigma_2 = \cdots = \sigma_k$$

$$H_\alpha: \mu_1 \leq [\mu_2, \dots, \mu_k] \text{ and } \sigma_1 \leq [\sigma_2, \dots, \sigma_k] \text{ (At least one inequality is strict)}$$

The L_1 test is the sum of the standardized test statistics for two tests. The first test is Fligner-Wolfe test statistic FW, and the second is the modified Ansari-Bradley test statistic AB.

$$L_1 = \frac{FW^* + AB^*}{\sqrt{2}}$$

where the FW^* represents the standardized test statistic for Fligner-Wolfe test statistic and AB^* represents the standardized test statistic for Ansari-Bradley test statistics.

The second test is given by:

$$L_2 = \frac{FW + AB - E(FW + AB)}{\sqrt{var(FW) + var(AB)}}$$

where the sum of the mean is given by $E(FW + AB)= E(FW) + E(AB)$ and the null standard deviation is $\sqrt{var(FW) + var(AB)}$. When the null hypothesis is true, the asymptotic distribution of L_1 and L_2 are standard normal distributions.

In their research, they found that L_2 has the highest powers when the change is only in location parameters. When the change is only in scale parameters, L_1 has the highest powers. When both the location and scale parameters are different, the test statistic that has higher powers changes depending on the underlying distribution. For both normal distribution and t-distribution with 3 degrees of freedom (symmetric distributions), L_1 has higher powers while L_2 has higher powers for the exponential distribution (skewed).

2.11. Fligner-Wolfe & Mann-Whitney Test

Along with the Fligner-Wolfe and Ansari-Bradley combination test, Alsubie and Magel (2020) proposed other three tests M_1, M_2 and M_3 for the simple tree alternative for location and scale testing. These tests are a combination of the Fligner-Wolfe test for detecting location changes and the modified Moses test for detecting scale changes. The hypotheses are

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k, \text{ and } \sigma_1 = \sigma_2 = \cdots = \sigma_k$$

$$H_\alpha: \mu_1 \leq [\mu_2, \dots, \mu_k] \text{ and } \sigma_1 \leq [\sigma_2, \dots, \sigma_k] \text{ (At least one inequality is strict)}$$

The first test M_1 is the sum of the standardized test statistics for two tests Fligner-Wolfe test statistic (FW) and the modified Moses test statistics (M).

$$Z_1 = \frac{FW - E(\mathrm{FW})}{\sqrt{var(\mathrm{FW})}}$$

$$Z_2 = \frac{M - E(\mathrm{M})}{\sqrt{var(\mathrm{M})}}$$

$$M_1 = \frac{Z_1 + Z_2}{\sqrt{2}}$$

where the Z_1 represents the standardized test statistic for Fligner-Wolfe test statistic and Z_2 represents the standardized test statistic for Moses test statistics.

The second test is given by:

$$M_2 = \frac{FW + \mathrm{M} - E(\mathrm{FW} + \mathrm{M})}{\sqrt{var(\mathrm{FW}) + var(\mathrm{M})}}$$

The sum of the null distribution of the mean is given by $E(FW + \mathrm{M}) = E(FW) + E(M)$ and the null standard deviation is $\sqrt{var(\mathrm{FW}) + var(\mathrm{M})}$.

The third test is given by:

$$M_3 = \frac{FW + 3\mathrm{M} - E(\mathrm{FW} + 3\mathrm{M})}{\sqrt{var(\mathrm{FW} + 3\mathrm{M})}}$$

When the null hypothesis is true, the asymptotic distribution of M_1, M_2 and M_3 are standard normal distributions.

Alsubie and Magel (2020) found that M_2 has the highest powers when the change is only in location parameters. When the change is only in scale parameters, L_1 has the highest powers. When both the location and scale parameters are different, the test statistic that has higher powers changes depending on the underlying distribution. For both normal distribution and t-distribution with 3 degrees of freedom (symmetric distributions), L_1 has higher powers while M_2 has higher powers for the exponential distribution (skewed), and some alpha values did not hold for tests for the exponential distribution.

CHAPTER 3. PROPOSED TESTS

Leinhardt & Wasserman (1979) present a 1975 New York Times published about the life expectancy and per capita income (in 1974 US dollars) for 105 nations classified into some categories: higher income, middle income and low income. Researchers expect that income and life expectancy are positively correlated, and with the life length increase, the lifespan becomes more unstable. In order to test if the average of life expectancy and the instability of the life are different assuming it is nondecreasing with income, we can use an ordered alternative test.

3.1. Proposed Tests

Alsubie and Magel(2020) proposed several nonparametric tests for the simple tree alternative to test for differences in location and scale. In this research, we will provide a method to test both location and scale for the nondecreasing ordered alternative. The proposed tests combine two tests together: one is to test the equality of location and the other one is to test the equality of scale against ordered alternatives between more than 2 populations. In our research, we use an ordered alternatives location test to test the equality of location and Moses test's technique to translate the scale test to a location test and then apply the corresponding location tests (see Figure 1).

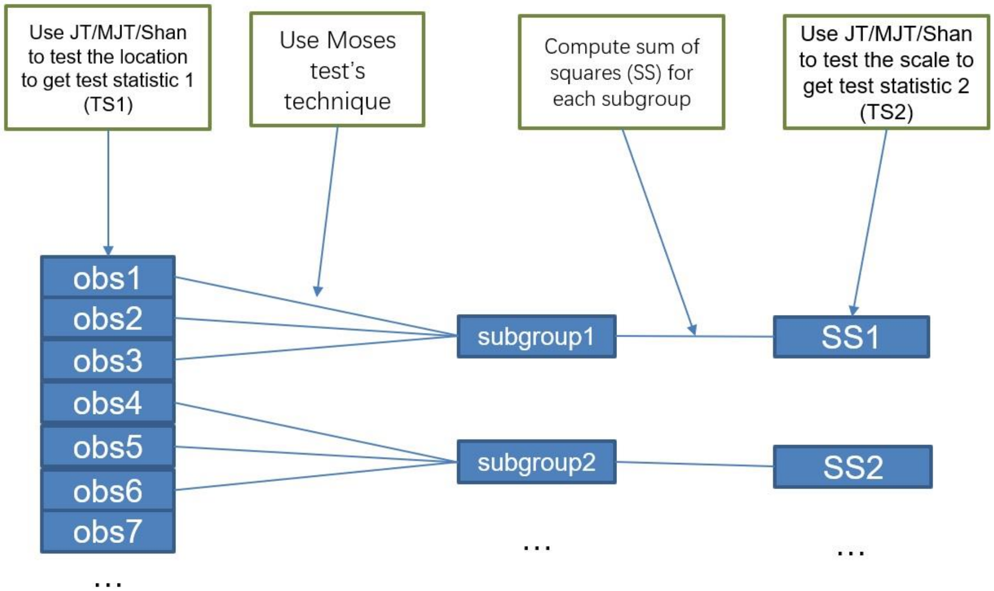

Figure 3.1. The main processes for the proposed tests

Specifically, the hypotheses are

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \; and \; \sigma_1 = \sigma_2 = \cdots = \sigma_k.$$

and

$$H_a: \mu_1 \leq \mu_2 \leq \cdots \leq \mu_k \; and \; \sigma_1 \leq \sigma_2 \leq \cdots \leq \sigma_k, \text{ at least one of the inequalities is strict}$$

3.1.1. Jonckheere-Terpstra (JT) Test + Moses Test

In order to test the equality of location and scale parameters at the same time, we use the JT test to test the equality of location parameters and the methods of the Moses test to test the equality of scale parameters.

For $k (k \geq 3)$ populations $1, 2, \ldots, k$,

$$JT_1 = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} U_{ij}.$$

where U_{ij} is the number of pairs of observations (a, b) for which x_{ia} is less than x_{jb}, $x_{ia} (or \; x_{jb})$ denotes the $ath (or \; bth)$ sample observation for $ith (or \; jth)$ population $i (or \; j)$.

Using the Moses technique, divide all observations in each sample randomly into sub-samples of equal size. For each sub-sample obtain the sum of the squared deviations of observations from their mean. Let $m_1, m_2, \dots, m_k$ be the number of sup-groups for each population. The JT test is applied using the sum of squares associated with each of the sub-samples within each of the original samples as the new sets of observations. This is referred to as JT_2. Because we use the sum of squares for each subgroup to calculate JT_2, the sample sizes for each sample are less than the initial sample sizes from the populations.

There are two ways to standardize the test statistic:

JM_1: standardize first:

$$JM_1 = \frac{JT_1^* + JT_2^*}{\sqrt{2}}$$

where $JT_1^* = \frac{JT_1 - E(JT_1)}{\sqrt{var(JT_1)}}$, and $JT_2^* = \frac{JT_2 - E(JT_2)}{\sqrt{var(JT_2)}}$.

The expected value and the variance given are similar to the expected value under the null distribution for Jonckheere-Terpstra (JT) test statistic:

$$E_0(JT_1) = \frac{N^2 - \sum_{i=1}^{k} n_i^2}{4} \text{ and } var_0(JT_1) = \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)}{72}$$

where $N = n_1 + n_2 + \cdots + n_k$ ($n_1, n_2, \dots, n_k$ are the sample sizes for each population). For JT_2, replace the sample sizes by $m_1, m_2, \dots, m_k$ instead of the $n_1, n_2, \dots, n_k$ ($m_1, m_2, \dots, m_k$ are the number of sup-groups for each population).

Both JT_1^* and JT_2^* have an asymptotic standard normal distribution under H_0(Terpstra, 1952). When H_0 is true, the asymptotic distribution of JT_1^* + JT_2^* should be normal with a mean of zero and a variance of two. As a result, the asymptotic distribution of the proposed test (JM_1) under H_0 is a standard normal distribution.

The asymptotic distribution of the test statistic is used. H_0 is rejected for a large value which is $JM_1 \geqslant Z\alpha$ at the α level of significance where $Z\alpha$ is the (1- α) 100% of the standard normal distribution. If the test is performed at a 5% level of significance, then $Z\alpha$= 1.645.

JM_2: standardize last:

$$JM_2 = \frac{JT_1 + JT_2 - [E(JT_1) + E(JT_2)]}{\sqrt{var(JT_1) + var(JT_2)}}$$

The sum of the null distribution of the mean is given by $E(JT_1)$ + $E(JT_2)$ and the null standard deviation is $\sqrt{var(JT_1) + var(JT_2)}$. When the null hypothesis is true, the asymptotic distribution of JM_2 is also a standard normal distribution.

3.1.2. Modified Jonckheere-Terpstra (MJT) Test + Moses Test

In order to test the equality of location and scale parameters at the same time, we use the MJT test to test the equality of location parameters and the methods of the Moses test to test the equality of scale parameters.

For $k(k \geq 3)$ populations $1, 2, \ldots, k$,

$$MJT_1 = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j - i)U_{ij}$$

where U_{ij} is the number of pairs of observations (a, b) for which x_{ia} is less than x_{jb}, $x_{ia}(or\ x_{jb})$ denotes the $ath(or\ bth)$ sample observation for $ith(or\ jth)$ population $i(or\ j)$.

Using the Moses technique, divide all observations in each sample randomly into sub-samples of equal size. For each sub-sample obtain the sum of the squared deviations of observations from their mean. Let m_i, m_j, m_l be the number of sup-groups for each population. The JT test is applied using the sum of squares associated with each of the sub-samples within each of the original samples as the new sets of observations. This is referred to as MJT_2. Because we use sum of squares for each subgroup to calculate MJT_2, the sample sizes for each sample are less than the initial sample sizes from the populations.

Also, we can standardize the test statistic MJM_1 first:

$$MJM_1 = \frac{MJT_1^* + MJT_2^*}{\sqrt{2}}$$

where $MJT_1^* = \frac{MJT_1 - E(MJT_1)}{\sqrt{var(MJT_1)}}$, and $MJT_2^* = \frac{MJT_2 - E(MJT_2)}{\sqrt{var(MJT_2)}}$

and MJM_2: standardize the test statistic last

$$MJM_2 = \frac{MJT_1 + MJT_2 - [E(MJT_1) + E(MJT_2)]}{\sqrt{var(MJT_1) + var(MJT_2)}}$$

The expected value and the variance given are similar to the expected value under the null distribution for modified Jonckheere-Terpstra test statistic:

$$E(MJT_1) = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)E(U_{ij})$$

$$var(MJT_1) = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} (j-i)^2 var(U_{ij}) + \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} \sum_{l=j+1}^{k} Cov(U_{ij}, U_{il})$$

$$E_0(U_{ij}) = \frac{1}{2} n_i n_j, \forall i \neq j$$

$$var_0(U_{ij}) = \frac{1}{12} n_i n_j (n_i + n_j + 1), \forall i \neq j$$

$$Cov(U_{ij}, U_{il}) = Cov(U_{ji}, U_{li}) = \frac{1}{12} n_i n_j n_l, if\ all\ i, j, and\ l\ are\ different$$

$$Cov(U_{ij}, U_{li}) = Cov(U_{ji}, U_{il}) = -\frac{1}{12} n_i n_j n_l, if\ all\ i, j, and\ l\ are\ different$$

$$Cov(U_{ij}, U_{lm}) = 0, if\ all\ i, j, and\ l\ are\ different$$

where n_i, n_j, n_l are the sample sizes for population i, j, and l. For MJT_2, replace the sample sizes by m_i, m_j, m_l instead of the n_i, n_j, n_l (m_i, m_j, m_l are the number of sup-groups for each population).

Both MJT_1^* and MJT_2^* have an asymptotic standard normal distribution under H_0(Neuhauser, Liu, and Hothorn, 1998). When H_0 is true, the asymptotic distribution of $MJT_1^* + MJT_2^*$ should be normal with a mean of zero and a variance of two. As a result, the asymptotic distribution of MJM_1 under H_0 is a standard normal distribution. The sum of the null distribution of the mean is given by $E(MJT_1)$ + $E(MJT_2)$ and the null standard deviation is $\sqrt{var(MJT_1) + var(MJT_2)}$. When the null hypothesis is true, the asymptotic distribution of MJM_2 is also a standard normal distribution.

3.1.3. Shan Test + Moses Test

In order to test the equality of location and scale parameters at the same time, we use the Shan test to test the equality of location parameters and the methods of the Moses test to test the equality of scale parameters.

For $k(k \geq 3)$ populations $1, 2, \ldots, k$, first we combine all the observations together. Then rank all the observations low to high.

Incorporating the actual differences is given as

$$S_1 = \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} D_{ij},$$

where $D_{ij} = \sum_{l=1}^{n_i} \sum_{m=1}^{n_j} Z_{ijlm}, Z_{ijlm} = (R_{jm} - R_{il}) I(x_{jm} > x_{il})$, $x_{il}(or\ x_{jm})$ denotes the $lth(or\ mth)$ sample observation for $ith(or\ jth)$ population $i(or\ j)$, and $R_{il}(or\ R_{jm})$ denotes the rank of the observation $x_{il}(or\ x_{jm})$ in the combined data.

Then, using the Moses test's technique, divide all observations in each sample randomly into sub-samples of equal size. For each sub-sample obtain the sum of the squared deviations of observations from their mean, combine all the sum of squares together and rank them from low to high. Let $m_1, m_2, \ldots, m_k$ be the number of sup-groups for each population. The Shan test is applied using the rank of sum of squares associated with each of the sub-samples within each of the original samples as the new sets of observations. This is referred to as S_2. Because we use the rank of the sum of squares for each subgroup to calculate S_2, the sample sizes for each sample are less than the initial sample sizes from the populations.

We have SM_1: standardize the test statistic first

$$SM_1 = \frac{S_1^* + S_2^*}{\sqrt{2}}$$

where $S_1^* = \frac{S_1 - E(S_1)}{\sqrt{var(S_1)}}$, and $S_2^* = \frac{S_2 - E(S_2)}{\sqrt{var(S_2)}}$.

and SM_2: standardize the test statistic last:

$$SM_2 = \frac{S_1 + S_2 - [E(S_1) + E(S_2)]}{\sqrt{var(S_1) + var(S_2)}}$$

The expected value and the variance given are similar to the expected value under the null distribution for Shan test statistic:

$$E_0(S_1) = \frac{N+1}{6} \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} n_i n_j \ ,$$

and

$$var_0(S_1) = \left(\frac{N^2+N}{12} - \frac{(N+1)^2}{36}\right) \sum_{i=1}^{k-1} \sum_{j=i+1}^{k} n_i n_j + 2\left[\sum_{i=1}^{k-1} n_i \binom{\sum_{j=i+1}^{k} n_j}{2} + \sum_{i=2}^{k} n_i \binom{\sum_{j=1}^{i-1} n_j}{2}\right] CovA +$$

$$2\left(\sum_{i=1}^{k-2} \sum_{j=i+1}^{k-1} \sum_{l=j+1}^{k} n_i n_j n_l\right) CovB,$$

where $CovA = \frac{2N^2+N-1}{90}$, and $CovB = \frac{-7N^2-11N-4}{360}$, n_i, n_j are the sample sizes for population i and j, $N = n_1 + n_2 + \cdots + n_k$. For S_2, replace the sample sizes by $m_1, m_2, \dots, m_k$ instead of the $n_1, n_2, \dots, n_k$ ($m_1, m_2, \dots, m_k$ are the number of sup-groups for each population).

Both S_1^* and S_2^* have an asymptotic standard normal distribution under H_0(Shan, 2014). When H_0 is true, the asymptotic distribution of S_1^* + S_2^* should be normal with a mean of zero and a variance of two. As a result, the asymptotic distribution of SM_1 under H_0 is a standard normal. The sum of the null distribution of the mean is given by $E(S_1)$ + $E(S_2)$ and the null standard deviation is $\sqrt{var(S_1) + var(S_2)}$.

When the null hypothesis is true, the asymptotic distribution of SM_2 is also a standard normal distribution.

3.2. Example

An example of life expectancy for different income level nations is given below. Suppose research is conducted to investigate the life expectancy for different income level nations. For this example, we only use the three categories: "low income", "middle income" and "higher income". It can be expected that higher income nations have longer life expectancy and higher life instability. So, we order the income level for the samples from low to high to test whether the average life length and instability were nondecreasing with income level. All observations are given below in the table.

Table 3.1. Life expectancy (in years) of nations with different income levels

Low income	Middle income	Higher income
44	63	79
54	68	60
45	54	69
50	69	65
54	59	60
45	63	69
48	57	68
56	65	72
56	62	71
42	54	70
47	60	68
50	63	66

$$H_0: \mu_1 = \mu_2 = \mu_3 \ and \ \sigma_1 = \sigma_2 = \sigma_3.$$

$H_a: \mu_1 \leq \mu_2 \leq \mu_3 \ and \ \sigma_1 \leq \sigma_2 \leq \sigma_3$, at least one of the inequalities is strict

To apply Moses test's technique, we need to divide all observations into sub-samples of equal size randomly. For this example, we use sub-group sample=3.

Table 3.2. Life expectancy (in years) of nations with different income levels with subgroup

Subgroup	Low income	Middle income	Higher income
1	44	63	79
1	54	68	60
1	45	54	69
2	50	69	65
2	54	59	60
2	45	63	69
3	48	57	68
3	56	65	72
3	56	62	71
4	42	54	70
4	47	60	68
4	50	63	66

Then calculate the sum of squares for each subgroup.

Table 3.3. The sum of squares of life expectancy of each subgroup

Subgroup	Low income	Middle income	Higher income
1	60.67	100.67	180.67
2	40.67	50.67	40.67
3	42.67	32.67	8.67
4	32.67	42	8

JT+Moses Test Statistic:

Use the original data (Table 3.1) to calculate the statistic of the JT test JT_1:

$$JT_1 = \sum_{i=1}^{2} \sum_{j=i+1}^{3} U_{ij} = U_{12} + U_{23} + U_{13} = 138 + 120.5 + 144 = 402.5$$

Then standardize it:

$$E(JT_1) = \frac{N^2 - \sum_{i=1}^{3} n_i^2}{4} = \frac{36^2 - 12^2 \times 3}{4} = 216$$

$$var(JT_1) = \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)}{72} = 1183.02$$

$$JT_1^* = \frac{JT_1 - E(JT_1)}{\sqrt{var(JT_1)}} = 5.4223$$

where $N = 36, n_1 = n_2 = n_3 = 12.$

Use the data of Table 3.3 to calculate the statistic JT_2:

$$JT_2 = \sum_{i=1}^{2} \sum_{j=i+1}^{3} U_{ij} = U_{12} + U_{23} + U_{13} = 9.5 + 5 + 5.5 = 20$$

Then standardize it:

$$E(JT_2) = \frac{N^2 - \sum_{i=1}^{k} n_i^2}{4} = \frac{12^2 - 4^2 \times 3}{4} = 24$$

$$var(JT_2) = \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)}{72} = 46.30$$

$$JT_2^* = \frac{JT_2 - E(JT_2)}{\sqrt{var(JT_2)}} = -0.5878$$

Because we divided observations into sub-samples of equal size randomly, so $N = 12, n_1 = n_2 = n_3 = 4.$

Finally, calculate the statistic JM_1 and JM_2:

$$JM_1 = \frac{JT_1^* + JT_2^*}{\sqrt{2}} = 3.4185\ (p - value = 0.0003)$$

$$JM_2 = \frac{JT_1 + JT_2 - [E(JT_1) + E(JT_2)]}{\sqrt{var(JT_1) + var(JT_2)}} = 5.2051\ (p - value < 0.00001)$$

Both JM_1 and JM_2 are greater than 1.645, so we reject H_0 at a 5% level of significance.

