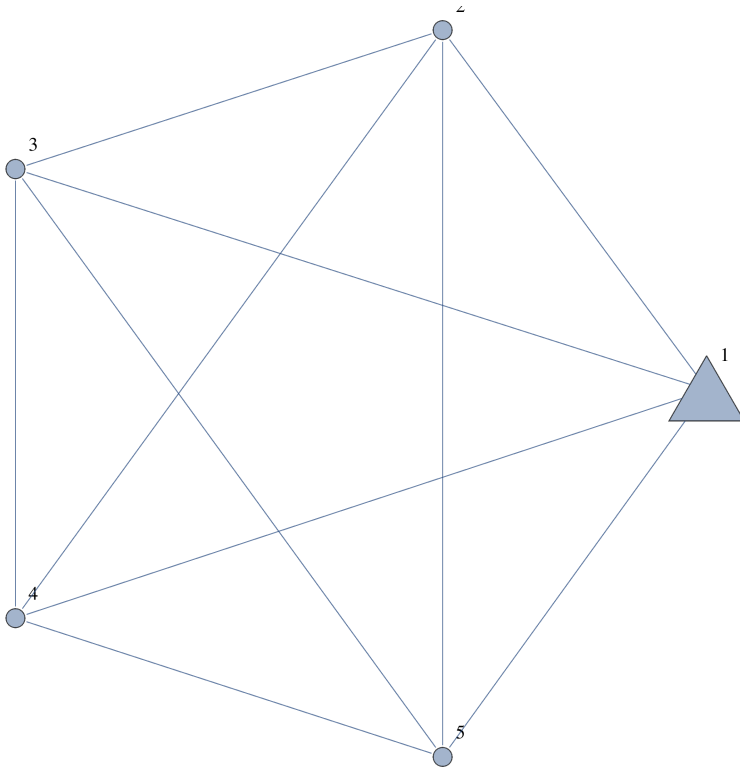# The K_5 with arbitrary weight

Start of the game
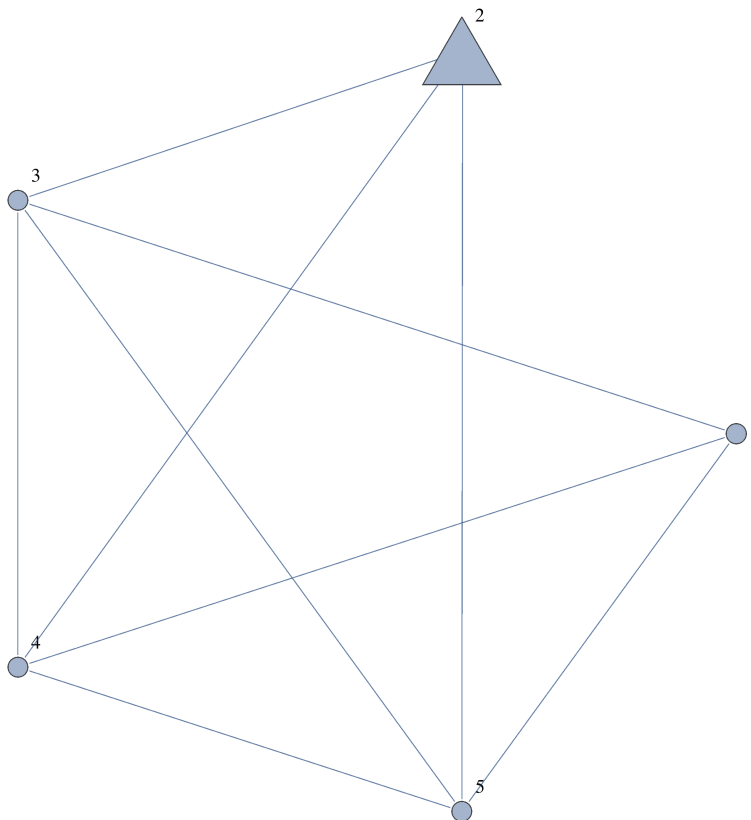
- **Player 1's Turn**

```
Graph[{1, 2, 3, 4, 5},
 {1 ↔ 2, 1 ↔ 3, 1 ↔ 4, 1 ↔ 5, 2 ↔ 3, 2 ↔ 4, 2 ↔ 5, 3 ↔ 4, 3 ↔ 5, 4 ↔ 5},
 VertexLabels -> "Name", VertexShapeFunction → { 1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
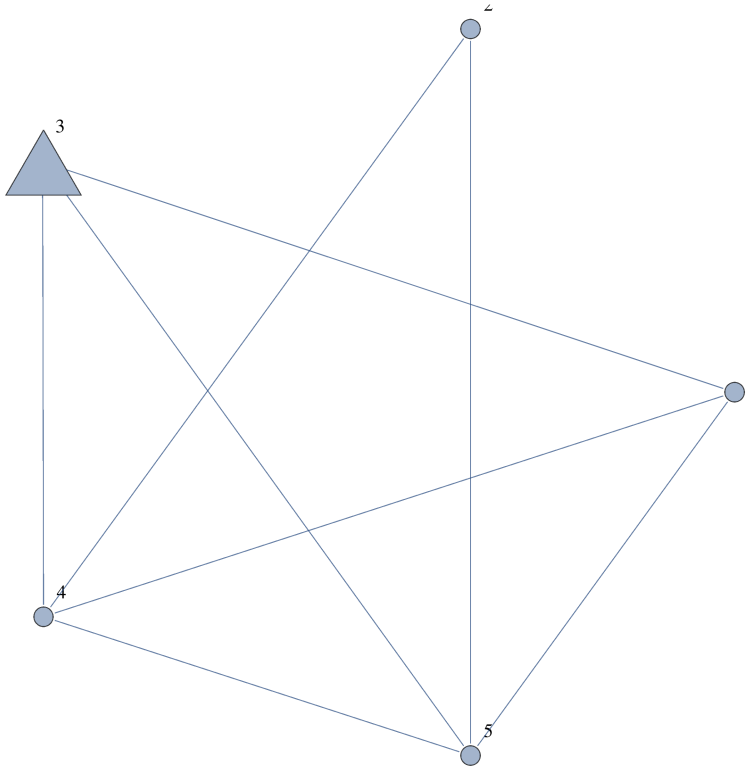
- **Player 2's Turn**

```
Graph[{1, 2, 3, 4, 5}, {1 ↔ 3, 1 ↔ 4, 1 ↔ 5, 2 ↔ 3, 2 ↔ 4, 2 ↔ 5, 3 ↔ 4, 3 ↔ 5, 4 ↔ 5},
  VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
  VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
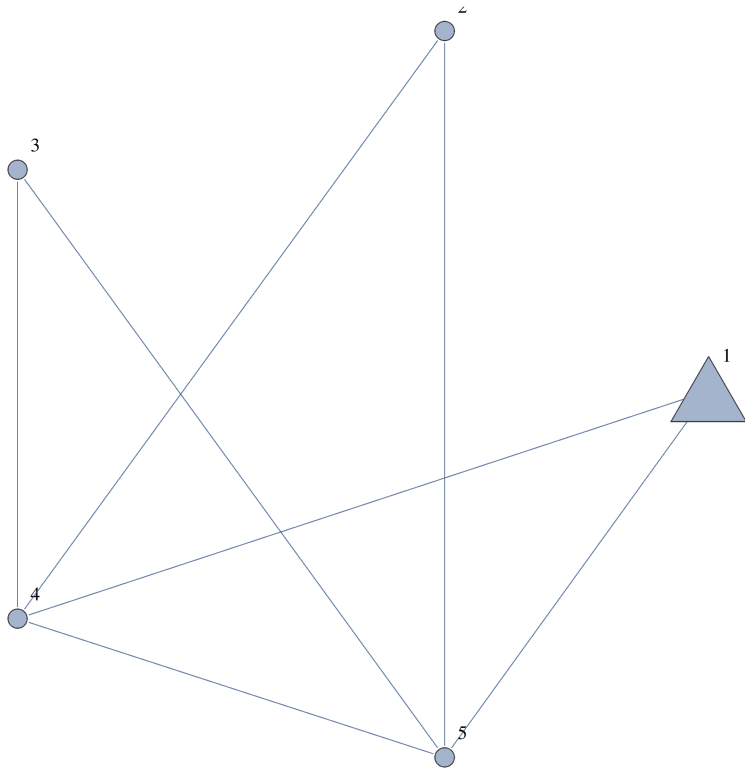
- **Player 1's Turn**

