A STUDY OF DEGRADATION MODELING AND LIFETIME ESTIMATION OF CAPACITORS

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The Supervisory Committee certifies that this ***disquisition*** complies with North Dakota State University's regulations and meets the accepted standards for the degree of

MASTER OF SCIENCE

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ABSTRACT

The degradation of capacitors under accelerated stress conditions occurs in a monotonic and non-linear fashion. Several efforts have been made to model the degradation behavior of capacitor considering either physics-of-failure models or statistical models and subsequently estimate its reliability and lifetime parameters. But most of these models fail to reflect the physical properties of the degradation path, which varies according to several intrinsic and extrinsic factors. These factors introduce random and temporal uncertainty among the population of capacitors. The Gamma stochastic process can model both type of uncertainties among the population of capacitors. In this thesis, we model the capacitor degradation by non-homogeneous Gamma stochastic process in which both the model parameters (shape and scale) are dependent on stress variables. The model parameters are estimated using the maximum likelihood estimation approach. Two case studies have been presented and the life of capacitors has been estimated using Gamma process model.

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DEDICATION

This thesis is dedicated to my parents, Manoj and Vandana – my greatest heroes.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

Capacitors are commonly used in a variety of electronic applications in a broad range of industries. They are extensively used in the energy industry for energy storage applications. With the depletion of fossil fuels and increase in carbon footprints, the thrust towards research and development in renewable energy usages has been increasing in the present time. The boom in renewable energy generation expected during the coming years will drive demand for capacitors used for a number of critical purposes, including power conversion functions in the fast-growing solar and wind segments. The capacitors used for harvesting energy are generally electrolytic and film capacitors which undergo harsh stress conditions such as higher level of voltage, temperature, and humidity. These operating conditions severely limit the useful life of capacitors. Studies have been carried out to estimate life of capacitor under various stress conditions, but the approach lacks combined effect of sample-to-sample variability and temporal variability of degradation behavior. This study deals with the shortcoming of commonly used approaches to estimate remaining useful life of capacitors under various stress conditions and the advantage of Gamma process-based model. The advantage of Gamma process model to estimate lifetime of capacitors addressing sample-to-sample and temporal variability as a function of stress is motivation behind the work presented in this thesis.

1.1. Motivation

Capacitors in power electronics are used for a wide variety of applications, including energy storage, ripple voltage filtering, and DC voltage smoothing. The two major types of capacitors used in power electronic systems are aluminum electrolytic capacitors and metallized film capacitors. The state of health, or life, of these capacitors depends on stress factors like temperature, voltage, ripple current, charge-discharge, and humidity. Various degradation measures such as capacitance, equivalent series resistance, dissipation factor, and insulation resistance have been used to monitor the degradation state of capacitors. To capture the degradation behavior in a shorter time, several acceleration models are used to replicate the specific failure behavior. The acceleration models are usually based on the physics underlying a particular failure mechanism. Some common acceleration models used in capacitor lifetime prediction are the Arrhenius law for temperature, inverse power law for voltage and humidity stresses, and Eyring law for capturing the interaction of other stress factors with temperature stress. The major limitation of acceleration models is their inability to track the capacitor degradation with time. That is, the acceleration models cannot predict the state, or health, of a capacitor at a particular time. Hence, there is a need for data-driven or statistical degradation models to ascertain the capacitor's actual state of health at a given time and subsequently estimate their reliability and remaining useful life. Several efforts have been made to model degradation behavior of the capacitor considering either physics-of-failure models or statistical models and subsequently estimate its reliability and lifetime parameters. But most of these models fail to reflect the physical properties of the degradation path, which varies according to several intrinsic and extrinsic factors. These factors introduce random and temporal uncertainty among the population of capacitors. The Gamma stochastic process can model both type of uncertainties among the population of capacitors. This thesis first reviews the behavior and the degradation modeling of these capacitors for reliability assessment and lifetime analysis. Then, we model the capacitor degradation by non-homogeneous Gamma stochastic process in which both the model parameters (shape and scale) are dependent on stress variables.

1.2. Research Objectives

To estimate the lifetime and reliability of capacitor, we must first understand the associated degradation mechanism and find a condition or health indicator that can describe the degradation process. Hence, the present work must be reviewed in the field of condition monitoring and degradation mechanism of capacitors. This should be followed by the review of models used to capture the degradation of capacitors and estimate their lifetime. Estimating life of capacitors under various stress conditions is not new and one can find many papers devoted to this aim. However, with the available data, finding a model that is best suited and gives very close results to real life of capacitors under different working conditions is a real challenge. Still, one can choose few guidelines to select optimum model to estimate life of capacitors. The research objectives are set with this in mind and are as follows:

1) Review models used for lifetime estimation of capacitors available in current literature.
2) The proposed model should address the limitations of the present models in literature.
3) The model should be based on health conditions of the capacitors rather than end-of-life data.
4) The model which provides higher accuracy of estimation with relatively smaller sample size should be considered.
5) The model should take care of sample-to-sample variability and temporal variability. The model should be dynamic (i.e. time dependent) and should give functional dependence of health indicators on various stresses such as temperature, humidity, voltage, etc.
6) Select relevant papers where the capacitors life has been estimated using conventional techniques and suggest a model which addresses the above objectives and delivers the

lifetime estimates which are better than the existing models in terms of closeness of the results to experimental values or field data.

1.3. Thesis Structure

This thesis consists of four chapters. After the introduction of the background and listing the motivation and objectives of the research in this section, Chapter 2 reviews the failure causes, modes, and mechanisms of two major types of capacitors used in power electronic systems - metallized film capacitors and electrolytic capacitors. The cause of failure of capacitors is identified along with the associated failure mechanisms and the major health indicators are defined. Then, the review of the degradation modeling related to these capacitors is presented, and both physics-of-failure and data-driven degradation models for reliability and lifetime estimation are discussed. Based on the analysis of the present degradation models for end-of-life estimation of capacitors, the shortcomings and research gap are identified.

In Chapter 3, a lifetime estimation model for capacitors based on the extensive literature review is proposed. It has been emphasized that the Gamma stochastic modeling is well-suited to model capacitor degradation under varied operating conditions along with sample-to-sample and temporal variation. Two case studies have been presented to this effect. The capacitor lifetime values obtained using Gamma stochastic model have been compared to the traditionally used models and it is shown that the obtained lifetime estimates are more accurate, even with a smaller sample size.

Chapter 4 highlights the conclusions of this thesis work. Then, the scope of further studies and future research directions are presented.

2. LITERATURE REVIEW[1]

2.1. Failure Analysis of Capacitors

Capacitors could fail due to various factors like manufacturing and design defects, material wear out, operating temperature, voltage, current, humidity and mechanical stress. Wear-out failures signify the end of useful life of a product, and this section is mainly concerned with the wear-out failures in DC-link capacitors. Electrolyte evaporation is the primary wear-out mechanism in electrolytic capacitors and is caused by high temperatures within the capacitor core. In the case of metallized film capacitors, self-healing or localized dielectric breakdown due to overvoltage is the main wear-out mechanism. These failure mechanisms and their root causes, along with the other causes of capacitor failure are discussed in detail in the following section.

2.1.1. Metallized Film Capacitors

Metallized film capacitors consist of dielectric films with a metallic coating on the surface. The electrodes are composed of very thin layers (20 to 100 nm) of metal, usually aluminum or zinc, evaporated onto the surface of the polymer film. The most common dielectric films that are used in the industry are mainly polypropylene (PP) or polyethylene terephthalate (PET). In special applications, dielectrics such as polyethylene naphthalene or polyphenylene sulfide are used for high-temperature operating conditions (up to 150°C). PET film capacitors have higher capacitance per volume than PP film capacitors due to their high dielectric constant and availability in thin gauges. They also have a better mechanical resistance and a slightly higher exploitation temperature than PP film capacitors, with operating temperatures up to

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[1] Sections of this chapter appeared in Gupta, A., Yadav, O.P., DeVoto, D. and Major, J. “A review of degradation behavior and modeling of capacitors.” In *International Electronic Packaging Technical Conference and Exhibition*, Vol. 51920, p. V001T04A004, American Society of Mechanical Engineers (2018). A. Gupta had the primary responsibility of conducting the survey of related research and review the literature. A. Gupta was the primary developer of the conclusions that are advanced here. A. Gupta also drafted and revised all versions of this chapter.

125°C. The negative point is that the loss factor is ten times larger, which means a ten-fold increase in temperature elevation for the same rated power. PP film has superior electrical characteristics. PP film capacitors exhibit very little capacitance change over time with an applied voltage, making them ideal for applications where a stable level of capacitance is needed. The temperature and frequency dependencies of electrical parameters for PP film capacitors are very low. These capacitors may be operated up to 100°C. The dissipation factor of PP film capacitors is smaller than that of other film capacitors. Also, the PP film material absorbs less moisture than PET film [1-3].

Metallized film capacitors have a unique self-healing ability. If there is a micro-void or defect in the dielectric film and the capacitor is connected to a voltage of a sufficient level (electric-field stress), a glow discharge occurs in the micro-void. This discharge generates heat due to a high current density in the breakdown region, which causes evaporation of the electrode layer around the void. The damaged part is disconnected from the capacitor and cannot degrade its performance. This is accompanied by a slight decrease in the capacitance [4]. Metallized film capacitors have this significant advantage of self-healing over general film capacitors or non-metallized film capacitors. In non-metallized film capacitors, when an overvoltage is applied and small localized dielectric breakdown occurs, the exposed electrodes make contact with each other and short, rendering the capacitor useless. In case of metallized films, the combination of the electrode foil thinness and the high energy density in the fault area causes the foils to vaporize and the capacitor stays in operation and avoids catastrophic failure. This phenomenon is termed as “self-healing” [5]. The self-healing or clearing process in metallized film capacitors depends on three factors, namely, the working voltage, mechanical pressure between the winding layers, and metallization thickness (resistivity) [6]. Accumulation of these self-healing events over time, causes a “soft” failure of the metallized film capacitor when the capacitance value decreases by a certain percentage of the initial capacitance. The definition of this soft failure (end-of-life criterion) varies from manufacturer to manufacturer and between applications. With time, the pressure inside the capacitor casing increases due to production of vaporized metal by self-healing events. This can eventually cause a catastrophic failure either through the pressure, increasing enough to burst the capacitor casing, or the vaporized metal concentration increasing enough to become conductive and cause a flashover event [7, 8]. Upon an increase in the working voltage, the self-healing time increases, and the heating of adjacent insulator layers becomes significant, which can be destructive for the structure of the capacitor [9]. Li et al. [10] investigated the influence of temperature and voltage on the life of metallized film capacitors and the temperature dependence of this self-healing characteristic. It is found that a catastrophic failure is more likely to happen under elevated working temperature and voltages due to the decrease in breakdown electric field strength of the capacitor with a rise in temperature. During the self-healing process, there is an energy discharge of stored energy. This discharge of energy under the same breakdown voltage becomes larger under higher temperature conditions.

Temperature alone does not seem to play a major part in the degradation of metallized film capacitors. At elevated temperatures, the mechanical properties of dielectric films change and their volume increases due to thermal expansion. Hence, thermal stress occurs due to the difference between coefficients of the thermal expansion of the dielectric film and electrode metallization on the film, which can influence the structure of the metal film. Research work and experiments conducted in [11] point out significant changes in the non-linearity of current-voltage characteristics of metallized PP film capacitors during accelerated thermal aging, as compared to negligible change in capacitance. These changes are proposed to be due to the deterioration in quality of the electrode metallization from temperature stresses. Apart from voltage and temperature, another important stress factor especially in case of DC link capacitors is the ripple current. By definition, a ripple current is an AC current through the capacitor due to incomplete suppression of the alternating waveform within the power supply. Root mean square (RMS) is a method of denoting the AC waveform as DC value that will produce the same heating effect, or power dissipation in the circuit. The ability of a capacitor to withstand a continuous RMS current depends on the frequency of the electrical stress and thermal power dissipation of the component. If the power generated by the capacitor body is greater than the power that can be dissipated by its surface, increased self-heating may occur. Self-heating increases the capacitor temperature, leading to a reduction of the breakdown voltage, and in the worst cases, even melting of the capacitor [1]. The self-heating temperature of a capacitor depends on equivalent series resistance (ESR), on the current across the capacitor, and on the thermal resistance between the case and the ambient temperature. The combination of high ripple current and voltage leads to electrochemical corrosion of the thin electrode metallization, gradually converting aluminum to aluminum oxide. The corrosion process involves the migration of oxygen and/or moisture of the polymer to the polymer/metallization interface. It is dependent on temperature, electrode thickness, stress, and frequency [2, 12, 13]. Metallized film capacitors using PP as the dielectric are less affected by the ripple current than ones with a PET dielectric.

The effect of AC voltage and temperature on the degradation of metallized film capacitors is studied in [14] and a life model involving all the main stressing factors (peak voltage, RMS value, and temperature) is proposed, using multivariate regression. It is found that the peak voltage has the most significant effect on capacitor aging. Although thermal aging contributes to degradation, its significance is less than the degradation due to peak voltage and

RMS voltage. The detachment of the “sprayed ends” or the schoopage from the capacitor roll is commonly found in pulsed power applications, and it normally results in an open-circuit failure [15]. The combined effect of electrical, thermal, and mechanical stresses gradually leads to the degradation of contact and eventually, the detachment of the sprayed ends. The high current peaks are mainly responsible for these effects, independent of the waveform energy content. As the contact is degraded, the ESR value, and consequently, the loss factor (tan δ) will greatly increase according to the number of pulses applied to the capacitor [16].

