A COMPARISON OF THE ANSARI-BRADLEY TEST AND THE MOSES TEST FOR THE VARIANCES

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ABSTRACT

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This paper is aimed to compare the powers and significance levels of two well known nonparametric tests: the Ansari-Bradley test and the Moses test in both situations where the equal-median assumption is satisfied and where the equal-median assumption is violated. R-code is used to generate the random data from several distributions: the normal distribution, the exponential distribution, and the t-distribution with three degrees of freedom. The power and significance level of each test was estimated for a given situation based on 10,000 iterations. Situations with the equal samples of size 10, 20, and 30, and unequal samples of size 10 and 20, 20 and 10, and 20 and 30 were considered for a variety of different location parameter shifts. The study shows that when two location parameters are equal, generally the Ansari-Bradley test is more powerful than the Moses test regardless of the underlying distribution; when two location parameters are different, the Moses is generally preferred. The study also shows that when the underlying distribution is symmetric, the Moses test with large subset size k generally has higher power than the test with smaller k; when the underlying distribution is not symmetric, the Moses test with larger k is more powerful for relatively small sample sizes and the Moses test with medium k has higher power for relatively large sample sizes.

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CHAPTER 1.

INTRODUCTION

A question that the researcher frequently encounters has to do with the equality of two population parameters that measure dispersion. In parametric statistical inference, the F test is often used to test the null hypothesis that two population dispersion parameters are equal. In the parametric case, the measures of dispersion are the two population variances, usually designated σ_1^2 and σ_2^2. However, the F test is not very reliable when the populations of interest are not normally distributed.

Several alternative dispersion tests have been proposed over the years. This research paper is related to further investigating two distribution-free nonparametric alternatives to the F test for testing the equality of variances from two populations. The first one is Ansari-Bradley test (Ansari and Bradley (1960)). The Ansari-Bradley test assumes that the two unknown population medians are equal. The second test considered is the Moses test (Moses (1963)), which does not depend on this equal-median assumption.

In this research, we would like to investigate what happens to the significance levels of the Ansari-Bradley test in situations where the medians of the two populations are not equal. We would also like to investigate how the powers of the Ansari-Bradley test compare to the powers of the Moses test in situations where the medians are not equal and the significance levels of the Ansari-Bradley test remain less than or equal to the stated significance level.

In addition to the above research, we also want to investigate the Moses test further. In Moses test, the researcher decided how many subsets, m, of each of equal size k to randomly decide the observations into before further applying Moses test. Is it better to use more subsets with fewer observations in them or fewer subsets with more observation in them? The researcher gets to decide on this. Thus, one subdivision may lead to significant results where another does not. As to how to choose the subset size k, Shorack (1969) recommends that k be as large as possible, but not greater than 10, and that the number of subsets for each sample be large enough to permit meaningful results from the application of the location test. Therefore, another goal in this paper is to determine what k is better, if possible, and to investigate to see if the recommendation as to what k value to use varies with the underlying distribution type.

CHAPTER 2.

REVIEW OF LITERATURE

A nonparametric test that is often used to test for equality of variances is the Ansari-Bradley test (Ansari and Bradley (1960)). An example in using the Ansari-Bradley test may be found in Daniel (1990). Castagliola (1996) introduced an algorithm for computing statistic of Ansari-Bradley test. Dinneen and Blakesley (1976) introduced a method for generating the null distribution of the Ansari-Bradley test statistic when sample sizes are small.

The Ansari-Bradley test is a nonparametric test designed to test for differences in dispersion based on two independent samples. It has the assumption that the location parameters of the two populations are equal. The null hypothesis is given as H_0: $\sigma_1 = \sigma_2$ and the three alternative hypotheses are the following: H_{a1}: $\sigma_1 \neq \sigma_2$, H_{a2}: $\sigma_1 \leq \sigma_2$, and H_{a3}: $\sigma_1 > \sigma_2$, where σ_1 and σ_2 are dispersion parameters of populations 1 and 2, respectively.

In calculating the Ansari-Bradley test, we first combine observations from the two samples together. The combined sample size will be denoted by n′ where n′ is the sum of n_1 and n_2 with n_1 and n_2 denoting the sample sizes from populations 1 and 2, respectively. The observations in the combined sample are arranged in order from smallest to largest. We next assign ranks to observations in the combined sample by the following method and keeping track of which population the observation came from: the smallest and largest observations get a rank of 1; the second smallest and second largest observations get a rank of 2; and we continue assigning ranks in this way until all the measurements in the combined sample get a rank. Let R_i denote the rank of the i^{th} measurement in the first sample. The test statistic T is then the sum of the ranks of all the measurements in the first sample, i.e., $T=\sum R_i$.

After we obtained the test statistic, we may use the following decision rule to decide whether to reject the null hypothesis $\sigma_1 = \sigma_2$ or not: for the two-sided alternative hypothesis $\sigma_1 \neq \sigma_2$, either a sufficiently large or a sufficiently small value of T will cause rejection of H_0. Therefore, for our chosen level of significance α, we reject H_0 if T is either greater than or equal to the larger critical value or less than the lower critical value. Critical values for small samples are given in Daniel (1990). When H_{a2} is used, we reject the null hypothesis for large values of the test statistic and for H_{a3} we reject the null hypothesis for small values of the test statistic.

When the sample sizes exceed those found in the table (Daniel, 1990) for the Ansari-Bradley test, we may compute

$$T^* = \frac{T-[n_1(n_1+n_2+2)/4]}{\sqrt{n_1 n_2 (n_1+n_2+2)(n_1+n_2-2)/[48(n_1+n_2-1)]}}$$

if n_1+n_2 is even, and

$$T^* = \frac{T-[n_1(n_1+n_2+1)^2/4(n_1+n_2)]}{\sqrt{n_1 n_2 (n_1+n_2+1)[3+(n_1+n_2)^2]/48(n_1+n_2)^2}}$$

if n_1+n_2 is odd.

The asymptotic null distribution of T* is the standard normal distribution.

The decision rule for the large-sample approximation is: we reject H_0 if $T^* \geq Z_{(\alpha/2)}$ or $T^* < -Z_{(\alpha/2)}$ for the alternative hypothesis $\sigma_1 = \sigma_2$; and we reject H_0 if $T^* \geq Z_\alpha$ or if $T^* < -Z_\alpha$ for the alternative hypotheses $\sigma_1 < \sigma_2$ and $\sigma_1 > \sigma_2$, respectively, where Z_α is the value found in the standard normal table.

Another nonparametric test for testing the equality of dispersion parameters was proposed by Moses (1963). Unlike the Ansari-Bradley test, the Moses test does not assume equality of location parameters. An example of applying the Moses test is given in Daniel, 1990.

The hypotheses for Moses test is the same as that for Ansari-Bradley test. Let's denote σ_1 and σ_2 as the dispersion parameters of the two populations respectively. The null hypothesis is $\sigma_1 = \sigma_2$, and the three alternative hypotheses are: $\sigma_1 \neq \sigma_2$; $\sigma_1 < \sigma_2$; and $\sigma_1 > \sigma_2$.

In order to calculate the test statistic for the Moses test, first of all, we divide the first and second samples up into m_1 and m_2 subsamples of equal size k, respectively. If there are any observations left over, they are discarded. There has been a recommendation made by Shorack (1969) that k be as large as possible, but not greater than 10, so that m_1 and m_2 are still of a reasonable size to perform the Mann-Whitney test (Mann and Whitney, 1947). For each of the first m_1 subsets, the sample mean is calculated, the distance between each observation and the sample mean is found and then squared, and these squared values are added up. The values $C_1, C_2, \dots, C_{m1}$ are used to denote these sum of squared values for each of the m_1 subsets in the first sample. $D_1, D_2, \dots, D_{m2}$ denote the same for the subsets in the second sample. Next, we apply the Mann-Whitney test (Mann and Whitney, 1947). In other words, we combine the m_1 subsamples of C's and m_2 subsamples of D's and rank all observations in the combined set from smallest to largest. We then sum the ranks of the observations from m_1 subsamples (that is, the C's).

The test statistic is then $T = S - m_1(m_1+1)/2$, where S is the sum of the ranks assigned to the sums of squares computed from the subsamples of X's, that is, the sum of the ranks assigned to the C's.