(0)

```
Graph[{1, 2, 3, 4, 5}, {1 ↔ 3, 1 ↔ 4, 1 ↔ 5, 2 ↔ 4, 2 ↔ 5, 3 ↔ 4, 3 ↔ 5, 4 ↔ 5},
 VertexLabels -> "Name", VertexShapeFunction → {3 -> "Triangle"},
 VertexSize → {3 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
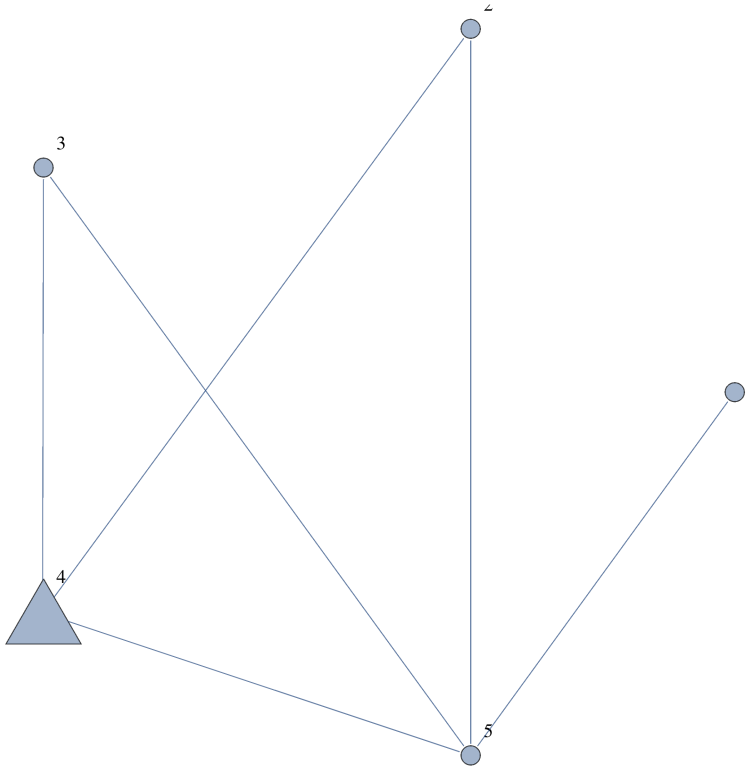

- **Player 2's Turn**

(0)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 4, 1 ⟷ 5, 2 ⟷ 4, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 1's Turn**

(00)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 4, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {4 -> "Triangle"},
 VertexSize → {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(00)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
  VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
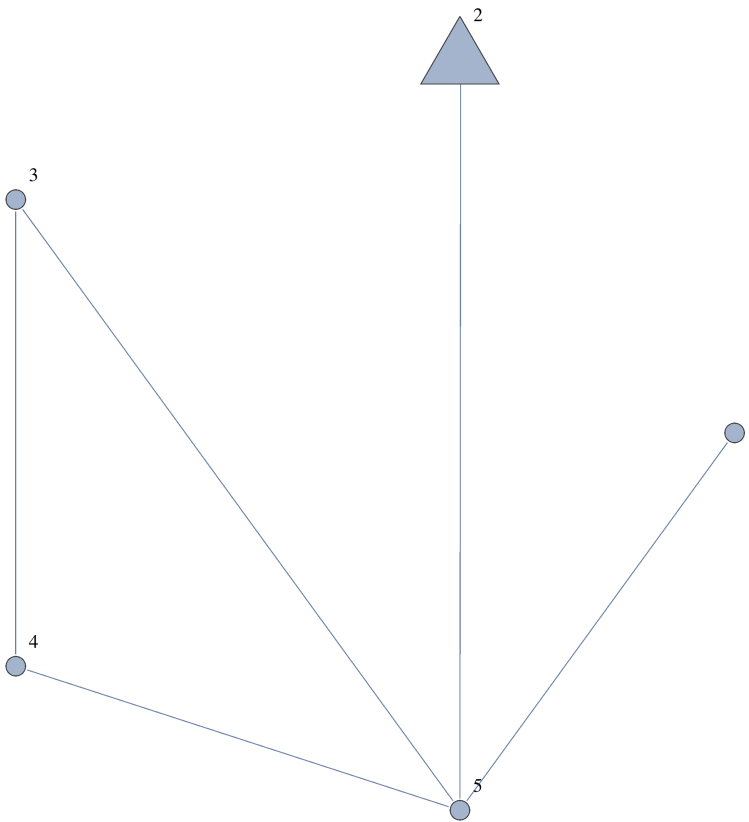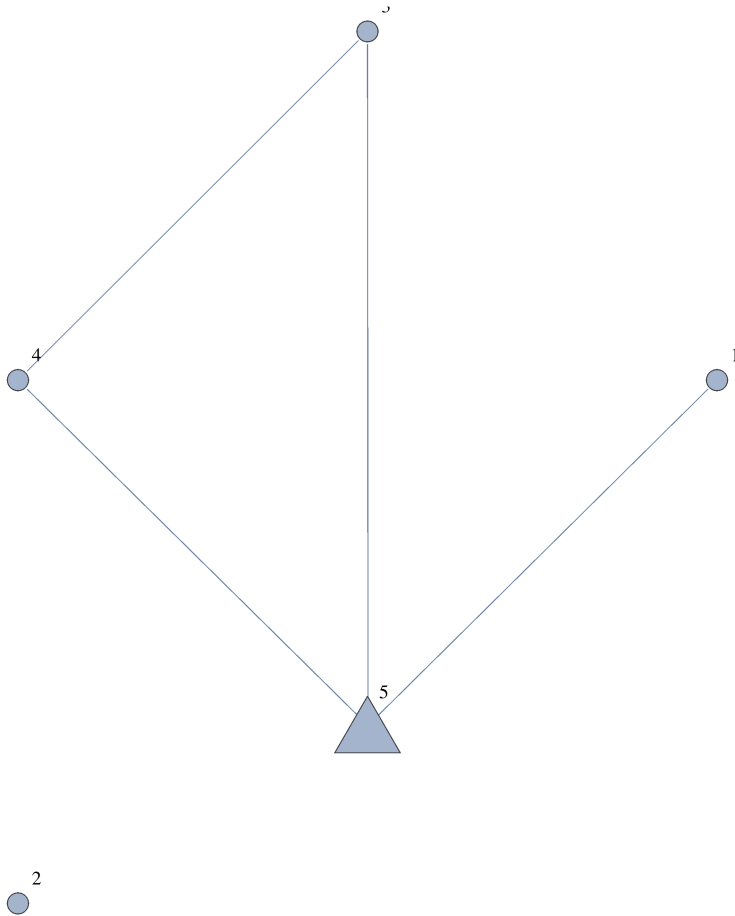

- **Player 1's Turn**

(000)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
 VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
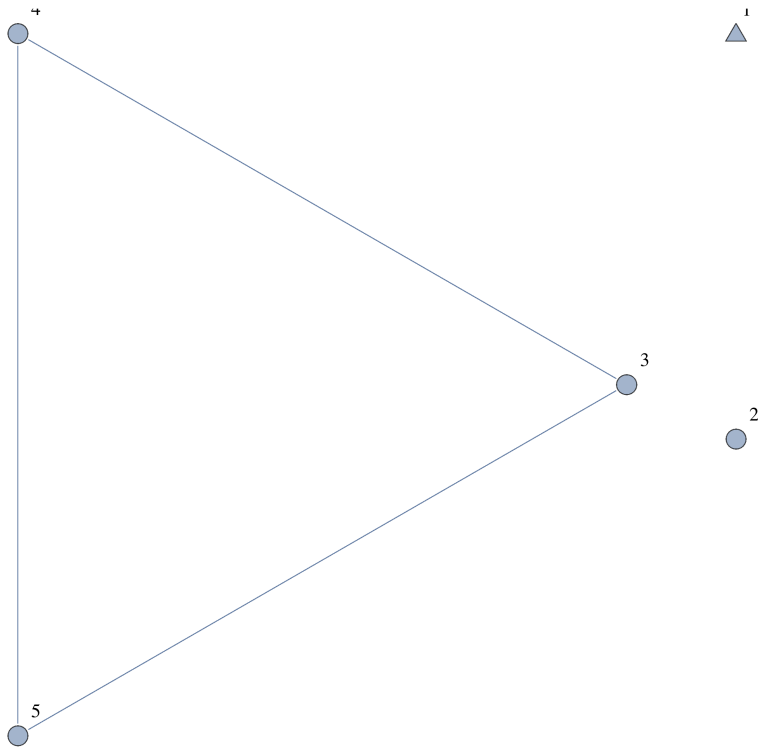

■ **Player 2's Turn**

(000)

```
Graph[{1, 2, 3, 4, 5}, {3 ↔ 4, 3 ↔ 5, 4 ↔ 5},
 VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
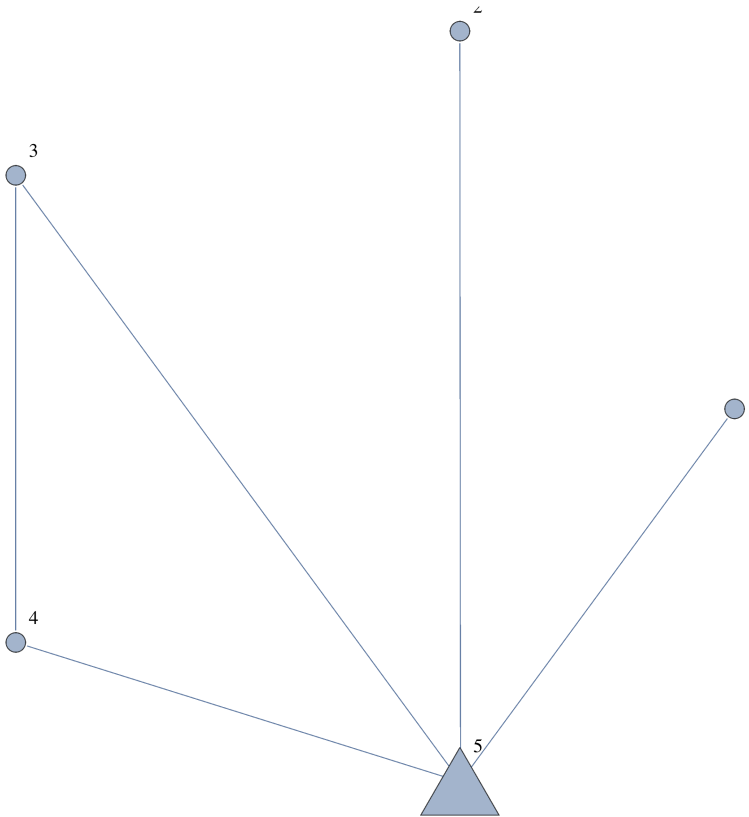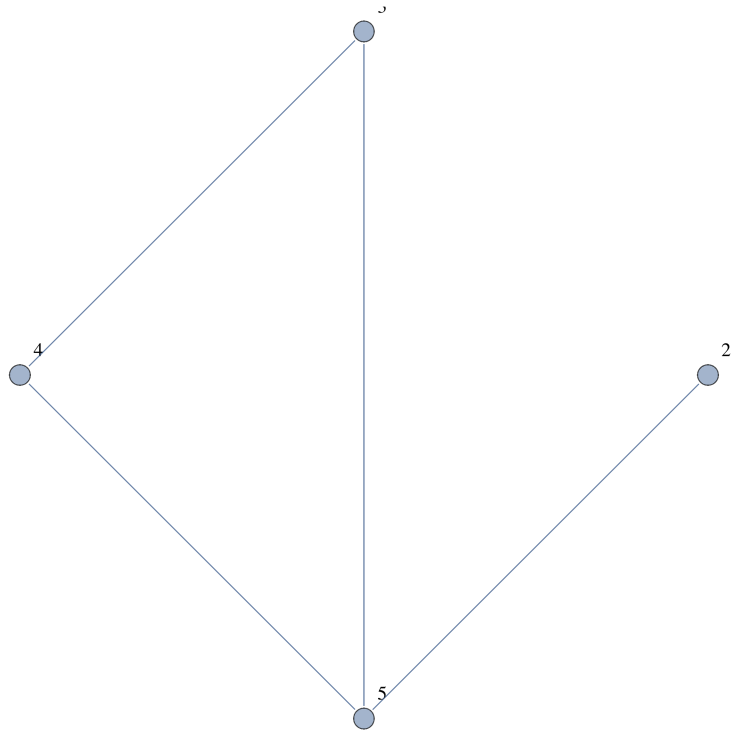

Player 2 lost!

- **Player 1's Turn**

(00+)

```
Graph[{1, 2, 3, 4, 5}, {1 ↔ 5, 2 ↔ 5, 3 ↔ 4, 3 ↔ 5, 4 ↔ 5},
 VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
 VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(00+)

```
Graph[{1, 2, 3, 4, 5}, {2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```

Player 2 lost!