Modified JT+Moses Test Statistic:

Use the original data (Table 3.1) to calculate the statistic of the MJT test MJT_1:

$$MJT_1 = \sum_{i=1}^{2} \sum_{j=i+1}^{3} (j-i)U_{ij} = U_{12} + U_{23} + 2U_{13} = 546.5$$

Then standardize it:

$$E(MJT_1) = E(U_{12} + U_{23} + 2U_{13}) = 72 + 72 + 144 = 288$$

$$var(MJT_1) = \mathrm{var}(U_{12}) + \mathrm{var}(U_{23}) + 4\mathrm{var}(U_{13}) + 4Cov(U_{12}, U_{13}) + 2Cov(U_{12}, U_{23}) + 4Cov(U_{13}, U_{23})$$

$$= 300 + 300 + 1200 + 576 - 288 + 576 = 2664$$

$$MJT_1^* = \frac{MJT_1 - E(MJT_1)}{\sqrt{var(MJT_1)}} = 5.008$$

Use the data of Table 3.3 to calculate the statistic MJT_2:

$$MJT_2 = \sum_{i=1}^{2} \sum_{j=i+1}^{3} (j-i)U_{ij} = U_{12} + U_{23} + 2U_{13} = 25.5$$

Then standardize it:

$$E(MJT_2) = E(U_{12} + U_{23} + 2U_{13}) = 8 + 8 + 16 = 32$$

$$var(MJT_2) = \mathrm{var}(U_{12}) + \mathrm{var}(U_{23}) + 4\mathrm{var}(U_{13}) + 4Cov(U_{12}, U_{13}) + 2Cov(U_{12}, U_{23}) + 4Cov(U_{13}, U_{23})$$

$$= 12 + 12 + 48 + 21.3333 - 10.6667 + 21.3333 = 104$$

$$MJT_2^* = \frac{MJT_2 - E(MJT_2)}{\sqrt{var(MJT_2)}} = -0.6373$$

Finally, calculate the statistic MJM_1 and MJM_2:

$$MJM_1 = \frac{MJT_1^* + MJT_2^*}{\sqrt{2}} = 3.091\ (p - value = 0.001)$$

$$MJM_2 = \frac{MJT_1 + MJT_2 - [E(MJT_1) + E(MJT_2)]}{\sqrt{var(MJT_1) + var(MJT_2)}} = 4.7899\ (p - value < 0.00001)$$

Both MJM_1 and MJM_2 are greater than 1.645, so we reject H_0 at a 5% level of significance.

Shan+Moses Test Statistic:

First, to apply the Shan test, we need to combine data and rank them.

Table 3.4. The rank for life expectancy (in years) of nations with different income level

Subgroup	Low income	Rank	Middle income	Rank	Higher income	Rank
1	44	2	63	22	79	36
1	54	10.5	68	28	60	18
1	45	3.5	54	10.5	69	31
2	50	7.5	69	31	65	24.5
2	54	10.5	59	16	60	18
2	45	3.5	63	22	69	31
3	48	6	57	15	68	28
3	56	13.5	65	24.5	72	35
3	56	13.5	62	20	71	34
4	42	1	54	10.5	70	33
4	47	5	60	18	68	28
4	50	7.5	63	22	66	26

Use the rank data of Table 3.4 to calculate the statistic of the Shan test S_1:

$$S_1 = \sum_{i=1}^{2} \sum_{j=i+1}^{3} D_{ij} = 1878 + 3102 + 1346 = 6326$$

Then standardize it:

$$E(S_1) = \frac{N+1}{6} \sum_{i=1}^{2} \sum_{j=i+1}^{3} n_i n_j = 2664$$

$$\mathrm{var}(S_1) = \left(\frac{N^2+N}{12} - \frac{(N+1)^2}{36}\right) \sum_{i=1}^{2} \sum_{j=i+1}^{3} n_i n_j + 2\left[\sum_{i=1}^{2} n_i \left(\sum_{j=i+1}^{3} n_j \atop 2\right) + \sum_{i=2}^{3} n_i \left(\sum_{j=1}^{i-1} n_j \atop 2\right)\right] CovA$$

$$+ 2\left(\sum_{i=1}^{1} \sum_{j=i+1}^{2} \sum_{l=j+1}^{3} n_i n_j n_l\right) CovB = 3588 + 479164.8 - 90931.2 = 391821.6$$

$$S_1^* = \frac{S_1 - E(S_1)}{\sqrt{var(S_1)}} = 5.8502$$

where $N = 36, n_1 = n_2 = n_3 = 12$, $CovA = \frac{2N^2+N-1}{90} = 29.1889$, and $CovB = \frac{-7N^2-11N-4}{360} = -26.3111$.

To apply Moses test's technique, we use the table below:

Table 3.5. The rank for the sum of squares of life expectancy of each subgroup

Subgroup	Low income	Rank	Middle income	Rank	Higher income	Rank
1	60.67	10	100.67	11	180.67	12
2	40.67	5.5	50.67	9	40.67	5.5
3	42.67	8	32.67	3.5	8.67	2
4	32.67	3.5	42	7	8	1

Use the data of Table 3.5 to calculate the statistic S_2:

$$S_2 = \sum_{i=1}^{2} \sum_{j=i+1}^{3} D_{ij} = 32 + 23 + 19.5 = 74.5$$

Then standardize it:

$$E(S_2) = \frac{N+1}{6} \sum_{i=1}^{2} \sum_{j=i+1}^{3} n_i n_j = 104$$

$$\text{var}(S_2) = \left(\frac{N^2+N}{12} - \frac{(N+1)^2}{36}\right) \sum_{i=1}^{2} \sum_{j=i+1}^{3} n_i n_j + 2\left[\sum_{i=1}^{2} n_i \left(\frac{\sum_{j=i+1}^{3} n_j}{2}\right) + \sum_{i=2}^{3} n_i \left(\frac{\sum_{j=1}^{i-1} n_j}{2}\right)\right] CovA$$
$$+ 2\left(\sum_{i=1}^{1} \sum_{j=i+1}^{2} \sum_{l=j+1}^{3} n_i n_j n_l\right) CovB = 46.6667 + 1807.2889 - 406.7556 = 1447.2$$

where $CovA = \frac{2N^2+N-1}{90} = 3.3222$, and $CovB = \frac{-7N^2-11N-4}{360} = -3.1778$, because we divided observations into sub-samples of equal size randomly, so $N = 12, n_1 = n_2 = n_3 = 4$.

$$S_2^* = \frac{S_2 - E(S_2)}{\sqrt{var(S_2)}} = -0.7754$$

Finally, calculate the statistic SM_1 and SM_2:

$$SM_1 = \frac{S_1^* + S_2^*}{\sqrt{2}} = 3.5884 \ (p - value = 0.00017)$$

$$SM_2 = \frac{S_1 + S_2 - [E(S_1) + E(S_2)]}{\sqrt{var(S_1) + var(S_2)}} = 5.7924 \ (p - value < 0.00001)$$

Both SM_1 and SM_2 are greater than 1.645, so we reject H_0 at a 5% level of significance.

The above example shows that SM_2 has the largest test statistic and lowest p-value of all six tests. And it is noted that this is only for this example. A simulation study will be conducted to determine how well the proposed tests maintain their significance levels. Powers will be estimated for the proposed tests under a variety of conditions for three, four and five populations. Several types of variable parameters will be considered: when the location parameters are different, and the scale parameters are equal; when the location parameters are equal, and the scale parameters are different; when the location and scale parameters are both different; when the sample sizes of populations are equal or different; we will also consider two different subgroup sample sizes for the Moses test.

CHAPTER 4. SIMULATION STUDY

A simulation study was conducted to compare the six proposed tests and was implemented in SAS version 9.4. The properties of the proposed test statistics were compared assuming random samples followed standard normal distribution, t-distribution with 3 degrees of freedom, or exponential distribution with the mean and variance equal to one for control. The function RAND was used to generate random samples from a specific distribution in SAS. In order to state the starting point "seed", the "Call streaminit" function was used before using the RAND function. The syntax for this function is

Call streaminit (seed)

In this research, seed = 0 was used to instruct RAND to use the system clock.

The call function for the normal distribution is

RAND ('Normal', μ, σ)

where μ is the mean, and σ is the standard deviation. We can change the variables μ and σ to obtain some random samples with the same or different locations and scales.

The call function for the t-distribution is

RAND ('T', 3)

where "3" is the degrees of freedom. We use function RAND ('T', 3)*a+b to create the random samples. By changing variables a and b, we can obtain random samples with the same or different locations and scales. We can assign the same values to a and different values to b to create random samples with the same scales and different locations. For example, we can create three random samples with the same scales and different locations with a=1, 1, 1 and b=0, 1 and 2. We can assign the different values to a and same values to b to create random samples with the different scales and same locations. For example, we can create three random samples with the different scales and same locations with a=1, 2, 3 and b=0, 0 and 0. By assigning different values to a and b, we can create some random samples with different locations and scales. For example, we can create three random samples with the different scales and locations with a=1, 2, 3 and b=0, 1 and 2.

The call function for the exponential distribution is

RAND (‘Exponential’, 1).

This function generates a random number from an exponential distribution with the mean and variance equal to one. We use function RAND (‘Exponential’, 1)*a+b to create the random samples. By changing variables a and b, we can obtain random samples with the same or different locations and scales. We can assign the same values to a and different values to b to create random samples with the same scales and different locations. For example, we can create three random samples with the same scales and different locations with a=1, 1, 1 and b=0, 1 and 2. We can assign different values to a to make the scales of the random samples different, and then assign different values to b to make the locations of the random samples equal to create random samples with the different scales and same locations. For example, we can create three random samples with the different scales and same locations with a=1, 2, 3 and b=0, -1 and -2. By assigning different values to a and b, we can create some random samples with different locations and scales. For example, we can create three random samples with the different scales and locations with a=1, 2, 3 and b=0, 1 and 2.

The six proposed tests were compared in two steps. The first step was to get the estimates of the alpha values of the proposed test statistics. Replications of 5000 samples were used for all simulations. The stated alpha values for the proposed test statistics were 0.05. The alpha values were estimated by the total number of times the null hypothesis was rejected when the null hypothesis was true and then dividing by 5000. The second step was to compare the powers of the test statistics under various conditions. Powers were estimated by the total number of times the proposed tests were rejected divided by 5000.

4.1. Simulation Outline

The proposed test statistics were examined in the cases of 3, 4, and 5 populations. Random samples were initially taken from 3, 4 and 5 identical populations. The alpha values were estimated by counting the number of times the null hypothesis was rejected and then dividing by 5000. The alpha values were estimated and compared to the stated alpha values for each simulation conducted.

The aspect of interest in this simulation was estimating and comparing the powers of all the proposed tests. Therefore, three varying conditions were assumed. First, the location parameters were different, and the scale parameters were equal. Second, the location parameters were equal, and the scale parameters were different. Lastly, both the location parameters and the scale parameters were different. The combination of sample sizes 18 and 30 were used for all populations. Two different subgroup sample sizes (3 and 6) were used for each scenario when applying the Moses test's technique.

4.2. Power Calculations

To compare the powers of the proposed tests, different numbers of populations, different combinations of location parameters and scale parameters, different combinations of sample sizes, and different subgroup sample sizes were considered.

For the case with three populations, three varying conditions were assumed—the location parameters were different, and the scale parameters were equal; the location parameters were equal, and the scale parameters were different; both the location parameters and the scale parameters were different. Different treatment effects were denoted by shifts in the locations and scales for each treatment. Various configurations of treatment effects were examined—when scale parameters were the same, and all the location parameters were different, but equally spaced; when all the location parameters were different, but not equally spaced; the first location parameter was the same as the second but the last one was different; the second location parameter was the same as the last but the first one was different. The same protocol was applied in the situation when location parameters were the same and all the scale parameters were different. The same protocol was also applied in the situation when location and scale parameters were all different.

For the case with four populations, three varying conditions were assumed—the location parameters were different, and the scale parameters were equal; the location parameters were equal, and the scale parameters were different; both the location parameters and the scale parameters were different. Different treatment effects were denoted by shifts in the locations and scales for each treatment. Various configuration of treatments effects was examined— when scale parameters were the same, and all the location parameters were different, but equally spaced; when all the location parameters were different, but not equally spaced; the first three location parameters were the same but the last one was different; the last three location parameters were the same but the first one was different. The same protocol was applied in the situation when location parameters were the same, and all the scale parameters were different. The same protocol was also applied in the situation when location and scale parameters were all different.

For the case with five populations, three varying conditions were assumed— the location parameters were different, and the scale parameters were equal; the location parameters were equal, and the scale parameters were different; both the location parameters and the scale parameters were different. Different treatment effects were denoted by shifts in the locations and scales for each treatment. Various configuration of treatments effects was examined— when scale parameters were the same, and all the location parameters were different, but equally spaced; when all the location parameters were different, but not equally spaced; the first four location parameters were the same but the last one was different; the last four location parameters were the same but the first one was different. The same protocol was applied in the situation when location parameters were the same, and all the scale parameters were different. The same protocol was also applied in the situation when location and scale parameters were all different.

The sample size considered in the simulations are 18 and 30, and different combinations of sample sizes 18 and 30 were used for all populations. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique.

CHAPTER 5. RESULTS

In this chapter, the results of the simulation study will be presented. These results are separated by distributions. The distributions are normal, t-distribution with 3 degrees of freedom, and exponential. The tables show the estimated powers and significance levels for the proposed tests. Test statistics with the subscript of 1 represent those standardized first. Test statistics with the subscript of 2 represent those standardized last. The symbol μ_i is the mean (location) parameter for ith population, and σ_i is the standard deviation (scale) parameter for the ith population.

5.1. Results of the Normal Distribution

5.1.1. Three Treatments

Tables 1-18 present the results of the simulation study for three treatments under the normal distribution. The sample sizes considered in the simulations are 18 and 30. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique. Tables 1, 2, 7, 8, 13 and 14 give estimated powers when the location parameters are different but scale parameters are equal. Tables 3, 4, 9, 10, 15 and 16 give estimated powers when the location parameters are equal but scale parameters are different. Tables 5, 6, 11, 12, 17 and 18 give estimated powers when the location and scale parameters are all different. It appears that all of the proposed tests maintained their alpha values and they were around 0.05 (Tables 1, 2, 7, 8, 13 and 14). When the populations have unequal location parameters and equal scale parameters, the standardize last tests have higher estimated powers than all the standardize first tests (Tables 1, 2, 7, 8, 13 and 14). When the populations have equal location parameters and unequal scale parameters, the standardize first tests have higher estimated powers than all the standardize last tests (Tables 3, 4, 9, 10, 15 and 16). When the populations have unequal location parameters and unequal scale parameters, the standardize first tests tend to have the higher estimated powers (Tables 5, 6, 11, 12, 17 and 18). Tables 7-12 show the results of simulations with unequal sample sizes. In these tables, the sample sizes for the first and second populations are 18 and the sample size for the third population is 30. Tables 13-18 also show the results of simulations when the sample sizes are unequal. In these tables, the sample size for the first population is 30 and the sample sizes for the second and third populations are 18. Results were found to be similar as to which test statistic had higher powers in the situations for both equal and unequal sample sizes.

When location parameters are equal and scale parameters are unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 3 and 4, for example). When location and scale parameters are both unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 5 and 6, for example). For the situations of unequal location parameters and equal scale parameters, the SM_2 test tends to have the highest estimated powers (Tables 1, 2, 7, 8, 13 and 14). For the situations of equal location parameters and unequal scale parameters, the SM_1 test generally has the highest estimated powers (Tables 3, 4, 9, 10 and 15). For the situations of unequal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 5, 11, 12, 17 and 18). The only exception to this is when the scale parameters of the two treatment populations are the same but different from the control group, and when the location parameters are all equal. In this case, the SM_1 and SM_2 tests tend to have lower estimated powers than the other tests (Tables 4, 10 and 16).

Table 5.1. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0508	0.0536	0.0490	0.0506	0.0476	0.0492
0	1	0.5	1	1	1	0.8294	0.9814	0.8236	0.9766	0.8318	0.9815
0	1	0.5	1	0.5	1	0.3628	0.5816	0.3756	0.5734	0.3610	0.5812
0	1	0	1	1	1	0.8130	0.9746	0.8258	0.9748	0.8420	0.9794
0	1	0.75	1	1	1	0.8168	0.9750	0.8168	0.9780	0.8260	0.9778
0	1	0.2	1	1	1	0.8266	0.9780	0.8140	0.9780	0.8282	0.9786

Table 5.2. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0501	0.0513	0.0521	0.0516	0.0489	0.0496
0	1	0.5	1	1	1	0.8152	0.9824	0.8232	0.9828	0.8538	0.9786
0	1	0.5	1	0.5	1	0.3446	0.5754	0.3768	0.5832	0.3682	0.5902
0	1	0	1	1	1	0.8002	0.9778	0.8138	0.9800	0.8484	0.9842
0	1	0.75	1	1	1	0.8034	0.9802	0.8108	0.9792	0.8376	0.9872
0	1	0.2	1	1	1	0.8206	0.9838	0.8120	0.9830	0.8196	0.9814

Table 5.3. Percentage of Rejection for k=3 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.8060	0.1660	0.7844	0.1604	0.8476	0.0962
0	1	0	2	0	2	0.4886	0.0954	0.4876	0.1018	0.5504	0.0592
0	1	0	1	0	2	0.5116	0.1352	0.5058	0.1294	0.5564	0.0878
0	1	0	2.5	0	3	0.7818	0.1470	0.7732	0.1504	0.8450	0.0880
0	1	0	1.5	0	3	0.8046	0.1850	0.7860	0.1728	0.8380	0.1018

Table 5.4. Percentage of Rejection for k=3 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.7588	0.0878	0.7306	0.0796	0.7650	0.0708
0	1	0	2	0	2	0.4322	0.0542	0.4678	0.0630	0.4060	0.0430
0	1	0	1	0	2	0.4996	0.0904	0.4922	0.0818	0.4722	0.0696
0	1	0	2.5	0	3	0.6728	0.0716	0.6860	0.0754	0.6878	0.0640
0	1	0	1.5	0	3	0.7398	0.0982	0.7288	0.0892	0.7470	0.0760

Table 5.5. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9872	0.7718	0.9846	0.7576	0.9902	0.6348
0	1	0.5	2	0.5	2	0.8264	0.4836	0.8404	0.4866	0.8608	0.3842
0	1	0	1	1	2	0.9560	0.8462	0.9612	0.8642	0.9624	0.7990
0	1	0.75	2.5	1	3	0.9852	0.7638	0.9860	0.7464	0.9902	0.6348
0	1	0.2	1.5	1	3	0.9848	0.7592	0.9832	0.7522	0.9884	0.6310

Table 5.6. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9830	0.6390	0.9782	0.6142	0.9832	0.5896
0	1	0.5	2	0.5	2	0.7850	0.3586	0.8380	0.3658	0.7744	0.3358
0	1	0	1	1	2	0.9488	0.7792	0.9524	0.7908	0.9526	0.7568
0	1	0.75	2.5	1	3	0.9774	0.6226	0.9806	0.5908	0.9806	0.5610
0	1	0.2	1.5	1	3	0.9752	0.6344	0.9784	0.6090	0.9772	0.5718

Table 5.7. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0524	0.0556	0.0503	0.0528	0.0493	0.0514
0	1	0.5	1	1	1	0.7156	0.9388	0.7088	0.9352	0.7230	0.9468
0	1	0.5	1	0.5	1	0.2504	0.4108	0.2872	0.4488	0.2634	0.4170
0	1	0	1	1	1	0.7740	0.9072	0.7610	0.9036	0.8290	0.9142
0	1	0.75	1	1	1	0.6766	0.9026	0.6742	0.9218	0.6914	0.9160
0	1	0.2	1	1	1	0.7682	0.9630	0.7400	0.9566	0.7902	0.9636

