OUTPATIENT APPOINTMENT SCHEDULING STUDY: UTILIZATION PROJECTION, NO-SHOW PREDICTION, AND CAPACITY ALLOCATION

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ABSTRACT

Long waiting times could result in many negative effects, such as low capacity utilization, high patient's no-show rates, and loss of social benefits, which will also lead to a waste of public resources. Therefore, to better utilize healthcare resource and serve the community, my dissertation will focus on three objectives:

- To study the relationship between appointment utilization and indirect waiting time (IWT).
- To predict the patient's no-shows without profiling them.
- To develop an optimization model for appointment capacity allocation.

To achieve these objectives, multiple models and approaches have been developed in this dissertation. For the first model, two mixture distribution models, including a beta geometric (BG) and a discrete Weibull (BdW) model were carried out to project the appointment utilization over IWT. The results indicated that appointment utilization is positively related to the IWT but tends to fluctuate after the first couple of weeks. Two mixture distribution models were also proved to be more accurate for projecting the appointment utilization when compared with commonly used curve-fitting models.

For the second objective, a conditional inference tree model was applied to predict the patient's no-show probability and classified no-show probability without profiling patients. This model was also compared with the general linear model and typically used logistic model, the result showed that using the conditional inference tree model with classified data will lead to a more accurate prediction and higher R-squared value.

For the final objective, three optimization methods and two scheduling strategies were examined. The proposed solution of capacity allocation provided a more robust, flexible, and

efficient allocation plan for outpatient appointments, which significantly improved the average daily profit and capacity utilization rate.

By completing those three objectives, this dissertation did not only provide a more accurate way to monitor and predict outpatient appointments but also proposed a more practical and efficient appointment capacity allocation strategy. This will help our society save healthcare resources, reduce unnecessary costs for the healthcare providers, and provide better healthcare services to the community.

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1. INTRODUCTION

The importance of the health care industry is increasing for most developed countries, especially for the United States whose health expenditure is highest all over the world. According to a CMS report (Keehan et al., 2017), the health care industry represents approximately 18% of the gross domestic product (GDP) of the United States in 2015 and will keep rising at a rate of 5.6 percent annually over the next decade. Nevertheless, patients in the United States are suffering from high health care costs and long wait times. To strengthen the competitiveness in the market, health care organizations not only need to improve the quality of medical care but also must improve service efficiency. As one of the most important operational systems for the delivery of health care, appointment scheduling is studied in this dissertation. In the introduction, the background of appointment scheduling in health care is covered followed by an overview of medical appointment scheduling procedures. Then, access delays of health care appointments will be discussed. What’s more, research objectives and research questions will be covered in the introduction.

1.1. Appointment Scheduling in Health Care

Appointment scheduling is one of the most important processes for health care services, and it is the first step for any health organization/system to monitor and measure patient access and patient workflow. A good appointment scheduling system should be efficient and accessible. It should also be able to smooth workflow, deal with unscheduled emergencies and satisfy patient and provider preferences while matching supply and demand (Gupta & Denton, 2008).

Before the1970s, people must go to a medical office and wait for hours for health care services. With the improvements in living standards and the development of technologies, people ask for better health care services and start to lose patience, many people have busy lives and do not want to waste their time waiting in a medical office. Under such circumstances, many appointment scheduling methods and techniques have been developed to shorten the treatment process and reduce patient waiting time.

At present, there are generally two ways for scheduling an appointment. One is to schedule manually using an appointment book, the other is to schedule electronically using a computer. Most of these appointments are made through a telephone or network provided by a medical institution, and the scheduling services are usually free. Appointment books are a ledger of workdays divided into multiple time intervals that display a weekly schedule for one physician or multiple physicians' schedules for a single day to enable the medical assistant to reserve specified lengths of time for patient treatment. When compared with an appointment book, computer scheduling has many advantages: it is more flexible, editable, and powerful. The medical assistant can easily adjust, edit, or change appointments. And the computer program itself can set up repeated appointments and a recall system, not to mention it is much more powerful and convenient to store the data and information. Therefore, appointment scheduling software is becoming more and more popular nowadays.

No matter which way of scheduling is used, a daily appointment schedule must be created. The daily appointment schedule is a list of the patients to be seen for that day. It is not only used as a reference to the patient's medical records and other personal information for that day but also shows the work arrangements of the physicians who are on duty. Usually, it should be printed out and posted in a specific area that patients cannot access to keep the confidentiality of patient information.

Although the use of IT technology made appointment scheduling much easier and faster, some structural problems cannot be solved with the original scheduling method. Thus, several different scheduling methods were developed for medical appointment scheduling. So far, there are mainly seven types of schedules for general medical offices (Bonewit-West et al., 2016), include time-specified, wave, modified wave, double booking, open booking, clustering/categorization, and multiple offices. Table 1 presents the details for each type of scheduling method.

Table 1. Types of Scheduling Methods in the Health Care Industry

Name	Description	Goal/Usage
Time-Specified (Stream) Scheduling	For this type of scheduling, each patient is given a specific appointment time based on status and needs. It could also be referred to as fixed appointment scheduling or single booking.	The goal of this type of schedule is to keep a steady patient flow with the shortest waiting time for patients.
Wave Scheduling	This method assumes that not everyone will be on time, therefore it assigns multiple patients at the beginning of each hour. The patients will be seen in the order in which they arrive.	The goal of this scheduling method is to make sure there is always a patient to be seen and try to reduce the waiting time for other patients at the same time.
Modified Wave Scheduling	This is a modification of the wave schedule. Wave scheduling can be changed in several ways based on status and needs. The medical assistant may schedule patients at regular intervals within a given hour and keep the rest of the hour open for other special cases.	This method is used to deal with special circumstances. For example, the medical assistant may schedule the first half of the hour for regular appointments and keep the second half-hour for walk-in patients or one appointment that takes longer.
Double Booking	Double booking means two patients are scheduled in a single time slot for the same provider.	Double booking is designed to reduce the impact of no show. Sometimes, it may be used when a patient has an acute illness or injury, or when one patient can be scheduled around a patient who is undergoing a procedure.
Open Booking	Unlike stream scheduling, open booking means patients are told to walk in during a time range of the day instead of a specific time. Usually, patients are seen based on a first come first served policy.	Open booking is used when there is a constant stream of patients or when the clinic is not busy. The disadvantage is that patients often experience long waiting times since patient flow is hard to predict.
Clustering/Categorization	This method groups patients with similar symptoms or treatment procedures within the same period of the day or on the same day of the week.	This type of schedule is usually used for physical examinations, diagnostic procedures, and pregnancy/ gynecology tests.
Multiple Offices	For this type of scheduling, physicians are assigned to patients in more than one office. Appointments may be scheduled for each office separately or distributed through a central system.	This is a special case when the patients must be transferred from office to office. The medical record may also be transported if there is no corresponding medical record in place.

1.2. Appointment Scheduling Procedures

A complete appointment scheduling cycle usually includes six procedures (Bonewit-West et al., 2016): setting up the appointment matrix, making an appointment, managing the appointment scheduling, completing a referral form for managed care, scheduling inpatient or outpatient diagnostic tests, and scheduling inpatient or outpatient admissions. Figure 1 shows the general procedures for appointment scheduling.

Under each procedure, processes need to be followed. For example, to set up an appointment matrix, the medical assistant needs to follow certain steps. First, the medical assistant needs to identify the time when the physicians and offices are not available to the patients, such as lunch and breaks, or when the clinic is closed. Then he/she must take the scheduling system, the physician’s preference and needs, and facilities and equipment requirements into consideration and find a balance between them, such as how long the appointment intervals should be, what time does the physician prefer to have his lunch/breaks, and when are the facilities and equipment available.

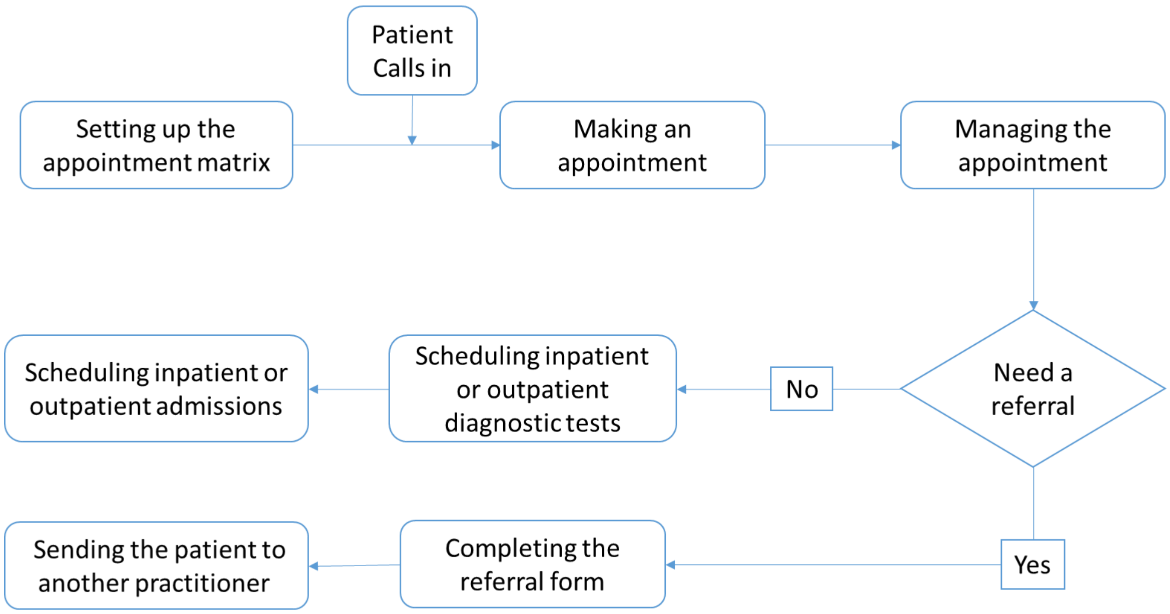

Figure 1. General Scheduling Procedures for an Appointment

Making an appointment is one of the most important procedures. In this procedure, patient information is obtained. Based on the information and the patient's preference and needs, the medical assistant will arrange an appointment with a date and time for the patient. A typical process to make an appointment is shown in Figure 2.

As for the management of the appointment schedule, there are generally five things the medical assistant can do: review appointments, cancel appointments, change appointments, indicate missed appointments, and document canceled or missed appointments.

A referral is not a necessary procedure for a patient appointment, but it usually happens under the following circumstances (Bonewit-West et al., 2016):

- A patient needs consultation on a specific disease or condition from a specialist.

- A patient needs a particular therapy from a provider, like physical therapy and occupational therapy.
- A patient needs community services from a provider, such as home health care.

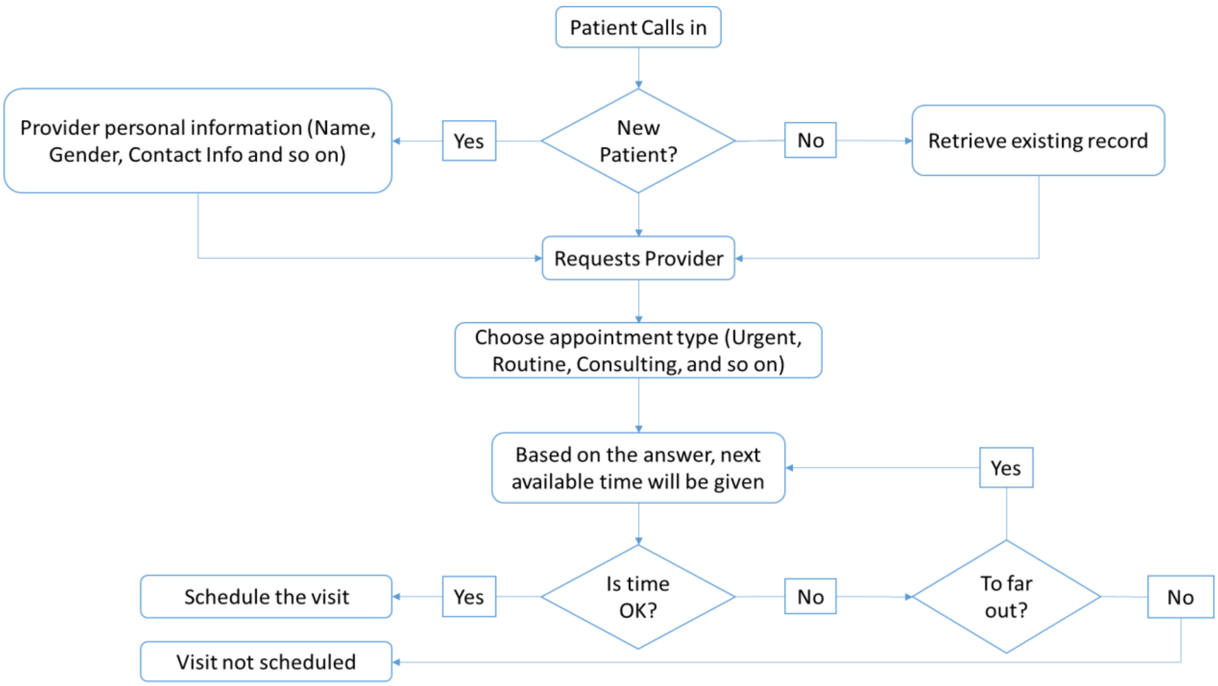

Figure 2. A Flowchart to Schedule an Appointment for a Patient

Scheduling inpatient or outpatient diagnostic tests or procedures is a complex task, the medical assistant needs to communicate with patients, physicians, the facility department, and the patient's insurance company to make sure they reach an agreement. The medical assistant needs to deal with many conflicts to set up such a test or procedure. Similar to the previous procedure, scheduling inpatient/outpatient admissions requires much effort and resources. The medical assistant must assemble the patient's demographic and insurance information and gather the patient's medical record from the physician so that the time and the reason for the admission can be decided. The assistant also needs to call the patient’s insurance company to obtain pre-authorization and call the admission department to schedule the admission. Then necessary information and documentation to all concerned should be provided. The medical assistant also has to inform and assist the patient to prepare and deal with all kinds of tests, procedures, and situations. Finally, all necessary information, data, and results in the patient’s medical record should be properly documented.

1.3. Appointment Delays in Health Care

There are generally three types of appointments: primary care appointments, specialty clinic appointments, and elective surgery appointments. No matter which type of appointment a patient is scheduled to, there are two types of access delays (Gupta & Denton, 2008). One is IWT, which is the difference between the time a patient requests an appointment and the time of that scheduled appointment. For example, if an appointment is made on Monday to receive care on Friday of the same week, the IWT would be four days. Another is direct waiting time, which is the difference between the time the patient arrives and the time when he is seen by the physician. Long access times can be caused by many reasons, such as cancelation and no-shows, the designation of providers, and the length of the intervals. The focus of this dissertation is on the IWT, so the direct waiting time will not be considered.

1.3.1. Indirect Waiting Time (IWT)

A survey released by Merritt Hawkins (Miller, 2017) indicates that the average time to schedule a new patient-physician appointment in 15 large-sized metropolitan areas has increased significantly since 2014. According to the survey, it takes an average of 24.1 days to schedule a new patient-physician appointment in 15 of the largest cities in the United States, up 30% from 2014. Among them, Boston is experiencing the highest average physician appointment wait time of 52.4 days to schedule an appointment for a new patient. This survey also includes the average new patient-physician appointment wait times of 15 mid-sized metropolitan areas in 2017. Among them, Yakima has the longest average physician appointment wait time of 48.8 days. Although it is shorter than Boston, it takes an average of 32 days to schedule a new patient-physician appointment in mid-sized metro areas, which is 32.8 percent longer than the average wait time in the 15 major metro markets. Average physician appointment wait times for new patients in 15 large metro markets and 15 mid-sized markets of 2017 are listed in Table 2.

Table 2. Average IWT for a New Patient-physician Appointment, 2017 (Miller, 2017)

Average Wait Time in Days, 2017					
Large-sized Metro Area	**All Days Per 5 Specialties**	**Average Per 5 Specialties**	**Mid-sized Metro Area**	**All Days Per 5 Specialties**	**Average Per 5 Specialties**
Boston	262	52.4	Yakima	244	48.8
Philadelphia	184	36.8	Cedar Rapids	212	42.4
Portland	140	28.0	Albany	198	39.6
Seattle	140	28.0	Manchester	197	39.4
Denver	133	26.6	Evansville	193	38.6
Los Angeles	121	24.2	Hartford	184	36.8
Detroit	110	22.0	Savannah	177	35.4
San Diego	108	21.6	Fort Smith	162	32.4
Atlanta	102	20.4	Fargo	161	32.2
Houston	98	19.6	Odessa	147	29.4
Minneapolis	87	17.4	Temecula	128	25.6
New York	85	17.0	Dayton	123	24.6
Miami	82	16.4	Lafayette	121	24.2
Washington, D.C.	80	16.0	Hampton	96	19.2
Dallas	74	14.8	Billings	54	10.8
Total	120.4	24.1	Total Average	159.8	32.0

Compared with the United States, some countries experience longer waiting times. Commonwealth Fund (Osborn & Squires, 2016) conducted a survey that comparing health policies of adults in 11 developed countries in 2016. The results show that U.S. adults are more likely than adults in the other 10 countries to skip the needed care because of the cost even though America did well for the accessibility of health care. According to the survey, in 2015 about 33% of U.S. adults did not request health care services when needed because of the cost barriers as shown in Figure 3. On the other hand, only 6% of the U.S. adults waited two months or longer to see a specialist, while about 30% of the Canadian adults had such an experience. The comparison is presented in Figure 4.

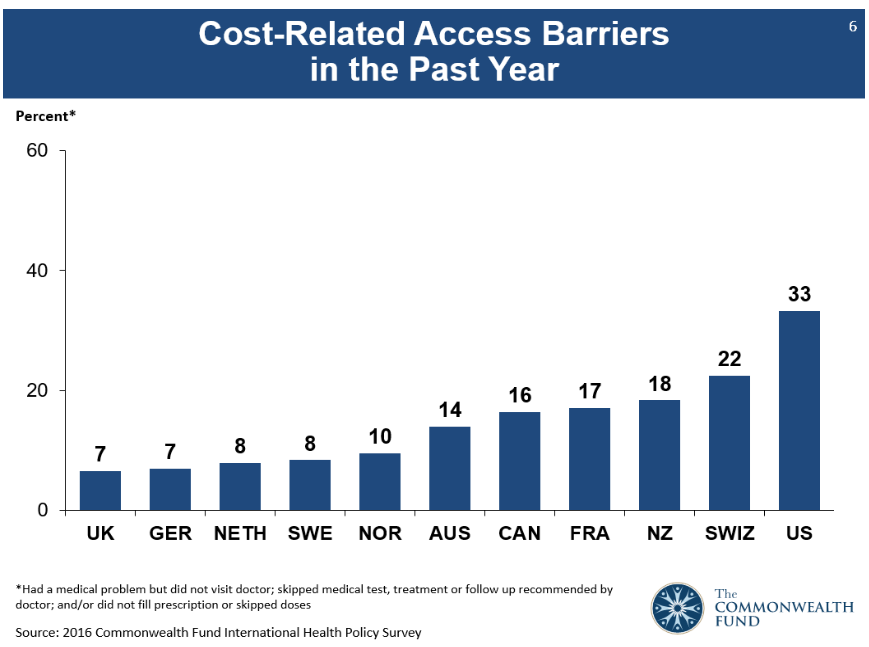

Figure 3. Cost-related Access Barriers in 2016 (Osborn & Squires, 2016)

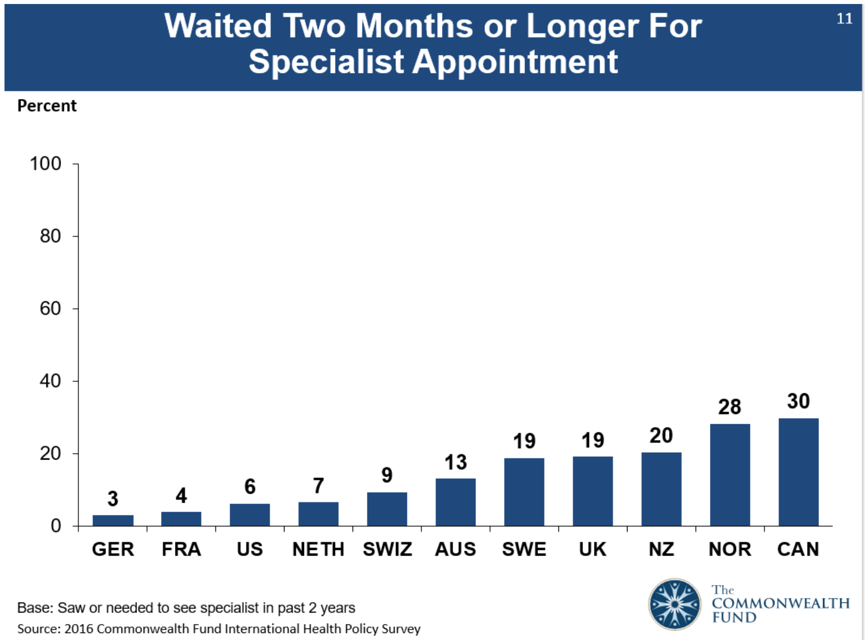

Figure 4. Percent of Patients Who Waited Two Months or Longer for Specialist Appointment (Osborn & Squires, 2016)

Long IWT could result in many negative effects. From the patient's perspective, long waiting times will cause worse general health perceptions, reduce patient's quality of life, and raise levels of anxiety (Oudhoff et al., 2007). What is worse is that longer waits often make the patients sicker. As a consequence, patients will have lower satisfaction and response with more negative reactions such as cancellations and no-shows (Ansell et al., 2017). According to Ryu and Lee (2017), indirect waiting times are positively correlated with no-show rates. However, shortening wait times may sometimes reduce a provider's revenue because longer appointment waits sometimes led to more costs from the patient's side (or insurance company) and therefore more profit for the provider. Another reason some health systems do not shorten IWT is due to the fact that reducing waiting time requires investment in systems. Even though long waiting times cause no-shows and patient dissatisfaction, reducing IWT is challenging. Although some health systems still choose to invest in shortening the IWT because this improves their competitiveness and efficiency, many choose to lower no-shows without reducing IWT on purpose (Ryu & Lee, 2017).

1.3.2. Appointment No-Shows

Appointment no-show is one of the most common and challenging issues for appointment scheduling. As shown in Figure 5, people do not show up for reasons in a variety of aspects (Mohamed et al., 2016). Forgetting the appointments is the primary reason for no-shows because the appointment date is scheduled too long from the scheduled date (Sharp & Hamilton, 2001; Cosgrove, 1990). Although there are generally four ways to remind the patient of the appointment, i.e., text message, phone call, direct mail, and email; medical offices sometimes fail to remind the patients (Hardy et al., 2001). Another big reason for no-shows is due to the cost barriers, especially in the U.S. (Osborn & Squires, 2016). Sometimes patients miss appointments because of emotional barriers, such as a negative perception toward the provider, frustration with a discomfort experience of the last visit, and fear of seeing a doctor or getting treatment (Salameh et al., 2012). Often, appointment no-shows are caused by misunderstanding, for example, patients may think that the provider disrespects their time or needs (Lacy et al., 2004). Also, patients may misunderstand the consequence of missed appointments due to the lack of communication (Lacy et al., 2004). Other patients missed their appointments due to logistical issues including trouble getting off work, daycare, horrible weather conditions, and lack of transportation (Sharp & Hamilton, 2001; Bean & Talaga, 1992). Besides, patients usually choose to skip appointments if they felt better (Lacy et al., 2004).

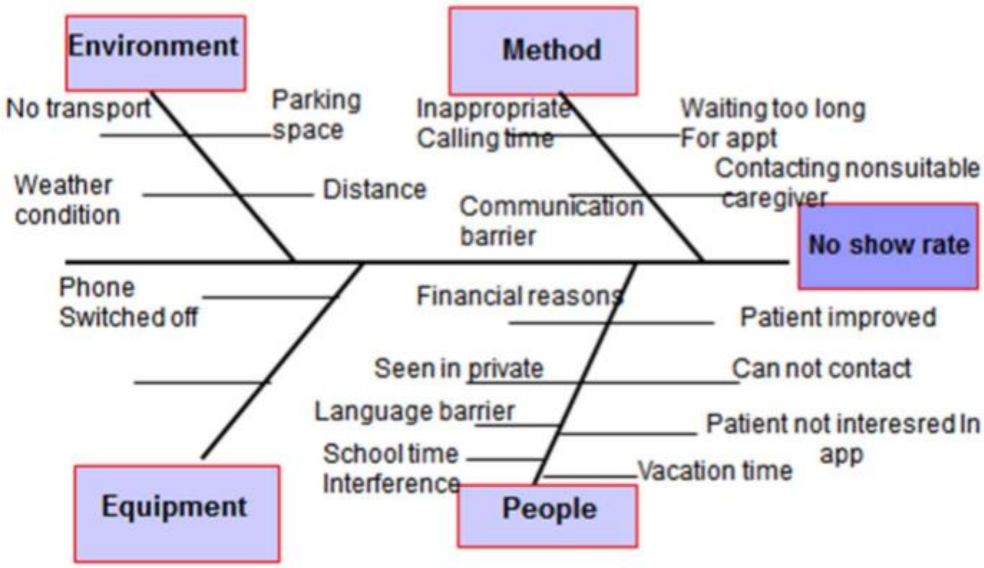

Figure 5. Reasons of Appointment No-shows (Mohamed et al., 2016)

No-show rates in community practices ranging from 5% to 55%, depending on the country, health care system, and type of practice (Hixon et al., 1999; George & Rubin, 2003). Figure 6 shows several average rates according to the type of practice. It should be noted that patients with mental disease are more likely to not show up for a scheduled appointment. Unfortunately, these patients are also the ones who gain the most by showing up (CROSSCHX, 2017). Missed appointments are not only harmful to patients' health but also very costly to providers. It is estimated that missed appointments cost the U.S. health care system more than $150 billion every year, and each no-show slot costs a physician $200 on average (Gier, 2017). Sometimes, no-shows could result in a daily loss of more than $1000 for one practice (Berg et al., 2013). Medical practices, researchers, and clinics have developed several tactics trying to decrease the possibility of no-shows (Hardy et al., 2001). While all these tactics have proven to be successful in reducing the number of no-shows, text messages seem to be the more cost-effective approach (Downer et al., 2005). Also, overbooking and missed appointment fee are often used to reduce the effect of no-shows. However, these can cause negative feedback and revenue loss. For instance, when all patients show up, overbooking may result in long direct wait times and increase the operational cost while decreasing patient satisfaction. While missed appointment fees do reduce no-show rates, they also lead to a negative perception of the provider as the patient feels punished for a service he does not use (McLean et al., 2016). Patient no-shows are a major problem for the current health care system for both providers and patients. A predictably low no-show rate can reduce wait times and increase clinic efficiency, thereby improving the quality of health care. However, it is difficult to find a solution that accommodates both sides, it is a trade-off. One day, people may find a perfect solution to eliminate patient no-shows, which is convenient and effective for the patients, and economically viable for the providers.

