PROPOSED NONPARAMETRIC TESTS FOR THE SIMPLE TREE ALTERNATIVE FOR LOCATION AND SCALE TESTING IN A MIXED DESIGN

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By

Fouad Ali Khalawi

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SUPERVISORY COMMITTEE:

Dr. Rhonda Magel
Chair

Dr. Ronald Degges

Dr. Curt Doetkott

Approved:

04/15/2022 — Date

Dr. Rhonda Magel — Department Chair

ABSTRACT

Six nonparametric tests are proposed for the mixed design consisting of a randomized complete block design (RCBD) and a completely randomized design (CRD). The proposed tests are designed to test for differences in location and/or scale for a simple tree alternative. The tests are a combination of the Fligner-Wolfe test, modified Page's test, and modified Ansari-Bradley test. A simulation study is conducted to determine how well the proposed tests maintain their significance levels. Powers of the six proposed tests are estimated under a variety of cases: changing the underlying distribution, changing the number of treatments, increasing the variance between the CRD and RCBD portion, changing the proportions of the number of blocks in the RCBD to the sample size for each treatment in the CRD, and changing the parameter arrangements. A recommendation for which test has higher power depends on whether the underlying distribution is symmetric or asymmetric.

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DEDICATION

This dissertation is dedicated to my family.

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CHAPTER 1. INTRODUCTION

The situation in which the researchers want to compare control with several treatments has been used in many scientific fields, particularly in medical experiments. For example, some researchers are interested in comparing different treatments (therapies) with control (standard therapy) or placebo in clinical trials to determine whether at least one of the treatments is better than the control. The simple tree alternative test is the most appropriate hypothesis in such situations. The simple tree alternative is given by:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \quad (1.1)$$

$$H_a: \mu_1 \leq [\mu_2, \mu_3, \ldots \mu_k] \text{ (at least one strict inequality)}$$

where μ_i is the location parameter of the i^{th} population with $i = 1,2, \ldots, k$ and k is the total number of populations. Population one (i=1) indicates the control population, while populations 2 through k are the treatment populations.

The simple tree alternative is used to test the null hypothesis of no differences among the treatments and the control against a one-sided alternative that at least one of the treatments is different (greater) than the control using a single distribution-free test. That is different than multiple comparison procedures; these multiple comparisons procedures make k individual inferential statements comparing each treatment to the control using experimentwise error rate for the family of statements.

Recently, many test statistics have been developed for testing the difference in location parameters when the data are a mixture of a randomized complete block design (RCBD) and a completely randomized design (CRD). This mixed design occurs when researchers start with a randomized complete block design, but they might have problems because they cannot get enough homogenous experimental units or there is some reason beyond their control such as subjects dropping out or moving away. Since researchers do not want to discard data, they end up evaluating the leftover observations as a CRD. In this case, this design will be known as a mixed design of RCBD and CRD. Dubnicka, Blair, and Hettmansperger (2002) proposed a test statistic for a mixed design consisting of paired and independent two-sample data. Dubnicka et al. (2002) combined the Wilcoxon-signed rank test statistic for paired samples and the Mann-Whitney test statistic for two independent samples. Dubnicka et al. (2002) research was extended to other cases; see Magel et al. (2010); Magel and Ndungu (2013); Olet and Magel (2017); Alsuhabi and Magel (2020).

As an example where the mixed design of RCBD and CRD would occur, researchers would like to examine five different drugs (treatments) on subjects (experiment units). Researchers may want to block on countries because they feel subjects from various countries may be different. First, researchers need five subjects from each country and use RCBD blocking on country. Then, within each country, five subjects are randomly assigned to different drugs. However, before the experiment was completed, some subjects dropped out and there was no time to collect new subjects. Thus, researchers decided to shift to CRD, where subjects are assigned randomly to the different drugs regardless of their nationality. The data from this design will be called a mixed design of RCBD and CRD. Table 1.1 and Table 1.2 illustrate the mixed design of RCBD and CRD.

Table 1.1. Randomized complete block design.

Counties	Drugs				
	1	2	3	4	5
1	S1	S2	S3	S4	S5
2	S1	S2	S3	S4	S5
3	S1	S2	S3	S4	S5
4	S1	S2	S3	S4	S5
5	S1	S2	S3	S4	S5

“S” indicates the subjects

Table 1.2. Completely randomized design.

Drugs				
1	2	3	4	5
S1	S2	S3	S4	S5
S6	S7	S8	S9	S10
S11	S12	S13	S14	S15
S16	S17	S18	S19	S20

"S" indicates the subjects

In some cases, researchers want to make inferences about the difference in the means (location) and variances (scale) of populations simultaneously. Lepage (1971) initiated the most commonly used nonparametric test for the location-scale problem by combining the Wilcoxon rank-sum and Ansari-Bradley's test statistics. Duran et al. (1976) developed a test based on Lepage's test but using Mood's test for the scale parameter. Other researchers have proposed a similar type of Lapage test, for example, Murakami (2007); Neuhauser, Leuchs, and Ball (2011); Marozzi (2013); Mukherjee and Marozzi (2018).

This research presents new nonparametric tests for a simple tree alternative for location and scale testing in a mixed design of RCBD and CRD. The null and alternative hypotheses for this research are given below:

$$\mathrm{H_0}: \mu_1 = \mu_2 = \cdots = \mu_k \quad , and \ \ \mathrm{H_0}: \sigma_1 = \sigma_2 = \cdots = \sigma_k \qquad (1.2)$$

$\mathrm{H_a}: \mu_1 \leq [\mu_2, \mu_3, \dots \mu_k]$ $\ and/or$ $\mathrm{H_a}: \sigma_1 \leq [\sigma_2, \dots, \sigma_k]$ with at least one strict inequality

where μ_i is the location parameter of population i and σ_i is the scale parameter of population i with $i = 1,2, \dots, k$ and k is the total number of populations. Population one $i = 1$ is indicated to the control population, while populations 2 through k are the treatment populations. We propose new nonparametric tests of the null hypothesis of no differences among the treatments and the control against a one-sided alternative that at least one of the treatments is different (yields a greater response) either on means or variance or both than the control.

The following chapters of this dissertation are organized in the following manner: Chapter 2 introduces a literature review of the previous studies; Chapter 3 introduces the proposed tests and an illustrative example; Chapter 4 is devoted to the simulation study for this research; Chapter 5 presents the simulation study results described in Chapter 4; Chapter 6 discusses the conclusion and future work of this research.

CHAPTER 2. LITERATURE REVIEW

2.1. Nonparametric Tests for Location Problem

2.1.1. Mann-Whitney

The Mann-Whitney test is a nonparametric test for testing the differences in location parameters for two independent samples (Mann and Whitney, 1947). The null and alternative hypotheses are given below:

$$H_0: \mu_1 = \mu_2 \tag{2.1}$$

$$H_{a1}: \mu_1 \neq \mu_2, H_{a2}: \mu_1 > \mu_2, H_{a3}: \mu_1 < \mu_2$$

where μ_1 is the location parameter of population one, and μ_2 is the location parameter of population two.

To compute the Mann-Whitney test, compare each observation in the first sample with each observation in the second sample. If the observation from the first sample is less than the observation in the second sample, assign 1; otherwise, 0. Add up the 0's and 1's and call the sum U.

$$U = \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} \phi(X_i, Y_j), \tag{2.2}$$

where

$$\phi(X_i, Y_j) = \begin{cases} 1, & \text{if } X_i < Y_j, \\ 0, & \text{otherwise.} \end{cases}$$

Mann-Whitney showed that the Mann-Whitney test (U) and the Wilcoxon rank-sum test (W) (Wilcoxon 1945) are equivalent. To compute the Wilcoxon rank-sum test, arrange all observations from smallest to largest, then assign a rank. The Wilcoxon rank-sum test is the sum of the ranks assigned to the observations from the first sample. That is,

$$W = \sum_{j=1}^{n_1} R_j \tag{2.3}$$

where R_j is the ranks assigned to the first sample, while n_1 and n_2 are the numbers of observations in the first sample and second sample, respectively.

The Mann-Whitney test (U) can be rewritten as follows:

$$U = W - \frac{n_1(n_1+1)}{2} \tag{2.4}$$

For large-sample approximation, when either the first sample size or the second sample is greater than twenty, the central limit theorem is applied. Therefore, the standardized version of the Mann-Whitney test is:

$$Z_U = \frac{U - \frac{n_1 n_2}{2}}{\sqrt{\frac{n_1 n_2 (n_1 + n_2 + 1)}{12}}} \tag{2.5}$$

Under the null hypothesis, (Z_U) has an asymptotically standard normal distribution. We reject the null hypothesis as follows:

- When H_{a1}: $\mu_1 \neq \mu_2$, the null hypothesis is rejected if $|Z_U| \geq Z_{\alpha/2}$.
- When H_{a2}: $\mu_1 > \mu_2$, the null hypothesis is rejected if $Z_U \geq Z_\alpha$.
- When H_{a3}: $\mu_1 < \mu_2$, the null hypothesis is rejected if $Z_U \leq -Z_\alpha$.

2.1.2. Kruskal-Wallis

The Kruskal-Wallis test is a nonparametric test for the general alternative for testing the differences in location parameters when more than two samples are tested in the CRD (Kruskal and Wallis 1953). The null and alternative hypotheses are given below:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k \tag{2.6}$$

$$H_a: \text{at least one of the two } \mu_i's \text{ are different}$$

where μ_i is the location parameter of the i^{th} population and k is the number of treatments.

To compute the Kruskal-Wallis test (KW), arrange all observations from smallest to largest, then assign a rank to each observation. Let r_{ij} be the rank of the observation x_{ij} and n_i be the number of observations in each treatment, and k be the number of treatments. The Kruskal-Wallis test (KW) is:

$$KW = \frac{12}{N(N+1)} \sum_{i=1}^{k} \frac{R_i^2}{n_i} - 3(N+1) \quad (2.7)$$

where $R_i = \sum_{i=1}^{n_i} r_{ij}$ and $N = n_1 + n_2 + \cdots + n_k, and\ i = 1,2, \ldots K, j = 1,2, \ldots n_i$.

For large n_i, the Kruskal-Wallis test (KW) has an asymptotic Chi-square distribution with k-1 degrees of freedom. Hence, the null hypothesis is rejected when the computed value of (KW) exceeds the upper value of Chi-square with k-1 degrees of freedom.

2.1.3. Jonckheere-Terpstra

The Jonckheere-Terpstra test is a nonparametric test for the nondecreasing ordered alternative in the CRD for location testing (Jonckheere 1954 and Terpstra 1952). The null and alternative hypotheses are given below:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k \quad (2.8)$$

$$H_a: \mu_1 \leq \mu_2 \leq \mu_3 \ldots \leq \mu_k \quad \text{with at least one strict inequality}$$

where μ_i is the location parameter of the i^{th} population and k is the number of treatments. The test statistic for Jonckheere-Terpstra is based on the summation of $k(k-1)/2$ Mann-Whitney statistics. Namely, it can be expressed as:

$$J = \sum_{i<j} U_{ij} \qquad i = 1,2, \ldots, k-1 \ , j = 1,2, \ldots, k \quad (2.9)$$

where U_{ij} counts the number of ordered pairs (a,b) where a is from sample i and b is from sample j in which a less than b.

Under H_0, the expected value and variance of the Jonckheere-Terpstra test J are, respectively,

$$E(J) = \frac{N^2 - \sum_{i=1}^{k} n_i^2}{4} \text{ and } Var(J) = \frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)}{72} \tag{2.10}$$

The standardized version of the Jonckheere-Terpstra test is:

$$Z_J = \frac{J - E(J)}{\sqrt{Var(J)}} = \frac{J - \frac{N^2 - \sum_{i=1}^{k} n_i^2}{4}}{\sqrt{\frac{N^2(2N+3) - \sum_{i=1}^{k} n_i^2(2n_i+3)}{72}}} \tag{2.11}$$

Under H_0, the standardized version of the Jonckheere-Terpstra test (Z_J) has an asymptotic standard normal distribution. The null hypothesis is rejected if $Z_J \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

2.1.4. Fligner-Wolfe

The Fligner-Wolfe test is a nonparametric test for testing the differences among the control group and other treatment groups (Fligner and Wolfe, 1982). The null and alternative hypotheses are given below:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \tag{2.12}$$

$$H_a: \mu_1 \leq [\mu_2, \dots, \mu_k], \text{with at least one strict inequality}$$

where μ_i is the location parameter of the i^{th} population with $i = 1,2, \dots, k$ and k is the total number of populations. Population one (i=1) indicates the control population, while populations 2 through k are the treatment populations.

To compute the Fligner-Wolfe test statistic (T_{L1}), arrange all observations from smallest to largest, then assign a rank to each observation. Sum the ranks that belong to the combined treatment sample. Let r_{ij} be the rank of the observation x_{ij}, where $i = 1,2,\dots,K, j = 1,2,\dots,n_i$, and n_i be the number of observations in each treatment, and k be the number of treatments. The Fligner-Wolfe test statistic is:

$$T_{L1} = \sum_{i=2}^{k}\sum_{j=1}^{n_i} r_{ij}. \tag{2.13}$$

The Fligner-Wolfe test also can be written in the form of Mann-Whitney U statistics. The expected value and variance of the Fligner-Wolfe test (T_{L1}) when the null hypothesis is true are, respectively,

$$E(T_{L1}) = \frac{n_t(N+1)}{2} \text{ and } Var(T_{L1}) = \frac{n_c n_t(N+1)}{12} \tag{2.14}$$

where n_c and n_t are the numbers of observations in the control sample and combined treatment sample respectively and $N = n_c + n_t$.

The standardized version of the Fligner-Wolfe test is:

$$Z_{L1} = \frac{T_{L1} - E(T_{L1})}{\sqrt{Var(T_{L1})}} \tag{2.15}$$

Under H_0, the standardized version of the Fligner-Wolfe test (Z_{L1}) has an asymptotic standard normal distribution. The null hypothesis is rejected when (Z_{L1}) exceeds Z_α at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

2.1.5. Page

Page's test is a nonparametric test for testing the nondecreasing ordered alternative in a randomized complete block design (Page, 1963). The null and alternative hypotheses are given below:

$$H_0: \mu_1 = \mu_2 = \mu_3 = \cdots = \mu_k \tag{2.16}$$

$$H_a: \mu_1 \le \mu_2 \le \mu_3 \dots \le \mu_k \quad \text{with at least one strict inequality}$$

where μ_i is the location parameter of the i^{th} population and k is the number of treatments.

To compute Page test statistic (L), arrange observations within each block from smallest to largest, then assign a rank. Let R_j be the sum of the ranks for j^{th} treatment. The Page test statistic (L) is:

$$L = \sum_{j=1}^{k} jR_j = R_1 + 2R_j + \cdots + kR_j. \tag{2.17}$$

Under H_0, the expected value and variance of Page test statistic (L) are, respectively,

$$E(L) = \frac{n(k+1)^2}{4} \text{ and } Var(L) = \frac{nk^2(k+1)(k^2-1)}{144} \tag{2.18}$$

The standardized version of the Page test is:

$$Z_L = \frac{L - E(L)}{\sqrt{Var(L)}} \tag{2.19}$$

where n is the number of blocks and k is the number of treatments.

Under H_0, the standardized version of the Page test (Z_L) has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_L \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

2.1.6. Modified Page's Test

The modified Page's test was designed for a simple tree alternative in a randomized complete block design for location problems by Olet and Magel (2017). The null and alternative hypotheses for modified Page's test for simple tree alternative are given below:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \tag{2.20}$$

$$H_a: \mu_1 \leq [\mu_2, \mu_3, \ldots \mu_k] \text{ with at least one strict inequality}$$

where μ_i is the location parameter of the i^{th} population with $i = 1,2, \dots, k$ and k is the total number of populations. Population one, $i = 1$, indicates the control population, while populations 2 through k are the treatment populations.

To compute the modified Page's test (T_{L2}), within each block, observations are arranged from least to greatest. Let R_j be the sum of the ranks for j^{th} treatment and R_1 indicates the sum of the ranks in the control sample. The modified Page's test is given below:

$$T_{L2} = R_1 + 2\sum_{j=2}^{k} R_j = R_1 + 2[R_2 + R_3 + \cdots R_k] \tag{2.21}$$

Under H_0, the expected value and variance of the modified Page's test (T_{L2}) are, respectively,

$$E(T_{L2}) = n_b E[L_1] = n_b\left(k^2 + \frac{k-1}{2}\right) \tag{2.22}$$

and

$$Var(T_{L2}) = n_b Var(L_1) = n_b\left(\frac{k^2-1}{12}\right)$$

where (L_1) is modified Page's test for one block with k treatment and n_b is the number of blocks. The standardized version of the modified Page's test is:

$$Z_{L2} = \frac{T_{L2} - E(T_{L2})}{\sqrt{Var(T_{L2})}} \tag{2.23}$$

Under H_0, the standardized version of the modified Page's test (Z_{L2}) has an asymptotic standard normal distribution. The null hypothesis is rejected only when $Z_{L2} \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

2.2. Nonparametric Tests for Scale Problem

2.2.1. Ansari-Bradley Test

The Ansari-Bradley test is a two-sample nonparametric test for testing the difference in scale parameters (Ansari and Bradley, 1960). The null and alternative hypotheses for the Ansari-Bradley test are:

$$H_0: \sigma_1 = \sigma_2 \tag{2.24}$$

$$H_{a1}: \sigma_1 \neq \sigma_2, H_{a2}: \sigma_1 > \sigma_2, H_{a3}: \sigma_1 < \sigma_2$$

where σ_1 is the scale parameter of the population one, and σ_2 is the scale parameter of the population two.

To obtain the Ansari-Bradley test statistic (AB), arrange all observations from smallest to largest. Assign a rank of 1 to the smallest and largest observations, assign a rank of 2 to the second smallest and second-largest observation, and so on. Let R_i be the rank of i^{th} observation in the first sample, The Ansari-Bradley test statistic is the sum of the ranks of the observation in the first sample and is given by:

$$AB = \sum_{i=1}^{n_1} R_i \tag{2.25}$$

When the null hypothesis is true, the expected value and variance of AB are, respectively,

If $N = n_1 + n_2$ is an even number:

$$E(AB) = \frac{n_1(N+2)}{4} \tag{2.26}$$

and

$$Var(AB) = \frac{n_1 n_2 (N+2)(N-2)}{48(N-1)}$$

If $N = n_1 + n_2$ is an odd number:

$$E(AB) = \frac{n_1(N+1)^2}{4N} \tag{2.27}$$

and

$$Var(AB) = \frac{n_1 n_2 (N+1)(3+N^2)}{48N^2}$$

where n_1 and n_2 are the numbers of observations in the first sample and second sample respectively and $N = n_1 + n_2$. The standardized version of the Ansari-Bradley test is:

$$Z_{AB} = \frac{AB - E(AB)}{\sqrt{Var(AB)}} \tag{2.28}$$

Under H_0, The standardized version of the Ansari-Bradley test (Z_{AB}) has an asymptotic standard normal distribution. We compare (Z_{AB}) for significance with a standard normal distribution. As following;

- When $H_{a1}: \sigma_1 \neq \sigma_2$, the null hypothesis is rejected if $|Z_{AB}| \geq Z_{\alpha/2}$.
- When $H_{a2}: \sigma_1 > \sigma_2$, the null hypothesis is rejected if $Z_{AB} \geq Z_\alpha$.
- When $H_{a3}: \sigma_1 < \sigma_2$, the null hypothesis is rejected if $Z_{AB} \leq -Z_\alpha$.

2.2.2. Modified Ansari-Bradley Test for CRD

Alsubie and Magel (2020) introduced a modified version of the AB test for CRD for the simple tree alternative. The null and alternative hypotheses for the modified Ansari-Bradley test for simple tree alternative are given below:

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k \tag{2.29}$$

$$H_a: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k] \text{ with at least one strict inequality}$$

where σ_i is the scale parameter of the i^{th} population with $i = 1,2, \ldots, k$ and k is the total number of populations. Population one, $i = 1$, indicates the control population, while populations 2 through k are the treatment populations.

Let n_c refer to the number of observations from the control population and n_t indicates the number of observations from the combined treatment populations where $N = n_c + n_t$. To calculate the modified AB test, arrange all observations in order from smallest to largest. Then, assign a rank of 1 to the smallest and largest observations, and assign a rank of 2 to the second smallest and second-largest observation, and so on. Let R_i be the rank of the i^{th} observation in the control sample, the modified Ansari-Bradley test for the simple tree alternative in CRD is the sum of the ranks in the control sample and given by:

$$T_{S1} = \sum_{i=1}^{n_c} R_i \tag{2.30}$$

When the null hypothesis is true, the expected value and variance of (T_{S1}) are, respectively,

If $N = n_c + n_t$ is an even number:

$$E(T_{S1}) = \frac{n_c\,(N+2)}{4} \tag{2.31}$$

and

$$Var(T_{S1}) = \frac{n_c\,n_t(N+2)(N-2)}{48(N-1)}$$

If $N = n_c + n_t$ is an odd number:

$$E(T_{S1}) = \frac{n_c\,(N+1)^2}{4N} \tag{2.32}$$

and

$$Var(T_{S1}) = \frac{n_c n_t(N+1)(3+N^2)}{48N^2}$$

The standardized version of modified Ansari-Bradley for CRD is:

$$Z_{S1} = \frac{T_{S1} - E(T_{S1})}{\sqrt{Var(T_{S1})}} \tag{2.33}$$

Under H_0, the standardized version of modified Ansari-Bradley for CRD (Z_{S1}) has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_{S1} \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

2.3. Nonparametric Tests for Either Location or Scale Problem

2.3.1. Lepage's Test

Lepage's test is a two-sample nonparametric test for testing whether there are differences in either the location parameters or the scale parameters (Lepage, 1971). The null and alternative hypotheses for Lepage's test are:

$$H_0: \mu_1 = \mu_2 \text{ and } \sigma_1 = \sigma_2 \tag{2.34}$$

$$H_a: \mu_1 \neq \mu_2 \text{ and / or } \sigma_1 \neq \sigma_2$$

To compute Lepage's test, compute the Wilcoxon rank-sum test (W) is given in Equation (2.3) and the Ansari-Bradley test (AB) is given in Equation (2.25). The expected value and variance for both Wilcoxon rank-sum test (W) and the Ansari-Bradley test are obtained. The Lepage's test is then defined by:

$$D = \frac{[W - E(W)]^2}{Var(W)} + \frac{[AB - E(AB)]^2}{Var(AB)} = (Z_W)^2 + (Z_{AB})^2 \tag{2.35}$$

Under H_0, and as $\min(n_1, n_2) \rightarrow \infty$, Lepage test statistic (D) has a Chi-square distribution with two degrees of freedom. The null hypothesis H_0 is rejected if Lepage's test statistic (D) exceeds the upper α value of the Chi-square distribution with 2 degrees of freedom $\chi^2_{\alpha,2}$.

2.3.2. Alsubie and Magel

Alsubie and Magel developed two nonparametric tests for the simple tree alternative for location and scale testing in the CRD (Alsubie and Magel, 2020). The null and alternative hypotheses are given below:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \quad \text{,and } H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k \tag{2.36}$$

$$H_a: \mu_1 \leq [\mu_2, \mu_3, \ldots \mu_k] \quad \text{and/or } H_a: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k] \text{ with at least one strict inequality}$$

where μ_i is the location parameter of the i^{th} population and σ_i is the scale parameter of the i^{th} population.

The two tests are a combination of the Fligner-Wolfe test for detecting location changes and the modified Ansari-Bradley for detecting scale changes. The first proposed test (A_1) is the sum of the standardized test statistic for Fligner-Wolfe test statistic (Z_{L1}) is given in Equation (2.15) and standardized test statistics for modified Ansari-Bradley test statistics (Z_{S1}) is given in Equation (2.33) and then re-standardizing this test statistic.

$$A_1 = \frac{Z_{L1} + Z_{S1}}{\sqrt{2}} \tag{2.37}$$

The second proposed test (A_2) is the sum of the Fligner-Wolfe test statistic (T_{L1}) is given in Equation (2.13) and modified Ansari-Bradley test statistic (T_{S1}) is given in Equation (2.30) and then standardizing this test statistic.

$$A_2 = \frac{(T_{L1} + T_{S1}) - \left(E(T_{L1}) + E(T_{S1})\right)}{\sqrt{var(T_{L1}) + var(T_{S1})}} \tag{2.38}$$

where $E(T_{L1})$ and $Var(T_{L1})$ are the expected value and variance of the Fligner Wolfe test (T_{L1}) and $E(T_{S1})$ and $Var(T_{S1})$ are the expected value and variance of the modified Ansari-Bradley test (T_{S1}).

Under H_0, both A_1 and A_2 have asymptotic standard normal distributions. The null hypothesis H_0 is rejected when the value of the standardizing test is greater than Z_α at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

The simulation results (Alsubie and Magel, 2020) show that the second proposed test (A2) has the highest powers when the change is only in location parameters. The first proposed test (A1) has the highest powers when the change is only in scale parameters. When the location and scale parameters are different, the test statistic with higher powers changes depending on the underlying distribution.

2.4. Nonparametric Tests for Mixed Design Problems

2.4.1. Dubnicka, Blair and Hettmansperger

Dubnicka, Blair, and Hettmansperger developed a nonparametric test for testing the difference in means when data are a mixture of paired observations and independent (Dubnicka, Blair, and Hettmansperger, 2002). The null and alternative hypotheses are:

$$H_0: \Delta = 0 \tag{2.39}$$

$$H_a: \Delta > 0$$

where $\Delta = \mu_1 - \mu_2$the difference between first and second population means.

The proposed test statistic, which sums the Wilcoxon Signed-Rank Statistic (S) by Wilcoxon (1945) for paired data and the Mann-Whitney Statistic (U) for unpaired data in Equation (2.2), is given in Equation (2.40).

$$T = S + U \tag{2.40}$$

The expected value and variance of T are obtained using the expected value and variance for both the Wilcoxon Signed-Rank Test and Mann-Whitney Test. Under H_0,

$$E(T) = E(S) + E(U) = \frac{n(n+1)}{4} + \frac{n_1 n_2}{2} \quad (2.41)$$

and

$$Var(T) = Var(S) + Var(U) = \frac{n(n+1)(2n+1)}{24} + \frac{n_1 n_2 (n_1 + n_2 + 1)}{12}$$

where n is the number of paired observations and n_1, n_2 are the number of observations in the first sample and second sample in the independent samples, respectively. The standardized version of the Dubnicka, Blair, and Hettmansperger test is:

$$Z_T = \frac{T - E(T)}{\sqrt{Var(T)}} \quad (2.42)$$

Under the null hypothesis and as the total sample size go to infinity, (Z_T) has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_T \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

2.4.2. Olet and Magel

Olet and Magel proposed and compared six nonparametric tests for the simple tree alternative for the RCBD and CRD mixed design for location testing (Olet and Magel, 2017). The null and alternative hypotheses are given below:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \quad (2.43)$$

$$H_a: \mu_1 \leq [\mu_2, \mu_3, \ldots \mu_k] \text{ with at least one strict inequality}$$

where μ_i is the location parameter of the i^{th} population with $i = 1,2,\ldots,k$ and k is the total number of populations. Population one (i=1) indicates the control population, while populations 2 through k are the treatment populations.

The six tests are a combination of the Fligner-Wolfe test and the modified Page's test. The first proposed test (Z_1) is the sum of Fligner-Wolfe test statistic (T_{L1}) given in Equation (2.13) and modified Page's test statistics ($\mathrm{T_{L2}}$) given in Equation (2.21) and then standardizing this test statistic.

$$\mathrm{Z_I} = \frac{(\mathrm{T_{L1}} + \mathrm{T_{L2}}) - (E(\mathrm{T_{L1}}) + E(\mathrm{T_{L2}}))}{\sqrt{(\mathrm{Var}(\mathrm{T_{L1}}) + \mathrm{Var}(\mathrm{T_{L2}}))}} \tag{2.44}$$

The second proposed test ($\mathrm{Z_{II}}$) is the sum of standardized Fligner-Wolfe test statistic ($\mathrm{Z_{L1}}$) given in Equation (2.15) and standardized modified Page's test statistic $\mathrm{Z_{L2}}$ given in Equation (2.23) and then re-standardizing this test statistic.

$$\mathrm{Z_{II}} = \frac{\mathrm{Z_{L1}} + \mathrm{Z_{L2}}}{\sqrt{2}} \tag{2.45}$$

The third proposed test ($\mathrm{Z_{III}}$) is similar to the first proposed test except that it differs by the weights used.

$$\mathrm{Z_{III}} = \frac{\left(\frac{n_a}{n}\mathrm{T_{L1}} + \frac{n_b}{n}\mathrm{T_{L2}}\right) - \left(\frac{n_a}{n}E(\mathrm{T_{L1}}) + \frac{n_b}{n}\mathrm{E}(\mathrm{T_{L2}})\right)}{\sqrt{\left(\frac{n_a^2}{n}\mathrm{Var}(\mathrm{T_{L1}}) + \frac{n_b^2}{n}\mathrm{Var}(\mathrm{T_{L2}})\right)}} \tag{2.46}$$

where $\mathrm{n_a}$ is the sample size for each treatment under the CRD portion, $\mathrm{n_b}$ is the number of blocks in the RCBD portion, and $\mathrm{n} = \mathrm{n_a} + \mathrm{n_b}$.

The fourth proposed test is (Z_{IV}) is similar to the first proposed test except that it differs by the weights used.

$$\mathrm{Z_{IV}} = \frac{\left(\frac{n_b}{n}\mathrm{T_{L1}} + \frac{n_a}{n}\mathrm{T_{L2}}\right) - \left(\frac{n_b}{n}E(\mathrm{T_{L1}}) + \frac{n_a}{n}\mathrm{E}(\mathrm{T_{L2}})\right)}{\sqrt{\left(\frac{n_b^2}{n}\mathrm{Var}(\mathrm{T_{L1}}) + \frac{n_a^2}{n}\mathrm{Var}(\mathrm{T_{L2}})\right)}} \tag{2.47}$$

where $\mathrm{n_a}$ is the sample size for each treatment under the CRD portion, $\mathrm{n_b}$ is the number of blocks in the RCBD portion, and $\mathrm{n} = \mathrm{n_a} + \mathrm{n_b}$.

The fifth proposed test (Z_V) is similar to the second proposed test except that it differs by the weights used.

$$Z_V = \frac{\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2}}{\sqrt{\operatorname{Var}\left(\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2}\right)}} = \frac{\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2}}{\sqrt{\left(\frac{n_b^2 + n_a^2}{n^2}\right)}} \qquad (2.48)$$

where n_a is the sample size for each treatment under the CRD portion, n_b is the number of blocks in the RCBD portion, and $n = n_a + n_b$.

The sixth proposed test (Z_{VI}) is similar to the second proposed test except that it differs by the weights used.

$$Z_{VI} = \frac{\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2}}{\sqrt{\operatorname{Var}\left(\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2}\right)}} = \frac{\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2}}{\sqrt{\left(\frac{n_a^2 + n_b^2}{n^2}\right)}} \qquad (2.49)$$

where n_a is the sample size for each treatment under the CRD portion, n_b is the number of blocks in the RCBD portion, and $n = n_a + n_b$.

These tests have asymptotic standard normal distributions and the null hypothesis H_0 is rejected when the value of the test statistics are greater than Z_α. The simulation results indicate that the second and sixth proposed tests have the highest powers overall.

CHAPTER 3. PROPOSED TESTS

This chapter introduces nonparametric tests for the simple tree alternative for location and/or scale testing in a mixed design. The mixed design being considered is a combination of the randomized complete block design (RCBD) and the completely randomized design (CRD). The null and alternative hypotheses for these proposed tests are given in Chapter One, Equation (1.2).

The proposed tests are a combination of the Fligner-Wolfe test, modified Page's test, and modified Ansari-Bradley test. We consider the Fligner-Wolfe test to detect location change in CRD, the modified Page's test to detect location change in RCBD, and the modified Ansari-Bradley test for CRD to detect scale change in CRD. We will propose a new version of the modified Ansari-Bradley test for a simple tree alternative for RCBD to detect scale changes in the RCBD portion (See Figure 3.1).

We combined tests to detect the location and scale parameters changes because when only scale parameters change and we use only location tests, which are the Fligner-Wolfe test and modified Page's test, to detect the scale change, the rejection percentage (power) will be low. Also, the rejection percentage will be low if we only use scale tests, which are the modified Ansari-Bradley test for CRD and RCBD, to detect the location change when only location parameters change.

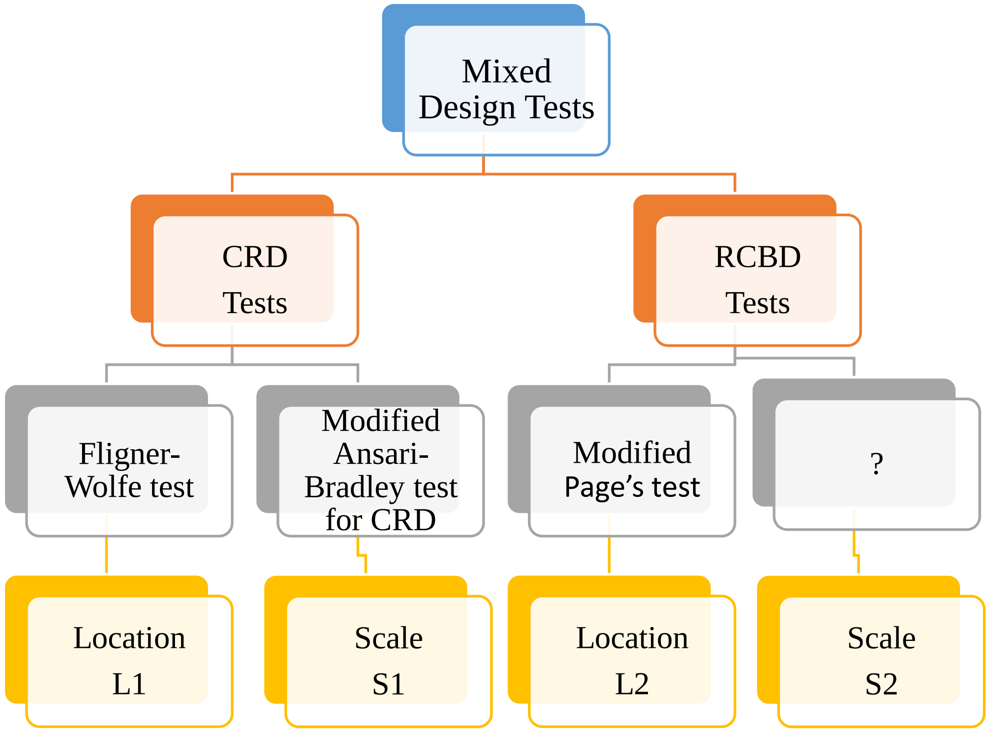

Figure 3.1. Location and scale tests in a mixed design.

3.1. Modified Ansari-Bradley Test for RCBD

Chapter Two discussed the Ansari-Bradley test for testing the two-sample dispersion problem and the modified Ansari-Bradley test for a simple tree alternative in a completely randomized design. However, we will propose a new version of the modified AB test for a simple tree alternative in randomized complete block design RCBD. In this case of RCBD, we assume there is only one observation for each treatment in each block. The null and alternative hypotheses for modified Ansari-Bradley test for RCBD for simple tree alternative are given below:

$$H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k \tag{3.1}$$

$$H_a: \sigma_1 \leq [\sigma_2, \dots, \sigma_k] \text{ with at least one strict inequality}$$

where σ_i is the scale parameter of the i^{th} population with $i = 1,2, \dots, k$ and k is the total number of populations. Population one, $i = 1$, indicates the control population, while populations 2 through k are the treatment populations.

To compute the modified Ansari-Bradley for a randomized complete block design (RCBD), arrange observations from smallest to largest separately within each block. Next, assign a rank of 1 to both the smallest and largest observations, assign a rank of 2 to the second smallest and second-largest observations, and continue in the manner separately within each block. Let C_j be the rank of observation receiving the control in j^{th} block. The modified Ansari-Bradley test for RCBD is the sum of the ranks in the control sample and given by:

$$T_{S2} = \sum_{j=1}^{n_b} C_j \tag{3.2}$$

where n_b is the number of blocks.

Below, a small hypothetical example illustrates how we rank and calculate the modified Ansari-Bradley test for the RCBD portion. Table 3.1 shows four populations where the first population is the control and the other three populations are the treatments and three blocks. At the same time, numbers inside the parentheses indicate the rank of the observations.

Table 3.1. Hypothetical example when k=4.

Block	Treatments			
	Treatment 1 (control)	Treatment 2	Treatment 3	Treatment 4
1	30 (1)	39 (1)	35 (2)	33 (2)
2	35 (2)	33 (1)	40 (1)	34 (2)
3	36 (2)	30 (1)	32 (2)	37 (1)

$T_{S2} = \sum_{j=1}^{n_b} C_j = 1 + 2 + 2 = 5$

Under H_0, the expected value and variance of T_{S2} are, respectively, If k is an even number:

$$E(T_{S2}) = n_b\left(\frac{(k+2)}{4}\right) \tag{3.3}$$

and

$$Var(T_{S2}) = n_b\left(\frac{(k-1)(k+2)(k-2)}{48(k-1)}\right)$$

If k is an odd number:

$$E(T_{S2}) = n_b\left(\frac{(k+1)^2}{4k}\right) \tag{3.4}$$

and

$$Var(T_{S2}) = n_b\left(\frac{(k-1)(k+1)(3+k^2)}{48k^2}\right)$$

The standardized version of modified Ansari-Bradley for RCBD is:

$$Z_{S2} = \frac{T_{S2} - E(T_{S2})}{\sqrt{Var(T_{S2})}} \tag{3.5}$$

where k is the number of treatments, and n_b is the number of blocks. Under the null hypothesis, the asymptotic distribution of Z_{S2} is standard normal distribution. The null hypothesis is rejected when $Z_{S2} \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

3.2. Proposed Mixed Design Tests

We propose standardized versions of the test statistic. In some proposed tests, we sum the standardized tests first, then re-standardized, while other tests are developed by adding the test statistics first and then standardized. Two situations are considered for these tests: a combination without applying any weight or applying weights.

3.2.1. Proposed Test One

The first proposed test for the mixed design Z_1 will be developed using the standardized Fligner-Wolfe test for CRD Z_{L1} given in (2.15), the standardized modified Page's test for RCBD Z_{L2} given in (2.23), the standardized modified Ansari-Bradley test for CRD Z_{S1} given in (2.33), and the standardized modified Ansari-Bradley test for RCBD Z_{S2} given in (3.5). We added these standardized tests together and then re-standardized them. As previously mentioned, under the null hypothesis, all these standardized tests have an asymptotic standard normal distribution. Thus, The asymptotic distribution of the summation of these tests $Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2}$ under the null hypothesis is normal with mean 0 and variance 4. The proposed test one can be written as follows:

$$Z_1 = \frac{(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2}) - E(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}{\sqrt{\mathrm{Var}(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}}$$

$$Z_1 = \frac{(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}{\sqrt{(1 + 1 + 1 + 1)}} = \frac{(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}{\sqrt{(4)}} \qquad (3.6)$$

Under H_0, The proposed test one has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_1 \geq Z_\alpha$ at the α level of significance where Z_α is the $(1 - \alpha)100\%$ of the standard normal distribution.

3.2.2. Proposed Test Two

The second proposed test for mixed design Z_2 will be developed using the Fligner-Wolfe test for CRD T_{L1} given in (2.13), the modified Page's test for RCBD T_{L2} given in (2.21), the modified Ansari-Bradley test for CRD T_{S1} given in (2.30), and the modified Ansari-Bradley test for RCBD T_{S2} given in (3.2). We added these unstandardized tests together and then standardized them. The proposed test two can be written as follows:

$$Z_2 = \frac{(T_{L1} + T_{L2} + T_{S1} + T_{S2}) - E(T_{L1} + T_{L2} + T_{S1} + T_{S2})}{\sqrt{Var(T_{L1} + T_{L2} + T_{S1} + T_{S2})}}$$

$$Z_2 = \frac{(T_{L1} + T_{L2} + T_{S1} + T_{S2}) - \left(E(T_{L1}) + E(T_{L2}) + E(T_{S1}) + E(T_{S2})\right)}{\sqrt{Var(T_{L1}) + Var(T_{L2}) + Var(T_{S1}) + Var(T_{S2})}} \qquad (3.7)$$

where $E(T_{L1})$ and $Var(T_{L1})$ are the expected value and variance of the Fligner Wolfe test T_{L1} given in (2.14), $E(T_{L2})$ and $Var(T_{L2})$ are the expected value and variance of the modified Page's test T_{L2} given in (2.12), $E(T_{S1})$ and $Var(T_{S1})$ are the expected value and variance of the modified Ansari-Bradley test T_{S1} for CRD given in ((2.31) and (2.32)), and $E(T_{S2})$ and $Var(T_{S2})$ are the expected value and variance of the modified Ansari-Bradley test T_{S2} for RCBD given in ((3.3) and (3.4)).

Under H_0, the proposed test two has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_2 \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

3.2.3. Proposed Test Three

The third proposed test for mixed design Z_3 will be developed using the similar way that we used to develop the first proposed test Z_1 except we will add weights. We consider the sample size for each treatment n_a under the CRD portion along with the number of blocks n_b under the RCBD portion as weight. The weight $\frac{n_a}{n}$ is used in standardized tests for CRD (Z_{L1} and Z_{S1}), and the weight $\frac{n_b}{n}$ is used in standardized tests for RCBD (Z_{L2} and Z_{S2}). The weights are assigned to standardized test statistics and then re-standardized. Proposed test three can be written as follows:

$$Z_3 = \frac{\left(\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2} + \frac{n_a}{n} Z_{S1} + \frac{n_b}{n} Z_{S2}\right) - E\left(\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2} + \frac{n_a}{n} Z_{S1} + \frac{n_b}{n} Z_{S2}\right)}{\sqrt{Var\left(\frac{n_a}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_a}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right)}}$$

$$Z_3 = \frac{\left(\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2} + \frac{n_a}{n} Z_{S1} + \frac{n_b}{n} Z_{S2}\right)}{\sqrt{\left(\frac{n_a^2}{n^2} + \frac{n_b^2}{n^2} + \frac{n_a^2}{n^2} + \frac{n_b^2}{n^2}\right)}} \quad (3.8)$$

where n is the sum of the sample size for each treatment n_a under the CRD portion and the number of blocks n_b under the RCBD portion $n = n_a + n_b$.

Under H_0, The proposed test three has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_3 \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

When the sample size for each treatment n_a under the CRD portion and the number of blocks n_b under the RCBD portion are equal, then the equal weight is assigned to all standardized tests. Thus, the proposed test three is equal to the proposed test one. However, If the sample size for each treatment n_a under the CRD portion greater than the number of blocks n_b under the RCBD portion, the standardized Fligner-Wolfe test and the standardized modified Ansari-Bradley test for CRD are assigned greater weight than the standardized modified Page's test and the standardized modified Ansari-Bradley test for RCBD. Also, If the sample size for each treatment n_a under the CRD portion less than the number of blocks n_b under the RCBD portion, then the standardized Fligner-Wolfe test and the standardized modified Ansari-Bradley test for CRD are assigned less weight than the standardized modified Page's test and the standardized modified Ansari-Bradley test for RCBD.

3.2.4. Proposed Test Four

The fourth proposed test for mixed design Z_4 will be developed using the similar way that we used to develop the second proposed test Z_2 except we will add weight. We consider the sample size for each treatment n_a under the CRD portion along with the number of blocks n_b

under the RCBD portion as weight. The weight $\frac{n_a}{n}$ is used in CRD tests (T_{L1} and T_{S1}), and the weight $\frac{n_b}{n}$ is used in RCBD tests (T_{L2} and T_{S2}). The weights are assigned to test statistics and then standardized. Proposed test four can be written as follows:

$$Z_4 = \frac{\left(\frac{n_a}{n} T_{L1} + \frac{n_b}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right) - E\left(\frac{n_a}{n} T_{L1} + \frac{n_b}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right)}{\sqrt{Var\left(\frac{n_a}{n} T_{L1} + \frac{n_b}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right)}}$$

$$Z_4 = \frac{\left(\frac{n_a}{n} T_{L1} + \frac{n_b}{n} T_{L2} + \frac{n_a}{n} T_{S1} + \frac{n_b}{n} T_{S2}\right) - \left(\frac{n_a}{n} E(T_{L1}) + \frac{n_b}{n} E(T_{L2}) + \frac{n_a}{n} E(T_{S1}) + \frac{n_b}{n} E(T_{S2})\right)}{\sqrt{\frac{n_a^2}{n^2} Var(T_{L1}) + \frac{n_b^2}{n^2} Var(T_{L2}) + \frac{n_a^2}{n^2} Var(T_{S1}) + \frac{n_b^2}{n^2} Var(T_{S2})}} \quad (3.9)$$

where $n = n_a + n_b$.

Under H_0, The proposed test four has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_4 \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

If $n_a = n_b$, then the equal weight is assigned to all tests. Thus, the proposed test four is equal to the proposed test two. If $n_a > n_b$, the Fligner-Wolfe test and the modified Ansari-Bradley test for CRD are assigned greater weight than modified Page's test and the modified Ansari-Bradley test for RCBD. If $n_a < n_b$, the Fligner-Wolfe test and the modified Ansari-Bradley test for CRD are assigned less weight than the modified Page's test and the modified Ansari-Bradley test for RCBD.

3.2.5. Proposed Test Five

The fifth proposed test for mixed design Z_5 is similar to proposed test three Z_3 with different weight values. The weight $\frac{n_b}{n}$ is used in standardized tests for CRD (Z_{L1} and Z_{S1}), and the weight $\frac{n_a}{n}$ is used in standardized tests for RCBD (Z_{L2} and Z_{S2}). Proposed test five is given below:

$$Z_5 = \frac{\left(\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_b}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right) - E\left(\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_b}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right)}{\sqrt{Var\left(\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_b}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right)}}$$

$$Z_5 = \frac{\left(\frac{n_b}{n} Z_{L1} + \frac{n_a}{n} Z_{L2} + \frac{n_b}{n} Z_{S1} + \frac{n_a}{n} Z_{S2}\right)}{\sqrt{\left(\frac{n_b^2}{n^2} + \frac{n_a^2}{n^2} + \frac{n_b^2}{n^2} + \frac{n_a^2}{n^2}\right)}} \tag{3.10}$$

where $n = n_a + n_b$.

Under H_0, The proposed test five has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_5 \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

If $n_a = n_b$, then the equal weight is assigned to all standardized tests. Thus, the proposed test five is equal to the proposed test one. If $n_a > n_b$, then the standardized modified Page's test and the standardized modified Ansari-Bradley test for RCBD are assigned greater weight than the standardized Fligner-Wolfe test and the standardized modified Ansari-Bradley test for CRD. If $n_a < n_b$, then the standardized modified Page's test and the standardized modified Ansari-Bradley test for RCBD are assigned less weight than the standardized Fligner-Wolfe test and the standardized modified Ansari-Bradley test for CRD.

3.2.6. Proposed Test Six

The sixth proposed test for mixed design Z_6 is similar to proposed test four Z_4 with different weight values. The weight $\frac{n_b}{n}$ is used in tests for CRD (T_{L1}, and T_{S1}), and the weight $\frac{n_a}{n}$ is used in tests for RCBD (T_{L2}, and T_{S2}). Proposed test six is given below:

$$Z_6 = \frac{\left(\frac{n_b}{n}\ T_{L1} + \frac{n_a}{n}\ T_{L2} + \frac{n_b}{n}\ T_{S1} + \frac{n_a}{n}\ T_{S2}\right) - E\left(\frac{n_b}{n}\ T_{L1} + \frac{n_a}{n}\ T_{L2} + \frac{n_a}{n}\ T_{S1} + \frac{n_b}{n}\ T_{S2}\right)}{\sqrt{Var\left(\frac{n_b}{n}\ T_{L1} + \frac{n_a}{n}\ T_{L2} + \frac{n_a}{n}\ T_{S1} + \frac{n_b}{n}\ T_{S2}\right)}}$$

$$Z_6 = \frac{\left(\frac{n_b}{n}\ T_{L1} + \frac{n_a}{n}\ T_{L2} + \frac{n_b}{n}\ T_{S1} + \frac{n_a}{n}\ T_{S2}\right) - \left(\frac{n_a}{n} E(T_{L1}) + \frac{n_a}{n}\ E(T_{L2}) + \frac{n_a}{n} E(T_{S1}) + \frac{n_a}{n}\ E(T_{S2})\right)}{\sqrt{\frac{n_b^2}{n^2} Var(T_{L1}) + \frac{n_a^2}{n^2}\ Var(T_{L2}) + \frac{n_b^2}{n^2} Var(T_{S1}) + \frac{n_a^2}{n^2}\ Var(T_{S2})}}$$

(3.11)

Under H_0, The proposed test six has an asymptotic standard normal distribution. The null hypothesis is rejected when $Z_6 \geq Z_\alpha$ at the α level of significance where Z_α is the $(1-\alpha)100\%$ of the standard normal distribution.

If $n_a = n_b$, then equal weight is assigned to all tests. Thus, proposed test six is equal to the proposed test two. If $n_a > n_b$, then the modified Page's test and the modified Ansari-Bradley test for RCBD are assigned greater weight than the Fligner-Wolfe test and the modified Ansari-Bradley test for CRD. If $n_a < n_b$, then the modified Pages test and the modified Ansari-Bradley test for RCBD are assigned less weight than the Fligner-Wolfe test and the modified Ansari-Bradley test for CRD.

3.3. Example

This section provides a real-life example to illustrate how each proposed test is being calculated. Consider the data set in Lim and Wolfe (1997) corresponding to an experimental design discussed in Heffner et al. (1974). The experiment was performed to study the effect of the drug d-amphetamine sulfate on the behavior of rats. The response variable was the lever press rate, defined as the total number of lever presses to obtain water divided by the elapsed time in seconds. Five different dosages were considered in the experiment (0.0, 0.5,1,1.5, and 2) in milligrams of drug per kilogram of weight of the rat, where the first dosage (0.0) was a saline solution. The control was the first drug dosage (0.0), while the other drug dosages were the treatments.

We assume that the researchers start with a randomized complete block design (RCBD) and end up with a completely randomized design (CRD) due to missing data or unexpected reasons. Thus, the data are collected using two different designs. The first is the RCBD, where similar or homogeneous rats are grouped in six blocks, and then within each block, five rats are randomly assigned to different drug dosages. The second design is the CRD, where twenty rats are randomly selected and assigned to different drug dosages. Table 3.2 contains all the observations for both RCBD and CRD portions, where an "X" denotes that no observation is taken. Table 3.3 and Table 3.4 represent the results of the RCBD and CRD separately.

Table 3.2. Drug effect data.

Rat	Dosage (mg/kg)				
	0.0	0.5	1	1.5	2
1	0.60	0.80	0.82	0.81	0.50
2	0.51	0.61	0.79	0.78	0.77
3	0.62	0.83	0.84	0.80	0.52
4	0.60	0.95	0.91	0.94	0.70
5	0.92	0.82	1.04	1.13	1.03
6	0.63	0.93	1.02	0.96	0.64
7	0.84	X	0.98	0.99	X
8	0.96	0.74	X	1.19	X
9	1.01	X	1.27	X	1
10	0.95	1.26	X	1.25	1.06
11	X	1.23	1.30	1.22	1.24
12	X	1.20	1.18	X	1.05

We want to test the hypothesis that there is no difference in mean and/or variance between the control (zero dosage) and treatments (drug dosage level) versus the alternative hypothesis that there is at least one of the treatments that differ in mean and/or variance from the control. We assume that the treatment means and variances are greater than the control if they are different. The null and alternative hypotheses are:

$$H_0: \mu_1 = \mu_2 = \cdots = \mu_k \quad \text{, and } \; H_0: \sigma_1 = \sigma_2 = \cdots = \sigma_k$$

$$H_a: \mu_1 \leq [\mu_2, \mu_3, \ldots \mu_k] \;\; \text{and/or } H_a: \sigma_1 \leq [\sigma_2, \ldots, \sigma_k] \text{ with at least one strict inequality.}$$

In this case μ_i is the population mean of i^{th} dosage level and σ_i is the population variance of i^{th} dosage level with $i = 1,2,3,4,$ and 5. The dosage one, $i = 1$, indicates the control, while dosages 2 through 5 are the treatments populations.

Table 3.3. Randomized complete block design data.

Rat	Dosage (mg/kg)				
	0.0	0.5	1	1.5	2
1	0.60	0.80	0.82	0.81	0.50
2	0.51	0.61	0.79	0.78	0.77
3	0.62	0.83	0.84	0.80	0.52
4	0.60	0.95	0.91	0.94	0.70
5	0.92	0.82	1.04	1.13	1.03
6	0.63	0.93	1.02	0.96	0.64

Table 3.4. Completely randomized design data.

Dosage (mg/kg)				
0.0	0.5	1	1.5	2
0.84	0.74	0.98	0.99	1
0.96	1.26	1.27	1.19	1.06
1.01	1.23	1.30	1.25	1.24
0.95	1.20	1.18	1.22	1.05

The six proposed tests combine the Fligner-Wolfe test, modified Page's test, modified Ansari-Bradley test for CRD, and modified Ansari-Bradley for RCBD. We will calculate these test statistics based on the data given in Tables 3.3 and 3.4.

1. Fligner-Wolfe test

Details of the Fligner-Wolfe test were given in Chapter 2.1.4.

Table 3.5. Ranks of observation in CRD for the Fligner-Wolfe test.

Dosage (mg/kg)				
0.0	0.5	1	1.5	2
0.84 (2)	0.74 (1)	0.98 (5)	0.99 (6)	1 (7)
0.96 (4)	1.26 (18)	1.27 (19)	1.19 (12)	1.06 (10)
1.01 (8)	1.23(15)	1.30 (20)	1.25 (17)	1.24 (16)
0.95 (3)	1.20 (13)	1.18 (11)	1.22 (14)	1.05 (9)

Recall, the Fligner-Wolfe test statistic is:

$T_{L1} = \sum_{i=2}^{k} \sum_{j=1}^{n_i} r_{ij}$.

where r_{ij} represents the rank of the i^{th}observation in the j^{th} sample of the combined sample.

$T_{L1} = \sum_{i=2}^{5} \sum_{j=1}^{n_i} r_{ij}$ = [1+18+15+13+5+19+20+11+6+12+17+14+7+10+16+9] = 193

The expected value and variance of Fligner-Wolfe test T_{L1} are, respectively,

$$E(T_{L1}) = \frac{n_t(N+1)}{2} = \frac{16(20+1)}{2} = \frac{n_2(N+1)}{2} = \frac{336}{2} = 168$$

$$Var(T_{L1}) = \frac{n_c n_t(N+1)}{12} = \frac{4 * 16(20+1)}{12} = \frac{64 * 21}{12} = \frac{1344}{12} = 112$$

The standardized version of the Fligner-Wolfe test is:

$$Z_{L1} = \frac{T_{L1} - E(T_{L1})}{\sqrt{Var(T_{L1})}} = \frac{193 - 168}{\sqrt{112}} = 2.36228$$

2. Modified Page's test

Details of the modified Page's test were given in Chapter 2.1.6.

Table 3.6. Ranks of observation in RCBD for modified Page's test test.

Block	Dosage (mg/kg)				
	0.0	0.5	1	1.5	2
1	0.60 (2)	0.80 (3)	0.82 (5)	0.81 (4)	0.50 (1)
2	0.51 (1)	0.61 (2)	0.79 (5)	0.78 (4)	0.77 (3)
3	0.62 (2)	0.83 (4)	0.84 (5)	0.80 (3)	0.52 (1)
4	0.60 (1)	0.95 (5)	0.91 (3)	0.94 (4)	0.70 (2)
5	0.92 (2)	0.82 (1)	1.04 (4)	1.13 (5)	1.03 (3)
6	0.63 (1)	0.93 (3)	1.02 (5)	0.96 (4)	0.64 (2)
R_j	$R_1 = 9$	$R_2 = 18$	$R_3 = 27$	$R_4 = 24$	$R_5 = 12$

Recall the modified Page's test statistic is given below:

$$T_{L2} = R_1 + 2\sum_{j=2}^{k} R_j = R_1 + 2[R_2 + R_3 + \cdots R_k]$$

In our case, k=5, so the modified Page's test statistics is given by:

$$T_{L2} = R_1 + 2\sum_{j=2}^{5} R_j = R_1 + 2[R_2 + R_3 + R_4 + R_5]$$

$$T_{L2} = R_1 + 2[R_2 + R_3 + R_4 + R_5]$$

By using Table 3.6.

$$T_{L2} = 9 + 2[18 + 27 + 24 + 12] = 9 + 2[81] = 9 + 162 = 171$$

The expected value and variance of the modified Page's test T_{L2} are, respectively,

$$E(T_{L2}) = n_b\left(k^2 + \frac{k-1}{2}\right) = 6\left(5^2 + \frac{5-1}{2}\right) = 6(25 + 2) = 162$$

and

$$Var(T_{L2}) = n_b\left(\frac{k^2 - 1}{12}\right) = 6\left(\frac{5^2 - 1}{12}\right) = 6\left(\frac{24}{12}\right) = 12$$

The standardized version of the modified Page's test statistic is:

$$Z_{L2} = \frac{T_{L2} - E(T_{L2})}{\sqrt{Var(T_{L2})}} = \frac{171 - 162}{\sqrt{12}} = \frac{9}{\sqrt{12}} = 2.59808$$

3. Modified Ansari-Bradley test for CRD

Details of the modified Ansari-Bradley test for CRD were given in Chapter 2.2.1.

Table 3.7. Ranks of observation in CRD for modified Ansari-Bradley test for CRD.

		Dosage (mg/kg)		
0.0	0.5	1	1.5	2
0.84 (2)	0.74 (1)	0.98 (5)	0.99 (6)	1 (7)
0.96 (4)	1.26 (3)	1.27 (2)	1.19 (9)	1.06 (10)
1.01 (8)	1.23(6)	1.30 (1)	1.25 (4)	1.24 (5)
0.95 (3)	1.20 (8)	1.18 (10)	1.22 (7)	1.05 (9)

Recall the modified Ansari-Bradley test statistic for CRD is:

$$T_{S1} = \sum_{i=1}^{n_c} R_i$$

where R_i is the rank of the i^{th} control observation in the combined ranking method defined in Chapter 2.

$T_{S1} = [2 + 4 + 8 + 3] = 17$

The expected value and variance of T_{S1} are, respectively,

$$E(T_{S1}) = \frac{n_c\,(N + 2)}{4} = \frac{4\,(20 + 2)}{4} = \frac{88}{4} = 22$$

$$Var(T_{S1}) = \frac{n_c\,n_t(N + 2)(N - 2)}{48(N - 1)} = \frac{4 * 16(20 + 2)(20 - 2)}{48(20 - 1)} = \frac{4 * 16(22)(18)}{48(19)} = \frac{25344}{912}$$

$$= 27.7895$$

The standardized version of the modified Ansari-Bradley test statistic for CRD is:

$$Z_{S1} = \frac{T_{S1} - E(T_{S1})}{\sqrt{Var(T_{S1})}} = \frac{17 - 22}{\sqrt{27.7895}} = -0.94848$$

4. Modified Ansari-Bradley test for RCBD

Details of the modified Ansari-Bradley test for RCBD were given in Chapter 3.1.

Table 3.8. Ranks of observation in RCBD for modified Ansari-Bradley test for RCBD.

Block	Dosage (mg/kg)				
	0.0	0.5	1	1.5	2
1	0.60 (2)	0.80 (3)	0.82 (1)	0.81 (2)	0.50 (1)
2	0.51 (1)	0.61 (2)	0.79 (1)	0.78 (2)	0.77 (3)
3	0.62 (2)	0.83 (2)	0.84 (1)	0.80 (3)	0.52 (1)
4	0.60 (1)	0.95 (1)	0.91 (3)	0.94 (2)	0.70 (2)
5	0.92 (2)	0.82 (1)	1.04 (2)	1.13 (1)	1.03 (3)
6	0.63 (1)	0.93 (3)	1.02 (1)	0.96 (2)	0.64 (2)

The modified Ansari-Bradley test statistic for RCBD is:

$$T_{S2} = \sum_{j=1}^{n_b} C_j = \sum_{j=1}^{6} C_j = [2 + 1 + 2 + 1 + 2 + 1] = 9$$

where C_j represents the rank of the control observation in block j where observations are ranked as given in Chapter 3.1.

The expected value and variance of T_{S2} are, respectively,

where k=5 is an odd number:

$$E(T_{S2}) = n_b\left(\frac{(k+1)^2}{4k}\right) = 6\left(\frac{(5+1)^2}{4*5}\right) = 6\left(\frac{36}{20}\right) = 10.8$$

$$Var(T_{S2}) = n_b\left(\frac{(k-1)(k+1)(3+k^2)}{48k^2}\right) = 6\left(\frac{(5-1)(5+1)(3+5^2)}{48*5^2}\right) = 6\left(\frac{(4)(6)(28)}{48*25}\right)$$

$$= 6 * 0.56 = 3.36$$

The standardized version of the modified Ansari-Bradley test statistic for RCBD is:

$$Z_{S2} = \frac{T_{S2} - E(T_{S2})}{\sqrt{Var(T_{S2})}} = \frac{9 - 10.8}{\sqrt{3.36}} = -0.98198$$

We will now calculate the value of each of the proposed tests. We are rejecting the null hypothesis for a large value of test statistics. The P-value associated with each test statistic will be found for these particular cases.

5. Proposed test one

$$Z_1 = \frac{(Z_{L1} + Z_{L2} + Z_{S1} + Z_{S2})}{\sqrt{(4)}} = \frac{(2.36228 + 2.59808 - 0.94848 - 0.98198)}{\sqrt{(4)}} = \frac{3.0299}{\sqrt{(4)}}$$

$$Z_1 = 1.51495$$

$$\text{P} - \text{value} = 1 - \text{P}(Z_1 \le 1.51495) \approx 1 - \text{P}(Z_1 \le 1.51) = 1 - 0.9345 = 0.0655$$

6. Proposed test two

$$Z_2 = \frac{(T_{L1} + T_{L2} + T_{S1} + T_{S2}) - \left(E(T_{L1}) + E(T_{L2}) + E(T_{S1}) + E(T_{S2})\right)}{\sqrt{Var(T_{L1}) + Var(T_{L2}) + Var(T_{S1}) + Var(T_{S2})}}$$

$$Z_2 = \frac{(193 + 171 + 17 + 9) - (168 + 162 + 22 + 10.8)}{\sqrt{112 + 12 + 27.7895 + 3.36}} = \frac{(390) - (362.8)}{\sqrt{155.1495}} = 2.18370$$

$$\text{P} - \text{value} = 1 - \text{P}(Z_2 \le 2.18370) \approx 1 - \text{P}(Z_2 \le 2.18) = 1 - 0.9854 = 0.0146$$

7. Proposed test three

$$Z_3 = \frac{\left(\frac{n_a}{n} Z_{L1} + \frac{n_b}{n} Z_{L2} + \frac{n_a}{n} Z_{S1} + \frac{n_b}{n} Z_{S2}\right)}{\sqrt{\left(\frac{n_a^2}{n^2} + \frac{n_b^2}{n^2} + \frac{n_a^2}{n^2} + \frac{n_b^2}{n^2}\right)}}$$

$$Z_3 = \frac{\left(\frac{4}{10} * 2.36228 + \frac{6}{10} * 2.59808 - \frac{4}{10} * 0.94848 - \frac{6}{10} * 0.98198\right)}{\sqrt{\left(\frac{16}{100} + \frac{36}{100} + \frac{16}{100} + \frac{36}{100}\right)}}$$

$$Z_3 = \frac{(0.944912 + 1.558848 - 0.379392 - 0.589188)}{\sqrt{(1.04)}} = \frac{(1.53518)}{\sqrt{(1.04)}} = 1.50537$$

$$\text{P} - \text{value} = 1 - \text{P}(Z_3 \le 1.50537) \approx 1 - \text{P}(Z_3 \le 1.51) = 1 - 0.9345 = 0.0655$$

8. Proposed test four

$$Z_4 = \frac{\left(\frac{n_a}{n}\ T_{L1} + \frac{n_b}{n}\ T_{L2} + \frac{n_a}{n}\ T_{S1} + \frac{n_b}{n}\ T_{S2}\right) - \left(\frac{n_a}{n} E(T_{L1}) + \frac{n_b}{n}\ E(T_{L2}) + \frac{n_a}{n} E(T_{S1}) + \frac{n_b}{n}\ E(T_{S2})\right)}{\sqrt{\frac{n_a^2}{n^2}\ Var(T_{L1}) + \frac{n_b^2}{n^2}\ Var(T_{L2}) + \frac{n_a^2}{n^2}\ Var(T_{S1}) + \frac{n_b^2}{n^2} Var(T_{S2})}}$$

$$Z_4 = \frac{\left(\frac{4}{10} * 193 + \frac{6}{10} * 171 + \frac{4}{10} * 17 + \frac{6}{10} * 9\right) - \left(\frac{4}{10} * 168 + \frac{6}{10} * 162 + \frac{4}{10} * 22 + \frac{6}{10}\ * 10.8\right)}{\sqrt{\frac{16}{100} * 112 + \frac{36}{100} * 12 + \frac{16}{100} * 27.7895 + \frac{36}{100} * 3.36}}$$

$$Z_4 = \frac{(77.2 + 102.6 + 6.8 + 5.4) - (67.2 + 97.2 + 8.8 + 6.48)}{\sqrt{17.92 + 4.32 + 4.44632 + 1.2096}}$$

$$Z_4 = \frac{(192) - (179.68)}{\sqrt{27.89592}} = 2.33260$$

$$P - value = 1 - P(Z_4 \leq 2.33260) \approx 1 - P(Z_4 \leq 2.33) = 1 - 0.9901 = 0.0099$$

9. Proposed test five

$$Z_5 = \frac{\left(\frac{n_b}{n}\ Z_{L1} + \frac{n_a}{n}\ \ Z_{L2} + \frac{n_b}{n}\ Z_{S1} + \frac{n_a}{n}\ \ Z_{S2}\right)}{\sqrt{\left(\frac{n_b^2}{n^2}\ + \frac{n_a^2}{n^2} + \frac{n_b^2}{n^2}\ + \frac{n_a^2}{n^2}\right)}}$$

$$Z_5 = \frac{\left(\frac{6}{10} * 2.36228 + \frac{4}{10} * 2.59808 - \frac{6}{10} * 0.94848\ - \frac{4}{10} * 0.98198\right)}{\sqrt{\left(\frac{36}{100} + \frac{16}{100} + \frac{36}{100} + \frac{16}{100}\right)}}$$

$$Z_5 = \frac{(1.417368 + 1.039232 - 0.569088 - 0.392792)}{\sqrt{(1.04)}} = \frac{(1.49472)}{\sqrt{(1.04)}} = 1.46569$$

$$P - value = 1 - P(Z_5 \leq 1.46569) \approx 1 - P(Z_5 \leq 1.47) = 1 - 0.9292 = 0.0708$$

10. Proposed test six

$$Z_6 = \frac{\left(\frac{n_b}{n}\ T_{L1} + \frac{n_a}{n}\ \ T_{L2} + \frac{n_b}{n}\ T_{S1} + \frac{n_a}{n}\ \ T_{S2}\right) - \left(\frac{n_a}{n} E(T_{L1}) + \frac{n_a}{n}\ \ E(T_{L2}) + \frac{n_a}{n} E(T_{S1}) + \frac{n_a}{n}\ E(T_{S2})\right)}{\sqrt{\frac{n_b^2}{n^2} Var(T_{L1}) + \frac{n_a^2}{n^2}\ Var(T_{L2}) + \frac{n_b^2}{n^2} Var(T_{S1}) + \frac{n_a^2}{n^2}\ \ Var(T_{S2})}}$$

$$Z_6 = \frac{\left(\frac{6}{10} * 193 + \frac{4}{10} * 171 + \frac{6}{10} * 17 + \frac{4}{10} * 9\right) - \left(\frac{6}{10} * 168 + \frac{4}{10} * 162 + \frac{6}{10} * 22 + \frac{4}{10}\ * 10.8\right)}{\sqrt{\frac{36}{100} * 112 + \frac{16}{100} * 12 + \frac{36}{100} * 27.7895 + \frac{16}{100} * 3.36}}$$

$$Z_6 = \frac{(115.8 + 68.4 + 10.2 + 3.6) - (100.8 + 64.8 + 13.2 + 4.32)}{\sqrt{40.32 + 1.92 + 10.00422 + 0.5376}} = \frac{(198) - (183.12)}{\sqrt{52.78182}}$$

$$Z_6 = 2.04815$$

$$\text{P} - \text{value} = 1 - \text{P}(Z_6 \leq 2.04815) \approx 1 - \text{P}(Z_6 \leq 2.05) = 1 - 0.9798 = 0.0202$$

In this example, proposed tests two, four, and six would have rejected the null hypothesis at a 5 % significant level. Proposed test four had the lowest p-value.

CHAPTER 4. SIMULATION STUDY

This chapter describes the procedure used in the simulation study to estimate and compare the powers of proposed tests. The simulation study was performed using SAS version 9.4. The random variables were generated from different distributions using the RAND function in the data step. The seed value for the RAND function was set by using the STREAMINIT subroutine. In this study, the seed was set to be zero, meaning each code run would produce a different data set (Bailer, 2010).

In this study, the significant level α for each proposed test statistic was estimated and compared to the stated alpha value, which was 0.05. The significant level of each proposed test was calculated by counting the number of times that the null hypothesis was rejected under H_0 then divided by the number of replications, which was 10,000 samples. If the estimated alpha value was approximately 0.05, the power of tests was compared to each other. The power of each proposed test was estimated by counting the number of times that the null hypothesis was rejected under H_a then divided by the number of replications, which was 10,000 samples.

The powers of the six proposed tests were estimated under various cases: changing the number of treatments; in this study, three, four, and five treatments were considered, changing the underlying distribution, changing the proportions of the number of blocks in the RCBD to the sample size for each treatment in the CRD, increasing the variance ratio between the CRD and RCBD portions, and changing the parameter arrangements.