Table 5.8. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0520	0.0541	0.0513	0.0530	0.0481	0.0503
0	1	0.5	1	1	1	0.7112	0.9364	0.7125	0.9254	0.7269	0.9521
0	1	0.5	1	0.5	1	0.2417	0.4201	0.2843	0.4347	0.2748	0.4270
0	1	0	1	1	1	0.7647	0.9023	0.7591	0.8945	0.8340	0.9052
0	1	0.75	1	1	1	0.6641	0.9133	0.6639	0.9141	0.7036	0.9147
0	1	0.2	1	1	1	0.7534	0.9570	0.7458	0.9520	0.8154	0.9622

Table 5.9. Percentage of Rejection for k=3 Populations; Normal Distribution with equal means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.6566	0.1308	0.6634	0.1202	0.6668	0.0600
0	1	0	2	0	2	0.3242	0.0726	0.3324	0.0864	0.3230	0.0476
0	1	0	1	0	2	0.4794	0.1184	0.4444	0.1086	0.4784	0.0574
0	1	0	2.5	0	3	0.5960	0.1086	0.6198	0.1044	0.6278	0.0614
0	1	0	1.5	0	3	0.7056	0.1534	0.6876	0.1332	0.7218	0.0720

Table 5.10. Percentage of Rejection for k=3 Populations; Normal Distribution with equal means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.6448	0.1074	0.6529	0.0632	0.6656	0.0502
0	1	0	2	0	2	0.3049	0.0697	0.3088	0.0489	0.2157	0.0413
0	1	0	1	0	2	0.4587	0.1059	0.4510	0.0524	0.2964	0.0473
0	1	0	2.5	0	3	0.5816	0.1022	0.6081	0.0553	0.6056	0.0509
0	1	0	1.5	0	3	0.6947	0.1477	0.6812	0.0628	0.6944	0.0522

Table 5.11. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9578	0.6226	0.9552	0.6578	0.9430	0.4782
0	1	0.5	2	0.5	2	0.6130	0.3294	0.6688	0.3524	0.6180	0.2464
0	1	0	1	1	2	0.9512	0.8242	0.9422	0.8244	0.9498	0.7622
0	1	0.75	2.5	1	3	0.9312	0.6098	0.9358	0.6170	0.9302	0.6164
0	1	0.2	1.5	1	3	0.9652	0.6828	0.9628	0.6800	0.9622	0.7038

Table 5.12. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9471	0.5913	0.9561	0.5320	0.9124	0.4257
0	1	0.5	2	0.5	2	0.6022	0.3040	0.6649	0.2389	0.5879	0.1956
0	1	0	1	1	2	0.9464	0.7953	0.9359	0.7154	0.9285	0.7146
0	1	0.75	2.5	1	3	0.9096	0.5824	0.9410	0.5006	0.9123	0.5721
0	1	0.2	1.5	1	3	0.9231	0.6659	0.9654	0.5728	0.9477	0.6582

Table 5.13. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0514	0.0541	0.0509	0.0521	0.0485	0.0507
0	1	0.5	1	1	1	0.7272	0.9376	0.7112	0.9378	0.7382	0.9488
0	1	0.5	1	0.5	1	0.3170	0.5466	0.3402	0.5294	0.3534	0.5564
0	1	0	1	1	1	0.6116	0.8680	0.6674	0.9032	0.6358	0.8952
0	1	0.75	1	1	1	0.7598	0.9578	0.7412	0.9484	0.7652	0.9610
0	1	0.2	1	1	1	0.6730	0.9080	0.6856	0.9240	0.6852	0.9152

Table 5.14. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and equal variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0517	0.0536	0.0512	0.0516	0.0479	0.0499
0	1	0.5	1	1	1	0.7154	0.9256	0.7201	0.9472	0.7325	0.9354
0	1	0.5	1	0.5	1	0.3199	0.5433	0.3359	0.5326	0.3554	0.5520
0	1	0	1	1	1	0.6038	0.8545	0.6643	0.9120	0.6301	0.8856
0	1	0.75	1	1	1	0.7391	0.9474	0.7381	0.9527	0.7963	0.9571
0	1	0.2	1	1	1	0.6537	0.8988	0.6891	0.9388	0.6922	0.9089

Table 5.15. Percentage of Rejection for k=3 Populations; Normal Distribution with equal means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.7032	0.1616	0.6670	0.1606	0.7832	0.1248
0	1	0	2	0	2	0.4666	0.1100	0.4564	0.1168	0.5712	0.0836
0	1	0	1	0	2	0.3592	0.1122	0.3770	0.1218	0.4096	0.0942
0	1	0	2.5	0	3	0.7090	0.1572	0.6730	0.1612	0.8142	0.1356
0	1	0	1.5	0	3	0.6442	0.1684	0.6452	0.1604	0.7298	0.1216

Table 5.16. Percentage of Rejection for k=3 Populations; Normal Distribution with equal means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.6897	0.1576	0.6582	0.1459	0.6818	0.1076
0	1	0	2	0	2	0.4523	0.1015	0.4610	0.0644	0.4364	0.0657
0	1	0	1	0	2	0.3478	0.1096	0.3661	0.0653	0.2945	0.0746
0	1	0	2.5	0	3	0.6344	0.1549	0.6651	0.0877	0.6457	0.1154
0	1	0	1.5	0	3	0.6268	0.1523	0.6328	0.0893	0.6023	0.1079

Table 5.17. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9576	0.6748	0.9478	0.6716	0.9690	0.6092
0	1	0.5	2	0.5	2	0.7880	0.4730	0.7762	0.4522	0.8416	0.4308
0	1	0	1	1	2	0.8210	0.6738	0.8596	0.7090	0.8550	0.6396
0	1	0.75	2.5	1	3	0.9668	0.6840	0.9510	0.6616	0.9816	0.6550
0	1	0.2	1.5	1	3	0.9330	0.6254	0.9302	0.6142	0.9468	0.5528

Table 5.18. Percentage of Rejection for k=3 Populations; Normal Distribution with different means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9445	0.6540	0.9456	0.5539	0.9476	0.5538
0	1	0.5	2	0.5	2	0.7526	0.4399	0.7631	0.3570	0.8274	0.3874
0	1	0	1	1	2	0.8020	0.6584	0.8549	0.6059	0.8333	0.5791
0	1	0.75	2.5	1	3	0.9564	0.6731	0.9543	0.5427	0.9640	0.6114
0	1	0.2	1.5	1	3	0.9166	0.6088	0.9322	0.5046	0.9357	0.5010

5.1.2. Four Treatments

Tables 19-36 give the results of the simulation study for four treatments under the normal distribution. The sample sizes considered in the simulations are 18 and 30. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique. Tables 19, 20, 25, 26, 31 and 32 give estimated powers when the location parameters are different but scale parameters are equal. Tables 21, 22, 27, 28, 33 and 34 give estimated powers when the location parameters are equal but scale parameters are different. Tables 23, 24, 29, 30, 35 and 36 give estimated powers when the location and scale parameters are all different. It can be seen that all the proposed tests maintained their alpha values and they were around 0.05 (Tables 19, 20, 25, 26, 31 and 32). When the populations have unequal location parameters and equal scale parameters, the standardize last tests have higher estimated powers than all the standardize first tests (Tables 19, 20, 25, 26, 31 and 32). When the populations have equal location parameters and unequal scale parameters, the standardize first tests have higher estimated powers than all the standardize last tests (Tables 19, 20, 25, 26, 31 and 32). When the populations have unequal location parameters and unequal scale parameters, the standardize first tests tend to have the higher estimated powers (Tables 23, 24, 29, 30, 35 and 36). Tables 25-30 show the results of simulations with unequal sample sizes. In these tables, the sample sizes for the first and second population are 18 and the sample sizes for the third and fourth populations are 30. Tables 31-36 also show the results of simulations when the sample sizes are unequal. In these tables, the sample sizes for the first and second populations are 30 and the sample sizes for the third and fourth populations are 18. Results were found to be similar as to which test statistic had higher powers in the situations for both equal and unequal sample sizes.

When location parameters are equal and scale parameters are unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 21 and 22, for example). When location and scale parameters are both unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 23 and 24, for example). For the situations of unequal location parameters and equal scale parameters for both equal and unequal sample sizes, the SM_2 test has the highest estimated powers (Tables 19, 20, 25, 26, 31 and 32). For the situations of equal location parameters and unequal scale parameters, when the subgroup sample size is 6, the MJM_1 test has the highest estimated powers (Tables 22, 28 and 34). When the subgroup sample size is 3, the SM_1 test has the highest estimated powers (Tables 21, 27 and 33). Because when the subgroup sample size is 3, the tests have higher estimated powers and the SM_1 test have the highest estimated powers or close to the highest estimated powers in sub-samples of size 6, we recommend use the SM_1 test for this situation. For the situations of unequal location parameters and unequal scale parameters, the SM_1 test generally has the highest estimated powers (Tables 23, 24, 29, 35 and 36). The exception to this is when all but one of the scale parameters are equal, and the location parameters are all equal. In this case, the SM_1 and SM_2 tests tend to have lower estimated powers than the other tests (Tables 22 and 28).

Table 5.19. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0486	0.0513	0.0476	0.0501	0.0489	0.0462
0	1	0.3	1	0.6	1	0.9	1	0.7986	0.9668	0.7922	0.9636	0.8032	0.9698
0	1	0.5	1	0.5	1	0.5	1	0.3450	0.5388	0.3360	0.5356	0.3266	0.5654
0	1	0	1	0	1	0.5	1	0.3398	0.5414	0.3324	0.5198	0.3362	0.5250
0	1	0.5	1	0.6	1	1.1	1	0.8233	0.9738	0.8112	0.9894	0.8904	0.9906
0	1	0.2	1	0.7	1	0.9	1	0.7906	0.9568	0.8202	0.9796	0.8328	0.9846

Table 5.20. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0495	0.0543	0.0491	0.0507	0.0513	0.0527
0	1	0.3	1	0.6	1	0.9	1	0.8028	0.9748	0.7802	0.9698	0.8152	0.9606
0	1	0.5	1	0.5	1	0.5	1	0.3544	0.5422	0.3480	0.5254	0.3138	0.5516
0	1	0	1	0	1	0.5	1	0.3426	0.5516	0.3478	0.5250	0.3426	0.5222
0	1	0.5	1	0.6	1	1.1	1	0.8378	0.9731	0.8262	0.9906	0.9020	0.9946
0	1	0.2	1	0.7	1	0.9	1	0.8064	0.9588	0.8236	0.9746	0.8442	0.9792

Table 5.21. Percentage of Rejection for k=4 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.9388	0.2070	0.9130	0.1984	0.9506	0.0986
0	1	0	2	0	2	0	2	0.4594	0.0862	0.4502	0.0880	0.4850	0.0534
0	1	0	1	0	1	0	2	0.4768	0.1358	0.4522	0.1182	0.4920	0.0826
0	1	0	2.5	0	3	0	3.5	0.8624	0.1644	0.8480	0.1608	0.8968	0.0864
0	1	0	1.5	0	3	0	4.5	0.9650	0.2418	0.9480	0.2168	0.9732	0.1258

Table 5.22. Percentage of Rejection for k=4 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.8278	0.0838	0.8786	0.1176	0.8180	0.0732
0	1	0	2	0	2	0	2	0.4328	0.0494	0.4472	0.0492	0.3006	0.0472
0	1	0	1	0	1	0	2	0.4470	0.0936	0.4406	0.0706	0.3078	0.0644
0	1	0	2.5	0	3	0	3.5	0.7148	0.0744	0.7774	0.0858	0.7196	0.0624
0	1	0	1.5	0	3	0	4.5	0.8434	0.1102	0.9048	0.1332	0.8422	0.1000

Table 5.23. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8256	0.6806	0.8292	0.6768	0.8640	0.5744
0	1	0.5	2	0.5	2	0.5	2	0.7882	0.4508	0.7896	0.4346	0.8030	0.3316
0	1	0	1	0	1	0.5	2	0.7608	0.4612	0.7618	0.4466	0.7874	0.3662
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.5526	0.3312	0.5684	0.3288	0.5694	0.1940
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.5692	0.3436	0.5616	0.3306	0.5676	0.1902

Table 5.24. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.7940	0.6178	0.7830	0.5334	0.7970	0.5292
0	1	0.5	2	0.5	2	0.5	2	0.7888	0.3442	0.7552	0.3138	0.7166	0.2832
0	1	0	1	0	1	0.5	2	0.7610	0.3602	0.7286	0.3732	0.7776	0.3240
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.5234	0.2686	0.6310	0.2732	0.5298	0.1202
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.5282	0.2774	0.6308	0.2874	0.5264	0.1310

Table 5.25. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0472	0.0541	0.0486	0.0491	0.0478	0.0486
0	1	0.3	1	0.6	1	0.9	1	0.6888	0.9078	0.6932	0.9080	0.6834	0.9108
0	1	0.5	1	0.5	1	0.5	1	0.2288	0.3432	0.2618	0.3380	0.2278	0.3576
0	1	0	1	0	1	0.5	1	0.3520	0.5486	0.3150	0.5016	0.3372	0.5586
0	1	0.5	1	0.6	1	1.1	1	0.7356	0.9598	0.7738	0.9144	0.7886	0.9668
0	1	0.2	1	0.7	1	0.9	1	0.7008	0.9198	0.7244	0.9412	0.7228	0.9444

Table 5.26. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0483	0.0527	0.0486	0.0494	0.0476	0.0522
0	1	0.3	1	0.6	1	0.9	1	0.6986	0.9284	0.6842	0.9082	0.6973	0.9161
0	1	0.5	1	0.5	1	0.5	1	0.2358	0.3656	0.2546	0.3339	0.2292	0.3676
0	1	0	1	0	1	0.5	1	0.3551	0.5474	0.3012	0.5025	0.3422	0.5496
0	1	0.5	1	0.6	1	1.1	1	0.7321	0.9428	0.7643	0.9167	0.7608	0.9655
0	1	0.2	1	0.7	1	0.9	1	0.6956	0.9134	0.7134	0.9366	0.7480	0.9330

Table 5.27. Percentage of Rejection for k=4 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.8316	0.1482	0.8468	0.1528	0.8560	0.0612
0	1	0	2	0	2	0	2	0.2830	0.0764	0.3230	0.0752	0.2702	0.0400
0	1	0	1	0	1	0	2	0.4858	0.1396	0.4422	0.1240	0.5028	0.0774
0	1	0	2.5	0	3	0	3.5	0.6848	0.1224	0.7082	0.1208	0.7064	0.0536
0	1	0	1.5	0	3	0	4.5	0.9150	0.1956	0.9092	0.1838	0.9168	0.0764

Table 5.28. Percentage of Rejection for k=4 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.8231	0.1029	0.8012	0.0958	0.7948	0.0514
0	1	0	2	0	2	0	2	0.2759	0.4579	0.2845	0.0377	0.1629	0.0537
0	1	0	1	0	1	0	2	0.4710	0.0924	0.4017	0.0678	0.3208	0.0673
0	1	0	2.5	0	3	0	3.5	0.6753	0.0843	0.6650	0.0717	0.5342	0.0431
0	1	0	1.5	0	3	0	4.5	0.9034	0.1576	0.8694	0.1134	0.7894	0.0566

Table 5.29. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.7162	0.5724	0.7422	0.5834	0.7492	0.4606
0	1	0.5	2	0.5	2	0.5	2	0.5294	0.2706	0.5974	0.3292	0.5164	0.1952
0	1	0	1	0	1	0.5	2	0.7924	0.4804	0.7302	0.4320	0.7930	0.3506
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4236	0.1022	0.4520	0.1332	0.4644	0.0906
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4530	0.1706	0.4666	0.1838	0.4724	0.1640

Table 5.30. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.6586	0.5067	0.7049	0.4576	0.6968	0.4081
0	1	0.5	2	0.5	2	0.5	2	0.4863	0.2347	0.5734	0.2157	0.5037	0.1444
0	1	0	1	0	1	0.5	2	0.7717	0.4289	0.7236	0.3230	0.7896	0.3030
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.3865	0.1272	0.4446	0.0668	0.4134	0.0963
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4079	0.2148	0.4358	0.0766	0.4120	0.1184

Table 5.31. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0478	0.0539	0.0476	0.0483	0.0487	0.0502
0	1	0.3	1	0.6	1	0.9	1	0.6980	0.9046	0.6872	0.9090	0.6810	0.9186
0	1	0.5	1	0.5	1	0.5	1	0.3562	0.5638	0.3208	0.5150	0.3472	0.5560
0	1	0	1	0	1	0.5	1	0.2170	0.3490	0.2528	0.3956	0.2152	0.3426
0	1	0.5	1	0.6	1	1.1	1	0.7946	0.9435	0.7846	0.9600	0.7794	0.9624
0	1	0.2	1	0.7	1	0.9	1	0.7598	0.9127	0.7328	0.9422	0.7254	0.9402

Table 5.32. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0475	0.0496	0.0506	0.0484	0.0496	0.0507
0	1	0.3	1	0.6	1	0.9	1	0.7061	0.9245	0.6789	0.9084	0.7053	0.9252
0	1	0.5	1	0.5	1	0.5	1	0.3691	0.5864	0.3174	0.5182	0.3792	0.5716
0	1	0	1	0	1	0.5	1	0.2300	0.3500	0.2459	0.4044	0.2395	0.3330
0	1	0.5	1	0.6	1	1.1	1	0.8154	0.9587	0.7731	0.9643	0.8105	0.9585
0	1	0.2	1	0.7	1	0.9	1	0.7433	0.9172	0.7214	0.9570	0.7424	0.9339

Table 5.33. Percentage of Rejection for k=4 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.8842	0.1982	0.8576	0.1958	0.9204	0.1280
0	1	0	2	0	2	0	2	0.4878	0.0888	0.4514	0.0928	0.5368	0.0666
0	1	0	1	0	1	0	2	0.3158	0.1104	0.3542	0.1172	0.3360	0.0778
0	1	0	2.5	0	3	0	3.5	0.8404	0.1542	0.7718	0.1624	0.8802	0.1110
0	1	0	1.5	0	3	0	4.5	0.8958	0.2288	0.8668	0.2160	0.9344	0.1404

Table 5.34. Percentage of Rejection for k=4 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = 30$, $n_3 = n_4 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.7751	0.1586	0.8149	0.1211	0.7758	0.1108
0	1	0	2	0	2	0	2	0.3759	0.0537	0.4159	0.0404	0.3620	0.0487
0	1	0	1	0	1	0	2	0.2129	0.0743	0.3141	0.0607	0.2209	0.0582
0	1	0	2.5	0	3	0	3.5	0.7364	0.1146	0.7314	0.0889	0.7117	0.0908
0	1	0	1.5	0	3	0	4.5	0.7799	0.0784	0.8245	0.1449	0.8069	0.1267

Table 5.35. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = 30$, $n_3 = n_4 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.7368	0.5818	0.7298	0.5890	0.7380	0.5170
0	1	0.5	2	0.5	2	0.5	2	0.8078	0.4652	0.7592	0.4292	0.8446	0.3722
0	1	0	1	0	1	0.5	2	0.5276	0.3118	0.5902	0.3554	0.5702	0.2474
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4992	0.2164	0.4896	0.2002	0.4998	0.1732
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4528	0.1452	0.4750	0.1590	0.4648	0.0908

Table 5.36. Percentage of Rejection for k=4 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.7221	0.5142	0.7050	0.4713	0.7166	0.4616
0	1	0.5	2	0.5	2	0.5	2	0.8024	0.4033	0.7624	0.3340	0.8304	0.3288
0	1	0	1	0	1	0.5	2	0.5298	0.2576	0.5812	0.2523	0.5485	0.1869
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4700	0.2067	0.4522	0.0813	0.4822	0.1296
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4118	0.1440	0.4442	0.0949	0.4413	0.0790

5.1.3. Five Treatments

Tables 37-54 outline the results of the simulation study for five treatments under the normal distribution. The sample sizes considered in the simulations are 18 and 30. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique. Tables 37, 38, 43, 44, 49 and 50 give estimated powers when the location parameters are different but scale parameters are equal. Tables 39, 40, 45, 46, 51 and 52 give estimated powers when the location parameters are equal but scale parameters are different. Tables 41, 42, 47, 48, 53 and 54 give estimated powers when the location and scale parameters are all different. All the proposed tests maintained their alpha values and they were around 0.05 (Tables 37, 38, 43, 44, 49 and 50). When the populations have unequal location parameters and equal scale parameters, the standardize last tests have higher estimated powers than all the standardize first tests (Tables 37, 38, 43, 44, 49 and 50). When the populations have equal location parameters and unequal scale parameters, the standardize first tests have higher estimated powers than all the standardize last tests (Tables 39, 40, 45, 46, 51 and 52). When the populations have unequal location parameters and unequal scale parameters, the standardize first tests tend to have the higher estimated powers (Tables 41, 42, 47, 48, 53 and 54). Tables 43-48 show the results of simulations with unequal sample sizes. In these tables, the sample size for the first, second and third populations are 18 and the sample sizes for the fourth and fifth populations are 30. Tables 49-54 also show the results of simulations when the sample sizes are unequal. In these tables, the sample sizes for the first, second and third populations are 30 and the sample sizes for the fourth and fifth populations are 18. Results were found to be similar as to which test statistic had higher powers in the situations for both equal and unequal sample sizes.