After we obtained the test statistic, we may use the following decision rule to decide whether to reject the null hypothesis $\sigma_1 = \sigma_2$ or not based on a given level of significance α: for the two-sided alternative hypothesis $\sigma_1 \neq \sigma_2$, if the computed value of T is less than $w_{\alpha/2}$ or greater than $w_{1-\alpha/2}$, we would reject H_0, where $w_{\alpha/2}$ is the critical value of T given in the table (Daniel, 1990) set up for the Mann-Whitney test and $w_{1-\alpha/2}$ is given by $w_{1-\alpha/2} = m_1m_2 - w_{\alpha/2}$; and for the alternative hypothesis $\sigma_1 < \sigma_2$ and $\sigma_1 > \sigma_2$, we reject H_0 if the computed T is less than w_α or is greater than $w_{1-\alpha}$, respectively, where w_α is the critical value of T obtained in the table (Daniel, 1990) set up for the Mann-Whitney test and $w_{1-\alpha}$ is given by $w_{1-\alpha} = m_1m_2 - w_\alpha$.

For large values of m_1 and m_2, the large approximation for the Mann-Whitney Test may be used. This is given by the following:

$$z = \frac{T - m_1m_2/2}{\sqrt{m_1m_2(m_1+m_2+1)/12}}$$

where T is the sum of the ranks for the first sample; m_1 and m_2 are the numbers of observations in the two new samples based on the C's and D's, respectively (Daniel, 1990).

The asymptotic null distribution of z is a standard normal distribution.

The decision rule for the large-sample approximation is: for alternative hypothesis $\sigma_1 < \sigma_2$, reject H_0 when $z < -Z_\alpha$; for alternative hypothesis $\sigma_1 > \sigma_2$, reject H_0 when $z > Z_\alpha$; and for alternative hypothesis $\sigma_1 = \sigma_2$, reject H_0 when $z < -Z_{\alpha/2}$ or $z > Z_{\alpha/2}$.

CHAPTER 3.

DESCRIPTION OF SIMULATION STUDY

A simulation study was conducted to compare the estimated significance levels and estimated powers of the Ansari-Bradley test and the Moses test when the Moses test was based on differing number of subsamples. Recall, the Ansari-Bradley test has the assumption that all of the location parameters are equal. The Moses test does not have this assumption, but different results could be obtained from the same two samples using the Moses test based on how many subsamples the researcher chose to use and how the samples were randomized.

Three underlying population distributions were considered in this study. These were the normal distribution, the exponential distribution, and the t-distribution with three degrees of freedom. The normal distribution was selected because it occurs often. The t-distribution with 3 degrees of freedom was selected because it is a symmetric distribution like the normal, but it does have thicker tails than a standard normal. The exponential distribution was chosen because it is not symmetric.

To begin with, significant levels were estimated when $\sigma_1 = \sigma_2$ by simulating 10,000 sets of observations, calculating all of the test statistics for each test, counting the number of times each test rejected, and dividing this number by 10,000 to get the estimated significance levels of each test. When estimating significance levels, we considered situations in which the location parameters were equal, situations where the location parameters were not equal, but not very different and situations where the location parameters were quite different.

Powers were estimated for all of the tests, under all distributions considered, by counting the number of times each test rejected divided by 10,000. Situations were considered in which the location parameters were the same and where they were different. In estimating the powers, the variances were always different. It is noted that the powers of the tests considered can only be compared when their level of significance is at a below the stated level of significance.

A variety of equal and unequal sample sizes were considered. The study included equal sample sizes of 10, 20, and 30 and unequal sample sizes of 10, 20 and 10, 10 as well as 20, 30.

For equal samples of size 10 and unequal samples of sizes 10 and 20 and 20 and 10, we considered 2 and 3 subsets for the Moses test (k=2 or k=3). When we had equal samples of sizes 20 or 30, and we had unequal samples of sizes 20 and 30, we considered 3, 4, and 6 subsets for the Moses test (k=3, 4, 6). All programs were written in R.

In situations where we wanted to compare powers of the tests for equal means, but different variances when the initial population was a t-distribution with 3 degrees of freedom, two random samples would initially be generated from t-distributions with 3 degrees of freedom and every observation in the second sample would get multiplied by the same number. If the second population had a variance of four times the first population, every element in the second sample would get multiplied by 2. When we also wanted to shift the location parameters in the case of a t-distribution, we would multiply each value in the second sample by a number and then add on a different number to each observation. The code rt(n, df) was used to generate values from a t-distribution where n was the sample size and df was the degrees of freedom.

In situations where we wanted to compare powers of the tests for equal means but different variances when the underlying populations were exponential, we generated two random samples using rexp(n_1, $1/\lambda_1$) and rexp(n_2, $1/\lambda_2$) where n_1 and n_2 were different, and λ_1 and λ_2 are the means. The appropriate constant was then subtracted from each value in the second sample so that the location parameters would be equal. In situations we considered when the location parameters were different, we also used rexp(n_1, $1/\lambda_1$) and rexp(n_2, $1/\lambda_2$) to generate two random samples and then subtracted an appropriate value from observations in the second sample.

When the random samples were from normal distributions, rnorm(n, mean, variance) was used to generate the appropriate samples where n is the sample size. The same value of the mean was used in generating the two samples when the location parameters were equal. Different values of the mean were used to generate samples from populations with different location parameters.

Results are given in Chapter 4. Conclusions are given in Chapter 5.

CHAPTER 4.

RESULTS

This chapter gives the estimated powers and significance levels that we found in our simulation study for the Ansari-Bradley test and for the Moses test using differing numbers of subset sizes. The results are divided up by distributions. The distributions included in the study were normal, t-distribution with three degrees of freedom and the exponential.

Results for the normal distribution

Results for the normal distribution are given in Tables 1-26. When the two location parameters were equal, both the Ansari-Bradley test and Moses tests had significance level of around 0.05. This is true for all the sample sizes we considered, including equal sample sizes of 10, 20 and 30 and unequal sample sizes of 10 and 20, 20 and 10, and 20 and 30. In the situation where the two location parameters were different, significance levels for Moses test were always around 0.05. Significance levels for the Ansari-Bradley test were decreasing as the two location parameters were getting more and more different except for unequal samples of sizes 20 and 10. In this case, significance levels for the Ansari-Bradley test were increasing as the two location parameters were getting further apart from each other. Powers could not be compared and this result also brings into questions other cases because these results were different from the cases of unequal samples of size 10 and 20.

Next, we consider the power results for normal distribution. When the equal-median assumption is satisfied, the Ansari-Bradley test generally had higher powers than Moses test did (including all the subset sizes k we considered) for most of the sample sizes. For instance, in the situation of equal samples of size 20 and both of the location parameters were 0, when the standard deviations were 1 and 1.5, the Ansari-Bradley test had the estimated power of 0.3686, which was higher than the estimated power of 0.3415 for Moses test with subset size k=4 (see Table 2); the Ansari-Bradley test also had the highest estimated powers when the standard deviations were 1 and 2 and 1 and 2.5. However, there were some special cases. For equal samples of size 30 and equal location parameters of 0, when the standard deviations were 1 and 1.5, the Ansari-Bradley test had the estimated power of 0.4976, and the Moses test with subset size k=6 had an estimated power of 0.5174 (see Table 7). Moreover, for unequal sample sizes of 20 and 30 and equal location parameters of 0, when the two standard deviations were 1 and 1.5, the Ansari-Bradley test had an estimated power of 0.4235 and the Moses test with k=6 had an estimated power of 0.5067; when standard deviations were 1 and 2, the Ansari-Bradley test had an estimated power of 0.7910 and the Moses test with k=6 had an estimated power of 0.8590; when standard deviations were 1 and 2.5, the Ansari-Bradley test had an estimated power of 0.9378 and the Moses test with k=6 had an estimated power of 0.9595 (see Table 17).

When the equal-median assumption was violated, in all the three situations where the standard deviation of the second population is 1.5, 2 and 2.5 times that of the first population, as the two location parameters were becoming more and more different, the estimated power of the Ansari-Bradley test is decreasing, while the estimated power of Moses test hardly has any change (see Tables 3-5, Tables 8-10, Tables 13-15, Tables 18-20, and Tables 23-25). As an example of this, when there were equal sample sizes of 20 and the second population had a standard deviation 1.5 times that of the first population, as the location parameter for the second sample is 0.5, 1, 1.5, 2 and 2.5 more than the location parameter for the first sample, the estimated powers of the Ansari-Bradley test were 0.3394, 0.2723, 0.1807, 0.0907 and 0.0302, respectively. The test is conservative as one can tell by examining the estimated significance levels (see Table 1). When the difference between two location parameters was 0.5, the estimated power of the Ansari-Bradley test is 0.3394, which is about the same as the estimated power of 0.3378 for Moses test with subset size k=6. However, when the difference became 1, the estimated power of the Ansari-Bradley test was 0.2723 and the Moses test with subset size k=6 had an estimated power of 0.3354. The Ansari-Bradley test was obviously less powerful than the Moses test once the location parameter difference reached 1. The same result held for even larger differences such as 1.5, 2 and 2.5. In comparing the results for Moses test with different subset sizes k, generally, the larger the k was, the higher estimated power the test had. There were a few cases however, such as when we had equal sample sizes of 20 in which we chose different subset sizes of 3, 4 and 6 where the test with k=4 is almost as powerful as that with k=6 when the second population had a standard deviation 1.5 or 2.5 times the standard deviation of the first population and had a higher estimated power than that with k=6 when the standard deviation of the second population is 2 times that of the first one (see Table 3, 5 and 4).