The degradation of metallized film capacitors is a concern in applications exposed to high humidity environments. The metallized electrode layer in metallized film capacitors is very thin, typically less than 50 nm, which is susceptible to corrosion due to the ingress of atmospheric moisture. The normal protective surface oxide formation may be comparable in thickness to that of the metal film. Corrosion usually progresses from the ends of the capacitor cylinder to the middle, related to the ingress of atmospheric moisture. Corrosion of metallized electrodes leads to an uneven distribution of current, which causes localized heating and eventually leads to catastrophic failure [17]. Regarding the effect of geometry of metallized film capacitors with the same specifications but different heights, experiments conducted by [18] show that long capacitors deteriorate faster than plate-shaped ones, due to higher heating under the same electrical stress.

2.1.2. Electrolytic Capacitors

Electrolytic capacitors are composed of an electrolyte-impregnated paper layer sandwiched between two highly roughened metal foils (usually aluminum). Grown directly onto the anode foil is a thin, high-quality oxide, which acts as a dielectric possessing excellent breakdown voltage characteristics. One of the primary functions of electrolytic capacitors is the smoothing of voltage ripple and storing electrical energy. Electrolytic capacitors provide high capacitance values, high volumetric efficiency, and an excellent price over performance ratio. However, the electrolytic capacitor has the shortest lifespan of the components in power electronics. Electrolytic capacitors tend to degrade and fail much faster under high electrical or thermal stress operating conditions than metallized film capacitors. The failures in electrolytic capacitors can be categorized into catastrophic and wear-out failures. The wear-out of electrolytic capacitors leads to the drift in the values of two important parameters, capacitance and ESR. The drift in these parameters ultimately leads to failure of the capacitor. Defined industry standards specify the end-of-life of an electrolytic capacitor under thermal stress if the capacitance value decreases by 10% and the ESR value increases by 250% or more from its initial rated value. Similarly, under electrical stress operations, the end-of-life is defined by its ESR increasing by 280% - 300% over and capacitance decreasing by 20% below its initial condition values. The ratings are different for electrical and thermal stress conditions because thermal stress conditions are considered for situations when the capacitor is in storage, and not in operation [19, 20].

The primary failure mechanism of an electrolytic capacitor is the evaporation of the electrolyte due to thermal overstress. A capacitance decrease and an ESR increase are caused by the loss of electrolyte, by diffusion (as vapor) through the sealing material in the wear-out failure period. If the electrolyte vapor pressure within the capacitor increases, by high temperatures for example, the diffusion rate increases. Factors that can increase the capacitor temperature, such as ambient temperature and ripple current, accelerate capacitor wear-out. Hence, estimating the temperature of the capacitor element (capacitor core) is of vital importance. The actual temperature of the capacitor element can be greater than the ambient temperature due to the temperature rise in applications with ripple current. Wear-out can also be accelerated by high internal pressure caused by gas generation from excessive leakage current or attack of the cathode foil by electrolyte (oxide breakdown, which is mainly observed under electrical overstress conditions) [21-26]. Table 1 summarizes the major failure causes, mechanisms and modes of aluminum electrolytic capacitors and metallized film capacitors, mainly concerned with the field aging or application phase of capacitors.

Table 1. Failure causes, mechanisms, and modes of capacitors

Root Cause	Capacitor Type	Major Failure Mechanism	Major Failure Modes	References
High Ambient Temperature	Electrolytic	Electrolyte evaporation; increase in pressure inside the capacitor	Capacitance loss, ESR increase; vent opening, in case of excess internal pressure	21, 22, 23, 25, 26 and 52
	Metallized Film	Deterioration of quality of electrode metallization	Change in the non-linearity of current-voltage characteristics	7, 10 and 11
Over Voltage Stress	Electrolytic	Degradation of oxide film, anode foil capacitance drops	Increased leakage current flow, capacitance loss, ESR increase	21, 26 and 27
	Metallized Film	Self-healing (“soft failure”)	Capacitance loss, ESR increase	1, 5, 6, 7, 8 and 9
Excess Ripple Current	Electrolytic	Electrolyte evaporation	Capacitance loss, ESR increase	21 23 25 26 and 52
	Metallized Film	Electrochemical corrosion; increased core temperature	Capacitance loss, ESR increase	2, 12, 13, 16 and 43
Continuous Charge/ Discharge Cycle (Pulsed Discharge)	Electrolytic	Cathode foil capacitance decrease due to formation of additional dielectric layer; electrolyte evaporation	Capacitance loss; vent opening, in case of excess internal pressure due to gas generated during oxide layer formation	21 and 27
	Metallized Film	Self-healing; electrochemical corrosion	Capacitance loss, ESR increase; detachment of the sprayed ends (schoopage) under high electric stress	15, 16, 44 and 45
Humidity	Metallized Film	Electrochemical corrosion; dielectric loss due to moisture absorption by film	Capacitance loss, ESR increase	1, 17 and 37

In summary, the life of aluminum electrolytic capacitors depends on environmental and electrical factors. Environmental factors include temperature, humidity, atmospheric pressure, and vibration. Electrical factors include operating voltage, ripple current, and charge-discharge duty cycle. Among these factors, temperature (ambient temperature and internal heating resulting from ripple current) is the most critical to the life of aluminum electrolytic capacitors, whereas conditions such as vibration, shock, and humidity will have little effect on the actual life of the capacitor. Voltage within the allowed operating range has little effect on the actual life expectancy of a capacitor. When the applied voltage is above the rated voltage of the capacitor, the leakage current flow increases at a much faster rate, thus increasing the amount of heat generated [27]. Because a capacitor is essentially an electrochemical device, increased temperatures accelerate the chemical reaction rates within the capacitor (usually a 10°C rise in temperature will double the chemical reaction rate) [28].

2.2. Degradation Modeling

2.2.1. Metallized Film Capacitors

2.2.1.1. Physics-of-Failure-Based Models

Acceleration models predict time-to-failure as a function of stress. Knowledge of the time to reach failure at an accelerated stress level, in conjunction with a calibrated acceleration factor, can be used to predict the equivalent time-to-failure at a different operating stress level. These models are usually based on the physics underlying a failure mechanism. Some common acceleration models used in capacitor lifetime prediction are the Arrhenius and Eyring models for temperature-dependent stresses, and the inverse power law for voltage and humidity stresses.

Arrhenius proposed a chemical kinetics model that showed the temperature dependence of reaction rates. Dakin proposed a lifetime model for electrical insulations, based on the Arrhenius model [29]. Capacitor aging as a function of the temperature follows an Arrhenius law, which is basically an exponential law [1, 30, 31]. The activation energy of a thermal process, as given in the Arrhenius law, is a stress- dependent quantity [30]. The experimental observation is that simultaneously applied stresses normally produce a degradation effect which is slightly lower than that derived by a simple multiplicative law. Therefore, an interaction term must be added. Eyring theory generalizes Arrhenius law to many factors besides the temperature, and captures the interaction of other stress factors with the temperature stress given in Eq. (1):

$$\tau_{Eyring}(T_i, S_1, S_2) = A_E T_i^{\alpha} \exp\left(\frac{E_a}{kT_i} + \left(B_E + \frac{C_E}{T_i}\right) S_{1i} + \left(D_E + \frac{E_E}{T_i}\right) S_{2i}\right) \tag{1}$$

where τ_{Eyring} is the useful life of the component, S_{1i} and S_{2i} are the supplementary considered stresses, A_E, B_E, C_E, D_E, E_E and α are the different constants [32]. Simoni [33] and Ramu [34] proposed combined models for thermal and electrical stress, derived from Eyring theory, which take into account the effect of application of electrical stress on the thermal degradation process. The combined-stress life model considering inverse-power electrical life law is given in Eq. (2) as:

$$L = L_o \left[\frac{E}{E_o}\right]^{-(n-bT)} \exp\left(-BT\right) \tag{2}$$

In Eq. (2), n, b and B are the model parameters, L_o is the life at temperature Θ_o, E_o is the value of electrical stress below which electrical aging can be neglected, $T\left(= \frac{1}{\Theta_o} - \frac{1}{\Theta}\right)$ is the so-called conventional thermal stress and Θ is the absolute temperature [35].

Albertini et al. [36] investigate a thermal life model for capacitors subjected to both constant and time-varying temperatures. The constant-temperature life models are based on the Arrhenius model and an Arrhenius–Miner Model is proposed for lifetime calculation under time-varying temperature stress. However, the life estimation by the proposed model is far from accurate when compared to the experimental results, and the temperatures to which capacitors are subjected in the experiment are much higher than the maximum recommended operating temperature (category temperature) of the capacitors.

Regarding voltage and humidity dependency, an inverse power law or an exponential law is generally used [1] [35-38]. An example of a degradation life model simply obtained by multiplying the aging rates of each stress is given in Eq. (3) as:

$$L = L_o \times \left(\frac{RH}{RH_o}\right)^{-n_3} \times \left(\frac{V}{V_o}\right)^{-n_1} \times 2^{\frac{T_o - T}{n_2}} \tag{3}$$

where L and L_0 are the lifetime under the use condition and reference condition, respectively. V and V_0 are the voltages at use condition and reference condition, respectively. T and T_0 are the temperatures at use condition and reference condition, respectively. RH and RH_0 are the relative humidity under the use condition and reference condition, respectively. The exponents for voltage, temperature and humidity stresses are denoted by n_1, n_2 and n_3, respectively.

2.2.1.2. Data-Driven Models

Acceleration models are most often considered to predict the operating lifetime of capacitors under specific environmental conditions (temperature, voltage and humidity). But the major limitation with these models is the inability to track the capacitor degradation with time. That is, the acceleration models cannot predict the state of health of the capacitor at a particular time. Hence, there is a need for data-driven or statistical degradation models to estimate the capacitor’s actual state-of health at a given time and to estimate the remaining useful life (RUL). In non-destructive degradation analysis, once the degradation information has been recorded, the measurements are extrapolated to the pre-defined failure level to estimate the failure time. The common degradation models used to perform extrapolation of data are linear, exponential, power, and logarithmic. In destructive degradation analysis, theoretical population models known as lifetime distribution models are used to describe unit lifetimes, which are probability density functions defined over a range of time. The corresponding cumulative distribution function gives the probability that a randomly selected unit will fail by a specified time. The

Weibull model is the most commonly used life distribution model, especially for capacitors. For a selected distribution, a degradation model is typically used to represent the change of the location or log-location parameter with time.

Sometimes there are probabilistic arguments based on the physics of the failure mode that tend to justify the choice of model. Other times, the model is used solely because of its empirical success in fitting actual failure data [39]. In most cases, the end-of-life criterion for metallized capacitors is defined as a 5% capacitance loss of the initial capacitance [40]. As discussed before, the capacitor degradation rate increases after the capacitance or ESR reaches the specified end-of-life criteria. Hence, the estimation of the capacitor degradation indicators (e.g., capacitance, ESR, tan δ) is of vital importance. Soliman and Wang [41] review the existing condition-monitoring technologies and present the algorithms involved.

Makdessi et al. [42] investigate accelerated aging tests at different temperatures and voltage stresses for metallized film capacitors. The self-healing process is assumed to be a non-homogeneous phenomenon that follows an exponential probabilistic law. A capacitance degradation model is proposed where the model parameters are related to the capacitor at specific voltage and temperature conditions. The model parameters are a function of voltage and temperature and determined by fitting the curve to the experimental points at the applied aging test conditions. The results for the capacitor estimated lifetime are compared with the estimated lifetime given by the Eyring law (1). There is good agreement between the expected lifetimes from both the methods. In [12], a capacitance degradation model under ripple currents is proposed where the model parameter is a constant associated with the characteristic of each type of capacitor related to the gas rate diffusion through the oxide layer.

Hua Li et al. [43] discuss the degradation of metallized film capacitors stressed under a high-temperature, high-humidity environment with an AC voltage. Capacitance and ESR are monitored throughout the experiments and the electro-chemical corrosion phenomenon in two types of metallized films are compared. It is found that the moisture from the surrounding environment takes time to penetrate the capacitor. Before this occurs, the capacitance loss is linear and mainly due to the pre-existing moisture in air trapped between the layers inside the capacitors. The capacitance then decreases faster due to the moisture penetrating effect. The transition time from the first stage to the latter is called as the ingress time of moisture. The capacitance change with time before and after the ingress of moisture is given by linear and power degradation models, respectively.

In [37], a set of metallized film capacitors undergo accelerated testing under constant temperature but varying humidity levels. The time-to-failure of the failed capacitors is fitted into a Weibull distribution curve and *B1* life, *B10* life, and mean time to failure (MTTF) is estimated under the required confidence intervals and the model parameter n_3 (humidity derating exponent factor) for the lifetime equation (3) is estimated. Li et al. [44] introduce new life prediction models to capture the degradation of metallized film capacitors in pulsed power systems. The capacitance loss is the accumulative result of self-healing in pulsed electrical shots and is divided into two parts: natural loss and sudden loss. The traditional life prediction models are the Weibull distribution model and the regression model, whose parameters are determined by the least squares method. The least-squares method is only appropriate when the capacitance loss is uniform and steady, and the lifetime obtained by the regression model strongly depends on the curve-fitting function (for instance. linear, quadratic, cubic). The Birnbaum–Saunders distribution (also known as the fatigue life distribution) is introduced, where the capacitance loss resulting from a fixed number of shots is assumed to be an independent distribution having non-negative random variables. It should obey the normal distribution, whose parameters are first calculated based on experimental data by the method-of-moment estimation algorithm, then used to calculate the lifetime expectancy and variance. However, this life-prediction model does not consider the sudden loss that occurs during lifetime testing of metallized film capacitors. Hence, the Poisson distribution is introduced to analyze the sudden capacitance loss, in which the number of random faults resulting in sudden drop in one shot obeys the law of Poisson distribution. Both the steady capacitance loss and sudden capacitance loss caused by random faults are assumed to be independent, identically distributed, and nonnegative random variables and should obey the law of normal distribution, whose parameters are dependent on the number of shots. Research conducted by Shin et al. [45] uses a similar approach in which the steady or the natural loss is captured by a random variable deterioration method and the sudden loss is captured by a Gamma process deterioration method. The deterioration rate of the random variable deterioration model is obtained by using the curve-fitting algorithm, and the shape function and scale parameter of the Gamma process is obtained using an expectation-maximization algorithm.