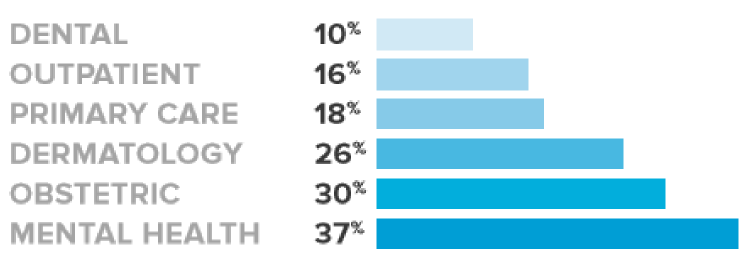

Figure 6. Average No-Show Rates According to the Type of Practice (CROSSCHX, 2017)

1.4. Research Objectives

The following research objectives have been identified:

- To study the relationship between appointment utilization and IWT.
- To predict the probability of no-shows for patients without profiling them.
- To develop an optimization model for capacity allocation under different scenarios.

1.5. Research Questions

As mentioned above, there are three research objectives this dissertation needs to achieve. For each objective, there are few research questions needed to be answered.

For the first objective, the research questions are: What kinds of methods or models could be used to project patient persistence with the appointments? Which method/model is the best among them for projection? And what is the trend of patient retention in the schedule as the IWT increases?

In terms of the second objective, three questions need to be considered: 1. Which variables should be selected as predictors? 2. Which models or approaches should be used to predict the daily no-show probability for a random patient? 3. Which method is the best or most sui for this objective?

As for the last objective, the first question is how do same-day demand and pre-schedule demand distributed? With the demand distributions identified, how to find an optimal number for the daily capacity? Based on the daily capacity, how many appointments should be scheduled for same-day appointments and pre-scheduled appointments each day?

2. UTILIZATION PROJECTION

In this study, appointment utilization refers to the percent of scheduled appointment slots that have been utilized, more specifically, the percent of the scheduled meetings that have been conducted between patients and physicians. To study this topic, this chapter covers the previous literature, research methodology, and data mining process of utilization projection for clinic appointment scheduling. For the first section, the literature reviews previous works that have been done in the related field, and most of the studies are based on the recent data regarding a domestic clinic or hospital in the United States. The second section talks about the research methodology used in this dissertation. The basic ideas, variables, and functions are introduced in this section. The third section discusses the data mining process of this dataset to better project the appointment utilization rate for this scheduling system. Besides, it demonstrates how to implement them in an Excel spreadsheet, to make them more accessible to healthcare professionals. As for the results, the in this chapter is threefold. First, it compares two proposed mixture distribution models to curve-fitting regression models in projecting outpatient appointment utilization. Second, the BG model provides insight to analyze the distribution of the probability of a risk event that prevents appointment from utilization. Last but not least, a competing risk analysis will be conducted to compare the survival probability and cumulative incidence between different appointment statuses. A summary concluding the status of research gaps and the significance of the study is presented at the end of this chapter.

2.1. Literature Review

Patient no-shows are a well-known problem in the health care industry. Many researchers have studied the cost of no-shows and provided some good advice to reduce no-shows. On the other hand, research on appointment utilization is not very common, especially on the relationship between appointment utilization and IWT. Projecting utilization rate should be done ahead of carrying out any action to improve appointment utilization because studying appointment retention helps identify the reasons and patterns of no-shows and cancellation trends so that better decisions can be made. Therefore, it is very important for health care organizations to accurately project appointment utilization to control costs and improve service.

2.1.1. Background

One of the main factors affecting appointment utilization, and one over which care providers may have some control, is IWT. It is well accepted that the longer the IWT of an appointment, the less likely the appointment will be utilized. That is, the more likely the appointment will be canceled (by either the patient or the care provider) or the patient will simply not show up to the appointment (a “no-show”).

The process by which appointment utilization is determined for an individual patient can be modeled in the following way. Initially, the patient intends to keep the appointment. However, each day within the IWT, there is a probability (Θ) that an event occurs which will result in appointment no-show or cancellation. If such an event occurs before the appointment date, the appointment will not be utilized. As Liu, Ziya, and Kulkarni (2010) mentioned, such a model should allow the value of Θ to vary between individuals. Modeled in this way, appointment utilization is analogous to models of customer retention in subscription services, or a patient's adherence to medication.

2.1.2. Beta-Geometric (BG) Model & Discrete Weibull (BdW) Model

Similar to the analysis of customer retention and treatment adherence over time, this study intends to understand the relationship between appointment statuses and IWT. To that end, four "curve-fitting" regression models, a beta-geometric (BG) model, and a discrete Weibull (BdW) model, which is an extension of the BG model, were studied in projecting the appointment utilization for an outpatient clinic.

The BG model has been proving to be a robust model for projecting cohort-level retention rates and related patterns into the future. Therefore, it is widely used in studies focused on projecting customer behavior. For example, Fader and Hardie (2007) used it to project customer retention and customer lifetime value. In particular, they demonstrated its superiority to common curve-fitting regression models at estimating the probability of customers leaving a business over time (known as customer churn). In contrast, the application of the BG model in health care is still rare, it only having been used to forecast patients' persistence to medication intake and refill, and patient churn on a medical test (Lee, Fader, & Hardie, 2007).

Compared with the BG model, whose individual propensity to churn does not change over time, the BdW model allows individual-level churn probabilities to increase or decrease over time (Fader, Hardie, Liu, Davin, & Steenburgh, 2018). Regression models are a familiar tool for most health care professionals and such accessibility makes them a logical choice for attempting to explain the relationship between IWT and appointment utilization. However, such models are ineffective for projecting a similar phenomenon, such as the aforementioned customer retention and patient persistence. Therefore, this study compared the regression models with the lesser-known BG model and BdW model, which are based on the previously described process in which each day can lead to an event preventing appointment utilization and has been shown effective in predicting similar phenomena.

2.1.3. Contributions

The application of BG/BdW in health care has mainly focused on forecasting the patient churn for medical treatments or tests. Meanwhile, there is not any research that fits the BG/BdW model to patient no-shows or appointment utilization. Therefore, this study fills that gap by applying BG/BdW to solve the outpatient appointment problem because the patient no-show problem can also be considered as a patient persistency problem. Instead of predicting no-shows or appointment utilization rates at a single point, we can project how many scheduled patients remain in the system over time.

Another contribution of this study is that although patient no-shows have been well addressed in the literature, it is rare to see studies considering heterogeneous appointment utilization rates which depends on realistic factors like indirect waiting time. The two models in this study allow for heterogeneity in the risk events to stop patients from utilizing appointments.

In particular, the BdW model allows for changes in individual propensity in the churn process. Therefore, work in this direction is recommended.

Furthermore, this study has studied the relationship of different appointment statuses with the indirect waiting time. Other than commonly studied appointment no-shows, this study also studied appointment utilization, appointment cancellation, and appointments canceled by the physicians. The results can help answer two questions: a) “which appointment status would lead to more risk of patient churning as the indirect waiting time increases?" and b) “ How would these appointment statuses change over time?”. By answering these questions, it enriches understanding of the relationship between appointment status and indirect waiting time and how appointments “survive” from appointment risks.

2.2. Methodology & Model

One of the most straightforward and accessible ways to quantify the relationship between IWT and appointment utilization is with curve-fitting regression models. As expected, appointment utilization decrease as the IWT increases and not necessarily in a linear way. Therefore, a variety of regression models shown below were taken into consideration:

$$\text{Linear: } y = a_{lin} + b_{lin}t \quad (1)$$

$$\text{Exponential: } \ln(y) = a_e + b_e t \quad (2)$$

$$\text{Quadratic: } y = a_q + b_1 t + b_2 t^2 \quad (3)$$

$$\text{Logistic: } \ln(y/(1-y)) = a_{log} + b_{log}t \quad (4)$$

Here y is the appointment utilization rate, t is the IWT, the α parameters are constants/intercepts, and the β parameters are coefficients. These curves can be fit to data using standard regression procedures.

In addition to the traditional regression models, this study also introduced the BG model for quantifying the relationship between appointment utilization and waiting time. Specifically, the BG model considers that for each day of the IWT there is a probability Θ that an event occurs which prevents the appointment from being utilized. This probability Θ can differ across patients but is assumed that in the patients' population it is distributed as the Beta distribution with positive parameters α and β:

$$f\,(\Theta = \theta) = [\theta^{\alpha\text{-}1}\,(1\text{-}\theta)^{\beta\text{-}1}] / \,B(\alpha, \beta) \tag{5}$$

where B(α, β) is the classic beta function. Given the parameter θ, for a given IWT τ, the probability that an event causing the appointment to not be utilized first occurs on day T follows the geometric distribution:

$$P(T = t \mid \Theta = \theta) = \theta\,(1\text{-}\theta)^t, \text{ for } t \leq \tau \tag{6}$$

Combining (5) and (6) provides the following probability that no event occurs within T days:

$$P(T = t) = B(\alpha+1, \beta+t) \,/\, B(\alpha, \beta), \text{ for } t \leq \tau \tag{7}$$

In practice, these values can be computed recursively without recourse to the beta function as

$$P(T = 0) = \alpha \,/\, (\alpha+\beta) \tag{8}$$

$$P(T = t) = (\beta+t\text{-}1) * P(T = t\text{-}1) / (\alpha+\beta+t), \text{ for } t \leq \tau \tag{9}$$

Note that in the BG model, for a given patient, the probability Θ that an event occurs which prevents the appointment from being utilized does not change over time. Some researchers suspect that there should be a strong dependence between this probability Θ and the IWT (Moe & Fader, 2009). In this case, the Weibull distribution, which allows for the probability Θ to increase or decrease over time (Rinne, 2009), is possibly a better choice than the geometric distribution. Further, it can be argued that the IWT t should be treated as the integer part of a continuous lifetime, the BG model will then be replaced with a beta-discrete-Weibull (BdW) model (Fader, Hardie, Liu, Davin, & Steenburgh, 2018). According to Rinne (2009), the cumulative distribution function of the Weibull distribution is

$$F(t) = 1 - \exp(-\lambda t^c), \lambda, c > 0 \tag{10}$$

where c is the “shape” parameter and λ is the “scale” parameter. Then, the survival function that the patient will not experience a disruption to appointment utilization before time t is

$$S(t)=1 - F(t) = \exp(-\lambda t^c) \tag{11}$$

Letting $\exp(-\lambda) = 1-\theta$, it follows the corresponding survival function in a discrete-time setting:

$$S(t \mid \theta, c) = (1 - \theta)\ t^c, t = 0, 1, 2, ..., \tau \tag{12}$$

Assuming heterogeneity in θ is characterized by a beta distribution with positive parameters α and β as in (5), then by combing (5) and (12) provides the following survival function:

$$S(t \mid \alpha, \beta, c) = \int_0^1 S(t \mid \theta, c)*f(\theta \mid \alpha, \beta)\, d\theta = B(\alpha, \beta + t^c)/ B(\alpha, \beta),\ t = 0, 1, 2, \ldots, \tau \tag{13}$$

Therefore, the probability that no event occurs within T days is:

$$P(T = t \mid \alpha, \beta, c) = S(t-1 \mid \alpha, \beta, c) - S(t \mid \alpha, \beta, c) = [B(\alpha, \beta + (t-1)^c) - B(\alpha, \beta + t^c)]/ B(\alpha, \beta) \tag{14}$$

where $t \le \tau$.

Including a constant term a_s, as in the regression models, we can now project the appointment utilization for appointments with a IWT of t as

$$y = a_s + P(T = t) \tag{15}$$

2.3. Data Description & Model Fitting

2.3.1. Data Description

The data was originally obtained from an outpatient clinic practicing family medicine in the state of New York, it consists of 19 variables over 12 months from July 2016 to June 2017. During this period, the data of each working day is collected, and each day there are many samples recorded. Some of these data are redundant and duplicate, thus data cleaning is needed. The general description of this dataset is shown in Table A1 in Appendix A. Many variables in this dataset are categorical. Some of them are nominal variables, such as DOW (day of the week), session name, and appointment status. Some of them are ordinal variables, such as appointment date, scheduled date, and appointment time. Besides, there are some binary variables, 1 means true and 0 means false in the context. For each appointment scheduled at the clinic, it includes the date on which the appointment was scheduled (“Scheduled Date”), the date of the appointment (“Appointment Date”), and the time difference in days between the scheduled date and the appointment Date (Appointment Date-Scheduled date) is the IWT.

The data also included the “Appointment Status” which indicated whether or not the appointment was utilized and, if not, why. This resulted in four categories for Appointment Status: “ARR”, “BMP”, “CAN”, and “NOS”. Status “ARR” indicates that the appointment was utilized as arranged. Status “BMP” indicates that the appointment was canceled by the physician, as opposed to the patient. Status “CAN” indicates that the appointment was canceled by the patient. Finally, “NOS” means that the patient was a no-show and did not show up to the appointment. In determining appointment utilization, only status “ARR” indicates that an appointment has been utilized, where “BMP”, “CAN”, and “NOS” are all risk events, which won’t be considered as appointment utilized. Although some canceled appointments were rescheduled, there is no information indicates which canceled appointments had been utilized. Hence, these four appointment statuses were used to calculate the appointment utilization rate for each IWT as the percentage of appointments that were utilized as arranged (“ARR”). Similarly, the no-show rate, cancellation rate, and “BMP” rate based on the appointment statuses can be calculated.

Of the total 114,029 observations in the database, 63.07% of appointments were utilized, 22.83% of appointments were canceled by the patients, 11.63% of appointments were no-show, and the rest of 2.47% of appointments were canceled by the physicians. The IWTs ranged from 0 (same-day appointments) to 184 days. However, many IWT values only contain very few observations in the data (e.g., there was only one appointment with a 184-day IWT). Therefore, to ensure a reasonable number of observations for each IWT used in our analysis, this study restricted the analysis to IWT with at least 30 observations. This resulted in 113,887 appointment records across 97 different IWTs ranging from 0 to 98 days.

Also, an open resource dataset (https://github.com/cbrands/investigate-a-datase) was found from GitHub. This dataset includes information from over 100,000 medical appointments in Brazil on patient no-shows. Unlike the data obtained from the medical center, this dataset does not include appointment statuses, however, it has information about no-shows and thus the appointments without no-shows can still be treated as utilized appointments. After processing, the revised data will only include the variables used in this study as shown in Table 3. Similar to the dataset obtained from the medical center in New York state, the IWTs were restricted with at least 30 observations.

Table 3. Useful Open-Source Data Variables and Description

Variable	**Description**
Scheduled Day	The date when the patient is scheduled.
Appointment Day	The date of the appointment.
No-Show	Appointment no-show status, 1 means no-show, 0 means otherwise.
IWT	The time difference between appointment date and scheduled date.

2.3.2. Model Fitting

Given the data collected, it needs to fit the four regression models and two mixture distribution models to the Appointment Utilization over time. The regression models can be fit using any standard statistical package, such as Excel’s Data Analysis add-in. The results for the regression models appear in Table A2. For fitting the BG model, the values of α and β need to be chosen. To our knowledge, there is no standard software package for calculating these values. However, one can select values of as, α and β geared at minimizing the mean square error (MSE) by using Excel's Solver. To that end, the recursive relationship of equations (8) and (9) in the Excel spreadsheet must be implemented first.

Figure A1 and Figure A2 in Appendix A demonstrated the implementation of the recursive relationship with initial values of a_s=0.5, α=1, and β=2. Figure A1 shows the equations that have been entered into Excel and Figure A2 shows the corresponding values to which they equate. Once the recursion is set up in the spreadsheet, Excel Solver can be used to select better values for as, α and β. Figure A3 shows how the minimization of the MSE was set up in Excel Solver. Note that no bounds on the variables were used with the Solver, requiring that the corresponding checkbox be unchecked in the options pane for the GRG Nonlinear Solver. The corresponding results along with the parameters for the models will be discussed in the next section.

Similar to the BG model, Figure A4 represents the equations of the BdW model with initial values of a_s=0.5, α=1, β=2, and c=1. Once the recursion is set up in the spreadsheet, Excel Solver can be used to optimize the values for all parameters as, α, β, and c as in Figure A4. The corresponding results along with the parameters for the models will be discussed in the next section.

2.4. Outputs & Results

2.4.1. Comparison Results for Appointment Utilization

As shown in Table A2, the BdW model provides the best fit with an R^2 of 0. 4670 and MSE of 0.0015. The BG model is second best with R^2 of 0.4565 and MSE of 0.0016, followed by the exponential regression model with R^2 of 0.3068 and MSE of 0.0021. The worst model to fit the data is the quadratic regression model with R^2 of -1.567 and MSE of 0.0077. The negative R^2 indicates the quadratic regression model provides a worse prediction than just taking the average appointment utilization, which is a horizontal line. In addition to those two measurements, the mean absolute error (MAE) and the root mean squared error (RMSE) were also calculated to better interpret the model's accuracy in case there are outliers as shown in Appendix A. To visualize the difference between the models, the curves fit by the regression models and two mixture distribution models against the actual appointment utilization rates were plotted in Figure 7 and a more detailed Figure A5. From the plots, it is clear that the advantage of the BG and BdW models is their ability to capture both the high and decreasing appointment utilization rates for short IWT and the lower and flatter appointment utilization rates for longer IWT. The other models all fail to properly capture the effect of short IWT. Besides, a residual plot is generated for each estimation as shown in Figure 8. This plot also indicates a great accuracy of BG and BdW models for utilization projection for short IWT.

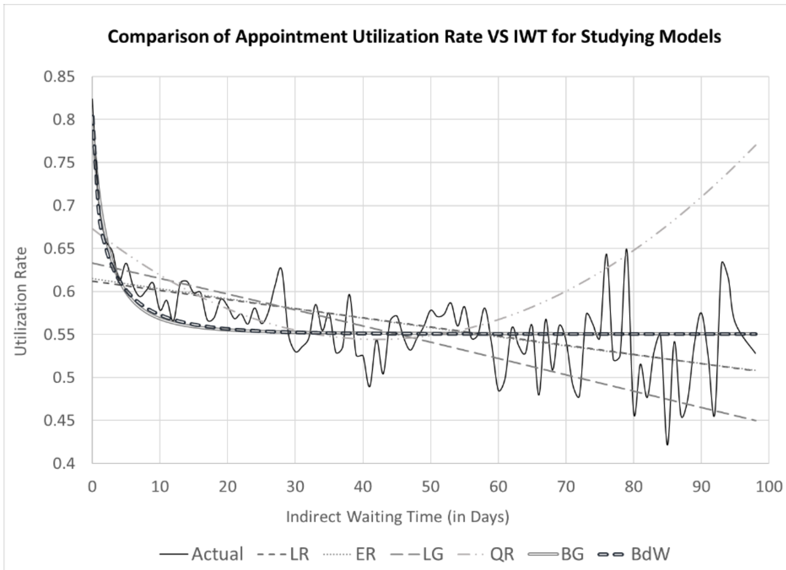

Figure 7. Model Estimates Versus Actual Utilization Rate

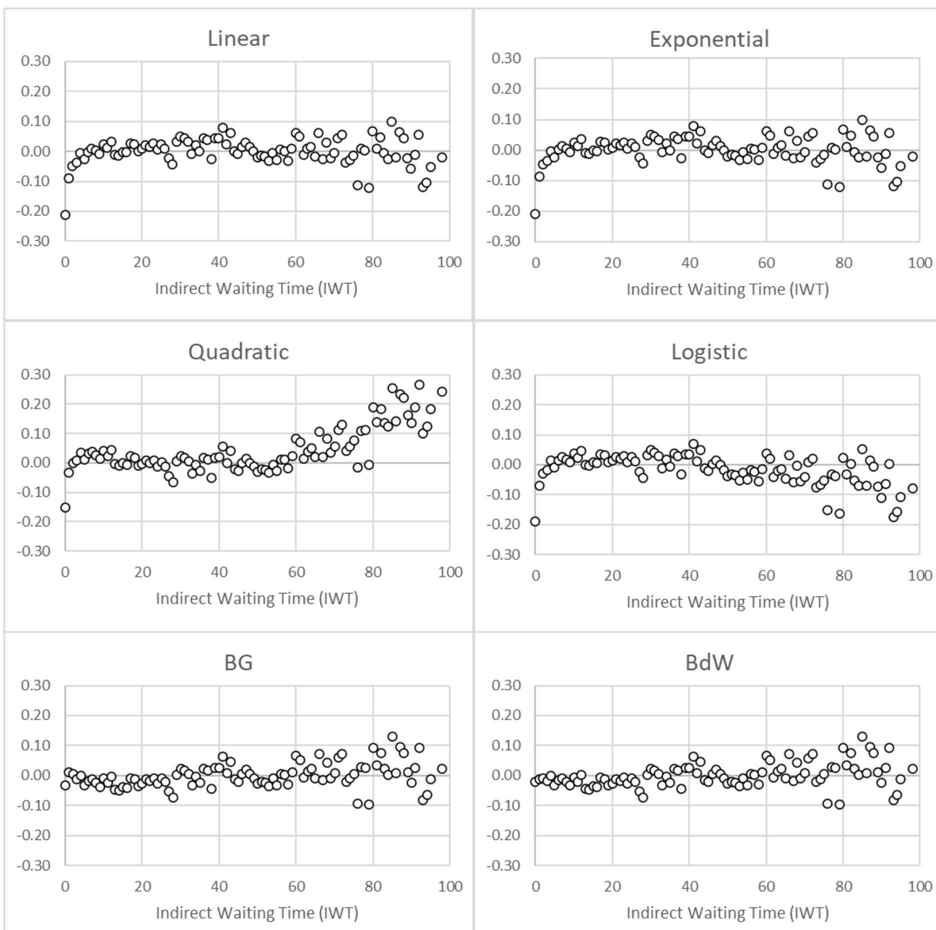

Figure 8. Estimation Residuals for Different Models

2.4.2. Distribution of Probability Parameter Θ

In addition to its benefits for using IWT to predict appointment utilization, the BG model also provides insight into the patient population allowing us to analyze the distribution of the parameter Θ. Note that the BG model was preferred as opposed to the BdW model for analyzing Θ as it provides a more direct interpretation of Θ without complications of switching between the continuous and discrete times or accounting for the shape parameter c. Recall that for a randomly selected patient Θ follows a beta distribution with parameters α and β on the interval [0, 1]. For the fit values of $\alpha = 1.8014$ and $\beta = 5.6545$, the corresponding probability density function (PDF) and cumulative density function (CDF) for Θ are shown in Figure 9 and Figure 10. From Figure 9 it can be seen that the distribution of Θ across patients is highly skewed to the right, indicating that the majority of patients have a fairly low value of Θ but with a significant number of outliers having large values of Θ. This means that most patients have a fairly low probability of an event preventing them from utilizing an appointment, but some outlier patients have a considerably larger likelihood of an event resulting in them not utilizing an appointment. This suggests that identifying outlier patients could help manage no-shows and cancellations. Similarly, Figure 10 shows that values of Θ greater than 40% are quite rare.

On the other hand, the expected IWT of an appointment would not be utilized E(T) equals1/Θ since it follows a geometric distribution as mentioned in equation (6) in section 2.3. Using the CDF of Θ, we can compute the CDF of 1/Θ, the expected time until an event results in an unutilized appointment, as shown in Figure 11. For instance, F(1/Θ = 20) equals 94.07% in the means that 94.07% of patients have an expected time until an event which leads to an unutilized appointment within 20 days or less. Reworded in terms of theta instead of 1/theta, it means that 94.07% of patients have a greater than ϴ=1/20=5% chance per day of an unutilized appointment.

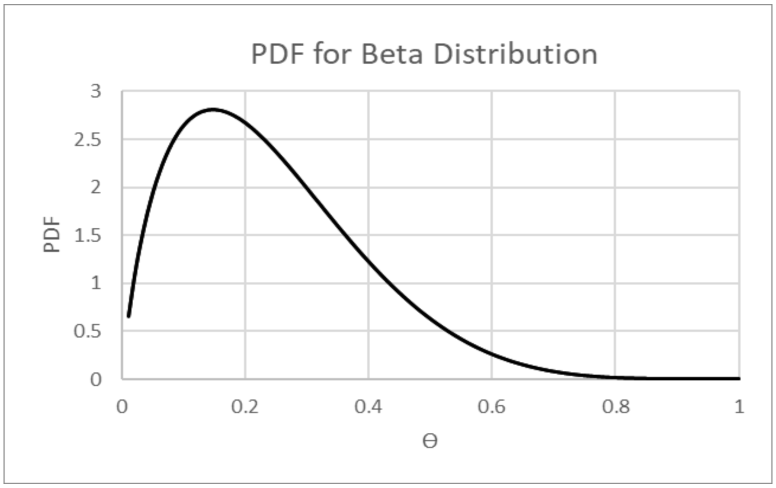

Figure 9. Probability Density Function for ϴ

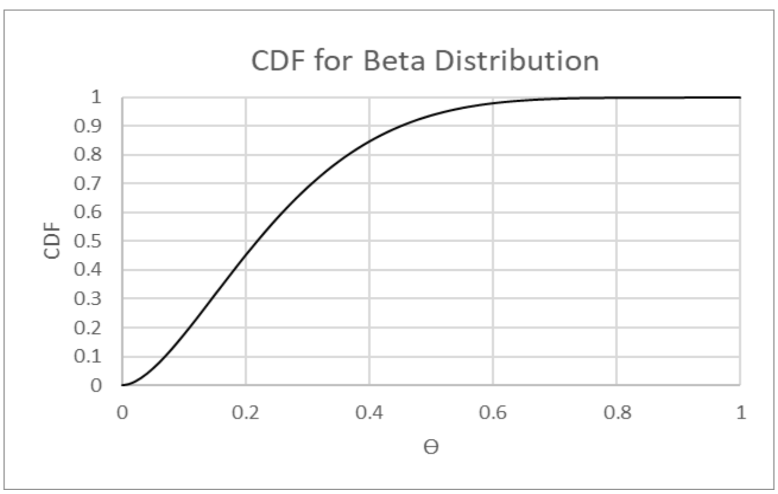

Figure 10. Cumulative Density Function for ϴ

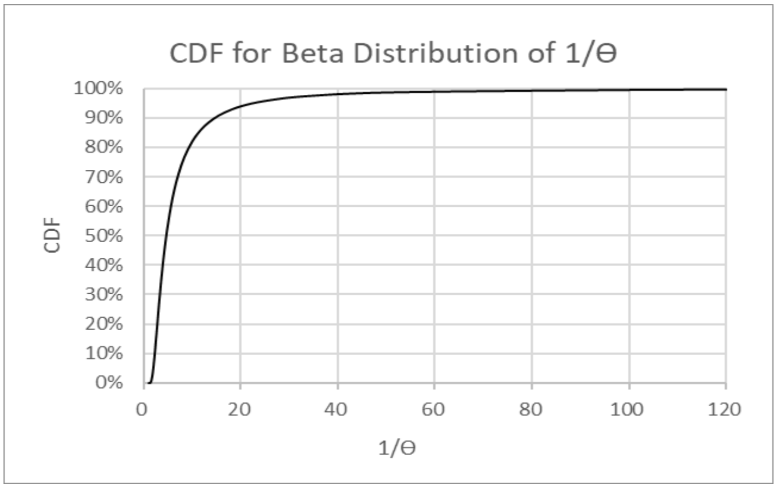

Figure 11. Cumulative Density Function for 1/ ϴ

2.4.3. Comparison Results between Different Appointment Statuses

In this section, the models were further explored at the different appointment statuses (NOS, CAN, and BMP) which correspond to an appointment not being utilized. A competing risk analysis was performed to establish an overall picture of how each of the three statuses contributes to unutilized appointments. Furthermore, the same mixture distribution and curve-fitting models were applied to each appointment status to determine the relationship with the IWT.