- **Player 1's Turn**

(0+)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 4, 1 ⟷ 5, 2 ⟷ 4, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {4 -> "Triangle"},
 VertexSize → {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
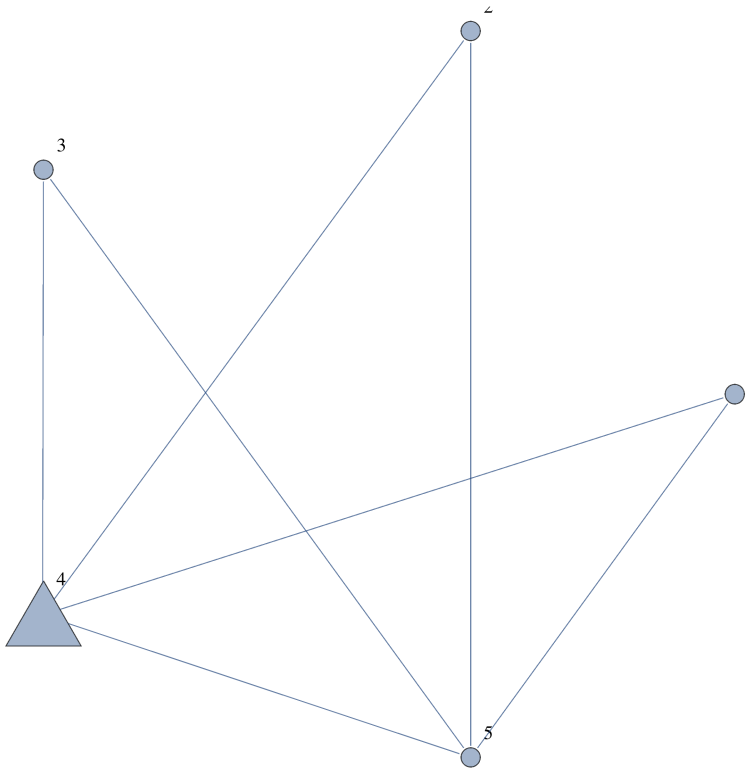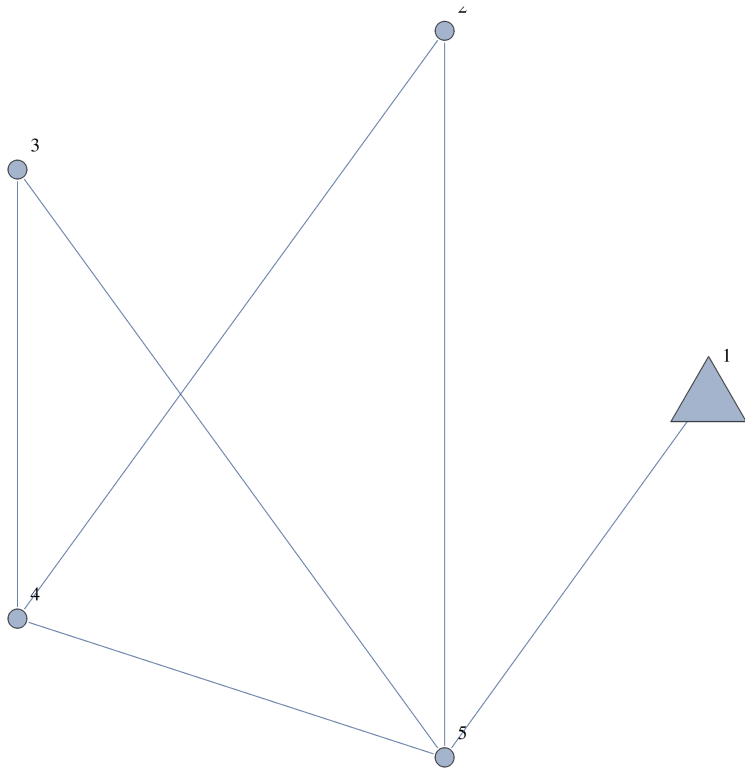

- **Player 2's Turn**

(0+)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 4, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
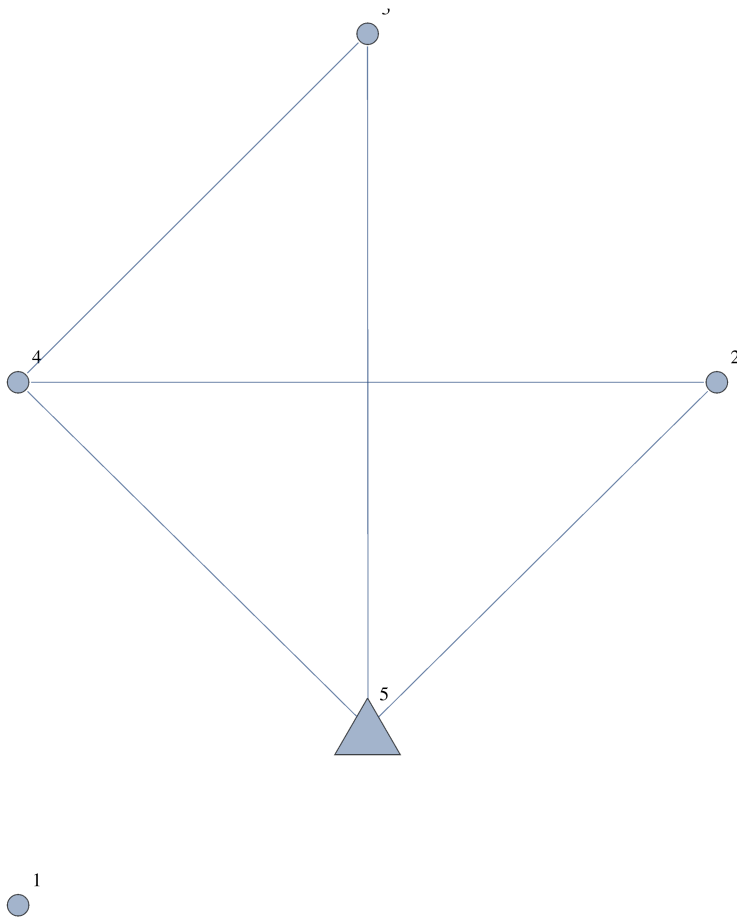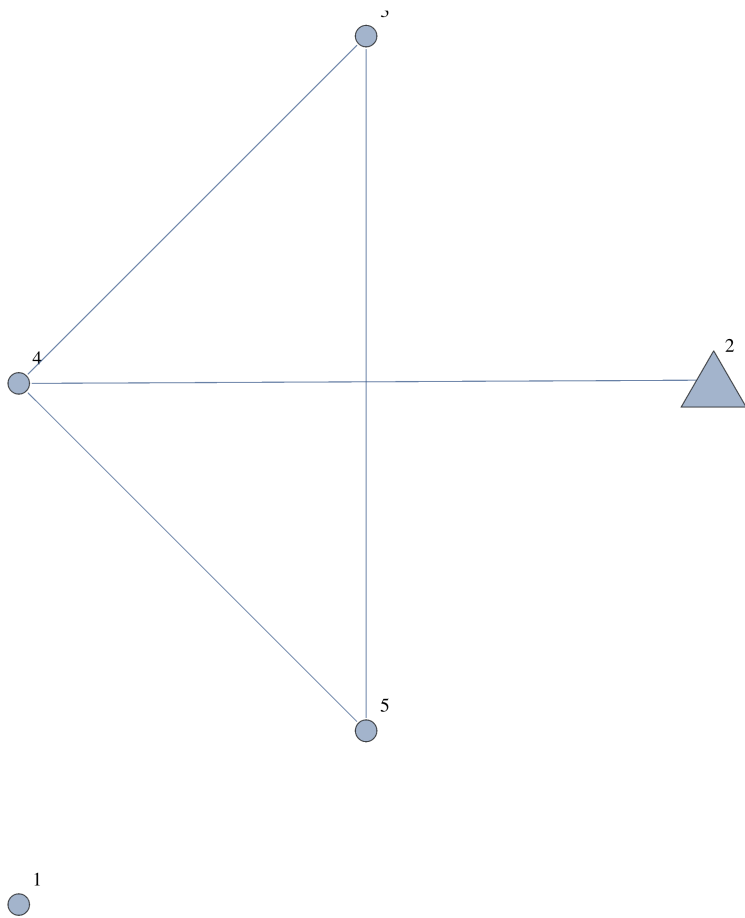

- **Player 1's Turn**

(0+0)

```
Graph[{1, 2, 3, 4, 5}, {2 ↔ 4, 2 ↔ 5, 3 ↔ 4, 3 ↔ 5, 4 ↔ 5},
  VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
  VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
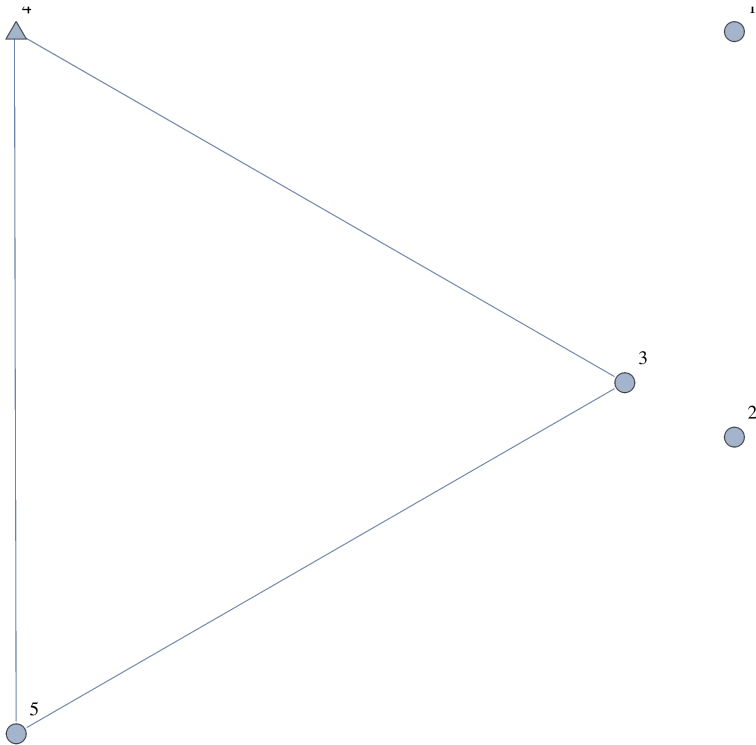

- **Player 2's Turn**

(0+0)

```
Graph[{1, 2, 3, 4, 5}, {2 ⟷ 4, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
 VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 1's Turn**

(0+00)

```
Graph[{1, 2, 3, 4, 5}, {3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {4 -> "Triangle"},
 VertexSize → {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
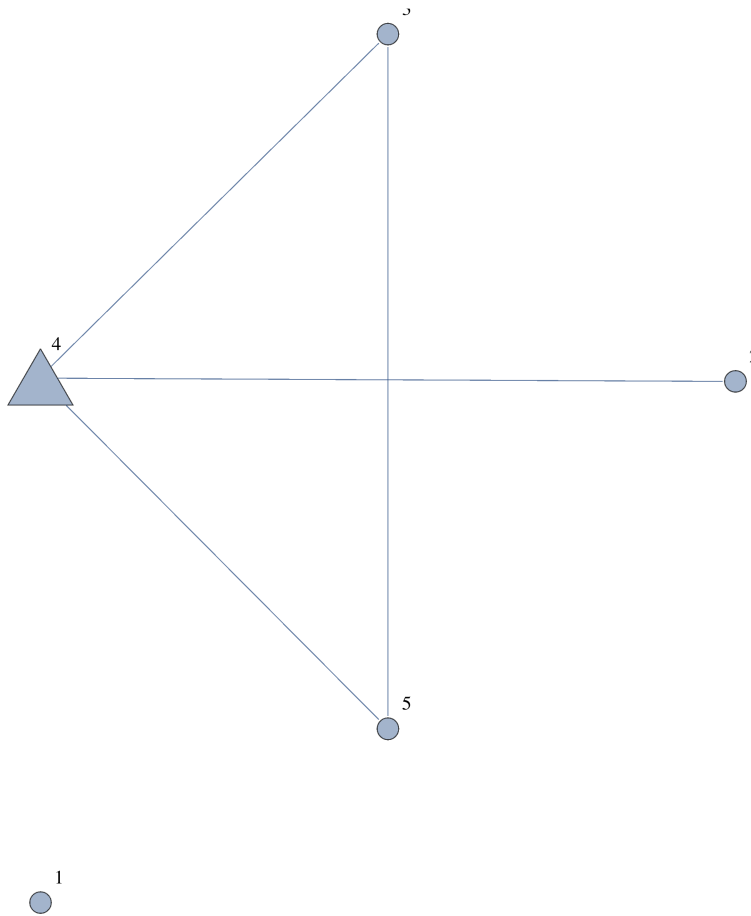

What remains is a K_3. Since the first player to play a K_3 wins, Player 1 will win this game. Hence, Player 2 lost!
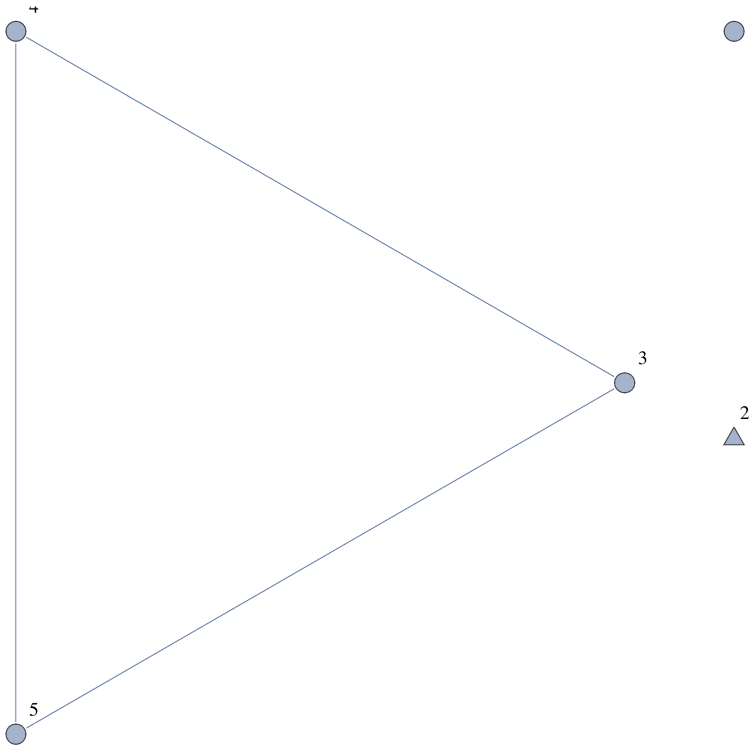
- **Player 1's Turn**

(0+0+)