Table 1. Estimated Significance Levels for n1=20, n2=20; normal-distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1	S1: 0 &1 S2: 0.5&1	S1: 0 & 1 S2: 1 & 1	S1: 0 &1 S2: 1.5&1	S1: 0 & 1 S2: 2 & 1	S1: 0 & 1 S2: 2.5 & 1
Ansari-Bradley	0.0484	0.0383	0.0238	0.0085	0.0020	0.0003
Moses(k=3)	0.0472	0.0433	0.0437	0.0483	0.0486	0.0454
Moses(k=4)	0.0469	0.0473	0.0474	0.0463	0.0473	0.0445
Moses(k=6)	0.0495	0.0490	0.0541	0.0504	0.0490	0.0481

Table 2. Estimated Powers 1 for n1=20, n2=20; normal-distributions

Estimated Powers			
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1.5	S1: 0 & 1 S2: 0 & 2	S1: 0 & 1 S2: 0 & 2.5
Ansari-Bradley	0.3686	0.7060	0.8808
Moses(k=3)	0.2911	0.5719	0.7627
Moses(k=4)	0.3415	0.6704	0.8396
Moses(k=6)	0.3305	0.6357	0.8171

Table 3. Estimated Powers 2 for n1=20, n2=20; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 1.5	S1: 0 & 1 S2: 1 & 1.5	S1: 0 & 1 S2: 1.5 & 1.5	S1: 0 & 1 S2: 2 & 1.5	S1: 0 & 1 S2: 2.5 & 1.5
Ansari-Bradley	0.3394	0.2723	0.1807	0.0907	0.0312
Moses(k=3)	0.2832	0.2859	0.2872	0.2819	0.2869
Moses(k=4)	0.3488	0.3436	0.3439	0.3343	0.3406
Moses(k=6)	0.3378	0.3354	0.3362	0.3377	0.3371

Table 4. Estimated Powers 3 for n1=20, n2=20; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2	S1: 0 & 1 S2: 1 & 2	S1: 0 & 1 S2: 1.5 & 2	S1: 0 & 1 S2: 2 & 2	S1: 0 & 1 S2: 2.5 & 2
Ansari-Bradley	0.6860	0.6276	0.5123	0.3719	0.2310
Moses(k=3)	0.5634	0.5618	0.5721	0.5777	0.5713
Moses(k=4)	0.6631	0.6696	0.6701	0.6796	0.6678
Moses(k=6)	0.6335	0.6401	0.6453	0.6334	0.6359

Table 5. Estimated Powers 4 for n1=20, n2=20; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2.5	S1: 0 & 1 S2: 1 & 2.5	S1: 0 & 1 S2: 1.5 & 2.5	S1: 0 & 1 S2: 2 & 2.5	S1: 0 & 1 S2: 2.5 & 2.5
Ansari-Bradley	0.8640	0.8340	0.7621	0.6499	0.5123
Moses(k=3)	0.7613	0.7612	0.7525	0.7559	0.7560
Moses(k=4)	0.8425	0.8387	0.8365	0.8432	0.8359
Moses(k=6)	0.8196	0.8230	0.8215	0.8193	0.8172

Table 6. Estimated Significance Levels for n1=30, n2=30; normal-distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1	S1: 0 & 1 S2: 0.5 & 1	S1: 0 & 1 S2: 1 & 1	S1: 0 & 1 S2: 1.5 & 1	S1: 0 & 1 S2: 2 & 1	S1: 0 & 1 S2: 2.5 & 1
Ansari-Bradley	0.0492	0.0434	0.0281	0.0074	0.0019	0.0019
Moses(k=3)	0.0523	0.0540	0.0507	0.0501	0.0501	0.0572
Moses(k=4)	0.0477	0.0449	0.0470	0.0512	0.0512	0.0499
Moses(k=6)	0.0523	0.0516	0.0511	0.0479	0.0479	0.0482

Table 7. Estimated Powers 1 for n1=30, n2=30; normal-distributions

Estimated Powers			
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1.5	S1: 0 & 1 S2: 0 & 2	S1: 0 & 1 S2: 0 & 2.5
Ansari-Bradley	0.4976	0.8692	0.9724
Moses(k=3)	0.4596	0.8059	0.9338
Moses(k=4)	0.4515	0.8174	0.9442
Moses(k=6)	0.5174	0.8622	0.9600

Table 8. Estimated Powers 2 for n1=30, n2=30; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 1.5	S1: 0 & 1 S2: 1 & 1.5	S1: 0 & 1 S2: 1.5 & 1.5	S1: 0 & 1 S2: 2 & 1.5	S1: 0 & 1 S2: 2.5 & 1.5
Ansari-Bradley	0.4712	0.3894	0.2640	0.1325	0.0447
Moses(k=3)	0.4494	0.4459	0.4481	0.4450	0.4539
Moses(k=4)	0.4675	0.4594	0.4527	0.4523	0.4475
Moses(k=6)	0.4979	0.5116	0.5085	0.5023	0.5005

Table 9. Estimated Powers 3 for n1=30, n2=30; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2	S1: 0 & 1 S2: 1 & 2	S1: 0 & 1 S2: 1.5 & 2	S1: 0 & 1 S2: 2 & 2	S1: 0 & 1 S2: 2.5 & 2
Ansari-Bradley	0.8502	0.7974	0.6968	0.5446	0.3501
Moses(k=3)	0.7944	0.7992	0.7978	0.8006	0.8057
Moses(k=4)	0.8189	0.8179	0.8128	0.8193	0.8209
Moses(k=6)	0.8611	0.8561	0.8602	0.8615	0.8589

Table 10. Estimated Powers 4 for n1=30, n2=30; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2.5	S1: 0 & 1 S2: 1 & 2.5	S1: 0 & 1 S2: 1.5 & 2.5	S1: 0 & 1 S2: 2 & 2.5	S1: 0 & 1 S2: 2.5 & 2.5
Ansari-Bradley	0.9659	0.9478	0.9108	0.8320	0.7029
Moses(k=3)	0.9389	0.9420	0.9433	0.9403	0.9421
Moses(k=4)	0.9424	0.9432	0.9449	0.9430	0.9460
Moses(k=6)	0.9642	0.9596	0.9632	0.9583	0.9629

Table 11. Estimated Significance Levels for n1=10, n2=10; normal-distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1	S1: 0 & 1 S2: 0.5 & 1	S1: 0 & 1 S2: 1 & 1	S1: 0 & 1 S2: 1.5 & 1	S1: 0 & 1 S2: 2 & 1	S1: 0 & 1 S2: 2.5 & 1
Ansari-Bradley	0.0613	0.0492	0.0334	0.0120	0.0030	0.0004
Moses(k=2)	0.0473	0.0465	0.0488	0.0471	0.0503	0.0458
Moses(k=3)	0.0507	0.0500	0.0507	0.0527	0.0486	0.0506

Table 12. Estimated Powers 1 for n1=10, n2=10; normal-distributions

Estimated Powers			
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1.5	S1: 0 & 1 S2: 0 & 2	S1: 0 & 1 S2: 0 & 2.5
Ansari-Bradley	0.2406	0.4412	0.6101
Moses(k=2)	0.1475	0.2799	0.3869
Moses(k=3)	0.1873	0.3289	0.4656

Table 13. Estimated Powers 2 for n1=10, n2=10; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 1.5	S1: 0 & 1 S2: 1 & 1.5	S1: 0 & 1 S2: 1.5 & 1.5	S1: 0 & 1 S2: 2 & 1.5	S1: 0 & 1 S2: 2.5 & 1.5
Ansari-Bradley	0.2175	0.1794	0.1169	0.0646	0.0265
Moses(k=2)	0.1501	0.1512	0.1521	0.1465	0.1536
Moses(k=3)	0.1784	0.1747	0.1759	0.1786	0.1768