Competing risk methods are widely used in healthcare research, particularly in cancer studies, where multiple potential outcomes like death and other failure events will occur (Dignam, Zhang, & Kocherginsky, 2012). In traditional survival analysis, the reasons for death are usually irrelevant to the analysis. In a competing risk survival analysis, each death event is reviewed. A similar situation applies to this study, there are three types of events that lead to unutilized appointments: no-show, appointment cancellations by patients, and appointment cancelations by physicians. The setting is similar to death research, where competing risks occur when patients experience one or more events that lead to death (Noordzij et al., 2013). As shown in Figure 12, the actual rates of the three failure events were compared. Generally speaking, the patient cancelation rate is higher than the other two events, and the no-show rate is higher than the physician cancelation rate. This indicates that most patients will call for cancelation rather than just not showing up. Also, as IWT increases, the physicians are more likely to cancel the appointments, but this trend does not apply to other failure events. For the other two failure events, the rate will increase dramatically in the first couple of days and then fluctuate as the IWT increases, which is not necessarily an increasing trend.

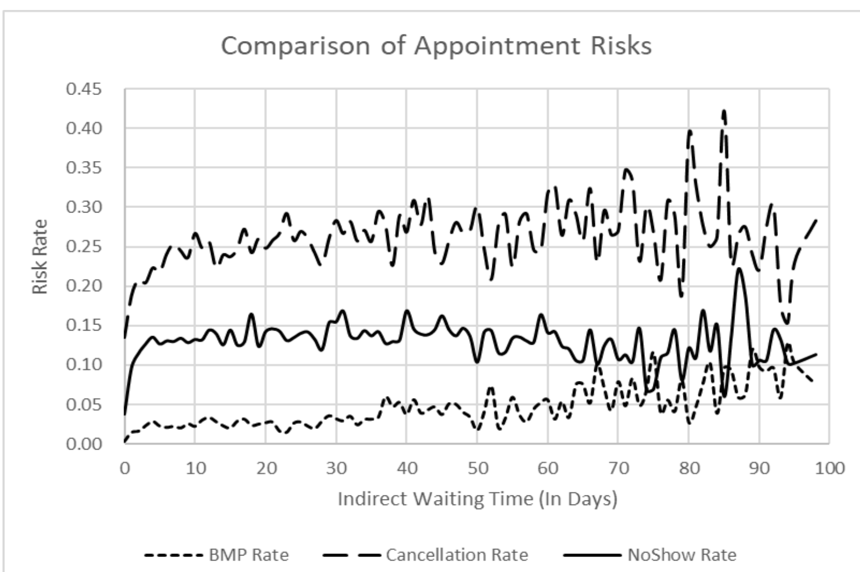

Figure 12. Comparison of Actual Rates of Appointment Cancelation, No-show, and BMP

Also, a survival analysis using SAS 9.4 for Windows was conducted. The primary reason for appointment non-utilization was appointments canceled by patients, which accounted for 61.81% of total non-utilized appointments, followed by appointment no-shows, which was about 31.49% of total non-utilized appointments, and the physician cancelations only accounted for 2.47% of the total non-utilized appointments. Most of the appointments were dropped in the first couple of weeks, which does not necessarily indicate a high drop rate as most appointments were scheduled with short IWT.

To identify the relationship between IWT and the three failure events, the same models were applied to project the other appointment statuses. As the appointment utilization decreases with IWT, the corresponding failure rates increase as the IWT increase. In this case, a constant term was used to minus the estimated appointment utilization. Accordingly, the functions for the different models can then be expressed as:

$$\text{Linear: } y = a_{lin} - b_{lin}t \tag{16}$$

$$\text{Exponential: } \ln(y) = a_e - b_e t \tag{17}$$

$$\text{Quadratic: } y = a_q - b_1 t - b_2 t^2 \tag{18}$$

$$\text{Logistic: } \ln(y/(1-y)) = a_{log} - b_{log}t \tag{19}$$

$$\text{BG \& BdW: } y = a_s - P(T = t) \tag{20}$$

The model evaluation metrics of all models for different appointment statuses are listed in Tables A3-A6. The results indicated that the BdW model outperforms other models for predicting utilization, no-show, and cancellation rate due to its relatively high R^2 values and low

MSE, MAE, and RMSE. This demonstrates that the BdW model is a better choice for studying the relationship between IWT and patient's appointment behaviors, showing the promise of the BdW model for representing such relationships in healthcare. However, for appointments canceled by the physicians (BMP), the BdW model does not perform well in estimating its rate. This indicated that there is a significant difference between the patient's behavior and physician's behavior in appointment cancelation, as a result, other models may be more appropriate for projecting BMP.

2.4.4. Numerical Test with Open Source Data

To further investigate the robustness of this model and test the generalizability of the approach, this study repeated the analysis of open-source data described in the former section. As seen in Table 4, the BdW model provides one of the best fits with the largest R^2, smallest MSE of 0.0022, and second smallest MAE and RMSE. However, in this case, the quadratic regression model also provides a good fit with the second-best with R^2 and MSE, but the best MAE and RMSE, specifically edging out the performance of the BG model. The worst model to fit the data is the exponential regression model, R^2 of 0, and MSE of 0.0035. This indicates the exponential regression model essentially used the average of the appointment utilization rate. Figure 13 and Figure A6 show the curves fitting of the regression models and two mixture distribution models against the actual appointment utilization rates. From the plot, it is noticed that the BG model and BdW model outperformed the curve-fitting models in predicting appointment utilization rates for short IWTs, while the quadratic model picks up a slightly increasing trend for the long

IWTs. These results suggest that different contexts (e.g., therapeutic areas) may result in different relationships between IWT and appointment utilization, requiring the use of a variety of models.

Table 4. Comparison of Model Measurements for Open Source Data

	Models					
Parameters	**LR**	**ER**	**QR**	**Logistic**	**BG**	**BdW**
R^2	0.010511	0.000000	0.351637	0.009929	0.318176	0.373882
MSE	0.003462	0.003499	0.002269	0.003464	0.002386	0.002191
MAE	0.044046	0.043612	0.034182	0.044014	0.038047	0.035153
RMSE	0.058841	0.059152	0.047630	0.058858	0.048844	0.046806

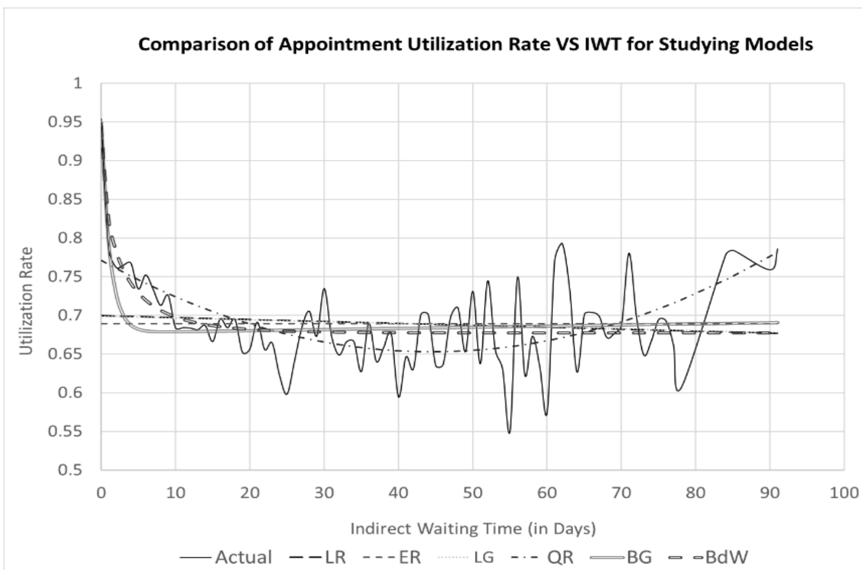

Figure 13. Model Estimates Versus Actual Utilization Rate for Open-Source Data

2.5. Summary & Discussion

Many articles have discussed how to predict no-shows, most of them focus on estimating the individual no-show probability and identifying the correlation between appointment no-shows and patient characteristics and behaviors. However, very few studies have focused on appointment utilization and its relationship with IWT. Therefore, the contribution of this paper is that the mixture distribution model, which was originally designed for customer behavior studies, is implemented to project appointment utilization for the first time. Besides, it has shown that BG and BdW models outperform curve-fitting regression models for using the in projecting appointment utilization. The BG model also allowed for analysis of the patient population's reliability about utilizing appointments. Specifically, we can see that the likelihood of a patient not utilizing an appointment is skewed across the patient population. The results indicated that most patients have a relatively low probability of not utilizing an appointment, but there are some outliers with high no-show or cancelation probability in the population.

Though specialized routines for implementing the mixture distribution models are not commonly available as are those for regression models, this study has demonstrated how the mixture distribution model can be implemented using commonly available spreadsheet tools. Regarding predicting appointment utilization, the skewed distribution of Θ across the patient population suggests pursuing approaches that identify outlier patients. This study further analyzed the three failures resulting in unutilized appointments. The results show that cancelations by patients to be the greatest cause of unutilized appointments, followed by patient no-shows and physician cancelations, respectively. Also, it is easy to see that the mixture distribution models worked well at predicting patient cancelations and no-shows, but not so well for physician cancelations. Finally, to test the generalizability of the results, an analysis on open-source data was repeated, and the mixture distributions proved to work well, but at this time, a quadratic regression model also shows a good projection accuracy. Taken together, these results highlight the promise of mixed distribution models in predicting no-show rates but also demonstrate the need for a variety of models to be used as patterns can change in different contexts.

3. NO-SHOW PREDICTION

Appointment no-show is one of the most common issues for appointment scheduling, and no-show prediction is one of the most challenging topics among no-show studies. Therefore, many studies have been conducted to seek the methods able to predict the probability of no-shows and the factors that influence patients' behavioral patterns. Almost all of these studies are based on data that have major impacts on patient no-shows. However, it is not always easy to obtain data due to business, privacy, or ethical issues. What's more, inequality in appointment scheduling is also one of the issues that should be taken into consideration. Therefore, it is better not to include the patient's demographic and personal information like race and age scheduling a patient to ensure equality. In this case, some recommend or popular predictive models, such as logistic regression, might not work under this circumstance.

Therefore, this chapter will focus on the literature review, methodologies, and models of appointment no-show prediction. In the first section, the literature review covers the background, factors, and models used in various studies. The second section talks about the research methodology used in this dissertation, and three models are introduced in this section. The third section discusses the outputs of the three models introduced in this study, the results are compared, and some conclusions can be generated. Lastly, a discussion of the research gaps and the contributions of the study are presented at the end of this chapter.

3.1. Literature Review

3.1.1. Background

No matter how perfect the system is, there are still no-shows. What's worse, it is very harmful to both patients and providers, as Gupta and Denton (2008) said, "Late cancellations and no-shows can lead to poor resource utilization, lower revenues and longer patient waiting times". Thus, how to accurately predict appointment no-shows becomes extremely important. No-shows can be predicted through data mining, which is a process of finding patterns and relationships between variables in large data sets using methods from machine learning, statistics, and database systems. B1 in Appendix B summarized some of the modeling patients' no-shows in recent years. In general, the factors used to predict the no-shows can be classified into the following categories:

- Patients' demographic information, such as race, age, gender, and income level.
- Appointment information, such as appointment time, attendance records, etc.
- Clinical information, like clinical characteristics and prescriptions.
- Provider's information, such as provider type and gender.
- Environmental factors, such as weather, distance, and transportation.

Although there are other types of factors that could be used to predict no-shows, most of the factors belong to the five categories mentioned above. Dantas et.al. (2018) have conducted a systematic literature review of the no-show in appointment scheduling, 105 articles and review papers were studied, and they found 46 factors were assessed in previous studies. Among those factors, the following have been identified as significant determinants of appointment no-shows: patient’s age; socioeconomic status; place of residence; ownership of private insurance, lead time, and prior no-show history (Dantas et. al., 2018).

From the factors above, it is easy to find that patients’ demographic information is extremely useful for no-show prediction. However, this violates the principle of equality if healthcare organizations schedule a patient differently based on one’s demographic information. Inequality in healthcare access is a common issue all around the world. Researchers have discovered that in the United States, people with low income have worse healthcare access when compared with others (Dickman, Himmelstein, & Woolhandler, 2017). This situation is not realized in the United States, pro-rich inequity in healthcare, especially for outpatient care, is a common issue worldwide (Allin, 2008; Liu, Hsiao, & Eggleston, 1999; Dror, Koren, & Steinberg, 2006). Other articles also addressed the inequalities of healthcare access among different races (Thomson, 1997; Lorence, Park, & Fox, 2006), ages (Fitzpatrick et al., 2004), gender (Namasivayam, Osuorah, Syed, & Antai, 2012), place of residence (Hartley, 2004), education (Plug et al., 2012), cultures and languages (Ngo-Metzger, et al., 2003), and people with disabilities (Lagu, Iezzoni, & Lindenauer, 2014). Under these circumstances, many researchers believe predicting patient no-show rates and making corresponding appointment schedules based on patient's demographic information is unethical and result in inequality in accessing healthcare.

Apart from the equality principle, patient's privacy and data security are becoming more and more important due to the development of electronic health records (EHRs) and big data technologies, which significantly improved the efficiency of data collecting and quantity of data collected (Kamble, Gunasekaran, Goswami, & Manda, 2018). EHRs contain information that is highly sensitive, such that leakage of this data violates the patient's privacy rights and may lead to serious consequences (Kaletsch & Sunyaevwhere, 2011). For this reason, it is not always easy to obtain demographic information, and it might violate patient's privacy rights to require patient's confidential data when making a healthcare appointment (Hong, Patrick, & Gillis, 2008). Similar to clinical information, it is either unethical to treat patients differently based on one's clinical condition, or it is hard to obtain enough information due to privacy protection and data security. In terms of the environmental factors, they are not only hard to collect and record, but also out of providers' control. Due to these reasons, this study focused on predictive model construction using limited data, which does not include any personally identifiable information that can be used to profile patients.

3.1.2. Model Comparison

As shown in B1, the most popular approach used to project appointment no-shows is the multivariate logistic regression model because the model has several advantages (Tufféry, 2011). First, logistic regression can deal with both independent variables and dependent variables that are not normally distributed. Second, it can handle nonlinear effects so that the independent variables and dependent variables do not have to be linearly related. Also, it can be used to generate probabilities of different levels for each observation in the response variable and thus can be used to classify new observations. Further, it has fewer requirements for variables, no homogeneity of variance assumption is needed, and independents can be either interval or unbounded. Last but not the least, the algorithm is easy to implement, computation is easy and fast, and low storage resources are needed. Simply speaking, it is a model that benefits data matching and easy to use.

Nevertheless, a major disadvantage of logistic regression is that it requires all observations to be independent of each other; otherwise, the model will tend to overweight the significance of them (Tufféry, 2011). Because of this, this study also proposed a general linear model (GLM) with interaction terms using a logit link function, which converts the probabilities of a binary response variable to a continuous scale so that it can be modeled with linear regression. With this model, the effects of each factor and interactions between factors can then be identified. However, it increases the complexity of interpretation of interactions in a GLM due to the effect of the link function, and misinterpretation is a central issue (Tsai & Gill, 2013).

Another issue of logistic regression is that it relies heavily on indicator selection (Tufféry, 2011). If the analysis includes the wrong factors or misses some important explanatory variables, which may occur in this study, the model will have litter predictive value. In other words, without enough features, the logistic regression model tends to underfit data and leads to a large bias. Further, logistic regression is sensitive to extreme values and missing values of variables (Tufféry, 2011). For these reasons, a decision tree model was carried out because it is a nonlinear model, which is less likely to underfit data when dealing with non-linear data, and it is robust to outliers and missing values. Besides, it is easier to interpret the results. Nevertheless, a large decision tree could indicate a pruning strategy that does not accurately capture the data statistics (Long, Griffith, Selker, & Dagostino, 1993). Thence, choosing the right algorithm to prune the tree for the decision tree model is very important for our study.

The most common decision tree algorithms are Classification and Regression Trees (CART) and C4.5 (an algorithm developed by Ross Quinlan in 1993), both tend to select variables with many possible splits or many missing values, which makes them easily overfit the data (Hothorn, Hornik, & Zeileis, 2006). Therefore, a so-called conditional inference trees (CTree) model was used to build our decision trees. Unlike CART and C4.5, Ctree uses significance test procedures to measure the association between responses and covariates rather than selecting the degerminators that maximize an information measure of node impurity (Hothorn, Hornik, & Zeileis, 2015). In this case, a pruning tree is not necessary for this algorithm; instead, selecting an appropriate alpha value and minimum split size is required.

3.1.3. Contributions

The main contributions of this study can be summarized from three aspects.

First, it demonstrated the ability to predict no-shows without profiling patients. Unlike previous studies, which require demographic information for prediction, this study used only appointment information for no-show prediction, providing a way to help the provider make a better appointment schedule and treat all patients equally at the same time. Although it is uncertain if the proposed prediction models can be generalized to other clinic settings, the methods and the analytic process used in this study can be extended and adapted.

Second, it is the first time a conditional inference tree (Ctree) model was used in predicting appointment no-shows. Unlike other decision tree models, Ctree uses recursive partitioning of the dependent variable based on the value of correlations, which avoids variable selection bias towards variables with many possible splits or missing values. The trees not only give the expected probability of no-shows like other classification and regression algorithms but also help us to better interpret the results with a tree plot. Hence from a practical perspective, this is critical as it can help identify where the high no-shows occur and determine which part(s) in the design and policies should be changed to reduce appointment no-shows.

Last but not the least, unlike many other studies, this study explored the use of different machine learning algorithms with different machine learning tools to build predictive models from large-size data. By comparing different predictive modeling methodologies, we can reveal different information from them, learn the weaknesses and advantages of each model, and select the right model for different objectives, as well as investigate the impact of a variety of appointment factors on patient no-shows.

3.2. Methodology & Model

3.2.1. Methodology Development

As shown in Figure 14, a Wald chi-square test was performed with a significance level of $\alpha = 0.05$ to identify the significant factors. Furthermore, a Tukey-Kramer method was used to perform pairwise multiple comparisons between the different levels of the factors. The Tukey-Kramer test is more balanced than other commonly used tests like Dunnett's test and Dunn's test, neither too reckless nor too strict, and can be applied to unbalanced data (Lee & Lee, 2018). By doing this, the dimension of the data and the complexity of the computation were successfully reduced.

The classified data was then used in a multivariate logistic regression model because of the advantages mentioned in section 3.1.2. Further, to study the interactions between the factors, a GLM univariate analysis with interactions was proposed. Also, to diminish the impact of underfitting, missing values, and outliers, a decision tree model was carried out. Then the prediction results were compared with two evaluation metrics: Root Mean Squared Error (RMSE) and R-squared (R^2). The most accurate outputs were then be selected and exported for further analysis and interpretation.

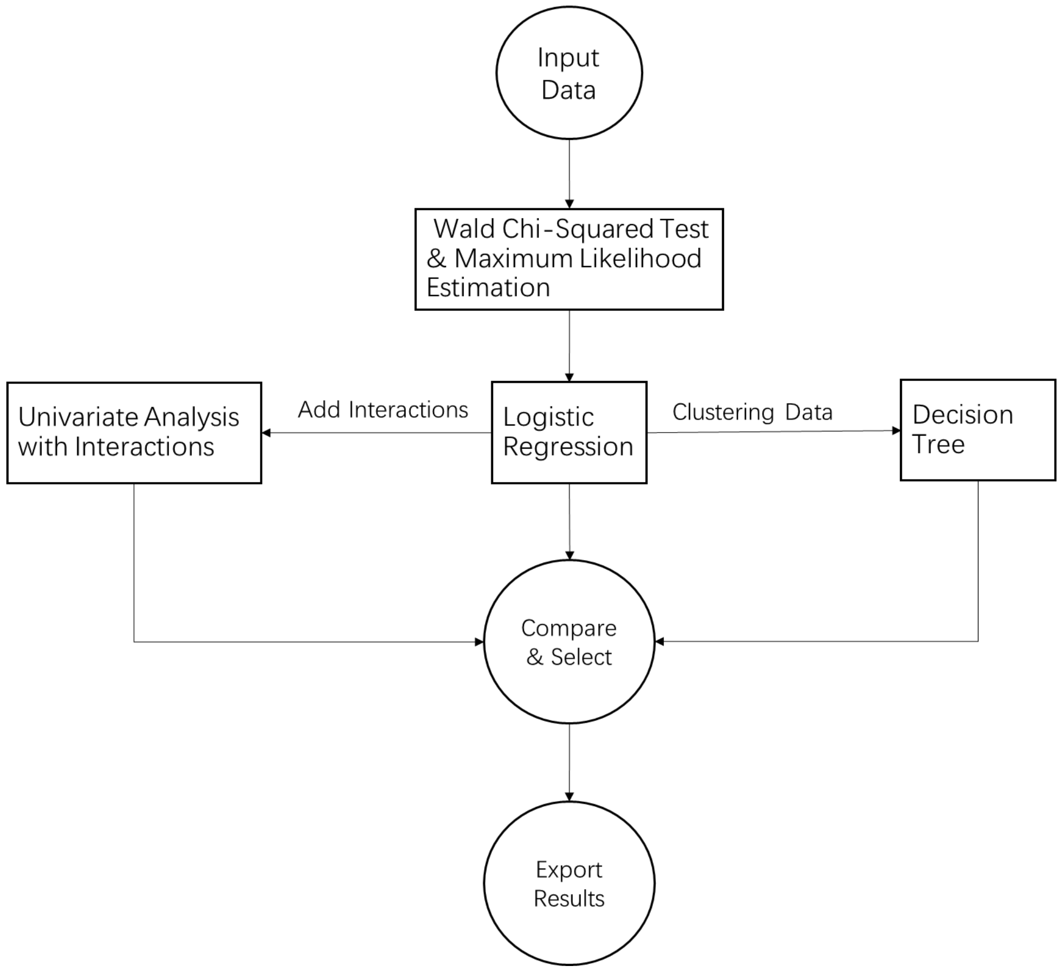

Figure 14. Flowchart of the Methodology Development

3.2.2. Model Formula

3.2.2.1. Logistic Regression Model

The formula for the logistic regression model is:

$$\text{Logit}(P) = \ln\left(\frac{P}{1-P}\right) = b_0+b_1X_1+\ldots+b_kX_k \text{ or } P = \frac{1}{1+e^{-(b_0+b_1x_1+\cdots+b_kx_k)}} \tag{21}$$

where P is the estimated no-show probability of an observation, b_0 is the intercept, and b_1, b_2, …, b_k are coefficients for different indicators X_1, X_2, …, X_k.

3.2.2.2. General Linear Model (GLM) with Interactions

The formula of GLM with interactions (two-way) is:

$$P = b_0+b_1X_1+\ldots+b_kX_k+ b_{k+1}X_1{*}X_2+ b_{k+2}X_1{*}X_3+$$

$$\ldots+ b_{2k-1}X_1{*}X_k+b_{2k}X_2{*}X_3+\ldots+ b_{\frac{k\text{^}2+k}{2}}X_{k-1}{*}\, X_k \quad (22)$$

where P is the estimated no show probability of an observation, b_0 is the intercept, b_k is the coefficient of a significant factor X_k, and b_{2k-1}, …, $b_{\frac{k\text{^}2+k}{2}}$ are coefficients of the interaction terms.

3.2.2.3. Conditional Inference Trees (Ctree) Model

The conditional class probabilities (Hothorn, Hornik, & Zeileis, 2006):

$$P(Y=y|X=x)=\left(\textstyle\sum_{i=1}^{n} w_i\,(x)\right)^{-1}\textstyle\sum_{i=1}^{n} w_i\,(x)I(Y_i=y),\ y=1,\ \ldots,J. \quad (23)$$

where J is the number of levels of the nominal response variable, $\sum_{i=1}^{n} w_i\,(x)$ denotes the sum of the case weights, and $I(Y_i=y)$ denotes the indicator function.

In this study, the number of levels J is 2 (0s and 1s). The case weight is a vector indicating if an observation is being included in a decision tree node. Therefore, the sum of case weights in a terminal node is the number of observations in that node.

3.3. Data Processing & Model Fitting

3.3.1. Data Processing

As listed in A1, the original dataset included two date variables, “Appointment Date” and “Scheduled Date”, which contained too many levels for reliable analysis. To reduce the dimension of these two variables, the date information was clustered based on the month of the date. As a result, there were only 12 levels in each variable instead of hundreds. The new variables are called “APT Month” and “SHD Month” correspondingly. After cleaning the redundant and invalid data, 11 factors were selected for predicting no-shows. To test which factors have significant effects on appointment no-shows, a Wald chi-square test was conducted. As shown in Table 5, 7 out of 11 of selected factors were proved to have significant effects on appointment no-shows. Meanwhile, the appointment time, at which month an appointment is scheduled, the appointment session, and the scheduling provider designation are not significant factors.

Table 5. Wald Chi-square Test of Effects

Effect	**DF**	**Wald Chi-Square**	**Pr>ChiSq**
APT_Month	11	37.4776	<.0001
SHD_Month	11	13.2856	0.2751
Appt_Time	94	105.3936	0.1983
DOW	5	72.6108	<.0001
SessionName	2	0.0121	0.9940
IWT	1	52.1392	<.0001
Duration_of_Appt	9	69.4297	<.0001
Is_Same_Day_Appointm	1	761.9221	<.0001
Location	6	43.0491	<.0001
Provider_Type	5	43.8920	<.0001
Scheduling_Provider_Designation	4	3.3265	0.5047

However, there are still too many levels in some factors that will lead to three major problems. First, with too many levels in the nominal data, the complexity of calculation will be extremely high, and therefore, requires more time and storage for computation. Second, there will be more outliers and noise in data, which leads to poor performance in predictive modeling. Last but not the least, it increases the difficulty of interpretation. Therefore, to reduce the levels of these factors, a Tukey-Kramer test for all significant factors was conducted. The results are shown in Figure 15.