4.1. Distributions

The study considered the following underlying distributions: normal, exponential, and t distribution with three degrees of freedom. The normal distribution was considered because it frequently occurs in nature. The t distribution with three degrees of freedom was also used. It is symmetric distribution like the normal, but it has heavier tails than normal. The exponential distribution was selected because it is an asymmetric distribution that often occurs and is right-tailed skewed.

The call function (X= Rand ("Normal", 0,1)) was used to generate the random sample from a standard normal distribution with a mean equal to zero and a standard deviation equal to one. The call function (X= Rand ("Exponential",1)) was used to generate the random sample from a standard exponential distribution with a mean equal to one and variance equal to one. Finally, the call function (X= Rand ("T",3)) was used to generate the random sample from t-distribution with three degrees of freedom. More details are illustrated in Tables 4.1 to 4.3.

4.2. Sample Sizes

This study considered three cases related to the number of blocks in the RCBD portion and the sample size for each treatment in the CRD portion. Powers and significance levels for the proposed tests were estimated in each case. We started with the number of blocks in the RCBD portion , n_b, is equal to the sample size for each treatment in the CRD portion, n_a. In this case, we considered n_b and n_a both equal 10. Then, the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion, $n_b = 10, n_a = 5$. Finally, we considered where the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion, $n_b = 5, n_a = 10$.

4.3. The Variance Between the CRD and RCBD Portion

The data used in this study was generated from a mixed design consisting of a CRD and RCBD portion. The RCBD is commonly used to reduce error variation; hence the error variance associated with RCBD data is smaller than that associated with CRD data. We want to decide whether or not it makes a difference in which test statistic to use when the error variance in both the RCBD data and the CRD data is about the same and when the CRD data have a larger error variance. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion. Second, the variance of the CRD portion is two times the variance of the RCBD portion. Third, the variance of the CRD portion is four times the variance of the RCBD portion. Forth, the variance of the CRD portion is eight times the variance of the RCBD portion. The Rand functions used to generate the random sample are listed in Tables 4.1 to 4.3.

4.4. Parameter Arrangements

Powers for the proposed tests were estimated under a variety of location and scale parameter arrangements. We started with changing the location parameters and keeping the scale parameters equal to each other. Next, we changed the scale parameters and kept the location parameters equal to each other. Lastly, we changed both the location and scale parameters. The following parameter arrangements were considered in the simulation study:

- For three populations:

  First, when only the location parameters change, the arrangements considered were the following:

1. The second and third populations have the same location parameters that are different than the first population (control).
2. The first population (control) and second population have the same location parameters that are different than the third population.
3. The three populations have different and equally spaced location parameters.
4. The three populations have different and unequally spaced location parameters.

Second, when only the scale parameters change, the arrangements considered were the following:

1. The second and third populations have the same scale parameters that are different than the first population (control).
2. The first population (control) and second population have the same scale parameters that are different than the third population.
3. The three populations have different and equally spaced scale parameters.
4. The three populations have different and unequally spaced scale parameters.

Finally, when both the location and scale parameters change, the arrangements considered were the following:

1. The second and third populations have the same location and scale parameters that are different than the first population (control).
2. The first population (control) and second population have the same location and scale parameters that are different than the third population.
3. The three populations have different and unequally spaced location and scale parameters.

- For Four populations:

First, when only the location parameters change, the arrangements considered were the following:

1. The last three populations have the same location parameters that are different than the first population (control).
2. The first three populations have the same location parameters that are different than the fourth population.

3. The first population (control) and second population have the same location parameters that are different than the third and fourth populations.
4. The four populations have different and equally spaced location parameters.
5. The four populations have different and unequally spaced location parameters.

Second, when only the scale parameters change, the arrangements considered were the following:

1. The last three populations have the same scale parameters that are different than the first population (control).
2. The first population (control) and second population have the same scale parameters that are different than the third and fourth populations.
3. The four populations have different and equally spaced scale parameters.
4. The four populations have different and unequally spaced scale parameters.

Finally, when both the location and scale parameters change, the arrangements considered were the following:

1. The last three populations have the same location and scale parameters that are different than the first population (control).
2. The first population (control) and second population have the same location and scale parameters that are different than the third and fourth populations.
3. The four populations have different and unequally spaced location and scale parameters.

- For five populations:

First, when only the location parameters change, the arrangements considered were the following:

1. The last four populations have the same location parameters that are different than the first population (control).
2. The first three populations have the same location parameters that are different than the fouth and fifth population.
3. The first population (control) and second population have the same location parameters that are different than the last three populations.
4. The five populations have different and equally spaced location parameters.
5. The five populations have different and unequally spaced location parameters.

Second, when only the scale parameters change, the arrangements considered were the following:

1. The last four populations have the same scale parameters that are different than the first population (control).
2. The first population (control) and second population have the same scale parameters that are different than the last three populations.
3. The five populations have different and equally spaced scale parameters.
4. The five populations have different and unequally spaced scale parameters.

Finally, when both the location and scale parameters change, the arrangements considered were the following:

1. The last four populations have the same location and scale parameters that are different than the first population (control).
2. The first population (control) and second population have the same location and scale parameters that are different than the last three populations.
3. The five populations have different and unequally spaced location and scale parameters.

Table 4.1. Generate the random sample from a normal distribution; a and b are non-negative constants.

The variance between the RCBD and CRD portion	Change parameters	The Rand functions in the RCBD portion	The Rand functions in the CRD portion
CRD=RCBD	Only location	X= Rand ("Normal", 0,1) $Y = a + X$	X= Rand ("Normal", 0,1) $Y = a + X$
	Only scale	X= Rand ("Normal", 0,1) $Y = b * X$	X= Rand ("Normal", 0,1) $Y = b * X$
	Both location and scale	X= Rand ("Normal", 0,1) $Y = a + b * X$	X= Rand ("Normal", 0,1) $Y = a + b * X$
CRD=2RCBD	Only location	X= Rand ("Normal", 0,1) $Y = a + X$	X= Rand ("Normal", 0, $\sqrt{2}$) $Y = a + X$
	Only scale	X= Rand ("Normal", 0,1) $Y = b * X$	X= Rand ("Normal", 0, $\sqrt{2}$) $Y = b * X$
	Both location and scale	X= Rand ("Normal", 0,1) $Y = a + b * X$	X= Rand ("Normal", 0, $\sqrt{2}$) $Y = a + b * X$
CRD=4RCBD	Only location	X= Rand ("Normal", 0,1) $Y = a + X$	X= Rand ("Normal", 0, $\sqrt{4}$) $Y = a + X$
	Only scale	X= Rand ("Normal", 0,1) $Y = b * X$	X= Rand ("Normal", 0, $\sqrt{4}$) $Y = b * X$
	Both location and scale	X= Rand ("Normal", 0,1) $Y = a + b * X$	X= Rand ("Normal", 0, $\sqrt{4}$) $Y = a + b * X$
CRD=8RCBD	Only location	X= Rand ("Normal", 0,1) $Y = a + X$	X= Rand ("Normal", 0, $\sqrt{8}$) $Y = a + X$
	Only scale	X= Rand ("Normal", 0,1) $Y = b * X$	X= Rand ("Normal", 0, $\sqrt{8}$) $Y = b * X$
	Both location and scale	X= Rand ("Normal", 0,1) $Y = a + b * X$	X= Rand ("Normal", 0, $\sqrt{8}$) $Y = a + b * X$

Table 4.2. Generate the random sample from an exponential distribution; a and b are non-negative constants.

The variance between the RCBD and CRD portion	Change parameters	The Rand functions in the RCBD portion	The Rand functions in the CRD portion
CRD=RCBD	Only location	X= Rand ("Exponential",1) $Y = a + X$	X= Rand ("Exponential",1) $Y = a + X$
	Only scale	X= Rand ("Exponential",1) $Y = b * X - (b - 1)$	X= Rand ("Exponential",1) $Y = b * X - (b - 1)$
	Both location and scale	X= Rand ("Exponential",1) $Y = b * X$	X= Rand ("Exponential",1) $Y = b * X$
CRD=2RCBD	Only location	X= Rand ("Exponential",1) $Y = a + X$	X= Rand ("Exponential", $\sqrt{2}$) $- (\sqrt{2} - 1)$ $Y = a + X$
	Only scale	X= Rand ("Exponential",1) $Y = b * X - (b - 1)$	X= Rand ("Exponential", $\sqrt{2}$) $- (\sqrt{2} - 1)$ $Y = b * X - (b - 1)$
	Both location and scale	X= Rand ("Exponential",1) $Y = b * X$	X= Rand ("Exponential", $\sqrt{2}$) $- (\sqrt{2} - 1)$ $Y = b * X$
CRD=4RCBD	Only location	X= Rand ("Exponential",1) $Y = a + X$	X= Rand ("Exponential", $\sqrt{4}$) $- (\sqrt{4} - 1)$ $Y = a + X$
	Only scale	X= Rand ("Exponential",1) $Y = b * X - (b - 1)$	X= Rand ("Exponential", $\sqrt{4}$) $- (\sqrt{4} - 1)$ $Y = b * X - (b - 1)$
	Both location and scale	X= Rand ("Exponential",1) $Y = b * X$	X= Rand ("Exponential", $\sqrt{4}$) $- (\sqrt{4} - 1)$ $Y = b * X$
CRD=8RCBD	Only location	X= Rand ("Exponential",1) $Y = a + X$	X= Rand ("Exponential", $\sqrt{8}$) $- (\sqrt{8} - 1)$ $Y = a + X$
	Only scale	X= Rand ("Exponential",1) $Y = b * X - (b - 1)$	X= Rand ("Exponential", $\sqrt{8}$) $- (\sqrt{8} - 1)$ $Y = b * X - (b - 1)$
	Both location and scale	X= Rand ("Exponential",1) $Y = b * X$	X= Rand ("Exponential", $\sqrt{8}$) $- (\sqrt{8} - 1)$ $Y = b * X$

Table 4.3. Generate the random sample from a t distribution; a and b are non-negative constants.

The variance between the RCBD and CRD portion	Change parameters	The Rand functions in the RCBD portion	The Rand functions in the CRD portion
CRD=RCBD	Only location	X= Rand ("T",3) $Y = a + X$	X= Rand ("T",3) $Y = a + X$
	Only scale	X= Rand ("T",3) $Y = b * X$	X= Rand ("T",3) $Y = b * X$
	Both location and scale	X= Rand ("T",3) $Y = a + b * X$	X= Rand ("T",3) $Y = a + b * X$
CRD=2RCBD	Only location	X= Rand ("T",3) $Y = a + X$	X= Rand ("T",3) $Y = a + X$
	Only scale	X= Rand ("T",3) $Y = b * X$	X= Rand ("T", 3) $* \sqrt{2}$ $Y = b * X$
	Both location and scale	X= Rand ("T",3) $Y = a + b * X$	X= Rand ("T", 3) $* \sqrt{2}$ $Y = a + b * X$
CRD=4RCBD	Only location	X= Rand ("T",3) $Y = a + X$	X= Rand ("T", 3) $* \sqrt{4}$ $Y = a + X$
	Only scale	X= Rand ("T",3) $Y = b * X$	X= Rand ("T", 3) $* \sqrt{4}$ $Y = b * X$
	Both location and scale	X= Rand ("T",3) $Y = a + b * X$	X= Rand ("T", 3) $* \sqrt{4}$ $Y = a + b * X$
CRD=8RCBD	Only location	X= Rand ("T",3) $Y = a + X$	X= Rand ("T", 3) $* \sqrt{8}$ $Y = a + X$
	Only scale	X= Rand ("T",3) $Y = b * X$	X= Rand ("T", 3) $* \sqrt{8}$ $Y = b * X$
	Both location and scale	X= Rand ("T",3) $Y = a + b * X$	X= Rand ("T", 3) $* \sqrt{8}$ $Y = a + b * X$

CHAPTER 5. RESULTS

This chapter discusses the simulation study results for the six proposed tests that we discussed in Chapter 4. We assumed the underlying distributions were normal, exponential, and t distribution with three degrees of freedom. Tables include the estimated powers and significance levels for proposed tests. The sample size for each treatment in the CRD portion, n_a, and the number of blocks in the RCBD portion, n_b, were titled in each table.

5.1. Results for the Normal Distribution

5.1.1. Three Treatments

Tables 5.1-5.18 show the simulation study results for three treatments (k=3) under the normal distribution. In the tables, (μ, σ) represent the mean and the standard deviation of the random variable generated from a normal distribution. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion (Table 5.1 to Table 5.9). Second, the variance of the CRD portion is two times the variance of the RCBD portion (Table 5.10 to Table 5.18). Third, the variance of the CRD portion is four times the variance of the RCBD portion (see Appendix A). Forth, the variance of the CRD portion is eight times the variance of the RCBD portion (see Appendix A).

We estimated the alpha values for the proposed tests and tabled them in the first row of the table, where all populations have the same location and scale parameters (0,1) (0,1) (0,1). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (5.1, 5.4, 5.7, 5.10, 5.13, and

5.16). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters.

We started with the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location parameters and the same scale parameters, the proposed test two, Z_2, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Next, we considered the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Finally, we considered the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. Similarly, the proposed test three, Z_3, has the highest powers when the populations have different location and scale parameters.

In cases where the variance of the CRD portion is either four or eight times the variance of the RCBD portion, the proposed tests that have the highest powers will be approximately the same as the cases where the variance of the CRD portion is two times the variance of the RCBD portion. The only exception to this is when the number of blocks in the RCBD portion is equal to the sample size for each treatment in the CRD portion, and the populations have different location parameters and the same scale parameters, the proposed test one, Z_1, has the highest powers (see Appendix A).

Table 5.1. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in RCBD = CRD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0517	0.0509	0.0517	0.0509	0.0517	0.0509
(0,1) (1,1) (1,1)	0.5041	**0.6766**	0.5041	**0.6766**	0.5041	**0.6766**
(0,1) (0,1) (1,1)	**0.3696**	0.3294	**0.3696**	0.3294	**0.3696**	0.3294
(0,1) (0.5,1) (1,1)	0.4204	**0.5060**	0.4204	**0.5060**	0.4204	**0.5060**
(0,1) (1,1) (1.5,1)	0.6335	**0.8290**	0.6335	**0.8290**	0.6335	**0.8290**
(0,1) (1.5,1) (2,1)	0.7535	**0.9673**	0.7535	**0.9673**	0.7535	**0.9673**
(0,1) (1.5,1) (1.75,1)	0.7235	**0.9484**	0.7235	**0.9484**	0.7235	**0.9484**

Table 5.2. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in RCBD =CRD; K=3, $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	**0.3187**	0.1749	**0.3187**	0.1749	**0.3187**	0.1749
(0,1) (0,1) (0,3)	**0.2266**	0.1158	**0.2266**	0.1158	**0.2266**	0.1158
(0,1) (0,2) (0,3)	**0.4153**	0.2267	**0.4153**	0.2267	**0.4153**	0.2267
(0,1) (0,2.5) (0,5)	**0.5711**	0.3092	**0.5711**	0.3092	**0.5711**	0.3092
(0,1) (0,2) (0,5)	**0.5016**	0.2697	**0.5016**	0.2697	**0.5016**	0.2697
(0,1) (0,3) (0,3.5)	**0.5420**	0.3046	**0.5420**	0.3046	**0.5420**	0.3046
(0,1) (0,6) (0,8)	**0.7336**	0.4438	**0.7336**	0.4438	**0.7336**	0.4438

Table 5.3. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	**0.7449**	0.6998	**0.7449**	0.6998	**0.7449**	0.6998
(0,1) (0,1) (1,3)	**0.4272**	0.2826	**0.4272**	0.2826	**0.4272**	0.2826
(0,1) (0,2) (1,3)	**0.6306**	0.4358	**0.6306**	0.4358	**0.6306**	0.4358
(0,1) (0.5,2.5) (1,5)	**0.7748**	0.5756	**0.7748**	0.5756	**0.7748**	0.5756
(0,1) (0.25,2) (0.5,5)	**0.6430**	0.4224	**0.6430**	0.4224	**0.6430**	0.4224
(0,1) (1,3) (1.5,3.5)	**0.8525**	0.7466	**0.8525**	0.7466	**0.8525**	0.7466
(0,1) (1.5,6) (1.75,8)	**0.8955**	0.7381	**0.8955**	0.7381	**0.8955**	0.7381

Table 5.4. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0502	0.0491	0.0509	0.0540	0.0510	0.0499
(0,1) (1,1) (1,1)	0.3852	0.5451	0.3678	**0.6465**	0.3422	0.4627
(0,1) (0,1) (1,1)	0.2852	0.2648	0.2843	**0.3344**	0.2505	0.2248
(0,1) (0.5,1) (1,1)	0.3168	0.3841	0.3077	**0.4791**	0.2791	0.3246
(0,1) (1,1) (1.5,1)	0.4651	0.6653	0.4445	**0.7726**	0.4041	0.5722
(0,1) (1.5,1) (2,1)	0.5372	0.8679	0.4937	**0.9373**	0.4643	0.7844
(0,1) (1.5,1) (1.75,1)	0.5211	0.8383	0.4783	**0.9181**	0.4513	0.7435

Table 5.5. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in RCBD =CRD; K=3, $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	**0.2213**	0.1242	0.1984	0.1430	0.2207	0.1136
(0,1) (0,1) (0,3)	**0.1710**	0.0998	0.1583	0.1161	0.1588	0.0939
(0,1) (0,2) (0,3)	0.2776	0.1496	0.2342	0.1713	**0.2786**	0.1346
(0,1) (0,2.5) (0,5)	0.3637	0.1920	0.2956	0.2170	**0.3868**	0.1779
(0,1) (0,2) (0,5)	0.3339	0.1777	0.2791	0.2021	**0.3432**	0.1629
(0,1) (0,3) (0,3.5)	0.3452	0.1846	0.2891	0.2097	**0.3654**	0.1702
((0,1) (0,6) (0,8)	0.4836	0.2650	0.3749	0.2886	**0.5249**	0.2476

Table 5.6. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	0.5658	0.5249	0.5107	**0.6125**	0.5485	0.4565
(0,1) (0,1) (1,3)	**0.3079**	0.2095	0.2906	0.2538	0.2844	0.1835
(0,1) (0,2) (1,3)	**0.4459**	0.2911	0.3943	0.3461	0.4359	0.2572
(0,1) (0.5,2.5) (1,5)	0.5573	0.3909	0.4801	0.4496	**0.5649**	0.3447
(0,1) (0.25,2) (0.5,5)	0.4487	0.2750	0.3839	0.3211	**0.4508**	0.2465
(0,1) (1,3) (1.5,3.5)	**0.6612**	0.5488	0.5831	0.6245	0.6584	0.4834
(0,1) (1.5,6) (1.75,8)	0.7432	0.5117	0.6199	0.5972	**0.7787**	0.4932

Table 5.7. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in RCBD =CRD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0530	0.0481	0.0511	0.0474	0.0543	0.0495
(0,1) (1,1) (1,1)	0.4127	0.6460	0.4410	0.6280	0.3243	**0.6757**
(0,1) (0,1) (1,1)	0.2955	0.3094	0.2995	0.2968	0.2517	**0.3276**
(0,1) (0.5,1) (1,1)	0.3456	0.4804	0.3579	0.4620	0.2814	**0.5058**
(0,1) (1,1) (1.5,1)	0.5081	0.7972	0.5437	0.7783	0.3888	**0.8231**
(0,1) (1.5,1) (2,1)	0.6152	0.9574	0.6637	0.9497	0.4473	**0.9667**
(0,1) (1.5,1) (1.75,1)	0.5758	0.9342	0.6242	0.9244	0.4184	**0.9495**

Table 5.8. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in RCBD =CRD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	0.2745	0.1706	**0.3241**	0.1665	0.2039	0.1748
(0,1) (0,1) (0,3)	0.2036	0.1198	**0.2196**	0.1171	0.1666	0.1246
(0,1) (0,2) (0,3)	0.3665	0.2289	**0.4361**	0.2228	0.2617	0.2347
(0,1) (0,2.5) (0,5)	0.4969	0.2993	**0.6007**	0.2937	0.3403	0.3071
(0,1) (0,2) (0,5)	0.4469	0.2645	**0.5369**	0.2564	0.3100	0.2710
(0,1) (0,3) (0,3.5)	0.4823	0.284	**0.5792**	0.2758	0.3289	0.2920
(0,1) (0,6) (0,8)	0.6586	0.4342	**0.7873**	0.4241	0.4421	0.4405

Table 5.9. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	0.6463	0.6623	**0.7117**	0.6460	0.4953	0.6883
(0,1) (0,1) (1,3)	0.3513	0.2697	**0.3834**	0.2608	0.2770	0.2799
(0,1) (0,2) (1,3)	0.5437	0.4024	**0.6239**	0.3914	0.3962	0.4208
(0,1) (0.5,2.5) (1,5)	0.6946	0.5665	**0.7904**	0.5494	0.5124	0.5836
(0,1) (0.25,2) (0.5,5)	0.5626	0.3948	**0.6556**	0.3825	0.3994	0.4098
(0,1) (1,3) (1.5,3.5)	0.7780	0.7225	**0.8487**	0.7080	0.6022	0.7431
(0,1) (1.5,6) (1.75,8)	0.9998	0.8440	0.9986	0.7846	**0.9999**	0.9272

Table 5.10. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_1	Z_1	Z_1	Z_1	Z_1
(0,1) (0,1) (0,1)	0.0535	0.0508	0.0535	0.0508	0.0535	0.0508
(0,1) (1,1) (1,1)	0.4264	**0.4837**	0.4264	0.4837	0.4264	0.4837
(0,1) (0,1) (1,1)	**0.3007**	0.2257	0.3007	0.2257	0.3007	0.2257
(0,1) (0.5,1) (1,1)	**0.3424**	0.3403	0.3424	0.3403	0.3424	0.3403
(0,1) (1,1) (1.5,1)	0.5386	**0.6238**	0.5386	0.6238	0.5386	0.6238
(0,1) (1.5,1) (2,1)	0.6621	**0.8374**	0.6621	0.8374	0.6621	0.8374
(0,1) (1.5,1) (1.75,1)	0.6268	**0.7919**	0.6268	0.7919	0.6268	0.7919

Table 5.11. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	**0.3192**	0.1716	**0.3192**	0.1716	**0.3192**	0.1716
(0,1) (0,1) (0,3)	**0.2246**	0.1268	**0.2246**	0.1268	**0.2246**	0.1268
(0,1) (0,2) (0,3)	**0.4156**	0.2277	**0.4156**	0.2277	**0.4156**	0.2277
(0,1) (0,2.5) (0,5)	**0.5605**	0.3073	**0.5605**	0.3073	**0.5605**	0.3073
(0,1) (0,2) (0,5)	**0.5108**	0.2749	**0.5108**	0.2749	**0.5108**	0.2749
(0,1) (0,3) (0,3.5)	**0.5437**	0.3039	**0.5437**	0.3039	**0.5437**	0.3039
(0,1) (0,6) (0,8)	**0.7274**	0.4404	**0.7274**	0.4404	**0.7274**	0.4404

Table 5.12. Estimated power of tests for mixed design under the normal distribution with different means and different variances; the variance CRD=2RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	**0.7138**	0.5675	**0.7138**	0.5675	**0.7138**	0.5675
(0,1) (0,1) (1,3)	**0.3898**	0.2306	**0.3898**	0.2306	**0.3898**	0.2306
(0,1) (0,2) (1,3)	**0.5961**	0.3714	**0.5961**	0.3714	**0.5961**	0.3714
(0,1) (0.5,2.5) (1,5)	**0.7488**	0.5032	**0.7488**	0.5032	**0.7488**	0.5032
(0,1) (0.25,2) (0.5,5)	**0.6365**	0.3843	**0.6365**	0.3843	**0.6365**	0.3843
(0,1) (1,3) (1.5,3.5)	**0.8288**	0.6561	**0.8288**	0.6561	**0.8288**	0.6561
(0,1) (1.5,6) (1.75,8)	**0.8806**	0.6721	**0.8806**	0.6721	**0.8806**	0.6721

Table 5.13. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0524	0.0504	0.0511	0.0541	0.0519	0.0512
(0,1) (1,1) (1,1)	0.3262	0.4023	0.3289	**0.5176**	0.2787	0.3282
(0,1) (0,1) (1,1)	0.2474	0.2014	0.2637	**0.2726**	0.2001	0.1658
(0,1) (0.5,1) (1,1)	0.2736	0.2887	0.2795	**0.3772**	0.2283	0.2339
(0,1) (1,1) (1.5,1)	0.4173	0.5266	0.4123	**0.6631**	0.3544	0.4328
(0,1) (1.5,1) (2,1)	0.4938	0.7193	0.4663	**0.8468**	0.4234	0.6073
(0,1) (1.5,1) (1.75,1)	0.4689	0.6702	0.4458	**0.8034**	0.4084	0.5578

Table 5.14. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	**0.2164**	0.1290	0.1906	0.1442	0.2158	0.1203
(0,1) (0,1) (0,3)	**0.1701**	0.1020	0.1589	0.1179	0.1614	0.0953
(0,1) (0,2) (0,3)	0.2759	0.1508	0.2333	0.1721	**0.2850**	0.1412
(0,1) (0,2.5) (0,5)	0.3647	0.1941	0.2991	0.2226	**0.3770**	0.1793
(0,1) (0,2) (0,5)	0.3385	0.1756	0.2841	0.2023	**0.3441**	0.1631
(0,1) (0,3) (0,3.5)	0.3486	0.1834	0.2911	0.2066	**0.3676**	0.1683
(0,1) (0,6) (0,8)	0.4783	0.2574	0.3726	0.2816	**0.5179**	0.2453

Table 5.15. Estimated power of tests for mixed design under the normal distribution with different means and different variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	**0.5238**	0.4252	0.4868	0.5224	0.4935	0.3631
(0,1) (0,1) (1,3)	**0.2835**	0.1835	0.276	0.2273	0.2559	0.1561
(0,1) (0,2) (1,3)	**0.4171**	0.2629	0.3694	0.3114	0.4060	0.2287
(0,1) (0.5,2.5) (1,5)	**0.5381**	0.3428	0.469	0.4020	0.5351	0.2978
(0,1) (0.25,2) (0.5,5)	**0.4288**	0.2525	0.3704	0.2977	0.4271	0.2225
(0,1) (1,3) (1.5,3.5)	**0.6375**	0.4804	0.5665	0.5674	0.6324	0.4135
(0,1) (1.5,6) (1.75,8)	0.7255	0.4552	0.6043	0.5497	**0.7535**	0.4315

Table 5.16. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance CRD=2RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0539	0.0505	0.0539	0.0490	0.0542	0.0505
(0,1) (1,1) (1,1)	0.3311	0.4444	0.3326	0.4254	0.2711	**0.4763**
(0,1) (0,1) (1,1)	**0.2297**	0.2082	0.2183	0.1970	0.2158	0.2239
(0,1) (0.5,1) (1,1)	0.2722	0.3248	0.2727	0.3110	0.2384	**0.3495**
(0,1) (1,1) (1.5,1)	0.4197	0.5787	0.4267	0.5568	0.3463	**0.6180**
(0,1) (1.5,1) (2,1)	0.5184	0.7982	0.5406	0.7741	0.3885	**0.8303**
(0,1) (1.5,1) (1.75,1)	0.4994	0.7553	0.5232	0.7364	0.3790	**0.7927**

Table 5.17. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=2RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	0.2801	0.1780	**0.3267**	0.1726	0.2093	0.1810
(0,1) (0,1) (0,3)	0.2048	0.1247	**0.2184**	0.1217	0.1679	0.1288
(0,1) (0,2) (0,3)	0.3560	0.2250	**0.4303**	0.2189	0.2479	0.2333
(0,1) (0,2.5) (0,5)	0.4941	0.3014	**0.5933**	0.2931	0.3419	0.3089
(0,1) (0,2) (0,5)	0.4526	0.2683	**0.5429**	0.2610	0.3176	0.2753
(0,1) (0,3) (0,3.5)	0.4848	0.2818	**0.5840**	0.2752	0.3352	0.2920
(0,1) (0,6) (0,8)	0.6659	0.4354	**0.7972**	0.4263	0.4535	0.4441

Table 5.18. Estimated power of tests for mixed design under the normal distribution with different means and different variances; the variance in CRD=2RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	0.6062	0.5393	**0.6580**	0.5210	0.4719	0.5681
(0,1) (0,1) (1,3)	0.3323	0.2276	**0.3526**	0.2184	0.2696	0.2398
(0,1) (0,2) (1,3)	0.5208	0.3654	**0.5911**	0.3529	0.3919	0.3787
(0,1) (0.5,2.5) (1,5)	0.6544	0.482	**0.7465**	0.4675	0.4827	0.5001
(0,1) (0.25,2) (0.5,5)	0.5925	0.3724	**0.6648**	0.3584	0.4449	0.3923
(0,1) (1,3) (1.5,3.5)	0.7522	0.6232	**0.8262**	0.6075	0.5809	0.6504
(0,1) (1.5,6) (1.75,8)	1	0.8027	0.9994	0.7254	1	0.9043

5.1.2. Four Treatments

Tables 5.19-5.36 show the simulation study results for four treatments (k=4) under the normal distribution. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion (Table 5.19 to Table 5.27). Second, the variance of the CRD portion is two times the variance of the RCBD portion (Table 5.28 to Table 5.36). Third, the variance of the CRD portion is four times the variance of the RCBD portion (see Appendix A). Forth, the variance of the CRD portion is eight times the variance of the RCBD portion (see Appendix A).

We estimated the alpha values for the proposed tests and tabled them in the first row of the table, where all populations have the same location and scale parameters (0,1) (0,1) (0,1) (0,1). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (5.19, 5.22, 5.25, 5.28, 5.31, and 5.34). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters.

The first case we considered is when the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location parameters and the same scale parameters, the proposed test two, Z_2, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

The second case we considered is when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

The last case we considered is when the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. Similarly, the proposed test three, Z_3, has the highest powers when the populations have different location and scale parameters.

In cases where the variance of the CRD portion is either four or eight times the variance of the RCBD portion, the proposed tests that have the highest powers will be approximately the same as the cases where the variance of the CRD portion is two times the variance of the RCBD portion. The only exception to this is when the number of blocks in the RCBD portion is equal to the sample size for each treatment in the CRD portion, and the populations have different location parameters and the same scale parameters, the proposed test one, Z_1, has the highest powers (see Appendix A).

Table 5.19. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 10$.

(M_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0506	0.0487	0.0506	0.0487	0.0506	0.0487
(0,1) (1,1) (1,1) (1,1)	0.4895	**0.7016**	0.4895	**0.7016**	0.4895	**0.7016**
(0,1) (0,1) (0,1) (1,1)	**0.2573**	0.2222	**0.2573**	0.2222	**0.2573**	0.2222
(0,1) (0,1) (1,1) (1,1)	0.4579	**0.4815**	0.4579	**0.4815**	0.4579	**0.4815**
(0,1) (0.5,1) (1,1) (1.5,1)	0.5587	**0.7113**	0.5587	**0.7113**	0.5587	**0.7113**
(0,1) (1,1) (1.5,1) (2,1)	0.6215	**0.9133**	0.6215	**0.9133**	0.6215	**0.9133**
(0,1) (1,1) (1.5,1) (1.75,1)	0.5903	**0.8921**	0.5903	**0.8921**	0.5903	**0.8921**

Table 5.20. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 10$.

(M_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	**0.3908**	0.1755	**0.3908**	0.1755	**0.3908**	0.1755
(0,1) (0,1) (0,3) (0,3)	**0.3884**	0.1664	**0.3884**	0.1664	**0.3884**	0.1664
(0,1) (0,2) (0,3) (0,4)	**0.6142**	0.2707	**0.6142**	0.2707	**0.6142**	0.2707
(0,1) (0,2.5) (0,5) (0,7.5)	**0.7767**	0.3848	**0.7767**	0.3848	**0.7767**	0.3848
(0,1) (0,2) (0,5) (0,1)	**0.3847**	0.1619	**0.3847**	0.1619	**0.3847**	0.1619
(0,1) (0,3) (0,3.5) (0,2)	**0.6004**	0.2749	**0.6004**	0.2749	**0.6004**	0.2749
(0,1) (0,4) (0,6) (0,8)	**0.8417**	0.4517	**0.8417**	0.4517	**0.8417**	0.4517

Table 5.21. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 10$.

(M_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	**0.8130**	0.7337	**0.8130**	0.7337	**0.8130**	0.7337
(0,1) (0,1) (0,3) (1,3)	**0.5667**	0.3046	**0.5667**	0.3046	**0.5667**	0.3046
(0,1) (0,2) (1,3) (1,4)	**0.8355**	0.5756	**0.8355**	0.5756	**0.8355**	0.5756
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	**0.9235**	0.6974	**0.9235**	0.6974	**0.9235**	0.6974
(0,1) (0.25,2) (0.5,5) (0.75,1)	**0.7299**	0.5076	**0.7299**	0.5076	**0.7299**	0.5076
(0,1) (1,3) (1.5,3.5) (2,2)	**0.9264**	0.8784	**0.9264**	0.8784	**0.9264**	0.8784
(0,1) (1,4) (1.5,6) (1.75,8)	**0.9536**	0.7866	**0.9536**	0.7866	**0.9536**	0.7866

Table 5.22. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(M_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0521	0.0581	0.0522	0.0539	0.0509	0.0549
(0,1) (1,1) (1,1) (1,1)	0.3621	0.5841	0.3508	**0.6770**	0.3214	0.4845
(0,1) (0,1) (0,1) (1,1)	0.2104	0.1941	0.2056	**0.2188**	0.1855	0.1612
(0,1) (0,1) (1,1) (1,1)	0.3658	0.4141	0.3592	**0.4932**	0.3184	0.3324
(0,1) (0.5,1) (1,1) (1.5,1)	0.4262	0.5939	0.4140	**0.6918**	0.3726	0.4876
(0,1) (1,1) (1.5,1) (2,1)	0.4492	0.8175	0.4327	**0.9014**	0.3939	0.7024
(0,1) (1,1) (1.5,1) (1.75,1)	0.4364	0.7853	0.4168	**0.8755**	0.3789	0.6623

Table 5.23. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(M_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	**0.2732**	0.1364	0.2450	0.1363	0.2648	0.1208
(0,1) (0,1) (0,3) (0,3)	**0.2742**	0.1286	0.2399	0.1322	0.2585	0.1080
(0,1) (0,2) (0,3) (0,4)	**0.4143**	0.1841	0.3571	0.1873	0.4089	0.1604
(0,1) (0,2.5) (0,5) (0,7.5)	0.5413	0.2329	0.4560	0.2377	**0.5497**	0.2055
(0,1) (0,2) (0,5) (0,1)	**0.2664**	0.1198	0.2477	0.1268	0.2472	0.1035
(0,1) (0,3) (0,3.5) (0,2)	**0.4080**	0.1753	0.3512	0.1798	0.4021	0.1502
(0,1) (0,4) (0,6) (0,8)	0.6108	0.2749	0.5160	0.2823	**0.6266**	0.2356

Table 5.24. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(M_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	0.6414	0.5817	0.5997	**0.6644**	0.6009	0.4844
(0,1) (0,1) (0,3) (1,3)	**0.4041**	0.2156	0.3696	0.2420	0.3720	0.1787
(0,1) (0,2) (1,3) (1,4)	**0.6349**	0.3984	0.5685	0.4371	0.6157	0.3291
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	**0.7553**	0.4844	0.6743	0.5310	0.7414	0.4122
(0,1) (0.25,2) (0.5,5) (0.75,1)	**0.5698**	0.3399	0.5312	0.4167	0.5303	0.2949
(0,1) (1,3) (1.5,3.5) (2,2)	0.7976	0.7340	0.7566	**0.8141**	0.7587	0.6307
(0,1) (1,4) (1.5,6) (1.75,8)	**0.8198**	0.5803	0.7426	0.6318	0.8109	0.4991

Table 5.25. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in RCBD =CRD; K=4; $n_b = 5, n_a = 10$.

$(M_1, \sigma_1)\ (\mu_2, \sigma_2)\ (\mu_3, \sigma_3)\ (\mu_4, \sigma_4)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0510	0.0536	0.0530	0.0555	0.0517	0.0551
(0,1) (1,1) (1,1) (1,1)	0.3871	0.6506	0.4027	0.6409	0.3111	**0.6868**
(0,1) (0,1) (0,1) (1,1)	0.2114	0.2083	0.2151	0.2091	0.1853	**0.2225**
(0,1) (0,1) (1,1) (1,1)	0.3677	0.4449	0.3736	0.4387	0.3078	**0.4741**
(0,1) (0.5,1) (1,1) (1.5,1)	0.4314	0.6616	0.4571	0.6514	0.3549	**0.6972**
(0,1) (1,1) (1.5,1) (2,1)	0.4898	0.8857	0.5285	0.8762	0.3767	**0.9103**
(0,1) (1,1) (1.5,1) (1.75,1)	0.4846	0.8658	0.5177	0.8565	0.3723	**0.8932**

Table 5.26. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 5, n_a = 10$.

$(M_1, \sigma_1)\ (\mu_2, \sigma_2)\ (\mu_3, \sigma_3)\ (\mu_4, \sigma_4)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	0.3338	0.1663	**0.3767**	0.1682	0.2411	0.1738
(0,1) (0,1) (0,3) (0,3)	0.3267	0.1584	**0.3610**	0.1589	0.2514	0.1684
(0,1) (0,2) (0,3) (0,4)	0.5196	0.2595	**0.6015**	0.2624	0.3638	0.2703
(0,1) (0,2.5) (0,5) (0,7.5)	0.6982	0.3788	**0.7992**	0.3816	0.4978	0.3925
(0,1) (0,2) (0,5) (0,1)	0.3183	0.1460	**0.3581**	0.1473	0.2393	0.1533
(0,1) (0,3) (0,3.5) (0,2)	0.5157	0.2468	**0.5888**	0.2487	0.3705	0.2644
(0,1) (0,4) (0,6) (0,8)	0.7721	0.4472	**0.8695**	0.4506	0.5553	0.4628

Table 5.27. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 5, n_a = 10$.

$(M_1, \sigma_1)\ (\mu_2, \sigma_2)\ (\mu_3, \sigma_3)\ (\mu_4, \sigma_4)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	0.7119	0.6988	**0.7611**	0.6917	0.5692	0.7287
(0,1) (0,1) (0,3) (1,3)	0.4715	0.2789	**0.5132**	0.2775	0.3604	0.2969
(0,1) (0,2) (1,3) (1,4)	0.7380	0.5448	**0.8083**	0.5408	0.5659	0.5691
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	0.8532	0.6715	**0.9119**	0.6662	0.6762	0.6966
(0,1) (0.25,2) (0.5,5) (0.75,1)	0.6186	0.4768	**0.6726**	0.4710	0.4848	0.5032
(0,1) (1,3) (1.5,3.5) (2,2)	0.8507	0.8488	**0.8896**	0.8397	0.7133	0.8707
(0,1) (1,4) (1.5,6) (1.75,8)	0.9024	0.7701	**0.9468**	0.7651	0.7486	0.7928

Table 5.28. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 10$.

(M_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0517	0.0505	0.0517	0.0505	0.0517	0.0505
(0,1) (1,1) (1,1) (1,1)	0.4213	**0.5019**	0.4213	**0.5019**	0.4213	**0.5019**
(0,1) (0,1) (0,1) (1,1)	**0.2154**	0.1604	**0.2154**	0.1604	**0.2154**	0.1604
(0,1) (0,1) (1,1) (1,1)	**0.3852**	0.3311	**0.3852**	0.3311	**0.3852**	0.3311
(0,1) (0.5,1) (1,1) (1.5,1)	0.4772	**0.5156**	0.4772	**0.5156**	0.4772	**0.5156**
(0,1) (1,1) (1.5,1) (2,1)	0.5667	**0.7502**	0.5667	**0.7502**	0.5667	**0.7502**
(0,1) (1,1) (1.5,1) (1.75,1)	0.5475	**0.7244**	0.5475	**0.7244**	0.5475	**0.7244**

Table 5.29. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 10$.

(M_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	**0.4032**	0.1797	**0.4032**	0.1797	**0.4032**	0.1797
(0,1) (0,1) (0,3) (0,3)	**0.3880**	0.1655	**0.3880**	0.1655	**0.3880**	0.1655
(0,1) (0,2) (0,3) (0,4)	**0.6199**	0.2793	**0.6199**	0.2793	**0.6199**	0.2793
(0,1) (0,2.5) (0,5) (0,7.5)	**0.7851**	0.3941	**0.7851**	0.3941	**0.7851**	0.3941
(0,1) (0,2) (0,5) (0,1)	**0.3860**	0.1654	**0.3860**	0.1654	**0.3860**	0.1654
(0,1) (0,3) (0,3.5) (0,2)	**0.7851**	0.3941	**0.7851**	0.3941	**0.7851**	0.3941
(0,1) (0,4) (0,6) (0,8)	**0.8437**	0.4564	**0.8437**	0.4564	**0.8437**	0.4564

Table 5.30. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance CRD=2RCBD; K=4; $n_b = 10, n_a = 10$.

(M_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	**0.7806**	0.6115	**0.7806**	0.6115	**0.7806**	0.6115
(0,1) (0,1) (0,3) (1,3)	**0.5455**	0.2636	**0.5455**	0.2636	**0.5455**	0.2636
(0,1) (0,2) (1,3) (1,4)	**0.8128**	0.4929	**0.8128**	0.4929	**0.8128**	0.4929
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	**0.9086**	0.6220	**0.9086**	0.6220	**0.9086**	0.6220
(0,1) (0.25,2) (0.5,5) (0.75,1)	**0.6955**	0.4136	**0.6955**	0.4136	**0.6955**	0.4136
(0,1) (1,3) (1.5,3.5) (2,2)	**0.9126**	0.7867	**0.9126**	0.7867	**0.9126**	0.7867
(0,1) (1,4) (1.5,6) (1.75,8)	**0.9493**	0.7280	**0.9493**	0.7280	**0.9493**	0.7280

Table 5.31. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(M_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0499	0.0508	0.0465	0.0499	0.0455	0.0457
(0,1) (1,1) (1,1) (1,1)	0.3229	0.4373	0.3225	**0.5455**	0.2756	0.3396
(0,1) (0,1) (0,1) (1,1)	0.1805	0.1576	**0.1902**	0.1840	0.1532	0.1296
(0,1) (0,1) (1,1) (1,1)	0.3081	0.3131	0.3227	**0.3887**	0.2573	0.2439
(0,1) (0.5,1) (1,1) (1.5,1)	0.3719	0.4548	0.3814	**0.5652**	0.3116	0.3529
(0,1) (1,1) (1.5,1) (2,1)	0.4313	0.6709	0.4194	**0.8019**	0.3712	0.5419
(0,1) (1,1) (1.5,1) (1.75,1)	0.4201	0.6431	0.4128	**0.7718**	0.358	0.5125

Table 5.32. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(M_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	**0.2676**	0.1286	0.2364	0.1298	0.2564	0.1104
(0,1) (0,1) (0,3) (0,3)	**0.2778**	0.1291	0.2490	0.1334	0.2591	0.1089
(0,1) (0,2) (0,3) (0,4)	**0.4154**	0.1851	0.3537	0.1891	0.4085	0.1600
(0,1) (0,2.5) (0,5) (0,7.5)	0.5507	0.2396	0.4634	0.2474	**0.5576**	0.2043
(0,1) (0,2) (0,5) (0,1)	**0.2685**	0.1285	0.2454	0.1318	0.2557	0.1078
(0,1) (0,3) (0,3.5) (0,2)	**0.3979**	0.1730	0.3443	0.1768	0.3917	0.1506
(0,1) (0,4) (0,6) (0,8)	0.6020	0.2712	0.5007	0.2754	**0.6207**	0.2336

Table 5.33. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(M_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	**0.6252**	0.4957	0.5944	0.5878	0.5686	0.3944
(0,1) (0,1) (0,3) (1,3)	**0.3797**	0.2004	0.3546	0.2233	0.3433	0.1647
(0,1) (0,2) (1,3) (1,4)	**0.6172**	0.3569	0.5646	0.4062	0.5867	0.2872
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	**0.7332**	0.4332	0.6574	0.4875	0.7199	0.3581
(0,1) (0.25,2) (0.5,5) (0.75,1)	**0.5316**	0.2704	0.5041	0.3466	0.4752	0.2322
(0,1) (1,3) (1.5,3.5) (2,2)	**0.7833**	0.6546	0.7425	0.7549	0.7505	0.5395
(0,1) (1,4) (1.5,6) (1.75,8)	**0.8004**	0.5166	0.7317	0.5800	0.7898	0.4325

Table 5.34. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance CRD=2RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0485	0.0491	0.0487	0.0516	0.0478	0.0507
(0,1) (1,1) (1,1) (1,1)	0.3273	0.4623	0.3295	0.4529	0.2799	**0.5027**
(0,1) (0,1) (0,1) (1,1)	**0.1676**	0.1428	0.1573	0.1428	0.1596	0.1577
(0,1) (0,1) (1,1) (1,1)	0.2932	0.2935	0.2848	0.2888	0.2663	**0.3232**
(0,1) (0.5,1) (1,1) (1.5,1)	0.3672	0.457	0.3616	0.4482	0.3211	**0.4969**
(0,1) (1,1) (1.5,1) (2,1)	0.4571	0.7095	0.4713	0.6933	0.3601	**0.7509**
(0,1) (1,1) (1.5,1) (1.75,1)	0.4272	0.6755	0.4454	0.6619	0.3475	**0.7184**

Table 5.35. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=2RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	0.3393	0.1690	**0.3845**	0.1726	0.2438	0.1801
(0,1) (0,1) (0,3) (0,3)	0.3346	0.1624	**0.3744**	0.1649	0.2498	0.1713
(0,1) (0,2) (0,3) (0,4)	0.5290	0.2634	**0.6160**	0.2654	0.3773	0.2772
(0,1) (0,2.5) (0,5) (0,7.5)	0.6995	0.3706	**0.7962**	0.3729	0.4976	0.3864
(0,1) (0,2) (0,5) (0,1)	0.3260	0.1540	**0.3600**	0.1558	0.2483	0.1624
(0,1) (0,3) (0,3.5) (0,2)	0.5099	0.2598	**0.5884**	0.2638	0.3615	0.2701
(0,1) (0,4) (0,6) (0,8)	0.7662	0.4435	**0.8613**	0.4462	0.5518	0.4584

Table 5.36. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=2RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	0.6864	0.5830	**0.7293**	0.5732	0.5487	0.6199
(0,1) (0,1) (0,3) (1,3)	0.4445	0.2398	**0.4832**	0.2393	0.3405	0.2575
(0,1) (0,2) (1,3) (1,4)	0.7213	0.4684	**0.7885**	0.4640	0.5655	0.4970
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	0.8452	0.6062	**0.9036**	0.6010	0.6675	0.6331
(0,1) (0.25,2) (0.5,5) (0.75,1)	0.5713	0.3787	**0.6105**	0.3736	0.4571	0.4085
(0,1) (1,3) (1.5,3.5) (2,2)	0.8389	0.7533	**0.8822**	0.7429	0.6985	0.7887
(0,1) (1,4) (1.5,6) (1.75,8)	0.8995	0.7056	**0.9455**	0.7018	0.7351	0.7320

5.1.3. Five Treatments

Tables 5.37-5.54 show the simulation study results for five treatments (k=5) under the normal distribution. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion (Table 5.37 to Table 5.45). Second, the variance of the CRD portion is two times the variance of the RCBD portion (Table 5.46 to Table 5.54). Third, the variance of the CRD portion is four times the variance of the RCBD portion (see Appendix A). Forth, the variance of the CRD portion is eight times the variance of the RCBD portion (see Appendix A).

We estimated the alpha values for the proposed tests and tabled them in the first row of the table, where all populations have the same location and scale parameters (0,1) (0,1) (0,1) (0,1) (0,1). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (5.37, 5.40, 5.43, 5.46, 5.49, and 5.53). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters.

We started with the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location parameters and the same scale parameters, the proposed test two, Z_2, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Next, we considered the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Finally, we considered the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. Similarly, the proposed test three, Z_3, has the highest powers when the populations have different location and scale parameters.

In cases where the variance of the CRD portion is either four or eight times the variance of the RCBD portion, the proposed tests that have the highest powers will be approximately the same as the cases where the variance of the CRD portion is two times the variance of the RCBD portion. The only exception to this is when the number of blocks in the RCBD portion is equal to the sample size for each treatment in the CRD portion, and the populations have different location parameters and the same scale parameters, the proposed test one, Z_1, has the highest powers (see Appendix A).

Table 5.37. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4) (μ_5,σ_5)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0530	0.0503	0.0530	0.0503	0.0530	0.0503
(0,1) (1,1) (1,1) (1,1) (1,1)	0.4668	**0.6857**	0.4668	**0.6857**	0.4668	**0.6857**
(0,1) (0,1) (0,1) (1,1) (1,1)	**0.3847**	0.3579	**0.3847**	0.3579	**0.3847**	0.3579
(0,1) (0,1) (1,1) (1,1) (1,1)	0.4689	**0.5485**	0.4689	**0.5485**	0.4689	**0.5485**
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.6361	**0.8341**	0.6361	**0.8341**	0.6361	**0.8341**
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.5944	**0.9547**	0.5944	**0.9547**	0.5944	**0.9547**
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.5946	**0.9431**	0.5946	**0.9431**	0.5946	**0.9431**

Table 5.38. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	**0.4429**	0.1848	**0.4429**	0.1848	**0.4429**	0.1848
(0,1) (0,1) (0,3) (0,3) (0,3)	**0.5208**	0.2004	**0.5208**	0.2004	**0.5208**	0.2004
(0,1) (0,2) (0,3) (0,4) (0,5)	**0.7449**	0.3297	**0.7449**	0.3297	**0.7449**	0.3297
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	**0.8940**	0.4744	**0.8940**	0.4744	**0.8940**	0.4744
(0,1) (0,2) (0,5) (0,1) (0,4.5)	**0.5551**	0.2173	**0.5551**	0.2173	**0.5551**	0.2173
(0,1) (0,3) (0,3.5) (0,2) (0,6)	**0.7593**	0.3389	**0.7593**	0.3389	**0.7593**	0.3389
(0,1) (0,4) (0,6) (0,8) (0,10)	**0.9239**	0.5348	**0.9239**	0.5348	**0.9239**	0.5348

Table 5.39. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	**0.8456**	0.7680	**0.8456**	0.7680	**0.8456**	0.7680
(0,1) (0,1) (0,3) (1,3) (1,3)	**0.7424**	0.4409	**0.7424**	0.4409	**0.7424**	0.4409
(0,1) (0,2) (1,3) (1,4) (1,5)	**0.9205**	0.6716	**0.9205**	0.6716	**0.9205**	0.6716
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	**0.9673**	0.7860	**0.9673**	0.7860	**0.9673**	0.7860
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	**0.8575**	0.6009	**0.8575**	0.6009	**0.8575**	0.6009
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	**0.9530**	0.9011	**0.9530**	0.9011	**0.9530**	0.9011
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	**0.9790**	0.8478	**0.9790**	0.8478	**0.9790**	0.8478

Table 5.40. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0515	0.0461	0.0518	0.0509	0.0486	0.0472
(0,1) (1,1) (1,1) (1,1) (1,1)	0.3564	0.5453	0.3446	**0.6597**	0.3073	0.4715
(0,1) (0,1) (0,1) (1,1) (1,1)	0.3028	0.2798	0.3019	**0.3564**	0.2612	0.2440
(0,1) (0,1) (1,1) (1,1) (1,1)	0.3714	0.4283	0.3607	**0.5317**	0.3210	0.3659
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.4784	0.6858	0.4668	**0.8125**	0.4080	0.5918
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.4305	0.8559	0.4193	**0.9381**	0.3535	0.7564
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.4409	0.8415	0.4295	**0.9263**	0.3683	0.7476

Table 5.41. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	**0.3048**	0.1209	0.2745	0.1456	0.2846	0.1101
(0,1) (0,1) (0,3) (0,3) (0,3)	**0.3479**	0.1301	0.3194	0.1542	0.3213	0.1140
(0,1) (0,2) (0,3) (0,4) (0,5)	**0.5413**	0.1934	0.4673	0.2381	0.5229	0.1706
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	0.6826	0.2601	0.5938	0.3143	**0.6857**	0.2327
(0,1) (0,2) (0,5) (0,1) (0,4.5)	**0.3924**	0.1414	0.3545	0.1752	0.3655	0.1255
(0,1) (0,3) (0,3.5) (0,2) (0,6)	**0.5368**	0.1912	0.4744	0.2331	0.5209	0.1709
(0,1) (0,4) (0,6) (0,8) (0,10)	0.7301	0.2995	0.6296	0.3544	**0.7405**	0.2677

Table 5.42. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	0.6870	0.5834	0.6521	**0.7020**	0.6312	0.5034
(0,1) (0,1) (0,3) (1,3) (1,3)	**0.5699**	0.3079	0.5269	0.3873	0.5328	0.2597
(0,1) (0,2) (1,3) (1,4) (1,5)	**0.7547**	0.4538	0.6876	0.5497	0.7280	0.3906
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	**0.8485**	0.5524	0.782	0.6479	0.8371	0.4807
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	**0.6840**	0.4265	0.6337	0.5296	0.6456	0.3631
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	0.8418	0.7392	0.8065	**0.8433**	0.7969	0.6562
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	**0.8867**	0.6297	0.8304	0.7244	0.8697	0.5523

Table 5.43. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0545	0.0516	0.0525	0.0518	0.0532	0.0541
(0,1) (1,1) (1,1) (1,1) (1,1)	0.3823	0.6612	0.3918	0.6481	0.3100	**0.6951**
(0,1) (0,1) (0,1) (1,1) (1,1)	0.3095	0.3366	0.3123	0.3302	0.2559	**0.3597**
(0,1) (0,1) (1,1) (1,1) (1,1)	0.3847	0.5153	0.3937	0.5051	0.3225	**0.5499**
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.5027	0.8022	0.5184	0.7894	0.4173	**0.8349**
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.6209	0.9411	0.5902	0.9303	0.5555	**0.9589**
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.4533	0.9221	0.4757	0.9123	0.3528	**0.9426**

Table 5.44. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	0.3694	0.1694	**0.4107**	0.1679	0.2756	0.1799
(0,1) (0,1) (0,3) (0,3) (0,3)	0.4249	0.1876	**0.4689**	0.1887	0.3138	0.2007
(0,1) (0,2) (0,3) (0,4) (0,5)	0.6526	0.3074	**0.7440**	0.3059	0.4662	0.3249
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	0.8186	0.4528	**0.8954**	0.4478	0.6081	0.4773
(0,1) (0,2) (0,5) (0,1) (0,4.5)	0.4576	0.1995	**0.5166**	0.1978	0.3340	0.2146
(0,1) (0,3) (0,3.5) (0,2) (0,6)	0.6519	0.3132	**0.7402**	0.3106	0.4734	0.3328
(0,1) (0,4) (0,6) (0,8) (0,10)	0.8674	0.5112	**0.9336**	0.5098	0.6524	0.5366

Table 5.45. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	0.7411	0.7325	**0.7872**	0.7202	0.6062	0.7631
(0,1) (0,1) (0,3) (1,3) (1,3)	0.6462	0.4188	**0.7003**	0.4117	0.4983	0.4460
(0,1) (0,2) (1,3) (1,4) (1,5)	0.8393	0.6273	**0.8968**	0.6172	0.6714	0.6621
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	0.9259	0.758	**0.9629**	0.7501	0.7885	0.7849
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	0.7537	0.5714	**0.8053**	0.5607	0.5936	0.6056
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	0.8927	0.8730	**0.9196**	0.8623	0.7732	0.8981
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	0.9733	0.8444	**0.9796**	0.8276	0.919	0.8780

Table 5.46. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0522	0.0528	0.0522	0.0528	0.0522	0.0528
(0,1) (1,1) (1,1) (1,1) (1,1)	0.4188	**0.5083**	0.4188	**0.5083**	0.4188	**0.5083**
(0,1) (0,1) (0,1) (1,1) (1,1)	**0.3085**	0.2482	**0.3085**	0.2482	**0.3085**	0.2482
(0,1) (0,1) (1,1) (1,1) (1,1)	**0.4043**	0.3911	**0.4043**	0.3911	**0.4043**	0.3911
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.5617	**0.6567**	0.5617	**0.6567**	0.5617	**0.6567**
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.5915	**0.8375**	0.5915	**0.8375**	0.5915	**0.8375**
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.5749	**0.8189**	0.5749	**0.8189**	0.5749	**0.8189**

Table 5.47. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	**0.4462**	0.1813	**0.4462**	0.1813	**0.4462**	0.1813
(0,1) (0,1) (0,3) (0,3) (0,3)	**0.5189**	0.2020	**0.5189**	0.2020	**0.5189**	0.2020
(0,1) (0,2) (0,3) (0,4) (0,5)	**0.7475**	0.3276	**0.7475**	0.3276	**0.7475**	0.3276
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	**0.8940**	0.4753	**0.8940**	0.4753	**0.8940**	0.4753
(0,1) (0,2) (0,5) (0,1) (0,4.5)	**0.5712**	0.2234	**0.5712**	0.2234	**0.5712**	0.2234
(0,1) (0,3) (0,3.5) (0,2) (0,6)	**0.7585**	0.3320	**0.7585**	0.3320	**0.7585**	0.3320
(0,1) (0,4) (0,6) (0,8) (0,10)	**0.9298**	0.5402	**0.9298**	0.5402	**0.9298**	0.5402

Table 5.48. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance CRD=2RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	**0.8238**	0.6500	**0.8238**	0.6500	**0.8238**	0.6500
(0,1) (0,1) (0,3) (1,3) (1,3)	**0.7194**	0.3735	**0.7194**	0.3735	**0.7194**	0.3735
(0,1) (0,2) (1,3) (1,4) (1,5)	**0.9077**	0.5844	**0.9077**	0.5844	**0.9077**	0.5844
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	**0.9641**	0.7226	**0.9641**	0.7226	**0.9641**	0.7226
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	**0.8283**	0.5041	**0.8283**	0.5041	**0.8283**	0.5041
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	**0.9582**	0.8427	**0.9582**	0.8427	**0.9582**	0.8427
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	**0.9794**	0.8103	**0.9794**	0.8103	**0.9794**	0.8103

Table 5.49. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0527	0.0488	0.0527	0.0520	0.0528	0.0529
(0,1) (1,1) (1,1) (1,1) (1,1)	0.3155	0.4221	0.3239	**0.5500**	0.2649	0.3464
(0,1) (0,1) (0,1) (1,1) (1,1)	0.2556	0.2065	0.2727	**0.2749**	0.202	0.1723
(0,1) (0,1) (1,1) (1,1) (1,1)	0.3199	0.3231	0.3276	**0.4318**	0.2617	0.2666
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.439	0.5516	0.4429	**0.6978**	0.3644	0.4478
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.4366	0.7236	0.4274	**0.8620**	0.374	0.6151
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.4293	0.7031	0.4174	**0.8388**	0.3675	0.5897

Table 5.50. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	**0.3089**	0.1232	0.2836	0.1494	0.2910	0.1131
(0,1) (0,1) (0,3) (0,3) (0,3)	**0.3536**	0.1268	0.3227	0.1566	0.3273	0.1168
(0,1) (0,2) (0,3) (0,4) (0,5)	**0.5411**	0.1926	0.4690	0.2391	0.5262	0.1730
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	**0.6724**	0.2550	0.5846	0.3133	0.6692	0.2260
(0,1) (0,2) (0,5) (0,1) (0,4.5)	**0.3936**	0.1446	0.3548	0.1792	0.3680	0.1286
(0,1) (0,3) (0,3.5) (0,2) (0,6)	**0.5408**	0.1928	0.4701	0.2411	0.5261	0.1698
(0,1) (0,4) (0,6) (0,8) (0,10)	0.7217	0.2940	0.6243	0.3467	**0.7299**	0.2626

Table 5.51. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	**0.6597**	0.5008	0.6261	0.6247	0.6000	0.4153
(0,1) (0,1) (0,3) (1,3) (1,3)	**0.6694**	0.4840	0.5911	0.5447	0.6617	0.4414
(0,1) (0,2) (1,3) (1,4) (1,5)	**0.7255**	0.3777	0.6769	0.4808	0.6876	0.3208
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	**0.8405**	0.5020	0.7788	0.6025	0.8235	0.4338
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	**0.6413**	0.3203	0.6172	0.4347	0.5771	0.2540
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	**0.8444**	0.6824	0.8113	0.802	0.8067	0.5854
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	**0.8789**	0.5726	0.8221	0.6801	0.866	0.4973

Table 5.52. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0513	0.0520	0.0537	0.0523	0.0514	0.0533
(0,1) (1,1) (1,1) (1,1) (1,1)	0.3275	0.4704	0.3315	0.4575	0.2734	**0.5108**
(0,1) (0,1) (0,1) (1,1) (1,1)	0.2509	0.2325	0.2332	0.2288	0.2282	**0.2535**
(0,1) (0,1) (1,1) (1,1) (1,1)	0.3056	0.3536	0.2946	0.3425	0.2775	**0.3851**
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.4469	0.6112	0.4496	0.5931	0.3696	**0.6526**
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.4526	0.7874	0.4696	0.7728	0.3454	**0.8289**
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.4488	0.7782	0.4652	0.7600	0.3524	**0.8178**

Table 5.53. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4) (μ_5,σ_5)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	0.3734	0.1732	**0.4197**	0.1739	0.2789	0.1835
(0,1) (0,1) (0,3) (0,3) (0,3)	0.4183	0.1846	**0.4720**	0.1834	0.3098	0.1993
(0,1) (0,2) (0,3) (0,4) (0,5)	0.6550	0.3159	**0.7433**	0.3121	0.4722	0.3366
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	0.8121	0.4415	**0.8979**	0.4373	0.6126	0.4671
(0,1) (0,2) (0,5) (0,1) (0,4.5)	0.4624	0.2026	**0.5219**	0.2028	0.3425	0.2199
(0,1) (0,3) (0,3.5) (0,2) (0,6)	0.6557	0.3166	**0.7452**	0.3154	0.4747	0.3364
(0,1) (0,4) (0,6) (0,8) (0,10)	0.8645	0.5077	**0.9377**	0.5040	0.6612	0.5323

Table 5.54. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4) (μ_5,σ_5)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	0.7135	0.6099	**0.7564**	0.5969	0.5824	0.651
(0,1) (0,1) (0,3) (1,3) (1,3)	0.6086	0.3520	**0.6512**	0.3462	0.4786	0.3828
(0,1) (0,2) (1,3) (1,4) (1,5)	0.8224	0.5542	**0.8802**	0.5460	0.6582	0.5895
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	0.9175	0.6876	**0.9580**	0.6801	0.7669	0.7199
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	0.7186	0.4619	**0.7647**	0.4528	0.5754	0.4988
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	0.9010	0.8074	**0.9298**	0.7937	0.7733	0.8387
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	0.9428	0.7654	**0.9705**	0.7559	0.8144	0.7979

5.2. Results for the Exponential Distribution

5.2.1. Three Treatments

Tables 5.55-5.72 show the simulation study results for three treatments (k=3) under the exponential distribution. In the tables, (μ, σ^2) represent the mean and variance of the random variable generated from an exponential distribution. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion (Table 5.55 to Table 5.63). Second, the variance of the CRD portion is two times the variance of the RCBD portion (Table 5.64 to Table 5.72). Third, the variance of the CRD portion is four times the variance of the RCBD portion (see

Appendix B). Forth, the variance of the CRD portion is eight times the variance of the RCBD portion (see Appendix B).

We estimated the alpha values for the proposed tests and tabled them in the first row of the tables, where all populations have the same location and scale parameters (1,1) (1,1) (1,1). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (5.55,5.58,5.61,5.64,5.67 and 5.70). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters.

We started with the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location parameters and the same scale parameters, the proposed test two, Z_2, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. In another scenario, when the populations have different location and scale parameters and the variance in the CRD portion is equal to the variance in the RCBD portion, the proposed test two, Z_2, has the highest powers. Otherwise, the proposed test one, Z_1, has the highest powers.

Next, we considered the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test four, Z_4, has the highest powers.

Finally, we considered the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. In another scenario, when the populations have different location and scale parameters and the variance in the CRD portion is equal to the variance in the RCBD portion, the proposed test six, Z_6, has the highest powers. Otherwise, the proposed test three, Z_3, has the highest powers.

In cases where the variance of the CRD portion is either four or eight times the variance of the RCBD portion, the proposed tests that have the highest powers are approximately the same as those where the variance of the CRD portion is two times the variance of the RCBD portion. The only exception to this is when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion and the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers (see Appendix B).