When location parameters are equal and scale parameters are unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 39 and 40, for example). When location and scale parameters are both unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 41 and 42, for example). For the situations of unequal location parameters and equal scale parameters, the SM_2 test has the highest estimated powers (Tables 37, 38, 43, 44, 49 and 50). For the situations of equal location parameters and unequal scale parameters and when the subgroup sample size is 6, the JM_1 test has the highest estimated powers (Tables 40, 46 and 52). When the subgroup sample size is 3, the SM_1 test has the highest estimated powers (Tables 39, 45 and 51). Because when the subgroup sample size is 3, the tests have higher estimated powers and the SM_1 test have the highest estimated powers or close to the highest estimated powers in sub-samples of size 6, we recommend use the SM_1 test for this situation. For the situations of unequal location parameters and unequal scale parameters and when the subgroup sample size is 6, the JM_1 test has the highest estimated powers (Tables 42, 48 and 54). When the subgroup sample size is 3, the SM_1 test has the highest estimated powers (Tables 41, 47 and 53). Because when the subgroup sample size is 3, the tests have higher estimated powers and the SM_1 test have the highest estimated powers or close to the highest estimated powers in sub-samples of size 6, we recommend use the SM_1 test for this situation. The exception to this is when all but one of the location or scale parameters are equal, the SM_1 and SM_2 tests sometimes tend to have lower estimated powers than the other tests (Tables 40, 42, 46 and 48).

Table 5.37. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0464	0.0507	0.0481	0.0511	0.0490	0.0477
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9676	0.9998	0.9596	0.9984	0.9628	0.9994
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.3172	0.5044	0.3218	0.4972	0.2964	0.4710
0	1	0	1	0	1	0	1	0.5	1	0.3160	0.4960	0.3260	0.4872	0.2938	0.4716
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.3178	0.4994	0.3288	0.5180	0.3240	0.5214
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.3150	0.5046	0.3320	0.5122	0.3006	0.5158

Table 5.38. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0490	0.0527	0.0521	0.0504	0.0473	0.0497
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9682	0.9990	0.9674	0.9942	0.9848	0.9964
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.3218	0.5106	0.3196	0.5070	0.2836	0.4734
0	1	0	1	0	1	0	1	0.5	1	0.3234	0.5110	0.3138	0.4924	0.3002	0.4722
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.3258	0.5084	0.3102	0.4994	0.3256	0.5274
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.3306	0.5206	0.3176	0.4978	0.3120	0.5260

Table 5.39. Percentage of Rejection for k=5 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.9838	0.2614	0.9770	0.2382	0.9854	0.1062
0	1	0	2	0	2	0	2	0	2	0.4178	0.0876	0.4290	0.0948	0.4292	0.1064
0	1	0	1	0	1	0	1	0	2	0.4314	0.1272	0.4294	0.1242	0.4390	0.0780
0	1	0	2.5	0	3	0	3.5	0	5	0.9606	0.2332	0.9432	0.2234	0.9636	0.1030
0	1	0	1.5	0	3	0	4.5	0	5	0.9904	0.2838	0.9846	0.2540	0.9936	0.1134

Table 5.40. Percentage of Rejection for k=5 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.9802	0.1000	0.9308	0.1574	0.9528	0.0808
0	1	0	2	0	2	0	2	0	2	0.3970	0.0520	0.3756	0.0560	0.2448	0.0489
0	1	0	1	0	1	0	1	0	2	0.4184	0.0938	0.3994	0.0766	0.2548	0.0598
0	1	0	2.5	0	3	0	3.5	0	5	0.9456	0.0826	0.8894	0.1484	0.9064	0.0790
0	1	0	1.5	0	3	0	4.5	0	5	0.9820	0.1026	0.9374	0.1704	0.9326	0.0876

Table 5.41. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.9666	0.8758	0.9608	0.8812	0.9756	0.7930
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.7490	0.4112	0.7424	0.4194	0.7478	0.2866
0	1	0	1	0	1	0	1	0.5	2	0.7258	0.4234	0.7276	0.4164	0.7088	0.3216
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.7858	0.5644	0.7774	0.5378	0.7972	0.4018
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.8336	0.6080	0.8314	0.6004	0.8482	0.4608

Table 5.42. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.9414	0.8230	0.9790	0.7378	0.8192	0.7478
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.7394	0.3070	0.7216	0.3986	0.6352	0.2382
0	1	0	1	0	1	0	1	0.5	2	0.7164	0.3460	0.7198	0.3430	0.6148	0.2794
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.7458	0.4910	0.7766	0.3588	0.6888	0.3652
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.7846	0.5338	0.7686	0.4252	0.7596	0.4480

Table 5.43. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0476	0.0491	0.0468	0.0496	0.0507	0.0497
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.8638	0.9942	0.8814	0.9962	0.9046	0.9962
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.2120	0.2962	0.2378	0.3490	0.1902	0.3224
0	1	0	1	0	1	0	1	0.5	1	0.3190	0.5262	0.3006	0.4646	0.3050	0.5262
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.2654	0.4102	0.2654	0.4096	0.2700	0.4896
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.2826	0.4108	0.2814	0.4182	0.2822	0.4984

Table 5.44. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0491	0.0520	0.0499	0.0487	0.0463	0.0451
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.8834	0.9996	0.9092	0.9864	0.9185	0.9999
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.2360	0.3056	0.2356	0.3249	0.2016	0.3324
0	1	0	1	0	1	0	1	0.5	1	0.3257	0.5366	0.2984	0.4655	0.3300	0.5172
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.2888	0.4192	0.2468	0.3910	0.2822	0.4883
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.2782	0.4268	0.2670	0.4038	0.2774	0.4970

Table 5.45. Percentage of Rejection for k=5 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.8798	0.2082	0.8806	0.1862	0.9072	0.0698
0	1	0	2	0	2	0	2	0	2	0.2440	0.0654	0.2818	0.0720	0.2476	0.0456
0	1	0	1	0	1	0	1	0	2	0.4592	0.1316	0.4140	0.1192	0.4590	0.0666
0	1	0	2.5	0	3	0	3.5	0	5	0.8300	0.1862	0.8548	0.1784	0.8636	0.0686
0	1	0	1.5	0	3	0	4.5	0	5	0.9036	0.2130	0.9182	0.2060	0.9444	0.0702

Table 5.46. Percentage of Rejection for k=5 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.8664	0.0863	0.8610	0.0128	0.8660	0.0600
0	1	0	2	0	2	0	2	0	2	0.2310	0.0489	0.1942	0.0581	0.1403	0.0493
0	1	0	1	0	1	0	1	0	2	0.4328	0.0735	0.4090	0.1190	0.2770	0.0565
0	1	0	2.5	0	3	0	3.5	0	5	0.8157	0.1273	0.8098	0.0695	0.7214	0.0581
0	1	0	1.5	0	3	0	4.5	0	5	0.8958	0.1327	0.8972	0.0861	0.8170	0.0504

Table 5.47. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.8542	0.7750	0.8784	0.7786	0.8832	0.6454
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.4672	0.2384	0.5470	0.2886	0.4602	0.1804
0	1	0	1	0	1	0	1	0.5	2	0.7450	0.4568	0.7080	0.4198	0.7666	0.3378
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.7108	0.4336	0.7184	0.3606	0.7484	0.4360
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.7724	0.4752	0.7716	0.3124	0.8058	0.5576

Table 5.48. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.7745	0.7253	0.6866	0.6528	0.6971	0.5929
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.4577	0.1844	0.5262	0.1751	0.2370	0.1296
0	1	0	1	0	1	0	1	0.5	2	0.6650	0.4259	0.6702	0.3108	0.6708	0.2902
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.6508	0.3460	0.6792	0.2704	0.6362	0.3224
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.7214	0.4094	0.7344	0.2516	0.6806	0.4222

Table 5.49. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0494	0.0486	0.0503	0.0480	0.0517	0.0488
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.8586	0.9942	0.8962	0.9960	0.9094	0.9948
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.3428	0.5328	0.3014	0.4780	0.3236	0.5298
0	1	0	1	0	1	0	1	0.5	1	0.2120	0.3146	0.2372	0.3170	0.2178	0.3072
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.2222	0.4182	0.2734	0.4364	0.2612	0.4548
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.2626	0.4094	0.2668	0.4196	0.2604	0.4680

Table 5.50. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0487	0.0490	0.0511	0.0497	0.0504	0.0498
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.8576	0.9995	0.9072	0.9734	0.9237	0.9814
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.3651	0.5437	0.3214	0.4830	0.3256	0.5454
0	1	0	1	0	1	0	1	0.5	1	0.2219	0.3151	0.2156	0.3160	0.2221	0.2976
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.2218	0.4272	0.2548	0.4178	0.2623	0.4409
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.2550	0.4254	0.2524	0.4052	0.2574	0.4617

Table 5.51. Percentage of Rejection for k=5 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.9572	0.2398	0.9344	0.2368	0.9630	0.1360
0	1	0	2	0	2	0	2	0	2	0.4668	0.0954	0.4110	0.0996	0.5060	0.0606
0	1	0	1	0	1	0	1	0	2	0.2888	0.1078	0.3346	0.1360	0.2968	0.0752
0	1	0	2.5	0	3	0	3.5	0	5	0.9306	0.2122	0.8926	0.2084	0.9536	0.1140
0	1	0	1.5	0	3	0	4.5	0	5	0.9662	0.2564	0.9512	0.2530	0.9816	0.1494

Table 5.52. Percentage of Rejection for k=5 Populations; Normal Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.9531	0.0976	0.9168	0.0781	0.9184	0.1188
0	1	0	2	0	2	0	2	0	2	0.4379	0.0657	0.4026	0.0482	0.3312	0.0427
0	1	0	1	0	1	0	1	0	2	0.2633	0.0684	0.2668	0.0587	0.1817	0.0556
0	1	0	2.5	0	3	0	3.5	0	5	0.9157	0.0549	0.8998	0.0405	0.8851	0.0938
0	1	0	1.5	0	3	0	4.5	0	5	0.9436	0.0643	0.9344	0.0783	0.8941	0.1357

Table 5.53. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.9253	0.7846	0.9308	0.5923	0.9369	0.7146
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.8022	0.4411	0.7840	0.4136	0.8368	0.4454
0	1	0	1	0	1	0	1	0.5	2	0.4986	0.3097	0.4762	0.4293	0.5083	0.4319
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.7788	0.4955	0.7444	0.4535	0.7867	0.5520
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.7680	0.4602	0.7780	0.4287	0.7842	0.5112

Table 5.54. Percentage of Rejection for k=5 Populations; Normal Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.8852	0.5864	0.9284	0.5662	0.8726	0.6500
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.7844	0.4200	0.7212	0.4082	0.8048	0.3088
0	1	0	1	0	1	0	1	0.5	2	0.4824	0.2604	0.4590	0.3838	0.4840	0.1724
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.7188	0.4360	0.7052	0.4158	0.6856	0.3590
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.7170	0.4094	0.7408	0.4248	0.7072	0.3132

5.2. Results of the T Distribution

5.2.1. Three Treatments

Tables 55-72 present the results of the simulation study for three treatments under the T distribution. The sample sizes considered in the simulations are 18, and 30. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique. Tables 55, 56, 61, 62, 67 and 68 give estimated powers when the location parameters are different but scale parameters are equal. Tables 57, 58, 63, 64, 69 and 70 give estimated powers when the location parameters are equal but scale parameters are different. Tables 59, 60, 65, 66, 71 and 72 give estimated powers when the location and scale parameters are all different. It appears that all the proposed tests maintained their alpha values and they were around 0.05 (Tables 55, 56, 61, 62, 67 and 68). When the populations have unequal location parameters and equal scale parameters, the standardize last tests have higher estimated powers than all the standardize first tests (Tables 55, 56, 61, 62, 67 and 68). When the populations have equal location parameters and unequal scale parameters, the standardize first tests have higher estimated powers than all standardize last tests (Tables 57, 58, 63, 64, 69 and 70). When the populations have unequal location parameters and unequal scale parameters, the standardize first tests tend to have the higher estimated powers (Tables 59, 60, 65, 66, 71 and 72). Tables 61-66 show the results of simulations with unequal sample sizes. In these tables, the sample sizes for the first and second populations are 18 and the sample size for the third population is 30. Tables 67-72 also show the results of simulations when the sample sizes are unequal. In these tables, the sample size for the first population is 30 and the sample sizes for the second and third populations are 18. Results were found to be similar as to which test statistic had higher powers in the situations for both equal and unequal sample sizes.

When location parameters are equal and scale parameters are unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 57 and 58, for example). When location and scale parameters are both unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 59 and 60, for example). For the situations of unequal location parameters and equal scale parameters, the SM_2 test tends to have the highest estimated powers (Tables 55, 56, 61, 62, 67 and 68). For the situations of equal location parameters and unequal scale parameters, the SM_1 test generally has the highest estimated powers (Tables 57, 58, 63, 64 and 69). For the situations of unequal location parameters and unequal scale parameters and when the subgroup sample size is 6, the $MJ\mathrm{M}_1$ test generally has the highest estimated powers (Tables 60 and 66). When the subgroup sample size is 3, the SM_1 test has the highest estimated powers (Tables 59, 65 and 71). Because when the subgroup sample size is 3, the tests have higher estimated powers and the

SM_1 test have the highest estimated powers or close to the highest estimated powers in sub-samples of size 6, we recommend use the SM_1 test for this situation. The exception to this is when all but one of the location or scale parameters are equal, the SM_1 and SM_2 tests sometimes tend to have lower estimated powers than the other tests (Tables 63, 64 and 72).

Table 5.55. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0487	0.0496	0.0471	0.0480	0.0475	0.0488
0	1	0.5	1	1	1	0.9616	0.9996	0.9582	0.9986	0.9712	0.9994
0	1	0.5	1	0.5	1	0.5550	0.7970	0.5490	0.8048	0.5660	0.8162
0	1	0	1	1	1	0.9450	0.9988	0.9500	0.9992	0.9750	0.9994
0	1	0.75	1	1	1	0.9532	0.9990	0.9460	0.9954	0.9640	0.9992
0	1	0.2	1	1	1	0.9604	0.9986	0.9568	0.9916	0.9726	0.9998

Table 5.56. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0464	0.0483	0.0488	0.0449	0.0466	0.0478
0	1	0.5	1	1	1	0.9474	0.9960	0.9578	0.9958	0.9932	0.9964
0	1	0.5	1	0.5	1	0.5368	0.8112	0.5502	0.8346	0.5532	0.8186
0	1	0	1	1	1	0.9322	0.9972	0.9480	0.9944	0.9914	0.9900
0	1	0.75	1	1	1	0.9398	0.9933	0.9400	0.9906	0.9756	0.9952
0	1	0.2	1	1	1	0.9544	0.9991	0.9548	0.9946	0.9714	0.9999

Table 5.57. Percentage of Rejection for k=3 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.2944	0.1224	0.3140	0.0968	0.2998	0.1664
0	1	0	2	0	2	0.1700	0.1022	0.1832	0.1230	0.1794	0.1496
0	1	0	1	0	2	0.1886	0.0920	0.1836	0.1380	0.1756	0.1586
0	1	0	2.5	0	3	0.3034	0.0788	0.3180	0.1044	0.3202	0.1588
0	1	0	1.5	0	3	0.3058	0.0984	0.3058	0.1146	0.3224	0.1644

Table 5.58. Percentage of Rejection for k=3 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.2672	0.1142	0.2602	0.1160	0.2684	0.1410
0	1	0	2	0	2	0.1136	0.1010	0.1634	0.1042	0.1650	0.1334
0	1	0	1	0	2	0.1766	0.0872	0.1700	0.0740	0.1956	0.1404
0	1	0	2.5	0	3	0.1944	0.1034	0.2308	0.0894	0.2830	0.1348
0	1	0	1.5	0	3	0.2810	0.1116	0.2486	0.0910	0.2914	0.1386

Table 5.59. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9848	0.9890	0.9870	0.9894	0.9954	0.9820
0	1	0.5	2	0.5	2	0.7352	0.7538	0.7210	0.7584	0.7258	0.7146
0	1	0	1	1	2	0.9748	0.9950	0.9780	0.9942	0.9834	0.9754
0	1	0.75	2.5	1	3	0.9856	0.9902	0.9854	0.9892	0.9918	0.9852
0	1	0.2	1.5	1	3	0.9846	0.9848	0.9854	0.9858	0.9938	0.9780

Table 5.60. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9754	0.8562	0.9606	0.8460	0.9784	0.9368
0	1	0.5	2	0.5	2	0.6938	0.6288	0.7086	0.6376	0.6394	0.6262
0	1	0	1	1	2	0.9876	0.9280	0.9892	0.9208	0.9736	0.9532
0	1	0.75	2.5	1	3	0.9778	0.8490	0.9800	0.8336	0.9622	0.9114
0	1	0.2	1.5	1	3	0.9830	0.8600	0.9836	0.8426	0.9626	0.9188

Table 5.61. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0504	0.0518	0.0493	0.0483	0.0474	0.0480
0	1	0.5	1	1	1	0.8566	0.9632	0.8434	0.9582	0.8624	0.9646
0	1	0.5	1	0.5	1	0.4254	0.7315	0.4606	0.6802	0.4684	0.6454
0	1	0	1	1	1	0.8231	0.9626	0.8852	0.9880	0.9620	0.9900
0	1	0.75	1	1	1	0.7324	0.9220	0.8034	0.9432	0.8294	0.9340
0	1	0.2	1	1	1	0.8546	0.9437	0.8828	0.9782	0.9346	0.9822

Table 5.62. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0523	0.0549	0.0513	0.0493	0.0496	0.0488
0	1	0.5	1	1	1	0.8522	0.9608	0.8471	0.9484	0.8463	0.9699
0	1	0.5	1	0.5	1	0.4167	0.7208	0.4577	0.6561	0.4998	0.6554
0	1	0	1	1	1	0.8138	0.9577	0.8833	0.9789	0.9870	0.9810
0	1	0.75	1	1	1	0.7399	0.9127	0.7931	0.9155	0.8416	0.9327
0	1	0.2	1	1	1	0.8398	0.9377	0.8886	0.9736	0.9598	0.9808

Table 5.63. Percentage of Rejection for k=3 Populations; T Distribution with equal means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.1638	0.0753	0.1930	0.0566	0.2190	0.1502
0	1	0	2	0	2	0.0935	0.0579	0.0459	0.0576	0.0980	0.0538
0	1	0	1	0	2	0.1349	0.0689	0.1222	0.0572	0.0976	0.0582
0	1	0	2.5	0	3	0.1300	0.0637	0.1646	0.0584	0.1930	0.1422
0	1	0	1.5	0	3	0.2170	0.0710	0.2074	0.0850	0.2462	0.1446

Table 5.64. Percentage of Rejection for k=3 Populations; T Distribution with equal means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.1520	0.0519	0.1525	0.0504	0.1622	0.1204
0	1	0	2	0	2	0.0742	0.0550	0.1523	0.0501	0.0693	0.0575
0	1	0	1	0	2	0.1142	0.0564	0.1288	0.0510	0.0744	0.0581
0	1	0	2.5	0	3	0.1156	0.0573	0.1529	0.0507	0.1592	0.1317
0	1	0	1.5	0	3	0.2061	0.0653	0.2010	0.0554	0.2088	0.1148

Table 5.65. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9531	0.8422	0.9576	0.8896	0.9582	0.8254
0	1	0.5	2	0.5	2	0.6291	0.6124	0.5494	0.6242	0.6830	0.5768
0	1	0	1	1	2	0.9433	0.9132	0.9590	0.9544	0.9708	0.9586
0	1	0.75	2.5	1	3	0.9471	0.8510	0.9352	0.8598	0.9618	0.9668
0	1	0.2	1.5	1	3	0.9636	0.8324	0.9650	0.9136	0.9876	0.9753

Table 5.66. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9424	0.8109	0.9485	0.7638	0.9076	0.7729
0	1	0.5	2	0.5	2	0.6183	0.5870	0.5455	0.5107	0.5529	0.5260
0	1	0	1	1	2	0.9285	0.8843	0.9427	0.8454	0.9095	0.9010
0	1	0.75	2.5	1	3	0.9355	0.8236	0.9004	0.7434	0.9239	0.8725
0	1	0.2	1.5	1	3	0.9415	0.8155	0.9576	0.8064	0.9531	0.8843

