

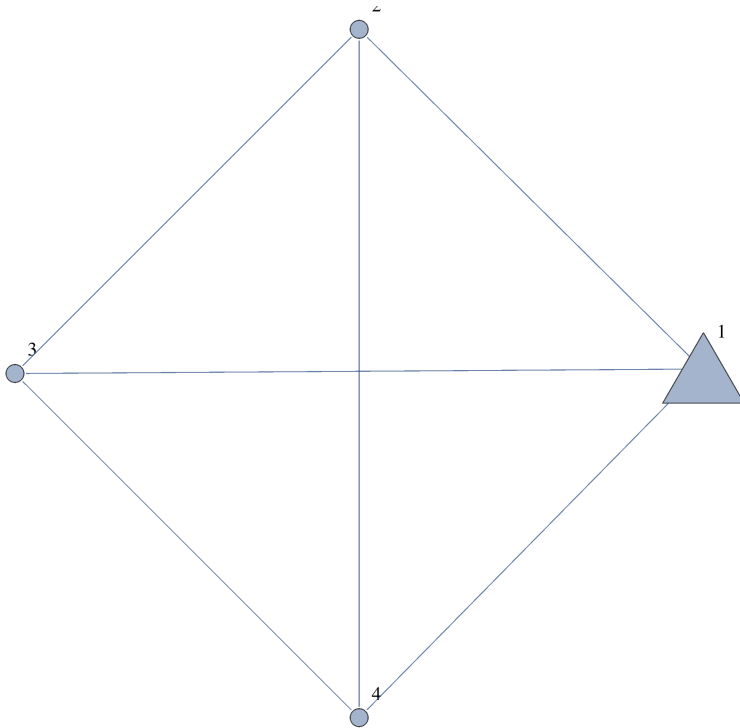
# The $K_4$ with arbitrary weight

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## Start of the game

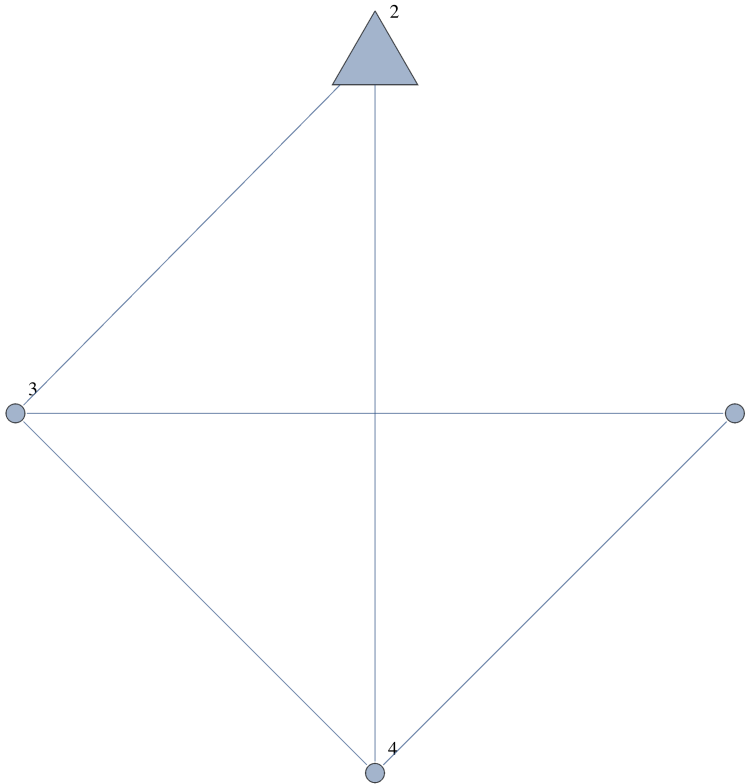
### ■ Player 1's Turn

```
Graph[{1, 2, 3, 4}, {1 ↔ 2, 1 ↔ 3, 1 ↔ 4, 2 ↔ 3, 2 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {1 -> "Triangle"},  
VertexSize -> {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```



### ■ Player 2's Turn

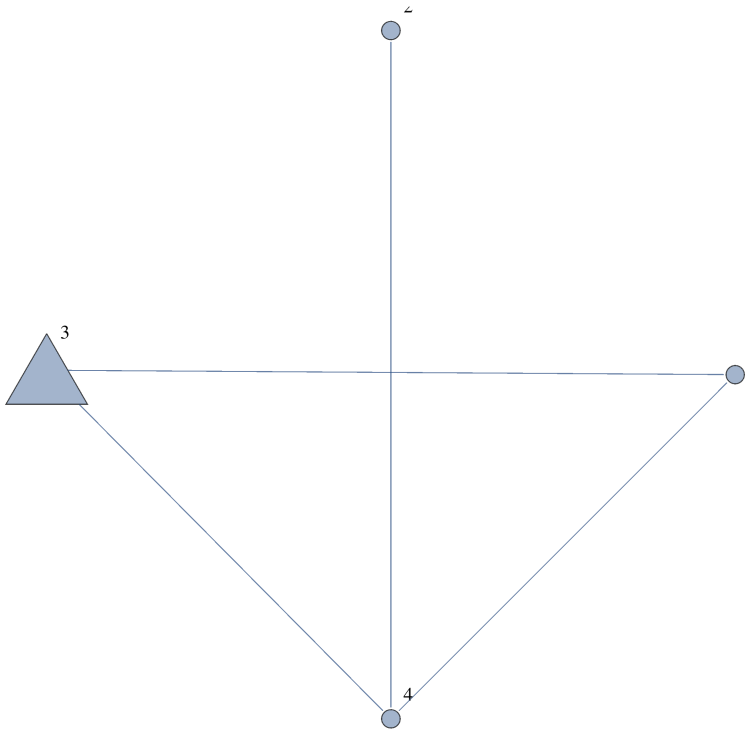
```
Graph[{1, 2, 3, 4}, {1 ↔ 3, 1 ↔ 4, 2 ↔ 3, 2 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {2 -> "Triangle"},  
VertexSize -> {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```