Table 5.55. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0489	0.0505	0.0489	0.0505	0.0489	0.0505
(1,1) (2,1) (2,1)	0.1753	**0.5769**	0.1753	**0.5769**	0.1753	**0.5769**
(1,1) (1,1) (2,1)	**0.3461**	0.3329	**0.3461**	0.3329	**0.3461**	0.3329
(1,1) (1.5,1) (2,1)	0.1877	**0.4451**	0.1877	**0.4451**	0.1877	**0.4451**
(1,1) (2,1) (2.5,1)	0.2759	**0.7266**	0.2759	**0.7266**	0.2759	**0.7266**
(1,1) (2.5,1) (3,1)	0.3847	**0.9003**	0.3847	**0.9003**	0.3847	**0.9003**
(1,1) (2.5,1) (2.75,1)	0.3259	**0.8651**	0.3259	**0.8651**	0.3259	**0.8651**

Table 5.56. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	**0.1289**	0.0433	**0.1289**	0.0433	**0.1289**	0.0433
(1,1) (1,1) (1,3)	**0.1280**	0.0521	**0.1280**	0.0521	**0.1280**	0.0521
(1,1) (1,2) (1,3)	**0.1520**	0.0487	**0.1520**	0.0487	**0.1520**	0.0487
(1,1) (1,5) (1,7.5)	**0.1760**	0.0580	**0.1760**	0.0580	**0.1760**	0.0580
(1,1) (1,2) (1,5)	**0.1641**	0.0521	**0.1641**	0.0521	**0.1641**	0.0521
(1,1) (1,3) (1,3.5)	**0.1689**	0.0505	**0.1689**	0.0505	**0.1689**	0.0505
(1,1) (1,6) (1,8)	**0.2202**	0.0720	**0.2202**	0.0720	**0.2202**	0.0720

Table 5.57. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	0.4517	**0.4737**	0.4517	**0.4737**	0.4517	**0.4737**
(1,1) (1,1) (3,9)	**0.4349**	0.3633	**0.4349**	0.3633	**0.4349**	0.3633
(1,1) (2,4) (3,9)	0.5946	**0.6413**	0.5946	**0.6413**	0.5946	**0.6413**
(1,1) (2.5,6.25) (5,25)	0.7599	**0.8453**	0.7599	**0.8453**	0.7599	**0.8453**
(1,1) (2,4) (5,25)	0.7478	**0.7931**	0.7478	**0.7931**	0.7478	**0.7931**
(1,1) (3,9) (3.5,12.25)	0.7324	**0.8101**	0.7324	**0.8101**	0.7324	**0.8101**
(1,1) (6,36) (8,64)	0.8644	**0.9766**	0.8644	**0.9766**	0.8644	**0.9766**

Table 5.58. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0497	0.0515	0.0495	0.0537	0.0498	0.0504
(1,1) (2,1) (2,1)	0.1445	0.4737	0.1503	**0.5854**	0.1108	0.3939
(1,1) (1,1) (2,1)	0.2938	0.2911	0.3069	**0.3798**	0.2450	0.2363
(1,1) (1.5,1) (2,1)	0.1724	0.3683	0.1818	**0.4618**	0.1356	0.3063
(1,1) (2,1) (2.5,1)	0.2100	0.6051	0.2121	**0.7261**	0.1558	0.5071
(1,1) (2.5,1) (3,1)	0.2481	0.7944	0.2357	**0.8930**	0.1740	0.6941
(1,1) (2.5,1) (2.75,1)	0.2037	0.7440	0.1997	**0.8526**	0.1429	0.6418

Table 5.59. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	0.0832	0.0340	0.0719	0.0312	**0.0936**	0.0372
(1,1) (1,1) (1,3)	**0.1007**	0.0437	0.0908	0.0479	0.0979	0.0450
(1,1) (1,2) (1,3)	0.0888	0.0353	0.0697	0.0321	**0.1011**	0.0381
(1,1) (1,5) (1,7.5)	0.1062	0.0308	0.0741	0.0265	**0.1339**	0.0393
(1,1) (1,2) (1,5)	0.0953	0.0346	0.0762	0.0312	**0.1144**	0.0402
(1,1) (1,3) (1,3.5)	0.0944	0.0316	0.0722	0.0284	**0.1134**	0.0369
(1,1) (1,6) (1,8)	0.1047	0.0348	0.0752	0.0280	**0.1379**	0.0439

Table 5.60. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	0.2947	0.2722	0.2959	**0.3602**	0.2569	0.2270
(1,1) (1,1) (3,9)	0.4301	0.4617	0.3867	**0.5191**	0.4121	0.4179
(1,1) (2,4) (3,9)	0.4512	0.4954	0.4310	**0.5951**	0.4152	0.4268
(1,1) (2.5,6.25) (5,25)	0.6652	0.8729	0.6063	**0.9367**	0.6162	0.7962
(1,1) (2,4) (5,25)	0.5755	0.6399	0.5412	**0.7445**	0.5247	0.5489
(1,1) (3,9) (3.5,12.25)	0.5302	0.6195	0.4948	**0.7223**	0.4895	0.5377
(1,1) (6,36) (8,64)	0.6844	0.8940	0.6294	**0.9517**	0.6254	0.8224

Table 5.61. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in RCBD =CRD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0534	0.0504	0.0500	0.0496	0.0524	0.0507
(1,1) (2,1) (2,1)	0.1251	0.5573	0.1170	0.5343	0.1113	**0.5878**
(1,1) (1,1) (2,1)	0.2724	0.3147	0.2536	0.2989	0.2497	**0.3375**
(1,1) (1.5,1) (2,1)	0.1504	0.4153	0.1359	0.3985	0.1389	**0.4412**
(1,1) (2,1) (2.5,1)	0.1962	0.7046	0.1878	0.6846	0.1627	**0.7343**
(1,1) (2.5,1) (3,1)	0.2552	0.8860	0.2655	0.8691	0.1722	**0.9077**
(1,1) (2.5,1) (2.75,1)	0.2020	0.8412	0.2084	0.8230	0.1348	**0.8674**

Table 5.62. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in RCBD =CRD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	0.1349	0.0484	**0.1633**	0.0491	0.0982	0.0461
(1,1) (1,1) (1,3)	0.1171	0.0513	**0.1258**	0.0513	0.0998	0.0510
(1,1) (1,2) (1,3)	0.1471	0.0491	**0.1854**	0.0498	0.1037	0.0466
(1,1) (1,5) (1,7.5)	0.1742	0.0591	**0.2371**	0.0591	0.1107	0.0558
(1,1) (1,2) (1,5)	0.1744	0.0587	**0.2271**	0.0595	0.1180	0.0575
(1,1) (1,3) (1,3.5)	0.1651	0.0571	**0.2248**	0.0573	0.1095	0.0533
(1,1) (1,6) (1,8)	0.2236	0.0782	**0.3134**	0.0778	0.1268	0.0760

Table 5.63. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	0.3785	0.4654	0.4128	0.4516	0.2973	**0.4871**
(1,1) (1,1) (3,9)	0.3578	0.3473	0.3733	0.3347	0.2902	**0.3687**
(1,1) (2,4) (3,9)	0.4881	0.5997	0.5249	0.5814	0.3784	**0.6273**
(1,1) (2.5,6.25) (5,25)	0.6617	0.8255	0.7157	0.8084	0.5132	**0.8485**
(1,1) (2,4) (5,25)	0.6322	0.7665	0.6760	0.7494	0.4987	**0.7906**
(1,1) (3,9) (3.5,12.25)	0.6123	0.7874	0.6632	0.7700	0.4751	**0.8117**
(1,1) (6,36) (8,64)	0.7576	0.9697	0.8104	0.9633	0.5728	**0.9770**

Table 5.64. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0532	0.0513	0.0532	0.0513	0.0532	0.0513
(1,1) (2,1) (2,1)	0.1436	**0.3972**	0.1436	**0.3972**	0.1436	**0.3972**
(1,1) (1,1) (2,1)	**0.2534**	0.2232	**0.2534**	0.2232	**0.2534**	0.2232
(1,1) (1.5,1) (2,1)	0.1634	**0.3034**	0.1634	**0.3034**	0.1634	**0.3034**
(1,1) (2,1) (2.5,1)	0.2198	**0.5412**	0.2198	**0.5412**	0.2198	**0.5412**
(1,1) (2.5,1) (3,1)	0.2792	**0.7372**	0.2792	**0.7372**	0.2792	**0.7372**
(1,1) (2.5,1) (2.75,1)	0.2276	**0.6805**	0.2276	**0.6805**	0.2276	**0.6805**

Table 5.65. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	**0.1262**	0.0443	**0.1262**	0.0443	**0.1262**	0.0443
(1,1) (1,1) (1,3)	**0.1346**	0.0512	**0.1346**	0.0512	**0.1346**	0.0512
(1,1) (1,2) (1,3)	**0.1507**	0.0464	**0.1507**	0.0464	**0.1507**	0.0464
(1,1) (1,5) (1,7.5)	**0.1699**	0.0521	**0.1699**	0.0521	**0.1699**	0.0521
(1,1) (1,2) (1,5)	**0.1705**	0.0496	**0.1705**	0.0496	**0.1705**	0.0496
(1,1) (1,3) (1,3.5)	**0.1621**	0.0512	**0.1621**	0.0512	**0.1621**	0.0512
(1,1) (1,6) (1,8)	**0.2177**	0.0684	**0.2177**	0.0684	**0.2177**	0.0684

Table 5.66. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance CRD=2RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	**0.4726**	0.3408	**0.4726**	0.3408	**0.4726**	0.3408
(1,1) (1,1) (3,9)	**0.4128**	0.2424	**0.4128**	0.2424	**0.4128**	0.2424
(1,1) (2,4) (3,9)	**0.6195**	0.4596	**0.6195**	0.4596	**0.6195**	0.4596
(1,1) (2.5,6.25) (5,25)	**0.7972**	0.6546	**0.7972**	0.6546	**0.7972**	0.6546
(1,1) (2,4) (5,25)	**0.7658**	0.5917	**0.7658**	0.5917	**0.7658**	0.5917
(1,1) (3,9) (3.5,12.25)	**0.7687**	0.6195	**0.7687**	0.6195	**0.7687**	0.6195
(1,1) (6,36) (8,64)	**0.9222**	0.8516	**0.9222**	0.8516	**0.9222**	0.8516

Table 5.67. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0501	0.0471	0.0513	0.0509	0.0495	0.0471
(1,1) (2,1) (2,1)	0.1300	0.3595	0.1345	**0.4754**	0.1059	0.2874
(1,1) (1,1) (2,1)	0.2367	0.2218	0.2688	**0.3073**	0.1813	0.176
(1,1) (1.5,1) (2,1)	0.1469	0.2799	0.1605	**0.3772**	0.1182	0.2251
(1,1) (2,1) (2.5,1)	0.1818	0.4689	0.1926	**0.608**	0.1365	0.3754
(1,1) (2.5,1) (3,1)	0.2049	0.6453	0.2031	**0.7866**	0.1513	0.5249
(1,1) (2.5,1) (2.75,1)	0.1746	0.5942	0.1741	**0.7421**	0.1301	0.4833

Table 5.68. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	0.0830	0.0350	0.0709	0.0331	**0.0930**	0.0374
(1,1) (1,1) (1,3)	**0.1046**	0.0466	0.0993	0.0517	0.0975	0.0474
(1,1) (1,2) (1,3)	0.0890	0.0344	0.0715	0.0300	**0.1035**	0.0383
(1,1) (1,5) (1,7.5)	0.1067	0.0367	0.0758	0.0307	**0.1346**	0.0432
(1,1) (1,2) (1,5)	0.0965	0.0312	0.0803	0.0287	**0.1145**	0.0373
(1,1) (1,3) (1,3.5)	0.0971	0.0327	0.0720	0.0296	**0.1142**	0.0373
(1,1) (1,6) (1,8)	0.1062	0.0384	0.0746	0.0303	**0.1405**	0.0446

Table 5.69. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=2RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	0.3465	0.2756	0.3224	**0.3617**	0.3219	0.2253
(1,1) (1,1) (3,9)	0.3176	0.2190	0.3153	**0.2923**	0.2776	0.1795
(1,1) (2,4) (3,9)	0.4604	0.3743	0.4248	**0.4846**	0.4315	0.3038
(1,1) (2.5,6.25) (5,25)	0.6259	0.5365	0.5771	**0.6704**	0.5935	0.4422
(1,1) (2,4) (5,25)	0.5841	0.4816	0.5404	**0.6098**	0.5503	0.3900
(1,1) (3,9) (3.5,12.25)	0.5838	0.4991	0.5297	**0.6316**	0.5527	0.4161
(1,1) (6,36) (8,64)	0.7620	0.7265	0.6964	**0.8502**	0.7343	0.6269

Table 5.70. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance CRD=2RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0501	0.0484	0.0505	0.0484	0.0508	0.0486
(1,1) (2,1) (2,1)	0.1057	0.3699	0.0992	0.3496	0.0943	**0.4013**
(1,1) (1,1) (2,1)	0.2015	0.2121	0.1743	0.2004	0.2057	**0.2309**
(1,1) (1.5,1) (2,1)	0.1154	0.2829	0.1037	0.2675	0.1161	**0.3033**
(1,1) (2,1) (2.5,1)	0.147	0.4965	0.1352	0.4731	0.1293	**0.5314**
(1,1) (2.5,1) (3,1)	0.1766	0.6884	0.1699	0.6639	0.1317	**0.7291**
(1,1) (2.5,1) (2.75,1)	0.1565	0.6479	0.1485	0.6200	0.1132	**0.6877**

Table 5.71. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=2RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	0.1329	0.0468	**0.1670**	0.0469	0.0947	0.0455
(1,1) (1,1) (1,3)	0.1218	0.0476	**0.1279**	0.0463	0.1057	0.0486
(1,1) (1,2) (1,3)	0.1572	0.0571	**0.1995**	0.0569	0.1046	0.0557
(1,1) (1,5) (1,7.5)	0.1793	0.0603	**0.2406**	0.0603	0.1151	0.0584
(1,1) (1,2) (1,5)	0.1734	0.058	**0.2269**	0.0582	0.1141	0.0564
(1,1) (1,3) (1,3.5)	0.1719	0.0572	**0.2312**	0.0586	0.1103	0.0551
(1,1) (1,6) (1,8)	0.2170	0.0733	**0.3142**	0.0738	0.1231	0.0691

Table 5.72. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=2RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,2) (2,2)	0.3975	0.3209	**0.4304**	0.3059	0.309	0.3412
(1,1) (1,1) (3,3)	0.3254	0.2297	**0.3262**	0.2189	0.278	0.244
(1,1) (2,2) (3,3)	0.5274	0.4422	**0.5721**	0.4260	0.4085	0.4739
(1,1) (2.5,2.5) (5,5)	0.6961	0.6183	**0.7527**	0.5940	0.5401	0.6533
(1,1) (2,2) (5,5)	0.6653	0.5644	**0.7129**	0.5413	0.5256	0.5988
(1,1) (3,3) (3.5,3.5)	0.6671	0.5917	**0.7211**	0.5712	0.5099	0.6256
(1,1) (6,6) (8,8)	0.8611	0.8216	**0.8939**	0.8068	0.6965	0.8479

5.2.2. Four Treatments

Tables 5.73-5.90 show the simulation study results for four treatments (k=4) under the exponential distribution. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion (Table 5.73 to Table 5.81). Second, the variance of the CRD portion is two times the variance of the RCBD portion (Table 5.82 to Table 5.90). Third, the variance of the CRD portion is four times the variance of the RCBD portion (see Appendix B). Forth, the variance of the CRD portion is eight times the variance of the RCBD portion (see Appendix B).

We estimated the alpha values for the proposed tests and tabled them in the first row of the table where all populations have the same location and scale parameters (1,1) (1,1) (1,1)

(1,1). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (5.73, 5.76, 5.79, 5.82, 5.85, and 5.88). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters.

We started with the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location parameters and the same scale parameters, the proposed test two, Z_2, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. In another scenario, when the populations have different location and scale parameters and the variance in the CRD portion is equal to the variance in the RCBD portion, the proposed test two, Z_2, has the highest powers. Otherwise, the proposed test one, Z_1, has the highest powers.

Next, we considered the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test four, Z_4, has the highest powers.

Finally, we considered the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location

parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. In another scenario, when the populations have different location and scale parameters and the variance in the CRD portion is equal to the variance in the RCBD portion, the proposed test six, Z_6, has the highest powers. Otherwise, the proposed test three, Z_3, has the highest powers.

In cases where the variance of the CRD portion is either four or eight times the variance of the RCBD portion, the proposed tests that have the highest powers are approximately the same as those where the variance of the CRD portion is two times the variance of the RCBD portion. The only exception to this is when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion and the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers (see Appendix B).

Table 5.73. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0504	0.0507	0.0504	0.0507	0.0504	0.0507
(1,1) (2,1) (2,1) (2,1)	0.1157	**0.5775**	0.1157	**0.5775**	0.1157	**0.5775**
(1,1) (1,1) (1,1) (2,1)	**0.2531**	0.2389	**0.2531**	0.2389	**0.2531**	0.2389
(1,1) (1,1) (2,1) (2,1)	0.2915	**0.4365**	0.2915	**0.4365**	0.2915	**0.4365**
(1,1) (1.5,1) (2,1) (2.5,1)	0.2246	**0.6134**	0.2246	**0.6134**	0.2246	**0.6134**
(1,1) (2,1) (2.5,1) (3,1)	0.2201	**0.8257**	0.2201	**0.8257**	0.2201	**0.8257**
(1,1) (2,1) (2.5,1) (2.75,1)	0.1849	**0.7852**	0.1849	**0.7852**	0.1849	**0.7852**

Table 5.74. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	**0.1484**	0.0323	**0.1484**	0.0323	**0.1484**	0.0323
(1,1) (1,1) (1,3) (1,3)	**0.1664**	0.0384	**0.1664**	0.0384	**0.1664**	0.0384
(1,1) (1,2) (1,3) (1,4)	**0.1902**	0.0371	**0.1902**	0.0371	**0.1902**	0.0371
(1,1) (1,2.5) (1,5) (1,7.5)	**0.2137**	0.0403	**0.2137**	0.0403	**0.2137**	0.0403
(1,1) (1,2) (1,5) (1,1)	**0.1751**	0.0397	**0.1751**	0.0397	**0.1751**	0.0397
(1,1) (1,3) (1,3.5) (1,2)	**0.1833**	0.0355	**0.1833**	0.0355	**0.1833**	0.0355
(1,1) (1,4) (1,6) (1,8)	**0.2311**	0.0442	**0.2311**	0.0442	**0.2311**	0.0442

Table 5.75. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	0.4894	**0.5034**	0.4894	**0.5034**	0.4894	**0.5034**
(1,1) (1,1) (3,9) (3,9)	**0.5938**	0.5537	**0.5938**	0.5537	**0.5938**	0.5537
(1,1) (2,4) (3,9) (4,16)	0.7040	**0.7744**	0.7040	**0.7744**	0.7040	**0.7744**
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.7979	**0.9308**	0.7979	**0.9308**	0.7979	**0.9308**
(1,1) (2,4) (5,25) (1,1)	**0.6369**	0.5715	**0.6369**	0.5715	**0.6369**	0.5715
(1,1) (3,9) (3.5,12.25) (2,4)	0.6822	**0.7505**	0.6822	**0.7505**	0.6822	**0.7505**
(1,1) (4,16) (6,36) (8,64)	0.7899	**0.9636**	0.7899	**0.9636**	0.7899	**0.9636**

Table 5.76. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0525	0.0584	0.0516	0.0524	0.0542	0.0554
(1,1) (2,1) (2,1) (2,1)	0.0932	0.5066	0.0987	**0.6013**	0.0774	0.4064
(1,1) (1,1) (1,1) (2,1)	0.2159	0.2178	0.2238	**0.2511**	0.1838	0.1773
(1,1) (1,1) (2,1) (2,1)	0.2489	0.3873	0.2618	**0.4739**	0.1982	0.3009
(1,1) (1.5,1) (2,1) (2.5,1)	0.1792	0.5438	0.1899	**0.6433**	0.1477	0.4413
(1,1) (2,1) (2.5,1) (3,1)	0.1483	0.7342	0.1541	**0.8343**	0.1126	0.6141
(1,1) (2,1) (2.5,1) (2.75,1)	0.1291	0.7012	0.1402	**0.8017**	0.1017	0.5776

Table 5.77. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	0.0968	0.0232	0.0795	0.0213	**0.1066**	0.0293
(1,1) (1,1) (1,3) (1,3)	**0.1125**	0.0270	0.0984	0.0266	0.1102	0.0318
(1,1) (1,2) (1,3) (1,4)	0.1013	0.0181	0.0729	0.0160	**0.1135**	0.0252
(1,1) (1,2.5) (1,5) (1,7.5)	0.1054	0.0177	0.0769	0.0136	**0.1331**	0.0257
(1,1) (1,2) (1,5) (1,1)	0.1150	0.0297	0.0990	0.0294	**0.1155**	0.0349
(1,1) (1,3) (1,3.5) (1,2)	0.1031	0.0199	0.0770	0.0169	**0.1168**	0.0261
(1,1) (1,4) (1,6) (1,8)	0.1176	0.0199	0.0791	0.0161	**0.1440**	0.0279

Table 5.78. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	0.3726	0.4102	0.3606	**0.4728**	0.3333	0.3327
(1,1) (1,1) (3,9) (3,9)	0.4643	0.4481	0.4518	**0.5257**	0.4063	0.3597
(1,1) (2,4) (3,9) (4,16)	0.5354	0.6453	0.5093	**0.7332**	0.4858	0.5369
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.6321	0.8276	0.5985	**0.9061**	0.5702	0.7131
(1,1) (2,4) (5,25) (1,1)	0.4916	0.4590	0.4757	**0.5405**	0.4328	0.3693
(1,1) (3,9) (3.5,12.25) (2,4)	0.5224	0.6172	0.4967	**0.7064**	0.4749	0.5113
(1,1) (4,16) (6,36) (8,64)	0.6104	0.8792	0.5709	**0.9415**	0.5512	0.7816

Table 5.79. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in RCBD =CRD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0522	0.0469	0.0502	0.0485	0.0522	0.0475
(1,1) (2,1) (2,1) (2,1)	0.0924	0.5427	0.0897	0.5309	0.0851	**0.5781**
(1,1) (1,1) (1,1) (2,1)	0.2085	0.2143	0.1993	0.2123	0.1867	**0.2319**
(1,1) (1,1) (2,1) (2,1)	0.2277	0.3990	0.2142	0.3920	0.2130	**0.4334**
(1,1) (1.5,1) (2,1) (2.5,1)	0.1809	0.5746	0.1708	0.5640	0.1568	**0.6129**
(1,1) (2,1) (2.5,1) (3,1)	0.1583	0.7828	0.1561	0.7692	0.1242	**0.8216**
(1,1) (2,1) (2.5,1) (2.75,1)	0.1288	0.7563	0.1250	0.7458	0.1065	**0.7942**

Table 5.80. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	0.1385	0.0360	**0.1729**	0.0384	0.0920	0.0351
(1,1) (1,1) (1,3) (1,3)	0.1536	0.0421	**0.1817**	0.0457	0.1150	0.0434
(1,1) (1,2) (1,3) (1,4)	0.1697	0.0343	**0.2262**	0.0372	0.1044	0.0351
(1,1) (1,2.5) (1,5) (1,7.5)	0.2054	0.0389	**0.2889**	0.0421	0.1191	0.0389
(1,1) (1,2) (1,5) (1,1)	0.1553	0.0409	**0.1762**	0.0421	0.1128	0.0409
(1,1) (1,3) (1,3.5) (1,2)	0.1732	0.0364	**0.2286**	0.0386	0.1058	0.0347
(1,1) (1,4) (1,6) (1,8)	0.2257	0.0458	**0.3252**	0.0507	0.1253	0.0438

Table 5.81. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	0.4011	0.4727	0.4277	0.4653	0.3239	**0.5067**
(1,1) (1,1) (3,9) (3,9)	0.4945	0.5149	0.5163	0.5062	0.4043	**0.5475**
(1,1) (2,4) (3,9) (4,16)	0.5921	0.7403	0.6282	0.7300	0.4731	**0.7719**
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.6895	0.9125	0.7326	0.9060	0.5424	**0.9331**
(1,1) (2,4) (5,25) (1,1)	0.5148	0.5349	0.5424	0.5279	0.4201	**0.5705**
(1,1) (3,9) (3.5,12.25) (2,4)	0.5676	0.7090	0.6002	0.6976	0.4490	**0.7412**
(1,1) (4,16) (6,36) (8,64)	0.6711	0.9484	0.7201	0.9432	0.5178	**0.9616**

Table 5.82. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0522	0.0467	0.0522	0.0467	0.0522	0.0467
(1,1) (2,1) (2,1) (2,1)	0.0984	**0.4107**	0.0984	**0.4107**	0.0984	**0.4107**
(1,1) (1,1) (1,1) (2,1)	**0.1969**	0.1639	**0.1969**	0.1639	**0.1969**	0.1639
(1,1) (1,1) (2,1) (2,1)	0.2357	**0.3073**	0.2357	**0.3073**	0.2357	**0.3073**
(1,1) (1.5,1) (2,1) (2.5,1)	0.1821	**0.4425**	0.1821	**0.4425**	0.1821	**0.4425**
(1,1) (2,1) (2.5,1) (3,1)	0.1776	**0.6411**	0.1776	**0.6411**	0.1776	**0.6411**
(1,1) (2,1) (2.5,1) (2.75,1)	0.1534	**0.5961**	0.1534	**0.5961**	0.1534	**0.5961**

Table 5.83. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	**0.1531**	0.0348	**0.1531**	0.0348	**0.1531**	0.0348
(1,1) (1,1) (1,3) (1,3)	**0.1708**	0.0384	**0.1708**	0.0384	**0.1708**	0.0384
(1,1) (1,2) (1,3) (1,4)	**0.1856**	0.0371	**0.1856**	0.0371	**0.1856**	0.0371
(1,1) (1,2.5) (1,5) (1,7.5)	**0.2159**	0.0372	**0.2159**	0.0372	**0.2159**	0.0372
(1,1) (1,2) (1,5) (1,1)	**0.1751**	0.0404	**0.1751**	0.0404	**0.1751**	0.0404
(1,1) (1,3) (1,3.5) (1,2)	**0.1793**	0.0290	**0.1793**	0.0290	**0.1793**	0.0290
(1,1) (1,4) (1,6) (1,8)	**0.2335**	0.0468	**0.2335**	0.0468	**0.2335**	0.0468

Table 5.84. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance CRD=2RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	**0.5221**	0.3569	**0.5221**	0.3569	**0.5221**	0.3569
(1,1) (1,1) (3,9) (3,9)	**0.5839**	0.3741	**0.5839**	0.3741	**0.5839**	0.3741
(1,1) (2,4) (3,9) (4,16)	**0.7636**	0.6033	**0.7636**	0.6033	**0.7636**	0.6033
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	**0.8826**	0.7898	**0.8826**	0.7898	**0.8826**	0.7898
(1,1) (2,4) (5,25) (1,1)	**0.6173**	0.3822	**0.6173**	0.3822	**0.6173**	0.3822
(1,1) (3,9) (3.5,12.25) (2,4)	**0.7362**	0.5698	**0.7362**	0.5698	**0.7362**	0.5698
(1,1) (4,16) (6,36) (8,64)	**0.9091**	0.8625	**0.9091**	0.8625	**0.9091**	0.8625

Table 5.85. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0505	0.0538	0.0499	0.0501	0.0476	0.0507
(1,1) (2,1) (2,1) (2,1)	0.0911	0.4003	0.0911	**0.4975**	0.0813	0.3037
(1,1) (1,1) (1,1) (2,1)	0.1800	0.1748	0.1972	**0.2069**	0.1438	0.1391
(1,1) (1,1) (2,1) (2,1)	0.2108	0.3065	0.2371	**0.3943**	0.1621	0.2338
(1,1) (1.5,1) (2,1) (2.5,1)	0.1638	0.4317	0.1751	**0.5393**	0.1339	0.3350
(1,1) (2,1) (2.5,1) (3,1)	0.1408	0.5897	0.1470	**0.7301**	0.1173	0.4654
(1,1) (2,1) (2.5,1) (2.75,1)	0.1207	0.5655	0.1273	**0.6942**	0.1002	0.4391

Table 5.86. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	0.0943	0.0237	0.0745	0.0198	**0.1035**	0.0291
(1,1) (1,1) (1,3) (1,3)	0.1170	0.0265	0.1016	0.0268	**0.1184**	0.0315
(1,1) (1,2) (1,3) (1,4)	0.0995	0.0193	0.0769	0.0162	**0.1147**	0.0247
(1,1) (1,2.5) (1,5) (1,7.5)	0.1145	0.0187	0.0827	0.0145	**0.1393**	0.0240
(1,1) (1,2) (1,5) (1,1)	0.1178	0.0286	0.1021	0.0274	**0.1192**	0.0352
(1,1) (1,3) (1,3.5) (1,2)	0.1033	0.0173	0.0797	0.0173	**0.1183**	0.0247
(1,1) (1,4) (1,6) (1,8)	0.1160	0.0177	0.0791	0.0140	**0.1452**	0.0251

Table 5.87. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	0.3809	0.3112	0.3587	**0.3812**	0.3461	0.2437
(1,1) (1,1) (3,9) (3,9)	**0.4548**	0.3336	0.4393	0.4194	0.3999	0.2530
(1,1) (2,4) (3,9) (4,16)	0.5853	0.5116	0.5358	**0.6211**	0.5501	0.3965
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.7205	0.6972	0.6577	**0.8161**	0.6962	0.5628
(1,1) (2,4) (5,25) (1,1)	**0.4836**	0.3360	0.4756	0.4347	0.4171	0.2539
(1,1) (3,9) (3.5,12.25) (2,4)	0.5721	0.4886	0.5234	**0.6029**	0.5290	0.3765
(1,1) (4,16) (6,36) (8,64)	0.7334	0.7564	0.6541	**0.8709**	0.7208	0.6297

Table 5.88. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance CRD=2RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0500	0.0512	0.0501	0.0548	0.0507	0.0517
(1,1) (2,1) (2,1) (2,1)	0.0800	0.3818	0.0825	0.3732	0.0696	**0.4179**
(1,1) (1,1) (1,1) (2,1)	0.1553	0.1482	0.1370	0.1474	0.1557	**0.1615**
(1,1) (1,1) (2,1) (2,1)	0.1716	0.2739	0.1512	0.2656	0.1707	**0.3006**
(1,1) (1.5,1) (2,1) (2.5,1)	0.1452	0.4023	0.1316	0.3920	0.1392	**0.4450**
(1,1) (2,1) (2.5,1) (3,1)	0.1264	0.6011	0.1224	0.5854	0.1076	**0.6486**
(1,1) (2,1) (2.5,1) (2.75,1)	0.1088	0.5538	0.1076	0.5366	0.0921	**0.6001**

Table 5.89. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=2RCBD; K=4; $\mathrm{n_b} = 5, \mathrm{n_a} = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	0.1430	0.0344	**0.1788**	0.0369	0.0964	0.0341
(1,1) (1,1) (1,3) (1,3)	0.1475	0.0385	**0.1681**	0.0410	0.1074	0.0398
(1,1) (1,2) (1,3) (1,4)	0.1725	0.0360	**0.2310**	0.0393	0.1020	0.0349
(1,1) (1,2.5) (1,5) (1,7.5)	0.2102	0.0409	**0.2917**	0.0449	0.1174	0.0394
(1,1) (1,2) (1,5) (1,1)	0.1544	0.0392	**0.1747**	0.0419	0.1145	0.0406
(1,1) (1,3) (1,3.5) (1,2)	0.1711	0.0379	**0.2285**	0.0412	0.1067	0.0379
(1,1) (1,4) (1,6) (1,8)	0.2204	0.0476	**0.3169**	0.0511	0.1220	0.0456

Table 5.90. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=2RCBD; K=4; $\mathrm{n_b} = 5, \mathrm{n_a} = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	0.4293	0.3404	**0.4725**	0.3358	0.3381	0.3707
(1,1) (1,1) (3,9) (3,9)	0.4863	0.3396	**0.5075**	0.3319	0.3978	0.3740
(1,1) (2,4) (3,9) (4,16)	0.6580	0.5573	**0.7134**	0.5465	0.5101	0.6011
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.8091	0.7550	**0.8594**	0.7422	0.6407	0.7986
(1,1) (2,4) (5,25) (1,1)	0.5025	0.3485	**0.5231**	0.3405	0.4144	0.3792
(1,1) (3,9) (3.5,12.25) (2,4)	0.6380	0.5229	**0.6853**	0.5113	0.5012	0.5643
(1,1) (4,16) (6,36) (8,64)	0.8352	0.8200	**0.8827**	0.8075	0.6689	0.8548

5.2.3. Five Treatments

Tables 5.91-5.108 show the simulation study results for five treatments (k=5) under the exponential distribution. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion (Table 5.91 to Table 5.99). Second, the variance of the CRD portion is two times the variance of the RCBD portion (Table 5.100 to Table 5.108). Third, the variance of the CRD portion is four times the variance of the RCBD portion (see Appendix B). Forth, the variance of the CRD portion is eight times the variance of the RCBD portion (see Appendix B).

We estimated the alpha values for the proposed tests and tabled them in the first row of the table where all populations have the same location and scale parameters (1,1) (1,1) (1,1) (1,1)

(1,1). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (5.91, 5.94, 5.97, 5.100, 5.103, and 5.106). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters.

We started with the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location parameters and the same scale parameters, the proposed test two, Z_2, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. In another scenario, when the populations have different location and scale parameters and the variance in the CRD portion is equal to the variance in the RCBD portion, the proposed test two, Z_2, has the highest powers. Otherwise, the proposed test one, Z_1, has the highest powers.

Next, we considered the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test four, Z_4, has the highest powers.

Finally, we considered the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location

parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. In another scenario, when the populations have different location and scale parameters and the variance in the CRD portion is equal to the variance in the RCBD portion, the proposed test six, Z_6, has the highest powers. Otherwise, the proposed test three, Z_3, has the highest powers.

In cases where the variance of the CRD portion is either four or eight times the variance of the RCBD portion, the proposed tests that have the highest powers are approximately the same as those where the variance of the CRD portion is two times the variance of the RCBD portion. The only exception to this is when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion and the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers (see Appendix B).

Table 5.91. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0522	0.0500	0.0522	0.0500	0.0522	0.0500
(1,1) (2,1) (2,1) (2,1) (2,1)	0.1062	**0.5707**	0.1062	**0.5707**	0.1062	**0.5707**
(1,1) (1,1) (1,1) (2,1) (2,1)	0.2905	**0.3456**	0.2905	**0.3456**	0.2905	**0.3456**
(1,1) (1,1) (2,1) (2,1) (2,1)	0.2103	**0.4688**	0.2103	**0.4688**	0.2103	**0.4688**
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.2654	**0.7290**	0.2654	**0.7290**	0.2654	**0.7290**
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.2156	**0.8753**	0.2156	**0.8753**	0.2156	**0.8753**
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.2073	**0.8569**	0.2073	**0.8569**	0.2073	**0.8569**

Table 5.92. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	**0.1745**	0.0333	**0.1745**	0.0333	**0.1745**	0.0333
(1,1) (1,1) (1,3) (1,3) (1,3)	**0.2005**	0.0324	**0.2005**	0.0324	**0.2005**	0.0324
(1,1) (1,2) (1,3) (1,4) (1,5)	**0.2137**	0.0276	**0.2137**	0.0276	**0.2137**	0.0276
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	**0.2497**	0.0340	**0.2497**	0.0340	**0.2497**	0.0340
(1,1) (1,2) (1,5) (1,1) (1,4.5)	**0.2182**	0.0328	**0.2182**	0.0328	**0.2182**	0.0328
(1,1) (1,3) (1,3.5) (1,2) (1,6)	**0.2238**	0.0266	**0.2238**	0.0266	**0.2238**	0.0266
(1,1) (1,4) (1,6) (1,8) (1,10)	**0.2539**	0.0334	**0.2539**	0.0334	**0.2539**	0.0334

Table 5.93. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	0.5125	**0.5225**	0.5125	**0.5225**	0.5125	**0.5225**
(1,1) (1,1) (3,9) (3,9) (3,9)	**0.6576**	0.6473	**0.6576**	0.6473	**0.6576**	0.6473
(1,1) (2,4) (3,9) (4,16) (5,25)	0.7424	**0.8444**	0.7424	**0.8444**	0.7424	**0.8444**
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.7820	**0.9566**	0.7820	**0.9566**	0.7820	**0.9566**
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	**0.7438**	0.7387	**0.7438**	0.7387	**0.7438**	0.7387
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	0.7497	**0.8502**	0.7497	**0.8502**	0.7497	**0.8502**
(1,1) (4,16) (6,36) (8,64) (10,100)	0.7481	**0.9740**	0.7481	**0.9740**	0.7481	**0.9740**

Table 5.94. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0539	0.0504	0.0519	0.0518	0.053	0.0516
(1,1) (2,1) (2,1) (2,1) (2,1)	0.0985	0.4565	0.1027	**0.5670**	0.0841	0.3935
(1,1) (1,1) (1,1) (2,1) (2,1)	0.2427	0.2785	0.2536	**0.3649**	0.1975	0.2353
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1745	0.3885	0.1824	**0.4817**	0.1408	0.3245
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.2068	0.6085	0.2107	**0.7258**	0.1618	0.5228
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1471	0.7592	0.1555	**0.8677**	0.1078	0.6704
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1503	0.7412	0.1577	**0.8543**	0.1111	0.6493

Table 5.95. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	0.1077	0.0226	0.0914	0.0206	**0.1145**	0.0253
(1,1) (1,1) (1,3) (1,3) (1,3)	0.1193	0.0228	0.1052	0.0236	**0.1200**	0.0250
(1,1) (1,2) (1,3) (1,4) (1,5)	0.1168	0.0151	0.0914	0.0144	**0.1319**	0.0180
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	0.1330	0.0180	0.0958	0.0133	**0.1553**	0.0201
(1,1) (1,2) (1,5) (1,1) (1,4.5)	**0.1359**	0.0232	0.1170	0.0224	0.1322	0.0228
(1,1) (1,3) (1,3.5) (1,2) (1,6)	0.1222	0.0166	0.0943	0.0149	**0.1358**	0.0192
(1,1) (1,4) (1,6) (1,8) (1,10)	0.1327	0.0145	0.0953	0.0129	**0.1607**	0.0180

Table 5.96. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	0.3892	0.4023	0.3761	**0.4973**	0.3489	0.348
(1,1) (1,1) (3,9) (3,9) (3,9)	0.5100	0.5043	0.4922	**0.6133**	0.4520	0.4307
(1,1) (2,4) (3,9) (4,16) (5,25)	0.5768	0.6892	0.5560	**0.7970**	0.5141	0.5978
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.6060	0.8459	0.5846	**0.9332**	0.5309	0.7541
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	0.5854	0.5755	0.5624	**0.6921**	0.5231	0.4897
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	0.5965	0.7005	0.5703	**0.8162**	0.5291	0.6105
(1,1) (4,16) (6,36) (8,64) (10,100)	0.5632	0.8846	0.5322	**0.9561**	0.4874	0.8015

Table 5.97. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0515	0.0522	0.0516	0.0538	0.0517	0.0546
(1,1) (2,1) (2,1) (2,1) (2,1)	0.0933	0.5237	0.0892	0.5130	0.0863	**0.5604**
(1,1) (1,1) (1,1) (2,1) (2,1)	0.2262	0.3117	0.2120	0.3050	0.2068	**0.3398**
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1537	0.4351	0.1464	0.4267	0.1384	**0.4710**
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.1965	0.6935	0.1949	0.6798	0.1674	**0.7293**
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1511	0.8558	0.1426	0.8437	0.1219	**0.8821**
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1447	0.8307	0.139	0.8189	0.1171	**0.8617**

Table 5.98. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	0.1585	0.0303	**0.1920**	0.0323	0.1120	0.0313
(1,1) (1,1) (1,3) (1,3) (1,3)	0.1740	0.0335	**0.2064**	0.0354	0.1227	0.0340
(1,1) (1,2) (1,3) (1,4) (1,5)	0.2034	0.0297	**0.2689**	0.0315	0.1238	0.0292
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	0.2362	0.0324	**0.3221**	0.0348	0.1381	0.0324
(1,1) (1,2) (1,5) (1,1) (1,4.5)	0.1859	0.0363	**0.2228**	0.0379	0.1345	0.0373
(1,1) (1,3) (1,3.5) (1,2) (1,6)	0.2016	0.0317	**0.2685**	0.0339	0.1202	0.0316
(1,1) (1,4) (1,6) (1,8) (1,10)	0.2388	0.0311	**0.3321**	0.0332	0.1356	0.0305

Table 5.99. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	0.4233	0.4999	0.4480	0.4904	0.3389	**0.5256**
(1,1) (1,1) (3,9) (3,9) (3,9)	0.5408	0.6080	0.5677	0.5971	0.4407	**0.6432**
(1,1) (2,4) (3,9) (4,16) (5,25)	0.6282	0.8096	0.6589	0.7980	0.5035	**0.8412**
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.6554	0.9371	0.6879	0.9303	0.5184	**0.9549**
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	0.6246	0.6947	0.6482	0.6825	0.5083	**0.7298**
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	0.6353	0.8157	0.6619	0.8038	0.5078	**0.8481**
(1,1) (4,16) (6,36) (8,64) (10,100)	0.6131	0.9605	0.6481	0.9544	0.4751	**0.9737**

Table 5.100. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0515	0.0510	0.0515	0.0510	0.0515	0.0510
(1,1) (2,1) (2,1) (2,1) (2,1)	0.1043	**0.4063**	0.1043	**0.4063**	0.1043	**0.4063**
(1,1) (1,1) (1,1) (2,1) (2,1)	0.2140	**0.2368**	0.2140	**0.2368**	0.2140	**0.2368**
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1714	**0.3288**	0.1714	**0.3288**	0.1714	**0.3288**
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.2140	**0.5563**	0.2140	**0.5563**	0.2140	**0.5563**
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1930	**0.7216**	0.1930	**0.7216**	0.1930	**0.7216**
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1808	**0.7012**	0.1808	**0.7012**	0.1808	**0.7012**

Table 5.101. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	**0.1610**	0.0271	**0.1610**	0.0271	**0.1610**	0.0271
(1,1) (1,1) (1,3) (1,3) (1,3)	**0.2039**	0.0331	**0.2039**	0.0331	**0.2039**	0.0331
(1,1) (1,2) (1,3) (1,4) (1,5)	**0.2192**	0.0295	**0.2192**	0.0295	**0.2192**	0.0295
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	**0.2465**	0.0335	**0.2465**	0.0335	**0.2465**	0.0335
(1,1) (1,2) (1,5) (1,1) (1,4.5)	**0.2228**	0.0345	**0.2228**	0.0345	**0.2228**	0.0345
(1,1) (1,3) (1,3.5) (1,2) (1,6)	**0.2278**	0.0292	**0.2278**	0.0292	**0.2278**	0.0292
(1,1) (1,4) (1,6) (1,8) (1,10)	**0.2583**	0.0310	**0.2583**	0.0310	**0.2583**	0.0310

Table 5.102. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance CRD=2RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	**0.5387**	0.3656	**0.5387**	0.3656	**0.5387**	0.3656
(1,1) (1,1) (3,9) (3,9) (3,9)	**0.6823**	0.4679	**0.6823**	0.4679	**0.6823**	0.4679
(1,1) (2,4) (3,9) (4,16) (5,25)	**0.8296**	0.6988	**0.8296**	0.6988	**0.8296**	0.6988
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	**0.9257**	0.8729	**0.9257**	0.8729	**0.9257**	0.8729
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	**0.7671**	0.5389	**0.7671**	0.5389	**0.7671**	0.5389
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	**0.8376**	0.7063	**0.8376**	0.7063	**0.8376**	0.7063
(1,1) (4,16) (6,36) (8,64) (10,100)	**0.9220**	0.9011	**0.9220**	0.9011	**0.9220**	0.9011

Table 5.103. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0517	0.0498	0.0512	0.0519	0.0504	0.0520
(1,1) (2,1) (2,1) (2,1) (2,1)	0.0956	0.3612	0.0946	**0.4671**	0.0852	0.3005
(1,1) (1,1) (1,1) (2,1) (2,1)	0.2026	0.2188	0.2265	**0.2953**	0.1543	0.1847
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1487	0.2883	0.1659	**0.3918**	0.1215	0.2354
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.1857	0.4830	0.1941	**0.6220**	0.1483	0.3927
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1567	0.6356	0.1577	**0.7781**	0.1234	0.5331
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1476	0.6153	0.1483	**0.7628**	0.1241	0.5138

Table 5.104. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	0.1038	0.0186	0.0898	0.0184	**0.1094**	0.0212
(1,1) (1,1) (1,3) (1,3) (1,3)	**0.1235**	0.0226	0.1098	0.0238	0.1217	0.0233
(1,1) (1,2) (1,3) (1,4) (1,5)	0.1279	0.0194	0.0927	0.0158	**0.1384**	0.0225
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	0.1337	0.0155	0.0986	0.0127	**0.1543**	0.0187
(1,1) (1,2) (1,5) (1,1) (1,4.5)	**0.1419**	0.0255	0.1215	0.0262	0.1386	0.0277
(1,1) (1,3) (1,3.5) (1,2) (1,6)	0.1201	0.0166	0.0939	0.0139	**0.1334**	0.0199
(1,1) (1,4) (1,6) (1,8) (1,10)	0.1350	0.0149	0.0949	0.0116	**0.1604**	0.0163

Table 5.105. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	**0.4133**	0.2958	0.3929	0.4051	0.3668	0.2384
(1,1) (1,1) (3,9) (3,9) (3,9)	**0.5217**	0.3701	0.5017	0.4995	0.4658	0.2964
(1,1) (2,4) (3,9) (4,16) (5,25)	0.6671	0.569	0.6076	**0.7227**	0.6383	0.4651
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.7630	0.7401	0.6926	**0.8718**	0.7412	0.6266
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	**0.6090**	0.4321	0.5806	0.5846	0.5432	0.3392
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	0.6676	0.5688	0.6162	**0.7184**	0.6307	0.4651
(1,1) (4,16) (6,36) (8,64) (10,100)	0.7521	0.7702	0.6668	**0.8917**	0.7375	0.6646

Table 5.106. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0516	0.0501	0.0530	0.0518	0.0477	0.0505
(1,1) (2,1) (2,1) (2,1) (2,1)	0.0877	0.3876	0.0848	0.3784	0.0814	**0.4254**
(1,1) (1,1) (1,1) (2,1) (2,1)	0.1668	0.2134	0.1422	0.2070	0.1733	**0.2332**
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1289	0.2953	0.1220	0.2898	0.1253	**0.3277**
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.1591	0.5123	0.1526	0.4999	0.1426	**0.5550**
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1302	0.6752	0.1309	0.6602	0.1055	**0.7202**
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1296	0.6574	0.129	0.6408	0.1049	**0.7018**

Table 5.107. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	0.1611	0.0317	**0.1956**	0.0323	0.1101	0.0308
(1,1) (1,1) (1,3) (1,3) (1,3)	0.1743	0.0325	**0.2029**	0.034	0.1256	0.0337
(1,1) (1,2) (1,3) (1,4) (1,5)	0.2066	0.0307	**0.2725**	0.0332	0.1266	0.0306
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	0.2066	0.0307	**0.2725**	0.0332	0.1266	0.0306
(1,1) (1,2) (1,5) (1,1) (1,4.5)	0.2406	0.0349	**0.3227**	0.0364	0.1346	0.0347
(1,1) (1,3) (1,3.5) (1,2) (1,6)	0.1924	0.0352	**0.2316**	0.0367	0.1369	0.0359
(1,1) (1,4) (1,6) (1,8) (1,10)	0.2018	0.0301	**0.2669**	0.0315	0.1259	0.0299

Table 5.108. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	0.4532	0.3407	**0.4892**	0.333	0.3638	0.3723
(1,1) (1,1) (3,9) (3,9) (3,9)	0.5630	0.4129	**0.5972**	0.4006	0.4557	0.4545
(1,1) (2,4) (3,9) (4,16) (5,25)	0.7367	0.6493	**0.7911**	0.6335	0.5742	0.6956
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.8456	0.8254	**0.8884**	0.8115	0.6743	0.8593
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	0.6498	0.4942	**0.6851**	0.4791	0.5223	0.5383
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	0.7397	0.6581	**0.7965**	0.6428	0.5812	0.7067
(1,1) (4,16) (6,36) (8,64) (10,100)	0.8572	0.8633	**0.9020**	0.8489	0.6738	0.8907

5.3. Results for the T Distribution

5.3.1. Three Treatments

Tables 5.109-5.126 show the simulation study results for three treatments (k=3) under the t distribution. In the tables, (μ, σ) represent the mean and the standard deviation of the random variable generated from a t distribution, where $\sigma = \sqrt{\frac{v}{v-2}}$ and v is the degree of freedom. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion (Table 5.109 to Table 5.117). Second, the variance of the CRD portion is two times the variance of the RCBD portion (Table 5.118 to Table 5.126). Third, the variance of the CRD portion is four times the variance of the RCBD portion (see Appendix C). Forth, the variance of the CRD portion is eight times the variance of the RCBD portion (see Appendix C).

We estimated the alpha values for the proposed tests and tabled them in the first row of the table where all populations have the same location and scale parameters (0,σ) (0,σ) (0,σ). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (5.109, 5.112, 5.115, 5.118, 5.121, and 5.124). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters

We started with the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location parameters and the same scale parameters, the proposed test two, Z_2, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Next, we considered the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Finally, we considered the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. Similarly, the proposed test three, Z_3, has the highest powers when the populations have different location and scale parameters.

In cases where the variance of the CRD portion is either four or eight times the variance of the RCBD portion, the proposed tests that have the highest powers will be approximately the same as the cases where the variance of the CRD portion is two times the variance of the RCBD portion. The only exception to this is when the number of blocks in the RCBD portion is equal to the sample size for each treatment in the CRD portion, and the populations have different location parameters and the same scale parameters, the proposed test one, Z_1, has the highest powers (see Appendix C).

Table 5.109. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0497	0.0487	0.0497	0.0487	0.0497	0.0487
(0,σ) (1,σ) (1,σ)	0.3931	**0.5206**	0.3931	**0.5206**	0.3931	**0.5206**
(0,σ) (0,σ) (1,σ)	**0.2689**	0.2460	**0.2689**	0.2460	**0.2689**	0.2460
(0,σ) (0.5,σ) (1,σ)	0.3245	**0.3808**	0.3245	**0.3808**	0.3245	**0.3808**
(0,σ) (1,σ) (1.5,σ)	0.4929	**0.6428**	0.4929	**0.6428**	0.4929	**0.6428**
(0,σ) (1.5,σ) (2,σ)	0.6013	**0.8351**	0.6013	**0.8351**	0.6013	**0.8351**
(0,σ) (1.5,σ) (1.75,σ)	0.5645	**0.7907**	0.5645	**0.7907**	0.5645	**0.7907**

Table 5.110. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	**0.2584**	0.1510	**0.2584**	0.1510	**0.2584**	0.1510
(0,σ) (0,σ) (0,3σ)	**0.1900**	0.1077	**0.1900**	0.1077	**0.1900**	0.1077
(0,σ) (0,2σ) (0,3σ)	**0.3436**	0.1899	**0.3436**	0.1899	**0.3436**	0.1899
(0,σ) (0,2.5σ) (0,5σ)	**0.4979**	0.2680	**0.4979**	0.2680	**0.4979**	0.2680
(0,σ) (0,2σ) (0,5σ)	**0.4479**	0.2391	**0.4479**	0.2391	**0.4479**	0.2391
(0,σ) (0,3σ) (0,3.5σ)	**0.4598**	0.2552	**0.4598**	0.2552	**0.4598**	0.2552
(0,σ) (0,6σ) (0,8σ)	**0.6913**	0.4098	**0.6913**	0.4098	**0.6913**	0.4098

Table 5.111. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	**0.6413**	0.5814	0.6413	0.5814	0.6413	0.5814
(0,σ) (0,σ) (0,3σ)	**0.3614**	0.2396	0.3614	0.2396	0.3614	0.2396
(0,σ) (0,2σ) (1,3σ)	**0.5222**	0.3505	0.5222	0.3505	0.5222	0.3505
(0,σ) (0.5,2.5σ) (1,5σ)	**0.6891**	0.4937	0.6891	0.4937	0.6891	0.4937
(0,σ) (0.25,2σ) (0.5,5σ)	**0.5705**	0.3579	0.5705	0.3579	0.5705	0.3579
(0,σ) (1,3σ) (1.5,3.5σ)	**0.7796**	0.6601	0.7796	0.6601	0.7796	0.6601
(0,σ) (1.5,6σ) (1.75,8σ)	**0.8632**	0.6965	0.8632	0.6965	0.8632	0.6965

Table 5.112. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0491	0.0447	0.0531	0.0517	0.0465	0.0446
(0,σ) (1,σ) (1,σ)	0.3022	0.4028	0.2913	**0.4958**	0.2727	0.3469
(0,σ) (0,σ) (1,σ)	0.2268	0.2184	0.2240	**0.2674**	0.2076	0.1890
(0,σ) (0.5,σ) (1,σ)	0.2556	0.2963	0.2546	**0.3699**	0.2276	0.2557
(0,σ) (1,σ) (1.5,σ)	0.3728	0.5177	0.3582	**0.6174**	0.3302	0.4347
(0,σ) (1.5,σ) (2,σ)	0.4467	0.6893	0.4179	**0.7937**	0.3911	0.5994
(0,σ) (1.5,σ) (1.75,σ)	0.4224	0.6484	0.3990	**0.7588**	0.3718	0.5602

Table 5.113. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	0.1810	0.1118	0.1611	0.1240	**0.1825**	0.1058
(0,σ) (0,σ) (0,3σ)	**0.1494**	0.0938	0.1403	0.1080	0.1443	0.0872
(0,σ) (0,2σ) (0,3σ)	**0.2390**	0.1318	0.2024	0.1475	0.2379	0.1227
(0,σ) (0,2.5σ) (0,5σ)	0.4343	0.2356	0.3462	0.2608	**0.4653**	0.2152
(0,σ) (0,2σ) (0,5σ)	0.2916	0.1602	0.2470	0.1835	**0.2982**	0.1468
(0,σ) (0,3σ) (0,3.5σ)	0.2922	0.1625	0.2479	0.1845	**0.3053**	0.1507
(0,σ) (0,6σ) (0,8σ)	0.4466	0.2500	0.3539	0.2708	**0.4869**	0.2367

Table 5.114. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	0.4698	0.4171	0.4287	**0.4981**	0.4500	0.3622
(0,σ) (0,σ) (0,3σ)	**0.2650**	0.1818	0.2477	0.2203	0.2438	0.1614
(0,σ) (0,2σ) (1,3σ)	**0.3646**	0.2347	0.3183	0.2811	0.3514	0.2061
(0,σ) (0.5,2.5σ) (1,5σ)	0.4762	0.3203	0.4123	0.3755	**0.4827**	0.2882
(0,σ) (0.25,2σ) (0.5,5σ)	**0.3903**	0.2319	0.3298	0.2746	0.3883	0.2047
(0,σ) (1,3σ) (1.5,3.5σ)	**0.5829**	0.4666	0.5021	0.5413	0.5790	0.4118
(0,σ) (1.5,6σ) (1.75,8σ)	0.6973	0.4586	0.5810	0.5416	**0.7295**	0.4406

Table 5.115. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in RCBD =CRD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0509	0.0524	0.0512	0.0518	0.0515	0.0531
(0,σ) (1,σ) (1,σ)	0.3226	0.4915	0.3462	0.4745	0.2621	**0.5181**
(0,σ) (0,σ) (1,σ)	0.2312	0.2429	0.2342	0.2347	0.1992	**0.2570**
(0,σ) (0.5,σ) (1,σ)	0.2668	0.3691	0.2809	0.3577	0.2176	**0.3887**
(0,σ) (1,σ) (1.5,σ)	0.4003	0.6238	0.4258	0.6034	0.3197	**0.6565**
(0,σ) (1.5,σ) (2,σ)	0.4771	0.8113	0.5022	0.7965	0.3629	**0.8356**
(0,σ) (1.5,σ) (1.75,σ)	0.4499	0.7584	0.4781	0.7396	0.3487	**0.7882**

Table 5.116. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in RCBD =CRD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	0.2338	0.1451	**0.2690**	0.1410	0.1744	0.1501
(0,σ) (0,σ) (0,3σ)	0.1781	0.1115	**0.1900**	0.1080	0.1471	0.1152
(0,σ) (0,2σ) (0,3σ)	0.3101	0.1856	**0.3614**	0.1825	0.2241	0.1909
(0,σ) (0,2.5σ) (0,5σ)	0.4297	0.2649	**0.5233**	0.2615	0.2921	0.2711
(0,σ) (0,2σ) (0,5σ)	0.3952	0.2346	**0.4714**	0.2283	0.2798	0.2431
(0,σ) (0,3σ) (0,3.5σ)	0.4154	0.2491	**0.5046**	0.2414	0.2857	0.2555
(0,σ) (0,6σ) (0,8σ)	0.6314	0.4010	**0.7501**	0.3905	0.4182	0.4108

Table 5.117. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in RCBD =CRD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	0.5472	0.5386	**0.6036**	0.5209	0.4157	0.5621
(0,σ) (0,σ) (0,3σ)	0.3028	0.2371	**0.3363**	0.2280	0.2381	0.2485
(0,σ) (0,2σ) (1,3σ)	0.4658	0.3488	**0.5372**	0.3391	0.3398	0.3610
(0,σ) (0.5,2.5σ) (1,5σ)	0.6084	0.4702	**0.7042**	0.4571	0.4370	0.4853
(0,σ) (0.25,2σ) (0.5,5σ)	0.4982	0.3488	**0.5826**	0.3397	0.3575	0.3617
(0,σ) (1,3σ) (1.5,3.5σ)	0.6967	0.6372	**0.7748**	0.6241	0.5271	0.6597
(0,σ) (1.5,6σ) (1.75,8σ)	0.9998	0.8051	0.9982	0.7308	**1**	0.9081

Table 5.118. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=2RCBD; K=3; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0509	0.0492	0.0509	0.0492	0.0509	0.0492
(0,σ) (1,σ) (1,σ)	0.3287	**0.3526**	0.3287	**0.3526**	0.3287	**0.3526**
(0,σ) (0,σ) (1,σ)	**0.2208**	0.1725	**0.2208**	0.1725	**0.2208**	0.1725
(0,σ) (0.5,σ) (1,σ)	**0.2717**	0.2594	**0.2717**	0.2594	**0.2717**	0.2594
(0,σ) (1,σ) (1.5,σ)	0.4111	**0.4660**	0.4111	**0.4660**	0.4111	**0.4660**
(0,σ) (1.5,σ) (2,σ)	0.5291	**0.6650**	0.5291	**0.6650**	0.5291	**0.6650**
(0,σ) (1.5,σ) (1.75,σ)	0.4905	**0.6107**	0.4905	**0.6107**	0.4905	**0.6107**

Table 5.119. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=2RCBD; K=3; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	**0.2638**	0.1495	**0.2638**	0.1495	**0.2638**	0.1495
(0,σ) (0,σ) (0,3σ)	**0.1918**	0.1063	**0.1918**	0.1063	**0.1918**	0.1063
(0,σ) (0,2σ) (0,3σ)	**0.3434**	0.1939	**0.3434**	0.1939	**0.3434**	0.1939
(0,σ) (0,2.5σ) (0,5σ)	**0.4833**	0.2737	**0.4833**	0.2737	**0.4833**	0.2737
(0,σ) (0,2σ) (0,5σ)	**0.4384**	0.2368	**0.4384**	0.2368	**0.4384**	0.2368
(0,σ) (0,3σ) (0,3.5σ)	**0.4619**	0.2626	**0.4619**	0.2626	**0.4619**	0.2626
(0,σ) (0,6σ) (0,8σ)	**0.6899**	0.4258	**0.6899**	0.4258	**0.6899**	0.4258

Table 5.120. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance CRD=2RCBD; K=3; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	**0.5954**	0.4554	**0.5954**	0.4554	**0.5954**	0.4554
(0,σ) (0,σ) (0,3σ)	**0.3259**	0.1943	**0.3259**	0.1943	**0.3259**	0.1943
(0,σ) (0,2σ) (1,3σ)	**0.4922**	0.3090	**0.4922**	0.3090	**0.4922**	0.3090
(0,σ) (0.5,2.5σ) (1,5σ)	**0.6660**	0.4420	**0.6660**	0.4420	**0.6660**	0.4420
(0,σ) (0.25,2σ) (0.5,5σ)	**0.5404**	0.3279	**0.5404**	0.3279	**0.5404**	0.3279
(0,σ) (1,3σ) (1.5,3.5σ)	**0.7476**	0.5594	**0.7476**	0.5594	**0.7476**	0.5594
(0,σ) (1.5,6σ) (1.75,8σ)	**0.8486**	0.6299	**0.8486**	0.6299	**0.8486**	0.6299

Table 5.121. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=2RCBD; K=3; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0513	0.0496	0.0537	0.0536	0.0481	0.0482
(0,σ) (1,σ) (1,σ)	0.2590	0.3097	0.2617	**0.3996**	0.2194	0.2572
(0,σ) (0,σ) (1,σ)	0.1904	0.1624	0.1929	**0.2116**	0.1630	0.1368
(0,σ) (0.5,σ) (1,σ)	0.2151	0.2309	0.2232	**0.2972**	0.1820	0.1919
(0,σ) (1,σ) (1.5,σ)	0.3253	0.3949	0.3244	**0.5077**	0.2798	0.3223
(0,σ) (1.5,σ) (2,σ)	0.3991	0.5548	0.3867	**0.6886**	0.3452	0.4573
(0,σ) (1.5,σ) (1.75,σ)	0.3823	0.5169	0.3762	**0.6451**	0.3294	0.4214

Table 5.122. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=2RCBD; K=3; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	0.1861	0.1104	0.1646	0.1302	**0.1866**	0.1032
(0,σ) (0,σ) (0,3σ)	**0.1503**	0.0938	0.1398	0.1097	0.1456	0.0897
(0,σ) (0,2σ) (0,3σ)	0.2292	0.1327	0.1975	0.1493	**0.2339**	0.1254
(0,σ) (0,2.5σ) (0,5σ)	0.4301	0.2345	0.3414	0.2593	**0.4563**	0.2186
(0,σ) (0,2σ) (0,5σ)	0.2833	0.1537	0.2444	0.1718	**0.2916**	0.1430
(0,σ) (0,3σ) (0,3.5σ)	0.3016	0.1631	0.2510	0.1830	**0.3127**	0.1545
(0,σ) (0,6σ) (0,8σ)	0.4481	0.2449	0.3602	0.2715	**0.4881**	0.2297

Table 5.123. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=2RCBD; K=3; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	**0.4424**	0.3494	0.4092	0.4373	0.4135	0.2952
(0,σ) (0,σ) (0,3σ)	**0.2487**	0.1650	0.2339	0.2004	0.2249	0.1449
(0,σ) (0,2σ) (1,3σ)	**0.3409**	0.2145	0.3025	0.2558	0.3357	0.1952
(0,σ) (0.5,2.5σ) (1,5σ)	**0.4602**	0.2931	0.3975	0.3436	0.4556	0.2575
(0,σ) (0.25,2σ) (0.5,5σ)	**0.3748**	0.2198	0.3224	0.2573	0.3723	0.1971
(0,σ) (1,3σ) (1.5,3.5σ)	**0.5602**	0.4027	0.5007	0.4899	0.5469	0.3494
(0,σ) (1.5,6σ) (1.75,8σ)	0.6792	0.4143	0.5704	0.5011	**0.7119**	0.3942

Table 5.124. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance CRD=2RCBD; K=3; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0518	0.0493	0.0497	0.0477	0.0536	0.0495
(0,σ) (1,σ) (1,σ)	0.2596	0.3338	0.2663	0.3188	0.2174	**0.3588**
(0,σ) (0,σ) (1,σ)	**0.1887**	0.1694	0.1819	0.1615	0.1737	0.1804
(0,σ) (0.5,σ) (1,σ)	0.2219	0.2521	0.2194	0.2397	0.1956	**0.2700**
(0,σ) (1,σ) (1.5,σ)	0.3299	0.4450	0.3356	0.4226	0.2746	**0.4746**
(0,σ) (1.5,σ) (2,σ)	0.4130	0.6293	0.4315	0.6054	0.3316	**0.6586**
(0,σ) (1.5,σ) (1.75,σ)	0.3889	0.5878	0.4081	0.5639	0.3079	**0.6216**

Table 5.125. Estimated power of tests for mixed design under the T distribution with same means and different variances; the variance in CRD=2RCBD; K=3; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	0.2339	0.1528	**0.2731**	0.1482	0.1762	0.1547
(0,σ) (0,σ) (0,3σ)	0.17000	0.1072	**0.1842**	0.1043	0.1421	0.1100
(0,σ) (0,2σ) (0,3σ)	0.3118	0.1883	**0.3693**	0.1807	0.2230	0.1942
(0,σ) (0,2.5σ) (0,5σ)	0.4376	0.2668	**0.5278**	0.2599	0.3080	0.2721
(0,σ) (0,2σ) (0,5σ)	0.3900	0.2407	**0.4683**	0.2343	0.2738	0.2487
(0,σ) (0,3σ) (0,3.5σ)	0.4139	0.2541	**0.5010**	0.2465	0.2919	0.2581
(0,σ) (0,6σ) (0,8σ)	0.6250	0.4006	**0.7497**	0.3925	0.4208	0.4118

Table 5.126. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=2RCBD; K=3; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	0.5019	0.4365	**0.5513**	0.4196	0.3864	0.4609
(0,σ) (0,σ) (0,3σ)	0.2729	0.1917	**0.2915**	0.1872	0.2279	0.2028
(0,σ) (0,2σ) (1,3σ)	0.4300	0.3080	**0.4903**	0.2997	0.3212	0.3165
(0,σ) (0.5,2.5σ) (1,5σ)	0.5800	0.4199	**0.6682**	0.4077	0.4235	0.4385
(0,σ) (0.25,2σ) (0.5,5σ)	0.4829	0.3167	**0.5541**	0.3063	0.3506	0.3297
(0,σ) (1,3σ) (1.5,3.5σ)	0.6583	0.5303	**0.7372**	0.5137	0.4937	0.5530
(0,σ) (1.5,6σ) (1.75,8σ)	1	0.7637	0.9984	0.6787	1	0.8820

5.3.2. Four Treatments

Tables 5.127-5.144 show the simulation study results for four treatments (k=4) under the t distribution. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion (Table 5.127 to Table 5.135). Second, the variance of the CRD portion is two times the variance of the RCBD portion(Table 5.136 to Table 5.144). Third, the variance of the CRD portion is four times the variance of the RCBD portion (see Appendix C). Forth, the variance of the CRD portion is eight times the variance of the RCBD portion (see Appendix C).

We estimated the alpha values for the proposed tests and tabled them in the first row of the table where all populations have the same location and scale parameters (0,σ) (0,σ) (0,σ) (0,σ). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (5.127, 5.130, 5.133, 5.136, 5.139, and 5.142). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters.

We started with the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location parameters and the same scale parameters, the proposed test two, Z_2, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Next, we considered the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Finally, we considered the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location

parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. Similarly, the proposed test three, Z_3, has the highest powers when the populations have different location and scale parameters.

In cases where the variance of the CRD portion is either four or eight times the variance of the RCBD portion, the proposed tests that have the highest powers will be approximately the same as the cases where the variance of the CRD portion is two times the variance of the RCBD portion. The only exception to this is when the number of blocks in the RCBD portion is equal to the sample size for each treatment in the CRD portion, and the populations have different location parameters and the same scale parameters, the proposed test one, Z_1, has the highest powers (see Appendix B).

Table 5.127. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 10.$

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0517	0.0512	0.0517	0.0512	0.0517	0.0512
(0,σ) (1,σ) (1,σ) (1,σ)	0.3853	**0.5352**	0.3853	**0.5352**	0.3853	**0.5352**
(0,σ) (0,σ) (0,σ) (1,σ)	**0.1954**	0.1733	**0.1954**	0.1733	**0.1954**	0.1733
(0,σ) (0,σ) (1,σ) (1,σ)	0.3494	**0.3627**	0.3494	**0.3627**	0.3494	**0.3627**
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	0.4393	**0.5367**	0.4393	**0.5367**	0.4393	**0.5367**
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.5156	**0.7581**	0.5156	**0.7581**	0.5156	**0.7581**
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.4953	**0.7263**	0.4953	**0.7263**	0.4953	**0.7263**

Table 5.128. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	**0.3199**	0.1509	**0.3199**	0.1509	**0.3199**	0.1509
(0,σ) (0,σ) (0,3σ) (0,3σ)	**0.3235**	0.1476	**0.3235**	0.1476	**0.3235**	0.1476
(0,σ) (0,2σ) (0,3σ) (0,4σ)	**0.5145**	0.2262	**0.5145**	0.2262	**0.5145**	0.2262
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	**0.7055**	0.3376	**0.7055**	0.3376	**0.7055**	0.3376
(0,σ) (0,2σ) (0,5σ) (0,σ)	**0.3234**	0.1457	**0.3234**	0.1457	**0.3234**	0.1457
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	**0.4978**	0.2236	**0.4978**	0.2236	**0.4978**	0.2236
(0,σ) (0,4σ) (0,6σ) (0,8σ)	**0.7950**	0.4054	**0.7950**	0.4054	**0.7950**	0.4054

Table 5.129. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	**0.7029**	0.6118	**0.7029**	0.6118	**0.7029**	0.6118
(0,σ) (0,σ) (0,3σ) (1, 3σ)	**0.4742**	0.2606	**0.4742**	0.2606	**0.4742**	0.2606
(0,σ) (0,2σ) (1,3σ) (1,4σ)	**0.7416**	0.4698	**0.7416**	0.4698	**0.7416**	0.4698
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	**0.8747**	0.6173	**0.8747**	0.6173	**0.8747**	0.6173
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	**0.6306**	0.4124	**0.6306**	0.4124	**0.6306**	0.4124
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	**0.8704**	0.7831	**0.8704**	0.7831	**0.8704**	0.7831
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	**0.9297**	0.7312	**0.9297**	0.7312	**0.9297**	0.7312

Table 5.130. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0522	0.0535	0.0529	0.0512	0.0530	0.0480
(0,σ) (1,σ) (1,σ) (1,σ)	0.3041	0.4543	0.2968	**0.5279**	0.2666	0.3718
(0,σ) (0,σ) (0,σ) (1,σ)	0.1639	0.1595	0.1615	**0.1734**	0.1489	0.1327
(0,σ) (0,σ) (1,σ) (1,σ)	0.2829	0.3157	0.2766	**0.3616**	0.2492	0.2580
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	0.3462	0.4549	0.3448	**0.5381**	0.2987	0.3666
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.3887	0.6435	0.3827	**0.7497**	0.3361	0.5291
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.3712	0.6169	0.3647	**0.7175**	0.3252	0.5070

Table 5.131. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	**0.2219**	0.1208	0.198	0.1208	0.2133	0.1049
(0,σ) (0,σ) (0,3σ) (0,3σ)	**0.2330**	0.1178	0.2121	0.1188	0.2166	0.1038
(0,σ) (0,2σ) (0,3σ) (0,4σ)	**0.3478**	0.1573	0.2987	0.1616	0.3449	0.1368
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	0.4742	0.2077	0.3993	0.2057	**0.4762**	0.1810
(0,σ) (0,2σ) (0,5σ) (0,σ)	**0.2312**	0.1066	0.2121	0.1101	0.2136	0.0942
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	**0.3320**	0.1543	0.2876	0.1573	0.3243	0.1354
(0,σ) (0,4σ) (0,6σ) (0,8σ)	0.5527	0.2426	0.4589	0.2463	**0.5639**	0.2127

Table 5.132. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	**0.5518**	0.4811	0.5092	0.5478	0.5122	0.3947
(0,σ) (0,σ) (0,3σ) (1, 3σ)	**0.3387**	0.1939	0.3057	0.2100	0.3117	0.1633
(0,σ) (0,2σ) (1,3σ) (1,4σ)	**0.5470**	0.3391	0.4961	0.3721	0.5276	0.2843
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	**0.6937**	0.4351	0.6153	0.4794	0.6806	0.3627
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	**0.4554**	0.2621	0.4223	0.3146	0.4168	0.2311
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	**0.7206**	0.6369	0.6707	0.7143	0.6855	0.5327
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	**0.7636**	0.5208	0.6814	0.5677	0.7588	0.4425

Table 5.133. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in RCBD =CRD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0529	0.0497	0.0533	0.0516	0.0518	0.0517
(0,σ) (1,σ) (1,σ) (1,σ)	0.3062	0.4982	0.3165	0.4890	0.2524	**0.5298**
(0,σ) (0,σ) (0,σ) (1,σ)	0.1728	0.1649	0.1730	0.1660	0.1476	**0.1765**
(0,σ) (0,σ) (1,σ) (1,σ)	0.2906	0.3461	0.2976	0.3405	0.2427	**0.3731**
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	0.3535	0.5043	0.3635	0.4985	0.3010	**0.5368**
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.4173	0.7238	0.4368	0.7117	0.3359	**0.7619**
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.3914	0.6880	0.4083	0.6761	0.3189	**0.7253**

Table 5.134. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	0.2713	0.1457	**0.3088**	0.1473	0.2031	0.1546
(0,σ) (0,σ) (0,3σ) (0,3σ)	0.2780	0.1444	**0.3097**	0.1469	0.2172	0.1505
(0,σ) (0,2σ) (0,3σ) (0,4σ)	0.4455	0.2256	**0.5164**	0.2270	0.3147	0.2376
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	0.6252	0.3310	**0.7188**	0.3318	0.4378	0.3466
(0,σ) (0,2σ) (0,5σ) (0,σ)	0.2754	0.1410	**0.3041**	0.1447	0.2106	0.1471
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	0.4273	0.2156	**0.4969**	0.2184	0.3084	0.2278
(0,σ) (0,4σ) (0,6σ) (0,8σ)	0.7157	0.3963	**0.8182**	0.3978	0.5084	0.4107

Table 5.135. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in RCBD =CRD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	0.6081	0.5762	**0.6553**	0.5707	0.4834	0.6051
(0,σ) (0,σ) (0,3σ) (1, 3σ)	0.3961	0.2454	**0.4376**	0.2481	0.3053	0.2602
(0,σ) (0,2σ) (1,3σ) (1,4σ)	0.6517	0.4557	**0.7202**	0.4544	0.4982	0.4781
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	0.8004	0.5901	**0.8698**	0.5853	0.6208	0.6160
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	0.5292	0.3778	**0.5671**	0.3732	0.4132	0.4020
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	0.7861	0.7509	**0.8299**	0.7429	0.6408	0.7796
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	0.8694	0.7029	**0.9234**	0.6988	0.6995	0.7253

Table 5.136. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0520	0.0510	0.0520	0.0510	0.0520	0.0510
(0,σ) (1,σ) (1,σ) (1,σ)	0.3236	**0.3710**	0.3236	**0.3710**	0.3236	**0.3710**
(0,σ) (0,σ) (0,σ) (1,σ)	**0.1690**	0.1304	**0.1690**	0.1304	**0.1690**	0.1304
(0,σ) (0,σ) (1,σ) (1,σ)	**0.2789**	0.2484	**0.2789**	0.2484	**0.2789**	0.2484
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	0.3706	**0.3915**	0.3706	**0.3915**	0.3706	**0.3915**
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.4581	**0.5802**	0.4581	**0.5802**	0.4581	**0.5802**
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.4339	**0.5445**	0.4339	**0.5445**	0.4339	**0.5445**

Table 5.137. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	**0.3189**	0.1520	0.3189	0.1520	0.3189	0.1520
(0,σ) (0,σ) (0,3σ) (0,3σ)	**0.3228**	0.1456	0.3228	0.1456	0.3228	0.1456
(0,σ) (0,2σ) (0,3σ) (0,4σ)	**0.5177**	0.2356	0.5177	0.2356	0.5177	0.2356
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	**0.7117**	0.3412	0.7117	0.3412	0.7117	0.3412
(0,σ) (0,2σ) (0,5σ) (0,σ)	**0.3302**	0.1478	0.3302	0.1478	0.3302	0.1478
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	**0.4958**	0.2227	0.4958	0.2227	0.4958	0.2227
(0,σ) (0,4σ) (0,6σ) (0,8σ)	**0.7882**	0.405	0.7882	0.405	0.7882	0.405

Table 5.138. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance CRD=2RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	**0.6788**	0.4947	0.6788	0.4947	0.6788	0.4947
(0,σ) (0,σ) (0,3σ) (1, 3σ)	**0.4453**	0.2178	0.4453	0.2178	0.4453	0.2178
(0,σ) (0,2σ) (1,3σ) (1,4σ)	**0.7168**	0.4061	0.7168	0.4061	0.7168	0.4061
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	**0.8617**	0.5455	0.8617	0.5455	0.8617	0.5455
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	**0.5792**	0.3260	0.5792	0.3260	0.5792	0.3260
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	**0.8588**	0.6833	0.8588	0.6833	0.8588	0.6833
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	**0.9183**	0.6644	0.9183	0.6644	0.9183	0.6644

Table 5.139. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0532	0.0533	0.0505	0.0524	0.0499	0.0496
(0,σ) (1,σ) (1,σ) (1,σ)	0.2774	0.3499	0.2823	**0.4394**	0.2358	0.2699
(0,σ) (0,σ) (0,σ) (1,σ)	0.1503	0.1382	**0.1565**	0.1535	0.1309	0.1152
(0,σ) (0,σ) (1,σ) (1,σ)	0.2374	0.2443	0.2423	**0.2969**	0.2034	0.1939
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	0.3009	0.3609	0.3055	**0.4460**	0.2578	0.2813
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.3624	0.5246	0.3612	**0.6397**	0.3147	0.4177
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.3472	0.5007	0.3449	**0.6163**	0.2934	0.3895

Table 5.140. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	**0.2208**	0.1134	0.1991	0.1185	0.2157	0.0980
(0,σ) (0,σ) (0,3σ) (0,3σ)	**0.2333**	0.1179	0.2129	0.1192	0.2213	0.1005
(0,σ) (0,2σ) (0,3σ) (0,4σ)	**0.3514**	0.1600	0.3044	0.1655	0.3408	0.1399
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	0.4801	0.2067	0.4110	0.2152	**0.4844**	0.1790
(0,σ) (0,2σ) (0,5σ) (0,σ)	**0.2339**	0.1128	0.2107	0.1163	0.2207	0.0975
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	**0.3381**	0.1530	0.2879	0.1535	0.3293	0.1321
(0,σ) (0,4σ) (0,6σ) (0,8σ)	0.5538	0.2395	0.4603	0.2438	**0.5597**	0.2093

Table 5.141. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=2RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	**0.5148**	0.3928	0.488	0.4683	0.4696	0.3114
(0,σ) (0,σ) (0,3σ) (1, 3σ)	**0.3272**	0.1710	0.3030	0.1886	0.2965	0.1421
(0,σ) (0,2σ) (1,3σ) (1,4σ)	**0.5129**	0.2914	0.4707	0.331	0.4862	0.2407
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	**0.6625**	0.3790	0.5960	0.4241	0.6425	0.3125
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	**0.4314**	0.2194	0.4088	0.2737	0.3818	0.1904
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	**0.7045**	0.5546	0.6633	0.6454	0.6545	0.4488
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	**0.7475**	0.4613	0.6693	0.5147	0.7386	0.3824

Table 5.142. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance CRD=2RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0502	0.0493	0.0497	0.0507	0.0492	0.0507
(0,σ) (1,σ) (1,σ) (1,σ)	0.2693	0.3497	0.2714	0.3426	0.232	**0.3812**
(0,σ) (0,σ) (0,σ) (1,σ)	**0.1429**	0.1218	0.1338	0.1217	0.1354	0.1302
(0,σ) (0,σ) (1,σ) (1,σ)	0.2306	0.2328	0.2277	0.2294	0.2124	**0.2566**
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	0.2952	0.3542	0.2858	0.3459	0.2593	**0.3873**
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.3632	0.5456	0.3674	0.5335	0.3028	**0.5887**
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.3494	0.5077	0.3444	0.4979	0.2959	**0.5485**

Table 5.143. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=2RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	0.2713	0.1409	**0.3042**	0.1434	0.2013	0.1469
(0,σ) (0,σ) (0,3σ) (0,3σ)	0.2786	0.1436	**0.3139**	0.1463	0.2065	0.1491
(0,σ) (0,2σ) (0,3σ) (0,4σ)	0.4445	0.2249	**0.5142**	0.2275	0.3143	0.2387
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	0.6275	0.3268	**0.7232**	0.3301	0.4398	0.3396
(0,σ) (0,2σ) (0,5σ) (0,σ)	0.2723	0.1369	**0.3025**	0.1388	0.2085	0.1438
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	0.4243	0.2137	**0.4937**	0.2160	0.3035	0.2249
(0,σ) (0,4σ) (0,6σ) (0,8σ)	0.7186	0.3927	**0.8134**	0.3951	0.5120	0.4098

Table 5.144. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=2RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	0.5622	0.4534	**0.6017**	0.4456	0.4511	0.4849
(0,σ) (0,σ) (0,3σ) (1, 3σ)	0.3781	0.2169	**0.4113**	0.2175	0.2860	0.2304
(0,σ) (0,2σ) (1,3σ) (1,4σ)	0.6189	0.392	**0.6791**	0.3898	0.4701	0.4142
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	0.7810	0.5242	**0.8488**	0.5217	0.6072	0.5540
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	0.4790	0.3052	**0.5115**	0.3039	0.3816	0.3275
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	0.7620	0.6532	**0.8107**	0.6438	0.6173	0.6863
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	0.8519	0.6271	**0.9093**	0.6245	0.6884	0.6547

5.3.3. Five Treatments

Tables 5.145-5.162 show the simulation study results for five treatments (k=5) under the t distribution. Tables are grouped based on the variance ratio between the CRD and the RCBD portions. Four cases were considered related to the variance among the CRD portion and the RCBD portion. First, the variance of the CRD portion is equal to the variance of the RCBD portion (Table 5.145 to Table 5.153). Second, the variance of the CRD portion is two times the variance of the RCBD portion (Table 5.136 to Table 5.144). Third, the variance of the CRD portion is four times the variance of the RCBD portion (see Appendix C). Forth, the variance of the CRD portion is eight times the variance of the RCBD portion (see Appendix C).