As we can see, variables clustered based on the statistical tests indicated that some of the levels belong to the same group, and it might tell us some important information about no-shows. For example, appointments in March had the lowest no-show rate, while appointments in May, June, and July had the highest no-show rates. One possible assumption is the appointment no-shows are probably associated with the weather or holidays. For the day of the week, Saturday has the highest no-show rate, followed by Monday and Friday. That’s likely because Saturdays are on the weekend, and Mondays and Fridays are the first days and the last days of the working days. Also, appointments with durations of 30 mins or 60 mins seem to have the highest no-show rates, it is probably due to different types of appointments. Besides, different provider types and locations might affect the no-show rate. Most importantly, there is a significant difference between same-day appointments and pre-scheduled appointments in terms of no-shows. This is obvious because patients are likely to get treated right away other than waiting a couple of days or weeks.

Nevertheless, the above assumptions need to be testified, but the reason for such a test in this study is to reduce the levels in data. The revised dataset and the clustered levels were described in Table 6. Compare with the original dataset, this new dataset could lead to more s coefficient estimates and better prediction performances.

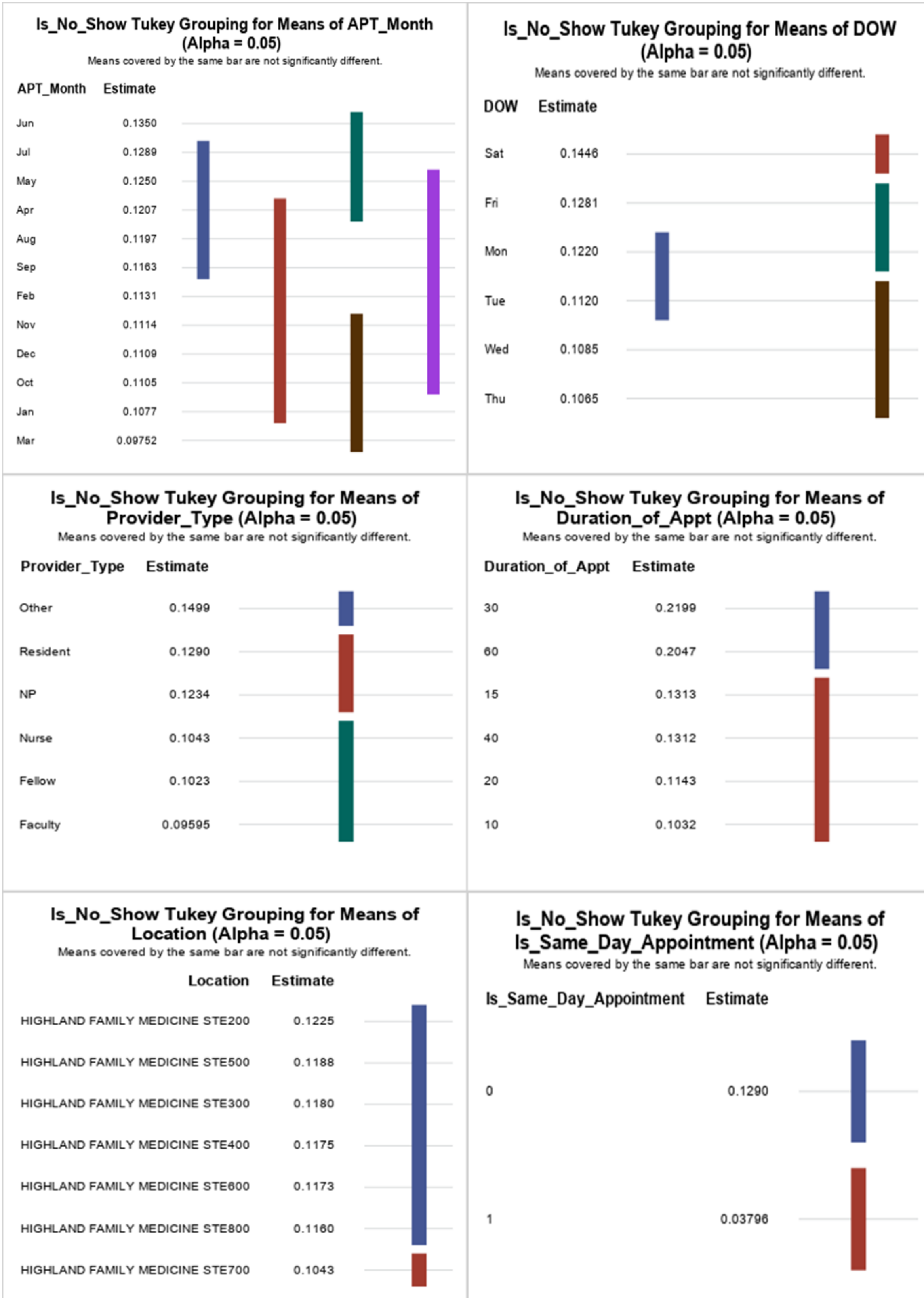

Figure 15. Tukey-Kramer Test Results for Significant Categorical Factors

Table 6. Revised Dataset and Clustered Levels

Variables	Levels
No_Show (Dependent Variable)	0,1
Month (Appointment Month)	Mar, May/Jun/Jul, & Other Months
DOW (Day of the week)	Sat, Mon/Fri, & Other DOWs
IWT (Indirect waiting time)	NA (Ordinal Data)
Duration (Duration of appointment)	30/60, Other Durations
Sameday (Same day appointment?）	0,1
Location (Hospital Suite Location)	STE 700, Other STEs
Provider (Provider Type)	Nurse/Fellow/Faculty, Resident/NP, Other

3.3.2. Model Fitting

To estimate the no-show probability (the expected probability of the value 1 in the dependent variable) in the logistic regression, a classification threshold was set to 0.5. In this case, an observation can be classified as a no-show if the predicted probability is larger than the threshold. To generate the output, a statistical tool SAS 9.4 was used for this model.

As for the GLM model with interactions, a test of subject effects is required to identify which interaction terms would likely impact appointment no-shows. As shown in Table 7, any term that has a p-value (Sig.) larger than 0.05 will be considered non-significant. Based on the result in the , the final GLM model should have a structure like the following :

Is No-show = Intercept + DOW + Location + Sameday + Provider + Month + DOW * Provider + DOW * Sameday + Duration * Location + Duration * Provider + Duration * Sameday + Provider * IWT + Location * Month + Location * Provider +Provider * Month + Sameday * Provider

To identify the coefficients for each term in this model and calculate the corresponding no-show probabilities for each observation, the GLM procedure in SPSS was used in this study.

Table 7. Tests of Between-Subjects Effect

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	204.638[a]	51	4.013	39.716	.000
Intercept	2.142	1	2.142	21.201	.000
DOW	.818	2	.409	4.049	.017
Duration	.068	1	.068	.669	.413
Location	1.332	1	1.332	13.183	.000
Sameday	.514	1	.514	5.086	.024
Provider	1.346	2	.673	6.662	.001
Month	.974	2	.487	4.823	.008
IWT	.052	1	.052	.519	.471
DOW * Duration	.080	2	.040	.394	.675
DOW * IWT	.235	2	.118	1.164	.312
DOW * Location	.065	2	.033	.322	.725
DOW * Month	.687	4	.172	1.700	.147
DOW * Provider	1.552	4	.388	3.841	.004
DOW * Sameday	2.123	2	1.062	10.509	.000
Duration * IWT	.003	1	.003	.028	.867
Duration * Location	2.183	1	2.183	21.608	.000
Duration * Month	.583	2	.291	2.885	.056
Duration * Provider	1.834	2	.917	9.079	.000
Duration * Sameday	1.845	1	1.845	18.260	.000
Location * IWT	.172	1	.172	1.705	.192
Month * IWT	.190	2	.095	.939	.391
Provider * IWT	4.247	2	2.123	21.018	.000
Sameday * IWT	.000	0	.	.	.
Location * Month	.848	2	.424	4.196	.015
Location * Provider	5.254	2	2.627	26.002	.000
Location * Sameday	.001	1	.001	.010	.920
Provider * Month	1.107	4	.277	2.738	.027
Sameday * Month	.310	2	.155	1.533	.216
Sameday * Provider	.973	2	.487	4.817	.008

In terms of the decision tree predictive model, Ctree uses significance test procedures to measure the association between responses and covariates rather than selecting the degerminators that maximize an information measure of node impurity (Hothorn, Hornik, & Zeileis, 2015). In this case, a pruning tree is not necessary for this algorithm; instead, selecting an appropriate alpha value and minimum split size is required. Therefore, a commonly used alpha value of 0.05 and a minimum split size of 30 were selected in the dissertation. The tool used to generate the results is RStudio.

3.4. Outputs & Results

3.4.1. Coefficient Estimates & Output Interpretation

3.4.1.1. Logistic Regression Model

Table 8 is the analysis of the maximum likelihood estimates (MLEs). The “Estimate” column in Table 8 represents the coefficient estimates (b_k) for the logistic regression model. Knowing these coefficients, the predicted probabilities of no-shows can now be generated. Moreover, this value also shows significance for the factors with more than two levels below <0.05. Any level that has a p-value smaller than 0.05 indicates there is a significant difference between this level and the base level. A positive value for an MLE indicates a higher no-show rate and a negative MLE value indicates a lower no-show rate.

Table 8. Analysis of Maximum Likelihood Estimates

Parameter		**DF**	**Estimate**	**Standard Error**	**Wald ChiSq**	**Pr>ChiSq**
Intercept		1	-1.1933	0.0850	196.9999	<.0001
Month	Mar	1	-0.1909	0.0349	29.8868	<.0001
Month	May/Jun/Jul	1	0.1337	0.0212	39.9176	<.0001
DOW	Other DOWs	1	-0.1394	0.0192	52.8764	<.0001
DOW	Sat	1	0.2348	0.0584	16.1415	<.0001
IWT		1	0.00345	0.000473	53.2394	<.0001
Duration	Other Duration	1	-0.6096	0.0835	53.2930	<.0001
Sameday	1	1	-1.2658	0.0439	833.1074	<.0001
Location	STE700	1	-0.1512	0.0273	30.7776	<.0001
Provider	Nurse/Fellow/Faculty	1	-0.3305	0.0206	256.4369	<.0001
Provider	Other	1	0.0315	0.0556	0.3219	0.5705

To better compare the base level and other levels in the factors, odds ratio point estimates were calculated. As shown in Table 9, the no-show rates in March are significantly lower than the no-show rates in other months (baseline). The average no-show rate in March is about 0.826 times of average no-show rates in other months, whereas average no-show rates in May, June, and July are about 1.143 times of average no-show rates in other months. Similarly, for the day of the week, Saturday has significantly higher no-show rates, while Tuesday, Wednesday, and Thursday have significantly lower no-show rates compared to Friday and Monday. For the duration of the appointment, patients are more likely to miss an appointment with a 30-minute or 60-minute duration. In terms of the location factor, suite 700 has lower no-shows than other locations. In the factor “Provider_Type”, the value “Nurse/Fellow/Faculty” has a significantly lower no-show rate.

Table 9. Odds Ratio Estimation

Effect	Point Estimate	95% Wald Confidence Limits	
Month: Mar vs Other Months	0.826	0.772	0.885
Month: May/Jun/Jul vs Other Months	1.143	1.097	1.191
DOW: Other DOWs vs Mon/Fri	0.870	0.838	0.903
DOW: Sat vs Mon/Fri	1.265	1.128	1.418
IWT	1.003	1.003	1.004
Duration: Other Duration vs 30/60	0.544	0.462	0.640
Sameday: 1 vs 0	0.282	0.259	0.307
Location: STE700 vs Other STEs	0.860	0.815	0.907
Provider: Nurse/Fellow/Faculty vs Resident/NP	0.719	0.690	0.748
Provider: Other vs Resident/NP	1.032	0.926	1.151

3.4.1.2. General Linear Model (GLM) with Interactions

The coefficients of all terms including those non-significant factors and interactions are shown in Table B2. To predict the no-show probabilities, use the value in “B” column in Table B2 as the coefficients for significant terms as indicated in Table 7. The parameters were computed using an alpha value of 0.05, and any parameter of 0 indicates it is either the baseline or redundant. With coefficients identified, the no-show probabilities can finally be predicted.

3.4.1.3. Conditional Inference Trees (Ctree) Model

Unlike the previous two models, the Ctree model uses significance test procedures to measure the association between responses and covariates. Therefore, there is no need to estimate the coefficients. Instead, a decision tree is required. It is a supervised machine learning algorithm that uses a binary tree graph (each node has two branches) to assign each data sample to a target value in the tree leaf. The outputs are presented in a tree plot shown in Figure B1. For better visualization, it was separated into 4 parts as shown in Figure B2-B5.

To interpret this plot, take terminal node 4 in the plot as shown in Figure B2 as an example. The sum of case weights is 8265, which means there are 8265 observations in this terminal node. The y=(0.962,0.038) indicates the average rate of appointment no-shows is 0.038 in this node, where the provider types belong to other and resident or NP, same-day appointments (IWT ≤ 0), and the appointment durations are not 30 mins or 60 mins. Similarly, the average no-show rates for all nodes in the tree plot can be obtained. An observation that falls in a terminal node will then assigned with a no-show probability that equals the average no-show rates in that node.

3.4.2. Model Evaluation & Comparison

As shown in Table B1, the most common evaluation method used for this type of study is the area under the ROC curve (AUC-ROC), which measures the percentage of correct classification. However, models using the original dataset returned relatively low AUC-ROC values, which indicated poor performances at classifying the appointment no-shows. For example, the logistic regression model based on the original data gave an AUC-ROC value of 0.6129, which indicates that it is unlikely to distinguish whether an individual would show up or not based on the non-demographic features obtained in this dissertation.

For this reason, binary classification was not the objective of this study. Instead, it aimed to predict the no-show probabilities of groups of individuals with the same clinical records.

Therefore, the coefficient of determination, known as R-squared (R^2), was used to evaluate the performance of the predictive models. The R^2 could be interpreted as the percent of the variation in the response variable explained by the model. One weakness of R^2 is that it tells nothing about the prediction error. In this case, another evaluation method called Root Mean Squared Error (RMSE) was also applied for evaluation. The reason why we chose RMSE instead of Mean Squared Error (MSE) is that the scale of the RMSE is the same as the scale of the targets. As a result, RMSE can be more accurately represent the average error between actual no-show probability and prediction.

To reduce the impact of outliers and noise, only groups with at least 30 observations were taken into the calculation sample sizes equal to or greater than 30 based on the Central Limit Theorem (CLT). The comparison results were presented in Table 10. The R^2 values indicated that the conditional interference trees model fit the data best, but the logistic regression model has the lowest RMSE value among the three. This means although the decision tree model explained more variance in the dependent variable, the logistic regression model gave a more accurate prediction than the decision tree model. It is hard to conclude which one of the two models is better, but both models performed better than the GLM model with interactions.

Nevertheless, the decision tree plot is much easier to interpret and use, especially for healthcare providers who do not have a strong statistical background, people can easily interpret the result and make corresponding appointment plans and schedules. For example, as shown in Figure B2, the same-day appointments with 30 or 60 minus durations have a significantly higher rate of no-shows than the same-day appointments of other durations. In this case, providers could overbook the same-day appointments with 30 or 60 minus duration and schedule as many same-day appointments with other duration as possible.

What's more, the decision tree model provided a simpler way to cluster all observations in a node as shown in Figure B1 rather than considering every single observation in the node. For example, considering appointments with an IWT less than 4 days as a whole part instead of IWTs from 0 to 4 days. In this case, the actual no-show rate will then equal the overall no-shows divided by the total number of appointments for all appointments with an IWT less than 4 days, and there is no need to compare the predicted probability for appointments with IWTs of 0 to 4 days. Using this way of appointment clustering, the evaluation results would improve remarkably. The clustered IWT decision model will then return an R^2 value of 0.9208 and an RMSE value of 0.0300. An advantage of this method is that it significantly simplified the calculations and improved the prediction accuracy. However, it does not change the fact that, when compared to the estimated probabilities to the no-show rates of all possible scenarios, this decision tree model did not perform better than the logistic regression model.

Table 10. Comparison of R-Squared and RMSE for Different Models

(For N>=30) Evaluation Parameters	**Logistic Regression**	**GLM with Interactions**	**Conditional Inference Trees**
RMSE	0.0392	0.0436	0.0414
R-Squared	0.5490	0.3650	0.5547

3.5. Summary & Discussion

This chapter focused on how to predict patients' no-show probabilities without profiling the patients. Although demographic and private information is extremely helpful for predicting patient behavior, they are sometimes unavailable and violate the right of patients. To relieve the impact of appointment no-shows, this dissertation proposed several models to predict appointment no-show probability based on mostly non-human related clinical information, such as appointment date, appointment duration, and IWT. These models are not meant to classify appointments, but rather, were built to predict no-show probabilities for a group of patients that share similar clinical information. Simply speaking, this study was trying to predict no-show rates for appointments with different clinical characteristics without profiling the patients. Providers can then make a corresponding schedule strategy to improve the healthcare accessibility for every patient, without considering their race, gender, age, income, education level, etc. In this way, patients are ensured to get equal rights to schedule an appointment, and at the same time, appointment no-shows can be reduced.

For future studies, it would be interesting to develop a survey to find out why these factors could impact appointment no-shows. Is it because of the weather? Maybe because of school and work schedule? Or different types of appointments? These could help us to better understand the impacts of nonhuman-related factors on appointment no-shows and make a better scheduling plan. What's more, health equity is another major issue in the current healthcare system. Unlike health equality, which provides equal opportunities for everyone to access healthcare services, health equity focuses on how to make sure no particular subset of groups is leftover or at a particular disadvantage. For example, providing accessible and affordable transportation services for the homeless, the handicapped, and the low-income patients so that they can access healthcare services. Besides, the decision tree plot model provided a good idea of clustering the appointments based on different scenarios. Based on the no-show rate for each scenario, providers are able to optimize and allocate the health care resources to better serve the people and communities.

4. CAPACITY ALLOCATION

In this chapter, a real dataset from a medical center with a no-show rate of around 12% was analyzed to reduce the waste of healthcare resources and improve capacity utilization. Two scheduling strategies are studied and compared to maximize the provider's profit. Also, a traditional single-stage stochastic program and a two-stage newsvendor model are built for comparison. Instead of only identifying the optimal number of open-access appointments for daily appointment capacity like other studies, this study provides a more comprehensive allocation of healthcare resources. Further, the sensitivity of the corresponding factors of an appointment on optimal capacity, average profit, and the maximum number of appointments that can be scheduled is investigated.

The first section of this chapter covers the literature review of outpatient appointment allocation. models of appointment no-show prediction, including some basic background of outpatient capacity allocation and commonly used models for optimizing appointment scheduling. In the second section, a profit formula is introduced. Further, several optimization methodologies are introduced, and two scheduling strategies are described. The third section displays the results and outputs of different optimization methods and scheduling strategies, followed by a sensitivity analysis for the best optimization method and scheduling strategy. Last but not the least, research contributions and potential future studies are discussed at the end of this chapter.

4.1. Literature Review

4.1.1. Background

Many studies have been conducted to seek an advanced access scheduling plan to minimize the impact of patient no-shows and waste of healthcare resources. The key to a successful advanced scheduling plan is to match the daily appointment demand with the healthcare provider's capacity. Recognition of the importance of an efficient healthcare system is rapidly growing in many developed countries. Nevertheless, many patients are still suffering from high healthcare costs and long wait times. This leads to patient no-shows and waste of healthcare resources, which will then conversely increase healthcare costs and patient wait times. To improve patient access to healthcare services and lower costs, many researchers have devoted themselves to develop new admission policies and scheduling methods to better allocate healthcare resources.

Capacity allocation in the outpatient appointment system (OAS) is a process dealing with how available slots should be distributed to different patient groups. Patients are classified into different groups based on many factors, such as appointment demand, patient type, no-show probability, priority level, revenue from each patient group, and preferences of patients and physicians. If patient characteristics are known, then these classifications can be used for prioritizing, scheduling, sequencing, and adjusting the appointment intervals (Cayirli & Veral, 2003). For example, Qu et al. (2012) developed a mean-variance model and an efficient solution procedure to select the percentage for open appointments in an open-access scheduling system to increase the average number of patients seen while also reducing the variability by classifying the patients by different appointment types. While Nguyen, Sivakumar, and Graves (2015) introduced a deterministic model to identify the optimal capacity between two patient groups, including those on their first visit and returning patients. Depending on the different objectives, the factors taken into consideration are different. One of the main tactical objectives for an OAS is how to assign the available slots to fulfill the appointments requested by the patients. For this objective, a variety of modeling approaches can be used.

4.1.2. Model Comparison

As shown in Table 11, stochastic optimization and stochastic dynamic programming are widely used to solve OAS problems because of their ability to deal with uncertainty and randomness. In particular, single-stage and two-stage stochastic programs are the most popular stochastic optimization models for OAS problems. Single-stage stochastic programs deal with problems with a random objective function or constraints where a decision is implemented without subsequent recourse (Luo et al., 2012). For a two-stage stochastic program, decision variables are divided into the first stage and second stage variables. First stage variables are decided without full information of the random parameters. Once the full information is received and uncertainty is discovered, further adjustments can be made through second-stage variables (Al-Qahtani & Elkamel, 2010).

Another useful model for dealing with general parametric demand distributions and unknown parameter values is the newsvendor model (Ding et al., 2002). It was originally an operations management and applied economics model used to determine optimal inventory levels, but more and more researchers have started to use it for scheduling problems in healthcare. For example, Strum, Vargas, and May (1999) applied a newsvendor model to determine the operating room schedule duration to allocate for surgical subspecialties to minimize costs associated with underutilization and overutilization of operating room time. Similar works include Houdenhoven et al. (2007), Denton, Viapiano, and Vogl (2006), and Olivares, Terwiesch, and Cassorla (2008). The Newsvendor model has also been used to solve appointment scheduling optimization problems for decades. Weiss (1990) proved that when there are only two patients, the optimal scheduling problem is similar to a simple newsvendor problem. Green, Savin, and Wang (2006) proposed a newsvendor policy in their research to achieve the most profi allocation of scheduled and nonscheduled examination slots.

Table 11. Commonly Used Models for Outpatient Appointment Capacity Allocation

Reference	**Access Policy**	**Objective**	**Model**	**Solution Method**
Qu et al., 2007.	Pre-Schedule & Sameday	Max the number of patients consulted.	Single-stage stochastic model	Sensitivity Analysis & Analytical Method
Qu et al., 2012.	Pre-Schedule & Sameday	Min. variability in the number of patients consulted.	Single-stage stochastic model	Sensitivity Analysis & Analytical Method
Qu et al., 2013.	Traditional	Min difference of service times between clinic sessions	Mixed-integer program	Monte Carlo Simulation & Computer packages
Hannebauer & Muller, 2001.	Traditional	Min the cumulated weighted between assignments & derivation of appointment's starting time.	Distributed constraint optimization model	Computer packages & Simulation
Luo et al., 2012.	Traditional	Max the profit between the revenue and the costs.	Single-stage stochastic model	Analytical Method & Simulation
Samorani & LaGanga, 2015.	Pre-Schedule & Sameday	Max the profit between the revenue and the costs.	Two-stage stochastic model	Column-generation & Heuristic
Rezaeiahari & Khasawneh, 2020.	Traditional	Min deviations from a patient's preferred start day & the flow time of patients at the clinic.	Two-stage stochastic model	Simulation-based tabu search & simulated annealing
Dobson et al., 2011.	Pre-Schedule & Sameday	Max the profit between the revenue and the costs.	Single-stage stochastic model/ Queuing theory	Analytical Method & Heuristic
Creemers et al., 2012.	Pre-Schedule & Same day	Min the total expected weighted waiting time of a single patient.	Single-stage stochastic model/ Queuing theory	Heuristic
Balasubramanian et al., 2013	Pre-Schedule & Sameday	Max the number of same-day patients consulted and continuity of care.	Stochastic dynamic programming	Heuristics & Simulation
Wang et al., 2018	Traditional	Max the revenue based on the number of patients consulted.	Dynamic programming	Computer packages/ Simulation

4.1.3. Contributions

The primary contribution of this study comes from its unique combination of optimization modeling and scheduling strategy. This combination covered the study of outpatient appointment scheduling (OAS) decisions from all classification groups introduced by Ahmadi-Javid, Jalali, and Klassen (2017), which included the access policy, the number of resources, and the capacity allocation. Also, overbooking is taken into consideration.

The contribution is also distinctive in its ability to deal with the daily fluctuations in patient demand, which may lead to poor capacity utilization because of patient appointment-booking preferences, which prevent clinics from applying extensive use of open-access policies. In this study, the proposed strategy suggests a flexible number of slots to cover as many pre-scheduled appointments as possible, meanwhile it uses an open-access policy for same-day appointments and a certain number of overbookings to cover appointment no-shows.

Furthermore, this study has a significant contribution to balancing and utilizing healthcare resources (appointment capacity). The level of resources is a key factor for the provider's profit. A high level of resources may increase costs due to waste of capacity, but a low level of resources may increase patient waiting time due to overbooking and loss of profit for excessive rejecting of patient demand, thus impacting the quality of healthcare and the provider's revenue. This proposed approach solved this conflict through two aspects. First, it identified an optimal capacity to balance the level of resources. Second, the scheduling strategy improved capacity utilization by assigning an appropriate number of slots for same-day and pre-scheduled appointments.

Other contributions include its representation of the demand classification and the use of the newsvendor model for optimal capacity identification. An empirical probability was used in this study, which ensures the accuracy of demand classification and optimal capacity identification because no assumption is required on the distribution of patient demand. Moreover, the application of the newsvendor model in identifying optimal capacity for outpatient appointments is also an interesting aspect. It is easy to use, and the results proved the model has a high accuracy by giving a near-optimal solution.

4.2. Methodology

4.2.1. Problems Statement & Assumptions

In an advanced access scheduling system, there are generally two types of appointments, which are pre-scheduled appointments and open access appointments. For pre-scheduled appointments, the patient can be scheduled days to months in advance, while open access appointments or same-day appointments can usually receive an appointment within 12–72 hours (Qu et al., 2007). Therefore, appointments with a maximum IWT of 2 days were considered as same-day appointments in this study. Also, walk-ins were treated as same-day appointments since the sample size of walk-in patients in this dataset is extremely small. According to Kodjababian (2013), the number of no-shows increases as the IWT increases. This has also been testified to by the previous two chapters in this dissertation. As a result, more pre-scheduled appointments will result in more no-shows. On the other hand, there will be no enough capacity or demand for same-day appointments. Both risks decrease the expected profit. Hence, it is necessary to allocate the capacity and find the optimal number of slots for each type of demand to find a balance.

In this study, it is assumed that the appointments are independent of each provider regardless of the appointment policies and types of providers. Further, it is assumed the decision a patient makes is independent of other patients' decisions. Finally, it is assumed that the daily capacity (the number of appointment slots available each day), denoted by C, is fixed.

4.2.2. Optimization Methods

In this study, the appointments scheduled by providers are independent of each other, and there is no association between patients' attendance. Also, the unit revenue and the unit fixed cost for each appointment are consistent, donated by R and F, and P_i is the daily profit on day i during the study period.