Table 5.67. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0510	0.0494	0.0483	0.0501	0.0489	0.0477
0	1	0.5	1	1	1	0.8333	0.9533	0.8458	0.9608	0.8776	0.9666
0	1	0.5	1	0.5	1	0.4973	0.7135	0.5136	0.7608	0.5584	0.7848
0	1	0	1	1	1	0.7532	0.9720	0.7916	0.9276	0.7688	0.9110
0	1	0.75	1	1	1	0.8344	0.9432	0.8704	0.9698	0.9032	0.9790
0	1	0.2	1	1	1	0.7624	0.9611	0.8284	0.9456	0.8296	0.9738

Table 5.68. Percentage of Rejection for k=3 Populations; T Distribution with different means and equal variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0501	0.0498	0.0473	0.0510	0.0479	0.0486
0	1	0.5	1	1	1	0.8215	0.9413	0.8547	0.9702	0.8819	0.9532
0	1	0.5	1	0.5	1	0.4702	0.6802	0.5093	0.7640	0.5904	0.8004
0	1	0	1	1	1	0.7454	0.9585	0.7885	0.9364	0.7631	0.9014
0	1	0.75	1	1	1	0.8137	0.9328	0.8673	0.9741	0.9343	0.9751
0	1	0.2	1	1	1	0.7431	0.9519	0.8319	0.9604	0.8466	0.9765

Table 5.69. Percentage of Rejection for k=3 Populations; T Distribution with equal means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.2306	0.0963	0.1966	0.1970	0.2354	0.0950
0	1	0	2	0	2	0.0870	0.0754	0.1799	0.0880	0.1962	0.0798
0	1	0	1	0	2	0.1056	0.0964	0.1448	0.0704	0.1488	0.0850
0	1	0	2.5	0	3	0.2539	0.0746	0.2178	0.0952	0.2894	0.1164
0	1	0	1.5	0	3	0.1459	0.0976	0.1650	0.0822	0.2142	0.0842

Table 5.70. Percentage of Rejection for k=3 Populations; T Distribution with equal means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.2221	0.0923	0.1878	0.1878	0.2092	0.0778
0	1	0	2	0	2	0.0727	0.0669	0.1645	0.0756	0.1214	0.0619
0	1	0	1	0	2	0.0942	0.0938	0.1239	0.0639	0.0663	0.0654
0	1	0	2.5	0	3	0.1793	0.0723	0.2099	0.0717	0.1509	0.0962
0	1	0	1.5	0	3	0.1285	0.0815	0.1526	0.0711	0.1067	0.0705

Table 5.71. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9573	0.8750	0.9502	0.9034	0.9642	0.9264
0	1	0.5	2	0.5	2	0.7623	0.6459	0.6568	0.7240	0.7066	0.7012
0	1	0	1	1	2	0.8156	0.9437	0.8764	0.8390	0.8760	0.8360
0	1	0.75	2.5	1	3	0.9694	0.8860	0.9504	0.9044	0.9732	0.9436
0	1	0.2	1.5	1	3	0.9272	0.8671	0.9324	0.8478	0.9422	0.9089

Table 5.72. Percentage of Rejection for k=3 Populations; T Distribution with different means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.9542	0.8542	0.9380	0.7857	0.9428	0.9010
0	1	0.5	2	0.5	2	0.7269	0.6128	0.6437	0.6288	0.6924	0.6878
0	1	0	1	1	2	0.7966	0.9283	0.8317	0.7359	0.7543	0.7355
0	1	0.75	2.5	1	3	0.9590	0.8751	0.9237	0.7855	0.9556	0.9263
0	1	0.2	1.5	1	3	0.9108	0.8505	0.9044	0.7382	0.9187	0.8971

5.2.2. Four Treatments

Tables 73-90 give the results of the simulation study for four treatments under the T distribution. The sample sizes considered in the simulations are 18 and 30. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique. Tables 73, 74, 79, 80, 85 and 86 give estimated powers when the location parameters are different but scale parameters are equal. Tables 75, 76, 81, 82, 87 and 88 give estimated powers when the location parameters are equal but scale parameters are different. Tables 77, 78, 83, 84, 89 and 90 give estimated powers when the location and scale parameters are all different. All the proposed tests maintained their alpha values and they were around 0.05 (Tables 73, 74, 79, 80, 85 and 86). When the populations have unequal location parameters and equal scale parameters, the standardize last tests have higher estimated powers than all the standardize first tests (Tables 73, 74, 79, 80, 85 and 86). When the populations have equal location parameters and unequal scale parameters, the standardize first tests have higher estimated powers than all the standardize last tests (Tables 75, 76, 81, 82, 87 and 88). When the populations have unequal location parameters and unequal scale parameters, the standardize first tests tend to have the higher estimated powers (Tables 77, 78, 83, 84, 89 and 90). Tables 79-84 show the results of simulations with unequal sample sizes. In these tables, the sample sizes for the first and second populations are 18 and the sample sizes for the third and fourth populations are 30. Tables 85-90 also show the results of simulations when the sample sizes are unequal. In these tables, the sample sizes for the first and second populations are 30 and the sample sizes for the third and fourth populations are 18. Results were found to be similar as to which test statistic had higher powers in the situations for both equal and unequal sample sizes.

When location parameters are equal and scale parameters are unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 75 and 76, for example). When location and scale parameters are both unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 77 and 78, for example). For the situations of unequal location parameters and equal scale parameters, the SM_2 test tends to have the highest estimated powers (Tables 73, 74, 79, 80, 85 and 86). For the situations of equal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 75, 76, 81, 82, 87 and 88). For the situations of unequal location parameters and unequal scale parameters and when the subgroup sample size is 6, the $M\mathrm{JM}_1$ test generally has the highest estimated powers (Tables 78 and 90. When the subgroup sample size is 3, the SM_1 test has the highest estimated powers (Tables 77, 83 and 89). Because when the subgroup sample size is 3, the tests have higher estimated powers and the SM_1 test have the highest estimated powers or close to the highest estimated powers in sub-samples of size

6, we recommend use the SM_1 test for this situation. The exception to this is when all but one of the location or scale parameters are equal, the SM_1 and SM_2 tests sometimes tend to have lower estimated powers than the other tests (Tables 76 and 82).

Table 5.73. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0507	0.0503	0.0486	0.0481	0.0493	0.0470
0	1	0.3	1	0.6	1	0.9	1	0.9823	0.9733	0.9268	0.9866	0.9326	0.9914
0	1	0.5	1	0.5	1	0.5	1	0.5346	0.7631	0.5094	0.7670	0.5316	0.7576
0	1	0	1	0	1	0.5	1	0.4216	0.6731	0.4566	0.5442	0.4492	0.7374
0	1	0.5	1	0.6	1	1.1	1	0.9856	0.9999	0.9404	0.9999	0.9999	0.9999
0	1	0.2	1	0.7	1	0.9	1	0.9999	0.9999	0.9630	0.9999	0.9772	0.9999

Table 5.74. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0536	0.0521	0.0494	0.0501	0.0513	0.0487
0	1	0.3	1	0.6	1	0.9	1	0.9865	0.9813	0.9148	0.9928	0.9546	0.9884
0	1	0.5	1	0.5	1	0.5	1	0.5440	0.7765	0.5214	0.7968	0.5188	0.7600
0	1	0	1	0	1	0.5	1	0.4244	0.6833	0.4720	0.5494	0.4756	0.7380
0	1	0.5	1	0.6	1	1.1	1	0.9860	0.9999	0.9954	0.9999	0.9999	0.9956
0	1	0.2	1	0.7	1	0.9	1	0.9598	0.9999	0.9664	0.9999	0.9999	0.9978

Table 5.75. Percentage of Rejection for k=4 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.3569	0.0976	0.4426	0.1348	0.4728	0.0688
0	1	0	2	0	2	0	2	0.2150	0.0734	0.1458	0.0592	0.2140	0.0438
0	1	0	1	0	1	0	2	0.2341	0.0967	0.1300	0.0668	0.2412	0.0534
0	1	0	2.5	0	3	0	3.5	0.3580	0.0733	0.3928	0.0948	0.4020	0.0572
0	1	0	1.5	0	3	0	4.5	0.3699	0.1033	0.4678	0.1386	0.4976	0.0884

Table 5.76. Percentage of Rejection for k=4 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.3459	0.0628	0.4082	0.0540	0.4102	0.0934
0	1	0	2	0	2	0	2	0.1884	0.0596	0.1428	0.0504	0.1304	0.0876
0	1	0	1	0	1	0	2	0.2043	0.0730	0.1184	0.0642	0.1430	0.0852
0	1	0	2.5	0	3	0	3.5	0.3104	0.0625	0.3222	0.0698	0.3548	0.0832
0	1	0	1.5	0	3	0	4.5	0.3483	0.0827	0.4246	0.0850	0.4266	0.0626

Table 5.77. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8092	0.7234	0.9086	0.8316	0.9912	0.9216
0	1	0.5	2	0.5	2	0.5	2	0.6680	0.5021	0.7064	0.6702	0.8566	0.6620
0	1	0	1	0	1	0.5	2	0.8084	0.5112	0.7566	0.6586	0.7440	0.5626
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.6238	0.5576	0.7208	0.5678	0.6530	0.5856
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.6280	0.5692	0.7616	0.5638	0.6690	0.5856

Table 5.78. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.7912	0.7053	0.8854	0.7652	0.8764	0.7922
0	1	0.5	2	0.5	2	0.5	2	0.5440	0.4826	0.6858	0.5856	0.6136	0.5816
0	1	0	1	0	1	0.5	2	0.7244	0.4637	0.7054	0.5032	0.5204	0.4986
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.5410	0.4950	0.6304	0.4834	0.5814	0.5118
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.5470	0.5030	0.6330	0.4308	0.5918	0.5264

Table 5.79. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0484	0.0523	0.0490	0.0487	0.0484	0.0499
0	1	0.3	1	0.6	1	0.9	1	0.8329	0.9286	0.8278	0.9410	0.8228	0.9653
0	1	0.5	1	0.5	1	0.5	1	0.4010	0.4560	0.4352	0.4194	0.4328	0.4538
0	1	0	1	0	1	0.5	1	0.3976	0.4544	0.4392	0.4260	0.4702	0.4532
0	1	0.5	1	0.6	1	1.1	1	0.7136	0.9848	0.9030	0.9858	0.9266	0.9999
0	1	0.2	1	0.7	1	0.9	1	0.8644	0.9530	0.8672	0.9628	0.8672	0.9999

Table 5.80. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0496	0.0514	0.0494	0.0494	0.0478	0.0529
0	1	0.3	1	0.6	1	0.9	1	0.8329	0.9153	0.8588	0.9312	0.8567	0.9339
0	1	0.5	1	0.5	1	0.5	1	0.4010	0.4538	0.4280	0.4653	0.4242	0.4660
0	1	0	1	0	1	0.5	1	0.3976	0.4532	0.4254	0.4569	0.4552	0.4554
0	1	0.5	1	0.6	1	1.1	1	0.7083	0.9678	0.8935	0.9681	0.9088	0.9835
0	1	0.2	1	0.7	1	0.9	1	0.8594	0.9367	0.8562	0.9582	0.8924	0.9916

Table 5.81. Percentage of Rejection for k=4 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.3290	0.0943	0.3764	0.0892	0.4882	0.0814
0	1	0	2	0	2	0	2	0.1143	0.0711	0.1365	0.0764	0.1480	0.1436
0	1	0	1	0	1	0	2	0.1569	0.0953	0.1200	0.0726	0.1220	0.1182
0	1	0	2.5	0	3	0	3.5	0.2670	0.0720	0.2930	0.0548	0.4516	0.0844
0	1	0	1.5	0	3	0	4.5	0.2866	0.0964	0.4290	0.1056	0.4412	0.0890

Table 5.82. Percentage of Rejection for k=4 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.3263	0.0921	0.3308	0.0822	0.3370	0.0756
0	1	0	2	0	2	0	2	0.1057	0.0685	0.1220	0.0689	0.1093	0.1073
0	1	0	1	0	1	0	2	0.1428	0.0943	0.1095	0.0664	0.0814	0.1081
0	1	0	2.5	0	3	0	3.5	0.2534	0.0710	0.2798	0.0557	0.2894	0.0839
0	1	0	1.5	0	3	0	4.5	0.2743	0.0933	0.2692	0.0852	0.2738	0.0792

Table 5.83. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.6844	0.6432	0.8152	0.7446	0.8473	0.8078
0	1	0.5	2	0.5	2	0.5	2	0.5814	0.3694	0.6010	0.4780	0.6030	0.5256
0	1	0	1	0	1	0.5	2	0.5140	0.5377	0.5620	0.4470	0.6861	0.5470
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.2934	0.5103	0.3684	0.5740	0.5404	0.5381
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.2743	0.5104	0.3644	0.5380	0.5996	0.5033

Table 5.84. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.6473	0.6075	0.7673	0.6894	0.7653	0.6538
0	1	0.5	2	0.5	2	0.5	2	0.4030	0.3243	0.4940	0.4575	0.4748	0.3513
0	1	0	1	0	1	0.5	2	0.3861	0.4964	0.4704	0.4130	0.4927	0.4994
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.2560	0.3434	0.3278	0.4514	0.4395	0.3822
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.2178	0.3202	0.3104	0.4688	0.4514	0.3594

Table 5.85. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0498	0.0479	0.0496	0.0493	0.0507	0.0516
0	1	0.3	1	0.6	1	0.9	1	0.9075	0.9264	0.8218	0.9420	0.8204	0.9721
0	1	0.5	1	0.5	1	0.5	1	0.4116	0.7844	0.4942	0.7464	0.5522	0.7950
0	1	0	1	0	1	0.5	1	0.4633	0.3584	0.4770	0.5200	0.4482	0.5312
0	1	0.5	1	0.6	1	1.1	1	0.9634	0.9804	0.9138	0.9814	0.9974	0.9999
0	1	0.2	1	0.7	1	0.9	1	0.8754	0.9588	0.8756	0.9638	0.8698	0.9999

Table 5.86. Percentage of Rejection for k=4 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0518	0.0496	0.0488	0.0513	0.0520	0.0526
0	1	0.3	1	0.6	1	0.9	1	0.9056	0.9120	0.8135	0.9514	0.8447	0.9730
0	1	0.5	1	0.5	1	0.5	1	0.4245	0.8176	0.4908	0.7496	0.5842	0.8000
0	1	0	1	0	1	0.5	1	0.4763	0.5322	0.4701	0.5288	0.4725	0.5488
0	1	0.5	1	0.6	1	1.1	1	0.9742	0.9808	0.9023	0.9857	0.9785	0.9865
0	1	0.2	1	0.7	1	0.9	1	0.9089	0.9452	0.8642	0.9786	0.8868	0.9825

Table 5.87. Percentage of Rejection for k=4 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = 30$, $n_3 = n_4 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.3276	0.0943	0.3872	0.1322	0.4033	0.0982
0	1	0	2	0	2	0	2	0.1029	0.0756	0.1649	0.0640	0.1618	0.0628
0	1	0	1	0	1	0	2	0.2088	0.0983	0.2320	0.0658	0.2448	0.0686
0	1	0	2.5	0	3	0	3.5	0.2785	0.0729	0.3166	0.0964	0.3554	0.0918
0	1	0	1.5	0	3	0	4.5	0.2754	0.0986	0.3566	0.1378	0.4188	0.1030

Table 5.88. Percentage of Rejection for k=4 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = 30$, $n_3 = n_4 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.1942	0.0547	0.2294	0.0575	0.3280	0.0810
0	1	0	2	0	2	0	2	0.0910	0.0574	0.0919	0.0516	0.0870	0.0649
0	1	0	1	0	1	0	2	0.1859	0.0622	0.1762	0.0593	0.1703	0.0690
0	1	0	2.5	0	3	0	3.5	0.2745	0.0533	0.2743	0.0529	0.2869	0.0716
0	1	0	1.5	0	3	0	4.5	0.2595	0.0518	0.2894	0.0667	0.2913	0.0893

Table 5.89. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8326	0.5818	0.8208	0.7322	0.8642	0.7332
0	1	0.5	2	0.5	2	0.5	2	0.6896	0.5794	0.7010	0.6398	0.7026	0.6996
0	1	0	1	0	1	0.5	2	0.6023	0.4869	0.6554	0.6070	0.6438	0.5912
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.5018	0.4184	0.5156	0.4890	0.5648	0.4914
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4470	0.3869	0.5464	0.4772	0.5662	0.4602

Table 5.90. Percentage of Rejection for k=4 Populations; T Distribution with different means and different variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.7379	0.5142	0.7474	0.7031	0.7118	0.7088
0	1	0.5	2	0.5	2	0.5	2	0.6542	0.5175	0.6430	0.6058	0.6354	0.6392
0	1	0	1	0	1	0.5	2	0.5045	0.4327	0.6180	0.5823	0.5695	0.4833
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4726	0.3887	0.4782	0.3516	0.4738	0.3212
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4060	0.2857	0.5156	0.3464	0.5267	0.3944

5.2.3. Five Treatments

Tables 91-108 give the results of the simulation study for five treatments under the T distribution. The sample sizes considered in the simulations are 18 and 30. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique. Tables 91, 92, 97, 98, 103 and 104 give estimated powers when the location parameters are different but scale parameters are equal. Tables 93, 94, 99, 100, 105 and 106 give estimated powers when the location parameters are equal but scale parameters are different. Tables 95, 96, 101, 102, 107 and 108 give estimated powers when the location and scale parameters are all different. All the proposed tests maintained their alpha values and they were around 0.05 (Tables 91, 92, 97, 98, 103 and 104). When the populations have unequal location parameters and equal scale parameters, the standardize last tests have higher estimated powers than all the standardize first tests (Tables 91, 92, 97, 98, 103 and 104). When the populations have equal location parameters and unequal scale parameters, the standardize first tests have higher estimated powers than all the standardize last tests (Tables 93, 94, 99, 100, 105 and 106). When the populations have unequal location parameters and unequal scale parameters, the standardize first tests tend to have the higher estimated powers (Tables 95, 96, 101, 102, 107 and 108). Tables 97-102 show the results of simulations with unequal sample sizes. In these tables, the sample size for the first, second and third populations are 18 and the sample sizes for the fourth and fifth populations are 30. Tables 103-108 also show the results of simulations when the sample sizes are unequal. In these tables, the sample sizes for the first, second and third populations are 30 and the sample sizes for the fourth and fifth populations are 18. Results were found to be similar as to which test statistic had higher powers in the situations for both equal and unequal sample sizes.

When location parameters are equal and scale parameters are unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 93 and 94, for example). When location and scale parameters are both unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 95 and 96, for example). For the situations of unequal location parameters and equal scale parameters, the SM_2 test tends to have the highest estimated powers (Tables 91, 92, 97, 98, 103 and 104). For the situations of equal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 93, 94, 99, 100, 105 and 106). For the situations of unequal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 95, 96, 101, 102, 107 and 108). The exception to this is when all but one of the scale parameters are equal and the location parameters are all equal. In this case, the SM_1 test sometimes tends to have lower estimated power than the other tests (Table 94).