```
Graph[{1, 2, 3, 4, 5}, {2 ⟷ 4, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {4 -> "Triangle"},
  VertexSize → {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(0+0+)

```
Graph[{1, 2, 3, 4, 5}, {3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
 VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
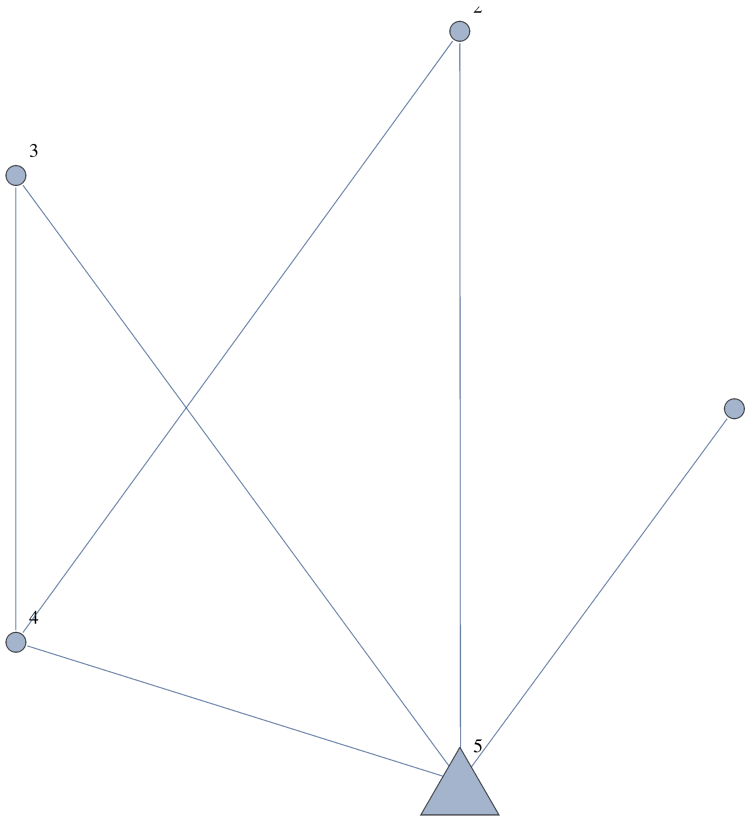
Player 2 lost!

- **Player 1's Turn**

(0++)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 4, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
  VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(0++)

```
Graph[{1, 2, 3, 4, 5}, {2 ⟷ 4, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
  VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
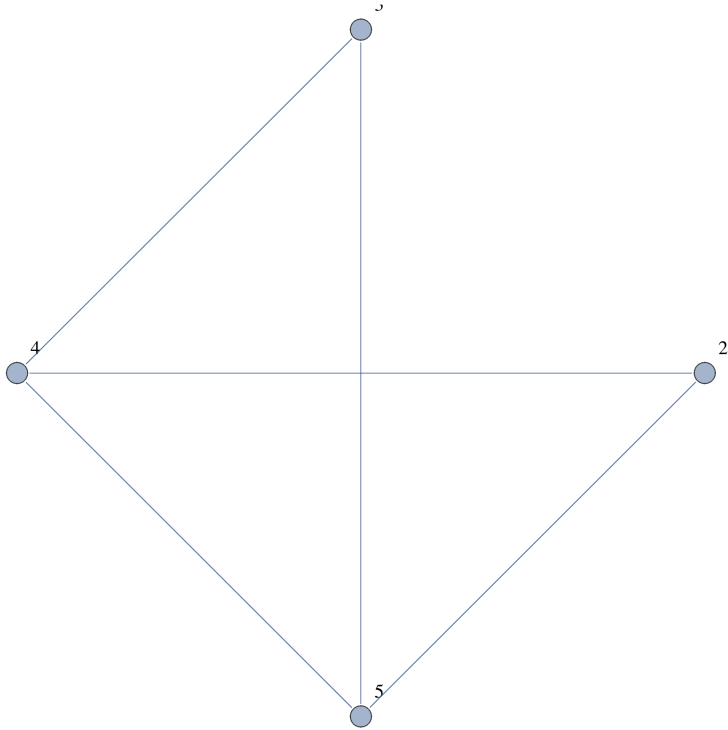

Player 2 lost!

- **Player 1's Turn**

(+)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 4, 1 ⟷ 5, 2 ⟷ 3, 2 ⟷ 4, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {3 -> "Triangle"},
  VertexSize → {3 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
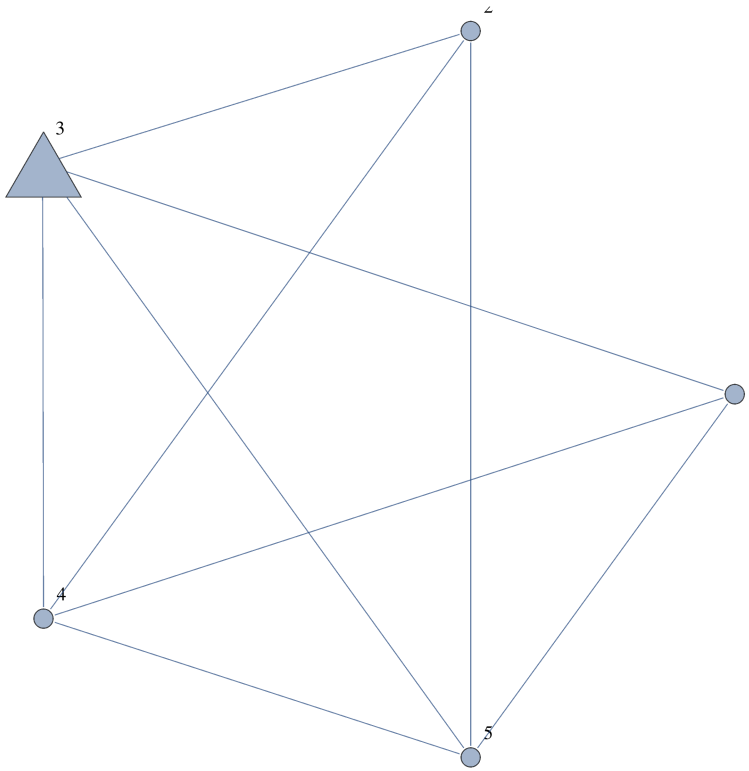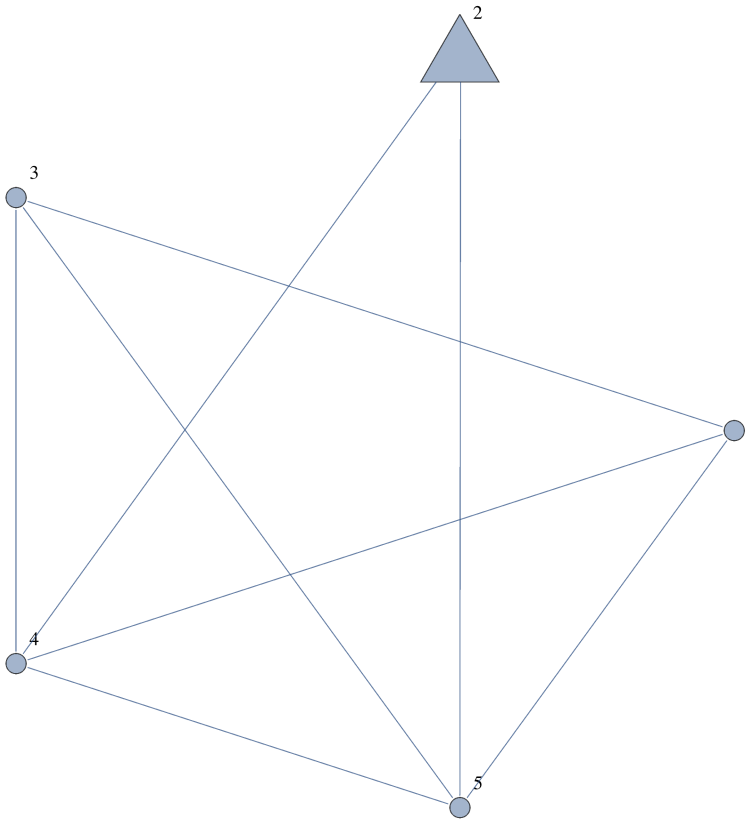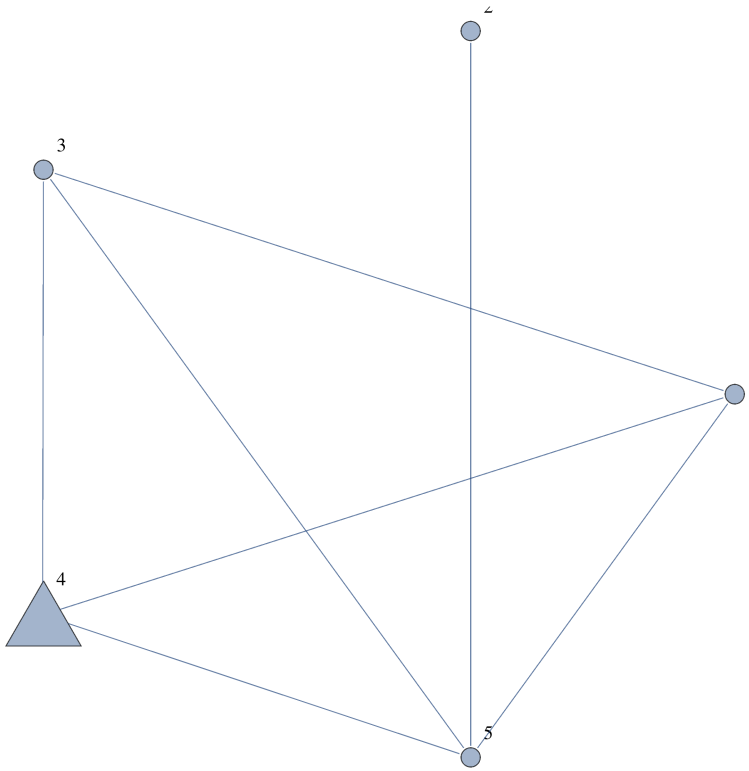