Table 14. Estimated Powers 3 for n1=10, n2=10; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2	S1: 0 & 1 S2: 1 & 2	S1: 0 & 1 S2: 1.5 & 2	S1: 0 & 1 S2: 2 & 2	S1: 0 & 1 S2: 2.5 & 2
Ansari-Bradley	0.4259	0.3754	0.2978	0.2066	0.1331
Moses(k=2)	0.2731	0.2718	0.2826	0.2884	0.2853
Moses(k=3)	0.3377	0.3352	0.3336	0.3290	0.3337

Table 15. Estimated Powers 4 for n1=10, n2=10; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2.5	S1: 0 & 1 S2: 1 & 2.5	S1: 0 & 1 S2: 1.5 & 2.5	S1: 0 & 1 S2: 2 & 2.5	S1: 0 & 1 S2: 2.5 & 2.5
Ansari-Bradley	0.6038	0.5459	0.4697	0.3880	0.2805
Moses(k=2)	0.3840	0.3829	0.3939	0.3823	0.3848
Moses(k=3)	0.4675	0.4663	0.4719	0.4648	0.4697

Table 16. Estimated Significance Levels for n1=20, n2=30; normal-distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1	S1: 0 & 1 S2: 0.5 & 1	S1: 0 & 1 S2: 1 & 1	S1: 0 & 1 S2: 1.5 & 1	S1: 0 & 1 S2: 2 & 1	S1: 0 & 1 S2: 2.5 & 1
Ansari-Bradley	0.0481	0.0349	0.0122	0.0013	0.0000	0.0000
Moses(k=3)	0.0498	0.0462	0.0479	0.0479	0.0493	0.0470
Moses(k=4)	0.0536	0.0503	0.0530	0.0547	0.0506	0.0480
Moses(k=6)	0.0481	0.0478	0.0441	0.0444	0.0471	0.0479

Table 17. Estimated Powers 1 for n1=20, n2=30; normal-distributions

Estimated Powers			
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1.5	S1: 0 & 1 S2: 0 & 2	S1: 0 & 1 S2: 0 & 2.5
Ansari-Bradley	0.4235	0.7910	0.9378
Moses(k=3)	0.3371	0.6797	0.8645
Moses(k=4)	0.4075	0.7579	0.9152
Moses(k=6)	0.5067	0.8590	0.9595

Table 18. Estimated Powers 2 for n1=20, n2=30; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 1.5	S1: 0 & 1 S2: 1 & 1.5	S1: 0 & 1 S2: 1.5 & 1.5	S1: 0 & 1 S2: 2 & 1.5	S1: 0 & 1 S2: 2.5 & 1.5
Ansari-Bradley	0.3701	0.2413	0.0939	0.0201	0.0021
Moses(k=3)	0.3395	0.2413	0.3372	0.3382	0.3343
Moses(k=4)	0.4026	0.4031	0.4067	0.4082	0.4132
Moses(k=6)	0.5070	0.5062	0.5042	0.5019	0.5129

Table 19. Estimated Powers 3 for n1=20, n2=30; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2	S1: 0 & 1 S2: 1 & 2	S1: 0 & 1 S2: 1.5 & 2	S1: 0 & 1 S2: 2 & 2	S1: 0 & 1 S2: 2.5 & 2
Ansari-Bradley	0.7569	0.6395	0.4279	0.2068	0.0653
Moses(k=3)	0.6679	0.6794	0.6758	0.6754	0.6694
Moses(k=4)	0.7666	0.7705	0.7575	0.7630	0.7586
Moses(k=6)	0.8591	0.8581	0.8557	0.8579	0.8607

Table 20. Estimated Powers 4 for n1=20, n2=30; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2.5	S1: 0 & 1 S2: 1 & 2.5	S1: 0 & 1 S2: 1.5 & 2.5	S1: 0 & 1 S2: 2 & 2.5	S1: 0 & 1 S2: 2.5 & 2.5
Ansari-Bradley	0.9222	0.8591	0.7352	0.5348	0.2950
Moses(k=3)	0.8594	0.8659	0.8583	0.8665	0.8626
Moses(k=4)	0.9162	0.9155	0.9153	0.9164	0.9177
Moses(k=6)	0.9616	0.9587	0.9632	0.9606	0.9620

Table 21. Estimated Significance Levels for n1=10, n2=20; normal-distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1	S1: 0 & 1 S2: 0.5 & 1	S1: 0 & 1 S2: 1 & 1	S1: 0 & 1 S2: 1.5 & 1	S1: 0 & 1 S2: 2 & 1	S1: 0 & 1 S2: 2.5 & 1
Ansari-Bradley	0.0492	0.0349	0.0119	0.0014	0.0001	0.0000
Moses(k=2)	0.0478	0.0512	0.0528	0.0494	0.0513	0.0485
Moses(k=3)	0.0464	0.0468	0.0470	0.0506	0.0467	0.0488

Table 22. Estimated Powers 1 for n1=10, n2=20; normal-distributions

Estimated Powers			
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1.5	S1: 0 & 1 S2: 0 & 2	S1: 0 & 1 S2: 0 & 2.5
Ansari-Bradley	0.2637	0.5450	0.7413
Moses(k=2)	0.1758	0.3482	0.4912
Moses(k=3)	0.2044	0.4047	0.5622

Table 23. Estimated Powers 2 for n1=10, n2=20; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 1.5	S1: 0 & 1 S2: 1 & 1.5	S1: 0 & 1 S2: 1.5 & 1.5	S1: 0 & 1 S2: 2 & 1.5	S1: 0 & 1 S2: 2.5 & 1.5
Ansari-Bradley	0.2329	0.1366	0.0507	0.0094	0.0013
Moses(k=2)	0.1822	0.1751	0.1786	0.1750	0.1742
Moses(k=3)	0.1976	0.1954	0.1917	0.1997	0.2004

Table 24. Estimated Powers 3 for n1=10, n2=20; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2	S1: 0 & 1 S2: 1 & 2	S1: 0 & 1 S2: 1.5 & 2	S1: 0 & 1 S2: 2 & 2	S1: 0 & 1 S2: 2.5 & 2
Ansari-Bradley	0.5003	0.3726	0.2093	0.0846	0.0243
Moses(k=2)	0.3434	0.3410	0.3462	0.3369	0.3404
Moses(k=3)	0.3951	0.3981	0.4009	0.3958	0.3974

Table 25. Estimated Powers 4 for n1=10, n2=20; normal-distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0.5 & 2.5	S1: 0 & 1 S2: 1 & 2.5	S1: 0 & 1 S2: 1.5 & 2.5	S1: 0 & 1 S2: 2 & 2.5	S1: 0 & 1 S2: 2.5 & 2.5
Ansari-Bradley	0.7023	0.5934	0.4363	0.2560	0.1180
Moses(k=2)	0.4981	0.4967	0.4984	0.4990	0.4996
Moses(k=3)	0.5680	0.5599	0.5594	0.5653	0.5640

Table 26. Estimated Significance Levels for n1=20, n2=10; normal-distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & 1 S2: 0 & 1	S1: 0 & 1 S2: 0.5 & 1	S1: 0 & 1 S2: 1 & 1	S1: 0 & 1 S2: 1.5 & 1	S1: 0 & 1 S2: 2 & 1	S1: 0 & 1 S2: 2.5 & 1
Ansari-Bradley	0.0502	0.0574	0.0846	0.1542	0.3295	0.6264
Moses(k=2)	0.0487	0.0479	0.0497	0.0501	0.0543	0.0487
Moses(k=3)	0.0462	0.0511	0.0491	0.0470	0.0469	0.0485

Results for the t-distribution with 3 degrees of freedom

Results for the t-distribution with 3 degrees of freedom are given in Tables 27-42.The comparison results of significance level for the t-distribution with 3 degrees of freedom were similar to that of normal distribution. When the two location parameters were equal, both the Ansari-Bradley test and Moses test had significance levels of around 0.05. When they became different, significance levels for the Moses test were always around 0.05, while the Ansari-Bradley test had decreasing significance levels for all the sample sizes other than unequal sample sizes of 20 and 10, for which the significance levels were increasing. Therefore, we do not consider the unequal sample of sizes 20 and 10 in power comparisons for the Ansari-Bradley test.

The power comparison results for the t-distribution with 3 degrees of freedom are as follows. When the equal-median assumption was satisfied, the Ansari-Bradley test always has the highest estimated power for all the sample size we considered, including equal sample of sizes 10, 20 and 30, as well as unequal sample of sizes 10 and 20, and 20 and 30. When two location parameters are different in both situations where the second population had 2 times and 3 times the standard deviation of the first sample, and as the difference between the two location parameters was becoming larger, the estimated power of the Ansari-Bradley test was decreasing while the estimated power for the Moses test hardly had any variation; the Ansari-Bradley test eventually became less powerful than the Moses test for large differences between the location parameters. For the comparison among Moses tests with different subset sizes k, we see that the test with larger k generally had higher estimated powers. However, for equal samples of size 30, the estimated powers for the test with k=4 were close to that for the test with k=6.