Table 5.91. Percentage of Rejection for k=5 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0484	0.04963	0.0471	0.0504	0.0497	0.0470
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9946	0.9999	0.9999	0.9999	0.9999	0.9999
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5413	0.5546	0.4952	0.5286	0.5014	0.6994
0	1	0	1	0	1	0	1	0.5	1	0.4125	0.5437	0.4502	0.5116	0.4268	0.5874
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.4901	0.5074	0.4282	0.5174	0.4320	0.5502
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.5243	0.5202	0.4342	0.5144	0.4450	0.5912

Table 5.92. Percentage of Rejection for k=5 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0497	0.0507	0.0489	0.0494	0.0493	0.0481
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9952	0.9991	0.9999	0.9999	0.9999	0.9999
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5459	0.5608	0.4930	0.5584	0.4886	0.7018
0	1	0	1	0	1	0	1	0.5	1	0.4199	0.5587	0.4380	0.5168	0.4532	0.5880
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.4981	0.5164	0.4096	0.4988	0.4636	0.5562
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.5399	0.5362	0.4198	0.5000	0.4864	0.5914

Table 5.93. Percentage of Rejection for k=5 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.3679	0.2964	0.4066	0.3746	0.4376	0.3764
0	1	0	2	0	2	0	2	0	2	0.2433	0.1294	0.2246	0.1660	0.2582	0.1468
0	1	0	1	0	1	0	1	0	2	0.2146	0.1677	0.2072	0.1728	0.2582	0.1488
0	1	0	2.5	0	3	0	3.5	0	5	0.3648	0.1266	0.3880	0.1574	0.4388	0.1738
0	1	0	1.5	0	3	0	4.5	0	5	0.3758	0.1456	0.4044	0.1758	0.4780	0.1760

Table 5.94. Percentage of Rejection for k=5 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.3043	0.1350	0.3604	0.2938	0.4050	0.2510
0	1	0	2	0	2	0	2	0	2	0.1225	0.0938	0.1712	0.0772	0.1262	0.0493
0	1	0	1	0	1	0	1	0	2	0.1016	0.1343	0.1772	0.0752	0.1260	0.0506
0	1	0	2.5	0	3	0	3.5	0	5	0.3448	0.1260	0.3042	0.1424	0.4016	0.1498
0	1	0	1.5	0	3	0	4.5	0	5	0.3674	0.1247	0.3472	0.1322	0.4370	0.1502

Table 5.95. Percentage of Rejection for k=5 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.9989	0.9932	0.9999	0.9632	0.9999	0.9508
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.6146	0.5341	0.6230	0.6912	0.6328	0.6170
0	1	0	1	0	1	0	1	0.5	2	0.6027	0.5680	0.7444	0.5464	0.7298	0.5180
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.5862	0.2908	0.5984	0.3588	0.6488	0.3934
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.5334	0.2336	0.6562	0.4252	0.6636	0.4562

Table 5.96. Percentage of Rejection for k=5 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.9686	0.9404	0.9714	0.8565	0.9728	0.9247
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.5050	0.4299	0.6022	0.5704	0.6000	0.5686
0	1	0	1	0	1	0	1	0.5	2	0.5933	0.4906	0.6366	0.4730	0.6562	0.4758
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.5462	0.2174	0.5376	0.2798	0.5804	0.3568
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.4844	0.2094	0.5534	0.3500	0.5550	0.4434

Table 5.97. Percentage of Rejection for k=5 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0493	0.0482	0.0490	0.0476	0.0517	0.0487
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9855	0.9968	0.9999	0.9999	0.9999	0.9999
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5170	0.3465	0.3636	0.5538	0.4952	0.5508
0	1	0	1	0	1	0	1	0.5	1	0.4237	0.5793	0.4292	0.5506	0.4380	0.5420
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.2134	0.3736	0.2648	0.4090	0.3780	0.5562
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.2262	0.4064	0.2836	0.4204	0.3966	0.6082

Table 5.98. Percentage of Rejection for k=5 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 18, n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0513	0.0496	0.0494	0.0485	0.0501	0.0497
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9951	0.9990	0.9999	0.9999	0.9999	0.9999
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5110	0.3559	0.3614	0.5497	0.5266	0.5608
0	1	0	1	0	1	0	1	0.5	1	0.4304	0.53897	0.4170	0.5515	0.4630	0.5330
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.2768	0.3826	0.2462	0.3904	0.3902	0.5549
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.2218	0.4224	0.2692	0.4060	0.4218	0.6068

Table 5.99. Percentage of Rejection for k=5 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 18, n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.3251	0.1314	0.4568	0.1462	0.4594	0.1400
0	1	0	2	0	2	0	2	0	2	0.2116	0.1024	0.2496	0.0568	0.2474	0.0918
0	1	0	1	0	1	0	1	0	2	0.1872	0.1769	0.2368	0.0452	0.2782	0.0974
0	1	0	2.5	0	3	0	3.5	0	5	0.3249	0.1210	0.4384	0.1426	0.4388	0.1494
0	1	0	1.5	0	3	0	4.5	0	5	0.3441	0.1258	0.4042	0.1480	0.4288	0.1428

Table 5.100. Percentage of Rejection for k=5 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.3117	0.1095	0.3906	0.1108	0.4182	0.1302
0	1	0	2	0	2	0	2	0	2	0.1986	0.0859	0.2077	0.0493	0.2099	0.0855
0	1	0	1	0	1	0	1	0	2	0.1608	0.1188	0.2068	0.0676	0.2038	0.0973
0	1	0	2.5	0	3	0	3.5	0	5	0.3106	0.1021	0.3546	0.1035	0.3966	0.0889
0	1	0	1.5	0	3	0	4.5	0	5	0.3263	0.1155	0.3570	0.1239	0.3614	0.0730

Table 5.101. Percentage of Rejection for k=5 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.9150	0.8126	0.9870	0.8556	0.9926	0.8784
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.3796	0.3074	0.4522	0.3108	0.5108	0.4652
0	1	0	1	0	1	0	1	0.5	2	0.7568	0.4524	0.7834	0.4678	0.7876	0.5342
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.5706	0.3667	0.5894	0.2706	0.6278	0.3140
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.5886	0.3198	0.5952	0.3124	0.6760	0.4176

Table 5.102. Percentage of Rejection for k=5 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.9029	0.8053	0.9038	0.8241	0.9123	0.8401
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.3579	0.2856	0.3200	0.2987	0.4566	0.3200
0	1	0	1	0	1	0	1	0.5	2	0.6124	0.4259	0.6756	0.3588	0.7126	0.4866
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.5267	0.3248	0.5286	0.2508	0.5871	0.2976
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.5508	0.2739	0.5324	0.2732	0.6012	0.3530

Table 5.103. Percentage of Rejection for k=5 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0474	0.0476	0.0493	0.0486	0.0508	0.0481
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9786	0.9981	0.9999	0.9999	0.9999	0.9999
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.4362	0.5891	0.4970	0.7612	0.5286	0.7582
0	1	0	1	0	1	0	1	0.5	1	0.4362	0.4671	0.3220	0.3316	0.3308	0.3230
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.4410	0.3578	0.4728	0.4358	0.4692	0.5522
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.4282	0.4418	0.4690	0.4218	0.3848	0.5996

Table 5.104. Percentage of Rejection for k=5 Populations; T Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0485	0.0492	0.0487	0.0476	0.0498	0.0481
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9676	0.9768	0.9739	0.9875	0.9764	0.9865
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.4585	0.6000	0.4948	0.7644	0.5206	0.7738
0	1	0	1	0	1	0	1	0.5	1	0.4461	0.4976	0.3098	0.3404	0.3551	0.3134
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.4806	0.3668	0.4542	0.4172	0.4803	0.5483
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.4606	0.4578	0.4546	0.4074	0.4018	0.5933

Table 5.105. Percentage of Rejection for k=5 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.3576	0.1679	0.3926	0.1724	0.4152	0.3062
0	1	0	2	0	2	0	2	0	2	0.2453	0.1456	0.2195	0.1418	0.2310	0.1568
0	1	0	1	0	1	0	1	0	2	0.1457	0.1423	0.1479	0.1238	0.1464	0.1460
0	1	0	2.5	0	3	0	3.5	0	5	0.3298	0.1594	0.3984	0.2480	0.4288	0.2948
0	1	0	1.5	0	3	0	4.5	0	5	0.3647	0.1964	0.4014	0.2712	0.4660	0.3120

Table 5.106. Percentage of Rejection for k=5 Populations; T Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.3535	0.1257	0.3464	0.1177	0.3706	0.2890
0	1	0	2	0	2	0	2	0	2	0.2164	0.1159	0.1661	0.1294	0.2062	0.1389
0	1	0	1	0	1	0	1	0	2	0.1202	0.1191	0.1179	0.1173	0.1287	0.1264
0	1	0	2.5	0	3	0	3.5	0	5	0.3149	0.1021	0.3446	0.2245	0.3603	0.2746
0	1	0	1.5	0	3	0	4.5	0	5	0.3421	0.1043	0.3342	0.2185	0.3585	0.2983

Table 5.107. Percentage of Rejection for k=5 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.8615	0.7864	0.9508	0.8950	0.9746	0.9078
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.6003	0.2980	0.6454	0.3206	0.6698	0.3792
0	1	0	1	0	1	0	1	0.5	2	0.4837	0.3511	0.5008	0.3624	0.5250	0.4288
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.6214	0.2380	0.5629	0.2735	0.6772	0.3506
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.6112	0.2511	0.6447	0.3287	0.7026	0.3086

Table 5.108. Percentage of Rejection for k=5 Populations; T Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.8316	0.7346	0.9232	0.8822	0.9321	0.8645
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.5181	0.2791	0.6046	0.2254	0.6618	0.3358
0	1	0	1	0	1	0	1	0.5	2	0.4299	0.2204	0.4930	0.2593	0.5093	0.3683
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.5814	0.1975	0.5021	0.2312	0.6083	0.2436
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.5622	0.2019	0.6019	0.3026	0.6196	0.2066

5.3. Results of the Exponential Distribution

5.3.1. Three Treatments

Tables 109-126 present the results of the simulation study for three treatments under the exponential distribution. The sample sizes considered in the simulations are 18 and 30. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique. Tables 109, 110, 115, 116, 121 and 122 give estimated powers when the location parameters are different but scale parameters are equal. Tables 111, 112, 117, 118, 123 and 124 give estimated powers when the location parameters are equal but scale parameters are different. Tables 113, 114, 119, 120, 125 and 126 give estimated powers when the location and scale parameters are all different. It is noted that all the tests do not maintain their alpha values (Tables 109, 110, 115, 116, 121 and 122), they are greater than the stated alpha value so that we cannot use the tests. The standardize first tests have higher alpha values than all standardize last tests (around 0.02). When the populations have unequal location parameters and equal scale parameters, the standardize last tests have higher estimated powers than all the standardize first tests (Tables 109, 110, 115, 116, 121 and 122). When the populations have equal location parameters and unequal scale parameters, the standardize first tests have higher estimated powers than all the standardize last tests (Tables 111, 112, 117, 118, 123). When the populations have unequal location parameters and unequal scale parameters, the standardize first tests tend to have the higher estimated powers (Tables 113, 114, 119, 120, 125 and 126). Tables 115-120 show the results of simulations with unequal sample sizes. In these tables, the sample sizes for the first and second populations are 18 and the sample size for the third population is 30. Tables 121-126 also show the results of simulations when the sample sizes are unequal. In these tables, the sample size for the first population is 30 and the sample sizes for the second and third populations are 18. Results were found to be similar as to which test statistic had higher powers in a situation for both equal and unequal sample sizes.

When location parameters are equal and scale parameters are unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 111 and 112, for example). When location and scale parameters are both unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 113 and 114, for example). For the situations of unequal location parameters and equal scale parameters, the SM_2 test tends to have the highest estimated powers (Tables 109, 110, 115, 116, 121 and 122). For the situations of equal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 111, 112, 117, 118, 123 and 124). For the situations of unequal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 113, 114, 119, 120, 125 and 126).

Table 5.109. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0922	0.0664	0.0906	0.0734	0.0842	0.0650
0	1	0.5	1	1	1	0.9240	0.9292	0.9118	0.9480	0.9332	0.9494
0	1	0.5	1	0.5	1	0.8680	0.7992	0.8804	0.8068	0.8858	0.8380
0	1	0	1	1	1	0.8892	0.9984	0.8896	0.9978	0.9232	0.9988
0	1	0.75	1	1	1	0.8988	0.9872	0.9028	0.9472	0.9246	0.9992
0	1	0.2	1	1	1	0.9160	0.9980	0.9072	0.90366	0.9316	0.9992

Table 5.110. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0924	0.0656	0.0987	0.0746	0.0885	0.0614
0	1	0.5	1	1	1	0.9298	0.9334	0.8816	0.9366	0.9230	0.9404
0	1	0.5	1	0.5	1	0.8764	0.7968	0.8676	0.8030	0.8796	0.8494
0	1	0	1	1	1	0.8854	0.9996	0.8968	0.9984	0.9562	0.9952
0	1	0.75	1	1	1	0.9100	0.9832	0.9052	0.9416	0.9330	0.9994
0	1	0.2	1	1	1	0.9098	0.9901	0.9114	0.9242	0.9552	0.9964

Table 5.111. Percentage of Rejection for k=3 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.1692	0.0052	0.1754	0.0050	0.2710	0.0058
0	1	0	2	0	2	0.0926	0.0026	0.1172	0.0056	0.1618	0.0032
0	1	0	1	0	2	0.1098	0.0040	0.1434	0.0024	0.1844	0.0036
0	1	0	2.5	0	3	0.1642	0.0032	0.1860	0.0036	0.2718	0.0064
0	1	0	1.5	0	3	0.1668	0.0022	0.1878	0.0032	0.2570	0.0056

Table 5.112. Percentage of Rejection for k=3 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.1220	0.0018	0.1216	0.0010	0.2070	0.0016
0	1	0	2	0	2	0.0862	0.0016	0.0974	0.0012	0.1298	0.0015
0	1	0	1	0	2	0.0978	0.0012	0.1298	0.0013	0.1274	0.0016
0	1	0	2.5	0	3	0.1248	0.0012	0.1288	0.0016	0.2490	0.0016
0	1	0	1.5	0	3	0.1220	0.0016	0.1306	0.0014	0.2072	0.0022

Table 5.113. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.8978	0.8686	0.9044	0.8412	0.9228	0.8950
0	1	0.5	2	0.5	2	0.5296	0.5110	0.5574	0.5310	0.5630	0.5380
0	1	0	1	1	2	0.9216	0.8858	0.9168	0.8876	0.9902	0.9290
0	1	0.75	2.5	1	3	0.8948	0.8356	0.8944	0.8388	0.9278	0.9052
0	1	0.2	1.5	1	3	0.8914	0.8354	0.8928	0.8372	0.9246	0.8904

Table 5.114. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.8784	0.8058	0.8980	0.7978	0.8980	0.8776
0	1	0.5	2	0.5	2	0.4582	0.4160	0.5350	0.4102	0.4816	0.4646
0	1	0	1	1	2	0.9044	0.9188	0.9080	0.9142	0.9192	0.9180
0	1	0.75	2.5	1	3	0.8770	0.7944	0.8890	0.8832	0.8956	0.8540
0	1	0.2	1.5	1	3	0.8618	0.8106	0.8910	0.8740	0.8992	0.8654

Table 5.115. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0933	0.0721	0.0975	0.0754	0.0876	0.0641
0	1	0.5	1	1	1	0.8102	0.9565	0.7970	0.9566	0.8544	0.9646
0	1	0.5	1	0.5	1	0.4556	0.6488	0.4920	0.6822	0.4882	0.6672
0	1	0	1	1	1	0.8502	0.9862	0.8248	0.9866	0.9102	0.9894
0	1	0.75	1	1	1	0.7586	0.9220	0.7602	0.9410	0.7900	0.9340
0	1	0.2	1	1	1	0.8576	0.9724	0.8332	0.9752	0.8936	0.9816

Table 5.116. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0949	0.0756	0.0895	0.0774	0.0856	0.0634
0	1	0.5	1	1	1	0.8058	0.9541	0.8007	0.9468	0.8583	0.9699
0	1	0.5	1	0.5	1	0.4469	0.6581	0.4891	0.6581	0.5196	0.6772
0	1	0	1	1	1	0.8409	0.9813	0.8229	0.9775	0.9352	0.9804
0	1	0.75	1	1	1	0.7661	0.9327	0.7499	0.9333	0.7522	0.9327
0	1	0.2	1	1	1	0.8428	0.9764	0.8390	0.9706	0.8188	0.9802

Table 5.117. Percentage of Rejection for k=3 Populations; Exponential Distribution with equal means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.0902	0.0025	0.1498	0.0034	0.1810	0.0038
0	1	0	2	0	2	0.0718	0.0012	0.1068	0.0032	0.1182	0.0038
0	1	0	1	0	2	0.0776	0.0022	0.1266	0.0036	0.1404	0.0036
0	1	0	2.5	0	3	0.0816	0.0032	0.1440	0.0036	0.1636	0.0032
0	1	0	1.5	0	3	0.0878	0.0031	0.1542	0.0034	0.1796	0.0034

Table 5.118. Percentage of Rejection for k=3 Populations; Exponential Distribution with equal means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.0720	0.0014	0.1393	0.0014	0.1664	0.0012
0	1	0	2	0	2	0.0611	0.0011	0.1032	0.0013	0.1041	0.0015
0	1	0	1	0	2	0.0669	0.0013	0.1232	0.0016	0.1132	0.0015
0	1	0	2.5	0	3	0.0660	0.0016	0.1323	0.0017	0.1512	0.0017
0	1	0	1.5	0	3	0.0731	0.0011	0.1478	0.0018	0.1698	0.0014

Table 5.119. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.8484	0.7894	0.8750	0.8414	0.8678	0.7662
0	1	0.5	2	0.5	2	0.3162	0.3868	0.3858	0.3968	0.2952	0.3752
0	1	0	1	1	2	0.9168	0.9638	0.8978	0.9478	0.9534	0.9164
0	1	0.75	2.5	1	3	0.8508	0.7816	0.8442	0.8094	0.9094	0.8352
0	1	0.2	1.5	1	3	0.8718	0.8590	0.8724	0.8650	0.9465	0.8642

Table 5.120. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances ($n_1 = 18, n_2 = 18, n_3 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.8177	0.7581	0.8459	0.7156	0.8572	0.7137
0	1	0.5	2	0.5	2	0.3054	0.3614	0.3819	0.2833	0.2744	0.2651
0	1	0	1	1	2	0.9020	0.9349	0.8915	0.8388	0.9258	0.8951
0	1	0.75	2.5	1	3	0.8292	0.7542	0.8294	0.6930	0.8651	0.8173
0	1	0.2	1.5	1	3	0.8597	0.8421	0.8450	0.7578	0.9009	0.8497

Table 5.121. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0913	0.0746	0.0948	0.0736	0.0886	0.0617
0	1	0.5	1	1	1	0.8218	0.9553	0.7994	0.9592	0.8396	0.9666
0	1	0.5	1	0.5	1	0.5522	0.7846	0.5450	0.7628	0.5782	0.8066
0	1	0	1	1	1	0.6878	0.8870	0.7312	0.9262	0.7170	0.9104
0	1	0.75	1	1	1	0.8418	0.9772	0.8272	0.9676	0.8638	0.9790
0	1	0.2	1	1	1	0.7624	0.9274	0.7788	0.9426	0.7886	0.9332

Table 5.122. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and equal variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0.0925	0.0716	0.0931	0.0740	0.0856	0.0622
0	1	0.5	1	1	1	0.8100	0.9433	0.8083	0.9686	0.8639	0.9532
0	1	0.5	1	0.5	1	0.5251	0.7513	0.5407	0.7660	0.6102	0.8222
0	1	0	1	1	1	0.6800	0.8735	0.7281	0.9350	0.7413	0.9008
0	1	0.75	1	1	1	0.8211	0.9668	0.8241	0.9719	0.8949	0.9751
0	1	0.2	1	1	1	0.7431	0.9182	0.7823	0.9574	0.8056	0.9269

Table 5.123. Percentage of Rejection for k=3 Populations; Exponential Distribution with equal means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.0936	0.0038	0.1534	0.0031	0.1788	0.0036
0	1	0	2	0	2	0.0760	0.0027	0.1008	0.0036	0.1664	0.0033
0	1	0	1	0	2	0.0726	0.0021	0.1092	0.0016	0.1616	0.0034
0	1	0	2.5	0	3	0.0914	0.0034	0.1072	0.0022	0.1600	0.0034
0	1	0	1.5	0	3	0.0936	0.0034	0.1518	0.0032	0.1876	0.0033

Table 5.124. Percentage of Rejection for k=3 Populations; Exponential Distribution with equal means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0.0871	0.0018	0.1446	0.0015	0.1634	0.0017
0	1	0	2	0	2	0.0563	0.0013	0.0954	0.0012	0.1366	0.0013
0	1	0	1	0	2	0.0540	0.0016	0.0983	0.0013	0.1351	0.0014
0	1	0	2.5	0	3	0.0832	0.0016	0.0993	0.0013	0.1511	0.0013
0	1	0	1.5	0	3	0.0910	0.0015	0.1394	0.0014	0.1619	0.0017

Table 5.125. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.8682	0.8416	0.8676	0.8552	0.8972	0.8738
0	1	0.5	2	0.5	2	0.4912	0.5304	0.4932	0.4966	0.5596	0.5188
0	1	0	1	1	2	0.7866	0.8134	0.8152	0.8324	0.8308	0.8216
0	1	0.75	2.5	1	3	0.8864	0.8558	0.8594	0.8540	0.9425	0.8866
0	1	0.2	1.5	1	3	0.8396	0.8016	0.8398	0.7992	0.9655	0.8488

Table 5.126. Percentage of Rejection for k=3 Populations; Exponential Distribution with different means and different variances ($n_1 = 30, n_2 = 18, n_3 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.5	2	1	3	0.8551	0.8208	0.8654	0.7375	0.8624	0.8418
0	1	0.5	2	0.5	2	0.4558	0.4973	0.4801	0.4014	0.5046	0.4762
0	1	0	1	1	2	0.7676	0.7980	0.7605	0.7293	0.7999	0.7703
0	1	0.75	2.5	1	3	0.8660	0.8449	0.8627	0.7351	0.8790	0.8589
0	1	0.2	1.5	1	3	0.8232	0.7850	0.8118	0.6896	0.9153	0.7437

5.3.2. Four Treatments

Tables 127-144 present the results of the simulation study for four treatments under the exponential distribution. The sample sizes considered in the simulations are 18 and 30. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique. Tables 127, 128, 133, 134, 139 and 140 give estimated powers when the location parameters are different but scale parameters are equal. Tables 129, 130, 135, 136, 141 and 142 give estimated powers when the location parameters are equal but scale parameters are different. Tables 131, 132, 137, 138, 143 and 144 give estimated powers when the location and scale parameters are all different. It is noted that all the tests do not maintain their alpha values (Tables 127, 128, 133, 134, 139 and 140), they are greater than the stated alpha value so that we cannot use the tests. The standardize first tests have higher alpha values than all standardize last tests (around 0.02). When the populations have unequal location parameters and equal scale parameters, the standardize last tests have higher estimated powers than all the standardize first tests (Tables 129, 130, 135, 136, 141 and 142). When the populations have equal location parameters and unequal scale parameters, the standardize first tests have higher estimated power than all standardize last tests (Tables 131, 132, 137, 138, 143 and 144). When the populations have unequal location parameters and unequal scale parameters, the standardize first tests tend to have the higher estimated power (Tables 129, 130, 135, 136, 141 and 142). Tables 133-138 show the results of simulations with unequal sample sizes. In these tables, the sample sizes for the first and second populations are 18 and the sample sizes for the third and fourth populations are 30. Tables 139-144 also show the results of simulations when the sample sizes are unequal. In these tables, the sample sizes for the first and second populations are 30 and the sample sizes for the third and fourth populations are 18. Results were found to be similar as to which test statistic had higher powers in the situations for both equal and unequal sample sizes.