Three cases were considered where the number of blocks in the RCBD is equal, larger, and smaller than the sample size for each treatment in the CRD portion. When the number of blocks in the RCBD portion is equal to the sample size for each treatment in the CRD portion, the weights will not affect, so the proposed test one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results.

We estimated the alpha values for the proposed tests and tabled them in the first row of the table where all populations have the same location and scale parameters (0,σ) (0,σ) (0,σ) (0,σ) (0,σ). The estimated alpha values for all proposed tests were around 0.05, meaning that all proposed tests maintained their alpha values, see the first row of Tables (5.145, 5.148, 5.151, 5.154, 5.157, and 5.160). Moreover, we estimated and compared the powers of all proposed tests under different combinations of location parameters and scale parameters.

We started with the number of blocks in the RCBD portion equal to the sample size for each treatment in the CRD portion. In this case, the weights will not affect, so the results of proposed tests one, three, and five are similar. Likewise, the proposed tests two, four, and six have the same results. When the populations have different location parameters and the same scale parameters, the proposed test two, Z_2, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Next, we considered we consider when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers.

Finally, we consider the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion. When the populations have different location parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. Similarly, the proposed test three, Z_3, has the highest powers when the populations have different location and scale parameters.

In cases where the variance of the CRD portion is either four or eight times the variance of the RCBD portion, the proposed tests that have the highest powers will be approximately the same as the cases where the variance of the CRD portion is two times the variance of the RCBD portion. The only exception to this is when the number of blocks in the RCBD portion is equal to the sample size for each treatment in the CRD portion, and the populations have different location parameters and the same scale parameters, the proposed test one, Z_1, has the highest powers (see Appendix C).

Table 5.145. Estimated power of tests for mixed design under the T distribution with different means and same variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0486	0.0507	0.0486	0.0507	0.0486	0.0507
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.3722	**0.5274**	0.3722	**0.5274**	0.3722	**0.5274**
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	**0.2880**	0.2726	**0.2880**	0.2726	**0.2880**	0.2726
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	0.3620	**0.4225**	0.3620	**0.4225**	0.3620	**0.4225**
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.4989	**0.6659**	0.4989	**0.6659**	0.4989	**0.6659**
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.5193	**0.8244**	0.5193	**0.8244**	0.5193	**0.8244**
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.5035	**0.8076**	0.5035	**0.8076**	0.5035	**0.8076**

Table 5.146. Estimated power of tests for mixed design under the T distribution with same means and different variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	**0.3549**	0.1576	**0.3549**	0.1576	**0.3549**	0.1576
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	**0.4405**	0.1776	**0.4405**	0.1776	**0.4405**	0.1776
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	**0.6645**	0.2848	**0.6645**	0.2848	**0.6645**	0.2848
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	**0.8431**	0.4120	**0.8431**	0.4120	**0.8431**	0.4120
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	**0.4829**	0.1912	**0.4829**	0.1912	**0.4829**	0.1912
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	**0.6759**	0.2866	**0.6759**	0.2866	**0.6759**	0.2866
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	**0.8952**	0.4843	**0.8952**	0.4843	**0.8952**	0.4843

Table 5.147. Estimated power of tests for mixed design under the T distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	**0.7431**	0.6308	**0.7431**	0.6308	**0.7431**	0.6308
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	**0.6471**	0.3668	**0.6471**	0.3668	**0.6471**	0.3668
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	**0.8497**	0.5769	**0.8497**	0.5769	**0.8497**	0.5769
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	**0.9482**	0.7148	**0.9482**	0.7148	**0.9482**	0.7148
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	**0.7657**	0.5064	**0.7657**	0.5064	**0.7657**	0.5064
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	**0.9154**	0.8291	**0.9154**	0.8291	**0.9154**	0.8291
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	**0.9643**	0.7969	**0.9643**	0.7969	**0.9643**	0.7969

Table 5.148. Estimated power of tests for mixed design under the T distribution with different means and same variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0498	0.0474	0.0536	0.0499	0.0523	0.0507
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.2883	0.4145	0.2900	**0.5093**	0.2541	0.3526
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	0.2401	0.2243	0.2364	**0.2784**	0.2108	0.1965
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	0.2901	0.3222	0.2885	**0.4070**	0.2493	0.2741
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.3825	0.5289	0.3789	**0.6456**	0.3322	0.4549
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.3906	0.6898	0.3838	**0.8021**	0.3299	0.5961
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.3804	0.6691	0.3731	**0.7905**	0.3202	0.5761

Table 5.149. Estimated power of tests for mixed design under the T distribution with same means and different variances; the variance in RCBD =CRD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	**0.2548**	0.1127	0.2303	0.1326	0.2403	0.1023
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	**0.2932**	0.1180	0.2693	0.1431	0.2772	0.1092
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	**0.4636**	0.1713	0.4033	0.2075	0.4415	0.1573
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	**0.6257**	0.2378	0.5447	0.2904	0.6245	0.2126
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	**0.3371**	0.1295	0.3041	0.1591	0.3179	0.1158
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	**0.4610**	0.1685	0.4014	0.2071	0.4457	0.1576
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	0.6728	0.2644	0.5798	0.3157	**0.6773**	0.2372

Table 5.150. Estimated power of tests for mixed design under the T distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	**0.5830**	0.4769	0.5490	0.5785	0.5356	0.4139
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	**0.4772**	0.2541	0.4427	0.3179	0.4433	0.2201
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	**0.6647**	0.3709	0.6013	0.4578	0.6352	0.3250
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	**0.8021**	0.4829	0.7339	0.5795	0.7828	0.4214
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	**0.5935**	0.3504	0.5477	0.4418	0.5510	0.3018
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	**0.7856**	0.6527	0.7483	0.7644	0.7436	0.5701
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	**0.8511**	0.5677	0.7849	0.665	0.8361	0.5025

Table 5.151. Estimated power of tests for mixed design under the T distribution with different means and same variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0496	0.0523	0.0496	0.0533	0.0508	0.0535
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.2995	0.5039	0.3090	0.4972	0.2491	**0.5371**
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	0.2429	0.2513	0.2456	0.2482	0.2092	**0.2695**
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	0.3031	0.3845	0.3087	0.3783	0.2542	**0.4161**
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.3872	0.6240	0.3996	0.6126	0.3208	**0.6596**
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.3935	0.7856	0.4027	0.7734	0.3172	**0.8201**
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.3916	0.7754	0.4075	0.7625	0.3153	**0.8070**

Table 5.152. Estimated power of tests for mixed design under the T distribution with same means and different variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	0.2981	0.1500	**0.3335**	0.1503	0.2294	0.1589
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	0.3555	0.1686	**0.3994**	0.1681	0.2652	0.1803
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	0.5662	0.2646	**0.6507**	0.2651	0.4088	0.2792
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	0.7622	0.3942	**0.8480**	0.3940	0.5538	0.4149
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	0.3955	0.1847	**0.4480**	0.1833	0.2885	0.1986
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	0.5630	0.2636	**0.6435**	0.2624	0.4073	0.2787
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	0.8201	0.4605	**0.9016**	0.4584	0.6092	0.4860

Table 5.153. Estimated power of tests for mixed design under the T distribution with different means and variances; the variance in RCBD =CRD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	0.6374	0.6007	**0.6795**	0.5910	0.5085	0.6338
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	0.5411	0.3496	**0.5904**	0.3437	0.4127	0.3726
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	0.7635	0.5425	**0.8234**	0.5363	0.5912	0.5756
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	0.8833	0.6890	**0.9332**	0.6807	0.7227	0.7160
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	0.6539	0.4762	**0.7131**	0.4702	0.5076	0.5078
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	0.8511	0.8016	**0.8839**	0.7902	0.7195	0.8295
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	0.9259	0.7761	**0.9590**	0.7687	0.7876	0.8017

Table 5.154. Estimated power of tests for mixed design under the T distribution with different means and same variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0502	0.0509	0.0502	0.0509	0.0502	0.0509
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.3220	**0.3792**	0.3220	**0.3792**	0.3220	**0.3792**
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	**0.2409**	0.2010	**0.2409**	0.2010	**0.2409**	0.2010
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	**0.3027**	0.2896	**0.3027**	0.2896	**0.3027**	0.2896
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.4470	**0.5015**	0.4470	**0.5015**	0.4470	**0.5015**
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.4813	**0.6674**	0.4813	**0.6674**	0.4813	**0.6674**
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.4761	**0.6467**	0.4761	**0.6467**	0.4761	**0.6467**

Table 5.155. Estimated power of tests for mixed design under the T distribution with same means and different variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	**0.3582**	0.1560	0.3582	0.1560	0.3582	0.1560
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	**0.4346**	0.1795	0.4346	0.1795	0.4346	0.1795
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	**0.6591**	0.2774	0.6591	0.2774	0.6591	0.2774
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	**0.8510**	0.4188	0.8510	0.4188	0.8510	0.4188
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	**0.4910**	0.1936	0.4910	0.1936	0.4910	0.1936
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	**0.6574**	0.2831	0.6574	0.2831	0.6574	0.2831
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	**0.8961**	0.4823	0.8961	0.4823	0.8961	0.4823

Table 5.156. Estimated power of tests for mixed design under the T distribution with different means and variances; the variance CRD=2RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	**0.7090**	0.5219	**0.7090**	0.5219	**0.7090**	0.5219
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	**0.6136**	0.3098	**0.6136**	0.3098	**0.6136**	0.3098
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	**0.8368**	0.5028	**0.8368**	0.5028	**0.8368**	0.5028
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	**0.9392**	0.6507	**0.9392**	0.6507	**0.9392**	0.6507
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	**0.7336**	0.4148	**0.7336**	0.4148	**0.7336**	0.4148
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	**0.9218**	0.7566	**0.9218**	0.7566	**0.9218**	0.7566
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	**0.9694**	0.7425	**0.9694**	0.7425	**0.9694**	0.7425

Table 5.157. Estimated power of tests for mixed design under the T distribution with different means and same variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0507	0.0487	0.0530	0.0498	0.0507	0.0494
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.2621	0.3281	0.2724	**0.4311**	0.2292	0.2717
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	0.1954	0.1714	0.2052	**0.2231**	0.1652	0.1496
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	0.2555	0.2577	0.2632	**0.3350**	0.2183	0.2181
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.3540	0.4288	0.3627	**0.5524**	0.3024	0.3513
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.3764	0.5722	0.3732	**0.7117**	0.3191	0.4722
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.3697	0.5493	0.3676	**0.6899**	0.3162	0.4571

Table 5.158. Estimated power of tests for mixed design under the T distribution with same means and different variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	**0.2492**	0.1113	0.2266	0.1305	0.2341	0.1018
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	**0.2934**	0.1173	0.2684	0.1392	0.2745	0.1086
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	**0.4614**	0.1724	0.4087	0.2091	0.4452	0.1576
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	**0.6243**	0.2334	0.5448	0.2848	0.6167	0.2072
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	**0.3360**	0.1247	0.3021	0.1495	0.3132	0.1132
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	**0.4678**	0.1749	0.4080	0.2147	0.4491	0.1567
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	0.6801	0.2705	0.5872	0.3207	**0.6824**	0.2409

Table 5.159. Estimated power of tests for mixed design under the T distribution with different means and variances; the variance in CRD=2RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	**0.5516**	0.3884	0.5282	0.5019	0.4964	0.3291
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	**0.4479**	0.2153	0.4172	0.2784	0.4076	0.1835
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	**0.6550**	0.3318	0.6010	0.4231	0.6175	0.2804
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	**0.7902**	0.4316	0.7195	0.5360	0.7685	0.3673
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	**0.5593**	0.2912	0.5265	0.3823	0.5121	0.2437
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	**0.7883**	0.5846	0.7479	0.7124	0.7424	0.4944
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	**0.8494**	0.5098	0.7863	0.6148	0.8294	0.4398

Table 5.160. Estimated power of tests for mixed design under the T distribution with different means and same variances; the variance CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0500	0.0493	0.0501	0.0507	0.0521	0.0510
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.2611	0.3515	0.2553	0.3422	0.2276	**0.3855**
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	0.1949	0.1867	0.1903	0.1830	0.1802	**0.1998**
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	0.2488	0.2665	0.2398	0.2615	0.2241	**0.2923**
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.3489	0.4620	0.3439	0.4517	0.2990	**0.4991**
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.3722	0.6340	0.3821	0.6172	0.3073	**0.6768**
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.3812	0.6087	0.3881	0.5955	0.3046	**0.6534**

Table 5.161. Estimated power of tests for mixed design under the T distribution with same means and different variances; the variance in CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	0.2986	0.1465	**0.3323**	0.1455	0.2280	0.1555
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	0.3611	0.1710	**0.4007**	0.1699	0.2727	0.1793
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	0.5668	0.2637	**0.6395**	0.2630	0.4077	0.2828
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	0.7597	0.3913	**0.8457**	0.3883	0.5554	0.4115
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	0.3947	0.1780	**0.4453**	0.1792	0.2955	0.1891
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	0.5697	0.2658	**0.6439**	0.2651	0.4140	0.2825
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	0.8191	0.4621	**0.9012**	0.4600	0.6100	0.4830

Table 5.162. Estimated power of tests for mixed design under the T distribution with different means and variances; the variance in CRD=2RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	0.5980	0.4831	**0.6402**	0.4715	0.479	0.5209
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	0.5100	0.2897	**0.5555**	0.2834	0.3927	0.3145
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	0.7345	0.4599	**0.7993**	0.4527	0.5740	0.4963
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	0.8723	0.6068	**0.9257**	0.5963	0.7062	0.6422
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	0.6240	0.3917	**0.6737**	0.3826	0.4924	0.4212
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	0.8358	0.7064	**0.8726**	0.6940	0.6934	0.7405
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	0.9219	0.7209	**0.9592**	0.7098	0.7722	0.7510

CHAPTER 6. CONCLUSION

We proposed six nonparametric tests for a simple tree alternative for testing the location-scale problem when the data are a mixture of a randomized complete block design (RCBD) and a completely randomized design (CRD). The tests combined the Fligner-Wolfe test, modified Pages test, and modified Ansari-Bradley test for CRD and RCBD. The simulation study showed that all proposed tests maintained their alpha values. Since the tests all maintained their alpha values, we compared them on the basis of estimated powers.

We will first consider the case where the underlying distributions are symmetric (normal and T). When the number of blocks in the RCBD portion is equal to the sample size for each treatment in the CRD portion, the proposed test one, Z_1, has the highest powers in the majority of cases. The only exception to this is when the variance in the CRD portion is equal or two times the variance in the RCBD portion, and the populations have different location parameters and the same scale parameters. In this situation, the proposed test two, Z_2, has the highest powers.

On the other hand, when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion, we found that when the populations have different location parameters and the same scale parameters, the proposed test four, Z_4, has the highest powers. However, when the populations have the same location parameters and different scale parameters, and the number of treatments is four or five, the proposed test one, Z_1, has the highest powers. While when the number of treatments is three, the proposed test five, Z_5, has the highest powers. In another scenario, when the populations have different location and scale parameters, the proposed test one, Z_1, has the highest powers. Figure 6.1 compares the powers of proposed tests when the number of blocks in the RCBD portion increase and the sample size for each treatment in the CRD portion is held constant ($n_a = 5$), and the populations have different location and scale parameters for the normal distribution. The remainder of the graphs can be found in Appendix D for cases where the variance of the CRD portion is larger than the variance of the RCBD portion, and the number of treatments is 3, 4, and 5.

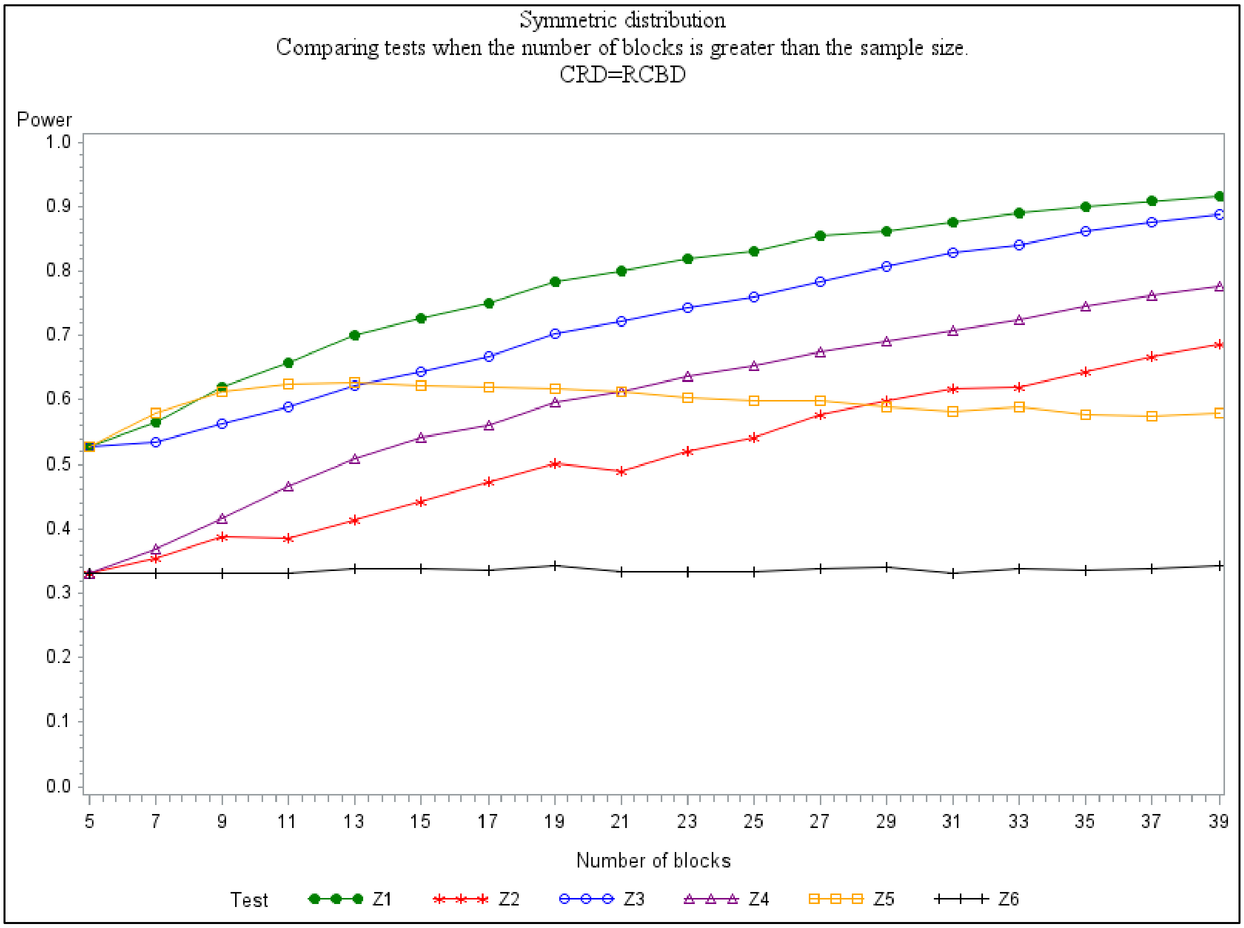

Figure 6.1. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=RCBD; K=4; n_a=5, and n_b=5,7,9,…,39.

On the contrary, when the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion, we found that the proposed tests three, Z_3, and six, Z_6, have the highest powers in the majority of the cases. In particular, when populations have different location parameters and the same scale parameters, the proposed test six, Z_6, has the highest powers. Otherwise, the proposed test three, Z_3, has the highest powers. Figure 6.2 compares the powers of proposed tests when the number of blocks in the RCBD portion is held constant ($n_b = 5$) and the sample size for each treatment in the CRD portion increase, and the populations have different location and scale parameters for the normal distribution. The remainder of the graphs can be found in Appendix for cases where the variance of the CRD portion is larger than the variance of the RCBD portion, and the number of treatments is 3, 4, and 5.

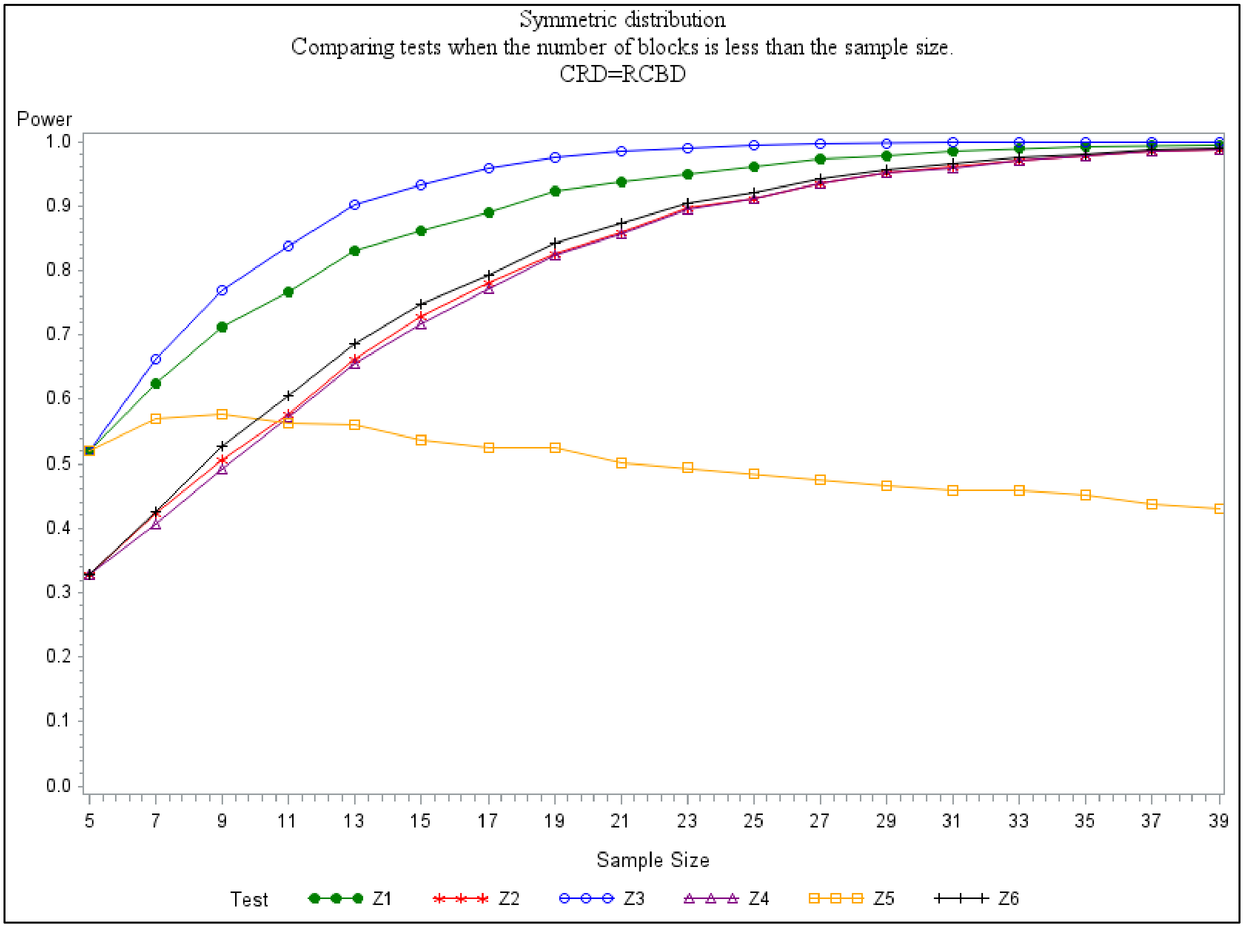

Figure 6.2. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=RCBD; K=4; n_a=5,7,9,…,39, and n_b=5.

Next, we will consider the case where the underlying distribution is asymmetric (exponential). When the number of blocks in the RCBD portion is equal to the sample size for each treatment in the CRD portion, we found that when the populations have different location parameters and same scale parameters, the proposed test two, Z_2, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test one, Z_1, has the highest powers. When the populations have different location and scale parameters and the variance in the CRD portion is equal to the variance in the RCBD portion, the proposed test two, Z_2, has the highest powers. Otherwise, the proposed test one, Z_1, has the highest powers.

However, when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion, we found that when the populations have different location parameters and same scale parameters, the proposed test four, Z_4, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test five, Z_5, has the highest powers. When the populations have different location and scale parameters and the variance of the CRD portion is equal or two times the variance of the RCBD portion, the proposed test four, Z_4, has the highest powers. While, when the variance of the CRD portion is four or eight times the variance of the RCBD portion, the proposed test one, Z_1, has the highest powers. Figure 6.3 compares the powers of proposed tests when the number of blocks in the RCBD portion increase and the sample size for each treatment in the CRD portion is held constant ($n_a = 5$), and the populations have different location and scale parameters for the exponential distribution. The remainder of the graph can be found in Appendix E for cases where the variance of the CRD portion is larger than the variance of the RCBD portion, and the number of treatments is 3, 4, and 5.

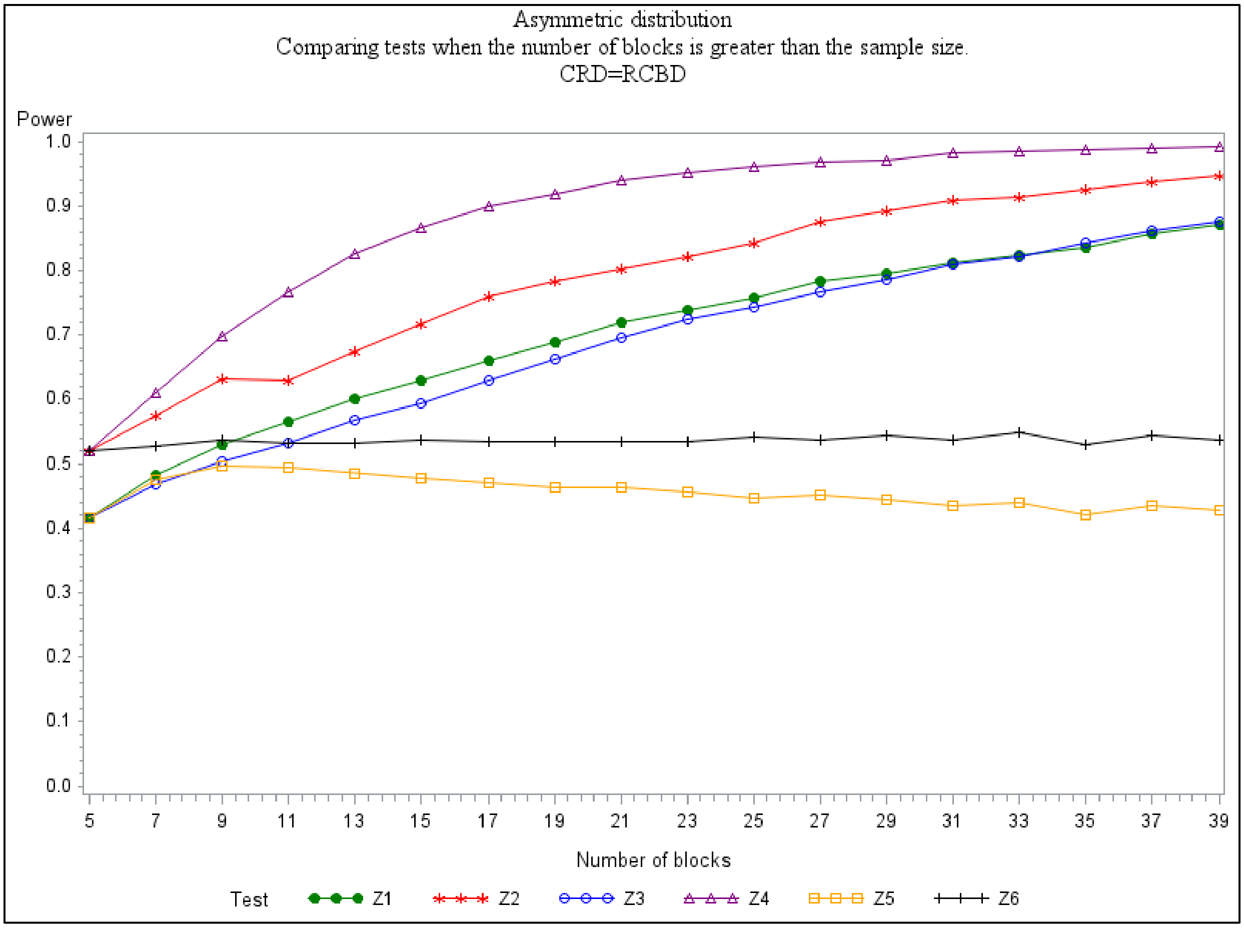

Figure 6.3. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=RCBD; K=4; n_a=5, and n_b=5,7,9,…,39.

Contrastingly, when the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion, we found that when the populations have different location parameters and same scale parameters, the proposed test six, Z_6, has the highest powers. When the populations have the same location parameters and different scale parameters, the proposed test three, Z_3, has the highest powers. When the populations have different location and scale parameters and the variance in the CRD portion is equal to the variance in the RCBD portion, the proposed test six, Z_6, has the highest powers. Otherwise, the proposed test three, Z_3, has the highest powers. Figure 6.4 compares the powers of proposed tests when the number of blocks in the RCBD portion is held constant ($n_b = 5$) and the sample size for each treatment in the CRD portion increase, and the populations have different location and scale parameters for the exponential distribution. The remainder of the graph can be found in Appendix E for cases where the variance of the CRD portion is larger than the variance of the RCBD portion, and the number of treatments is 3, 4, and 5.

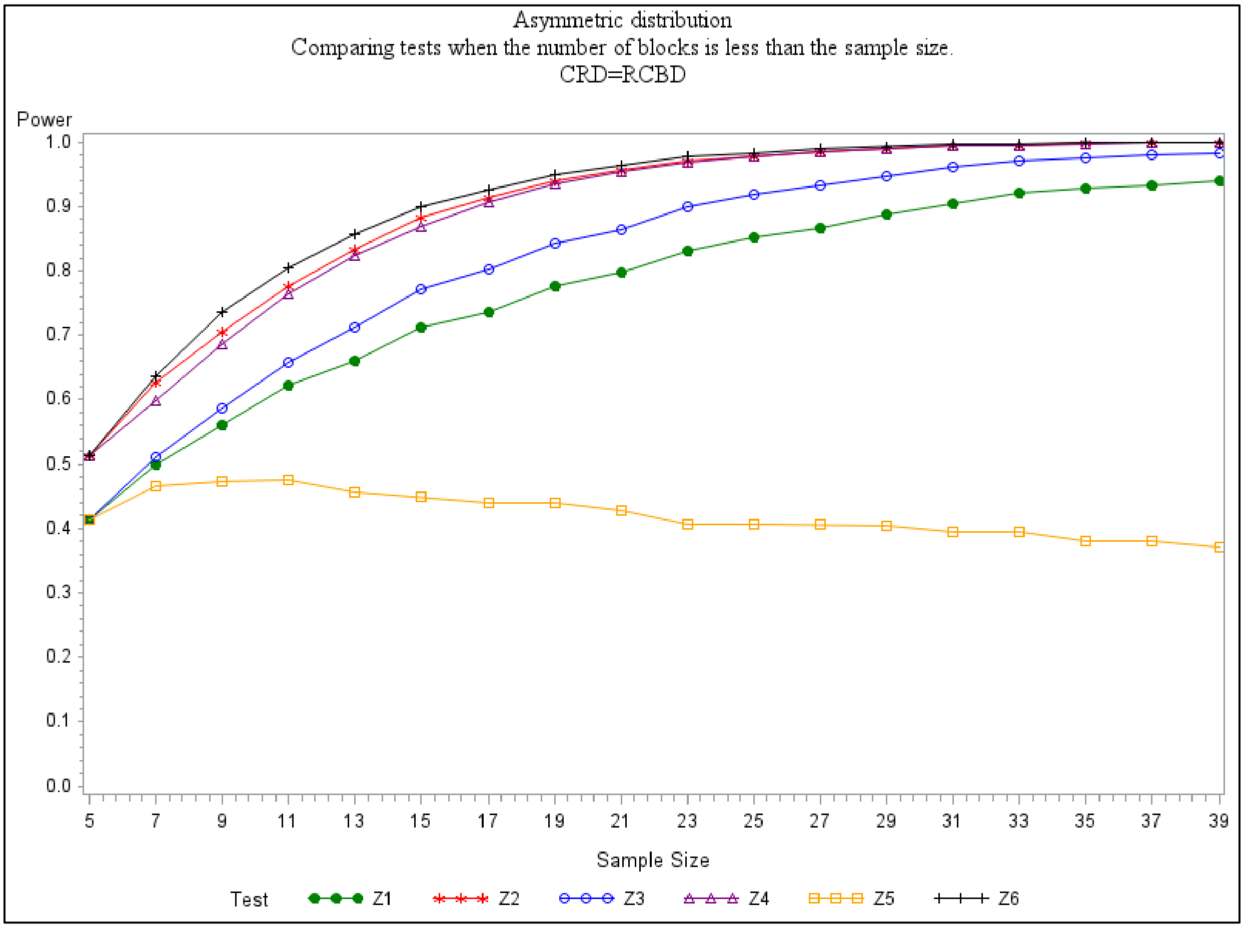

Figure 6.4. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=RCBD; K=4; n_a=5,7,9,…,39, and n_b=5.

In the light of our findings, the overall recommendation for location and scale testing is to use the proposed test one, Z_1, and proposed test three, Z_3, if the observations are assumed to come from an approximately symmetric distribution. The proposed test one, Z_1, is recommended when the number of blocks of the RCBD portion is equal to or greater than the sample size for each treatment in the CRD portion. However, if the number of blocks for the RCBD portion is less than the sample size for each treatment in the CRD portion, the proposed test three, Z_3, is recommended. On the other hand, if the observations are assumed to come from an asymmetric distribution, and the variance in the CRD portion is equal to the variance in the RCBD portion, the overall recommendation is to use the proposed test two, Z_2, proposed test four, Z_4, and proposed test six, Z_6. The proposed test two, Z_2, is recommended when the sample size for each treatment in the CRD portion is equal to the number of blocks in the RCBD portion. The proposed test four, Z_4, is recommended when the number of blocks in the RCBD portion is greater than the sample size for each treatment in the CRD portion. When the number of blocks in the RCBD portion is less than the sample size for each treatment in the CRD portion, the proposed test six, Z_6, is recommended. However, when the variance in the CRD portion is larger than the variance in the RCBD portion, the overall recommendation is approximately similar to the recommendation that we mentioned when the observations are assumed to come from symmetric distribution. The summary of results can be found in Appendix F.

For future study, this research might be extended to develop a new test for multiple comparison tests (Post Hoc Test) for location and scale problems in a mixed design. Also, this research might consider different arrangements of the sample sizes, number of blocks, and increasing the number of treatments. Moreover, we might extend the comparison between the proposed tests using other distributions.

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APPENDIX A. ESTIMATED POWERS FOR NORMAL DISTRIBUTION

Table A1. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0505	0.0514	0.0505	0.0514	0.0505	0.0514
(0,1) (1,1) (1,1)	**0.3497**	0.3273	**0.3497**	0.3273	**0.3497**	0.3273
(0,1) (0,1) (1,1)	**0.2489**	0.1606	**0.2489**	0.1606	**0.2489**	0.1606
(0,1) (0.5,1) (1,1)	**0.2863**	0.2395	**0.2863**	0.2395	**0.2863**	0.2395
(0,1) (1,1) (1.5,1)	**0.4375**	0.4311	**0.4375**	0.4311	**0.4375**	0.4311
(0,1) (1.5,1) (2,1)	0.5606	**0.6249**	0.5606	**0.6249**	0.5606	**0.6249**
(0,1) (1.5,1) (1.75,1)	0.5265	**0.5732**	0.5265	**0.5732**	0.5265	**0.5732**

Table A2. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	**0.3175**	0.1731	**0.3175**	0.1731	**0.3175**	0.1731
(0,1) (0,1) (0,3)	**0.2318**	0.1216	**0.2318**	0.1216	**0.2318**	0.1216
(0,1) (0,2) (0,3)	**0.4128**	0.2203	**0.4128**	0.2203	**0.4128**	0.2203
(0,1) (0,2.5) (0,5)	**0.5581**	0.3068	**0.5581**	0.3068	**0.5581**	0.3068
(0,1) (0,2) (0,5)	**0.5068**	0.2727	**0.5068**	0.2727	**0.5068**	0.2727
(0,1) (0,3) (0,3.5)	**0.5512**	0.3005	**0.5512**	0.3005	**0.5512**	0.3005
(0,1) (0,6) (0,8)	**0.7321**	0.4496	**0.7321**	0.4496	**0.7321**	0.4496

Table A3. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	**0.6543**	0.4557	0.6543	0.4557	0.6543	0.4557
(0,1) (0,1) (1,3)	**0.3636**	0.2014	0.3636	0.2014	0.3636	0.2014
(0,1) (0,2) (1,3)	**0.5698**	0.3333	0.5698	0.3333	0.5698	0.3333
(0,1) (0.5,2.5) (1,5)	**0.7327**	0.4594	0.7327	0.4594	0.7327	0.4594
(0,1) (0.25,2) (0.5,5)	**0.6157**	0.3505	0.6157	0.3505	0.6157	0.3505
(0,1) (1,3) (1.5,3.5)	**0.8059**	0.5727	0.8059	0.5727	0.8059	0.5727
(0,1) (1.5,6) (1.75,8)	**0.8754**	0.6304	0.8754	0.6304	0.8754	0.6304

Table A4. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0508	0.0519	0.0532	0.0563	0.0504	0.0533
(0,1) (1,1) (1,1)	0.2899	0.3097	0.3051	**0.4296**	0.2358	0.2396
(0,1) (0,1) (1,1)	0.2179	0.1623	**0.2386**	0.2319	0.1718	0.1341
(0,1) (0.5,1) (1,1)	0.2403	0.2309	0.2627	**0.3218**	0.1918	0.1834
(0,1) (1,1) (1.5,1)	0.3477	0.3874	0.3633	**0.5362**	0.2848	0.3030
(0,1) (1.5,1) (2,1)	0.4247	0.5523	0.4244	**0.7263**	0.3547	0.4366
(0,1) (1.5,1) (1.75,1)	0.4066	0.5149	0.4079	**0.6845**	0.3392	0.4057

Table A5. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	**0.2147**	0.1235	0.1898	0.1436	0.2127	0.1152
(0,1) (0,1) (0,3)	**0.1656**	0.0984	0.1566	0.1161	0.1569	0.0918
(0,1) (0,2) (0,3)	0.2653	0.1458	0.2290	0.1629	**0.2703**	0.1327
(0,1) (0,2.5) (0,5)	0.3643	0.1921	0.3046	0.2152	**0.3788**	0.1798
(0,1) (0,2) (0,5)	0.3351	0.1777	0.2772	0.2020	**0.3450**	0.1641
(0,1) (0,3) (0,3.5)	0.3509	0.1879	0.2898	0.2108	**0.3666**	0.1733
(0,1) (0,6) (0,8)	0.4818	0.2660	0.3775	0.2919	**0.5270**	0.2488

Table A6. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	**0.5095**	0.3730	0.4786	0.4844	0.4714	0.3081
(0,1) (0,1) (1,3)	**0.2703**	0.1560	0.2641	0.2046	0.2340	0.1319
(0,1) (0,2) (1,3)	**0.4025**	0.2355	0.3630	0.2875	0.3845	0.2048
(0,1) (0.5,2.5) (1,5)	**0.5180**	0.3114	0.4436	0.3752	0.5094	0.2723
(0,1) (0.25,2) (0.5,5)	**0.4186**	0.2379	0.3581	0.2854	0.4168	0.2080
(0,1) (1,3) (1.5,3.5)	**0.6091**	0.4222	0.5510	0.5211	0.5903	0.3529
(0,1) (1.5,6) (1.75,8)	0.7136	0.4039	0.5965	0.5017	**0.7426**	0.3796

Table A7. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=4RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0505	0.0510	0.0505	0.0497	0.0498	0.0506
(0,1) (1,1) (1,1)	0.2695	0.2972	0.2536	0.2790	0.2420	**0.3259**
(0,1) (0,1) (1,1)	0.1862	0.1503	0.1695	0.1409	**0.1891**	0.1647
(0,1) (0.5,1) (1,1)	0.2160	0.2165	0.2040	0.2054	0.2025	**0.2347**
(0,1) (1,1) (1.5,1)	0.3378	0.3954	0.3300	0.3732	0.2914	**0.4332**
(0,1) (1.5,1) (2,1)	0.4202	0.5725	0.4279	0.5469	0.3328	**0.6190**
(0,1) (1.5,1) (1.75,1)	0.4065	0.5425	0.4096	0.5131	0.3184	**0.5843**

Table A8. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=4RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	0.2713	0.1672	**0.3167**	0.1635	0.2004	0.1705
(0,1) (0,1) (0,3)	0.1963	0.1242	**0.2171**	0.1213	0.1586	0.1271
(0,1) (0,2) (0,3)	0.3649	0.2186	**0.4330**	0.2136	0.2532	0.2218
(0,1) (0,2.5) (0,5)	0.4988	0.3068	**0.6005**	0.2988	0.3430	0.3135
(0,1) (0,2) (0,5)	0.4470	0.2655	**0.5391**	0.2595	0.3116	0.2742
(0,1) (0,3) (0,3.5)	0.4788	0.3021	**0.5787**	0.2956	0.3299	0.3068
(0,1) (0,6) (0,8)	0.6638	0.4420	**0.7960**	0.4315	0.4442	0.4514

Table A9. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=4RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	0.5615	0.4426	**0.6102**	0.4245	0.4404	0.4688
(0,1) (0,1) (1,3)	0.3050	0.1875	**0.3198**	0.1794	0.2503	0.1980
(0,1) (0,2) (1,3)	0.5016	0.3353	**0.5636**	0.3224	0.3773	0.3517
(0,1) (0.5,2.5) (1,5)	0.6411	0.4355	**0.7291**	0.4218	0.4769	0.4549
(0,1) (0.25,2) (0.5,5)	0.5322	0.3352	**0.6158**	0.3244	0.3859	0.3490
(0,1) (1,3) (1.5,3.5)	0.7266	0.5471	**0.8003**	0.5288	0.5589	0.5745
(0,1) (1.5,6) (1.75,8)	0.9999	0.7630	0.9991	0.6765	1	0.8836

Table A10. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0513	0.0542	0.0513	0.0542	0.0513	0.0542
(0,1) (1,1) (1,1)	**0.2955**	0.2308	**0.2955**	0.2308	**0.2955**	0.2308
(0,1) (0,1) (1,1)	**0.2261**	0.1345	**0.2261**	0.1345	**0.2261**	0.1345
(0,1) (0.5,1) (1,1)	**0.2454**	0.1769	**0.2454**	0.1769	**0.2454**	0.1769
(0,1) (1,1) (1.5,1)	**0.3624**	0.2970	**0.3624**	0.2970	**0.3624**	0.2970
(0,1) (1.5,1) (2,1)	**0.4668**	0.4412	**0.4668**	0.4412	**0.4668**	0.4412
(0,1) (1.5,1) (1.75,1)	**0.4334**	0.4033	**0.4334**	0.4033	**0.4334**	0.4033

Table A11. Estimated power of tests for mixed design under the normal distribution with same means and variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	**0.3129**	0.1693	**0.3129**	0.1693	**0.3129**	0.1693
(0,1) (0,1) (0,3)	**0.2306**	0.1254	**0.2306**	0.1254	**0.2306**	0.1254
(0,1) (0,2) (0,3)	**0.4122**	0.2181	**0.4122**	0.2181	**0.4122**	0.2181
(0,1) (0,2.5) (0,5)	**0.5603**	0.3115	**0.5603**	0.3115	**0.5603**	0.3115
(0,1) (0,2) (0,5)	**0.4978**	0.2702	**0.4978**	0.2702	**0.4978**	0.2702
(0,1) (0,3) (0,3.5)	**0.5409**	0.2964	**0.5409**	0.2964	**0.5409**	0.2964
(0,1) (0,6) (0,8)	**0.7245**	0.4390	**0.7245**	0.4390	**0.7245**	0.4390

Table A12. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	**0.6265**	0.3837	**0.6265**	0.3837	**0.6265**	0.3837
(0,1) (0,1) (1,3)	**0.3555**	0.1829	**0.3555**	0.1829	**0.3555**	0.1829
(0,1) (0,2) (1,3)	**0.5584**	0.3044	**0.5584**	0.3044	**0.5584**	0.3044
(0,1) (0.5,2.5) (1,5)	**0.7140**	0.4184	**0.7140**	0.4184	**0.7140**	0.4184
(0,1) (0.25,2) (0.5,5)	**0.5928**	0.3282	**0.5928**	0.3282	**0.5928**	0.3282
(0,1) (1,3) (1.5,3.5)	**0.7909**	0.5072	**0.7909**	0.5072	**0.7909**	0.5072
(0,1) (1.5,6) (1.75,8)	**0.8593**	0.5819	**0.8593**	0.5819	**0.8593**	0.5819

Table A13. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0523	0.0502	0.0526	0.0537	0.0531	0.0511
(0,1) (1,1) (1,1)	0.2482	0.2470	0.2762	**0.3611**	0.1969	0.1853
(0,1) (0,1) (1,1)	0.1987	0.1358	**0.2286**	0.2023	0.1501	0.1086
(0,1) (0.5,1) (1,1)	0.2128	0.1862	0.2335	**0.2683**	0.1665	0.1496
(0,1) (1,1) (1.5,1)	0.3079	0.3024	0.3411	**0.4485**	0.2366	0.2266
(0,1) (1.5,1) (2,1)	0.3597	0.4233	0.3808	**0.6070**	0.2899	0.3134
(0,1) (1.5,1) (1.75,1)	0.3488	0.4047	0.3614	**0.5748**	0.2810	0.2991

Table A14. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	**0.2147**	0.1223	0.1881	0.1396	0.2111	0.1149
(0,1) (0,1) (0,3)	**0.1689**	0.1016	0.1573	0.1158	0.1645	0.0975
(0,1) (0,2) (0,3)	0.2773	0.1502	0.2317	0.1727	**0.2839**	0.1399
(0,1) (0,2.5) (0,5)	0.3635	0.1958	0.2967	0.2200	**0.3806**	0.1800
(0,1) (0,2) (0,5)	0.3307	0.1764	0.2732	0.1993	**0.3440**	0.1642
(0,1) (0,3) (0,3.5)	0.3555	0.1894	0.2938	0.2131	**0.3700**	0.1736
(0,1) (0,6) (0,8)	0.4812	0.2633	0.3759	0.2846	**0.5249**	0.2456

Table A15. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	**0.4735**	0.3152	0.4552	0.4249	0.4281	0.2524
(0,1) (0,1) (1,3)	**0.2683**	0.1577	0.2674	0.2045	0.2355	0.1321
(0,1) (0,2) (1,3)	**0.3934**	0.2196	0.3631	0.2711	0.3690	0.1892
(0,1) (0.5,2.5) (1,5)	**0.5129**	0.3003	0.4582	0.3715	0.5049	0.2552
(0,1) (0.25,2) (0.5,5)	**0.4108**	0.2262	0.356	0.2699	0.4021	0.1989
(0,1) (1,3) (1.5,3.5)	**0.5965**	0.3780	0.5431	0.4781	0.5674	0.3125
(0,1) (1.5,6) (1.75,8)	0.7023	0.3766	0.5853	0.4738	**0.7333**	0.3540

Table A16. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=8RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1)	0.0507	0.0514	0.0505	0.0515	0.0532	0.0517
(0,1) (1,1) (1,1)	0.2229	0.2087	0.2055	0.1938	0.2131	**0.2327**
(0,1) (0,1) (1,1)	0.1640	0.1150	0.1418	0.1079	**0.1720**	0.1288
(0,1) (0.5,1) (1,1)	0.1880	0.1531	0.1660	0.1421	**0.1868**	0.1728
(0,1) (1,1) (1.5,1)	0.2820	0.2709	0.2594	0.2517	0.2583	**0.3043**
(0,1) (1.5,1) (2,1)	0.3359	0.3945	0.3285	0.3695	0.2793	**0.4402**
(0,1) (1.5,1) (1.75,1)	0.3173	0.3615	0.3055	0.3389	0.2631	**0.4027**

Table A17. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=8RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2)	0.2751	0.1664	**0.3204**	0.1607	0.2032	0.1732
(0,1) (0,1) (0,3)	0.2050	0.1219	**0.2190**	0.1201	0.1681	0.1253
(0,1) (0,2) (0,3)	0.3571	0.2167	**0.4236**	0.2112	0.2617	0.2225
(0,1) (0,2.5) (0,5)	0.4995	0.3084	**0.6084**	0.3015	0.3386	0.3163
(0,1) (0,2) (0,5)	0.4413	0.2692	**0.5337**	0.2613	0.3105	0.2730
(0,1) (0,3) (0,3.5)	0.4810	0.2899	**0.5807**	0.2830	0.3265	0.2931
(0,1) (0,6) (0,8)	0.6663	0.4369	**0.7942**	0.4279	0.4489	0.4442

Table A18. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=8RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2)	0.5198	0.3579	**0.5576**	0.3398	0.4183	0.3878
(0,1) (0,1) (1,3)	0.3005	0.1782	**0.3138**	0.1712	0.2469	0.1887
(0,1) (0,2) (1,3)	0.4763	0.2882	**0.5348**	0.2746	0.3628	0.3029
(0,1) (0.5,2.5) (1,5)	0.6234	0.4047	**0.7094**	0.3898	0.4646	0.4248
(0,1) (0.25,2) (0.5,5)	0.5190	0.3076	**0.5970**	0.3003	0.3783	0.3207
(0,1) (1,3) (1.5,3.5)	0.7018	0.4814	**0.7704**	0.4643	0.5383	0.5103
(0,1) (1.5,6) (1.75,8)	0.9998	0.7226	0.9993	0.6322	0.9999	0.8577

Table A19. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0513	0.0493	0.0513	0.0493	0.0513	0.0493
(0,1) (1,1) (1,1) (1,1)	**0.3504**	0.3481	**0.3504**	0.3481	**0.3504**	0.3481
(0,1) (0,1) (0,1) (1,1)	**0.1780**	0.1195	**0.1780**	0.1195	**0.1780**	0.1195
(0,1) (0,1) (1,1) (1,1)	**0.3238**	0.2300	**0.3238**	0.2300	**0.3238**	0.2300
(0,1) (0.5,1) (1,1) (1.5,1)	**0.3942**	0.3481	**0.3942**	0.3481	**0.3942**	0.3481
(0,1) (1,1) (1.5,1) (2,1)	0.4927	**0.5653**	0.4927	**0.5653**	0.4927	**0.5653**
(0,1) (1,1) (1.5,1) (1.75,1)	0.4679	**0.5271**	0.4679	**0.5271**	0.4679	**0.5271**

Table A20. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	**0.3955**	0.1818	**0.3955**	0.1818	**0.3955**	0.1818
(0,1) (0,1) (0,3) (0,3)	**0.3974**	0.1682	**0.3974**	0.1682	**0.3974**	0.1682
(0,1) (0,2) (0,3) (0,4)	**0.6044**	0.2702	**0.6044**	0.2702	**0.6044**	0.2702
(0,1) (0,2.5) (0,5) (0,7.5)	**0.7823**	0.3871	**0.7823**	0.3871	**0.7823**	0.3871
(0,1) (0,2) (0,5) (0,1)	**0.3831**	0.1603	**0.3831**	0.1603	**0.3831**	0.160
(0,1) (0,3) (0,3.5) (0,2)	**0.5937**	0.2687	**0.5937**	0.2687	**0.5937**	0.2687
(0,1) (0,4) (0,6) (0,8)	**0.8465**	0.4591	**0.8465**	0.4591	**0.8465**	0.4591

Table A21. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	**0.7539**	0.5059	**0.7539**	0.5059	**0.7539**	0.5059
(0,1) (0,1) (0,3) (1,3)	**0.5194**	0.2326	**0.5194**	0.2326	**0.5194**	0.2326
(0,1) (0,2) (1,3) (1,4)	**0.7973**	0.4425	**0.7973**	0.4425	**0.7973**	0.4425
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	**0.9024**	0.5735	**0.9024**	0.5735	**0.9024**	0.5735
(0,1) (0.25,2) (0.5,5) (0.75,1)	**0.6558**	0.3406	**0.6558**	0.3406	**0.6558**	0.3406
(0,1) (1,3) (1.5,3.5) (2,2)	**0.9043**	0.6885	**0.9043**	0.6885	**0.9043**	0.6885
(0,1) (1,4) (1.5,6) (1.75,8)	**0.9464**	0.6739	**0.9464**	0.6739	**0.9464**	0.6739

Table A22. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0490	0.0526	0.0481	0.0502	0.0486	0.0485
(0,1) (1,1) (1,1) (1,1)	0.2884	0.3492	0.3059	**0.4639**	0.2376	0.2584
(0,1) (0,1) (0,1) (1,1)	0.1560	0.1278	**0.1741**	0.1532	0.1267	0.1001
(0,1) (0,1) (1,1) (1,1)	0.2599	0.2313	0.2836	**0.3024**	0.1979	0.1726
(0,1) (0.5,1) (1,1) (1.5,1)	0.3308	0.3550	0.3555	**0.4793**	0.2591	0.2617
(0,1) (1,1) (1.5,1) (2,1)	0.3835	0.5354	0.3889	**0.6933**	0.3227	0.4050
(0,1) (1,1) (1.5,1) (1.75,1)	0.3538	0.4958	0.3641	**0.6511**	0.2933	0.3677

Table A23. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	**0.2696**	0.1255	0.2388	0.1302	0.2557	0.1091
(0,1) (0,1) (0,3) (0,3)	**0.2678**	0.1277	0.2411	0.1309	0.2511	0.1080
(0,1) (0,2) (0,3) (0,4)	**0.4083**	0.1836	0.3536	0.1866	0.4054	0.1576
(0,1) (0,2.5) (0,5) (0,7.5)	0.5454	0.2327	0.4575	0.2396	**0.5484**	0.1977
(0,1) (0,2) (0,5) (0,1)	**0.2635**	0.1207	0.2457	0.1255	0.2433	0.1028
(0,1) (0,3) (0,3.5) (0,2)	**0.3974**	0.1741	0.3444	0.1781	0.3887	0.1476
(0,1) (0,4) (0,6) (0,8)	0.6172	0.2814	0.5096	0.2827	**0.6355**	0.2451

Table A24. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	**0.5916**	0.4219	0.5678	0.5242	0.5273	0.3250
(0,1) (0,1) (0,3) (1,3)	**0.3758**	0.1816	0.3533	0.2127	0.3382	0.1453
(0,1) (0,2) (1,3) (1,4)	**0.5940**	0.3190	0.5470	0.3643	0.5586	0.2530
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	**0.7159**	0.3965	0.6455	0.4489	0.7040	0.3246
(0,1) (0.25,2) (0.5,5) (0.75,1)	**0.5010**	0.2424	0.4769	0.3084	0.4439	0.2087
(0,1) (1,3) (1.5,3.5) (2,2)	**0.7618**	0.5715	0.7241	0.6865	0.7108	0.4538
(0,1) (1,4) (1.5,6) (1.75,8)	**0.7959**	0.4732	0.7187	0.5409	0.7813	0.3904

Table A25. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=4RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0532	0.0522	0.0524	0.0546	0.0499	0.0534
(0,1) (1,1) (1,1) (1,1)	0.2631	0.3034	0.2535	0.2932	0.2363	**0.3412**
(0,1) (0,1) (0,1) (1,1)	**0.1437**	0.1086	0.1329	0.1076	0.1407	0.1212
(0,1) (0,1) (1,1) (1,1)	**0.2426**	0.2070	0.2198	0.2021	0.2344	0.2296
(0,1) (0.5,1) (1,1) (1.5,1)	0.3003	0.3117	0.2872	0.3028	0.2720	**0.3500**
(0,1) (1,1) (1.5,1) (2,1)	0.3800	0.5091	0.3781	0.4928	0.3188	**0.5574**
(0,1) (1,1) (1.5,1) (1.75,1)	0.3559	0.4655	0.3528	0.4494	0.3004	**0.5160**

Table A26. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	0.3304	0.1719	**0.3783**	0.1741	0.2439	0.1783
(0,1) (0,1) (0,3) (0,3)	0.3213	0.1599	**0.3622**	0.1637	0.2372	0.1688
(0,1) (0,2) (0,3) (0,4)	0.5253	0.266	**0.6059**	0.2682	0.3753	0.2779
(0,1) (0,2.5) (0,5) (0,7.5)	0.6959	0.3807	**0.7957**	0.3809	0.4929	0.3937
(0,1) (0,2) (0,5) (0,1)	0.3223	0.1547	**0.3591**	0.1581	0.244	0.1654
(0,1) (0,3) (0,3.5) (0,2)	0.5099	0.2500	**0.5865**	0.2525	0.3684	0.2607
(0,1) (0,4) (0,6) (0,8)	0.7715	0.4448	**0.8663**	0.4476	0.5509	0.4617

Table A27. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=4RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	0.6371	0.4572	**0.6692**	0.4464	0.5243	0.4954
(0,1) (0,1) (0,3) (1,3)	0.3588	0.2206	**0.4238**	0.2233	0.2570	0.2347
(0,1) (0,2) (1,3) (1,4)	0.6902	0.3996	**0.7510**	0.3961	0.5407	0.4272
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	0.8254	0.5385	**0.8914**	0.5341	0.6490	0.5684
(0,1) (0.25,2) (0.5,5) (0.75,1)	0.5407	0.3070	**0.5647**	0.3024	0.436	0.3355
(0,1) (1,3) (1.5,3.5) (2,2)	0.8232	0.6546	**0.8653**	0.6427	0.6845	0.6953
(0,1) (1,4) (1.5,6) (1.75,8)	0.891	0.6447	**0.9417**	0.6393	0.7287	0.6719

Table A28. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0522	0.0501	0.0522	0.0501	0.0522	0.0501
(0,1) (1,1) (1,1) (1,1)	**0.3024**	0.2466	**0.3024**	0.2466	**0.3024**	0.2466
(0,1) (0,1) (0,1) (1,1)	**0.1626**	0.1058	**0.1626**	0.1058	**0.1626**	0.1058
(0,1) (0,1) (1,1) (1,1)	**0.2701**	0.1693	**0.2701**	0.1693	**0.2701**	0.1693
(0,1) (0.5,1) (1,1) (1.5,1)	**0.3366**	0.2482	**0.3366**	0.2482	**0.3366**	0.2482
(0,1) (1,1) (1.5,1) (2,1)	**0.4002**	0.3874	**0.4002**	0.3874	**0.4002**	0.3874
(0,1) (1,1) (1.5,1) (1.75,1)	**0.3928**	0.3653	**0.3928**	0.3653	**0.3928**	0.3653

Table A29. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	**0.3947**	0.1707	**0.3947**	0.1707	**0.3947**	0.1707
(0,1) (0,1) (0,3) (0,3)	**0.3965**	0.1666	**0.3965**	0.1666	**0.3965**	0.1666
(0,1) (0,2) (0,3) (0,4)	**0.6168**	0.2773	**0.6168**	0.2773	**0.6168**	0.2773
(0,1) (0,2.5) (0,5) (0,7.5)	**0.7809**	0.3904	**0.7809**	0.3904	**0.7809**	0.3904
(0,1) (0,2) (0,5) (0,1)	**0.3907**	0.1658	**0.3907**	0.1658	**0.3907**	0.1658
(0,1) (0,3) (0,3.5) (0,2)	**0.5932**	0.2601	**0.5932**	0.2601	**0.5932**	0.2601
(0,1) (0,4) (0,6) (0,8)	**0.8479**	0.4632	**0.8479**	0.4632	**0.8479**	0.4632

Table A30. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	**0.7251**	0.4158	**0.7251**	0.4158	**0.7251**	0.4158
(0,1) (0,1) (0,3) (1,3)	**0.5143**	0.2212	**0.5143**	0.2212	**0.5143**	0.2212
(0,1) (0,2) (1,3) (1,4)	**0.7772**	0.3944	**0.7772**	0.3944	**0.7772**	0.3944
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	**0.8914**	0.5178	**0.8914**	0.5178	**0.8914**	0.5178
(0,1) (0.25,2) (0.5,5) (0.75,1)	**0.6314**	0.2844	**0.6314**	0.2844	**0.6314**	0.2844
(0,1) (1,3) (1.5,3.5) (2,2)	**0.8916**	0.5995	**0.8916**	0.5995	**0.8916**	0.5995
(0,1) (1,4) (1.5,6) (1.75,8)	**0.9371**	0.6245	**0.9371**	0.6245	**0.9371**	0.6245

Table A31. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0498	0.0545	0.0510	0.0515	0.0497	0.0495
(0,1) (1,1) (1,1) (1,1)	0.2553	0.2814	0.2833	**0.3945**	0.2037	0.2039
(0,1) (0,1) (0,1) (1,1)	0.1488	0.1189	**0.1651**	0.1405	0.1164	0.0945
(0,1) (0,1) (1,1) (1,1)	0.2394	0.2003	**0.2696**	0.2616	0.1829	0.1503
(0,1) (0.5,1) (1,1) (1.5,1)	0.2915	0.2866	0.3224	**0.4019**	0.2237	0.1992
(0,1) (1,1) (1.5,1) (2,1)	0.3391	0.4229	0.3576	**0.5842**	0.2713	0.3005
(0,1) (1,1) (1.5,1) (1.75,1)	0.3169	0.3952	0.3406	**0.5524**	0.2572	0.2839

Table A32. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	**0.2793**	0.1362	0.2495	0.1398	0.2667	0.1187
(0,1) (0,1) (0,3) (0,3)	**0.2655**	0.1269	0.2443	0.1288	0.2528	0.1076
(0,1) (0,2) (0,3) (0,4)	**0.4087**	0.1811	0.3522	0.1841	0.4057	0.1547
(0,1) (0,2.5) (0,5) (0,7.5)	0.5421	0.2347	0.4503	0.2399	**0.5480**	0.2019
(0,1) (0,2) (0,5) (0,1)	**0.2708**	0.1248	0.2458	0.1272	0.2493	0.1072
(0,1) (0,3) (0,3.5) (0,2)	**0.3955**	0.1727	0.3412	0.177	0.3932	0.1494
(0,1) (0,4) (0,6) (0,8)	0.6032	0.2745	0.5027	0.2771	**0.6280**	0.2365

Table A33. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	**0.5591**	0.3629	0.5428	0.4642	0.4887	0.2687
(0,1) (0,1) (0,3) (1,3)	**0.3612**	0.1740	0.3427	0.1994	0.3252	0.1416
(0,1) (0,2) (1,3) (1,4)	**0.5886**	0.2905	0.5412	0.3479	0.5470	0.2299
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	**0.7068**	0.3683	0.6434	0.4275	0.6825	0.2917
(0,1) (0.25,2) (0.5,5) (0.75,1)	**0.4899**	0.2118	0.4726	0.2804	0.4243	0.1779
(0,1) (1,3) (1.5,3.5) (2,2)	**0.7477**	0.5013	0.7153	0.6380	0.6877	0.3795
(0,1) (1,4) (1.5,6) (1.75,8)	**0.7775**	0.443	0.7074	0.5128	0.7603	0.3604