- **Player 2's Turn**

(+)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 4, 1 ⟷ 5, 2 ⟷ 4, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
 VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
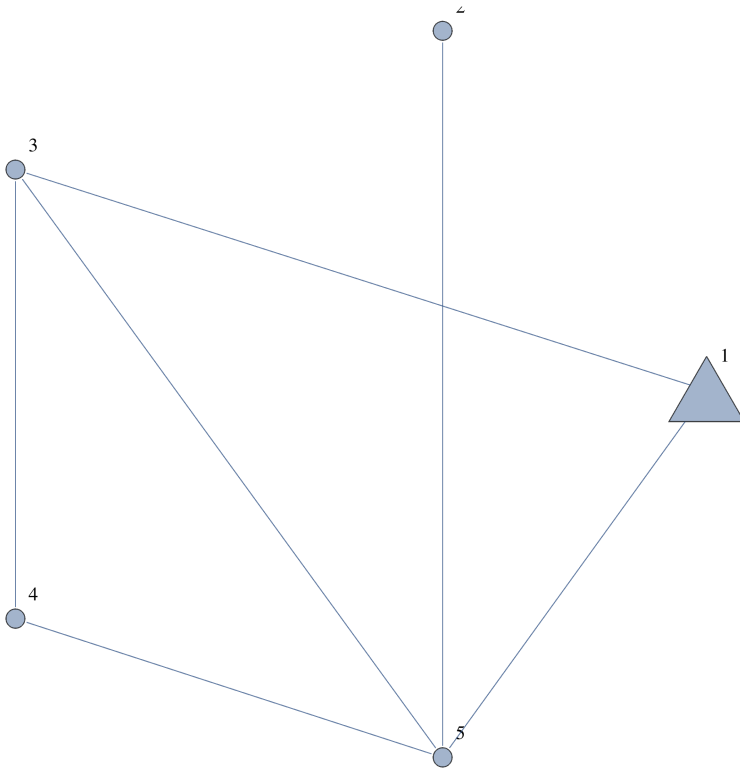

- **Player 1's Turn**

(+0)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 4, 1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {4 -> "Triangle"},
 VertexSize → {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(+0)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


Player 2 has two nonisomorphic options, to v_3 and v_5.

■ **Player 1's Turn**

(+00_3)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {3 -> "Triangle"},
 VertexSize → {3 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
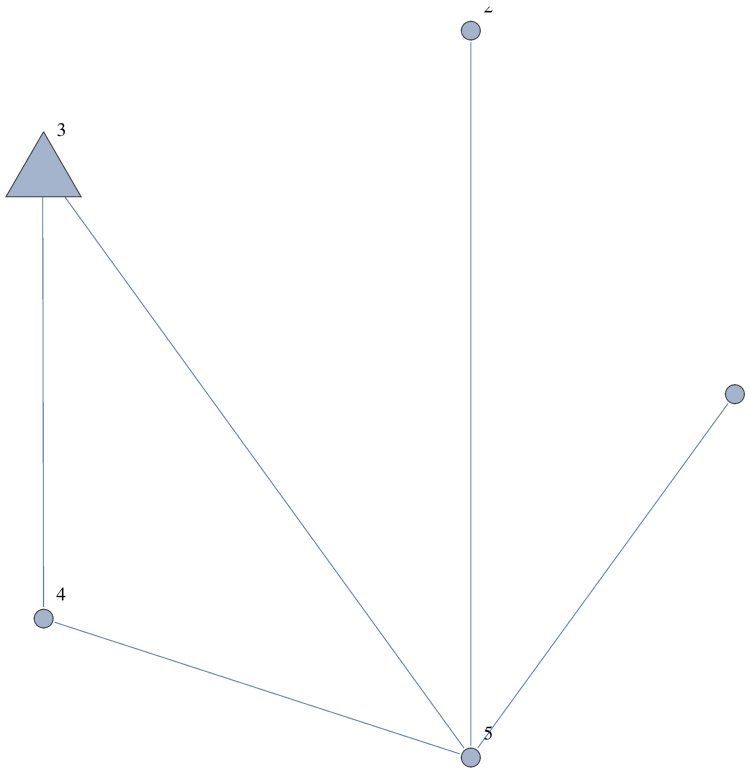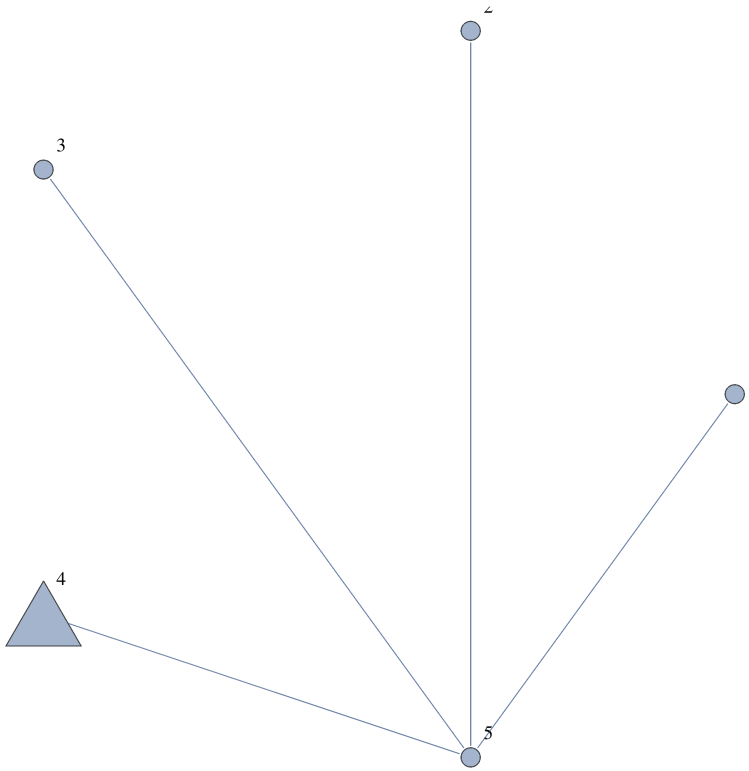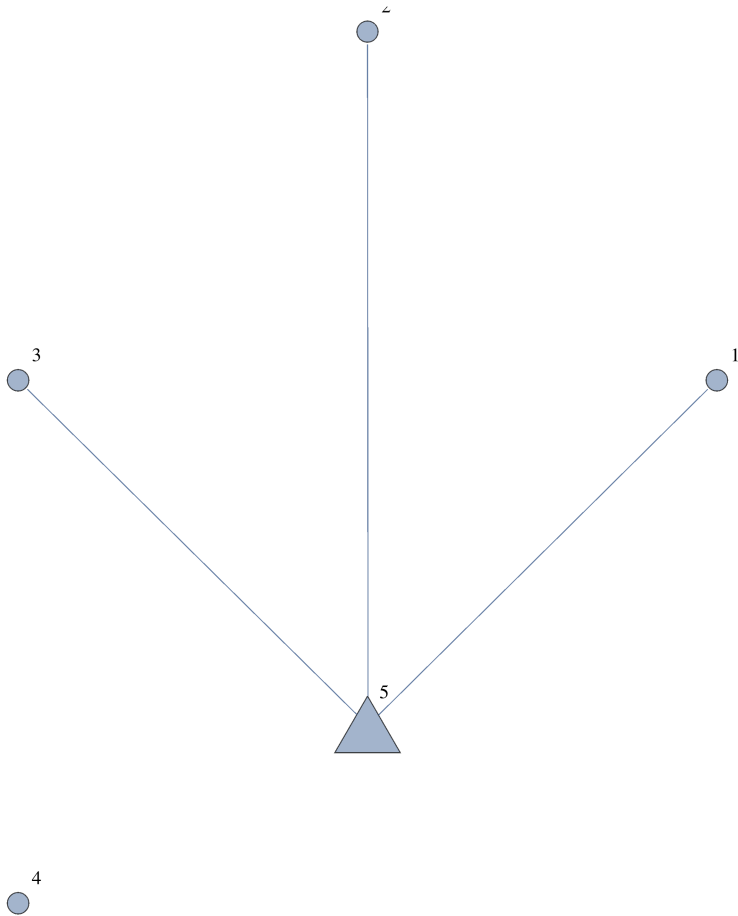

- **Player 2's Turn**

(+00_3)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {4 -> "Triangle"},
  VertexSize → {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
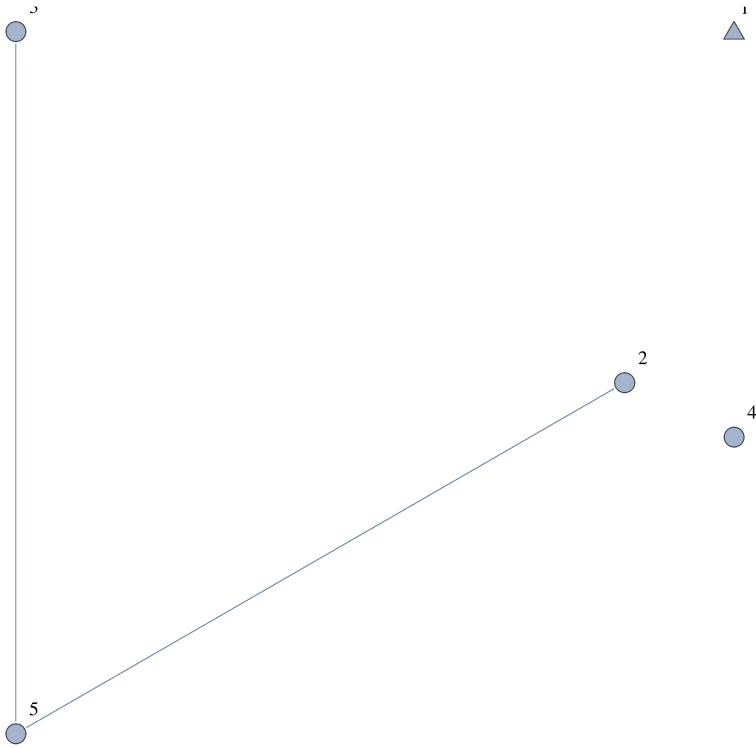

- **Player 1's Turn**

(+00_30)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
 VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