Table 27. Estimated Significance Levels for n1=20, n2=20; t-distributions with 3 degrees of freedom

Estimated Significance Levels					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & σ^2	S1: 0 & σ^2 S2: 1 & σ^2	S1: 0 & σ^2 S2: 1.5 & σ^2	S1: 0 & σ^2 S2: 2 & σ^2	S1: 0 & σ^2 S2: 3 & σ^2
Ansari-Bradley	0.0475	0.0429	0.0394	0.0298	0.0147
Moses(k=3)	0.0469	0.0470	0.0466	0.0466	0.0456
Moses(k=4)	0.0482	0.0489	0.0445	0.0445	0.0491
Moses(k=6)	0.0487	0.0515	0.0469	0.0469	0.0490

Table 28. Estimated Powers 1 for n1=20, n2=20; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $4\sigma^2$	S1: 0 & σ^2 S2: 1 & $4\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $4\sigma^2$	S1: 0 & σ^2 S2: 2 & $4\sigma^2$	S1: 0 & σ^2 S2: 3 & $4\sigma^2$
Ansari-Bradley	0.5937	0.5471	0.4911	0.4245	0.2746
Moses(k=3)	0.4409	0.4271	0.4378	0.4468	0.4375
Moses(k=4)	0.4852	0.4780	0.4823	0.4824	0.4802
Moses(k=6)	0.6148	0.6254	0.6147	0.6132	0.6160

Table 29. Estimated Powers 2 for n1=20, n2=20; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $9\sigma^2$	S1: 0 & σ^2 S2: 1 & $9\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $9\sigma^2$	S1: 0 & σ^2 S2: 2 & $9\sigma^2$	S1: 0 & σ^2 S2: 3 & $9\sigma^2$
Ansari-Bradley	0.8922	0.8758	0.8357	0.7866	0.6613
Moses(k=3)	0.8922	0.7538	0.7529	0.7490	0.7462
Moses(k=4)	0.7877	0.7898	0.7933	0.7909	0.7462
Moses(k=6)	0.9016	0.9078	0.9044	0.7909	0.9130

Table 30. Estimated Significance Levels for n1=30, n2=30; t-distributions with 3 degrees of freedom

Estimated Significance Levels					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & σ^2	S1: 0 & σ^2 S2: 1 & σ^2	S1: 0 & σ^2 S2: 1.5 & σ^2	S1: 0 & σ^2 S2: 2 & σ^2	S1: 0 & σ^2 S2: 3 & σ^2
Ansari-Bradley	0.0504	0.0453	0.0362	0.0291	0.0136
Moses(k=3)	0.0566	0.0562	0.0515	0.0546	0.0136
Moses(k=4)	0.0488	0.0471	0.0488	0.0437	0.0469
Moses(k=6)	0.0461	0.0446	0.0476	0.0506	0.0487

Table 31. Estimated Powers 1 for n1=30, n2=30; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $4\sigma^2$	S1: 0 & σ^2 S2: 1 & $4\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $4\sigma^2$	S1: 0 & σ^2 S2: 2 & $4\sigma^2$	S1: 0 & σ^2 S2: 3 & $4\sigma^2$
Ansari-Bradley	0.7592	0.7097	0.6496	0.5731	0.3815
Moses(k=3)	0.6626	0.6563	0.6595	0.6609	0.6596
Moses(k=4)	0.6353	0.6201	0.6230	0.6261	0.6264
Moses(k=6)	0.6199	0.6200	0.621	0.6276	0.6154

Table 32. Estimated Powers 2 for n1=30, n2=30; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $9\sigma^2$	S1: 0 & σ^2 S2: 1 & $9\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $9\sigma^2$	S1: 0 & σ^2 S2: 2 & $9\sigma^2$	S1: 0 & σ^2 S2: 3 & $9\sigma^2$
Ansari-Bradley	0.9795	0.9631	0.9480	0.9214	0.8205
Moses(k=3)	0.9357	0.9421	0.9338	0.9389	0.9351
Moses(k=4)	0.9217	0.9193	0.9163	0.9172	0.9149
Moses(k=6)	0.9038	0.9032	0.9043	0.9024	0.9039

Table 33. Estimated Significance Levels for n1=10, n2=10; t-distributions with 3 degrees of freedom

Estimated Significance Levels					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & σ^2	S1: 0 & σ^2 S2: 1 & σ^2	S1: 0 & σ^2 S2: 1.5 & σ^2	S1: 0 & σ^2 S2: 2 & σ^2	S1: 0 & σ^2 S2: 3 & σ^2
Ansari-Bradley	0.0529	0.0486	0.0431	0.0313	0.0144
Moses(k=2)	0.0486	0.0506	0.0476	0.0528	0.0481
Moses(k=3)	0.0487	0.0506	0.0504	0.0486	0.0503

Table 34. Estimated Powers 1 for n1=10, n2=10; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $4\sigma^2$	S1: 0 & σ^2 S2: 1 & $4\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $4\sigma^2$	S1: 0 & σ^2 S2: 2 & $4\sigma^2$	S1: 0 & σ^2 S2: 3 & $4\sigma^2$
Ansari-Bradley	0.3670	0.3247	0.2999	0.2629	0.1658
Moses(k=2)	0.2329	0.2441	0.2383	0.2329	0.2379
Moses(k=3)	0.2617	0.2595	0.2679	0.2623	0.2637

Table 35. Estimated Powers 2 for n1=10, n2=10; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $9\sigma^2$	S1: 0 & σ^2 S2: 1 & $9\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $9\sigma^2$	S1: 0 & σ^2 S2: 2 & $9\sigma^2$	S1: 0 & σ^2 S2: 3 & $9\sigma^2$
Ansari-Bradley	0.6253	0.6034	0.5561	0.5129	0.3915
Moses(k=2)	0.4187	0.4257	0.4214	0.4231	0.4216
Moses(k=3)	0.4557	0.4633	0.4702	0.4534	0.4725

Table 36. Estimated Significance Levels for n1=20, n2=30; t-distributions with 3 degrees of freedom

Estimated Significance Levels					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & σ^2	S1: 0 & σ^2 S2: 1 & σ^2	S1: 0 & σ^2 S2: 1.5 & σ^2	S1: 0 & σ^2 S2: 2 & σ^2	S1: 0 & σ^2 S2: 3 & σ^2
Ansari-Bradley	0.0484	0.0237	0.0106	0.0031	0.0001
Moses(k=3)	0.0450	0.0474	0.0445	0.0490	0.0470
Moses(k=4)	0.0522	0.0541	0.0529	0.0536	0.0565
Moses(k=6)	0.0475	0.0449	0.0475	0.0470	0.0445

Table 37. Estimated Powers 1 for n1=20, n2=30; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $4\sigma^2$	S1: 0 & σ^2 S2: 1 & $4\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $4\sigma^2$	S1: 0 & σ^2 S2: 2 & $4\sigma^2$	S1: 0 & σ^2 S2: 3 & $4\sigma^2$
Ansari-Bradley	0.6702	0.5617	0.4366	0.2930	0.0909
Moses(k=3)	0.5122	0.5277	0.5238	0.5239	0.5147
Moses(k=4)	0.5590	0.5740	0.5774	0.5767	0.5643
Moses(k=6)	0.6150	0.6225	0.6243	0.6214	0.6179

Table 38. Estimated Powers 2 for n1=20, n2=30; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $9\sigma^2$	S1: 0 & σ^2 S2: 1 & $9\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $9\sigma^2$	S1: 0 & σ^2 S2: 2 & $9\sigma^2$	S1: 0 & σ^2 S2: 3 & $9\sigma^2$
Ansari-Bradley	0.9462	0.9071	0.8501	0.7606	0.4908
Moses(k=3)	0.8465	0.8402	0.8372	0.8466	0.8496
Moses(k=4)	0.8718	0.8823	0.8712	0.8523	0.8496
Moses(k=6)	0.9064	0.8823	0.8985	0.8989	0.9020

Table 39. Estimated Significance Levels for n1=10, n2=20; t-distributions with 3 degrees of freedom

Estimated Significance Levels					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & σ^2	S1: 0 & σ^2 S2: 1 & σ^2	S1: 0 & σ^2 S2: 1.5 & σ^2	S1: 0 & σ^2 S2: 2 & σ^2	S1: 0 & σ^2 S2: 3 & σ^2
Ansari-Bradley	0.0517	0.0209	0.0086	0.0026	0.0000
Moses(k=2)	0.0498	0.0492	0.0494	0.0528	0.0475
Moses(k=3)	0.0483	0.0535	0.0462	0.0466	0.0500