When location parameters are equal and scale parameters are unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 129 and 130, for example). When location and scale parameters are both unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 131 and 132, for example). For the situations of unequal location parameters and equal scale parameters, the SM_2 test tends to have the highest estimated powers (Tables 127, 128, 133, 134, 139 and 140). For the situations of equal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 129, 130, 135, 136, 141 and 142). For the situations of unequal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 131, 132, 137, 138, 143 and 144).

Table 5.127. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0884	0.0726	0.0914	0.0712	0.0815	0.0649
0	1	0.3	1	0.6	1	0.9	1	0.9270	0.9745	0.8804	0.9850	0.8946	0.9814
0	1	0.5	1	0.5	1	0.5	1	0.5294	0.7668	0.5408	0.7690	0.5514	0.7794
0	1	0	1	0	1	0.5	1	0.5400	0.5604	0.3962	0.5428	0.3974	0.5368
0	1	0.5	1	0.6	1	1.1	1	0.9686	0.9932	0.8972	0.9999	0.9690	0.9999
0	1	0.2	1	0.7	1	0.9	1	0.9689	0.9762	0.9134	0.9982	0.9662	0.9970

Table 5.128. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0849	0.0732	0.0931	0.0736	0.0842	0.0679
0	1	0.3	1	0.6	1	0.9	1	0.9312	0.9725	0.8684	0.9912	0.9166	0.9784
0	1	0.5	1	0.5	1	0.5	1	0.5388	0.7802	0.5528	0.7988	0.5386	0.7818
0	1	0	1	0	1	0.5	1	0.5428	0.5506	0.4116	0.5480	0.4238	0.5374
0	1	0.5	1	0.6	1	1.1	1	0.9646	0.9925	0.9122	0.9999	0.9691	0.9999
0	1	0.2	1	0.7	1	0.9	1	0.9572	0.9782	0.9168	0.9963	0.9436	0.9859

Table 5.129. Percentage of Rejection for k=4 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.3150	0.0512	0.3040	0.0430	0.3740	0.0482
0	1	0	2	0	2	0	2	0.0924	0.0234	0.0798	0.0282	0.0964	0.0360
0	1	0	1	0	1	0	2	0.1064	0.0246	0.0898	0.0388	0.1200	0.0216
0	1	0	2.5	0	3	0	3.5	0.3268	0.0256	0.2608	0.0440	0.3236	0.0480
0	1	0	1.5	0	3	0	4.5	0.3304	0.0290	0.3498	0.0472	0.3922	0.0496

Table 5.130. Percentage of Rejection for k=4 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.2040	0.0320	0.2496	0.0330	0.2614	0.0372
0	1	0	2	0	2	0	2	0.0658	0.0214	0.0668	0.0207	0.0880	0.0308
0	1	0	1	0	1	0	2	0.0766	0.0276	0.0782	0.0344	0.0842	0.0318
0	1	0	2.5	0	3	0	3.5	0.1792	0.0244	0.2002	0.0352	0.2464	0.0392
0	1	0	1.5	0	3	0	4.5	0.2088	0.0266	0.2466	0.0304	0.2612	0.0338

Table 5.131. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.7284	0.6894	0.7490	0.6604	0.8624	0.7188
0	1	0.5	2	0.5	2	0.5	2	0.5140	0.4212	0.5066	0.4790	0.4604	0.4802
0	1	0	1	0	1	0.5	2	0.5166	0.4886	0.5174	0.5700	0.5574	0.5540
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4726	0.4772	0.4032	0.3476	0.4870	0.4544
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4756	0.4758	0.4400	0.3844	0.4838	0.4496

Table 5.132. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.6968	0.6171	0.7028	0.6170	0.8372	0.7018
0	1	0.5	2	0.5	2	0.5	2	0.5046	0.4146	0.4822	0.3582	0.4120	0.3938
0	1	0	1	0	1	0.5	2	0.4768	0.4576	0.4742	0.4966	0.5242	0.5152
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4134	0.4146	0.3658	0.3102	0.4348	0.4132
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4046	0.4096	0.4092	0.3536	0.4284	0.4246

Table 5.133. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0863	0.0735	0.0928	0.0703	0.0805	0.0647
0	1	0.3	1	0.6	1	0.9	1	0.8172	0.9355	0.7814	0.9394	0.7848	0.9386
0	1	0.5	1	0.5	1	0.5	1	0.4132	0.5812	0.4666	0.6214	0.4526	0.6578
0	1	0	1	0	1	0.5	1	0.5522	0.5676	0.3788	0.5246	0.4184	0.5738
0	1	0.5	1	0.6	1	1.1	1	0.9209	0.9592	0.8598	0.9549	0.8872	0.9781
0	1	0.2	1	0.7	1	0.9	1	0.9101	0.9492	0.8176	0.9598	0.8262	0.9624

Table 5.134. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0847	0.0717	0.0956	0.0743	0.0796	0.0681
0	1	0.3	1	0.6	1	0.9	1	0.8270	0.9261	0.7724	0.9296	0.8067	0.9430
0	1	0.5	1	0.5	1	0.5	1	0.4202	0.6036	0.4594	0.6373	0.4540	0.6418
0	1	0	1	0	1	0.5	1	0.5453	0.5764	0.3650	0.5255	0.4007	0.5682
0	1	0.5	1	0.6	1	1.1	1	0.9174	0.9222	0.8503	0.9572	0.9091	0.9698
0	1	0.2	1	0.7	1	0.9	1	0.9049	0.9328	0.8066	0.9552	0.8458	0.9719

Table 5.135. Percentage of Rejection for k=4 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.2780	0.0276	0.2486	0.0454	0.3438	0.0376
0	1	0	2	0	2	0	2	0.0440	0.0164	0.0410	0.0234	0.0442	0.0164
0	1	0	1	0	1	0	2	0.0554	0.0184	0.0382	0.0198	0.0536	0.0136
0	1	0	2.5	0	3	0	3.5	0.2508	0.0264	0.2846	0.0456	0.3070	0.0394
0	1	0	1.5	0	3	0	4.5	0.2804	0.0228	0.2686	0.0464	0.3534	0.0442

Table 5.136. Percentage of Rejection for k=4 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.1907	0.0229	0.1922	0.0248	0.2596	0.0304
0	1	0	2	0	2	0	2	0.0411	0.0175	0.0399	0.0185	0.0385	0.0165
0	1	0	1	0	1	0	2	0.0406	0.0183	0.0353	0.0172	0.0522	0.0120
0	1	0	2.5	0	3	0	3.5	0.1603	0.0245	0.1778	0.0293	0.2751	0.0392
0	1	0	1.5	0	3	0	4.5	0.1688	0.0252	0.1712	0.0202	0.2562	0.0305

Table 5.137. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.6190	0.4917	0.6620	0.5570	0.7486	0.5940
0	1	0.5	2	0.5	2	0.5	2	0.3552	0.3410	0.3144	0.3736	0.3936	0.3240
0	1	0	1	0	1	0.5	2	0.5482	0.5078	0.5858	0.5554	0.5596	0.5418
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.3591	0.2630	0.3640	0.1592	0.3836	0.3094
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.3580	0.2268	0.3742	0.1268	0.4067	0.3244

Table 5.138. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = 18,\ n_3 = n_4 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.5596	0.5260	0.6847	0.5412	0.6961	0.5634
0	1	0.5	2	0.5	2	0.5	2	0.2895	0.2051	0.3304	0.2601	0.2732	0.1635
0	1	0	1	0	1	0.5	2	0.4854	0.4563	0.5092	0.4464	0.4383	0.4942
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.3299	0.1880	0.2566	0.1218	0.3415	0.2893
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.3170	0.1710	0.2034	0.1060	0.3699	0.2611

Table 5.139. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0843	0.0712	0.0902	0.0741	0.0825	0.0659
0	1	0.3	1	0.6	1	0.9	1	0.8264	0.9223	0.7754	0.9004	0.8824	0.9264
0	1	0.5	1	0.5	1	0.5	1	0.5406	0.8018	0.5256	0.7484	0.5720	0.8062
0	1	0	1	0	1	0.5	1	0.3172	0.3680	0.3166	0.4186	0.2964	0.4578
0	1	0.5	1	0.6	1	1.1	1	0.9299	0.9629	0.8706	0.9405	0.9008	0.9737
0	1	0.2	1	0.7	1	0.9	1	0.9191	0.9321	0.8260	0.9308	0.9288	0.9582

Table 5.140. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0.0824	0.0743	0.0920	0.0719	0.0830	0.0699
0	1	0.3	1	0.6	1	0.9	1	0.8345	0.9022	0.7671	0.9098	0.8867	0.9130
0	1	0.5	1	0.5	1	0.5	1	0.5535	0.8244	0.5222	0.7516	0.6040	0.8218
0	1	0	1	0	1	0.5	1	0.3302	0.3690	0.3097	0.4274	0.3207	0.4482
0	1	0.5	1	0.6	1	1.1	1	0.9026	0.9481	0.8591	0.9548	0.9091	0.9698
0	1	0.2	1	0.7	1	0.9	1	0.9126	0.9266	0.8146	0.9256	0.9458	0.9519

Table 5.141. Percentage of Rejection for k=4 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.2004	0.0224	0.2378	0.0426	0.2594	0.0392
0	1	0	2	0	2	0	2	0.0408	0.0160	0.0474	0.0210	0.0504	0.0228
0	1	0	1	0	1	0	2	0.0446	0.0178	0.0498	0.0230	0.0508	0.0232
0	1	0	2.5	0	3	0	3.5	0.2048	0.0254	0.2210	0.0460	0.3032	0.0480
0	1	0	1.5	0	3	0	4.5	0.2612	0.0560	0.3110	0.0442	0.3358	0.0498

Table 5.142. Percentage of Rejection for k=4 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0.1713	0.0218	0.2059	0.0195	0.2314	0.0204
0	1	0	2	0	2	0	2	0.0489	0.0151	0.0455	0.0158	0.0528	0.0165
0	1	0	1	0	1	0	2	0.0455	0.0169	0.0483	0.0143	0.0505	0.0220
0	1	0	2.5	0	3	0	3.5	0.1808	0.0242	0.2142	0.0221	0.2575	0.0292
0	1	0	1.5	0	3	0	4.5	0.2453	0.0244	0.2963	0.0187	0.2647	0.0305

Table 5.143. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.6396	0.5011	0.6496	0.5726	0.8050	0.6428
0	1	0.5	2	0.5	2	0.5	2	0.4736	0.4356	0.4762	0.4736	0.5218	0.5010
0	1	0	1	0	1	0.5	2	0.4834	0.3392	0.5458	0.4788	0.5368	0.4386
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4214	0.3380	0.4542	0.3488	0.4607	0.4048
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.3536	0.2935	0.4112	0.3706	0.4635	0.3168

Table 5.144. Percentage of Rejection for k=4 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = 30,\ n_3 = n_4 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.6049	0.4335	0.6048	0.5549	0.7496	0.6214
0	1	0.5	2	0.5	2	0.5	2	0.4382	0.4037	0.4694	0.3784	0.5076	0.4576
0	1	0	1	0	1	0.5	2	0.4156	0.2850	0.5068	0.3757	0.5151	0.3781
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4022	0.3083	0.4168	0.3114	0.4172	0.3871
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.3126	0.2523	0.3804	0.2398	0.4433	0.2817

5.3.3. Five Treatments

Tables 145-162 present the results of the simulation study for five treatments under the exponential distribution. The sample sizes considered in the simulations are 18 and 30. Subgroup sample sizes considered are 3 and 6 for the Moses test's technique. Tables 145, 146, 151, 152, 157 and 158 give estimated powers when the location parameters are different but scale parameters are equal. Tables 147, 148, 153, 154, 159 and 160 give estimated powers when the location parameters are equal but scale parameters are different. Tables 149, 150, 155, 156, 161 and 162 give estimated powers when the location and scale parameters are all different. It is noted that all the tests do not maintain their alpha values (Tables 147, 148, 153, 154, 159 and 160), they are greater than the stated alpha value so that we cannot use the tests. The standardize first tests have higher alpha values than all standardize last tests (around 0.02). When the populations have unequal location parameters and equal scale parameters, the standardize last tests have higher estimated powers than all the standardize first tests (Tables 147, 148, 153, 154, 159 and 160). When the populations have equal location parameters and unequal scale parameters, the standardize first tests have higher estimated powers than all the standardize last tests (Tables 147, 148, 153, 154, 159 and 160). When the populations have unequal location parameters and unequal scale parameters, the standardize first tests tend to have the higher estimated powers (Tables 147, 148, 153, 154, 159 and 160). Tables 151-156 show the results of simulations with unequal sample sizes. In these tables, the sample size for the first, second and third populations are 18 and the sample sizes for the fourth and fifth populations are 30. Tables 157-162 also show the results of simulations when the sample sizes are unequal. In these tables, the sample sizes for the first, second and third populations are 30 and the sample sizes for the fourth and fifth populations are 18. Results were found to be similar as to which test statistic had higher powers in the situations for both equal and unequal sample sizes.

When location parameters are equal and scale parameters are unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 147 and 148, for example). When location and scale parameters are both unequal, and when the subgroup sample size increases from 3 to 6, the estimated powers for testing location and scale parameters become lower (compare Tables 149 and 150, for example). For the situations of unequal location parameters and equal scale parameters, the SM_2 test tends to have the highest estimated powers (Tables 145, 146, 151, 152, 157 and 158). For the situations of equal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 147, 148, 153, 154, 159 and 160). For the situations of unequal location parameters and unequal scale parameters, the SM_1 test has the highest estimated powers (Tables 149, 150, 155, 156, 161 and 162).

Table 5.145. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0907	0.0713	0.0894	0.0720	0.0833	0.0657
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5224	0.7224	0.5266	0.7206	0.5212	0.7412
0	1	0	1	0	1	0	1	0.5	1	0.3922	0.4150	0.3898	0.5102	0.3750	0.5868
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.4721	0.5268	0.4636	0.5442	0.4928	0.6454
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.4137	0.5396	0.4104	0.5372	0.5378	0.6516

Table 5.146. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0935	0.0741	0.0867	0.0734	0.0845	0.0682
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9905	0.9991	0.9999	0.9957	0.9984	0.9969
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5270	0.7486	0.5244	0.7304	0.5014	0.7236
0	1	0	1	0	1	0	1	0.5	1	0.3996	0.4300	0.3776	0.5154	0.3770	0.5874
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.4801	0.5358	0.4450	0.5256	0.4930	0.6488
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.4293	0.5556	0.4160	0.5228	0.5084	0.6380

Table 5.147. Percentage of Rejection for k=5 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.2770	0.0556	0.3680	0.0576	0.4088	0.0558
0	1	0	2	0	2	0	2	0	2	0.0518	0.0348	0.0586	0.0386	0.0667	0.0316
0	1	0	1	0	1	0	1	0	2	0.0596	0.0360	0.0670	0.0392	0.0606	0.0318
0	1	0	2.5	0	3	0	3.5	0	5	0.2880	0.0544	0.3560	0.0524	0.3904	0.0514
0	1	0	1.5	0	3	0	4.5	0	5	0.2926	0.0510	0.3864	0.0504	0.4126	0.0572

Table 5.148. Percentage of Rejection for k=5 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.2534	0.0458	0.3218	0.0468	0.3762	0.0596
0	1	0	2	0	2	0	2	0	2	0.0310	0.0208	0.0352	0.0302	0.0478	0.0341
0	1	0	1	0	1	0	1	0	2	0.0366	0.0274	0.0370	0.0284	0.0436	0.0314
0	1	0	2.5	0	3	0	3.5	0	5	0.2680	0.0462	0.3022	0.0474	0.3132	0.0526
0	1	0	1.5	0	3	0	4.5	0	5	0.2842	0.0402	0.3392	0.0468	0.3816	0.0586

Table 5.149. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.9999	0.8772	0.9999	0.8806	0.9999	0.8604
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.4222	0.3886	0.4594	0.3638	0.4550	0.4154
0	1	0	1	0	1	0	1	0.5	2	0.6714	0.5630	0.6832	0.5398	0.6954	0.5128
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.7058	0.6626	0.7338	0.6048	0.7622	0.6068
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.7400	0.6098	0.7224	0.6712	0.7402	0.6628

Table 5.150. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.8920	0.8471	0.8988	0.8565	0.8924	0.8547
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.4026	0.3644	0.4386	0.3430	0.4122	0.3670
0	1	0	1	0	1	0	1	0.5	2	0.6020	0.4856	0.6554	0.4664	0.6018	0.4706
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.6658	0.4892	0.6730	0.5258	0.6938	0.5702
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.6710	0.4356	0.6696	0.5960	0.6616	0.6500

Table 5.151. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 18, n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0913	0.0721	0.0886	0.0716	0.0866	0.0654
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9461	0.9943	0.9449	0.9977	0.9880	0.9926
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.4172	0.5342	0.3950	0.5558	0.3374	0.5792
0	1	0	1	0	1	0	1	0.5	1	0.3952	0.5452	0.3688	0.5492	0.3316	0.5690
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.3588	0.5020	0.3112	0.5282	0.3036	0.5975
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.3112	0.4418	0.3654	0.5976	0.3284	0.5534

Table 5.152. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 18, n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0927	0.0756	0.0810	0.0794	0.0871	0.0642
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9557	0.9997	0.9527	0.9879	0.9999	0.9999
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.4412	0.5436	0.3928	0.5417	0.3464	0.5826
0	1	0	1	0	1	0	1	0.5	1	0.4019	0.5556	0.3566	0.5501	0.3112	0.5724
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.3454	0.4930	0.3074	0.5096	0.3274	0.5812
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.3156	0.4258	0.3510	0.5832	0.3242	0.5426

Table 5.153. Percentage of Rejection for k=5 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.2230	0.0424	0.2982	0.0408	0.3306	0.0406
0	1	0	2	0	2	0	2	0	2	0.1520	0.0174	0.1228	0.0206	0.1558	0.0334
0	1	0	1	0	1	0	1	0	2	0.1574	0.0204	0.0966	0.0184	0.1306	0.0304
0	1	0	2.5	0	3	0	3.5	0	5	0.2024	0.0774	0.2764	0.0724	0.2904	0.0830
0	1	0	1.5	0	3	0	4.5	0	5	0.2658	0.0702	0.3462	0.0734	0.3634	0.0860

Table 5.154. Percentage of Rejection for k=5 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.2096	0.0295	0.2520	0.0378	0.3160	0.0304
0	1	0	2	0	2	0	2	0	2	0.1250	0.0139	0.1202	0.0181	0.1315	0.0303
0	1	0	1	0	1	0	1	0	2	0.1310	0.0177	0.0866	0.0140	0.1314	0.0343
0	1	0	2.5	0	3	0	3.5	0	5	0.1881	0.0615	0.2226	0.0715	0.2546	0.0735
0	1	0	1.5	0	3	0	4.5	0	5	0.2480	0.0701	0.2990	0.0775	0.3028	0.0758