■ **Player 2's Turn**

(+00_30)

```
Graph[{1, 2, 3, 4, 5}, {2 ⟷ 5, 3 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
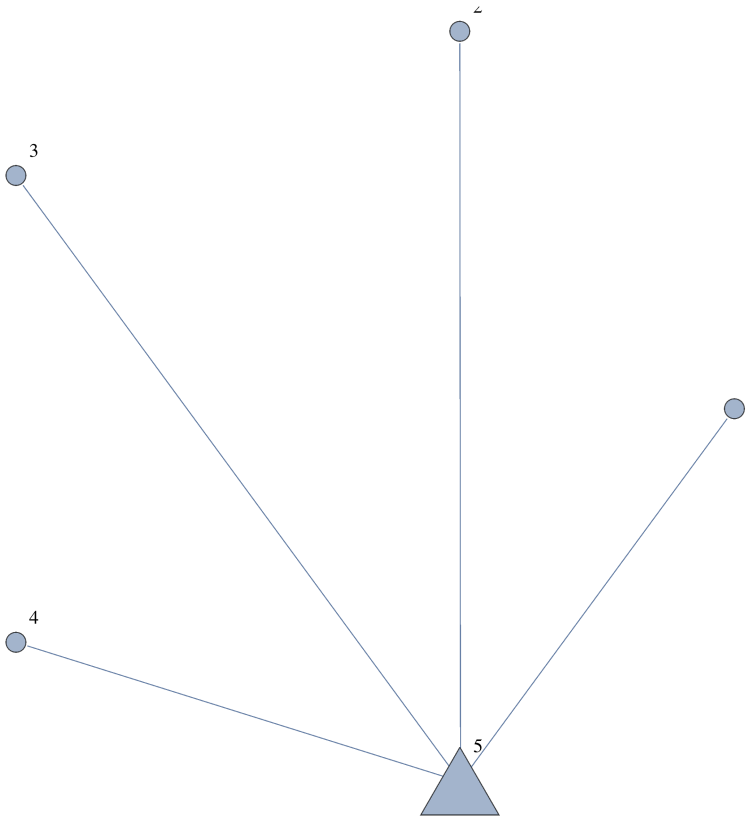

Player 2 lost!

- **Player 1's Turn**

(+00_3+)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
  VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(+00_3+)

```
Graph[{1, 2, 3, 4, 5}, {2 ⟷ 5, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
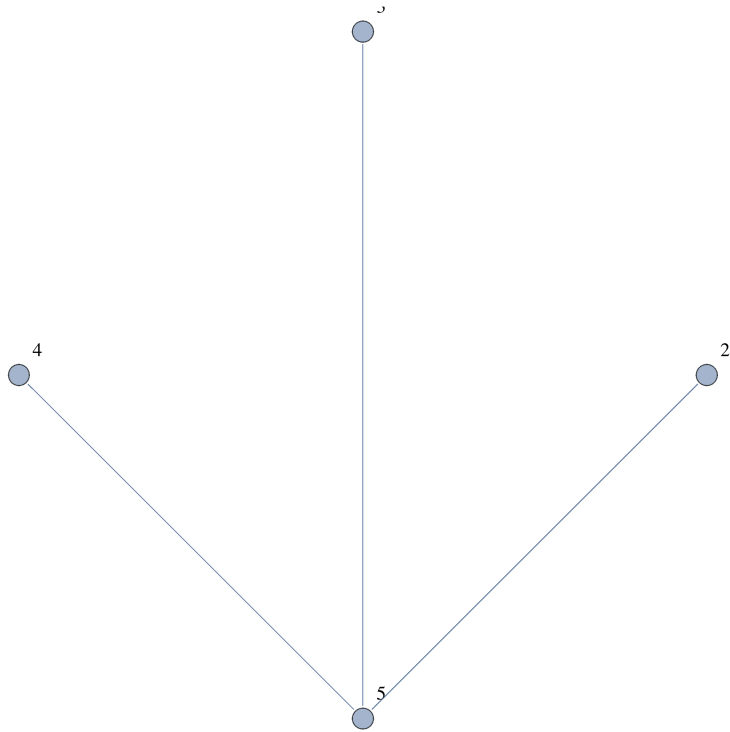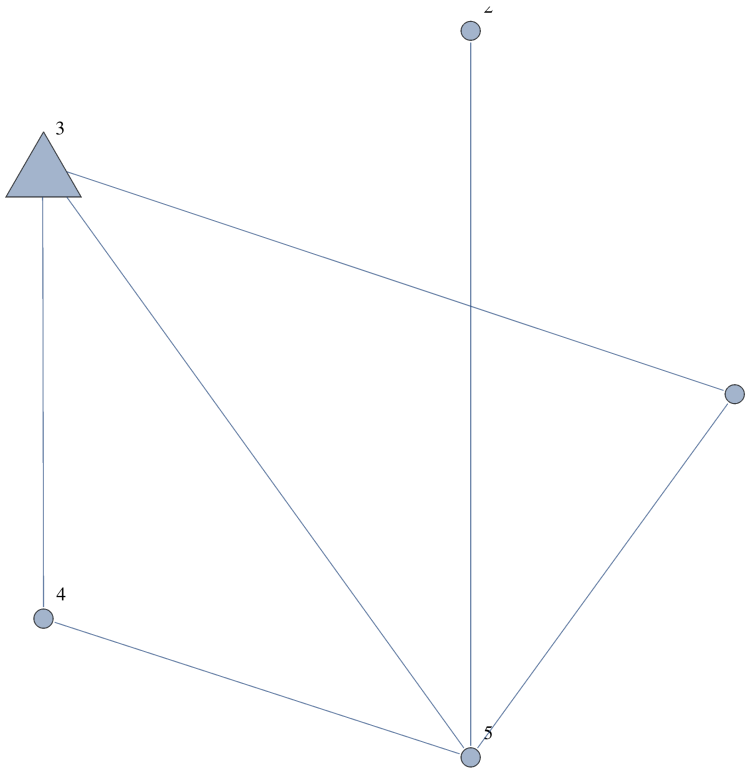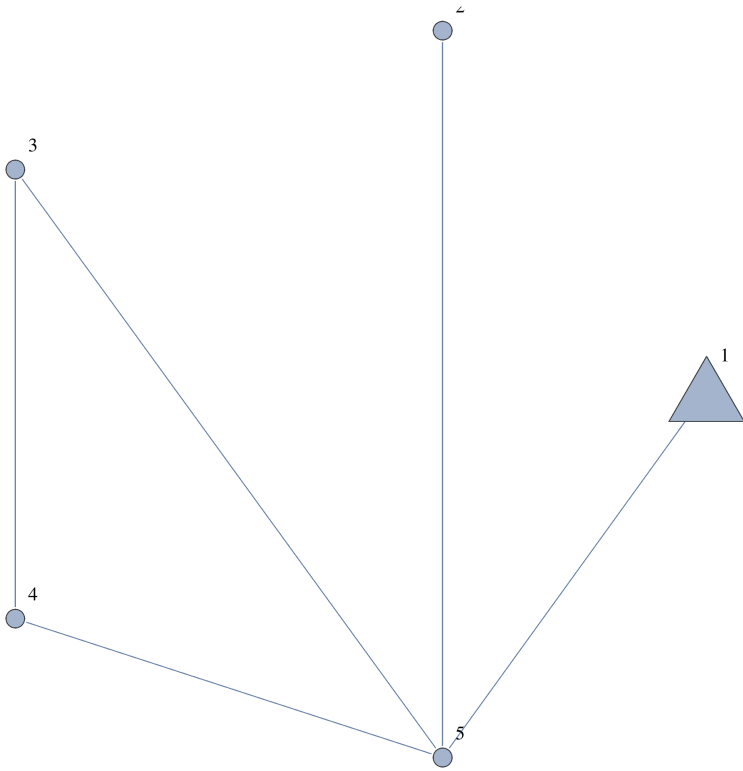

Player 2 lost!

- **Player 1's Turn**

(+0+_3)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {3 -> "Triangle"},
  VertexSize → {3 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
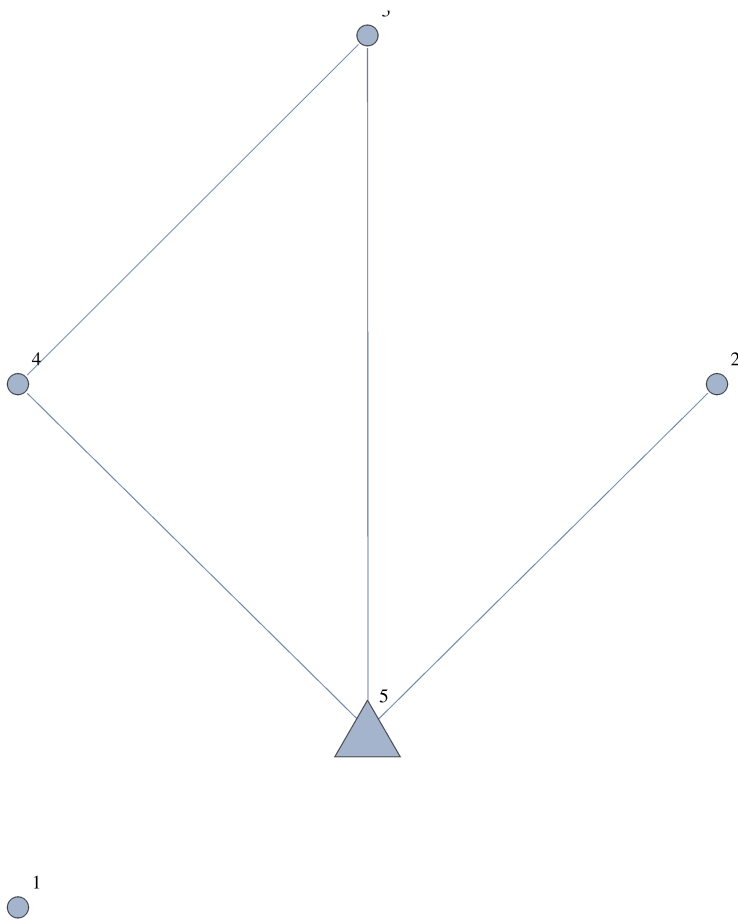

- **Player 2's Turn**

(+0+_3)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
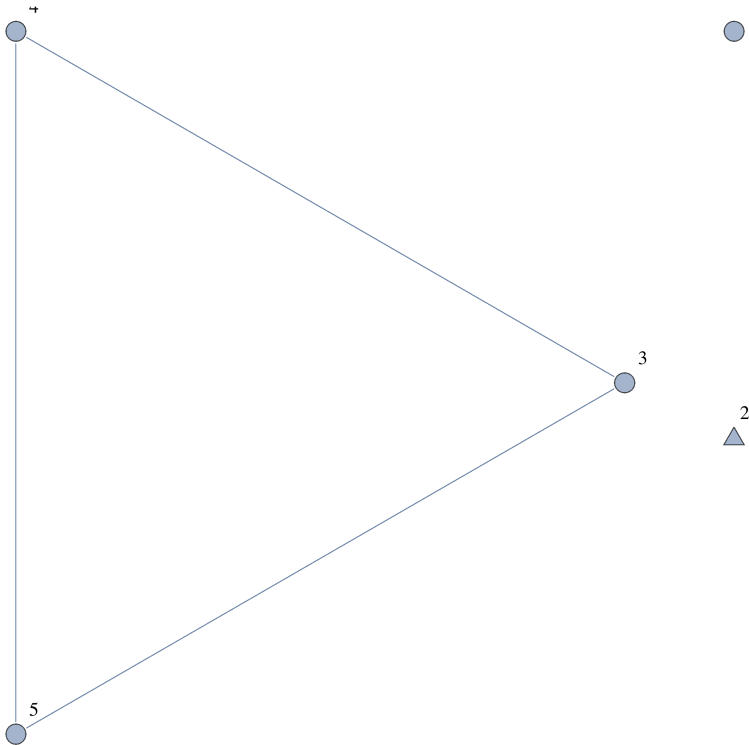

- **Player 1's Turn**

(+0+_30)

```
Graph[{1, 2, 3, 4, 5}, {2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
 VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(+0+_30)