2.2.2. Electrolytic Capacitors

2.2.2.1. Physics-of-Failure-Based Models

The Arrhenius model is used for the temperature lifetime dependency of electrolytic capacitors. Gualous and Gallay [46] assumed that the capacitor life is proportional to the inverse reaction rate of the process and proposed a modified Arrhenius equation (Eq. (4)).

$$L = B * e^{\left(\frac{E_A}{kT_{op}}\right)} \tag{4}$$

In Eq. (4), L is the expected capacitor life (in hours) for an operating temperature T_{op} (in Kelvin), k is the Boltzmann constant, E_A is the activation energy, and B is a material specific parameter.

In industry, the rule of thumb related to life of electrolytic capacitor is given in Eq. (5):

$$L = L_o * 2^{\left(\frac{T_o - T_{op}}{10}\right)} \tag{5}$$

where L is the expected life of the capacitor at the operating temperature T_{op}, and L_o is the expected life at the rated temperature, T_o. This means that for every 10°C increase in operating temperature, the life of the electrolytic capacitor reduces by half [23, 28, 47]. The temperature acceleration factor is approximately two over an ambient temperature range from 60°C to 95°C. However, according to the Arrhenius equation, the reciprocal of T is directly proportional to the logarithm of lifetime, which means that, strictly speaking, there is the temperature range where the theory of lifetime reducing by half for every 10°C rise is not applicable. Especially for capacitors whose maximum operating temperature is 105°C or higher, the temperature acceleration factor needs to be modified depending on temperature ranges of the lifetime estimation [28].

Dehbi and Wondrak [48] slightly modified the model given by Eq. (5) to account for the applied voltage. Jánó and Pitica [49] combined the Dehbi and Wondrak's [48] and Gualous and Gallay's [46] models and proposed the following (Eq. (6)):

$$L = B * e^{\left(\frac{E_A}{kT_{op}}\right)} * \left(\frac{V_o}{V}\right)^n \tag{6}$$

in which V_o is the maximum rated voltage, V is the operating voltage and n is the capacitor type parameter (0 for axial, 1 for radial). This suggests that only radial capacitors are affected by working voltages.

The most common monitoring methods for degradation of capacitor with operational time include ESR-based monitoring. Changes in chemical and physical properties of the capacitor electrolyte will affect the value of ESR [50, 51]. The conductivity of the capacitor electrolyte increases because of a rise in the temperature, hence the value of the ESR will decrease with increasing temperature, accompanied by a rise in the capacitance value. Gasperi [52] proposed an algorithm to estimate the ESR of an electrolytic capacitor. A commonly used rule-of-thumb is that a capacitor reaches the end of life when it has lost about 40% of its electrolyte, that is, when the ESR increases to roughly 2.7 times of the initial value. Gasperi related the volume of electrolyte to the ESR of the capacitor as Eq. (7):

$$\frac{ESR}{ESR_o} = \left(\frac{V_{ol-i}}{V_{ol}}\right)^2 \tag{7}$$

where, ESR_o is the initial ESR, V_{ol} is the volume of electrolyte, and $V_{ol\text{-}i}$ is the initial volume of electrolyte.

The rate of electrolyte loss is assumed to be directly proportional to the vapor pressure of the electrolyte. Since vapor pressure is temperature dependent, the core temperature of the capacitor critically affects the vapor pressure inside the capacitor and knowing the core temperature (ambient temperature + heat rise by ripple current), the vapor pressure of the capacitor can be estimated by an exponential relation between the vapor pressure of the electrolyte and the core temperature [52].

Kulkarni et al. [22] presented the capacitance and ESR degradation models as a function of electrolyte volume loss, by analyzing the geometry of the capacitor (Eq. (8.1) and Eq. (8.2)).

$$C(t) = \left[\frac{2\varepsilon_R\varepsilon_o}{d_C}\right]\left[\frac{V_{ol-i} - V_{dol}(t)}{j_{eo}\, t\, w_e}\right] \tag{8.1}$$

$$ESR(t) = \left[\frac{\rho_E d_C P_E}{2}\right]\left[\frac{j_{eo}\, t\, w_e}{V_{ol-i} - V_{dol}(t)}\right] \tag{8.2}$$

In these models, t is the aging time, V_{dol} is the dispersion volume at time t, ρ_E is electrolyte resistivity, d_C is cathode oxide layer thickness, $\mathcal{E}_R$ is relative dielectric constant, $\mathcal{E}_o$ is permittivity of free space, w_e is the volume of ethylene glycol molecules, j_{eo} is evaporation rate, and P_E is the correlation factor related to electrolyte spacer porosity and average electrolyte pathway. It is clearly seen by the models that, as the electrolyte volume decreases, the capacitance decreases and the ESR increases.

2.2.2.2. Data-Driven Models

In the research work by Kulkarni et al. [53], the capacitor degradation parameters are observed under nominal conditions. ESR drift as a function of time is determined by the Arrhenius model, given in Eq. (9):

$$\frac{1}{ESR_t} = \frac{1}{ESR_o}(1 - k.t.\exp\left(\frac{-E}{T+273}\right)) \tag{9}$$

where t is the operating time, ESR_t is the ESR value at t, T is the temperature in degree Celsius at which the capacitor operates, ESR_0 is the initial ESR value when t is equal to 0, k is a constant which depends on the design and the construction of the capacitor and E is the activation energy defined by Boltzmann's constant. A third-degree regression model, determined to be the best fit by least squares, is proposed for the average capacitance degradation with time. In [54], capacitance loss with time under overvoltage conditions is captured by an exponential degradation model and the model parameters are estimated by a non-linear least-squares regression algorithm.

The values of capacitance and the ESR vary both with time and the ambient temperature. Abdennadher et al. [55] proposed an aging algorithm for real-time condition monitoring of capacitors to determine the RUL of electrolytic capacitors used in uninterruptible power supplies. For a given ambient temperature of the capacitor, the total lifetime is given by the

Arrhenius equation. Also, at that particular temperature, the ideal value of ESR or capacitance are obtained by curve-fitting equations. The actual or degraded value of capacitance or ESR is obtained from the experiment at a given time, which gives the operating time of capacitor at those values by curve-fitting equations. Subtracting the operating time - obtained by curve-fitting equations – from the total lifetime obtained by Arrhenius equation yields the RUL. Similar degradation modeling is completed in [56], where the relationship between temperature and the instantaneous value of ESR and capacitance is modeled by an inverse exponential and linear degradation model, respectively, and the model parameters are calculated with a least-square fitting method. When the ambient temperature and the degradation rate are constant, the evolution of ESR and capacitance with time are given by exponential and linear degradation models, respectively. Based on these statistical equations, a life prediction model for electrolytic capacitors operating at a constant temperature has been proposed to find the RUL as defined by the minimum residual life of either capacitance or ESR end-of-life criteria, whichever is shorter.

Under thermal and voltage stresses, the temperature of the capacitor hotspot continuously increases. More recently, Sun et al. [57] proposed a hybrid degradation model by combining the physics-of-failure model with a similar statistical model of capacitance and ESR degradation with time at constant capacitor temperature, where model parameters are assumed to follow the Arrhenius equation, as given in Eqs. (10.1) – (10.4).

$$C(t) = C_o.(1 + A.t) \tag{10.1}$$

$$ESR(t) = ESR_o.e^{B.t} \tag{10.2}$$

$$A(t) = A_o.e^{\frac{-E_{a1}}{(k.T)}} \tag{10.3}$$

$$B(t) = B_o.e^{\frac{-E_{a2}}{(k.T)}} \tag{10.4}$$

Here, C_o and ESR_o are the initial capacitance and ESR, respectively, A and B describe temperature-dependent degradation rates, A_0 and B_0 are base degradation rates, E_{a1} and E_{a2} are the activation energies, and κ is the Boltzmann constant. These models are then combined and integrated to obtain variable-temperature capacitance and ESR degradation models.

In [58], accelerated life tests on aluminum electrolytic capacitors under temperature and voltage stress are conducted to study the effect of applied voltage and ambient temperature on the capacitor. The degradation of the capacitor is studied by measuring its capacitance loss at random times over the test duration and a regression equation representing the degradation pattern is developed, which is used to predict the time to failure of each capacitor. Weibull analysis for the predicted failure times of all the capacitors at each stress level (voltage, temperature) is carried out and the MTTF of the population at that stress level is calculated. Analysis of variance (ANOVA) is also carried out in MINITAB software to determine which of the two stress factors has more effect on the life of capacitor and how the interaction effect between these two stresses impacts the life of the capacitor. It is found that the applied voltage affects the mean life most, followed by ambient temperature and interaction effect. A regression equation that correlates the life of a capacitor with both stress levels is obtained that can be used to predict the life when the capacitor is used beyond the experimental stress levels. Table 2 summarizes the most significant research work associated with the various stress types in capacitors.

Table 2. Research work associated with the various stress types in capacitors

Stress Type	Ref.	Model Used	Major Contribution
		Metallized Film Capacitors	
Temperature	[36]	Arrhenius model	Life model proposed for time-varying thermal stress, using Miner's cumulative damage theory
DC Voltage, Temperature	[42]	Exponential regression equation, Eyring law	Capacitance degradation with time; constant high temperature and voltage. Lifetime estimated by regression model and Eyring law
Ripple Current	[12]	Regression equation, based on underlying physics-of-failure	Electrochemical corrosion introduced as a failure mechanism in presence of ripple current. Regression equation proposed for capacitance loss with time based on theory of the oxidation kinetics.
Pulsed Discharge	[44]	Birnbaum–Saunders distribution (fatigue life distribution), Poisson distribution	Standard life prediction models (regression, Weibull) are compared with the newly introduced Birnbaum–Saunders distribution (to model steady capacitance loss) and Poisson distribution (to model sudden capacitance loss) for pulsed power applications.
	[45]	Random variable deterioration method, Gamma process deterioration model	Steady capacitance loss is modeled by random variable deterioration method and sudden loss is modeled by a Gamma process deterioration method.
AC Voltage, Temperature, Humidity	[43]	Regression equation	Capacitance and ESR modeling according to the "ingress time" of moisture
Humidity, Temperature	[37]	Weibull distribution, combined model (Arrhenius model + inverse power law for humidity)	Time to failure is fitted into a Weibull distribution curve and B1 life, B10 life and MTTF are estimated under the required confidence intervals and the model parameter for the lifetime equation is estimated for all lifetime definitions.
		Electrolytic Capacitors	
Temperature	[52]	Arrhenius equation	Physics-of-failure model to predict ESR change with electrolyte loss is proposed. Lifetime of capacitors at different temperatures is calculated using Arrhenius law, and the results are compared to lifetimes obtained via an approximation law used in industry.
	[55]	Regression equation, Arrhenius equation	For a given ambient temperature of the capacitor, the total lifetime is given by the Arrhenius equation. The operating time of the capacitor is obtained using regression equations characterizing capacitance and ESR with time and temperature, respectively. Remaining useful life is estimated by subtracting total lifetime with the operating time obtained by regression equations.
	[57]	Regression equation, Arrhenius equation	Hybrid degradation modeling approach is proposed in which the parameters of regression equations characterizing capacitance and ESR with time, follow Arrhenius law. Increase in output current is considered as a failure criterion. Time to failure can be calculated at variable-temperature operating conditions.
Temperature, Voltage	[49]	Combined model (Arrhenius + inverse power)	A new prediction method for capacitor lifetime is introduced. The lifetime obtained is found to be more accurate than the previous models. This prediction methodology suggests that only radial capacitors are actually affected by working voltages.
Voltage	[54]	Regression equation	A remaining useful life prediction methodology for electrolytic capacitors based on Kalman filter is presented.
Nominal Conditions	[53]	Regression equation, Arrhenius equation	Under nominal conditions (below rated voltage and temperature), ESR drift as a function of time is determined by Arrhenius model (exponential model). A third-degree regression model is proposed for the average capacitance degradation with time.

2.3. Discussion

Although electrolytic capacitors are a popular choice for the DC-link applications due to high capacitance values, they generally have a low reliability and a short shelf life, mainly due to their sensitivity to temperature variations. Metallized polymer film capacitors, especially the capacitors with polypropylene as a dielectric, can be considered as a good substitute for electrolytic capacitors in fault-tolerant applications because of their low temperature and frequency dependency, and very little change of capacitance with time under voltage applications due to their unique self-healing ability. The insulation resistance as a failure indicator has been largely neglected in the existing research, but it severely affects the capacitor degradation, contributing to phenomenon like thermal runaway, especially in high-DC-voltage conditions. Hence, both capacitance loss and decrease in insulation resistance should be considered to access the complete health of metallized film capacitor. There is a need for real-time capacitor degradation models that take into account the operating conditions (ambient temperature, voltage, ripple current, humidity, frequency, time, etc.) for more accurate stress analysis. Also, an investigation into the interaction effect among various stressors on the lifetime of capacitors needs to be conducted [30]. There is a need for hybrid models, which combine the physics-of-failure and data-driven models, for capturing the capacitor degradation more realistically. This is particularly needed in the case of metallized film capacitors, as little research has been published in this area apart from the work of Makdessi et al. [12, 42]. Also, regression-based degradation modeling seems to be the most widely used among the data-driven approaches. But regression-based models require large sample sets to increase their predictive accuracy. In regression analysis, many variables are susceptible to noise in the data or certain unnecessary information, therefore reducing the prediction power of regression analysis.