Table A34. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=8RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1)	0.0475	0.0514	0.0476	0.0525	0.0479	0.0529
(0,1) (1,1) (1,1) (1,1)	0.2274	0.2231	0.2114	0.2167	0.2158	**0.2516**
(0,1) (0,1) (0,1) (1,1)	0.1264	0.0893	0.1121	0.0889	**0.1291**	0.1000
(0,1) (0,1) (1,1) (1,1)	0.2018	0.1471	0.1757	0.1441	**0.2049**	0.1665
(0,1) (0.5,1) (1,1) (1.5,1)	0.2523	0.2222	0.2273	0.2114	0.2419	**0.2558**
(0,1) (1,1) (1.5,1) (2,1)	0.3145	0.3466	0.2993	0.3329	0.2806	**0.3935**
(0,1) (1,1) (1.5,1) (1.75,1)	0.2940	0.3185	0.2774	0.3062	0.2646	**0.3662**

Table A35. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=8RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2)	0.3395	0.1682	**0.3711**	0.1704	0.2477	0.1795
(0,1) (0,1) (0,3) (0,3)	0.3222	0.1551	**0.3600**	0.1582	0.2419	0.1665
(0,1) (0,2) (0,3) (0,4)	0.5283	0.2518	**0.6125**	0.2526	0.3741	0.2646
(0,1) (0,2.5) (0,5) (0,7.5)	0.6972	0.3802	**0.7978**	0.3845	0.4982	0.3940
(0,1) (0,2) (0,5) (0,1)	0.3173	0.1546	**0.3597**	0.1592	0.2335	0.1633
(0,1) (0,3) (0,3.5) (0,2)	0.5064	0.2429	**0.5823**	0.2468	0.3511	0.2570
(0,1) (0,4) (0,6) (0,8)	0.7744	0.4514	**0.8662**	0.4509	0.5621	0.4718

Table A36. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=8RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2)	0.6104	0.3858	**0.6375**	0.3767	0.4997	0.4256
(0,1) (0,1) (0,3) (1,3)	0.3480	0.1926	**0.3999**	0.1949	0.2548	0.2076
(0,1) (0,2) (1,3) (1,4)	0.6685	0.3691	**0.7342**	0.3637	0.5250	0.3945
(0,1) (0.5,2.5) (1,5) (1.5,7.5)	0.8137	0.4968	**0.8759**	0.4927	0.6423	0.5253
(0,1) (0.25,2) (0.5,5) (0.75,1)	0.5135	0.2627	**0.5288**	0.2582	0.4226	0.2904
(0,1) (1,3) (1.5,3.5) (2,2)	0.8017	0.5633	**0.8345**	0.5478	0.6691	0.6111
(0,1) (1,4) (1.5,6) (1.75,8)	0.8765	0.5900	**0.9329**	0.5825	0.7158	0.6220

Table A37. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4) (μ_5,σ_5)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0507	0.0490	0.0507	0.0490	0.0507	0.0490
(0,1) (1,1) (1,1) (1,1) (1,1)	**0.3629**	0.3582	**0.3629**	0.3582	**0.3629**	0.3582
(0,1) (0,1) (0,1) (1,1) (1,1)	**0.2551**	0.1764	**0.2551**	0.1764	**0.2551**	0.1764
(0,1) (0,1) (1,1) (1,1) (1,1)	**0.3427**	0.2676	**0.3427**	0.2676	**0.3427**	0.2676
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	**0.4829**	0.4619	**0.4829**	0.4619	**0.4829**	0.4619
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.5320	**0.6579**	0.5320	**0.6579**	0.5320	**0.6579**
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.5049	**0.6242**	0.5049	**0.6242**	0.5049	**0.6242**

Table A38. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4) (μ_5,σ_5)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	**0.4453**	0.1788	**0.4453**	0.1788	**0.4453**	0.1788
(0,1) (0,1) (0,3) (0,3) (0,3)	**0.5201**	0.2021	**0.5201**	0.2021	**0.5201**	0.2021
(0,1) (0,2) (0,3) (0,4) (0,5)	**0.7589**	0.3291	**0.7589**	0.3291	**0.7589**	0.3291
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	**0.8889**	0.4683	**0.8889**	0.4683	**0.8889**	0.4683
(0,1) (0,2) (0,5) (0,1) (0,4.5)	**0.5655**	0.2186	**0.5655**	0.2186	**0.5655**	0.2186
(0,1) (0,3) (0,3.5) (0,2) (0,6)	**0.7489**	0.3298	**0.7489**	0.3298	**0.7489**	0.3298
(0,1) (0,4) (0,6) (0,8) (0,10)	**0.9250**	0.5278	**0.9250**	0.5278	**0.9250**	0.5278

Table A39. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4) (μ_5,σ_5)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	**0.7947**	0.5458	**0.7947**	0.5458	**0.7947**	0.5458
(0,1) (0,1) (0,3) (1,3) (1,3)	**0.6997**	0.3324	**0.6997**	0.3324	**0.6997**	0.3324
(0,1) (0,2) (1,3) (1,4) (1,5)	**0.8935**	0.5405	**0.8935**	0.5405	**0.8935**	0.5405
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	**0.9605**	0.6648	**0.9605**	0.6648	**0.9605**	0.6648
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	**0.8002**	0.4336	**0.8002**	0.4336	**0.8002**	0.4336
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	**0.9547**	0.7709	**0.9547**	0.7709	**0.9547**	0.7709
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	**0.9778**	0.7538	**0.9778**	0.7538	**0.9778**	0.7538

Table A40. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4) (μ_5,σ_5)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0497	0.0500	0.0490	0.0511	0.0507	0.0526
(0,1) (1,1) (1,1) (1,1) (1,1)	0.2957	0.3393	0.3088	**0.4625**	0.2461	0.2670
(0,1) (0,1) (0,1) (1,1) (1,1)	0.2346	0.1759	**0.2583**	0.2415	0.1805	0.1441
(0,1) (0,1) (1,1) (1,1) (1,1)	0.2813	0.2594	0.3095	**0.3575**	0.2261	0.2048
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.3891	0.4249	0.4175	**0.5831**	0.3101	0.3295
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.3997	0.5770	0.3941	**0.7480**	0.3372	0.4595
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.3945	0.5618	0.3969	**0.7339**	0.3368	0.4474

Table A41. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4) (μ_5,σ_5)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	**0.3111**	0.1216	0.2777	0.1477	0.2883	0.1116
(0,1) (0,1) (0,3) (0,3) (0,3)	**0.3592**	0.1355	0.3235	0.1615	0.3304	0.1209
(0,1) (0,2) (0,3) (0,4) (0,5)	**0.5414**	0.1924	0.4676	0.2367	0.5253	0.1725
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	0.6806	0.2527	0.5890	0.3114	**0.6819**	0.2225
(0,1) (0,2) (0,5) (0,1) (0,4.5)	**0.3850**	0.1396	0.3419	0.1695	0.3556	0.1257
(0,1) (0,3) (0,3.5) (0,2) (0,6)	**0.5351**	0.1908	0.4713	0.2361	0.5210	0.1667
(0,1) (0,4) (0,6) (0,8) (0,10)	0.7264	0.2852	0.6317	0.3456	**0.7296**	0.2570

Table A42. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4) (μ_5,σ_5)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	**0.6407**	0.4302	0.6134	0.5674	0.5776	0.3432
(0,1) (0,1) (0,3) (1,3) (1,3)	**0.5217**	0.2347	0.4974	0.3161	0.4684	0.1977
(0,1) (0,2) (1,3) (1,4) (1,5)	**0.7085**	0.3495	0.6513	0.4517	0.6742	0.2906
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	**0.8347**	0.4523	0.7704	0.5607	0.8116	0.3772
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	**0.6328**	0.3132	0.6068	0.4266	0.5684	0.2499
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	**0.8375**	0.6032	0.8041	0.7513	0.7943	0.4983
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	**0.8778**	0.5247	0.8187	0.6456	0.8667	0.4465

Table A43. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=4RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0543	0.0504	0.0543	0.0508	0.0525	0.0536
(0,1) (1,1) (1,1) (1,1) (1,1)	0.2805	0.3285	0.2696	0.3170	0.2502	**0.3610**
(0,1) (0,1) (0,1) (1,1) (1,1)	**0.2013**	0.1631	0.1808	0.1580	0.1969	0.1808
(0,1) (0,1) (1,1) (1,1) (1,1)	0.2554	0.2342	0.2358	0.2256	0.2435	**0.2616**
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.3599	0.4108	0.3382	0.3952	0.3187	**0.4591**
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.3946	0.5908	0.4012	0.5735	0.3150	**0.6460**
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.3822	0.5782	0.3912	0.5630	0.3064	**0.6267**

Table A44. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=4RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	0.3738	0.1753	**0.4189**	0.1751	0.2821	0.1888
(0,1) (0,1) (0,3) (0,3) (0,3)	0.4299	0.1836	**0.4764**	0.1830	0.3165	0.1986
(0,1) (0,2) (0,3) (0,4) (0,5)	0.6633	0.3166	**0.7495**	0.3141	0.4806	0.3381
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	0.8124	0.4431	**0.8982**	0.4404	0.6048	0.4708
(0,1) (0,2) (0,5) (0,1) (0,4.5)	0.4731	0.2138	**0.5289**	0.2112	0.3447	0.2291
(0,1) (0,3) (0,3.5) (0,2) (0,6)	0.6554	0.3033	**0.7416**	0.3012	0.4726	0.3257
(0,1) (0,4) (0,6) (0,8) (0,10)	0.8683	0.5118	**0.9372**	0.5083	0.6597	0.5367

Table A45. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=4RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	0.6804	0.4915	**0.7134**	0.4774	0.5533	0.5365
(0,1) (0,1) (0,3) (1,3) (1,3)	0.5858	0.3084	**0.6236**	0.3029	0.4575	0.3343
(0,1) (0,2) (1,3) (1,4) (1,5)	0.8026	0.4874	**0.8613**	0.4778	0.6407	0.5252
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	0.9146	0.635	**0.9572**	0.6243	0.7621	0.6744
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	0.6888	0.3977	**0.7319**	0.3875	0.5553	0.4324
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	0.8934	0.7264	**0.9229**	0.711	0.7602	0.7692
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	0.9457	0.7216	**0.9739**	0.7104	0.8145	0.7576

Table A46. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0513	0.0511	0.0513	0.0511	0.0513	0.0511
(0,1) (1,1) (1,1) (1,1) (1,1)	**0.3037**	0.2451	**0.3037**	0.2451	**0.3037**	0.2451
(0,1) (0,1) (0,1) (1,1) (1,1)	**0.2268**	0.1303	**0.2268**	0.1303	**0.2268**	0.1303
(0,1) (0,1) (1,1) (1,1) (1,1)	**0.2847**	0.1828	**0.2847**	0.1828	**0.2847**	0.1828
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	**0.4018**	0.3363	**0.4018**	0.3363	**0.4018**	0.3363
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.4487	**0.4744**	0.4487	**0.4744**	0.4487	**0.4744**
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.4533	**0.4605**	0.4533	**0.4605**	0.4533	**0.4605**

Table A47. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	**0.4335**	0.1705	**0.4335**	0.1705	**0.4335**	0.1705
(0,1) (0,1) (0,3) (0,3) (0,3)	**0.5022**	0.1996	**0.5022**	0.1996	**0.5022**	0.1996
(0,1) (0,2) (0,3) (0,4) (0,5)	**0.7533**	0.3332	**0.7533**	0.3332	**0.7533**	0.3332
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	**0.8930**	0.4667	**0.8930**	0.4667	**0.8930**	0.4667
(0,1) (0,2) (0,5) (0,1) (0,4.5)	**0.5671**	0.2266	**0.5671**	0.2266	**0.5671**	0.2266
(0,1) (0,3) (0,3.5) (0,2) (0,6)	**0.7567**	0.3294	**0.7567**	0.3294	**0.7567**	0.3294
(0,1) (0,4) (0,6) (0,8) (0,10)	**0.9253**	0.5249	**0.9253**	0.5249	**0.9253**	0.5249

Table A48. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	**0.7663**	0.4434	**0.7663**	0.4434	**0.7663**	0.4434
(0,1) (0,1) (0,3) (1,3) (1,3)	**0.6864**	0.2974	**0.6864**	0.2974	**0.6864**	0.2974
(0,1) (0,2) (1,3) (1,4) (1,5)	**0.8869**	0.4789	**0.8869**	0.4789	**0.8869**	0.4789
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	**0.9618**	0.6303	**0.9618**	0.6303	**0.9618**	0.6303
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	**0.7836**	0.3781	**0.7836**	0.3781	**0.7836**	0.3781
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	**0.9488**	0.6812	**0.9488**	0.6812	**0.9488**	0.6812
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	**0.9779**	0.7122	**0.9779**	0.7122	**0.9779**	0.7122

Table A49. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0506	0.0474	0.0488	0.0471	0.0519	0.0514
(0,1) (1,1) (1,1) (1,1) (1,1)	0.2596	0.2651	0.2841	**0.3906**	0.2082	0.2054
(0,1) (0,1) (0,1) (1,1) (1,1)	0.1964	0.1428	**0.2279**	0.2040	0.1529	0.1155
(0,1) (0,1) (1,1) (1,1) (1,1)	0.2585	0.2108	0.2971	**0.3086**	0.1953	0.1633
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.3475	0.3366	0.3870	**0.4986**	0.2677	0.2538
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.3580	0.4565	0.3665	**0.6466**	0.2963	0.3520
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.3528	0.4354	0.3621	**0.6244**	0.2869	0.3318

Table A50. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	**0.3097**	0.1284	0.2759	0.1509	0.2831	0.1187
(0,1) (0,1) (0,3) (0,3) (0,3)	**0.3581**	0.1336	0.3245	0.1652	0.3345	0.1225
(0,1) (0,2) (0,3) (0,4) (0,5)	**0.5460**	0.1970	0.4723	0.2413	0.5292	0.1741
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	0.6830	0.2643	0.5927	0.3215	**0.6843**	0.2338
(0,1) (0,2) (0,5) (0,1) (0,4.5)	**0.3897**	0.1439	0.3503	0.1770	0.3605	0.1280
(0,1) (0,3) (0,3.5) (0,2) (0,6)	**0.5458**	0.1955	0.4732	0.2394	0.5301	0.1752
(0,1) (0,4) (0,6) (0,8) (0,10)	0.7333	0.2979	0.6347	0.3566	**0.7386**	0.2648

Table A51. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	**0.6152**	0.3625	0.6057	0.5116	0.5374	0.2847
(0,1) (0,1) (0,3) (1,3) (1,3)	**0.5187**	0.2259	0.4917	0.3051	0.4591	0.1838
(0,1) (0,2) (1,3) (1,4) (1,5)	**0.7020**	0.3172	0.6548	0.4236	0.6604	0.2561
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	**0.8351**	0.4194	0.7706	0.5421	0.8071	0.3493
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	**0.6142**	0.2893	0.5952	0.3942	0.5432	0.2335
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	**0.8226**	0.5316	0.7926	0.6924	0.7750	0.4171
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	**0.8689**	0.4796	0.8099	0.6042	0.8511	0.3951

Table A52. Estimated power of tests for mixed design under the normal distribution with different means and same variances; the variance in CRD=8RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)$$(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,1) (0,1) (0,1) (0,1)	0.0524	0.0504	0.0515	0.0523	0.0540	0.0523
(0,1) (1,1) (1,1) (1,1) (1,1)	0.2294	0.2242	0.2109	0.2136	0.2174	**0.2549**
(0,1) (0,1) (0,1) (1,1) (1,1)	0.1746	0.1228	0.1525	0.1193	**0.1789**	0.1381
(0,1) (0,1) (1,1) (1,1) (1,1)	0.2141	0.1701	0.1932	0.1629	**0.2150**	0.1944
(0,1) (0.5,1) (1,1) (1.5,1) (2,1)	0.3128	0.2892	0.2808	0.276	0.2918	**0.3324**
(0,1) (1,1) (1.5,1) (2,1) (2.5,1)	0.3366	0.4225	0.3262	0.4052	0.2814	**0.4754**
(0,1) (1,1) (1.5,1) (1.75,1) (2.5,1)	0.3265	0.3973	0.3184	0.3799	0.2703	**0.4439**

Table A53. Estimated power of tests for mixed design under the normal distribution with same means and different variances; the variance in CRD=8RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)$$(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (0,2) (0,2) (0,2) (0,2)	0.3729	0.1678	**0.4123**	0.1671	0.2767	0.1806
(0,1) (0,1) (0,3) (0,3) (0,3)	0.4202	0.1888	**0.4721**	0.1880	0.3141	0.2034
(0,1) (0,2) (0,3) (0,4) (0,5)	0.6615	0.3128	**0.7467**	0.3105	0.4758	0.3359
(0,1) (0,2.5) (0,5) (0,7.5) (0,10)	0.8175	0.4501	**0.9001**	0.4463	0.6084	0.4754
(0,1) (0,2) (0,5) (0,1) (0,4.5)	0.4605	0.1980	**0.5187**	0.1967	0.3360	0.2152
(0,1) (0,3) (0,3.5) (0,2) (0,6)	0.6595	0.3190	**0.7474**	0.3159	0.4761	0.3428
(0,1) (0,4) (0,6) (0,8) (0,10)	0.8611	0.5110	**0.9353**	0.5067	0.6540	0.5366

Table A54. Estimated power of tests for mixed design under the normal distribution with different means and variances; the variance in CRD=8RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)$$(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,1) (1,2) (1,2) (1,2) (1,2)	0.6489	0.4058	**0.6755**	0.3934	0.5380	0.4485
(0,1) (0,1) (0,3) (1,3) (1,3)	0.5654	0.2777	**0.6091**	0.2689	0.4453	0.3064
(0,1) (0,2) (1,3) (1,4) (1,5)	0.7933	0.4502	**0.8489**	0.4398	0.6308	0.4900
(0,1) (0.5,2.5) (1,5) (1.5,7.5) (2,10)	0.9055	0.5930	**0.9506**	0.5831	0.7517	0.6281
(0,1) (0.25,2) (0.5,5) (0.75,1) (1,4.5)	0.6661	0.3400	**0.6990**	0.3290	0.5358	0.3747
(0,1) (1,3) (1.5,3.5) (2,2) (2.5,6)	0.8807	0.6461	**0.9143**	0.6282	0.7468	0.6920
(0,1) (1,4) (1.5,6) (1.75,8) (2.5,10)	0.9401	0.6756	**0.9694**	0.6649	0.801	0.7112

APPENDIX B. ESTIMATED POWERS FOR EXPONENTIAL DISTRIBUTION

Table B1. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0515	0.0511	0.0515	0.0511	0.0515	0.0511
(1,1) (2,1) (2,1)	0.1234	**0.2791**	0.1234	**0.2791**	0.1234	**0.2791**
(1,1) (1,1) (2,1)	**0.2160**	0.1583	**0.2160**	0.1583	**0.2160**	0.1583
(1,1) (1.5,1) (2,1)	0.1461	**0.2248**	0.1461	**0.2248**	0.1461	**0.2248**
(1,1) (2,1) (2.5,1)	0.1805	**0.3790**	0.1805	**0.3790**	0.1805	**0.3790**
(1,1) (2.5,1) (3,1)	0.2157	**0.5415**	0.2157	**0.5415**	0.2157	**0.5415**
(1,1) (2.5,1) (2.75,1)	0.1833	**0.4888**	0.1833	**0.4888**	0.1833	**0.4888**

Table B2. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	**0.1314**	0.0465	**0.1314**	0.0465	**0.1314**	0.0465
(1,1) (1,1) (1,3)	**0.1291**	0.0539	**0.1291**	0.0539	**0.1291**	0.0539
(1,1) (1,2) (1,3)	**0.1463**	0.0499	**0.1463**	0.0499	**0.1463**	0.0499
(1,1) (1,5) (1,7.5)	**0.1658**	0.0537	**0.1658**	0.0537	**0.1658**	0.0537
(1,1) (1,2) (1,5)	**0.1678**	0.0510	**0.1678**	0.0510	**0.1678**	0.0510
(1,1) (1,3) (1,3.5)	**0.1696**	0.0534	**0.1696**	0.0534	**0.1696**	0.0534
(1,1) (1,6) (1,8)	**0.2147**	0.0669	**0.2147**	0.0669	**0.2147**	0.0669

Table B3. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	**0.4523**	0.2424	**0.4523**	0.2424	**0.4523**	0.2424
(1,1) (1,1) (3,9)	**0.3698**	0.1693	**0.3698**	0.1693	**0.3698**	0.1693
(1,1) (2,4) (3,9)	**0.5915**	0.3267	**0.5915**	0.3267	**0.5915**	0.3267
(1,1) (2.5,6.25) (5,25)	**0.7783**	0.4779	**0.7783**	0.4779	**0.7783**	0.4779
(1,1) (2,4) (5,25)	**0.7436**	0.4278	**0.7436**	0.4278	**0.7436**	0.4278
(1,1) (3,9) (3.5,12.25)	**0.7467**	0.4614	**0.7467**	0.4614	**0.7467**	0.4614
(1,1) (6,36) (8,64)	**0.9296**	0.7006	**0.9296**	0.7006	**0.9296**	0.7006

Table B4. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0505	0.0497	0.0522	0.0537	0.0506	0.0500
(1,1) (2,1) (2,1)	0.1263	0.2938	0.1333	**0.4024**	0.1074	0.2258
(1,1) (1,1) (2,1)	0.2117	0.1832	0.2516	**0.2597**	0.1532	0.1405
(1,1) (1.5,1) (2,1)	0.1340	0.2271	0.1507	**0.3175**	0.1097	0.1778
(1,1) (2,1) (2.5,1)	0.1685	0.3614	0.1799	**0.5036**	0.1278	0.2759
(1,1) (2.5,1) (3,1)	0.1733	0.4940	0.1765	**0.6580**	0.1317	0.3789
(1,1) (2.5,1) (2.75,1)	0.1559	0.4539	0.1523	**0.6197**	0.1193	0.3513

Table B5. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	0.0882	0.0363	0.0707	0.0332	**0.0991**	0.0409
(1,1) (1,1) (1,3)	**0.1003**	0.0447	0.0943	0.0489	0.0960	0.0443
(1,1) (1,2) (1,3)	0.0863	0.0315	0.0675	0.0294	**0.1011**	0.0363
(1,1) (1,5) (1,7.5)	0.1048	0.0384	0.0732	0.0305	**0.1394**	0.0446
(1,1) (1,2) (1,5)	0.0965	0.0335	0.0740	0.0289	**0.1155**	0.0386
(1,1) (1,3) (1,3.5)	0.0914	0.0341	0.068	0.0289	**0.1134**	0.0405
(1,1) (1,6) (1,8)	0.1091	0.0363	0.0749	0.0307	**0.1411**	0.0437

Table B6. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	**0.3287**	0.2093	0.3128	0.2925	0.2972	0.1684
(1,1) (1,1) (3,9)	0.2949	0.1679	**0.3034**	0.2423	0.2443	0.1321
(1,1) (2,4) (3,9)	**0.4375**	0.2828	0.4093	0.4019	0.4015	0.2158
(1,1) (2.5,6.25) (5,25)	**0.5946**	0.4047	0.5509	0.5544	0.5555	0.3158
(1,1) (2,4) (5,25)	**0.5588**	0.3604	0.5297	0.5071	0.5126	0.2813
(1,1) (3,9) (3.5,12.25)	**0.5526**	0.3774	0.5099	0.5163	0.5132	0.2888
(1,1) (6,36) (8,64)	**0.7528**	0.5835	0.6757	0.7509	0.7225	0.4712

Table B7. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=4RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0527	0.0523	0.0505	0.0515	0.0542	0.0521
(1,1) (2,1) (2,1)	0.0869	0.2514	0.0817	0.2348	0.0805	**0.2748**
(1,1) (1,1) (2,1)	0.1570	0.1456	0.1261	0.1365	**0.1701**	0.1631
(1,1) (1.5,1) (2,1)	0.1066	0.2062	0.0937	0.1952	0.1098	**0.2262**
(1,1) (2,1) (2.5,1)	0.1170	0.3349	0.1097	0.3127	0.1078	**0.3691**
(1,1) (2.5,1) (3,1)	0.1388	0.4905	0.1348	0.4630	0.1107	**0.5339**
(1,1) (2.5,1) (2.75,1)	0.1148	0.4434	0.1142	0.4156	0.0899	**0.4894**

Table B8. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=4RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	0.1285	0.0465	**0.1582**	0.0465	0.0913	0.0452
(1,1) (1,1) (1,3)	0.1185	0.0494	**0.1299**	0.0493	0.0996	0.0484
(1,1) (1,2) (1,3)	0.1424	0.0493	**0.1846**	0.0490	0.0969	0.0475
(1,1) (1,5) (1,7.5)	0.1777	0.0588	**0.2376**	0.0594	0.1136	0.0568
(1,1) (1,2) (1,5)	0.1701	0.0568	**0.2209**	0.0575	0.1148	0.0528
(1,1) (1,3) (1,3.5)	0.1729	0.0549	**0.2315**	0.0555	0.1118	0.0532
(1,1) (1,6) (1,8)	0.2122	0.0715	**0.3091**	0.0729	0.1264	0.0691

Table B9. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=4RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	0.3706	0.2192	**0.3989**	0.2077	0.2971	0.2408
(1,1) (1,1) (3,9)	**0.2996**	0.1606	0.2961	0.1506	0.2677	0.1787
(1,1) (2,4) (3,9)	0.4908	0.2993	**0.5355**	0.2829	0.3897	0.3278
(1,1) (2.5,6.25) (5,25)	0.6851	0.4459	**0.7393**	0.4207	0.5308	0.4820
(1,1) (2,4) (5,25)	0.6312	0.3878	**0.6789**	0.3656	0.5036	0.4238
(1,1) (3,9) (3.5,12.25)	0.6417	0.4161	**0.6983**	0.3937	0.4927	0.4528
(1,1) (6,36) (8,64)	0.8646	0.6537	**0.9044**	0.6277	0.6844	0.6945

Table B10. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0525	0.0513	0.0525	0.0513	0.0525	0.0513
(1,1) (2,1) (2,1)	0.1249	**0.2141**	0.1249	**0.2141**	0.1249	**0.2141**
(1,1) (1,1) (2,1)	**0.1893**	0.1243	**0.1893**	0.1243	**0.1893**	0.1243
(1,1) (1.5,1) (2,1)	0.1328	**0.1673**	0.1328	**0.1673**	0.1328	**0.1673**
(1,1) (2,1) (2.5,1)	0.1549	**0.2708**	0.1549	**0.2708**	0.1549	**0.2708**
(1,1) (2.5,1) (3,1)	0.1808	**0.3840**	0.1808	**0.3840**	0.1808	**0.3840**
(1,1) (2.5,1) (2.75,1)	0.1641	**0.3507**	0.1641	**0.3507**	0.1641	**0.3507**

Table B11. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	**0.1316**	0.0473	**0.1316**	0.0473	**0.1316**	0.0473
(1,1) (1,1) (1,3)	**0.1297**	0.0528	**0.1297**	0.0528	**0.1297**	0.0528
(1,1) (1,2) (1,3)	**0.1425**	0.0422	**0.1425**	0.0422	**0.1425**	0.0422
(1,1) (1,5) (1,7.5)	**0.1729**	0.0516	**0.1729**	0.0516	**0.1729**	0.0516
(1,1) (1,2) (1,5)	**0.1623**	0.0527	**0.1623**	0.0527	**0.1623**	0.0527
(1,1) (1,3) (1,3.5)	**0.1580**	0.0535	**0.1580**	0.0535	**0.1580**	0.0535
(1,1) (1,6) (1,8)	**0.2079**	0.0695	**0.2079**	0.0695	**0.2079**	0.0695

Table B12. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	**0.4187**	0.1675	**0.4187**	0.1675	**0.4187**	0.1675
(1,1) (1,1) (3,9)	**0.3555**	0.1381	**0.3555**	0.1381	**0.3555**	0.1381
(1,1) (2,4) (3,9)	**0.5636**	0.2287	**0.5636**	0.2287	**0.5636**	0.2287
(1,1) (2.5,6.25) (5,25)	**0.7462**	0.3478	**0.7462**	0.3478	**0.7462**	0.3478
(1,1) (2,4) (5,25)	**0.7047**	0.2975	**0.7047**	0.2975	**0.7047**	0.2975
(1,1) (3,9) (3.5,12.25)	**0.7103**	0.3252	**0.7103**	0.3252	**0.7103**	0.3252
(1,1) (6,36) (8,64)	**0.9024**	0.5382	**0.9024**	0.5382	**0.9024**	0.5382

Table B13. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0511	0.0490	0.0512	0.0526	0.0518	0.0507
(1,1) (2,1) (2,1)	0.1155	0.2299	0.1240	**0.3416**	0.0998	0.1772
(1,1) (1,1) (2,1)	0.1876	0.1498	**0.2325**	0.2234	0.1275	0.1171
(1,1) (1.5,1) (2,1)	0.1321	0.1881	0.1456	**0.2767**	0.1025	0.1446
(1,1) (2,1) (2.5,1)	0.1483	0.2865	0.1614	**0.4240**	0.1172	0.2159
(1,1) (2.5,1) (3,1)	0.1565	0.3779	0.1634	**0.5504**	0.121	0.2828
(1,1) (2.5,1) (2.75,1)	0.1452	0.3531	0.1449	**0.5147**	0.1170	0.2591

Table B14. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	0.0885	0.0343	0.0720	0.0319	**0.0969**	0.0368
(1,1) (1,1) (1,3)	**0.0953**	0.0436	0.0929	0.0486	0.0941	0.0451
(1,1) (1,2) (1,3)	0.0841	0.0325	0.0635	0.0279	**0.1030**	0.0356
(1,1) (1,5) (1,7.5)	0.1079	0.0380	0.0752	0.0296	**0.1388**	0.0441
(1,1) (1,2) (1,5)	0.0966	0.0342	0.0711	0.0313	**0.1143**	0.0376
(1,1) (1,3) (1,3.5)	0.0925	0.0316	0.0711	0.0271	**0.1106**	0.0370
(1,1) (1,6) (1,8)	0.1070	0.0358	0.0719	0.0308	**0.1402**	0.0417

Table B15. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	**0.3115**	0.1672	0.3051	0.2499	0.2722	0.1290
(1,1) (1,1) (3,9)	0.2737	0.1406	**0.2929**	0.2086	0.2231	0.1081
(1,1) (2,4) (3,9)	**0.4079**	0.2214	0.3967	0.3294	0.3564	0.1639
(1,1) (2.5,6.25) (5,25)	**0.5573**	0.3086	0.5286	0.4682	0.4981	0.2251
(1,1) (2,4) (5,25)	**0.5184**	0.2846	0.5048	0.4327	0.4603	0.2052
(1,1) (3,9) (3.5,12.25)	**0.5146**	0.2956	0.4857	0.4345	0.4658	0.2176
(1,1) (6,36) (8,64)	**0.6957**	0.4587	0.6370	0.6460	0.6527	0.3405

Table B16. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=8RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1)	0.0494	0.0501	0.0495	0.0498	0.0516	0.0511
(1,1) (2,1) (2,1)	0.0864	0.1882	0.0822	0.1756	0.0838	**0.2115**
(1,1) (1,1) (2,1)	0.1392	0.1158	0.1095	0.1077	**0.1646**	0.1290
(1,1) (1.5,1) (2,1)	0.1001	0.1477	0.0887	0.1388	0.1059	**0.1633**
(1,1) (2,1) (2.5,1)	0.1081	0.2352	0.0978	0.2158	0.1019	**0.2641**
(1,1) (2.5,1) (3,1)	0.1149	0.3392	0.1090	0.3140	0.0909	**0.3805**
(1,1) (2.5,1) (2.75,1)	0.1054	0.3122	0.1022	0.2885	0.0812	**0.3514**

Table B17. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=8RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2)	0.1274	0.0495	**0.1606**	0.0498	0.0925	0.0460
(1,1) (1,1) (1,3)	0.1230	0.0507	**0.1297**	0.0500	0.1085	0.0520
(1,1) (1,2) (1,3)	0.1444	0.0488	**0.1910**	0.0500	0.0992	0.0484
(1,1) (1,5) (1,7.5)	0.1715	0.0550	**0.2360**	0.0544	0.1139	0.0519
(1,1) (1,2) (1,5)	0.1695	0.0562	**0.2194**	0.0571	0.1117	0.0543
(1,1) (1,3) (1,3.5)	0.1719	0.0569	**0.2341**	0.0584	0.1078	0.0548
(1,1) (1,6) (1,8)	0.2165	0.0739	**0.3114**	0.0742	0.1250	0.0705

Table B18. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=8RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4)	0.3457	0.1576	**0.3664**	0.1479	0.2811	0.1764
(1,1) (1,1) (3,9)	**0.2749**	0.1194	0.2611	0.1101	0.2490	0.1335
(1,1) (2,4) (3,9)	0.4636	0.2091	**0.4932**	0.1952	0.3685	0.2362
(1,1) (2.5,6.25) (5,25)	0.6281	0.2995	**0.6723**	0.2815	0.4866	0.3387
(1,1) (2,4) (5,25)	0.5744	0.2762	**0.6120**	0.2586	0.4622	0.3101
(1,1) (3,9) (3.5,12.25)	0.5929	0.2831	**0.6372**	0.2628	0.4576	0.3190
(1,1) (6,36) (8,64)	0.8236	0.4755	**0.8677**	0.4466	0.6439	0.5256

Table B19. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0511	0.0502	0.0511	0.0502	0.0511	0.0502
(1,1) (2,1) (2,1) (2,1)	0.0994	**0.3049**	0.0994	**0.3049**	0.0994	**0.3049**
(1,1) (1,1) (1,1) (2,1)	**0.1649**	0.1257	**0.1649**	0.1257	**0.1649**	0.1257
(1,1) (1,1) (2,1) (2,1)	0.1931	**0.2160**	0.1931	**0.2160**	0.1931	**0.2160**
(1,1) (1.5,1) (2,1) (2.5,1)	0.1585	**0.3149**	0.1585	**0.3149**	0.1585	**0.3149**
(1,1) (2,1) (2.5,1) (3,1)	0.1441	**0.4606**	0.1441	**0.4606**	0.1441	**0.4606**
(1,1) (2,1) (2.5,1) (2.75,1)	0.1329	**0.4338**	0.1329	0.4338	0.1329	0.4338

Table B20. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	**0.1545**	0.0309	**0.1545**	0.0309	**0.1545**	0.0309
(1,1) (1,1) (1,3) (1,3)	**0.1724**	0.0389	**0.1724**	0.0389	**0.1724**	0.0389
(1,1) (1,2) (1,3) (1,4)	**0.1801**	0.0329	**0.1801**	0.0329	**0.1801**	0.0329
(1,1) (1,2.5) (1,5) (1,7.5)	**0.2150**	0.0368	**0.2150**	0.0368	**0.2150**	0.0368
(1,1) (1,2) (1,5) (1,1)	**0.1734**	0.0440	**0.1734**	0.0440	**0.1734**	0.0440
(1,1) (1,3) (1,3.5) (1,2)	**0.1794**	0.0350	**0.1794**	0.0350	**0.1794**	0.0350
(1,1) (1,4) (1,6) (1,8)	**0.2309**	0.0437	**0.2309**	0.0437	**0.2309**	0.0437

Table B21. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	**0.5071**	0.2411	**0.5071**	0.2411	**0.5071**	0.2411
(1,1) (1,1) (3,9) (3,9)	**0.5683**	0.2618	**0.5683**	0.2618	**0.5683**	0.2618
(1,1) (2,4) (3,9) (4,16)	**0.7449**	0.4267	**0.7449**	0.4267	**0.7449**	0.4267
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	**0.8937**	0.6340	**0.8937**	0.6340	**0.8937**	0.6340
(1,1) (2,4) (5,25) (1,1)	**0.5742**	0.2529	**0.5742**	0.2529	**0.5742**	0.2529
(1,1) (3,9) (3.5,12.25) (2,4)	**0.7239**	0.4106	**0.7239**	0.4106	**0.7239**	0.4106
(1,1) (4,16) (6,36) (8,64)	**0.9205**	0.7220	**0.9205**	0.722	**0.9205**	0.7220

Table B22. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0515	0.0527	0.0534	0.0506	0.0504	0.0512
(1,1) (2,1) (2,1) (2,1)	0.0977	0.3291	0.0965	**0.4315**	0.0905	0.2467
(1,1) (1,1) (1,1) (2,1)	0.1553	0.1415	**0.1820**	0.1741	0.1243	0.1102
(1,1) (1,1) (2,1) (2,1)	0.1839	0.2401	0.2196	**0.3189**	0.1358	0.1744
(1,1) (1.5,1) (2,1) (2.5,1)	0.1450	0.3379	0.1627	**0.4459**	0.1162	0.2522
(1,1) (2,1) (2.5,1) (3,1)	0.1376	0.4775	0.1378	**0.6201**	0.1148	0.3580
(1,1) (2,1) (2.5,1) (2.75,1)	0.1171	0.4491	0.1193	**0.5887**	0.1033	0.3312

Table B23. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	0.0937	0.0252	0.0748	0.0207	**0.1087**	0.0329
(1,1) (1,1) (1,3) (1,3)	0.1116	0.0273	0.0997	0.0285	**0.1148**	0.0335
(1,1) (1,2) (1,3) (1,4)	0.0960	0.0193	0.0722	0.0159	**0.1137**	0.0244
(1,1) (1,2.5) (1,5) (1,7.5)	0.1139	0.0168	0.0804	0.0128	**0.1422**	0.0242
(1,1) (1,2) (1,5) (1,1)	**0.1146**	0.0296	0.0967	0.0287	0.1113	0.0343
(1,1) (1,3) (1,3.5) (1,2)	0.0977	0.0163	0.0702	0.0139	**0.1158**	0.0237
(1,1) (1,4) (1,6) (1,8)	0.1195	0.0186	0.0789	0.0149	**0.1521**	0.0263

Table B24. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	**0.3804**	0.2327	0.3662	0.3122	0.3315	0.1693
(1,1) (1,1) (3,9) (3,9)	**0.4347**	0.2517	0.4314	0.3395	0.3609	0.1834
(1,1) (2,4) (3,9) (4,16)	**0.5693**	0.3879	0.5293	0.5201	0.5261	0.2779
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.7062	0.5489	0.6490	**0.7162**	0.6835	0.3997
(1,1) (2,4) (5,25) (1,1)	**0.4448**	0.2540	0.4462	0.3484	0.3735	0.1813
(1,1) (3,9) (3.5,12.25) (2,4)	**0.5558**	0.3732	0.5153	0.5058	0.5161	0.2655
(1,1) (4,16) (6,36) (8,64)	0.7455	0.6239	0.6621	**0.7850**	0.7334	0.4725

Table B25. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=4RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0538	0.0492	0.0524	0.0507	0.0528	0.0501
(1,1) (2,1) (2,1) (2,1)	0.0835	0.2768	0.0858	0.2691	0.0725	**0.3067**
(1,1) (1,1) (1,1) (2,1)	0.1293	0.1092	0.1058	0.1086	**0.1361**	0.1209
(1,1) (1,1) (2,1) (2,1)	0.1468	0.1903	0.1243	0.1844	0.1629	**0.2133**
(1,1) (1.5,1) (2,1) (2.5,1)	0.1234	0.2811	0.1078	0.2722	0.1177	**0.3154**
(1,1) (2,1) (2.5,1) (3,1)	0.1086	0.4252	0.1067	0.4093	0.0923	**0.4718**
(1,1) (2,1) (2.5,1) (2.75,1)	0.0989	0.393	0.0992	0.3792	0.0846	**0.4384**

Table B26. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	0.1449	0.0374	**0.1814**	0.039	0.0964	0.0369
(1,1) (1,1) (1,3) (1,3)	0.1479	0.0407	**0.1686**	0.0432	0.1083	0.0414
(1,1) (1,2) (1,3) (1,4)	0.1750	0.0365	**0.2346**	0.0397	0.1057	0.0356
(1,1) (1,2.5) (1,5) (1,7.5)	0.2112	0.0396	**0.2929**	0.0427	0.1181	0.0384
(1,1) (1,2) (1,5) (1,1)	0.1501	0.0393	**0.1740**	0.0422	0.1115	0.040
(1,1) (1,3) (1,3.5) (1,2)	0.1779	0.0395	**0.2339**	0.0416	0.1086	0.0376
(1,1) (1,4) (1,6) (1,8)	0.2260	0.0446	**0.3244**	0.0499	0.1228	0.0442

Table B27. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=4RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	0.4181	0.2290	**0.4476**	0.2238	0.3331	0.2560
(1,1) (1,1) (3,9) (3,9)	**0.4116**	0.1956	0.4064	0.1891	0.3491	0.2238
(1,1) (2,4) (3,9) (4,16)	0.6392	0.3759	**0.6867**	0.3629	0.5025	0.4185
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.8061	0.5709	**0.8607**	0.5502	0.6327	0.6270
(1,1) (2,4) (5,25) (1,1)	0.4720	0.2275	**0.4756**	0.2223	0.3992	0.2591
(1,1) (3,9) (3.5,12.25) (2,4)	0.6210	0.3642	**0.6718**	0.3513	0.4856	0.4101
(1,1) (4,16) (6,36) (8,64)	0.8548	0.6712	**0.9125**	0.6527	0.6695	0.7210

Table B28. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0528	0.0529	0.0528	0.0529	0.0528	0.0529
(1,1) (2,1) (2,1) (2,1)	0.0923	**0.2279**	0.0923	**0.2279**	0.0923	**0.2279**
(1,1) (1,1) (1,1) (2,1)	**0.1472**	0.0975	**0.1472**	0.0975	**0.1472**	0.0975
(1,1) (1,1) (2,1) (2,1)	**0.1762**	0.1622	**0.1762**	0.1622	**0.1762**	0.1622
(1,1) (1.5,1) (2,1) (2.5,1)	0.1516	**0.2382**	0.1516	**0.2382**	0.1516	**0.2382**
(1,1) (2,1) (2.5,1) (3,1)	0.1319	**0.3311**	0.1319	**0.3311**	0.1319	**0.3311**
(1,1) (2,1) (2.5,1) (2.75,1)	0.1212	**0.3163**	0.1212	**0.3163**	0.1212	**0.3163**

Table B29. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	**0.1500**	0.0342	**0.1500**	0.0342	**0.1500**	0.0342
(1,1) (1,1) (1,3) (1,3)	**0.1687**	0.0404	**0.1687**	0.0404	**0.1687**	0.0404
(1,1) (1,2) (1,3) (1,4)	**0.1834**	0.0363	**0.1834**	0.0363	**0.1834**	0.0363
(1,1) (1,2.5) (1,5) (1,7.5)	**0.2210**	0.0377	**0.2210**	0.0377	**0.2210**	0.0377
(1,1) (1,2) (1,5) (1,1)	**0.1677**	0.0407	**0.1677**	0.0407	**0.1677**	0.0407
(1,1) (1,3) (1,3.5) (1,2)	**0.1833**	0.0375	**0.1833**	0.0375	**0.1833**	0.0375
(1,1) (1,4) (1,6) (1,8)	**0.2311**	0.0432	**0.2311**	0.0432	**0.2311**	0.0432

Table B30. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	**0.4732**	0.1674	**0.4732**	0.1674	**0.4732**	0.1674
(1,1) (1,1) (3,9) (3,9)	**0.5332**	0.1854	**0.5332**	0.1854	**0.5332**	0.1854
(1,1) (2,4) (3,9) (4,16)	**0.7199**	0.2911	**0.7199**	0.2911	**0.7199**	0.2911
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	**0.8626**	0.4442	**0.8626**	0.4442	**0.8626**	0.4442
(1,1) (2,4) (5,25) (1,1)	**0.5388**	0.1795	**0.5388**	0.1795	**0.5388**	0.1795
(1,1) (3,9) (3.5,12.25) (2,4)	**0.6986**	0.2805	**0.6986**	0.2805	**0.6986**	0.2805
(1,1) (4,16) (6,36) (8,64)	**0.9062**	0.5356	**0.9062**	0.5356	**0.9062**	0.5356

Table B31. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0485	0.0525	0.0493	0.0500	0.0469	0.0491
(1,1) (2,1) (2,1) (2,1)	0.0930	0.2691	0.0900	**0.3661**	0.0855	0.1946
(1,1) (1,1) (1,1) (2,1)	0.1419	0.1208	**0.1681**	0.1526	0.1031	0.0916
(1,1) (1,1) (2,1) (2,1)	0.1734	0.2035	0.2128	**0.2784**	0.1278	0.1437
(1,1) (1.5,1) (2,1) (2.5,1)	0.1359	0.2757	0.1542	**0.3839**	0.1106	0.1992
(1,1) (2,1) (2.5,1) (3,1)	0.1280	0.3799	0.1274	**0.5249**	0.1086	0.2682
(1,1) (2,1) (2.5,1) (2.75,1)	0.1152	0.3714	0.1163	**0.5104**	0.1011	0.2669

Table B32. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	0.0960	0.0195	0.0767	0.0191	**0.1041**	0.0262
(1,1) (1,1) (1,3) (1,3)	**0.1180**	0.0279	0.0978	0.0275	0.1169	0.0341
(1,1) (1,2) (1,3) (1,4)	0.1074	0.0186	0.0795	0.0155	**0.1245**	0.0261
(1,1) (1,2.5) (1,5) (1,7.5)	0.1198	0.0193	0.086	0.0153	**0.1445**	0.0255
(1,1) (1,2) (1,5) (1,1)	0.1149	0.0286	0.1011	0.0279	**0.1156**	0.0347
(1,1) (1,3) (1,3.5) (1,2)	0.1027	0.0189	0.0779	0.0154	**0.1177**	0.0248
(1,1) (1,4) (1,6) (1,8)	0.1196	0.0172	0.0807	0.0126	**0.1511**	0.0252

Table B33. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	**0.3559**	0.1799	0.3432	0.2589	0.3082	0.1251
(1,1) (1,1) (3,9) (3,9)	0.4039	0.1984	**0.4141**	0.2912	0.3346	0.1406
(1,1) (2,4) (3,9) (4,16)	**0.5406**	0.2956	0.509	0.4355	0.4888	0.1989
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	**0.6765**	0.4212	0.6206	0.6095	0.6292	0.2814
(1,1) (2,4) (5,25) (1,1)	0.4199	0.1966	**0.4283**	0.2868	0.3352	0.1340
(1,1) (3,9) (3.5,12.25) (2,4)	**0.5095**	0.2758	0.4829	0.4021	0.4516	0.1850
(1,1) (4,16) (6,36) (8,64)	**0.6956**	0.4872	0.621	0.6805	0.6769	0.3296

Table B34. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=8RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1)	0.0526	0.0468	0.0496	0.0486	0.0520	0.0472
(1,1) (2,1) (2,1) (2,1)	0.0752	0.2009	0.0777	0.1948	0.0699	**0.2287**
(1,1) (1,1) (1,1) (2,1)	0.1149	0.096	0.0944	0.0939	**0.1229**	0.1054
(1,1) (1,1) (2,1) (2,1)	0.1293	0.1430	0.1054	0.1384	0.1421	**0.1623**
(1,1) (1.5,1) (2,1) (2.5,1)	0.1110	0.2051	0.0987	0.1961	0.1126	**0.2368**
(1,1) (2,1) (2.5,1) (3,1)	0.1013	0.3014	0.101	0.2894	0.0867	**0.3432**
(1,1) (2,1) (2.5,1) (2.75,1)	0.0900	0.2835	0.0884	0.2701	0.0766	**0.3229**

Table B35. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=8RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2)	0.2243	0.0435	**0.2383**	0.0416	0.1737	0.0463
(1,1) (1,1) (1,3) (1,3)	0.2157	0.0426	**0.2174**	0.0418	0.1797	0.0445
(1,1) (1,2) (1,3) (1,4)	0.2460	0.0465	**0.2826**	0.0442	0.1695	0.0466
(1,1) (1,2.5) (1,5) (1,7.5)	0.2773	0.0475	**0.3441**	0.0471	0.1839	0.0469
(1,1) (1,2) (1,5) (1,1)	0.2129	0.0464	**0.2165**	0.0455	0.1781	0.0499
(1,1) (1,3) (1,3.5) (1,2)	0.2395	0.0423	**0.2817**	0.0417	0.1715	0.0435
(1,1) (1,4) (1,6) (1,8)	0.2874	0.0480	**0.3675**	0.0468	0.1783	0.0473

Table B36. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=8RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4)	0.3878	0.1522	**0.4057**	0.1469	0.3177	0.1756
(1,1) (1,1) (3,9) (3,9)	**0.3956**	0.1433	0.3901	0.1382	0.3440	0.1657
(1,1) (2,4) (3,9) (4,16)	0.6002	0.2530	**0.6422**	0.2410	0.4758	0.2936
(1,1) (2.5,6.25) (5,25) (7.5,56.25)	0.7662	0.3957	**0.8305**	0.3784	0.6016	0.4532
(1,1) (2,4) (5,25) (1,1)	**0.4288**	0.1569	0.4287	0.1504	0.3651	0.1841
(1,1) (3,9) (3.5,12.25) (2,4)	0.5875	0.2470	**0.6273**	0.2360	0.4670	0.2864
(1,1) (4,16) (6,36) (8,64)	0.8235	0.4796	**0.8855**	0.4603	0.6359	0.5401

Table B37. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0533	0.0534	0.0533	0.0534	0.0533	0.0534
(1,1) (2,1) (2,1) (2,1) (2,1)	0.1022	**0.3025**	0.1022	**0.3025**	0.1022	**0.3025**
(1,1) (1,1) (1,1) (2,1) (2,1)	**0.1873**	0.1732	**0.1873**	0.1732	**0.1873**	0.1732
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1523	**0.2457**	0.1523	**0.2457**	0.1523	**0.2457**
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.1904	**0.4068**	0.1904	**0.4068**	0.1904	**0.4068**
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1645	**0.5461**	0.1645	**0.5461**	0.1645	**0.5461**
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1626	**0.5326**	0.1626	**0.5326**	0.1626	**0.5326**

Table B38. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	**0.1656**	0.0275	**0.1656**	0.0275	**0.1656**	0.0275
(1,1) (1,1) (1,3) (1,3) (1,3)	**0.2105**	0.0335	**0.2105**	0.0335	**0.2105**	0.0335
(1,1) (1,2) (1,3) (1,4) (1,5)	**0.2181**	0.0294	**0.2181**	0.0294	**0.2181**	0.0294
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	**0.2558**	0.0289	**0.2558**	0.0289	**0.2558**	0.0289
(1,1) (1,2) (1,5) (1,1) (1,4.5)	**0.2167**	0.0356	**0.2167**	0.0356	**0.2167**	0.0356
(1,1) (1,3) (1,3.5) (1,2) (1,6)	**0.2240**	0.0312	**0.2240**	0.0312	**0.2240**	0.0312
(1,1) (1,4) (1,6) (1,8) (1,10)	**0.2613**	0.0316	**0.2613**	0.0316	**0.2613**	0.0316

Table B39. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	**0.5361**	0.2508	**0.5361**	0.2508	0.5361	0.2508
(1,1) (1,1) (3,9) (3,9) (3,9)	**0.6561**	0.3118	**0.6561**	0.3118	**0.6561**	0.3118
(1,1) (2,4) (3,9) (4,16) (5,25)	**0.8373**	0.5255	**0.8373**	0.5255	**0.8373**	0.5255
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	**0.9412**	0.7329	**0.9412**	0.7329	**0.9412**	0.7329
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	**0.7307**	0.3562	**0.7307**	0.3562	**0.7307**	0.3562
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	**0.8435**	0.5268	**0.8435**	0.5268	**0.8435**	0.5268
(1,1) (4,16) (6,36) (8,64) (10,100)	**0.9521**	0.7939	**0.9521**	0.7939	**0.9521**	0.7939

Table B40. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2) (μ_5, σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0505	0.0489	0.0534	0.0537	0.0500	0.0514
(1,1) (2,1) (2,1) (2,1) (2,1)	0.1007	0.2933	0.0986	**0.3994**	0.0921	0.2396
(1,1) (1,1) (1,1) (2,1) (2,1)	0.1749	0.1779	0.2068	**0.2515**	0.1274	0.1467
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1403	0.2347	0.1542	**0.3255**	0.1144	0.1902
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.1724	0.3843	0.1901	**0.5228**	0.1350	0.3040
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1404	0.4924	0.1428	**0.6601**	0.1167	0.3948
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1395	0.4764	0.1438	**0.6409**	0.1183	0.3780

Table B41. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2) (μ_5, σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	0.1067	0.0212	0.0922	0.0196	**0.1112**	0.0252
(1,1) (1,1) (1,3) (1,3) (1,3)	0.1354	0.0231	0.1157	0.0224	**0.1323**	0.0268
(1,1) (1,2) (1,3) (1,4) (1,5)	0.1166	0.0172	0.0884	0.0141	**0.1294**	0.0201
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	0.1314	0.0163	0.0924	0.0120	**0.1510**	0.0190
(1,1) (1,2) (1,5) (1,1) (1,4.5)	0.1356	0.0232	0.1176	0.0254	**0.1352**	0.0249
(1,1) (1,3) (1,3.5) (1,2) (1,6)	0.1171	0.0164	0.0913	0.0141	**0.1282**	0.0183
(1,1) (1,4) (1,6) (1,8) (1,10)	0.1325	0.0159	0.0948	0.013	**0.1573**	0.0198

Table B42. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2) (μ_5, σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	**0.3976**	0.2189	0.3814	0.3223	0.3514	0.1672
(1,1) (1,1) (3,9) (3,9) (3,9)	**0.5031**	0.2737	0.4903	0.4112	0.4448	0.1997
(1,1) (2,4) (3,9) (4,16) (5,25)	**0.6652**	0.4324	0.6077	0.6203	0.6278	0.3237
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.7805	0.6111	0.6968	**0.7971**	0.7684	0.4802
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	**0.5756**	0.3123	0.5568	0.4725	0.5064	0.2306
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	**0.6680**	0.4344	0.6143	0.6223	0.6310	0.3237
(1,1) (4,16) (6,36) (8,64) (10,100)	0.7932	0.6604	0.6879	**0.8335**	0.7945	0.5263

Table B43. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=4RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2) (μ_5, σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0504	0.0486	0.0500	0.0509	0.0516	0.0509
(1,1) (2,1) (2,1) (2,1) (2,1)	0.0814	0.2815	0.0822	0.2722	0.0724	**0.3097**
(1,1) (1,1) (1,1) (2,1) (2,1)	0.1484	0.1566	0.1199	0.1527	0.1614	**0.1732**
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1183	0.2182	0.1064	0.2119	0.1161	**0.2452**
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.1392	0.3636	0.1270	0.3493	0.1313	**0.4052**
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1171	0.4977	0.1195	0.4797	0.0982	**0.5441**
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1124	0.4797	0.1105	0.4623	0.0922	**0.5277**

Table B44. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=4RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2) (μ_5, σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	0.1519	0.0307	**0.1764**	0.0324	0.1084	0.0314
(1,1) (1,1) (1,3) (1,3) (1,3)	0.1702	0.0352	**0.1967**	0.0378	0.1251	0.0370
(1,1) (1,2) (1,3) (1,4) (1,5)	0.1923	0.0281	**0.2480**	0.0300	0.1200	0.0288
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	0.2213	0.0321	**0.3059**	0.0345	0.1282	0.0320
(1,1) (1,2) (1,5) (1,1) (1,4.5)	0.1839	0.0357	**0.2180**	0.0370	0.1385	0.0370
(1,1) (1,3) (1,3.5) (1,2) (1,6)	0.1992	0.0293	**0.2550**	0.0307	0.1284	0.0297
(1,1) (1,4) (1,6) (1,8) (1,10)	0.2365	0.0340	**0.3317**	0.0363	0.1340	0.0341

Table B45. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=4RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1, σ_1^2) (μ_2, σ_2^2) (μ_3, σ_3^2) (μ_4, σ_4^2) (μ_5, σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	0.4423	0.2281	**0.4720**	0.2206	0.3513	0.2546
(1,1) (1,1) (3,9) (3,9) (3,9)	0.5363	0.2792	**0.5655**	0.2661	0.4379	0.3173
(1,1) (2,4) (3,9) (4,16) (5,25)	0.7344	0.4654	**0.7955**	0.4456	0.5742	0.5188
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.8852	0.6808	**0.9352**	0.6604	0.7016	0.7351
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	0.6113	0.3180	**0.6442**	0.3044	0.4966	0.3610
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	0.7419	0.4722	**0.8062**	0.4521	0.5853	0.5288
(1,1) (4,16) (6,36) (8,64) (10,100)	0.9051	0.7422	**0.9474**	0.7234	0.7099	0.7932

Table B46. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0526	0.0504	0.0526	0.0504	0.0526	0.0504
(1,1) (2,1) (2,1) (2,1) (2,1)	0.1017	**0.2333**	0.1017	**0.2333**	0.1017	**0.2333**
(1,1) (1,1) (1,1) (2,1) (2,1)	**0.1657**	0.1312	**0.1657**	0.1312	**0.1657**	0.1312
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1440	**0.1891**	0.1440	**0.1891**	0.1440	**0.1891**
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.1647	**0.2957**	0.1647	**0.2957**	0.1647	**0.2957**
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1417	**0.3936**	0.1417	**0.3936**	0.1417	**0.3936**
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1420	**0.3909**	0.1420	**0.3909**	0.1420	**0.3909**

Table B47. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	**0.1733**	0.0286	**0.1733**	0.0286	**0.1733**	0.0286
(1,1) (1,1) (1,3) (1,3) (1,3)	**0.2064**	0.0341	**0.2064**	0.0341	**0.2064**	0.0341
(1,1) (1,2) (1,3) (1,4) (1,5)	**0.2143**	0.0284	**0.2143**	0.0284	**0.2143**	0.0284
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	**0.2460**	0.0292	**0.2460**	0.0292	**0.2460**	0.0292
(1,1) (1,2) (1,5) (1,1) (1,4.5)	**0.2218**	0.0373	**0.2218**	0.0373	**0.2218**	0.0373
(1,1) (1,3) (1,3.5) (1,2) (1,6)	**0.2247**	0.0307	**0.2247**	0.0307	**0.2247**	0.0307
(1,1) (1,4) (1,6) (1,8) (1,10)	**0.2495**	0.0327	**0.2495**	0.0327	**0.2495**	0.0327

Table B48. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	**0.5157**	0.1748	**0.5157**	0.1748	**0.5157**	0.1748
(1,1) (1,1) (3,9) (3,9) (3,9)	**0.6152**	0.2091	**0.6152**	0.2091	**0.6152**	0.2091
(1,1) (2,4) (3,9) (4,16) (5,25)	**0.8123**	0.3597	**0.8123**	0.3597	**0.8123**	0.3597
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	**0.9269**	0.5391	**0.9269**	0.5391	**0.9269**	0.5391
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	**0.6945**	0.2435	**0.6945**	0.2435	**0.6945**	0.2435
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	**0.8199**	0.3702	**0.8199**	0.3702	**0.8199**	0.3702
(1,1) (4,16) (6,36) (8,64) (10,100)	**0.9436**	0.6114	**0.9436**	0.6114	**0.9436**	0.6114

Table B49. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0505	0.0509	0.0528	0.0539	0.0506	0.0510
(1,1) (2,1) (2,1) (2,1) (2,1)	0.0931	0.2434	0.0930	**0.3494**	0.0888	0.1934
(1,1) (1,1) (1,1) (2,1) (2,1)	0.1718	0.1529	0.2049	**0.2198**	0.1286	0.1273
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1364	0.2025	0.1504	**0.2938**	0.1116	0.1608
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.1544	0.3129	0.1695	**0.4496**	0.1227	0.2396
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1351	0.3995	0.1351	**0.5637**	0.1161	0.3140
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1345	0.3894	0.1340	**0.5566**	0.1137	0.2999

Table B50. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	0.1084	0.0209	0.0921	0.0194	**0.1104**	0.0226
(1,1) (1,1) (1,3) (1,3) (1,3)	0.1293	0.0251	0.1126	0.0232	**0.1293**	0.0276
(1,1) (1,2) (1,3) (1,4) (1,5)	0.1215	0.0182	0.0961	0.0164	**0.1372**	0.0214
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	0.1304	0.0145	0.0961	0.0132	**0.1526**	0.0186
(1,1) (1,2) (1,5) (1,1) (1,4.5)	0.1360	0.0258	0.1177	0.0248	**0.1367**	0.0276
(1,1) (1,3) (1,3.5) (1,2) (1,6)	0.1255	0.0158	0.0987	0.0156	**0.1370**	0.0194
(1,1) (1,4) (1,6) (1,8) (1,10)	0.1352	0.0175	0.0961	0.0140	**0.1650**	0.0211

Table B51. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) (μ_4,σ_4^2) (μ_5,σ_5^2)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	**0.3735**	0.1653	0.3706	0.2635	0.3199	0.1201
(1,1) (1,1) (3,9) (3,9) (3,9)	**0.4668**	0.1977	0.4660	0.3315	0.3916	0.1414
(1,1) (2,4) (3,9) (4,16) (5,25)	**0.6239**	0.3227	0.5776	0.5149	0.5746	0.2258
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	**0.7346**	0.4571	0.6630	0.6856	0.7091	0.3231
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	0.5401	0.2364	**0.5416**	0.3941	0.4582	0.1629
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	**0.6319**	0.3318	0.5847	0.5238	0.5795	0.2304
(1,1) (4,16) (6,36) (8,64) (10,100)	**0.7642**	0.5152	0.6712	0.7450	0.7553	0.3759

Table B52. Estimated power of tests for mixed design under the exponential distribution with different means and same variances; the variance in CRD=8RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) $(\mu_4,\sigma_4^2)$$(\mu_5,\sigma_5^2)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,1) (1,1) (1,1) (1,1)	0.0509	0.0498	0.0491	0.0515	0.0530	0.0512
(1,1) (2,1) (2,1) (2,1) (2,1)	0.0779	0.2036	0.0785	0.1972	0.0721	**0.2277**
(1,1) (1,1) (1,1) (2,1) (2,1)	0.1262	0.1243	0.1036	0.1218	**0.1413**	0.1408
(1,1) (1,1) (2,1) (2,1) (2,1)	0.1087	0.1681	0.0969	0.1631	0.1113	**0.1877**
(1,1) (1.5,1) (2,1) (2.5,1) (3,1)	0.1234	0.2651	0.1153	0.2537	0.1199	**0.2979**
(1,1) (2,1) (2.5,1) (3,1) (3.5,1)	0.1095	0.3584	0.1037	0.3431	0.0929	**0.4027**
(1,1) (2,1) (2.5,1) (2.75,1) (3.5,1)	0.1118	0.3529	0.1109	0.3408	0.0953	**0.3941**

Table B53. Estimated power of tests for mixed design under the exponential distribution with same means and different variances; the variance in CRD=8RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) $(\mu_4,\sigma_4^2)$$(\mu_5,\sigma_5^2)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (1,2) (1,2) (1,2) (1,2)	0.1487	0.0294	**0.1916**	0.0302	0.1058	0.0300
(1,1) (1,1) (1,3) (1,3) (1,3)	0.1839	0.0342	**0.2171**	0.0361	0.1304	0.0347
(1,1) (1,2) (1,3) (1,4) (1,5)	0.1989	0.0292	**0.2688**	0.0310	0.1226	0.0286
(1,1) (1,2.5) (1,5) (1,7.5) (1,10)	0.2328	0.0361	**0.3269**	0.0382	0.1315	0.0347
(1,1) (1,2) (1,5) (1,1) (1,4.5)	0.1943	0.0354	**0.2295**	0.0370	0.1375	0.0377
(1,1) (1,3) (1,3.5) (1,2) (1,6)	0.1973	0.0278	**0.2630**	0.0296	0.1176	0.0278
(1,1) (1,4) (1,6) (1,8) (1,10)	0.2336	0.0339	**0.3310**	0.0368	0.1283	0.0320

Table B54. Estimated power of tests for mixed design under the exponential distribution with different means and variances; the variance in CRD=8RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1^2) (μ_2,σ_2^2) (μ_3,σ_3^2) $(\mu_4,\sigma_4^2)$$(\mu_5,\sigma_5^2)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(1,1) (2,4) (2,4) (2,4) (2,4)	0.4267	0.1584	**0.4463**	0.1525	0.3430	0.1822
(1,1) (1,1) (3,9) (3,9) (3,9)	0.4984	0.1907	**0.5156**	0.1810	0.4116	0.225
(1,1) (2,4) (3,9) (4,16) (5,25)	0.6928	0.3073	**0.7567**	0.2927	0.5452	0.3611
(1,1) (2.5,6.25) (5,25) (7.5,56.25) (10,100)	0.8439	0.4749	**0.9062**	0.4496	0.6644	0.5442
(1,1) (2,4) (5,25) (1,1) (4.5,20.25)	0.5746	0.2096	**0.5910**	0.1979	0.4747	0.2445
(1,1) (3,9) (3.5,12.25) (2,4) (6,36)	0.7122	0.3142	**0.7671**	0.2951	0.5602	0.3653
(1,1) (4,16) (6,36) (8,64) (10,100)	0.8789	0.5491	**0.9349**	0.5242	0.6752	0.6133

APPENDIX C. ESTIMATED POWERS FOR T DISTRIBUTION

Table C1. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0511	0.0453	0.0511	0.0453	0.0511	0.0453
(0,σ) (1,σ) (1,σ)	**0.2815**	0.2516	**0.2815**	0.2516	**0.2815**	0.2516
(0,σ) (0,σ) (1,σ)	**0.1879**	0.1349	**0.1879**	0.1349	**0.1879**	0.1349
(0,σ) (0.5,σ) (1,σ)	**0.2282**	0.1862	**0.2282**	0.1862	**0.2282**	0.1862
(0,σ) (1,σ) (1.5,σ)	**0.3521**	0.3224	**0.3521**	0.3224	**0.3521**	0.3224
(0,σ) (1.5,σ) (2,σ)	0.4417	**0.4705**	0.4417	**0.4705**	0.4417	**0.4705**
(0,σ) (1.5,σ) (1.75,σ)	0.4118	**0.4417**	0.4118	**0.4417**	0.4118	**0.4417**

Table C2. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	**0.2653**	0.1554	**0.2653**	0.1554	**0.2653**	0.1554
(0,σ) (0,σ) (0,3σ)	**0.2020**	0.1155	**0.2020**	0.1155	**0.2020**	0.1155
(0,σ) (0,2σ) (0,3σ)	**0.3449**	0.1922	**0.3449**	0.1922	**0.3449**	0.1922
(0,σ) (0,2.5σ) (0,5σ)	**0.4872**	0.2777	**0.4872**	0.2777	**0.4872**	0.2777
(0,σ) (0,2σ) (0,5σ)	**0.4452**	0.2375	**0.4452**	0.2375	**0.4452**	0.2375
(0,σ) (0,3σ) (0,3.5σ)	**0.4709**	0.2566	**0.4709**	0.2566	**0.4709**	0.2566
(0,σ) (0,6σ) (0,8σ)	**0.6924**	0.4212	**0.6924**	0.4212	**0.6924**	0.4212

Table C3. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	**0.5469**	0.3688	**0.5469**	0.3688	**0.5469**	0.3688
(0,σ) (0,σ) (0,3σ)	**0.3072**	0.1736	**0.3072**	0.1736	**0.3072**	0.1736
(0,σ) (0,2σ) (1,3σ)	**0.4774**	0.2715	**0.4774**	0.2715	**0.4774**	0.2715
(0,σ) (0.5,2.5σ) (1,5σ)	**0.6415**	0.3888	**0.6415**	0.3888	**0.6415**	0.3888
(0,σ) (0.25,2σ) (0.5,5σ)	**0.5336**	0.3024	**0.5336**	0.3024	**0.5336**	0.3024
(0,σ) (1,3σ) (1.5,3.5σ)	**0.7254**	0.4882	**0.7254**	0.4882	**0.7254**	0.4882
(0,σ) (1.5,6σ) (1.75,8σ)	**0.8422**	0.5841	**0.8422**	0.5841	**0.8422**	0.5841

Table C4. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0505	0.0481	0.0528	0.0524	0.0497	0.0495
(0,σ) (1,σ) (1,σ)	0.2344	0.2446	0.2503	**0.3373**	0.1966	0.1971
(0,σ) (0,σ) (1,σ)	0.1724	0.1318	**0.1873**	0.1805	0.1387	0.1073
(0,σ) (0.5,σ) (1,σ)	0.1961	0.1875	0.2088	**0.2514**	0.1637	0.1530
(0,σ) (1,σ) (1.5,σ)	0.2903	0.3158	0.3037	**0.4276**	0.2461	0.2467
(0,σ) (1.5,σ) (2,σ)	0.3536	0.4316	0.3568	**0.5787**	0.2925	0.3391
(0,σ) (1.5,σ) (1.75,σ)	0.3329	0.3993	0.3397	**0.5358**	0.2740	0.3100

Table C5. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	**0.1822**	0.1098	0.1635	0.1231	0.1807	0.1054
(0,σ) (0,σ) (0,3σ)	**0.1420**	0.0847	0.1362	0.0972	0.1336	0.0833
(0,σ) (0,2σ) (0,3σ)	0.2269	0.1321	0.2011	0.1481	**0.2314**	0.1246
(0,σ) (0,2.5σ) (0,5σ)	0.4221	0.2241	0.3339	0.2460	**0.4549**	0.2125
(0,σ) (0,2σ) (0,5σ)	0.2815	0.1525	0.2431	0.1732	**0.2881**	0.1394
(0,σ) (0,3σ) (0,3.5σ)	0.2970	0.1636	0.2448	0.1853	**0.3169**	0.1568
(0,σ) (0,6σ) (0,8σ)	0.4542	0.2470	0.3604	0.2714	**0.4870**	0.2306

Table C6. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	**0.4053**	0.2894	0.3879	0.3795	0.3701	0.2382
(0,σ) (0,σ) (0,3σ)	**0.2294**	0.1499	0.2228	0.1841	0.2112	0.1302
(0,σ) (0,2σ) (1,3σ)	**0.3343**	0.1960	0.2999	0.2389	0.3174	0.1742
(0,σ) (0.5,2.5σ) (1,5σ)	**0.4515**	0.2680	0.3971	0.3249	0.4430	0.2317
(0,σ) (0.25,2σ) (0.5,5σ)	**0.3601**	0.2109	0.3122	0.2476	0.3564	0.1873
(0,σ) (1,3σ) (1.5,3.5σ)	**0.5328**	0.3548	0.4776	0.4394	0.5087	0.3006
(0,σ) (1.5,6σ) (1.75,8σ)	0.6601	0.3739	0.5557	0.4601	**0.6926**	0.3525

Table C7. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=4RCBD; K=3; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0530	0.0501	0.0529	0.0485	0.0544	0.0520
(0,σ) (1,σ) (1,σ)	0.2196	0.2352	0.2087	0.2218	0.1971	**0.2567**
(0,σ) (0,σ) (1,σ)	0.1528	0.1284	0.1392	0.1223	**0.1534**	0.1386
(0,σ) (0.5,σ) (1,σ)	0.1774	0.1741	0.1690	0.1664	0.1699	**0.1900**
(0,σ) (1,σ) (1.5,σ)	0.2682	0.2957	0.2587	0.2780	0.2371	**0.3230**
(0,σ) (1.5,σ) (2,σ)	0.3327	0.4412	0.3361	0.4187	0.2801	**0.4783**
(0,σ) (1.5,σ) (1.75,σ)	0.3204	0.3987	0.3182	0.3787	0.2742	**0.4367**

Table C8. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=4RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	0.2325	0.1487	**0.2710**	0.1442	0.1772	0.1534
(0,σ) (0,σ) (0,3σ)	0.1741	0.1063	**0.1862**	0.1035	0.1465	0.1104
(0,σ) (0,2σ) (0,3σ)	0.3071	0.1841	**0.3581**	0.1796	0.2210	0.1880
(0,σ) (0,2.5σ) (0,5σ)	0.4349	0.2626	**0.5231**	0.2563	0.3032	0.2695
(0,σ) (0,2σ) (0,5σ)	0.3882	0.2288	**0.4674**	0.2240	0.2692	0.2364
(0,σ) (0,3σ) (0,3.5σ)	0.4064	0.2579	**0.4885**	0.2519	0.2762	0.2653
(0,σ) (0,6σ) (0,8σ)	0.6241	0.4014	**0.7509**	0.3951	0.4162	0.4067

Table C9. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=4RCBD; K=3; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	0.4520	0.3344	**0.4844**	0.3204	0.3554	0.3566
(0,σ) (0,σ) (0,3σ)	0.2545	0.1652	**0.2671**	0.1605	0.2161	0.1750
(0,σ) (0,2σ) (1,3σ)	0.4127	0.2696	**0.4727**	0.2607	0.3144	0.2814
(0,σ) (0.5,2.5σ) (1,5σ)	0.5662	0.3829	**0.6488**	0.3695	0.4241	0.3993
(0,σ) (0.25,2σ) (0.5,5σ)	0.4646	0.2912	**0.5396**	0.2812	0.3365	0.3058
(0,σ) (1,3σ) (1.5,3.5σ)	0.6348	0.4687	**0.7039**	0.4534	0.4803	0.4907
(0,σ) (1.5,6σ) (1.75,8σ)	0.9998	0.7218	0.9979	0.624	1	0.8542

Table C10. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 10$.