```
Graph[{1, 2, 3, 4, 5}, {3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
 VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
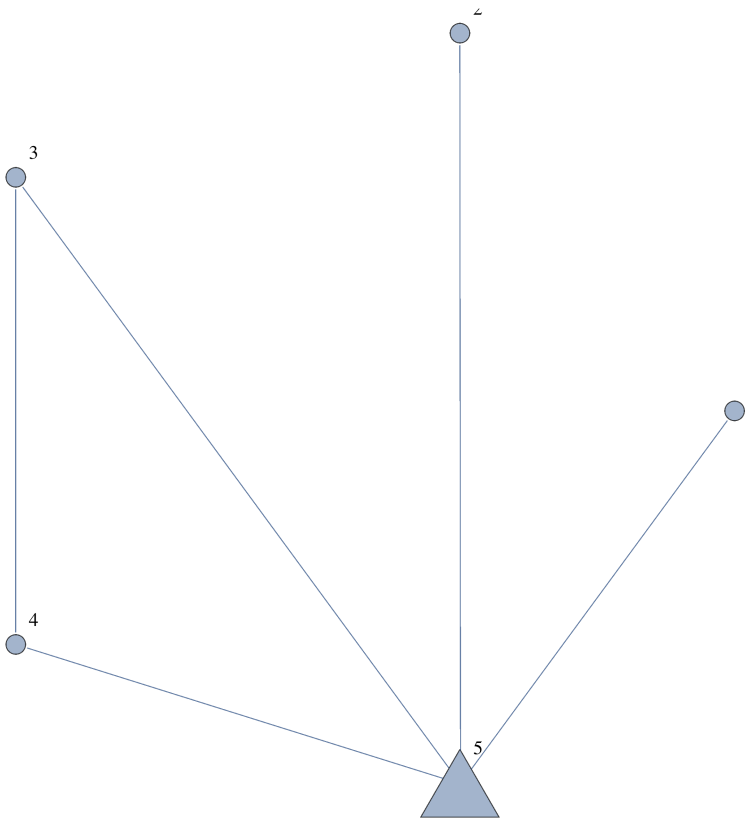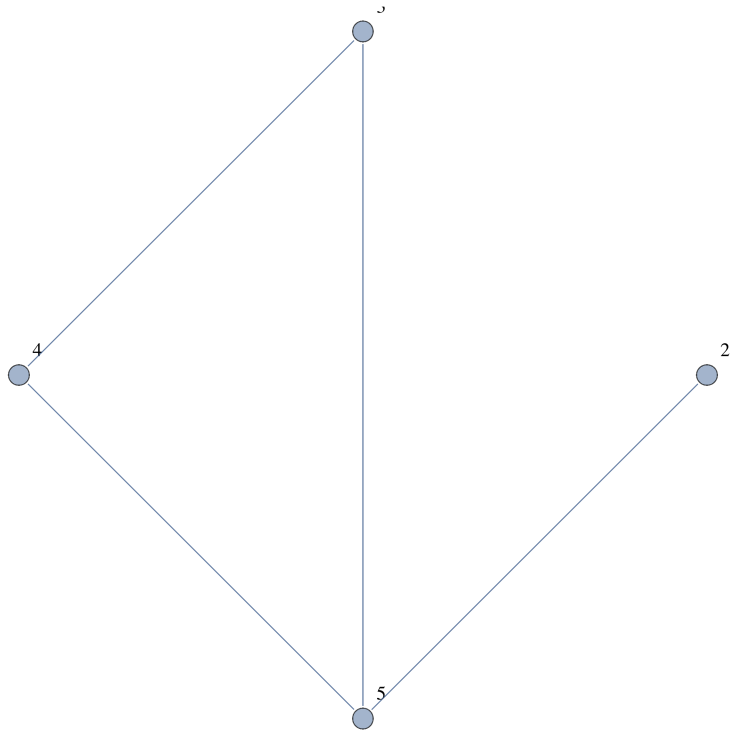

Player 2 lost!

- **Player 1's Turn**

(+0+_3+)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
 VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(+0+_3+)

```
Graph[{1, 2, 3, 4, 5}, {2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {1 -> "Triangle"},
 VertexSize → {1 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
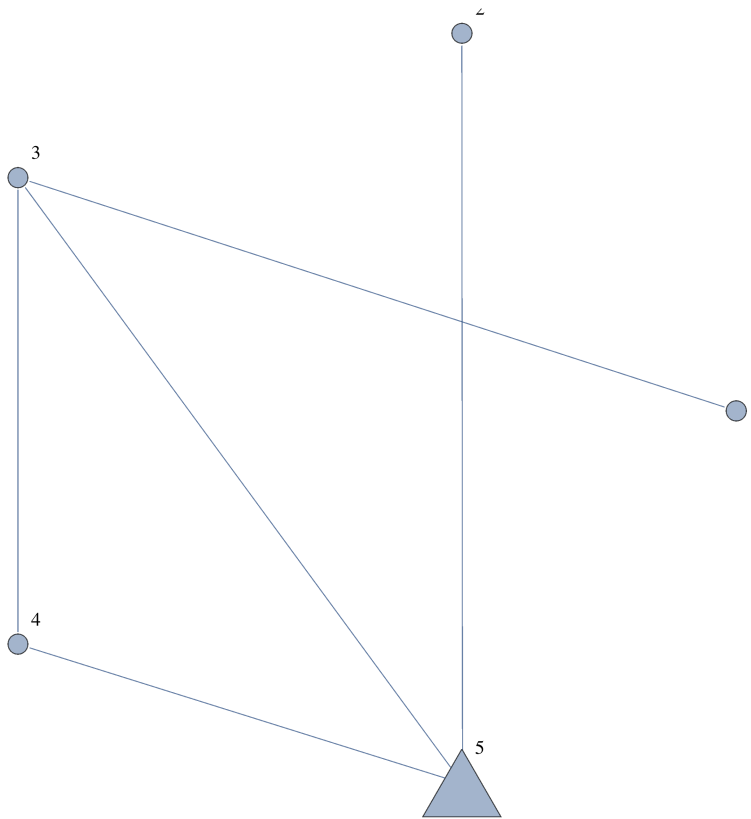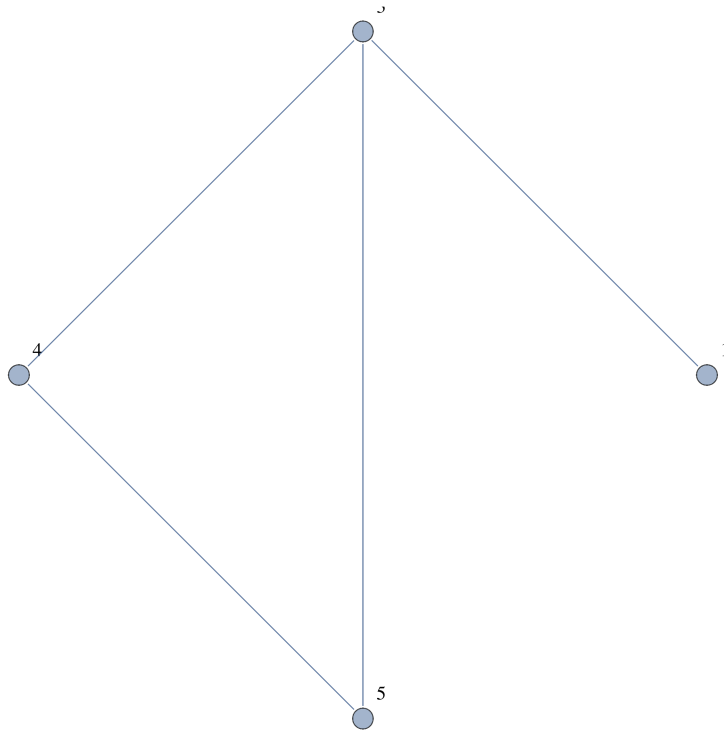

Player 2 lost!

- **Player 1's Turn**

(+00_5)

```
Graph[{1, 2, 3, 4, 5}, {1 ↔ 3, 2 ↔ 5, 3 ↔ 4, 3 ↔ 5, 4 ↔ 5},
 VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
 VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(+00_5)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
 VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
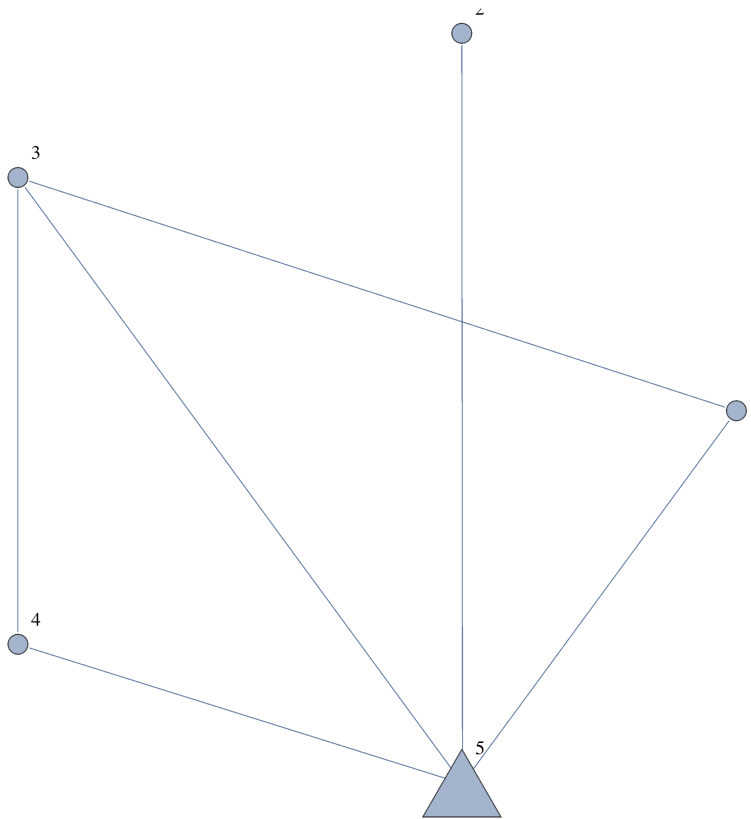

Player 2 lost!

- **Player 1's Turn**

(+0+_5)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
  VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


- **Player 2's Turn**

(+0+_5)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
 VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
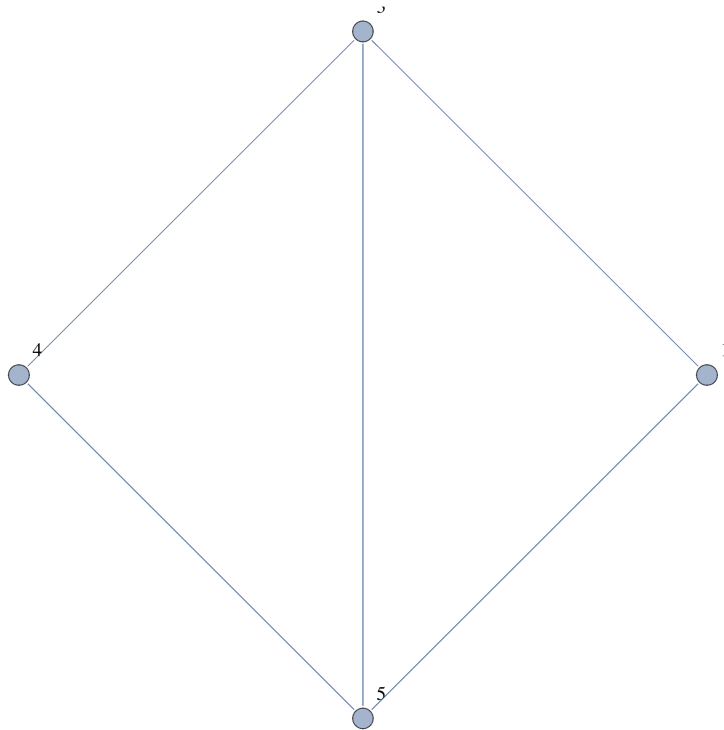

Player 2 lost!