### ■ Player 1's Turn

(0)

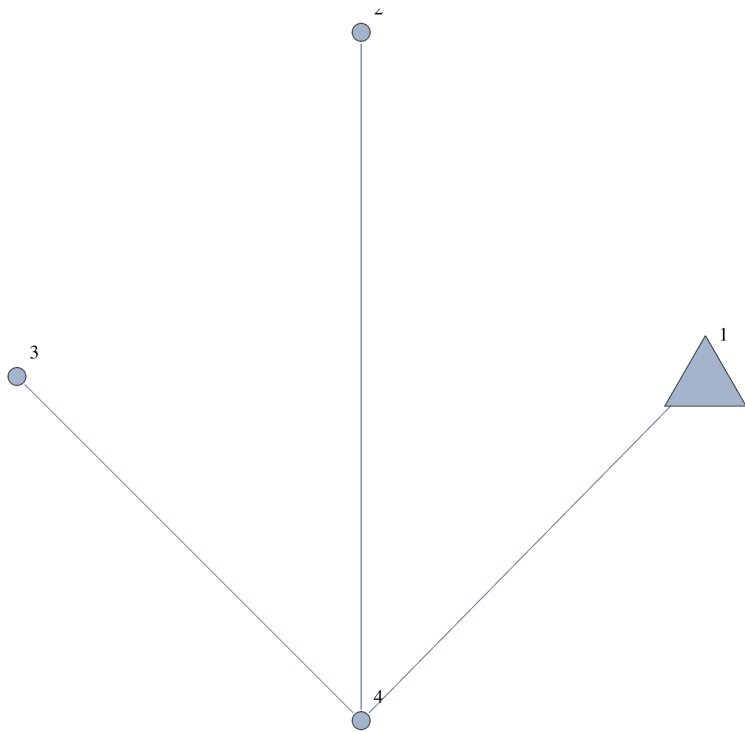
```
Graph[{1, 2, 3, 4}, {1 ↔ 3, 1 ↔ 4, 2 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {3 -> "Triangle"},  
VertexSize -> {3 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```



■ Player 2's Turn

(0)

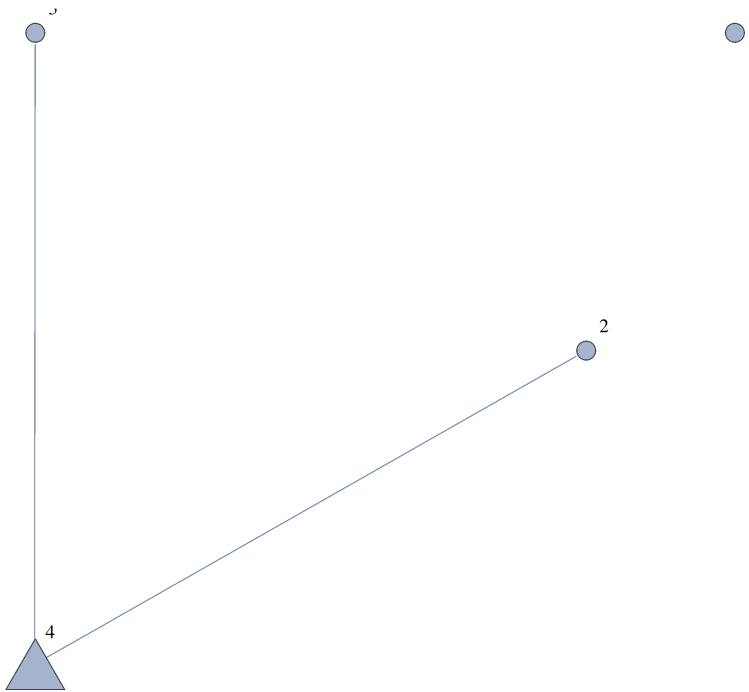
```
Graph[{1, 2, 3, 4}, {1 ↔ 4, 2 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {1 -> "Triangle"},  
VertexSize -> {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```