Data-driven models are becoming increasingly popular especially in cases where it is difficult to capture and understand the physics behind a degradation process. Among these models, the stochastic process models like Weiner and Gamma processes have been widely used in literature. Van Noortwijk [59] reviewed the application of the Gamma process in maintenance, its statistical properties, and parameter estimation methods. Bo Sun et al. [60] proposed an improved reliability analysis method based on the accelerated aging test of rubber O-rings in which the traditional accelerated model and the Gamma stochastic process model are combined. The shape parameter of the Gamma degradation model is represented by the Arrhenius model, while the scale parameter remains constant for different stress levels. Limon et al. [61] used the Gamma process to model the degradation behavior of LEDs under multiple stresses in which both Gamma parameters are considered to depend on stress variables.

The degradation path of a capacitor varies according to intrinsic factors (such as the inhomogeneity of the dielectric material) and extrinsic factors (including driving current, forward voltage, power dissipation, junction temperature). This leads to uncertainty caused by variation in these factors and temporal uncertainty among the population of capacitors. Due to the random uncertainty, the parameters of the deterioration process vary from sample-to-sample affecting the path of degradation process. Temporal uncertainty is associated with the evolution or progression of deterioration over time, which affects the rate of degradation. Gamma stochastic process can express temporal uncertainty as well as random uncertainty. It is best suited for degradation processes which are monotonic and unidirectional with tiny positive increments over time [62]. Under accelerated stress conditions, capacitor degradation indicators (capacitance and equivalent series resistance) most likely follow a non-linear trajectory with time. Thus, a non-homogeneous Gamma stochastic process is the best choice for modeling the degradation of capacitors as it captures the degradation path completely, considering both the variation with stress and the variation in time.

3. DEGRADATION MODELING OF CAPACITORS[2]

This chapter deals with the stochastic degradation modeling of electrolytic capacitors. Capacitance of the capacitor is considered as health indicator parameter, and it is recorded at various times under elevated stress conditions. Change in capacitance value for various samples as time progresses constitutes the data set. These data have been analyzed in realm of stochastic process where the sample-to-sample variability and temporal variability are accommodated into the model by making shape parameter time and stress dependent and scale parameter (related to rate of degradation) only stress dependent. It has been emphasized that in the presence of various models for estimating remaining useful life, the Gamma stochastic modeling is well-suited due to adaptive nature of Gamma distribution function which could take various shape and scale as a function of time and stress. Due to time and stress dependence of the Gamma distribution function, the model mimics the real-life situation better than the other models in which distribution is fixed in time and stress independent.

3.1. Introduction

In today's advanced world, we are surrounded by various technologically advanced products which barely need maintenance or whose working life is quite long. This has been possible due to advancement in material science and study of component life leading to a very reliable product development. Due to a long reliable life of the components, the historical failure data is difficult to collect, or the data is insufficient. In the industrial environment where various mechanical and electronic items are working coherently, failure of any one component will lead

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[2] Section 3.4.1 of this chapter contains text from: Gupta, A., Yadav, O.P., Roy, A., DeVoto, D. and Major, J. "Degradation modeling and reliability assessment of capacitors." In *International Electronic Packaging Technical Conference and Exhibition,* Vol. 59322, p. V001T06A018, American Society of Mechanical Engineers (2019). A. Gupta had the primary responsibility of proposing and presenting the degradation model and the lifetime estimation methodology for capacitors. A. Gupta was the primary developer of the conclusions that are advanced here. A. Gupta also drafted and revised all versions of this chapter.

to production loss and an increase in cost of production. In this scenario, it is required to study and diagnose the health of components in a short period of time so that deterioration in its health or performance can be captured. Generally, one or two parameters are monitored which reflects the health of the component. The deterioration or drift of these parameters carries the signature of the events which will ultimately happen in future and will cause failure of the component. While studying the component health and its failure at a later time, the path of deterioration differs from component to component, thus giving sample-to-sample variability. Besides this, temporal variability is also encountered.

While studying the life of capacitors which is a key electronics component, various models have been used such as physics-of-failure models, statistical models, stochastic models, or hybrid models. The goal of these models is to accurately predict RUL of the capacitor under study. The accuracy of the prediction depends on the choice of model and the number of samples. A preferred model is the one which accounts for sample-to-sample variation and temporal variability of the parameters. Generally, collection of data from a large sample set can accommodate these two variations but at the cost of time and money. Accelerated data testing is performed under elevated stress conditions which leads to faster health deterioration of the component under study. The accelerated data is ultimately used to calculate the RUL of the component under normal operating conditions.

For monitoring the health of capacitors, both capacitance and ESR values can be chosen as health indicators. In literature, we can find either ESR or capacitance or both have been used to monitor the health of a capacitor. In the case studies presented in this thesis, the data is derived from literature in which the component under study are electrolytic capacitors, and only the capacitance is considered as the major health indicator.

Among the various approaches to estimate life of capacitors, the physics-of-failure model uses physics principles and related equations for the processes taking place inside a capacitor which leads to the deterioration of the capacitance. The physics-of-failure equations does not account for all the effects responsible for decay of capacitance under varied stress conditions. These models are therefore not useful in estimating the life of capacitors as its lacks sample-to-sample variability and statistical nature of the deterioration process.

The data-driven model is a black box kind of approach where the exact process taking place inside the capacitor is not required to be known. However, to account for sample-to-sample variation and usage of capacitor under various conditions, a large data set under variety of accelerated stress conditions is generated. Various regression models are used to estimate the end-of-life of capacitors. The data set corresponding to end-of-life is used along with statistical distribution to estimate the RUL under normal operating conditions. The major drawback of these models is use of distributions which are fixed in time, or in other words, non-dynamic. Moreover, the effect of stress variables on the distribution function parameters is not considered. Such approach does not reflect dynamic nature of the deterioration process taking place inside a capacitor. To address such issues, the distribution representing the data should be adaptive and must have parameters which can change with time and stresses.

Recently, such a dynamic approach was applied to predict the life of LEDs [61]. However, a similar approach to estimate life of electrolytic capacitors has not been carried out. We therefore carry out similar approach to estimate RUL of electrolytic capacitors where the Gamma distribution has been used with both Gamma distribution parameters, shape parameter and scale parameter, as function of time and stress, and only stress respectively. In the present chapter, we will use this methodology i.e. Gamma stochastic model to estimate life of electrolytic capacitor and will show that this gives realistic RUL estimates compared to traditionally used data-driven model with static distribution function for lifetime estimation.

3.2. Gamma Stochastic Process

Stochastic processes are a natural choice for modeling the randomness in degradation processes caused by inherent randomness and environmental factors (covariates such as temperature and voltage etc.). For a stochastic process to be useful for degradation modeling, there are several requirements such as [63]:

1) It should have clear physical explanations.
2) It is easy to understand and use.
3) It should have good mathematical properties.
4) It can incorporate prior information and flexible to include the covariates and random effects.

The Gamma distribution can be used to represent monotonic capacitor degradation because it provides a flexible representation of variety of distribution shapes while utilizing only two parameters, the shape and scale parameter. Gamma distribution is good choice for describing the capacitor degradation due to variety of reasons [64]:

1) The first advantage of Gamma distribution is that it is bounded on the left at zero. This is important as negative degradation implies increase in capacitance with time and which is not the case where capacitance value decreases always in a monotonic way.
2) The Gamma distribution is positively skewed means that it has an extended tail to the right of the distribution. This basically mimics degradation of those sample capacitors (may be in a very small number) which hardly deteriorates.

3) Gamma distribution takes care of sample-to-sample variability (number of samples reaching a particular degradation value) via shape factor and the temporal variability (accounting for rate of degradation).
4) One can choose shape and scale parameters as time invariant or time variant. The time variant shape factor can adapt a linear or non-linear form in order to account linear or non-linear decrease in capacitance with time respectively. However, the scale factor which is normally considered constant can be also taken time dependent to allow change in the rate of the degradation with time.

Like the modeling approaches in the Wiener process-based methods, covariates can also be incorporated into the Gamma process by setting the shape and scale parameters as function of accelerated stress [65]. Lawless and Crowder [66] proposed a modified Gamma process in which the scale parameter follows the Gamma distribution to capture unit to unit variability. Further the scale parameter is assumed to be function of covariates to incorporate environmental or imposed stress factors.

3.3. Gamma Process Model for Degradation

The probability density function of Gamma distribution is defined as:

$$f_{a,b}(x) = \frac{1}{\Gamma(a\,\varphi(t))}\, b^{a\,\varphi(t)}\, x^{a\,\varphi(t)-1}\, e^{-bx}\, I_{x>0} \tag{11}$$

where $I_{x>0}$ is the indicator function, whose value is 1 for $x > 0$, and 0 for any other value outside its domain. $\Gamma(a) = \int_0^\infty u^{a-1}e^{-u}du$ is the corresponding Gamma function for shape parameter $a > 0$ and rate ($\frac{1}{scale}$) $b > 0$. The shape parameter describes the effect of stress on the performance of products and the scale parameter describes the influence of random factors such as environmental factors, human factors, material differences, etc. on the performance of products [67]. $\varphi(t)$ is a monotonically increasing function of time. If $\varphi(t) = t$, it becomes a homogeneous Gamma process and represents the linear degradation process. To capture the non-linear behavior of degradation with respect to time, a time transformation function is considered with $\varphi(t) = t^c$, where c represents the non-linear constant. This non-linear constant does not depend on the stress level. Let $A(t) = a.\varphi(t)$ be a non-decreasing, real valued shape function with $t \geq 0$ and the rate is b; a stochastic process X_t is a non-homogeneous Gamma process, such that:

1. $X_0 = 0$

2. $X_{t\geq 0}$ has independent increments.

3. Increments are Gamma distributed. Hence for $0 \leq w < t$, the distribution of $X_t - X_w$ follows the Gamma distribution $\Gamma(A(t) - A(w); b)$.

This is a general gamma process used in many degradation analyses where the shape factor is time dependent to account for change in degradation value with time, but scale factor is constant assuming that rate of the degradation is constant [67]. But in real life problems where the sample-to-sample variability and temporal variability exist, with fixed scale parameter, Gamma distribution does not simulate the real degradation path followed by sample as a function of time. Besides, the shape and scale parameters do not have explicit stress dependence. To take care of this aspect, Limon [61] modified the gamma distribution and expressed the time dependent shape parameter *A(t)* and rate b as follows:

$$A(t,S) = a(S).\varphi(t) = e^{\left(\gamma_0 + \sum_{i=1}^{h} \gamma_i S_i\right)}.\varphi(t) \tag{12}$$

$$b(S) = e^{(\delta_0 + \sum_{i=1}^{h} \delta_i S_i)} \tag{13}$$

Here, S represents the transformed stresses, h is the number of stress factors and, γ and δ are the corresponding stress coefficients. Both the Gamma parameters are dependent on stress variables and are expressed by Eyring law.

Consider an accelerated degradation test where x_{ijk} indicates the i^{th}observation of the j^{th}sample under the k^{th}stress level at the time t_{ijk}. If the degradation increment can be expressed by $\Delta x_{ijk} = x_{ijk} - x_{(i-1)jk}$, then the likelihood function of the degradation can be written as:

$$L\left(a_{ijk}, b_{ijk}\right) =$$

$$\prod_{i=1}^{n} \prod_{j=1}^{m} \prod_{k=1}^{p} \frac{b_{jk}^{a_{jk}(t_{ijk}^{c} - t_{(i-1)jk}^{c})}}{\Gamma(a_{jk}(t_{ijk}^{c} - t_{(i-1)jk}^{c}))} \Delta x_{ijk}^{a_{jk}\left(t_{ijk}^{c} - t_{(i-1)jk}^{c}\right) - 1 . e^{(-\Delta x_{ijk} b_{jk})}} \tag{14}$$

By maximizing the logarithm of the likelihood function of the observed deterioration increments, the maximum likelihood estimates of a and b can be obtained.

Substituting (12) and (13) in (14) and estimating the model coefficient parameters, the shape and scale parameters of the Gamma distribution can be estimated for normal operating conditions.

Now, failure occurs when the degradation reaches a pre-defined threshold value denoted by ω. The cumulative distribution function (CDF) of time-to-failure, denoted as ξ, as per the Gamma distribution is given as:

$$F(t) = \frac{\Gamma(at^{c}, \omega_b)}{\Gamma(at^{c})} \tag{15}$$

Here, $\Gamma(q,r) = \int_{r}^{\infty} r^{q-1} e^{-r} dr$ and $\omega_b = (\omega - x_0)b$ where x_o is the initial degradation value. This CDF, given by (15), is difficult to evaluate because of the Gamma function. An approximated CDF of ξ considering Birnbaum-Saunders distribution is given as:

$$F_{BS}(t) = \emptyset\left[\frac{1}{x}\left(\sqrt{\frac{t^{c}}{y}} - \sqrt{\frac{y}{t^{c}}}\right)\right] \tag{16}$$

where, $x = \frac{1}{\sqrt{\omega_b}}$, $y = \frac{\omega_b}{a}$ and $\emptyset(.)$ is the standard normal function [68].

The corresponding PDF at time to failure for non-linear Gamma degradation process is expressed as:

$$f(t) = \frac{1}{2\sqrt{2\pi}xy}\left[\left(\frac{y}{t^c}\right)^{1/2} + \left(\frac{y}{t^c}\right)^{3/2}\right] exp\left[-\frac{1}{2x^2}\left(\frac{t^c}{y} + \frac{y}{t^c} - 2\right)\right] \tag{17}$$

The mean lifetime, according to the Birnbaum-Saunders approximation for a Gamma process can be written as:

$$\xi = \left(\frac{\omega_b}{a} + \frac{1}{2a}\right)^{\frac{1}{c}} \tag{18}$$

The lifetime at operating conditions can be estimated using equation (18).