Table 40. Estimated Powers 1 for n1=10, n2=20; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $4\sigma^2$	S1: 0 & σ^2 S2: 1 & $4\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $4\sigma^2$	S1: 0 & σ^2 S2: 2 & $4\sigma^2$	S1: 0 & σ^2 S2: 3 & $4\sigma^2$
Ansari-Bradley	0.4332	0.3348	0.2310	0.1413	0.0349
Moses(k=2)	0.2998	0.3033	0.2993	0.3024	0.2977
Moses(k=3)	0.3142	0.3154	0.3136	0.3118	0.3200

Table 41. Estimated Powers 2 for n1=10, n2=20; t-distributions with 3 degrees of freedom

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & $9\sigma^2$	S1: 0 & σ^2 S2: 1 & $9\sigma^2$	S1: 0 & σ^2 S2: 1.5 & $9\sigma^2$	S1: 0 & σ^2 S2: 2 & $9\sigma^2$	S1: 0 & σ^2 S2: 3 & $9\sigma^2$
Ansari-Bradley	0.7587	0.6734	0.5719	0.4648	0.2387
Moses(k=2)	0.5568	0.5461	0.5427	0.5354	0.5555
Moses(k=3)	0.5590	0.5749	0.5687	0.5681	0.5803

Table 42. Estimated Significance Levels for n1=20, n2=10; t-distributions with 3 degrees of freedom

Estimated Significance Levels					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 0 & σ^2 S2: 0 & σ^2	S1: 0 & σ^2 S2: 1 & σ^2	S1: 0 & σ^2 S2: 1.5 & σ^2	S1: 0 & σ^2 S2: 2 & σ^2	S1: 0 & σ^2 S2: 3 & σ^2
Ansari-Bradley	0.0522	0.0897	0.1521	0.2417	0.5067
Moses(k=2)	0.0524	0.0472	0.0510	0.0517	0.0486
Moses(k=3)	0.0459	0.0505	0.0472	0.0480	0.0475

Results for the exponential distribution

Results for the exponential distribution are given in Tables 43-66. In the exponential distribution case, when two location parameters were the same, as is similar to the other two distributions, both tests had significance levels of around 0.05. In the situation where the two location parameters were not equal, significance levels of the Moses test were around 0.05, while that of the Ansari-Bradley test were decreasing as the difference between the two parameters became greater.

Power comparison results for the exponential distribution are as follows. When the two location parameters of the two populations were equal, the Ansari-Bradley test had higher estimated powers than the Moses test did. When the difference between the two location parameters was getting greater, the estimated power of the Ansari-Bradley test was decreasing while the estimated power of Moses remained almost the same; when the second population had a standard deviation of 1.5 and then 2 times the standard deviation of the first population, the Ansari-Bradley test eventually became less powerful than Moses test as the differences between the two location parameters became large. For instance, in the situation where the second population had a standard deviation of 1.5 times the first, when the difference between the two location parameters was 0.2, the estimated power of the Ansari-Bradley test was 0.5646, which was higher than 0.2935, the estimated power of Moses test with subset size k of 3. As the difference became 0.3, the Ansari-Bradley test still had the highest estimated power of 0.4167. However, when the difference was 0.4, the Ansari-Bradley test had an estimated power of 0.2479, which is less than 0.2870, the estimated power of Moses test with k equal to 3. As we compare the estimated powers of the Moses test using different subset sizes k, we see from the simulation results that, for equal samples of size 10 and unequal samples of sizes 10 and 20, and 20 and 10, the Moses test when k=3 is more powerful than the Moses test when k=2. The estimated powers for both tests when k=3 and k=2 were almost the same for the sample sizes of 10 and 20. However, for relatively large sample sizes including equal sample sizes of 20 and 30, and unequal sample sizes of 20 and 30, in which we chose k of 3, 4, and 6, the Moses test with k of 4 had the highest estimated power among the three for sample sizes of 20 and 20, and 20 and 30. For equal samples of size 30, the test with k equal to 3 seemed to have higher power than with k of 4 and 6; however, this estimated power difference was very small.

Table 43. Estimated Significance Levels for n1=20, n2=20; Exponential distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 &1 S2: 1 &1	S1: 1 &1 S2: 1.02 &1	S1: 1 &1 S2: 1.05 &1	S1: 1 &1 S2: 1.1 &1	S1: 1 &1 S2: 1.2 &1	S1: 1 &1 S2: 1.5 &1
Ansari-Bradley	0.0477	0.0425	0.0285	0.0187	0.0058	0.0000
Moses(k=3)	0.0478	0.0501	0.0459	0.045	0.0456	0.0467
Moses(k=4)	0.0491	0.0479	0.0476	0.045	0.0502	0.0457
Moses(k=6)	0.0508	0.0531	0.0502	0.049	0.0500	0.0462

Table 44. Estimated Powers 1 for n1=20, n2=20; Exponential distributions

Estimated Powers				
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.2^2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1 & 4^2
Ansari-Bradley	0.2503	0.6028	0.8745	0.9952
Moses(k=3)	0.0936	0.1915	0.3856	0.8279
Moses(k=4)	0.0978	0.2136	0.4200	0.8743
Moses(k=6)	0.1003	0.1907	0.3683	0.8092

Table 45. Estimated Powers 2 for n1=20, n2=20; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1.2 & 1.5^2	S1: 1 & 1 S2: 1.3 & 1.5^2	S1: 1 & 1 S2: 1.4 &1.5^2	S1: 1 & 1 S2: 1.5 & 1.5^2
Ansari-Bradley	0.6028	0.4179	0.3057	0.1868	0.0988
Moses(k=3)	0.1915	0.1879	0.1871	0.1885	0.1846
Moses(k=4)	0.2136	0.2112	0.2074	0.2153	0.2106
Moses(k=6)	0.1907	0.1897	0.2007	0.1926	0.1942

Table 46. Estimated Powers 3 for n1=20, n2=20; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1.5 & 2^2	S1: 1 & 1 S2: 1.7 & 2^2	S1: 1 & 1 S2: 1.8 & 2^2	S1: 1 & 1 S2: 2 & 2^2
Ansari-Bradley	0.8745	0.6619	0.4763	0.3626	0.1370
Moses(k=3)	0.3856	0.3762	0.3692	0.3742	0.3762
Moses(k=4)	0.4200	0.4224	0.4221	0.4131	0.4195
Moses(k=6)	0.3683	0.3606	0.3762	0.3636	0.3709

Table 47. Estimated Significance Levels for n1=30, n2=30; Exponential distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 &1 S2: 1 &1	S1: 1 & 1 S2: 1.02 & 1	S1: 1 &1 S2: 1.05 &1	S1: 1 &1 S2: 1.1 &1	S1: 1 &1 S2: 1.2 &1	S1: 1 &1 S2: 1.5 &1
Ansari-Bradley	0.0533	0.0398	0.0257	0.0116	0.0034	0.0000
Moses(k=3)	0.0535	0.0509	0.0505	0.0506	0.0527	0.0542
Moses(k=4)	0.0492	0.0515	0.0486	0.0520	0.0480	0.0457
Moses(k=6)	0.0483	0.0491	0.0454	0.0460	0.0501	0.0461

Table 48. Estimated Powers 1 for n1=30, n2=30; Exponential distributions

Estimated Powers				
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.2^2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1 & 4^2
Ansari-Bradley	0.3361	0.7717	0.9682	1.0000
Moses(k=3)	0.1302	0.2922	0.5765	0.9686
Moses(k=4)	0.1132	0.2691	0.5551	0.9634
Moses(k=6)	0.1183	0.2730	0.5680	0.9676

Table 49. Estimated Powers 2 for n1=30, n2=30; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1.2 & 1.5^2	S1: 1 & 1 S2: 1.3 & 1.5^2	S1: 1 & 1 S2: 1.4 & 1.5^2	S1: 1 & 1 S2: 1.5 & 1.5^2
Ansari-Bradley	0.7717	0.5646	0.4167	0.2479	0.1202
Moses(k=3)	0.2922	0.2935	0.2951	0.2870	0.2945
Moses(k=4)	0.2691	0.2748	0.2770	0.2732	0.2717
Moses(k=6)	0.2730	0.2750	0.2757	0.2845	0.2751