Treatments Effects	Proposed Tests					
(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0511	0.0520	0.0511	0.0520	0.0511	0.0520
(0,σ) (1,σ) (1,σ)	**0.2356**	0.1840	**0.2356**	0.1840	**0.2356**	0.1840
(0,σ) (0,σ) (1,σ)	**0.1606**	0.1080	**0.1606**	0.1080	**0.1606**	0.1080
(0,σ) (0.5,σ) (1,σ)	**0.1954**	0.1388	**0.1954**	0.1388	**0.1954**	0.1388
(0,σ) (1,σ) (1.5,σ)	**0.2930**	0.2307	**0.2930**	0.2307	**0.2930**	0.2307
(0,σ) (1.5,σ) (2,σ)	**0.3757**	0.3382	**0.3757**	0.3382	**0.3757**	0.3382
(0,σ) (1.5,σ) (1.75,σ)	**0.3496**	0.3082	**0.3496**	0.3082	**0.3496**	0.3082

Table C11. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 10$.

Treatments Effects	Proposed Tests					
(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	**0.2570**	0.1484	**0.2570**	0.1484	**0.2570**	0.1484
(0,σ) (0,σ) (0,3σ)	**0.1940**	0.1192	**0.1940**	0.1192	**0.1940**	0.1192
(0,σ) (0,2σ) (0,3σ)	**0.3432**	0.1930	**0.3432**	0.1930	**0.3432**	0.1930
(0,σ) (0,2.5σ) (0,5σ)	**0.4866**	0.2702	**0.4866**	0.2702	**0.4866**	0.2702
(0,σ) (0,2σ) (0,5σ)	**0.4340**	0.2346	**0.4340**	0.2346	**0.4340**	0.2346
(0,σ) (0,3σ) (0,3.5σ)	**0.4694**	0.2697	**0.4694**	0.2697	**0.4694**	0.2697
(0,σ) (0,6σ) (0,8σ)	**0.6884**	0.4127	**0.6884**	0.4127	**0.6884**	0.4127

Table C12. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 10$.

Treatments Effects	Proposed Tests					
(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	**0.5304**	0.3197	**0.5304**	0.3197	**0.5304**	0.3197
(0,σ) (0,σ) (0,3σ)	**0.2913**	0.1557	**0.2913**	0.1557	**0.2913**	0.1557
(0,σ) (0,2σ) (1,3σ)	**0.4626**	0.2568	**0.4626**	0.2568	**0.4626**	0.2568
(0,σ) (0.5,2.5σ) (1,5σ)	**0.6332**	0.3661	**0.6332**	0.3661	**0.6332**	0.3661
(0,σ) (0.25,2σ) (0.5,5σ)	**0.5252**	0.2922	**0.5252**	0.2922	**0.5252**	0.2922
(0,σ) (1,3σ) (1.5,3.5σ)	**0.7008**	0.4341	**0.7008**	0.4341	**0.7008**	0.4341
(0,σ) (1.5,6σ) (1.75,8σ)	**0.8276**	0.5455	**0.8276**	0.5455	**0.8276**	0.5455

Table C13. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

Treatments Effects	Proposed Tests					
(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0520	0.0487	0.0514	0.0527	0.0501	0.0486
(0,σ) (1,σ) (1,σ)	0.2070	0.1933	0.2297	**0.2834**	0.1649	0.1513
(0,σ) (0,σ) (1,σ)	0.1567	0.1200	**0.1774**	0.1632	0.1235	0.0962
(0,σ) (0.5,σ) (1,σ)	0.1736	0.1522	0.1941	**0.2193**	0.1428	0.1210
(0,σ) (1,σ) (1.5,σ)	0.2479	0.2428	0.2723	**0.3543**	0.1943	0.1836
(0,σ) (1.5,σ) (2,σ)	0.3093	0.3380	0.3300	**0.4877**	0.2528	0.2565
(0,σ) (1.5,σ) (1.75,σ)	0.2826	0.3108	0.3067	**0.4485**	0.2278	0.2324

Table C14. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

Treatments Effects	Proposed Tests					
(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	0.1797	0.1096	0.1614	0.1252	**0.1822**	0.1039
(0,σ) (0,σ) (0,3σ)	**0.1501**	0.0860	0.1423	0.1035	0.1385	0.0798
(0,σ) (0,2σ) (0,3σ)	0.2324	0.1349	0.2059	0.1502	**0.2343**	0.1251
(0,σ) (0,2.5σ) (0,5σ)	0.4229	0.2319	0.3410	0.2563	**0.4545**	0.2166
(0,σ) (0,2σ) (0,5σ)	0.2852	0.1542	0.2416	0.1735	**0.2867**	0.1432
(0,σ) (0,3σ) (0,3.5σ)	0.2940	0.1686	0.2467	0.1834	**0.3093**	0.1584
(0,σ) (0,6σ) (0,8σ)	0.4427	0.2366	0.3514	0.2629	**0.4733**	0.2206

Table C15. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

Treatments Effects	Proposed Tests					
(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	**0.3808**	0.2575	0.3726	0.3431	0.3394	0.2095
(0,σ) (0,σ) (0,3σ)	**0.2322**	0.1361	0.2240	0.1765	0.2045	0.1161
(0,σ) (0,2σ) (1,3σ)	**0.3343**	0.1823	0.3035	0.2324	0.3164	0.1611
(0,σ) (0.5,2.5σ) (1,5σ)	**0.4368**	0.2534	0.3865	0.3092	0.4283	0.2170
(0,σ) (0.25,2σ) (0.5,5σ)	**0.3443**	0.1915	0.3053	0.2250	0.3364	0.1700
(0,σ) (1,3σ) (1.5,3.5σ)	**0.5076**	0.3099	0.4656	0.3996	0.4799	0.2589
(0,σ) (1.5,6σ) (1.75,8σ)	0.6511	0.3383	0.5411	0.4302	**0.6769**	0.3206

Table C16. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=8RCBD; K=3; $n_b = 5, n_a = 10$.

Treatments Effects	Proposed Tests					
(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ)	0.0519	0.0514	0.0513	0.0504	0.0529	0.0541
(0,σ) (1,σ) (1,σ)	0.1827	0.1721	0.1656	0.1598	0.1743	**0.1886**
(0,σ) (0,σ) (1,σ)	0.1336	0.1013	0.1174	0.0961	**0.1388**	0.1108
(0,σ) (0.5,σ) (1,σ)	0.1514	0.1280	0.1335	0.1216	**0.1518**	0.1421
(0,σ) (1,σ) (1.5,σ)	0.2210	0.2043	0.2012	0.1904	0.2063	**0.2277**
(0,σ) (1.5,σ) (2,σ)	0.2769	0.2986	0.2651	0.2782	0.2472	**0.3356**
(0,σ) (1.5,σ) (1.75,σ)	0.2598	0.2801	0.2425	0.2596	0.2279	**0.3115**

Table C17. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

Treatments Effects	Proposed Tests					
(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ)	0.2245	0.1494	**0.2648**	0.1470	0.1726	0.1538
(0,σ) (0,σ) (0,3σ)	0.1815	0.1155	**0.1987**	0.1117	0.1502	0.1185
(0,σ) (0,2σ) (0,3σ)	0.3118	0.1929	**0.3618**	0.1893	0.2227	0.1983
(0,σ) (0,2.5σ) (0,5σ)	0.4370	0.2701	**0.5268**	0.2634	0.3015	0.2758
(0,σ) (0,2σ) (0,5σ)	0.4002	0.2356	**0.4792**	0.2294	0.2819	0.2427
(0,σ) (0,3σ) (0,3.5σ)	0.4019	0.2460	**0.4883**	0.2392	0.2812	0.2533
(0,σ) (0,6σ) (0,8σ)	0.6329	0.4022	**0.7574**	0.3936	0.4260	0.4148

Table C18. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=8RCBD; K=3; $n_b = 10, n_a = 5$.

Treatments Effects	Proposed Tests					
(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3)	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ)	0.4299	0.2875	**0.4590**	0.2719	0.3479	0.3116
(0,σ) (0,σ) (0,3σ)	0.2539	0.1555	**0.2624**	0.1488	0.2135	0.1662
(0,σ) (0,2σ) (1,3σ)	0.4013	0.2467	**0.4540**	0.2387	0.3034	0.2603
(0,σ) (0.5,2.5σ) (1,5σ)	0.5522	0.3532	**0.6316**	0.3404	0.4075	0.3695
(0,σ) (0.25,2σ) (0.5,5σ)	0.4530	0.2831	**0.5219**	0.2710	0.3245	0.2915
(0,σ) (1,3σ) (1.5,3.5σ)	0.6007	0.4027	**0.6716**	0.3873	0.4585	0.4248
(0,σ) (1.5,6σ) (1.75,8σ)	0.9999	0.6837	0.9982	0.5935	1	0.8313

Table C19. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0509	0.0505	0.0509	0.0505	0.0509	0.0505
(0,σ) (1,σ) (1,σ) (1,σ)	**0.2794**	0.2648	**0.2794**	0.2648	**0.2794**	0.2648
(0,σ) (0,σ) (0,σ) (1,σ)	**0.1508**	0.1039	**0.1508**	0.1039	**0.1508**	0.1039
(0,σ) (0,σ) (1,σ) (1,σ)	**0.2373**	0.1754	**0.2373**	0.1754	**0.2373**	0.1754
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	**0.3085**	0.2663	**0.3085**	0.2663	**0.3085**	0.2663
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.4001	**0.4244**	0.4001	**0.4244**	0.4001	**0.4244**
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.3646	**0.3885**	0.3646	**0.3885**	0.3646	**0.3885**

Table C20. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	**0.3157**	0.1467	**0.3157**	0.1467	**0.3157**	0.1467
(0,σ) (0,σ) (0,3σ) (0,3σ)	**0.3281**	0.1478	**0.3281**	0.1478	**0.3281**	0.1478
(0,σ) (0,2σ) (0,3σ) (0,4σ)	**0.5248**	0.2436	**0.5248**	0.2436	**0.5248**	0.2436
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	**0.7158**	0.3443	**0.7158**	0.3443	**0.7158**	0.3443
(0,σ) (0,2σ) (0,5σ) (0,σ)	**0.3343**	0.1509	**0.3343**	0.1509	**0.3343**	0.1509
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	**0.5015**	0.2286	**0.5015**	0.2286	**0.5015**	0.2286
(0,σ) (0,4σ) (0,6σ) (0,8σ)	**0.7994**	0.4081	**0.7994**	0.4081	**0.7994**	0.4081

Table C21. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	**0.6405**	0.4002	0.6405	0.4002	0.6405	0.4002
(0,σ) (0,σ) (0,3σ) (1, 3σ)	**0.4387**	0.2026	0.4387	0.2026	0.4387	0.2026
(0,σ) (0,2σ) (1,3σ) (1,4σ)	**0.6947**	0.3655	0.6947	0.3655	0.6947	0.3655
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	**0.8509**	0.5050	0.8509	0.5050	0.8509	0.5050
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	**0.5493**	0.2762	0.5493	0.2762	0.5493	0.2762
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	**0.8317**	0.5841	0.8317	0.5841	0.8317	0.5841
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	**0.9164**	0.6115	0.9164	0.6115	0.9164	0.6115

Table C22. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0480	0.0541	0.0464	0.0494	0.0482	0.0486
(0,σ) (1,σ) (1,σ) (1,σ)	0.2351	0.2739	0.2484	**0.3574**	0.1980	0.2106
(0,σ) (0,σ) (0,σ) (1,σ)	0.1267	0.1121	**0.1388**	0.1263	0.1075	0.0907
(0,σ) (0,σ) (1,σ) (1,σ)	0.2052	0.1930	0.2262	**0.2408**	0.1641	0.1488
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	0.2517	0.2704	0.2755	**0.3570**	0.2015	0.2005
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.3288	0.4158	0.3440	**0.5468**	0.2725	0.3122
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.3015	0.3943	0.3210	**0.5183**	0.2525	0.2948

Table C23. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	**0.2142**	0.1117	0.1895	0.1096	0.2063	0.0974
(0,σ) (0,σ) (0,3σ) (0,3σ)	**0.2377**	0.1148	0.2167	0.1174	0.2188	0.1017
(0,σ) (0,2σ) (0,3σ) (0,4σ)	**0.3417**	0.1586	0.2935	0.1598	0.3348	0.1370
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	0.4783	0.2091	0.4031	0.2100	**0.4824**	0.1773
(0,σ) (0,2σ) (0,5σ) (0,σ)	**0.2343**	0.1122	0.2117	0.1136	0.2153	0.0996
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	**0.3285**	0.1499	0.2782	0.1516	0.3270	0.1331
(0,σ) (0,4σ) (0,6σ) (0,8σ)	0.5569	0.2483	0.468	0.2576	**0.5717**	0.2149

Table C24. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=4RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) (μ_4,σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	**0.4777**	0.3305	0.4567	0.4044	0.4254	0.2574
(0,σ) (0,σ) (0,3σ) (1, 3σ)	**0.3111**	0.1616	0.2919	0.1787	0.2764	0.1331
(0,σ) (0,2σ) (1,3σ) (1,4σ)	**0.5092**	0.2690	0.4656	0.3094	0.4780	0.2115
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	**0.6499**	0.3397	0.5859	0.3895	0.623	0.2751
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	**0.4231**	0.1982	0.4101	0.2530	0.3701	0.1684
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	**0.6682**	0.4692	0.6340	0.5738	0.6123	0.3647
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	**0.7446**	0.4304	0.6714	0.4878	0.7300	0.3526

Table C25. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=4RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0502	0.0527	0.0516	0.0546	0.0514	0.0533
(0,σ) (1,σ) (1,σ) (1,σ)	0.2278	0.2524	0.2157	0.2454	0.2081	**0.2763**
(0,σ) (0,σ) (0,σ) (1,σ)	**0.1182**	0.0911	0.1072	0.0917	0.1173	0.0980
(0,σ) (0,σ) (1,σ) (1,σ)	**0.1901**	0.164	0.1766	0.1602	0.1834	0.1827
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	0.2381	0.2457	0.2294	0.2391	0.2268	**0.2732**
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.3012	0.3800	0.2985	0.3691	0.2627	**0.4203**
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.2941	0.3641	0.2873	0.3517	0.2565	**0.4036**

Table C26. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=4RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	0.2748	0.1505	**0.3131**	0.1527	0.2037	0.1594
(0,σ) (0,σ) (0,3σ) (0,3σ)	0.2793	0.1434	**0.3091**	0.1453	0.2097	0.1497
(0,σ) (0,2σ) (0,3σ) (0,4σ)	0.4451	0.2243	**0.5063**	0.2275	0.3164	0.2366
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	0.6261	0.3308	**0.7232**	0.3337	0.4429	0.3458
(0,σ) (0,2σ) (0,5σ) (0,σ)	0.2751	0.1415	**0.3027**	0.1436	0.2085	0.1478
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	0.4266	0.2124	**0.4961**	0.2144	0.3055	0.2211
(0,σ) (0,4 σ) (0,6σ) (0,8σ)	0.7146	0.393	**0.8193**	0.3963	0.5041	0.4077

Table C27. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=4RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	0.5235	0.3587	**0.5512**	0.3523	0.4248	0.3929
(0,σ) (0,σ) (0,3σ) (1, 3σ)	0.3618	0.1924	**0.3922**	0.1937	0.2833	0.2025
(0,σ) (0,2σ) (1,3σ) (1,4σ)	0.5812	0.3300	**0.6398**	0.3293	0.4489	0.3528
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	0.7680	0.4739	**0.8360**	0.4710	0.5874	0.4981
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	0.4570	0.2595	**0.4805**	0.2562	0.3662	0.2809
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	0.7323	0.5396	**0.7727**	0.5281	0.5984	0.5781
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	0.8446	0.5757	**0.9042**	0.5714	0.6769	0.6066

Table C28. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0521	0.0539	0.0521	0.0539	0.0521	0.0539
(0,σ) (1,σ) (1,σ) (1,σ)	**0.2477**	0.1960	**0.2477**	0.1960	**0.2477**	0.1960
(0,σ) (0,σ) (0,σ) (1,σ)	**0.1342**	0.0884	**0.1342**	0.0884	**0.1342**	0.0884
(0,σ) (0,σ) (1,σ) (1,σ)	**0.2062**	0.1329	**0.2062**	0.1329	**0.2062**	0.1329
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	**0.2659**	0.1938	**0.2659**	0.1938	**0.2659**	0.1938
(0,σ) (1,σ) (1.5,σ) (2,σ)	**0.3476**	0.3073	**0.3476**	0.3073	**0.3476**	0.3073
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	**0.3209**	0.2868	**0.3209**	0.2868	**0.3209**	0.2868

Table C29. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	**0.3202**	0.1490	**0.3202**	0.1490	**0.3202**	0.1490
(0,σ) (0,σ) (0,3σ) (0,3σ)	**0.3353**	0.1518	**0.3353**	0.1518	**0.3353**	0.1518
(0,σ) (0,2σ) (0,3σ) (0,4σ)	**0.5159**	0.2319	**0.5159**	0.2319	**0.5159**	0.2319
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	**0.7145**	0.3409	**0.7145**	0.3409	**0.7145**	0.3409
(0,σ) (0,2σ) (0,5σ) (0,σ)	**0.3358**	0.1504	**0.3358**	0.1504	**0.3358**	0.1504
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	**0.4959**	0.2232	**0.4959**	0.2232	**0.4959**	0.2232
(0,σ) (0,4 σ) (0,6σ) (0,8σ)	**0.7927**	0.4136	**0.7927**	0.4136	**0.7927**	0.4136

Table C30. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	**0.6089**	0.3368	**0.6089**	0.3368	**0.6089**	0.3368
(0,σ) (0,σ) (0,3σ) (1, 3σ)	**0.4189**	0.1867	**0.4189**	0.1867	**0.4189**	0.1867
(0,σ) (0,2σ) (1,3σ) (1,4σ)	**0.6703**	0.3217	**0.6703**	0.3217	**0.6703**	0.3217
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	**0.8333**	0.4602	**0.8333**	0.4602	**0.8333**	0.4602
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	**0.5228**	0.2408	**0.5228**	0.2408	**0.5228**	0.2408
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	**0.8169**	0.4922	**0.8169**	0.4922	**0.8169**	0.4922
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	**0.9012**	0.5542	**0.9012**	0.5542	**0.9012**	0.5542

Table C31. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0509	0.0536	0.0507	0.0507	0.0480	0.0498
(0,σ) (1,σ) (1,σ) (1,σ)	0.2115	0.2196	0.2330	**0.2992**	0.1656	0.1601
(0,σ) (0,σ) (0,σ) (1,σ)	0.1172	0.1002	**0.1276**	0.1171	0.0977	0.0795
(0,σ) (0,σ) (1,σ) (1,σ)	0.1851	0.1600	**0.2104**	0.2100	0.1455	0.1185
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	0.2392	0.2279	0.2666	**0.3112**	0.1822	0.1638
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.2866	0.3355	0.3000	**0.4611**	0.2315	0.2382
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.2653	0.3120	0.2866	**0.4318**	0.2147	0.2213

Table C32. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	**0.2215**	0.1155	0.1953	0.1176	0.2136	0.1010
(0,σ) (0,σ) (0,3σ) (0,3σ)	**0.2288**	0.1166	0.2024	0.1172	0.2199	0.1013
(0,σ) (0,2σ) (0,3σ) (0,4σ)	**0.3479**	0.1578	0.2995	0.1631	0.3352	0.1365
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	**0.4849**	0.2122	0.4078	0.2162	0.4838	0.1812
(0,σ) (0,2σ) (0,5σ) (0,σ)	**0.2344**	0.119	0.2121	0.1214	0.2186	0.1028
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	**0.3408**	0.1584	0.2943	0.1641	0.3347	0.1384
(0,σ) (0,4 σ) (0,6σ) (0,8σ)	0.5539	0.2467	0.4632	0.2452	**0.5599**	0.2147

Table C33. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=8RCBD; K=4; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	**0.4651**	0.2969	0.4546	0.3748	0.4068	0.2284
(0,σ) (0,σ) (0,3σ) (1, 3σ)	**0.3095**	0.1529	0.2911	0.1735	0.2781	0.1226
(0,σ) (0,2σ) (1,3σ) (1,4σ)	**0.4933**	0.2428	0.4525	0.2825	0.4526	0.1951
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	**0.6367**	0.3232	0.5743	0.3662	0.6035	0.2605
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	**0.4056**	0.1745	0.3970	0.2271	0.3501	0.1526
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	**0.6524**	0.4138	0.6279	0.5268	0.5857	0.3177
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	**0.7258**	0.3987	0.662	0.4587	0.7090	0.3233

Table C34. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=8RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,σ) (0,σ) (0,σ)	0.0535	0.0526	0.0500	0.0535	0.0519	0.0537
(0,σ) (1,σ) (1,σ) (1,σ)	0.1818	0.1690	0.1689	0.1649	0.1792	**0.1933**
(0,σ) (0,σ) (0,σ) (1,σ)	0.1074	0.0851	0.0998	0.0856	**0.1127**	0.0907
(0,σ) (0,σ) (1,σ) (1,σ)	0.1672	0.1271	0.1460	0.1234	**0.1674**	0.1436
(0,σ) (0.5,σ) (1,σ) (1.5,σ)	**0.2022**	0.1748	0.1850	0.1684	0.2009	0.2012
(0,σ) (1,σ) (1.5,σ) (2,σ)	0.2556	0.2570	0.2372	0.2478	0.2407	**0.2927**
(0,σ) (1,σ) (1.5,σ) (1.75,σ)	0.2442	0.2463	0.2252	0.2381	0.2312	**0.2789**

Table C35. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=8RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (0,2σ) (0,2σ) (0,2σ)	0.2662	0.1441	**0.2972**	0.1461	0.2014	0.1509
(0,σ) (0,σ) (0,3σ) (0,3σ)	0.2878	0.1442	**0.3168**	0.1480	0.2161	0.1520
(0,σ) (0,2σ) (0,3σ) (0,4σ)	0.4461	0.2243	**0.5208**	0.2263	0.3197	0.2343
(0,σ) (0,2.5σ) (0,5σ) (0,7.5σ)	0.6297	0.3349	**0.7326**	0.3392	0.4453	0.3470
(0,σ) (0,2σ) (0,5σ) (0,σ)	0.2776	0.1426	**0.3090**	0.1443	0.2136	0.1503
(0,σ) (0,3σ) (0,3.5σ) (0,2σ)	0.4288	0.2122	**0.4947**	0.2159	0.3075	0.2234
(0,σ) (0,4 σ) (0,6σ) (0,8σ)	0.7123	0.3985	**0.8136**	0.3998	0.5052	0.4157

Table C36. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=8RCBD; K=4; $n_b = 5, n_a = 10$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) (μ_4, σ_4)	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ) (1,2σ) (1,2σ) (1,2σ)	0.4926	0.3094	**0.5163**	0.3049	0.4052	0.3357
(0,σ) (0,σ) (0,3σ) (1, 3σ)	0.3540	0.1779	**0.3762**	0.1778	0.2795	0.1909
(0,σ) (0,2σ) (1,3σ) (1,4σ)	0.5765	0.3114	**0.6317**	0.3106	0.4441	0.3354
(0,σ) (0.5,2.5σ) (1,5σ) (1.5,7.5σ)	0.7469	0.4321	**0.8123**	0.4279	0.5729	0.4587
(0,σ) (0.25,2σ) (0.5,5σ) (0.75,σ)	0.4247	0.2189	**0.4327**	0.2177	0.3408	0.2386
(0,σ) (1,3σ) (1.5,3.5σ) (2,2σ)	0.7014	0.4458	**0.7366**	0.4355	0.5851	0.4879
(0,σ) (1,4σ) (1.5,6σ) (1.75,8σ)	0.8340	0.5246	**0.8915**	0.5192	0.6604	0.5571

Table C37. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0507	0.0493	0.0507	0.0493	0.0507	0.0493
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	**0.2810**	0.2727	**0.2810**	0.2727	**0.2810**	0.2727
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	**0.2042**	0.1479	**0.2042**	0.1479	**0.2042**	0.1479
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	**0.2599**	0.2075	**0.2599**	0.2075	**0.2599**	0.2075
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	**0.3733**	0.3503	**0.3733**	0.3503	**0.3733**	0.3503
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.4305	**0.4976**	0.4305	**0.4976**	0.4305	**0.4976**
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.4139	**0.4784**	0.4139	**0.4784**	0.4139	**0.4784**

Table C38. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	**0.3518**	0.1557	**0.3518**	0.1557	**0.3518**	0.1557
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	**0.4249**	0.1649	**0.4249**	0.1649	**0.4249**	0.1649
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	**0.6645**	0.2822	**0.6645**	0.2822	**0.6645**	0.2822
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	**0.8434**	0.4168	**0.8434**	0.4168	**0.8434**	0.4168
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	**0.4786**	0.1875	**0.4786**	0.1875	**0.4786**	0.1875
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	**0.6665**	0.2887	**0.6665**	0.2887	**0.6665**	0.2887
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	**0.8965**	0.4824	**0.8965**	0.4824	**0.8965**	0.4824

Table C39. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	**0.6720**	0.4079	**0.6720**	0.4079	**0.6720**	0.4079
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	**0.5853**	0.2768	**0.5853**	0.2768	**0.5853**	0.2768
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	**0.8119**	0.4377	**0.8119**	0.4377	**0.8119**	0.4377
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	**0.9364**	0.6089	**0.9364**	0.6089	**0.9364**	0.6089
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	**0.7058**	0.3509	**0.7058**	0.3509	**0.7058**	0.3509
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	**0.9121**	0.6639	**0.9121**	0.6639	**0.9121**	0.6639
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	**0.9633**	0.6983	**0.9633**	0.6983	**0.9633**	0.6983

Table C40. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0505	0.0468	0.0517	0.0497	0.0495	0.0519
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.2406	0.2555	0.2548	**0.3571**	0.1974	0.2087
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	0.1794	0.1423	**0.1993**	0.1915	0.1443	0.1214
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	0.2200	0.1973	0.2358	**0.2743**	0.1838	0.1628
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.3091	0.3268	0.3281	**0.4521**	0.2507	0.2599
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.3373	0.4533	0.3415	**0.6068**	0.2838	0.3613
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.3320	0.4267	0.3425	**0.5830**	0.2712	0.3389

Table C41. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	**0.2493**	0.1097	0.229	0.1295	0.2357	0.1040
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	**0.3089**	0.1261	0.2838	0.1511	0.2871	0.1167
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	**0.4595**	0.1738	0.4019	0.2100	0.4363	0.1553
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	**0.6209**	0.2367	0.5335	0.2839	0.6162	0.2147
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	**0.3392**	0.1297	0.3060	0.1563	0.3162	0.1225
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	**0.4683**	0.1770	0.4097	0.2109	0.4496	0.1624
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	0.6835	0.2729	0.5869	0.3235	**0.6872**	0.2433

Table C42. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=4RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1, σ_1) (μ_2, σ_2) (μ_3, σ_3) $(\mu_4, \sigma_4)(\mu_5, \sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	**0.5310**	0.3380	0.5109	0.4516	0.4733	0.2754
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	**0.4363**	0.1930	0.4111	0.2610	0.3861	0.1669
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	**0.6239**	0.3039	0.5822	0.3880	0.5800	0.2550
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	**0.7832**	0.3954	0.7239	0.5014	0.7559	0.3325
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	**0.5284**	0.2536	0.5079	0.3411	0.4767	0.2072
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	**0.7679**	0.5111	0.7296	0.6521	0.7180	0.4182
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	**0.8444**	0.4621	0.7805	0.5835	0.8226	0.3899

Table C43. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=4RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0496	0.0512	0.0499	0.0530	0.0513	0.0518
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.2249	0.2471	0.2150	0.2405	0.2039	**0.2730**
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	**0.1572**	0.1367	0.1468	0.1350	0.1535	0.1486
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	0.2041	0.1882	0.1899	0.1827	0.1965	**0.2085**
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.2838	0.3188	0.2705	0.3086	0.2588	**0.3531**
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.3292	0.4500	0.3220	0.4349	0.2771	**0.4937**
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.3166	0.4423	0.3211	0.4247	0.2663	**0.4870**

Table C44. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=4RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	0.3006	0.1515	**0.3369**	0.1520	0.2250	0.1611
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	0.3609	0.1659	**0.4032**	0.1666	0.2693	0.1772
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	0.5691	0.2708	**0.6477**	0.2684	0.4035	0.2872
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	0.7561	0.3979	**0.8483**	0.3942	0.5563	0.4208
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	0.3992	0.1859	**0.4527**	0.1862	0.3001	0.1977
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	0.5714	0.2676	**0.6500**	0.2659	0.4077	0.2853
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	0.8185	0.4533	**0.9037**	0.4508	0.6119	0.4773

Table C45. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=4RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	0.5640	0.3919	**0.5951**	0.3815	0.4586	0.4269
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	0.4929	0.2600	**0.5338**	0.2547	0.3878	0.2827
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	0.7142	0.4116	**0.7742**	0.4031	0.5576	0.4411
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	0.8646	0.5617	**0.9163**	0.5518	0.7034	0.5970
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	0.5859	0.3173	**0.6222**	0.3126	0.4650	0.3488
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	0.8259	0.6210	**0.8651**	0.6059	0.6899	0.6635
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	0.9176	0.6624	**0.9582**	0.6484	0.7722	0.6958

Table C46. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0521	0.0495	0.0521	0.0495	0.0521	0.0495
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	**0.2466**	0.1959	**0.2466**	0.1959	**0.2466**	0.1959
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	**0.1777**	0.1181	**0.1777**	0.1181	**0.1777**	0.1181
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	**0.2247**	0.1530	**0.2247**	0.1530	**0.2247**	0.1530
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	**0.3167**	0.2488	**0.3167**	0.2488	**0.3167**	0.2488
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	**0.3700**	0.3577	**0.3700**	0.3577	**0.3700**	0.3577
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	**0.3685**	0.3452	**0.3685**	0.3452	**0.3685**	0.3452

Table C47. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	**0.3547**	0.1506	**0.3547**	0.1506	**0.3547**	0.1506
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	**0.4332**	0.1801	**0.4332**	0.1801	**0.4332**	0.1801
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	**0.6539**	0.2793	**0.6539**	0.2793	**0.6539**	0.2793
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	**0.8511**	0.4149	**0.8511**	0.4149	**0.8511**	0.4149
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	**0.4885**	0.1994	**0.4885**	0.1994	**0.4885**	0.1994
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	**0.6664**	0.2837	**0.6664**	0.2837	**0.6664**	0.2837
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	**0.8942**	0.4797	**0.8942**	0.4797	**0.8942**	0.4797

Table C48. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)$$(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	**0.6535**	0.3485	**0.6535**	0.3485	**0.6535**	0.3485
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	**0.5860**	0.2495	**0.5860**	0.2495	**0.5860**	0.2495
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	**0.8064**	0.4012	**0.8064**	0.4012	**0.8064**	0.4012
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	**0.9348**	0.5617	**0.9348**	0.5617	**0.9348**	0.5617
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	**0.6888**	0.3159	**0.6888**	0.3159	**0.6888**	0.3159
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	**0.8968**	0.5794	**0.8968**	0.5794	**0.8968**	0.5794
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	**0.9594**	0.6516	**0.9594**	0.6516	**0.9594**	0.6516

Table C49. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0519	0.0476	0.0515	0.0498	0.0504	0.0509
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.2135	0.2054	0.2358	**0.3017**	0.1668	0.1622
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	0.1583	0.1172	**0.1779**	0.1591	0.1266	0.1006
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	0.2039	0.1647	0.2299	**0.2367**	0.1580	0.1320
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.2766	0.2671	0.3052	**0.3861**	0.2182	0.2035
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.3134	0.3689	0.3265	**0.5255**	0.2530	0.2837
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.3016	0.3469	0.3204	**0.5016**	0.2430	0.2623

Table C50. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	**0.2483**	0.1107	0.2272	0.1285	0.2372	0.1034
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	**0.2910**	0.1210	0.2669	0.1423	0.2744	0.1133
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	**0.4673**	0.1785	0.4100	0.2137	0.4497	0.1585
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	**0.6224**	0.2307	0.5338	0.2801	0.6166	0.2084
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	**0.3360**	0.1244	0.3020	0.1543	0.3153	0.1155
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	**0.4569**	0.1755	0.3963	0.2086	0.4441	0.1600
(0,σ)(0,4 σ)(0,6σ)(0,8σ)(0,10σ)	0.6715	0.2668	0.5780	0.3171	**0.6759**	0.2397

Table C51. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=8RCBD; K=5; $n_b = 10, n_a = 5$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	**0.4998**	0.2815	0.4965	0.3935	0.4287	0.2233
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	**0.4350**	0.1959	0.4153	0.2536	0.3860	0.1622
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	**0.6280**	0.2800	0.5768	0.3730	0.5799	0.2345
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	**0.7700**	0.3679	0.7078	0.4745	0.7384	0.3036
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	**0.5241**	0.2358	0.5006	0.3271	0.4606	0.1909
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	**0.7471**	0.4432	0.7157	0.5979	0.6873	0.3520
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	**0.8264**	0.4389	0.7683	0.5588	0.8086	0.3650

Table C52. Estimated power of tests for mixed design under the t distribution with different means and same variances; the variance in CRD=8RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,σ)(0,σ)(0,σ)(0,σ)	0.0533	0.0469	0.0496	0.0475	0.0544	0.0480
(0,σ)(1,σ)(1,σ)(1,σ)(1,σ)	0.1937	0.1837	0.1809	0.1772	0.1906	**0.2072**
(0,σ)(0,σ)(0,σ)(1,σ)(1,σ)	0.1384	0.1089	0.1215	0.1068	**0.1421**	0.1189
(0,σ)(0,σ)(1,σ)(1,σ)(1,σ)	0.1759	0.1467	0.1555	0.1419	**0.1768**	0.1635
(0,σ)(0.5,σ)(1,σ)(1.5,σ)(2,σ)	0.2527	0.2279	0.2261	0.2204	0.2352	**0.2603**
(0,σ)(1,σ)(1.5,σ)(2,σ)(2.5,σ)	0.2798	0.3276	0.2700	0.3144	0.2454	**0.3694**
(0,σ)(1,σ)(1.5,σ)(1.75,σ)(2.5,σ)	0.2746	0.3075	0.2650	0.2945	0.2430	**0.3471**

Table C53. Estimated power of tests for mixed design under the t distribution with same means and different variances; the variance in CRD=8RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(0,2σ)(0,2σ)(0,2σ)(0,2σ)	0.2989	0.1441	**0.3388**	0.1444	0.2240	0.1519
(0,σ)(0,σ)(0,3σ)(0,3σ)(0,3σ)	0.3475	0.1583	**0.3921**	0.1583	0.2618	0.1681
(0,σ)(0,2σ)(0,3σ) (0,4σ) (0,5σ)	0.5678	0.2711	**0.6472**	0.2704	0.4073	0.2879
(0,σ)(0,2.5σ)(0,5σ)(0,7.5σ)(0,10σ)	0.7657	0.3984	**0.8507**	0.3946	0.5603	0.4237
(0,σ)(0,2σ)(0,5σ)(0,σ)(0,4.5σ)	0.4012	0.1773	**0.4469**	0.1775	0.2983	0.1874
(0,σ)(0,σ)(0,3.5σ)(0,2σ)(0,6σ)	0.5677	0.2682	**0.6501**	0.2671	0.4126	0.2865
(0,σ)(0,4σ)(0,6σ)(0,8σ)(0,10σ)	0.8204	0.4612	**0.9007**	0.4588	0.6120	0.4868

Table C54. Estimated power of tests for mixed design under the t distribution with different means and variances; the variance in CRD=8RCBD; K=5; $n_b = 5, n_a = 10$.

(μ_1,σ_1) (μ_2,σ_2) (μ_3,σ_3) $(\mu_4,\sigma_4)(\mu_5,\sigma_5)$	Proposed Tests					
	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
(0,σ)(1,2σ)(1,2σ)(1,2σ)(1,2σ)	0.5285	0.3200	**0.5505**	0.3091	0.4423	0.3542
(0,σ)(0,σ)(0,3σ)(1, 3σ)(1, 3σ)	0.4753	0.2251	**0.5051**	0.2226	0.3739	0.2436
(0,σ)(0,2σ)(1,3σ)(1,4σ)(1,5σ)	0.6990	0.3802	**0.7562**	0.3721	0.5450	0.4113
(0,σ)(0.5,2.5σ)(1,5σ)(1.5,7.5σ)(2,10σ)	0.8599	0.5250	**0.9137**	0.5144	0.6928	0.5601
(0,σ)(0.25,2σ)(0.5,5σ)(0.75,σ)(1,4.5σ)	0.5699	0.2891	**0.6047**	0.2823	0.4523	0.3166
(0,σ)(1,3σ)(1.5,3.5σ)(2,2σ)(2.5,6σ)	0.8036	0.5389	**0.8408**	0.5238	0.6639	0.5849
(0,σ)(1,4σ)(1.5,6σ)(1.75,8σ)(2.5,10σ)	0.9036	0.6095	**0.9483**	0.5986	0.7562	0.6487

APPENDIX D. COMPARING THE POWERS OF PROPOSED TESTS FOR SYMMETRIC DISTRIBUTION (NORMAL DISTRIBUTION)

D.1. Increasing the Number of Blocks

K=3

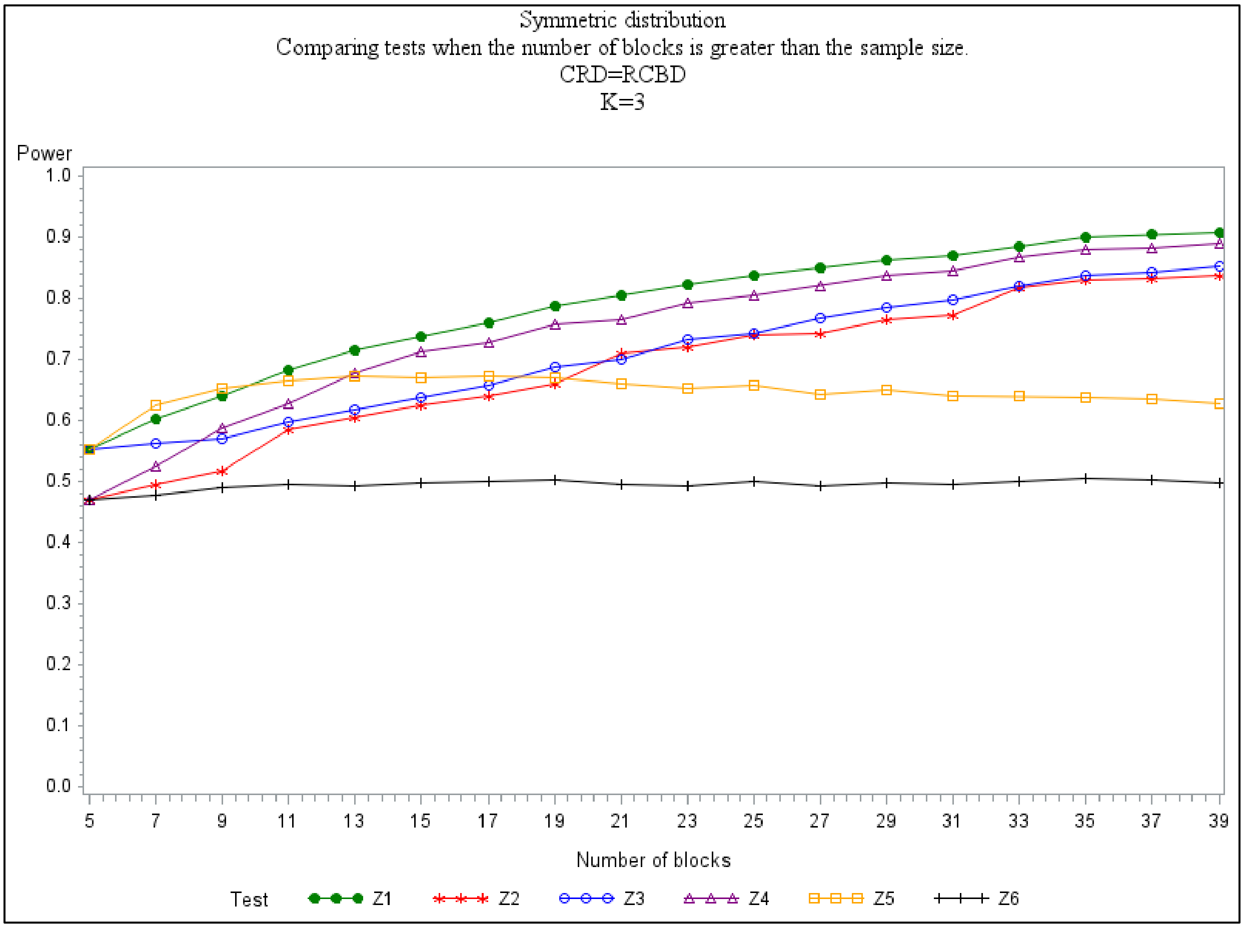

Figure D1. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=RCBD; K=3; n_a=5, and n_b=5,7,9,…,39.

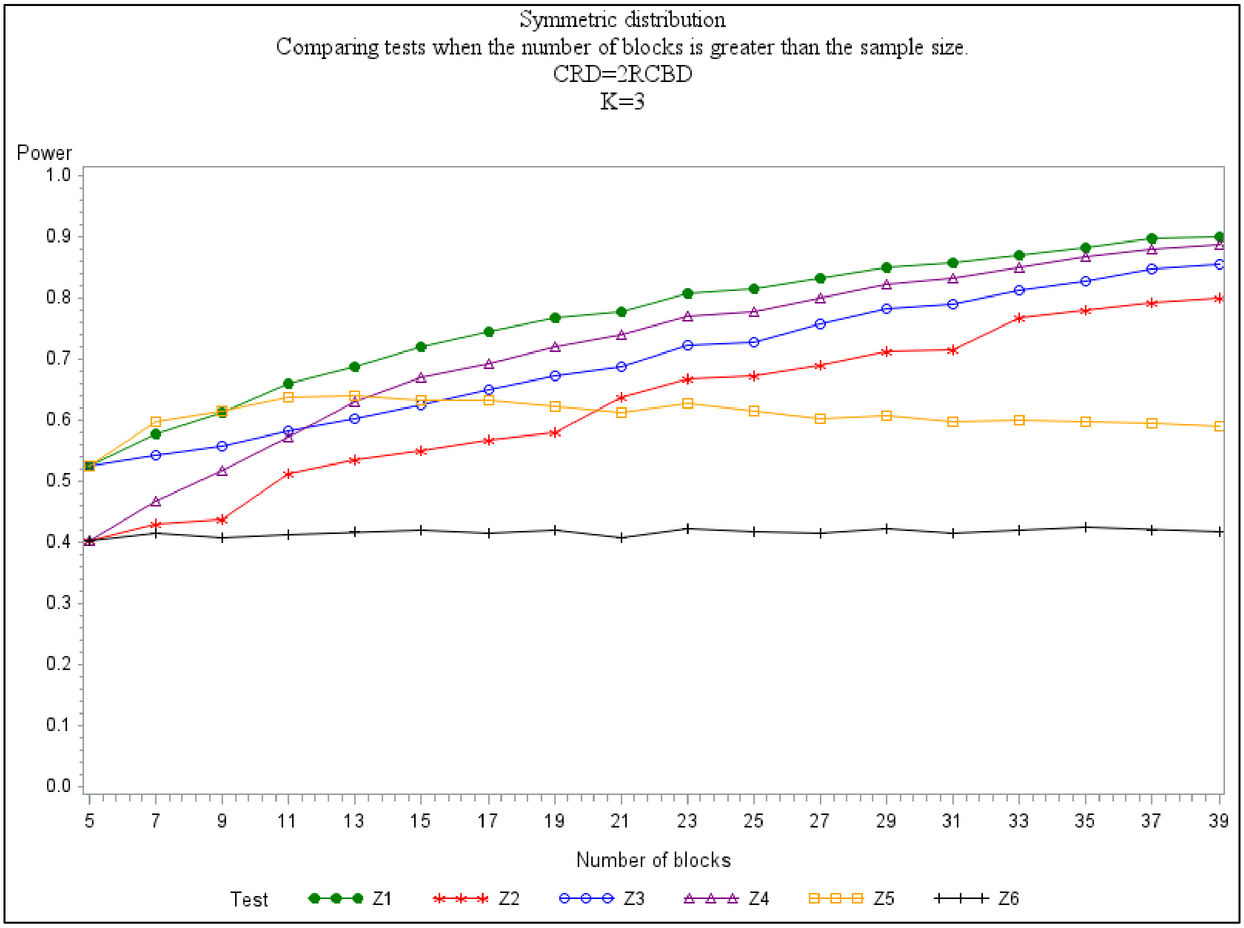

Figure D2. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=2RCBD; K=3; n_a=5, and n_b=5,7,9,…,39.

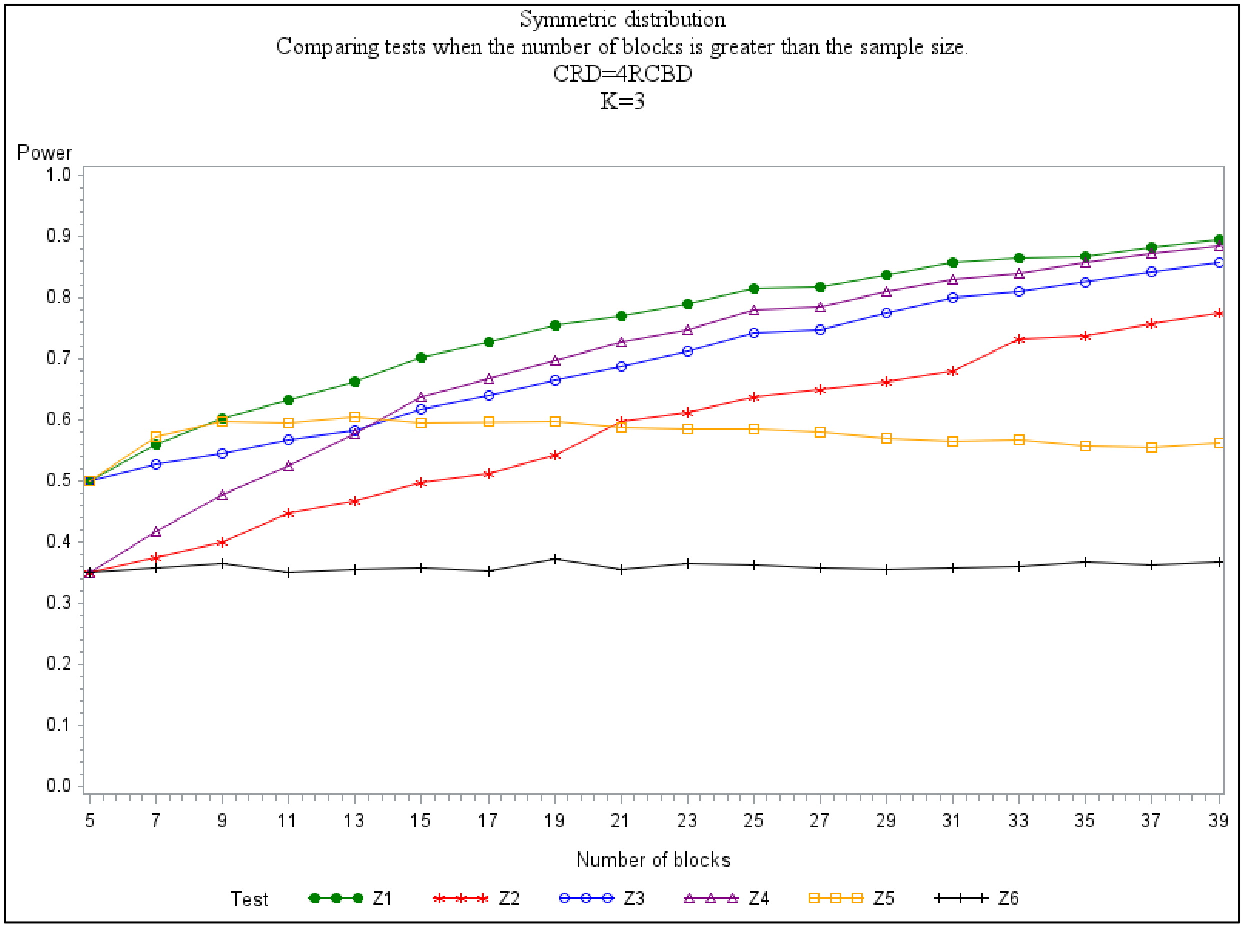

Figure D3. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=4RCBD; K=3; n_a=5, and n_b=5,7,9,…,39.

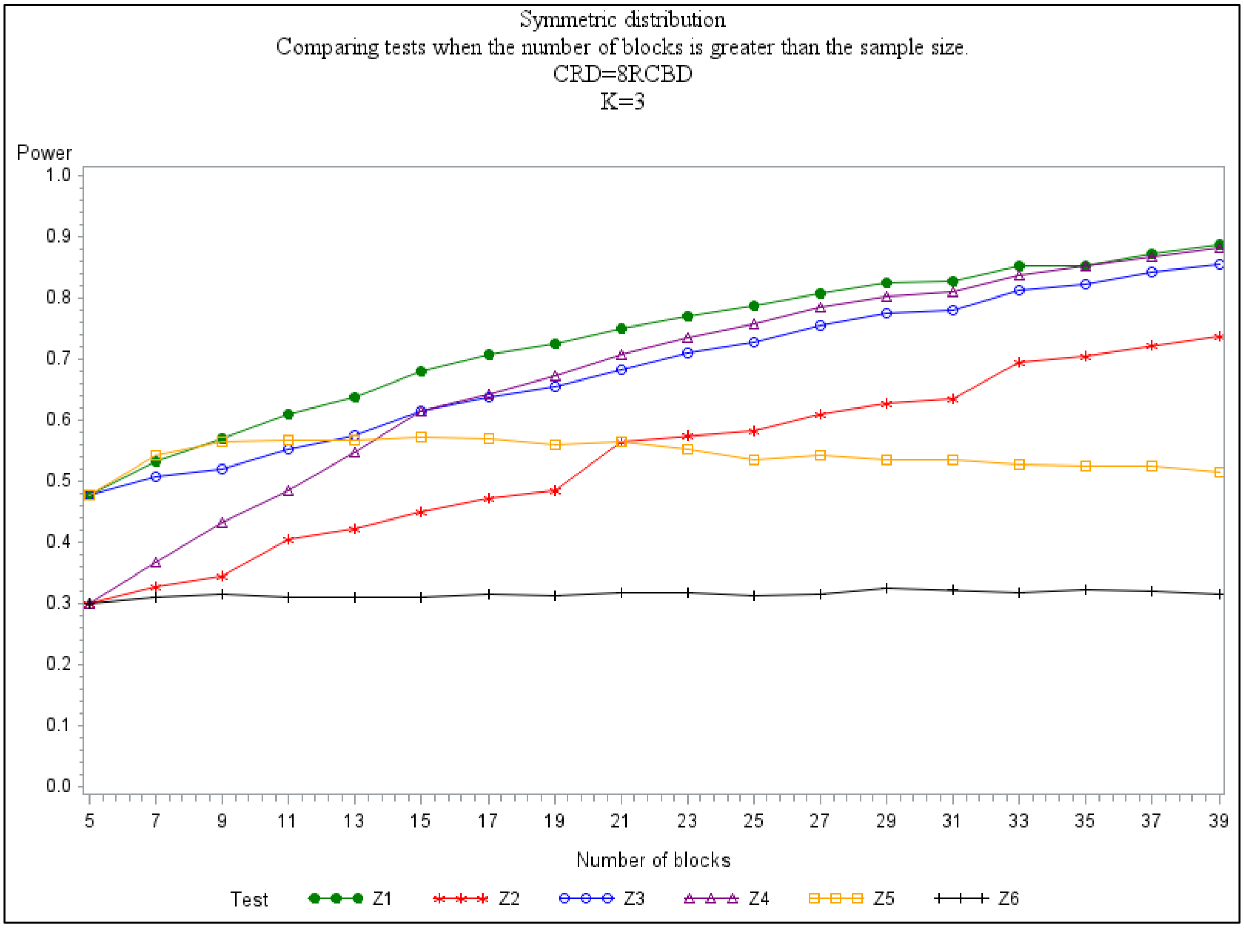

Figure D4. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=4RCBD; K=3; n_a=5, and n_b=5,7,9,…,39.

K=4

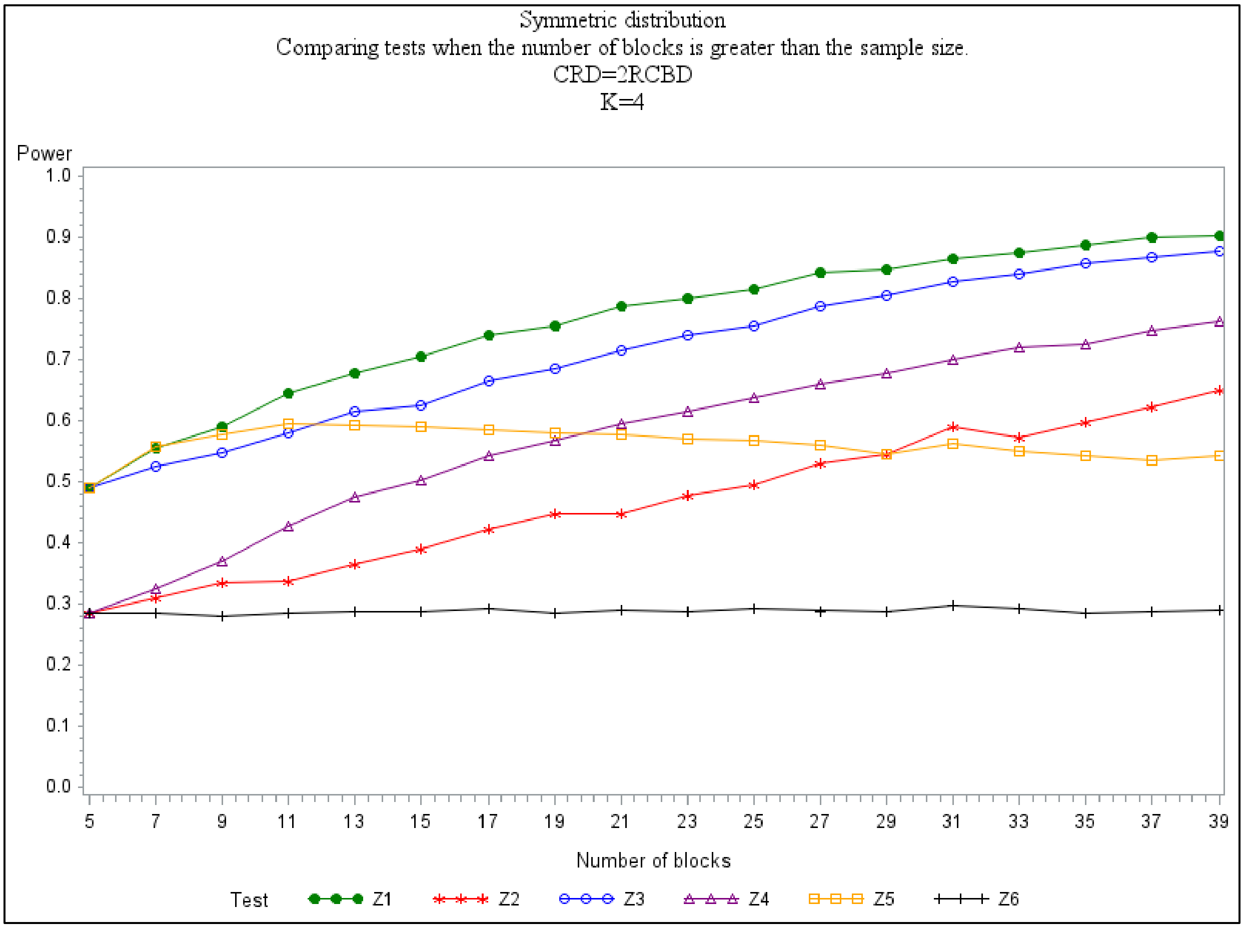

Figure D5. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=2RCBD; K=4; n_a=5, and n_b=5,7,9,…,39.

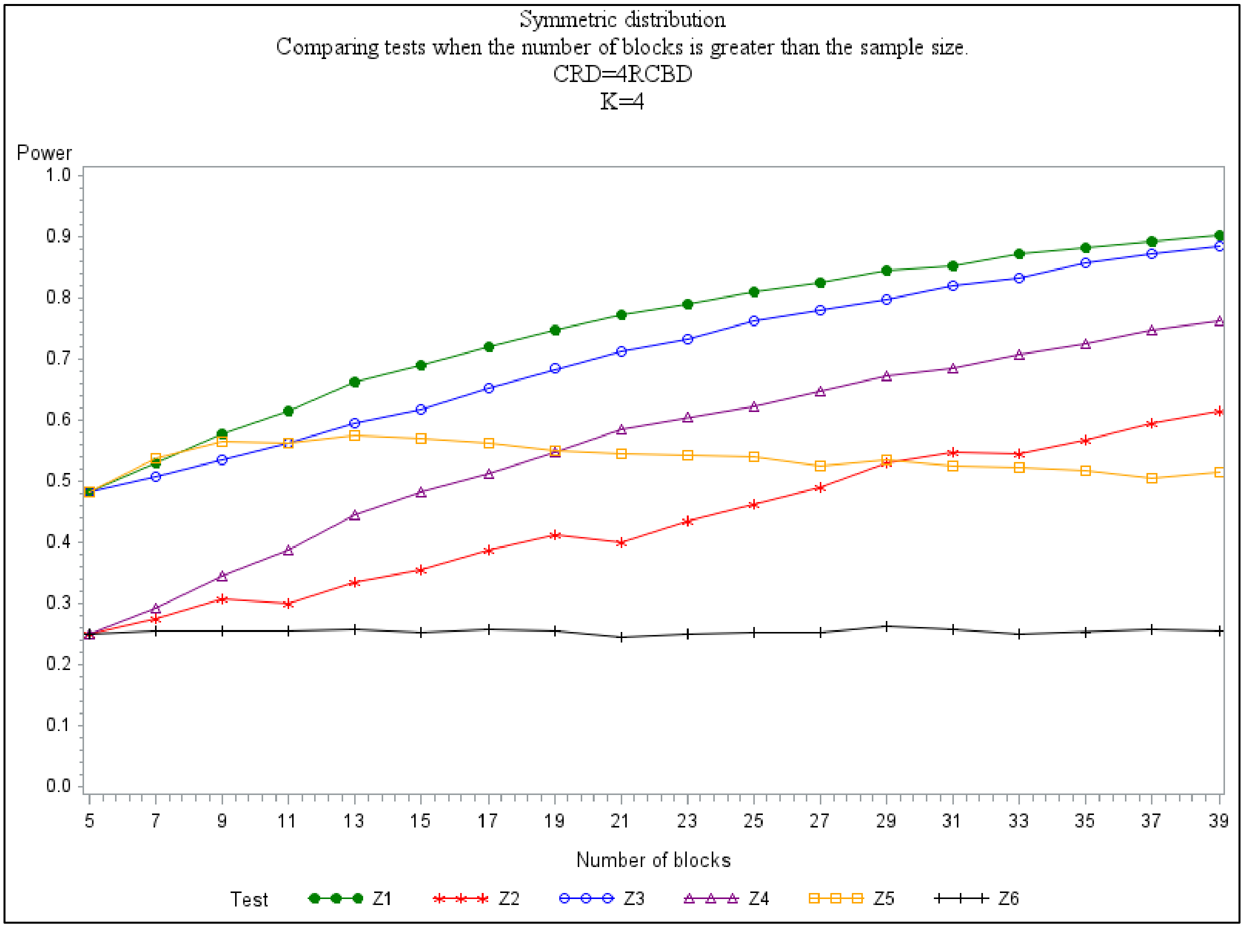

Figure D6. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=4RCBD; K=4; n_a=5, and n_b=5,7,9,…,39.

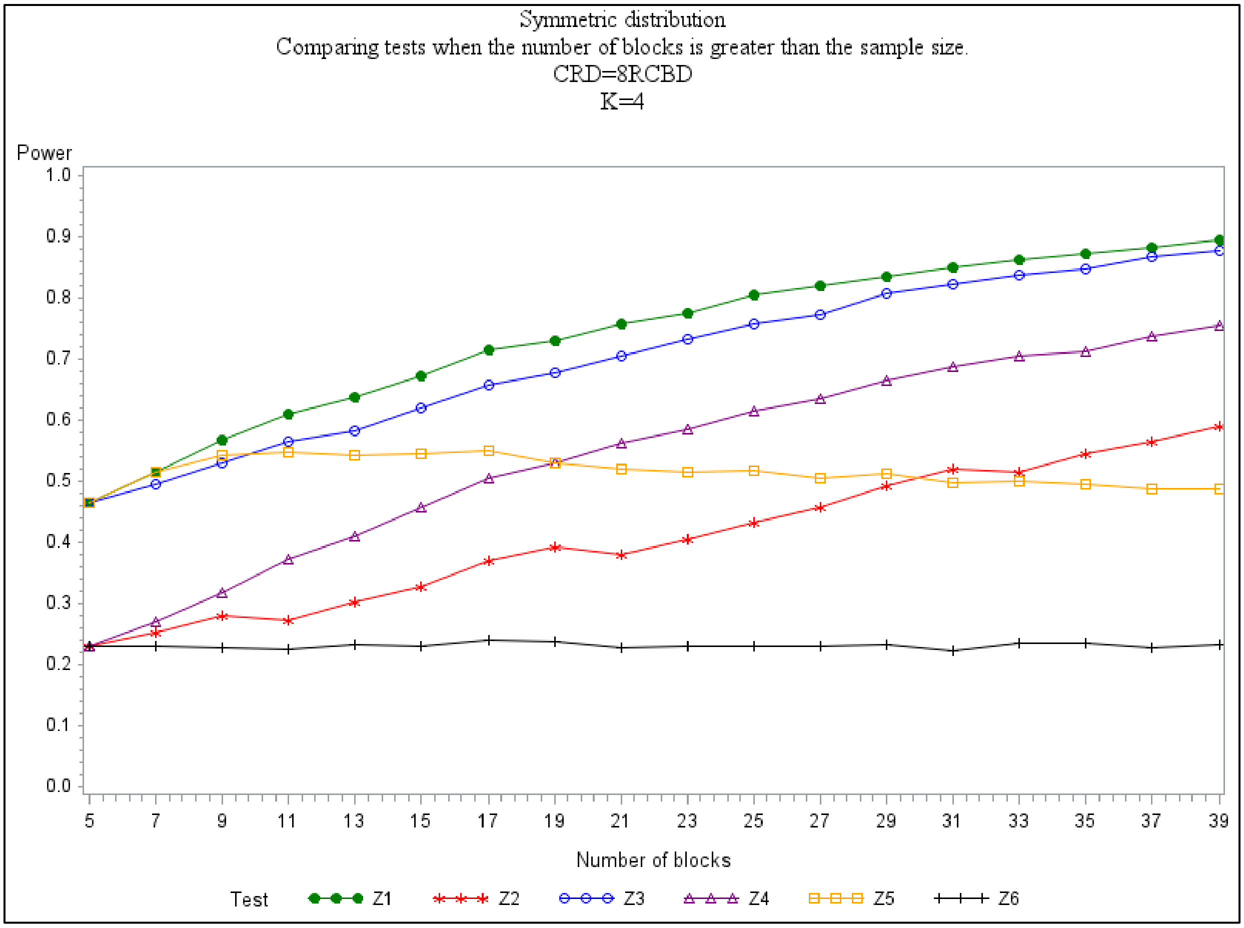

Figure D7. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=8RCBD; K=4; n_a=5, and n_b=5,7,9,…,39.

K=5

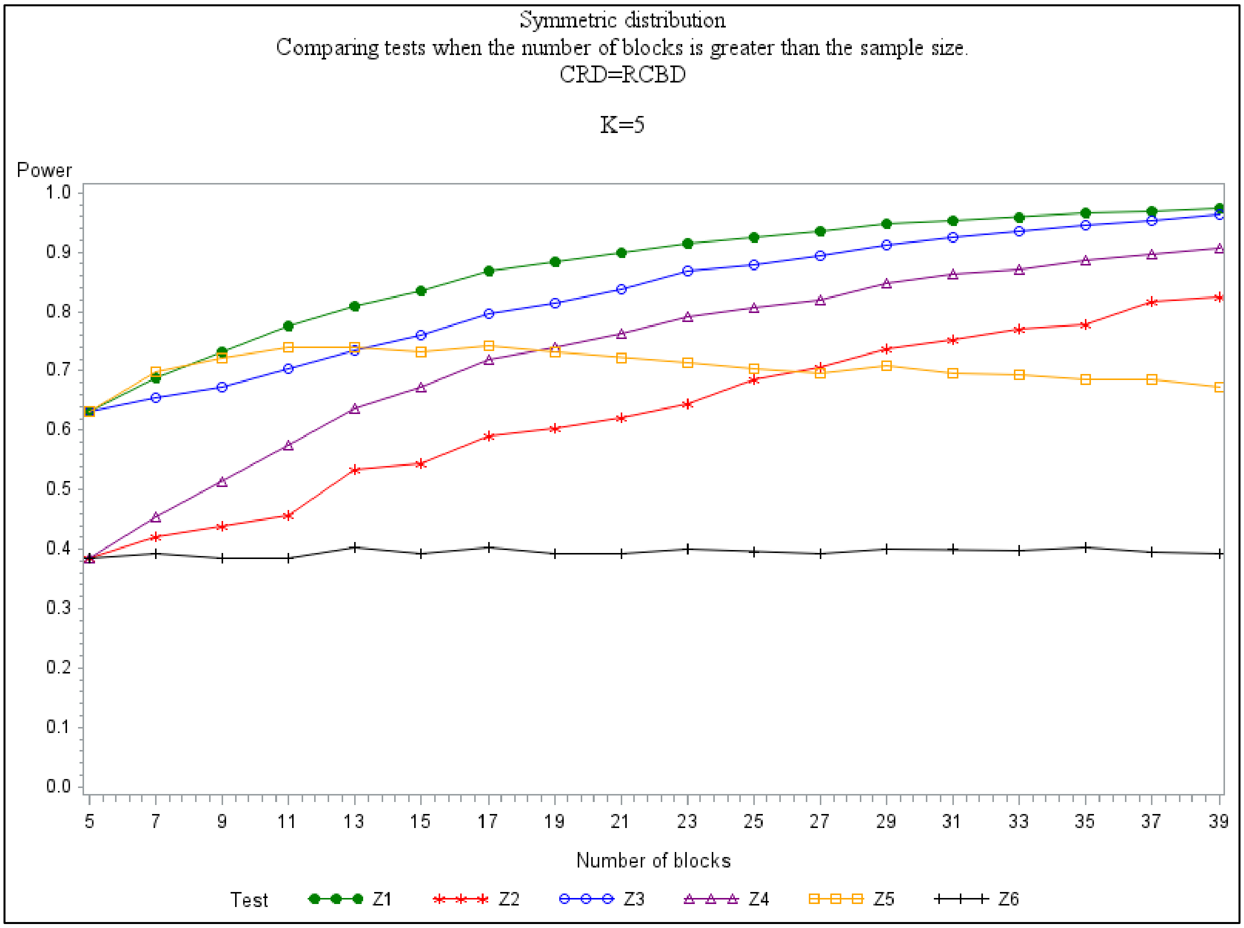

Figure D8. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=RCBD; K=5; n_a=5, and n_b=5,7,9,…,39.