■ **Player 1's Turn**

(00)

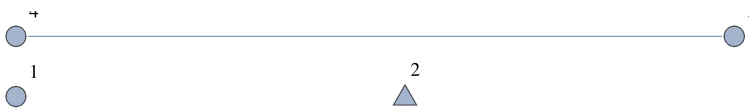
```
Graph[{1, 2, 3, 4}, {2 ↔ 4, 3 ↔ 4},
  VertexLabels -> "Name", VertexShapeFunction -> {4 -> "Triangle"},
  VertexSize -> {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```



#### ■ Player 2's Turn

(00)

```
Graph[{1, 2, 3, 4}, {3 ↔ 4}, VertexLabels -> "Name",
  VertexShapeFunction -> {2 -> "Triangle"},
  VertexSize -> {2 -> 0.35}, GraphLayout -> "CircularEmbedding"]
```

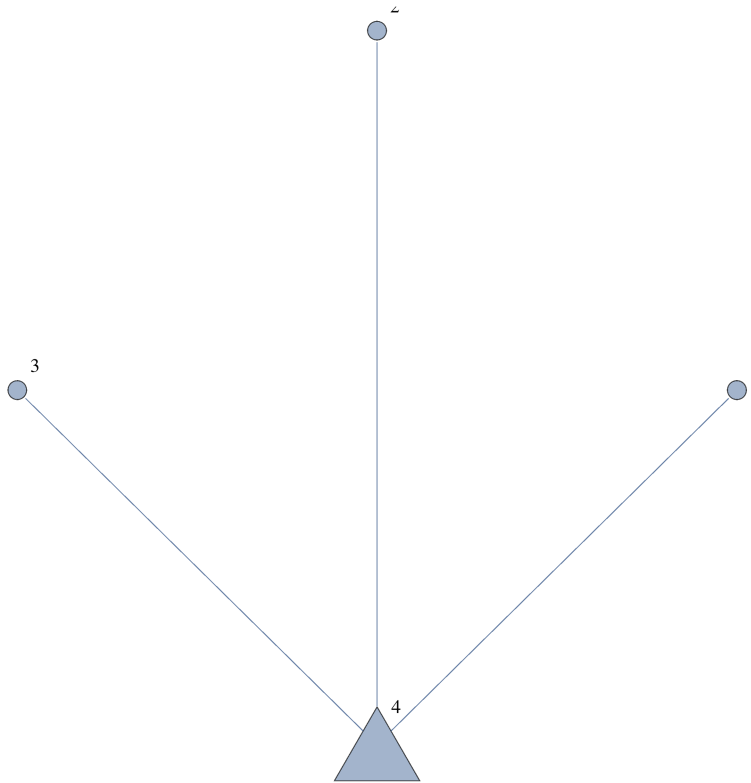


Player 2 lost!