3.4. Case Studies on Capacitor Degradation Data

The gamma degradation modeling approach is now applied to capture the degradation path of capacitors, both for the individual capacitors in a test population, and for the entire capacitor population, respectively. Then, lifetime of the capacitor population is estimated. Figure 1 illustrates the degradation path of capacitors under various stress levels and gives us a clear picture of how the degradation path of capacitors varies with different stress levels and time.

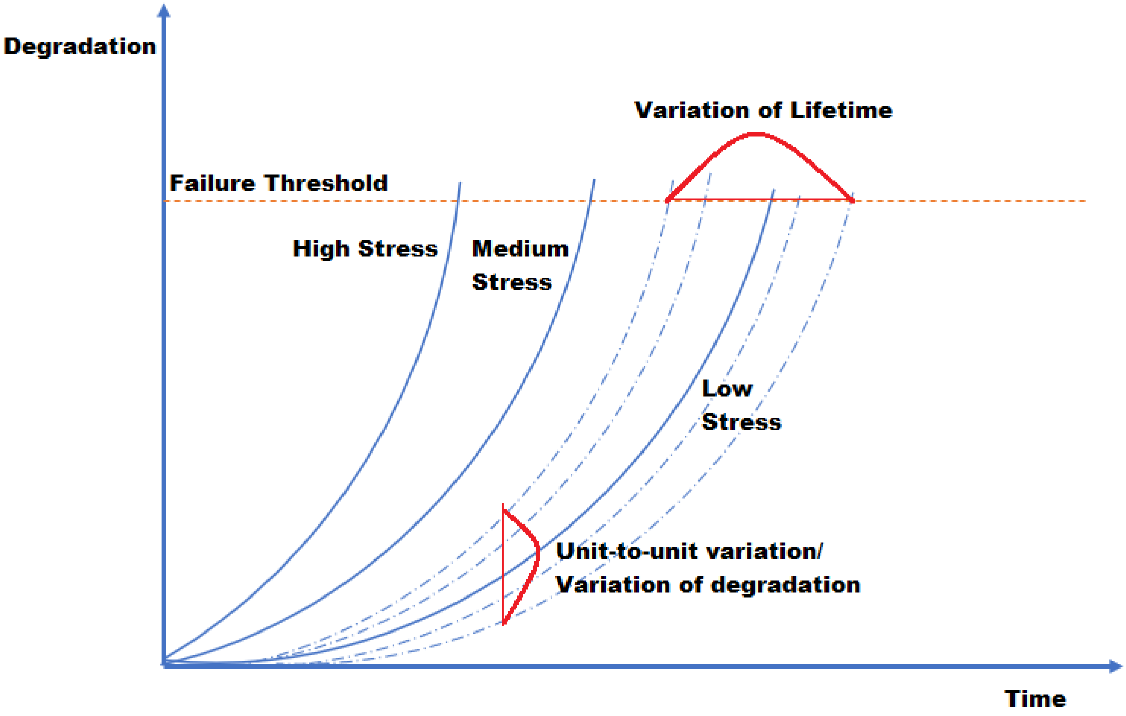

Figure 1. Degradation path of a capacitor

3.4.1. Case Study A

The experimental dataset is obtained from NASA Prognostics Data Repository, in which a set of six capacitors is considered [69]. Kulkarni et al. [70] presents the experimental setup used for the accelarated degradation of these capacitors under electrical overstress conditions. Electrolytic capacitors of 2200 µF capacitance, with a maximum rated voltage of 10V, maximum current rating of 1A and maximum operating temperature of 105^{o}C are used for the study. Capacitors are subjected to electrical stress at 10V.

The primary failure mechanism of an electrolytic capacitor is the evaporation of the electrolyte by diffusion through the sealing material of the capacitor, which leads to a decrease in the capacitance value. An electrolytic capacitor is considered as failed when the capacitance decreases by 20% of the initial value. Therefore, the initial percentage loss of capacitance of all samples is considered as x_o= 0 and ω = 20 is considered as failure threshold value for the capacitors. Table 3 shows the percentage decrease of the capacitance of the six capacitors with time, as given in [69].

Table 3. Percentage decrease of capacitance with time

	Capacitance Loss (%)					
Time (h)	**Cap 1**	**Cap 2**	**Cap 3**	**Cap 4**	**Cap 5**	**Cap 6**
0	0	0	0	0	0	0
24	0.45	0.78	0.64	0.83	0.61	0.44
47	1.24	1.69	1.58	1.86	1.56	1.55
71	1.60	2.34	2.20	2.48	2.10	1.99
94	2.95	3.86	3.58	3.89	3.53	3.25
116	4.96	6.23	6.45	6.24	6.00	5.99
139	7.23	9.44	9.32	9.03	8.59	7.54
149	9.65	11.89	11.54	11.37	10.87	9.76
161	11.96	14.96	14.69	14.75	13.65	12.68
171	15.85	19.64	19.05	20.04	18.23	17.23
194	17.45	21.68	21.05	22.68	20.80	22.04

Plotting these degradation values with time, it can be clearly seen that the degradation path of the capacitors is non-linear, as shown in Figure 2.

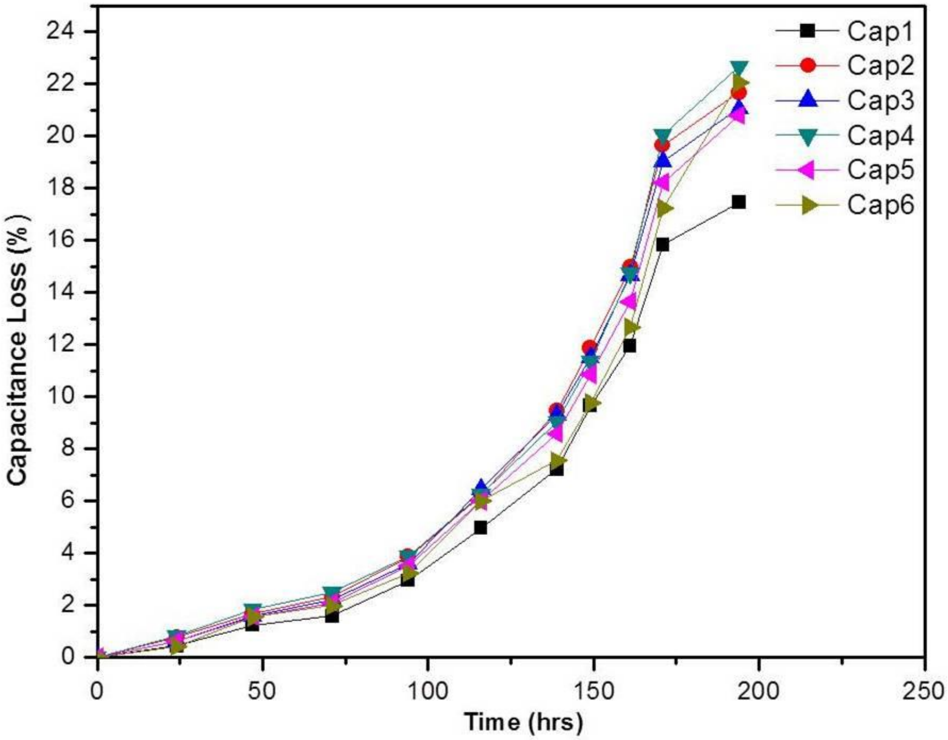

Figure 2. Percentage capacitance loss with time

In this situation, the assumption of the homogeneous Gamma process is limited. The non-homogeneous Gamma process is naturally more suitable for such degradation modeling, as the degradation process follows a non-linear trajectory [62]. To model the degradation path of capacitors by non-homogeneous Gamma process and obtain the process parameters, the likelihood function L(a,b) of the Gamma degradation process (given by equation 14) is minimized by using an optimization function (fmin) of scipy package in Python. The input data are collected in tabular format where degradation values for each sample are provided at various time points and stress levels. The optimization script calculated the likelihood function as an aggregation of the base function for each data point and then found the optimal values of the coefficients to yield the minimized value of the aggregated function. This process is repeated for both for the entire population of capacitors and for individual capacitor samples. The set of optimal coefficient values for each experiment are reported.

3.4.1.1. Results and Discussion

The obtained values of shape parameter (a), rate (b) and non-linear constant (c) for each of the six capacitors individually and for the entire capacitor population are reported in Table 4.

Table 4. Parameter values

Sample	a ($x10^{-3}$)	b	c
1	1.8023	1.5485	1.8253
2	2.6968	1.4576	1.7785
3	2.5499	1.5400	1.7939
4	1.8306	1.4000	1.8530
5	1.5520	1.7882	1.9143
6	0.3154	1.8584	2.2350
Total Population	1.6044	1.5052	1.8760

Once the model parameters are obtained, the corresponding remaining useful lifetimes of each sample at every time instance is calculated via equation (18). The time-to-failure for the entire capacitor population at the start of the experiment, i.e. the mean lifetime of the capacitor population, is found to be 191.3 hours. The mean lifetime of individual capacitor samples is calculated and compared to the estimated time-to-failure values reported in [70], as reported in Table 5 and subsequently plotted in Figure 3.

Table 5. Comparison of lifetime values (hours)

Sample	Estimated Lifetime (reference [70])	Calculated Mean Lifetime
1	175	210.9
2	172	187.2
3	173.9	190.3
4	179.2	183
5	176.5	191.5
6	179.4	186.9

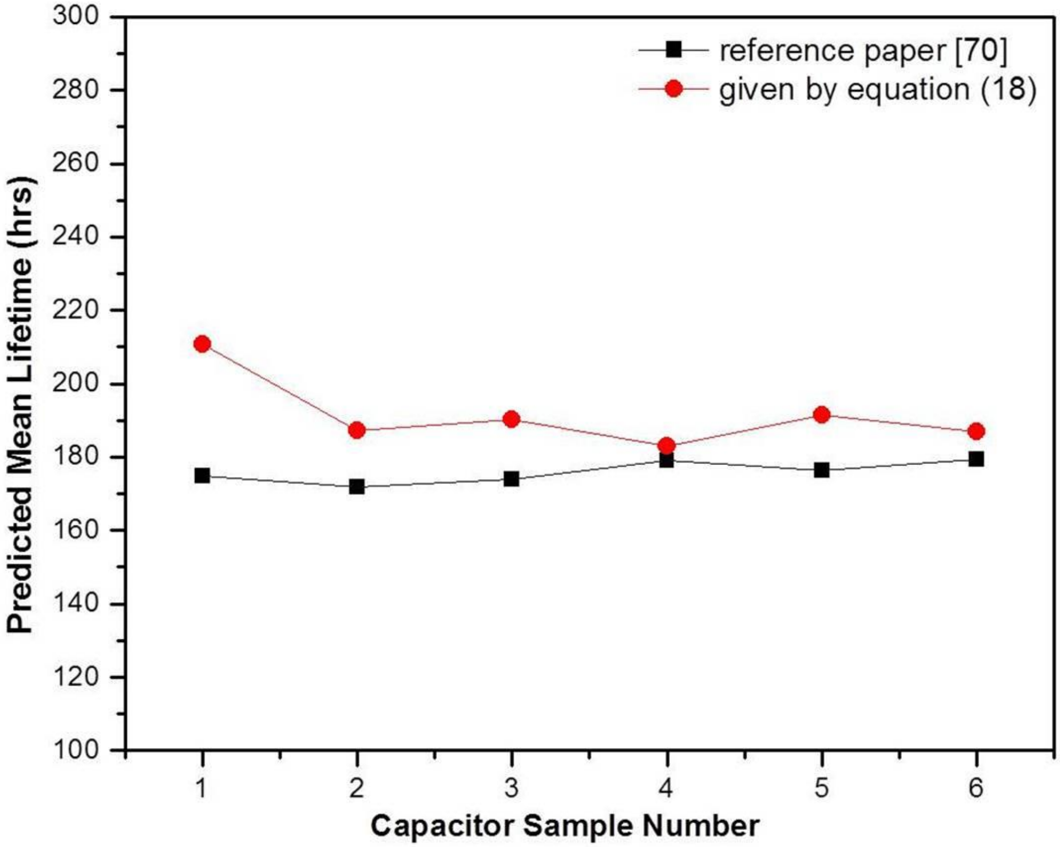

Figure 3. Predicted mean lifetimes

The average relative error between the calculated and the reference lifetime values is 8.9%. As apparent from Figure 3, the mean lifetime values considering the Gamma degradation model is very close to the predicted time-to-failure values of capacitor samples, as given in [70]. Since the available dataset has just one stress factor and a single stress level, only limited information can be inferred from the data. But Gamma stochastic process can analyze the non-linear degradation behavior of capacitors considering multiple stress variables, given an exhaustive dataset with multiple stress levels, and estimate the lifetime values at normal operating conditions for the entire sample population, as shown in the following case study.

3.4.2. Case Study B

In case study A, the data set for six capacitors was taken from [69] and only one stress i.e. 10 V was applied. Although reasonable lifetime estimates were obtained at an elevated stress level, but the true potential of stochastic modeling could not be explored, which is predicting the life of capacitors under normal operating conditions. We will now consider a test study on a population of electrolytic capacitors which are subjected to multiple stress levels of voltage and temperature.

The experimental dataset is obtained from [58]. Accelerated life tests on aluminum electrolytic capacitors under accelerated temperature and voltage stress are conducted. The specifications of the 100 μF capacitors used for the experiment are as follows:

Table 6. Capacitor specifications

Rating	100 μF/16 V
Cap tolerance	30% of the rated capacitance (100 μF)
Stated life	2000 hours at 16 V and +85°C

The capacitor is deemed to have failed if the capacitance drops to below 70 μF, as per the capacitor product data sheet.

The experiment included the capacitors being tested at three accelerated voltage levels (18, 20 and 22 V), at three different temperature settings (85°C, 95°C and 110°C). The capacitance of each capacitor is measured offline at random time intervals. The sample observation for two capacitors at each stress level (temperature and voltage), is given in table 7-9.