4.2.2.1. Single-Stage Stochastic Programming

In terms of the single-stage stochastic program, the expected daily profit can be expressed as:

$$
\begin{aligned}
P = {} & ((1 - \alpha_P)*\mathrm{Min}\{D_P, N_P\}+(1 - \alpha_S)*\mathrm{Min}\{D_S, N_S\})*R - F*C \\
& - \alpha_P*\mathrm{Min}\{D_P, N_P\}*L_P - \alpha_S*\mathrm{Min}\{D_S, N_S\}*L_S \\
& - \mathrm{Max}\{(1 - \alpha_P)*\mathrm{Min}\{D_P, N_P\}+(1 - \alpha_S)*\mathrm{Min}\{D_S, N_S\} - C, 0\}*\Theta \qquad (24)
\end{aligned}
$$

where,

Parameters:

R = Revenue from each serviced customer.

F = Fixed cost per one appointment worth of capacity.

L_P = Penalty cost associated with a preschedule no-show beyond lost revenue and unused capacity.

L_S = Penalty cost associated with a same-day no-show beyond lost revenue and unused capacity.

α_P = Actual no-show rate of prescheduled appointments.

α_S = Actual no-show rate for same-day appointments.

Ɵ = Penalty for each customer who shows up beyond capacity C (covers additional waiting time penalty).

Random Variables:

D_P = Number of requests for prescheduled appointments.

D_S = Number of requests for same-day appointments.

Decision Variables:

N_P = Maximum number of prescheduled appointments to schedule.

N_S = Maximum number of same-day appointments to schedule.

C = Provider's Capacity.

The first term $((1 – \alpha_P)*\text{Min}\{D_P, N_P\}+(1 – \alpha_s)*\text{Min}\{D_s, N_S\})*R$ is the revenue of the appointments that are expected to be utilized. The second term F*C is the total fixed cost for all available appointment slots regardless of the utilization. The $\alpha_P*\text{Min}\{D_{Pi}, N_P\}*L_P –\alpha_S*\text{Min}\{D_S, N_S\}*L_S$ refers to total penalty cost due to appointment no-shows. The last term $\text{Max}\{(1 – \alpha_P)*\text{Min}\{D_P, N_P\}+(1 – \alpha_S)*\text{Min}\{D_S, N_S\} – C, 0\}*\Theta$ is the total penalty cost for all patients that showed up but beyond the capacity.

With the formula of P identified, the objective of this model is then finding the optimal number for each type of appointment that should be reserved to maximize the average daily profit, which can be expressed as:

$$\text{Max } \overline{P} = \sum_{i=0}^{n} P_i \, / \, n \tag{25}$$

subject to $C = N_p + N_s$, where C, N_p, N_s are non-negative integers, and n is the number of observations in the dataset.

4.2.2.2. Two-Stage Newsvendor Model

This model is very similar to the last model, the difference is in the first stage. Unlike the single-stage stochastic program, this model identifies the optimal capacity C in the first stage and uses it in the second stage to find an optimal number of pre-scheduled appointments N_p. Assume this optimal daily capacity is C*, it can be expressed as follows in this study:

$$C^* = F^{-1}\left(\frac{R}{R+F}\right) \tag{26}$$

where F^{-1} denotes the inverse distribution function of demand and the critical fractile CR = $\frac{R}{R+F}$ is the ratio corresponding to the optimal quantity C*. In other words, CR = P(C ≤ C*).

In this case, if there is one observation whose cumulative probability matches the value of CR, then the optimal capacity is identified. Otherwise, find an observation that has a cumulative probability closest to and larger than CR in the demand empirical distribution, labeled as upper bound demand (C_U). Similarly, label the demand that has a cumulative probability closest to and smaller than CR as lower bound demand (C_L). Also, label the corresponding cumulative probabilities as P_U and P_L. Thus, the optimal quantity is:

$$C^* = C_U - \frac{P_u - \mathrm{CR}}{P_u - P_L} * (C_U - C_L) \text{ or } C_L + \frac{\mathrm{CR} - P_L}{P_u - P_L} * (C_U - C_L) \quad (27)$$

After the optimal capacity is identified, the corresponding number of slots for pre-scheduled and same-day appointments can be identified to maximize the overall profit. The objective function is similar to equation (25) except that the optimal capacity is not equal to the sum of N_s and N_p. Instead, the number of reserved same-day appointment slots N_s will equal $C^* - N_p + N_p * \alpha_p$ so that overbooking is considered to reduce the impact of no-shows from pre-scheduled appointments. The objective function is shown below:

$$\text{Max } \bar{P} = \sum_{i=0}^{n} P_i \,/\, n \quad (28)$$

subject to $N_s = C^* - N_p + N_p * \alpha_p$, where C^*, N_p, N_s are non-negative integers, and n is the number of observations in the dataset.

4.2.2.3. Two-Stage Newsvendor Model with Demand Classification

Different from the newsvendor model in the last section, this model adds one more step which classifies the appointment demands based on the distribution among different months and days of the week. The appointment demands were classified using Tukey's HSD test, the results are shown in Table C1.

Based on these results, the appointment demands can be classified into two groups (May/Jun & Other Months) by different months of the year and three groups (Mon/Tue/Wed, Thu/Fri, Sat) by different days of the week. As observed in Table C1, it is evident that there are fewer requests for appointments in May and June. Further, appointment requests on Saturdays are much less than on other days of the week, and appointment requests on Mondays, Tuesdays, and Wednesdays have the highest daily average. Given these test results, the appointment demands can be classified into six groups, donated as ξ_s. These six groups include May/June – Monday/Tuesday/Wednesday, Other Months –Monday/Tuesday/Wednesday, May/June – Thursday/Friday, Other Months – Thursday/Friday, May/June – Saturday, Other Months – Saturday. Label them from ξ_1 to ξ_6 correspondingly.

After the classification, apply the optimization function (28) for each group, and therefore, there will be an optimal average profit $\bar{P}_s$ for each group ξ_s. Hence, the overall average profit $\bar{P} = \sum_{s=1}^{6} \bar{P}_s * n_s / \sum_{s=1}^{6} n_s$, where n_s is the number of observations under each group.

4.2.3. Scheduling Strategies

Other than the three optimization methods, two scheduling strategies are examined in this chapter. One of the strategies is to try to identify two optimal values, N_p and N_s, based on the historical data. The logic behind it is to reserve a certain percentage of available appointment slots for potential same-day appointments. Another strategy is to keep the N_p and N_s flexible and try to identify their values based on the pre-scheduled appointment demands. The reason behind this strategy is simple. Since pre-scheduled demands can be known before the appointment date so that providers can adjust the corresponding N_{ps} for different values of D_p.

4.2.3.1. Fixed N_P & N_S

Theoretically speaking, it is better to fulfill the same-day appointments first because same-day appointments usually have a lower no-show rate. As many studies have revealed, it is recommended to hold appointments open if the provider has a very high demand for same-day appointments. However, this limitation of this suggestion is obvious. It is common sense to keep all appointments open if a high same-day appointment demand is given, but it is risky for the provider if the same-day appointment demand is low or fluctuating. Sometimes, the demand distribution does not follow any known distribution and cannot be predicted. Therefore, it is usually better to reserve a certain number of available slots for same-day appointments if the demand is not high enough or fluctuates wildly.

To identify the average daily profit using this strategy, fixed N_p and Ns values will be applied to each observation. In other words, N_p and Ns are consistent in each working day, regardless of the difference in daily demand. Nevertheless, there is a minor difference between the three optimization methods mentioned in the previous section. For the single-stage stochastic program, both N_p and N_s are the decision variables that need to be optimized, and the optimal capacity equals the sum of them. For the second optimization method, the optimal capacity C* is identified in the first stage, and only N_p needs to be optimized in the second stage. N_s can then be calculated based on constraints (28). The $C^* - N_p$ covers the appointment slots left in the capacity, and $N_p{*}\ \alpha_p$ covers the potential no-shows of the pre-scheduled appointments by overbooking the same-day appointments. As for the last optimization method, it is very similar to the second method except that fixed N_P and Ns are calculated under each group, and they could be different due to the different demand distribution of each group.

4.2.3.2. *Flexible N_P & N_S*

Unlike the first strategy, this scheduling strategy will allow a flexible N_S corresponding to the D_P. As shown in formula (24), the number of pre-scheduled appointments that are actually scheduled is $\min\{D_P, N_P\}$, denoted by A_P. And the number of same-day appointments that are actually scheduled is $\min\{D_S, N_S\}$, denoted by A_S. For a random value of A_P, we can maximize the profit by changing the A_S value according to (24) given the optimal capacity and the values of the parameters. As a result, an optimal A_S value can be identified for a given A_P value. Although the number of same-day appointment demands D_S is usually unknown and difficult to control, the maximum number of same-day appointments to be scheduled N_S is optimized when it equals this optimal A_S. This is because A_S is equal to the $\min\{D_S, N_S\}$, which makes sure the daily profit is optimal if the D_S value is equal to or larger than the optimized N_S value. To demonstrate this process, an enumerate method is carried out with parameters and optimal capacity values given in Table 12. Using values in Table 12, an example of corresponding optimal N_S values for a list of given A_P values is shown in Table C2.

Table 12. Given Values of Required Parameters and Capacity for Capacity Allocation

R	**F**	**L_P**	**L_S**	**θ**
10	3	3	3	13

After the optimal N_S is identified for a given A_P, the daily profit can be calculated for any given D_S. Assume there is a list of D_S values with n observations, then those demands can be denoted from D_1 to D_n, respective to the sequence of the observations in the list. Also, their corresponding daily profit can be labeled as P_1 to P_n. Therefore, the expected profit for each pair of A_P and N_S values can be calculated by summing the product of P_n with the corresponding probability of each D_n in the list. The results are presented in Table C3. Unlike the result in Table C2, Table C3 indicates that a higher N_P is probably a better choice. This is reasonable since the D_S can be small compared with the optimal N_S. In this case, scheduling more pre-scheduled appointments would be safer to maximize the expected profit in case of the waste of the same-day appointment capacity.

Similarly, let the maximum number of pre-scheduled appointments allowed N_P to equal the A_P value for the given optimal N_S in Table C3. And consider a list of D_P values with n observations, the expected profit for each pair of N_P and N_S values can be calculated by summing the product of profits for all D_P values with the corresponding probabilities of D_P in the list. The results are shown in Table C4, which indicates an optimal pair of N_P and N_S occurs when the N_P value equals the maximum D_P value in the list.

To sum up, the strategy with flexible N_P and N_S values tends to schedule the pre-scheduled appointment first and leave the rest of the capacity and overbooking slots for same-day appointments and pre-scheduled appointment no-shows. Since pre-scheduled appointment demand is known, if we can fulfill the pre-scheduled appointments (except no-shows) first and make the N_S open for the same-day appointments, it is probably better than just fixing N_S and N_P for all appointments.

4.3. Outputs & Results

To evaluate the performances of the optimization methods and scheduling strategies in this study, the average daily profit, the expected annual profit, the optimal capacity, and the sum of N_P and N_S are compared. Further, a sensitivity analysis was proposed to study the impact of the related parameters on these indicators.

To optimize profit, the first step is to identify the unit revenue R and the fixed cost F for each appointment slot. The problem with these two parameters is that they vary for different organizations, and largely depend on the types of appointments, thus, without knowing the information of the two parameters, dummy values were given to them in this study. The initial values for R and F were set to be 10 and 3, respectively. Also, the initial values of L_P, L_S, and θ were given as 3, 3, and 13 as indicated in Table12. Further, all parameters are considered consistent for all appointments in the dataset. Other than these parameters, α_p and α_s values used in this study were set to be the actual daily no-show rates of pre-scheduled and same-day appointments from the real dataset obtained from the medical center.

4.3.1. Performance Comparisons

The numerical results generated in this study are based on the real data obtained from the medical center. First, three optimization methods are compared. The results are shown in Table 13. The results indicate that the two-stage newsvendor model has better performance than the single-stage stochastic program. One reason is that the single-stage stochastic program identifies the optimal capacity by adding the N_P and N_S. In this case, overbooking is not taken into consideration. Meanwhile, the two-stage newsvendor model has identified the optimal capacity based on the distribution of the demand. Thus, in the second stage, only N_P needs to be identified, and N_S can be formulated to cover the no-shows of the pre-scheduled appointments. Thus, the sum of N_P and N_S can allow an overbooking over the capacity. The best optimization method is the two-stage newsvendor model with classified demands because the average daily profit improved significantly. This is because the demands are not balanced. Therefore, classifying the demands could improve the accuracy of the model that helps to build a better allocation of capacity. The detailed capacity allocation plans are presented in Table 14.

Table 13. Comparison of Three Optimization Methods

Methods	**N**	**Average Profit**	**Annual Profit**	**C**	**Np**	**Ns**	**Np+Ns**
Single Stochastic	306	$1,448	$442,955	458	325	133	458
Two-Stage Newsvendor	306	$1,519	$464,856	483	364	167	531
Newsvendor (Classified Group)	306	$1,809	$553,531	NA	NA	NA	NA

Table 14. Capacity Allocation for Two-Stage Newsvendor Model Under Each Group

Newsvendor (Classified Group)	**N**	**Average Profit**	**Total Profit**	**C**	**Np**	**Ns**	**Np+Ns**
Group 1: May/June+Mon/Tue/Wed	26	$1,804	$46,901	442	323	167	490
Group 2: May/June+Thu/Fri	18	$1,198	$21,569	320	274	87	361
Group 3: May/June+Sat	8	$137	$1,099	31	21	12	33
Group 4: Other+Mon/Tue/Wed	125	$2,575	$321,931	530	385	194	579
Group 5: Other+Thu/Fri	85	$1,809	$153,778	404	351	100	451
Group 6: Other+Sat	44	$188	$8,252	47	31	20	51

Since the newsvendor model with classified demand has the best performance, two strategies were applied using this optimization method. The results of the first strategy (Strategy 1) are discussed in Table 13 and Table 14. For the second strategy (Strategy 2), the results are indicated in Table 15. Comparing Table 14 with Table 15, it is easy to see that the second strategy slightly outperforms the first strategy with a higher average daily profit and allows for more appointments to be scheduled as shown in Figure C1 and Figure C2. Note that there is no fixed N_P and N_S for the second strategy, therefore only the maximum number of appointments that can be scheduled is indicated. This also indicated that Strategy 2 allows more overbooking than Strategy 1. The average daily profit for the newsvendor model using Strategy 2 is $1,838, and the total annual profit is $562,464. This also outperformed all other options as indicated in Figure 16.

Table 15. Capacity Allocation for the Proposed Optimization Method Using Strategy 2

Newsvendor (Classified Group)	**N**	**Average Profit**	**Total Profit**	**C**	**Max (Np+Ns)**
Group 1: May/June+Mon/Tue/Wed	26	$1,850	$48,099	442	512
Group 2: May/June+Thu/Fri	18	$1,230	$22,140	320	373
Group 3: May/June+Sat	8	$151	$1,205	31	35
Group 4: Other+Mon/Tue/Wed	125	$2,602	$325,237	530	602
Group 5: Other+Thu/Fri	85	$1,849	$157,171	404	465
Group 6: Other+Sat	44	$196	$8,612	47	55

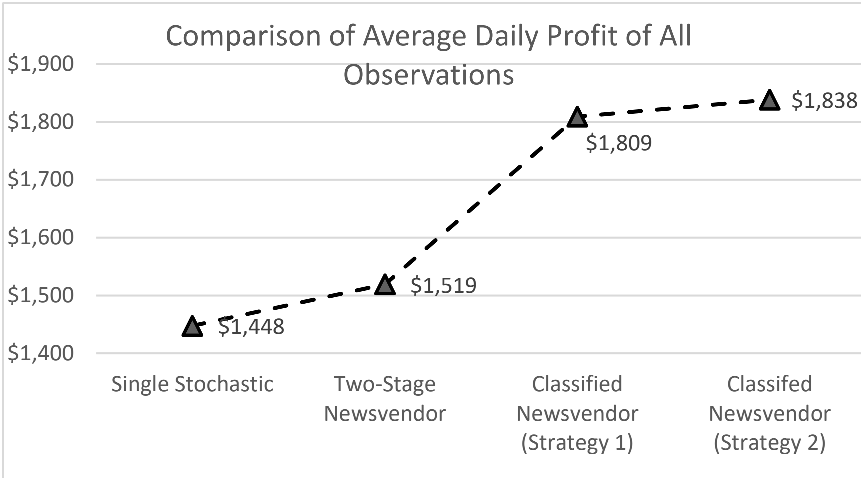

Figure 16. Comparison of Average Daily Profit for All Methods and Strategies

4.3.2. Sensitivity Analysis

To study the impact of the given parameters on the indicators, a sensitivity analysis is conducted. Since the combination of the third optimization method and the second strategy outperformed all other combinations, the sensitivity analysis is only conducted for this option. Also, for the sensitivity analysis, it is assumed that the change of unit revenue R and unit fixed cost F will not change how much a patient needs to pay for a visit because there have been many studies that have stated that the amount paid has a significant impact on patient no-shows (Osborn et al., 2016; Mohamed et al., 2016).

Take group 1 as an example, the sensitivity analysis results were demonstrated in Figures C3-C9. Comparing Figure C3 and Figure C4, it is clear that there is a strong positive correlation between unit revenue R and the average profit, and a strong negative correlation between fixed cost F and the average profit. However, the increase of the unit revenue R does not have a significant impact on the optimal capacity and the maximum number of appointments that can be scheduled, but the decrease of the unit revenue does. On the other hand, the fixed cost does have a strong negative correlation with both optimal capacity and the maximum number of appointments allowed. This is probably because demand is limited; therefore, there is no need to increase capacity and appointments scheduled very much to maximize the expected profit.

Comparing Figure C5 and Figure C6, both no-show rates have a strong positive correlation with the maximum number of appointments that can be scheduled but have a strong negative correlation with the average daily profit. This makes sense because a higher no-show rate will reduce the profit that can be generated, and therefore, to compensate for the loss, it is necessary to have more overbooking. However, the no-show rates won't impact the optimal capacity since the optimal capacity using the newsvendor model only depends on the unit revenue R and fixed cost F.

Comparing Figure C7 and Figure C8, it is interesting to see that they have no impact on the maximum appointments allowed. This probably indicates that the proposed strategy is very efficient at eliminating appointment no-shows. Both L_P and L_S will have a negative correlation with the average daily profit, but L_P has more impact on it. There are two explanations for this phenomenon. First, the no-show rate for the pre-scheduled appointments is usually higher than the no-show rate for the same-day appointments. Second, this strategy tends to schedule a large number of pre-scheduled appointments, which is explained in the previous section. Also, they do not have any impact on the optimal capacity in this model.

As for Figure C9, the increase of the penalty cost for each customer who shows up beyond capacity θ has almost no impact on the average daily profit and the maximum number of appointments that can be scheduled, but the decrease of θ does, especially on the maximum number of appointments allowed. This proves the proposed combination is robust to the appointment conflicts but sensitive to appointment no-shows. Hence, this combination of scheduling suggests a large amount of overbooking if θ is small to compensate for the impact of appointment no-shows. Again, θ does not have any impact on optimal capacity.

4.4. Summary & Discussion

In this study, three optimization methods and two scheduling strategies were compared. The proposed combination not only generated the highest average daily profit but also allowed the highest number of appointments to be scheduled. This indicates the flexibility of the combination. One of the major advantages is the high utilization of the capacity. Since the pre-scheduled appointments can be known before the appointment date, this proposed strategy will first schedule all pre-scheduled appointments until $(1-\alpha_P)*D_P$ equals the optimal capacity identified in the first stage using the newsvendor model. If $(1-\alpha_P)*D_P$ is less than the optimal capacity, same-day appointments can then be scheduled. The number of same-day appointments that can be scheduled N_S will be $(C^* - (1-\alpha_P)*D_P)/(1-\alpha_S)$. As a result, the maximum number of conflicts will be $\alpha_P*D_P + \alpha_S*D_S$ if and only if all scheduled appointments are utilized. In other words, the proposed combination is very efficient at utilizing the appointment capacity and very robust to appointment conflicts.

This study also classified the appointment demands into different groups based on empirical distributions instead of the Poisson distribution. The reason for this is because the distributions of demands in some groups do not fit any commonly known distribution type. Further, the appointment demands in this dataset are highly related to the office hours of the medical center, which explains the difference in appointment demands for different days of the week shown in Table C1. Therefore, it is reasonable to use empirical distributions for all demand distributions.

One drawback of this study is that all models did not consider the appointment costs (e.g., transportation cost and penalty cost paid by patients), from the patient's side, which would certainly impact the overall no-show rate and the total profit. Hence, for future studies, it is necessary to consider the cost from the patient's perspective as one of the decision variables. Further, it would be interesting to add a percentage (other than 100% in this study) of D_P that would be scheduled or set a bar (for example, schedule all pre-scheduled appointments if it is less than N for a given capacity. If the D_P is larger than N, leave it for same-day appointments). Also, a combination of these two strategies by setting a bar value N for N_P first and covering only a flexible percentage of the D_P if it exceeds N can be studied.

5. CONCLUSION

This dissertation has studied several traditional models and proposed new approaches for three objectives, which are projecting the relationship between appointment utilization and IWT, predicting the probability of no-shows for patients without profiling them, and developing an optimization approach and scheduling strategy for outpatient capacity allocation. The results indicated that the proposed methods generally outperformed the traditional ones. Further, the results in this dissertation corroborated the results and outputs reported in the previous literature. For example, it has been proved by many studies that longer IWT is likely to lead to more appointment no-shows.

The findings and methods in this study can be used to help providers to make more accurate predictions and improved allocation plans. In a traditional outpatient appointment system, predictions and optimizations are not applied in such a system. These clinics usually schedule patients based on previous experience or routine practice. As a result, many resources are wasted, and patients are dissatisfied. The proposed models and approaches could significantly reduce the waste of resources and improve the accessibility of healthcare services for patients.

Nevertheless, there are some limitations to this study. The first and biggest limitation is the dataset obtained. Although one of the objectives of this dissertation is to avoid profiling patients, it is unlikely to give a very accurate prediction of patient’s behavior on no-shows without demographic and personal information. Simply speaking, the dataset used in this dissertation missed some important information. Therefore, no-show classification becomes very challenging. Another limitation of this study is that the generalizability of the results and controls may not be met. This is due to the irregular distribution of the data and data groups being classified instead of randomly assigned. All of these will affect the interpretation of the results.

Because of these limitations, future studies should focus on three aspects. First, to collect more related information to enhance the models and approaches so that more accurate predictions and better optimizations could be generated. The second aspect is to discover more information from the current dataset and find a way to test the assumptions based on it. Both aspects are related to the dataset, but the study can also be improved by refining the current models or developing new approaches to generate more accurate predictions and better optimizations.

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APPENDIX A. SUPPLEMENT MATERIAL FOR UTILIZATION PROJECTION

Table A1. Sample Data Variables & Description.

Variable	Description
Appointment Date	The date of the appointment
Appt Time	Exact appointment time (e.g., 9:00 am)
DOW	Day of the week
Session Name	Including morning session (AM), afternoon session (PM), and evening session (EVE).
Appointment Date – Scheduled Date	Same as IWT, that is, the time difference between the appointment date and the scheduled date.
Scheduled Date	The date when the patient is scheduled
Appointment Status	There are four statuses of the appointment: patient no-show, appointment utilized, appointment canceled, appointment canceled by the physician.
Cancellation Reason	The reason why patients cancel his/her appointment
Canceled Date	The date of the cancellation
Duration_of_Appt	Duration of the appointment (vary from 10mins to 90 mins)
Is No Show?	Binary decision. 1 if yes, 0 otherwise
Is Same-Day Appointment?	Binary decision. 1 if yes, 0 otherwise
Is Same-Day Cancellation?	Binary decision. 1 if yes, 0 otherwise
Location	Location of the clinic
Provider Name	Name of the provider
Scheduling Provider Designation	The designation of provider that has been scheduled to patients.
Provider Type	The type of the provider
Patient #	A specific ID for each patient
Walk-in?	Binary decision. 1 if yes, 0 otherwise

Table A2. Parameters, R2, MSE, MAE, and RMSE for all Models

Model	Parameter	Value	Model	Parameter	Value
Linear Regression	a_{lin}	0.6301	BG Model	a_s	0.5507
	b_{lin}	-0.0018		alpha	1.8014
	R^2	0.2984		beta	5.6545
	MSE	0.0021		R^2	0.4565
	MAE	0.0320		MSE	0.0016
	RMSE	0.0458		MAE	0.0303
Exponential Regression	a_e	-0.4859		RMSE	0.0403
	b_e	-0.0020	BdW Model	a_s	0.5506
	R^2	0.3068		alpha	8314.1850
	MSE	0.0021		beta	24559.6700
	MAE	0.0319		c	0.7579
	RMSE	0.0455		R^2	0.4670
Quadratic Regression	a_q	0.6729		MSE	0.0016
	b_1	-0.0061		MAE	0.0298
	b_2	0.0001		RMSE	0.0399
	R^2	-1.5666			
	MSE	0.0077			
	MAE	0.0574			
	RMSE	0.0876			
Logistic Regression	a_{log}	0.5462			
	b_{log}	-0.0076			
	R^2	0.0247			
	MSE	0.0029			
	MAE	0.0388			
	RMSE	0.0540			

Table A3. R-squared (R^2) of All Models for Different Appointment Statuses

Comparison of R^2						
	Linear	**Exponential**	**Quadratic**	**Logistic**	**BG**	**BdW**
Utilization	0.298401	0.306845	-1.566643	0.024704	0.456543	0.467014
No-show	0.016449	0.015773	0.076598	0.015873	0.140357	0.149185
Cancellation	0.081063	0.076970	0.146271	0.078388	0.093917	0.225949
BMP	0.601708	0.626573	0.624874	0.000000	0.027520	0.365258

Table A4. Mean Square Error (MSE) of All Models for Different Appointment Statuses

Comparison of MSE						
	Linear	**Exponential**	**Quadratic**	**Logistic**	**BG**	**BdW**
Utilization	0.002095	0.002070	0.007665	0.002913	0.001623	0.001592
No-show	0.000609	0.000610	0.000572	0.000610	0.000533	0.000527
Cancellation	0.001651	0.001659	0.001534	0.001656	0.001628	0.001391
BMP	0.000291	0.000273	0.000274	0.000731	0.000711	0.000464

Table A5. Mean Absolute Error (MAE) of All Models for Different Appointment Statuses

Comparison of MAE						
	Linear	**Exponential**	**Quadratic**	**Logistic**	**BG**	**BdW**
Utilization	0.032035	0.031918	0.038796	0.057429	0.030267	0.029753
No-show	0.017028	0.017047	0.015774	0.017045	0.016430	0.016216
Cancellation	0.028979	0.029023	0.027972	0.029007	0.029310	0.025348
BMP	0.012924	0.012032	0.012053	0.021618	0.021259	0.015967

Table A6. Root Mean Squared Error (RMSE) of All Models for Different Appointment Statuses