#### ■ Player 1's Turn

(0+)

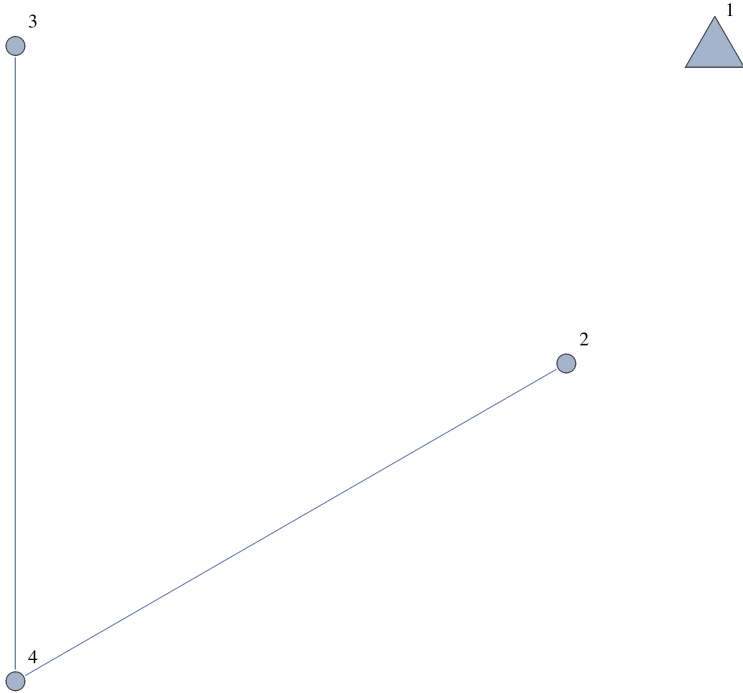
```
Graph[{1, 2, 3, 4}, {1 ↔ 4, 2 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {4 -> "Triangle"},  
VertexSize -> {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```



■ **Player 2's Turn**

(0+)

```
Graph[{1, 2, 3, 4}, {2 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {1 -> "Triangle"},  
VertexSize -> {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```

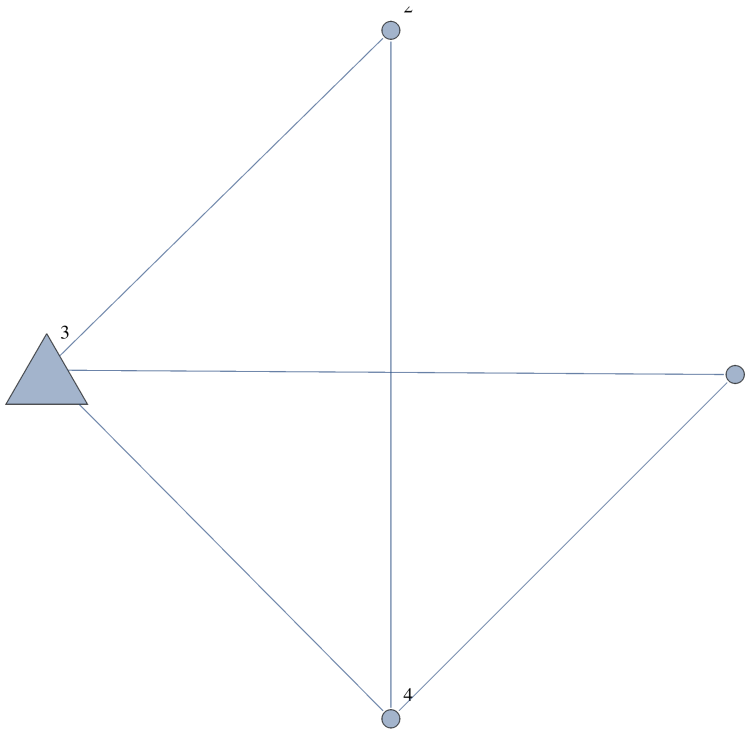


Player 2 lost!

### ■ Player 1's Turn

(+)