Table 7. Test observations: Exp no. 1 (85°C)

	Capacitance Measurement (μF)					
	18 V		20 V		22 V	
Time (hrs)	Cap. 1	Cap. 2	Cap. 3	Cap. 4	Cap. 5	Cap. 6
0	95.5	93.3	97.7	94.5	93.8	91.8
7.64	89.4	87.4	78.7	76.2	74.1	75.6
10.58	89.2	87.5	78.5	76	73.9	75.4
21.59	87.5	86	77	74.4	72.6	74
32.43	86.4	85	76.7	74.3	72.4	73.7
38.13	83.8	83.4	74.9	72.8	71.2	71.9

Table 8. Test observations: Exp no. 2 (95°C)

	Capacitance Measurement (μF)					
	18 V		20 V		22 V	
Time (hrs)	Cap. 7	Cap. 8	Cap. 9	Cap. 10	Cap. 11	Cap. 12
0	92.7	96.4	94	90.6	94.2	94.2
4.35	87.3	87.4	77.9	75.3	67.8	67.6
8.12	85.2	84.6	76.8	74.1	66.9	66.8
12.33	82.5	81.6	75.6	72.8	65.2	65.7
16.91	82.3	80.8	75.2	72.6	65	65.5
22.69	78.5	76.1	72.6	70.1	63.6	64

Table 9. Test observations: Exp no. 3 (110°C)

	Capacitance Measurement (μF)					
	18 V		20 V		22 V	
Time (hrs)	Cap. 13	Cap. 14	Cap. 15	Cap. 16	Cap. 17	Cap. 18
0	90.1	90.3	94.6	96.4	92.4	94.2
0.42	83.1	85.2	73.7	70.1	65.7	65.5
0.61	81.5	83.2	72.5	69.6	64.9	64.7
0.9	76.5	78.8	68	65.1	61.8	61.8
11.81	70.4	69.7	67.8	65.4	61.8	61.3
15.55	69.4	69.2	67.3	64.9	60.9	60.8

Plotting these degradation values with time, it can be clearly observed that the degradation path of capacitors is non-linear, as shown in Figure 4-6.

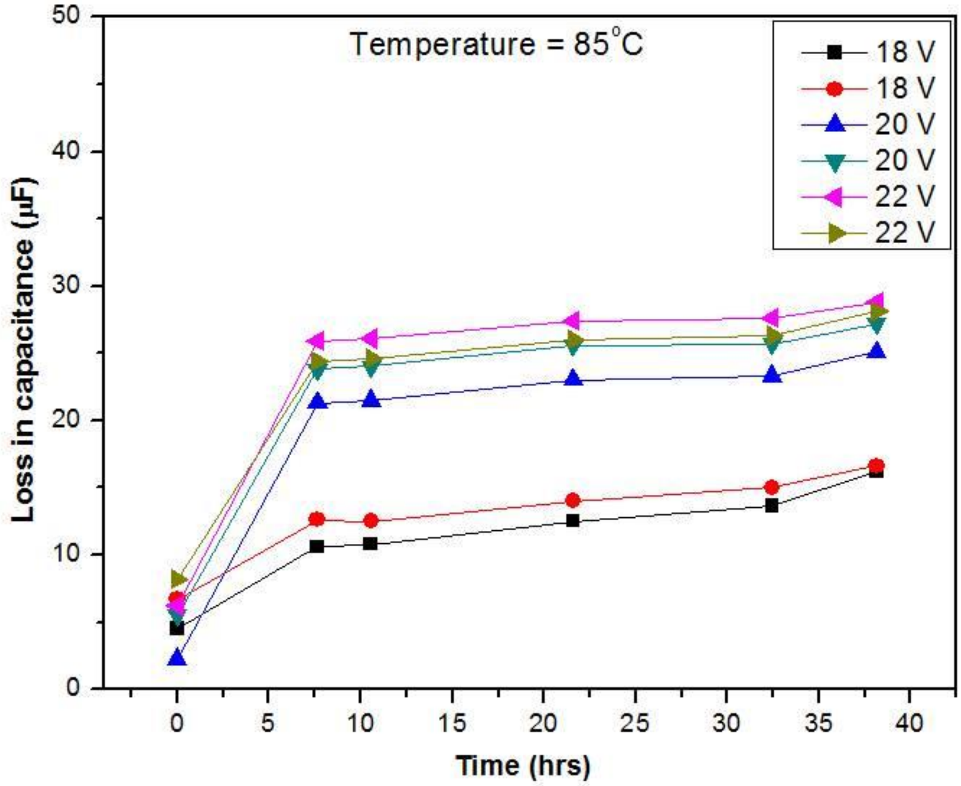

Figure 4. Loss in capacitance value at 85°C at various voltage levels

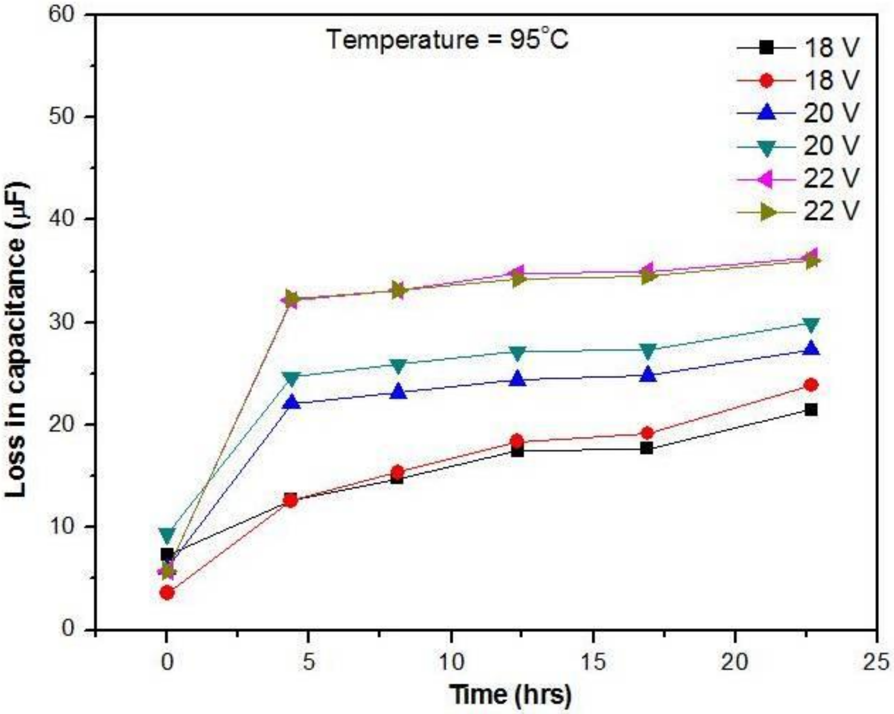

Figure 5. Loss in capacitance value at 95°C at various voltage levels

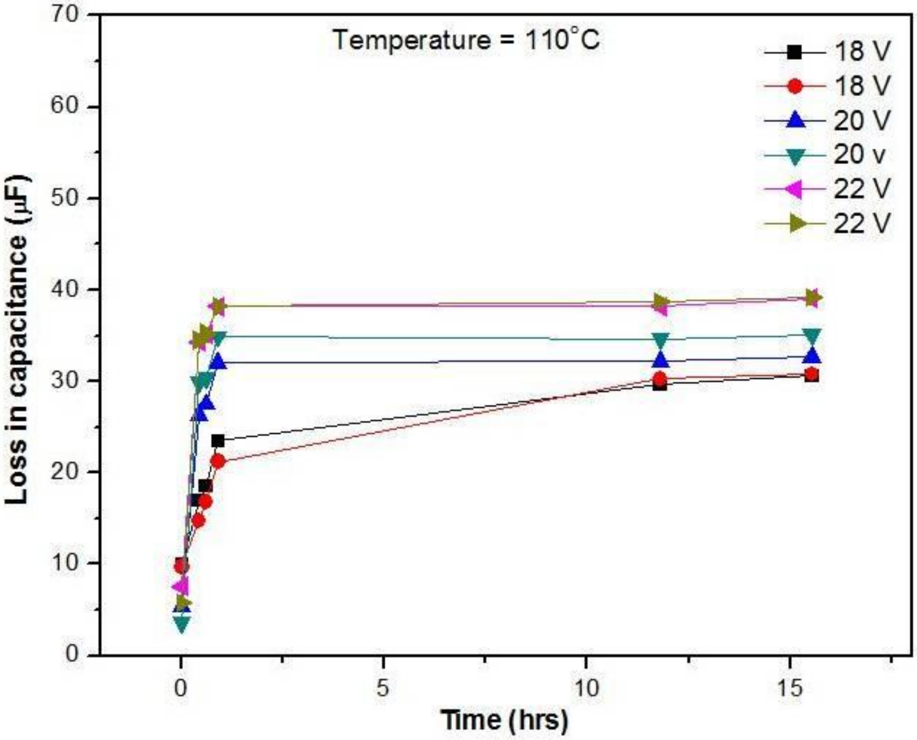

Figure 6. Loss in capacitance value at 110°C at various voltage levels

It is also observed that under higher stress values of temperature and voltage, the capacitor degrades faster and reaches the end-of-life in a shorter time. Observing the non-linear trajectory of degradation process, non-homogeneous Gamma process is chosen for degradation modeling. Stress levels are further transformed, as shown below [71].

$$S_1 = \frac{1/T_0 - 1/T}{1/T_0 - 1/T_M} \tag{19.1}$$

$$S_2 = \frac{\ln V - \ln V_0}{\ln V_M - \ln V_0} \tag{19.2}$$

Here, the subscripts '0' and 'M' in the above definitions denote the use and the maximum stress levels, respectively. Considering T_0 and T_M as 40°C and 110°C respectively, the values of S_1 at 85°C, 95°C and 110°C are calculated to be 0.688, 0.818 and 1, respectively. Similarly, considering V_0 and V_M as 16 V and 22 V respectively, the values of S_2 at 18 V, 20 V and 22 V are calculated as 0.37, 0.70 and 1, respectively. With three stress levels for each stress factor - temperature and voltage, the nine stress combinations (treatment levels) are represented by kth index and these values are tabulated in Table 10.

Table 10. Stress levels

Kth Index	**S_{1K} (Temperature)**	**S_{2K} (Voltage)**
1	0.688 (85°C)	0.37 (18 V)
2	0.688 (85°C)	0.70 (20 V)
3	0.688 (85°C)	1.00 (22 V)
4	0.818 (95°C)	0.37 (18 V)
5	0.818 (95°C)	0.70 (20 V)
6	0.818 (95°C)	1.00 (22 V)
7	1.00 (110°C)	0.37 (18 V)
8	1.00 (110°C)	0.70 (20 V)
9	1.00 (110°C)	1.00 (22 V)

3.4.2.1. Results and Discussion

The process parameters are obtained in a similar fashion as in case study A (section 3.4.1). Substituting all the degradation values Δx_{ijk} at every time point for each capacitor sample and the corresponding transformed stress values in the logarithm of the likelihood function (equation 14) and maximizing it, unknown constants: γ_0, γ_1, γ_2, δ_0, δ_1, δ_2 and c are estimated. The basic idea of optimization is to iteratively find value of unknown parameters that maximizes the log-likelihood. The optimization to find maximum of log-likelihood is performed by a code written using an optimization function (fmin) of scipy package in Python. The optimization script processed in Python software is provided in the Appendix section of this thesis for further understanding. An important feature of the maximum likelihood estimation is that it could find the unknowns with a small number of samples with sufficient accuracy [72]. This reduces number of samples to be tested thus reducing both time and money. The optimal values of the coefficients calculated by the optimization script for the entire population of capacitors are reported in Table 11.

Table 11. Parameter values

Coefficient	**Value**
γ_0	2.9267
γ_1	-0.9803
γ_2	-0.2122
δ_0	4.1996
δ_1	-3.9745
δ_2	-1.51075
c	0.30384

Now that the process parameters are estimated, the time-to-failure of any capacitor from a population, at any given stress level and any moment of time can be easily known.

The main advantage of the Gamma degradation modeling is the adaptive and dynamic nature of its shape and scale parameters. To estimate the mean lifetime of the capacitor population at normal operating conditions (16 V and 85^oC), the stress dependent shape and scale parameters are firstly estimated as a function of time under respective stress conditions. In Table

12, the estimated values of shape and scale parameters along with the mean degradation values of the capacitor population as a function of time are listed, at normal operating conditions.

Table 12. Mean degradation values of population at normal operating conditions

Time (hrs)	**A(t,S)**	**b(s)**	**1/b(s)**	**Mean degradation value (A*1/b)**
500	62.8342	4.3281	0.231	14.5175
1000	76.5641	4.3281	0.231	17.9207
1500	87.7333	4.3281	0.231	20.2703
2000	95.7472	4.3281	0.231	22.1218
2500	102.464	4.3281	0.231	23.6737
3000	108.3003	4.3281	0.231	25.0222

As observed from the table above, the shape parameter is continuously increasing with time showing the dynamic nature of Gamma model. The mean degradation is plotted in Figure 7. The non-linear variation of the mean degradation value as a function of time can be clearly observed in the figure.