Table 50. Estimated Powers 3 for n1=30, n2=30; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1.5 & 2^2	S1: 1 & 1 S2: 1.7 & 2^2	S1: 1 & 1 S2: 1.8 & 2^2	S1: 1 & 1 S2: 2 & 2^2
Ansari-Bradley	0.9682	0.8248	0.6309	0.4881	0.1846
Moses(k=3)	0.5765	0.5833	0.5863	0.5812	0.5760
Moses(k=4)	0.5551	0.5502	0.5446	0.5461	0.5544
Moses(k=6)	0.5680	0.5664	0.5593	0.5710	0.5683

Table 51. Estimated Significance Levels for n1=10, n2=10; Exponential distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 &1 S2: 1 &1	S1: 1 & 1 S2: 1.02 & 1	S1: 1 &1 S2: 1.05 &1	S1: 1 &1 S2: 1.1 &1	S1: 1 &1 S2: 1.2 &1	S1: 1 &1 S2: 1.5 &1
Ansari-Bradley	0.0546	0.0487	0.0388	0.0287	0.0126	0.0012
Moses(k=2)	0.0471	0.0457	0.0464	0.0490	0.0513	0.0459
Moses(k=3)	0.0506	0.0502	0.0505	0.0499	0.0466	0.0489

Table 52. Estimated Powers 1 for n1=10, n2=10; Exponential distributions

Estimated Powers				
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.2^2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1 & 4^2
Ansari-Bradley	0.1801	0.3762	0.6064	0.8737
Moses(k=2)	0.0741	0.1277	0.2056	0.4965
Moses(k=3)	0.0800	0.1323	0.2293	0.5276

Table 53. Estimated Powers 2 for n1=10, n2=10; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1.2 & 1.5^2	S1: 1 & 1 S2: 1.3 & 1.5^2	S1: 1 & 1 S2: 1.4 & 1.5^2	S1: 1 & 1 S2: 1.5 & 1.5^2
Ansari-Bradley	0.3762	0.2662	0.2115	0.1511	0.0903
Moses(k=2)	0.1277	0.1224	0.1233	0.1213	0.1242
Moses(k=3)	0.1323	0.1287	0.1242	0.1311	0.1326

Table 54. Estimated Powers 3 for n1=10, n2=10; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1.5 & 2^2	S1: 1 & 1 S2: 1.7 & 2^2	S1: 1 & 1 S2: 1.8 & 2^2	S1: 1 & 1 S2: 2 & 2^2
Ansari-Bradley	0.6064	0.4163	0.3059	0.2356	0.1032
Moses(k=2)	0.2056	0.2035	0.2120	0.2047	0.2077
Moses(k=3)	0.2293	0.2139	0.2255	0.2238	0.2223

Table 55. Estimated Significance Levels for n1=20, n2=30; Exponential distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 &1 S2: 1 &1	S1: 1 &1 S2: 1.02 &1	S1: 1 &1 S2: 1.05 &1	S1: 1 &1 S2: 1.1 &1	S1: 1 &1 S2: 1.2 &1	S1: 1 &1 S2: 1.5 &1
Ansari-Bradley	0.0471	0.0390	0.0300	0.0145	0.0044	0.0001
Moses(k=3)	0.0429	0.0472	0.0501	0.0478	0.0498	0.0481
Moses(k=4)	0.0523	0.0582	0.0552	0.0551	0.0510	0.0544
Moses(k=6)	0.0357	0.0382	0.0337	0.0356	0.0363	0.0339

Table 56. Estimated Powers 1 for n1=20, n2=30; Exponential distributions

Estimated Powers				
Median and σ S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.2^2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1 & 4^2
Ansari-Bradley	0.2882	0.6830	0.9291	0.9935
Moses(k=3)	0.1016	0.2182	0.4434	0.9183
Moses(k=4)	0.1157	0.2473	0.5100	0.9417
Moses(k=6)	0.0758	0.1784	0.3765	0.8627

Table 57. Estimated Powers 2 for n1=20, n2=30; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1.2 & 1.5^2	S1: 1 & 1 S2: 1.3 & 1.5^2	S1: 1 & 1 S2: 1.4 & 1.5^2	S1: 1 & 1 S2: 1.5 & 1.5^2
Ansari-Bradley	0.6830	0.4775	0.3510	0.2117	0.0986
Moses(k=3)	0.2182	0.2134	0.2146	0.2224	0.2200
Moses(k=4)	0.2473	0.2586	0.2483	0.2597	0.2568
Moses(k=6)	0.1784	0.1796	0.1810	0.1738	0.1699

Table 58. Estimated Powers 3 for n1=20, n2=30; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1.5 & 2^2	S1: 1 & 1 S2: 1.7 & 2^2	S1: 1 & 1 S2: 1.8 & 2^2	S1: 1 & 1 S2: 2 & 2^2
Ansari-Bradley	0.9291	0.7385	0.5217	0.3844	0.1196
Moses(k=3)	0.4434	0.4510	0.4400	0.4453	0.4442
Moses(k=4)	0.5100	0.4983	0.4986	0.4936	0.4950
Moses(k=6)	0.3765	0.3835	0.3758	0.3766	0.3791

Table 59. Estimated Significance Levels for n1=10, n2=20; Exponential distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 &1 S2: 1 &1	S1: 1 &1 S2: 1.02 &1	S1: 1 &1 S2: 1.05 &1	S1: 1 &1 S2: 1.1 &1	S1: 1 &1 S2: 1.2 &1	S1: 1 & 1 S2: 1.5 & 1
Ansari-Bradley	0.0473	0.0436	0.0360	0.0255	0.0116	0.0009
Moses(k=2)	0.0500	0.0485	0.0500	0.0491	0.0484	0.0473
Moses(k=3)	0.0478	0.0486	0.0455	0.0470	0.0467	0.0451

Table 60. Estimated Powers 1 for n1=10, n2=20; Exponential distributions

Estimated Powers				
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.2^2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1 & 4^2
Ansari-Bradley	0.1798	0.4535	0.7121	0.9205
Moses(k=2)	0.0822	0.1462	0.2514	0.6445
Moses(k=3)	0.0821	0.1456	0.2644	0.6428

Table 61. Estimated Powers 2 for n1=10, n2=20; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1.2 & 1.5^2	S1: 1 & 1 S2: 1.3 & 1.5^2	S1: 1 & 1 S2: 1.4 & 1.5^2	S1: 1 & 1 S2: 1.5 & 1.5^2
Ansari-Bradley	0.4535	0.3098	0.2244	0.1505	0.0786
Moses(k=2)	0.1462	0.1395	0.1363	0.1413	0.1465
Moses(k=3)	0.1456	0.1389	0.1417	0.1417	0.1416

Table 62. Estimated Powers 3 for n1=10, n2=20; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1.5 & 2^2	S1: 1 & 1 S2: 1.7 & 2^2	S1: 1 & 1 S2: 1.8 & 2^2	S1: 1 & 1 S2: 2 & 2^2
Ansari-Bradley	0.7121	0.5050	0.3316	0.2435	0.0890
Moses(k=2)	0.2514	0.2561	0.2581	0.2567	0.2507
Moses(k=3)	0.2644	0.2689	0.2657	0.2517	0.2673

Table 63. Estimated Significance Levels for n1=20, n2=10; Exponential distributions

Estimated Significance Levels						
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 &1 S2: 1 &1	S1: 1 &1 S2: 1.02 &1	S1: 1 &1 S2: 1.05 &1	S1: 1 &1 S2: 1.1 &1	S1: 1 &1 S2: 1.2 &1	S1: 1 & 1 S2: 1.5 & 1
Ansari-Bradley	0.0491	0.0455	0.0299	0.0188	0.0071	0.0009
Moses(k=2)	0.0477	0.0510	0.0471	0.0484	0.0518	0.0482
Moses(k=3)	0.0491	0.0501	0.0463	0.0476	0.0451	0.0451

Table 64. Estimated Powers 1 for n1=20, n2=10; Exponential distributions

Estimated Powers				
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.2^2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1 & 4^2
Ansari-Bradley	0.2152	0.4855	0.7331	0.9700
Moses(k=2)	0.0890	0.1538	0.2763	0.6261
Moses(k=3)	0.0866	0.1625	0.2963	0.6690

Table 65. Estimated Powers 2 for n1=20, n2=10; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 1.5^2	S1: 1 & 1 S2: 1.2 & 1.5^2	S1: 1 & 1 S2: 1.3 & 1.5^2	S1: 1 & 1 S2: 1.4 & 1.5^2	S1: 1 & 1 S2: 1.5 & 1.5^2
Ansari-Bradley	0.4855	0.3294	0.2445	0.1705	0.1006
Moses(k=2)	0.1538	0.1522	0.1494	0.1536	0.1586
Moses(k=3)	0.1625	0.1614	0.1548	0.1592	0.1597

Table 66. Estimated Powers 3 for n1=20, n2=10; Exponential distributions

Estimated Powers					
Median and σ^2 S1: sample 1 S2: sample 2	S1: 1 & 1 S2: 1 & 2^2	S1: 1 & 1 S2: 1.5 & 2^2	S1: 1 & 1 S2: 1.7 & 2^2	S1: 1 & 1 S2: 1.8 & 2^2	S1: 1 & 1 S2: 2 & 2^2
Ansari-Bradley	0.7331	0.5147	0.3778	0.3149	0.1540
Moses(k=2)	0.2763	0.2784	0.2802	0.2756	0.2776
Moses(k=3)	0.2963	0.2930	0.2937	0.2964	0.2898

CHAPTER 5.