Comparison of RMSE						
	Linear	**Exponential**	**Quadratic**	**Logistic**	**BG**	**BdW**
Utilization	0.045775	0.045499	0.053970	0.087552	0.040287	0.039897
No-show	0.024686	0.024695	0.023919	0.024693	0.023079	0.022960
Cancellation	0.040636	0.040726	0.039167	0.040695	0.040350	0.037295
BMP	0.017066	0.016525	0.016562	0.027041	0.026667	0.021544

	I	J	K	L	M	N	O
1	**as**	0.551712714364226	MSE	=AVERAGE(N7:N103)			
2	**alpha**	1.57043204451498	MAE	=AVERAGE(O7:O103)			
3	**beta**	2	RMSE	=SQRT(L1)			
4			R^2	=1-N104/G104			
5							
6	**Indirect Waiting Time**	**Appointment Utlization**	**P(T=t)**	**S(t)**	**y=as+P(T=t)**	**SE**	**AE**
7	0	0.823714519478885	=J1/(J1+J2)	=1	=$J$1+K7	=(J7-M7)^2	=ABS(J7-M7)
8	1	0.700638911788953	=($J$2+A8-1)*K7/($J$1+$J$2+A8)	=L7-K7	=$J$1+K8	=(J8-M8)^2	=ABS(J8-M8)
9	2	0.660058309037901	=($J$2+A9-1)*K8/($J$1+$J$2+A9)	=L8-K8	=$J$1+K9	=(J9-M9)^2	=ABS(J9-M9)
10	3	0.646023329798515	=($J$2+A10-1)*K9/($J$1+$J$2+A10)	=L9-K9	=$J$1+K10	=(J10-M10)^2	=ABS(J10-M10)
11	4	0.613047649504317	=($J$2+A11-1)*K10/($J$1+$J$2+A11)	=L10-K10	=$J$1+K11	=(J11-M11)^2	=ABS(J11-M11)
12	5	0.6330242510699	=($J$2+A12-1)*K11/($J$1+$J$2+A12)	=L11-K11	=$J$1+K12	=(J12-M12)^2	=ABS(J12-M12)
13	6	0.607514256960751	=($J$2+A13-1)*K12/($J$1+$J$2+A13)	=L12-K12	=$J$1+K13	=(J13-M13)^2	=ABS(J13-M13)
14	7	0.594220349187237	=($J$2+A14-1)*K13/($J$1+$J$2+A14)	=L13-K13	=$J$1+K14	=(J14-M14)^2	=ABS(J14-M14)
15	8	0.601536772777168	=($J$2+A15-1)*K14/($J$1+$J$2+A15)	=L14-K14	=$J$1+K15	=(J15-M15)^2	=ABS(J15-M15)
16	9	0.60987012987013	=($J$2+A16-1)*K15/($J$1+$J$2+A16)	=L15-K15	=$J$1+K16	=(J16-M16)^2	=ABS(J16-M16)
17	10	0.578604044357469	=($J$2+A17-1)*K16/($J$1+$J$2+A17)	=L16-K16	=$J$1+K17	=(J17-M17)^2	=ABS(J17-M17)
18	11	0.589711417816813	=($J$2+A18-1)*K17/($J$1+$J$2+A18)	=L17-K17	=$J$1+K18	=(J18-M18)^2	=ABS(J18-M18)

Figure A1. Equations for Recursion in Excel

I	J	K	L	M	N	O
as	0.551712714	MSE	0.001679			
alpha	1.570432045	MAE	0.031335			
beta	2	RMSE	0.04098			
		R^2	0.437691			
Indirect Waiting Time	**Appointment Utlization**	**P(T=t)**	**S(t)**	**y=as+P(T=t)**	**SE**	**AE**
0	0.823714519	0.259978831	1	0.811691546	0.0001	0.012
1	0.700638912	0.130768789	0.740021	0.682481503	0.0003	0.0182
2	0.660058309	0.081543057	0.609252	0.633255771	0.0007	0.0268
3	0.64602333	0.056840241	0.527709	0.608552955	0.0014	0.0375
4	0.61304765	0.04243357	0.470869	0.594146285	0.0004	0.0189
5	0.633024251	0.033188503	0.428436	0.584901217	0.0023	0.0481
6	0.607514257	0.026847933	0.395247	0.578560648	0.0008	0.029
7	0.594220349	0.022280994	0.368399	0.573993709	0.0004	0.0202
8	0.601536773	0.018865344	0.346118	0.570578059	0.001	0.031
9	0.60987013	0.016233334	0.327253	0.567946049	0.0018	0.0419
10	0.578604044	0.014155363	0.311019	0.565868077	0.0002	0.0127
11	0.589711418	0.012481471	0.296864	0.564194185	0.0007	0.0255

Figure A2. Values for Recursion in Excel

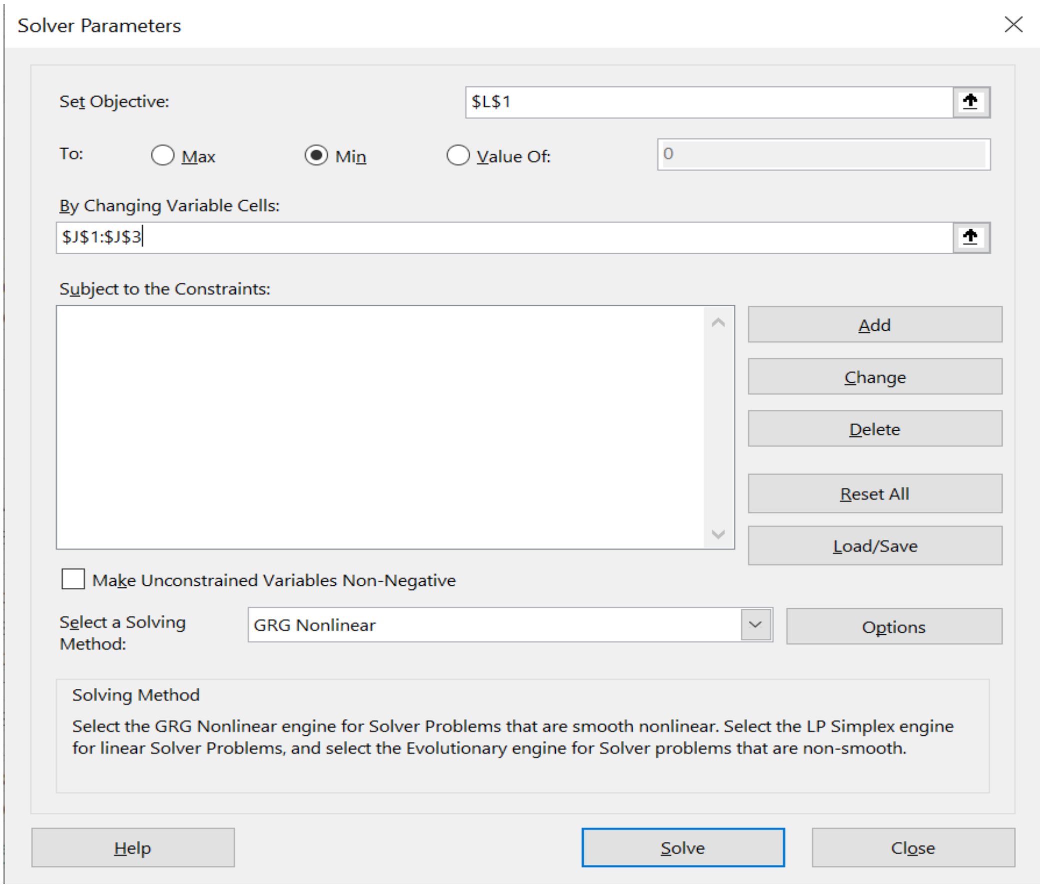

Figure A3. Entries for Excel Solver

	A	B	C	D	E
1	as	0.550639650881032	ln(B(alpha,beta)	=GAMMALN(B2)+GAMMALN(B3)-GAMMALN(B2+B3)	
2	alpha	17287.6271749978	MSE	=AVERAGE(F7:F103)	
3	beta	51066.2846551982	MAE	=AVERAGE(H7:H103)	
4	c	0.757900438237619	RMSE	=SQRT(D2)	
5			R^2	=1-F104/G104	
6	Indirect Waiting Time	Appointment Utlization	P(T=t)	S(t)	y=as+P(T=t)
7	0	0.823714519478885	=D7-D8	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A7^$B$4)-GAMMALN($B$2+$B$3+A7^$B$4)-$D$1)	=C7+$B$1
8	1	0.700638911788953	=D8-D9	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A8^$B$4)-GAMMALN($B$2+$B$3+A8^$B$4)-$D$1)	=C8+$B$1
9	2	0.660058309037901	=D9-D10	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A9^$B$4)-GAMMALN($B$2+$B$3+A9^$B$4)-$D$1)	=C9+$B$1
10	3	0.646023329798515	=D10-D11	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A10^$B$4)-GAMMALN($B$2+$B$3+A10^$B$4)-$D$1)	=C10+$B$1
11	4	0.613047649504317	=D11-D12	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A11^$B$4)-GAMMALN($B$2+$B$3+A11^$B$4)-$D$1)	=C11+$B$1
12	5	0.6330242510699	=D12-D13	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A12^$B$4)-GAMMALN($B$2+$B$3+A12^$B$4)-$D$1)	=C12+$B$1
13	6	0.607514256960751	=D13-D14	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A13^$B$4)-GAMMALN($B$2+$B$3+A13^$B$4)-$D$1)	=C13+$B$1
14	7	0.594220349187237	=D14-D15	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A14^$B$4)-GAMMALN($B$2+$B$3+A14^$B$4)-$D$1)	=C14+$B$1
15	8	0.601536772777168	=D15-D16	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A15^$B$4)-GAMMALN($B$2+$B$3+A15^$B$4)-$D$1)	=C15+$B$1
16	9	0.60987012987013	=D16-D17	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A16^$B$4)-GAMMALN($B$2+$B$3+A16^$B$4)-$D$1)	=C16+$B$1
17	10	0.578604044357469	=D17-D18	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A17^$B$4)-GAMMALN($B$2+$B$3+A17^$B$4)-$D$1)	=C17+$B$1
18	11	0.589711417816813	=D18-D19	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A18^$B$4)-GAMMALN($B$2+$B$3+A18^$B$4)-$D$1)	=C18+$B$1
19	12	0.56594427244582	=D19-D20	=EXP(GAMMALN($B$2)+GAMMALN($B$3+A19^$B$4)-GAMMALN($B$2+$B$3+A19^$B$4)-$D$1)	=C19+$B$1

Figure A4. Equations for Recursion of BdW Model in Excel

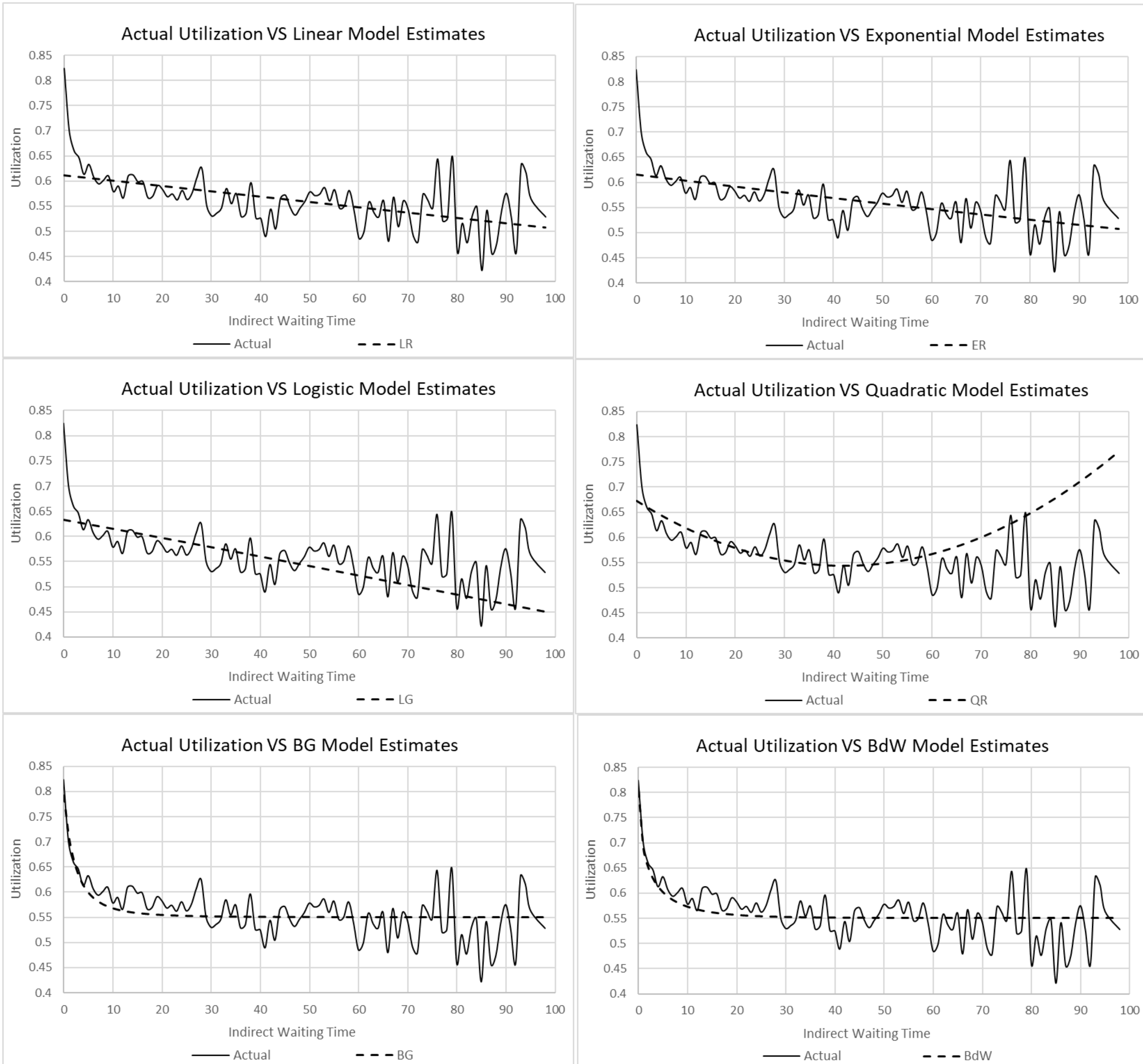

Figure A5. Model Estimates Versus Actual Appointment Utilization

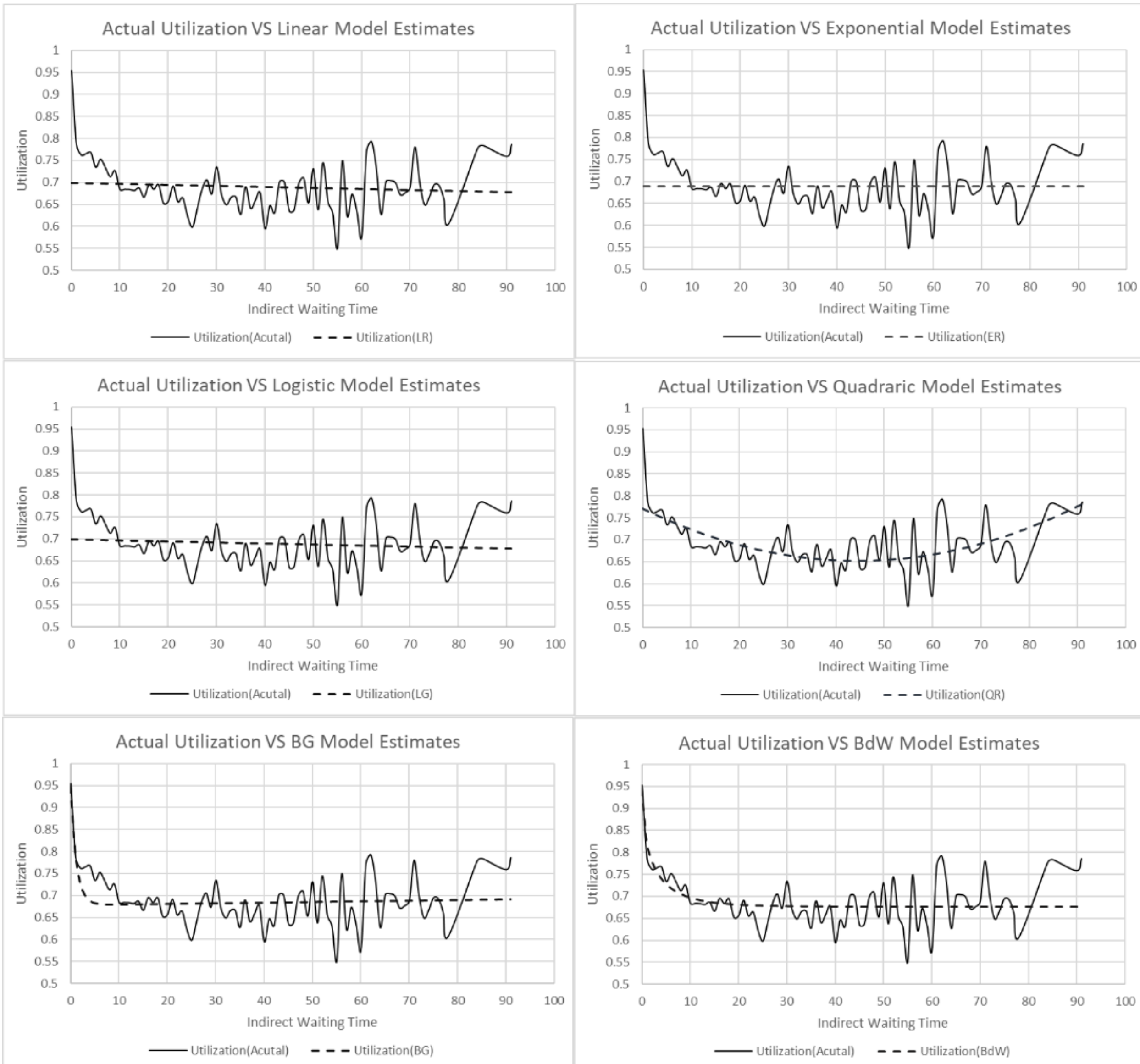

Figure A6. Model Estimates Versus Actual Appointment Utilization for Open-Source Data

APPENDIX B. SUPPLEMENT MATERIAL FOR NO-SHOW PREDICTION

Table B1. Summary of Patient No-show Prediction Studies in Recent Years.

Study	Model	Factors	Evaluation Methods
Mohammadi et al. (2018)	Multivariable logistic regression, Artificial neural network, naïve Bayes classifier.	Clinic operational and financial data, patients' demographic information and clinical characteristics.	Area under the ROC curve (AUC-ROC), Percentage of correct classification.
Daggy et al. (2010)	Multivariable logistic regression	Patients' demographic information, appointment details, appointment attendance records, and clinical characteristics	Area under the ROC curve (AUC-ROC), Hosmer–Lemeshow test.
Samuels et al. (2015)	Multivariable logistic regression	Patients' demographic information, appointment details, appointment attendance records, and providers' information.	Chi-square test and Fisher's exact test.
Kurasawa et al. (2016)	Multivariable logistic regression	Patients' clinical condition, demographic information, appointment details, medical information, etc.	Area under the ROC curve (AUC-ROC), F-measure.
Reid et al. (2015)	Multivariable logistic regression	Patients' demographic information, appointment attendance records, appointment type, clinical diagnoses, and clinical procedure information.	Area under the ROC curve (AUC-ROC)
Farro (2013)	Multivariate logistic regression	Patients' demographic factors, environmental factors, and patients' behavior.	R-squared (R2) statistic.
Alaeddini et al. (2015)	Hybrid probabilistic model based on logistic regression and empirical Bayesian inference.	Patients' social and demographic information, appointment attendance records, and clinical characteristics.	Mean Square Error (MSE), Area under the ROC curve (AUC-ROC).
Elvira et al. (2017)	Gradient boosting machine (GMB) model	Patients' social and demographic information, appointment details, and attendance records.	Area under the ROC curve (AUC-ROC), Confusion matrix.
Huang and Hanauer (2014)	Multivariate logistic regression	Patients' demographic information, appointment characteristics, insurance information.	Goodness of Fit test with Anderson-Darling (AD) statistic.

Table B2. Parameter Estimates of the GLM Model

Dependent Variable: No-show									
					95% Confidence Interval				
Parameter	**B**	**Std. Error**	**t**	**Sig.**	**Lower Bound**	**Upper Bound**	**Partial Eta Squared**	**Noncent. Parameter**	**Observed Power[b]**
Intercept	.012	.022	.547	.584	-.031	.056	.000	.547	.085
[Month=Jun]	-.002	.026	-.069	.945	-.053	.049	.000	.069	.051
[Month=Mar]	-.012	.025	-.463	.643	-.060	.037	.000	.463	.075
[Month=Other]	0[a]	.	.	.	.	.	.	.	.
[DOW=M/F]	.009	.022	.422	.673	-.034	.053	.000	.422	.071
[DOW=Other]	.010	.022	.436	.663	-.033	.053	.000	.436	.072
[DOW=Sat]	0[a]	.	.	.	.	.	.	.	.
[Duration=30/60]	.357	.079	4.549	.000	.203	.511	.000	4.549	.995
[Duration=Other]	0[a]	.	.	.	.	.	.	.	.
[Sameday=0]	.173	.018	9.347	.000	.137	.209	.001	9.347	1.000
[Sameday=1]	0[a]	.	.	.	.	.	.	.	.
[Location=Other]	-.006	.019	-.337	.736	-.043	.031	.000	.337	.063
[Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[Provider=Faculty]	.007	.015	.480	.632	-.023	.037	.000	.480	.077
[Provider=Other]	0[a]	.	.	.	.	.	.	.	.
IWT	.001	.000	2.613	.009	.000	.002	.000	2.613	.743
[DOW=M/F] * [Duration=30/60]	.052	.073	.710	.478	-.091	.195	.000	.710	.109
[DOW=M/F] * [Duration=Other]	0[a]	.	.	.	.	.	.	.	.
[DOW=Other] * [Duration=30/60]	.076	.073	1.039	.299	-.067	.218	.000	1.039	.180
[DOW=Other] * [Duration=Other]	0[a]	.	.	.	.	.	.	.	.
[DOW=Sat] * [Duration=30/60]	0[a]		.	.	.	.	.	.	.
[DOW=Sat] * [Duration=Other]	0[a]	.	.	.	.	.	.	.	.
[DOW=M/F] * IWT	-.001	.000	-2.273	.023	-.002	.000	.000	2.273	.623
[DOW=Other] * IWT	-.001	.000	-2.717	.007	-.002	.000	.000	2.717	.775
[DOW=Sat] * IWT	0[a]	.	.	.	.	.	.	.	.
[DOW=M/F] * [Location=Other]	.015	.018	.812	.417	-.021	.051	.000	.812	.128
[DOW=M/F] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[DOW=Other] * [Location=Other]	.016	.018	.907	.365	-.019	.052	.000	.907	.148
[DOW=Other] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[DOW=Sat] * [Location=Other]	0[a]	.	.	.	.	.	.	.	.
[DOW=Sat] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[Month=Jun] * [DOW=M/F]	.002	.024	.100	.920	-.044	.049	.000	.100	.051

Table B2. Parameter Estimates of the GLM Model (continued)

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval		Partial Eta Squared	Noncent. Parameter	Observed Power[b]
					Lower Bound	Upper Bound			
[Month=Jun] * [DOW=Other]	.005	.024	.221	.825	-.041	.052	.000	.221	.056
[Month=Jun] * [DOW=Sat]	0[a]	.	.	.	.	.	.	.	.
[Month=Mar] * [DOW=M/F]	.010	.023	.435	.664	-.035	.054	.000	.435	.072
[Month=Mar] * [DOW=Other]	-.006	.022	-.246	.805	-.050	.038	.000	.246	.057
[Month=Mar] * [DOW=Sat]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [DOW=M/F]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [DOW=Other]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [DOW=Sat]	0[a]	.	.	.	.	.	.	.	.
[DOW=M/F] * [Provider=Faculty]	.001	.014	.041	.968	-.027	.028	.000	.041	.050
[DOW=M/F] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[DOW=Other] * [Provider=Faculty]	.002	.014	.165	.869	-.025	.029	.000	.165	.053
[DOW=Other] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[DOW=Sat] * [Provider=Faculty]	0[a]	.	.	.	.	.	.	.	.
[DOW=Sat] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[DOW=M/F] * [Sameday=0]	-.049	.018	-2.784	.005	-.083	-.014	.000	2.784	.795
[DOW=M/F] * [Sameday=1]	0[a]	.	.	.	.	.	.	.	.
[DOW=Other] * [Sameday=0]	-.066	.017	-3.800	.000	-.100	-.032	.000	3.800	.967
[DOW=Other] * [Sameday=1]	0[a]	.	.	.	.	.	.	.	.
[DOW=Sat] * [Sameday=0]	0[a]	.	.	.	.	.	.	.	.
[DOW=Sat] * [Sameday=1]	0[a]	.	.	.	.	.	.	.	.
[Duration=30/60] * IWT	.000	.000	-.793	.428	-.001	.001	.000	.793	.125
[Duration=Other] * IWT	0[a]	.	.	.	.	.	.	.	.
[Duration=30/60] * [Location=Other]	-.003	.032	-.107	.915	-.067	.060	.000	.107	.051

Table B2. Parameter Estimates of the GLM Model (continued)

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	Partial Eta Squared	Noncent. Parameter	Observed Power[b]
[Duration=30/60] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[Duration=Other] * [Location=Other]	0[a]	.	.	.	.	.	.	.	.
[Duration=Other] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[Month=Jun] * [Duration=30/60]	-.016	.030	-.527	.598	-.075	.043	.000	.527	.082
[Month=Jun] * [Duration=Other]	0[a]	.	.	.	.	.	.	.	.
[Month=Mar] * [Duration=30/60]	-.005	.031	-.159	.874	-.066	.056	.000	.159	.053
[Month=Mar] * [Duration=Other]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [Duration=30/60]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [Duration=Other]	0[a]	.	.	.	.	.	.	.	.
[Duration=30/60] * [Provider=Faculty]	.023	.060	.388	.698	-.094	.141	.000	.388	.067
[Duration=30/60] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[Duration=Other] * [Provider=Faculty]	0[a]	.	.	.	.	.	.	.	.
[Duration=Other] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[Duration=30/60] * [Sameday=0]	-.540	.025	-21.794	.000	-.589	-.492	.004	21.794	1.000
[Duration=30/60] * [Sameday=1]	0[a]	.	.	.	.	.	.	.	.
[Duration=Other] * [Sameday=0]	0[a]	.	.	.	.	.	.	.	.
[Duration=Other] * [Sameday=1]	0[a]	.	.	.	.	.	.	.	.
[Location=Other] * IWT	.000	.000	2.132	.033	2.438E-5	.001	.000	2.132	.568
[Location=STE700] * IWT	0[a]	.	.	.	.	.	.	.	.
[Month=Jun] * IWT	3.784E-6	.000	.022	.982	.000	.000	.000	.022	.050
[Month=Mar] * IWT	.000	.000	.747	.455	.000	.000	.000	.747	.116
[Month=Other] * IWT	0[a]	.	.	.	.	.	.	.	.
[Provider=Faculty] * IWT	.000	.000	-3.559	.000	-.001	.000	.000	3.559	.945
[Provider=Other] * IWT	0[a]	.	.	.	.	.	.	.	.
[Sameday=0] * IWT	0[a]	.	.	.	.	.	.	.	.