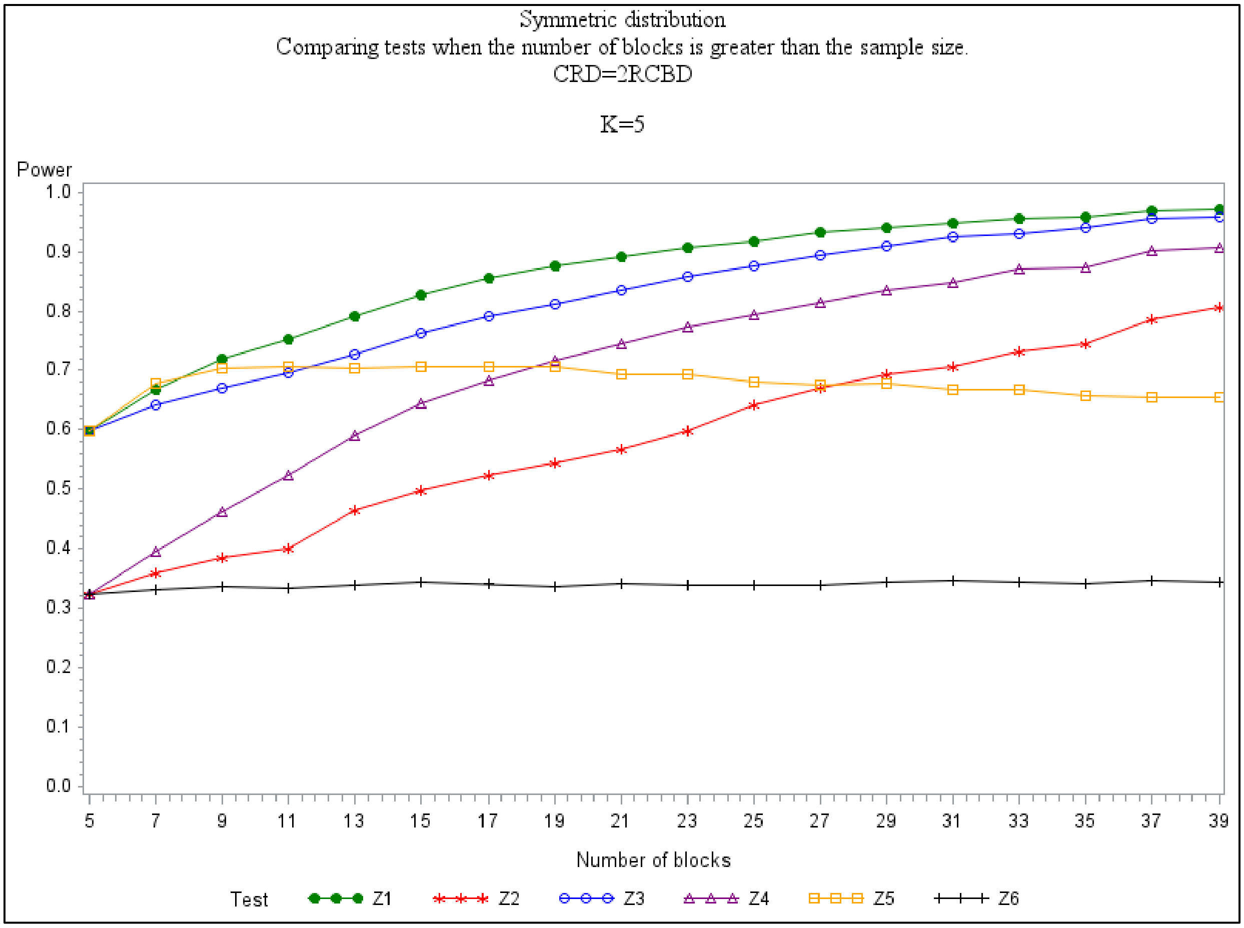

Figure D9. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=2RCBD; K=5; n_a=5, and n_b=5,7,9,…,39.

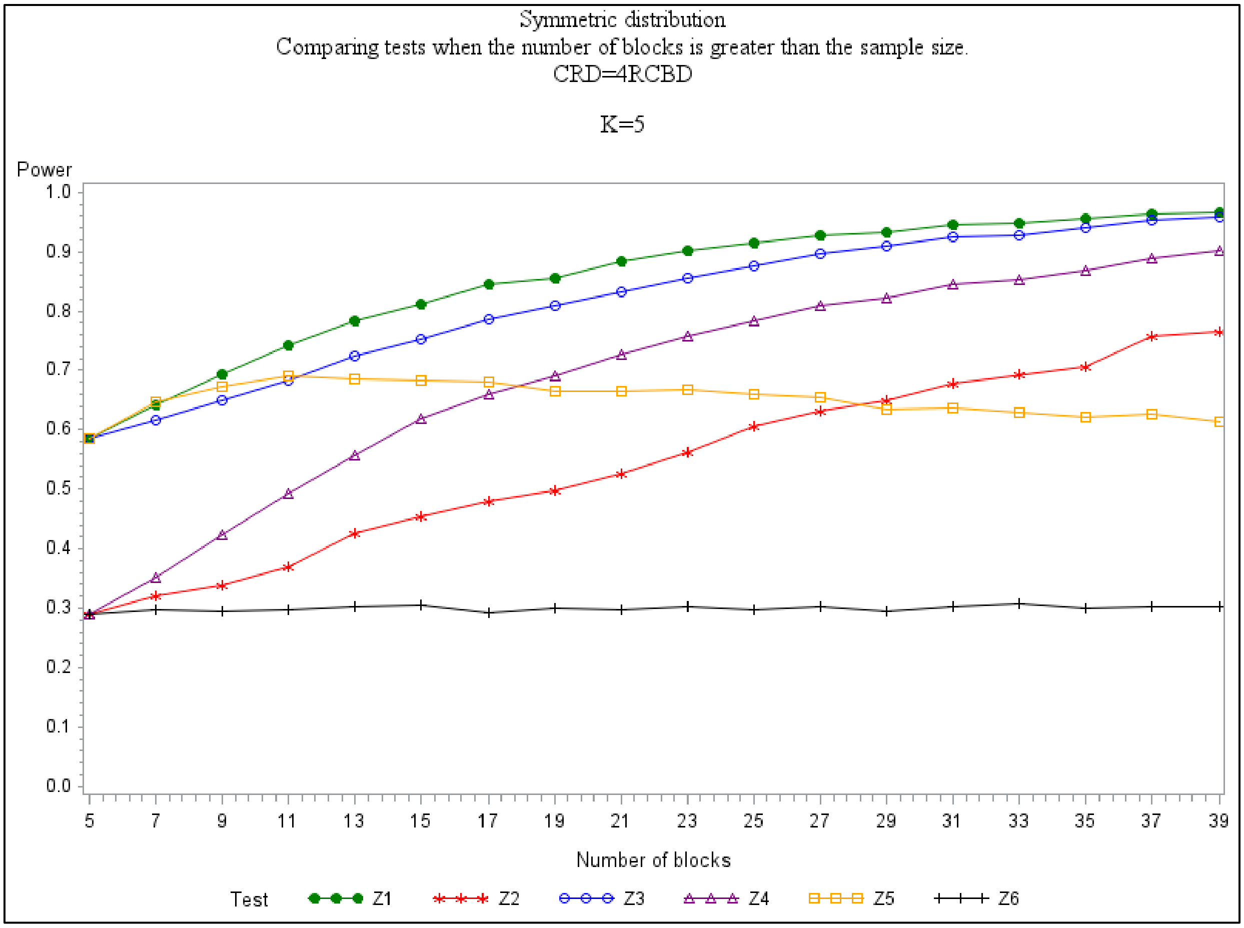

Figure D10. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=4RCBD; K=5; n_a=5, and n_b=5,7,9,…,39.

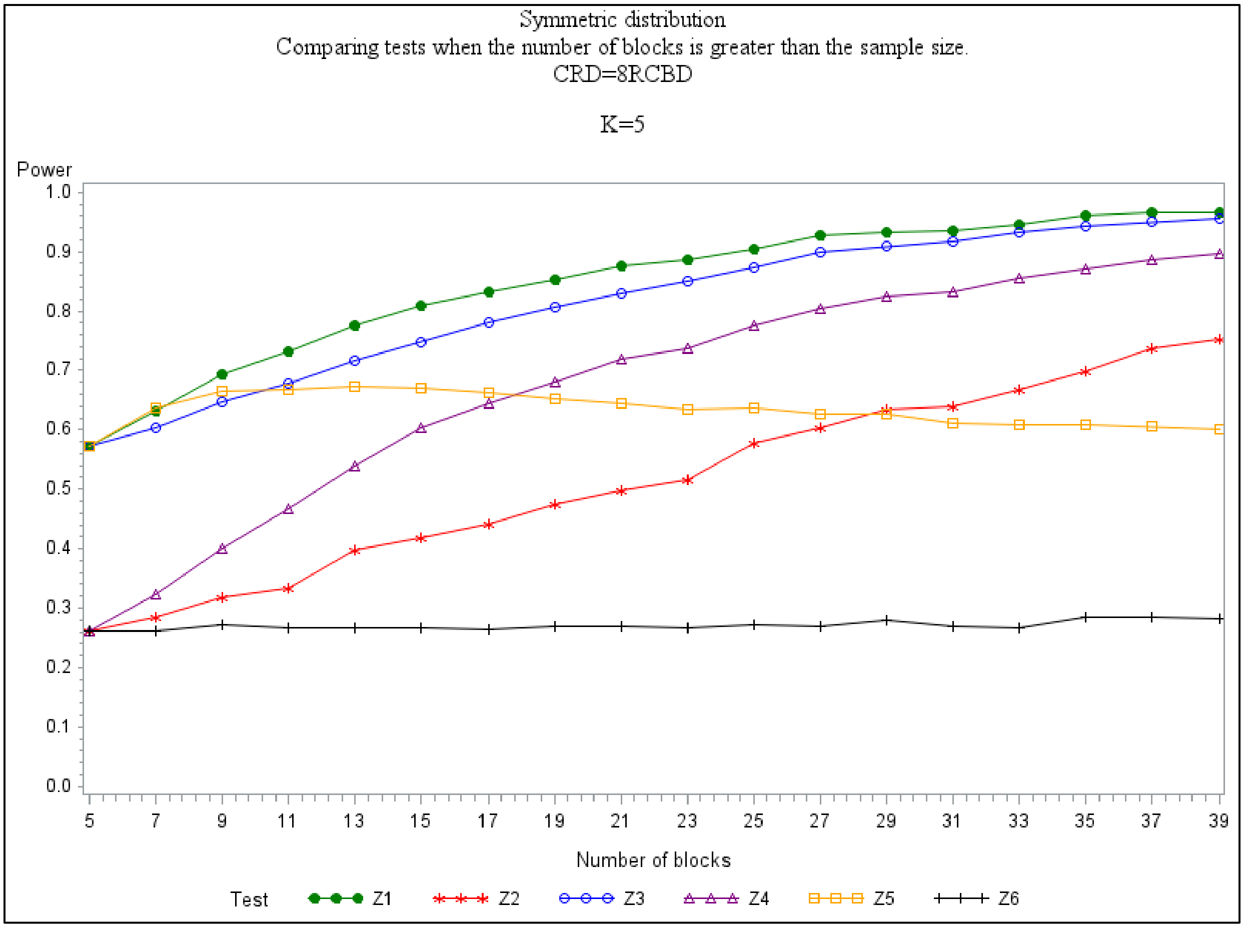

Figure D11. Estimated powers for proposed tests when the populations have different location and scale parameters for the normal distribution; CRD=8RCBD; K=5; n_a=5, and n_b=5,7,9,…,39.

D.2. Increasing the Sample Sizes

K=3

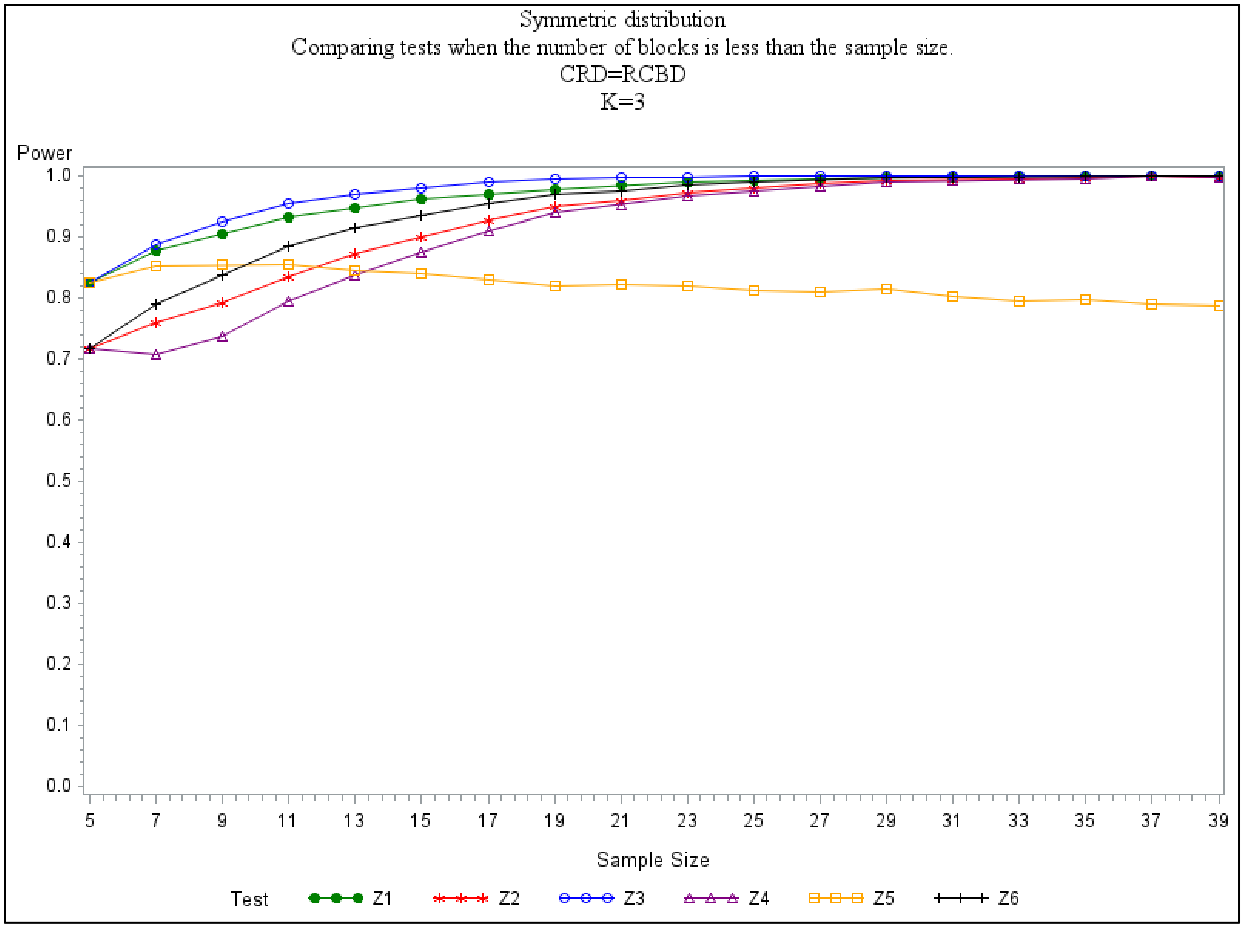

Figure D12. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

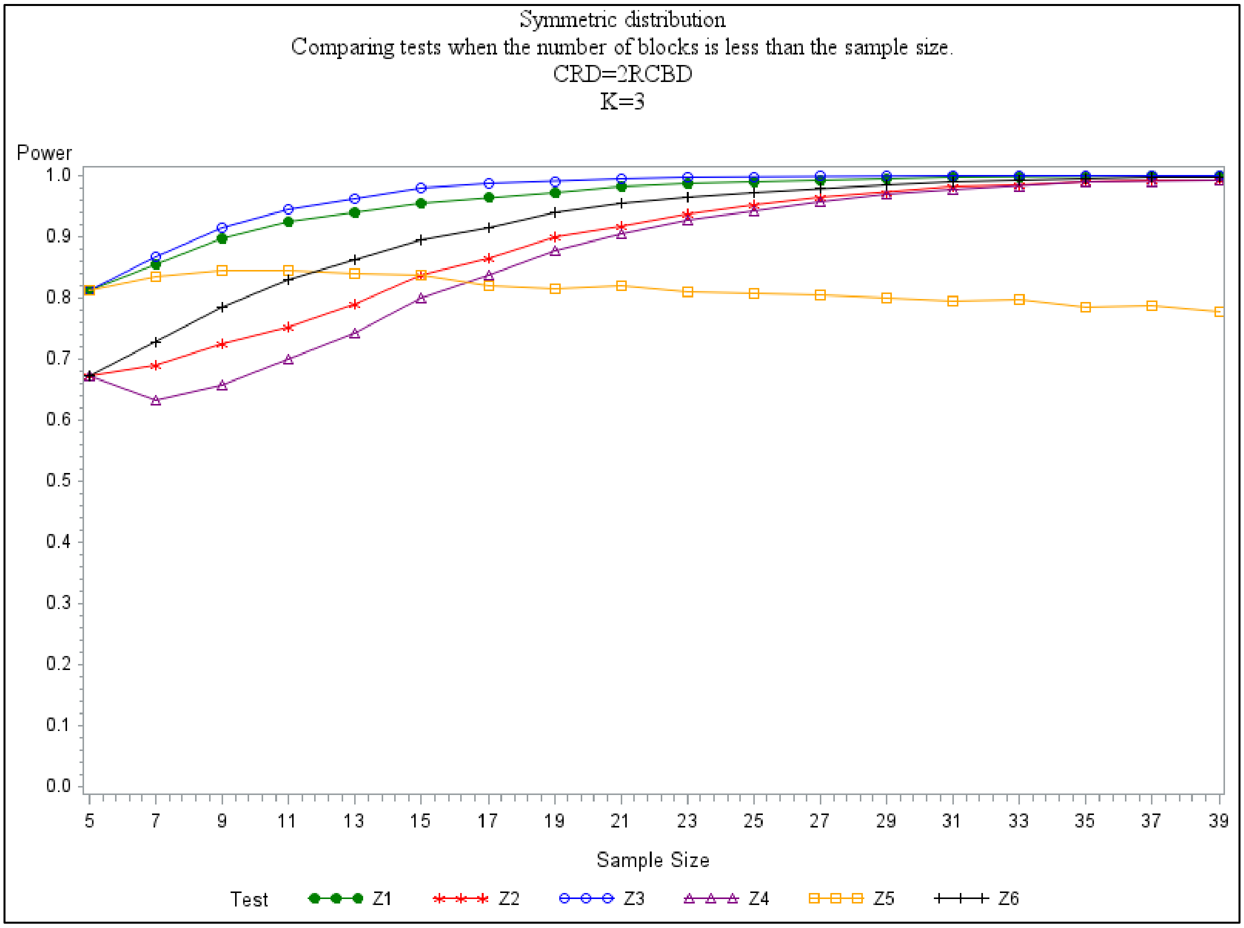

Figure D13. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=2RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

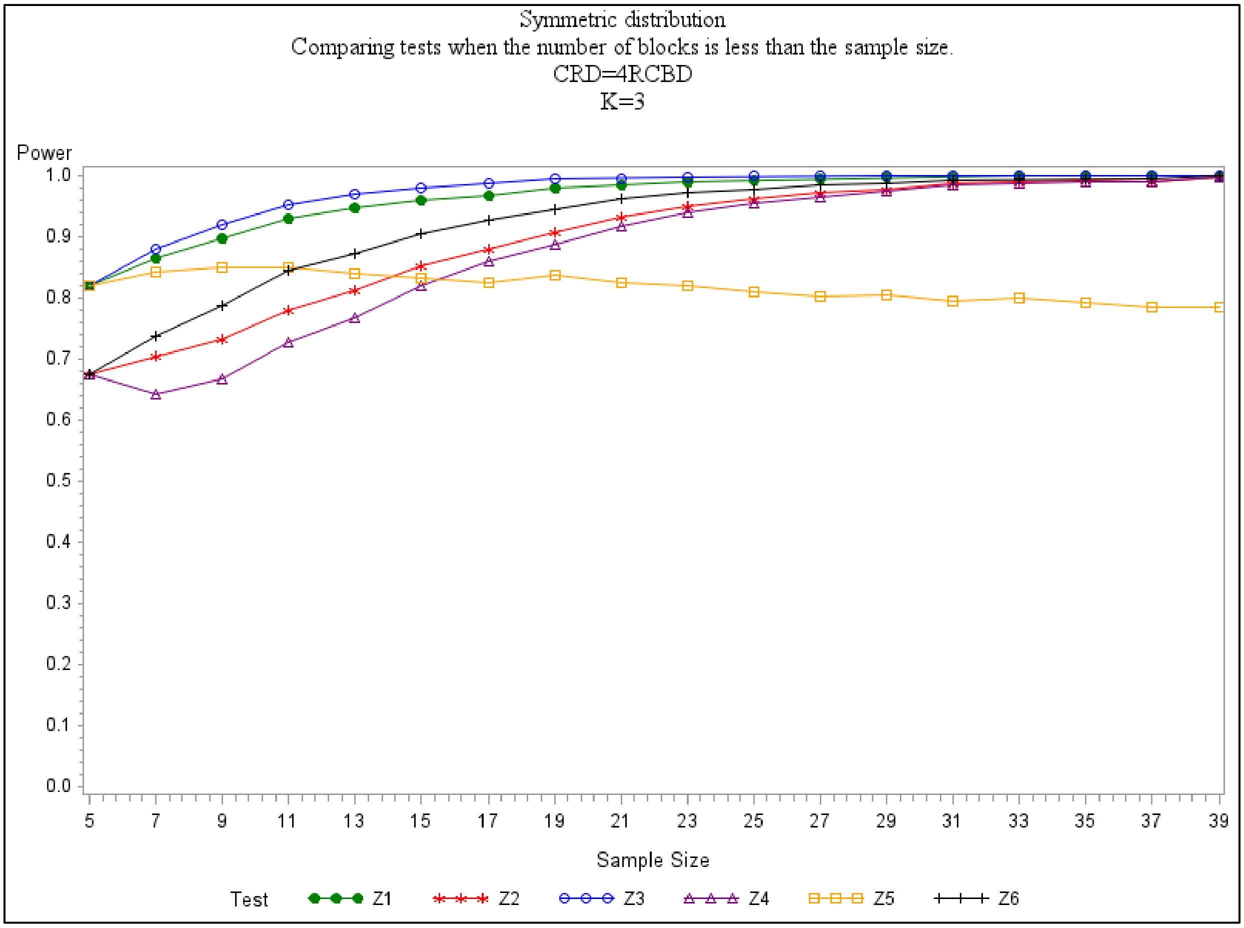

Figure D14. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=4RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

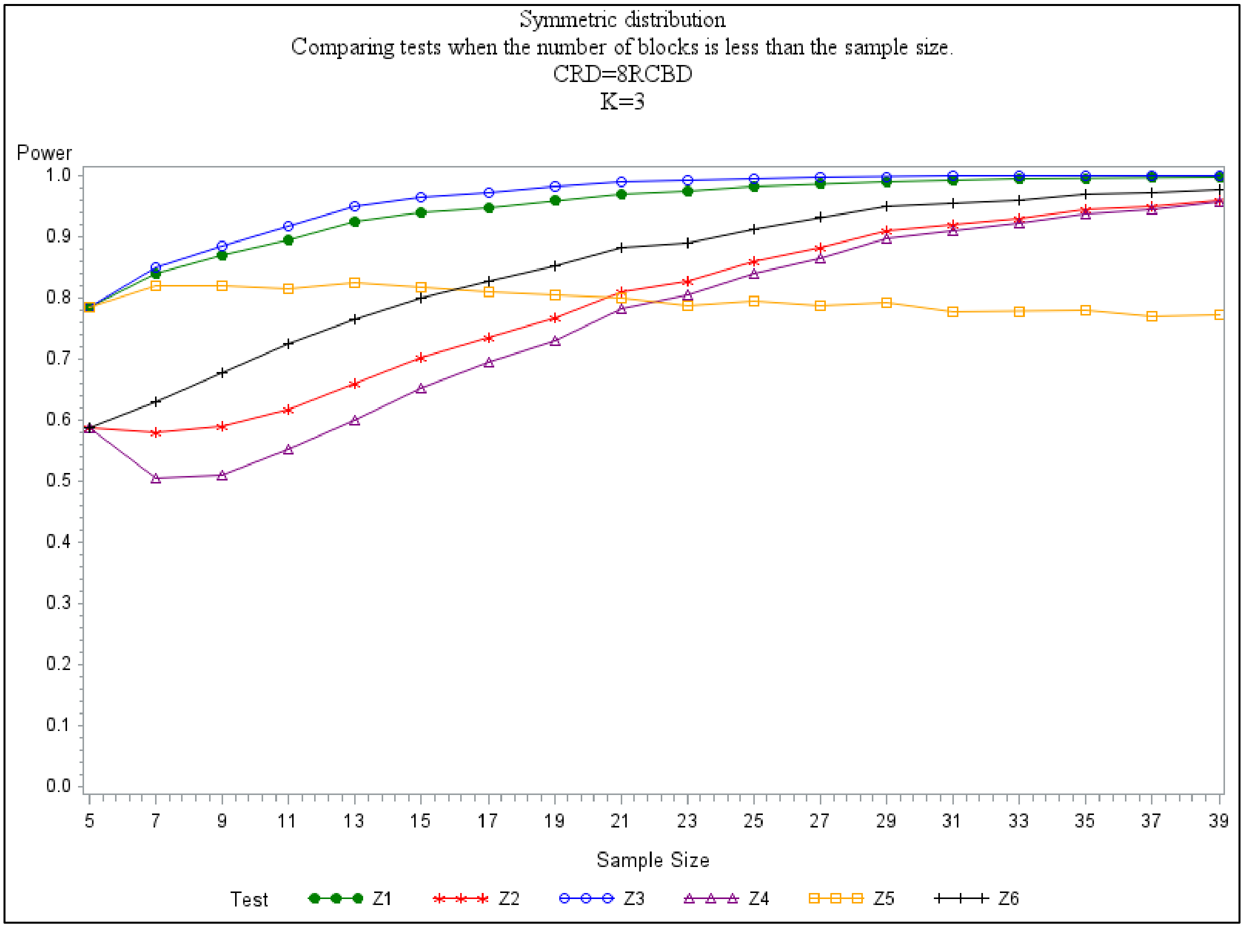

Figure D15. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=8RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

K=4

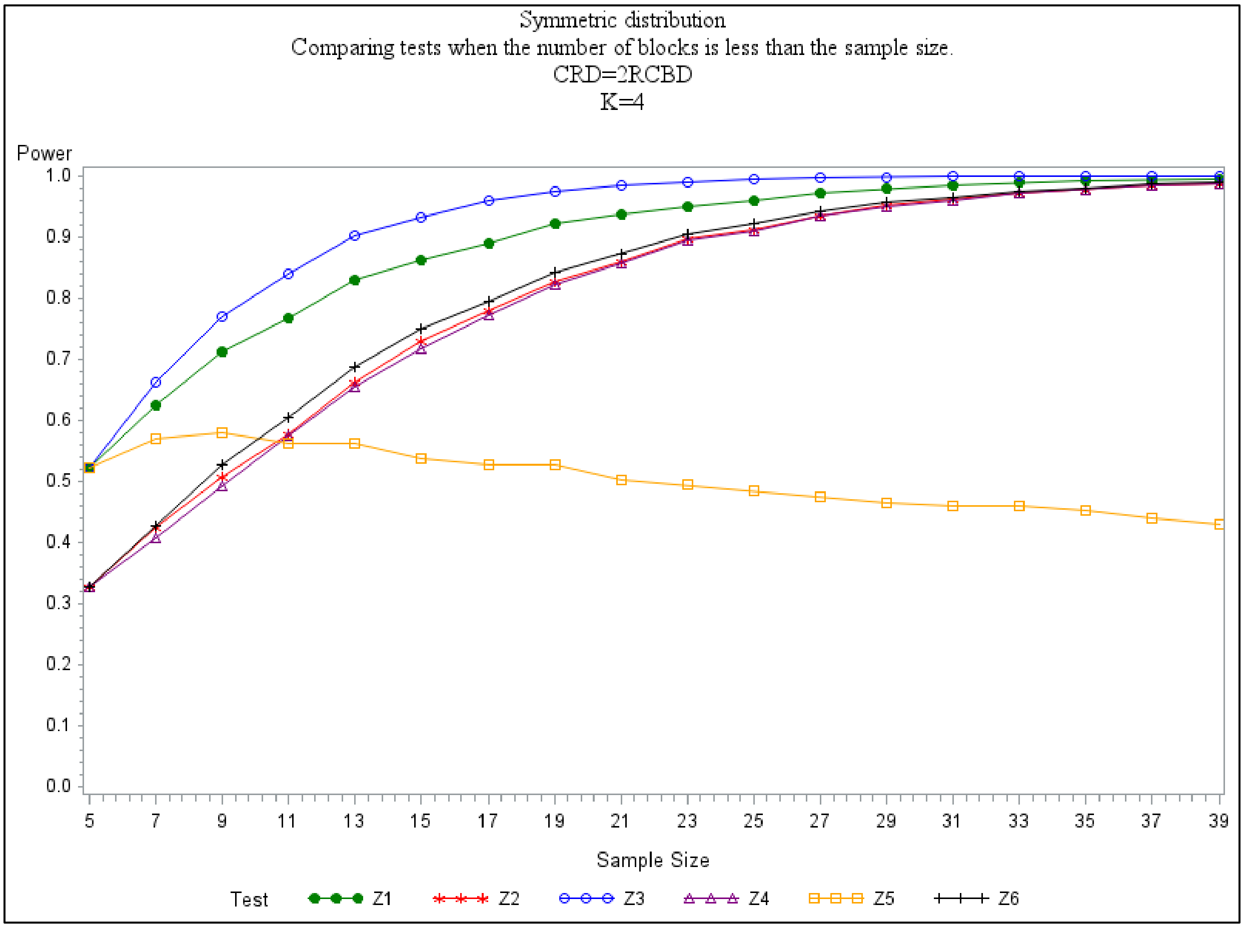

Figure D16. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=2RCBD; K=4; n_a=5,7,9,…,39, and n_b=5.

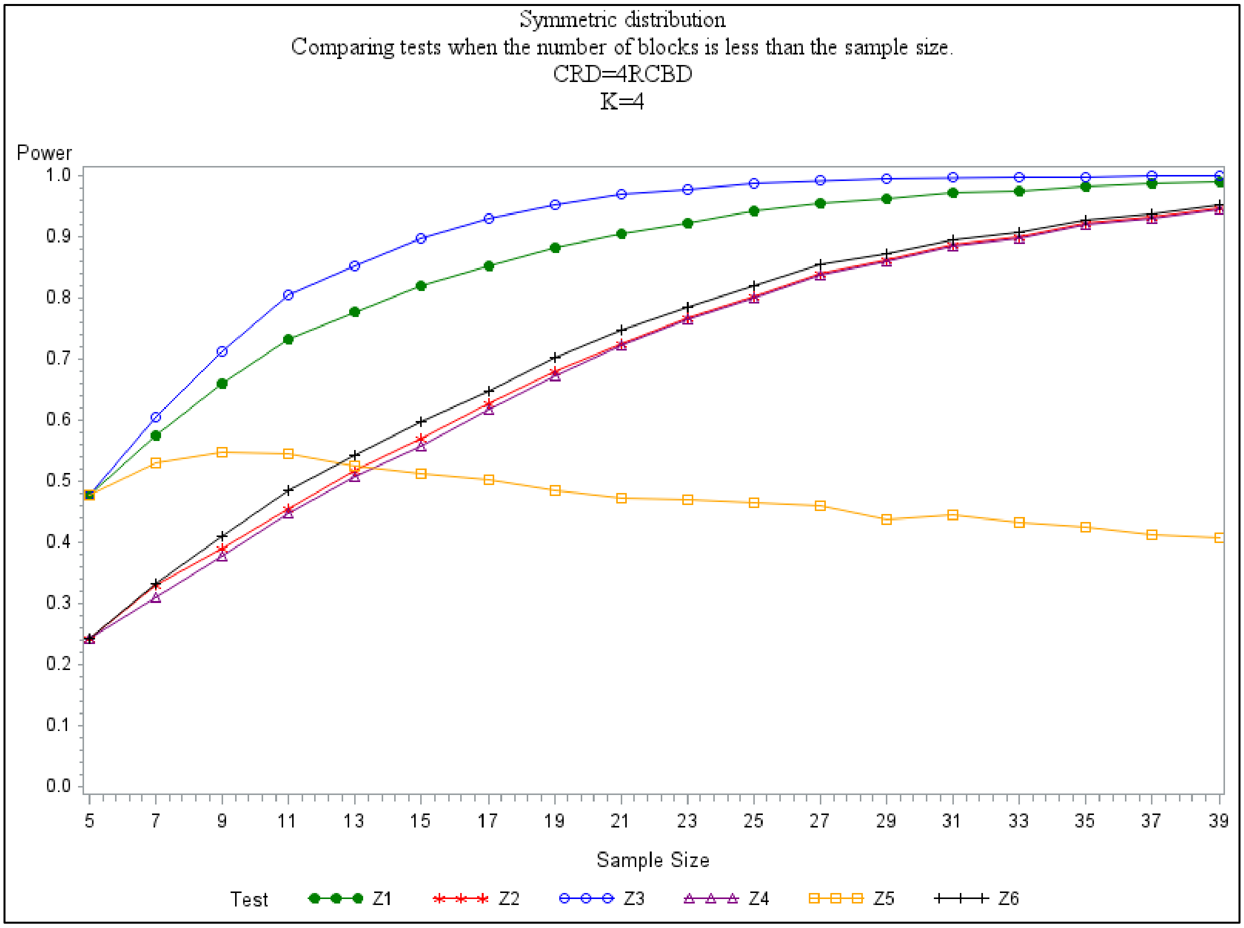

Figure D17. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=4RCBD; K=4; n_a=5,7,9,…,39, and n_b=5.

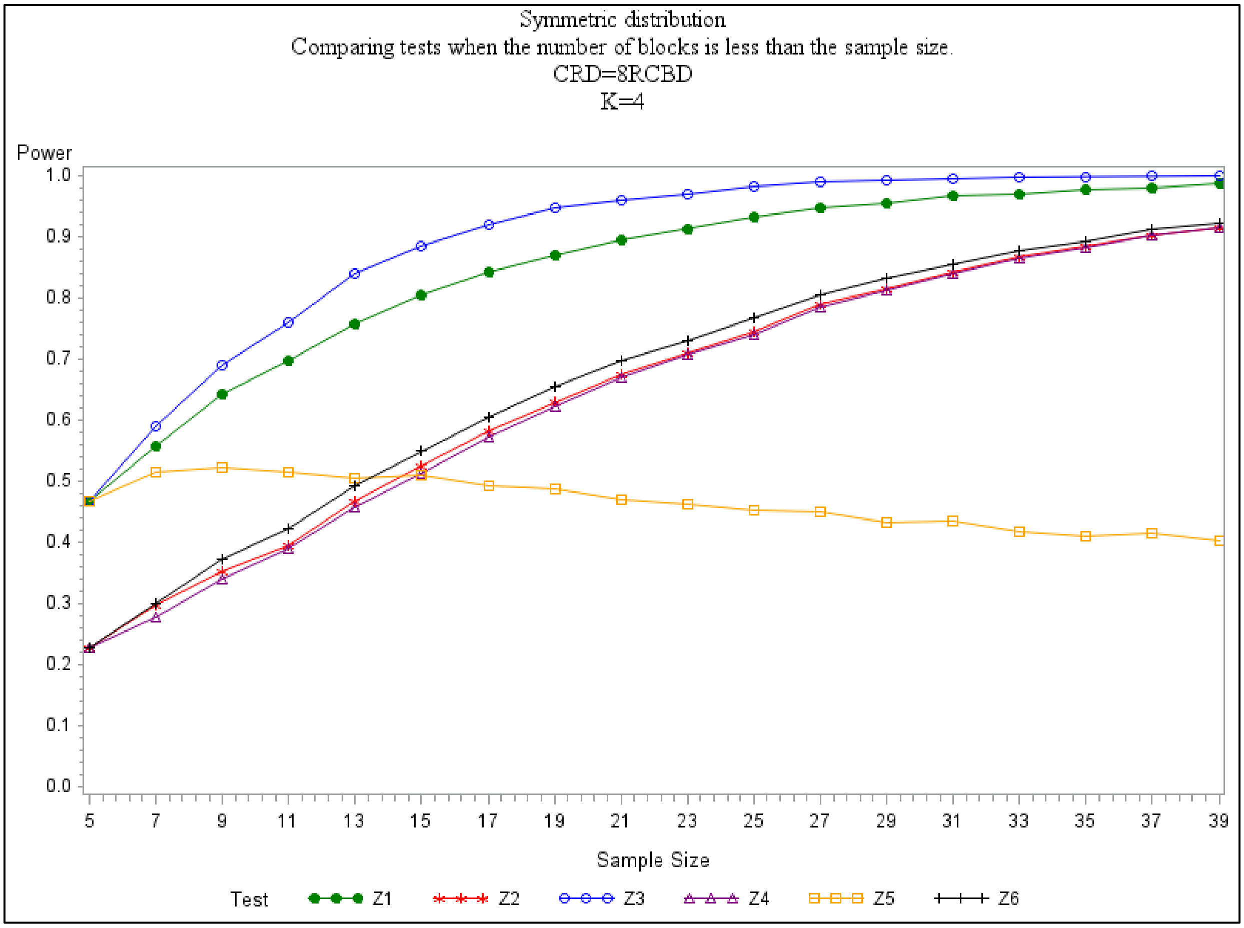

Figure D18. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=8RCBD; K=4; n_a=5,7,9,…,39, and n_b=5.

K=5

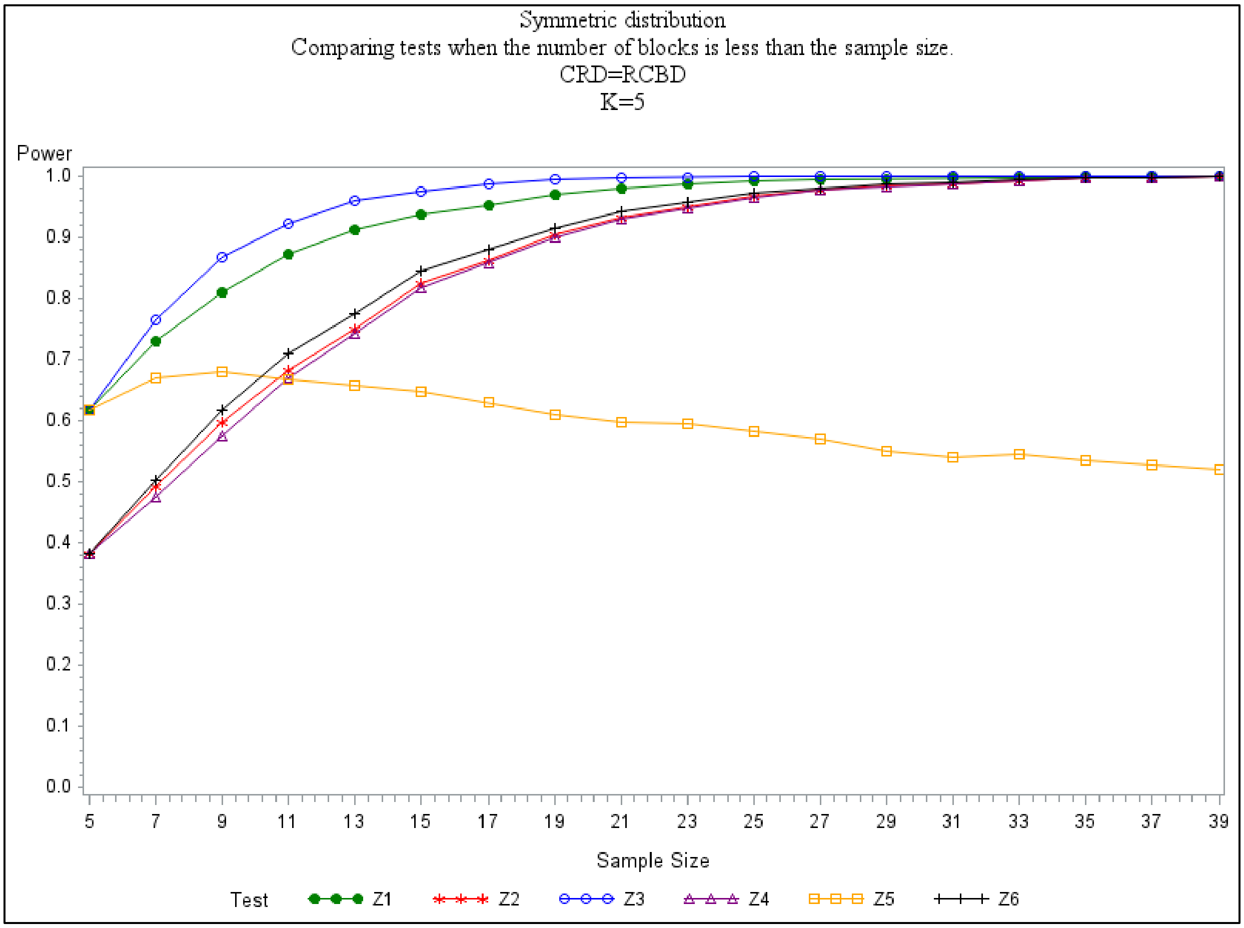

Figure D19. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=RCBD; K=5; n_a=5,7,9,…,39, and n_b=5.

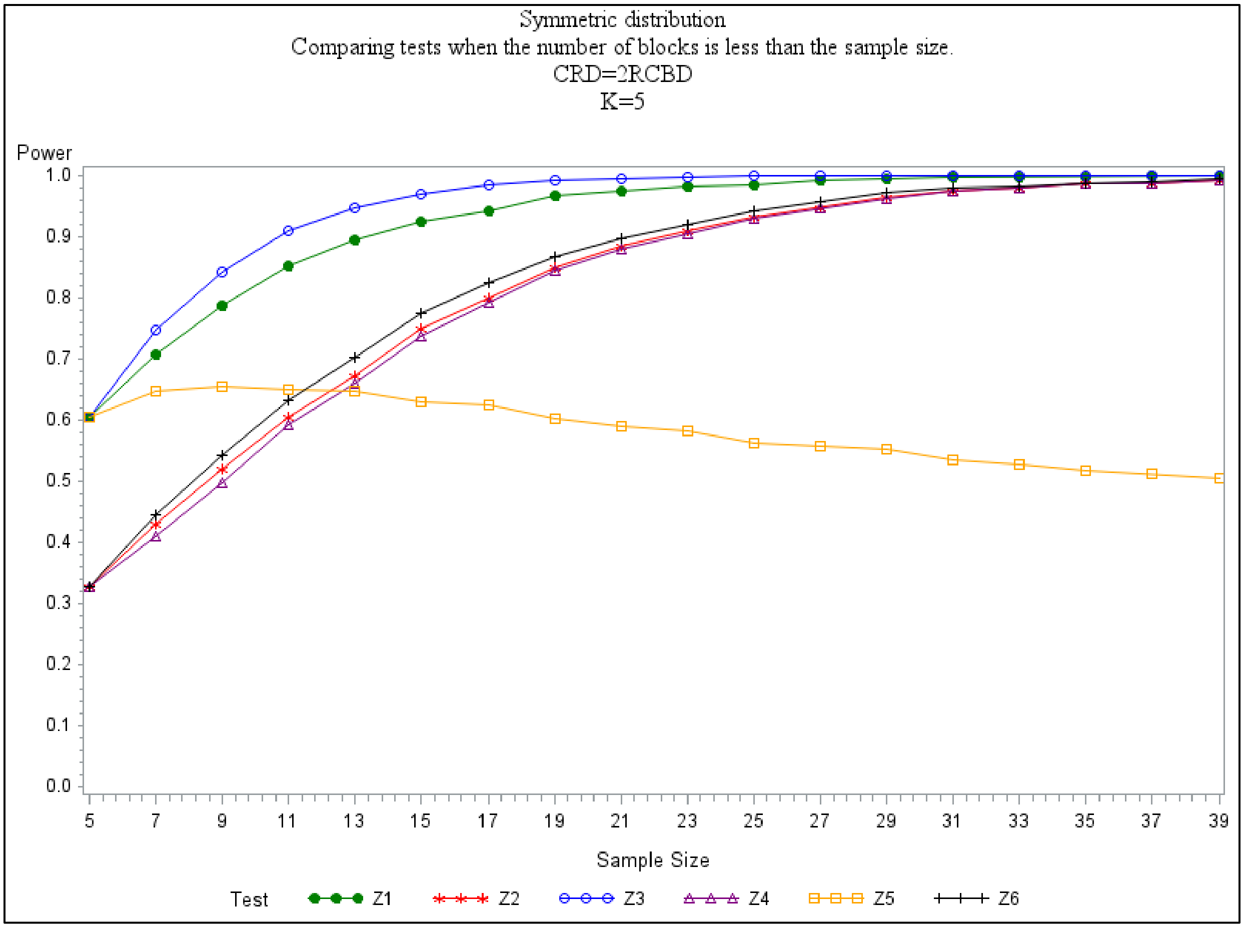

Figure D20. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=2RCBD; K=5; n_a=5,7,9,…,39, and n_b=5.

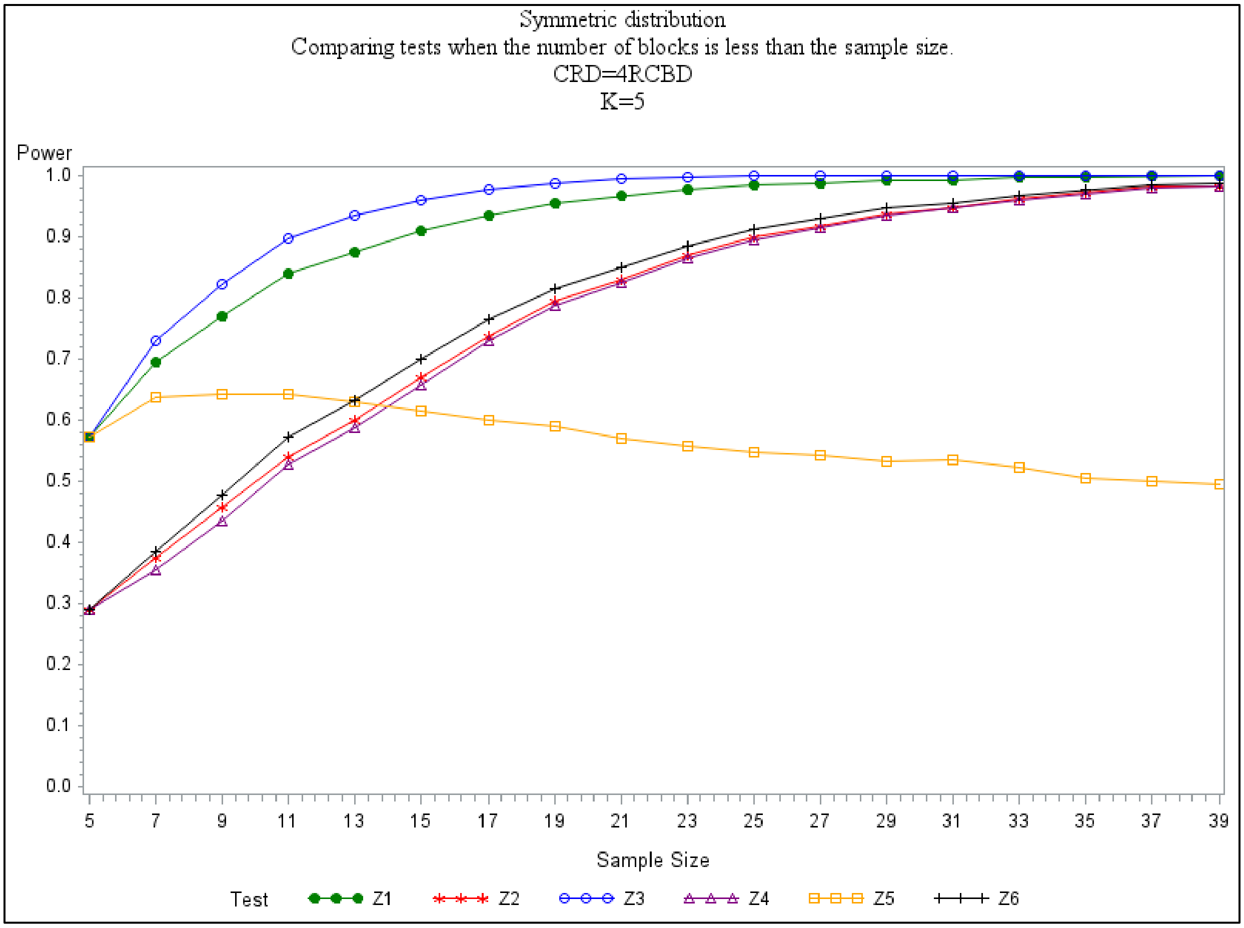

Figure D21. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=4RCBD; K=5; n_a=5,7,9,…,39, and n_b=5.

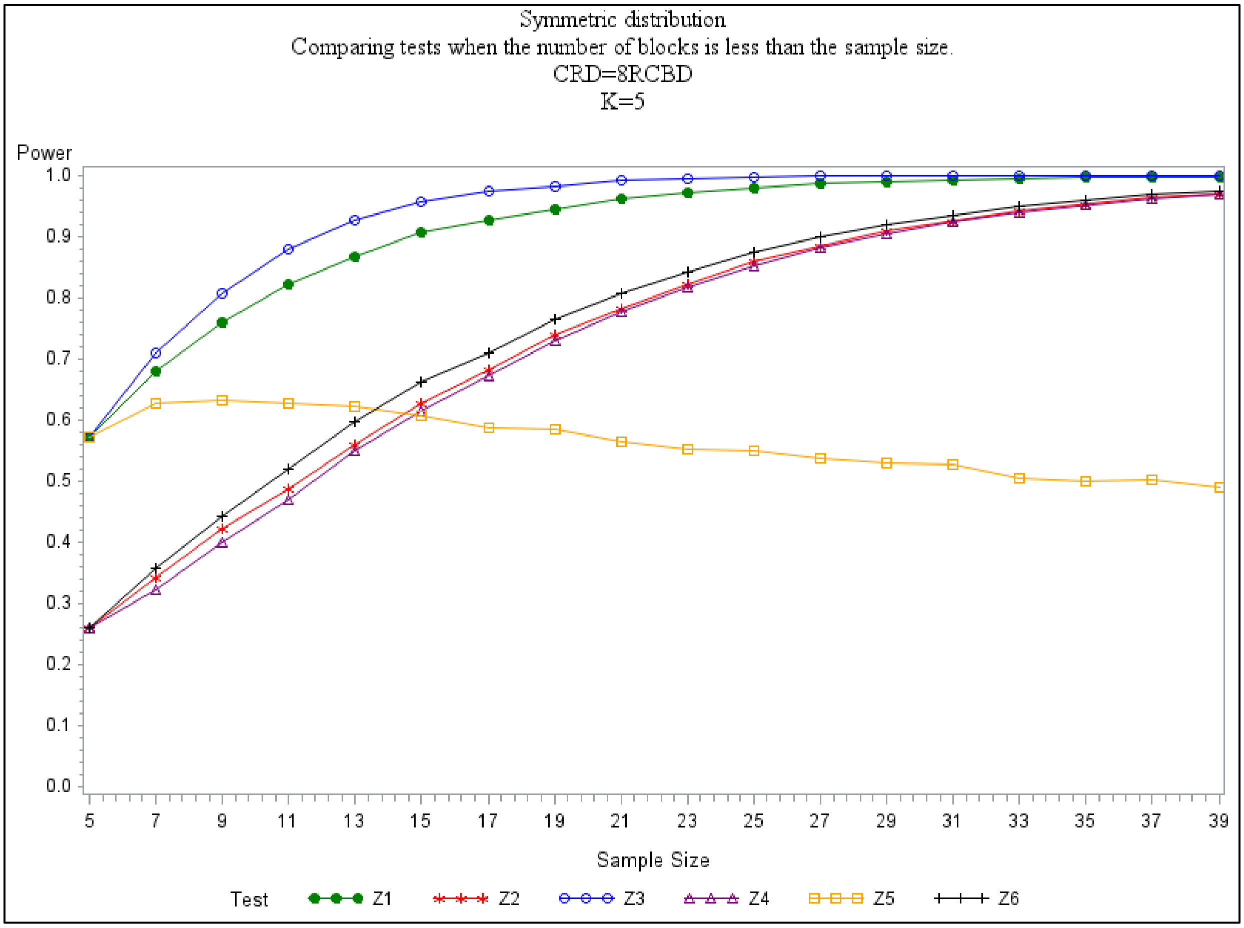

Figure D22. Estimated powers for proposed tests when the populations have different location and scale parameters for normal distribution; CRD=8RCBD; K=5; n_a=5,7,9,…,39, and n_b=5.

APPENDIX E. COMPARING THE POWERS OF PROPOSED TESTS FOR ASYMMETRIC DISTRIBUTION (EXPONENTIAL DISTRIBUTION)

E.1. Increasing the Number of Blocks

K=3

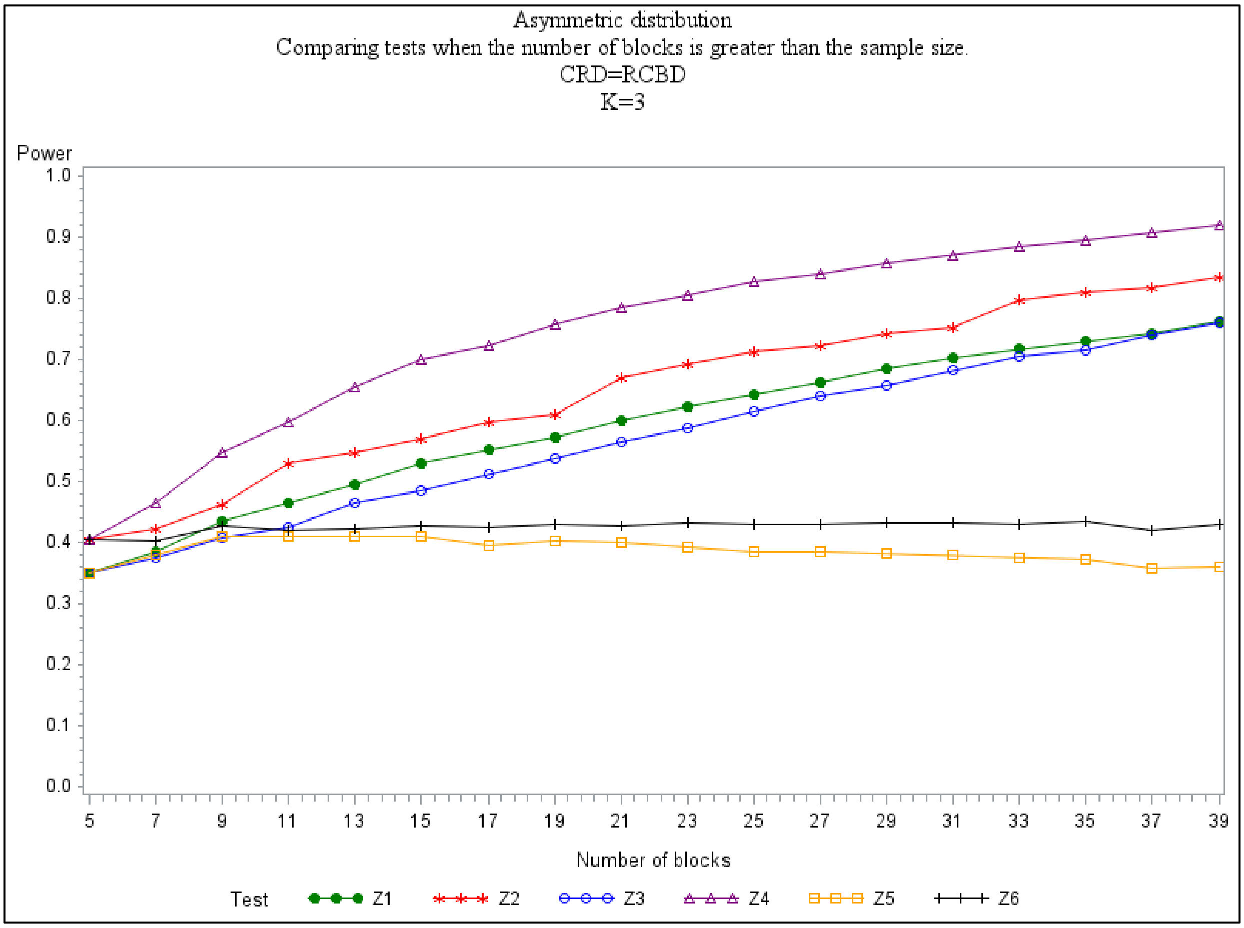

Figure E1. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=RCBD; K=3; n_a=5, and n_b=5,7,9,…,39.

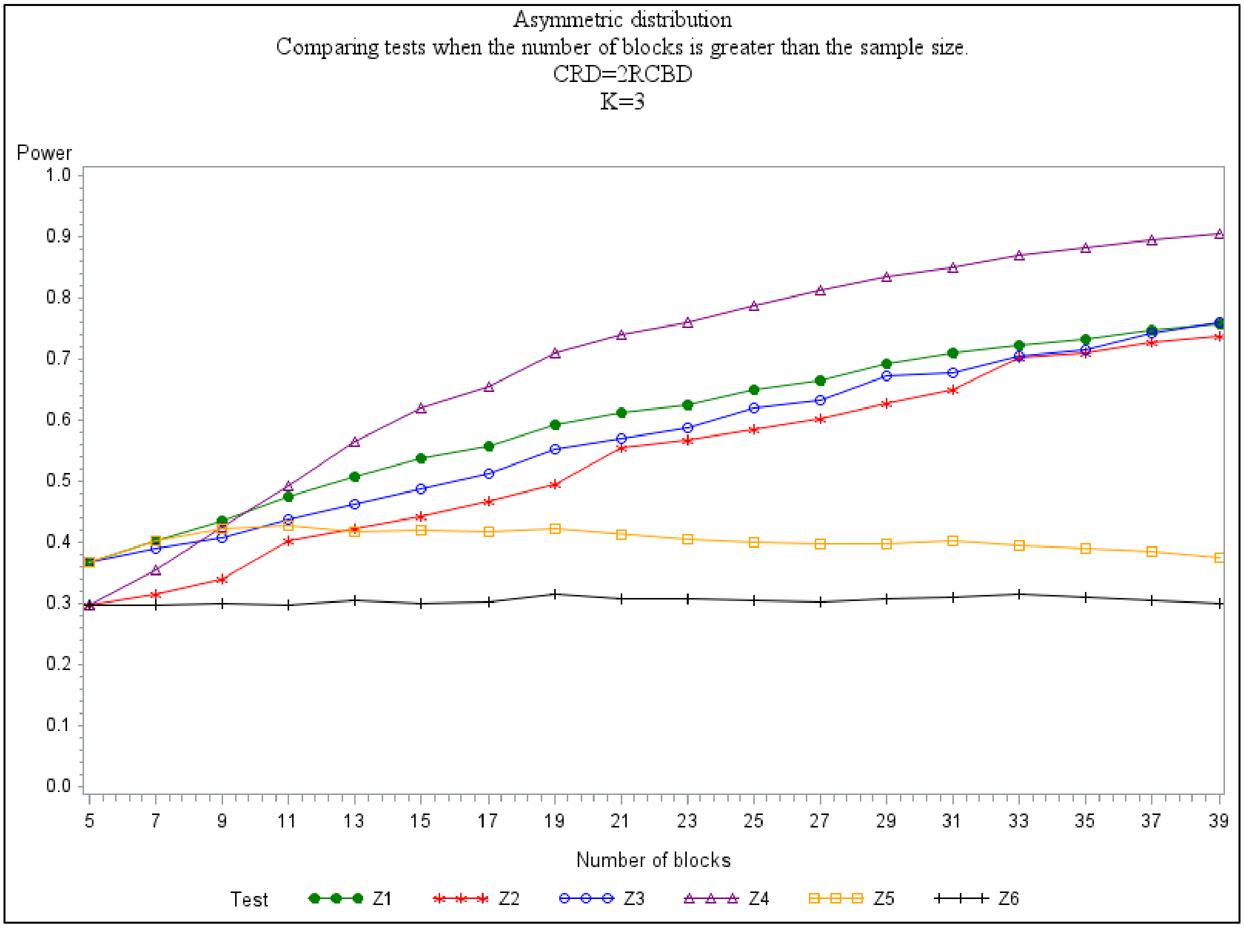

Figure E2. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=2RCBD; K=3; n_a=5, and n_b=5,7,9,…,39

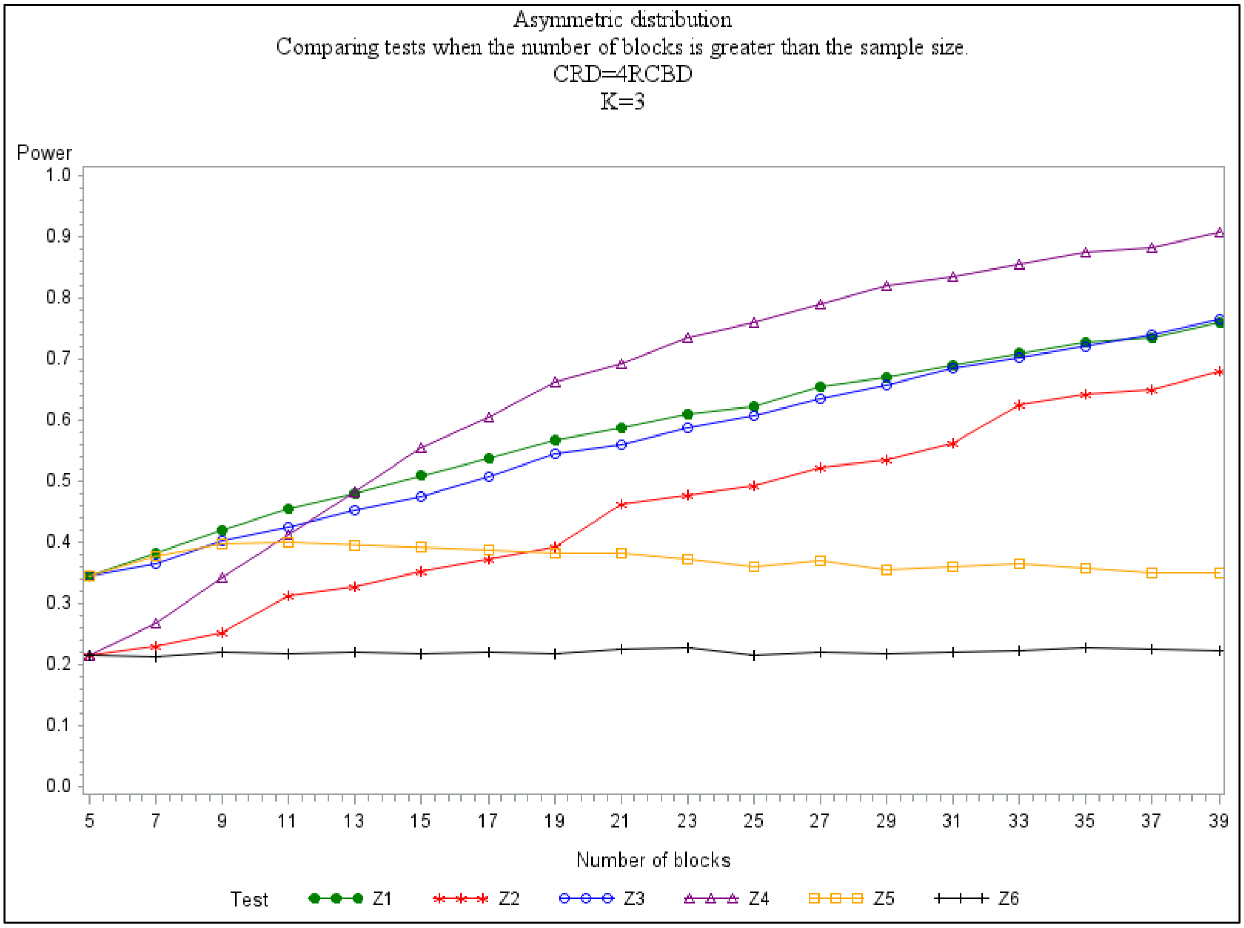

Figure E3. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=4RCBD; K=3; n_a=5, and n_b=5,7,9,…,39

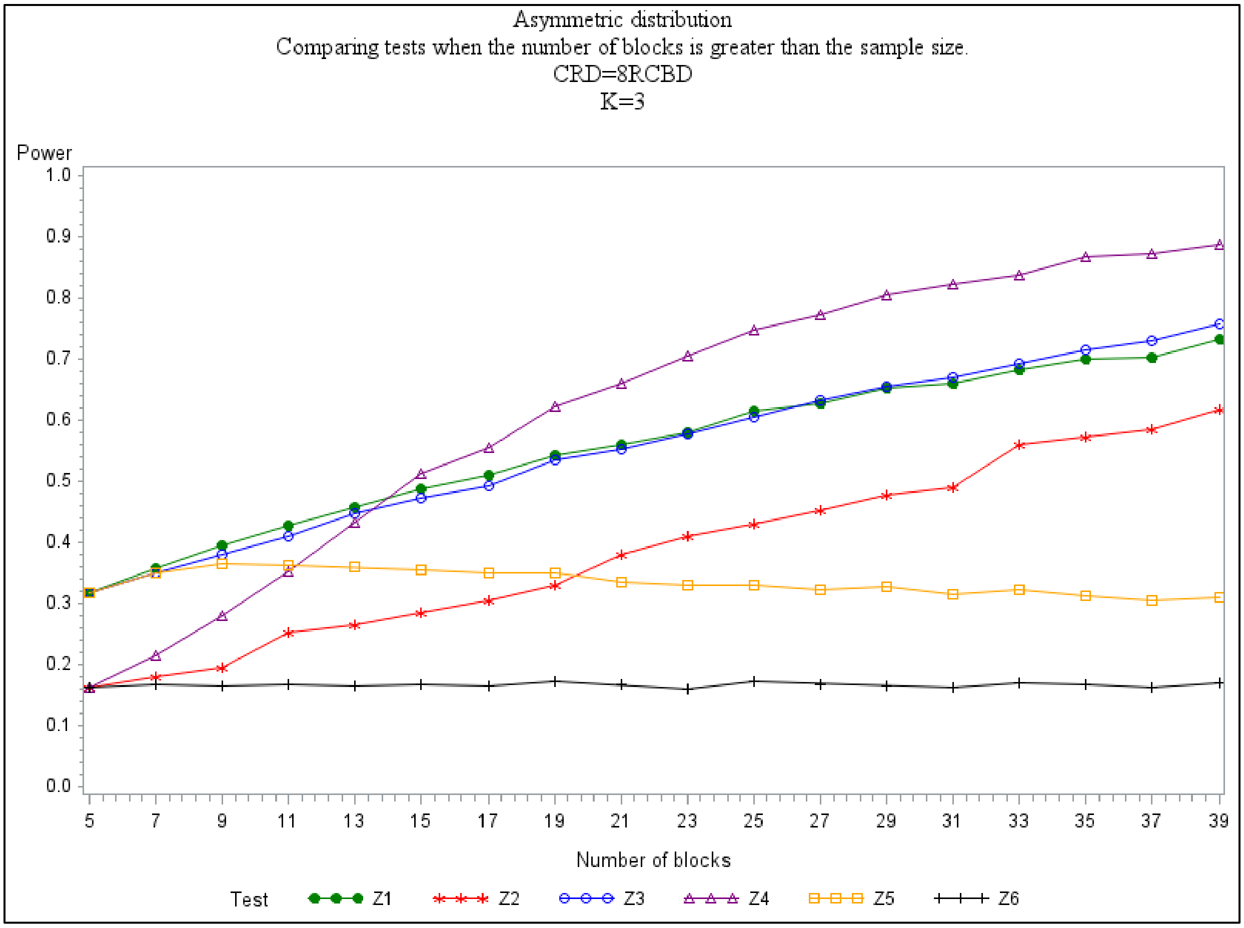

Figure E4. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=8RCBD; K=3; n_a=5, and n_b=5,7,9,…,39