CONCLUSIONS

When the two populations have equal location parameters, the Ansari- Bradley test is preferred to the Moses test as it is more powerful than the Moses test. When the two location parameters are different, it is better to use the Moses test than the Ansari-Bradley test. This is especially true when the difference between location parameters becomes large. The significance level of the Ansari-Bradley test becomes unstable or very small.

Next we discuss how to choose the subset size k when using the Moses test. If the underlying distribution is symmetric, the Moses test with larger subset size k seems more powerful than that with smaller k. If the underlying distribution is not symmetric, when the sample size is relatively small, such as the exponential distribution in our simulation study with equal sample sizes of 10 and unequal sample sizes of 20 and 10 and 10 and 20, the Moses test with larger k generally has higher powers than with smaller k. However, when the sample sizes are relatively large, it seems the Moses test with the medium k is the most powerful. For instance, in our situation where both sample sizes are 20 or 30, or the two sample sizes are 20 and 30 respectively, the Moses test with medium k of 4 is generally the most powerful among the three subset sizes k of 3, 4, and 6. Therefore, we may try a middle value of k when the underlying distribution is not symmetric and the sample sizes are relatively large.

This research only considered three types of underlying distributions: normal; exponential; and t-distribution. The sample sizes considered were various combinations of 10, 20, and 30. Results could vary with different distributions and sample sizes.

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APPENDIX. EXAMPLE R CODE FOR NORMAL DISTRIBUTIONS

****

The following R code generates random data from normal distributions with unequal sample samples of size 20 and 30 for the estimated powers of the Ansari-Bradley test and Moses test with subset size k of 3, 4 and 6. The means for the two populations are 0 and 1, and the variances for the two populations are 1 and 2.

****

 # R Code for the Ansari-Bradley Test

```
Tstar = rep(NA, m)
 m <- 10000
 for(j in 1:m){
 n1 <- 20
 n2 <- 30
 n <- n1+n2
 s1 <- rnorm(n1, 0, 1)
 s2 <- rnorm(n2, 0.5, 2)
 s <- append(s1, s2)
 r <- rank(s)
 for(i in 1:n1)
```

```
  if(r[i] > (n/2)) r[i] = n+1-r[i]
T <- sum(r[1:n1])
Nu <- T-(n1*(n+2)/4)
De <- sqrt(n1*n2*(n+2)*(n-2)/48/(n-1))
Tstar[j] <- Nu/ De
}
sum(Tstar >= qnorm(.95)) / m
```

 # R Code for the Moses Test with k=3

```
m <- 10000
 Tstar = rep(NA, m)
 for(p in 1:m) {
 n1 <- 20
 k <- 3
 s1 <- rnorm(n1, 0, 1)
 id3 <- sample(1:n1)
 id1 <- sample(id3, n1-2)
 ind1 <- rep(1:k, each = length(id1)/k)
 A1 <- matrix(rep(NA, length(id1)), ncol = k)
 for(i in 1:k){
   A1[,i] <- s1[id1[ind1==i]]
  }
 B1 <- matrix(rep(NA,length(id1)), ncol = k)
 C <- rep(0, length(id1)/k)
```

```
for(i in 1:(length(id1)/k)){
 for(j in 1:k){
  B1[i,j] = (A1[i,j] - mean(A1[i,]))*(A1[i,j] - mean(A1[i,]))
C[i] = sum(B1[i,])
}
}
n2 <- 30
s2 <- rnorm(n2, 0.5, 2)
id2 <- sample(1:n2)
ind2 <- rep(1:k, each = n2/k)
A2 <- matrix(rep(NA, 30), ncol = k)
for(i in 1:k){
 A2[,i] <- s2[id2[ind2==i]]
 }
B2 <- matrix(rep(NA,n2), ncol = k)
D <- rep(0, n2/k)
for(i in 1:(n2/k)){
 for(j in 1:k){
  B2[i,j] = (A2[i,j] - mean(A2[i,]))*(A2[i,j] - mean(A2[i,]))
D[i] = sum(B2[i,])
}
}
r <- rank(append(C, D))
c <- length(C)
```

```
d <- length(D)
S <- sum(r[1:(length(id1)/k)])
T = S - c*(c+1)/2
Nu = T-(c)*(d)/2
De = sqrt(c*d*(c+d+1)/12)
Tstar[p] = Nu/De
}
sum(Tstar < qnorm(.05)) / m
```

 # R Code for the Moses Test with k=4

```
m <- 10000
Tstar = rep(NA, m)
for(p in 1:m) {
n1 <- 20
k <- 4
s1 <- rnorm(n1, 0, 1)
id1 <- sample(1:n1)
ind1 <- rep(1:k, each = n1/k)
A1 <- matrix(rep(NA, n1), ncol = k)
for(i in 1:k){
  A1[,i] <- s1[id1[ind1==i]]
 }
B1 <- matrix(rep(NA,n1), ncol = k)
C <- rep(0, n1/k)
```

```
for(i in 1:(n1/k)){
 for(j in 1:k){
   B1[i,j] = (A1[i,j] - mean(A1[i,]))*(A1[i,j] - mean(A1[i,]))
C[i] = sum(B1[i,])
}
}
n2 <- 30
s2 <- rnorm(n2, 0.5, 2)
id4 <- sample(1:n2)
id2 <- sample(id4,n2-2)
ind2 <- rep(1:k, each = length(id2)/k)
A2 <- matrix(rep(NA, length(id2)), ncol = k)
for(i in 1:k){
  A2[,i] <- s2[id2[ind2==i]]
 }
B2 <- matrix(rep(NA,length(id2)), ncol = k)
D <- rep(0, length(id2)/k)
for(i in 1:( length(id2)/k)){
  for(j in 1:k){
    B2[i,j] = (A2[i,j] - mean(A2[i,]))*(A2[i,j] - mean(A2[i,]))
D[i] = sum(B2[i,])
}
}
r <- rank(append(C, D))
```

```
c <- length(C)

d <- length(D)

S <- sum(r[1:(n1/k)])

T = S - c*(c+1)/2

Nu = T-(c)*(d)/2

De = sqrt(c*d*(c+d+1)/12)

Tstar[p] = Nu/De

}

sum(Tstar < qnorm(.05)) / m
```

 # R Code for the Moses Test with k=6

```
m <- 10000

Tstar = rep(NA, m)

for(p in 1:m) {

n1 <- 30

k <- 6

s1 <- rnorm(n1, 0, 1)

id1 <- sample(1:n1)

ind1 <- rep(1:k, each = n1/k)

A1 <- matrix(rep(NA, 30), ncol = k)

for(i in 1:k){

  A1[,i] <- s1[id1[ind1==i]]

 }

B1 <- matrix(rep(NA,n1), ncol = k)
```

```
C <- rep(0, n1/k)
for(i in 1:(n1/k)){
 for(j in 1:k){
   B1[i,j] = (A1[i,j] - mean(A1[i,]))*(A1[i,j] - mean(A1[i,]))
C[i] = sum(B1[i,])
}
}
n2 <- 30
s2 <- rnorm(n2, 0.5, 2)
id2 <- sample(1:n2)
ind2 <- rep(1:k, each = n2/k)
A2 <- matrix(rep(NA, 30), ncol = k)
for(i in 1:k){
 A2[,i] <- s2[id2[ind2==i]]
 }
B2 <- matrix(rep(NA,n2), ncol = k)
D <- rep(0, n2/k)
for(i in 1:(n2/k)){
 for(j in 1:k){
   B2[i,j] = (A2[i,j] - mean(A2[i,]))*(A2[i,j] - mean(A2[i,]))
D[i] = sum(B2[i,])
}
}
r <- rank(append(C, D))
```

```
c <- length(C)

d <- length(D)

S <- sum(r[1:(n1/k)])

T = S - c*(c+1)/2

Nu = T-(c)*(d)/2

De = sqrt(c*d*(c+d+1)/12)

Tstar[p] = Nu/De

}

sum(Tstar < qnorm(.05)) / m
```