Table B2. Parameter Estimates of the GLM Model (continued)

Parameter	B	Std. Error	t	Sig.	95% Confidence Interval Lower Bound	95% Confidence Interval Upper Bound	Partial Eta Squared	Noncent. Parameter	Observed Power[b]
[Sameday=1] * IWT	0[a]	.	.	.	.	.	.	.	.
[Month=Jun] * [Location=Other]	.005	.009	.559	.576	-.013	.023	.000	.559	.087
[Month=Jun] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[Month=Mar] * [Location=Other]	.008	.009	.936	.349	-.009	.026	.000	.936	.155
[Month=Mar] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [Location=Other]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[Location=Other] * [Provider=Faculty]	-.002	.006	-.297	.767	-.013	.010	.000	.297	.060
[Location=Other] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[Location=STE700] * [Provider=Faculty]	0[a]	.	.	.	.	.	.	.	.
[Location=STE700] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[Sameday=0] * [Location=Other]	-.024	.008	-2.929	.003	-.040	-.008	.000	2.929	.834
[Sameday=0] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[Sameday=1] * [Location=Other]	0[a]	.	.	.	.	.	.	.	.
[Sameday=1] * [Location=STE700]	0[a]	.	.	.	.	.	.	.	.
[Month=Jun] * [Provider=Faculty]	.001	.007	.089	.929	-.013	.015	.000	.089	.051
[Month=Jun] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[Month=Mar] * [Provider=Faculty]	-.009	.007	-1.286	.198	-.023	.005	.000	1.286	.251
[Month=Mar] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [Provider=Faculty]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[Month=Jun] * [Sameday=0]	.012	.011	1.162	.245	-.008	.033	.000	1.162	.213
[Month=Jun] * [Sameday=1]	0[a]	.	.	.	.	.	.	.	.
[Month=Mar] * [Sameday=0]	-.017	.010	-1.696	.090	-.037	.003	.000	1.696	.396

Table B2. Parameter Estimates of the GLM Model (continued)

					95% Confidence Interval				
Parameter	B	Std. Error	t	Sig.	Lower Bound	Upper Bound	Partial Eta Squared	Noncent. Parameter	Observed Power[b]
[Month=Mar] * [Sameday=1]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [Sameday=0]	0[a]	.	.	.	.	.	.	.	.
[Month=Other] * [Sameday=1]	0[a]	.	.	.	.	.	.	.	.
[Sameday=0] * [Provider=Faculty]	.000	.006	.049	.961	-.012	.013	.000	.049	.050
[Sameday=0] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.
[Sameday=1] * [Provider=Faculty]	0[a]	.	.	.	.	.	.	.	.
[Sameday=1] * [Provider=Other]	0[a]	.	.	.	.	.	.	.	.

a. This parameter is set to zero because it is redundant.
b. Computed using alpha = .05

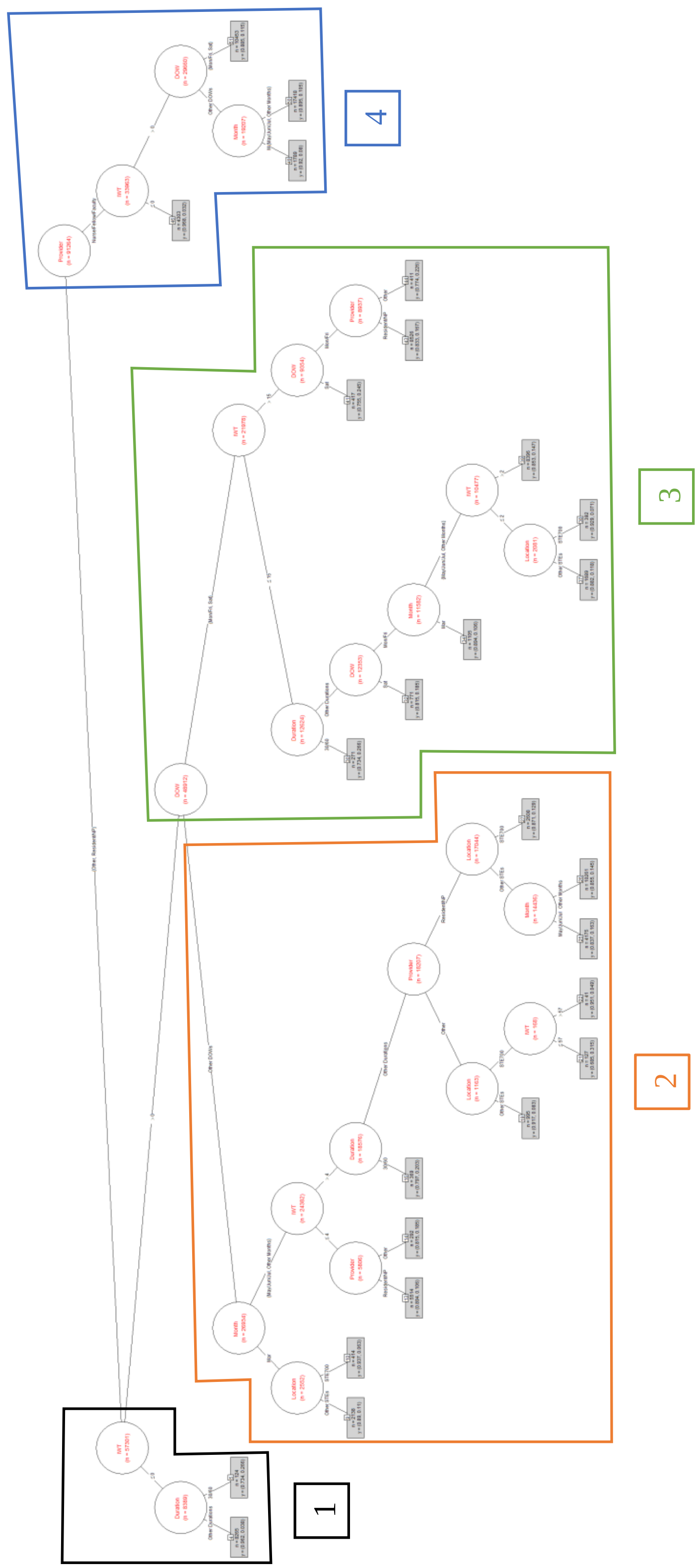

Figure B1. Decision Tree Plot (Ctree Model) for Appointment No-show Prediction

Where Provider Types ∈ (Resident, NP, and Other)

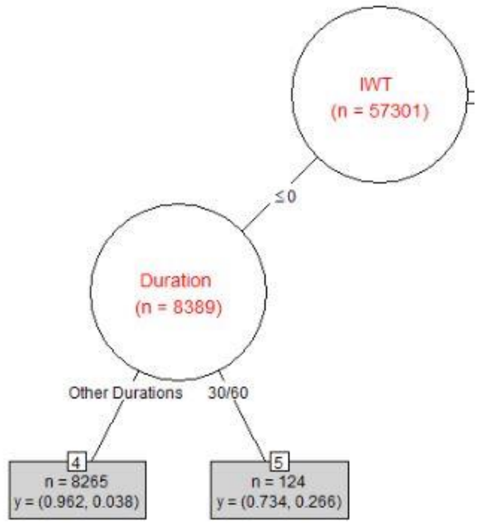

Figure B2. Part 1 of Decision Tree Plot Using Conditional Inference Trees Model

Where Provider Types ∈ (Resident, NP, and Other) & Day of week ∈ (Tuesday, Wednesday, and Thursday)

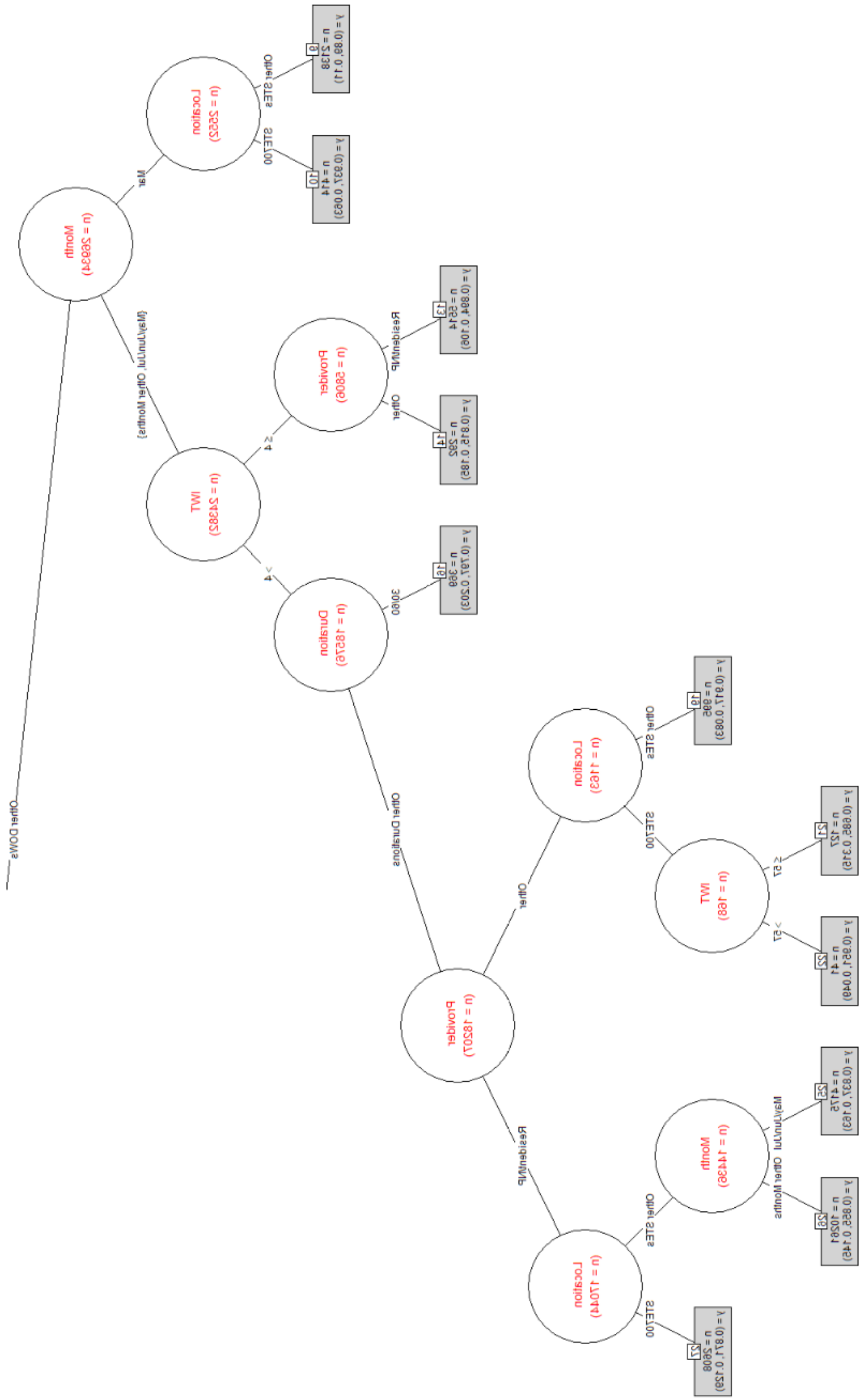

Figure B3. Part 2 of Decision Tree Plot Using Conditional Inference Trees Model

Where Provider Types ∈ (Resident, NP, and Other) & Day of week ∈ (Monday, Friday, and Saturday)

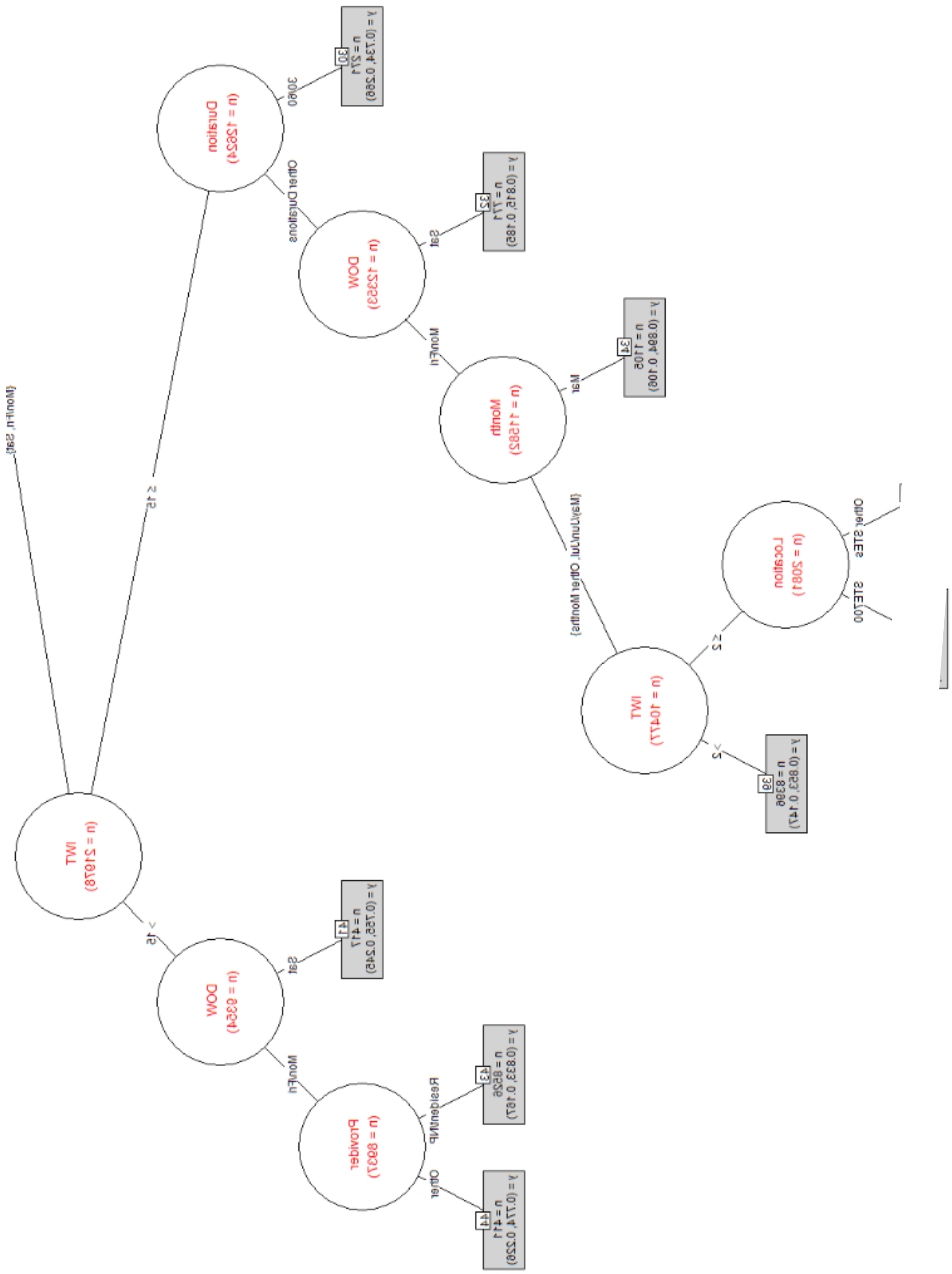

Figure B4. Part 3 of Decision Tree Plot Using Conditional Inference Trees Model

Where Provider Types ∈ (Nurse, Fellow, and Faculty)

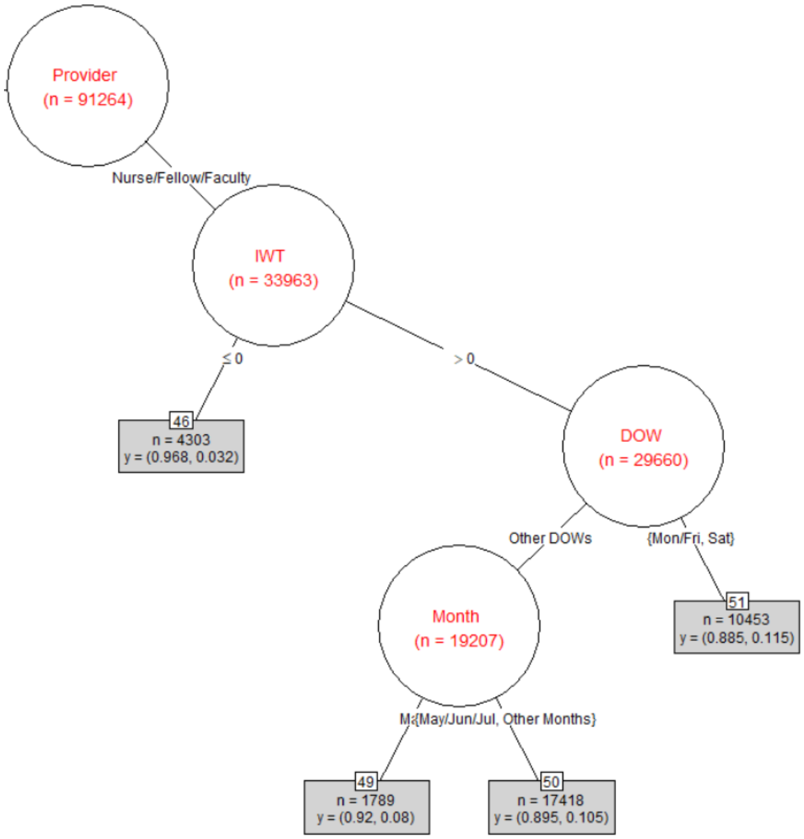

Figure B5. Part 4 of Decision Tree Plot Using Conditional Inference Trees Model

APPENDIX C. SUPPLEMENT MATERIAL FOR CAPACITY ALLOCATION

Table C1. Tukey's Studentized Range (HSD) Test for Demand

Means with the same letter are not significantly different.						**Means with the same letter are not significantly different.**			
Tukey Grouping			**Mean**	**N**	**Month**	**Tukey Grouping**	**Mean**	**N**	**DOW**
	A		426.96	25	Jan	A	447.46	56	Mon
B	A		373.34	29	Mar	A	396.02	62	Wed
B	A		365.75	24	Nov	A	389.92	64	Tue
B	A		362.76	25	Feb	B	319.15	62	Fri
B	A		362.68	25	Sep	B	296.9	60	Thu
B	A		348.88	26	Oct	C	34.64	59	Sat
B	A		345.15	26	Dec				
B	A		344.37	30	Aug				
B	A	C	322.85	26	Jul				
B	A	C	303.86	29	Apr				
B		C	226.63	46	May				
		C	188.25	52	Jun				

Table C2. The Expected Profit and Optimal Ns Given Ap

Ap	**Ns**	**Expected Profit if Ds ≥ Ns**	
30	456	$	2,968.71
117	375	$	2,950.05
167	329	$	2,939.31
196	302	$	2,933.08
225	275	$	2,926.85
274	230	$	2,916.33
295	210	$	2,911.82
323	184	$	2,905.80
353	156	$	2,899.35
391	121	$	2,891.18

Table C3. Expected Profit for Each Pair of Ap and Ns Given Ds.

		Ds										
Ap	Ns	98	119	128	138	148	164	173	194	205	212	Expected profit
30	456	$ (206.86)	$ (19.00)	$ 61.51	$ 150.97	$ 240.42	$ 383.55	$ 464.07	$ 651.92	$ 750.33	$ 812.95	$ 328.99
117	375	$ 496.29	$ 684.15	$ 764.66	$ 854.11	$ 943.57	$1,086.70	$1,167.21	$1,355.07	$1,453.47	$1,516.09	$ 1,032.13
167	329	$ 900.39	$1,088.25	$1,168.76	$1,258.22	$1,347.68	$1,490.81	$1,571.32	$1,759.18	$1,857.58	$1,920.20	$ 1,436.24
196	302	$1,134.78	$1,322.63	$1,403.15	$1,492.60	$1,582.06	$1,725.19	$1,805.70	$1,993.56	$2,091.96	$2,154.58	$ 1,670.62
225	275	$1,369.16	$1,557.02	$1,637.53	$1,726.98	$1,816.44	$1,959.57	$2,040.08	$2,227.94	$2,326.34	$2,388.96	$ 1,905.00
274	230	$1,765.18	$1,953.04	$2,033.55	$2,123.01	$2,212.47	$2,355.60	$2,436.11	$2,623.97	$2,722.37	$2,784.99	$ 2,301.03
295	210	$1,934.91	$2,122.77	$2,203.28	$2,292.73	$2,382.19	$2,525.32	$2,605.83	$2,793.69	$2,892.09	$2,904.73	$ 2,465.75
323	184	$2,161.21	$2,349.07	$2,429.58	$2,519.03	$2,608.49	$2,751.62	$2,832.13	$2,898.54	$2,898.54	$2,898.54	$ 2,634.68
353	156	$2,403.67	$2,591.53	$2,672.04	$2,761.50	$2,850.96	$2,891.92	$2,891.92	$2,891.92	$2,891.92	$2,891.92	$ 2,773.93
391	121	$2,710.79	$2,890.00	$2,883.53	$2,883.53	$2,883.53	$2,883.53	$2,883.53	$2,883.53	$2,883.53	$2,883.53	$ 2,866.90

Table C4. Expected Profit for Each Pair of Np and Ns Given Dp.

		Dp										
Np	Ns	117	167	196	196	225	274	295	323	353	295	Expected profit
30	456	$328.99	$328.99	$328.99	$328.99	$328.99	$328.99	$328.99	$328.99	$328.99	$328.99	$328.99
117	375	$1,032.13	$1,032.13	$1,032.13	$1,032.13	$1,032.13	$1,032.13	$1,032.13	$1,032.13	$1,032.13	$1,032.13	$1,032.13
167	329	$1,032.13	$1,436.24	$1,436.24	$1,436.24	$1,436.24	$1,436.24	$1,436.24	$1,436.24	$1,436.24	$1,436.24	$1,395.83
196	302	$1,032.13	$1,436.24	$1,670.62	$1,670.62	$1,670.62	$1,670.62	$1,670.62	$1,670.62	$1,670.62	$1,670.62	$1,583.33
225	275	$1,032.13	$1,436.24	$1,670.62	$1,670.62	$1,905.00	$1,905.00	$1,905.00	$1,905.00	$1,905.00	$1,905.00	$1,723.96
274	230	$1,032.13	$1,436.24	$1,670.62	$1,670.62	$1,905.00	$2,301.03	$2,301.03	$2,301.03	$2,301.03	$2,301.03	$1,921.98
295	210	$1,032.13	$1,436.24	$1,670.62	$1,670.62	$1,905.00	$2,301.03	$2,465.75	$2,465.75	$2,465.75	$2,465.75	$1,987.87
323	184	$1,032.13	$1,436.24	$1,670.62	$1,670.62	$1,905.00	$2,301.03	$2,465.75	$2,634.68	$2,634.68	$2,465.75	$2,021.65
353	156	$1,032.13	$1,436.24	$1,670.62	$1,670.62	$1,905.00	$2,301.03	$2,465.75	$2,634.68	$2,773.93	$2,465.75	$2,035.58
391	121	$1,032.13	$1,436.24	$1,670.62	$1,670.62	$1,905.00	$2,301.03	$2,465.75	$2,634.68	$2,773.93	$2,465.75	$2,035.58

Table C5. Empirical Distribution for Demands under Each Group

ξ_1: May/June - Mon/Tue/Wed			ξ_2: May/June - Thu/Fri			ξ_3: May/June – Sat		
Date	**Sum of Demand**	**CDF**	**Date**	**Sum of Demand**	**CDF**	**Date**	**Sum of Demand**	**CDF**
5/9/2016	5	0.0204	5/6/2016	1	0.0294	5/28/2016	1	0.0667
5/11/2016	26	0.0408	5/12/2016	32	0.0588	5/14/2016	2	0.1333
5/10/2016	40	0.0612	6/30/2017	34	0.0882	5/21/2016	5	0.2000
5/18/2016	43	0.0816	5/13/2016	35	0.1176	6/4/2016	8	0.2667
5/16/2016	49	0.1020	6/29/2017	43	0.1471	6/11/2016	10	0.3333
5/17/2016	58	0.1224	5/19/2016	55	0.1765	6/18/2016	13	0.4000
5/24/2016	59	0.1429	5/26/2016	60	0.2059	6/17/2017	16	0.4667
5/25/2016	90	0.1633	5/20/2016	67	0.2353	6/24/2017	16	0.5333
5/23/2016	92	0.1837	5/27/2016	75	0.2647	6/25/2016	18	0.6000
5/31/2016	98	0.2041	6/2/2016	104	0.2941	6/10/2017	27	0.6667
6/1/2016	98	0.2245	6/9/2016	112	0.3235	5/27/2017	31	0.7333
6/28/2017	101	0.2449	6/3/2016	114	0.3529	6/3/2017	31	0.8000
6/7/2016	122	0.2653	6/22/2017	132	0.3824	5/13/2017	32	0.8667
6/27/2017	127	0.2857	6/23/2017	141	0.4118	5/6/2017	43	0.9333
6/13/2016	128	0.3061	6/10/2016	151	0.4412	5/20/2017	44	1.0000
6/6/2016	145	0.3265	6/16/2016	156	0.4706			
6/8/2016	154	0.3469	6/23/2016	194	0.5000			
6/26/2017	180	0.3673	6/16/2017	202	0.5294			
6/14/2016	187	0.3878	6/17/2016	212	0.5588			
6/20/2016	233	0.4082	6/24/2016	218	0.5882			
6/15/2016	238	0.4286	6/15/2017	239	0.6176			
6/21/2017	241	0.4490	6/8/2017	252	0.6471			
6/20/2017	253	0.4694	6/30/2016	259	0.6765			
6/21/2016	254	0.4898	5/25/2017	302	0.7059			
6/22/2016	276	0.5102	6/9/2017	316	0.7353			
6/27/2016	280	0.5306	6/2/2017	318	0.7647			
6/19/2017	282	0.5510	5/12/2017	328	0.7941			
6/14/2017	290	0.5714	6/1/2017	337	0.8235			
6/28/2016	326	0.5918	5/18/2017	340	0.8529			
6/7/2017	335	0.6122	5/5/2017	349	0.8824			
6/13/2017	335	0.6327	5/26/2017	349	0.9118			
6/29/2016	346	0.6531	5/4/2017	352	0.9412			
6/12/2017	361	0.6735	5/19/2017	359	0.9706			
6/6/2017	377	0.6939	5/11/2017	385	1.0000			
5/17/2017	408	0.7143						
5/16/2017	412	0.7347						
5/10/2017	441	0.7551						
5/24/2017	442	0.7755						
5/23/2017	443	0.7959						
6/5/2017	447	0.8163						
5/31/2017	448	0.8367						
5/3/2017	449	0.8571						
5/9/2017	458	0.8776						
5/2/2017	491	0.8980						
5/30/2017	493	0.9184						
5/15/2017	500	0.9388						
5/8/2017	504	0.9592						
5/22/2017	533	0.9796						
5/1/2017	596	1.0000						