- **Player 1's Turn**

(++)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 4, 1 ⟷ 5, 2 ⟷ 4, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {4 -> "Triangle"},
 VertexSize → {4 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
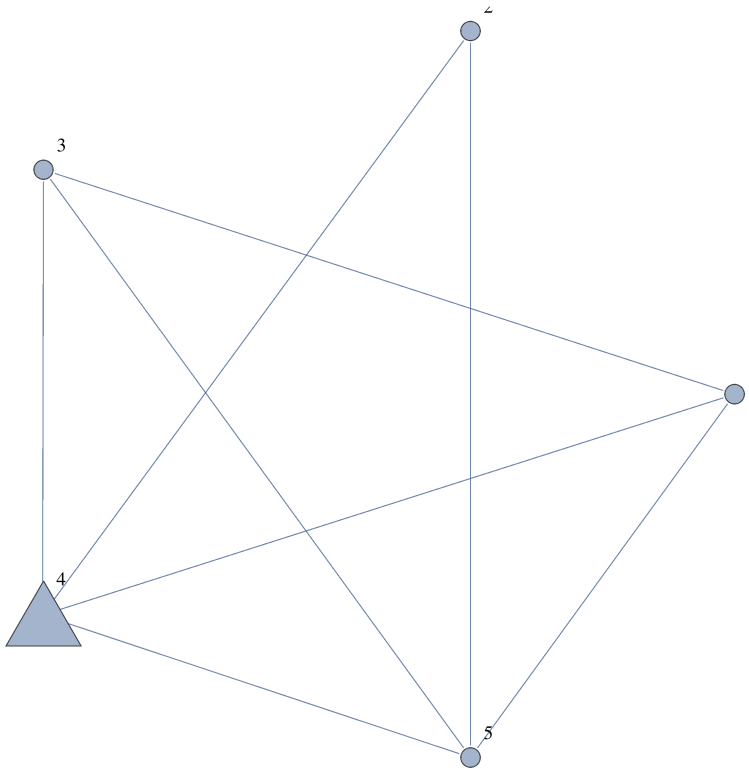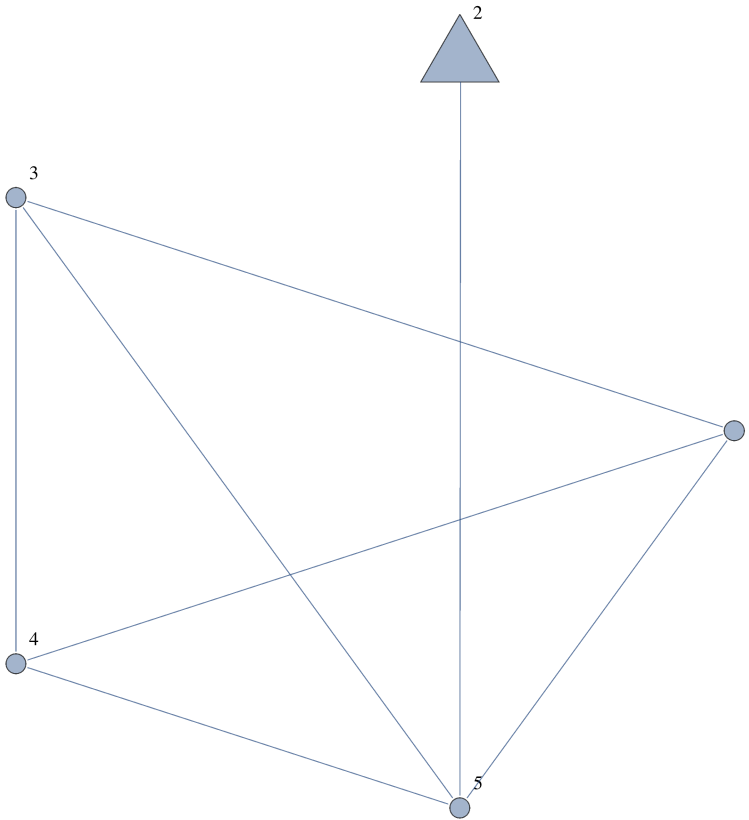

- **Player 2's Turn**

(++)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 4, 1 ⟷ 5, 2 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
 VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
 VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
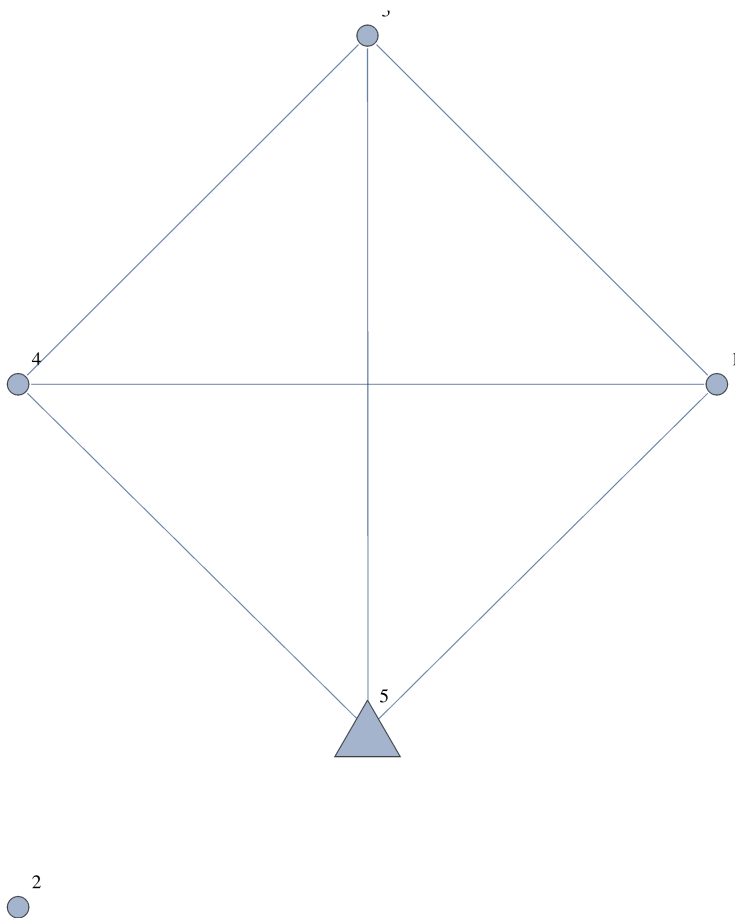

- **Player 1's Turn**

(++0)

```
Graph[{1, 2, 3, 4, 5}, {1 ↔ 3, 1 ↔ 4, 1 ↔ 5, 3 ↔ 4, 3 ↔ 5, 4 ↔ 5},
 VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
 VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```

What remains is the K_4. Since the first player to play a K_4 wins, Player 1 will win this game. Hence, Player 2 lost!

- **Player 1's Turn**

(+++)

```
Graph[{1, 2, 3, 4, 5}, {1 ↔ 3, 1 ↔ 4, 1 ↔ 5, 2 ↔ 5, 3 ↔ 4, 3 ↔ 5, 4 ↔ 5},
 VertexLabels -> "Name", VertexShapeFunction → {5 -> "Triangle"},
 VertexSize → {5 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```
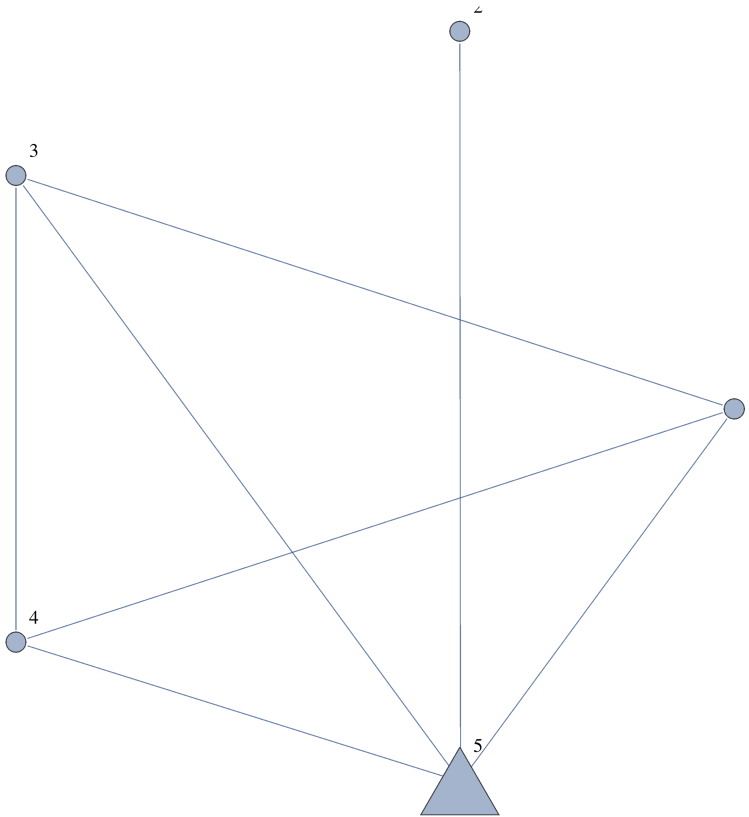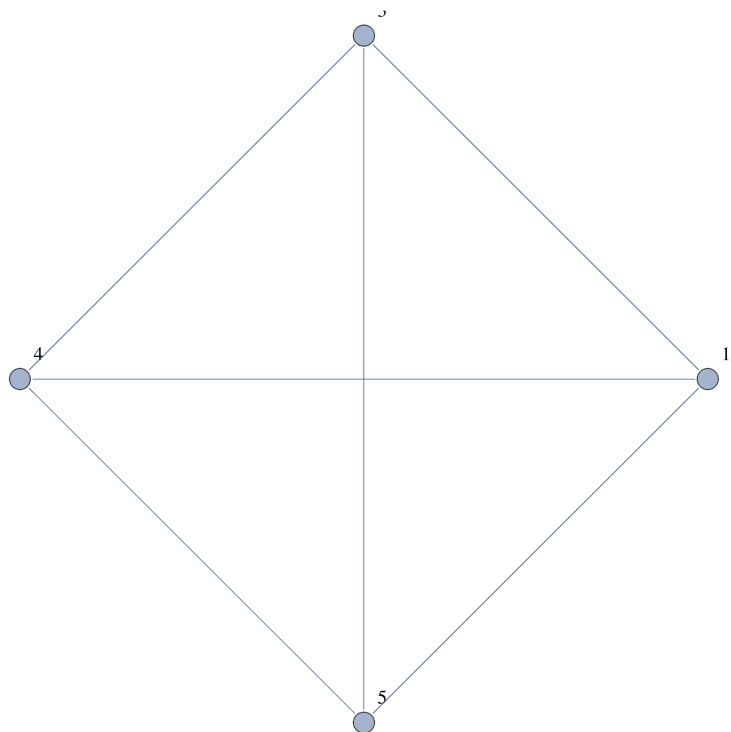

■ **Player 2's Turn**

(+++)

```
Graph[{1, 2, 3, 4, 5}, {1 ⟷ 3, 1 ⟷ 4, 1 ⟷ 5, 3 ⟷ 4, 3 ⟷ 5, 4 ⟷ 5},
  VertexLabels -> "Name", VertexShapeFunction → {2 -> "Triangle"},
  VertexSize → {2 -> 0.15}, GraphLayout -> "CircularEmbedding"]
```


Player 2 lost!