K=4

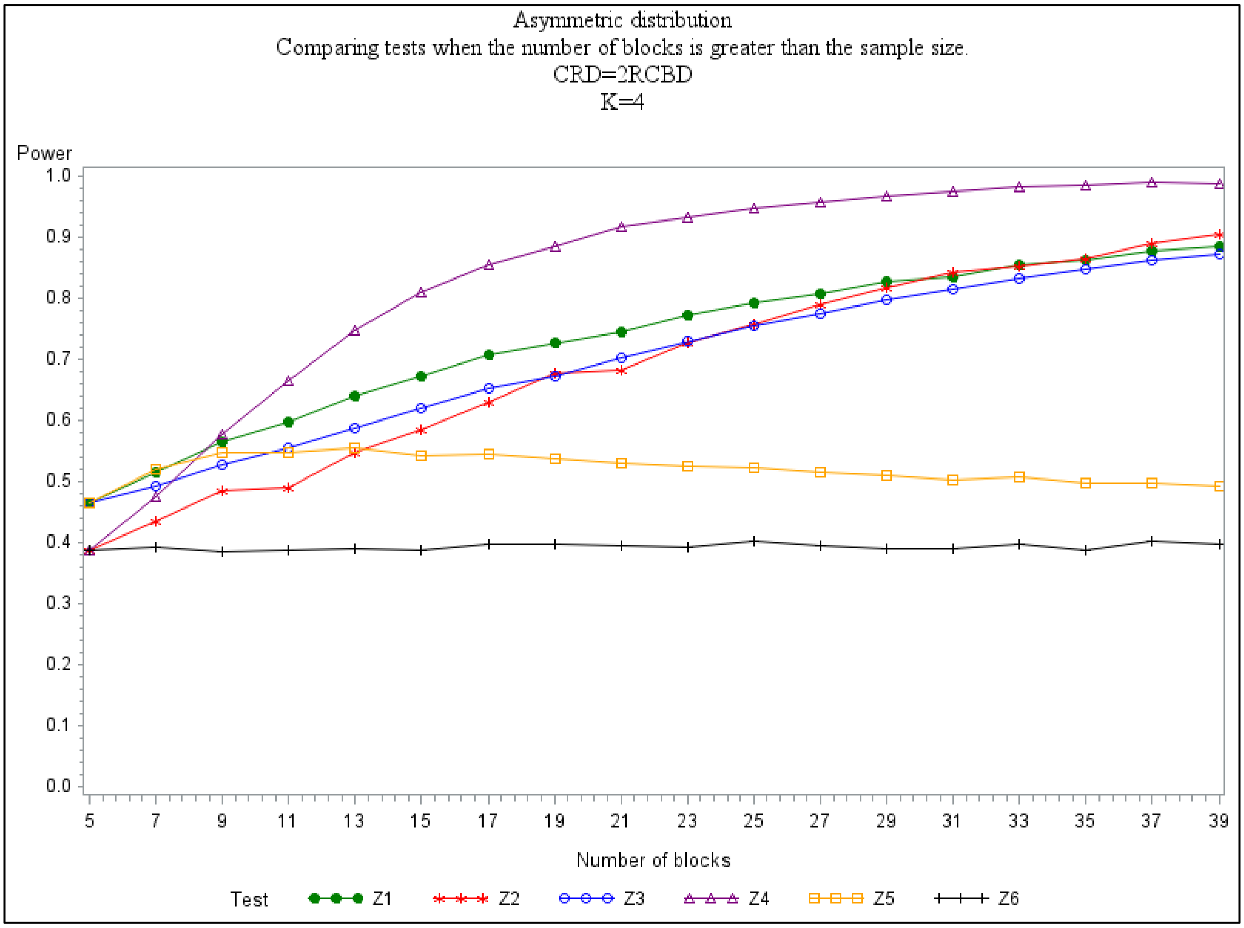

Figure E5. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=2RCBD; K=4; n_a=5, and n_b=5,7,9,…,39

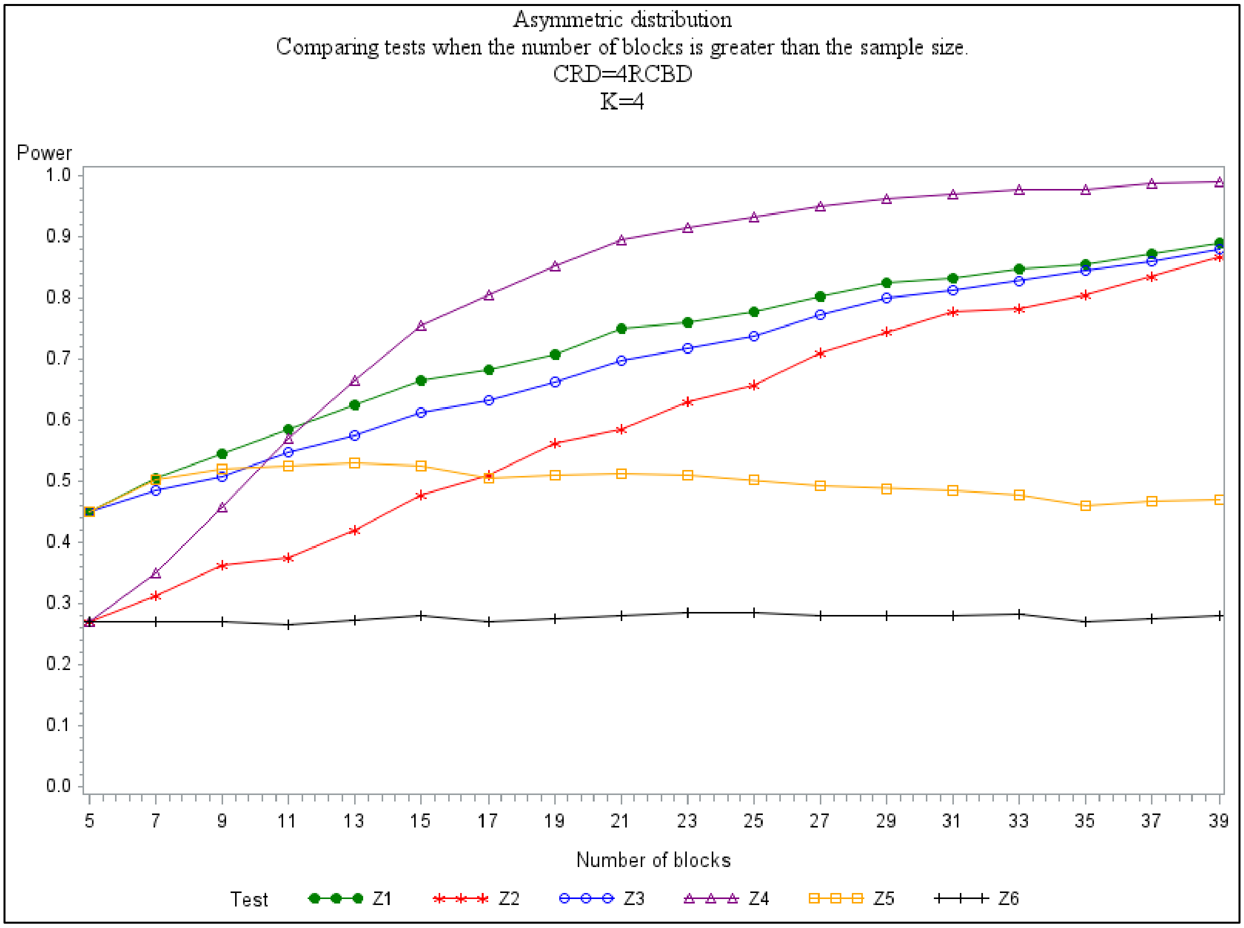

Figure E6. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=4RCBD; K=4; n_a=5, and n_b=5,7,9,…,39

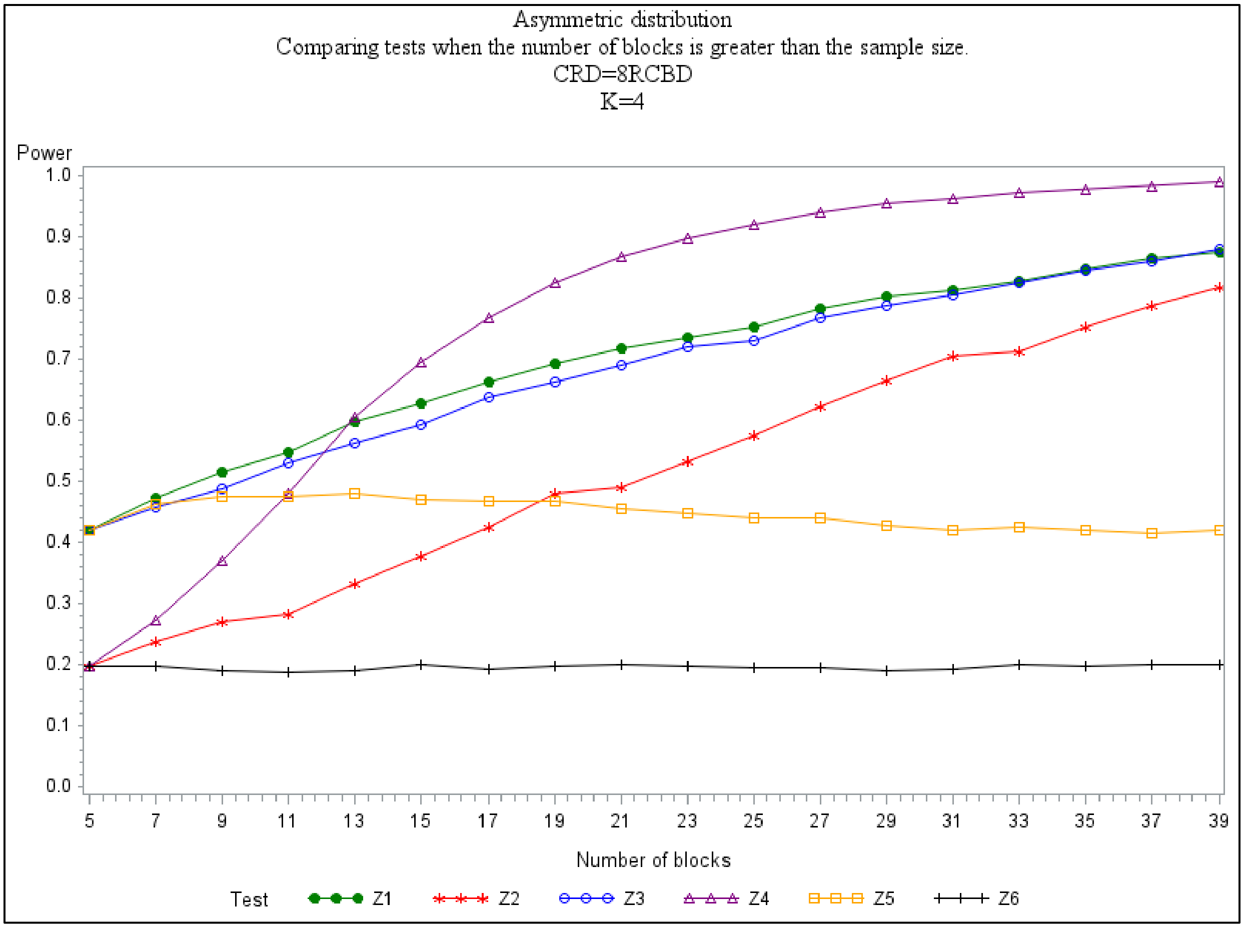

Figure E7. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=8RCBD; K=4; n_a=5, and n_b=5,7,9,…,39

K=5

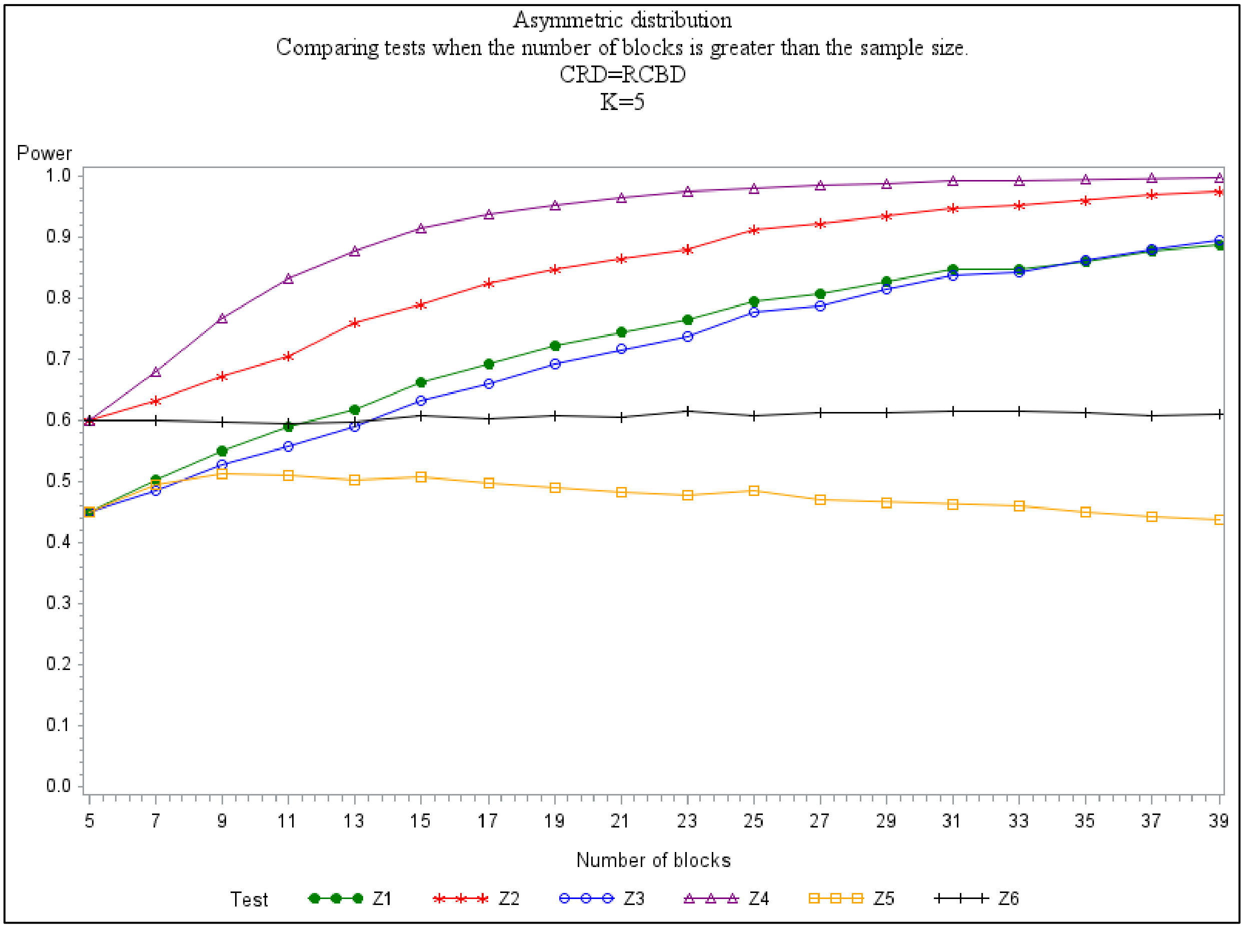

Figure E8. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=RCBD; K=5; n_a=5, and n_b=5,7,9,…,39

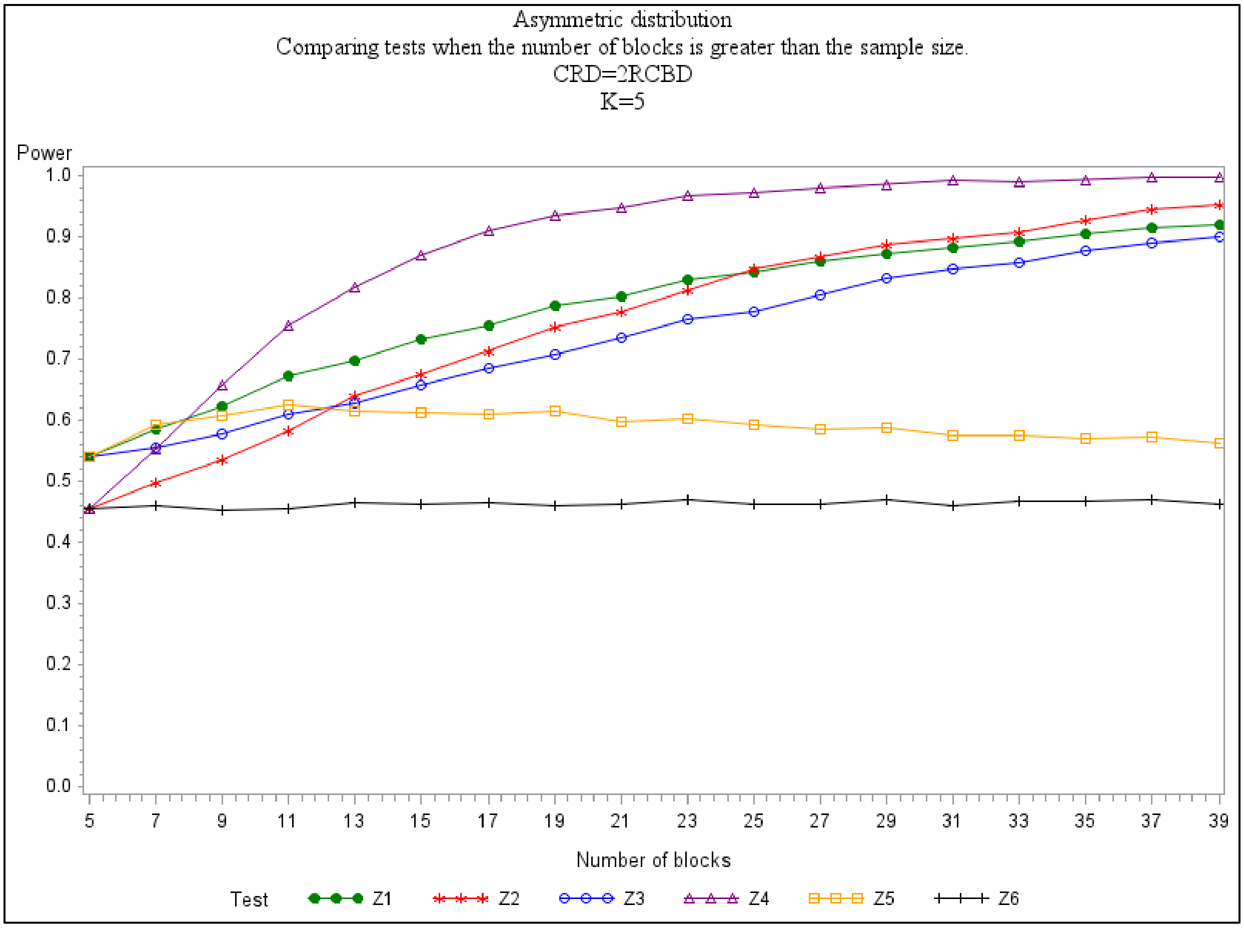

Figure E9. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=2RCBD; K=5; n_a=5, and n_b=5,7,9,…,39

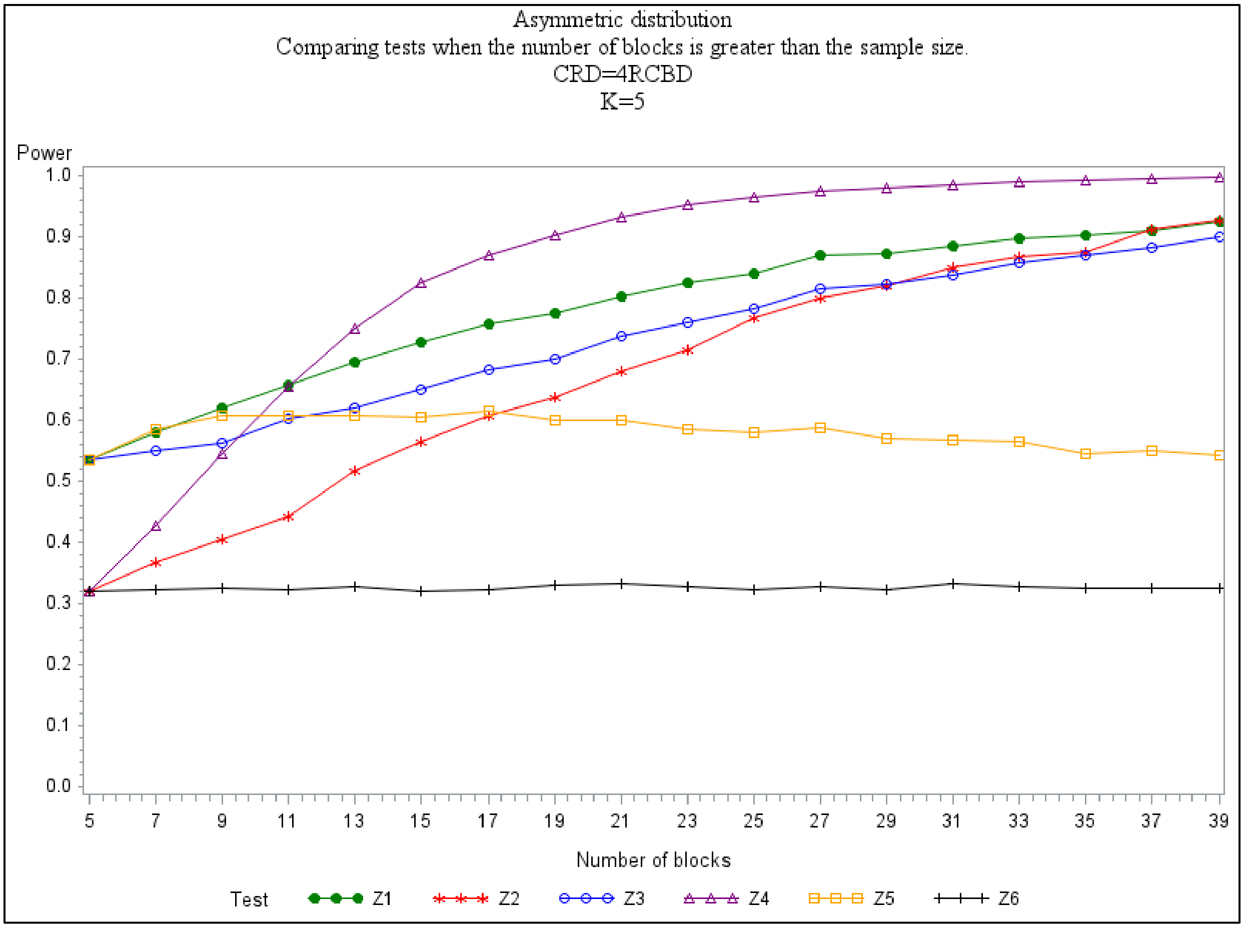

Figure E10. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=4RCBD; K=5; n_a=5, and n_b=5,7,9,…,39

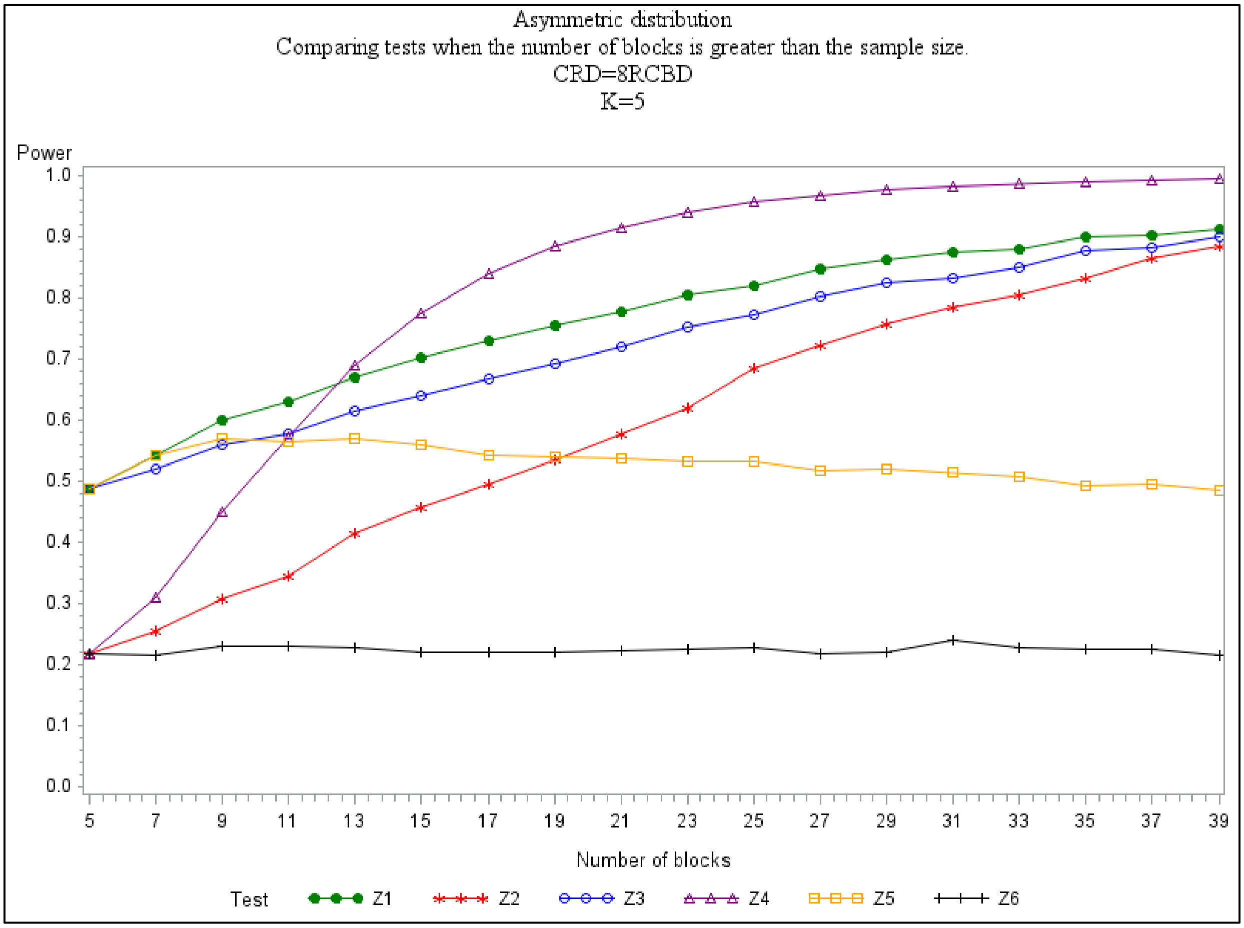

Figure E11. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=8RCBD; K=5; n_a=5, and n_b=5,7,9,…,39

E.2. Increasing the Sample Sizes

K=3

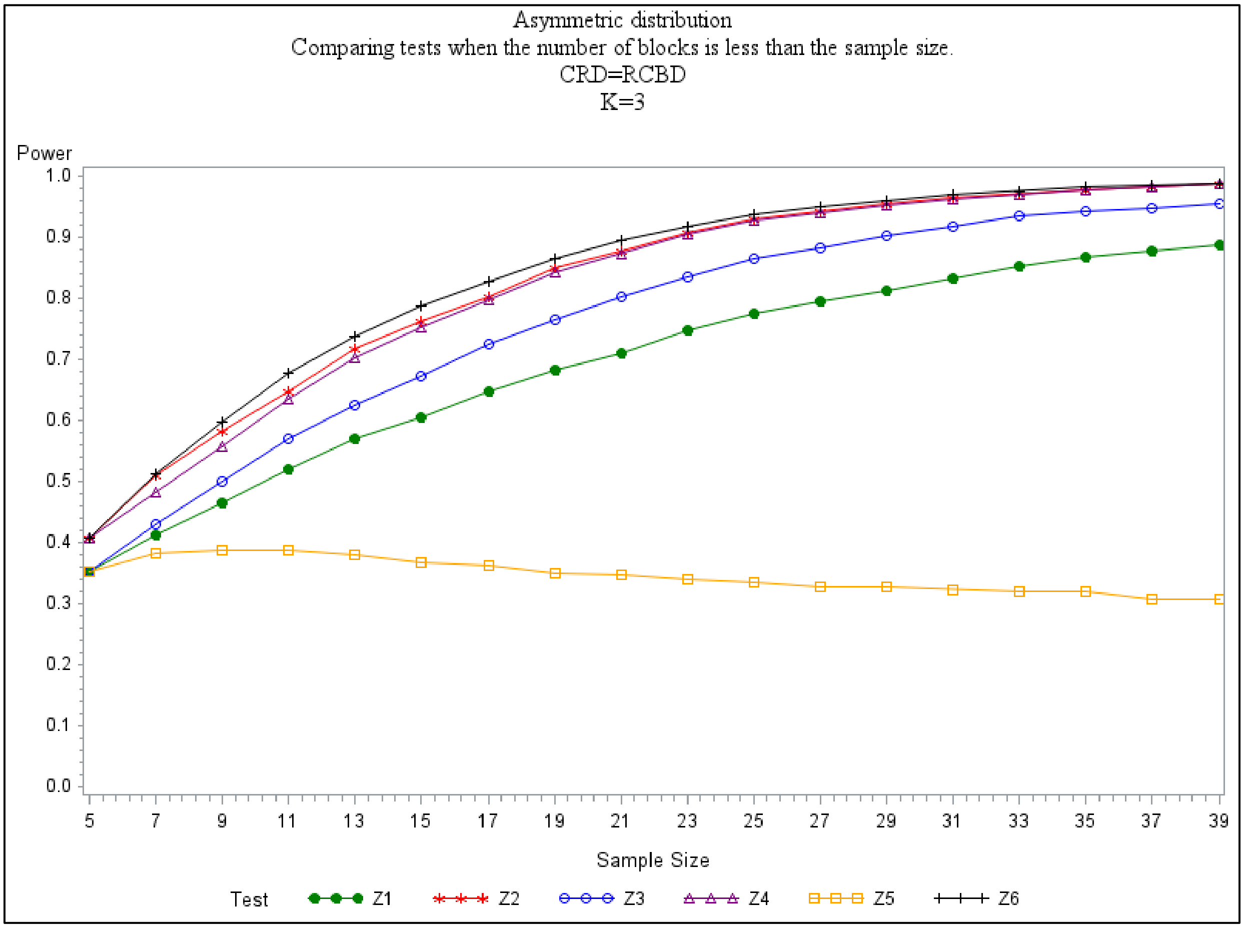

Figure E12. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

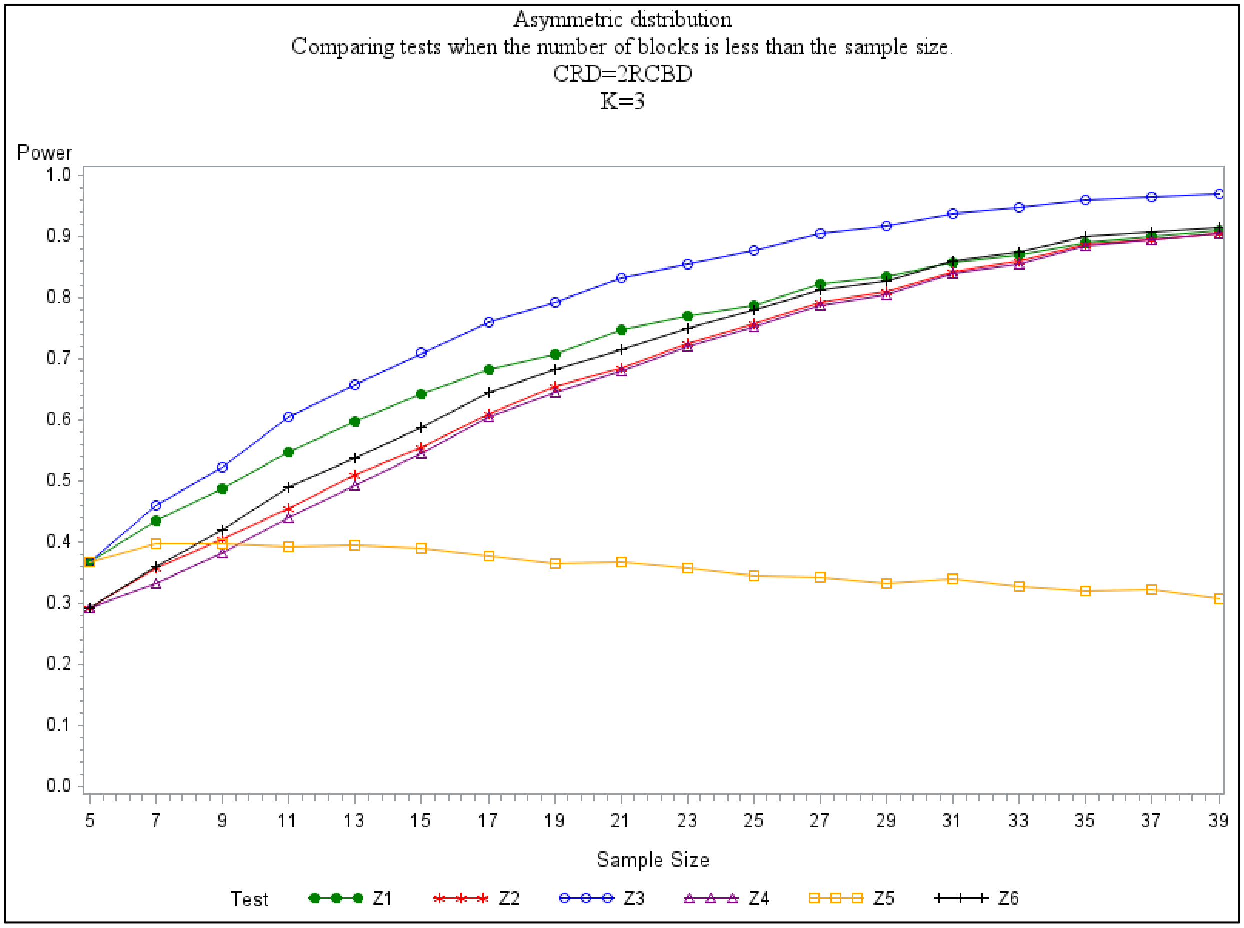

Figure E13. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=2RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

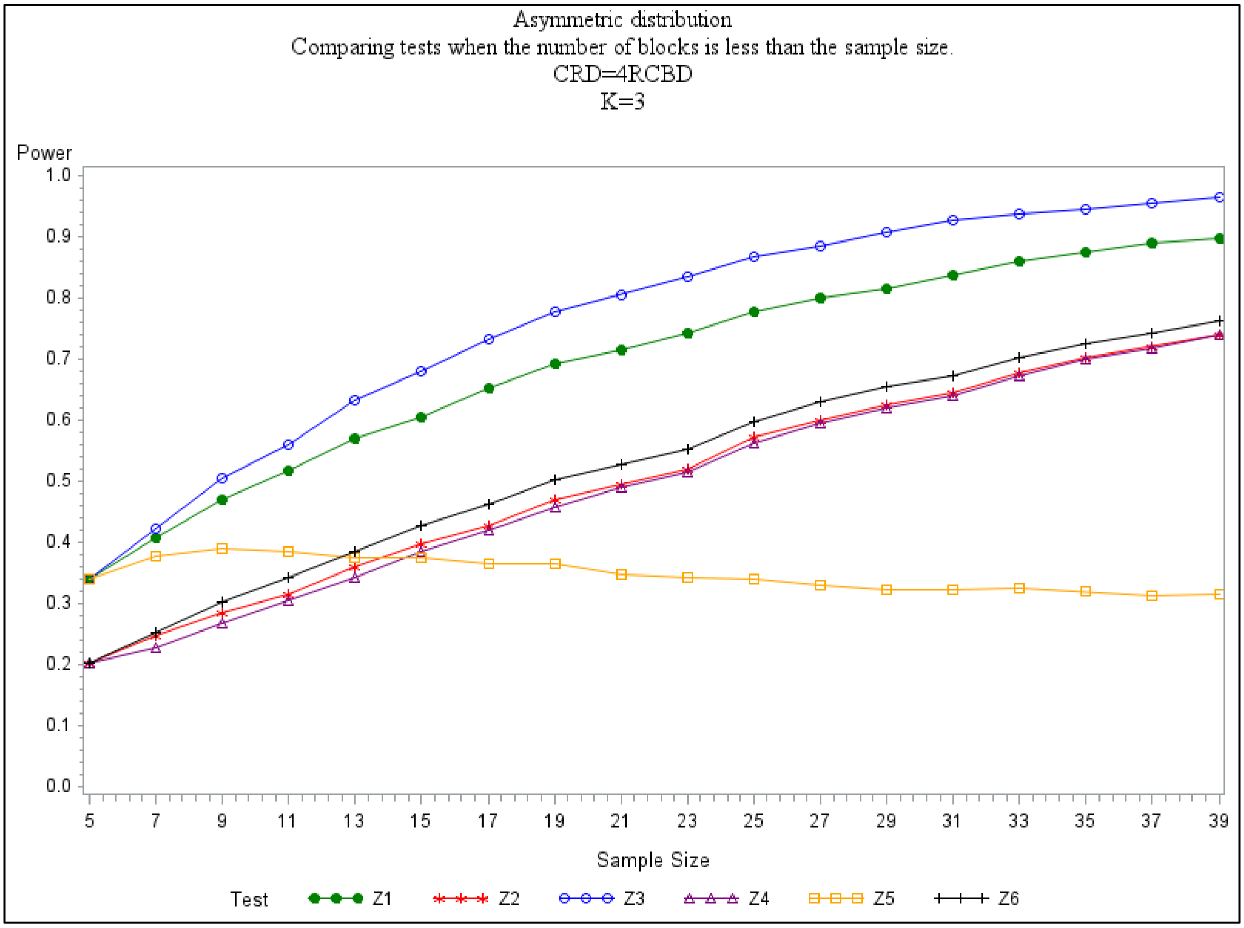

Figure E14. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=4RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

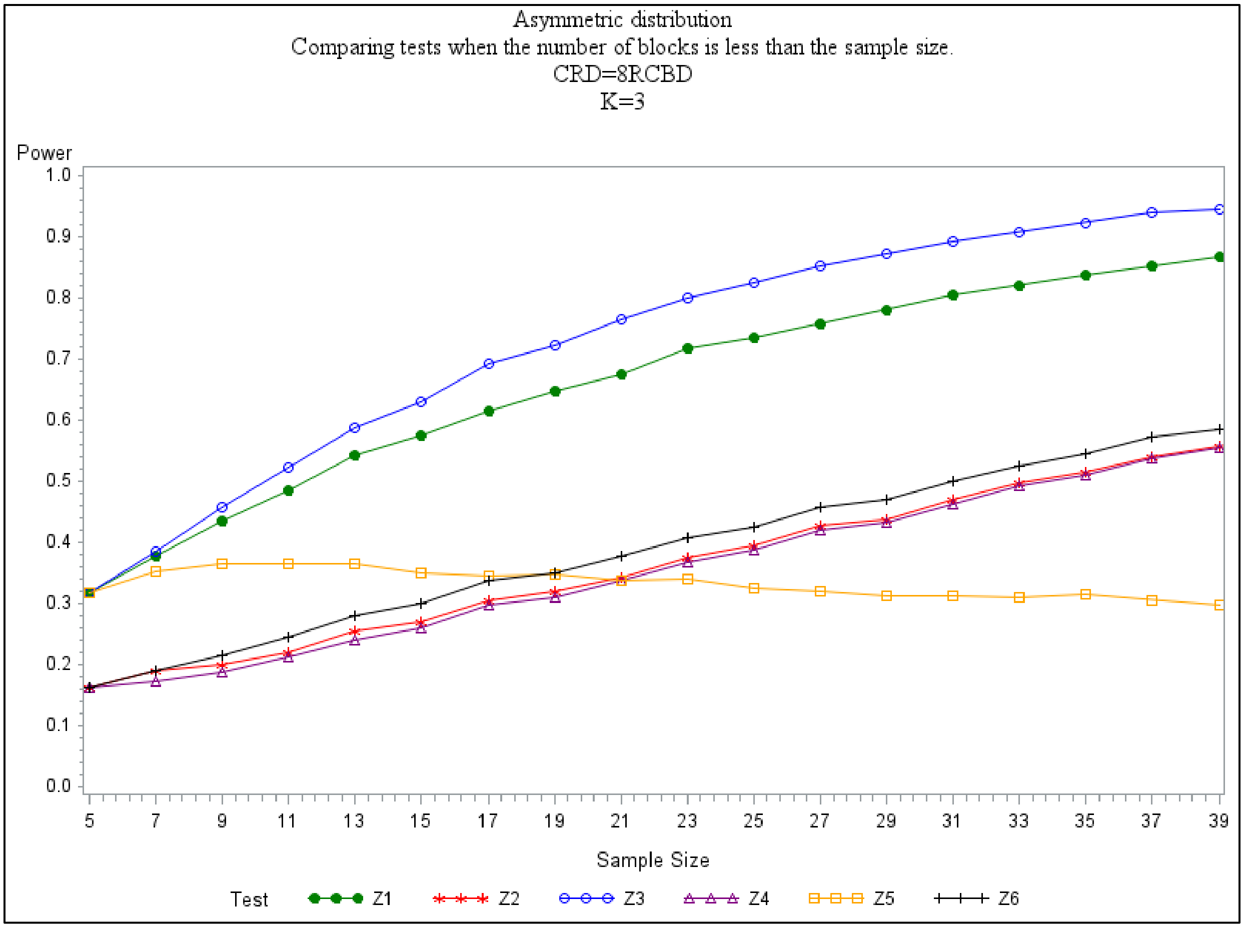

Figure E15. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=8RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

K=4

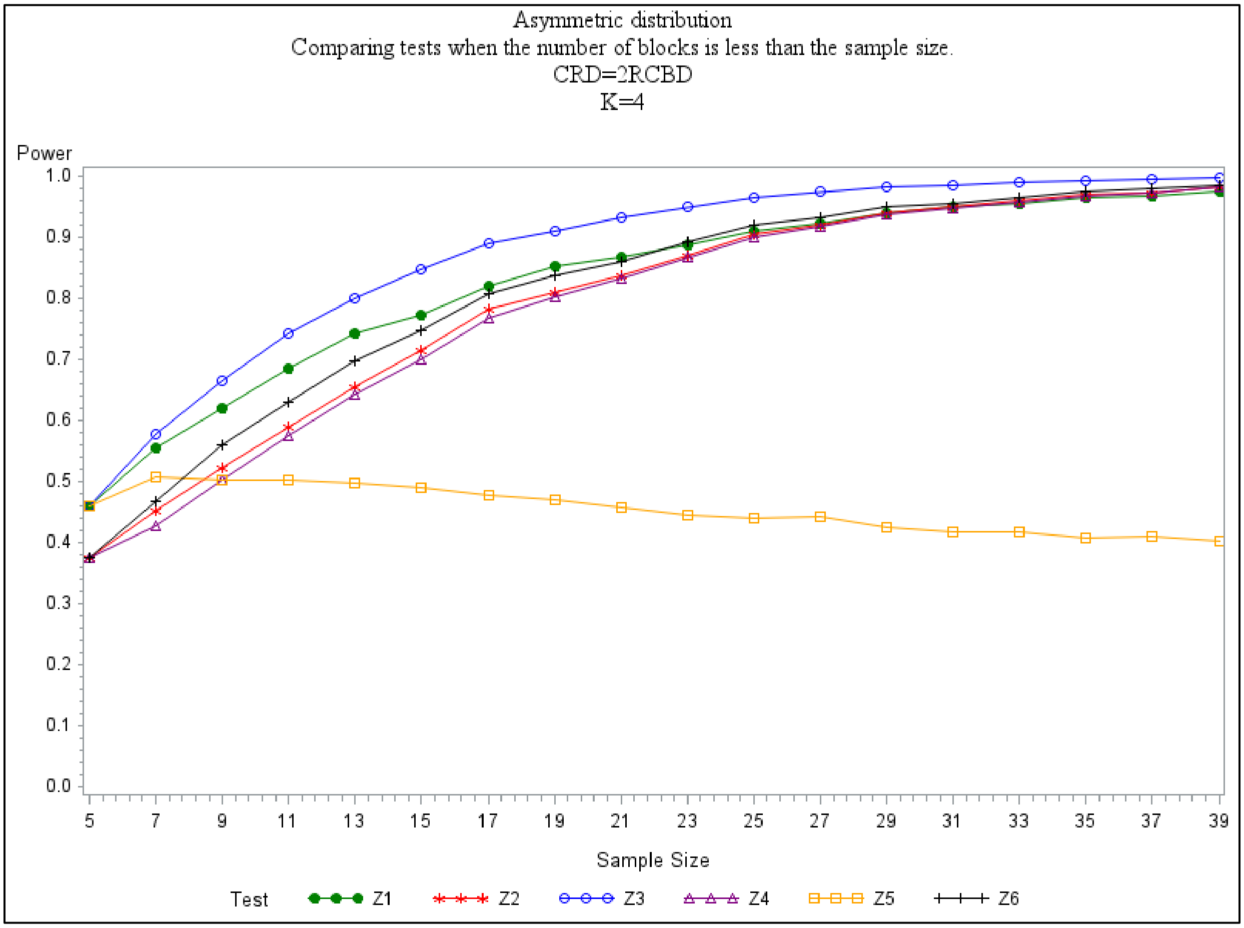

Figure E16. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=2RCBD; K=4; n_a=5,7,9,…,39, and n_b=5.

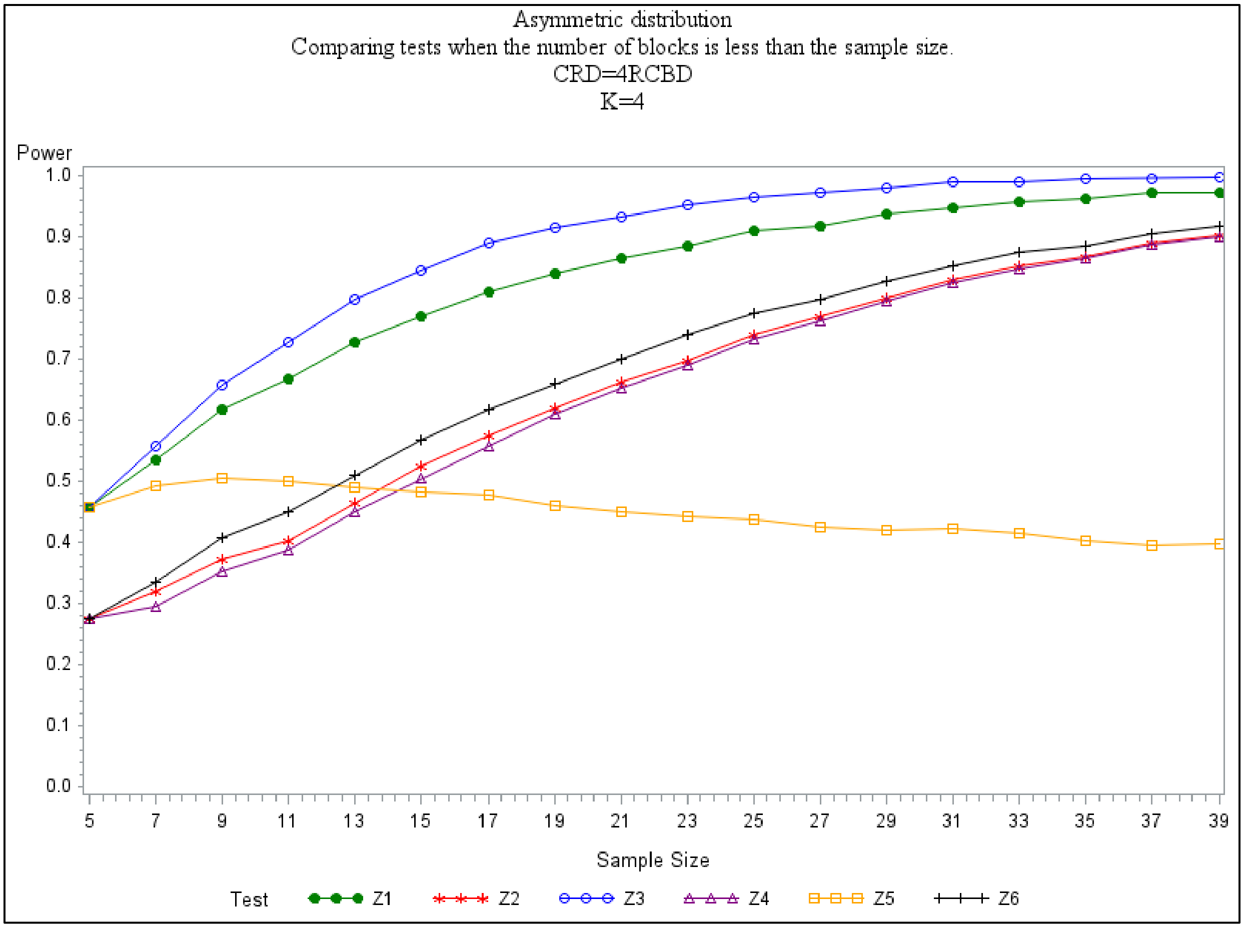

Figure E17. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=4RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

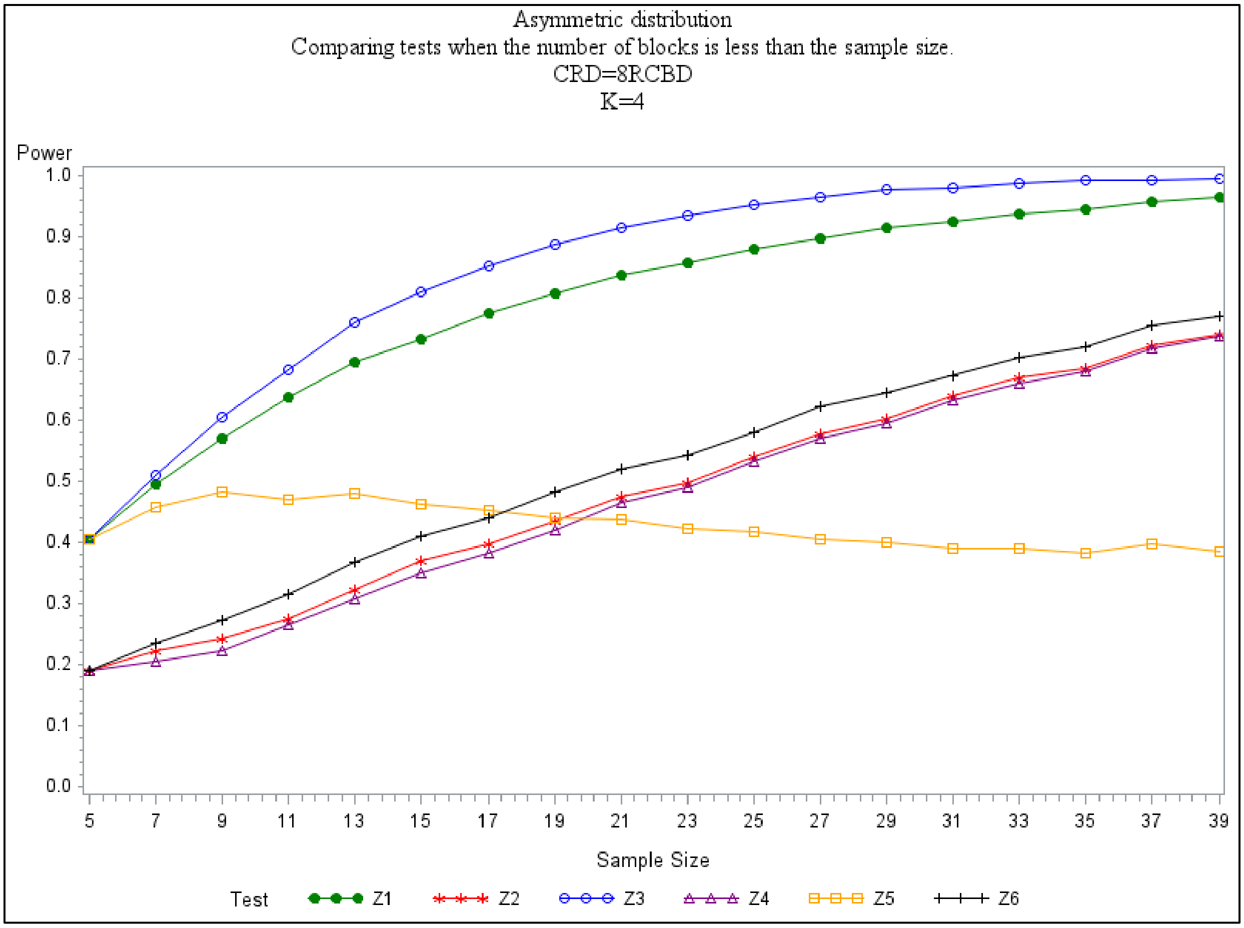

Figure E18. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=8RCBD; K=3; n_a=5,7,9,…,39, and n_b=5.

K=5

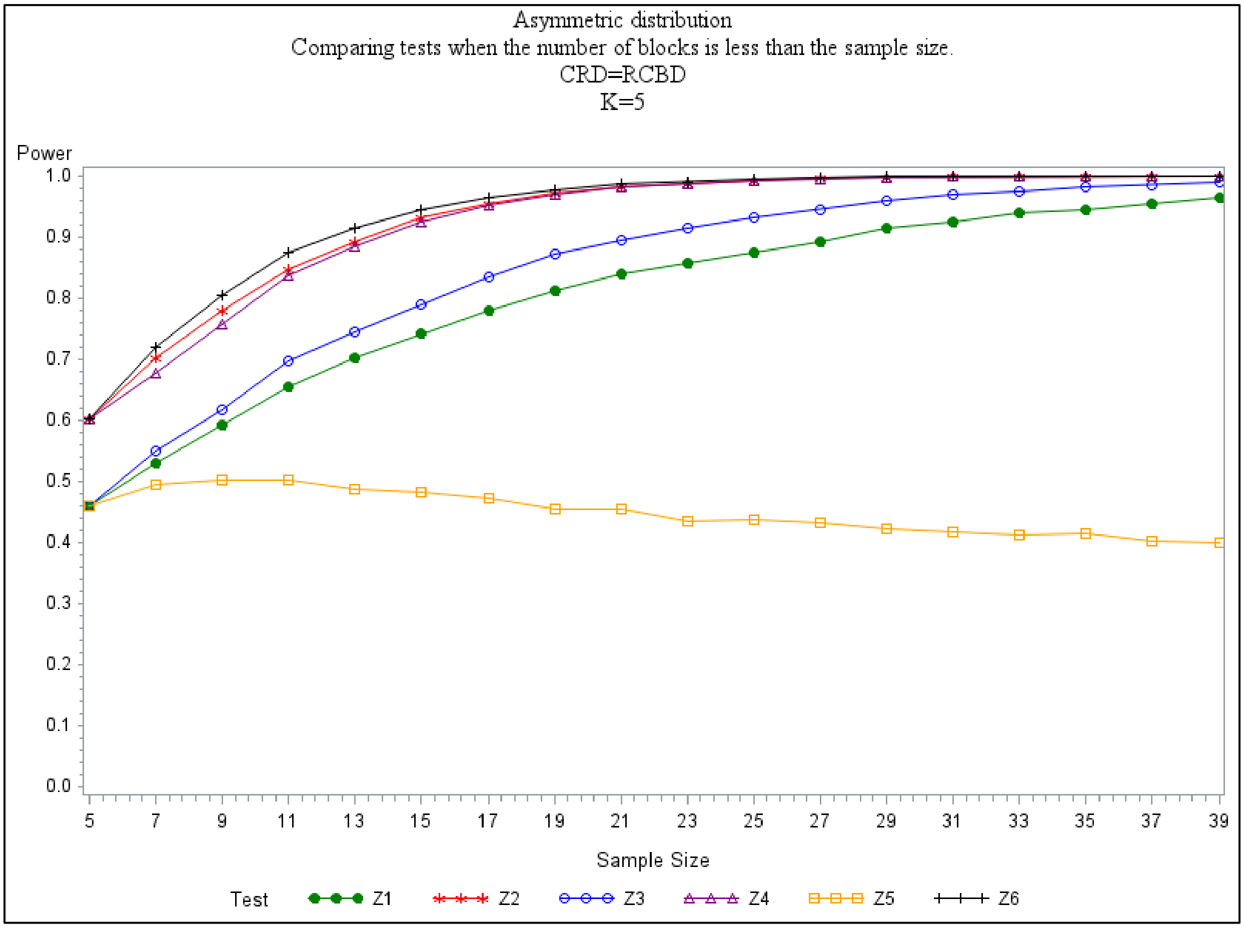

Figure E19. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=RCBD; K=5; n_a=5,7,9,…,39, and n_b=5.

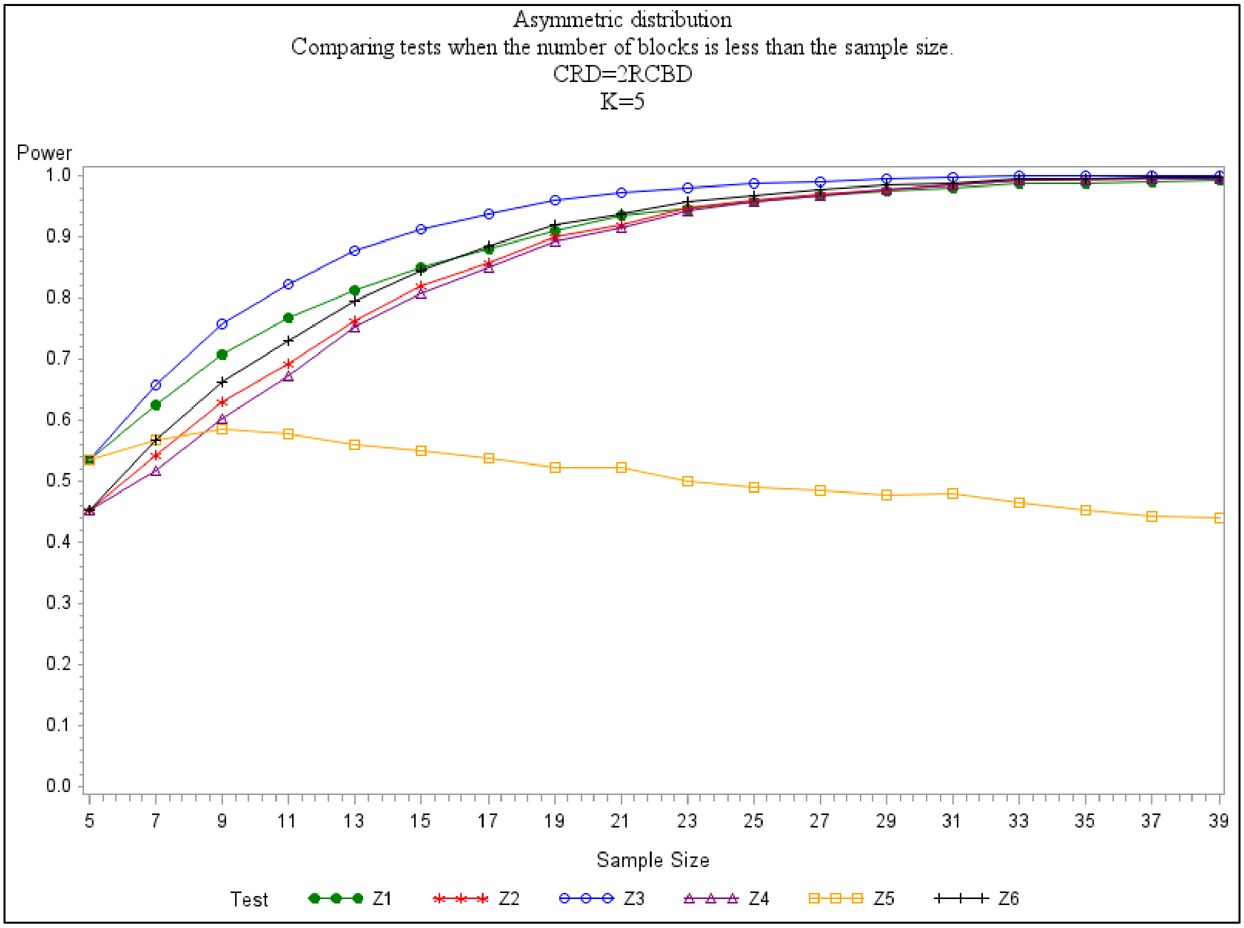

Figure E20. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=2RCBD; K=5; n_a=5,7,9,…,39, and n_b=5.

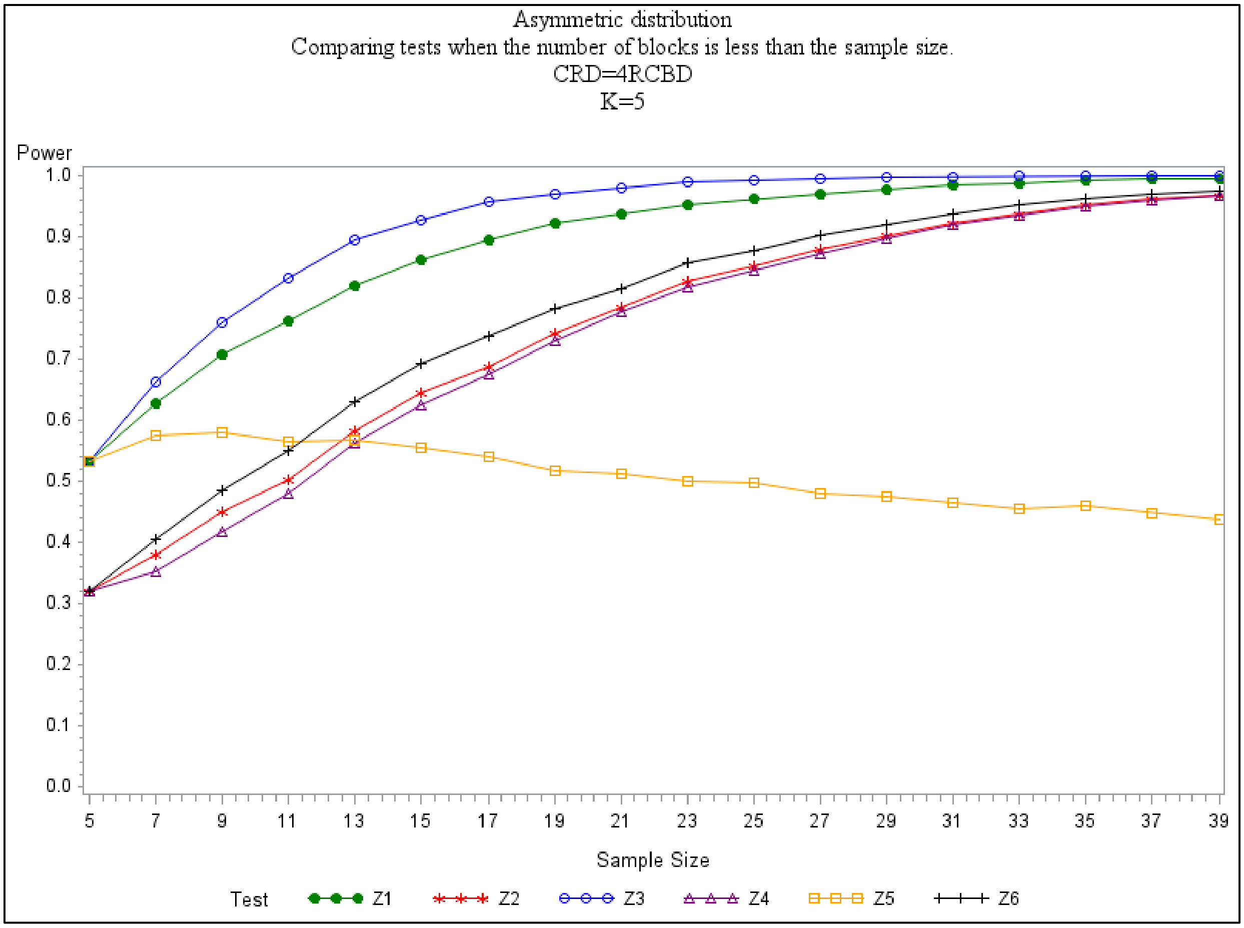

Figure E21. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=4RCBD; K=5; n_a=5,7,9,…,39, and n_b=5.

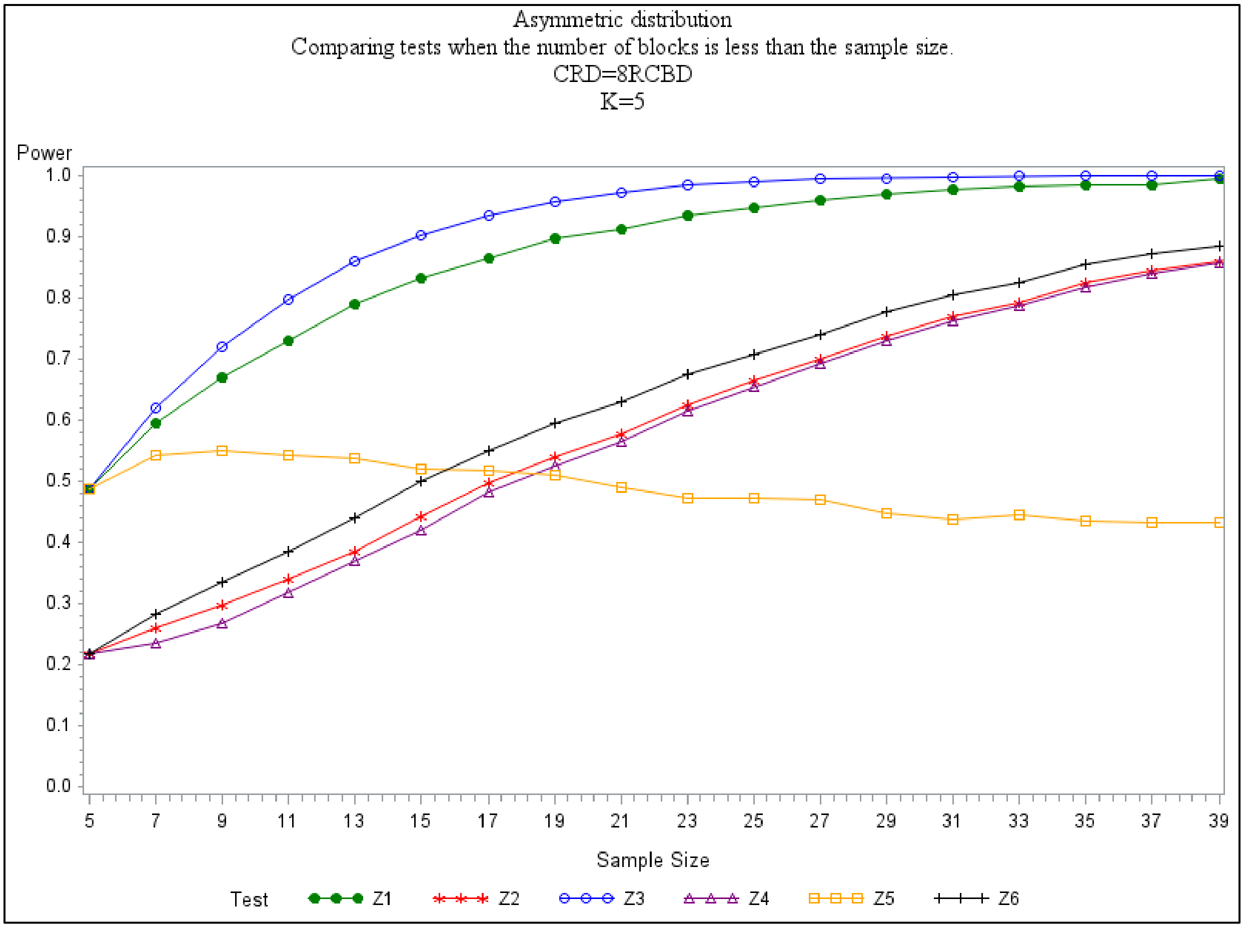

Figure E22. Estimated powers for proposed tests when the populations have different location and scale parameters for exponential distribution; CRD=8RCBD; K=5; n_a=5,7,9,…,39, and n_b=5.

APPENDIX F. RESULTS SUMMARY

Table F1. Results summary for normal and t distribution; k=4, k=5.

	$(n_a = 10, n_b = 10)$			$(n_a = 5, n_b = 10)$			$(n_a = 10, n_b = 5)$		
	MEAN	VAR	BOTH	MEAN	VAR	BOTH	MEAN	VAR	BOTH
CRD=RCBD	Z_2	Z_1	Z_1	Z_4	Z_1	Z_1	Z_6	Z_3	Z_3
CRD=2RCBD	Z_2	Z_1	Z_1	Z_4	Z_1	Z_1	Z_6	Z_3	Z_3
CRD=4RCBD	Z_1	Z_1	Z_1	Z_4	Z_1	Z_1	Z_6	Z_3	Z_3
CRD=8RCBD	Z_1	Z_1	Z_1	Z_4	Z_1	Z_1	Z_6	Z_3	Z_3

Table F2. Results summary for normal and t distribution; k=3.

	$(n_a = 10, n_b = 10)$			$(n_a = 5, n_b = 10)$			$(n_a = 10, n_b = 5)$		
	MEAN	VAR	BOTH	MEAN	VAR	BOTH	MEAN	VAR	BOTH
CRD=RCBD	Z_2	Z_1	Z_1	Z_4	Z_5	Z_1	Z_6	Z_3	Z_3
CRD=2RCBD	Z_2	Z_1	Z_1	Z_4	Z_5	Z_1	Z_6	Z_3	Z_3
CRD=4RCBD	Z_1	Z_1	Z_1	Z_4	Z_5	Z_1	Z_6	Z_3	Z_3
CRD=8RCBD	Z_1	Z_1	Z_1	Z_4	Z_5	Z_1	Z_6	Z_3	Z_3

Table F3. Results summary for exponential distribution; k=3, k=4, k=5.

	$(n_a = 10, n_b = 10)$			$(n_a = 5, n_b = 10)$			$(n_a = 10, n_b = 5)$		
	MEAN	VAR	BOTH	MEAN	VAR	BOTH	MEAN	VAR	BOTH
CRD=RCBD	Z_2	Z_1	Z_2	Z_4	Z_5	Z_4	Z_6	Z_3	Z_6
CRD=2RCBD	Z_2	Z_1	Z_1	Z_4	Z_5	Z_4	Z_6	Z_3	Z_3
CRD=4RCBD	Z_2	Z_1	Z_1	Z_4	Z_5	Z_1	Z_6	Z_3	Z_3
CRD=8RCBD	Z_2	Z_1	Z_1	Z_4	Z_5	Z_1	Z_6	Z_3	Z_3