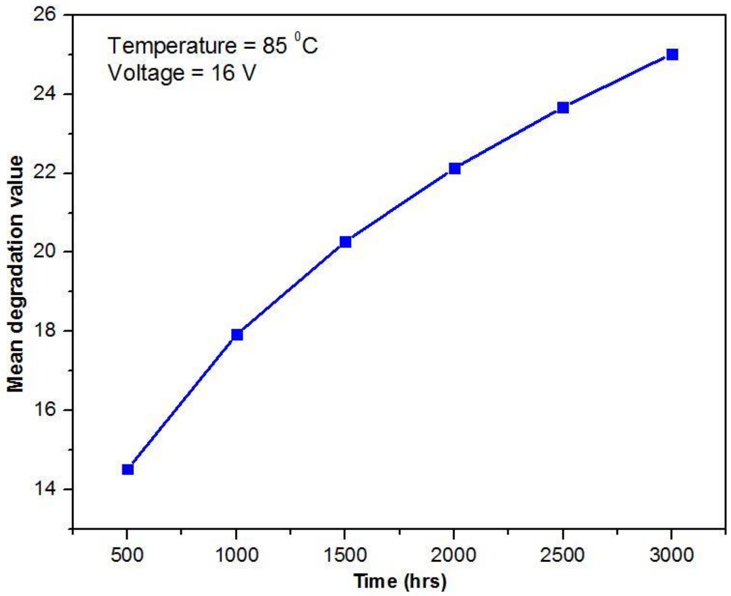

Figure 7. Mean degradation value as a function of time under normal operation

Using threshold value of degradation as 23, the PDF at time to failure under normal operating conditions using equation 17, is represented in Figure 8. It is interesting to note that using PDF as shown in Figure 8, the skewness ($\frac{2}{\sqrt{shape\ factor)}}$) after 500 hours is less than 0.5 so the distribution is almost similar to normal distribution [67]. In such cases, the mean and median are almost same and the time at which peak is attained is typically the mean value or MTTF. Similarly, the CDF and reliability at time to failure is calculated by using PDF and is represented in Figure 9. From this figure, the time at which 50% of the population fails i.e. median time can be easily estimated as 2450 hours. However, this is an approximate estimate and the exact time to failure i.e. MTTF can be estimated by equation (18) but it will be close to the median value.

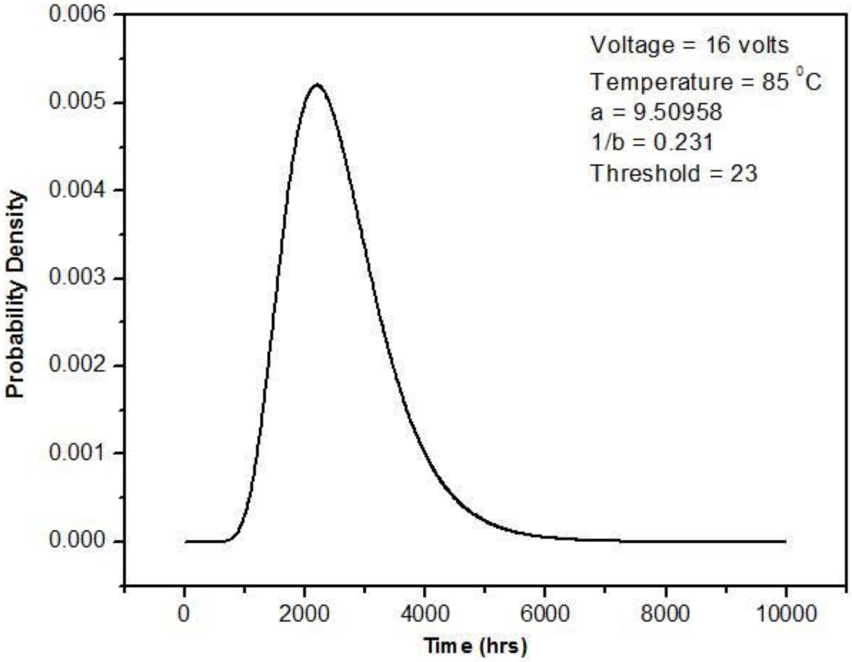

Figure 8. PDF at time to failure

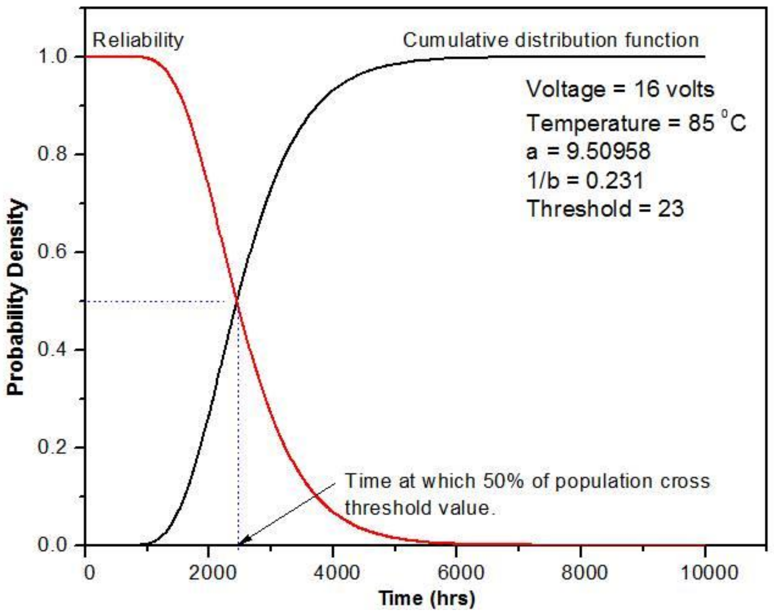

Figure 9. CDF and reliability as a function of time

Now, the effect of dynamic shape parameter on the PDF which relates the probability with the degradation of capacitance as time progresses is demonstrated in Figure 10. The various gamma distributions are shown in Figure 10. As the time progresses the shape of gamma distribution changes, thus accounting for the temporal variability of sample degradation.

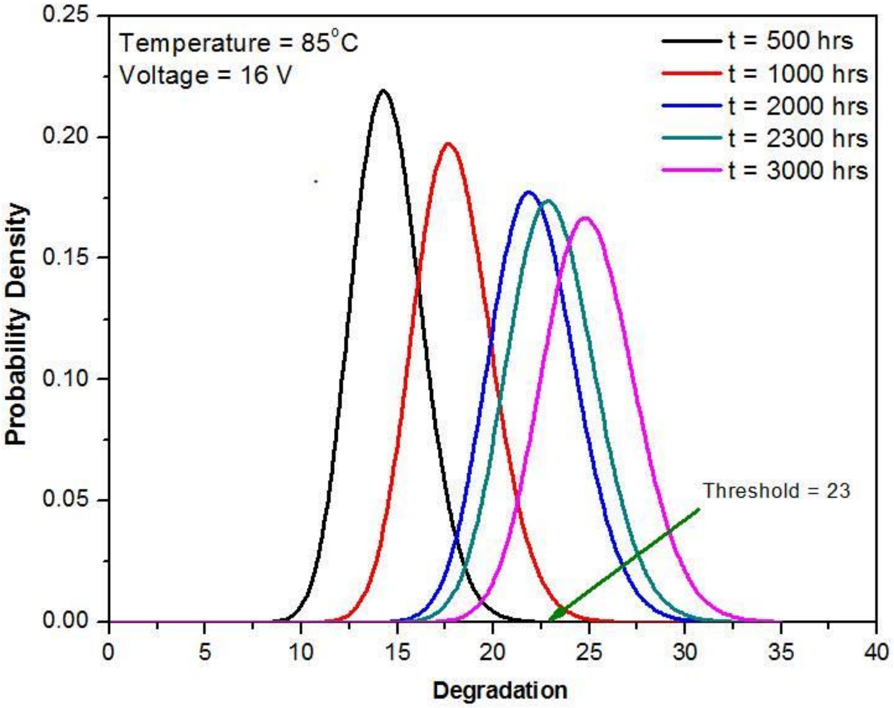

Figure 10. PDF at normal operating conditions at different time values

Another advantage of gamma degradation process modeling is the stress dependent shape and scale parameter. This feature highlights the adaptive nature of Gamma degradation process, thus accounting for temporal as well as sample-to-sample variability of degradation with different stress values. This is a distinct approach compared to other modeling approaches where distribution is considered either at time to failure, or during the process of degradation where the distribution is time and stress invariant. The time and stress dependency of Gamma degradation model makes the lifetime estimates more realistic [73]. The effect of stress on the PDF at given time is shown in Figure 11.

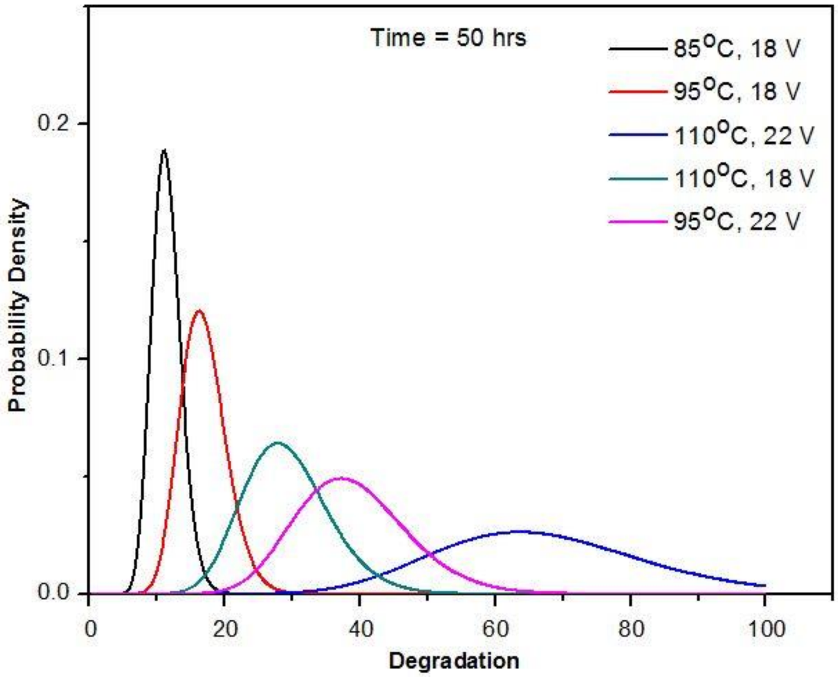

Figure 11. PDF at elevated stress conditions at different treatment levels

After analyzing various aspects of dynamic and adaptive nature of Gamma degradation process, now the time-to-failure for the entire capacitor population at the start of the experiment, i.e. the mean lifetime of the capacitor population at normal operating conditions (16 V and +85°C) is calculated. Using equation (18), the estimated life comes out to be 2310 hours. This is very close to the median life as estimated earlier in this section. The capacitor data handbook states the life of capacitor as 2000 hours. Hence, the estimation of lifetime using Gamma degradation process is very close to the reported values in the handbook.

In [58], the author calculates the mean lifetime of the capacitor population at stress conditions of 16 V and +85°C, using an acceleration model that is a combination of Arrhenius law and inverse power law. The capacitor mean lifetime at normal operating conditions is found to be 4003 hours. In regression-based estimation of life of capacitors in [58], the following steps are involved. After the degradation data at various treatment levels (temperature and voltage) are collected, non-linear equation is fit to the data at a given treatment level. Extrapolating the data to the threshold value of degradation, the failure times of the capacitors are obtained. Weibull distribution is fitted to the failure times, thus finding the shape and scale parameters at given treatment level. With the help of shape and scale parameters, MTTF can be estimated. Using combination of Arrhenius law and inverse power law, the life of capacitor at normal operating conditions is estimated.

The regression-based approach accounts for sample-to-sample variability by choosing a number of capacitors under a given treatment level, however that number may not be sufficient to represent the total sample-to-sample variability. The regression model requires a large data set to reach sufficient accuracy. Moreover, in regression analysis temporal variability cannot be taken into account [62, 73, 74]. Regression-based models are unable to capture the actual degradation process because of oversimplification of reality. These models consider the inherent degradation to be deterministic [74]. Some of the limitations of regression analysis can be overcome by using normally distributed error term or by considering regression coefficients as random variables with a certain distribution. But this distribution is neither adaptive nor dynamic so it cannot mimic real life degradation problems. This is probably the reason for overestimation of capacitor life using regression approach [73].

Gamma stochastic model gives a far more accurate prediction for capacitor lifetime. Also, it is noteworthy that Gamma stochastic model predicts more accurate lifetime with just a sample size of 18 capacitors, as compared to lifetime prediction using 90 capacitors by regression modeling in [58]. The number of samples and data points required for regression modeling to acquire sufficient accuracy are much more, compared to stochastic process modeling [65]. Moreover, adaptive nature of Gamma degradation model is more suitable to mimic real life data under sample and temporal variability under various stress conditions.

3.5. Conclusion

There is a need for hybrid models in the existing literature, which combine the physics-of-failure and data-driven models, for capturing the capacitor degradation more realistically. We attempt to bridge that gap by modeling the capacitor degradation by the non-homogeneous Gamma process where the process parameters are stress dependent and modeled by a generalized Eyring equation. The main advantage of using the Gamma stochastic process for degradation modeling is that, after the parameters are estimated, the time-to-failure of any capacitor from a population, at any given stress level and any moment of time can be easily known. The Gamma stochastic process also gives a more accurate lifetime estimation than the regression model, while using even lesser sample observation set than the latter.

In this thesis, a lifetime estimation model for capacitors is presented. Since the degradation path of capacitors is non-linear and monotonic, the non-homogeneous Gamma process is well-suited for such degradation modeling. The model parameters are estimated using the maximum-likelihood estimation approach in Python. The estimated mean lifetime is then presented for two different case studies.

4. CONCLUSIONS AND SCOPE OF FURTHER STUDIES

Power electronic converters are an integral part of many industries because of their undisputed performance in various applications. With the increase in demand of energy, power converters have become an important area of research in applications such as renewable energy generation and hybrid vehicles for energy conservation. The current research areas mainly focus on the condition monitoring of power converters to address their reliability in terms of availability, maintenance cost and safety in emerging applications. Figure 12 highlights the power electronics as a key element of various electronic systems.