```
Graph[{1, 2, 3, 4}, {1 ↔ 3, 1 ↔ 4, 2 ↔ 3, 2 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {3 -> "Triangle"},  
VertexSize -> {3 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```

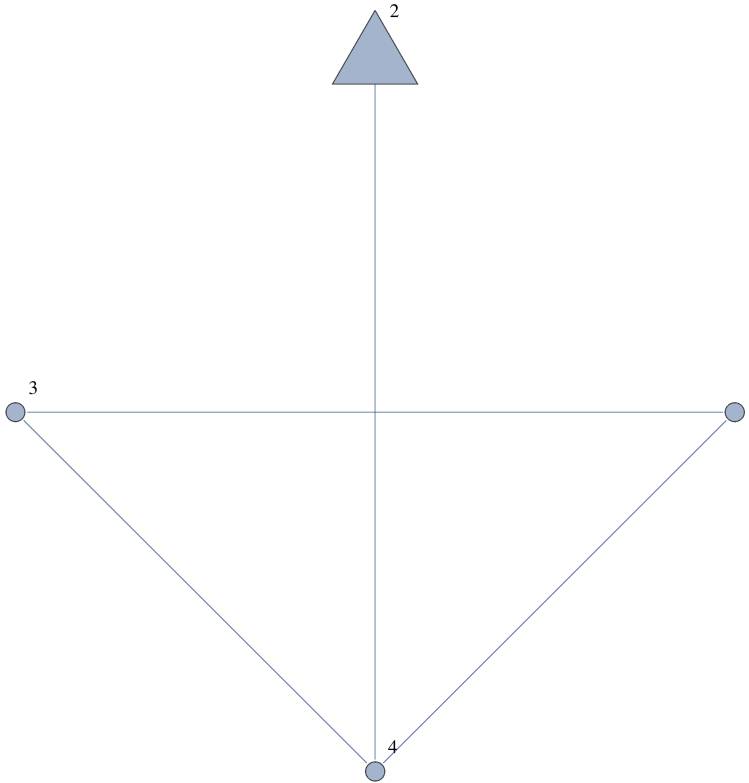


■ **Player 2's Turn**

(+)



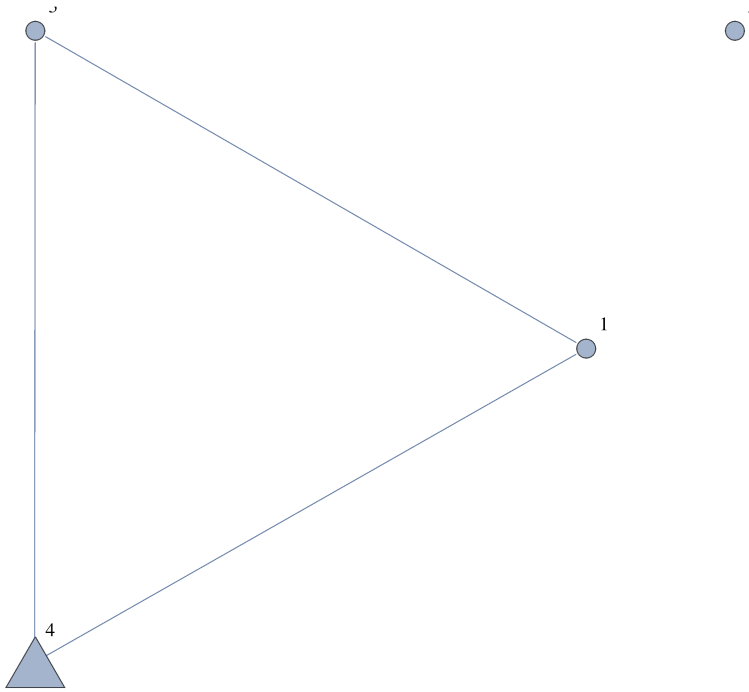
```
Graph[{1, 2, 3, 4}, {1 ↔ 3, 1 ↔ 4, 2 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {2 -> "Triangle"},  
VertexSize -> {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```



■ Player 1's Turn

(+0)

```
Graph[{1, 2, 3, 4}, {1 ↔ 3, 1 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {4 -> "Triangle"},  
VertexSize -> {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```