Table C5. Empirical Distribution for Demands under Each Group (continued)

ξ_4: Other - Mon/Tue/Wed			ξ_5: Other - Thu/Fri			ξ_6: Other– Sat		
Date	**Sum of Demand**	**CDF**	**Date**	**Sum of Demand**	**CDF**	**Date**	**Sum of Demand**	**CDF**
2/23/2016	1	0.0075	4/22/2016	1	0.0114	7/2/2016	17	0.0227
3/1/2016	1	0.0150	4/28/2016	1	0.0227	8/13/2016	19	0.0455
3/30/2016	1	0.0226	8/11/2017	1	0.0341	12/31/2016	23	0.0682
4/11/2016	1	0.0301	4/13/2017	59	0.0455	9/24/2016	26	0.0909
4/12/2016	1	0.0376	12/15/2016	263	0.0568	12/17/2016	29	0.1136
7/18/2017	1	0.0451	7/15/2016	288	0.0682	1/21/2017	29	0.1364
8/14/2017	1	0.0526	7/14/2016	296	0.0795	10/1/2016	30	0.1591
8/16/2017	1	0.0602	10/20/2016	296	0.0909	11/12/2016	30	0.1818
2/14/2017	267	0.0677	4/7/2017	300	0.1023	12/3/2016	30	0.2045
3/15/2017	324	0.0752	7/21/2016	303	0.1136	10/15/2016	31	0.2273
11/23/2016	348	0.0827	9/1/2016	305	0.1250	3/18/2017	33	0.2500
12/14/2016	387	0.0902	9/15/2016	305	0.1364	12/10/2016	35	0.2727
8/23/2016	395	0.0977	8/18/2016	311	0.1477	3/4/2017	35	0.2955
11/8/2016	414	0.1053	8/25/2016	316	0.1591	4/8/2017	35	0.3182
8/3/2016	417	0.1128	7/1/2016	319	0.1705	7/9/2016	36	0.3409
10/4/2016	417	0.1203	9/22/2016	320	0.1818	8/6/2016	36	0.3636
3/14/2017	417	0.1278	7/28/2016	321	0.1932	11/5/2016	37	0.3864
12/21/2016	418	0.1353	12/23/2016	323	0.2045	3/25/2017	37	0.4091
7/26/2016	420	0.1429	3/2/2017	323	0.2159	4/1/2017	38	0.4318
7/20/2016	424	0.1504	10/6/2016	325	0.2273	10/22/2016	40	0.4545
11/2/2016	424	0.1579	8/4/2016	326	0.2386	10/29/2016	40	0.4773
11/1/2016	428	0.1654	7/29/2016	328	0.2500	12/24/2016	40	0.5000
9/21/2016	430	0.1729	4/27/2017	335	0.2614	4/22/2017	40	0.5227
7/27/2016	431	0.1805	2/23/2017	339	0.2727	4/29/2017	41	0.5455
8/31/2016	431	0.1880	9/29/2016	340	0.2841	11/19/2016	42	0.5682
11/22/2016	433	0.1955	11/11/2016	340	0.2955	2/4/2017	42	0.5909
10/12/2016	435	0.2030	11/3/2016	341	0.3068	9/17/2016	43	0.6136
10/25/2016	437	0.2105	12/8/2016	341	0.3182	10/8/2016	43	0.6364
10/5/2016	441	0.2180	12/29/2016	342	0.3295	1/28/2017	43	0.6591
10/31/2016	442	0.2256	7/7/2016	343	0.3409	4/15/2017	43	0.6818
1/24/2017	445	0.2331	9/23/2016	344	0.3523	7/30/2016	44	0.7045
4/19/2017	445	0.2406	4/20/2017	345	0.3636	7/16/2016	45	0.7273
8/17/2016	446	0.2481	8/26/2016	347	0.3750	7/23/2016	46	0.7500
12/13/2016	446	0.2556	9/8/2016	348	0.3864	9/3/2016	47	0.7727
12/28/2016	446	0.2632	2/2/2017	349	0.3977	2/18/2017	47	0.7955
11/9/2016	448	0.2707	2/24/2017	349	0.4091	8/27/2016	48	0.8182
2/22/2017	449	0.2782	11/4/2016	351	0.4205	2/25/2017	49	0.8409
7/6/2016	450	0.2857	8/5/2016	352	0.4318	3/11/2017	49	0.8636
11/21/2016	453	0.2932	11/17/2016	352	0.4432	9/10/2016	50	0.8864
9/14/2016	454	0.3008	2/9/2017	352	0.4545	2/11/2017	51	0.9091
10/26/2016	455	0.3083	7/8/2016	353	0.4659	8/20/2016	52	0.9318
11/16/2016	455	0.3158	9/30/2016	356	0.4773	11/26/2016	52	0.9545
12/6/2016	456	0.3233	10/28/2016	358	0.4886	1/7/2017	62	0.9773
7/19/2016	459	0.3308	10/21/2016	360	0.5000	1/14/2017	62	1.0000
10/18/2016	460	0.3383	3/9/2017	360	0.5114			
9/27/2016	461	0.3459	11/10/2016	361	0.5227			
2/20/2017	461	0.3534	12/22/2016	362	0.5341			
11/15/2016	462	0.3609	12/1/2016	364	0.5455			
8/2/2016	463	0.3684	3/23/2017	367	0.5568			
10/24/2016	465	0.3759	7/22/2016	372	0.5682			
7/13/2016	466	0.3835	12/16/2016	372	0.5795			
1/31/2017	466	0.3910	10/27/2016	375	0.5909			
4/12/2017	466	0.3985	10/13/2016	379	0.6023			
2/21/2017	469	0.4060	4/6/2017	380	0.6136			
4/26/2017	472	0.4135	10/7/2016	381	0.6250			
8/9/2016	474	0.4211	8/19/2016	383	0.6364			
9/13/2016	474	0.4286	8/12/2016	384	0.6477			
10/11/2016	474	0.4361	10/14/2016	384	0.6591			
10/3/2016	475	0.4436	9/9/2016	387	0.6705			

Table C5. Empirical Distribution for Demands under Each Group (continued)

ξ_4: Other - Mon/Tue/Wed			ξ_5: Other - Thu/Fri		
Date	Sum of Demand	CDF	Date	Sum of Demand	CDF
10/10/2016	475	0.4511	4/21/2017	388	0.6818
2/1/2017	475	0.4586	11/18/2016	389	0.6932
8/24/2016	477	0.4662	4/28/2017	390	0.7045
11/29/2016	478	0.4737	3/30/2017	392	0.7159
3/8/2017	478	0.4812	3/16/2017	394	0.7273
7/18/2016	480	0.4887	2/3/2017	395	0.7386
3/7/2017	480	0.4962	12/9/2016	399	0.7500
4/18/2017	480	0.5038	1/20/2017	399	0.7614
9/28/2016	481	0.5113	9/2/2016	406	0.7727
8/30/2016	486	0.5188	4/14/2017	406	0.7841
12/5/2016	487	0.5263	12/30/2016	407	0.7955
3/1/2017	487	0.5338	3/17/2017	407	0.8068
8/16/2016	492	0.5414	12/2/2016	408	0.8182
8/10/2016	493	0.5489	1/26/2017	408	0.8295
12/7/2016	494	0.5564	2/16/2017	408	0.8409
11/14/2016	495	0.5639	8/11/2016	409	0.8523
9/26/2016	496	0.5714	2/10/2017	418	0.8636
11/30/2016	496	0.5789	3/10/2017	420	0.8750
2/8/2017	496	0.5865	1/5/2017	421	0.8864
7/12/2016	499	0.5940	1/27/2017	422	0.8977
2/7/2017	499	0.6015	2/17/2017	423	0.9091
12/12/2016	500	0.6090	9/16/2016	424	0.9205
9/19/2016	501	0.6165	1/19/2017	425	0.9318
1/17/2017	502	0.6241	1/6/2017	444	0.9432
4/4/2017	503	0.6316	1/13/2017	456	0.9545
4/5/2017	505	0.6391	3/3/2017	456	0.9659
9/7/2016	506	0.6466	1/12/2017	458	0.9773
3/29/2017	506	0.6541	3/31/2017	460	0.9886
8/22/2016	508	0.6617	3/24/2017	468	1.0000
10/19/2016	509	0.6692			
4/10/2017	509	0.6767			
8/29/2016	510	0.6842			
9/6/2016	511	0.6917			
12/20/2016	513	0.6992			
4/25/2017	515	0.7068			
4/3/2017	516	0.7143			
9/20/2016	519	0.7218			
2/28/2017	519	0.7293			
4/11/2017	521	0.7368			
1/16/2017	523	0.7444			
7/25/2016	525	0.7519			
3/28/2017	527	0.7594			
8/1/2016	528	0.7669			
9/12/2016	533	0.7744			
2/15/2017	533	0.7820			
4/17/2017	536	0.7895			
1/25/2017	538	0.7970			
4/24/2017	540	0.8045			
7/5/2016	541	0.8120			
12/27/2016	542	0.8195			
1/10/2017	542	0.8271			
10/17/2016	544	0.8346			
2/6/2017	545	0.8421			
11/7/2016	547	0.8496			
12/19/2016	547	0.8571			
2/13/2017	548	0.8647			
3/6/2017	548	0.8722			
3/21/2017	553	0.8797			
1/30/2017	555	0.8872			

Table C5. Empirical Distribution for Demands under Each Group (continued)

ξ_4: Other - Mon/Tue/Wed		
Date	**Sum of Demand**	**CDF**
3/13/2017	556	0.8947
1/3/2017	559	0.9023
1/11/2017	560	0.9098
3/22/2017	573	0.9173
1/4/2017	578	0.9248
1/18/2017	578	0.9323
1/23/2017	579	0.9398
8/15/2016	583	0.9474
3/27/2017	583	0.9549
2/27/2017	585	0.9624
7/11/2016	587	0.9699
3/20/2017	592	0.9774
11/28/2016	602	0.9850
1/9/2017	620	0.9925
8/8/2016	642	1.0000

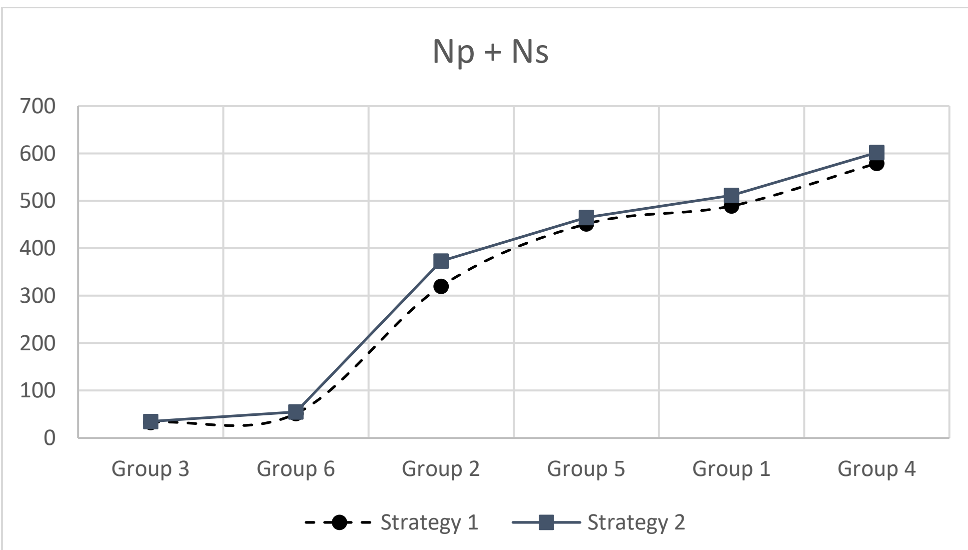

Figure C1. Comparison of Strategies on the Maximum Appointments Can Be Scheduled

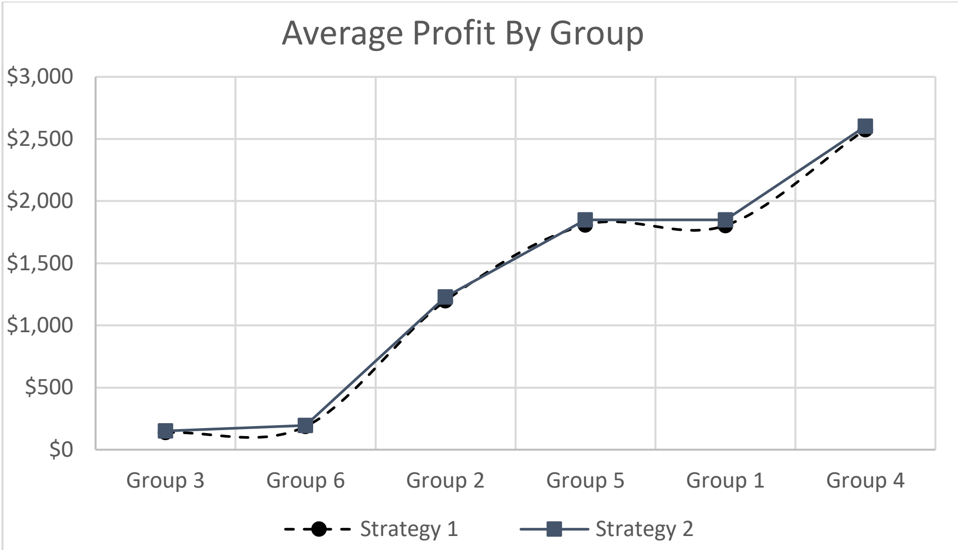

Figure C2. Comparison of Strategies on the Average Profit under Each Demand Group

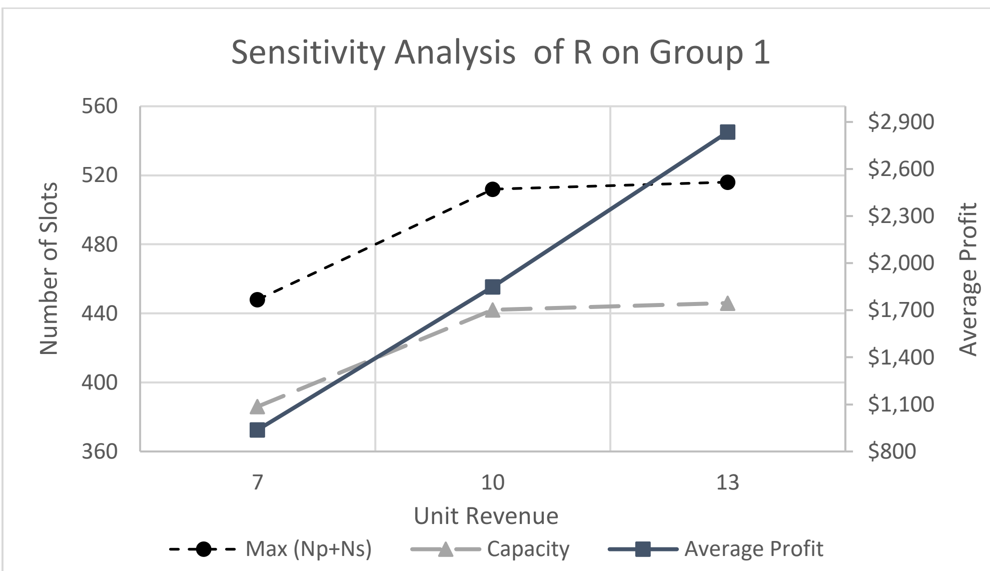

Figure C3. Sensitivity Analysis of Unit Revenue on Group 1

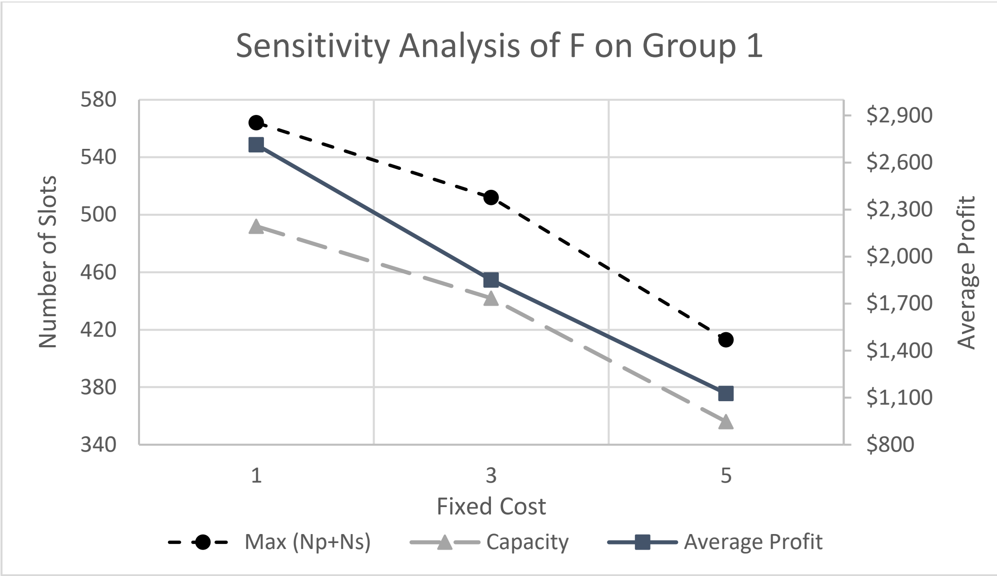

Figure C4. Sensitivity Analysis of Fixed Cost on Group 1

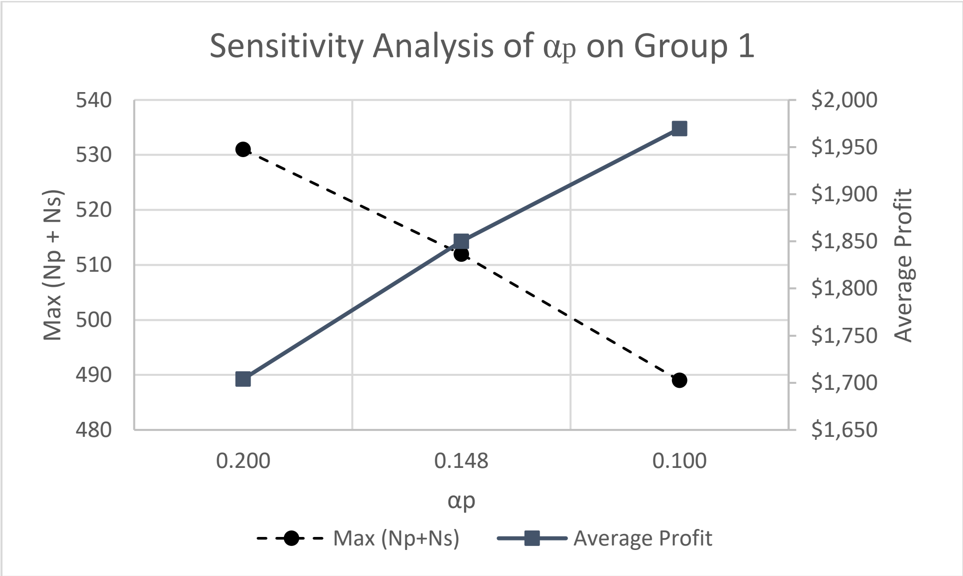

Figure C5. Sensitivity Analysis of α_P on Group 1

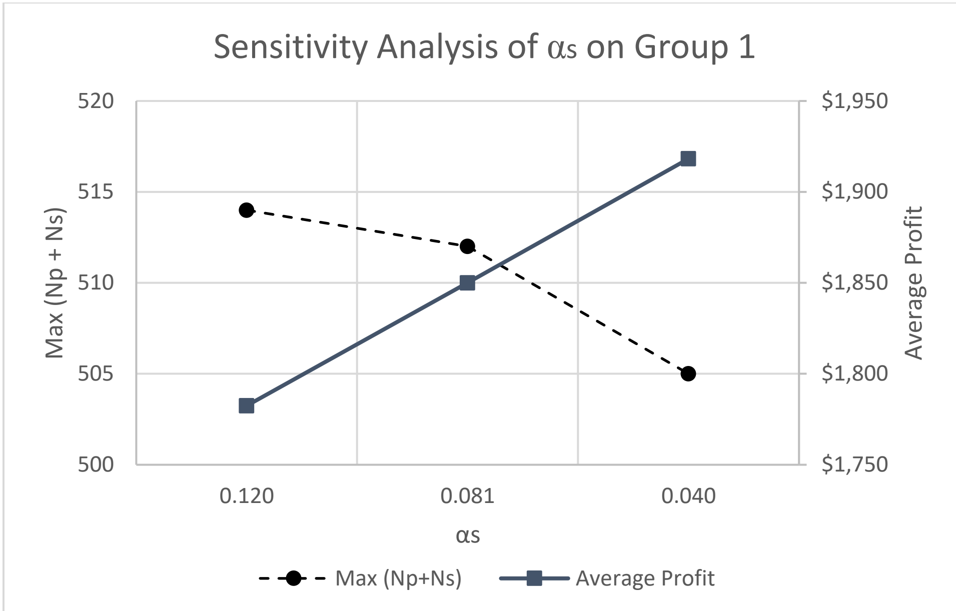

Figure C6. Sensitivity Analysis of α_S on Group 1

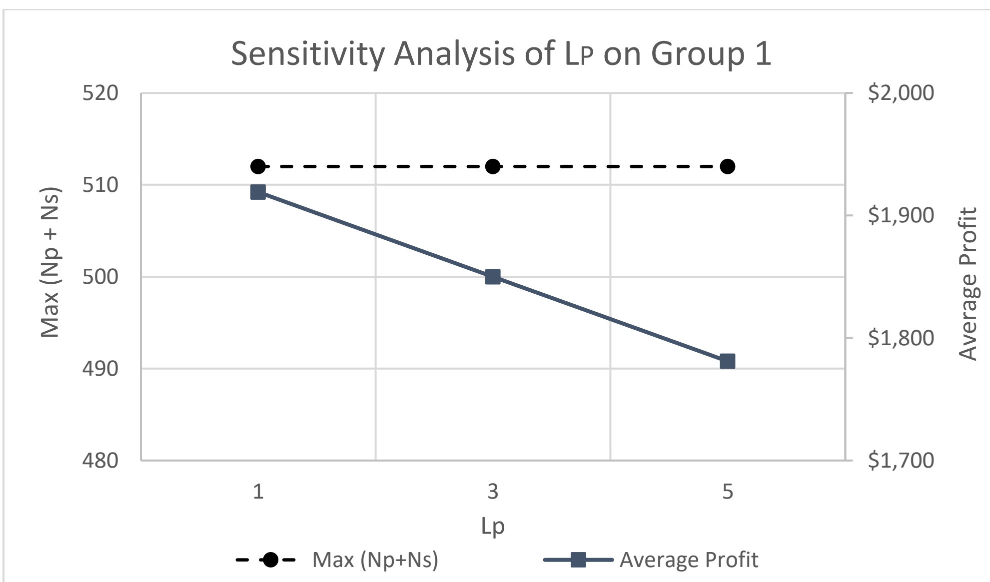

Figure C7. Sensitivity Analysis of L_P on Group 1

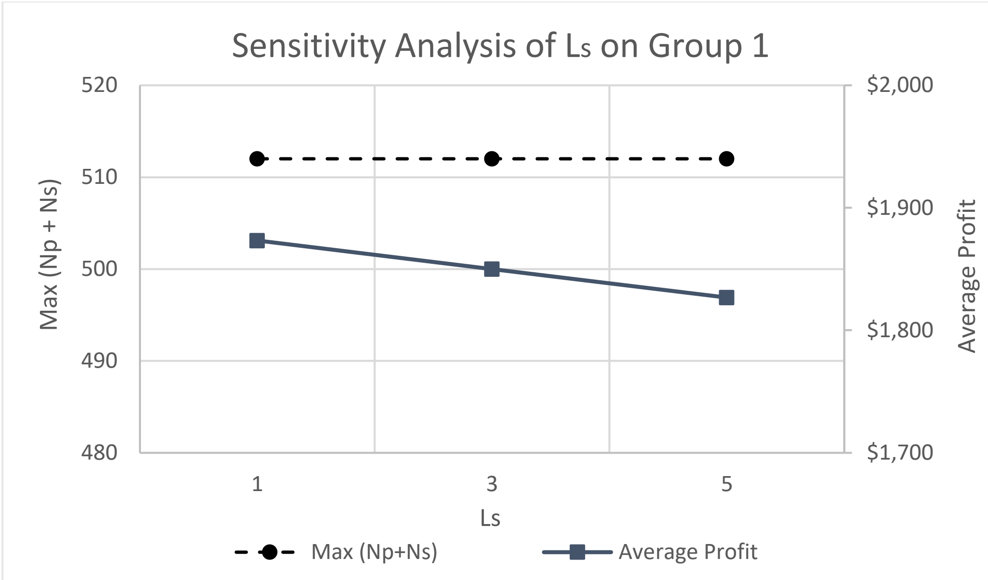

Figure C8. Sensitivity Analysis of L_S on Group 1

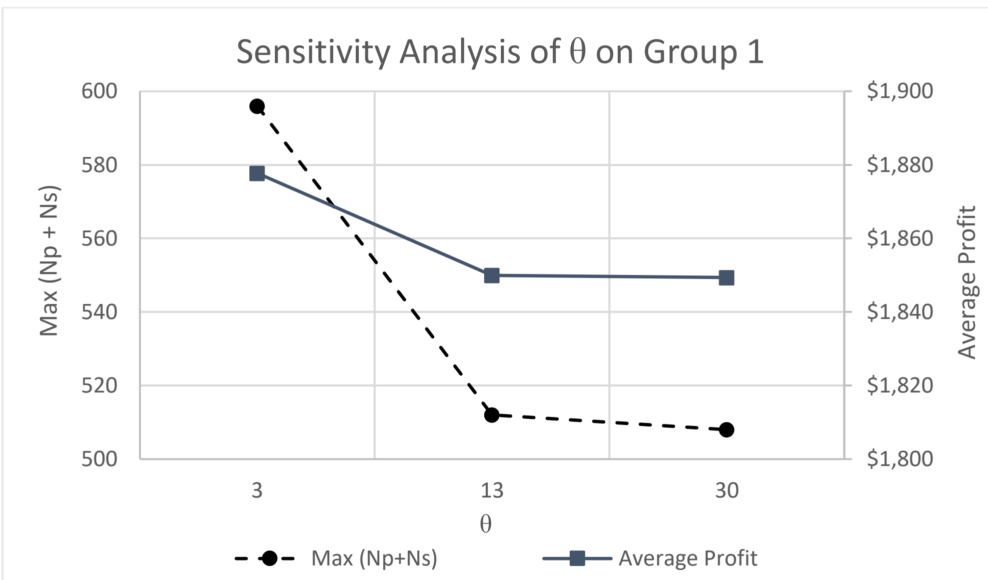

Figure C9. Sensitivity Analysis of θ on Group 1