Table 5.155. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.8148	0.7991	0.8030	0.7641	0.9526	0.7880
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.2704	0.2558	0.2772	0.2248	0.3374	0.3092
0	1	0	1	0	1	0	1	0.5	2	0.7106	0.5964	0.7222	0.4612	0.7332	0.5290
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.4863	0.3424	0.5320	0.3166	0.5412	0.3274
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.4264	0.3648	0.4606	0.3584	0.5826	0.3242

Table 5.156. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = 18,\ n_4 = n_5 = 30$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.7951	0.7494	0.7812	0.6383	0.9201	0.7219
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.2409	0.2018	0.2564	0.2113	0.2688	0.2584
0	1	0	1	0	1	0	1	0.5	2	0.6106	0.5655	0.6144	0.3522	0.6582	0.4814
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.3463	0.3566	0.3712	0.2768	0.4534	0.2410
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.3774	0.3010	0.3978	0.3192	0.5078	0.2596

Table 5.157. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0927	0.0753	0.0904	0.0756	0.0837	0.0649
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9509	0.9743	0.9397	0.9963	0.9674	0.9999
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5480	0.7108	0.5284	0.7032	0.5484	0.7200
0	1	0	1	0	1	0	1	0.5	1	0.2882	0.3336	0.2616	0.3302	0.2790	0.3224
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.3230	0.3772	0.3380	0.4310	0.2826	0.4948
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.3176	0.4612	0.3030	0.4740	0.2914	0.5462

Table 5.158. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and equal variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	1	0	1	0	1	0	1	0.0857	0.0776	0.0933	0.0754	0.0859	0.0683
0	1	0.3	1	0.6	1	0.9	1	1.2	1	0.9499	0.9629	0.9475	0.9839	0.9538	0.9865
0	1	0.5	1	0.5	1	0.5	1	0.5	1	0.5703	0.7217	0.5262	0.7064	0.5804	0.7356
0	1	0	1	0	1	0	1	0.5	1	0.2981	0.3641	0.2494	0.3390	0.3033	0.3128
0	1	0.15	1	0.2	1	0.35	1	0.4	1	0.3126	0.3862	0.3194	0.4496	0.2937	0.4909
0	1	0.04	1	0.2	1	0.24	1	0.4	1	0.3400	0.4772	0.3114	0.4884	0.3084	0.5399

Table 5.159. Percentage of Rejection for k=5 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.3504	0.0440	0.3540	0.0446	0.3864	0.0456
0	1	0	2	0	2	0	2	0	2	0.1708	0.0226	0.1356	0.0256	0.1994	0.0316
0	1	0	1	0	1	0	1	0	2	0.1130	0.0234	0.1656	0.0298	0.2012	0.0290
0	1	0	2.5	0	3	0	3.5	0	5	0.3530	0.0434	0.3664	0.0470	0.3804	0.0424
0	1	0	1.5	0	3	0	4.5	0	5	0.3684	0.0436	0.3834	0.0442	0.4006	0.0532

Table 5.160. Percentage of Rejection for k=5 Populations; Exponential Distribution with equal means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0	2	0	3	0	4	0	5	0.2463	0.0382	0.3078	0.0393	0.3436	0.0484
0	1	0	2	0	2	0	2	0	2	0.1219	0.0271	0.1322	0.0280	0.1346	0.0263
0	1	0	1	0	1	0	1	0	2	0.1085	0.0204	0.1256	0.0263	0.1363	0.0206
0	1	0	2.5	0	3	0	3.5	0	5	0.2381	0.0361	0.3126	0.0405	0.3383	0.0422
0	1	0	1.5	0	3	0	4.5	0	5	0.2458	0.0385	0.3362	0.0369	0.3399	0.0395

Table 5.161. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=3)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.8258	0.8105	0.8324	0.8287	0.9574	0.9371
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.4876	0.4274	0.5218	0.3932	0.4820	0.4776
0	1	0	1	0	1	0	1	0.5	2	0.4480	0.4400	0.4396	0.3558	0.4506	0.4236
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.6010	0.3693	0.5673	0.3228	0.6217	0.4570
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.5688	0.3781	0.6159	0.3786	0.6262	0.4132

Table 5.162. Percentage of Rejection for k=5 Populations; Exponential Distribution with different means and different variances ($n_1 = n_2 = n_3 = 30,\ n_4 = n_5 = 18$; subgroup sample size=6)

μ_1	σ_1	μ_2	σ_2	μ_3	σ_3	μ_4	σ_4	μ_5	σ_5	JM_1	JM_2	MJM_1	MJM_2	SM_1	SM_2
0	1	0.2	1.2	0.4	1.4	0.6	1.6	0.8	1.8	0.8059	0.8087	0.7906	0.7110	0.9045	0.8417
0	1	0.5	2	0.5	2	0.5	2	0.5	2	0.4054	0.3985	0.5010	0.2980	0.4240	0.4142
0	1	0	1	0	1	0	1	0.5	2	0.4642	0.4093	0.4018	0.2527	0.4149	0.3631
0	1	0.15	1.3	0.2	1.4	0.35	1.5	0.4	1.8	0.5410	0.2898	0.5281	0.2195	0.5906	0.2640
0	1	0.04	1.1	0.2	1.4	0.24	1.5	0.4	1.8	0.5178	0.3273	0.5787	0.2747	0.6092	0.3152

CHAPTER 6. CONCLUSION

For testing both location and scale, in this research we propose six new nonparametric tests for the monotonic ordered alternatives problem in k-sample ($k \geq 3$). These new tests use the Jonckheere-Terpstra Test, the Modified Jonckheere-Terpstra Test and the Shan Test combined with the Moses test's technique, respectively. Each combination has two different versions: standardize the test statistics first (JM_1, MJM_1 and SM_1) and standardize the test statistics last (JM_2, MJM_2 and SM_2).

Two subgroup sample sizes were used for the Moses test's technique. When the scale parameters were not equal, regardless of whether location parameters were equal or not, the tests had higher powers when the subgroup sample size was 3 instead of 6. The smaller subgroup sample size allowed larger sample sizes when applying Moses test's technique. Overall, we recommend using smaller subgroup sample sizes to keep more subgroups when testing for difference in scale.

For the symmetric distributions (Normal and T), all the proposed tests maintained their alpha values. When the difference was only in treatment location parameters, because the weight of test statistic pattern for the location parameters is much higher than the weight of test statistic pattern for scale parameters, the test which standardizes the individual patterns last has more power than the test which standardizes the individual patterns first. When the difference was only in treatment scale parameters, the test which standardizes the individual patterns first has more power than the test which standardizes the individual patterns last. For all tests, although the JM or MJM tests have slightly higher estimated powers for a few cases when the subgroup sample size is small, SM_1 and SM_2 have obvious higher power estimates in most cases (see Figures 6.1 and 6.2). So, we recommend using SM_1 and SM_2. When all but one of the location or scale parameters of the treatment populations are equal, the powers of SM_1 and SM_2 are not stable. Sometimes these tests have lower powers than the other tests when this is true.

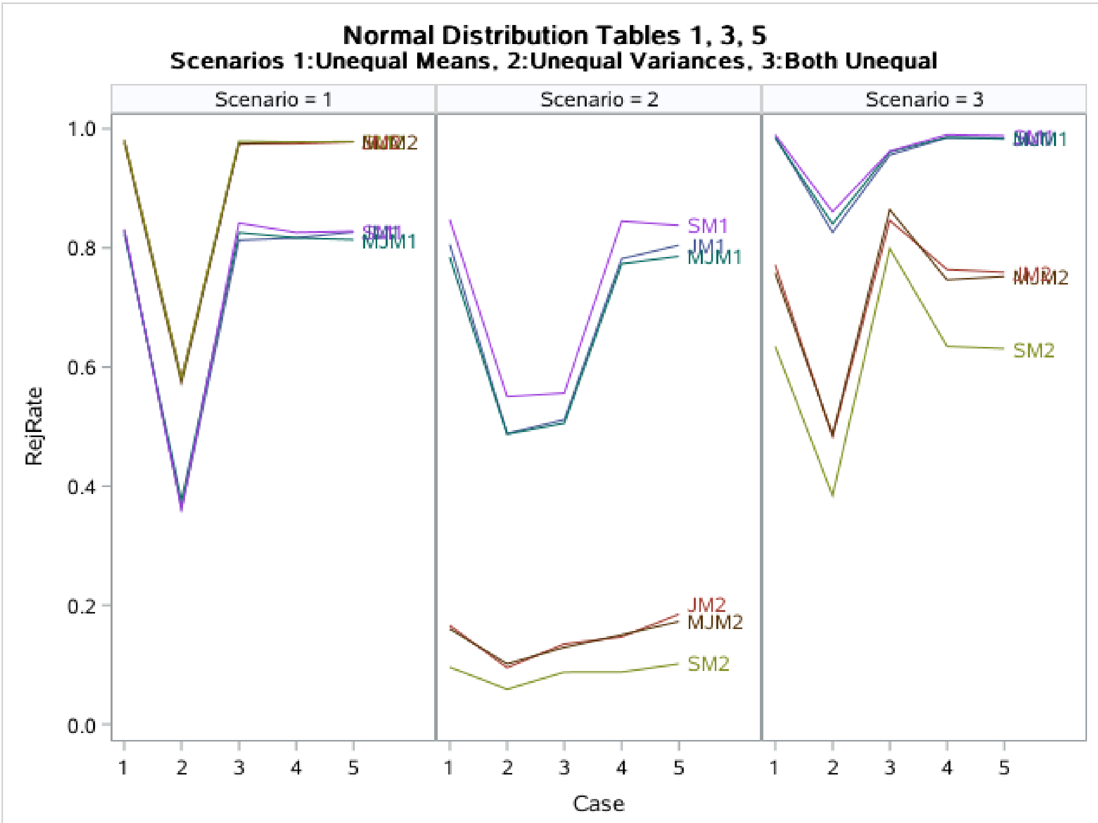

Figure 6.1. The estimated powers for normal distribution for Tables 1, 3 and 5

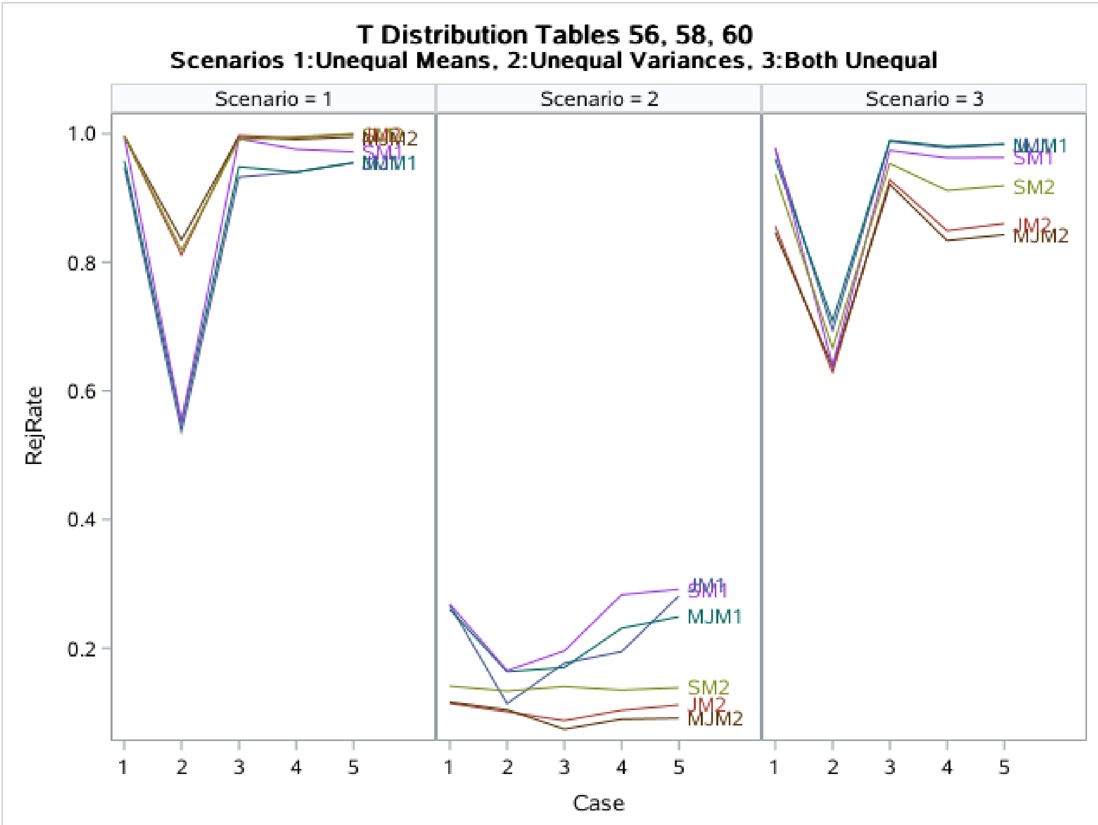

Figure 6.2. The estimated powers for T distribution for Tables 56, 58 and 60

For the non-symmetric distribution (exponential), none of the tests maintained their alpha values, they were greater than the stated alpha value. That is because the Jonckheere's test is the extension of the Mann-Whitney test, and for the non-symmetrical population, the Mann-Whitney and Moses tests are not independent (Hollander, 2013). The same situation happens to the Shan test is used. The tests which standardize the individual patterns last have lower alpha values than the tests which standardizes the individual patterns first, this difference is around 0.02. For the exponential distribution, when the difference was only in treatment scale parameters, the powers of all tests became too low to compare. Only the alpha values of the SM_2 test was close to 0.05, and SM_2 had the highest powers for testing different locations. So, we recommend to only use SM_2 test for the exponential distribution.

Overall, we recommend keeping the subgroup sample size small. If the distribution that one is sampling from is assumed to be approximately symmetric, and only the location parameters are different, SM_2 has the highest powers. When the scale parameters were not equal, regardless of whether location parameters were equal or not, SM_1 has the highest powers. So, we recommend using SM_2 if researcher ascertain that only the location parameters are different, otherwise we recommend using SM_1 for the test. If one expects the underlying distribution to be relatively skewed, only the alpha values of the SM_2 test is close to 0.05. So, we recommend using SM_2 for relatively skewed distribution.

Future work could reduce the interference effect of exponential distribution on the test results. Also, this research could be extended to the comparison between the proposed tests using another asymmetric distribution that less skew than the exponential distribution.

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APPENDIX

A.1. SAS code for Modified Jonckheere-Terpstra (MJT) test + Moses test for 3 populations with different location parameters and same scale parameters under normal distribution (all sample sizes are 30 and subgroup sample size is 3)

```
data w;
 call streaminit(0);
 do sample=1 to 5000;
 do group=1 to 10;
        do iter=1 to 3;
           trt='A'; Y=rand('Normal',0,1); output;
        end;
   end;
      do group=1 to 10;
        do iter=1 to 3;
           trt='B'; Y=rand('Normal',1,1); output;
          end;
        end;
      do group=1 to 10;
        do iter=1 to 3;
           trt='C'; Y=rand('Normal',2,1); output;
          end;
        end;
   end;
run;
```

```
proc means noprint data = w;
       by sample;
       class group trt;
       var Y;
       output out = counts N = group_size mean = subsample_mean var = sample_var;
run;

data counts;
      set counts;
      if _TYPE_=0 then delete;
      if _TYPE_=1 then delete;
      if _TYPE_=2 then delete;
run;

proc sort data = counts;
   by sample trt group;
run;

proc iml;
   use w;
   do sample=1 to 5000;
      p=( ((sample-1)*(90)+1) : (sample*(90)) );
      read point p var{Y} into fullsampl;

      x1=fullsampl[ 1:30,];
      x2=fullsampl[ 31:60,];
```

```
   x3=fullsampl[61:90,];


U12=0;
do i=1 to nrow(x1);
   do j=1 to nrow(x2);
      if x1[i,1]<x2[j,1] then U12=U12+1;
      if x1[i,1]=x2[j,1] then U12=U12+0.5;
   end;
end;


U13=0;
do i=1 to nrow(x1);
   do j=1 to nrow(x3);
      if x1[i,1]<x3[j,1] then U13=U13+1;
      if x1[i,1]=x3[j,1] then U13=U13+0.5;
     end;
end;


U23=0;
do i=1 to nrow(x2);
   do j=1 to nrow(x3);
      if x2[i,1]<x3[j,1] then U23=U23+1;
      if x2[i,1]=x3[j,1] then U23=U23+0.5;
     end;
  end;
```

```
   JT= U12+2*U13+ U23;

   JT_vec = sample || JT;
   JT_all = JT_all // JT_vec;

end;
SJT_all=(JT_all-(0.5*30*30+30*30+0.5*30*30))/sqrt(0.5*30*30*61+0.5*30*30*30);

use counts;
   do sample=1 to 5000;
      p=( ((sample-1)*(30)+1) : (sample*(30)) );
      read point p var{sample_var} into fullsampl;

      x1=fullsampl[ 1:10,];
      x2=fullsampl[ 11:20,];
      x3=fullsampl[ 21:30,];

   U12=0;
   do i=1 to nrow(x1);
      do j=1 to nrow(x2);
         if x1[i,1]<x2[j,1] then U12=U12+1;
         if x1[i,1]=x2[j,1] then U12=U12+0.5;
      end;
   end;

   U13=0;
```

```
  do i=1 to nrow(x1);
     do j=1 to nrow(x3);
        if x1[i,1]<x3[j,1] then U13=U13+1;
        if x1[i,1]=x3[j,1] then U13=U13+0.5;
      end;
  end;

  U23=0;
  do i=1 to nrow(x2);
     do j=1 to nrow(x3);
        if x2[i,1]<x3[j,1] then U23=U23+1;
        if x2[i,1]=x3[j,1] then U23=U23+0.5;
      end;
    end;

  MJT= U12+2*U13+ U23;

  MJT_vec = sample || MJT;
  MJT_all = MJT_all // MJT_vec;
end;
SMJT_all=(MJT_all-(0.5*10*10+10*10+0.5*10*10))/sqrt(0.5*10*10*21+0.5*10*10*10);
MMJT=(SJT_all+SMJT_all)/sqrt(2);

cname = {"Sample" "MMJT"};
create JTout from MMJT [colname=cname];
append from MMJT;
```

```
Quit;

data results;
     merge JTout end=eof;
        if MMJT>1.645 then Reject_T=1;
           else Reject_T=0;
        T_counter+Reject_T;
        if eof then do;
             power_T=T_counter/5000;
             file "power.txt" mod;
                put @1 power_T    ;
             end;
     run;
```

A.2. First few lines of the SAS code for the test for 3 populations with same location parameters and different scale parameters under T distribution (all sample sizes are 30 and subgroup sample size is 3)

```
data w;
  call streaminit(0);
  do sample=1 to 5000;
  do group=1 to 10;
        do iter=1 to 3;
           trt='A'; Y=rand('T',3)*0.5774; output;
/* let rand('T',3) multiply by square root of 1/3 to make the variance of the population equal to 1*/
        end;
```

```
   end;
       do group=1 to 10;
         do iter=1 to 3;
             trt='B'; Y=rand('T',3)*0.8165; output;
/* let rand('T',3) multiply by square root of 2/3 to make the variance of the population equal to 2,*/
           end;
         end;
       do group=1 to 10;
         do iter=1 to 3;
             trt='C'; Y=rand('T',3); output;
           end;
         end;
   end;
run;
```

A.3. First few lines of the SAS code for the test for 3 populations with different location and scale parameters under exponential distribution (all sample sizes are 30 and subgroup sample size is 3)

```
data w;
  call streaminit(0);
  do sample=1 to 5000;
  do group=1 to 10;
         do iter=1 to 3;
             trt='A'; Y=rand('EXPONENTIAL',1)-1; output;
/* let rand('EXPONENTIAL',1) minus 1 to make the mean of the population equal to 0*/
         end;
   end;
```

```
       do group=1 to 10;
        do iter=1 to 3;
          trt='B'; Y=rand('EXPONENTIAL',1)*1.4142-0.4142; output;
/* let rand('EXPONENTIAL',1) multiply by square root of 2 to make the variance of the population equal to
2, and then minus 0.4142 to make the mean of the population equal to 1*/


         end;
        end;
       do group=1 to 10;
        do iter=1 to 3;
          trt='C'; Y=rand('EXPONENTIAL',1)*1.7321+0.2679; output;
/* let rand('EXPONENTIAL',1) multiply by square root of 3 to make the variance of the population equal to
3, and then plus 0.2679 to make the mean of the population equal to 2*/
         end;
        end;
   end;
run;
```