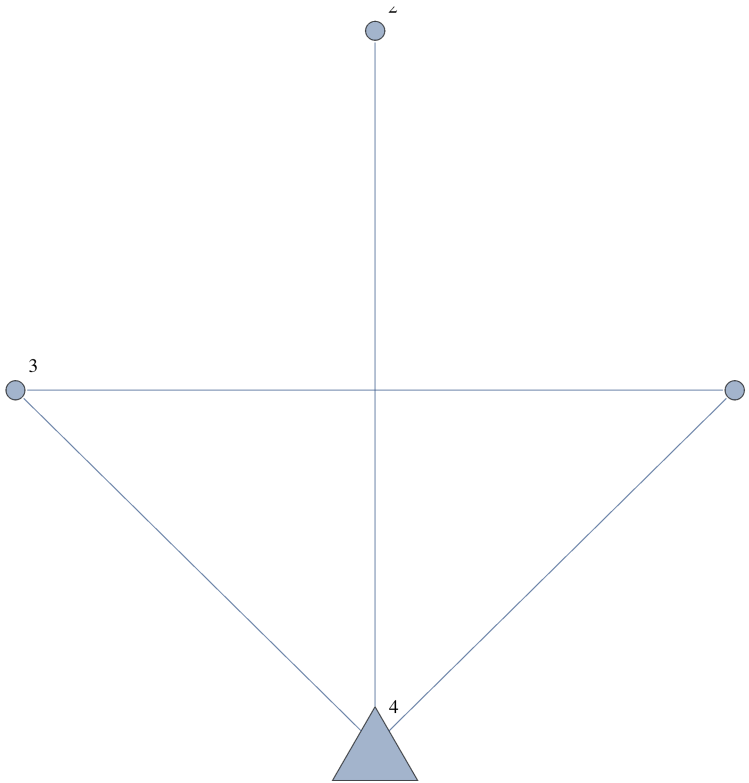


What remains is the  $K_3$ . Since the first player to play a  $K_3$  wins, Player 1 will win this game. Hence, Player 2 lost!

■ **Player 1's Turn**

(++)

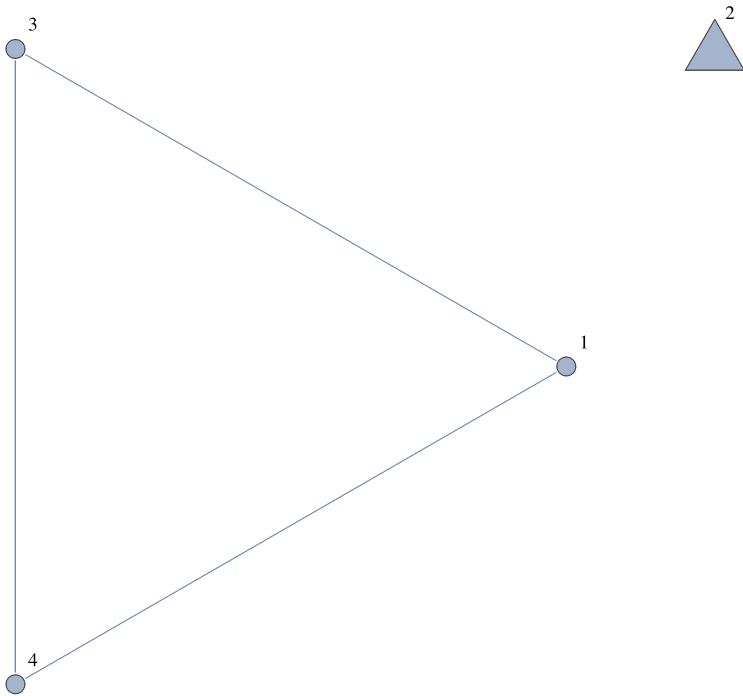
```
Graph[{1, 2, 3, 4}, {1 ↔ 3, 1 ↔ 4, 2 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {4 -> "Triangle"},  
VertexSize -> {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```



#### ■ Player 2's Turn

(++)

```
Graph[{1, 2, 3, 4}, {1 ↔ 3, 1 ↔ 4, 3 ↔ 4},  
VertexLabels -> "Name", VertexShapeFunction -> {2 -> "Triangle"},  
VertexSize -> {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```



Player 2 lost!