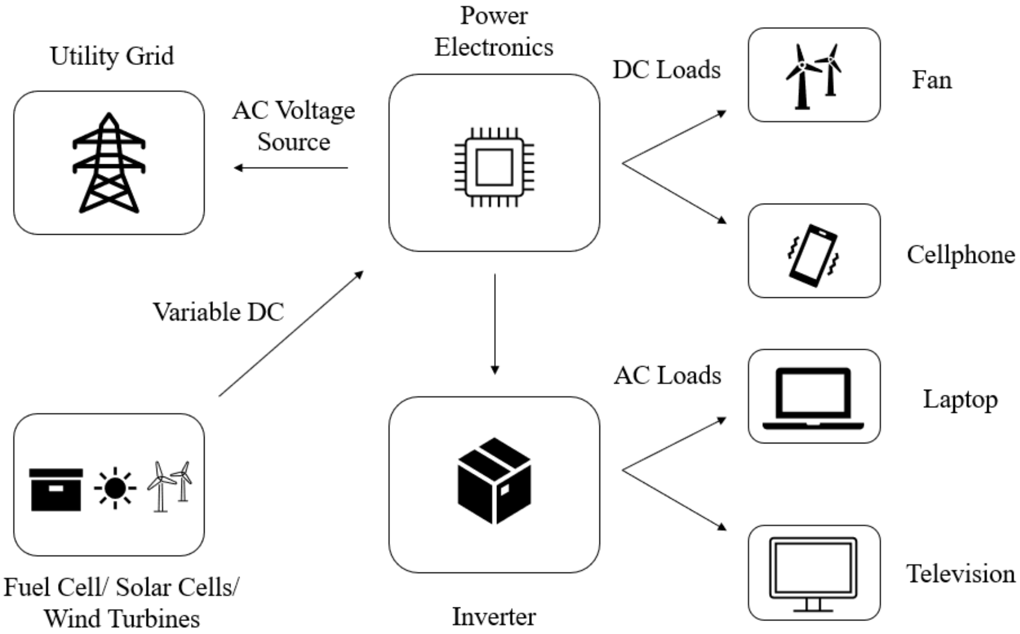

Figure 12. Power electronics as the key element of various electronic systems [75]

Capacitors are crucial components in power converters, playing significant roles such as filters, snubbers, and energy storage elements. Among various capacitors, aluminum electrolytic capacitors are preferred due to higher capacitance, volumetric efficiency, and better price over performance ratio. However, the aluminum electrolytic capacitors are the weakest link in terms of reliability as compared to other power electronics components. The majority of failures in the power electronic systems, as high as 30%, are due to the failure of capacitors [76]. With the increase in use of power converters in critical applications, condition monitoring of capacitors and their prognosis has become inevitable to make power converters a reliable electronic device.

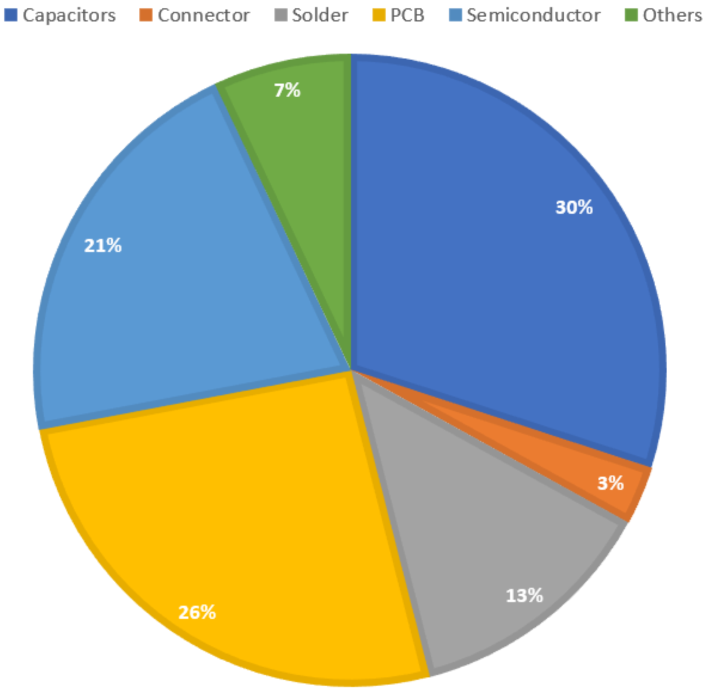

Figure 13. Failure distribution in power electronic systems [76]

The purpose of the work presented in this thesis is therefore related to reliability studies of various types of capacitors. Since this topic is not new and exhaustive research in this direction has already been done, so a review of various methodologies to estimate the lifetime of capacitors was carried out and the best methodology was identified which fulfills following criteria.

1) The method for lifetime estimation should be based on the health conditions (change of capacitance or ESR of capacitors) as a function of time rather than end-of-life data-based predictions. This will give future health conditions of the capacitors as a function of time, enabling better maintenance planning and implementation.

2) The number of samples used for lifetime estimation should be less and at the same time, sample to sample variability should be sufficiently addressed.
3) The health condition data as a function of time should be a reasonably small fraction of the total time (till time-to-failure) and should be capable of capturing the temporal variability of health condition. This will save time and money [77].
4) The statistical method used for parameter estimation should give accurate values of the parameters with less health condition data of the samples.

The most common methods for lifetime estimation are data-driven models which does not require the physical understanding of the degradation process. In these models, the health conditions such as degradation of capacitance with time of the samples are monitored and analyzed. The regression-based model is the most common method to estimate capacitor lifetime, but it suffers from several limitations such as:

1) The number of samples in order to get accurate estimates of the regression equation parameters is very large [65].
2) Error term used in regression equation to address sample to sample variability has a fixed distribution (generally normal distribution of random parameters).
3) Temporal variability of degradation process cannot be addressed [62,73,74].
4) The time for which the degradation data is collected is a large fraction of the total time up to end-of-life, making the estimation time-consuming and costly. Moreover, regression-based predictions are accurate only close to the time domain of the data collection and time-to-failure is generally much more than the observation time.

Due to these limitations, the regression-based predictions lack precision of lifetime estimation. We have therefore used stochastic-based degradation analysis of capacitors which has the following merits:

1) The degradation data of fewer samples is considered, and the sample-to-sample variability is addressed by choosing a time and stress dependent Gamma degradation distribution.
2) With a small sample size, the maximum likelihood parameter estimation method gives sufficiently accurate values [72].
3) The nature of Gamma distribution is such that it automatically considers temporal variability of the degradation of health indicator of samples into account [73].
4) Time and stress dependent degradation distribution captures the overall degradation behaviors as a function of time at very early stages of the accelerated stress degradation, thus RUL estimation does not require degradation data close to end-of-life. This makes the stochastic process RUL estimation accurate, fast, and efficient thus saving both time and money.

The Gamma-based degradation method has been successfully applied for the lifetime estimation of LEDs [61] but this methodology has not been implemented to estimate the lifetime of aluminum electrolytic capacitors. The main limitation of applying and validating a chosen methodology is the availability of degradation data of aluminum electrolytic capacitors as a function of time. Data generation requires elaborate experimental arrangements, and it is very costly and time-consuming. The degradation data used in this thesis for the first case study was obtained from Kulkarni et al. [69], but the data was not exhaustive and only a single stress factor and a single stress level was considered. However, the data was useful to validate the accuracy of lifetime estimates of capacitors obtained by using Gamma degradation model.

To highlight the true potential of Gamma degradation modeling and its ability to predict the lifetime values at normal operating conditions, a more comprehensive dataset with multiple stress factors and stress levels was required. Therefore, in the second case study, the data generated by Rathore et al. [58] was considered and the dynamic and adaptive Gamma degradation modeling had been applied for the lifetime estimation of capacitors. A Gamma process consists of Gamma distributed degradation events, whose evolution with time is governed by shape and scale parameters. The shape parameter was considered as time and stress dependent and the scale parameter only stress dependent. The non-linear variation of the cumulative degradation was addressed by temporal part of the shape parameter. The MTTF estimated by the Birnbaum-Saunders approximate PDF at time-to-failure is close to the time at which 50% of the capacitor population crosses a pre-defined capacitance loss value. At end-of-life of capacitors, the Gamma distribution of failure time is almost normally distributed as the skewness is less than 0.5 and the lifetime estimated from 50% criteria is close to the theoretically calculated value. Due to the merits of the stochastic degradation model, the estimated RUL was 2310 hours which is very close to handbook value i.e., 2000 hours. It is important to mention that the degradation data of only 18 capacitors was used as a sample set for Gamma degradation modeling, as compared to 90 capacitors data used by Rathore et al. [58], and the results obtained by Gamma degradation modeling are much better in terms of accuracy.

4.1. Conclusions

Accelerated degradation test is a useful technique to estimate the product lifetime at normal use stress level, especially for highly reliable products. Degradation analysis to estimate remaining useful life can be broadly divided into two categories namely physics-of-failure and statistical data-driven techniques. Physics-of-failure methods construct the models based on failure mechanism of the device. The success of physics-of-failure method relies on the exact degradation model description in terms of mathematical equations. The physical equations may describe the degradation process but cannot completely incorporate the uncertainties because of variation in material properties and environmental conditions. Thus physics-of-failure based models cannot accurately estimate the device failure working under varied environment and with sample variability. In a nutshell, physics-of-failure based models lack statistical properties of the degradation path.

The data-driven approach mainly collects the degradation data of devices and fits non-linear regression equation to predict pseudo-failure time. A suitable distribution is thus applied to the pseudo-failure time and RUL is estimated. These models include statistical properties of degradation path, but the chosen probability distribution is fixed in time, or in other words, it is assumed that the distribution (i.e. parameters of the distribution) does not change from initial stage to final stage when the component fails. However, a Gamma degradation model accommodates linear or non-linear monotonic variation in degradation parameter due to its time dependent shape parameter. Recently, the Gamma degradation model with both shape and scale parameter stress dependent, and shape parameter time dependent, has been applied to model degradation behavior of LEDs. This approach is well-suited to model the sample-to-sample variability as well as temporal variability. A similar approach has not been applied to predict the RUL of capacitors. The main advantage of this approach is the dynamic and adaptive nature of Gamma degradation model thus, its ability to analyze the degradation of capacitors under varied operating conditions along with sample-to-sample variation.

4.2. Scope of Further Studies

1) We have used capacitance degradation data from reference [58] and applied Gamma stochastic process to estimate lifetime of electrolytic capacitors. The estimation is better than regression-based technique (calculated lifetime using degradation data of 90 capacitors) used in the reference but the accuracy of our estimation can be further improved if the degradation data is available for more than 18 capacitors. One need to carry out more experiments and collect degradation data (but not as much as required in regression-based analysis) under various stress levels and apply the adaptive and dynamic Gamma stochastic process to further validate the significance of the technique used in this thesis.
2) In the present work, interaction between various stresses has been neglected. The interaction terms need to be taken in future studies to see the effect of interaction term on lifetime estimation.
3) The cause of deterioration in electrolytic capacitors is evaporation of electrolyte and both the case studies presented in this thesis are conducted on electrolytic capacitors. It will be interesting to see how well Gamma degradation modeling technique predicts RUL in case of film and ceramic capacitors where the cause of deterioration is different.
4) Beside temperature and voltage stress, humidity also affects deterioration of capacitance. With three major stresses, deterioration data need to be collected and analyzed with the help of Gamma process modeling. The accuracy of RUL estimation with three major stresses is rarely addressed in past and needs attention.
5) In literature, data-driven regression-based models are used to predict RUL of devices as these models are simple to implement but the number of samples required are more to

achieve accuracy of estimates. It will be useful if one can compare the time and cost of experiments for lifetime estimates using regression-based and Gamma degradation process models.

6) Another advantage of using Gamma stochastic modeling is a smaller number of test samples required to predict RUL. But a quantitative estimate regarding number of samples required to accurately estimate RUL is required to be carried out. This will help in optimization of lifetime estimation experiment in terms of cost and time.
7) The lifetime estimation of capacitors is not new, and many studies using various models have been carried out. If the degradation process fits the criteria of Gamma process and degradation data is available, one can compare the accuracy obtained with Gamma process model with the conventionally used models. We feel that the biggest advantage is dynamic and adaptive nature of Gamma process-based degradation modeling which takes care of sample-to-sample as well as temporal variability.

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APPENDIX. OPTIMIZATION SCRIPT FOR CASE STUDY B

```
import numpy as np
from scipy.optimize import minimize, fmin
import os
import pandas as pd
from scipy import special
import sys
os.getcwd()
os.listdir('../')
data_df = pd.read_csv('casestudy.csv')
data_df['time'] = pd.to_numeric(data_df['time'], downcast='integer')
print (data_df.head())
time_points = [0, 2, 4, 6, 8, 10, 12, 14, 16, 18, 20]
def obj(X):
        a0, a1, a2, b0, b1, b2, c = X
        L = 0
        for i, row in data_df.iterrows():
          s1 = row['S1']
          s2 = row['S2']
          dy = row['Delta_Y']
          t  = row['time']
          t_Prev = time_points[time_points.index(row['time'])-1]

          L += (-1)*np.exp(a0+a1*s1+a2*s2)*(t**c - t_Prev**c)*(b0+b1*s1+b2*s2) \
            + np.log(abs(special.Gamma(( np.exp(a0+a1*s1+a2*s2) * (t**c -
             t_Prev**c))))) \ - (np.exp(a0+a1*s1+a2*s2) * (t**c - t_Prev**c) -
             1)*np.log(abs(dy)) \+ dy*np.exp(b0+b1*s1+b2*s2)
       return L
import random
for i in range(100):

      initial_guess = [random.uniform(0, 1), random.uniform(0, 1), random.uniform(0,
      1), random.uniform(0, 1), random.uniform(0, 1), random.uniform(0, 1),
      random.uniform(0, 1)]
      print (initial_guess)
      result = fmin(obj, initial_guess, maxiter=10000)
      print (result)
sys.exit()
```